Design of Two-Way Slabs

LEARNING OUTCOMES

After reading this chapter, you should be able to

- describe the key features of reinforced concrete two-way slabs
- design two-way slabs for flexure in accordance with the CSA A23.3 design procedures
- evaluate shear resistance of two-way slabs considering one-way and two-way shear effects
- estimate immediate and long-term deflections in two-way slabs according to CSA A23.3 requirements

12.1 INTRODUCTION

This chapter builds on the fundamentals presented in Chapters 10 and 11, which explained how multiple span slabs and beams can be designed as continuous structures by the strategic placement of reinforcing steel in the top and bottom regions. Continuous reinforced concrete structures that span over several supports are characterized by greater flexural stiffness than single span structures with the same overall length. This could result in reduced member dimensions and gives reinforced concrete a significant economic advantage over other materials. The same advantage can apply to two-way slabs, which are continuous structures spanning in two directions. Two-way slabs are unique to reinforced concrete and are considered to be an advanced design topic for students and inexperienced engineers. Two-way slabs offer an economical design solution, characterized by minimal slab thickness and reduced building height, which is easy to form and fast to construct. As a result, two-way slabs are the most popular floor system for multi-storey building construction in Canada.

The intent of this chapter is to explain behaviour of two-way slab systems and assist readers in deriving simple, practical, and economical design solutions.

The design of two-way slabs encompasses the following four key steps (note the section numbers referenced in the brackets):

1. Selecting the slab thickness (Section 12.5.2)
2. Designing the slab for flexure according to one of following three procedures:
   a) Direct Design Method (Section 12.6)
   b) Equivalent Frame Method (Section 12.7.2) or
   c) Three-Dimensional Elastic Analysis (Section 12.7.3)
3. Designing the slab for shear (Section 12.9), and
4. Performing a deflection check, when required by CSA A23.3 (Section 12.10).

This chapter is divided into eleven sections. Section 12.2 provides an overview of various two-way slab systems and describes their main features. The key factors influencing the structural behaviour of two-way slabs subjected to flexure are discussed in Section 12.3.

Common definitions and terms associated with two-way slabs are explained in Section 12.4. General CSA A23.3 design provisions for two-way slabs, including minimum thickness requirements, are presented in Section 12.5. Section 12.6 explains the CSA A23.3 Direct Design Method, a statics-based procedure which can be used for flexural design of regular slab systems subjected to gravity loads. Section 12.7 discusses elastic analysis, which includes a two-dimensional Equivalent Frame Method and three-dimensional Finite Element Analysis. Section 12.8 outlines key concepts of the Yield Line Method. The application of these design methods is illustrated through a few examples. The same design problem is solved using different methods, and the results are compared to identify the advantages and disadvantages of each method. It is expected that the reader will be able to make an informed choice of the most appropriate method for a specific design application. Section 12.9 describes concepts and procedures related to design of two-way slabs for shear. Section 12.10 discusses deflection control and the relevant calculation procedures for two-way slabs. Finally, structural drawings and detailing of two-way slabs are discussed in Section 12.11.

12.2 TYPES OF TWO-WAY SLABS

12.2.1 Background

Most two-way slab systems in reinforced concrete buildings are floor and roof slabs supported by columns and/or walls. Various floor and roof systems typical in reinforced concrete construction were introduced in Section 1.3.1. These systems included both one-way and two-way slabs. One-way slabs transfer gravity loads in one direction, while two-way slabs are characterized by significant bending in two directions. The following two-way slab systems will be discussed in this chapter:

- Flat plates
- Flat slabs
- Waffle slabs
- Slabs with beams

The first two-way slab in reinforced concrete was a flat slab constructed in 1906 by C.A.P. Turner for the C.A. Bovey Building in Minneapolis, Minnesota in 1906 (Sozen and Siess, 1963). A significant amount of controversy and arguments accompanied the initial applications of the flat slabs. This was mostly due to their unique load path and structural design, which appeared strange to engineers who were used to designing timber and steel structures with one-way load paths. In spite of these challenges, flat slabs with column capitals and drop panels were widely used in the first half of the 20th century for industrial buildings with heavy loads and relatively long spans (drop panels and capitals are illustrated in Figure 12.2). After World War II, many buildings were built for lighter loading, such as residential occupancy, thus drop panels and capitals were not needed. The resulting system is called flat plate to distinguish it from flat slab.

Design provisions for two-way slabs were introduced in North American codes in 1941, through ACI 318-41 in the USA. Flat plate and flat slab systems are currently the most popular floor systems used for construction of multi-storey residential and office buildings in Canada. Waffle slabs and slabs with beams can also be used as economical light-weight solution for large spans.

Choice of the most appropriate slab system for a particular application depends on several factors, including span lengths (distance between the supports) and live loads. Economic considerations, in particular construction costs, are another important factor. In North America, formwork cost represents approximately 50% of the overall cost for two-way slab floor system. Concrete material, and its placing and finishing, account for 30% of the cost, while the remaining 20% of the cost is associated with the material and placing cost of the reinforcing steel. Hence, labour and material costs are key considerations for cost-efficient design of two-way slabs. For a detailed discussion on economic aspects of two-way slabs the reader is referred to PCA (2000).

In most residential and commercial building designs, the slab self-weight is a significant proportion of the total load. Hence, a potential increase in the overall construction cost due to a live load increase from 2.5 to 5.0 kPa may be only 5 to 10%.

12.2.2 Flat Plates

A *flat plate* is a floor system that consists of solid slabs reinforced in two directions and supported directly by the columns or walls, as shown in Figure 12.1a. This system is economical for short and medium spans ranging from 6 to 9 m (20 to 30 ft). The slab thickness usually ranges from 160 to 250 mm (6.5 to 10 inches). Flat plates are suitable for light loads (live loads up to 5.0 kPa), and the slab thickness is usually controlled by long-term deflections.

A slab layout is defined by column locations. Parallel column lines (gridlines) in perpendicular directions form a rectangular slab *panel* (or a bay), as shown in Figure 12.1b. A slab panel is defined by the span lengths l_1 and l_2, and the ratio of span lengths (l_1/l_2) is called the *aspect ratio*. The simplest and most optimal design uses a column layout on a square grid (corresponding to the aspect ratio of 1.0).

In some cases, both flat plates and flat slabs can have shallow beams along the perimeter of the slab. These beams are called *spandrel beams* (or edge beams). These spandrel beams are effective in increasing the slab stiffness at the edges and thereby reducing slab deflections. A typical spandrel beam section is shown in Figure 12.1c.

A few important advantages of a flat plate system are outlined below:

1. Due to the relatively thin structure, a flat plate system permits the construction of the maximum number of floors for a given building height. An additional benefit associated with the reduced floor height is an overall reduction in the building envelope (exterior walls) and partitions, utility shafts, and a significant reduction in seismic and wind loads.
2. Flat plate systems can be designed to satisfy most of the fire resistance requirements of the National Building Code of Canada without the need for additional fire protection. This is a significant advantage compared to floor systems in other materials, such as steel and wood.
3. Flat plate systems can be adapted to a non-uniform column layout, usually without a significant cost premium. This is a significant advantage as it can efficiently accommodate flexibility in architectural design. Flat plates require simple formwork, resulting in minimal construction effort in terms of time, labour, and cost. Flat plates in high-rise buildings are usually constructed using prefabricated formwork systems (fly-forms) that can be transported and lifted for reuse at the next floor level. In a high-rise building with flat plates, a construction crew can form, reinforce, and place concrete for each floor in very short (three- to four-day) cycles.

In conclusion, a flat plate system is very efficient and economical, and for that reason it is the most widely used floor system for high-rise building construction in Canada. Typical applications include residential buildings, offices, hotels, hospitals, and parking garages. A parking garage and a residential building, are shown in Figure 12.1d and e, respectively.

gure 12.1 Flat plate system:
a) isometric view; b) plan
view; c) vertical section;
d) parking garage, and
e) residential building under
construction
(Svetlana Brzev).

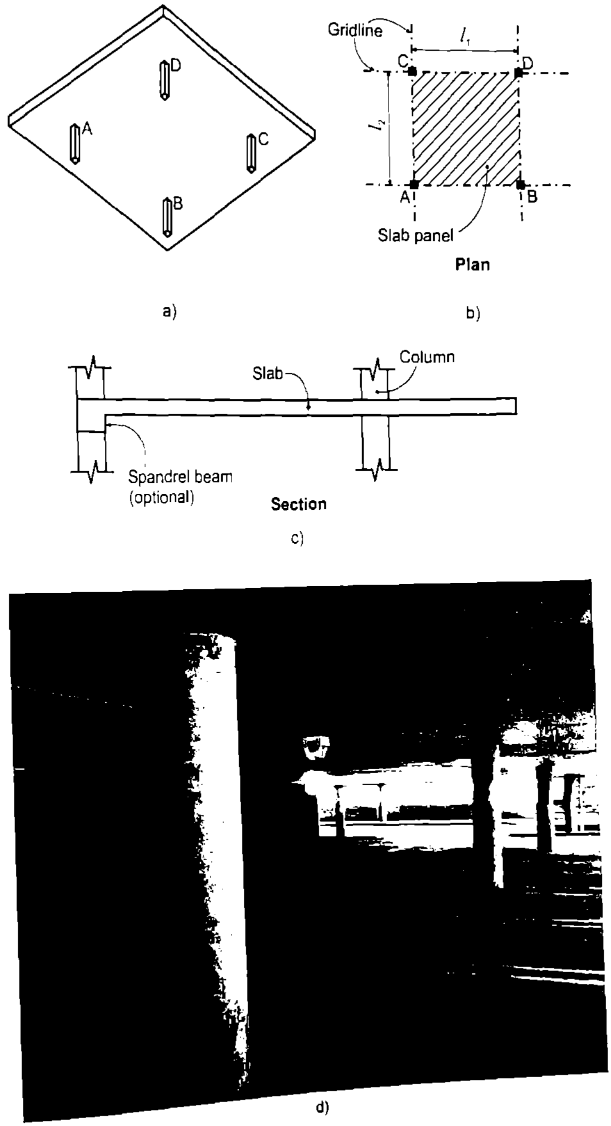


Figure 12.1 (*cont.*)

e)

12.2.3 Flat Slabs

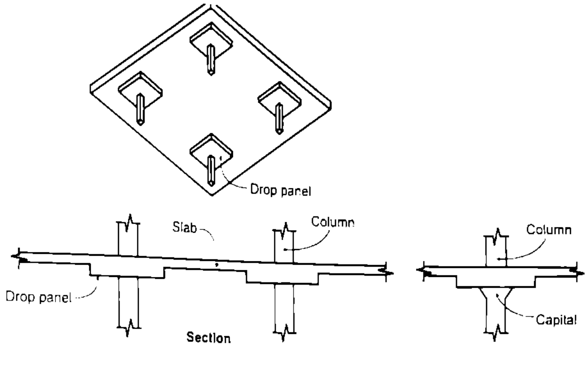
The span-to-thickness ratio for a flat plate is usually limited by permitted long-term deflections. Thin slabs could also have punching shear issues in column regions. The punching shear capacity can be increased by thickening the slab locally around the columns, by way of *drop panels*. A flat plate system with drop panels is called a *flat slab*, as shown in Figure 12.2a. Drop panels cause an increase in the slab stiffness in regions subjected to negative bending moments, and therefore help to control deflections in midspan regions. A smaller amount of top reinforcement is required in slabs with drop panels than in flat plates.

Drop panels are usually square in plan, with plan dimensions typically one-third of the span length in each direction of the slab. Thickness of a drop panel is often governed by the formwork dimensions (e.g. 89 mm thickness can be specified when 2" x 4" plank is used on the edge). In some flat slab designs, column capitals are provided with or without drop panels. A *capital* is an upper portion of the column, usually of conical shape and with larger cross-sectional dimensions than the remaining portion of the column, as shown in Figure 12.2b. Rectangular-shaped capitals (also known as shear caps) are used in some designs as they are easier to form. The designer may use capitals, drop panels, or both to address punching shear and deflection issues in floor slabs. Both drop panels and capitals are effective in providing additional punching shear capacity of the slab. Typical building applications are shown in Figure 12.2c and d.

12.2.4 Waffle Slabs

A *waffle slab* is a floor system that meets requirements for larger spans and loads at a reduced slab weight, while also providing an aesthetically pleasing ceiling. Waffle slab systems consist of evenly spaced concrete joists spanning in two directions — this system is also known as a two-way joist system. The joists are commonly formed by using standard pans or domes installed in the forms to produce a coffered soffit in the slab. The perpendicular orientation of the joists results in evenly spaced rectangular-shaped voids on the underside of the slab — this is the reason why the system is referred to as waffle slab. A plan view and a vertical section of waffle slab are shown in Figure 12.3a.

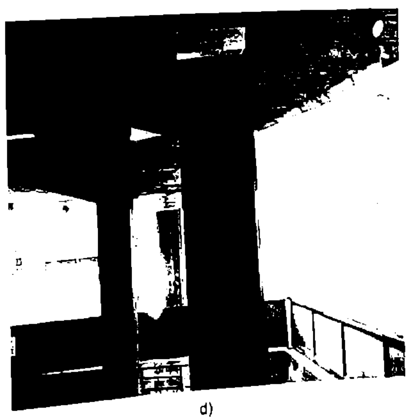
gure 12.2 Flat slab system: an isometric view and a vertical ction; b) column capital; c) a flat ab with capitals, and d) a flat ab with column capitals and drop anels etlana Brzev).



a)

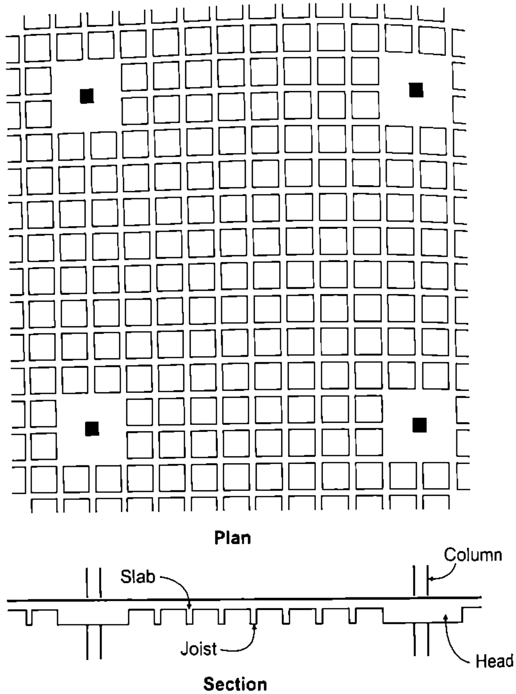
b)

c)



d)

Figure 12.3 Waffle slab system: a) plan and a vertical section, and b) en example of a building application.



a)

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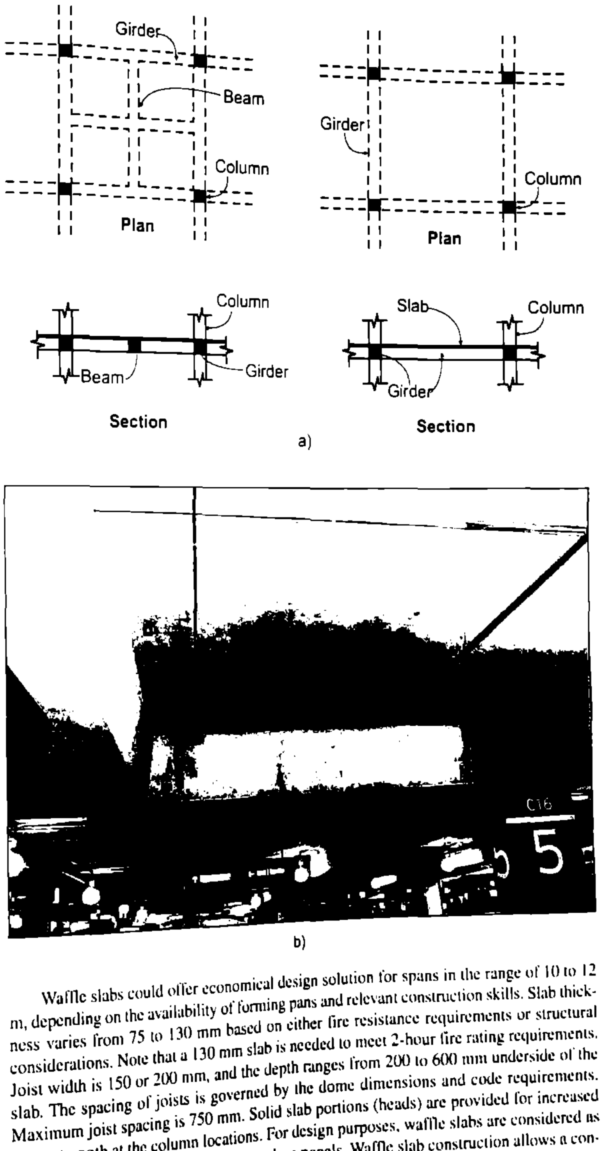


ure 12.4 Slab with beams: typical plans and vertical tions, and b) an example of lding application atiana Brzev).

Waffle slabs could offer economical design solution for spans in the range of 10 to 12 m, depending on the availability of forming pans and relevant construction skills. Slab thickness varies from 75 to 130 mm based on either fire resistance requirements or structural considerations. Note that a 130 mm slab is needed to meet 2-hour fire rating requirements. Joist width is 150 or 200 mm, and the depth ranges from 200 to 600 mm underside of the slab. The spacing of joists is governed by the dome dimensions and code requirements. Maximum joist spacing is 750 mm. Solid slab portions (heads) are provided for increased shear strength at the column locations. For design purposes, waffle slabs are considered as flat slabs with the solid heads acting as drop panels. Waffle slab construction allows a considerable dead load reduction compared to conventional flat slab construction since the volume of concrete used can be minimized due to the short span between the joists.

Waffle slabs are particularly advantageous when the use of long span and/or heavy loads is desired without the use of deepened drop panels, capitals or support beams. Typical applications include parking garages or warehouses. A parking garage with a waffle slab system is shown in Figure 12.3b.

High formwork and labour costs are the main reasons for limited applications of waffle slabs in contemporary concrete construction practice.

12.2.5 Slabs with Beams

In *slab with beams* system a solid slab panel is supported by beams on all four sides. With this system, when the ratio of the span lengths is two or more, load is predominately transferred by bending in the short direction and the panel essentially acts as a one-way slab. As the panel approaches a square shape, a significant load is transferred by bending in both orthogonal directions, and the panel should be treated as a two-way slab. Plan views and vertical sections for slab with beams are shown in Figure 12.4a. The presence of beams may require a greater storey height. A typical application of the slab with beams system (parking garage) is shown in Figure 12.4b.

12.3 BEHAVIOUR OF TWO-WAY SLABS SUBJECTED TO FLEXURE

12.3.1 Background

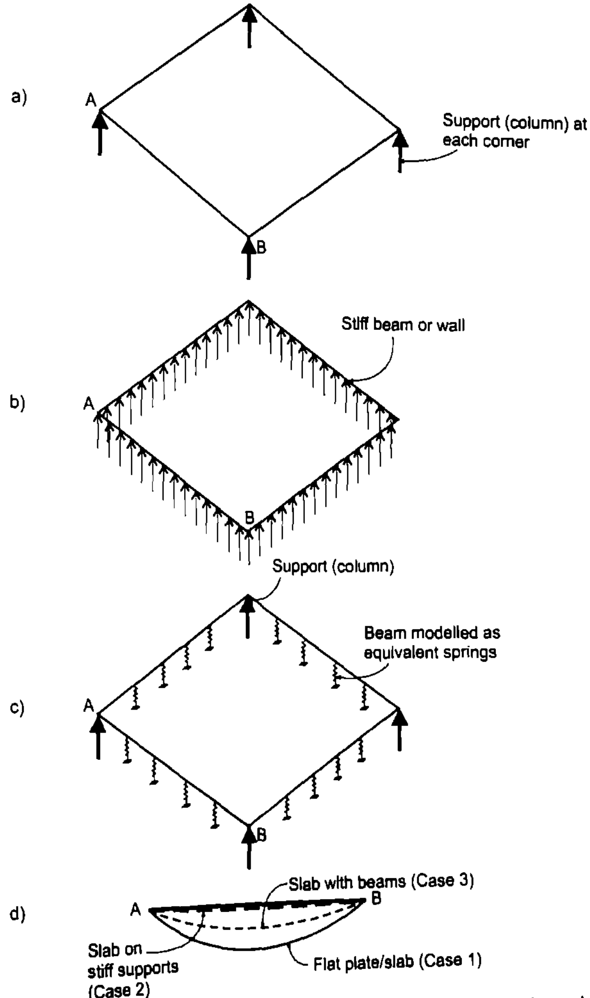
Before we proceed with explaining different design approaches for two-way slabs, it is important for the reader to understand how these systems behave under gravity loads. Flexural behaviour of two-way slabs is significantly influenced by the support conditions. The following three cases will be considered:

1) Flat plate/slab panels supported only by columns at the corners; note the supports shown with arrows (see Figure 12.5a).
2) Slab panels supported either by stiff beams or walls (slabs on stiff supports) have continuous supports along the edges, as shown in Figure 12.5b; note fixed supports on all sides.
3) Slab with beams on all sides - an intermediate case between 1) and 2), where a slab is supported by columns (shown with arrows) and beams (shown as spring supports), as illustrated in Figure 12.5c.

It is important to discuss the influence of support conditions upon the magnitude of deflections in two-way slabs for the three above described cases. The total slab deflection at mid span is the sum of the slab deflection at the edges plus the relative slab deflection between the edges and the midspan. Figure 12.5a shows a flat plate supported only by four columns. The midspan deflections are largest in this case since the deflections along the unrestrained edges between the columns will cause an increase in the slab deflection at midspan. On the other hand, deflections in the slab supported along all four edges by stiff beams or walls (Figure 12.5b) are smallest since the deflection is equal to zero along the edges. Finally, when a slab is supported by beams on all four sides (Figure 12.5c), the deflections in midspan regions fall in between the other two cases, because the beams are expected to deflect, but less than in the case of a flat plate. A conceptual diagram showing deflected shapes for the above discussed cases is presented in Figure 12.5d.

Consider two slabs characterized by the same thickness (180 mm) and span lengths (5 m square) that are subjected to the same uniform gravity load of 9.5 kPa. The only difference is in the support conditions: one of the slabs is supported by columns at the corners only (flat slab), while another is continuously supported at the edges by walls. In the first case, displacements are restrained by the columns (which act like sticks with pinned connections at the

jure 12.5 Two-way slabs with
ferent support conditions:
flat slab/plate; b) slab on stiff
pports; c) slab with beams on
sides, and d) deflected shapes
r two-way slabs.



top, similar to simple supports in beams), thereby allowing displacements and rotations at the four slab edges. In the latter case, walls provide restraints both for displacements and rotations and act like fixed supports. These are two extreme support conditions: the most flexible and the most rigid. The displacement contours (regions with equal displacements) are shown in Figure 12.6. The maximum displacements for the flat slab supported only by the columns and the slab on fixed supports are 15 mm and 1 mm, respectively. The values can be seen from the legend corresponding to displacement contours. (Note that the approaches for deflection calculations in two-way slabs will be discussed in detail in Section 12.10).

It can be seen from the above examples that two-way slabs deflect due to bending in two perpendicular directions. The deflected shape in each direction depends on the slab stiffness, the support conditions, and the magnitude and type of applied loads. For multi-span slabs, the deflected shape also depends on the properties of adjacent spans in each direction.

When subjected to uniform gravity load, a slab with several panels deforms into a series of shallow hills at the columns (negative or concave curvature), and bowl-shaped valleys at the center of each panel (positive or convex curvature). The sign of curvature (positive or negative) determines the placement of flexural reinforcement. For example, top steel is placed in regions of negative curvature, while bottom steel is placed in regions of positive curvature. Note that positive curvature corresponds to positive bending moment and vice

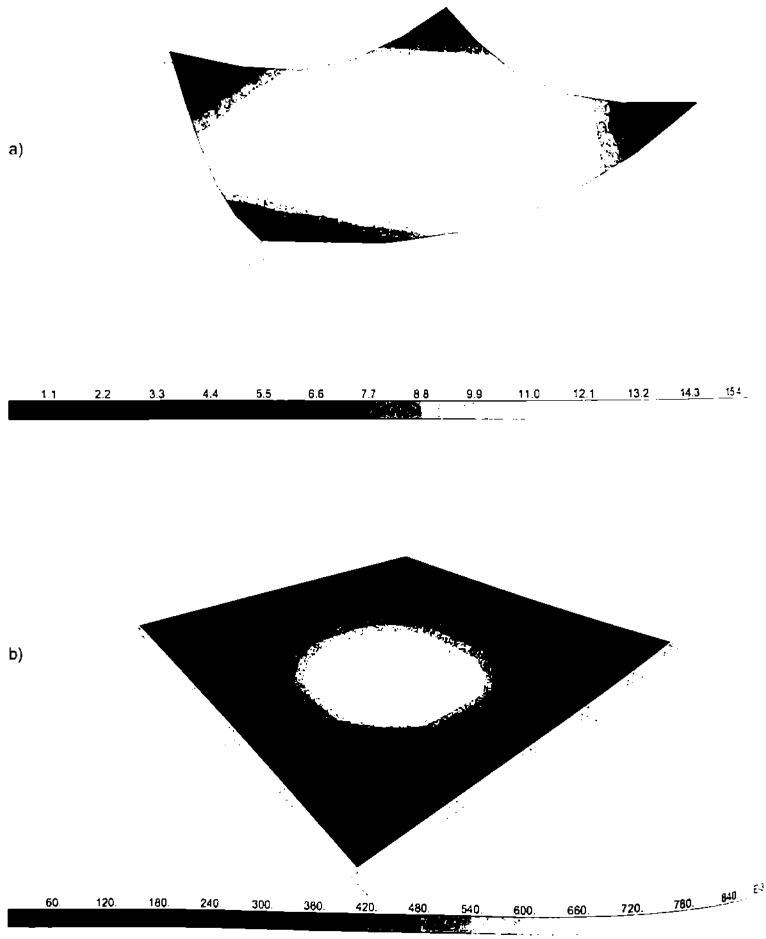
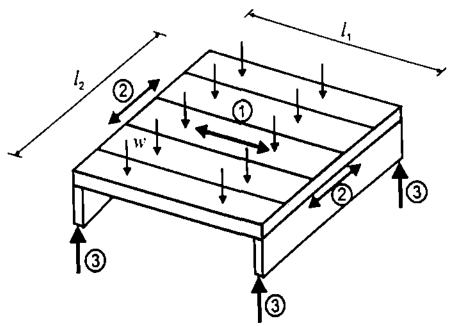


Figure 12.6 Displacements in two-way slabs: a) a flat plate supported on columns, and b) a slab on continuous supports along the edges (units: mm).

versa. Due to the double curvature in two-way slabs, both top and bottom flexural reinforcement must be provided in two orthogonal directions (usually in line with column gridlines). This makes two-way slab design significantly more complex compared to one-way slab and beam design.

12.3.2 Gravity Load Paths in Two-Way Slabs

Before proceeding with the design of a two-way slab, the designer must understand its load path. The concept of load path was first introduced in Section 1.3.3. Gravity load path in this context denotes the manner (or "the path") in which the gravity load is transferred from the slab to the supports. In reinforced concrete slabs, gravity load can be transferred to supports through either one-way or two-way load paths. "One-way" refers to a pre-determined path in one direction of the slab panel. Load distribution in one-way slabs is explained in Section 3.6. The concept of one-way load path can be explained by an example of a timber floor system consisting of parallel planks supported by the beams shown in Figure 12.7. When gravity load is applied on the floor, a plank (1) carries the load along its span and transfers it to the beams (2). These beams in turn transfer the plank reactions (2) onto the supporting columns (3). In this system, both the planks and the beams are treated as simply supported at their ends and the load transfer is achieved through a pre-determined one-way load path.



igure 12.7 One-way slab system omprised of parallel timber planks upported by two beams.

Reinforced concrete two-way slabs are characterized by load paths in two orthogonal directions. These load paths simultaneously transfer load from the slab to the supports. The proportion of load carried in each direction will depend on the relative flexural stiffness of the slab in each direction. To explain this concept, an example of a timber floor system with one-way load path will be expanded to idealize load path in two directions.

First, let us idealize the slab shown in Figure 12.8a as a system of timber planks laid in two orthogonal directions and simply supported by rigid walls. Consider load distribution for a system of two planks (A and B) which are laid on top of one another and connected at the intersection (see Figure 12.8b). The point load P is applied at the intersection of these two planks. Both planks jointly support the load, that is,

$$P = P_A + P_B$$

where P_A and P_B are the loads resisted by planks A and B, respectively. The planks have the same cross-sectional dimensions: 250 mm width by 50 mm depth, however their spans are different: plank A has a shorter span (l_A) while plank B has a longer span (l_B), say twice the span of plank A ($l_B = 2l_A$).

The proportion of load supported by each plank can be determined from the deflection compatibility at the point of intersection, that is, the deflections of the two planks at the point of intersection must be equal. The deflection for plank A at the intersection, δ_A, can be determined by treating the plank as a simply supported beam subjected to point load P_A, that is,

$$\delta_A = \frac{P_A \cdot l_A^3}{48 \cdot E \cdot I_A}$$

where E is the modulus of elasticity for timber, and I_A is the moment of inertia for plank A (refer to Table A.16 for deflection equation for a beam subjected to point load).

The maximum deflection for plank B at the intersection, δ_B, can be determined from the same equation as plank A, that is,

$$\delta_B = \frac{P_B \cdot l_B^3}{48 \cdot E \cdot I_B}$$

These deflections need to be equal based on compatibility requirements, thus

$$\delta_A = \delta_B$$

and

$$\frac{P_A \cdot l_A^3}{48 \cdot E \cdot I_A} = \frac{P_B \cdot l_B^3}{48 \cdot E \cdot I_B}$$

Since the planks have the same modulus of elasticity, E, and the same moment of inertia ($I_A = I_B$), the above equation can be simplified as follows

$$\frac{P_A}{P_B} = \frac{l_B^3}{l_A^3}$$

Since $l_B = 2l_A$, it follows that

$$\frac{P_A}{P_B} = 8$$

Since

$$P = P_A + P_B = \frac{9}{8} P_A$$

It can be concluded that 8/9 (approximately 89%) of the total load P is supported by plank A, that is,

$$P_A = \frac{8}{9} P$$

while the remaining 11% of the load P is supported by plank B.

This can be explained by a difference in flexural stiffness between planks A and B. Flexural stiffness is directly proportional to the moment of inertia (I), however it is inversely proportional to the third power of the span length (l^3). In this example, both planks have the same moment of inertia, however plank A has one-half of the span length of plank B. As a result, plank A has eight times larger stiffness compared to plank B, and thus carries a significant portion (about 90 %) of the total load P.

Also, note that that the resulting load on each plank support is quite different. Each plank A support carries approximately 45 % of the total load P, while the supports for plank B carry only 5% each.

Next, let us consider a modified version of the same example. Let us rotate plank B such that the longer cross-sectional dimension is placed in the vertical direction, while plank A

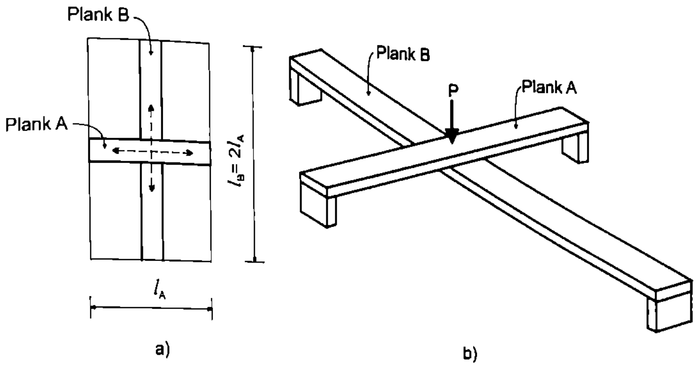


Figure 12.8 Two-way load path example: timber planks with the same stiffness on rigid supports: a) plan view, and b) isometric view.

remains the same as in the first example (see Figure 12.9). Planks A and B are now going to have different moments of inertia for bending about the horizontal axis (x-x), as follows

$$I_A = \frac{250 \cdot 50^3}{12} = 2.6 \cdot 10^6 \text{ mm}^4$$

and

$$I_B = \frac{250^3 \cdot 50}{12} = 65.0 \cdot 10^6 \text{ mm}^4$$

that is, $I_B = 25\, I_A$.

Let us estimate a fraction of the total load P resisted by each plank by following the same approach as in the previous example. As shown earlier, deflections at the point of intersection have to be equal due to the compatibility requirement, that is,

$$\delta_A = \delta_B$$

In this case, the planks have the same modulus of elasticity, E, however their moments of inertia and the spans are different, hence

$$\frac{P_A}{P_B} = \frac{l_B^3 \cdot I_A}{l_A^3 \cdot I_B}$$

Since $l_B = 2l_A$ and $I_B = 25\, I_A$, it follows that

$$\frac{P_A}{P_B} = 0.32$$

Therefore

$$P = P_A + P_B = 1.32 P_B$$

Hence,

$$P_A = 0.24P$$

and

$$P_B = 0.76P$$

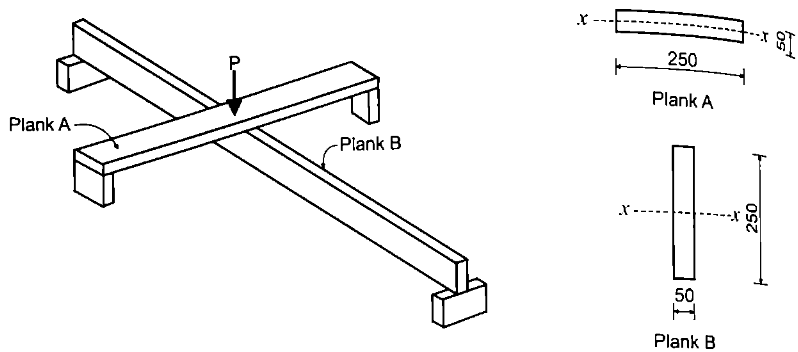


Figure 12.9 Two-way load path example: timber planks with rigid supports and different stiffness.

It can be concluded that only 24% of the total load P is supported by plank A, while the major portion of the load (76%) is supported by plank B. Plank B has 3 times higher flexural stiffness compared to plank A. As a result, plank B carries a major portion of the total load. Plank A now only supports 24% of the total load; this is a large decrease from 89% (see the previous example). The changes in support reactions are also significant, since 38% of the total load is transferred to each plank B support, and only 12% is transferred to each plank A support.

The above two examples show that, although planks A and B have equal spans, the support reactions are significantly different due to a change in relative plank stiffnesses in the two examples. The above examples have also shown that in structural systems with multiple load paths such as two-way slabs, the "stiffer" load path will always attract a larger fraction of the total load. For example, in two-way slabs with uniform thickness and different span lengths in two directions, a larger fraction of the total load will be supported by a slab panel in the shorter direction due to larger flexural stiffness in that direction, as discussed in the first example.

12.3.3 Distribution of Bending Moments in Two-Way Slabs

Statics-Based Approach for Estimating Bending Moments in Two-Way Slabs
When two-way slabs were first introduced in concrete construction practice at the beginning of the 20th century, initial design solutions were based solely on the statics-based approach which was originally proposed by J.R. Nichols in 1914 (Sozen and Siess, 1963). That approach may be useful to provide an insight into key parameters which influence moment distribution in two-way slabs, and it will be used in the following two examples.

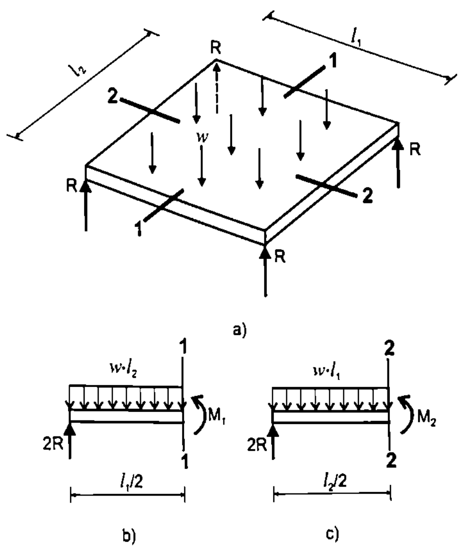
Let us consider a two-way flat slab panel supported at the corners shown in Figure 12.10a. The panel has rectangular shape (span l_1 is longer than span l_2), and it is subjected to uniformly distributed live area load, w. The four support reactions (R) are the same, that is,

$$R = \frac{w \cdot l_1 \cdot l_2}{4}$$

Next, let us idealize the slab as a beam with the span l_1. The beam is subjected to uniform line load ($w \cdot l_2$) which corresponds to the tributary width l_2. Bending moment on Section 1-1 at midspan can be determined from a free-body diagram of the slab shown in Figure 12.10b, as follows

$$M_1 = \frac{(w \cdot l_2) l_1^2}{8}$$

ure 12.10 Bending moments a flat slab panel: a) a typical nel; b) a free-body diagram at ction 1-1, and c) a free-body agram at Section 2-2.



Note that M_1 is equal to the maximum bending moment for a simply supported beam with the span l_1 subjected to uniform load $w \cdot l_2$. It is important to note that M_1 is a total bending moment which corresponds to the width l_2.

The same exercise can be performed to determine bending moment at Section 2-2 in other direction (see Figure 12.10c), as follows

$$M_2 = \frac{(w \cdot l_1)l_2^2}{8}$$

where M_2 is equal to the maximum bending moment in a simply supported beam with span l_2 and subjected to load $w \cdot l_1$.

The ratio of moments M_1 and M_2 is equal to

$$\frac{M_1}{M_2} = \frac{l_1}{l_2}$$

The above equation shows that bending moments at the midspan of a slab panel are proportional to the ratio of span lengths.

Statics-based approach will be also used to illustrate an approximate distribution of bending moments in different slab regions. Consider a square slab panel ABDE with span l supported by columns shown in Figure 12.11a. The structural action of the slab with a uniform load w is simulated by a grid of simply supported beams carrying the same uniform load, as shown in Figure 12.11b. The interior beams FG and JH simulate the midspan regions of the slab, while the perimeter beams simulate regions close to column gridlines. Because of compatibility the interior beams deflect the same amount at midspan.

First, let us consider the bending moment in an interior beam FG of unit width shown in Figure 12.11c. The maximum bending moment for a simply supported beam subjected to uniform load is equal to

$$M_{FG} = \frac{w \cdot l^2}{8}$$

Beam FG is supported by perimeter beams of unit width AB and DE, which are in turn supported by the columns at each corner. In addition to supporting tributary load from the slab, these perimeter beams also support the reactions from the beam FG, that is,

$$R_F = \frac{w \cdot l}{2}$$

Load distribution for a typical perimeter beam AB is shown in Figure 12.11d. It can be seen that the beam is subjected to uniform load w in addition to the point load R_F. The resulting maximum bending moment is equal to

$$M_{AB} = \frac{w \cdot l^2}{8} + \frac{R_F \cdot l}{4} = 2\left(\frac{w \cdot l^2}{8}\right)$$

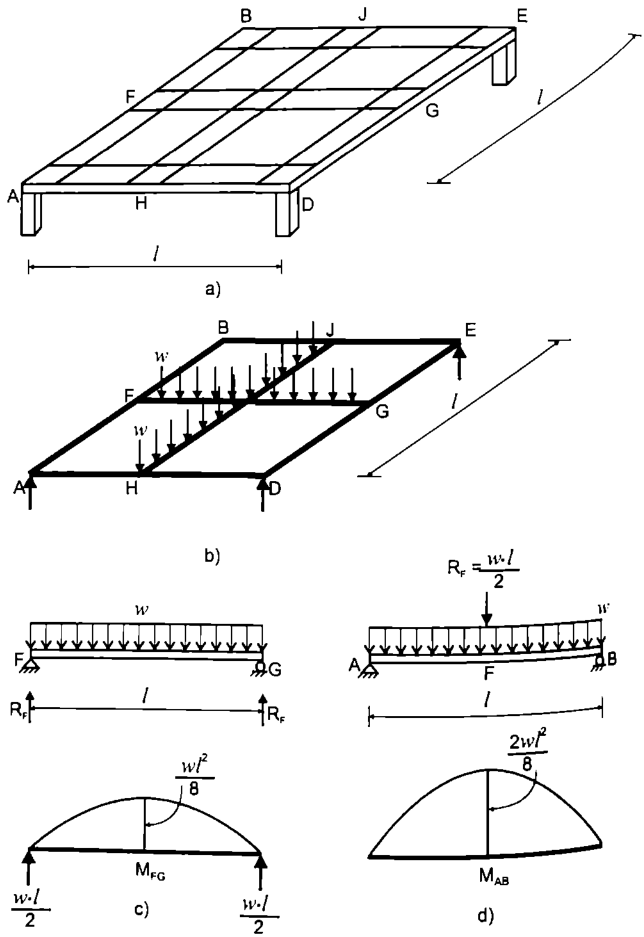


Figure 12.11 Flat plate/slab modelled as a beam grid:
a) an actual slab panel showing idealized beam strips;
b) an idealized beam grid;
c) moment distribution in interior beam FG, and
d) moment distribution in perimeter beam AB.

The ratio of maximum bending moments for a perimeter and an interior beam is equal to 2.0, that is,

$$\frac{M_{AB}}{M_{FG}} = 2.0$$

This example shows that the bending moments in perimeter beams are significantly higher than the corresponding moments in interior beams. As a result, bending moments along the column gridlines in two-way slabs are always higher than the bending moments in the midspan regions. The Direct Design Method procedure covered in Section 12.6 prescribes larger design bending moments for column strips (perimeter beam in this example) than middle strips (interior beam in this example).

The key points from the above discussion are summarized below:

1. Bending moments at midspan of a column-supported slab are directly proportional to the ratio of their spans: larger bending moments occur in the longer span.
2. Bending moments along column gridlines are always higher than bending moments at midspan regions of the slab.

Bending Moments in Slabs with Beams Relative beam-to-slab stiffness ratio is one of the key factors that influences moment distribution in two-way slabs with beams. The case of a slab with beams is an intermediate case, and the moment distribution falls in between the two extreme cases. In the first case, the slab does not have beams at support lines, thus the beam stiffness is equal to zero. As a result, the slab behaves similar to a flat slab, that is, the slab carries the entire load and resists bending moments. In the second case, where the slab has stiff supports on all sides, the stiffness of idealized beams along the support lines is extremely large and the behaviour is similar to slabs on stiff supports.

A parametric study on the distribution of bending moments between beams and slab for slabs with beams was performed by Park and Gamble (2000). Charts showing a bending moment distribution in slabs with beams depending on the relative beam-to-slab stiffness ratio (expressed through the α factor which is discussed in Section 12.4.5) and the span length ratio l_1/l_2 are presented in Figure 12.12. Two different span length ratios (l_1/l_2), that is, 1.0 and 2.0, were considered in the study. It can be seen from Figure 12.12a that the negative bending moment at the support is distributed between the slab and the beam. When beam stiffness exceeds 3.0, a major portion of the moment is resisted by the beams (more than 50%). Also, there is a more significant sharing of bending moments between the beams and the slab for square column grid ($l_1/l_2 = 1.0$) compared to rectangular grid ($l_1/l_2 = 2.0$). The same tendency was observed as related to the distribution of positive moments at midspan, as illustrated in Figure 12.12b.

12.3.4 Moment Redistribution in Cracked Two-Way Slabs

Similar to other reinforced concrete flexural members, two-way slabs are expected to experience cracking under service loads. The effect of cracking and reinforcement distribution on the behaviour of continuous reinforced concrete members is discussed in Section 10.7. This section discusses the effect of cracking on the behaviour of two-way flat slabs, which are also continuous systems, but their behaviour is more complex than beams and one-way slabs.

Before cracking, a two-way slab shows linear elastic behaviour, which is characterized by linear stress-strain relationship in concrete and steel. Cracking in the slab occurs when bending moments reach the cracking moment (M_{cr}) (see Section 4.2 for more details). In continuous flat slab and flat plate systems, cracking initially takes place in the vicinity of columns or supports, since these areas are typically characterized by largest bending moments (which happen to be negative moments). Once the slab cracks, there is a significant drop in flexural

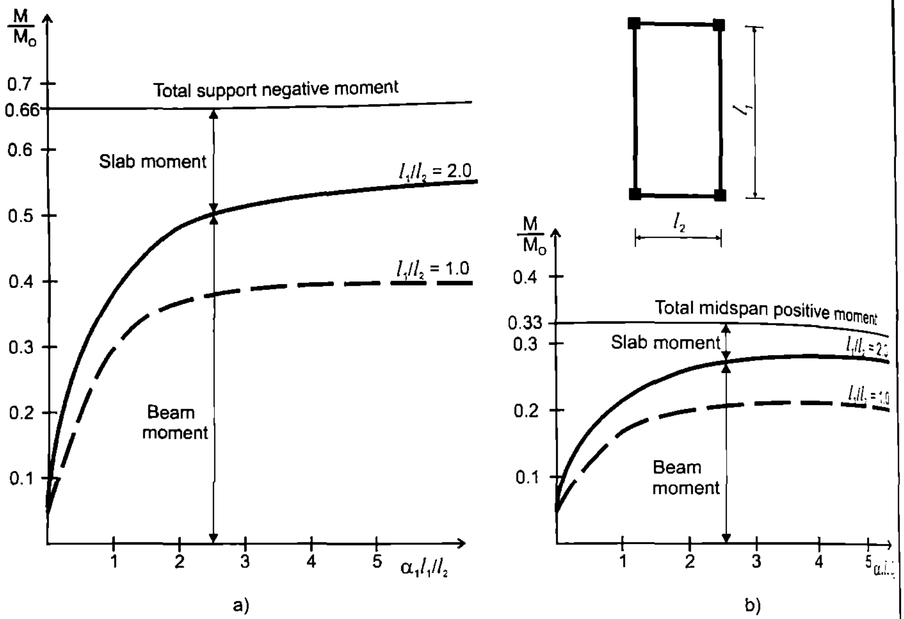


Figure 12.12 Bending moments versus relative beam-to-slab stiffness ratio in slabs with beams: a) negative bending moments at the supports, and b) positive bending moments at midspan.

stiffness in the cracked regions. This results in an increase in positive bending moments in the uncracked regions, which could cause cracking in the midspan regions and lead to further redistribution of bending moments within the slab. This phenomenon is known as *moment redistribution*: bending moments will redistribute from cracked to uncracked regions in continuous reinforced concrete beams and slabs. The concept of moment redistribution was introduced in Section 10.9 in relation to continuous beams and one-way slabs.

A significant moment redistribution in two-way slabs subjected to gravity loads was reported by Rangan and Hall (1984). They tested a few half-scale models of the end panel of a flat plate system, as illustrated in Figure 12.13a. The models were subjected to progressively increasing gravity loads, and the behaviour was monitored at three load levels: service load (5.4 kPa), factored design load (9.0 kPa), and ultimate load/failure (22.5 kPa). Transverse moment distribution in the vicinity of column A (Section 1-1) is shown in Figure 12.13b. The ultimate moment at the column (M_{ULT}) was approximately 2.6 times higher than the design moment (M_{DES}). This indicates a significant reserve in bending moment capacity beyond the design load level.

It can be also seen that the moment magnitudes are highest at the column face, and that there is a significant drop in moment values just beyond. Moment redistribution was also observed in the perpendicular span due to redistribution of bending moments in the primary span. Along Section 2-2 parallel with column line AB (see Figure 2.13c), a significant increase in bending moments was observed in columns A and B relative to the total moment (moment gradient) M_o. Bending moment at the column location was originally 0.33 M_o, but it increased to 0.50M_o after the onset of cracking. A reverse tendency was observed at the midspan - bending moment decreased from 0.66 M_o to 0.47M_o after the onset of cracking.

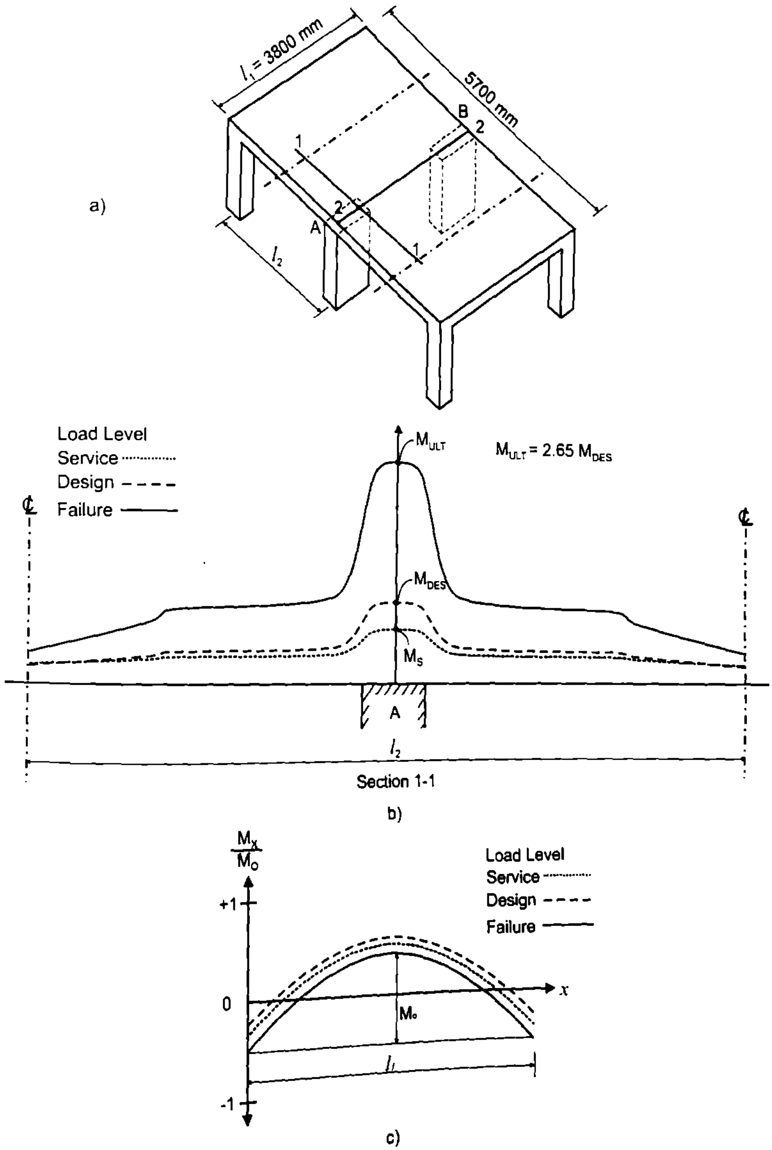


Figure 12.13 Moment redistribution in flat slabs: a) model layout; b) transverse moment distribution — Section 1-1 at the column A, and c) moment distribution in longitudinal direction — Section 2-2 (adapted from Rangan and Hall, 1984 with the permission of the American Concrete Institute).

It can be concluded that, due to significant redistribution of bending moments in two-way slabs, the designer has to ensure that the total flexural capacity between the column and midspan sections is equal to the total moment M_o. This is one of the key rules related to the design of continuous flexural members and two-way slabs and will be discussed in Section 12.6. The actual distribution of bending moments and reinforcement between the column and midspan sections is a secondary consideration, and it depends on design practices and standards followed in a specific design. Hence, when total moment M_o for a specific slab span is given, the designer can select a number of different reinforcement amounts for midspan and column regions, which would result in the same ultimate load capacity for the span.

12.3.5 Design for Flexure According to CSA A23.3

According to CSA A23.3 Cl.13.5.1, a two-way slab system can be designed using any procedure which satisfies conditions of equilibrium and compatibility, provided that the strength and serviceability requirements have been met. The following design procedures are outlined in CSA A 23.3 (Clauses 13.6 to 13.9):

- Direct Design Method
- Elastic Frame Method (also known as the Equivalent Frame Method)
- Elastic Plate Theory, and
- Theorems of Plasticity (e.g. the Yield Line Method).

The design of two-way slab systems must be performed considering both gravity and lateral loads. Some of the CSA A23.3 design procedures, like the Direct Design Method, can be used for gravity load analysis only, while others can be used both for gravity and lateral load analysis. Common design procedures will be explained in detail and illustrated by design examples later in this chapter.

12.4 DEFINITIONS

12.4.1 Design Strip

A23.3 Cl.13.1.2

A *design strip* is the portion of a slab system bound laterally by the centrelines of the panels on each side of the column (CSA A23.3 Cl.13.9.2.1). This concept is illustrated in Figure 12.14, which shows a partial plan of a two-way floor slab without beams (flat plate). A typical slab panel is enclosed by four columns (e.g. panel ABEF shown in Figure 12.14a). A portion of the slab at each floor level may be modelled as an equivalent beam, and its width is referred to as a design strip. Equivalent beams at each floor level along with the supporting columns constitute a frame. In a general design scenario, several frames in both directions, east-west (E-W) and north-south (N-S), need to be considered. A typical frame ABC in the E-W direction is shown in Figure 12.14a, and the frame DBF in the N-S direction is shown in Figure 12.14b. The design strips in each direction are shown hatched in the figure.

According to the CSA A23.3 notation, the longitudinal direction of the frame is referred to as direction 1, and the span in this direction is denoted as l_1 (see Figure 12.14). The other (transverse) direction is referred to as direction 2 and the corresponding span is denoted as l_2. Both l_1 and l_2 are measured centre to centre of the support (column). The width of the design strip is denoted as l_{2a}. In many instances, slab span lengths are variable. In that case, the width of design strip l_{2a} needs to be determined by considering average l_2 value for the two slab spans adjacent to the gridline under consideration.

In flat slab and flat plate systems, the design strip is divided into the column strip and the middle strip. The *column strip* is defined as having a width equal to one-half the smaller span length (l_1 or l_2), as shown hatched in Figure 12.15. Note the following two cases:

1) For $l_1 < l_2$, an interior column strip width is equal to $l_1 / 2$ (see Figure 12.15a), and
2.) For $l_1 \geq l_2$, an interior column strip width is equal to $l_2 / 2$ (see Figure 12.15b).

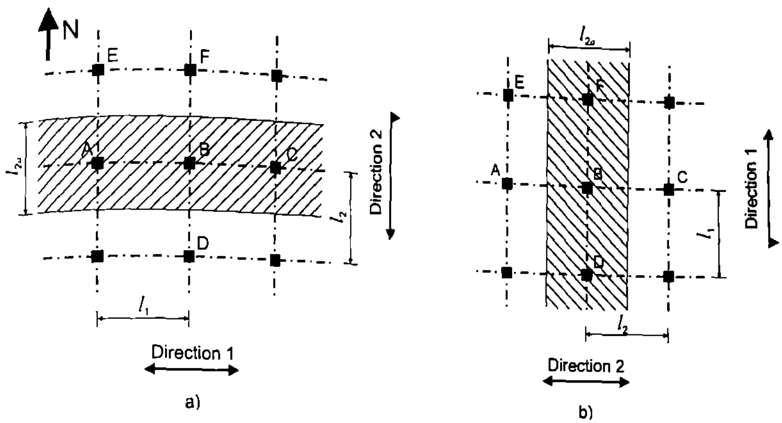


Figure 12.14 Design strip: a) frame ABC in the E-W direction, and b) frame DBF in the N-S direction.

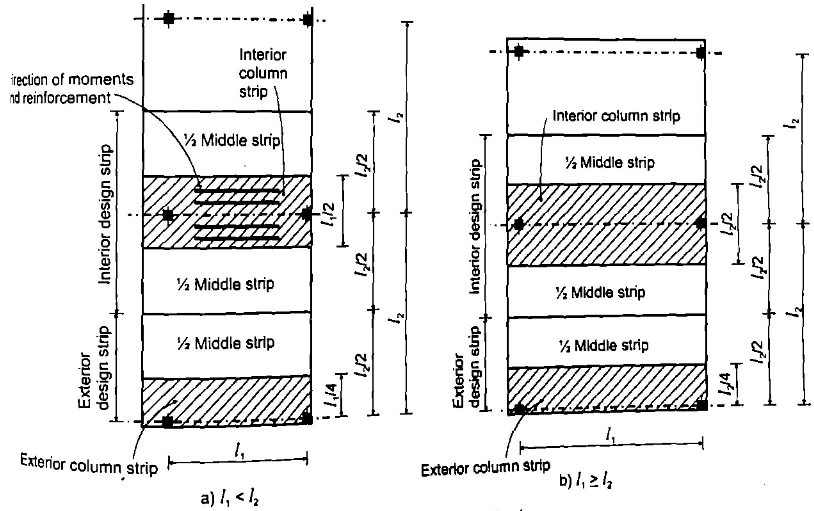


Figure 12.15 Design strip, column strip, and middle strip: a) $l_1 < l_2$, and b) $l_1 \geq l_2$ (courtesy of the Portland Cement Association).

The width of a column strip is determined using the smaller of l_1 or l_2, to account for the tendency for bending moment to concentrate about the column gridline when the span length of the design strip is smaller than its width.

The *middle strip* is a portion of the design strip outside the column strip. Each middle strip is bounded by two column strips.

Note that there are two types of design strips, depending on the frame location within a building plan: *interior* and *exterior* design strips, as shown in Figure 12.15. An interior design strip consists of a column strip and two half-middle strips, while an exterior strip consists of a column strip and one half-middle strip.

12.4.2 Band Width (b_b)

A23.3 Cl.2.3

The band width, b_b, is used for detailing of concentrated reinforcement near columns. It is a portion of the column strip in two-way slabs without beams, which extends by a distance of $1.5h_s$ past the sides of the column (where h_s denotes slab thickness), as shown in Figures 12.16a and b. In slabs with capitals and drop panels, b_b should extend a distance $1.5h$ past the column capital, where h is the overall slab depth at drop panel location (see Figure 12.16c).

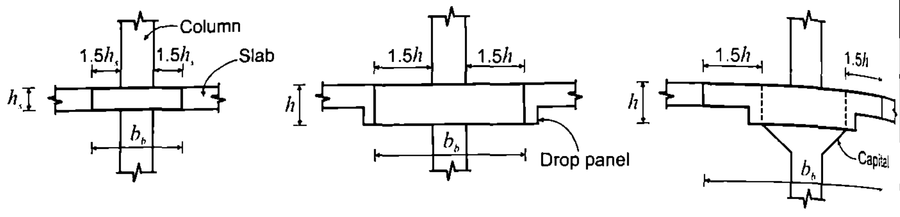


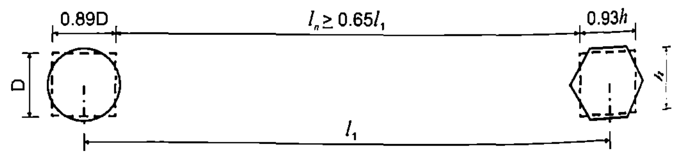
Figure 12.16 Band width b_b: a) flat plate; b) flat slab with drop panels, and c) flat slab with capitals and drop panels.

12.4.3 Clear Span

A23.3 Cl.13.9.2.3

Span denotes centre-to-centre distance between supports (e.g. l_1 span in Figure 12.17), while *clear span* denotes clear distance between supports (e.g. l_n span in Figure 12.17). CSA A23.3 Cl.13.9.2.3 requires that $l_n \geq 0.65l_1$. In some cases, slabs are supported by circular or polygonal-shaped columns. In this case, it is recommended that the supports are treated as square sections with equivalent cross-sectional area.

Figure 12.17 Clear span l_n for slabs supported by circular and polygonal columns
(adapted from CAC, 2005 with the permission of the Cement Association of Canada).



12.4.4 Effective Beam Section

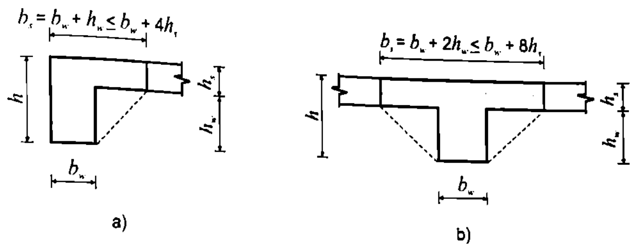
A23.3 Cl.13.8.2.7

In two-way slabs with beams, a portion of the slab acts together with the beam as an L- or a T-section, as shown in Figure 12.18 (note that the concept of T-beams was introduced in Section 3.7). L-sections are found in end spans (edge beam shown in Figure 12.18a), while T-sections are characteristic for typical interior spans (interior beam shown in Figure 12.18b). The effective flange width for two-way slabs (noted as b_f in the figure) is different from the effective flange width for T-beams in one-way slabs discussed in Section 5.8.

The effective beam section for the two-way slab design is shown shaded in Figure 12.18, where

h_s = slab thickness
h = overall beam depth
h_w = beam web depth (below the slab soffit)
b_f = effective flange width
b_w = beam web width

ure 12.18 Effective
im section: a) edge
im, and b) interior
im.



12.4.5 Beam-to-Slab Stiffness Ratio (α)

Consider an elevation of a two-way slab with beams as shown in Figure 12.19a. Beam cross-sectional dimensions, slab thickness, and spacing of adjacent beams will influence the relative stiffness distribution for beams and the slab. This is relevant for the design of two-way slabs with beams. The effect of beam stiffness on deflections and moment distribution in the slab can be taken into account through the beam-to-slab stiffness ratio, α. This section explains a procedure for finding α value for the design of two-way slabs with beams.

First, let us identify equivalent T- and L-beam sections, consisting of a beam web and a portion of the slab (note the sections shown shaded in the figure). These beams span between the column centers (in the direction perpendicular to the plane of the drawing). Flexural stiffness for a beam (k) can be determined from the following equation

$$k = \frac{4EI}{L} \qquad [12.1]$$

where L denotes the beam span, E denotes the modulus of elasticity of concrete, and I denotes the moment of inertia for the beam section.

Next, let us divide the slab into sections, where section width is defined by adjacent panel centrelines and its depth is equal to the slab thickness. Slab sections for an end span and a typical interior span are shown shaded in Figure 12.19b. Flexural stiffness for a slab section can be determined from Eqn 12.1, by computing the moment of inertia for a rectangular section as shown in Figure 12.19c. In cast-in-place concrete construction, beams and slabs are placed monolithically, thus their E and L values are equal (provided that the same concrete mix was used for the beam and slab pour, otherwise the E value could be different). As a result, Eqn 12.1 can be simplified as follows

$$\alpha = \frac{I_b}{I_s} \qquad [12.2]$$

where I_b denotes the beam moment of inertia and I_s denotes the slab moment of inertia. The dimensions of an effective beam section can be determined as shown in Figure 12.18. Note that an L-section is used to determine the moment of inertia for the edge beam, I_{be}, while a T-section is used for a typical interior beam (moment of inertia I_{bi}), as shown in Figure 12.19a.

The moment of inertia for a slab section can be determined as follows (see Figure 12.19c)

$$I_s = \frac{b_s \times h_s^3}{12}$$

where

$b_s = l_{be}$ section width for an end span (corresponding to the moment of inertia I_{se})

$b_s = l_{bi}$ section width for an interior span (corresponding to the moment of inertia I_{si})

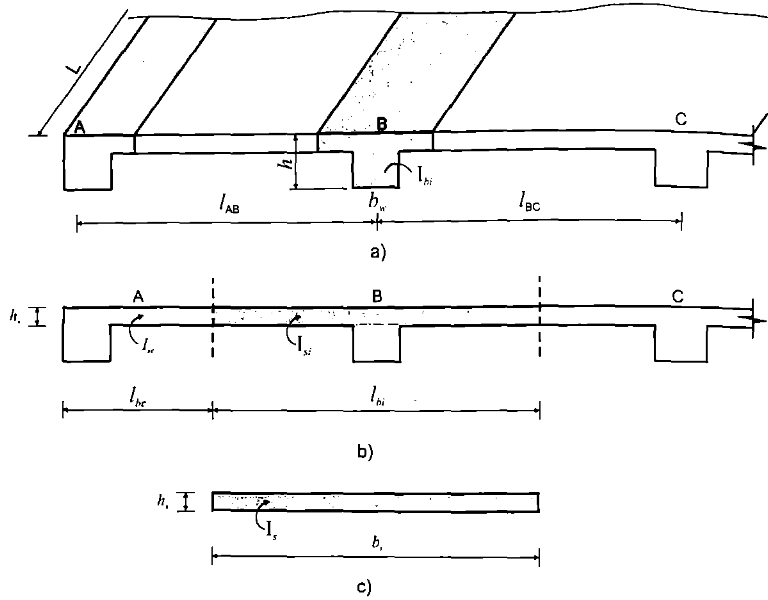


Figure 12.19 Beam and slab properties for an end span and an interior span: a) beam sections; b) slab properties, and c) a typical slab cross-section.

The beam moment of inertia, I_b, can be determined from the first principles, or an approximate value can be determined from the following simplified equation (CSA A23.3 Cl.13.2.5)

A23.3 Eq. 13.4

$$I_b = \frac{b_w h^3}{12}\left[2.5\left(1 - \frac{h_s}{h}\right)\right]$$

The corresponding α value can be determined by substituting the I_b expression into Eqn 12.1 as follows

$$\alpha = \frac{2.5 b_w}{b_s}\left(\frac{h}{h_s}\right)^3\left(1 - \frac{h_s}{h}\right) \qquad [12.3]$$

Alternatively, the beam moment of inertia can be determined from charts included in the Concrete Design Handbook (CAC, 2005).

12.5 GENERAL CSA A23.3 DESIGN PROVISIONS

12.5.1 Regular Two-Way Slab Systems

A23.3 Cl.2.2

Some of the CSA A23.3 design methods, namely the Equivalent Frame Method and the Direct Design Method, can be applied only to regular two-way slab systems. The reason is that most provisions related to these design methods are based on research studies performed on regular slab systems. A *regular* two-way slab system consists of approximately rectangular panels and carries primarily uniform gravity loading.

According to CSA A23.3, a regular slab system should meet the following geometric limitations illustrated in Figure 12.20:

(#a) within a panel, the ratio of longer to shorter span, centre-to-centre of supports, is not greater than 2.0, that is,

$l_1 / l_2 \leq 2.0$

(#b) for slab systems with beams between supports, the ratio of relative effective stiffnesses of beams in the two directions is restricted as follows:

$$0.2 \leq \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \leq 5.0$$

where α_1 and α_2 denote the beam-to-slab stiffness ratio for beams in directions 1 and 2 respectively (refer to Section 12.4.5 for an explanation of the beam-to-slab stiffness ratio)

(#c) column offsets are not greater than 20% of the span (in the direction of the offset) from either axis between the centrelines of successive columns; and

(#d) flexural reinforcement is placed on an orthogonal grid.

Note that the requirements #a, #c, and #d apply to flat plates and flat slabs, while an additional requirement (#b) applies to slabs with beams.

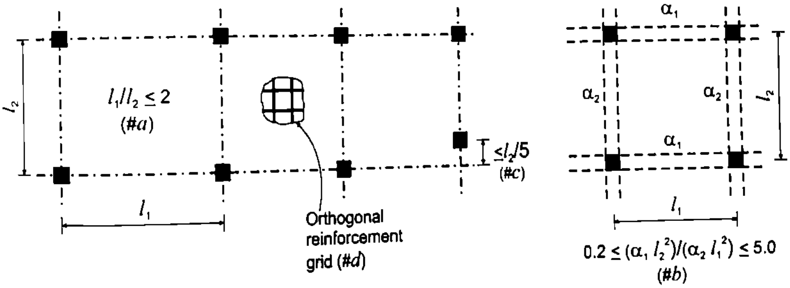


Figure 12.20 A summary of the CSA A23.3 requirements for regular two-way slab systems (#a to #d).

12.5.2 Minimum CSA A23.3 Slab Thickness Requirements for Deflection Control

A23.3 Cl.13.2

Minimum Slab Thickness The thickness of two-way slabs under normal loading conditions is often determined by deflection considerations. CSA A23.3 Cl.13.2 prescribes the minimum slab thickness, h_s, for the different types of two-way slabs (with and without beams), which will be discussed in this section. These minimum slab thickness/span ratios enable the designer to avoid detailed deflection calculations in routine designs; this is similar to the indirect approach for deflection control in flexural members discussed in Section 4.5.2. Note that the CSA A23.3 minimum thickness values are independent of design loading and concrete compressive strength (f_c'), and may lead to conservative design solutions in some cases. For example, the designer may be able to reduce thickness for slabs subjected to light loading (e.g. residential occupancies) by performing detailed deflection calculations. CSA A23.3 approaches for detailed deflection calculations are outlined in Section 12.10.

CSA A23.3 Cl.13.2.1 states that the minimum slab thickness, h_s, shall be based on serviceability requirements but should not be less than 120 mm. NBCC fire resistance requirements also limit the minimum slab thickness depending on the fire rating (see Section 1.8.4). For example, a minimum 130 mm thickness is required for a two-hour fire rating (Appendix D of NBCC 2015).

A23.3 Cl.13.2.3

Flat Plates The minimum thickness, h_s, for the slab without drop panels depends on the slab span and the steel yield strength, f_y. The minimum thickness can be determined from the following equation:

A23.3 Eq. 13.1

$$h_s \geq \frac{l_n(0.6 + f_y/1000)}{30} \qquad [12.4]$$

where

f_y = steel yield strength (MPa)

l_n = longer clear span

Two-way slabs usually have same or similar spans in two orthogonal directions. However, in some cases these spans are different, as illustrated in Figure 12.21. The clear span, l_n for slab thickness calculations (longer clear span) can be determined as follows

$l_n = \max(l_{n1}, l_{n2})$

For example, the clear span can be determined for the slab panel shown in Figure 12.21.b. Since

$l_{n1} > l_{n2}$

it follows that

$l_n = l_{n1}$

When Grade 400 steel is used, which is standard in Canada, f_y = 400 MPa, and the above equation can be simplified as follows:

$$h_s \geq \frac{l_n}{30} \qquad [12.5]$$

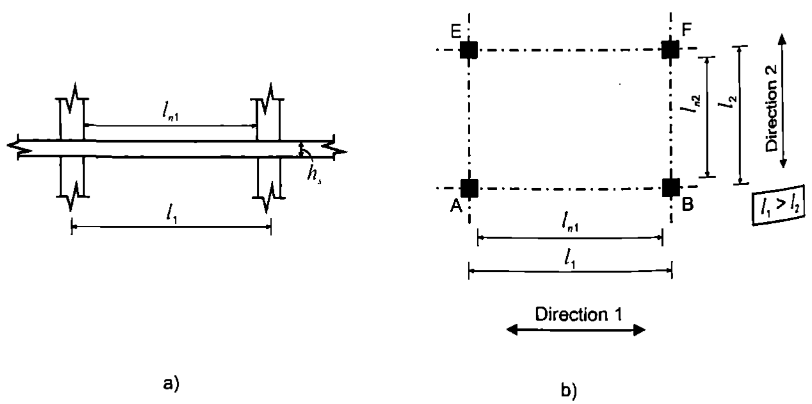


Figure 12.21 A typical two-way slab panel: a) elevation, and b) plan.

CSA A23.3 prescribes an additional requirement for end spans (discontinuous edges). In order to use the same equation, an edge beam should be provided with the stiffness ratio $\alpha \geq 0.80$. When an edge beam is not provided, the minimum slab thickness should be increased by 10 % for panels with discontinuous edge(s), that is, the slab thickness should be at least $1.1h_s$, where h_s is determined from Eqn 12.4. These requirements are illustrated in Figure 12.22.

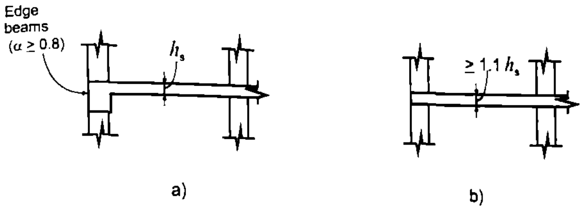


Figure 12.22 Additional requirements for end spans: a) a slab with edge beam, and b) a slab without edge beam.

A23.3 Cl.13.2.4

Flat Slabs (Slabs With Drop Panels) Flat slabs have drop panels, which are formed by thickening the bottom of the slab around the columns, as shown in Figure 12.23. Drop panels are effective in increasing slab stiffness in the areas around the columns; this results in smaller deflections compared to flat plates. For that reason, the minimum slab thickness, h_s, is somewhat reduced and it can be determined from the following equation

A23.3 Eq. 13.2

$$h_s \geq \frac{l_n(0.6 + f_y/1000)}{30} - \left(\frac{2x_d}{l_n}\right)\Delta_h \qquad [12.6]$$

where

Δ_h = additional thickness of the drop panel underneath the slab, and

x_d = drop panel overhang (dimension from the face of the column to the edge of drop panel).

The smaller of the values determined in the two directions should be used for the slab thickness calculation, that is, $x_d = \min(x_{d1}, x_{d2})$ (see Figure 12.23).
The following dimensional limits should be met

$\Delta_h \leq h_s$

and

$x_d \leq l_n/4$

Note that the maximum x_d value results in a decrease in slab depth by approximately 45 mm.

Additional requirement for slab thickness at discontinuous edges is the same as that related to flat plates (see Figure 12.22).

A23.3 Cl.13.2.5

Slabs with Beams Between All Supports The minimum thickness for slab panels with beams on all sides depends on the panel aspect ratio and the relative stiffness of beams in two directions. The minimum thickness, h_s, for slabs with beams between all supports is equal to

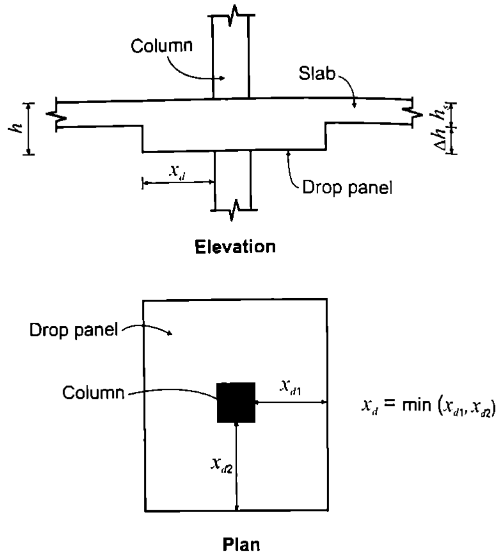
A23.3 Eq. 13.3

$$h_s \geq \frac{l_n(0.6 + f_y/1000)}{30 + 4\beta\alpha_m} \qquad [12.7]$$

where

β = ratio of clear spans in long and short directions; for example, $\beta = l_1/l_2$ for a panel shown in Figure 12.21, and

Figure 12.23 Drop panel dimensions.



α_m = average beam-to-slab stiffness ratio.

Note that

$1.0 \leq \beta \leq 2.0$, otherwise the slab should be treated as a one-way slab.

The beam-to-slab stiffness ratio, α_m, is an average value obtained considering all beams along the panel edges, and it can be determined using the procedures outlined in Section 12.4.5. For the given h_s value, the required α_m value can be determined from the following equation

$$\alpha_m = \frac{1}{4\beta}\left[\frac{l_n(0.6 + f_y/1000)}{h_s} - 30\right]$$

Relative stiffness of beams in the two orthogonal directions is an important parameter influencing the deflections in two-way slabs. For example, when beams in one direction are significantly stiffer, the slab tends to act as a one-way slab spanning between the stiffer beams (even if columns are located on an essentially square grid). Note that CSA A23.3 gives the following upper bound value for α_m:

$\alpha_m \leq 2.0$

Minimum slab thicknesses for various slab types and different values of the key parameters β and α_m are summarized in Table 12.1.

Table 12.1 Minimum Thickness for Two-Way Slab Systems (Grade 400 Reinforcement)

Two-way Slab System	α_m	β	Minimum h_s
Flat plate with edge beams	-	≤ 2.0	$l_n/30$
Flat plate without edge beams	-	≤ 2.0	$l_n/27$
Slab with beams between all supports	1.0	1.0	$l_n/34$
		2.0	$l_n/38$
	≤ 2.0	1.0	$l_n/38$
		2.0	$l_n/46$

12.6 DESIGN FOR FLEXURE ACCORDING TO THE DIRECT DESIGN METHOD

A23.3 Cl.13.9

This section describes the underlying concepts and design provisions for the Direct Design Method (DDM), a statics-based method for the design of two-way slabs adopted by CSA A23.3 and other codes. The application of this method will be demonstrated through two design examples.

12.6.1 Limitations

A23.3 Cl.3.9.1

CSA A23.3 prescribes that the DDM can be used when the following requirements have been met (see Figure 12.24):

#1 A slab must be regular (refer to Section 12.5.1 for more details on regular two-way slabs).
#2 There must be at least three continuous spans in each direction.
#3 The successive span lengths (centre-to-centre of supports) in each direction must not differ by more than one-third of the longer span.
#4 The DDM can be used only for gravity load analysis; gravity loads must be uniformly distributed over the entire slab panel.
#5 The factored live load must not exceed two times the factored dead load.

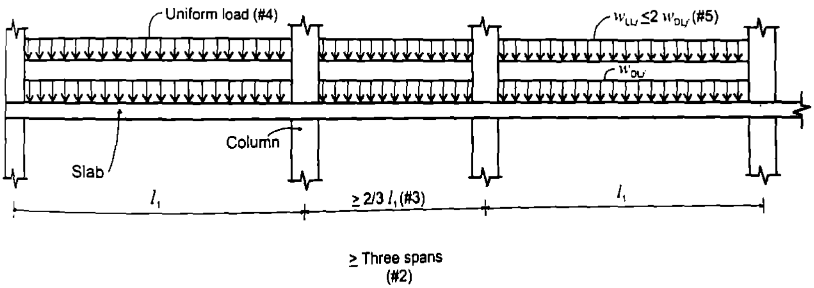


Figure 12.24 Limitations of the Direct Design Method.

Requirement #4 states that the DDM can be used only for gravity load analysis. The design considers a partial frame that consists of the design strip for the slab at a specific floor level and columns above and below that level, as shown in Figure 12.25 (the concept of design strip was introduced in Section 12.4.1). A similar model can be used to design slab at the top floor level. Note that lateral swaying of the frame is prevented by a roller support at the far end of the slab. Refer to Section 12.7.2 for discussion on frame models for gravity load analysis.

12.6.2 The Concept

A23.3 Cl.13.9.2

The distribution of bending moments in a two-way slab according to the DDM will be explained by an example. Let us first explain moment distribution in the longitudinal direction. Consider the span AB of a flat plate system shown in Figure 12.26a. The DDM is based on plane frame model and treats the slab as a wide beam with a width equal to the design strip. The slab is subjected to uniform area load w, which needs to be transformed into linear load $w \cdot l_{2a}$ (based on the design strip width l_{2a}), as shown in Figure 12.26b. A

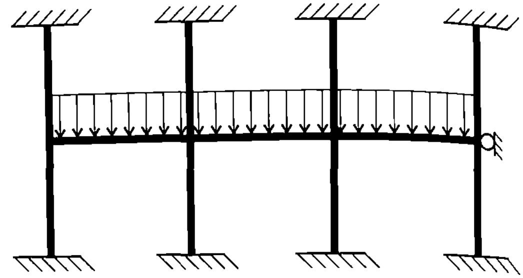


Figure 12.25 A gravity frame model for the DDM.

conceptual diagram illustrating moment distribution for span AB can be seen in Figure 12.26c. End moments M_A and M_B are negative due to the restraints (columns) at points A and B, while the midspan moment M_C is positive. Moment distribution is similar to that for continuous beams and slabs discussed in Section 10.2.2. According to DDM, magnitudes of bending moments at points A, B, and C depend on several factors, including the end support conditions and the type of slab system (slab on beams or flat plate/slab). However, the magnitude of moment gradient M_o is always equal to the sum of average value for bending moments at the supports A and B and the moment at the midspan C, as shown below

$$M_o = \frac{M_A + M_B}{2} + M_C$$

However, M_o is also equal to the maximum moment of an equivalent simply supported beam AB with the span l_n, subjected to uniform load $w \times l_{2a}$, that is,

$$M_o = \frac{(w \times l_{2a}) \times l_n^2}{8}$$

Note that bending moments are determined based on the clear span l_n (instead of the centre-to-centre span l_l). This is similar to the design approach for continuous beams and slabs presented in Chapter 10.

The above statement can be proven by considering a free-body diagram shown in Figure 12.26d. The beam support reaction at point A, R_A, is equal to

$$R_A = (w \times l_{2a}) \times l_n / 2$$

and the bending moment at point C is equal to

$$M_c = R_A \times l_n/2 - (w \times l_{2a})(l_n/2)(l_n/4) = \frac{(w \times l_{2a}) \times l_n^2}{8}$$

which is equal to the moment gradient M_o.

Note that the DDM provisions refer to moment gradient M_o as the *total factored static moment* (CSA A23.3 Cl.13.9.2). M_o can be determined from the following equation:

A23.3 Eq. 13.23

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8} \qquad [12.8]$$

where

w_f = factored load per unit area of the slab

l_{2a} = width of the design strip

l_n = clear span, that is, length of span measured face-to-face of supports (columns, capitals, brackets, or walls).

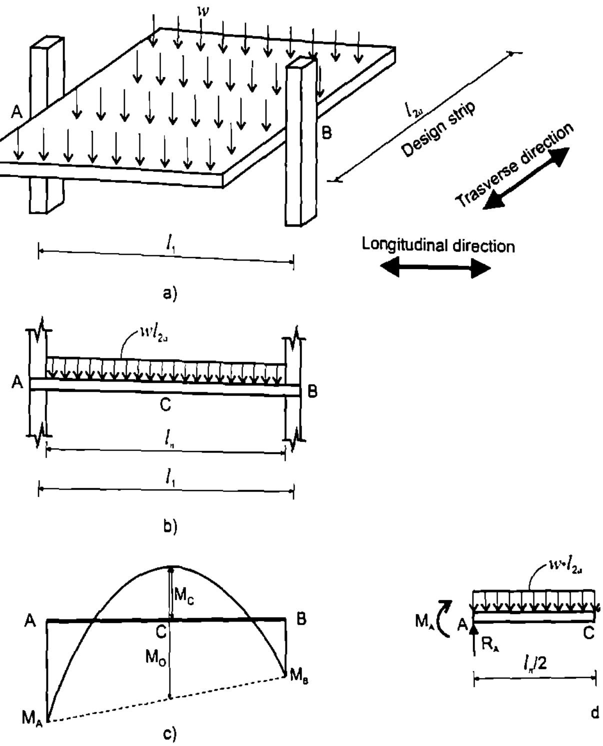


Figure 12.26 Bending moment distribution within a slab span: a) an isometric (3-D) model of the slab span; b) a linear (2-D) model; c) bending moment distribution, and d) a free-body diagram.

Next, let us discuss the transverse distribution of bending moments. Figure 12.27a shows the variation of bending moments in the transverse direction. Note that points A and B denote the column locations, however columns have been omitted from the drawing for clarity. It can be seen from the figure that bending moments at the support B are largest at the column location (moment M_i), and that the values drop towards the ends, that is, moment M_j is the smallest of all. It should be noted that the distribution is symmetrical with regards to the column gridline. Moment variation at the support B in the transverse direction is shown in Figure 12.27b. It can be seen that the bending moments vary in a nonlinear manner, but average bending moments can be used for design. Two different bending moment values are assigned: the larger value is assigned to a region close to the column lines (moment M_{CS}), and the smaller value is assigned to a portion of the slab close to the panel centreline (moment M_{MS}).

In flat plate and flat slab systems, these regions are called "column strip" and "middle strip", as shown in Figure 12.27c. Note that, in slab systems with beams, these regions are called "beam strip" (instead of "column strip") and "slab strip" (instead of "middle strip"). CSA A23.3 provisions for bending moment distribution in two-way slabs according to the DDM are outlined in the following sections.

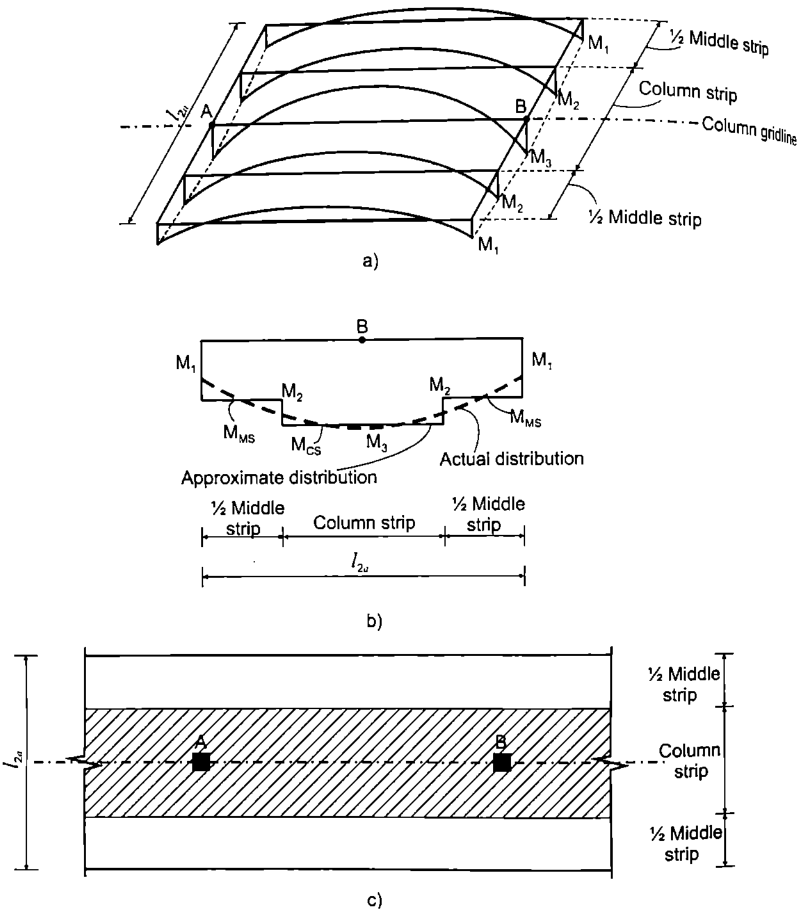


Figure 12.27 Transverse distribution of bending moments in a flat plate: a) an isometric view showing variation of bending moments; b) transverse distribution at support B, and c) column strip and middle strip.

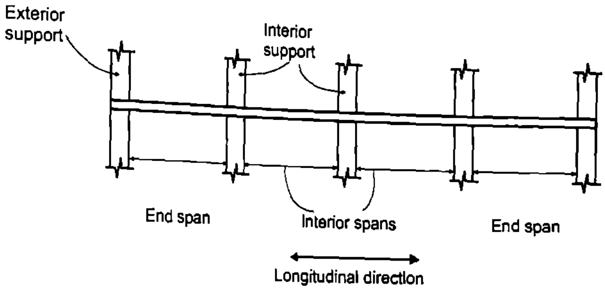
12.6.3 Bending Moment Distribution in Flat Plates and Flat Slabs

A23.3 Cl.13.9.3 and 13.11

This section discusses distribution of the total factored moment, M_o, within a specific span of a two-way slab, which is performed in the following two steps:

1) Distribute M_o between critical locations (supports and midspan) - this is referred to as *distribution in longitudinal direction*, and
2) For each critical location, distribute the moment obtained in the previous step to column strip and middle strip — this is referred to as *transverse distribution*.

The distribution of bending moments in two-way slabs depends on the location of a slab span within a frame. For example, bending moment values in end spans are different compared to interior spans. Note that an exterior column in the end span is called an "exterior support", while all other columns are referred to as "interior supports" (see Figure 12.28).



ure 12.28 Exterior and erior supports.

Bending moment distribution within a specific span of a flat slab depends on several factors, including the type of end supports (restrained/unrestrained), the type of slab (flat slab or flat plate), location of the span within a building (end span or interior span), and the location within a span (support or midspan). The sign of a bending moment depends on the location within a span — bending moments at the supports are negative, while the moments at the midspan are positive. Bending moment values are summarized in Tables 12.2 to 12.4. Note that most values refer to flat plates, while the values for flat slabs (where they are different) are shown in the notes beneath each table.

Table 12.2 shows a typical case: a flat plate supported by columns at all points of support (including the exterior supports). Note the labelling for moments at critical sections, e.g. M_1 denotes the bending moment at the exterior support. The same labelling scheme is used in all tables.

Note that the moment distribution in the end span depends on the type of exterior support. The slab may be cast with a continuously reinforced concrete wall — this is referred to as a "fully restrained exterior edge" (see Table 12.3). Alternatively, a slab end span may be supported by a support which enables rotation - this is referred to as an "unrestrained exterior edge" (see Table 12.4).

These tables show the bending moments values at critical sections within a span (supports and midspan), that is, in longitudinal direction. Moment values are expressed as a fraction of M_o (according to CSA A23.3 Cl.13.9.3). Subsequently, each of these moments needs to be distributed transversely to the column strip and the middle strip; the corresponding moment values are also included in the tables. CSA A23.3 Cl.13.11 prescribes a range of moment values for column and middle strips as a fraction of the bending moment at a critical section (support/midspan).

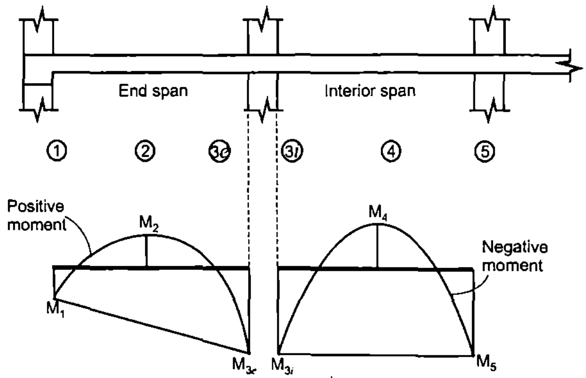
Bending moments at the support may be different for two adjacent slab spans, but the designer should design slab sections for the larger of the two moments (Cl.13.9.3.4). For example, a negative bending moment at the first interior support (Section 3e) is equal to -0.70 M_o while the moment at the same support corresponding to an interior span (Section 3i) is -0.65 M_o. The designer should use the moment with the larger absolute value (-0.70 M_o) for the design at this location; an alternative would be to adjust spans so that the end span is shorter.

Note that Cl.9.3.3 permits an increase or decrease in negative or positive factored moments by 15%, provided that the total static moment M_o for a span in the direction under consideration is not less than that required by Eqn 12.8. This is known as "moment balancing" and it will be illustrated through Example 12.1. Note that the sum of positive and negative moments within a span must remain equal to M_o.

According to Cl.13.11.2, a major portion of the transverse bending moment at a critical section (support/midspan) is assigned to the column strip. The remaining portion is distributed to slab sections on both sides of the column strip, also known as half middle strips. The column strip and two half middle strips constitute the design strips, as shown in Figure 12.15.

Transverse distribution of bending moments according to the DDM will be illustrated by the following example. Consider a typical interior span of a two-way flat plate shown in Figure 12.29a. The distribution of bending moments at the critical sections in the longitudinal direction for an interior span is illustrated in Figure 12.29b. The positive bending

Table 12.2 Flat slab or flat plate supported directly by columns (partially restrained exterior edge)

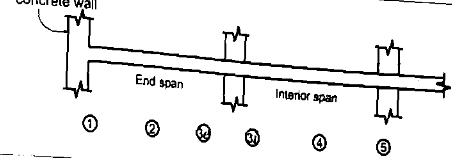


Type of span		End Span			Interior Span		
	Section Location	(1) Exterior	(2) Midspan	(3e) First Interior Support	(3i) Interior Support	(4) Midspan Support	(5) Interior Support
		M_1	M_2	M_{3e}	M_{3i}	M_4	M_5
	Sign	Negative	Positive	Negative	Negative	Positive	Negative
Longitudinal	Total Moment	$-0.26\ M_o$	$+0.52\ M_o$	$-0.70\ M_o$	$-0.65\ M_o$	$+0.35\ M_o$	$-0.65\ M_o$
Transverse	Column Strip Moment	$-0.26\ M_o$	$+(0.29$ to $0.34)\ M_o$	$-(0.49$ to $0.63)\ M_o$	$-(0.46$ to $0.59)\ M_o$	$+(0.19$ to $0.23)\ M_o$	$-(0.46$ to $0.59)\ M_o$*
	Middle Strip Moment	0	= Total Moment - Column Strip Moment				

Note

* = $-(0.49$ to $0.59)\ M_o$ for flat slabs (with drop panels)

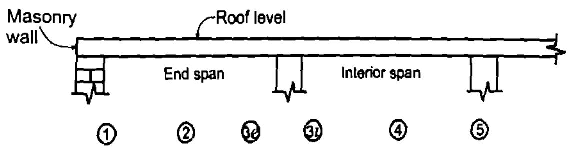
Table 12.3 Flat slab or flat plate with a fully restrained exterior edge



Type of span	End Span			Interior Span		
Section Location	(1) Exterior Support M_1	(2) Midspan M_2	(3e) First Interior Support M_{3e}	(3i) Interior Support M_{3i}	(4) Midspan M_4	(5) Interior Support M_5
Sign	Negative	Positive	Negative	Negative	Positive	Negative
Longitudinal Total Moment	$-0.65\ M_o$	$+0.35\ M_o$	$-0.65\ M_o$	$-0.65\ M_o$	$+0.35\ M_o$	$-0.65\ M_o$
Transverse Column Strip Moment	$-0.65\ M_o$	$+(0.19$ to $0.23)\ M_o$	$-(0.46$ to $0.59)\ M_o$	$-(0.46$ to $0.59)\ M_o$*	$+(0.19$ to $0.23)\ M_o$	$-(0.46$ to $0.59)\ M_o$
Middle Strip Moment	0	= Total Moment - Column Strip Moment				

Note
* = $-(0.49$ to $0.59)\ M_o$ for flat slabs (with drop panels)

Table 12.4 Flat slab or flat plate with an unrestrained exterior edge



Type of span	End Span			Interior Span		
Section Location	(1) Exterior Support M_1	(2) Midspan M_2	(3e) First Interior Support M_{3e}	(3i) Interior Support M_{3i}	(4) Midspan M_4	(5) Interior Support M_5
Sign	Negative	Positive	Negative	Negative	Positive	Negative
Longitudinal Total Moment	0	$+0.66\ M_o$	$-0.75\ M_o$	$-0.65\ M_o$	$+0.35\ M_o$	$-0.65\ M_o$
Transverse Column Strip Moment	0	$+(0.36$ to $0.43)\ M_o$	$-(0.53$ to $0.68)\ M_o$	$-(0.46$ to $0.59)\ M_o$*	$+(0.19$ to $0.23)\ M_o$	$-(0.46$ to $0.59)\ M_o$*
Middle Strip Moment	0	= Total Moment - Column Strip Moment				

Note
* = $-(0.49$ to $0.59)\ M_o$ for flat slabs (with drop panels)

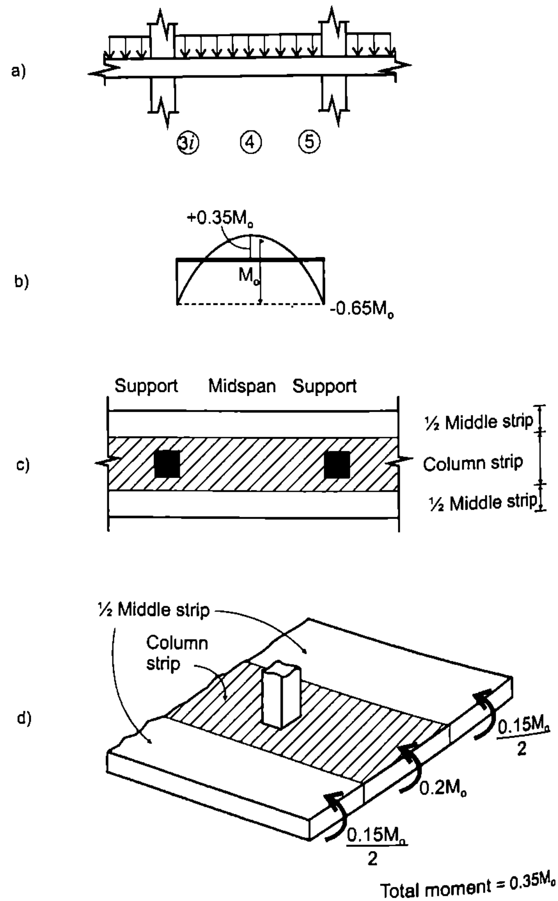
moment at the midspan (column 4 in Table 12.2) is equal to $+0.35\ M_o$. Transverse distribution of the midspan moment is shown in Figure 12.29d.

First, the designer needs to set the column strip moment. Let us set the value to $+0.2\ M_o$; this is within the permitted range $+(0.19$ to $0.23)\ M_o$ according to Table 12.2. The remaining portion of the transverse bending moment is equal to the difference between the column strip moment ($+0.2\ M_o$) and the total moment for that section ($+0.35\ M_o$), that is, ($+0.15\ M_o$). This bending moment is resisted by the two half middle strips, as shown in Figure 12.29c.

CSA A23.3 prescribes the following requirements regarding the bending moments at column locations:

a) Interior columns: according to Cl.13.11.2.7, slab band b_b should be designed to resist at least one-third of the total factored negative moment at the column location equal to $-0.65\ M_o$ (note that the band b_b was introduced in Section 12.4.2). This is illustrated in Figure 12.30a. The total moment for an interior column section (M_{3i}) is equal to

Figure 12.29 Transverse distribution of bending moments in a flat plate: a) a typical interior span; b) the moment distribution at critical sections (supports and midspan); c) column strip and half middle strips, and d) the transverse moment distribution at midspan - an isometric view.



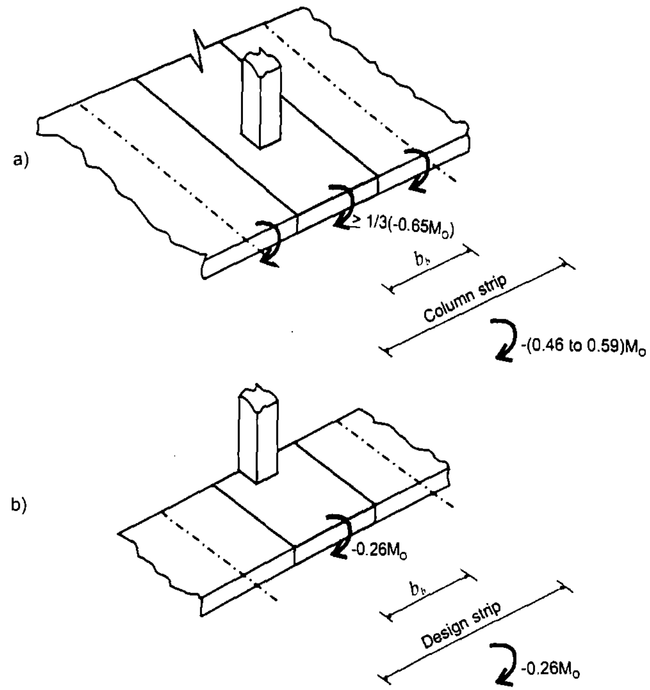


Figure 12.30 Moment distribution at columns: a) an interior column, and b) an exterior column.

(-0.65M_o) (see Table 12.2). One-third of that moment is assigned to band b_b centered at the column. The moment for the column strip is equal to $-(0.46$ to $0.59)M_o$. The remaining bending moment, equal to the difference between the column strip moment and the moment at band b_b, should be resisted by the remaining portion of the column strip outside band b_b.

b) Exterior columns: according to Cl.13.10.3, the total factored negative moment at an exterior column (M_t) equal to $-0.26\ M_o$ should be resisted by the band b_b. This is illustrated in Figure 12.30b.

12.6.4 Bending Moment Distribution in Slabs With Beams Between all Supports

A23.3 Cl.13.9.3 and 13.12

The distribution of transverse bending moments in slabs with beams between all of their supports is different than that in flat slabs and flat plates. The design strip is divided into a beam strip and a slab strip. The beam strip is located in the proximity of column lines, similar to the column strips in flat plates and flat slabs. The width of the beam strip is equal to the effective flange width, b_f, shown in Figure 12.18. The remaining portion of the design strip is called the slab strip, and it is divided into two half-strips, as shown in Figure 12.31a.

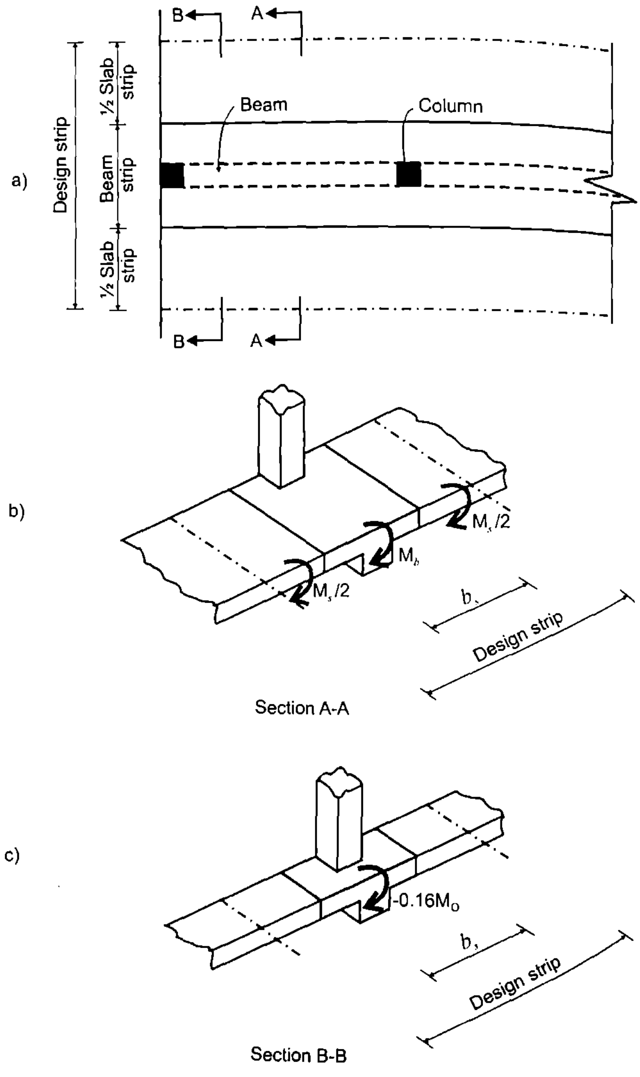


Figure 12.31 Design strip in a two-way slabs with beams: a) a plan view; b) an isometric view showing beam and slab strips and the corresponding bending moments, and c) moment distribution at the exterior column.

First, the bending moment (M) is determined at the critical section in the longitudinal direction (at the supports or the midspan), and the value is expressed as a fraction of the total factored moment, M_o. Next, a bending moment at each critical section is distributed to the beam and slab strips in transverse direction (see Figure 12.31b), that is,

$M = M_b + M_s$

where M_s is the moment for the slab strip, and M_b is the moment for the beam strip M_b can be determined from the following equation (Cl.13.12.2.1):

$$M_b = \left[\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)\right] \times M \qquad [12.9]$$

where

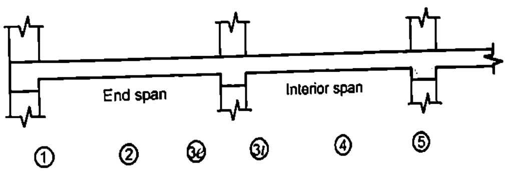
l_1 and l_2 = slab spans in the direction 1 (longitudinal direction in the plane of the frame) and 2 (transverse direction) respectively, and

α_1 = the beam-to-slab stiffness ratio in direction 1 (corresponding to l_1), as discussed in Section 12.4.5.

The above equations apply to all slab locations (supports and midspan), except for the exterior column, where 100% of the negative bending moment (M_1) is assigned to the beam strip, as shown in Figure 12.31c (Cl.13.12.2.2). Note that the beam should be designed to resist its self-weight and 100% of the concentrated or distributed loads applied directly to the beam (e.g. load due to a partition wall) (Cl.13.12.2.3).

Table 12.5 summarizes the bending moment values at critical sections for slabs with beams.

able 12.5 A slab with beams between all supports



	Type of span	End Span			Interior Span		
	Section Location	(1) Exterior Support M_1	(2) Midspan M_2	(3e) First Interior Support M_{3e}	(3i) Interior Support M_{3i}	(4) Midspan M_4	(5) Interior Support M_5
	Sign	Negative	Positive	Negative	Negative	Positive	Negative
Longitudinal	Total Moment M	$-0.16\ M_o$	$+0.59\ M_o$	$-0.70\ M_o$	$-0.65\ M_o$	$+0.35\ M_o$	$-0.65\ M_o$
	Beam Strip Moment M_b	$-0.16M_o$	$M_b = \left[\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)\right] \times M$ [12.9]				
Transverse	Slab Strip Moment M_s	0	$M_s = M - M_b$				

Bending moment distribution for the selected values of α_1 (0.5 and 1.0) and l_2/l_1 ratio (0.5, 1.0, and 2.0) has been summarized in the following tables. It can be seen from Table 12.6 that the bending moment in the beam strip significantly decreases with an increase in the l_2/l_1 ratio; a similar trend can be observed in Table 12.7. This is illustrated in Figure 12.32.

Table 12.6 Transverse distribution of bending moments in slabs with beams: $\alpha_1 = 0.5$

l_2/l_1	Beam strip (M_b)	Slab strip (M_s)
0.5	$0.52 \times M$	$0.48 \times M$
1.0	$0.42 \times M$	$0.58 \times M$
2.0	$0.20 \times M$	$0.80 \times M$

Table 12.7 Transverse distribution of bending moments in slabs with beams: $\alpha_1 = 1.0$

l_2/l_1	Beam strip (M_b)	Slab strip (M_s)
0.5	$0.64 \times M$	$0.36 \times M$
1.0	$0.51 \times M$	$0.49 \times M$
2.0	$0.26 \times M$	$0.74 \times M$

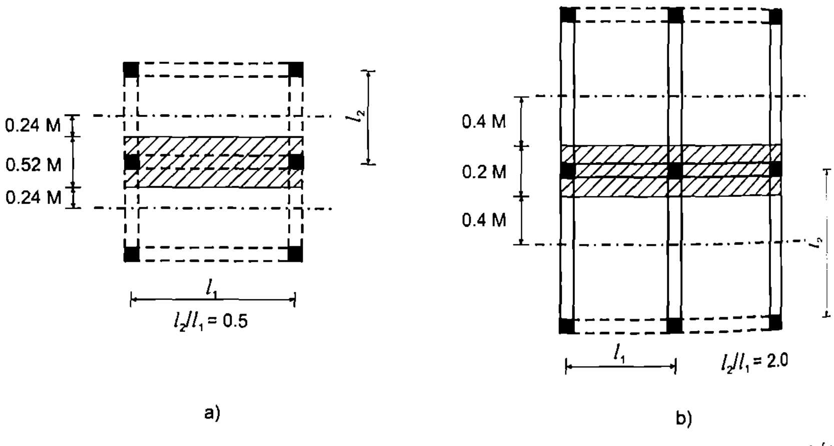


Figure 12.32 Variation in the transverse distribution of bending moment between the beam strip and the slab strip for $\alpha_1 = 0.5$: a) $l_2/l_1 = 0.5$, and b) $l_2/l_1 = 2.0$.

12.6.5 Unbalanced Moments

A23.3 Cl.13.9.4

One of the key issues associated with two-way slab systems is safety of slab-column connections. All loads carried by the slab converge on the column. This section explains the calculation procedure for unbalanced bending moments which occur due to an uneven distribution of live loads in adjacent slab spans.

Unbalanced moments in the slab are caused by an uneven distribution of live load in adjacent spans. This concept is illustrated in Figure 12.33. Slab span AB shown in Figure 12.33

is subjected to both dead and live load, while the adjacent span BC is subjected to dead load only. Bending moments at the interior column B due to loading in spans AB and BC (M_{BL} and M_{BR}) are shown in Figure 12.33b. The unbalanced bending moment M_u shown in Figure 12.33c is equal to the difference between the slab bending moments for spans AB and BC, that is,

$$M_u = M_{BL} - M_{BR}$$

The unbalanced moment is resisted by the connection and the column or wall above and below the slab, as follows (see Figure 12.33c)

$$M_u = M_{B_1} + M_{B_2}$$

where M_{B_1} and M_{B_2} are bending moments in the column above and below the connection due to the unbalanced moment. Note that M_u that is distributed to the column in proportion to the flexural stiffness ($4EI/h$), where E is modulus of elasticity, I is moment of inertia, and h is column height (centre-to-centre distance between the floor slabs). The column segments shown in Figure 12.33c have different heights (h_{B1} and h_{B2}) and moments of inertia (I_1 qnd I_2); this will result in different flexural stiffnesses and bending moments. When the column segments above and below the connection have the same height and corresponding stiffness, each segment will resist one-half of the unbalanced moment M_u. Transfer of unbalanced moments through the connection will be discussed in Section 12.9.3.

The unbalanced moments are intended to account for uneven live load in adjacent spans when the design is performed according to the DDM. This is not required for the EFM, because it is possible to apply pattern loading which considers the effect of uneven live load (see Section 12.7.2 for a discussion on pattern loading).

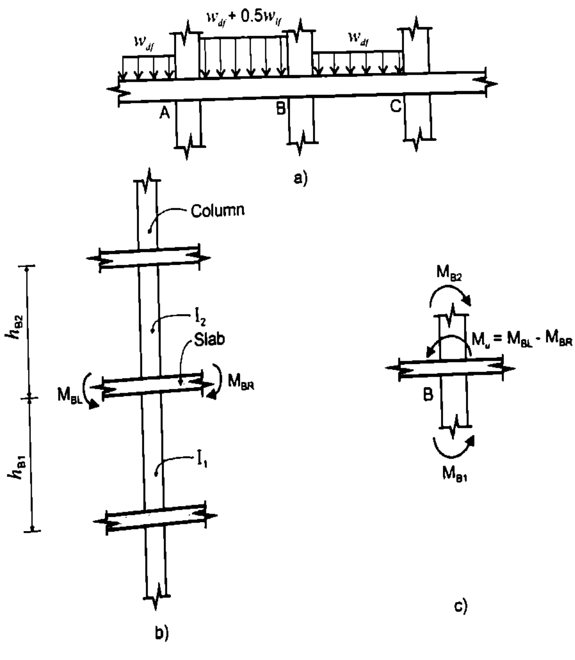


Figure 12.33 Unbalanced bending moments: a) an uneven load on adjacent slab spans; b) bending moments at connection B, and c) unbalanced bending moment transferred to the column.

Interior Columns

The unbalanced moment, M_u, for an interior column is computed assuming that the longer span adjacent to the column (l_n) is subjected to the factored dead load and half the factored live load, while the shorter span (l'_n) is subjected to the factored dead load only. The factored unbalanced moment at an interior column is equal to the difference between the bending moments at adjacent spans with different lengths, that is,

A23.3 Eq. 13.24

$$M_u = 0.07\left[(w_{df} + 0.5w_{lf})l_{2a}l_n^2 - w'_{df}l'_{2a}(l'_n)^2\right] \quad [12.10]$$

where

w_{df} = factored dead load per unit area for the longer span corresponding to clear span l_n in longitudinal direction and design strip l_{2a}

w'_{df} = factored dead load per unit area for the shorter span corresponding to clear span l'_n in longitudinal direction and design strip l'_{2a}

w_{lf} = factored live load per unit area (longer span only)

The coefficient 0.07 in Eqn 12.10 is approximately equal to 0.65 times 1/8 (note that 0.65 is the multiplier used in the DDM to obtain the bending moment at the interior support location, and 1/8 is multiplier in Eqn 12.8).

The notation used in the above equation is presented in Figures 12.34a and b. Bending moments at the support are shown in Figure 12.34c. Note that

$$M_{BL} = 0.07\left[(w_{df} + 0.5w_{lf})l_{2a}l_n^2\right]$$

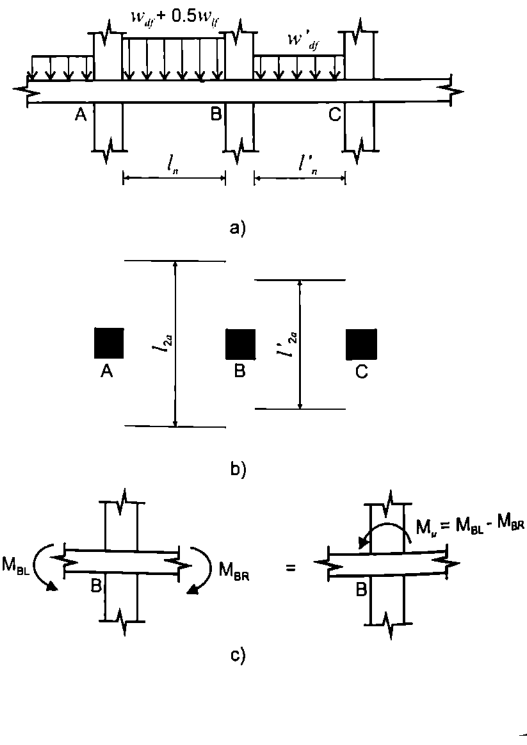


Figure 12.34 Unbalanced moment according to CSA A23.3: a) loading pattern causing an unbalanced moment at interior support B, b) plan length definitions, and c) free-body diagrams.

and

$$M_{BR} = 0.07\left[w'_{lf} l'_{2a} (l'_n)^2 \right]$$

When adjacent span lengths are equal and subjected to the same design live load of the same intensity, Eqn 12.10 can be simplified as follows

$$M_u = 0.07\left[(0.5 w_{lf}) l_{2a} l_n^2 \right]$$

Exterior Columns

The moment transferred at an exterior column is equal to the negative factored moment at the exterior support, thus the entire bending moment can be considered as unbalanced. In general, the slab capacity to transfer bending moments to the exterior columns is limited due to limited ability of the slab top reinforcement to develop flexural capacity at the edge. It is therefore recommended to reduce the design moments at the exterior columns to a minimum.

12.6.6 CSA A23.3 Reinforcement Requirements for Two-Way Slabs

It should be noted that the CSA A23.3 reinforcement requirements presented in this section apply to two-way slabs designed according to all design procedures.

Design of Flexural Reinforcement The amount of top and bottom reinforcement is determined considering the slab section with the width (b) and depth (d). The width depends on the location: it could be a column width, a slab width, or a band (b_b).

The design bending moment (M_f) corresponds to the section under consideration, and it is determined according to the procedures discussed earlier in this section. The required reinforcement area (A_s) can be found by applying the Direct Procedure discussed in Section 5.5.1 and Eqn 5.4 as follows

$$A_s = 0.0015 f'_c b \left(d - \sqrt{d^2 - \frac{3.85 M_f}{f'_c b}} \right) \qquad [5.4]$$

The required bar spacing (s) can be determined from the following equation

$$s \le A_b \frac{b}{A_s} \qquad [12.11]$$

where A_b denotes the bar area.

A23.3 Cl.13.10

Reinforcement Requirements for Flat Slabs and Flat Plates

Minimum reinforcement area (Cl.13.10.1)

CSA A23.3 prescribes the same minimum reinforcement area for two-way and one-way slabs (see Section 5.7.1), that is (Cl.7.8.1),

$$A_{s\,min} = 0.002\, A_g \qquad [5.16]$$

and

$$A_g = b \cdot h_s$$

where A_g is the gross cross-sectional area of a slab section, h_s is the slab thickness, and b is the width of the slab strip (e.g. column strip or middle strip in flat plates). The minimum amount of reinforcement is intended to control shrinkage and temperature effects in slabs.

Maximum amount of reinforcement (Cl.10.5.2)

This check is the same as for other reinforced concrete flexural members (see Section 5.6.1). In order to achieve the steel-controlled failure, the reinforcement ratio (ρ) should not exceed the balanced reinforcement ratio (ρ_b) presented in Table A.4, that is,

$$\rho \leq \rho_b$$

where

$$\rho = \frac{A_s}{b \cdot d}$$

[3.1]

Note that the flexural reinforcement in two-way slabs is often closer to the minimum amount prescribed by CSA A23.3.

Reinforcement spacing

Maximum permitted reinforcement spacing (s_{max}) depends on the slab thickness, location within the slab (support or midspan), and the sign of bending moment (positive or negative). A summary of the key CSA A23.3 reinforcement spacing requirements for two-way slabs is presented in Table 12.8.

Table 12.8 CSA A23.3 Reinforcement Spacing Limits for Two-Way Slabs

Location	Symbol	Maximum spacing* (s_{max})	Code Clause
General requirement - all locations	s	$\leq 5h_s$ ≤ 500 mm	7.8.1
Negative flexural reinforcement - band b_b	s_b	$\leq 1.5h_s$ ≤ 250 mm	13.10.4
Negative flexural reinforcement - outside the band b_b	s^-	$\leq 3h_s$ ≤ 500 mm	13.10.4
Positive flexural reinforcement	s^+	$\leq 3h_s$ ≤ 500 mm	13.10.4

Note
* = lesser of the alternative values

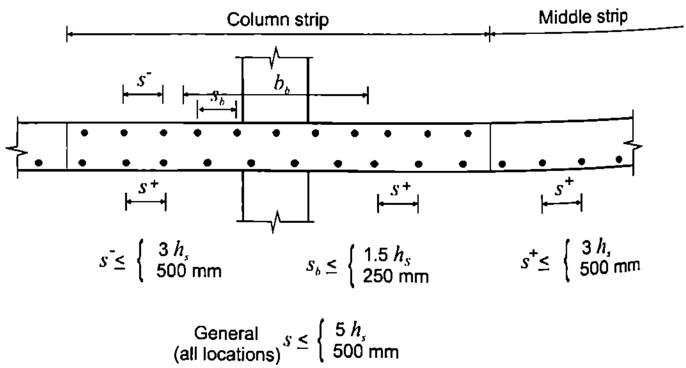


Figure 12.35 Reinforcement distribution at column locations.

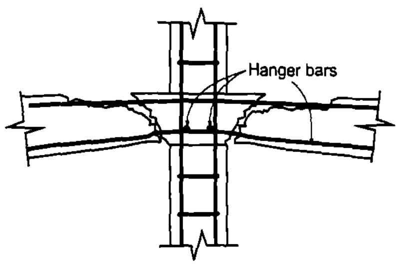
Reinforcement anchorage and curtailment (Cl.13.10.5 and 13.10.8)

The requirements regarding the anchorage of the reinforcement are included in Cl.13.10.5. The top reinforcing bars with either 90° or 180° hooks are placed at slab edges to control cracking in the slab. The bottom reinforcement, placed to resist tension due to positive moments, is straight and continuous between the columns. At interior columns, top reinforcing bars are distributed in each direction. These bars are generally cut off near one-third spans on each side of columns in regions where there are no negative moments. At the top floor or roof level, hooked dowels matching the column reinforcement are placed at the top of the slab. Guidance regarding reinforcement lengths and arrangements in two-way slabs is provided in Cl.13.10.8.

Structural integrity reinforcement (Cl.13.10.6)

Experience has shown that accidental overload of a column supporting a flat slab structure could result in the failure of the floor or roof structure and possibly lead to a building collapse. This phenomenon is known as progressive collapse and it was illustrated through a real-life case study at the beginning of this chapter. CSA A23.3 prescribes additional continuous bottom reinforcement at slab-to-column interface in flat slabs to help enhance the structural integrity in the event of accidental overload of an individual column. These reinforcing bars, known as *integrity reinforcement*, are intended to provide additional tension resistance in the slab after the failure takes place at the column location. The failure mechanism and conceptual layout of integrity reinforcement are illustrated in Figure 12.36.

Figure 12.36 Shear failure of a flat plate showing integrity reinforcement (hanger bars).



According to Cl.13.10.6.1, the total area of integrity reinforcement ($\sum A_{sb}$) connecting the slab or drop panel to the column or column capital can be calculated from the following equation

A23.3 Eq. 13.26

$$\sum A_{sb} = \frac{2V_{se}}{f_y} \qquad [12.12]$$

where V_{se} denotes the shear force transmitted to the column or column capital due to specified loads (see Figure 12.37), which can be determined as follows

$$V_{se} = w \cdot A$$

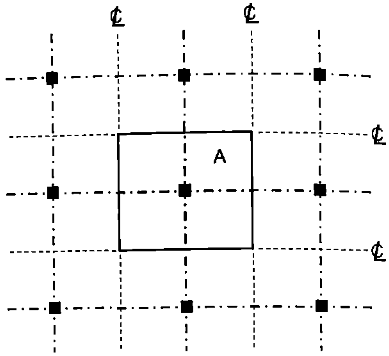
and

$$w = DL + LL$$

where A is the tributary slab area for two-way shear calculations (shown shaded in the figure), and DL and LL are specified dead and live load, respectively.

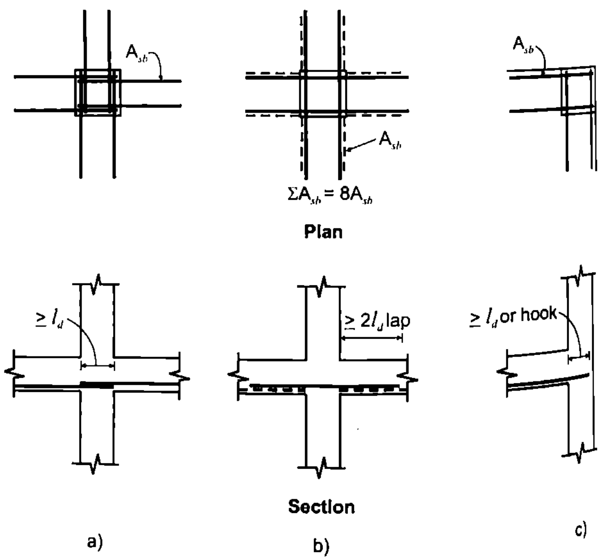
According to Cl.13.10.6.2, integrity reinforcement should consist of at least two bottom reinforcing bars that extend through the column core or column capital region in each span direction. It is essential for this reinforcement to be continuous through the column and to ensure that an adequate anchorage has been provided. The following alternative arrangements are prescribed by Cl.13.10.6.3 (see Figure 12.38):

Figure 12.37 Tributary area A for the design of integrity reinforcement.



a) bottom reinforcement can be extended through the column, and a Class A tension lap splice discussed in Section 9.9 can be used (Cl.13.10.6.3a);
b) additional bottom reinforcement can be placed over a column or column capital, such that an overlap of $2l_d$ is provided with the bottom reinforcement in adjacent spans, where l_d is bar development length discussed in Section 9.3 (Cl.13.10.6.3b), and
c) at discontinuous edges (end spans in the slab), bottom reinforcement needs to be extended and bent, hooked, or otherwise anchored over the supports such that the yield stress can develop at the face of column or column capital (Cl.13.10.6.3c).

Figure 12.38 CSA A23.3 provisions for integrity reinforcement: a) bottom reinforcement lap spliced through the column; b) additional bottom reinforcement placed over a column, and c) bottom reinforcement at a discontinuous edge (adapted from CAC, 2005 with the permission of the Cement Association of Canada).



A23.3 Cl.13.12

Reinforcement Requirements for Slabs with Beams Reinforcement in slabs with beams must comply with the same requirements as discussed in previous section regarding flat slabs and flat plates. In general, a slab strip resists a portion of the bending moment and the reinforcement should be uniformly distributed over the slab width.

When beams are relatively stiff, slabs will develop torsional moments at exterior corners at a 45° angle to the edges. These moments will cause tension both in the top and the bottom of the slab. The resulting cracking pattern in the slab is shown in Figure 12.39a. For further details on torsional moments in two-way slabs the reader is referred to Park and Gamble (2000).

Special *corner reinforcement* needs to be provided to resist these twisting moments in slabs with beams (Cl.13.12.5). The reinforcement should be designed to resist the maximum positive bending moment per unit width of the slab panel. The reinforcement should be provided within a band parallel to the diagonal in the top of the slab and a band perpendicular to the diagonal in the bottom of the slab (see Figure 12.39b). Alternatively, the reinforcement may be placed in two layers parallel to the edges of the slab in both the top and bottom of the slab. The reinforcement must extend at least one-fifth of the shorter span in each direction from the corner.

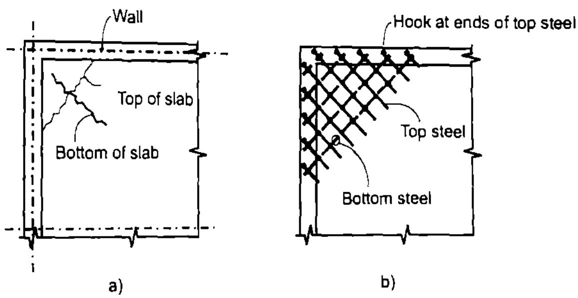


Figure 12.39 Corner reinforcement in slabs with beams: a) cracking pattern, and b) corner reinforcement.

12.6.7 Design Applications of the Direct Design Method

The design of two-way slabs for flexure according to the DDM was discussed in detail in this section. General design steps are outlined in Checklist 12.1. Although the steps have been presented in specific sequence, it is not necessary to follow the same sequence in all design situations. Two design examples illustrating the design of a flat plate and a slab with beams according to the DDM are presented next.

Checklist 12.1 Design of Two-Way Slabs for Flexure According to the Direct Design Method

Step	Description	Code Clause
1	Check whether the Direct Design Method can be used (see Section 12.6.1).	13.9.1
2	Select slab thickness — use the CSA A23.3 minimum thickness requirements outlined in Section 12.5.2 as a reference.	13.2
3	Identify the design strip for frame under consideration as outlined in Section 12.4.1.	
3a	Flat plates and flat slabs: determine the widths for the column and middle strips.	13.1.2 Commentary

(Continued)

Checklist 12.1 Continued

Step	Description	Code Clause
3b	Slabs with beams: determine the widths for the beam and slab strips. The width of the beam strip is equal to the flange width of the effective beam section (see Section 12.4.4).	13.8.2.7
4	Compute the total factored static moment M_o for the span. Determine the factored load for the span by treating the slab as a wide beam with the width equal to the design strip (see Section 12.6.2).	13.9.2
5	Distribute M_o in the longitudinal direction between critical sections (supports and midspan) within the span.	13.9.3
5a	Flat plates and flat slabs: refer to Tables 12.2 to 12.4 (see Section 12.6.3).	
5b	Slabs with beams: refer to Table 12.5 (see Section 12.6.4).	
6	Perform a transverse distribution of the bending moment for each critical section obtained in Step 5.	
6a	Flat plates and flat slabs: find bending moments for the column and middle strips (see Tables 12.2 to 12.4).	13.11
6b	Slabs with beams: find bending moments for the beam strip and the slab strip (see Table 12.5).	13.12
7	Design and detail flexural reinforcement for the slab. Determine the area of top and bottom reinforcement for various slab sections. Refer to the design procedures for rectangular beam sections outlined in Section 5.5.	
7a	Flat plates and flat slabs: distribute the reinforcement according to the CSA A23.3 reinforcement requirements summarized in Section 12.6.6 (in particular Table 12.8).	13.10
7b	Slabs with beams: refer to additional reinforcement requirements summarized in Section 12.6.6.	13.12
8	Slabs with beams and flat plates/slabs with edge beams: design the beams according to the CSA A23.3 design provisions for flexural members explained in Chapter 5.	

Example 12.1

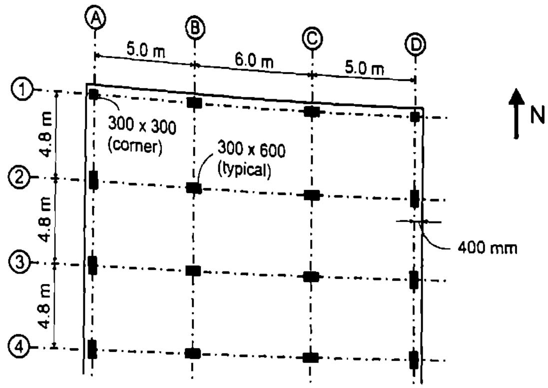
Two-Way Flat Plate - Direct Design Method

Consider a floor plan of a two-way slab system without beams (flat plate) shown in the following figure. The plan shows an intermediate floor level, and a typical storey height is 3.0 m. Column dimensions are 300 mm by 600 mm, except for the corner columns (300 mm by 300 mm), as shown in the figure. Edge (spandrel) beams will be provided. The slab is subjected to specified live load (LL) of 3.6 kPa, and superimposed dead load (DL_s) of 1.44 kPa, in addition to its self-weight. Consider only the effect of gravity loads for this design - lateral loads are to be resisted by shear walls, which are omitted from the drawing. Use 15M bars for slab reinforcement.

Use the CSA A23.3 Direct Design Method to determine design bending moments and the amount and distribution of reinforcement for an interior frame along gridline 2.

Given: $f'_c = 30$ MPa

$f_y = 400$ MPa



SOLUTION:

1. **Check whether the criteria for the CSA A23.3 Direct Design Method are satisfied (see Section 12.6.1):**
   CSA A23.3 Cl. 13.9.1 prescribes that the DDM can be used when the following requirements have been met:
   - #1 A slab is regular (see Section 12.5.1).
   - #2 There are three continuous spans in each direction.
   - #3 The successive span lengths, centre-to-centre of supports, in each direction must not differ more than one-third of the longer span.
     E-W direction: Span 2 – Span 1 = 6.0 m – 5.0 m = 1 m < 6.0 m/3 = 2.0 m
     N-S direction: all span lengths are equal (4.8 m)
   - #4 The slab is subjected to uniformly distributed gravity loads.
   - #5 The factored live load does not exceed two times the factored dead load (this will be confirmed in Step 3).

2. **Determine the required slab thickness based on deflection control requirements.**
   For two-way slab systems without beams, the minimum overall thickness (h_s) can be determined according to CSA A23.3 Cl.13.2.3 as follows (when Grade 400 reinforcement is used)

$$h_s \geq \frac{l_n}{30}$$

We need to determine the clear span for each span under consideration. Since l_n denotes the longer clear span, two clear span values need to be considered for each slab panel along gridline 2:

Span 1 (end span AB)

E-W direction: $l_n = 5.0 - (\frac{0.3}{2} + \frac{0.6}{2}) = 4.55$ m

N-S direction: $l_n = 4.8 - (\frac{0.3}{2} + \frac{0.6}{2}) = 4.35$ m

The longer span (E-W direction) governs, that is,

$l_{n1} = 4.55$ m

The required slab thickness is

$$h_s \geq \frac{l_{n1}}{30} = \frac{4550}{30} = 152 \text{ mm}$$

Since the design does not anticipate the provision of edge beams, CSA A23.3 Cl.13.2.3 requires the slab thickness to be increased by 10 %, that is,

$h_s > 1.1 \times 152 = 167$ mm

Span 2 (interior span BC)

E-W direction: $l_n = 6.0 - (\frac{0.6}{2} + \frac{0.6}{2}) = 5.4$ m

N-S direction: $l_n = 4.8 - (\frac{0.3}{2} + \frac{0.3}{2}) = 4.5$ m

Therefore, the span in E-W direction governs, that is,

$l_{n2} = 5.4$ m

The required slab thickness is

$$h_s \geq \frac{l_{n2}}{30} = \frac{5400}{30} = 180 \text{ mm}$$

In this case, the required thickness is larger for Span 2 (180 mm) than for Span 1 (152 mm). In practice the slab thickness should be uniform and the higher value should be used, that is,

$h_s = 180$ mm

3. **Find the factored design loads.**

a) Calculate the dead load acting on the slab.
First, calculate the slab's self-weight:

$$DL_w = h \times \gamma_w = 0.18 \text{ m} \times 24 \text{ kN/m}^3 = 4.32 \text{ kPa}$$

where $\gamma_w = 24 \text{ kN/m}^3$ is the unit weight for normal-density concrete.
The following superimposed dead load was given:

$DLs = 1.44$ kPa

Finally, the total factored dead load is equal to

$$w_{DL,f} = 1.25(DL_w + DL_s) = 1.25(4.32 + 1.44) = 7.2 \text{ kPa}$$

b) Calculate the factored live load:

$$w_{LL,f} = 1.5 \times LL_w = 1.5 \times 3.6 \text{ kPa} = 5.4 \text{ kPa}$$

c) The total factored load is

$$w_f = w_{DL,f} + w_{LL,f} = 7.2 + 5.4 = 12.6 \text{ kPa}$$

Note that the factored live load $w_{LL,f} = 5.4$ kPa is less than twice the factored dead load $2 \times w_{DL,f} = 2 \times 7.2 \text{ kPa} = 14.4 \text{ kPa}$. Therefore, the requirement #5 for the application of DDM has been met.

4. **Determine the widths for design strip, column strip, and middle strip.**

a) Design strip
The frame under consideration is laid along gridline 2; this is referred to as Direction 1, while the transverse direction is referred to as Direction 2. The corresponding spans are illustrated on the following sketch, that is,

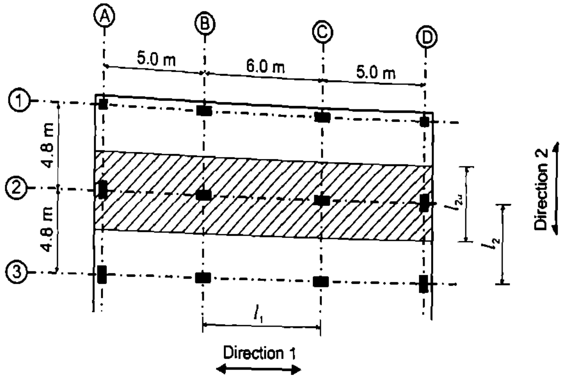
$l_1 = 6$ m (let us consider the longer span)

and

$l_2 = 4.8$ m

The design strip is denoted as l_{2a} (Direction 2). The width of the design strip is to be determined by taking an average value for the two spans adjacent to the gridline under consideration. In this case, the spans in Direction 2 are equal, thus

$l_{2a} = l_2 = 4.8$ m



b) Column strip and middle strip
The width of the column strip can now be determined, following the guidelines provided in Section 12.4.1. First, we need to compare the spans in both directions (1 and 2). For span BC, $l_2 = 4.8$ m and $l_1 = 6$ m, hence

$l_2 < l_1$

According to CSA A23.3, the shorter span (l_2) is used to find the width of the column strip. Note that the same conclusion would apply to span AB in E-W direction ($l_1 = 5$ m and $l_2 = 4.8$ m; hence $l_2 < l_1$).

As a result, the width of the column strip (l_c) is equal to

$l_c = l_2/2 = 4.8/2 = 2.4$ m

Note that the smaller of two spans is considered for the column strip width.
The middle strip (l_m) is a portion of the design strip outside the column strip, that is,

$l_m = l_{2a} - l_c = 4.8 - 2.4 = 2.4$ m

The column and middle strips are illustrated on the sketch below. Note that the middle strip is divided into two half-strips, shown cross-hatched on the following sketch.

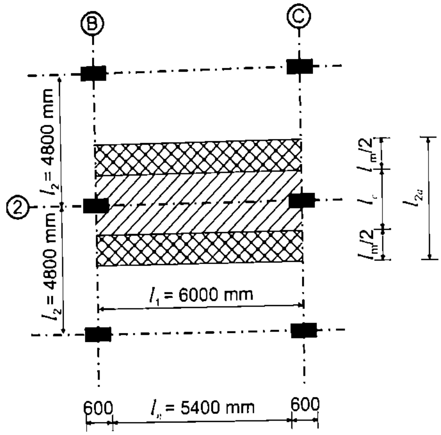
5. Find the factored bending moments in the slab.

a) Find the total factored static moment M_o.
The factored static moment can be determined from Eqn 12.8 as follows (see Section 12.6.2)

A23.3 Eq. 13.23

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8} \qquad [12.8]$$

where
$w_f = 12.6$ kPa is the total factored load
$l_{2a} = 4.8$ m is the width of the design strip



l_n is the clear span in the longitudinal direction (along gridline 2). Since the spans are different, this calculation needs to be performed for each span.

Span 1 (end span AB):

$$l_n = 5.0 - \left(\frac{0.3}{2} + \frac{0.6}{2}\right) = 4.55 \text{ m}$$

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8}$$

$$= \frac{12.6 \text{ kPa} \times 4.8 \text{ m} \times (4.55 \text{ m})^2}{8} = 156 \text{ kNm}$$

Span 2 (interior span BC):

$$l_n = 6.0 - \left(\frac{0.6}{2} + \frac{0.6}{2}\right) = 5.4 \text{ m}$$

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8}$$

$$= \frac{12.6 \text{ kPa} \times 4.8 \text{ m} \times (5.4 \text{ m})^2}{8} = 220 \text{ kNm}$$

b) Distribute the total factored static moment to critical locations in the longitudinal direction, and subsequently distribute the moment at each critical location transversely to column strip and middle strip.

Distribution of bending moments in the longitudinal direction will be performed according to Cl.13.9.3, while the transverse distribution of bending moments is performed according to Cl.13.11; these requirements are summarized in Table 12.2. Bending moments at critical locations in the longitudinal direction (supports and midspan) are expressed in terms of the total factored moment M_o, as shown on the following sketch. The calculations are summarized in the following tables.

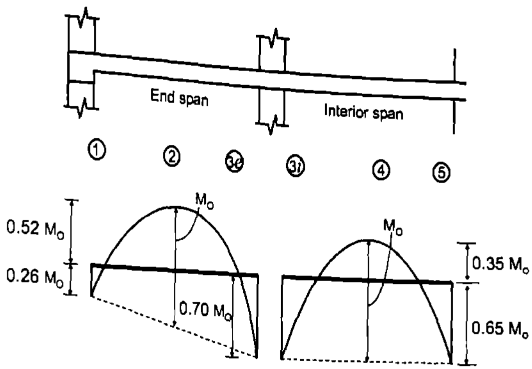


Table 12.9 Factored bending moments for Span 1 (End Span AB)

M_o = 156 kNm

		A Negative moment (kNm)	Midspan Positive moment (kNm)	B Negative moment (kNm)
Longitudinal	Bending moments at critical sections	$-0.26M_o$ $= (-0.26) \times 156$ $= -41$	$+0.52M_o$ $= (+0.52) \times 156$ $= +81$	$-0.70M_o$ $= (-0.70) \times 156$ $= -109$
Transverse column strip	CSA A23.3 Provisions	$-0.26M_o$	$+(0.29 \text{ to } 0.34)M_o$	$-(0.49 \text{ to } 0.63)M_o$
	Proposed value Design moment	$-0.26M_o$ -41	$+0.29M_o$ $+0.29 \times 156 = +45$	$-0.63M_o$ $-0.63 \times (156) = -98$
Transverse middle strip	Design moment	0	$=81-45 = +36$	$=-109-(-98) = -11$

Confirm that the sum of bending moments within a span is equal to M_o:

1. Average negative bending moment = $(-41-109)/2 = -75$ kNm
2. Sum of absolute values for positive and negative bending moments = 81+75 = 156 kNm = M_o

Table 12.10 Factored bending moments for Span 2 (Interior Span BC)

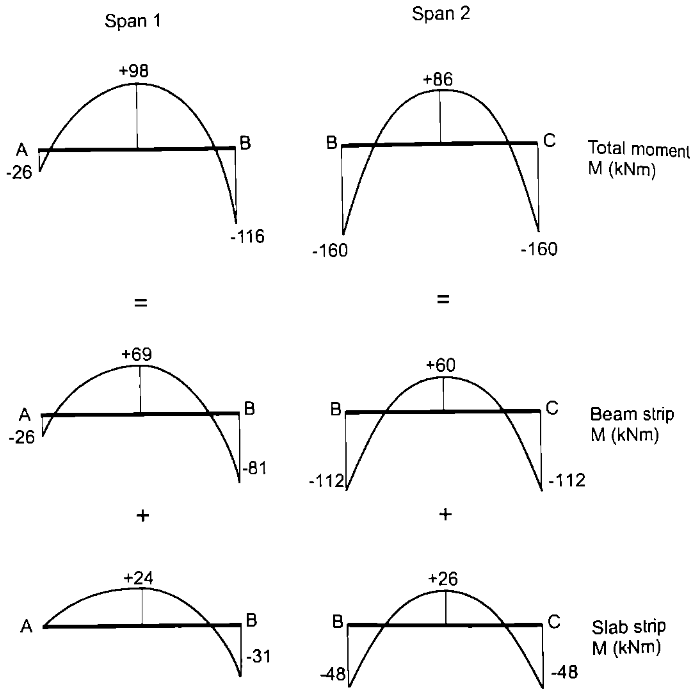
M_o = 220 kNm

		B Negative moment (kNm)	Midspan Positive moment (kNm)	C Negative moment (kNm)
Longitudinal	Bending moments at critical sections	$-0.65M_o$ $= (-0.65) \times 220$ $= -143$	$+0.35M_o$ $= (+0.35) \times 220$ $= +77$	$-0.65M_o$ $= (-0.65) \times 220$ $= -143$
Transverse column strip	CSA A23.4 Provisions	$-(0.46 \text{ to } 0.59)M_o$	$+(0.19 \text{ to } 0.23)M_o$	$-(0.46 \text{ to } 0.59)M_o$
	Proposed value Design moment	$-0.59M_o$ $-0.59 \times (220) = -130$	$+0.19M_o$ $+0.19 \times (220) = +42$	$-0.59M_o$ $-0.59 \times (220) = -130$
Transverse middle strip	Design moment	$=-143-(-130) = -13$	$=77-42 = +35$	$=-143-(-130) = -13$

Confirm that the sum of bending moments within a span is equal to M_o:

1. Average negative bending moment for Span 2 = (−143−143)/2=−143 kNm
2. Sum of absolute values for positive and negative bending moments = 77+143 = 220 kNm = M_o

Bending moment diagrams for the total moment in Span 1 and Span 2, as well as the moments in column strip and middle strip, are shown below.



It can be seen that the negative bending moment at support B has different values for Span 1 (−109 kNm) and Span 2 (−143 kNm). In practice, the top reinforcement at support B would need to be designed for the greater of the two moments from adjacent spans. The solution will proceed by following that approach. However, a more effective solution can be obtained if these two moments are made equal by increasing the moment for Span 1 and/or decreasing the moment for Span 2; this procedure is often referred to as "balancing" of bending moments. Since the balancing approach is somewhat more complex, it will be discussed in Step 10 (at the end of the example).

6. **Design the flexural reinforcement.**
The dimensions of the column strip and middle strip were found in Step 4. In this design, the column strip and the middle strip have the same width (2.4 m). The effective slab depth is

d = 180 − 25 − 15 = 140 mm

Note that the d value was determined considering a 25 mm average concrete cover, plus an average diameter for the two perpendicular layers of 15M bars. In design practice, this approach may be used to obtain a reasonable estimate of the moment resistance in each direction with an error in d value equal to half bar diameter.

The flexural reinforcement calculation can be presented in a tabular form. The key equations are summarized below.

a) The required reinforcement area can be found using the Direct Procedure (see Section 5.5.1) and Eqn 5.4 as follows

$$A_s = 0.0015 f_c' b \left(d - \sqrt{d^2 - \frac{3.85 M_f}{f_c' b}} \right) \qquad [5.4]$$

b) The required spacing can be determined from the following equation

$$s \le A_b \frac{b}{A_s}$$

Note that $A_b = 200 \text{ mm}^2$ for 15M bars.

c) Maximum reinforcement spacing (s_b) within the band width b_b

The reinforcement spacing requirements for negative reinforcement in column strips are outlined in Section 12.6.6. It follows that, within band width b_b, reinforcement spacing (s_b) is limited to the lesser of (Cl.13.10.4)

$s_b \le 1.5 h_s = 1.5 \times 180 = 270$ mm

or

$s_b \le 250$ mm

In this case,

$s_b \le 250$ mm governs.

d) Maximum reinforcement spacing - outside the band width b_b

Spacing for the reinforcement resisting negative bending moments in the column strip outside the band b_b is limited to the lesser of (Cl.13.10.4)

$s^- \le 3h_s = 3 \times 180 = 540$ mm

or

$s^- \le 500$ mm

In this case, $s^- \le 500$ mm governs.

Maximum spacing for the reinforcement resisting positive bending moments in the slab is the same as for the negative bending moments (Cl.13.10.4), that is, $s^+ \le 500$ mm.

e) The minimum reinforcement requirement (Cl.7.8.1)

$A_{s,min} = 0.002 A_g$

Note that the area A_g refers to the gross cross-sectional area for the section under consideration.

Column Strip Calculations

CSA A23.3 Cl.13.10.3 requires that the bending moment in the column strip of an exterior column be entirely resisted by the strip b_b centered at the column (see Figure 12.16). For an interior column, the band b_b should be designed to resist at least one-third of the total factored negative bending moment for the entire design strip (Cl.13.11.2.7). This is illustrated in Table 12.11. For exterior column A, design bending moment for the band b_b is equal to the total moment (−41 kNm). However, for interior columns (B and C), reinforcement within the band b_b is designed using one-third of the total bending moment (−143 kNm), that is, the design moment is −48 kNm.

Find the width for band b_b.

i) Exterior column (A):

$b_b = 600 + 3 \times 180 = 1140 \text{ mm} \cong 1200 \text{ mm}$

where 600 mm is the column cross-sectional dimension in transverse direction, and 180 mm is the slab thickness.

ii) Interior columns (B and C):

$b_b = 300 + 3 \times 180 = 840 \text{ mm} \cong 900 \text{ mm}$

where 300 mm is the column cross-sectional dimension in transverse direction, and 180 mm is the slab thickness.

Note that the reinforcement calculations have been performed considering two different slab sections:

a) The section with the width b_b and the overall depth h_s (for the column locations only), and
b) The section with the remaining width (column strip minus the band b_b) and the overall depth h_s.

Table 12.11 Column strip - factored bending moments and flexural reinforcement calculations

		Exterior column (A)	Midspan	Interior columns (B and C)
	Steel location	**Top**	**Bottom**	**Top**
	Total bending moment (kNm)	−41	+81*	−143**
	Bending moment - column strip (kNm)	−41	+45	−130
	Column strip width (mm)	2400	2400	2400
Within b_b	Band width b_b	1200		900
	Design moment M_f (kNm)	−41		=−143/3=−48
	Required reinforcement area A_s (mm²)	900		1088
	Required spacing s (mm)	267		165
	Max spacing (mm)	250		250
	Design reinforcement (area in mm²)	6−15M@250 (1200)		6−15M@150 (1200)
	Min reinforcement area $A_{s,min}$ (mm²)	432		324
Outside b_b	Strip width (mm)	0	2400	=2400−900=1500
	Design moment M_f (kNm)	0	+45	=−130−(−48)=−82
	Required reinforcement area A_s (mm²)	0	958	1880
	Required spacing s (mm)		500	158
	Max spacing (mm)		500	250
	Design reinforcement (area in mm²)	0	6−15M@400 (1200)	11−15M@150 (2200)
	Min reinforcement area $A_{s,min}$ (mm²)	432	864	540
	Design reinforcement - summary	6−15M@250 (centered over column within 1200 mm)	15M@4000 (2400)	15M@150 (uniform spacing for the entire column strip)

Notes:
* - Larger midspan moment selected (+81 kNm for Span 1 versus +77 kNm for Span 2)
** - Larger negative moment for support B selected (−143 kNm for Span 2 versus −109 kNm for Span 1)

Note that the maximum reinforcement requirement check has been omitted from the table, since it does not govern due to the small amount of flexural steel. For example, top reinforcement at exterior column A within the band b_b (1200 mm width) is equal to 6-15M. The corresponding reinforcement ratio is

$$\rho = \frac{A_s}{b \cdot d} = \frac{6 \cdot 200}{1200 \cdot 140} = 0.007 \qquad [3.1]$$

This is significantly less than the balanced reinforcement ratio (ρ_b) of 0.027 for $f'_c = 30$ MPa presented in Table A.4, thus $\rho < \rho_b$.

Middle Strip Calculations

Note that there is no need to provide reinforcement in the middle strip at an exterior column, according to CSA A23.3 Cl.13.10.3. The calculations are summarized in Table 12.12.

ble 12.12 Middle strip - factored bending moments and flexural reinforcement calculations

el location	Exterior column (A)	Midspan	Interior columns (B and C)
	Top	Bottom	Top
sign bending moment - middle strip M_f (kNm)	0	+36*	−13**
iddle strip width (mm)	2400	2400	2400
quired reinforcement area A_s (mm²)	0	762	271
equired spacing s (mm)		630	1774
ax spacing (mm)	500	500	500
in reinforcement area $A_{s\,min}$	864	864	864
esign reinforcement (area in mm²)	0	15M@400 (2400)	15M@400 (2400)

tes:

* Larger midspan moment selected (+36 kNm for Span 1 versus +35 kNm for Span 2)

** Larger negative moment for support B selected (−13 kNm for Span 2 versus −11 kNm for Span 1)

7. Find the factored moments for the columns.

a) Interior column (B)

The purpose of this calculation is to find unbalanced moments, which were explained in Section 12.6.5. Unbalanced moment for an interior column can be determined from the following equation

A23.3 Eq. 13.24

$$M_u = 0.07\left[(w_{df} + 0.5w_{lf})l_{2a}l_n^2 - w'_{df}l'_{2a}(l'_n)^2\right] \qquad [12.10]$$

Since both Span 1 and Span 2 need to be considered, it is necessary to identify the longer and the shorter span in the plane of the frame. Based on the calculation from Step 2, it follows that Span 2 is longer, that is,

$l_n = 5.4$ m (longer span) and $l_{2a} = 4.8$ m (corresponding transverse span)

whereas

$l'_n = 4.55$ m (shorter span) and $l'_{2a} = 4.8$ m (corresponding transverse span)

In this case, factored dead load is equal for both spans, that is,

$w_{df} = w'_{df} = 7.2$ kPa

and the factored live load is

$w_{lf} = 5.4$ kPa

thus

$$M_{fu} = 0.07\Big[(7.2\text{ kPa} + 0.5(5.4\text{ kPa}))(4.8\text{ m})\times(5.4\text{ m})^2 -$$

$$(7.2\text{ kPa})\times(4.8\text{ m})\times(4.55\text{ m})^2\Big] = 46.9\text{ kNm}$$

Note that this unbalanced moment is transferred to the column segments above and below the slab proportional to their stiffness. If column segments have the same geometric properties (cross-sectional dimension and storey height), one-half of the moment is transferred to each segment.

b) Exterior column (A)

CSA A23.3 Cl.13.10.3 requires that the entire exterior factored moment be transferred from the slab directly to the columns. Bending moment at the face of the support is equal to

$M_f = -41$ kNm

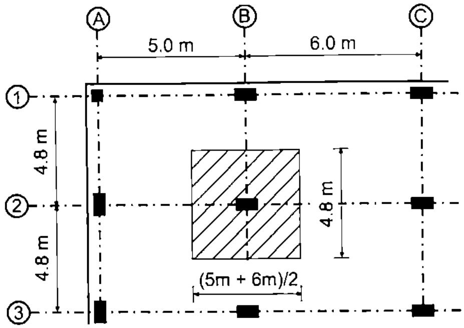
8. **Determine the slab integrity reinforcement.**

The purpose of integrity reinforcement is discussed in Section 12.6.6. According to CSA A23.3 Cl.13.10.6.1, the total area of bottom reinforcement (A_{sb}) connecting the slab, drop panel, or slab band to the column or column capital on all faces of the periphery of the column or column capital shall be at least equal to

A23.3 Eq. 13.26

$$\sum A_{sb} = \frac{2V_{se}}{f_y} \qquad [12.13]$$

where V_{se} is the shear force transmitted to the column or column capital due to specified loads, but should not be less than the shear corresponding to twice the self-weight of the slab. In this case, the tributary area for the shear design is determined for interior column B (see the sketch below)



$$A = (4.8\ \text{m}) \times \left(\frac{5\ \text{m} + 6\ \text{m}}{2}\right) = 26.4\ \text{m}^2$$

Consider the following loads:

Total specified load $w_1 = 4.32\ \text{kPa} + 1.44\ \text{kPa} + 3.6\ \text{kPa} = 9.36\ \text{kPa}$

Twice the self-weight $w = 2 \times 4.32\ \text{kPa} = 8.64\ \text{kPa}$

In this case, the total specified load is larger and it governs. Next, the shear force can be determined as follows

$V_{se} = w \times A = 9.36\ \text{kPa} \times 26.4\ \text{m}^2 = 247\ \text{kN}$

Finally, the required area of integrity reinforcement can be determined as follows

$$\sum A_{sb} = \frac{2 \times 247 \times 10^3\ \text{N}}{400\ \text{MPa}} = 1235\ \text{mm}^2$$

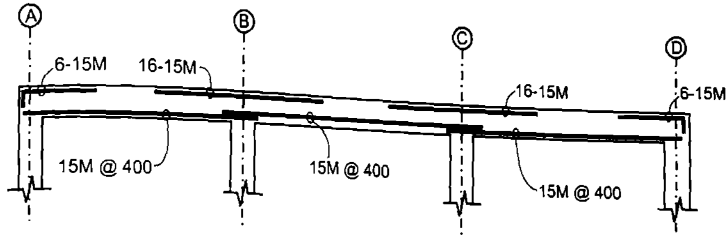
It is required to provide 8-15M bars (total area 1600 mm^2), that is, 4-15M bars in each direction.

9. **Present a design summary.**

A drawing summarizing the design solution is presented below. Note that the reinforcement should be laid out such that it is easy to construct. Rebar spacing should be specified using simple rounded numbers and it should preferably be repetitive. The same spacing should be used in both orthogonal directions to avoid confusion at the construction site. A good judgement is required to minimize potential construction errors and strike balance between labour, material usage, and cost.

In this design, the original calculations (omitted from this example) showed 15M@400 mm o.c. bottom steel for column strips and 15M@500 mm o.c. for middle strips. It may be simpler to place 15M@400 mm in each direction throughout

Although this solution adds a few extra rebars in the middle strips, it will result in labour savings since simple specification will result in a more efficient placement.

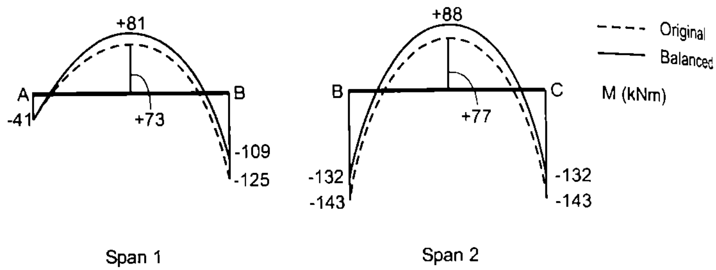


10. An alternative solution: balancing of bending moments at supports.
Balancing is achieved when bending moments at the support between two adjacent spans are made equal by increasing the moment for one span and/or decreasing the moment for other span. Balancing of bending moments is permitted by CSA A23.3 Cl.13.9.3.3 (see Section 12.6.3). To demonstrate the balancing process, let us increase the negative moment at support B in Span 1 by 15%, and decrease the negative moment at the same location in Span 2 by 15%. The balancing process is illustrated in the following table. Note that CSA A23.3 requires that the sum of positive and negative bending moments within a span must remain equal to M_o.

Table 12.13 Balancing of bending moments for Spans 1 and 2

		Span 1 = End Span AB M_o = 156 kNm			Span 2 = Interior Span BC M_o = 220 kNm		
		Support A	Midspan	Support B	Support B	Midpsan	Support C
		Negative moment (kNm) (1)	Positive moment (kNm) (2)	Negative moment (kNm) (3)	Negative moment (kNm) (4)	Positive moment (kNm) (5)	Negative moment (kNm) (6)
Original values	(a)	−41	+81	−109	−143	+77	−143
Moments varied by 15%	(b)			−109 × 1.15 = −125	−143/1.15 = −124		−143/1.15 = −124
Balanced moments	(c)	−41	156 − (41 + 125)/2 = +73	−125	−124	220 − (124 + 124)/2 = +96	−124
Balanced/original (d)/(a)	(d)	(−41)/ (−41) = 1.0	(+73)/(+81) = 0.90	(+125)/(+109) = 1.15	(−124)/(−143) = 0.87	(+96)/(+77) = 1.25	(−124)/(−143) = 0.87
(% difference)		(0%)	(−10%)	(+15%)	(−13%)	(+25% > 15%)	
Revised balanced moments	(e)	−41	156 − (41 + 125)/2 = +73	−125	−(220−88) = −132	+77(1.15) = +88	−132
Balanced/original (e)/(a)	(f)	(−41)/ (−41) = 1.0	(+73)/(+81) = 0.90	(+125)/(+109) = 1.15	(−132)/(−143) = 0.92	(+88)/(+77) = 1.14	(−132)/(−143) = 0.92
(% difference)		(0%)	(−10%)	(+15%)	(−8%)	(+14%)	(−8%)

It can be seen that the positive bending moment in Span 2 (column 5) exceeds the 15% limit, since the balanced value is 95 kNm compared to the original value of 77 kNm. Therefore, the moments need to be balanced once again. Let us set the positive moment in Span 2 to the upper limit, that is, a 15% increase (88 kNm), as shown in row (e). The corresponding negative moment is equal to -132 kNm, based on the total factored static moment of 220 kNm. The moment values in Span 1 are also slightly revised, as shown in row (e). The negative moment at support B is -125 kNm, which corresponds to the maximum increase of 15%. The corresponding positive moment is calculated in the same manner as before, and the resulting value is +73 kNm. All balanced moment values are within the 15% limit prescribed by CSA A23.3, as shown in row (f). Diagrams showing the original and balanced bending moments are shown below.



Example 12.2

Two-Way Slab with Beams - Direct Design Method

Consider a floor plan of a two-way slab system with beams shown in the following figure. Typical beam dimensions are 400 mm width by 600 mm overall depth. The plan shows an intermediate floor level, and a typical storey height is 3.0 m. Column dimensions are 400 mm by 400 mm. The slab is subjected to a specified live load (LL) of 3.6 kPa, and superimposed dead load (DL_s) of 1.44 kPa, in addition to its self-weight. Consider only the effect of gravity loads for this design - lateral loads are to be resisted by shear walls (not shown on the drawing).

Use the CSA A23.3 Direct Design Method to determine design bending moments for an interior frame along gridline 2.

Given: $f_c' = 30$ MPa

$f_y = 400$ MPa

SOLUTION: **1. Check whether the criteria for the CSA A23.3 Direct Design Method are satisfied.**

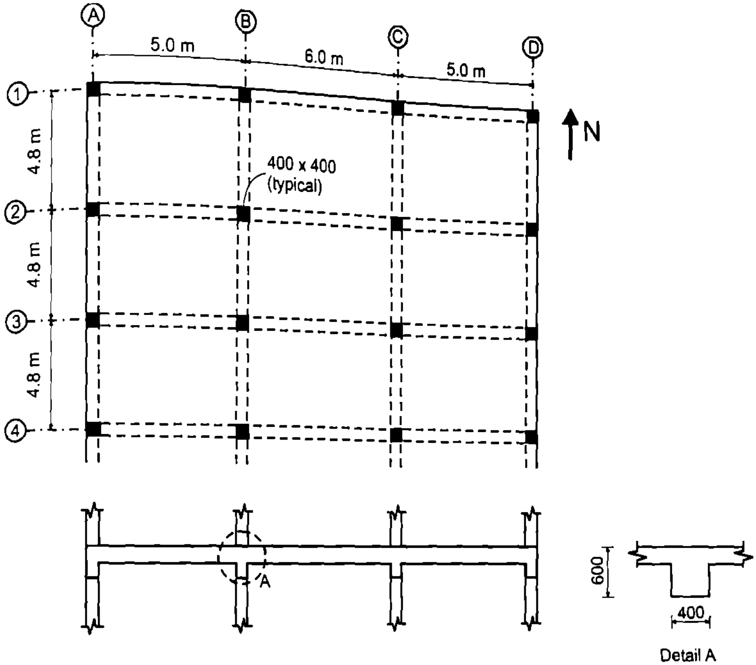
CSA A23.3 Cl. 13.9.1 prescribes that the DDM can be used when the following requirements have been met (see Section 12.6.1):

#1 A slab is regular (see Section 12.5.1).

Requirement b): for slab systems with beams between supports, the ratio of relative effective stiffnesses of beams in the directions 1 and 2 should be restricted as follows:

$$0.2 \le \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \le 5.0$$

where α_1 and α_2 denote the beam-to-slab stiffness ratio for beams in the directions 1 and 2, respectively. This requirement will be checked in Step 2.



#2 There are three continuous spans in each direction.

#3 The successive span lengths, centre-to-centre of supports, in each direction must not differ more than one-third of the longer span.

E-W direction: Span 2 – Span 1 = 6.0 m – 5.0 m = 1 m < 6.0/3=2.0 m

N-S direction: all span lengths are equal (4.8 m)

#4 The slab is subjected to uniformly distributed gravity loads.

#5 The factored live load does not exceed two times the factored dead load (this will be confirmed in Step 3).

2. **Determine the required slab thickness based on deflection control requirements.** For two-way slab systems with beams, the minimum overall thickness (h_s) can be determined according to CSA A23.3 Cl.13.2.5 as follows

A23.3 Eq. 13.3

$$h_s \geq \frac{l_n(0.6 + f_y/1000)}{30 + 4\beta\alpha_m} \qquad [12.7]$$

We need to determine clear span l_n. Since the longer clear span governs, let us consider only interior span BC:

E-W direction: $l_n = 6.0 - (\frac{0.4}{2} + \frac{0.4}{2}) = 5.6$ m

N-S direction: $l_n = 4.8 - (\frac{0.4}{2} + \frac{0.4}{2}) = 4.4$ m

Note that the clear span was determined considering 400 mm wide beams.

Therefore, the span in E-W direction governs, that is,

$l_{n2} = 5.6$ m

Next, let us find β for interior slab panel 23BC, that is,

$$\beta = \frac{l_1}{l_2} = \frac{6.0\ \text{m}}{4.8\ \text{m}} = 1.25$$

Note that l_1 and l_2 denote long and short span directions for the panel under consideration.

Next, let us assume α_m value of 2.0, since the actual value cannot be determined unless the slab thickness is given, thus

$\alpha_m = 2.0$ (this is the maximum permitted value per Cl.13.2.5)

The required slab thickness is

$$h_s \geq \frac{l_n(0.6 + f_y/1000)}{30 + 4\beta\alpha_m} = \frac{5600(0.6 + 400/1000)}{30 + 4\cdot(1.25)\cdot(2.0)} = 140\ \text{mm}$$

Let us round up the slab thickness, that is,

$h_s = 160$ mm

Note that a smaller thickness (say 150 mm) could have been used based on the slab thickness requirements.

Next, find the α_m value for the panel 23BC using the selected slab thickness. The simplified equation presented in Section 12.4.5 will be used for this purpose. We need to find the α value for both directions (1 and 2). Different slab section widths (l_{b1} and l_{b2}) will be used for directions 1 and 2 respectively.

For Direction 1: $l_{b1} = \dfrac{5000 + 6000}{2} = 5500$ mm (an average value for spans AB and BC)

thus

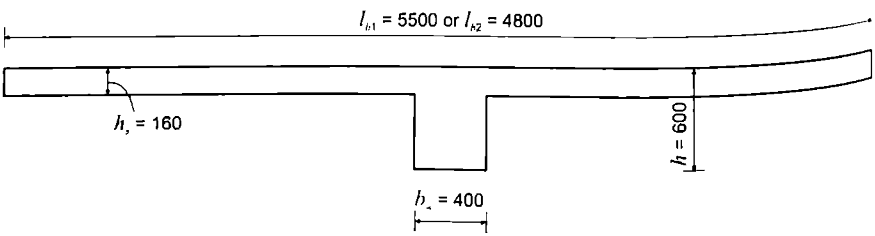
$$\alpha_1 = \frac{2.5b_w}{l_{b1}}\left(\frac{h}{h_s}\right)^3\left(1 - \frac{h_s}{h}\right) = \frac{2.5\cdot 400}{5500}\left(\frac{600}{160}\right)^3\left(1 - \frac{160}{600}\right) = 7.0 \qquad [12.3]$$

For Direction 2: $l_{b2} = 4800$ mm

thus

$$\alpha_2 = \frac{2.5b_w}{l_{b1}}\left(\frac{h}{h_s}\right)^3\left(1 - \frac{h_s}{h}\right) = \frac{2.5\cdot 400}{4800}\left(\frac{600}{160}\right)^3\left(1 - \frac{160}{600}\right) = 8.0 \qquad [12.3]$$

The beam and slab dimensions used in the above equation are shown on the sketch below.



The average beam-to-slab stiffness ratio for the entire panel can be calculated as follows:

$$\alpha_m = \frac{2(7.0+8.0)}{4} = 7.5$$

Since

$$\alpha_m = 7.5 > 2.0$$

We are going to use $\alpha_m = 2.0$ for the remaining calculations.
Finally, let us confirm that the CSA A23.3 requirement #1 b) for regular slabs with beams has been met. We need to find the effective beam stiffness in two directions, as follows

$$\frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} = \frac{7.0\cdot(4.8)^2}{8.0\cdot(6.0)^2} = 0.54$$

Since

$$0.2 < \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} = 0.54 < 5.0$$

It follows that the slab is regular according to CSA A23.3 Cl.2.2.

3. **Find the factored design loads.**

a) Calculate the dead load acting on the slab.
First, calculate the slab self-weight, based on the 160 mm slab thickness and 400 mm × 600 mm beams along gridline 2 that will be carried by the 4.8 m wide design strip:

$$DL_w = \left(0.16\,\text{m} + \frac{0.4\,\text{m}\cdot(0.6\,\text{m}-0.16\,\text{m})}{4.8\,\text{m}}\right)\times 24\,\text{kN/m}^3 = 4.72\ \text{kPa}$$

where $\gamma_w = 24\,\text{kN/m}^3$ is the unit weight of concrete.
The following superimposed dead load was given:

$DLs = 1.44$ kPa

Finally, the total factored dead load is equal to

$$w_{DL,f} = 1.25(DL_w + DL_s) = 1.25(4.72+1.44) = 7.7\ \text{kPa}$$

b) Calculate the factored live load, as follows

$$w_{LL,f} = 1.5\times LL_w = 1.5\times 3.6\ \text{kPa} = 5.4\ \text{kPa}$$

c) The total factored load is

$$w_f = w_{DL,f} + w_{LL,f} = 7.7+5.4 = 13.1\ \text{kPa}$$

Note that the factored live load $w_{LL,f} = 5.4$ kPa is less than twice the factored dead load, that is, $2 \times w_{DL,f} = 2 \times 7.7$ kPa $= 15.4$ kPa. Therefore, requirement #5 has been met.

4. **Determine the widths for design strip, beam strip, and slab strip.**

a) Design strip
The frame under consideration is laid along gridline 2; this is referred to as Direction 1, while transverse direction is referred to as Direction 2. The corresponding spans are illustrated on the following sketch, that is,

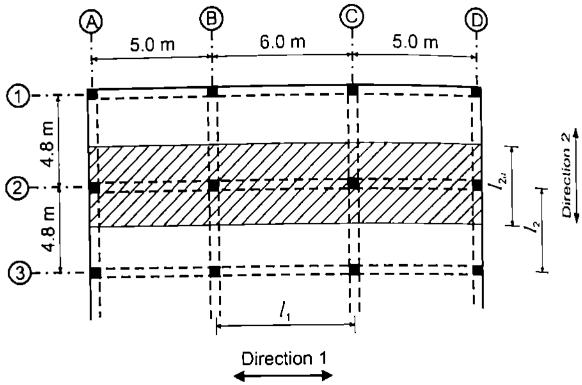
$l_1 = 6$ m (let us consider the largest span)

and

$l_2 = 4.8$ m

The design strip is denoted as l_{2a} (Direction 2). The design strip width is determined by taking an average value for the two spans adjacent to the gridline under consideration. In this case, the spans in Direction 2 are equal, thus

$l_{2a} = l_2 = 4.8$ m



b) Beam strip and slab strip

The width of the beam strip is determined based on the effective flange width b_s (see Section 12.4.3), that is,

$b_s = b_w + 2h_w = 400 + 2 \cdot 440 = 1280$ mm

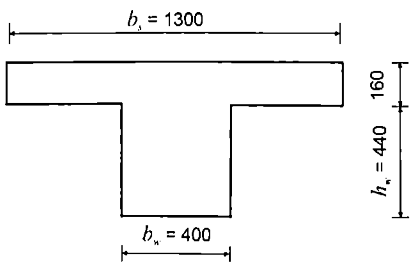
but

$b_s \leq b_w + 8h_s = 400 + 8 \cdot 160 = 1680$ mm

hence

$b_s = 1280 \text{ mm} \cong 1.3$ m

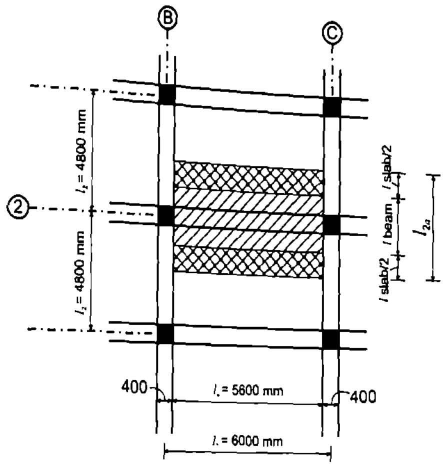
Dimensions of the equivalent beam section are shown on the following sketch.



The width of the slab strip is equal to

$l_{slab} = l_{2a} - l_{beam} = 4.8 - 1.3 = 3.5$ m

Beam and slab strips are illustrated on the following sketch. Note that the slab strip is divided into two half-strips, shown cross-hatched on the sketch.



5. Find the factored bending moments in the slab.

a) Find the total factored static moment M_o.

The factored static moment can be determined from the following equation (see Section 12.6.2)

A23.3 Eq. 13.23

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8} \quad [12.8]$$

where

w_f = 13.1 kPa total factored load

l_{2a} = 4.8 m width of the design strip

l_n is the clear span in the longitudinal direction (along gridline 2). Since the spans are different, this calculation needs to be performed for each span.

Span 1 (end span AB)

$$l_n = 5.0 - \left(\frac{0.4}{2} + \frac{0.4}{2}\right) = 4.6 \text{ m}$$

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8}$$

$$= \frac{13.1 \text{ kPa} \times 4.8 \text{ m} \times (4.6 \text{ m})^2}{8} = 166 \text{ kNm}$$

Span 2 (interior span BC)

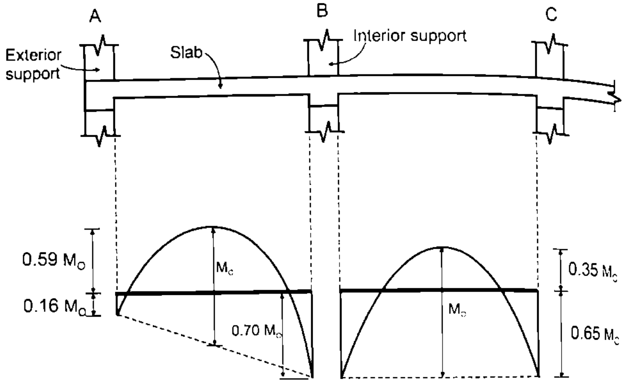
$$l_n = 6.0 - \left(\frac{0.4}{2} + \frac{0.4}{2}\right) = 5.6 \text{ m}$$

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8}$$

$$= \frac{13.1 \text{ kPa} \times 4.8 \text{ m} \times (5.6 \text{ m})^2}{8} = 246 \text{ kNm}$$

b) Distribute the total factored static moment at the critical locations in longitudinal direction.

The distribution of bending moments in longitudinal direction will be performed according to CSA A23.3 Cl.13.9.3; these requirements are summarized in Table 12.5 presented earlier in this chapter. Bending moments at critical locations in longitudinal direction (supports and midspan) are expressed in terms of the total factored moment M_o, as shown on the sketch below.



Distribution of bending moments in the transverse direction is performed according to Cl.13.12 (see Table 12.5). The beam strip moment can be calculated from the following equation:

$$M_b = \left[\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)\right] \times M \qquad [12.9]$$

For this design (considering interior span BC):

$\alpha_1 = 7.0$

$l_1 = 6.0$ m

$l_2 = 4.8$ m

Therefore,

$M_b = 0.70 \times M$

The slab strip moment is obtained as a difference between the total design moment for a particular location and the beam strip moment. The calculations are summarized in Tables 12.14 and 12.15.

Table 12.14 Factored bending moments for Span 1 (End Span AB)

		M_o = 166 kNm		
	Location	A	Midspan	C
Longitudinal direction	Bending moments at critical sections M	Negative moment (kNm) $-0.16M_o$ $= (-0.16) \times 166$ $= -26$	Positive moment (kNm) $+0.59M_o$ $= (+0.59) \times 166$ $= +98$	Negative moment (kNm) $-0.70M_o$ $= (-0.70) \times 166$ $= -116$
Transverse distribution	Beam strip moment $M_b = 0.70 \times M$	$-0.16M_o$ $= -26$	$= 0.70 \times (+98)$ $= +69$	$= 0.70 \times (-116)$ $= -81$
	Slab strip moment (total for the two halves)	0	$= 98 - 69 = +29$	$= -116-(-81) = -35$

Confirm that the sum of bending moments within a span is equal to M_o:

1. Average negative bending moment = (−26−116)/2=−71 kNm
2. Sum of absolute values for positive and negative bending moments = 98+71 = 169 kNm ≅ M_o

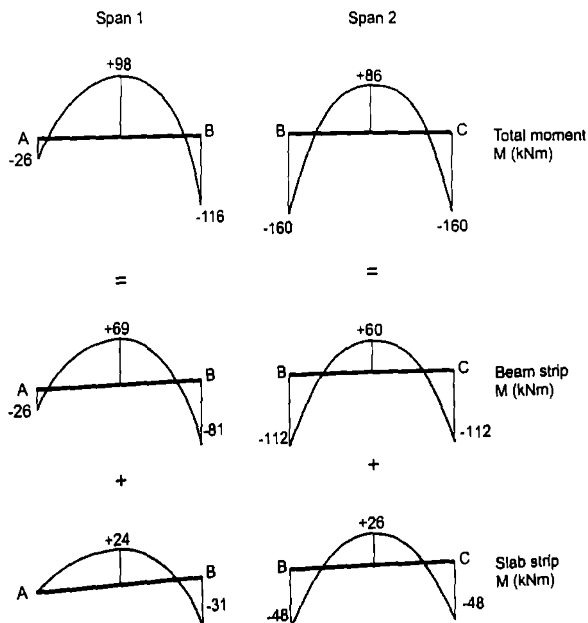
...le 12.15 Factored bending moments for Span 2 (Interior Span BC)

...ation		M_o = 246 kNm		
		B Negative moment (kNm)	Midspan Positive moment (kNm)	C Negative moment (kNm)
...ngitudinal	Bending moments at critical sections M	$-0.65M_o$ $= (-0.65) \times 246$ $= -160$	$+0.35M_o$ $= (+0.35) \times 246$ $= +86$	$-0.65M_o$ $= (-0.65) \times 246$ $= -160$
...ansverse	Beam strip moment $M_b = 0.70 \times M$	$= 0.70 \times (-160)$ $= -102$	$= 0.70 \times (+86)$ $= +60$	$= 0.70 \times (-160)$ $= -102$
	Slab strip moment (total for the two halves)	= −160−(−112) = −48	= 86−60 = +26	= −160−(−112) = −48

Confirm that the sum of bending moments within a span is equal to M_o:

1. Average negative bending moment = (−160−160)/2=−160 kNm
2. Sum of absolute values for positive and negative bending moments = 86+160 = 246 kNm ≅ M_o

Bending moment diagrams for the total moment in Span 1 and Span 2, as well as the moments for beam and slab strips, are shown next.



12.7 ELASTIC ANALYSIS

12.7.1 Background

This section presents design approaches which are based on elastic behaviour of reinforced concrete structures, hence the name Elastic Analysis. CSA A23.3 permits the following two elastic analysis approaches for design of reinforced concrete two-way slab systems:

- Equivalent Frame Method (referred to as "slab systems as elastic frames" by Cl.13.8), and
- Three-Dimensional Elastic Analysis (Cl.13.6).

According to the Equivalent Frame Method (EFM), an actual three-dimensional building structure consisting of slabs (with/without beams) and columns is divided into a series of parallel frames in longitudinal and transverse directions of the building. Each frame is modelled as a two-dimensional (2-D) structure called an *equivalent frame*. The design of two-way slab systems is based on the analysis of these equivalent frames in each principal direction, and the results are combined to create a design solution for an entire slab at the floor level.

According to the Three-Dimensional (3-D) Elastic Analysis, a floor system is idealized as a 3-D model, which takes into account properties of horizontal components (slabs and slab-column connections) and vertical components (columns and walls). This is a computer-based analysis procedure, and it is readily available due to the affordability of computer hardware and specialized software packages. In design practice, the use of 3-D elastic analysis for the design of two-way slabs is gaining popularity, especially for complex slabs with irregular (non-rectangular) plan shapes, or slabs characterized by large column and wall offsets relative to a rectangular grid.

It should be noted that the Elastic Analysis approaches presented in this section are based on the assumption of linear elastic behaviour of reinforced concrete structures, and that reinforced concrete is treated as a homogeneous material. Results of such analyses do not accurately simulate the behaviour of cracked slabs at service and ultimate load levels.

The Elastic Analysis approaches will be explained and illustrated by a few design examples.

12.7.2 Equivalent Frame Method

A23.3 Cl.13.8

Features A comparison of key features for the Equivalent Frame Method and the Direct Design Method is outlined in Table 12.16.

Table 12.16 Key Features of the Equivalent Frame Method and the Direct Design Method

	Equivalent Frame Method (EFM)	Direct Design Method (DDM)
1.	Reinforced concrete is treated as a homogeneous material with linear elastic stress-strain relationship. As discussed in Section 12.3.4, reinforced concrete structures show inelastic (nonlinear) behaviour once cracking has been initiated. Therefore, the results of an elastic analysis represent an approximation of actual structural behavior.	Same as the EFM.
2.	The slab must meet the requirements for regular two-way slabs outlined in CSA A23.3 Cl. 2.2 (see Section 12.5.1)	Same as the EFM.
3.	There are no restrictions with regards to the span dimensions between adjacent slab spans.	DDM contains constraints related to adjacent span lengths.
4.	Variations in slab thickness (moment of inertia) along the span have to be considered.	DDM considers only constant slab thickness.
5.	Bending moments in the frame are determined using an elastic analysis procedure.	DDM does not require analysis to be performed; empirical equations are used to find bending moments.

(continued)

le 12.16 (Continued)

Equivalent Frame Method (EFM)	Direct Design Method (DDM)
There are no restrictions regarding the load magnitude, however pattern loadings have to be considered when a live-to-dead load ratio is large.	DDM does not directly consider pattern loadings.
The sum of average negative bending moments and the maximum positive moment for a typical span should not exceed the total factored moment M_o.	Same as EFM.
The EFM can be used to perform lateral load analysis.	DDM cannot be used for the lateral load analysis.

A23.3 Cl.13.8.1

Equivalent Frames: Concept According to CSA 23.3 Cl. 13.8, a two-way slab system can be idealized as a series of parallel 2-D or "plane" frames in longitudinal and transverse directions of a building. CSA A23.3 refers to these frames as "elastic frames". The authors believe that the term "equivalent frame" is more appropriate and it will be used in this text (the same term is used in ACI 318 concrete design standard in the U.S.).

CSA A23.3 Cl.13.8.1 outlines provisions related to the frame geometry. Each equivalent frame is composed of line members intersecting at column and slab centrelines. The slab is modelled as a wide beam (referred to as "slab-beam"), and its width corresponds to the tributary portion of the slab extending on each side of the column midway between the adjacent frames.

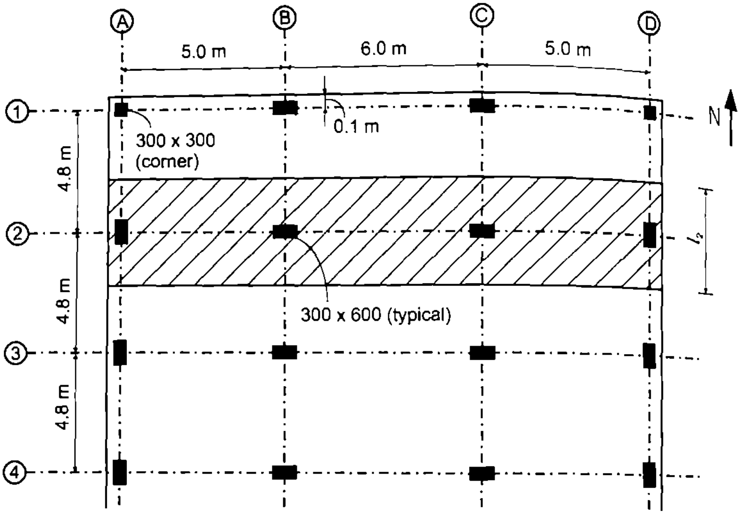
One of the first challenges a designer is faced with when implementing the EFM is how to isolate a 2-D frame from a 3-D building structure. This will be illustrated by an example. Figure 12.40a shows a floor plan of a building with a two-way flat slab system. An isometric view of the equivalent frame on gridline 2 is shown in Figure 12.40b. The frame is defined by columns laid along gridline 2. The slab is modelled as a wide beam (called slab-beam), which comprises a portion of the slab defined by vertical cutting planes located midway between the adjacent column gridlines 1-2 and 2-3. Gravity load acts over the slab-beam. A 2-D view of the frame is shown in Figure 12.40c. A notation related to frame span lengths was introduced in Section 12.4.1. For example, span lengths in the plane of the frame are labelled as l_1 (see span BC in Figure 12.40b), and the slab-beam width in the transverse direction is referred to as l_2; this is the same as slab design strip l_{2a} for the DDM, as discussed in Section 12.4.1.

A basic assumption of the equivalent frame model is that the width of the slab-beam is bounded by imaginary vertical cutting planes located midway between adjacent column gridlines on each side of the frame. In slab systems with uniform spans, the locations of these cutting planes generally correspond to the locations of zero shear forces and torsional moments. As a result, each equivalent frame in the transverse direction is comprised of a column strip bounded by two half-middle strips. However, this assumption carries some errors, especially for end spans, where the zero shear is located at more than half-way distance from the end column in the perpendicular direction.

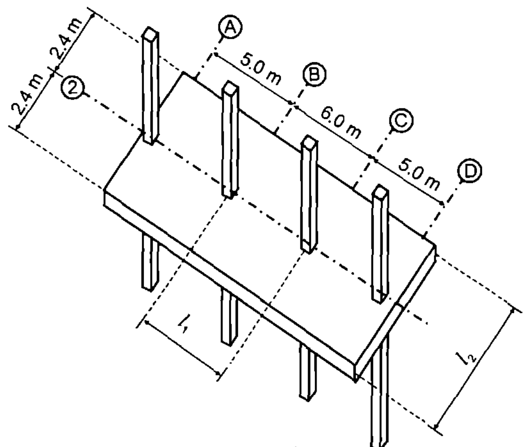
Structural analysis is performed to determine bending moment distribution in the frame. These frames are statically indeterminate systems, and internal forces can be obtained using established analysis procedures such as the moment distribution method or the Direct Stiffness Method.

Gross cross-sectional properties of slab-beams and columns are used for the frame model. For slabs with drop panels or slab bands, slab-beam properties need to reflect the gross cross-sectional dimensions of these members, including any variations along the span.

The results of elastic analysis are used to find bending moments and shear forces in longitudinal direction of the frame. However, transverse moment distribution is performed by applying empirical coefficients to find moments in the column strip and the middle strip (similar to the DDM).



a)



b)

Figure 12.40 Defining an equivalent frame: a) a partial floor plan; b) isometric view of an equivalent frame on Gridline 2, and c) a 2-D frame for gravity load analysis.

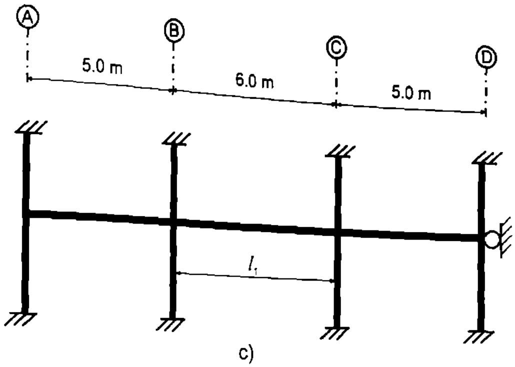


Figure 12.40 (*cont.*)

Each frame is analyzed separately, and the results for individual frames are superimposed to create the final design solution for building under consideration.

For gravity load analysis, each floor and roof slab can be analyzed separately. Far ends of the columns must be restrained by providing a roller support which prevents side sway of the frame (CSA A23.3 Cl.13.8.1.2). A frame model suitable for gravity load analysis is shown in Figure 12.40c.

A23.3 Cl.13.8.2 and 13.8.3

Modelling of Equivalent Frame Members An equivalent 2-D frame consists of slab-beam and column members. The EFM assumes rigid beam-column connections, that is, all members joined at a connection undergo the same rotation. A partial view of an equivalent 2-D frame is shown in Figure 12.41a. It can be seen from the figure that slab-beams look like beam elements in a regular beam and column frame, however these beams are very wide (their width is equal to the tributary slab width l_2).

Column members are more complex due to the presence of *attached torsional members*, that is, imaginary linear members which extend from the column in transverse direction, as shown in Figure 12.41b. The purpose of these members is to take into account reduced flexural stiffness of the columns at the locations of beam-column connections. This concept will be discussed in more detail later in this section.

Note that Direction 1 is along the plane of the frame, while Direction 2 denotes transverse direction (see Figure 12.41b).

An initial step in the frame analysis is to determine flexural stiffnesses for equivalent frame members. Appropriate stiffnesses must be estimated to simulate the behaviour of the actual slab system (CSA A23.3 Cl.13.8.1.5). Flexural stiffness is proportional to EI, which is a product of the modulus of elasticity (E) and the moment of inertia (I).

Geometric properties (cross-sectional dimensions) of frame members can be determined using either non-prismatic or prismatic approach according to CSA A23.3 (see Section 3.2 for the definition of prismatic and non-prismatic flexural members). Simply put, *prismatic* refers to constant cross-sectional properties along the member length, while *non-prismatic* refers to variable cross-sectional properties. Both approaches use gross cross-sectional properties of frame members (the effect of cracking is not taken into account).

A *non-prismatic approach* is used in conjunction with the moment distribution method, which has been used since the EFM was first introduced in North American design codes in the 1960s. This method was suitable for the analysis of simple equivalent frames by digital calculators. Each non-prismatic slab and column member with variable cross-sectional properties along the span can be modelled as a single member (CSA A23.3 Cl.13.8.2).

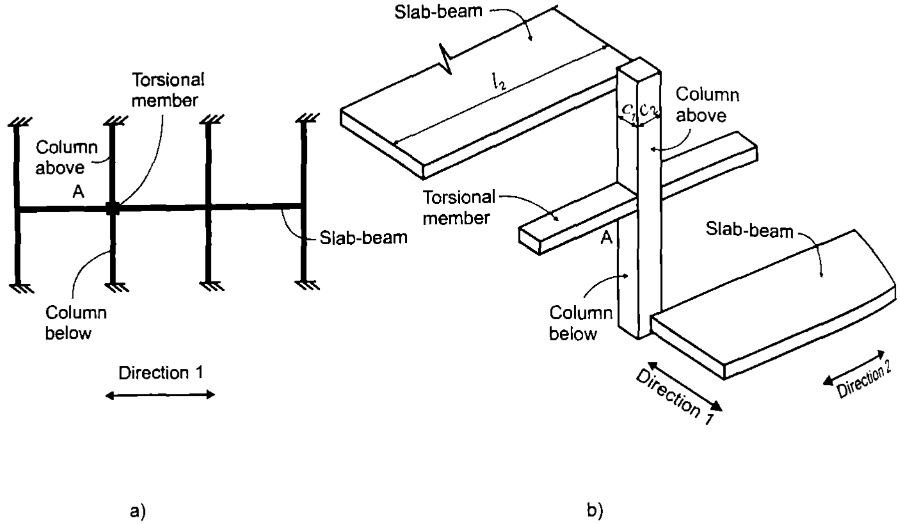


Figure 12.41 An equivalent frame: a) a 2-D view, and b) an isometric view.

Correction factors are applied to modify member stiffnesses, carry-over factors, and fixed end bending moments for each frame member. Once the adjusted properties for all frame members have been determined, the moment distribution method can be used to derive bending moments at each joint of the equivalent frame. A detailed discussion on moment distribution method is beyond the scope of this book, however the reader is referred to MacGregor and Bartlett (2000) for more information.

A *prismatic approach* considers members with constant cross-sectional properties (CSA A23.3 Cl.13.8.3). This approach is used in conjunction with the Direct Stiffness Method which is applied through computer-based frame analysis. Since the prismatic approach is currently more widely used in design practice, it will be explained in the next section.

Let us illustrate modelling of slab-beam members for the EFM applications. Consider a typical span of a flat slab shown in Figure 12.42a. For modelling purpose, span length (l_1) is equal to distance between the column centrelines (AB). Cross-sectional dimensions of a typical slab section (1-1) are shown in Figure 12.42b. The slab-beam section has thickness h_s, and its width l_2 is equal to design strip width shown in Figure 12.40b. The corresponding moment of inertia is equal to I_1.

A slab-beam could be modelled as a single prismatic member provided that column and slab-beam dimensions are relatively small. However, member dimensions in reinforced concrete structures are significant and should be considered in the structural model. Therefore, moment of inertia of the slab-beam member between the column face and the column centreline must be modified to account for the column and slab-beam dimensions. This is reflected by the moment of inertia I_2 which corresponds to slab section 2-2 shown in Figure 12.42b. Note that I_2 is obtained by modifying I_1 value by the multiplier prescribed by CSA A23.3 Cl.13.8.2.3. A variation in the EI value along the slab span is shown in Figure 12.42c. For the non-prismatic approach, a slab span is modelled as a single member shown in Figure 12.42d, and EI variation along the slab span is taken into account by design aids (tables) used in conjunction with the moment distribution method. Alternatively, a slab-beam can be modelled as a series of connected prismatic segments shown in Figure 12.42e.

Each portion of the slab-beam with variable cross-sectional properties is treated as a separate segment; in this case, there are three segments (1, 2 and 3).

The above described technique can be applied to flat slabs with drop panels and/or columns with capitals; however, the model will become more complex because it needs to take into account geometric properties of drop panels and column capitals.

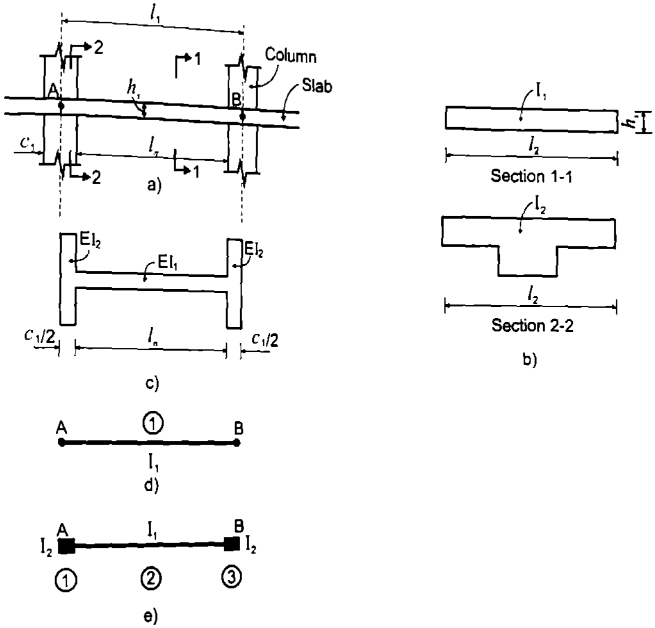


Figure 12.42 Modelling of slab-beam properties - elevation view: a) a typical slab span; b) cross-sections; c) variation of stiffness along the span; d) prismatic model, and e) non-prismatic model.

Figure 12.43 illustrates modelling of column properties for the EFM analysis. Figure 12.43a shows a typical column spanning between flat slabs at two adjacent floor levels. Column stiffness is determined using the column height l_c measured between the slab centrelines, as shown in the figure. Moment of inertia of the column outside the slab-column connection is based on the gross cross-sectional area of concrete (I_c) shown in Figure 12.43b. Note that the column moment of inertia is considered to be infinite in the slab-column connection region, which extends from the face of the slab-beam to the slab centreline at each end. A variation of EI along the column height is shown in Figure 12.43c. According to the non-prismatic approach, a column is modelled as a single member, as shown in Figure 12.43d, and the variation of column properties is accounted for by design aids used in conjunction with the moment distribution method. Alternatively, a prismatic approach can be used in conjunction with the Direct Stiffness Method, as shown in Figure 12.43e. It can be seen from the figure that there are three column segments (1, 2 and 3). The end segments (1 and 3) are assigned significantly larger (e.g. by an order of magnitude) moment of inertia values compared to I_c.

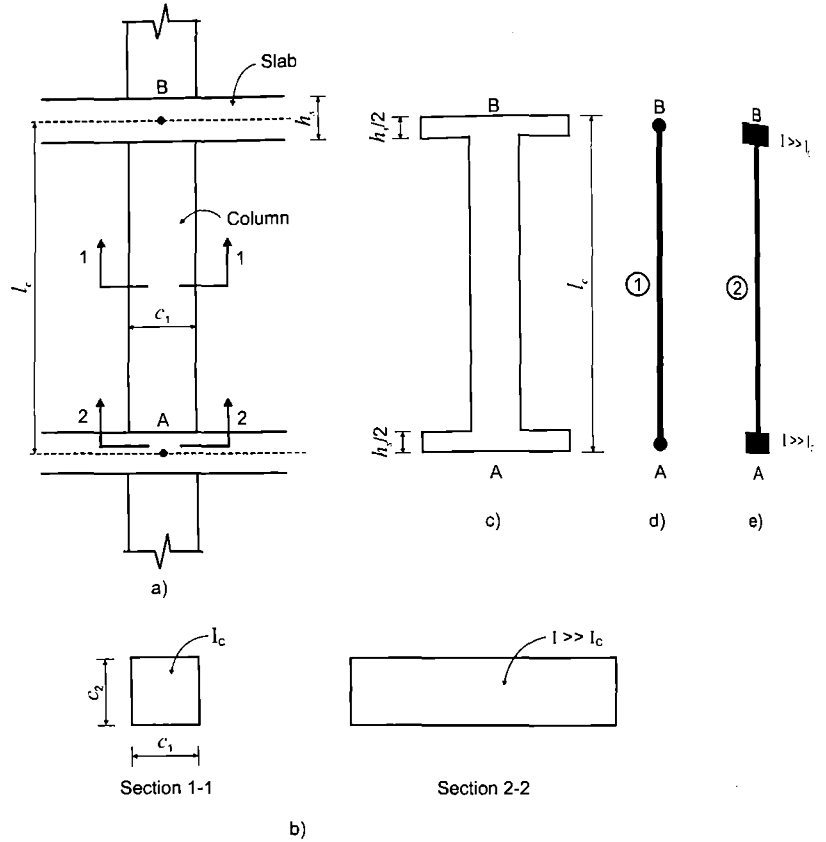
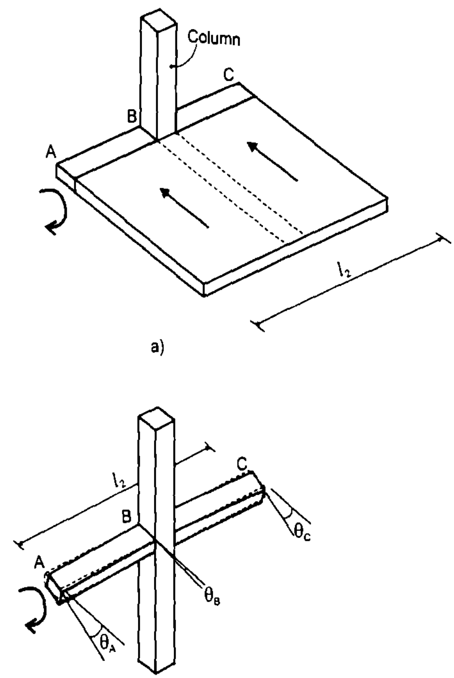


Figure 12.43 Modelling of column properties - an elevation view: a) a typical column; b) cross-sections; c) variation of stiffness along the column length; d) prismatic model, and e) non-prismatic model.

The equivalent frame models for slabs without beams are characterized by slab-beam members with significantly larger width compared to the columns; this has implications on the load transfer from the slab to the column.

Figure 12.44a shows gravity load path in the vicinity of a slab-to-column connection. The load in the vicinity of the column is transferred from the slab to the column directly through bending. However, load further away from the column centreline is transferred into the column by twisting of a slab strip on each side of the column. This has the effect of reducing flexural stiffness of the column relative to flexural stiffness of the slab. For analysis purposes, slab strip ABC (shown shaded in the figure) is considered as an attached torsional member, which accounts for a reduction in the column flexural stiffness. Figure 12.44b shows an isometric view of the attached torsional member ABC which has the length equal to the slab-beam width l_2. The rotation at the support (column) at point B is labelled as θ_2. It can be seen that the rotations at points A and C located at far ends of the slab section

(θ_A and θ_C respectively) are significantly larger than rotation θ_B at column face. This is due to the fact that the column has a minimal effect on slab rotations at far ends of the slab section.



…ure 12.44 Loads and deforma…ns in the slab-column connec…n region: a) load transfer from …e slab to the column, and …rotation of the attached …rsional member.

The above discussion is useful for understanding transfer of bending moments from the slab to the column. In a slab-and-column frame, bending moments from the slab are transferred directly to the column only over a rather narrow strip approximately equal to the column width, as shown in Figure 12.45a. The remaining bending moments in the slab must be transferred to the column through torsion of the attached torsional member, as shown in Figure 12.45b.

The rotational stiffness of the slab-to-column connection is a function of the torsional stiffness of the attached torsional member and the flexural stiffness of the columns framing into the connection from above and below. For analysis purposes, a column and the attached torsional members can be considered as an *equivalent column*, as shown in Figure 12.44b (CSA A23.3 Cl.13.8.2.5). The stiffness of an equivalent column (K_{ec}) can be determined from the following equation (CSA A23.3 Cl.13.8.2.6):

A23.3 Eq. 13.18

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

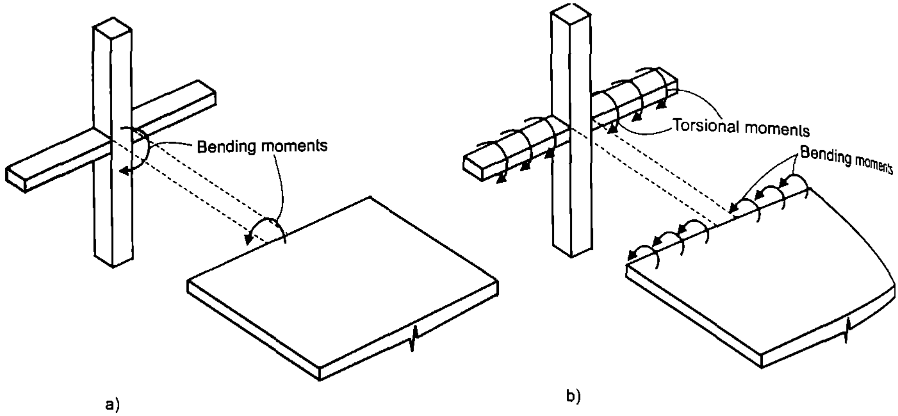


Figure 12.45 Transfer of moments from the slab to the column: a) bending moments at the column location, and b) torsional moments acting on the attached torsional member.

where $\sum K_c$ is the sum of flexural stiffness values for columns above and below the slab, and K_t is the stiffness of the torsional member. CSA A23.3 contains provisions for estimating the stiffness of attached torsional members (Cl. 13.8.2.8 to 13.8.2.10).

A23.3 Cl.13.8.3

Prismatic Approach for Modelling Frame Sections A prismatic approach considers members with constant cross-sectional properties. Variation in slab properties within a span can be accounted for by considering prismatic segments with different gross cross-sectional properties. This approach is used in conjunction with the Direct Stiffness Method and it is suitable for computer applications.

A slab span is divided into several (usually 10 to 20) slab-beam segments which are joined together. Each segment has constant cross-sectional properties, and it simulates the stiffness of a specific slab section. This approach can take into account the effect of drop panels by considering segments with appropriate cross-sectional properties at drop panel locations.

A column is usually modelled as a single member. A column capital can be modelled as an additional segment both in the slab and the column with appropriate stiffness properties. CSA 23.3 Cl. 13.8.3.3 accounts for a reduction in column stiffness due to the attached torsional member through the column stiffness modification factor (ψ), which can be obtained from the expressions presented in Table 12.17.

Table 12.17 Column Stiffness Modification Factor ψ (CSA A23.3 Cl.13.8.3.3)

Span Ratio	Slabs without beams ($\alpha_1 = 0$)	Slabs with beams
$l_2/l_1 \le 1.0$	$\psi = 0.3$	$\psi = 0.3 + 0.7\dfrac{\alpha_1 l_2}{l_1}$
$l_2/l_1 > 1.0$	$\psi = 0.6\left(\dfrac{l_2}{l_1} - 0.5\right)$	$\psi = 0.6\left(\dfrac{l_2}{l_1} - 0.5\right) + \left(1.3 - 0.6\dfrac{l_2}{l_1}\right)\dfrac{\alpha_1 l_2}{l_1}$

Notes:
l_1 and l_2 - slab spans in the plane of the frame (Direction 1) and transverse direction (Direction 2), respectively α_1 - beam-to-slab stiffness ratio in Direction 1.

CSA A23.3 Cl.13.8.3.3 sets a range for the stiffness modification factor values, as follows

$$0.3 \leq \psi \leq 1.0$$

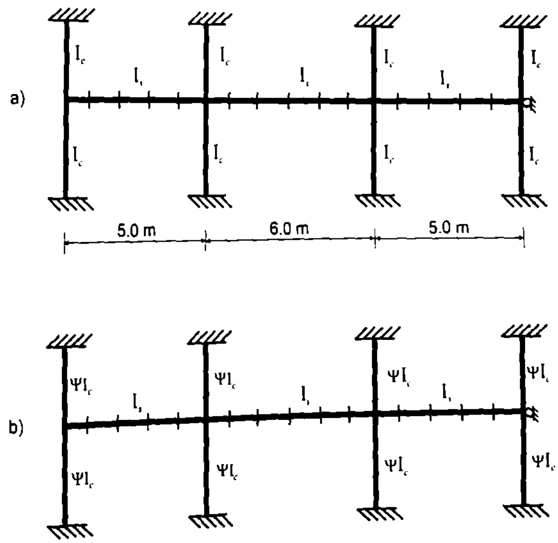
also

$$\frac{\alpha_1 l_2}{l_1} \leq 1.0$$

For flat slabs and flat plates, ψ values vary between 0.3 (when $l_2/l_1 \leq 1.0$) and 0.9 (when $l_2/l_1 = 2.0$). The ψ factor increases when the slab span in the transverse direction is significantly larger than the span in the longitudinal direction, because the column flexural stiffness is more significant and it attracts larger negative moments over the column strips.

To account for the effect of column flexural stiffness, the designer should use the product $\psi \cdot I_c$ (instead of I_c) for columns in an equivalent frame model. For example, consider the equivalent frame discussed earlier in this section (see Figure 12.40). Gross cross-sectional properties, that is, column moment of inertia (I_c) and slab-beam moment of inertia (I_s), are presented in Figure 12.46a, while modified frame properties are shown in Figure 12.46b. Each slab-beam span is divided into several segments with the same properties (moment of inertia I_s), but column properties need to be modified by the ψ factor.

Figure 12.46 Equivalent frame with prismatic member properties: a) actual frame properties, and b) modified frame properties used for design.



A23.3 Cl.13.8.5

Design Bending Moments at Supports The members of an equivalent frame span centre-to-centre between supports. Therefore, bending moments and shear forces obtained from the EFM analysis are given at member centerlines. However, in reality supports (columns) in a reinforced concrete structure have finite dimensions, hence bending moments derived at centerlines may not be realistic. There may be a significant difference between the negative bending moments at the column centreline and the face of the column. CSA A23.3 Cl.13.8.5.1 permits the use of reduced bending moments at the face of support for design purpose. The locations where bending moments are reduced are referred to as *critical sections* by CSA A23.3.

An example illustrating the reduction of bending moments at the face of an interior column is shown in Figure 12.47. Critical sections at the face of a column are shown in Figure 12.47a. Note that a critical section should be taken at the column face, but the distance should not exceed $0.175\ l_1$ from the column centreline (this limit is set for columns with rectangular cross-sections where one dimension is significant compared to the slab span). Actual bending moment values for the slab at the support (M_L and M_R) are shown in Figure 12.47b; note that these values are different because the balance of these bending moments is transferred to the column. Reduced bending moments at the critical sections (M_{RL} and M_{RR}) should be used to design the flexural reinforcement. The reduced moments at the face of the column can be calculated from the equation presented in Section 10.4.3.

For exterior columns with column capitals Cl. 13.8.5.2 permits the critical section to be located at a distance equal to one-half of the capital projection from the face of the column, as shown in Figure 12.48.

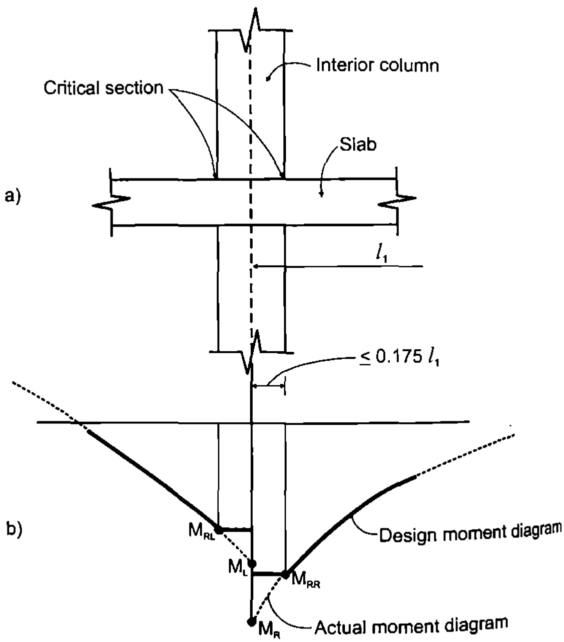
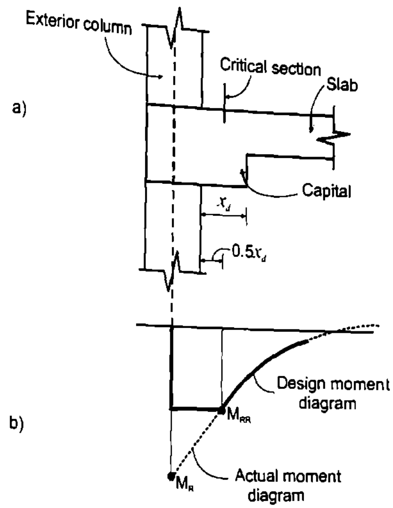


Figure 12.47 Bending moment diagram at the critical section at the face of an interior column: a) critical sections at the column face, and b) bending moment diagram.

A23.3 Cl.13.8.4

Arrangement of Live Load (Load Patterns) Live loads are transient loads, and a variation in their magnitude and arrangement may significantly influence bending moments and shear forces in reinforced concrete structures. Therefore, bending moments and shear envelopes obtained by considering variations in live load patterns must be taken into account by design (refer to Section 10.3 for more details on load patterns and moment envelopes). However, two-way reinforced concrete slabs usually have a significant self-weight, thus dead load often accounts for a major portion of the total factored load. In these cases, variations in the live load arrangement may not have a significant effect on the maximum bending moments and shear forces in the slab.

CSA A23.3 provisions related to the live load arrangement for design of two-way slabs according to the EFM consider the following three cases:



...gure 12.48 Bending moment ...agram at the critical section at ...e face of an exterior column with ...e capital: a) critical section, and ...bending moment diagram.

1) When the loading pattern is given, the frame should be analyzed considering that pattern (Cl. 13.8.4.1). For example, in a warehouse with moving crane loads on the slab, the crane load should be considered with or without the occupancy live loads to determine the maximum bending moments and shear forces.
2) When the specified live load is uniformly distributed and does not exceed three-quarters (75%) of the specified dead load, the designer only needs to consider the full factored load on all spans (Cl. 13.8.4.2). This should apply to most two-way slab building applications with residential, office, and/or retail occupancy.
3) When the specified live-to-dead load ratio in the slab exceeds 75%, several live load patterns need to be considered to determine the maximum factored bending moments (Cl. 13.8.4.3). These load patterns (LP1 to LP4) take into account partial factored live loads, as shown on an example of a two-way slab in Figures 12.49a to d. However, the factored moments should not be taken as less than those developed due to full factored live loads on all panels (Cl. 13.8.4.4); this is illustrated by load pattern LP5 in Figure 12.49e.

Note that, when the specified live-to-dead load ratio exceeds 75%, the designer may wish to increase the design dead load to bring the ratio down to below 75%. In this manner, the designer avoids the need to consider pattern loading in the design. The resulting design solution uses a higher than required load and it is slightly more conservative, which may be a good practical approach.

A23.3 Cl. 13.5.2 and 13.5.3

Buildings with Two-Way Slab Systems: Gravity and Lateral Load Analysis A structural analysis of a reinforced concrete building with a two-way floor system can be performed by modelling a 3-D structure as a series of idealized parallel 2-D frames in each direction, as discussed earlier in this section. Key considerations related to the analysis of 2-D frames for gravity and lateral loads will be discussed in this section. Note that it is required to perform the gravity and lateral load analyses separately, and the results should be combined (Cl. 13.5.3).

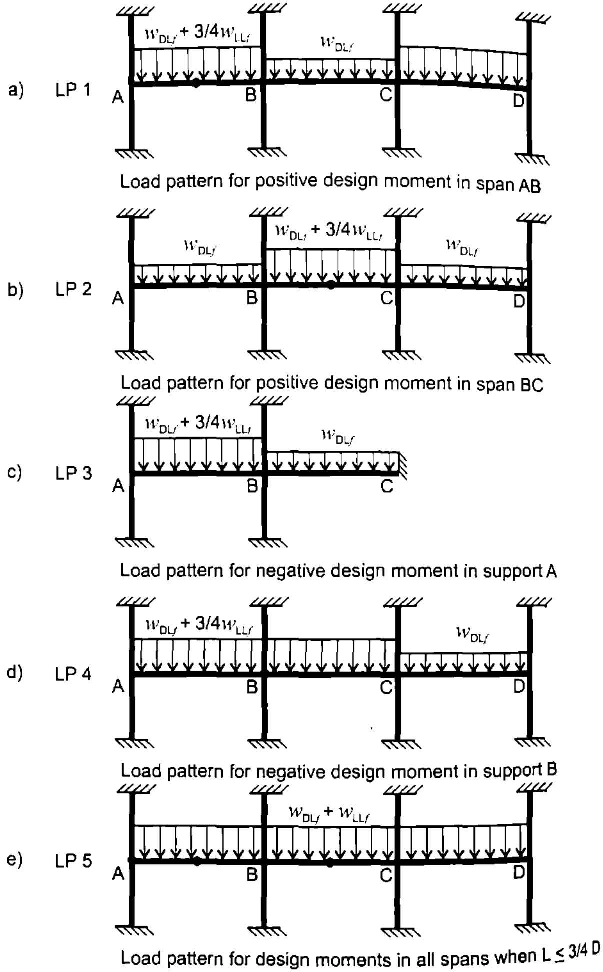
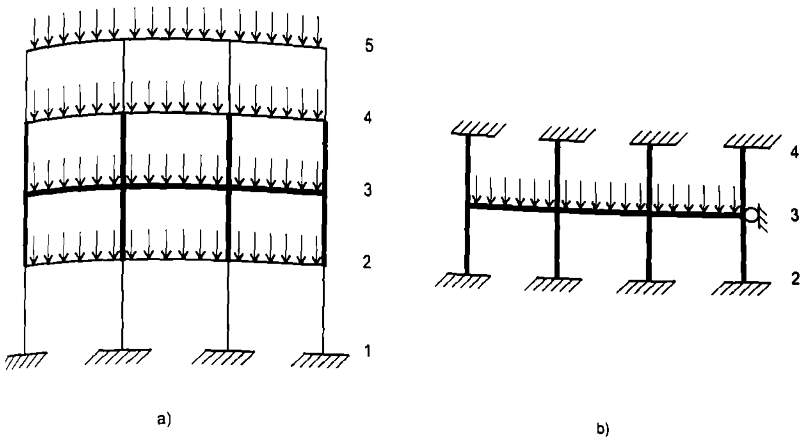


Figure 12.49 Load patterns for gravity load analysis of equivalent frames: a) positive design moment in span AB; b) positive design moment in span BC; c) negative design moment at support A; d) negative design moment at support B, and e) full factored load in all spans.

For gravity load analysis, the designer needs to consider only a portion of the frame, which consists of a slab at the floor level under consideration and the column segments above and below that level. Consider a 2-D frame shown in Figure 12.50a. It is not required to perform analysis of the entire frame for gravity loads - a partial frame can be used instead. An example of a partial frame for the slab at level 3 and the adjoining columns is shown in Figure 12.50b. Note that a lateral restraint in the form of a roller support need to be provided at the far end of the frame at the slab level to prevent lateral movements. Also, far ends of the columns are restrained by fixed supports. Gravity load analysis can be performed using either the Direct Design Method or the Equivalent Frame Method.



igure 12.50 Frame model for gravity load analysis: a) a complete frame, and b) a partial frame used for gravity load analysis.

The lateral load model will be explained on an example of a four-storey building with a flat plate floor system shown in Figure 12.51a. The building can be modelled as a series of four frames in N-S direction and three frames in the E-W direction. These frames are defined by passing vertical cutting-planes through the structure midway between the column gridlines. Next, consider a typical frame along gridline 2 in E-W direction (which is shown hatched in Figure 12.51a). An isometric view of the frame is shown in Figure 12.51b. The frame geometry is defined by column and slab properties. A 2-D view of the frame with the actual column and slab thicknesses is shown in Figure 12.51c; note that column and slab centrelines are shown dashed in the figure. Finally, a 2-D model of the frame used for the analysis is shown in Figure 12.51d.

When frames constitute the lateral load-resisting system, lateral load needs to be distributed between frames aligned in the same direction. For example, consider three frames in E-W direction (along gridlines 1, 2, and 3). The two exterior frames are supported by the walls along gridlines 1 and 3, while the interior frame along gridline 2 is supported by the columns. As a result, frames 1 and 3 will resist most of the lateral loads since their lateral stiffnesses are significantly larger than frame 2. Distribution of lateral loads in proportion to frame stiffness is appropriate when floors and the roof are relatively rigid compared to the vertical elements (columns or walls). In that case, a linked frame model can be used for lateral load analysis. An example of a linked frame model for E-W direction of the building is shown in Figure 12.51e. The model consists of three 2-D frames (gridlines 1, 2, and 3) which are connected by rigid pin-ended links. The entire lateral load is applied to the linked frame. Distribution of loads to individual frames is usually performed using structural analysis software.

The EFM can be used to perform lateral load analysis for buildings with a two-way slab systems, however the DDM cannot be used for that purpose. Cl.13.5.2 states that the lateral load analysis should take into account the effects of cracking and reinforcement on stiffness of frame members.

In a building, flat plates or flat slabs supported by columns are rarely designed as the primary lateral load-resisting system, and their application is not permitted in regions of high seismicity according to the NBCC 2015. In most cases, reinforced concrete shear walls are used as principal components of the seismic force-resisting system in buildings with flat plate or flat slab floor systems. However, lateral swaying of floor and roof

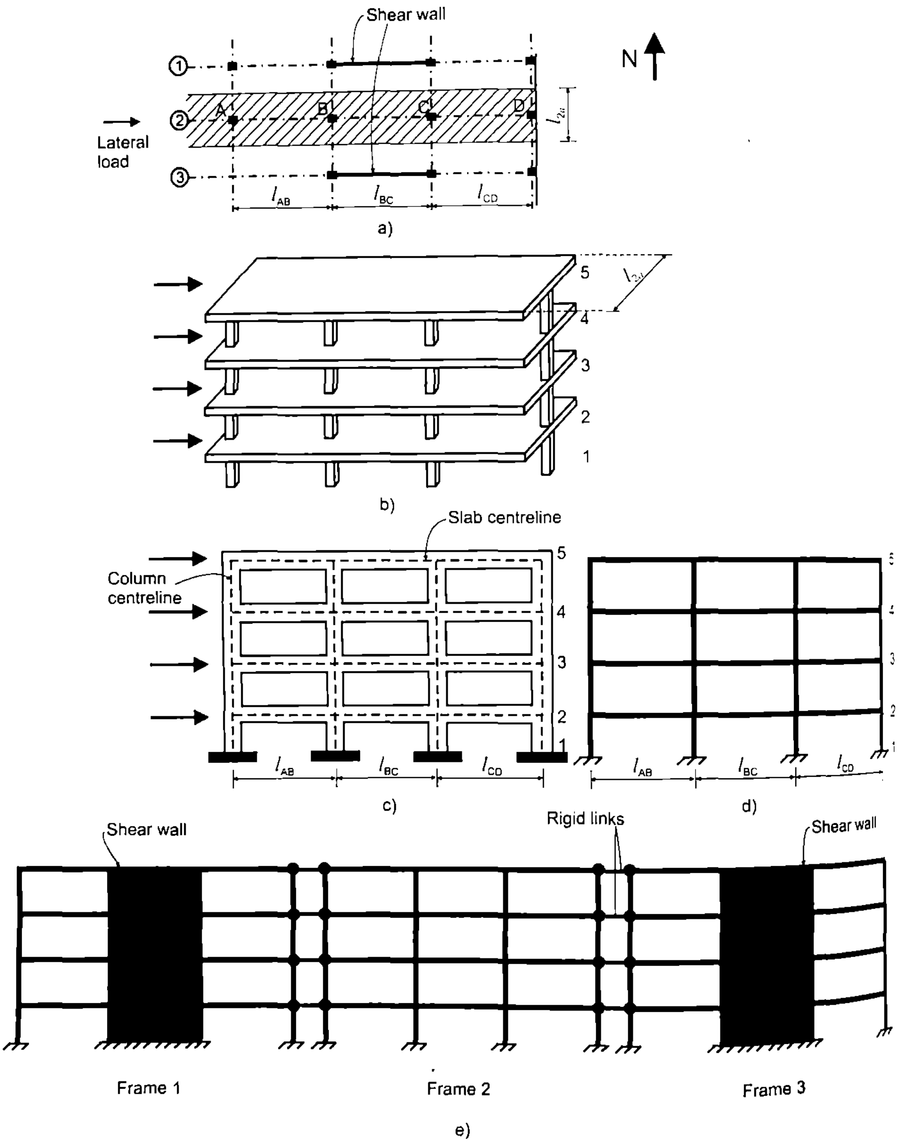


Figure 12.51 Frame models for lateral load analysis of two-way slabs: a) a typical building plan; b) an isometric view of a 2-D frame model; c) a 2-D frame showing actual column and slab dimensions; d) a 2-D model used for the lateral load analysis, and e) a linked frame model for lateral load analysis.

systems due to seismic or wind loading will have an effect upon the gravity load-bearing capacity of a slab system. When a building is subjected to lateral loading, lateral swaying will cause additional stresses in floor and roof slabs. The designer must carefully consider the effects of such lateral movements upon the punching shear capacity of flat plates and flat slabs.

The difference in lateral displacements between adjacent floors is called "inter-storey drift". Inter-storey drift for a frame is shown in Figure 12.52a. Note that inter-storey drift for floor levels i and j is denoted as Δ_{ij}, and the drift for floor levels j and k is denoted as Δ_{jk}. The total lateral drift for a building is equal to the lateral displacement at the roof level relative to the base of the building and it is denoted as Δ; this is relevant for seismic design. A magnitude of inter-storey drift at each floor level needs to be obtained from the seismic analysis of the primary lateral resisting structure. The designer needs to evaluate the effect of inter-storey drift on the floor and roof slabs by performing a frame analysis. A partial frame can be subjected to differential displacements equal to inter-storey drift values at adjacent floor levels, as shown in Figure 12.52b.

The designer needs to combine internal forces due to inter-storey drift with those due to gravity loading. Analysis of reinforced concrete buildings for lateral loads is outside the scope of this book.

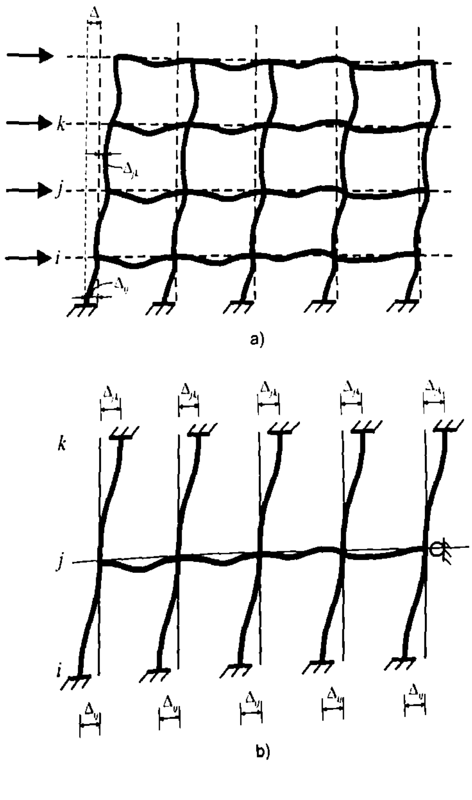


Figure 12.52 Lateral load analysis: a) inter-storey drift in the frame determined from lateral load analysis, and b) a partial frame subjected to lateral displacements at the supports.

Guidelines for Effective Modelling of Equivalent Frames Designers need to understand that an equivalent frame model is an approximation of the actual structural behavior. The EFM is an elastic analysis approach and it does not account for cracking and inelastic behaviour of reinforced concrete structures, hence the use of an overly accurate equivalent frame model may not be justified. Designers may keep in mind the following guidelines related to effective modelling of equivalent frames:

1. Bending moments are typically negative in support regions and positive in midspan regions of the slab. For a slab span subjected to uniformly distributed load, the sum of absolute values for positive bending moment at the midspan and average negative bending moment at the supports is equal to the moment gradient M_o (as discussed in Section 12.6.2).
2. It is important to estimate a reasonable range for maximum negative moments at the supports and maximum positive moments in midspan regions of the slab, and provide an adequate amount of reinforcement in these regions. A balance between the negative (top) reinforcement and the positive (bottom) reinforcement is required to minimize flexural cracking, a key design consideration for continuously reinforced concrete slab structures.
3. Continuously reinforced concrete slabs represent a cost-effective structural system, as discussed in Chapter 10. It is important to ensure an appropriate placement of reinforcement in order to take advantage of structural efficiency for a continuous span structure. The effective design of continuous slabs is also critical to minimize deflections. Section 10.2 discusses general concepts of continuously reinforced concrete structures; these concepts should be considered when optimizing a two-way slab system which has continuous spans in two directions.

The designer should keep in mind that two-way slabs are continuous structures, that is, statically indeterminate and redundant systems. Consequently, there may be several acceptable design solutions. For that reason, the designer may wish to perform a sensitivity analysis by varying positive and negative bending moments in each span to identify the most appropriate solution.

Furthermore, the designer can influence bending moment distribution in the slab either by increasing or decreasing column stiffness for modelling purposes. Two extreme solutions will be discussed below.

The first solution involves a model in which column stiffness is disregarded (equal to zero).The frame is modelled as a continuous beam system where the slab is supported by pin or roller supports instead of rigidly connected columns. In this case, positive bending moments in the slab and the corresponding amount of bottom reinforcement are expected to be larger compared to an alternative solution where column stiffness is considered in the analysis. This is due to the fact that the presence of rigidly connected columns increases the slab stiffness at the supports resulting in an increase in negative bending moments, and a corresponding decrease in the positive bending moments at midspan regions. Hence, the solution where column stiffness is ignored may result in an increase in positive bending moments at midspan regions and a decrease in negative bending moments at the slab support regions. This type of analysis is considered an acceptable approximation in design practice and is particularly suitable for simple hand calculations.

A more appropriate solution is to account for column stiffness by considering lengths and cross-sectional dimensions of columns above and below the floor slab under consideration. This solution would result in more realistic negative bending moment values and the consequent amount of top reinforcement in the support regions. However, large column stiffness could also attract larger unbalanced bending moments that would need to be transferred by the slab into the columns by means of two-way shear. This may be a concern since flat slabs typically have limited two-way shear capacity at the slab-to-column interface. When the two-way shear capacity in the slab is exceeded, cracking will occur in the vicinity of the slab-to-column interface resulting in a reduction in the stiffness of the slab-column region. Hence, in many cases it may be more appropriate to model the slab with a reduced column stiffness by arbitrarily increasing the column length, such that the unbalanced moment in the slab is reduced to match the slab shear capacity.

Design Applications of the Equivalent Frame Method The purpose of this section is to demonstrate design of two-way slabs without beams according to the EFM by three design examples.

Example 12.3

Two-Way Flat Plate (Slab without Beams) - Equivalent Frame Method

Consider a floor plan of a two-way slab system without beams designed in Example 12.1. The floor height is 3 m.

Use the CSA A23.3 Equivalent Frame Method to determine the design bending moments and reinforcement for an interior frame along gridline 2.

Given: $f_c' = 30$ MPa

$f_y = 400$ MPa

SOLUTION:

1. **Check whether the criteria for the CSA A23.3 Equivalent Frame Method (EFM) are satisfied.**

The EFM can be applied when a slab is regular, that is, when the following CSA A23.3 Cl. 2.2 provisions have been met (see Section 12.5.1 for discussion on regular slabs):

(#a) Within a panel, the ratio of longer to shorter span, centre-to-centre of supports, is not greater than 2.0. *In this case,* $l_1 / l_1 = 6.0\ \text{m} / 4.8\ \text{m} = 1.25 < 2.0$.

(#b) For slab systems with beams between supports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 5.0. *This is not applicable, since this is a slab without beams.*

(#c) Column offsets are not greater than 20% of the span (in the direction of the offset) from either axis between centerlines of successive columns. *There are no column offsets in this case.*

(#d) The reinforcement is placed in an orthogonal grid. *The reinforcement will be placed in an orthogonal grid.*

This is a regular slab according to CSA A23.3 Cl.2.2, therefore the EFM can be used for this design.

2. **Determine the required slab thickness.**

The slab thickness is the same as in Example 12.1, that is,

$h_s = 180$ mm

3. **Calculate the factored design loads.**

a) Calculate the dead load acting on the slab.

First, calculate the slab's self-weight:

$$DL_w = h_s \times \gamma_c = 0.18\ \text{m} \times 24\ \text{kN/m}^3 = 4.32\ \text{kPa}$$

where $\gamma_c = 24\ \text{kN/m}^3$ is the unit weight of concrete.

The superimposed dead load is given, that is,

$$DL_s = 1.44\ \text{kPa}$$

Finally, the total factored dead load is

$$w_{DL,f} = 1.25(DL_w + DL_s) = 1.25(4.32 + 1.44) = 7.2\ \text{kPa}$$

b) Calculate the factored live load:

$$w_{LL,f} = 1.5 \times LL_w = 1.5 \times 3.6\ \text{kPa} = 5.4\ \text{kPa}$$

c) The total factored load is

$$w_f^* = w_{DL,f} + w_{LL,f} = 7.2 + 5.4 = 12.6\ \text{kPa}$$

d) Check whether the pattern loading needs to be considered according to CSA A23.3 Cl.13.8.4, that is, check the ratio of specified live load and dead load.

The total specified dead load is

$$w_{DL} = DL_w + DL_s = 4.32 + 1.44 = 5.76 \text{ kPa}$$

The specified live load is

$$w_{LL} = 3.6 \text{ kPa}$$

The ratio of the specified live and dead load:

$$\frac{w_{LL}}{w_{DL}} = \frac{3.6 \text{ kPa}}{5.76 \text{ kPa}} = 0.63 < 0.75$$

Since the ratio is less than 0.75, pattern loading does not need to be considered. The frame needs to be designed for the effects of total factored dead and live load on all spans. Hence, the design load is

$$w_f^* = 12.6 \text{ kPa}$$

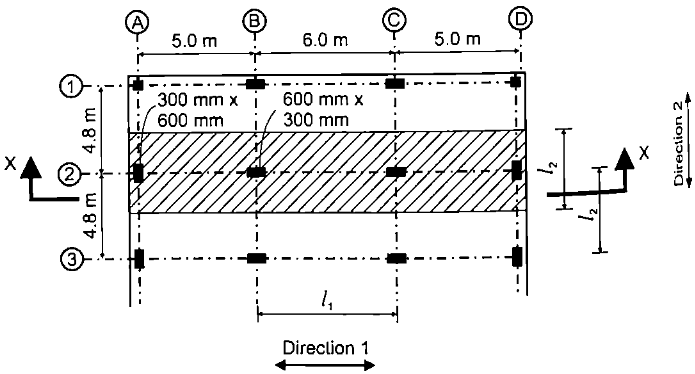
This is a uniformly distributed area load which could be used for 3-D analysis. However, we need to find the design load for 2-D frame analysis, which can be obtained when the tributary slab width ($l_2 = 4.8$ m) is taken into account, as follows

$$w_f = w_f^* \times l_2 = 12.6 \text{ kPa} \times 4.8 \text{ m} = 60.5 \text{ kN/m}$$

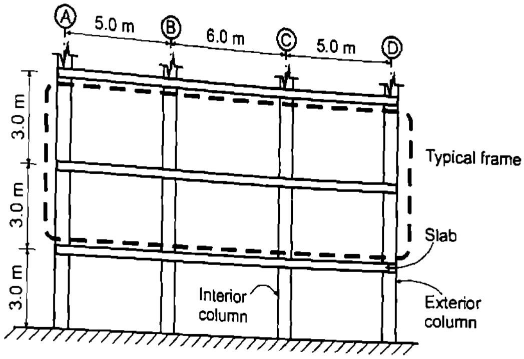
4. Develop a frame model.

a) Determine the frame geometry.

In order to model an actual 3-D building as a series of parallel 2-D frames, the designer first needs to find the frame width. In this case, the width (l_2) is 4.8 m, as discussed above. This is shown on the sketch, for a frame along gridline 2.

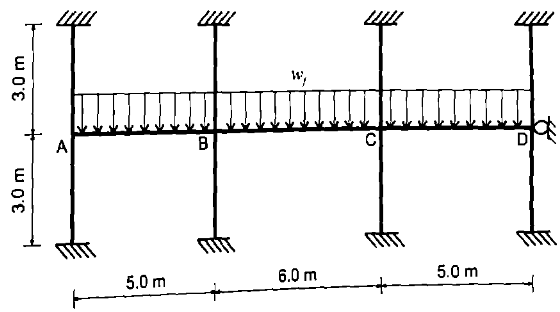


Next, it is necessary to sketch the frame and identify beam spans in the horizontal direction and column heights in the vertical direction. Note that the frame geometry is defined by slab and column centrelines. The following sketch shows Section X-X of the frame (note that actual dimensions of frame members are shown on the sketch).



Section X-X

Finally, the designer needs to prepare a linear (wire) drawing of the frame. For gravity load analysis, CSA A23.3 permits the designer to consider one floor level plus adjacent columns above and below the floor. The sketch below shows frame geometry (beam and column spans). The frame axes correspond to the centrelines of slabs and columns. The column ends are shown fixed (no rotation). Note that the sketch also shows uniform design load (w_f) on the slab.

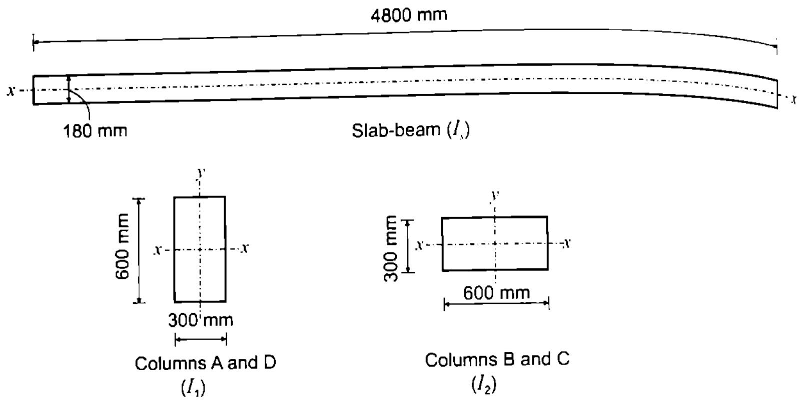


Frame Model

b) Determine cross-sectional properties for the frame members.
Typical cross-sections for frame members (slab-beams and columns) are illustrated on the following sketch.

i) Slab-beams
In the frame model, the slab is treated as a wide beam with the following gross cross-sectional properties: 4.8 m width (as discussed above) and 180 mm depth (equal to the slab thickness). Moment of inertia for the slab about axis x-x (see the sketch) is equal to

$$I_s = \frac{4800\ \text{mm} \times (180\ \text{mm})^3}{12} = 2.3 \times 10^9\ \text{mm}^4$$



For prismatic modelling, each span can be divided into ten or more segments, where each segment has the same moment of inertia (I_s).

ii) Columns

Column gross cross-sectional dimensions are the same (300 mm by 600 mm). However, note that the layout is different for exterior columns (A and D) and interior columns (B and C). For the frame along gridline B bending in the columns occurs about the axis y-y (as shown on the sketch), therefore moment of inertia values for the columns are different.

Columns A and D (moment of inertia I_1):

$$I_1 = \frac{600\ \text{mm} \times (300\ \text{mm})^3}{12} = 1.35 \times 10^9\ \text{mm}^4$$

Columns B and C (moment of inertia I_2):

$$I_2 = \frac{300\ \text{mm} \times (600\ \text{mm})^3}{12} = 5.4 \times 10^9\ \text{mm}^4$$

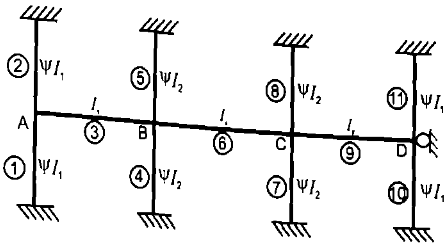
For prismatic modelling, Cl. 13.8.3.3 requires that the column moment of inertia be modified by the factor ψ. Since this is a slab without beams, $\alpha_1 = 0$ and $l_2/l_1 = 4.8$ m/6 m = 0.8 < 1.0, hence

$\psi = 0.3$

Therefore, the column moment of inertia values are $0.3 \cdot I_1$ and $0.3 \cdot I_2$.

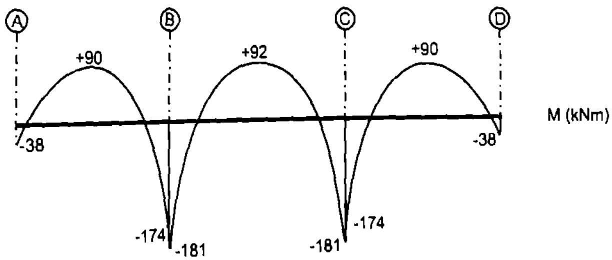
c) Sketch the final frame geometry and member properties.

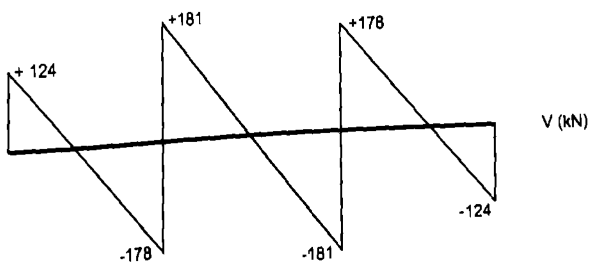
A sketch showing the final frame geometry and section properties is shown next. Note that the frame has 11 members. Slab-beam members are labelled as 3, 6, and 9, and the remaining members are columns. For analysis purposes, the designer can choose to divide the slab-beam into 10 to 20 segments, depending on the desired level of accuracy. Keep in mind that the EFM is an approximate method, hence it is not necessary to divide slab-beam members into too many segments.



5. **Determine the factored bending moments (M_f).**
This analysis was performed using a commercially available software package. The resulting bending moment and shear diagrams are shown next — note that the diagrams show only bending moments and shear forces in the slab-beam members.
Negative bending moments transferred from the slab into the columns are higher at end supports A and D (38 kNm) than at interior supports B and C. Negative moment at columns B and C is equal to 7 kNm, that is, the difference in bending moments between adjacent spans (−181 kNm and −174 kNm). In this design, the spans seem to be reasonably balanced and one would expect small unbalanced moments at the interior supports (shared equally between the columns above and below the slab).

6. **Calculate the reduced negative bending moments at the supports (CSA A23.3 Cl.13.8.5.1).**
First, let us calculate a reduced moment at the face of the support B for span AB. The procedure outlined in Section 10.4.3 will be followed (see Figure 10.13).





Reduced moment at the face of the support (M_1) will be calculated from the following equation

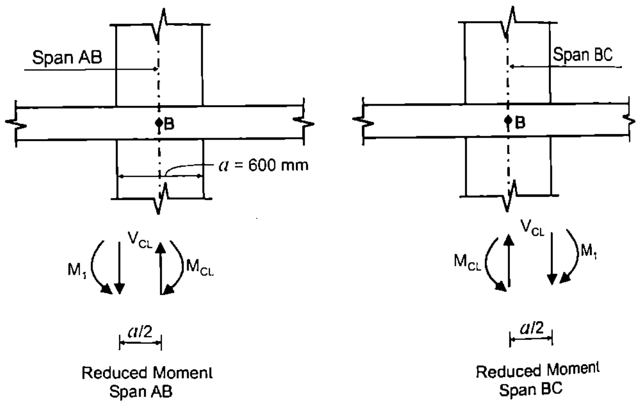
$$M_1 = M_{CL} - V_{CL} \times \frac{a}{2}$$

a) Moment at support B for span AB (see the sketch below):
Support width: $a = 600$ mm
Moment at point B: $M_{CL} = -174$ kNm
Shear force at point B: $V_{CL} = -178$ kN
The reduced moment is equal to

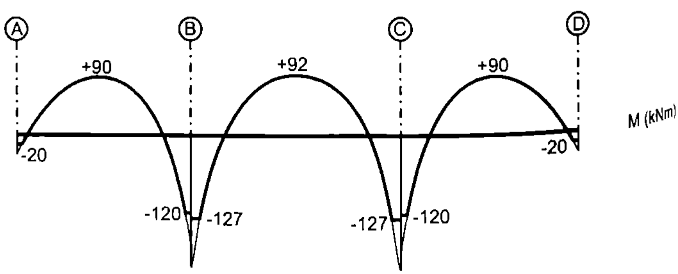
$$M_1 = M_{CL} - V_{CL} \times \frac{a}{2} = -174 - (-178 \times \frac{0.6}{2}) = -120 \text{ kNm}$$

b) Moment at support B for span BC (see the sketch below):
Support width: $a = 600$ mm
Moment at point B: $M_{CL} = -181$ kNm
Shear force at point B: $V_{CL} = 181$ kN
The reduced moment is equal to

$$M_1 = M_{CL} - V_{CL} \times \frac{a}{2} = -181 - (-181 \times \frac{0.6}{2}) = -127 \text{ kNm}$$



Below is a revised bending moment diagram with the reduced bending moments at the supports. Alternatively, bending moments at the critical sections could be obtained directly from the analysis software by placing nodes at the critical sections.



7. **Design the slab flexural reinforcement.**

a) Determine the effective depth (d).

The effective depth will be determined based on the following parameters:

Slab thickness: 180 mm

Concrete cover: 20 mm (see Table A.2)

Bar diameter: 15 mm (assume 15M bars)

For a flat slab, reinforcement is provided in two directions at the top and at the bottom. An average effective depth can be estimated as follows

$d = 180 - 20 - 15 = 145$ mm

However, note that a 25 mm concrete cover is often required for the bottom slab reinforcement since most slabs require a minimum two-hour fire rating. For that reason, let us use a slightly reduced average effective depth for both top and bottom reinforcement, that is,

$d = 140$ mm

b) The required area of reinforcement can be found according to the Direct Procedure (see Section 5.5.1) by using the following equation

$$A_s = 0.0015 f_c' b\left(d - \sqrt{d^2 - \frac{3.85 M_f}{f_c' b}}\right) \qquad [5.4]$$

For this case,

$b = l_2 = 4800$ mm

c) The minimum area of reinforcement is calculated from the following equation (CSA A23.3 Cl.7.8.1)

$A_{s\,min} = 0.002\,A_g$

d) The check for the maximum reinforcement ratio has been omitted from this design because the minimum reinforcement governs in most cases. If the check was to be performed, it would need to confirm that the reinforcement ratio for the slab-beam section under consideration is less than the balanced reinforcement ratio (see Table A.4), that is,

$\rho \le \rho_b$

e) The spacing of bottom reinforcement is limited to the lesser of (CSA A23.3 Cl.13.10.4) (see Table 12.8)

$s \le 3h_s = 3 \times 180 = 540$ mm

or

$s \le 500$ mm

In this case, $s = 500$ mm governs.

The results of these design calculations are summarized in Table 12.18. The slab is symmetrical, hence it is sufficient to perform calculations for spans AB and BC.

The top reinforcing steel at interior column locations needs to be designed to resist the greater of the two slab moments on either side of the column. For example, the top steel segment for support B should be designed for the bending moment of −127 kNm (span BC), which is larger than the moment of −120 kNm for span AB.

The designer may choose to distribute the top reinforcement over columns by following the same procedure outlined in Section 12.6 in relation to the Direct Design

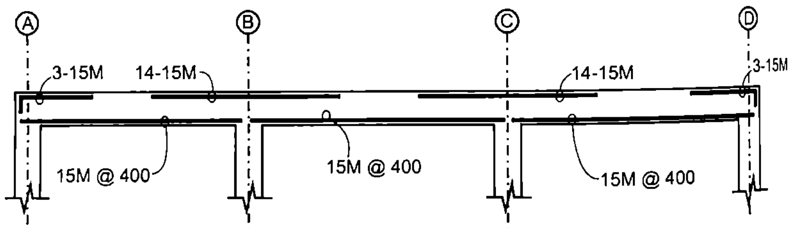
Table 12.18 Factored bending moments and the flexural reinforcement

Span	AB			BC		
Location	Top	Bottom	Top	Top	Bottom	Top
M_f (kNm)	−20	+90	−120	−127	+92	−127
A_s (mm²) [5.4]	415	1917	2744	2744	1961	2744
Top reinforcement	3–15M		14–15M			14–15M
Bottom reinforcement (required)		15M@500			15M@490	
Bottom reinforcement (design)		15M@400			15M@400	

Method. However, in practice it may not be required to place the top reinforcement in middle strip regions of two-way slabs with similar span lengths in each direction, i.e. within 10 to 15%. In that case, it may be more effective to place the entire top reinforcement in each direction within the column strip regions.

8. **Provide a design summary.**
The design summary is presented below. Note that the spacing of bottom reinforcement is 400 mm, although 500 mm spacing is adequate according to the design calculations. In this case, it is deemed appropriate to use 400 mm spacing for bottom reinforcement to satisfy the spacing requirements for both spans (AB and BC), and provide a reserve strength to allow for construction errors. For that reason, it is recommended to place reinforcing steel spaced in increments of 100 mm.

Note that this example does not include the calculation of cut-off points for the reinforcement, which is an important part of the design. Refer to Chapter 11 for examples related to detailing of flexural reinforcement in continuous slabs.



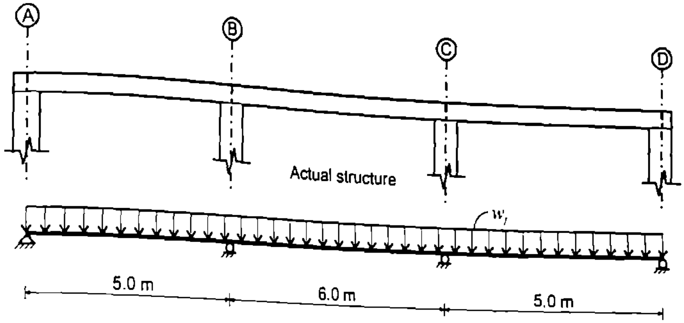
Example 12.4

Two-Way Flat Plate - A Simplified Solution Ignoring Column Stiffness

Consider the same slab discussed in Example 12.3, but ignore the column stiffness, that is, treat the slab as a continuous beam.
Determine bending moments and design the flexural reinforcement for the frame along gridline 2.

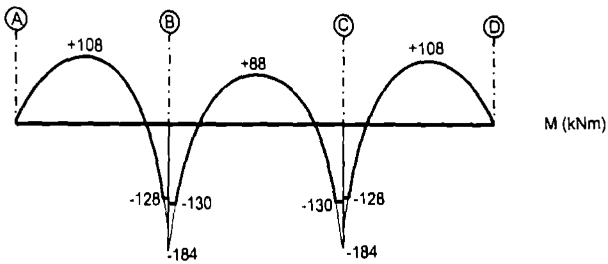
SOLUTION: In this case, we are going to treat the slab as a continuous beam, as shown on the following sketch. This system is easier to analyze than the frame system. The beam is going to be subjected to the same factored load as the frame in Example 12.3, that is,

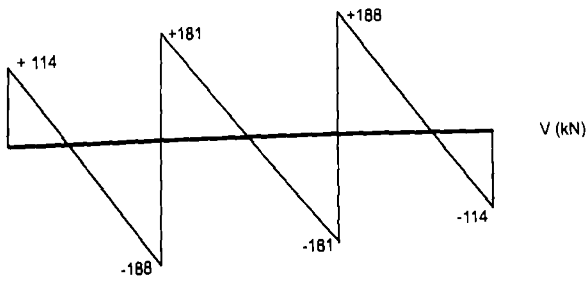
$$w_f = 60.5 \text{ kN/m}$$



Continuous beam model

The bending moment and shear force diagrams obtained from the structural analysis are shown below. Reduced moments at the supports are also shown on the diagram.





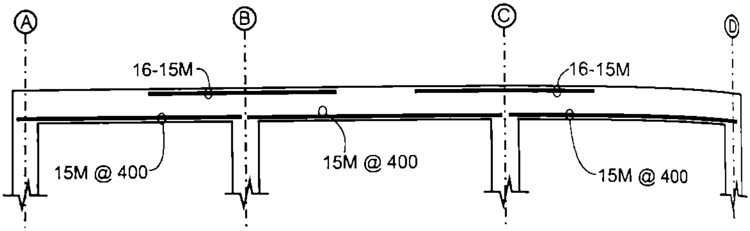
The required area and amount of reinforcement are summarized in Table 12.19. Note that the same procedure was used to design the reinforcement as in Example 12.3.

A design summary showing the reinforcement arrangement is shown on the next page.

A comparison of the bending moments obtained in this example with the values obtained in Example 12.3 indicates that the positive bending moment in the end span increased from 90 to 108 kNm (approximately 20%). However, the positive bending moment at the midspan has decreased by 4%. Note that there are no negative moments at the end supports due to the initial analysis assumption (zero column stiffness). There

Table 12.19 Factored bending moments and the flexural reinforcement

Span	AB			BC		
Location	Top	Bottom	Top	Top	Bottom	Top
M_f (kNm)	0	+108	−128	−130	+88	−130
A_s (mm²)	0	2316	2812		1873	2812
Top reinforcement			16–15M			16–15M
Bottom reinforcement (required)		15M@400			15M@500	
Bottom reinforcement (design)		15M@400			15M@400	



is a minimal increase in bending moment values at the interior columns. These observations are in line with the discussion presented earlier in this section: when column stiffness is ignored, bending moments in the slab will be higher than those obtained from the EFM analysis which takes into account column properties.

It can be seen that the amount of reinforcement is very similar to that obtained from the EFM analysis presented in Example 12.3. In both examples, the required bottom reinforcement is 15M@500. However, 15M@400 specification has been used for the design solution to achieve uniformity and avoid chances of construction errors. Therefore, the only difference between these examples is in the amount of top reinforcement: 16-15M for this example versus 14-15M for Example 12.3.

Both designs could be used in practice. This discussion is intended to illustrate a variety of available reinforcement arrangements for continuous slabs. Several solutions for top and bottom reinforcement are possible, and all of them should result in the same total factored moment M_o.

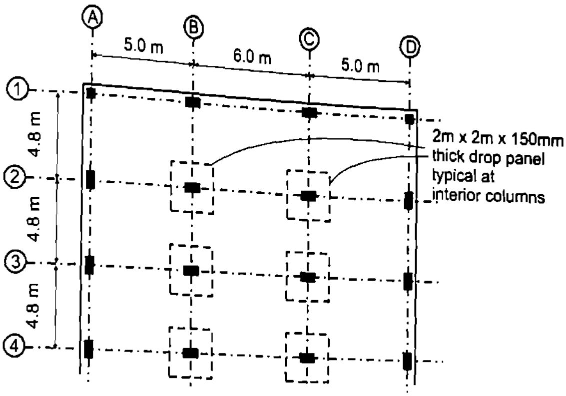
It can be concluded that the designs which take into account column stiffness result in higher negative moments at the interior supports and lower positive moments and deflections in midspan regions.

Example 12.5

Two-Way Flat Slab with Drop Panels - Equivalent Frame Method

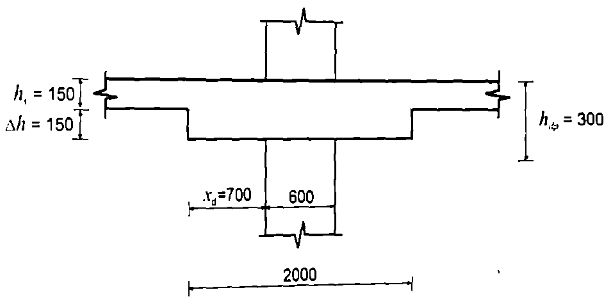
Consider the same slab as discussed in Example 12.3, but consider drop panels at the interior column locations, as shown on the following floor plan sketch. Use drop panels with square plan dimensions (2 m × 2 m), and 150 mm thickness.

Use the CSA A23.3 Equivalent Frame Method to determine the design bending moments and size and spacing of reinforcement for an interior frame along gridline 2.



SOLUTION: **1. Determine the slab thickness.**

The dimensions of a typical drop panel are shown on the sketch below.



Span BC is characterized by the largest length

$l_n = 5400$ mm

and it governs. Drop panel plan dimensions are 2 m square. Let us confirm that the CSA A23.3 Cl.13.2.4 requirement has been met

$$x_d \le \frac{l_n}{4} = \frac{5400}{4} = 1350 \text{ mm}$$

Since

$x_d = 700$ mm < 1350 mm OK

The proposed drop panel thickness is 150 mm, therefore

$\Delta_h = 150$ mm

Note that CSA A23.3 Cl.13.2.4 requires that

$\Delta_h \le h_s$

According to Cl.13.2.4, the minimum thickness for a slab with drop panels is equal to

$$h_s \ge \frac{l_n(0.6 + f_y/1000)}{30} - \left(\frac{2x_d}{l_n}\right)\Delta_h \qquad \text{(A23.3 Eq.13.2)}$$

$$= \frac{5400(0.6+400/1000)}{30} - \left(\frac{2 \cdot 700}{5400}\right) \cdot 150 = 141 \text{ mm}$$

Therefore, the slab thickness will be selected as follows

$h_s = 150$ mm

Note that the total slab thickness at drop panel locations is

$h_{dp} = h_s + \Delta_h = 150 + 150 = 300$ mm

Note that the slab thickness (150 mm) is less than the thickness used for flat plates without drop panels (180 mm) — see Example 12.3. This is a common practice for slab designs where drop panels are provided at the columns.

2. **Calculate the factored design loads.**

a) Calculate the dead load acting on the slab.
First, calculate the slab self-weight:

$$DL_w = h_s \times \gamma_w = 0.15 \text{ m} \times 24 \text{ kN/m}^3 = 3.6 \text{ kPa}$$

where $\gamma_w = 24 \text{ kN/m}^3$ is the unit weight of concrete.

Next, let us calculate the additional dead load due to drop panels. The self-weight for a 150 mm thick drop panel is as follows:

$$W_{dp} = (\Delta_h \times \gamma_w)(2 \text{ m} \times 2 \text{ m}) = (0.15 \text{ m} \times 24 \text{ kN/m}^3)(2 \text{ m} \times 2 \text{ m}) = 14.4 \text{ kN}$$

For simplicity, the load due to drop panels is distributed over a frame area. For example, the equivalent frame along the gridline 2 has 16 m length and 4.8 m width, and there are two drop panels in total. Therefore, dead load due to drop panels can be calculated as follows

$$DL_{dp} = \frac{2W_{dp}}{16 \text{ m} \times 4.8 \text{ m}} = 0.38 \text{ kPa}$$

The superimposed dead load is given, that is,

$DL_s = 1.44$ kPa

Finally, the total factored dead load is

$$w_{DL,f} = 1.25(DL_w + DL_{dp} + DL_s) = 1.25(3.6 + 0.38 + 1.44) = 6.8 \text{ kPa}$$

b) Calculate the factored live load:

$$w_{LL,f} = 1.5 \times LL_w = 1.5 \times 3.6 \text{ kPa} = 5.4 \text{ kPa}$$

c) The total factored area load is

$$w_f{}^* = w_{DL,f} + w_{LL,f} = 6.8 + 5.4 = 12.2 \text{ kPa}$$

The total specified dead load is equal to

$$w_{DL} = 3.6 + 0.38 + 1.44 = 5.42 \text{ kPa}$$

The specified live load is

$$w_{LL} = 3.6 \text{ kPa}$$

The ratio between specified live and dead load is

$$\frac{w_{LL}}{w_{DL}} = \frac{3.6 \text{ kPa}}{5.42 \text{ kPa}} = 0.66 < 0.75$$

Since the ratio is less than 0.75, pattern loading does not need to be considered, and the frame needs to be designed considering only total factored dead and live load on all spans.

d) The total factored load on the frame is

$w_f^* = 12.2$ kPa

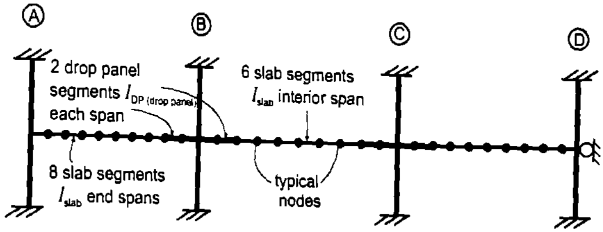
This is a uniform area load which could be used for 3-D analysis. However, we need to find the design load for 2-D frame analysis, which can be obtained when the tributary slab width ($l_2 = 4.8$ m) is taken into account, as follows

$w_f = w_f^* \times l_2 = 12.2 \text{ kPa} \times 4.8 \text{ m} = 58.6 \text{ kN/m}$

3. **Develop a frame model.**

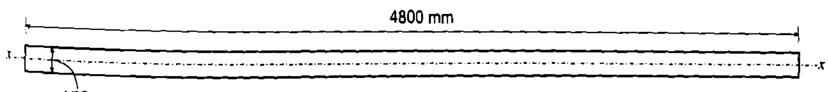
a) Determine the frame geometry.

A prismatic model is going to be used for the frame analysis. In this case, each slab span is divided into 10 segments. For example, span BC contains 4 segments modelling drop panels and 6 segments for the slab-beam. Note that different lengths can be used for slab and drop panel segments.

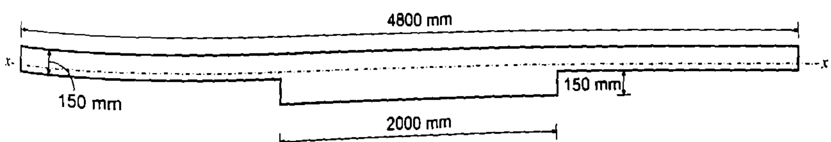


b) Determine cross-sectional properties for frame members

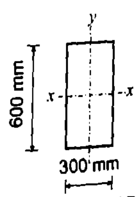
Typical cross-sections for frame members (slab-beams and columns) are illustrated on the sketch below.



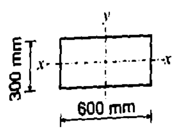
Slab-beam (I_s)



Drop panel (I_{dp})



Columns A and D (I_1)



Columns B and C (I_2)

i) Slab-beams
The slab is treated as a wide beam with the following gross cross-sectional properties: 4.8 m width and 150 mm depth. Moment of inertia for the slab about axis x-x (see the sketch) is equal to

$$I_s = \frac{4800\ \text{mm} \times (150\ \text{mm})^3}{12} = 1.35 \times 10^9\ \text{mm}^4$$

ii) Drop panels
A drop panel is modelled as a T-section. The centroid can be determined as follows:

$$y = \frac{4.8 \cdot 0.15 \cdot \frac{0.15}{2} + 2.0 \cdot 0.15 \cdot \left(0.15 + \frac{0.15}{2}\right)}{4.8 \cdot 0.15 + 2.0 \cdot 0.15} = 0.12\ \text{m} = 120\ \text{mm}$$

and the moment of inertia about axis x-x is equal to

$$I_{dp} = \frac{4.8 \cdot (0.15)^3}{12} + \frac{2.0 \cdot (0.15)^3}{12} + (4.8 \cdot 0.15)\left(\frac{0.15}{2} - 0.12\right)^2 +$$

$$(2.0 \cdot 0.15)\left(\frac{3 \cdot 0.15}{2} - 0.12\right)^2 = 6.7 \times 10^{-3}\ \text{m}^4 = 6.7 \times 10^9\ \text{mm}$$

ii) Columns
Column properties are identical to those in Example 12.3.
Columns A and D (moment of inertia I_1):

$$I_1 = \frac{600\ \text{mm} \times (300\ \text{mm})^3}{12} = 1.35 \times 10^9\ \text{mm}^4$$

Columns B and C (moment of inertia I_2):

$$I_2 = \frac{300\ \text{mm} \times (600\ \text{mm})^3}{12} = 5.4 \times 10^9\ \text{mm}^4$$

CSA A23.3 Cl.13.8.3.3 requires that a column's moment of inertia should be modified by the factor, ψ, which was determined in Example 12.3, as follows,

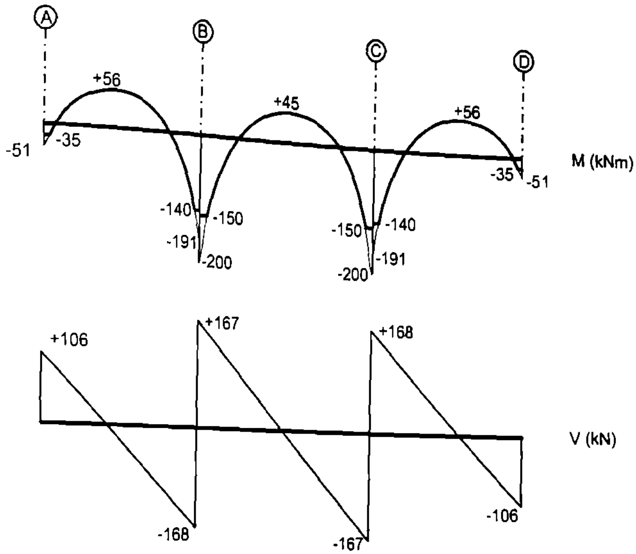
$\psi = 0.3$

Therefore, the column moment of inertia values are $0.3 \cdot I_1$ and $0.3 \cdot I_2$.

4. **Determine the factored bending moments (M_f).**
Bending moment and shear force diagrams for gridline 2 are shown on the following sketch.
The results show that the positive moments are significantly lower in both spans, however the negative moments in the slab and the columns are higher compared to Example 12.3.

5. **Design flexural reinforcement for the slab.**
From the bending moment diagrams it is possible to determine the required amount of reinforcing steel for the slab. The calculation procedure is outlined below.

a) Determine the effective depth (d).
The effective depth will be determined based on the following parameters:
Slab thickness: 150 mm
Concrete cover: 20 mm (see Table A.2)
Bar diameter: 15 mm (assume 15M bars)
The average effective depth can be estimated as follows:
$d = 150 - 20 - 15 = 115$ mm
The final effective depth for the slab (rounded down):
$d = 110$ mm



Note that the larger effective thickness needs to be considered at drop panel locations, since the overall thickness is 300 mm (slab plus drop panel). Therefore,

$d_{dp} = 300 - 20 - 15 = 265$ mm

The final rounded effective depth for drop panel locations:

$d_{dp} = 260$ mm

b) The required area of reinforcement can be found from the Direct Procedure using Eqn 5.4 as follows

$$A_s = 0.0015 f_c' b \left(d - \sqrt{d^2 - \frac{3.85 M_f}{f_c' b}} \right) \quad [5.4]$$

For this case,

$b = l_2 = 4800$ mm

c) The minimum area of reinforcement was calculated from the following equation (CSA A23.3 Cl.7.8.1)

$A_{s,min} = 0.002 A_g$

d) The spacing of bottom reinforcement is limited to the lesser of (CSA A23.3 Cl.13.10.4) (see Table 12.8)

$s \le 3h_s = 3 \times 150 = 450$ mm

or

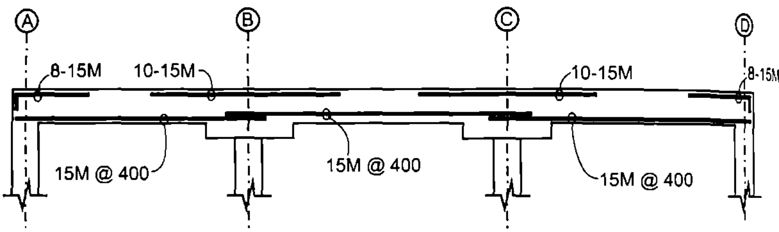
$s \le 500$ mm

In this case, $s = 450$ mm governs.

Table 12.20 Factored bending moments and the flexural reinforcement

Span	AB			BC		
Location	Top	Bottom	Top	Top	Bottom	Top
d (mm)	115	115	265	265	115	265
M_f (kNm)	−35	56	−140	−150	45	−150
A_s (mm²) [5.4]	895	1448	1578	1695	1157	1695
$A_{s,min}$ (mm²)	1440	1440	1440	1440	1440	1440
Top reinforcement (required)	5–15M		8–15M	9–15M		9–15M
Top reinforcement (design)	8–15M		10–15M	10–15M		10–15M
Bottom reinforcement (required)		15M@450			15M@450	
Bottom reinforcement (design)		15M@400			15M@400	

6. Provide a design summary.



Learning from Examples

The previous examples were useful to evaluate the effect of column stiffness (Examples 12.3 and 12.4) and drop panels (Examples 12.3 and 12.5) on the design of two-way slabs without beams according to the EFM using the prismatic modelling approach.

The effect of column stiffness is discussed below (based on Examples 12.3 and 12.4):

1. For slabs with relatively balanced spans, where interior spans are very similar and end spans are shorter than interior spans (70 to 90% of interior spans), bending moment values are usually not sensitive to column stiffness.
2. As a consequence of ignoring the column stiffness, the positive bending moment in the end span would increase. If the unbalanced moments at the interior columns are small, it may be reasonable to ignore column stiffness in the EFM analysis.
3. In general, a slab analysis where the effect of column stiffness is ignored generally leads to a design solution with larger amount of bottom reinforcement.

The effect of drop panels is discussed below (based on Examples 12.3 and 12.5):

1. There is a general decrease in reinforcing steel required in flat slab design with drop panels (Example 12.5) in comparison with flat plate design (Example 12.3). The amount of bottom reinforcement is governed by the maximum bar spacing requirements (CSA A23.3 Cl.13.10.4) - see Section 12.6.6 for more details.
2. The slab design with drop panels leads to an overall reduction in concrete volume by almost 20% compared to the flat plate design. Although some of the top reinforcement is reduced by 40%, an overall reduction in the amount of reinforcement

is approximately 10%. These potential cost savings are offset by additional labour and material required to form drop panels around each interior column. Flat plate construction is faster to form, thus it cuts down the construction time. The choice of a more efficient solution largely depends on the relative labour versus material costs for a particular geographic region. The choice of design solution (flat plate versus drop panels) is often driven by contractor's expertise, experience, and preference, as well as extra construction costs versus savings due to chances for a reduced construction schedule.

12.7.3 Three-Dimensional Elastic Analysis

A23.3 Cl.13.6

The Concept CSA A23.3 restricts applications of the DDM and 2-D EFM to regular two-way slabs which are in compliance with the requirements of Cl.2.2 (see Section 12.5.1). Neither of these methods can be used to design flat slabs with irregular shapes and non-rectangular column and/or wall grids, due to potentially significant errors associated with estimating internal forces and deflections in these structures. Irregular slab systems can be analyzed using the Finite Element Method (FEM), a numerical structural analysis method which enables a realistic determination of internal forces and deflections in complex 3-D structures. The FEM is a matrix method which idealizes the structure by modelling slabs and columns as a finite element mesh. In a typical FEM model of a building with two-way slabs, columns are modelled as linear (1-D) finite elements, while slabs are modelled as 2-D finite elements (called plate elements). Displacements within each element are expressed in terms of one or more degrees of freedom (displacements or slopes) specified at element nodal points. The element stiffness matrix is formed based on a displacement function and given stress-strain relationship for concrete and/or steel. The stiffness matrix of an entire slab is then assembled. The analysis is performed using standard matrix techniques for solving equilibrium equations. The results are in the form of displacements and internal forces (bending moments and shear forces), which can be used to proportion and detail the reinforcement. The FEM is based on the Elastic Plate Theory, and relevant provisions are outlined in CSA A23.3 Cl.13.6. Detailed coverage of the FEM is beyond the scope of this book, however the reader is referred to Zienkiewicz, Taylor and Zhu (2005); Bathe (1995); and Cook et al. (2001) for more information.

An analysis performed using the Finite Element Method will be referred to as the Finite Element Analysis (FEA) in this section. The basic terms associated with the FEA will be explained by an example of a flat plate/slab system consisting of a single panel supported by four columns at the corners and subjected to uniform load w, as shown in Figure 12.53a. For analysis purposes, the panel is subdivided into a mesh of finite elements, as shown in Figure 12.53b. Note that gridlines in the mesh are parallel with x and y axes which are set in two orthogonal directions of the slab.

Internal forces in the slab can be examined on a small rectangular element cut from the slab by planes parallel with column centrelines shown in Figure 12.54. Note that internal shear forces (N_{xy} and N_{yx}) and axial forces (P_x and P_y) in the plane of the slab are known as membrane forces (see Figure 12.54a). The effect of membrane forces is disregarded in the flexural analysis of two-way slabs. Therefore, in this discussion we are going to focus on bending moments m_x and m_y shown in Figure 12.54b. Along with bending moments, torsional moments m_{xy} and m_{yx} are also present. A plan view of the same element in the x-y plane is shown in Figure 12.54c, along with the bending and torsional moments. It is convenient to use moments in the x-y plane for the design of orthogonal reinforcement meshes. However, it may be of interest to consider the maximum bending moments in the slab, called *principal moments* (denoted as m_1 and m_2), as shown in Figure 12.54d.

FEA is suitable for computer-based applications, and it has become popular due to rapid advancements in computer hardware and software in the last few decades. FEA software packages, both general-purpose and specialized for slab analysis, are commercially available. Most of these packages are based on linear elastic analysis, however a few packages are able to simulate inelastic (or nonlinear) behaviour of reinforced concrete flat slabs in the post-cracking stage, e.g. ADAPT (2010) and SAFE (CSI).

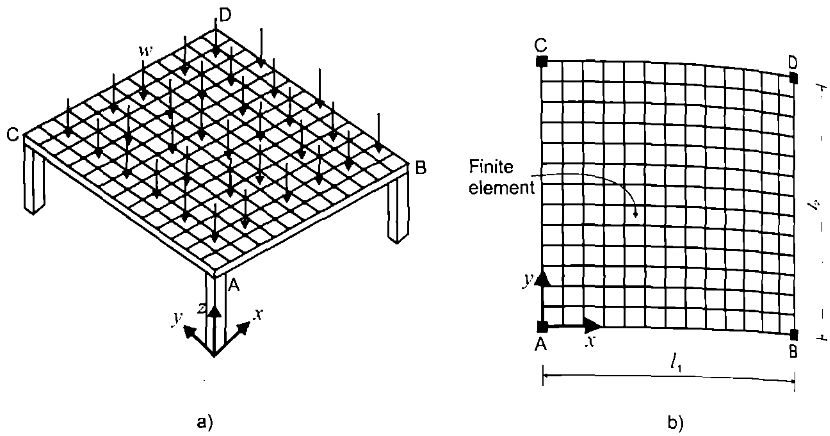
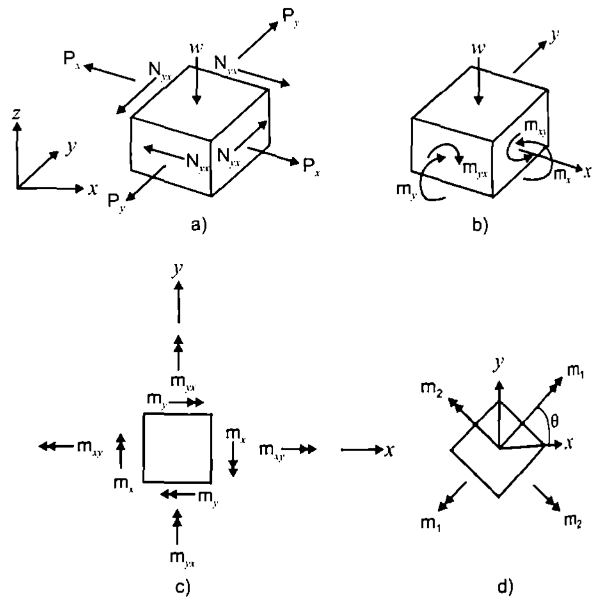


Figure 12.53 Flat plate/slab system: a) an isometric view, and b) a plan view (courtesy of Gelacio Juárez Luna).

Figure 12.54 Internal forces and moments on a typical slab element: a) internal shear and axial forces (membrane forces); b) internal bending and torsional moments - an isometric view; c) a plan view in x-y plane, and d) principal moments.



Design of two-way slabs according to the 3-D Elastic Analysis must consider the effect of bending moments and torsional effects. When reinforcement is placed as an orthogonal mat in the x- and y-direction, factored design bending moments (m_x) and (m_y) must be adjusted to account for the effect of torsional moment (m_{xy}) (CSA A23.3 Cl.13.6.4). For further details on torsional moments in two-way slabs the reader is referred to Park and Gamble (2000).

Example 12.6 illustrates an application of a 3-D FEA to a regular slab system and compares the results with the 2-D EFM.

xample 12.6

wo-Way Slab ithout Beams: D Elastic nalysis

Consider a floor plan of a two-way slab system without beams designed earlier in this chapter using both the Direct Design Method (Example 12.1) and the 2-D Equivalent Frame Method (Example 12.3). The slab thickness is 180 mm. The slab is subjected to superimposed dead load of 1.44 kPa and live load of 3.6 kPa, in addition to its self-weight.

Use 3-D Elastic Analysis (FEA) to determine the design bending moments and the reinforcement layout along gridline 2.

SOLUTION: First, the slab system is modelled as a finite element mesh shown in Figure 12.55. This example uses rectangular-shaped finite elements with the length and width approximately equal to 1 m, and their thickness is equal to the slab thickness (180 mm). Note that the mesh density, which is related to the number and size of finite elements, can affect the accuracy of numerical results for the model under consideration. For slabs with more complex geometry, a finer mesh with a larger number of smaller finite elements may improve the accuracy of calculated bending moments and shear forces in critical regions.

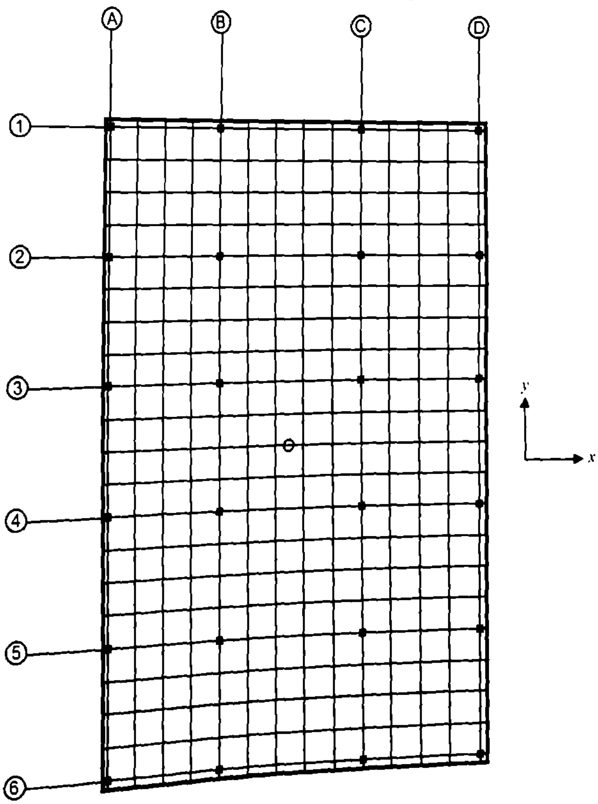


Figure 12.55 Finite element model of a two-way slab.

This FEA was carried out using a commercial software package widely used in design practice. The resulting bending moment distribution in each orthogonal direction is shown in Figure 12.56. The diagrams represent bending moment contours, which show distribution and magnitude of bending moments in the slab. Bending moment contours are indicated by different colours, and their magnitudes are indicated on the legend bar. Note that m_x denotes bending moments about the x-axis, which are used to design reinforcement in y-direction. For example, reinforcement along gridline B must be proportioned using m_x values shown in Figure 12.56a. Cumulative (positive/negative) bending moments along gridline B shown on the diagram are equal to the sum of moments across the tributary slab width between centrelines of spans AB and BC. Similarly, moments m_y about y-axis shown in Figure 12.56b are used to design reinforcement in x-direction, for example along gridline 2.

It can be seen from the diagrams that negative moments are concentrated in the column regions. Two-dimensional moment gradient profile is illustrated by a color contour diagram. It can be seen that the region of negative bending moments extends to approximately one-third of the span from the column centroid in each direction. It can also be seen that midspan regions along the column lines are characterized by very low or non-existent negative bending moment values. Note that the distribution of positive bending moments (marked by yellow-coloured contours) is uniform.

Aggregate bending moments shown in Figure 12.56 can be used to perform a comparison with other design methods, such as the 2-D EFM discussed in Section 12.7.2. Consider the moment distribution along gridline 2 shown in Figure 12.56b. The maximum aggregate negative bending moment is 181 kNm (column 2-B); this is equal to the value obtained from the EFM analysis (see Example 12.3). The maximum positive moment value for span AB is equal to 91.8 kNm (this is very close to the value of 90 kNm obtained from the EFM). Finally, the maximum aggregate positive moment for span BC is 91.1 kNm, which is very similar to the value of 92 kNm obtained from the EFM. Since both methods are based on an elastic analysis, the total bending moment values across a span width obtained from these two methods should be equal, subject only to minor deviations due to a slight difference in the models. This shows that the EFM can be considered as an acceptable design method for regular two-way slabs.

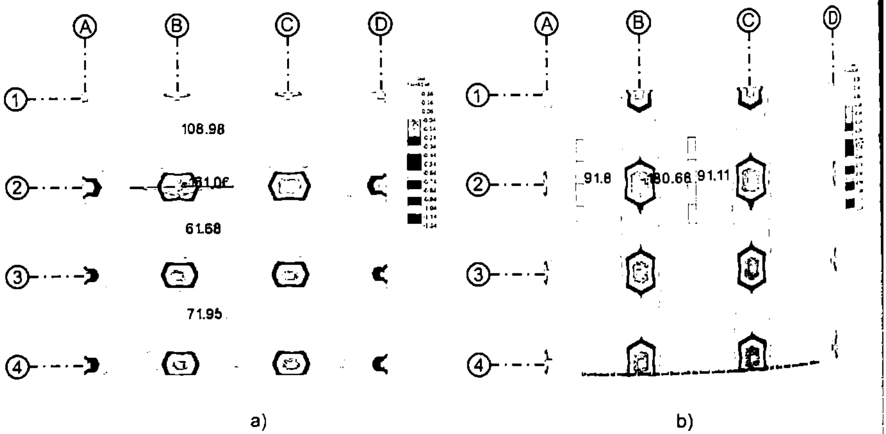


Figure 12.56 Bending moment contours for a regular slab: a) moment m_x, and b) moment m_y.

Moment contour diagrams obtained from the 3-D FEA show a continuous variation of bending moment values in horizontal plane. A reinforcement layout can be developed based on these moments. As with the EFM, the bending moments can be calculated at face of the supports (as described in Section 12.7.2). The values of design moments should be the same as that derived from the EFM. It is clear from these diagrams that the top reinforcement should be concentrated around the columns to resist negative bending moments, and that the distribution of the bottom reinforcement in midspan regions is almost uniform.

However, it is not realistic to place the reinforcing steel to match continouously variable moments across the slab supports. In practice, the designer only needs to consider the total (aggregate) negative moment over each column region in each direction, and to account for concentration of bending moments in the most critical zone within the column region. Figure 12.57 illustrates a practical solution for the top reinforcement layout based on the 3-D FEA. This type of reinforcement layout is known as "mat reinforcement". The reinforcement is concentrated in the column region, corresponding to the negative moment distribution on each side of the column, and its spacing is uniform in each direction. Reinforcement in the approximate middle half of the mat, corresponding to the critical column region, is more closely spaced, that is, rebar spacing is reduced by one-half compared to the remainder of the mat. Usually, the number of reinforcing bars is rounded up to the next even number. The aggregate moment capacity across the mat in each direction should not be less than the total moment across the negative moment region obtained by the 3-D FEA.

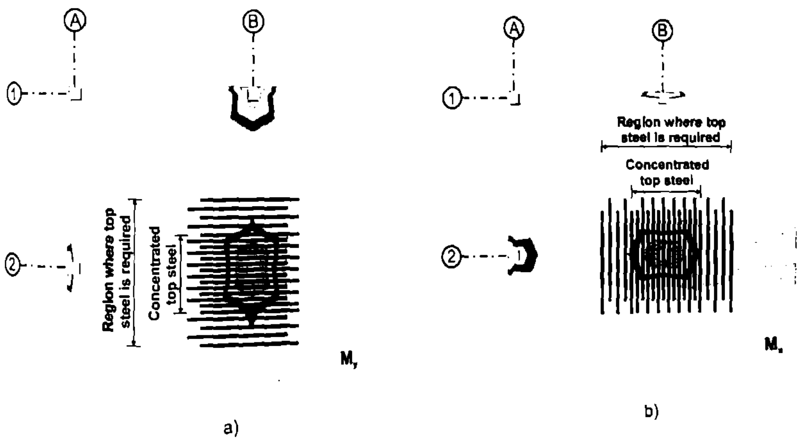


Figure 12.57 Moment and reinforcement distribution for column 2-B: a) gridline 2, and b) gridline B.

It is important to recognize that negative bending moments in midspan regions along each column line are either very low or nonexistent, thus it is acceptable to omit the top reinforcement in those regions (as shown in Figure 12.57). CSA A23.3 Cl.13.11.2.2 prescribes that a maximum 90% of the total negative bending moment at an interior column can be resisted by the column strip. To satisfy this requirement, the width of a top mat is always larger than the column strip, and a small fraction of the mat reinforcement is placed within the middle strip.

The design based on the FEA requires a provision of larger amount of top steel at the columns to suit higher negative moments, while the amount of reinforcement can be reduced or eliminated altogether in midspan regions due to low or nonexistent bending moment values. The bottom slab reinforcement can be uniformly distributed. Some designers prefer to provide additional bottom reinforcement at column centrelines to satisfy the CSA A23.3 requirement which states that 55% of the positive reinforcement should be provided within the column strip (Cl.13.11.2.2). Since the required amount of reinforcement is small, either the maximum bar spacing or the minimum area of reinforcement governs. As a result, it is possible to specify uniform spacing for the bottom reinforcement, which is the case in this example. The final reinforcement layout for this slab is shown in Figure 12.58.

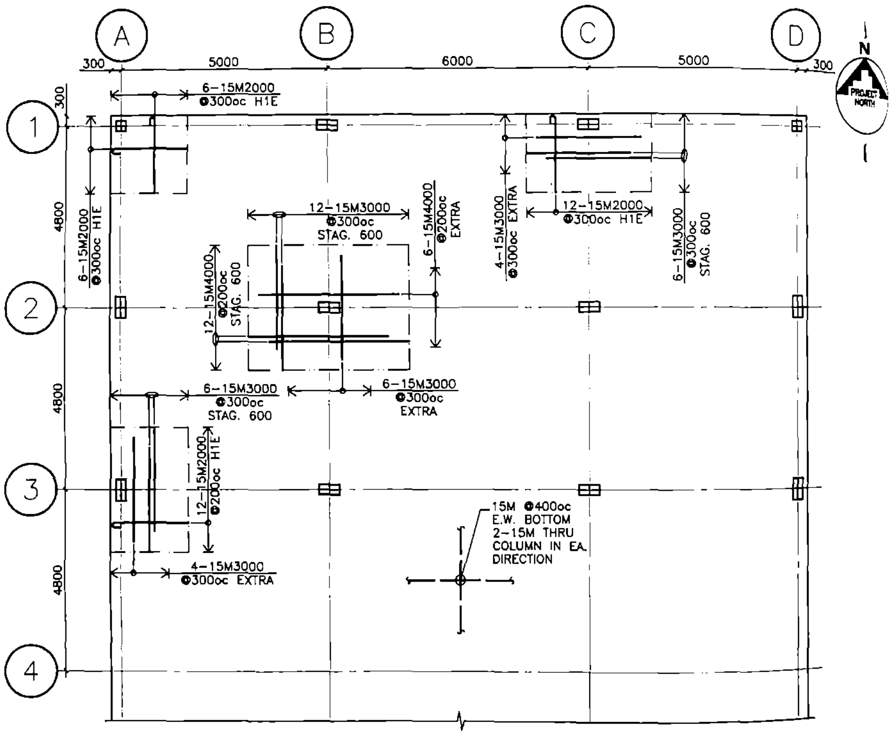


Figure 12.58 Reinforcement layout for the slab example based on the FEA.

As illustrated in Example 12.6, the total bending moments across each span obtained from the FEA should be comparable to other methods (EFM and DDM). The main difference is that a 3-D FEA model provides a moment variation in the slab in two directions, while a 2-D EFM model gives a variation in bending moment values in the plane of the frame; however, the EFM assumes that the transverse distribution of bending moments is uniform across the slab width. This assumption is not applicable to thin flat plates and could lead to significant design errors.

In conclusion, different design solutions will be obtained depending on the method of analysis used to determine bending moments. A design solution based on the 3-D FEA will likely result in the top steel concentrated only over the columns, while both the DDM and EFM will require for the top reinforcement to be provided throughout the span.

3-D Finite Element Analysis of Irregular Two-Way Slabs The behaviour of irregular two-way slabs is expected to be significantly affected by three-dimensional effects. For that reason, 3-D FEA should be used as a design tool to ensure a realistic prediction of actual moments in the slab. This analysis should produce a design solution that results in an appropriate amount and distribution of reinforcement. Application of 3-D FEA for design of irregular slabs will be discussed in this section.

Consider an office building with a complex floor plan and an irregular column grid shown in Figure 12.59. A 250 mm thick flat plate is used as the floor system. The CSA A23.3 requirements for regular two-way slab systems have not been met (Cl. 2.2). In spite of that, let us try to apply the EFM and check if it is workable for this design. First, we need to divide the slab into equivalent frames in each direction, but it is a challenging and a subjective exercise due to irregular slab shape and an irregular column and wall grid. Besides that, there is an additional column around the large opening on gridline 9 which makes the task of defining equivalent frames even more challenging.

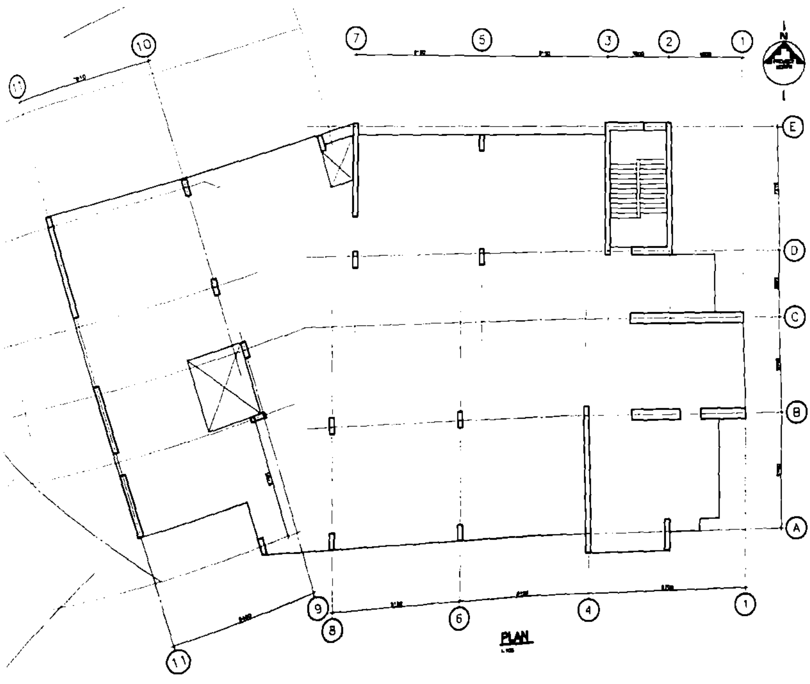


Figure 12.59 Floor plan of a building with an irregular column grid.

Figure 12.60 shows a possible plan view layout of equivalent frames. These frames are defined by the corresponding strips which are shown hatched on the figure. Note that the strip along gridline 5 appears to be the most regular. The strip along gridline 7 is non-rectangular, particularly at the north end of the slab where an opening is provided for a mechanical shaft. The strips along gridlines B and D are skewed and characterized by variable widths. The strip along gridline B is particularly complex, due to a large opening in the slab near column B-10 which cuts away a significant portion of the strip.

It is clear that a 2-D EFM analysis could lead to uncertain results in terms of the accuracy of design solution. In this example, 2-D equivalent frame models would not be able to represent a 3-D structure with sufficient accuracy, and the resulting bending moment values would not be representative of actual moments in the slab. CSA A23.3 does not permit an application of the 2-D EFM in this case. Therefore, 3-D FEA appears to be the most appropriate analysis method for irregular slabs like the one discussed in this example.

Behaviour of a complex flat plate system subjected to gravity loads can be understood on a conceptual level by studying a deflection contour diagram shown in Figure 12.61a. The contours shown on the diagram denote regions characterized by equal deflections. It can be seen that the deflections are smallest (or nonexistent) at column and wall locations, and are increasingly larger towards the midspan. Note that deflection values will depend on the span length and relative stiffness ratio between the supports and the slab.

The designer can get a sense for the deflection pattern in a complex flat plate slab by drawing an analogy with the deflection pattern of a deformed tent structure; this will be

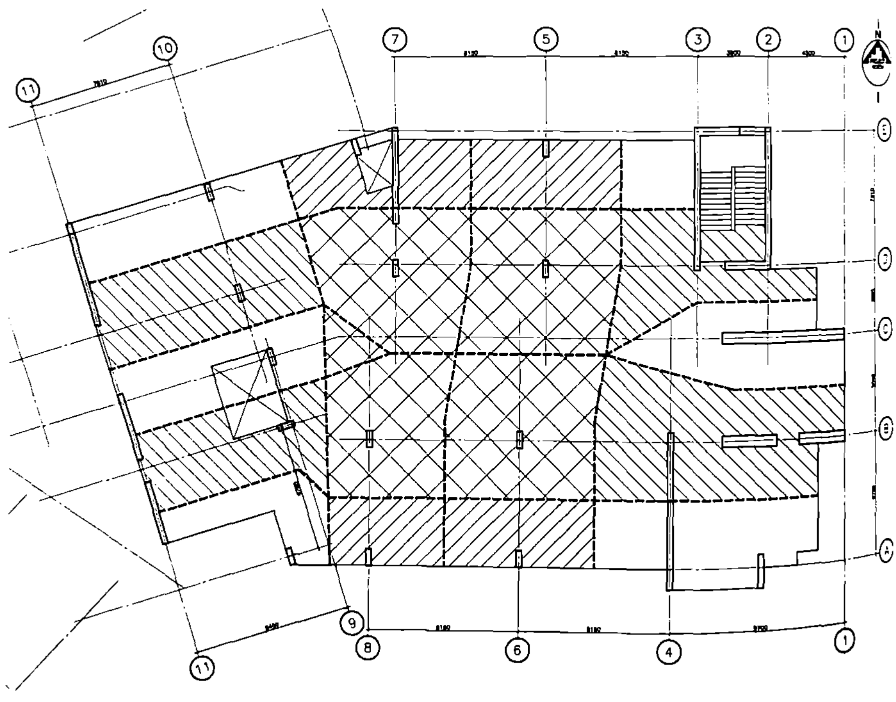
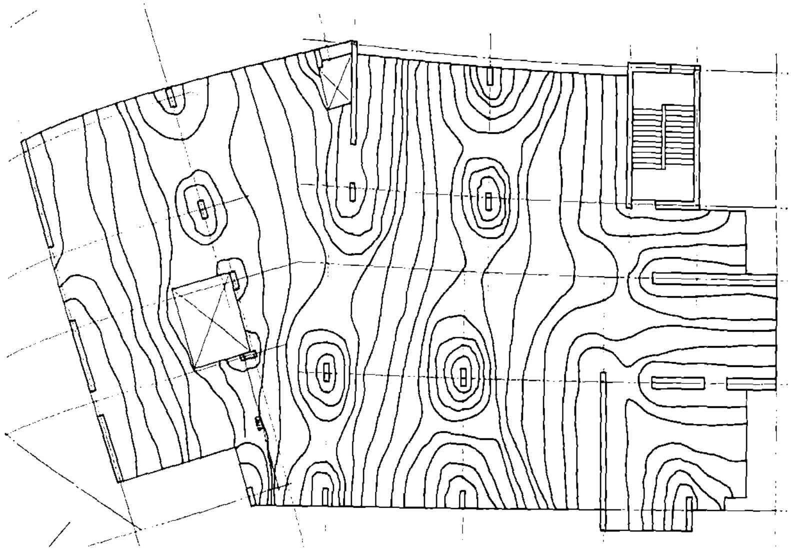
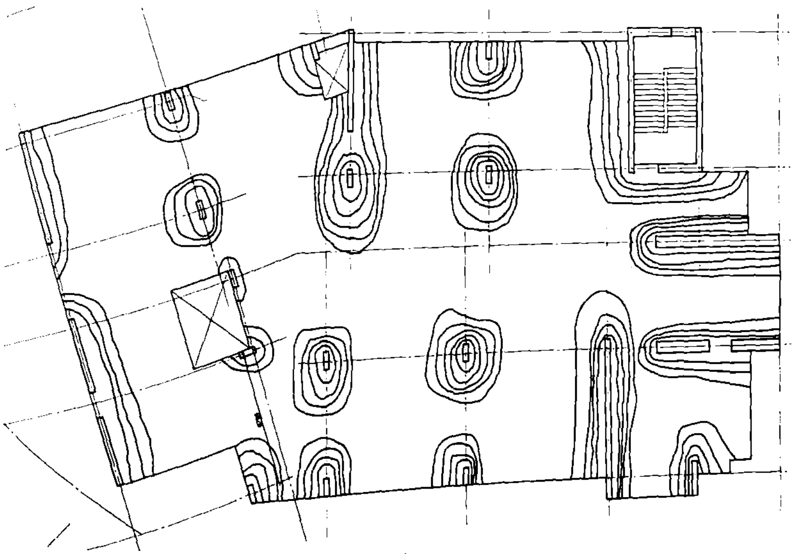


Figure 12.60 Floor plan of an irregular flat slab showing equivalent frames with dashed lines.



a)



b)

Figure 12.61 Tent analogy for a complex slab system: a) deflection contours, and b) bending moment contours.

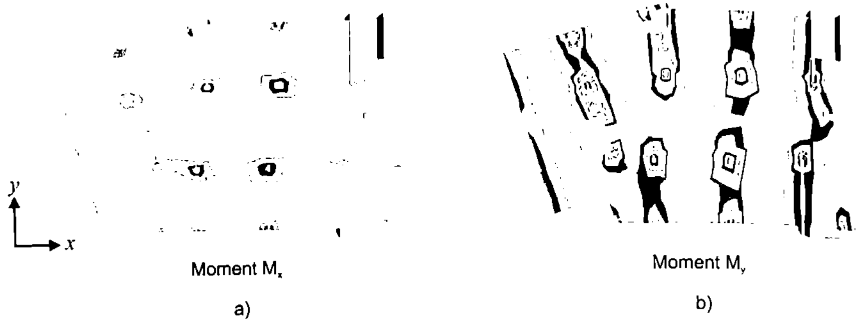


Figure 12.62 Bending moment contours for an irregular slab: a) moment m_x, and b) moment m_y.

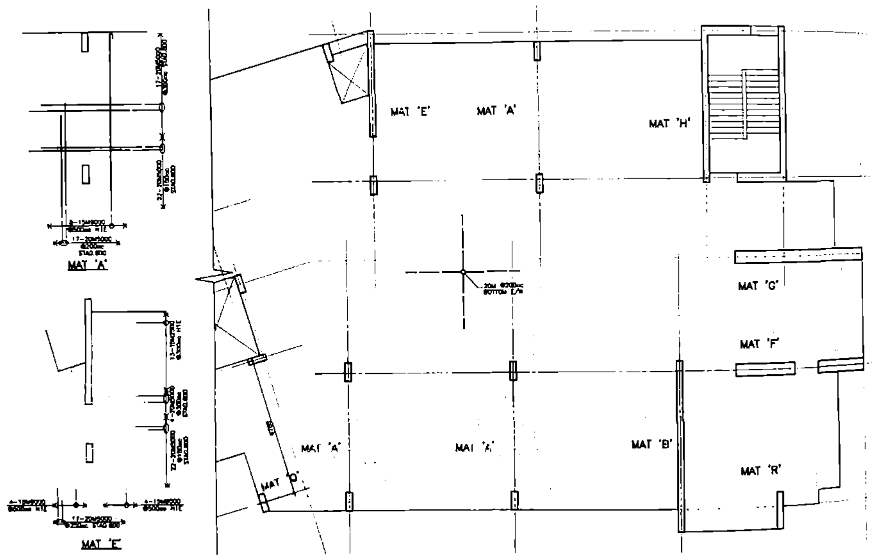


Figure 12.63 Reinforcement layout for the irregular flat slab example.

referred to as the Tent Analogy. Imagine that the floor structure is very flexible, like a large tent placed on top of the columns and walls. The tent material is resilient to tension and it is not going to tear apart under heavy load. The tent is expected to deform due to its self-weight in the most efficient shape, that is, in the form of two-way catenaries between the supports and volcano-like deflection contours around the supports. The Tent Analogy can be used to obtain qualitative deflection patterns for two-way slabs with a complex geometry.

Figure 12.61b shows contour lines of negative moments for this design. These negative moments need to be resisted by top steel at the supports, which can be placed in the form of reinforcement mats.

Bending moment diagrams obtained from the FEA are shown in Figure 12.62. Note that for short slab spans the negative moment (m_y) regions could extend to midspan in the y-direction (see Figure 12.62b). The corresponding top steel extends through adjacent spans, as seen from the reinforcement layout shown in Figure 12.63.

12.8 YIELD LINE METHOD

A23.3 Cl.13.7

12.8.1 Background

The Theorems of Plasticity include Hillerborg's strip method and the Yield Line Method (YLM). These methods are able to predict the ultimate load capacity of a slab: the YLM and the Hillerborg's strip method give an upper- and a lower-bound estimate respectively. The YLM was developed by a Danish engineer K.W. Johansen in 1943 and it will be discussed in this section.

The YLM is able to estimate a reserve in the load capacity of a two-way slab beyond the onset of flexural failure characterized by the maximum concrete compression strain of 0.0035 and yielding in the tension steel at a specific slab location, as defined by the Ultimate Limit States (ULS) design approach. In two-way slabs, the slab will not fail immediately after the initial onset of failure at the location of maximum bending moment as there are redundant load paths that would continue to support additional load. The YLM is able to determine the ultimate load-carrying capacity of a two-way slab prior to failure.

The intent of this section is to expose the reader to underlying concepts of the YLM and demonstrate its application through a design example. A detailed coverage of the YLM is beyond the scope of this book, however the reader is referred to other resources, such as Kennedy and Goodchild (2003) and Park and Gamble (2000).

12.8.2 The Concept

The YLM can more accurately predict ultimate load capacity for the two-way slabs with a ductile flexural behaviour subjected to gravity loading than the ULS design approach. According to the conventional ULS design approach, steel-controlled flexural failure at a specific location takes place at the bending moment corresponding to the concrete compression strain of 0.0035, when steel reinforcement is deformed well beyond the yield point. As a result, plastic hinges will form at such locations in reinforced concrete beams and one-way slabs - this is considered the ultimate (failure) stage. However, two-way slabs carry load in two orthogonal directions and bending moments vary in each direction. This unique characteristics allows for redundant load paths. The slab will initially carry load through the stiffest load path until yielding causes a drop in stiffness and the load is transferred to other (stiffer) load paths. Hence, a localized flexural failure that initially occurs in the regions with the largest bending moments is not considered the actual failure for a two-way slab. This is only an onset of failure, and the slab can continue to support load through other load paths. Upon further load increase, the bending moments in the yielded regions will remain constant, while other regions will support increasing bending moments until yielding takes place and plastic hinges are formed. As the loading continues to increase, cracks in adjacent plastic hinges join to form *yield lines*, and ultimately a *yield pattern* is formed by interconnecting yield lines. The slab's ultimate capacity will be reached when interconnected yield lines form the yield pattern which corresponds to the minimum slab load-carrying capacity. Fur-

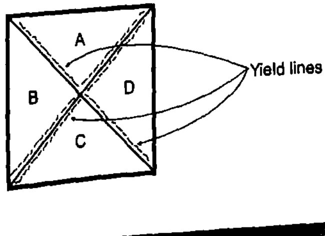


Figure 12.64 Yield pattern for a two-way slab simply supported on four sides.

ther load increase would cause the slab to collapse due to the loss of stability and failure across the yield lines. A yield pattern for a two-way slab simply supported on four sides is shown in Figure 12.64; note the yield lines that separate regions A, B, C, and D.

A yield pattern can be understood as a failure scenario for a particular slab. In general, there may be several yield patterns for a particular configuration of slab and loading, but the pattern that gives the least load at the ultimate stage (failure) governs and it is known as the *yield line solution*.

The ultimate slab capacity is determined using the Virtual Work Method. This method is based on the underlying principle that the work done externally and internally must balance. At failure, the energy exerted by external loads is equal to the internal energy dissipated by rotations about the yield lines. The method states that the internal work (IW) and the external work (EW) are equal, as follows

$$IW = EW \qquad [12.13]$$

Internal work (IW) is the work dissipated by internal moments on rotations along the yield lines, that is,

$$IW = \Sigma\,(m \cdot l \cdot \theta) \qquad [12.14]$$

while external work (EW) is induced by applied external loads on the slab, as follows

$$EW = \Sigma\,(P \cdot \delta) \qquad [12.15]$$

where

P = load(s) acting within a particular region
δ = the vertical displacement of the load(s) P on each region expressed as a fraction of unity
m = internal moment per unit length of yield line
l = length of the yield line
θ = the rotation of a region about its axis of rotation

It is important to note that the summation sign in the above equations denotes that both external and internal work are calculated for all regions of the slab under consideration.

The Virtual Work Method is illustrated by an example of a slab panel subjected to uniformly distributed load.

Figure 12.65a shows the slab which has developed a mechanism where the yielding of reinforcement and plastic rotations have occurred along the yield lines both at the supports and at the midspan.

Figure 12.65b shows a slab model which will be used for the design according to the YLM. Note that P_1 and P_2 are resultants of the uniform load w used for the external

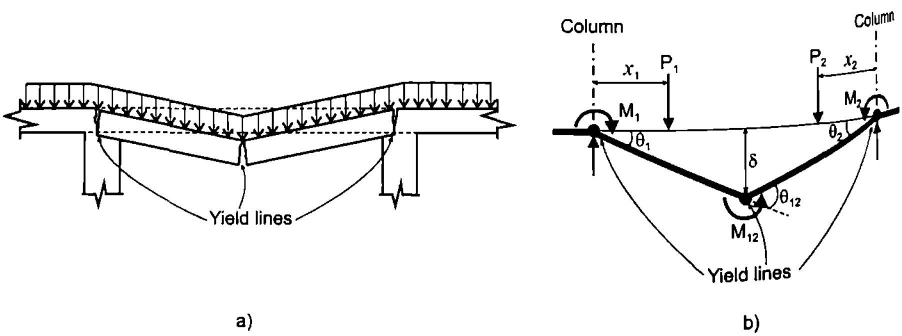


Figure 12.65 Plastic mechanism in a two-way slab: a) actual slab span, showing locations where the reinforcement has yielded and the plastic rotations have occurred, and b) a slab model.

work calculations, and x_1 and x_2 denote the distances of these resultants from adjacent supports.

It should be noted that the governing yield pattern for a slab under consideration requires the least amount of internal work prior to failure. The design according to the YLM consists of identifying one or more valid yield patterns and performing calculations to determine the governing yield pattern. An application of the YLM will be illustrated by the following example.

Example 12.7

Two-Way Flat Plate - Yield Line Method

Consider a floor plan of a flat plate slab system designed in Example 12.3. The factored design load (w_f) is 12.6 kPa.

Determine the ultimate load capacity for the slab by the YLM: a) for slab strip between gridlines 1 and 2, and b) for slab strip between gridlines 2 and 3.

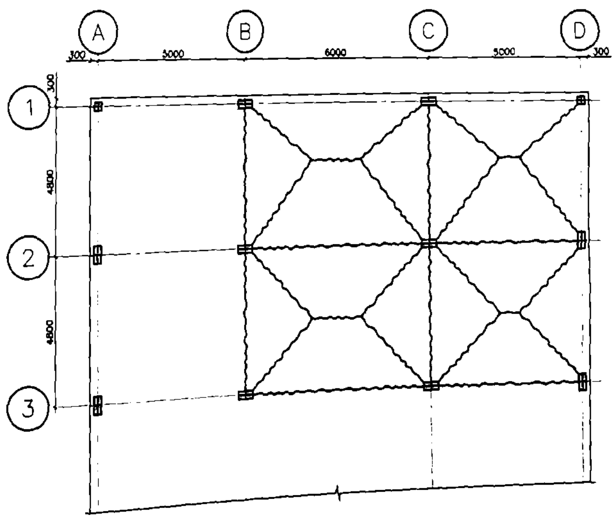
SOLUTION:

1. **Determine possible yield patterns.**

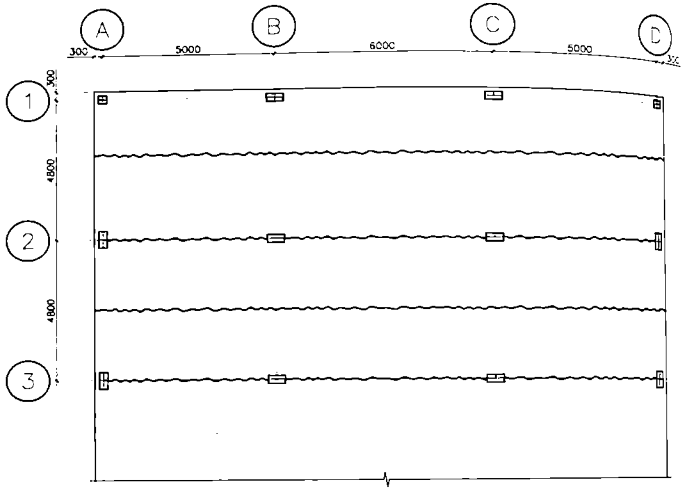
Several yield patterns are postulated to form and should be considered for design. Note that the positive bending yield lines at the end spans are not expected to form exactly at midspan due to lack of negative moment capacity at the slab edges. These yield lines tend to form closer towards the slab edge, away from the midspan.

Let us consider the following three yield patterns:

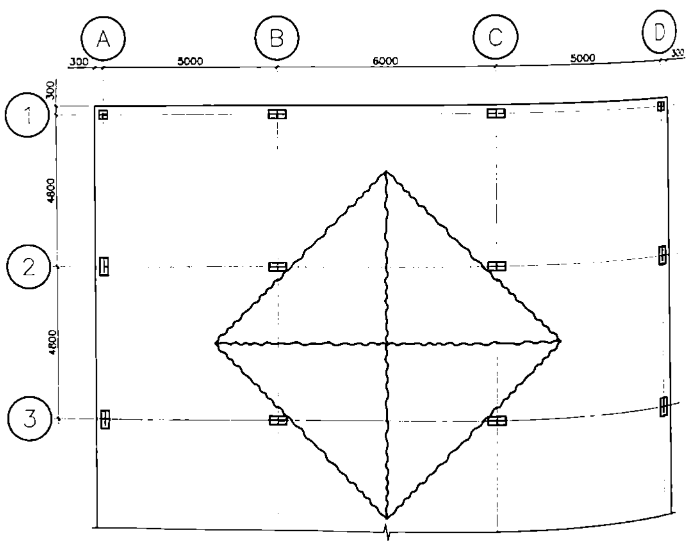
Yield Pattern 1 (YP1) – Positive bending yield lines are formed within each span, while negative bending yield lines are formed along interior gridlines; note that four patterns are shown on the diagram below, and they need to be checked individually.



Yield Pattern 2 (YP2) – Parallel straight yield lines are formed in both the midspan regions and along the gridlines, as shown on the diagram below; note that similar patterns could be identified in the perpendicular direction (two patterns are shown on the diagram, and each needs to be checked individually).



Yield Pattern 3 (YP3) – The largest rectangular yield pattern is formed within a set of interior columns, as shown on the diagram below.



The yield pattern that requires the largest moment resistance along the yield lines governs. To assess the reserve capacity for the previously designed slab areas, we are going to introduce the ratio of the Available Internal Work (*IWA*) to the applied External Work (*IWA/EW*). This ratio is greater than 1.0 for yield patterns that contain more reinforcement than required to satisfy the equal work principle.

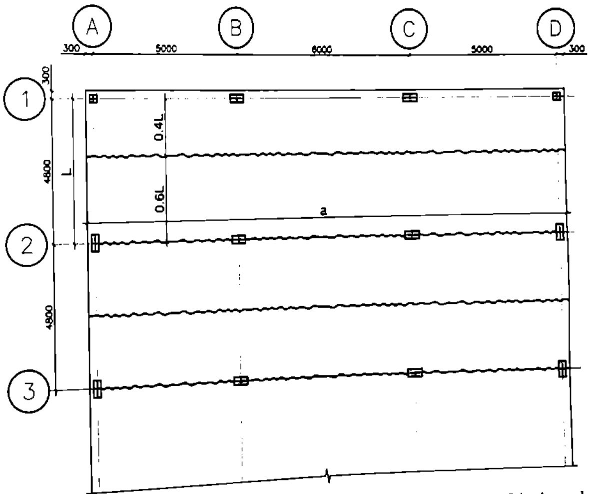
In general, all possible yield patterns should be considered to determine the *IWA/EW* ratio. For this example, calculations revealed that the governing yield pattern is pattern YP2 discussed above. Although this example shows only yield patterns for slab strips between gridlines 1 and 2 and 2 and 3, similar yield patterns in the perpendicular direction also need to be considered.

Based on the above discussion, we are going to proceed using the pattern YP2. In many cases it is not obvious which yield pattern produces the least *IWA/EW* ratio, hence this process may need to be repeated for each possible yield pattern.

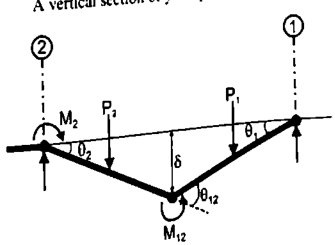
2. **Find the ultimate load capacity for the slab strip between gridlines 1 and 2.**

   a) Determine relevant dimensions for the yield pattern.

   For the end span 1-2, it is assumed that the positive steel yield line occurs at 0.4L from the slab edge (gridline 1), as shown on the diagram below. However, it can be shown that if the yield line is taken at midspan, the error is only about 3%, which is insignificant.



A vertical section of yield pattern (YP2) between gridlines 1 and 2 is shown below.



b) Compute the External Work (EW).

Let us first calculate rotations (θ) along the yield lines, as shown on the sketch above. Note that the displacement (δ) is assigned a unit value for ease of calculation. The rotations can be found from the above sketch, as follows:

$$\theta_1 = \frac{\delta}{0.4L} = \frac{1}{0.4L}$$

$$\theta_2 = \frac{\delta}{0.6L} = \frac{1}{0.6L}$$

$$\theta_{12} = \theta_1 + \theta_2 = \frac{1}{0.24L}$$

The design load is equal to

$w_f = 12.6$ kPa

The load resultants for regions 1 and 2 can be determined as a product of the slab area and load (w_f), that is,

$$P_l = (0.4L \cdot a) \cdot w_f$$

$$P_l = (0.6L \cdot a) \cdot w_f$$

Finally, the external work can be calculated as follows:

$$EW = \Sigma(P \cdot \delta) \qquad [12.15]$$

$$= P_l \frac{\delta}{2} + P_2 \frac{\delta}{2}$$

$$= \frac{1}{2}(0.4 + 0.6)L \cdot a \cdot w_f = \frac{L \cdot a \cdot w_f}{2}$$

$$= \frac{4.8\text{ m} \cdot (16\text{ m}) \cdot (12.6\text{ kPa})}{2} = 484\text{ kNm}$$

c) Compute the Available Internal Work (IWA).

In order to calculate the IWA, it is required to find the moment resistance along the yield lines.

i) Find the moment M_2.

The section considered for this calculation uses the total slab width, that is,

b = 16 m

and the effective depth based on average depth of two 15M bar layers, as follows

d = 180 mm − 20 mm − 15 mm = 145 mm

The reinforcement design (omitted from this example) requires four mats across the entire slab width between gridlines A and D: two interior mats with 18-15M bars, and two edge mats with 10-15M bars. In total, the top reinforcement consists of 56-15M bars. Hence,

$$A_s = 56 \times 200\text{ mm}^2 = 11200\text{ mm}^2$$

The factored moment resistance for a rectangular slab section will be determined from the procedure presented in Section 3.5, as follows

$$T_r = \phi_s A_s F_y = 0.85(11200\text{ mm}^2)(400\text{ MPa}) = 3808\text{ kN}$$

$$C_r = \phi_c \cdot f'_c \cdot a \cdot b = 0.65(30\text{ MPa})\,(16000\text{ mm})\,(a)$$

From the equation of equilibrium

$$T_r = C_r$$

it can be found that

$a = 12.22$ mm

thus

$$M_2 = T_r\left(d - \frac{a}{2}\right) = 3808\,\text{kN}\left(145\,\text{mm} - \frac{12.2\,\text{mm}}{2}\right) = 529\,\text{kNm}$$

ii) Find the moment M_{12}.

This moment is calculated considering the bottom reinforcement 15M@400 mm across 16 m slab length. Hence,

$$A_s = \frac{16000\,\text{mm}}{400\,\text{mm}} \times 200\,\text{mm}^2 = 8000\,\text{mm}^2$$

$T_r = \phi_s A_s F_y = 0.85\ (8000\ \text{mm}^2)(400\ \text{MPa}) = 2720\ \text{kN}$

$C_r = \varphi_c \cdot f_c' \cdot a \cdot b = 0.65\ (30\ \text{MPa})\ (16000\ \text{mm})\ (a)$

thus

$a = 9$ mm

and

$$M_{12} = 2720\,\text{kN}\left(145\,\text{mm} - \frac{9\,\text{mm}}{2}\right) = 382\,\text{kNm}$$

iii) Finally, available internal work (IWA) at the supports and the midspan can be calculated as follows

$$IWA = IW = \Sigma(m \cdot l \cdot \theta) \qquad [12.14]$$

$$= M_1\theta_1 + M_2\theta_2 + M_{12}\theta_{12}$$

where

$M_1 = 0$ at the slab edge

and

$l = 1$ (assume unit length for the yield line)

thus

$$IWA = M_2\theta_2 + M_{12}\theta_{12}$$

$$= M_2\left(\frac{1}{0.6L}\right) + M_{12}\left(\frac{1}{0.24L}\right)$$

$$= \frac{529\,\text{kNm}}{0.6(4.8\,\text{m})} + \frac{382\,\text{kNm}}{0.24(4.8\,\text{m})} = 515\,\text{kNm}$$

d) Find the IWA/EW ratio.

$$\frac{IWA}{EW} = \frac{515\ \text{kNm}}{484\ \text{kNm}} = 1.06$$

Therefore, for end span between gridlines 1 & 2, there is a 6% reserve capacity assuming that there is a zero negative moment capacity at the four edge columns. This reserve capacity translates to the actual ultimate load capacity, as follows

$w_u = 1.06 \cdot w_f = 1.06 \cdot 12.6 = 13.4$ kPa

3. Find the ultimate load capacity for the slab strip between gridlines 2 and 3.

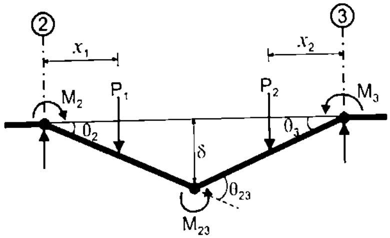
a) Find the relevant dimensions for the yield pattern.

The yield pattern is the same as above (YP2), and a vertical section is shown on next page.

b) Compute the External Work (EW).

In this case, the maximum deflection δ occurs at the midspan, hence

$$\theta_2 = \theta_3 = \frac{\delta}{0.5L} = \frac{1}{0.5L}$$



and

$$\theta_{23} = \theta_2 + \theta_3 = \frac{1}{0.25L}$$

The load resultants can be determined in the same manner as before, that is,

$$P_1 = P_2 = (0.5L \cdot a) \cdot w_f$$

$$EW = \Sigma(P \cdot \delta) \qquad [12.15]$$

$$= P_1 \frac{\delta}{2} + P_2 \frac{\delta}{2} = \frac{1}{2}(0.5 + 0.5)L \cdot a \cdot w_f = 484 \text{ kNm}$$

c) Compute the Available Internal Work (IWA).
The same top and bottom reinforcement is used here as for the previous slab section, hence

$$M_2 = M_3 = 529 \text{ kNm}$$

$$M_{23} = M_{12} = 382 \text{ kNm}$$

and

$l = 1$ (assume unit length for the yield line)

$$IWA = IW = \Sigma(m \cdot l \cdot \theta) \qquad [12.14]$$

$$= M_2\theta_2 + M_3\theta_3 + M_{23}\theta_{23} = 2\,M_2\theta_2 + M_{23}\theta_{23}$$

$$= 2 \cdot M_2\left(\frac{1}{0.5L}\right) + M_{12}\left(\frac{1}{0.25L}\right)$$

$$= \frac{2(529)}{0.5(4.8)} + \frac{382}{0.25(4.8)} = 759 \text{ kNm}$$

d) Find the *IWA/EW* ratio.

$$\frac{IWA}{EW} = \frac{759 \text{ kNm}}{484 \text{ kNm}} = 1.57$$

The ultimate load capacity is equal to

$$w_u = 1.57 \cdot w_f = 1.57 \cdot 12.6 = 19.8 \text{ kPa}$$

The above calculation shows a significant reserve in the load capacity of 57% for interior span 2-3. This is due to the flexural capacity provided by the top steel at both support lines, compared to only one support line for end span 1-2, thereby resulting in a significantly smaller reserve capacity of 6%. This example illustrates that interior spans in two-way slabs designed in accordance with an elastic analysis procedure have a significant reserve in the load capacity.

12.8.3 Practical Design According to the Yield Line Method

Although the YLM is most commonly applied for evaluating the ultimate load capacity of existing slabs, it is also becoming more popular as a primary tool for design of new slabs. Some of the advantages of the YLM compared to elastic analysis methods are summarized below:

1. The YLM offers a more economical design solution, since it takes into consideration a reserve capacity beyond the onset of flexural failure as defined by conventional methods.
2. The reinforcement solution is simpler, and the placement of steel tends to be more uniform and regularly arranged.
3. The YLM is a very appropriate design tool for slabs with complex configurations, and it gives the designer a better understanding of the overall structural capacity and failure mechanisms.

The YLM can be used for design of both regular and irregular slabs. The design is usually performed by hand calculations and the designer is required to derive a solution from the basic principles.

In general, application of the YLM is a trial and error approach. The designer usually needs to carry out a few iterations using different yield patterns to identify the one which results in the lowest load capacity for a particular slab panel. The complexity of yield patterns may initially present a challenge, and novice designers may not be confident in the resulting design solution. However, experienced designers may be able to intuitively eliminate the yield patterns that do not govern and consider only a few patterns that are likely to govern.

12.9 DESIGN FOR SHEAR

12.9.1 Background

Shear stresses in two-way slabs occur due to gravity loads and bending moments. These bending moments are caused by gravity loads (unbalanced moments) and/or lateral loads, and need to be transferred from the slab to the columns through slab-column connections. Slab areas in the vicinity of a slab-column connection are subjected to the highest shear stresses. Shear failure in two-way slabs is sudden, and it must be carefully considered in the design to avoid potentially catastrophic consequences of shear failure, particularly in flat slabs and flat plates. The slab shear resistance in two-way slabs with beams is usually not critical, but the beams must still be designed for shear. This section builds on the background knowledge from Chapter 6 related to the shear design of beams and one-way slabs, and it also provides new information specific to the shear design of two-way slabs.

The main objective of shear design in two-way slabs is to check the concrete shear resistance for one-way and two-way shear. When the factored shear stress exceeds the concrete shear resistance, it is required to either provide shear reinforcement or modify the design (e.g. provide drop panels).

12.9.2 Shear Design for Two-Way Slabs without Beams

A23.3 Cl.13.3

CSA A23.3 Shear Design Requirements The main CSA A23.3 shear design requirement for two-way slabs is the strength requirement. The maximum factored shear stress, v_f, should not exceed the factored shear stress resistance, v_r (Cl. 13.3.1), as follows:

$$v_f \le v_r \qquad [12.16]$$

and

$$v_r = v_c + v_s$$

where

v_c is the factored concrete shear stress, and

DID YOU KNOW?

A few cast-in-place reinforced concrete buildings with flat plate floor systems collapsed during construction in the last 50 years. The causes of failure were usually complex and could be attributed to several factors, but it appears that these collapses were primarily triggered by punching shear failures of flat plate floor slabs. For example, a 16-storey reinforced concrete building at 2000 Commonwealth Avenue in Boston, USA collapsed during its construction in January of 1971, killing 4 and injuring 30 construction workers (King and Delatte, 2004). Floor and roof systems consisted of cast-in-place flat plates supported by columns, and the slab thickness ranged from 190 to 230 mm. First, there was a punching shear failure in the roof slab at one of the column locations. The workers felt a drop in the roof slab of about 100 mm within a few seconds, which was followed by the complete collapse of the slab. This was followed by the total progressive collapse of the east side of the building, which left the floor plates stacked in the basement of the structure (see Figure 12.66). The most significant construction deficiencies were the lack of shoring under the roof slab, and low concrete strength. At the time of collapse, the concrete strength was reported to be in the range from 11 to 13 MPa (compared to 20 MPa design strength). No testing was done to confirm the strength before the shoring was removed. Also, actual loads on the roof were approximately 6.2 kPa, while structural plans specified allowable construction loading of only 1.4 kPa (note that increased loads were due to construction equipment and boilers stored on the roof). One of the main lessons from the collapse is that redundancy within structural design is essential to prevent progressive collapse. Slabs in the collapsed building did not have shear reinforcement or continuous integrity reinforcement through the columns, which is prescribed by CSA A23.3 and other codes to mitigate punching shear failure and prevent progressive collapse in these systems.

Figure 12.66 A view of the collapsed building with flat plate floor system in Boston, USA in 1971.

(from Boston Globe, January 26, 1971; republished with permission, courtesy of Getting Images/Boston Globe).

v_s is the factored shear stress in shear reinforcement (design of shear reinforcements is discussed in Section 12.9.4).

Note that the strength requirement is presented in the stress form, that is, it is required to compare stresses rather than forces. For example, the factored shear stress resistance, v_r, is expressed in stress units (e.g. MPa) while the factored shear force resistance, V_r, is expressed in force units (e.g. kN). The stress approach is suitable for checking the two-way shear resistance, whereas the force approach can be used to check the one-way shear resistance: this is similar to the shear checks for beams and one-way slabs discussed in Chapter 6. The shear resistances, v_r and V_r, will be determined from CSA A23.3 equations and discussed later in this section.

According to A23.3 Cl.13.3.2, two different shear mechanisms must be considered in two-way slabs without beams:

- One-way shear (Cl.13.3.6) and
- Two-way shear (Cl.13.3.3 to 13.3.5).

A23.3 Cl.13.3.6

One-Way Shear (Beam Shear) The one-way shear resistance for two-way slabs (often referred to as beam shear resistance) is determined in the same manner as for beams, one-way slabs, and footings specified in CSA A23.3 Cl.11.1 to 11.3 (see Section 6.5.4). Note that the one-way shear resistance usually does not govern in the design of flat plates or flat slabs, however, it should still be checked.

When the slab has sufficient thickness, shear reinforcement is not required (Cl.11.2.8.1), that is,

$$V_f \leq V_c$$

The factored shear force, V_f, is determined by considering the slab spanning as a wide beam, with the width corresponding to design strip b_w and span l_n, as shown in Figure 12.67a. The critical section is located at a distance d_v from the column face, as shown in Figure 12.67b. The design shear envelope is shown in Figure 12.67c. Note that both V_f and V_c can be determined based on a unit slab width (equal to 1 m), instead of the total design strip width, b_w. The unit shear forces are denoted as V_f' and V_c', and the final conclusion should be the

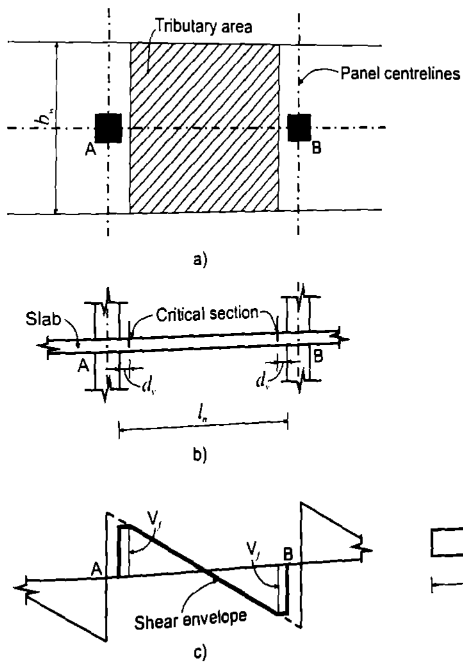


Figure 12.67 One-way shear design of two-way slabs: a) a plan view showing the tributary area; b) slab elevation showing the critical section; c) shear force envelope, and d) a typical slab cross-section for one-way shear design.

same, irrespective of whether the forces are calculated based on the unit width or the entire design strip width.

In two-way slabs with beams, the one-way shear resistance is provided by the beams (see Section 12.9.5 for more details).

The factored concrete shear resistance, V_c, can be determined from the following equation

A23.3 Eq. 11.6

$$V_c = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v \qquad [6.12]$$

where $\beta = 0.21$ (for slabs where $h_s \leq 350$ mm, A23.3 Cl.11.3.6.2)

A typical cross-section is shown in Figure 12.67d. Note that b_w is the width of the design strip, and d_v is the effective shear depth taken as the greater $0.9d$ of and $0.72h_s$.

When $h_s > 350$ mm, it is required to take into account the effect of depth on shear strength, thus β should be determined from A23.3 Eq.11.9 (refer to Table 6.1).

For corner columns, V_c is determined from Eqn 6.12 considering

$$b_w = b_o$$

where b_o denotes perimeter of the critical section. It can be seen from Figure 12.68 that the critical section for a corner column may be extended into the slab overhang (cantilevered portion beyond the column) for a length not exceeding d (Cl.13.3.6.2).

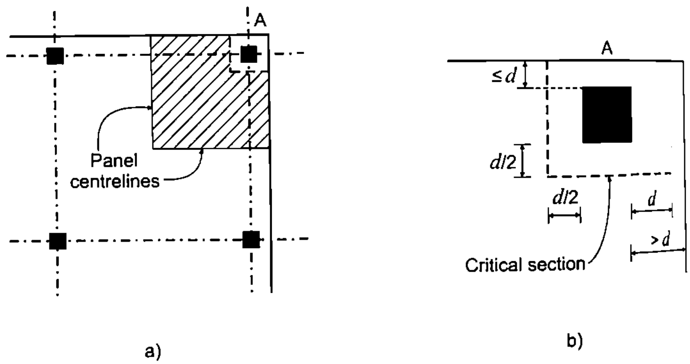


Figure 12.68 Tributary area for one-way shear design of corner columns.

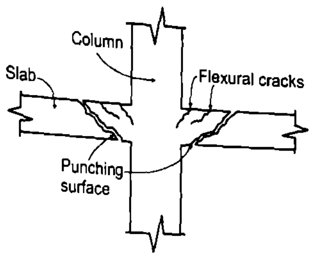
Two-Way Shear (Punching Shear)

A23.3 Cl.13.3.3 to 13.3.5

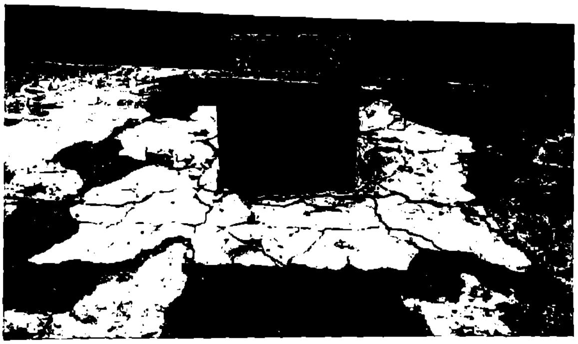
The mechanism

A two-way shear (punching shear) mechanism results in failure along the surface of a truncated pyramid (or a truncated cone) around the column. This failure mechanism takes place due to excessive gravity loads transmitted from the slab into the supporting columns. The mechanism develops when shear stresses on the area in the vicinity of column perimeter exceed the concrete shear strength.

The punching shear mechanism is illustrated in Figure 12.69a. Initially, circular and radial cracks develop on the top slab surface — these cracks are due to negative bending moments; the slab area in the vicinity of the column is subjected to significant bending moments which cause flexural stresses, as discussed earlier in this chapter. With a further load increase, diagonal tension cracks develop near the mid-depth of the slab and later propagate to the surface. It should be noted that these cracks first form at about one-half of the load corresponding to the punching shear failure, at a distance of approximately one-half of the igure 12.69 Punching shear ilure: a) a vertical slab section nowing the cracking pattern adapted from Ghali and Hammill, 992 with the permission of the merican Concrete Institute), and) an isometric view of the slab-olumn connection showing cracks t the top of the slab observed in n experimental study (courtesy of in-Yuan Cheng, Gustavo Parra-ontesinos, and Carol K. Shield).



a)



b)

slab depth from the column perimeter. A cracking pattern on the top slab surface characteristic of the punching shear mechanism is shown on an experimental specimen in Figure 12.69b.

The two-way shear mechanism is illustrated in Figure 12.70. Consider a two-way flat plate subjected to uniformly distributed gravity load, w. The slab tends to move uniformly downward due to the load, while the column (or other type of support) resists this movement. Shear (diagonal tension) stresses along the inclined planes are shown on slab element ABC in Figure 12.70a. Tensile stress, f_t, acts perpendicular to the inclined surface AC, while the shear stress, v, acts parallel with surfaces AB and BC. The cracking takes place when tensile stress in the slab reaches the concrete tensile resistance. The failed shape is in the form of a truncated pyramid, as shown on the isometric diagram in Figure 12.70a. For design purposes, CSA A23.3 permits the use of a simplified failure shape, geometrically similar to the column, as shown in Figure 12.70b. The critical section is located at a distance $d/2$ from the face of the column (note that d denotes the effective slab depth). The stress distribution is shown on slab element ABCD. Shear stress, v, acts downward, and it is transferred across the design shear surface with area $b_o \cdot d$, where b_o is the perimeter of the critical section (see isometric diagram in Figure 12.70b).

A23.3 Cl.13.3.3

The critical section for two-way shear

The critical section for two-way shear at an interior column is defined by four vertical sides, as shown in Figure 12.70b. The section should be taken at distance $d/2$ from the perimeter of the concentrated load or column (Cl. 13.3.3.1), as shown on an example of a flat plate in Figure 12.71a. The critical section is perpendicular to the plane of the slab and located so that its perimeter b_o is at a minimum. Note that the critical section is shown with dashed

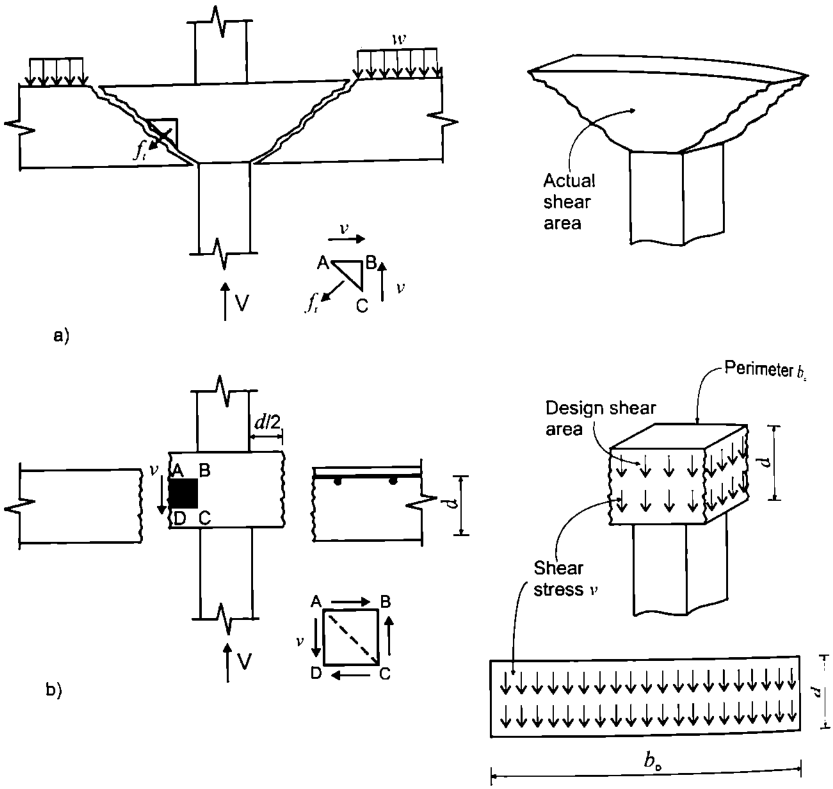


Figure 12.70 Two-way punching shear failure mechanism: a) actual failure surface (truncated pyramid), and b) a simplified failure surface model consisting of the vertical sides.

lines, and the tributary slab area for the factored shear force calculation is shown hatched on the drawing.

When a slab has variable thickness (for example, a flat slab with drop panels), shear failure can occur either through the thickened portion of the slab near the column, or through the slab portion outside the drop panel. As a result, there are two critical sections (Cl.13.3.3.2), as illustrated on an example shown in Figure 12.71b. Critical section 1 is located within the thicker portion (drop panel), at a distance $d_1/2$ from the face of the column (as shown with dashed lines in the figure). Critical section 2 is located in the slab at a distance $d_2/2$ from the end of drop panel (as shown with dashed and dotted lines in the figure); note that d_1 and d_2 denote the effective depth of thickened slab with drop panel and slab without drop panel, respectively. Figure 12.71c shows the notation related to the critical section shown in Figure 12.71a. Note that c_1 is column dimension in the direction of the span for which moments are determined, and c_2 is column dimension in the direction perpendicular to c_1. The perimeter of the critical section, b_o, can be determined as follows

$$b_o = 2(c_1 + d) + 2(c_2 + d) \qquad [12.17]$$

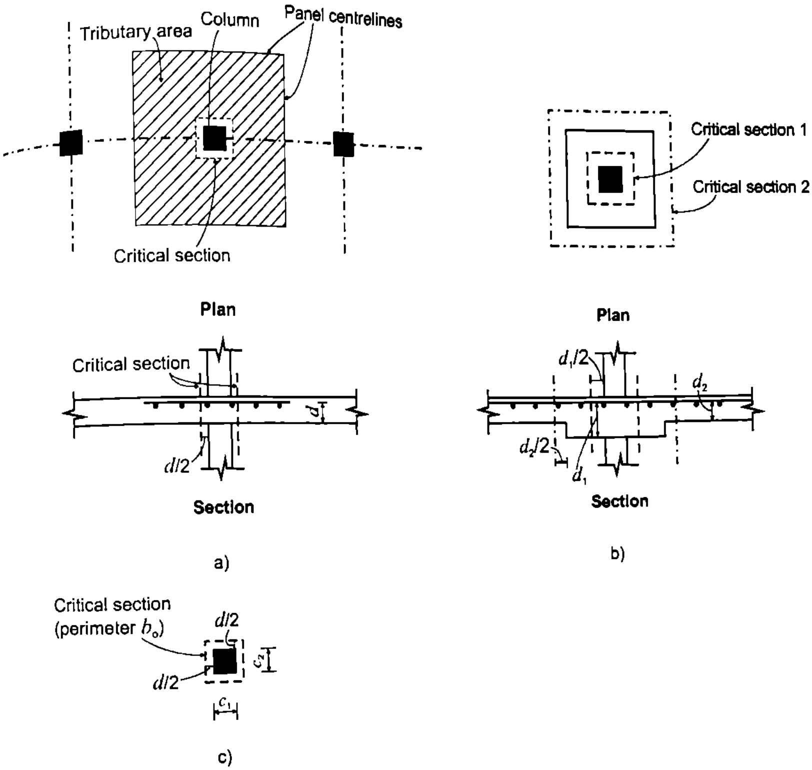


Figure 12.71 Critical section for two-way shear: a) a flat plate; b) a flat slab, and c) dimension notation.

The shape and size of the critical section depend on the location of the column within a building (Cl. 13.3.3.3). For a typical interior column of rectangular shape, critical section may be assumed to have four vertical sides, while for corner and edge columns this section may be assumed to have two or three vertical sides, respectively (see Figure 12.72).

The shape of a critical section also depends on the cross-sectional shape of the supporting column. Circular column sections may be found in many buildings. In theory, it would be possible for the critical section to have a circular shape, as shown in Figure 12.73a. However, the intent of Cl. 13.3.3.3 is that a critical section has straight surfaces. Experimental studies have shown that punching shear strength for circular columns exceeds the strengths for square columns with the same cross-sectional area. Therefore, it is conservative and analytically simpler to idealize circular columns as square columns with the same cross-sectional area (ACI, 1988); this is shown in Figure 12.73a. Critical sections for some irregular column shapes are shown in Figure 12.73b and c.

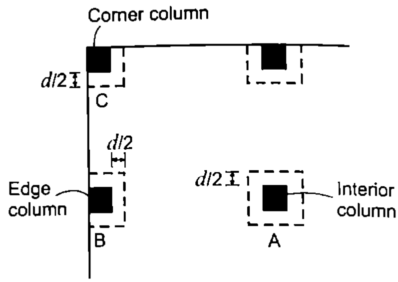


Figure 12.72 Critical sections for different column locations within a building.

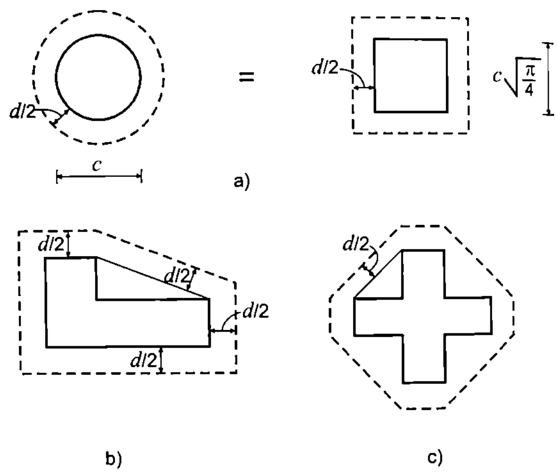


Figure 12.73 Critical section for different column shapes: a) circular; b) L-shaped, and c) cross-shaped (courtesy of the American Concrete Institute).

At edge and corner columns, where the slab cantilevers beyond the exterior face of the support, critical section may be assumed to have three straight sides and extend into the cantilevered portion of the slab (overhang), as shown in Figure 12.74. The critical section can be assumed to extend into the cantilevered portion of the slab for a distance not exceeding d (Cl.13.3.3.3). The objective of this provision is to conservatively determine the perimeter of the critical section, b_o.

Designers often need to deal with openings or slab edges located close to the columns (or other supports) in two-way slabs. One of the concerns regarding the openings is that shear flow in the slab is interrupted, hence the resulting shear capacity of the connection is reduced. This must be taken into account in the design.

Design provisions related to the critical perimeter at slab openings are outlined in Cl.13.3.3.4. Mechanical and electrical systems are often required to penetrate through the slab. Openings or holes placed in the vicinity of columns are often required due to architectural constraints. This presents an additional challenge to structural engineers when considering punching shear in critical regions around the columns. Vertical openings (holes) passing through the slab reduce shear strength when located within the intersecting column strips or at a distance closer than $10h_s$ to the column (where h_s denotes slab thickness). Horizontal openings (ducts) are often needed to provide space for electrical conduits. The effect of these openings can be disregarded, when their distance to the column exceeds $2h_s$

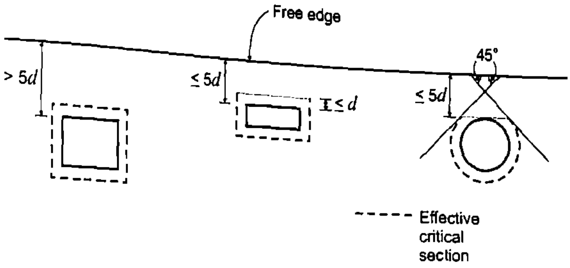


Figure 12.74 Critical section at the slab edge depending on the overhang length and the column shape
(adapted from CAC, 2005 with the permission of the Cement Association of Canada).

Also, the width should be limited to slab thickness h_s, and the depth to $h_s/3$. Holes located adjacent to all four sides of columns are not permitted. Note that CSA A23.3 does not include any of the aforementioned provisions related to slab openings.

Factored shear stress (v_f)

The factored shear stress, v_f, is determined by dividing the factored shear force, V_f, by the area over which the shear stresses are acting ($b_o \times d$), that is,

$$v_f = \frac{V_f}{b_o \times d} \qquad [12.18]$$

Note that $b_o \times d$ denotes the total area of vertical sides forming the critical section, as shown in Figure 12.70b.

The factored shear force, V_f, is the resultant of the factored gravity load, w_f, acting on the slab area, A, tributary to a column, that is,

$$V_f = w_f \cdot A$$

Tributary area is shown in Figure 12.75. Note that the area for an interior column is shown with a hatched pattern and the area for an edge column is shown with a cross-hatched pattern. Note that the area enclosed by the perimeter of the critical section is not included in the tributary area, because load acting over that area is transferred directly to the column.

Note that the tributary area is bound by lines of zero shear. For interior panels, these lines are assumed to pass through the centre of the panel, while for edge panels in flat plates, the lines of zero shear correspond to a distance of $0.45l_n$ from an exterior column, as shown

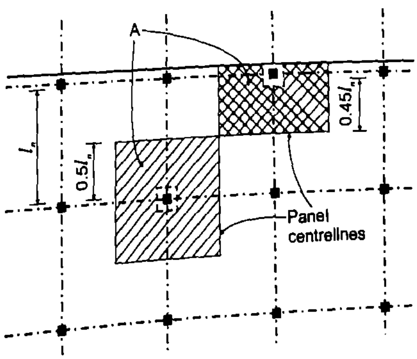


Figure 12.75 Tributary area for two-way shear design for an interior and an edge column.

in Figure 12.75 (note that l_n denotes the clear span). Most designers consider the lines of zero shear that coincide with the panel centrelines.

Note that v_f should be determined considering the full load on all spans, as well as any other loading patterns which might result in larger stresses (Cl.13.3.3.1).

A23.3 Cl.13.3.4

Factored shear resistance of concrete (v_c) for slabs without shear reinforcement

When a flat slab or flat plate is subjected to moderate gravity loads such that

$$v_f \le v_c$$

the entire shear resistance can be provided by the concrete, that is,

$$v_r = v_c$$

According to Cl.13.3.4.1, the factored shear resistance of concrete (v_c) should be taken as the <u>smallest</u> value obtained from the three equations outlined below.

i) The effect of column shape

A23.3 Eq. 13.5

$$v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19\lambda\phi_c\sqrt{f_c'} \text{ (MPa)} \qquad [12.19]$$

where

$\beta_c = h/b$ is the ratio of the long-side dimension (h) to the short-side dimension (b) of the column, the concentrated load, or reaction area (see Figure 12.76a). The A23.3 Eq. 13.5 takes into account the reduced shear strength in columns with elongated cross-sectional shape (where $\beta_c \ge 2$). As the column becomes more elongated, shear is mostly resisted along the short side, while the ultimate shear stress on the long side approaches the strength limit for beams or one-way slabs, that is, $0.19\lambda\phi_c\sqrt{f_c'}$. Figure 12.76b provides guidance on how to determine β_c for columns with irregular shapes.

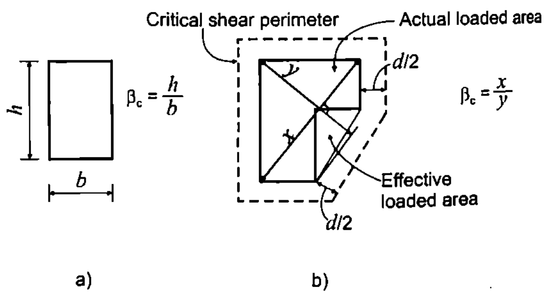


Figure 12.76 The effect of column shape: a) regular column, and b) irregular column (courtesy of the Cement Association of Canada).

ii) The effect of column location within a building

A23.3 Eq. 13.6

$$v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right)\lambda\phi_c\sqrt{f_c'} \text{ (MPa)} \qquad [12.20]$$

where

α_s = 4 for an interior column (see column A in Figure 12.72)
= 3 for an edge column (see column B in Figure 12.72)
= 2 for a corner column (see column C in Figure 12.72)

The A23.3 Eq. 13.6 may also govern when columns or capitals become very large. For example, this equation governs for an interior column when $b_o/d > 20$.

iii) Shear strength of plain concrete

A23.3 Eq. 13.7

$$v_c = 0.38\lambda\phi_c\sqrt{f'_c} \text{ (MPa)} \qquad [12.21]$$

Note that the A23.3 Eq. 13.7 is related to the shear strength of plain concrete. It gives a conservative estimate of v_c for interior and square-shaped columns. Note that, for normal density concrete ($\lambda = 1.0$), this equation can be simplified as follows

$$v_c = 0.25\sqrt{f'_c} \text{ } (MPa) \qquad [12.22]$$

The following two additional CSA A23.3 requirements, concrete compressive strength and size effect, need to be checked at this stage, as described below.

a) Concrete compressive strength (f'_c) limit

It can be seen from above equations that v_c depends on $\sqrt{f'_c}$. This is due to the fact that the tensile strength of concrete (modulus of rupture) is proportional to $\sqrt{f'_c}$, and shear failure is primarily controlled by the concrete tensile strength, as illustrated in Figure 12.70a. Note that CSA A23.3 Cl.13.3.4.2 sets a limit for f'_c, that is,

$$\sqrt{f'_c} \leq 8 \text{ (MPa)}$$

b) Size effect

When $d > 300$ mm, Cl.13.3.4.3 requires that the v_c value needs to be modified to account for the size effect, as follows

$$v_c \times \frac{1300}{1000 + d}$$

12.9.3 Combined Moment and Shear Transfer at Slab-Column Connections

A23.3 Cl.13.10.2 and 13.3.5

Unbalanced moments need to be transferred from the slab to the column through the slab-column connection (see Section 12.6.5 for discussion on unbalanced moments). The transfer mechanism is somewhat complex, and involves both flexure and shear.

Consider a portion of a two-way slab in the vicinity of an interior column shown in Figure 12.77a. The slab-column connection needs to transfer the factored moment M_u and the factored shear force V_f acting along the column axis. The transfer takes place through the following two mechanisms:

1. Flexure: a fraction of the unbalanced moment ($\gamma_f \times M_u$) is transferred through flexure in the slab along the strip b_b, as shown in Figure 12.77b, and
2. Shear: the remaining fraction of the unbalanced moment ($\gamma_v \times M_u$) is transferred by vertical shear stress (see Figure 12.77b). This stress is combined with punching shear stress caused by the shear force V_f, shown in Figure 12.77c.

The multiplier γ_v is required to find a fraction of the unbalanced moment transferred by shear ($\gamma_v \times M_u$), and it can be determined from the following equation (Cl.13.3.5.3)

A23.3 Eq. 13.8

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \qquad [12.23]$$

where

$b_1 = c_1 + d$

$b_2 = c_2 + d$

Note that b_1 denotes length of the critical section and c_1 is the dimension of the column along the span for which moments are determined (plane of the frame), and b_2 and c_2 are dimensions in the perpendicular direction (see Figure 12.77a).

The multiplier γ_f determines the remaining portion of the unbalanced moment ($\gamma_f \times M_u$) which is to be transferred through flexure, that is,

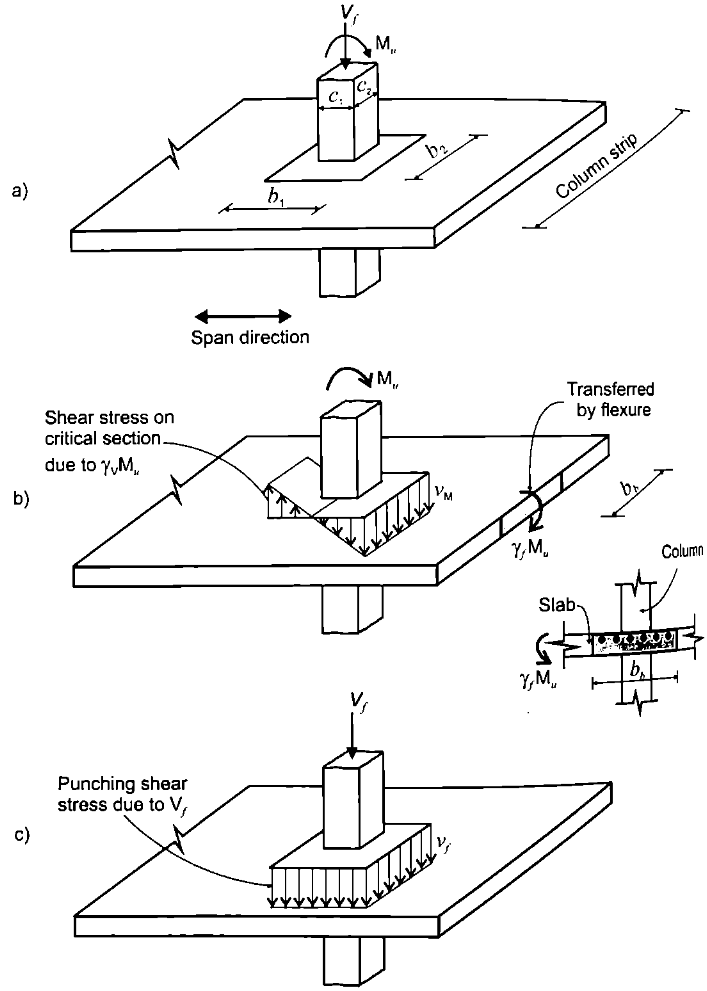


Figure 12.77 Shear and moment transfer: a) a portion of the slab at the slab-column connection showing the critical section; b) flexure and shear due to the moment M_u, and c) punching shear stresses due to the shear force V_f.

and

A23.3 Eq. 13.25

$$\gamma_f = 1 - \gamma_v$$

or

[12.24]

$$\gamma_f = \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$$ [12.25]

An increased amount of reinforcement within the strip b_b is prescribed to resist unbalanced moments at column locations (Clauses13.3.5.6 and 13.10.2); refer to Section 12.4.2 for an explanation of strip b_b.

It can be seen from the above equations that for square column shapes, 60 % of the unbalanced moment M_u is transmitted by conventional bending stresses at the column face $(\gamma_f \times M_u)$, while the remaining 40 % is transmitted by vertical shear stresses $(\gamma_v \times M_u)$; this ratio may vary depending on the column and slab geometry.

Shear stresses at the slab-column connection due to gravity load and the unbalanced moment are calculated according to Cl.13.3.5.5. The total factored shear stress at the perimeter of a critical slab section, v_{totalf}, can be determined as follows (see Figure 12.78a)

$$v_{totalf} = v_f + v_M$$ [12.26]

where v_f is the two-way (punching) shear stress due to V_f, and v_M is the shear stress due to unbalanced bending moment M_{fu} about axis y-y. It should be noted that, in a general case, bending moments may simultaneously occur in two directions (about axes x-x and y-y). In this case, the total shear stress needs to be expanded to account for an additional term, as follows

$$v_{totalf} = v_f + (v_M)_x + (v_M)_y$$ [12.27]

where $(v_M)_x$ and $(v_M)_y$ refer to vertical shear stresses due to bending moments about axes x-x and y-y respectively. Note that the designer needs to consider multiple moment directions for edge and corner columns.

Distribution of shear stresses at an interior and an exterior column is illustrated in Figure 12.78.

Uniformly distributed punching shear stress (v_f) at the centroid of critical section can be calculated from Eqn. 12.18.

Shear stress, v_M, due to the bending moment $(\gamma_v \times M_u)$ transferred by the eccentricity of shear varies linearly about the centroid of critical section (see Figure 12.78), which can be determined from the following equation (Cl.13.3.5.4):

$$v_M = \frac{(\gamma_v \times M_u) \times e}{J}$$ [12.28]

where

e = distance of the centroidal axis of the critical section perimeter to the point where shear stresses are being computed, and

J = property of the critical shear section analogous to the polar moment of inertia, equal to the sum of moments of inertia for the faces perpendicular to the centroidal axis C-C, plus the area of parallel faces times the square of the distance from those faces to axis C-C. Note that each face has its principal axes x-x and y-y; this is illustrated on an example of the face AA'BB' (see Figure 12.79a).

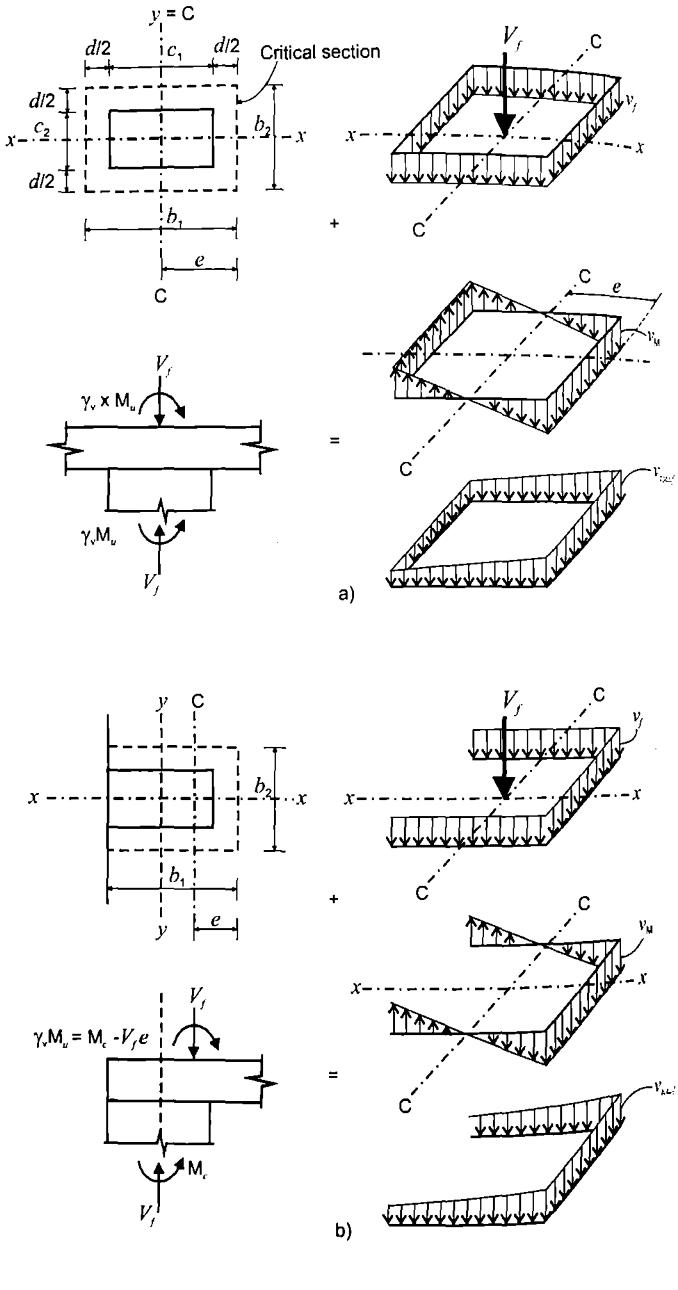
J value for an interior column can be determined from the following equation (using the notation shown in Figure 12.79a)

[12.29]

$$J = \frac{2(b_1 d^3)}{12} + \frac{2(d b_1^3)}{12} + 2(b_2 d)\left(\frac{b_1}{2}\right)^2$$

Note that the first and second term denote moments of inertia for the faces AA'BB' and EE'DD' respectively. The third term is equal to the product of area and the squared distance from axis C-C for the faces AA'DD' and BB'EE'. The J value calculated in this manner is

Figure 12.78 Shear stresses at the critical section: a) interior column, and b) exterior column.



valid for bending about axis C-C. A similar equation could be derived for the bending about the perpendicular axis.

The J value for an exterior/edge column can be determined in a similar manner as for the interior column (see Figure 12.79b), that is,

$$J = \frac{2(b_1 d^3)}{12} + \frac{2(d b_1^3)}{12} + 2(b_1 d)\left(\frac{b_1}{2} - e\right)^2 + (b_2 d)e^2 \qquad [12.30]$$

The first and the second term in Eqn 12.30 are the same as for the interior column. The third term denotes the product of the area for the faces AA'BB' and EE'DD' and the squared distance $(b_1/2 - e)$ between their respective centroids and axis C-C. The last term is equal to the product of the area of the face BB'EE' and squared distance (e) from that face to axis C-C.

The axis C-C runs through the shear centre of the column perimeter. In case of an interior column shown in Figure 12.79a, shear center coincides with the geometric centre of the area ABED. As a result, the distance (e) from axis C-C to the faces AA'DD' and BB'EE' is equal to $(b_1/2)$.

The location of the shear center for an exterior/edge column and its distance from the face BB'CC' can be found as a centroid of the areas for faces of the critical column perimeter, with the reference axis running along BE. Distance (e) from the face BB'EE' to axis C-C can be determined as follows (see Figure 12.79b):

$$e = \frac{2(b_1 d)(b_1/2)}{2(b_1 d) + (b_2 d)}$$

The derivation of an equation for J for a corner column is beyond the scope of this book (refer to MacGregor and Bartlett, 2000 for more details).

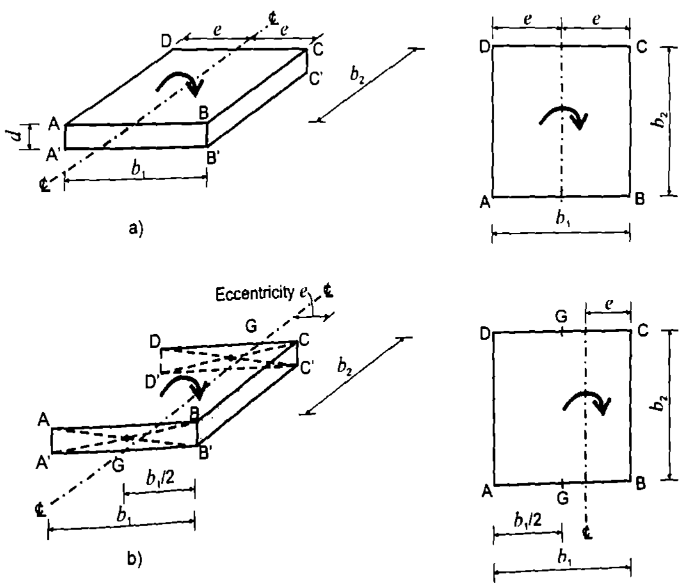


Figure 12.79 Calculation of the section property J: a) an interior column, and b) an exterior column.

12.9.4 Shear Reinforcement

A23.3 Cl.13.3.7

CSA A23.3 Reinforcement Requirements Shear reinforcement is required when the total factored shear stress, v_{totalf}, exceeds the shear resistance of concrete, v_c, that is,

$$v_{totalf} > v_c$$

Before proceeding with the design of shear reinforcement, the designer may wish to explore alternative approaches for increasing the concrete shear resistance, v_c, such as: i) increasing the slab thickness, ii) providing drop panels, iii) increasing the column size (this will result in an increase in the perimeter of the critical section), and/or iv) increasing the concrete compressive strength f_c'.

When shear reinforcement is provided, the slab shear resistance is determined as follows (Cl.13.3.7.3)

$$v_r = v_c + v_s$$

where v_s is the factored shear stress in shear reinforcement. Note that CSA A23.3 requires that a reduced concrete shear resistance be used for slabs with shear reinforcement.

The shear reinforcement needs to resist the shear stress beyond the concrete shear resistance, that is,

$$v_s \geq v_{totalf} - v_c$$

Note that CSA A23.3 sets the upper stress limit for shear resistance of two-way slabs with shear reinforcement, that is,

$$v_r \leq v_{max}$$

where v_{max} is the maximum allowed factored shear stress in accordance with Cl.13.3.7.4.

Vertical shear reinforcement can effectively increase the shear strength of slab-column connections in two-way slabs. CSA A23.3 Cl.13.3.7.1 permits the use of the following three types of shear reinforcement for slabs:

1. Headed shear reinforcement (shear studs), in the form of large headed studs welded to steel strips (see Figure 12.80a).
2. Stirrup reinforcement, usually in the form of closed vertical stirrups enclosing horizontal bars radiating outwards in two perpendicular directions from the support (see Figure 12.80b).
3. Shearheads, that is, cross-shaped elements constructed by welding rolled steel sections (W or channel sections) into a rigid unit embedded in the slab (see Figure 12.80c).

Shear studs are the most common type of shear reinforcement used for two-way slab construction in Canada, and the design will be discussed later in this section. Design of stirrup reinforcement is covered in Cl.13.3.9 and the procedure is similar to shear stud design. Design provisions for shearheads are not included in CSA A23.3, but the designer is referred to ACI 318M-02/ACI 318RM-02.

Anchorage of shear reinforcement in shallow slabs is critical for effective prevention of punching shear failure. Shear reinforcement controls the size of diagonal shear cracks, as shown in Figure 12.81. In order for a bar to be fully effective, it needs to develop its full yield strength, f_{yv}, at the intersection with the crack. Due to a shallow slab depth, the reinforcement needs to be effectively hooked at the ends. Anchorage provisions are outlined in Clauses 7.1.2 and 12.13.

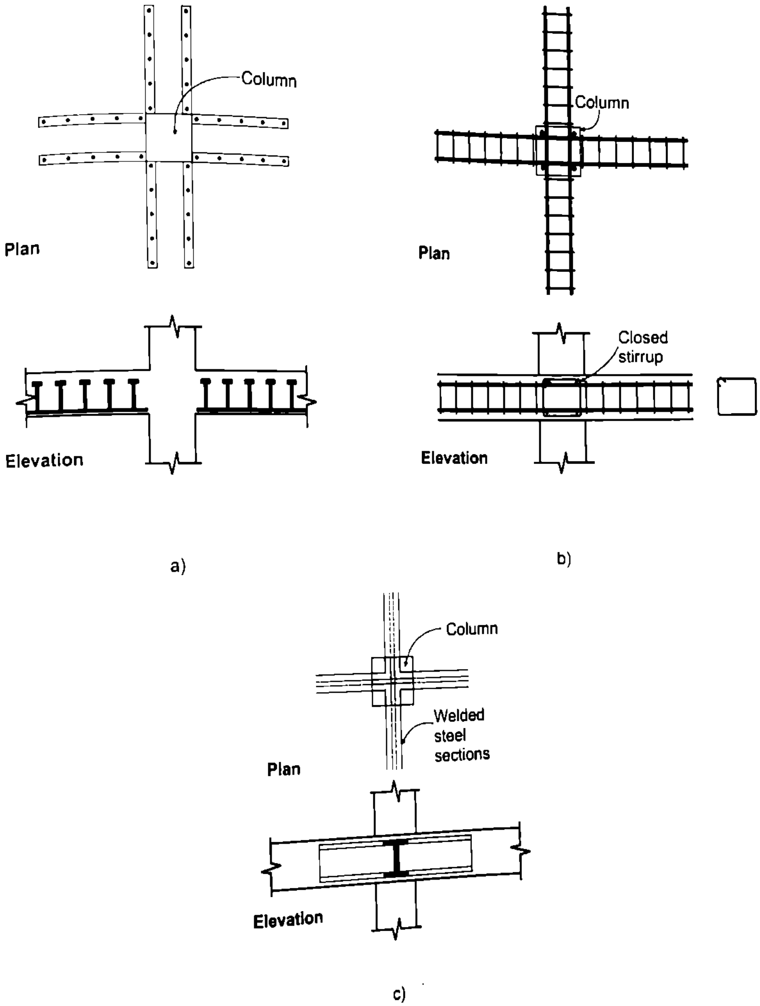


Figure 12.80 Types of shear reinforcement: a) shear studs; b) stirrups, and c) shearheads.

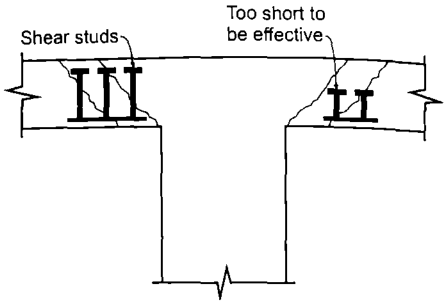
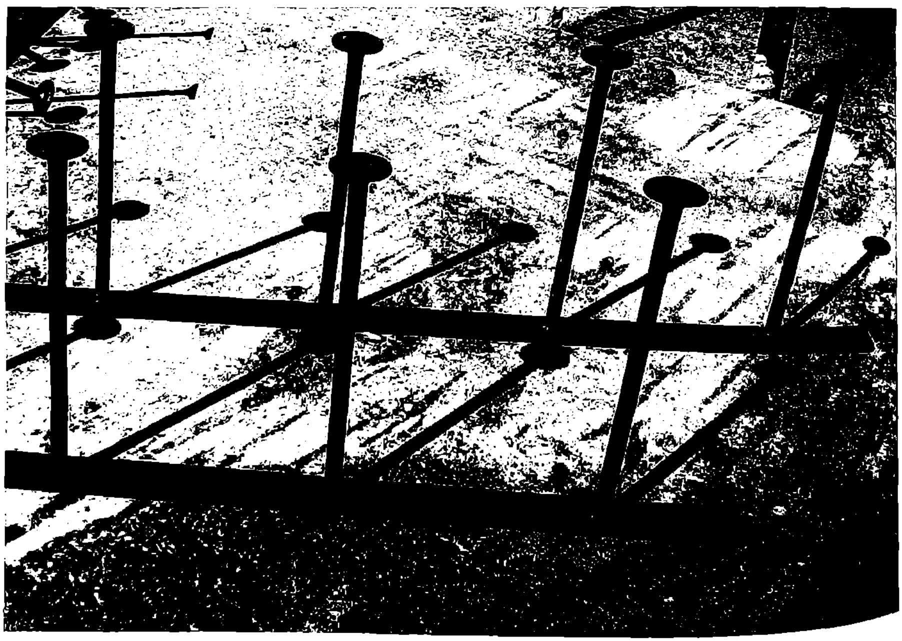


Figure 12.81 Shear reinforcement in a cracked slab section (adapted from Ghali and Hammill, 1992 with the permission of the American Concrete Institute).

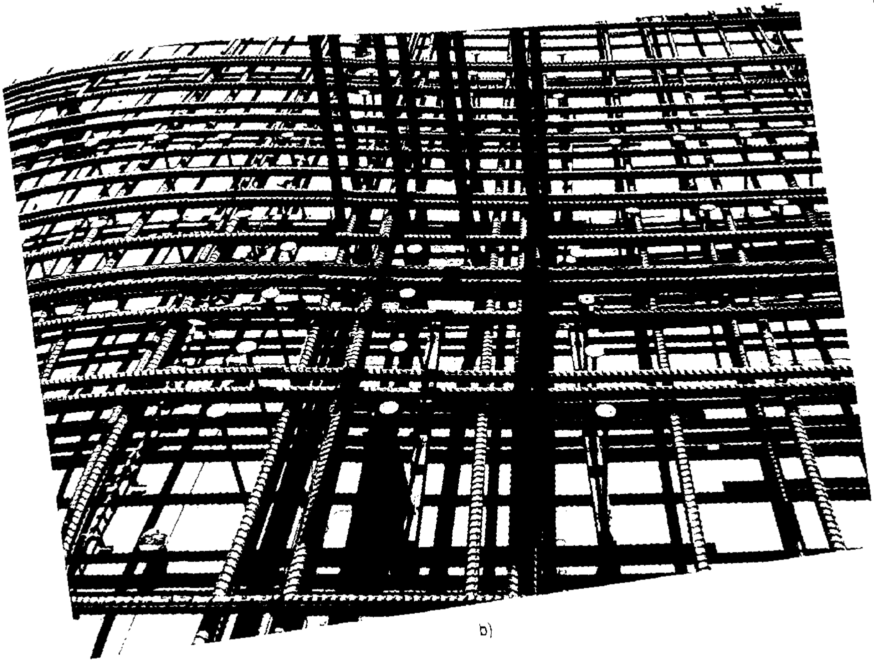
A23.3 Cl.13.3.8

Design of Slabs with Shear Stud Reinforcement Shear studs (headed shear reinforcement) consist of vertical rods with anchor heads at the top, and they are welded to a steel strip (also known as or stud rail) at the bottom, as shown in Figure 12.82a. Multiple stud rails are arranged in two perpendicular directions for square and rectangular columns, or in radial direction for circular columns. The stud rails are secured in position before the top and bottom flexural reinforcement is placed. The stud rail rests on bar chairs to maintain concrete cover. Figure 12.82b shows a two-way slab under construction with shear stud reinforcement arranged at a column location.



a)

Figure 12.82 Shear stud reinforcement: a) stud rails, and b) shear studs installed in a flat plate under construction (Svetlana Brzev).



b)

Figure 12.82 *(cont.)*

A23.3 Cl.13.3.8.2

Maximum factored shear stress

Shear studs can be used when the maximum factored shear stress in two-way slabs is less than the following limit

$$v_{fmax} \le v_{max} = 0.75\lambda\phi_c\sqrt{f'_c} \qquad [12.31]$$

A23.3 Cl.13.3.8.3

Concrete shear resistance

The concrete shear resistance, v_c, in the zone reinforced with shear stud reinforcement is less than that for slabs without shear reinforcement, and it should be determined from the following equation:

$$v_c = 0.28\lambda\phi_c\sqrt{f'_c} \qquad [12.32]$$

A23.3 Cl.13.3.8.5

Steel shear resistance

The factored steel shear resistance for stud reinforcement, v_s, is determined in the similar manner as that for beams, as follows

A23.3 Eq. 13.11

$$v_s = \frac{\phi_s f_{yv} A_v}{b_o s} \qquad [12.33]$$

where

s = spacing of studs

f_{yv} = 345 MPa specified yield strength of studs (a typical value used in practice); must be ≤ 400 MPa

A_{vs} = the sum of areas of all studs along the perimeter of a critical section (b_o); it can be calculated as a product of the total number of stud rails around a column and the cross-sectional area of one stud.

Eqn 12.33 can be presented in an alternative form:

$$v_s = \frac{V_s}{b_o \cdot d}$$

where

$$V_s = \frac{\phi_s f_{yv} A_{vs} d}{s}$$

is the resultant for steel shear resistance, corresponding to the reinforcement with area A_{vs} and spacing s (similar to Eqn. 6.9 when $\theta = 45°$).

A23.3 Cl.13.3.7.2

Critical sections

The shear stress needs to be checked both inside and outside the reinforced zone, thus there are the following two critical sections:

- Critical section 1 inside the reinforced zone at distance $d/2$ from the column face (same as for slabs without shear reinforcement), as per Cl.13.3.3.1, and
- Critical section 2 outside the shear-reinforced zone at distance $d/2$ from the outermost shear reinforcement.

A23.3 Cl.13.3.7.4

Length of reinforced zone

The reinforcement needs to be extended at least by a distance $2d$ from the face of the column. At the section where reinforcement is discontinued, it is required that v_{totalf} is less than the following limit:

$$v_{totalf} \leq 0.19\lambda\phi_c\sqrt{f'_c} \qquad [12.33]$$

A23.3 Cl.13.3.8.6

Spacing requirements

The following stud spacing requirements need to be followed:

a) For $v_f \leq 0.56\lambda\phi_c\sqrt{f'_c}$

$s_o \leq 0.4d$

and

$s \leq 0.75d$

where s_o denotes distance of the first stud from the column face.

b) For $v_f > 0.56\lambda\phi_c\sqrt{f'_c}$

$s_o \leq 0.4d$

and

$s \leq 0.5d$

c) The distance between adjacent stud rails in the direction parallel to the column face: $g \leq 2d$. This is not a CSA A23.3 requirement, but it is recommended by ACI (2008).

Stud layout and spacing requirements are illustrated in Figure 12.83. Note that the stud rails are placed at the column corners, and additional rails may need to be provided depending on column dimensions. When more than two stud rails are placed along a column face, they should be evenly spaced. The minimum number of stud rails is 8, 6, and 4 for interior, edge, and corner columns, respectively (DECON, 2009).

Detailing of shear studs is critical for their effectiveness in providing the punching shear resistance in two-way slabs. Detailed recommendations for shear studs are summarized below (see Figure 12.84):

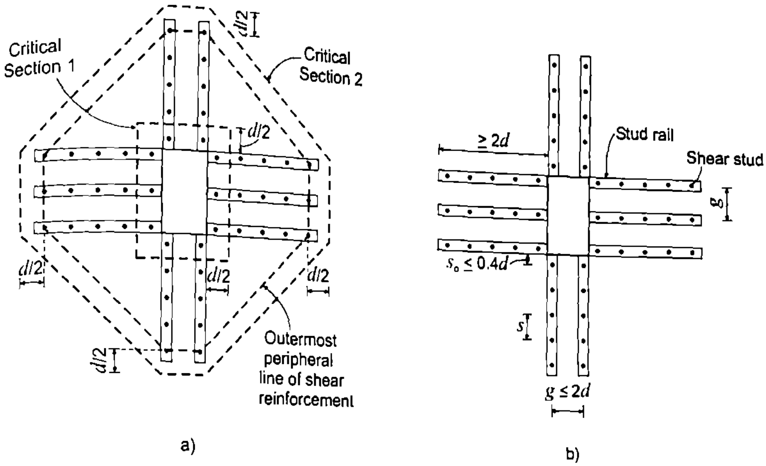


Figure 12.83 Stud arrangement for an interior column location: a) critical sections, and b) stud spacing.

1. Shear stud reinforcement should be located along concentric lines parallel to the perimeter of the column cross-section (Cl.13.3.8.4). Bottom stud rails should be aligned with the column faces in square or rectangular columns.
2. Shear studs must be mechanically anchored at each end by a plate or a head bearing against the concrete to develop bar yield strength (Cl.13.3.8.1). An effective anchorage can be achieved when the area of the top plate or the head is at least ten times the cross-sectional area of the bar.
3. The minimum concrete cover over the stud heads should be the same as the minimum cover for the flexural reinforcement (Cl.13.3.8.7). The concrete cover should not exceed the minimum cover plus one-half the bar diameter of the flexural reinforcement.

Stud rails with different specifications are commercially available (in terms of the number of studs per rail and the stud size). Overall stud height depends on the slab thickness. Stud

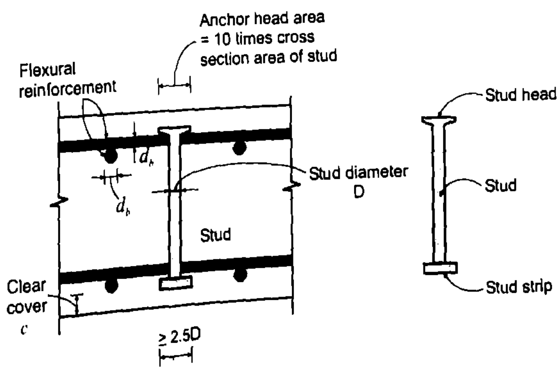


Figure 12.84 Location of shear stud reinforcement relative to flexural reinforcement (adapted from Ghali and Hammill, 1992 with the permission of the American Concrete Institute).

diameter ranges from 9.5 mm (3/8") to 19.1 mm (3/4"), and the rail thickness ranges from 4.8 mm (3/16") to 9.5 mm (3/8"). It has been found that 3/8" or 1/2" studs are most economical solutions for standard slabs. It is usually a good idea to keep the same stud diameter throughout the project, unless there is a wide range of slab thicknesses (DECON, 2009).

12.9.5 Shear Design of Two-Way Slabs with Beams

A23.3 Cl.13.4

Both beams and slab participate in transferring shear from slab to the columns in two-way slabs with beams. The fraction of load to be transferred by the beams depends i) on the beam-to-slab stiffness ratio for the beam under consideration, α_1, and ii) on the ratio of slab panel lengths, l_2/l_1. The following three scenarios need to be considered:

1. When, $\alpha_1 l_2/l_1 \geq 1.0$ beams are assumed to transfer the entire vertical load from the slab into the columns, that is, shear is resisted solely by the beams (Cl.13.4.1). It is not required to evaluate two-way shear resistance for the slab.

The beam shear resistance needs to be checked for all loads applied directly to the beam, plus the slab area enclosed by lines extending at 45° angle outward from the corners of the panel and the centrelines of adjacent panels on each side of the beam, as shown in Figures 12.85a and b. The beam shear resistance needs to be checked according to the design provisions for flexural members (beams and one-way slabs) prescribed by CSA A23.3 Cl.11 (see Section 6.5 for more details).

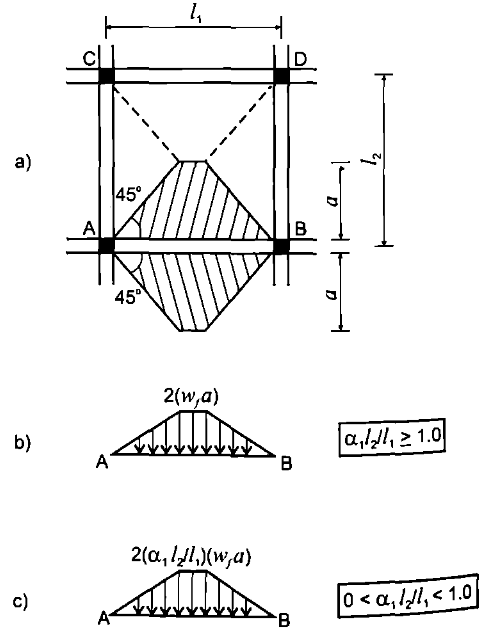


Figure 12.85 Tributary areas and loading in a two-way slab with beams: a) tributary area for an interior beam; b) beam loading when $\alpha_1 l_2/l_1 \geq 1.0$, and c) beam loading when $0 < \alpha_1 l_2/l_1 < 1.0$.

2. When $0 < \alpha_1 l_2/l_1 < 1.0$, the load distribution between the beams and the slab is determined by linear interpolation (Cl. 13.4.2). A typical interior beam needs to be designed for a fraction of the total load, $2(w_f \cdot a)$, expressed as a multiplier of $\alpha_1 l_2/l_1$ (see Figure 12.85c), while the slab needs to be designed for two-way shear considering the fraction of the reduced factored shear force, V_f^*, obtained by using the multiplier $(1 - \alpha_1 l_2/l_1)$, that is,

$$V_f^* = V_f(1 - \alpha_1 l_2/l_1) \qquad [12.34]$$

where V_f denotes the shear force corresponding to the two-way shear in the slab, as discussed in Section 12.9.2. Two-way shear resistance for a slab should be checked at the critical section with perimeter b_o, which needs to be reduced to account for the intersecting beams, as shown in Figure 12.86.

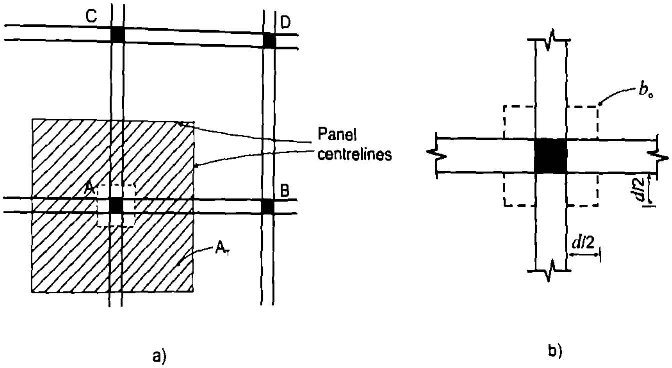


Figure 12.86 Distribution of load in a two-way slab when $0 < \alpha_1 l_2/l_1 < 1.0$: a) beam load, and b) critical section for two-way shear.

3. When $\alpha_1 l_2/l_1 = 0$, there are no beams, and the slab should be designed for shear as a two-way slab without beams discussed in Section 12.9.2.

One-way shear resistance for two-way slabs with beams should be checked in the same manner as for flat slabs and flat plates discussed in Section 12.9.2.

12.9.6 Design of Two-Way Slabs for Shear According to CSA A23.3: Summary and Design Examples

Key concepts related to the design of two-way slabs for shear have been presented in this section, and the key steps are outlined in Checklist 12.2. Although the steps are presented in a certain sequence, it is not necessary to follow the same sequence in all design situations. Three design examples will be presented to illustrate the application of the CSA A23.3 shear design provisions.

Checklist 12.2 Design of Two-Way Slabs for Shear

Step	Description	Code Clause
1	Check one-way shear resistance (Section 12.9.2.2).	13.3.6
1a	Determine the factored shear force (V_f) by treating the slab as a wide beam (see Figure 12.67).	
1b	Determine the factored concrete shear resistance: A23.3 Eq.11.6 $V_c = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v$ **[6.12]**	11.3.4
1c	Shear reinforcement is not required when $V_f \leq V_c$.	11.2.8.1
2	Check two-way (punching) shear resistance (see Section 12.9.2).	13.3
2a	Determine the location and properties of the critical section with perimeter b_o. The section should be taken at distance $d/2$ from the perimeter of the concentrated load or support (see Figure 12.71).	13.3.3
2b	Find the factored shear stress: $v_f = \dfrac{V_f}{b_o \times d}$ **[12.18]**	
2c	Find the concrete shear resistance based on the following three criteria (the smallest value governs): i) The effect of column shape A23.3 Eq.13.5 $v_c = \left(1+\dfrac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c}$ (MPa) **[12.19]** ii) The effect of column location within a building A23.3 Eq.13.6 $v_c = \left(\dfrac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c}$ (MPa) **[12.20]** iii) Shear strength of plain concrete A23.3 Eq.13.7 $v_c = 0.38\lambda\phi_c\sqrt{f'_c}$ (MPa) **[12.21]**	13.3.4
2d	Determine the shear stress, v_M, due to the a fraction of unbalanced bending moment ($\gamma_v \times M_u$) transferred through the slab-column connection (see Section 12.9.3) $v_M = \dfrac{(\gamma_v \times M_u)\times e}{J}$ **[12.28]**	13.10.2 13.3.5
2e	Find the total factored shear stress: $v_{total} = v_f + v_M$ **[12.26]**	
3	Design the slab shear reinforcement. Shear reinforcement is required when $v_{total} > v_c$	13.3.7
3a	Design the stud reinforcement (when required) - see Section 12.9.4. The shear studs can be used when the total factored shear stress is limited to $v_{total} \leq v_{max} = 0.75\lambda\phi_c\sqrt{f'_c}$ **[12.31]**	13.3.8

(Continued)

Checklist 12.2 Continued

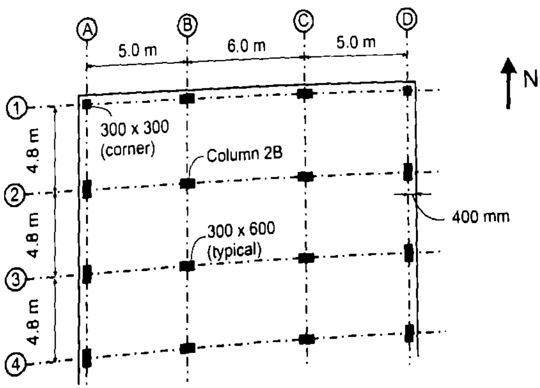
Step	Description	Code Clause
3b	Find the concrete shear resistance for slabs with shear stud reinforcement: $v_c = 0.28\lambda\phi_c\sqrt{f'_c}$ **[12.32]**	
3c	Determine the required steel shear resistance (v_s): $v_s \geq v_{f,max} - v_c$	
3d	Find the total required area of stud reinforcement (A_{vs}) from the following equation $\boxed{\text{A23.3 Eq.13.11}}$ $v_s = \frac{\phi_s f_{yv} A_{vs}}{b_o s}$ **[12.33]**	
3e	Check the following spacing requirements: • stud spacing (s) • distance of the first stud from the column face (s_o) • stud rail spacing (g)	
4	Check the two-way shear resistance for slabs with beams (see Section 12.9.5).	13.4
4a	When $\alpha_1 l_2/l_1 \geq 1.0$, beams provide the entire shear resistance for the floor system (refer to Section 6.7 for shear design of beams).	
4b	When $0 < \alpha_1 l_2/l_1 < 0$, shear is resisted both by the beams and the slab (use linear interpolation to find the design shear forces and stresses).	

Example 12.8

Two-Way Flat Plate - Shear Design

Consider a plan view of a two-way flat plate floor system designed in Example 12.1, shown in the sketch below. Use the slab thickness of 180 mm and the effective depth of 140 mm. The factored area load is w_f = 12.6 kPa.

Design the slab for shear according to the CSA A23.3 requirements. Consider only an interior column at the intersection of gridlines 2 and B. Disregard the effect of unbalanced moments.



Given: $f'_c = 30$ MPa
$f_y = 400$ MPa
$\lambda = 1.0$ (normal-density concrete)
$\varphi_c = 0.65$
$\phi_s = 0.85$

SOLUTION: 1. **Check the one-way shear resistance (A23.3 Cl.13.3.6).**

a) First, locate critical sections for the one-way shear design. The following two sections will be considered: 1-1 and 2-2 (one section in each horizontal direction of the building). Note that the critical sections are located at a distance d_v from the face of the column, as shown on the sketch below. Since $d = 140$ mm (given), let us determine d_v, which was previously defined as the effective shear depth taken as the greater of $0.9d$ and $0.72h_s$, that is,

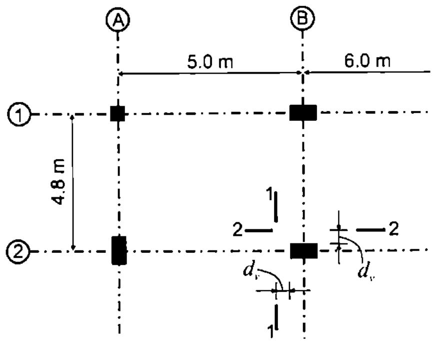
$d_v = 0.9d = 0.9 \times 140 \text{ mm} = 126 \text{ mm}$

or

$d_v = 0.72h = 0.72 \times 180 \text{ mm} = 130 \text{ mm}$

The larger value governs, that is,

$d_v = 130$ mm



b) Find the factored shear force V_f.

V_f is a design shear force at the critical section located at a distance d_v from the column face. Let us consider Section 1-1 for the slab span AB along gridline 2, as shown on the following sketch. The span is modelled as a wide beam with the width

$b_w = 4.8$ m

The clear span for span AB is

$l_n = 5.0 \text{ m} - 0.6 \text{ m}/2 - 0.3 \text{ m}/2 = 4.55 \text{ m}$

The factored area load $w_f = 12.6$ kPa needs to be transformed into the linear load w'_f acting on the wide beam, that is,

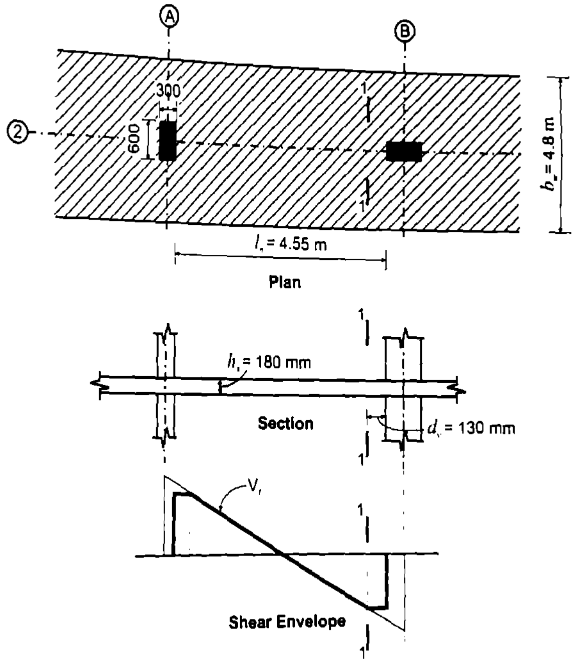
$$w'_f = w_f \cdot b_w = 12.6 \text{ kPa} \cdot 4.55 \text{ m} = 57.3 \text{ kN/m}$$

Next, the design shear force can be calculated as an internal shear force at the support of a continuous beam, that is,

$$V_f = w'_f \left(\frac{l_n - 2d_v}{2}\right) = 57.3 \text{ kN/m}\left(\frac{4.55 - 2 \cdot 0.13}{2}\right) \cong 123 \text{ kN}$$

Since Section 1-1 corresponds to the tributary width of 4.8 m, we can find the shear force per 1 m slab width, that is,

$V_f' = 123\ \text{kN}/4.8\ \text{m} = 26.0\ \text{kN/m}$



Calculations can be performed in a similar manner for Section 2-2. In this case, we are going to consider span 1-2 along gridline B. The clear span is

$l_n = 4.8\ \text{m} - 2(0.3\ \text{m}/2) = 4.5\ \text{m}$

and

$$b_w = \frac{5.0\ \text{m} + 6.0\ \text{m}}{2} = 5.5\ \text{m}$$

hence

$w_f' = w_f \cdot b_w = 12.6\ \text{kPa} \cdot 5.5\ \text{m} = 69.3\ \text{kN/m}$

Next, the design shear force can be calculated as an internal shear force at the support of a continuous beam, that is,

$$V_f = w_f'\left(\frac{l_n - 2d_v}{2}\right) = 69.3\ \text{kN/m}\left(\frac{4.5 - 2 \cdot 0.13}{2}\right) \cong 147\ \text{kN}$$

and the corresponding unit shear force is (based on the tributary width of 5.5 m)

$V_f' = 147\ \text{kN}/5.5\ \text{m} = 27\ \text{kN/m}$

The larger value governs, that is,

$V_f' = 27$ kN/m

c) Find the factored shear resistance V_c'.

The factored shear resistance (equal to the concrete shear resistance) should be determined from the following equation

A23.3 Eq. 11.6

$$V_c = \phi_c \lambda \beta \sqrt{f_c'}\, b_w d_v \qquad [6.12]$$

where

$b_w = 1000$ mm unit slab width (because V_f' was determined based on the same width)

$\beta = 0.21$ because $h_s = 180$ mm ≤ 350 mm (Cl.11.3.6.2)

Finally,

$$V_c' = 0.65 \times 1.0 \times 0.21\sqrt{30 \text{ MPa}}\,(1000 \text{ mm})(130 \text{ mm}) = 97 \text{ kN/m}$$

Since

$$V_f' = 27 \text{ kN/m} < V_c' = 97 \text{ kN/m}$$

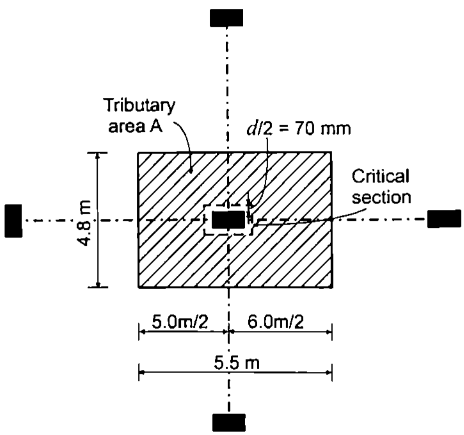
it can be concluded that the one-way shear resistance is satisfactory.

2. **Check the two-way shear resistance (CSA A23.3 Cl.13.3.4).**

a) Find the critical section.

The critical section is located at a distance $d/2 = 70$ mm from the face of the column, as shown on the sketch below. The perimeter of the critical section, b_o, is equal to:

$b_o = 2(300 \text{ mm} + 140 \text{ mm}) + 2(600 \text{ mm} + 140 \text{ mm}) = 2360$ mm



b) Find the factored shear force (V_f) and the factored shear stress (v_f).

V_f is determined as a product of the factored load, w_f, and the tributary area, A, shown hatched on the sketch, that is,

$$V_f = w_f \cdot A = 12.6 \text{ kPa} \times \left[\left(\frac{5.0 \text{ m} + 6.0 \text{ m}}{2}\right) \times (4.8 \text{ m}) - (0.3 \text{ m} + 0.14 \text{ m})(0.6 \text{ m} + 0.14 \text{ m})\right] = 329 \text{ kN}$$

Next, find the factored shear stress, v_f, as follows

$$v_f = \frac{V_f}{b_o \times d} = \frac{329 \times 10^3\,\text{N}}{2360\,\text{mm} \times 140\,\text{mm}} = 0.96\ \text{MPa} \qquad [12.18]$$

c) Find the concrete shear resistance, v_c.

Confirm that f_c' satisfies the requirements of CSA A23.3 Cl.13.3.4.2:

$\sqrt{f_c'} \le 8$ (MPa)

Since

$\sqrt{30} = 5.5\ \text{MPa} \le 8\ \text{MPa}$

Use the following three criteria:

i) The effect of column shape:

A23.3 Eq. 13.5

$$v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f_c'} \qquad [12.19]$$

where

$\beta_c = \frac{b_2}{b_1} = \frac{600\,\text{mm}}{300\,\text{mm}} = 2.0$ is the ratio of longer and shorter column cross-sectional dimension.

Therefore,

$$v_c = \left(1 + \frac{2}{2}\right) 0.19 \cdot 1.0 \cdot 0.65 \cdot \sqrt{30} = 1.35\ \text{MPa}$$

ii) The effect of column location within a building:

A23.3 Eq. 13.6

$$v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f_c'} \qquad [12.20]$$

where $\alpha_s = 4$ (interior column)

$$v_c = \left(\frac{4 \cdot 140}{2360} + 0.19\right) \cdot 1.0 \cdot 0.65 \cdot \sqrt{30} = 1.52\ \text{MPa}$$

iii) Shear strength of plain concrete:

Since normal-density concrete is used, let us use Eqn 12.22, that is,

$$v_c = 0.25\sqrt{f_c'} = 0.25\sqrt{30} = 1.37\ \text{MPa} \qquad [12.22]$$

The smallest v_c value governs, hence

$v_c = 1.35$ MPa

Since the effect of unbalanced moments is disregarded in this example, it follows that

$v_{totalf} = v_f$

and

$v_{totalf} = 0.96\ \text{MPa} < v_c = 1.3\ \text{MPa}$

It can be concluded that the two-way shear resistance is adequate based on the punching shear requirement. However, it is required to check the total shear stresses due to punching shear and the transfer of unbalanced moments. This will be performed in the next example.

If the two-way shear resistance is not satisfied, the designer can find the effective depth, d, which satisfies the two-way shear resistance requirements using the procedure outlined in Section 14.4.1 related to the two-way shear design of spread footings. Alternatively, the designer can assume a higher d value and repeat the calculations until all requirements have been satisfied.

Example 12.9

Two-Way Flat Plate - Shear and Moment Transfer

Consider the same slab-column connection as discussed in Example 12.8. *Design the slab for two-way shear according to the CSA A23.3 requirements, but consider the effect of unbalanced moments in the design.*

SOLUTION:

The objective of this example is to find the total shear stress due to the combined effect of punching shear and the unbalanced bending moment at the column (CSA A23.3 Cl.13.3.5.5).

The total shear stress at the perimeter of a critical slab section, v_{totalf}, can be determined as follows

$$v_{totalf} = v_f + v_M$$

where $v_f =$ 0.96 MPa is the punching shear stress determined in the previous step, and v_M is the shear stress due to unbalanced bending moment, M_{ufu}. An unbalanced moment for the interior column under consideration was determined in Example 12.1 (Step 7), as follows

$$M_{ufu} = 46.9 \text{ kNm}$$

The procedure for finding the v_M value is outlined below.

1. **Find the portion of unbalanced moment transferred from the slab by flexure ($\gamma_f \times M_{ufu}$).**

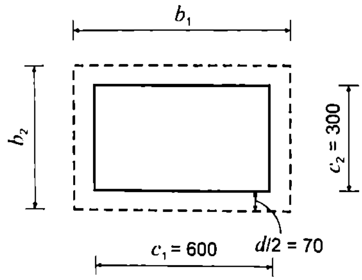
CSA A23.3 Cl.13.3.5 requires that a fraction of the unbalanced moment be transferred from the slab to the column by flexure. The corresponding moment is equal to

$$\gamma_f \times M_{ufu}$$

Note that γ_f depends on the dimensions of the critical section at the specific column location, that is,

$$\gamma_f = \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1+\frac{2}{3}\sqrt{\frac{740}{440}}} = 0.54 \qquad [12.25]$$

where (see the sketch below)



$b_1 = c_1 + d = 600 + 140 = 740$ mm

$b_2 = c_2 + d = 300 + 140 = 440$ mm

Therefore,

$\gamma_f \times M_{sfu} = 0.54 \times 46.9 = 25.3$ kNm

Find the required reinforcement corresponding to the bending moment above. For each 15M bar, it follows that

$$M_r \cong \phi_s A_b f_y (0.9d) = 0.85 \times 200 \text{ mm}^2 \times 400 \text{ MPa} \times (0.9 \times 140) = 8.6 \text{ kNm}$$

The required number of 15M bars is

$$\frac{25.3 \text{ kNm}}{8.6 \text{ kNm}} \cong 3.0$$

Use 3-15M bars. This is flexural reinforcement and it should be provided over the column within the band width b_b. However, the required flexural reinforcement provided at that location was previously determined as 6-15M@150 (see Example 12.1, Table 12.11), which is larger than the 3-15M bars determined from this calculation. In conclusion, there is no need to provide additional flexural reinforcement bars to ensure moment transfer from the slab to the columns. Note that this calculation should be performed as a part of the flexural design.

2. **Find the portion of unbalanced moment transferred from the slab by shear ($\gamma_v \times M_{sfu}$).**

The remaining portion of unbalanced moment is transferred from the slab by shear. Since

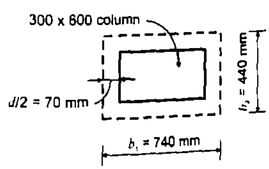
$\gamma_v = 1 - \gamma_f = 1 - 0.54 = 0.46$

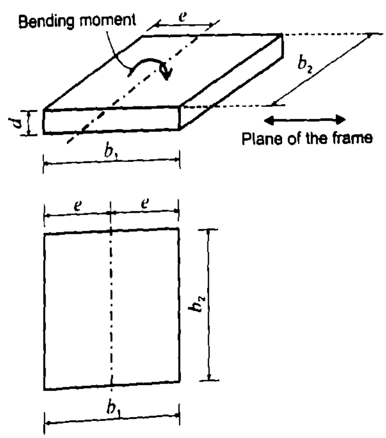
Therefore,

$\gamma_v \times M_{sfu} = 0.46 \times 46.9 = 21.6$ kNm

3. **Find the section property, J, for the interior column.**

The underlying equations were presented in Section 12.9.3, and the terms are illustrated on the sketch below.





$$J = \frac{2(b_1 d^3)}{12} + \frac{2(d b_1^3)}{12} + 2(b_2 d)\left(\frac{b_1}{2}\right)^2 \quad [12.29]$$

where

$b_1 = c_1 + d = 600 + 2 \cdot 70 = 740$ mm

$b_2 = c_2 + d = 300 + 2 \cdot 70 = 440$ mm

Therefore,

$$J = \frac{2(740 \cdot 140^3)}{12} + \frac{2(140 \cdot 740^3)}{12} + 2(440 \cdot 140)\left(\frac{740}{2}\right)^2 = 2.67 \cdot 10^{10}\ \text{mm}^4$$

iii) Find the shear stress v_M due to unbalanced moment $\gamma_v \times M_{afu}$.

Since this is a symmetrical section, the eccentricity is equal to

$$e = \frac{b_1}{2} = \frac{740}{2} = 370\ \text{mm}$$

Therefore,

$$v_M = \frac{(\gamma_V \times M_{fu}) \times e}{J} = \frac{(21.6 \cdot 10^6\ \text{Nmm})(370\ \text{mm})}{2.67 \cdot 10^{10}\ \text{mm}^4} = 0.3\ \text{MPa} \quad [12.28]$$

iv) Finally, find the total shear stress, v_{total}, that is,

$$v_{totalf} = v_f + v_M = 0.96 + 0.3 = 1.26\ \text{MPa} \quad [12.26]$$

Since

$v_{totalf} = 1.26 MPa < v_c = 1.35$ MPa

it follows that the two-way shear stress requirement has been satisfied.

Example 12.10

Two-Way Flat Plate - Design of Shear Stud Reinforcement

Consider a scenario where after the slab design discussed in Example 12.9 was completed, the total factored load had to be increased by 25%. Assume a proportional increase in the total factored shear stress (v_{totalf}), which was originally equal to 1.26 MPa. Perform shear design calculations using the same 180 mm slab thickness, an effective depth of 140 mm, and the same material properties. Use steel stud reinforcement to satisfy the shear resistance requirements if needed. Steel yield strength for stud reinforcement is f_{yr} = 345 MPa. Disregard the effect of unbalanced moments.

SOLUTION: Note that one-way shear will not be considered in this design, since it was checked in Example 12.9 and it did not govern the shear design.

1. **Find the critical sections for two-way shear design (CSA A23.3 Cl.13.3.7).**

   In two-way slabs with shear reinforcement, there are two critical sections (see Figure 12.83):

   i) Critical section 1 inside the reinforced zone at the distance $d/2$ from the column face (same as for slabs without shear reinforcement), and
   ii) Critical section 2 outside the shear-reinforced zone at a distance $d/2$ from the outermost shear reinforcement.

At this point, critical section 1 has been defined in Example 12.9 and the critical perimeter is equal to

$b_o = 2360$ mm

Properties of critical section 2 will be discussed later in this example.

2. **Find the factored shear stress (v_f) at critical section 1.**

Assume a 25% increase in the total factored shear stress from Example 12.9 (1.26 MPa), that is,

$v_{totalf} = 1.25(1.26 \text{ MPa}) = 1.56 \text{ MPa}$

and

$V_f = 1.25(329 \text{ kN}) = 411 \text{ kN}$

3. **Check whether shear reinforcement is required for this design.**

We need to check whether the slab shear resistance is still adequate, considering a 25% stress increase. The concrete shear resistance was determined in Example 12.9, as follows:

$v_c = 1.35 \text{ MPa}$

Since

$v_f = 1.56 \text{ MPa} > v_c$

It follows that shear reinforcement is required. Let us try to use stud reinforcement.

4. **Confirm that stud reinforcement can be used (CSA A23.3 Cl.13.3.8.2).**

Stud reinforcement can be used when the maximum total shear stress at critical section 1 is below the limit prescribed by CSA A23.3, that is,

$$v_f \le v_{max} = 0.75\lambda\phi_c\sqrt{f_c'} = 0.75 \cdot 1.0 \cdot 0.65\sqrt{30} = 2.7 \text{ MPa} \quad [12.31]$$

Since

$v_f = 1.63 \text{ MPa} < 2.7 \text{ MPa}$

it follows that stud reinforcement can be used for this design.

5. **Find the reduced concrete shear resistance for slabs with stud reinforcement according to CSA A23.3 Cl.13.3.8.3.**

$$v_c = 0.28\lambda\phi_c\sqrt{f_c'} = 0.28 \cdot 1.0 \cdot 0.65\sqrt{30} = 1.0 \text{ MPa} \quad [12.23]$$

Note that this v_c value should be used in the next steps. The v_c value of 1.35 MPa discussed in Step 3 can only be used for slabs without shear reinforcement.

6. **Design the stud reinforcement.**

a) Find the required steel shear resistance:

$v_s = v_f - v_c = 1.56 - 1.0 = 0.56 \text{ MPa}$

b) Find the required stud spacing (Cl.13.3.8.6):

First, let us check the level of design shear stress, as follows:

$v_f = 1.56 \text{ MPa}$

$v_f \le 0.56\lambda\phi_c\sqrt{f_c'} = 0.56 \cdot 1.0 \cdot 0.65\sqrt{30} = 2.0 \text{ MPa}$

thus

$v_f = 1.56 \text{ MPa} < 2.0 \text{ MPa}$

CSA A23.3 stud spacing requirements for this stress level are outlined below.

Distance of the first stud from the column face is equal to

$s_o \leq 0.4d = 0.4 \cdot 140 = 56$ mm

Typical stud spacing is

$s \leq 0.75d = 0.75 \cdot 140 = 105$ mm

Let us proceed with the following values:

$s_o = 55$ mm

and

$s = 105$ mm

c) Find the required area of stud reinforcement.

Since

$$v_s = \frac{\phi_s f_{yv} A_{vs}}{b_o s} \quad [12.33]$$

it follows that the required area of stud reinforcement is equal to

$$A_{vs} = \frac{v_s \cdot b_o \cdot s}{\phi_s f_{yv}} = \frac{0.56 \text{ MPa} \cdot 2360 \text{ mm} \cdot 105 \text{ mm}}{0.85 \cdot 345 \text{ MPa}} = 473 \text{ mm}^2$$

Let us assume 9.5 mm (3/8 inch) studs (the smallest size), and the corresponding area per stud is $A_{1s} = 71$ mm^2. The required number of studs (n) along the perimeter of the critical section is

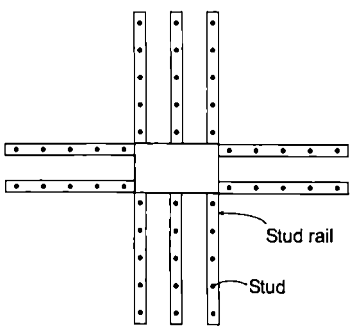
$$n = \frac{A_{vs}}{A_{1s}} = \frac{473}{71} = 6.6 \cong 7$$

7. **Develop a preliminary layout of stud reinforcement which meets the area and spacing requirements.**

The layout will be developed based on the spacing determined in the previous step (see Figure 12.82). First, we need to determine the layout of stud rails, based on the recommended spacing between rails (g), as follows:

$g \leq 2d = 2 \cdot 140 = 280$ mm

Since the column dimensions are 300 mm by 600 mm, let us use 3 stud rails perpendicular to the long side, and 2 rails perpendicular to the short side of the column. Therefore, we will have 10 stud rails in total, as shown on the sketch below.



8. **Check whether the shear resistance is adequate.**

The selected stud parameters are as follows

$n = 10$ and $A_{1s} = 71\ \text{mm}^2$

thus

$A_{vs} = n \cdot A_{1s} = 10 \cdot 71 = 710\ \text{mm}^2 > 473\ \text{mm}^2$

It can be concluded that the proposed stud layout is satisfactory. However, let us find the actual shear resistance, as follows

$v_r = v_c + v_s$

Since

A23.3 Eq. 13.11

$$v_s = \frac{\phi_s f_{yv} A_{vs}}{b_o s} = \frac{0.85 \cdot 345\ \text{MPa} \cdot 710\ \text{mm}^2}{2360\ \text{mm} \cdot 105\ \text{mm}} = 0.84\ \text{MPa} \qquad [12.33]$$

and

$v_c = 1.0\ \text{MPa}$

thus

$v_r = 1.0 + 0.84 = 1.84\ \text{MPa}$

Let us perform the checks to confirm that v_r is adequate:

$v_r = 1.84\ \text{MPa} > v_f = 1.56\ \text{MPa}$

and

$v_r = 1.84\ \text{MPa} < v_{max} = 2.7\ \text{MPa}$

Therefore, it can be concluded that the design is adequate.

9. **Find the required length for stud rails (CSA A23.3 Cl. 13.3.7.4).**

a) Estimate the required stud rail length.

CSA A23.3 prescribes that the reinforcement needs to be extended at least by a distance $2d$ from the face of the column. Therefore, the minimum length for a stud rail (l_s) is equal to

$l_s \geq 2d = 2 \cdot 140 = 280\ \text{mm}$

Let us assume that a typical stud rail has 6 studs at 105 mm spacing, and 55 mm end spacing, that is,

$l_s = 5s + 2s_o = 5 \cdot 105\ \text{mm} + 2 \cdot 55\ \text{mm} = 635\ \text{mm}$

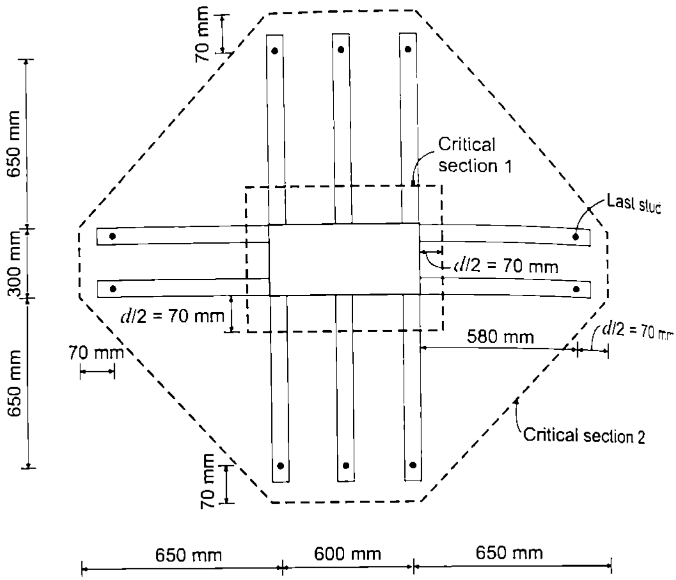
A typical stud rail is shown on the sketch below.

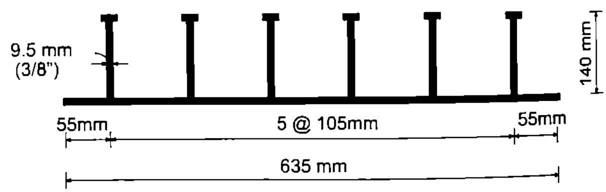
b) Check the shear stress requirement.

We need to confirm that the proposed stud length is adequate. CSA A23.3 requires that the reinforcement should be extended such that the design shear stress is below the following limit:

$v_f \leq 0.19\lambda\phi_c\sqrt{f_c'} = 0.19 \cdot 1.0 \cdot 0.65\sqrt{30} = 0.68\ \text{MPa}$

This requirement will be confirmed by finding the total shear stress (due to gravity load) at critical section 2, as shown on the following sketch.





Typical stud rail

i) Find the properties for critical section 2.

First, let us find the dimensions of critical section. The critical section is located at a distance d/2 from the outermost shear reinforcement (the furthest stud within in a rail), as shown on the sketch above. The perimeter of critical section 2 is equal to

$$b_o = 2(600\text{ mm} + 300\text{ mm}) + 4 \cdot 920\text{ mm} = 5480\text{ mm}$$

ii) Find the factored shear stress v_f due to the shear force V_f:

$$v_f = \frac{V_f}{b_o \times d} = \frac{411 \times 10^3\text{ N}}{5480\text{ mm} \times 140\text{ mm}} = 0.54\text{ MPa} \qquad [12.18]$$

iii) Confirm that the stress is within the permitted limits.

Since

$v_f = 0.54$ MPa < 0.68 MPa

It follows that the proposed stud length is adequate. However, if the effect of unbalanced moment was considered, the factored shear stress might exceed the shear resistance. In that case, it would be possible to increase the total stud area by adding one more rail along the 600 mm column dimension, and the total number of stud rails would increase from 10 to 12. Alternatively, longer stud rails could be used; this would result in an increase in the critical shear perimeter.

10. Provide a design summary.

Based on the design performed above, the following stud specification can be used:

Use 10 stud rails with overall height of 140 mm, 4.8 mm thickness, with 6-9.5 mm (3/8") diameter studs. The typical spacing is 105 mm on centre, and 55 mm end spacing.

12.10 DEFLECTIONS

A23.3 Cl.13.2.7

12.10.1 Background

Serviceability limit states ensure that the intended use and occupancy of a building are maintained, and include deflections, cracking, vibrations etc. Serviceability considerations for reinforced concrete flexural members are covered in Chapter 4. The discussion presented in this section is focused on deflections in two-way slabs. Deflections due to service loads must be limited to a tolerable amount. Excessive deflections can cause damage to non-structural elements such as partitions and glazing. Noticeable deflections appear unsafe and are not aesthetically pleasing.

The key factors influencing deflections in two-way slabs, such as concrete properties, creep and shrinkage, cracking, construction procedures and loads, and the placement of top reinforcement, will be discussed in this section. Other factors, such as slab geometry (span/thickness ratio), continuity, and support restraints, also influence deflections, and they are accounted for in deflection calculation procedures.

Concrete properties

The key concrete properties which influence deflections include the modulus of elasticity (E_c) and the modulus of rupture (f_r). Stiffness of an uncracked member increases in proportion to its modulus of elasticity, which is in turn directly proportional with the square root of the characteristic concrete compressive strength (f_c'), as discussed in Section 2.3.4. Key factors influencing the E_c value include aggregates, cement, silica fume, and admixtures. Lower water/cement ratio, a lower slump, and changes in concrete mix proportions can cause an increase in the modulus of elasticity (ACI 435R-95). Modulus of elasticity is a time-dependent property, and its value increases over time.

Modulus of rupture (f_r) denotes the concrete tensile strength, and it influences the onset of cracking since the cracking moment (M_{cr}) is directly proportional to f_r. The standard approach for determining the modulus of rupture reflects laboratory conditions, since the f_r value is determined from small specimens prepared and tested in a controlled environment. In practice, two-way slabs are constructed over large areas, under variable weather conditions and with a variation in the quality of workmanship. It is expected that the in-situ f_r values are significantly more variable compared to the laboratory conditions (Scanlon, 1999).

Creep and shrinkage

Time-dependent deflections in slabs are caused by creep and shrinkage. Shrinkage is a reduction in concrete volume over time due to the cement hydration, loss of moisture, and other factors, and it occurs independently of applied loading. Different types of shrinkage were discussed in Section 2.3.6. Shrinkage cracking in two-way slabs is mostly due to external and/or internal restraints. External supports, e.g. columns or walls, restrain the free horizontal movement in the slab. When shrinkage occurs, it can cause random cracking, thereby decreasing slab stiffness in cracked regions. Internal restraints are provided by flexural reinforcement. As the slab tries to shorten due to shrinkage, reinforcement placed on one side of the slab (top or bottom) shows a tendency to restrain the movement, thereby causing localized warping. This effect increases gradually and can cause progressive cracking in the slab over time. CSA A23.3-14 prescribes a reduced modulus of rupture for the cracking moment (M_{cr}) calculation in order to account for the effect of restraint shrinkage and other factors that cause cracking in two-way slabs at service load level (Cl.13.2.7).

Creep is demonstrated by an increase in concrete strains under sustained stresses, as discussed in Section 2.3.5. The effects of creep are more pronounced in reinforced concrete structures loaded at an early age, such as two-way flat slabs loaded at the construction stage. Creep-induced strains in concrete cause a significant increase in deflections due to sustained loads over time (by a factor of 2 or 3).

Cracking

Cracking is one of the key factors influencing deflections in two-way slabs. Slab deflections are sensitive to the extent of cracking, since two-way slabs are typically lightly reinforced (often requiring only minimum reinforcement for flexural strength). Scanlon (1999) concluded that slab sections with low flexural reinforcement ratios close to the minimum CSA A23.3 requirements are characterized by a significant difference between their cracked and gross stiffness. The cracked transformed moment of inertia, I_{cr}, is considerably lower than the gross moment of inertia, I_{gr}; often in the range of $I_{cr} = I_{gr} / 3$. Note that cracking is not uniform through the slab due to a variation in the bending moments. Regions with low bending moments remain uncracked and are characterized by significantly higher stiffness than the cracked regions.

This section outlines key methods for deflection calculations and their application is illustrated by a few examples.

12.10.2 CSA A23.3 Deflection Control Requirements

A23.3 Cl.13.2

Deflections in two-way slabs are required to remain within acceptable limits. CSA A23.3 prescribes two approaches for deflection control: indirect approach and detailed deflection calculations.

According to the *indirect approach*, the designer is permitted to select the minimum slab thickness that results in a robust design, thus detailed deflection calculations are not required (Cl.13.2.2). The minimum CSA A23.3 slab thickness requirements are explained in Section 12.5.2. Note that the indirect approach can be applied only to regular two-way slabs discussed in Section 12.5.1.

Detailed deflection calculations must be performed for slabs with the span-to-thickness ratio below the CSA A23.3 limit. According to Cl. 13.2.7, the deflections should be computed by taking into account the size and the shape of a slab panel, the support conditions, and the nature of restraints at the panel edges.

Both immediate and long-term deflections need to be considered in the design. *Immediate deflections* are initial deflections that occur as soon as the slab is constructed and the shoring is removed, while *long-term deflections* occur over time and may be caused by creep, shrinkage, and temperature strains. The CSA A23.3 procedures for estimating immediate and long-term deflections are discussed in Section 4.4.

The deflections in two-way slabs must be within the limits prescribed by CSA A23.3 Table 9.3. Note that CSA A23.3 prescribes the same deflection limits for one-way and two-way slabs.

Immediate slab deflections can be calculated using one of the following three methods:

1. *The Crossing Beam Method* is based on treating a two-way slab as an orthogonal one-way system, thus allowing the deflection calculations by beam analogy.
2. *The Equivalent Frame Method* uses a linear elastic analysis of 2-D frames. This method is discussed in detail in Section 12.7.2. An effective moment of inertia is used to account for the effect of cracking.
3. *The Finite Element Method* (FEM) is explained in Section 12.7.3, and it can be used to obtain both internal forces and deflections in two-way slabs. It is a computer-based method, and depending on the software capabilities it can be used either for linear elastic or nonlinear analysis which takes into account the effect of cracking.

For some types of slabs and support conditions, it is possible to obtain closed-form solutions for deflections based on the Elastic Plate Theory, which uses a partial differential equation for load-deflection response. The approach can be used to determine internal bending moments and shear forces. It is rarely used in practice due to the availability of computer-based methods such as the FEM.

Long-term deflections in two-way slabs can be estimated in the same manner as explained in Section 4.4.3. According to Cl.9.8.2.5, long-term deflections are obtained by multiplying the immediate deflection due to a sustained load by the factor ζ_s, given by

A23.3 Eq. 9.5

$$\zeta_s = 1 + \frac{s}{1 + 50\rho'} \qquad [4.15]$$

Where s is the time-dependent factor for creep deflection under sustained load (see Table 4.1), and ρ' is the compression reinforcement ratio (usually taken as 0 for two-way slabs). For practical applications in slabs without compression steel, the ζ_s factor ranges from 1.0 (load sustained for 3 months) to 3.0 (load sustained for at least 5 years).

12.10.3 The Crossing Beam Method for Deflection Calculations

The Concept The Crossing Beam Method (also known as the Wide Beam Method) is an approximate method for calculating deflections in compliance with the CSA A23.3 requirements. The method will be explained on an example of a slab panel with spans l_x and l_y supported by the columns shown in Figure 12.87. The column strip in x-direction and the middle strip in y-direction are considered as continuous wide beams. The widths for column and middle strips are denoted as l_c and l_m, respectively (see Section 12.4.1 for an explanation of column and middle strips).

The maximum deflection for the column strip in the x-direction is denoted as Δ_{cx}, as shown in Figure 12.88a. Similarly, the maximum deflection for middle strip in the y-direction is denoted as Δ_{my}, as shown in Figure 12.88b. The slab deflection (Δ_{max}) is obtained by adding the maximum deflections for the column strip and the middle strip, as shown in Figure 12.88c. The same procedure can be applied by considering deflections for the column strip in the y-direction (Δ_{cy}) and the middle strip in the x-direction (Δ_{mx}).

For square isotropic panels (where $l_y/l_x = 1.0$), the maximum slab deflection can be determined from the following equation:

$$\Delta_{max} = \Delta_{cx} + \Delta_{my} = \Delta_{cy} + \Delta_{mx} \qquad [12.35]$$

where

Δ_{cx} and Δ_{cy} are the column strip deflections for x- and y-directions, respectively, and

Δ_{mx} and Δ_{my} are the middle strip deflections for x- and y-directions, respectively.

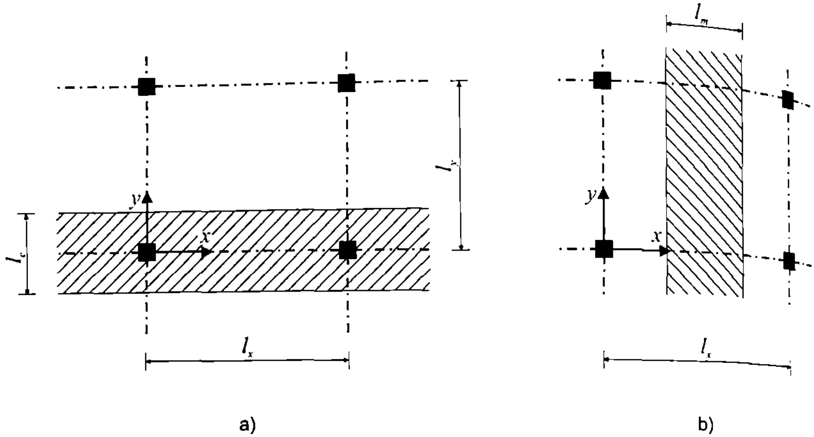


Figure 12.87 A slab panel for deflection calculations: a) column strip in x-direction, and b) middle strip in y-direction.

For rectangular panels, or for panels that have different properties in the two directions, an average maximum deflection is calculated by considering the maximum deflections in both directions, as follows:

$$\Delta_{max} = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} \qquad [12.36]$$

The calculated deflections should be compared with the deflection limits specified in CSA A23.3 Table 9.3. Note that the CSA A23.3 deflection limits for a slab panel should be calculated by considering the span length (l_n) measured diagonally between columns, that is,

$$l_n = \sqrt{l_{nx}^2 + l_{ny}^2}$$

Deflection Calculations for Column Strip and Middle Strip The column strip and the middle strip deflections can be determined according to the procedure for continuous beams explained in Section 4.6.3. A typical slab span considered for the deflection calculations is shown in Figure 12.89a. Note that the deflections are restrained at the supports; this is a simplifying assumption which does not reflect actual behaviour of a column-supported slab. The span is modelled as a continuous beam with span l_n (clear span), as shown in Figure 12.89b. The deflection (Δ) can be calculated from the following equation:

$$\Delta = k\left(\frac{5}{48}\right)\frac{M_m l_n^2}{E_c I_e} \qquad [4.12]$$

where the coefficient k can be determined as follows

$$k = 1.2 - 0.2\frac{M_o}{M_m} \qquad [4.17]$$

Deflection for a specific span depends on the end moments (M_1 and M_2) and the midspan moment (M_m), as shown in Figure 12.89c. Moment gradient (M_o) depends on bending moments at the supports and the midspan, that is,

$$M_o = M_m + (M_1 + M_2)/2 \qquad [12.37]$$

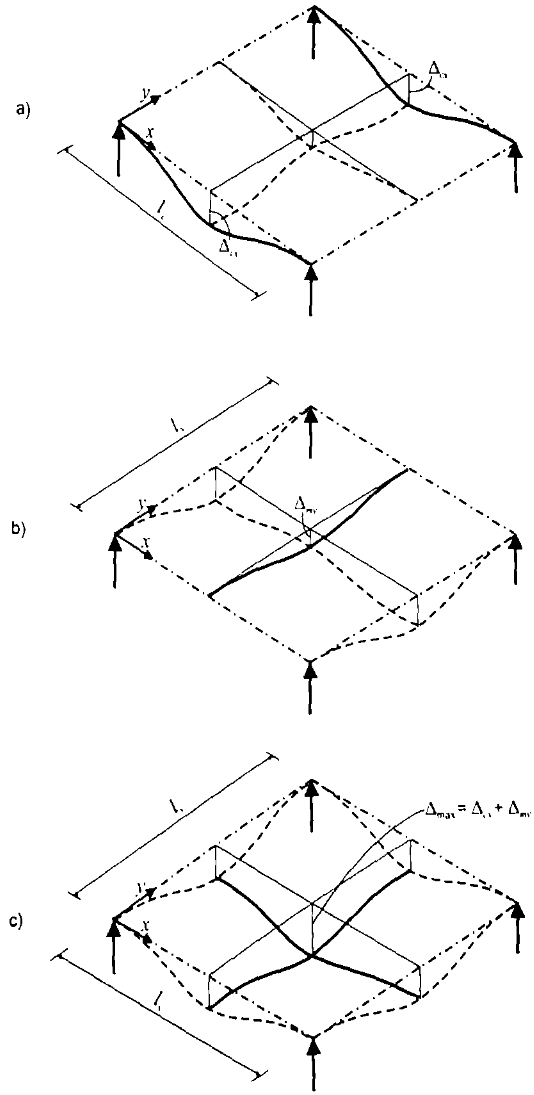


Figure 12.88 The Crossing Beam Method for deflection calculations: a) deflections in the column strip in x-direction; b) deflections in the middle strip in y-direction, and c) combined deflections (courtesy of the American Concrete Institute).

Note that the bending moments are obtained from the flexural design calculations using either the DDM or the EFM. The design for flexure is performed using the factored loads and the corresponding bending moments. However, bending moments for the deflection calculations are based on service level loads. Since the flexural design is performed using an elastic analysis, it is appropriate to perform a simple scaling of factored bending moments to reduce them to the service load level.

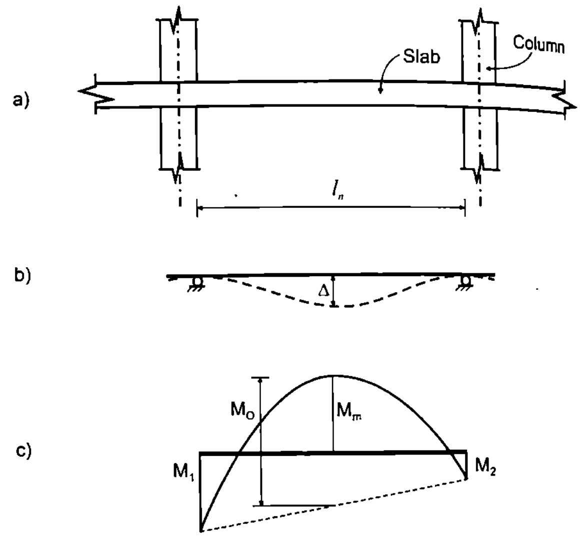


Figure 12.89 Bending moments for a slab strip: a) an elevation showing the slab geometry; b) deflected shape of a column strip or middle strip section, and c) a bending moment distribution.

The modulus of elasticity for concrete (E_c) can be determined from the following equation (see Section 2.3.4): $l_n = \sqrt{l_{nx}^2 + l_{ny}^2}$

A23.3 Eq. 8.2

$$E_c = 4500\sqrt{f_c'} \text{ (MPa)} \qquad [2.2]$$

The effective moment of inertia (I_e) is determined in the same manner as for flexural members discussed in Section 4.3.5, that is,

A23.3 Eq. 9.1

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g \qquad [4.11]$$

where

I_g = the moment of inertia of a gross concrete section

I_{cr} = the moment of inertia of the cracked section

Note that the moments of inertia I_g and I_{cr} are determined based on a wide beam section using either column strip or middle strip dimensions, as discussed above. The underlying equations are outlined in Sections 4.3.3 and 4.3.4.

The cracking moment (M_{cr}) is determined in the same manner as presented in Section 4.2, that is,

$$M_{cr} = \frac{f_r \cdot I_g}{y_t} \qquad [4.1]$$

It should be noted that CSA A23.3 Cl.13.2.7 prescribes the use of a reduced modulus of rupture (f_r^*) equal to one-half of the f_r value for deflection calculations in two-way slabs, that is,

$$f_r^* = 0.5 \cdot f_r = 0.3\lambda\sqrt{f_c'} \text{ (MPa)} \qquad [12.38]$$

where f_r denotes the modulus of rupture discussed in Section 2.3.2.

An average effective moment of inertia, $I_{e,avg}$, for continuous spans takes into account the moment of inertia at the supports and the midspan (Cl.9.8.2.4), as explained in Section 4.6.3. The following equations will be used for two-way slabs:

- For spans with two continuous ends (interior spans):

A23.3 Eq. 9.3

$$I_{e,avg} = 0.7I_{e,m} + 0.15(I_{e1} + I_{e2}) \qquad [4.18]$$

- For spans with one continuous end (end spans):

$$I_{e,avg} = 0.75I_{e,m} + 0.25I_{ec} \qquad [12.39]$$

Note that the latter equation was proposed by CAC (2005) for column-supported end spans where partial fixity is provided by an exterior column; this equation is different from A23.3 Eq.9.4 which was used for deflection calculations in continuous reinforced concrete flexural members (see Eqn 4.19 in Chapter 4).

The designer is permitted to use other I_e values provided that the computed deflections are in reasonable agreement with the results of comprehensive tests (Cl.13.2.7).

The bending moment (M_a) is determined at the *service load level*, and its value depends on the load for which the deflection has been computed: dead load moment (M_D), live load moment (M_L), or dead plus live load moment (M_{D+L}). In the absence of detailed calculations, the bending moment due to the construction load (M_{const}) can be taken as 2.0 to 2.2 times the slab dead load for deflection calculations in multi-storey buildings.

Alternatively, the designer is referred to ACI 435R-95 (2003) for more details on construction load calculations, and also Scanlon and Supernant (2011) for a practical deflection calculation procedure which takes into account the construction loading history, time-dependent concrete properties, and the effect of cracking.

The effective moment of inertia (I_e) is a significant factor influencing the deflection magnitudes, therefore it is critical to use realistic I_e values for deflection calculations. The effective moment of inertia should be calculated based on the load level under consideration. For example, the dead load deflection (Δ_D) should be calculated using the I_e value based on the dead load moment (M_D), while the deflection due to combined dead plus live load (Δ_{D+L}) should be calculated using the I_e value based on the total load moment (M_{D+L}). As a result, two different I_e values need to be used for deflection calculations, as illustrated in Figure 12.90a; this approach will be referred to as the Standard Procedure in this section. However, construction loads usually govern over the combined dead and live load for deflection calculations in multi-storey flat slabs. It is expected that cracking takes place due to construction loads and its effect should be accounted for in deflection calculations. This can be accomplished by using the I_e value corresponding to the bending moment at the construction load level (M_{const}) for immediate deflection calculations, as shown in Figure 12.90b; this approach will be referred to as the Alternative Procedure.

Immediate deflections for a typical span of a two-way slab can be computed according to the following procedure:

1. Calculate the dead load deflection (Δ_D) using the I_e value which corresponds to the dead load bending moment (M_D).
2. Calculate the deflection (Δ_{D+L}) due to the combined dead and live load using the I_e value which corresponds to the total bending moment (M_{D+L}).

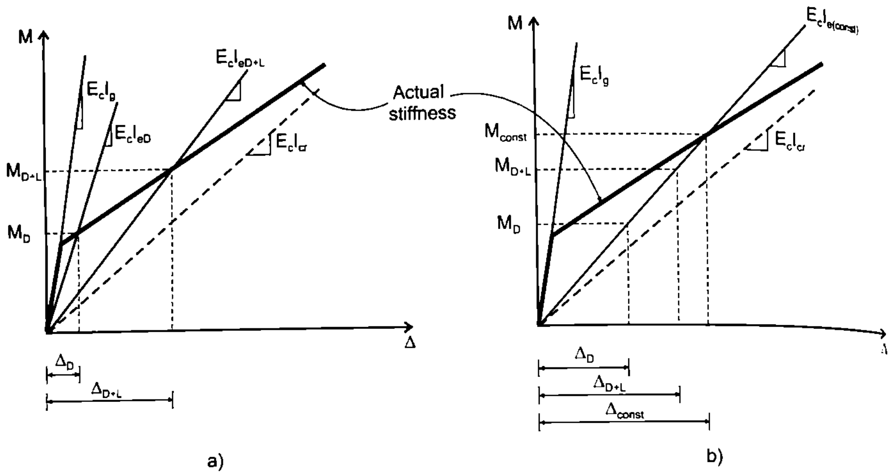


Figure 12.90 Effective moment of inertia for deflection calculations: a) Standard Procedure for flexural members, and b) Alternative Procedure for multi-storey flat slab construction
(adapted from ACI 435,1991 with the permission of the American Concrete Institute).

3. The live load deflection (Δ_L) is calculated as follows:

$$\Delta_L = \Delta_{D+L} - \Delta_D$$

Alternatively, the procedure can take into account the effect of construction loading, as follows:

1. Calculate the effective moment of inertia ($I_{e,const}$) corresponding to the bending moment at the construction load level (M_{const}).
2. Calculate the dead load deflection (Δ_D) due to bending moment (M_D) using the I_e value calculated in Step 1.
3. Calculate the deflection (Δ_{D+L}) due to combined dead and live loads using the bending moment (M_{D+L}) and the I_e value calculated in Step 1.
4. The live load deflection (Δ_L) is calculated as follows:

$$\Delta_L = \Delta_{D+L} - \Delta_D$$

Note that both procedures use service (specified) bending moments, and not factored bending moments which are used for the strength design (e.g. moment and shear resistance).

Once the immediate deflections have been determined, a long-term deflection (Δ_t) can be determined from the following equation:

$$\Delta_t = \zeta_t \cdot \Delta_D + \Delta_L \qquad [12.40]$$

where the first term denotes deflection due to a sustained load which is magnified by a multiplier, ζ_t, and the second term denotes the deflection due to a live load; note that ζ_t should be calculated from Eqn. 4.15. This is the most basic case, where only the dead load is considered as a sustained load. In reality, a fraction of the live load should also be considered as a sustained load, and the long-term deflection is determined according to the following procedure:

1. Find the deflection due to sustained load shortly after the construction is completed (e.g. installation of non-structural elements):

   $\Delta_{t1} = \zeta_{t1} \cdot (\Delta_D + \Delta_{LS})$

2. Find the deflection due to the sustained load corresponding to the maximum creep effects:

   $\Delta_{t2} = \zeta_{t2} \cdot (\Delta_D + \Delta_{LS})$

3. Finally, calculate the total long-term deflection due to the sustained load and the transient live load:

$$\Delta_t = (\Delta_{t2} - \Delta_{t1}) + (\Delta_l - \Delta_{LS}) \qquad [12.41]$$

where Δ_{LS} denotes deflection due to the sustained live load; this is usually expressed as a fraction of the live load, e.g. 20% of the live load could be sustained (this depends on the project requirements). See Section 4.4.3 for more details on long-term deflections.

The Crossing Beam Method for deflection calculations of two-way slabs is summarized in Checklist 12.3 and its application will be illustrated by two examples.

Checklist 12.3 Deflection Calculations for Two-Way Slabs According to the Crossing Beam Method

Step	Description	Code Clause
1	For a slab panel under consideration, select a column strip in one direction and a middle strip in the perpendicular direction (a flat plate/slab), or a beam strip in one direction and a slab strip in the perpendicular direction (a slab with beams). Perform the following calculations (steps 2 to 5) for each strip.	13.2.7
2	Find the moment of inertia values. Gross moment of inertia: $I_g = \frac{b \cdot h_s^3}{12}$ [4.2] Cracked moment of inertia: $I_{cr} = \frac{b \cdot y^3}{3} + n \cdot A_s \cdot (d-y)^2$ [4.10]	
3	Determine the cracking moment: $M_{cr} = \frac{f_r^* \cdot I_g}{y_t}$ [4.1] where (f_r^*) is a reduced modulus of rupture: $f_r^* = 0.5 \cdot f_r = 0.3\lambda\sqrt{f_c'}$ (MPa) [12.38]	13.2.7
4	Compute the effective moment of inertia: A23.3 Eq.9.1 $I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g$ [4.11]	9.8.2.3

(Continued)

Checklist 12.3 Continued

Step	Description	Code Clause
5	Determine the immediate deflections due to service loads: $\Delta = k\left(\frac{5}{48}\right)\frac{M_m l_n^2}{E_c I_e}$ [4.12] where $k = 1.2 - 0.2\frac{M_o}{M_m}$ [4.17] and $M_m = M_o + (M_1 + M_2)/2$ [12.37]	
5a	Find the dead load deflection (Δ_D) using bending moments (M_m) and (M_o), and the effective moment of inertia I_e due to dead load (D).	
5b	Find the total (dead plus live) load deflection (Δ_{D+L}) using bending moments (M_m) and (M_o), and the effective moment of inertia I_e due to the total load ($D + L$).	
5c	Finally, the live load deflection (Δ_L) is calculated as follows: $\Delta_L = \Delta_{D+L} - \Delta_D$	
6	Calculate immediate deflections for the slab panel by combining the column strip and the middle strip live load deflections (see Figure 12.88): $\Delta_{max} = \Delta_{cx} + \Delta_{my} = \Delta_{cy} + \Delta_{mx}$ [12.35]	
7	Calculate the maximum long-term deflections.	9.8.2.5
7a	Find the deflection due to sustained load shortly after the construction has been completed (e.g. after installation of non-structural elements): $\Delta_{t1} = \zeta_{t1} \cdot (\Delta_D + \Delta_{LS})$	
7b	Find the deflection due to sustained load corresponding to the maximum creep effects: $\Delta_{t2} = \zeta_{t2} \cdot (\Delta_D + \Delta_{LS})$	
7c	Calculate the total long-term deflection due to sustained load and transient live load: $\Delta_t = (\Delta_{t2} - \Delta_{t1}) + (\Delta_L + \Delta_{LS})$ [12.41]	
8	Check whether deflections are within the limits prescribed by CSA A23.3.	9.8.5.3 (Table 9.3)

Example 12.11

Two-Way Flat Plate - Deflection Calculations According to the Crossing Beam Method Using the Standard Procedure

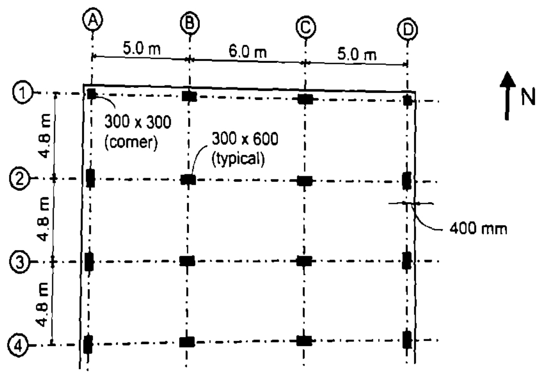
Consider a floor plan of a two-way slab system without beams (flat plate) which was designed for flexure in Example 12.1 (DDM) and Example 12.3 (EFM). Consider a design with 160 mm slab thickness (note that the previous examples used 180 mm slab thickness).

Use the Crossing Beam Method to find the immediate deflections for an end panel between column gridlines 1 and 2 in N-S direction, and gridlines B and C in E-W direction. Check whether immediate and long-term deflections are within the limits prescribed by CSA A23.3. Consider live load deflection limits for an occupancy where non-structural elements are not likely to be damaged by large deflections.

For long-term deflections, consider that non-structural elements have been installed after one month, and that 20% of the live load is sustained.

Use the effective moment of inertia at the following two levels: i) the dead load, and ii) the dead plus live load level.

Given: $f_c' = 30$ MPa



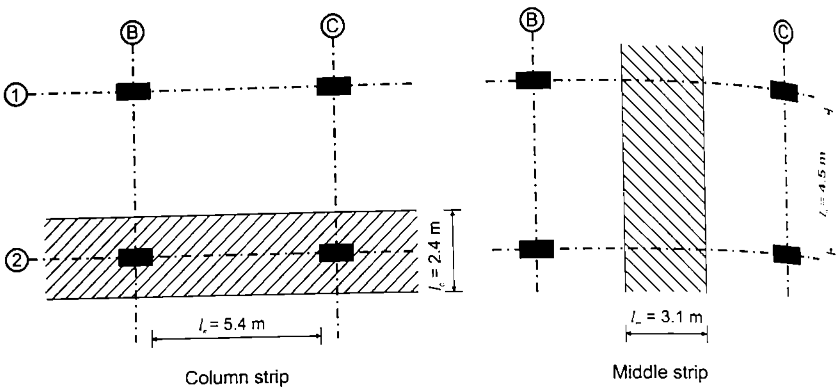
SOLUTION:

1. **Check the slab thickness requirements (CSA A23.3 Cl.13.2.3).**

   This check was performed in Example 12.1, and it was concluded that the minimum 180 mm thickness is required in order to satisfy the indirect approach for deflection control, that is, deflection calculations are not required provided that the thickness exceeds 180 mm. However, the slab thickness is reduced to 160 mm in this example and detailed deflection calculations are required. Since the slab satisfies the CSA A23.3 requirements for regular slabs specified by CSA A23.3 Cl.2.2, it is possible to use the Crossing Beam Method.

2. **Identify the design strips for deflection calculations.**

   It is necessary to calculate the deflections for an end panel between column gridlines 1 and 2 in N-S direction, and gridlines B and C in E-W direction. Let us consider column strip along gridline 2 (span BC), and a middle strip spanning between gridlines 1 and 2, as shown on the following sketch.



The width of column strip for gridline 2 is equal to

$l_c = 2.4$ m (as discussed in Example 12.1).

The width of middle strip for slab panel 23BC spanning between gridlines B and C can be determined as follows (see Figure 12.15):

$l_1 = 4.8$ m

$l_2 = (6.0 + 5.0)/2 = 5.5$ m design strip

Since $l_1 < l_2$, it follows that the column strip width is equal to

$l_c = l_1/2 = 4.8\text{ m}/2 = 2.4$ m

thus the width of middle strip is

$l_m = l_2 - l_c = 5.5 - 2.4 = 3.1$ m

3. **Determine the factored bending moments for the column strip and the middle strip.**

This step is based on Example 12.1, where the design was performed according to the DDM. However, the moments need to be recalculated due to different loads.

a) Perform the load analysis:

The slab's self-weight:

$DL_w = h \times \gamma_w = 0.16\text{ m} \times 24\text{ kN/m}^3 = 3.84\text{ kPa}$

The superimposed dead load:

$DL_s = 1.44$ kPa

Live load:

$LL_w = 3.6$ kPa

The total factored load:

$w_f = 1.25(DL_w + DL_s) + 1.5 \times LL_w = 12.0$ kPa

Note that the factored load is smaller than that used in Example 12.1 (12.6 kPa) due to a reduced slab thickness (slab thickness in this example is 160 mm, which is less than the 180 mm thickness used in Example 12.1).

b) Calculate the factored bending moments for the column strip.

The moments are going to be calculated according to the DDM. Refer to Step 5 in Example 12.1.

Clear span:

$$l_n = 6.0 - \left(\frac{0.6}{2} + \frac{0.6}{2}\right) = 5.4 \text{ m}$$

Total factored static moment:

A23.3 Eq. 13.23

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8} = \frac{12.0 \text{ kPa} \times 4.8 \text{ m} \times (5.4 \text{ m})^2}{8} = 210 \text{ kNm} \qquad [12.8]$$

Note that the total factored moment in this example is slightly less than the moment obtained in Example 12.1 (220 kNm) - again, this is due to the smaller slab thickness.

Table 12.21 Factored bending moments for the column strip (Interior Span BC)

		Interior Span BC: M_o = 210 kNm		
		B Negative moment M_1 (kNm)	Midspan Positive moment M_m (kNm)	C Negative moment M_2 (kNm)
Longitudinal direction	Bending moments at critical sections	$-0.65M_o$ $= (-0.65) \times 210$ $= -137$	$+0.35M_o$ $= (+0.35) \times 210$ $= +73$	$-0.65M_o$ $= (-0.65) \times 210$ $= -137$
Transverse distribution - column strip	CSA A23.3 Provisions	$-(0.46 \text{ to } 0.59)M_o$	$+(0.19 \text{ to } 0.23)M_o$	$-(0.46 \text{ to } 0.59)M_o$
	Proposed value Design moment	$-0.59M_o$ $-0.59 \times (210) = -124$	$+0.23M_o$ $+0.23 \times 210 = +48$	$-0.59M_o$ $-0.59 \times (210) = -124$
Transverse distribution - middle strip	Design moment	$=-137-(-124) = -13$	$=73-48 = +25$	$=-137-(-124) = -13$

c) Calculate factored bending moments for the middle strip.

Clear span:

$$l_n = 4.8 - \left(\frac{0.3}{2} + \frac{0.3}{2}\right) = 4.5 \text{ m}$$

Total factored static moment:

$$M_o = \frac{w_f \times l_{2a} \times l_n^2}{8} = \frac{12.0 \text{ kPa} \times 5.5 \text{ m} \times (4.5 \text{ m})^2}{8} = 167 \text{ kNm}$$

Table 12.22 Factored bending moments for the middle strip (Span 1-2)

		End Span 1-2: M_o = 167 kNm		
		B Negative moment M_1 (kNm)	Midspan Positive moment M_m (kNm)	C Negative moment M_2 (kNm)
Longitudinal direction	Bending moments at critical sections	$-0.26M_o$ $= (-0.26) \times 167$ $= -43$	$+0.52M_o$ $= (+0.52) \times 167$ $= +87$	$-0.70M_o$ $= (-0.70) \times 167$ $= -117$
Transverse distribution - column strip	CSA A23.3 Provisions Proposed value Design moment	$-0.26M_o$ $-0.26M_o$ -43	$+(0.29 \text{ to } 0.34)M_o$ $+0.3M_o$ $+0.3 \times 167 = +50$	$-(0.49 \text{ to } 0.63)M_o$ $-0.5M_o$ $-0.5 \times (167) = -84$
Transverse distribution - middle strip	Design moment	0	$=87-50 = +37$	$=-117-(-84) = -33$

4. Perform deflection calculations for the column strip and the middle strip.

The deflection calculation procedure as presented in this example can be presented in a tabular form. The key equations are summarized below.

a) Material properties

The modulus of elasticity of concrete (E_c):

$$E_c = 4500\sqrt{f_c'} = 4500\sqrt{30} = 24650 \text{ MPa}$$ [2.2]

Modular ratio:

$$n = \frac{E_s}{E_c} = \frac{200000}{24648} = 8.1$$ [4.7]

Reduced modulus of rupture (f_r^*):

$$f_r^* = 0.5 \cdot f_r = 0.3\lambda\sqrt{f_c'} = 0.3 \cdot 1.0 \cdot \sqrt{30} = 1.64 \text{ MPa}$$ [12.38]

b) Cross-sectional dimensions (see the sketch below)

Column strip:

b = 2400 mm

Middle strip:

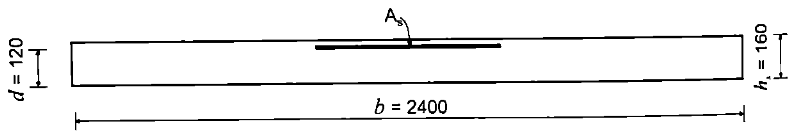
b = 3100 mm

Slab thickness:

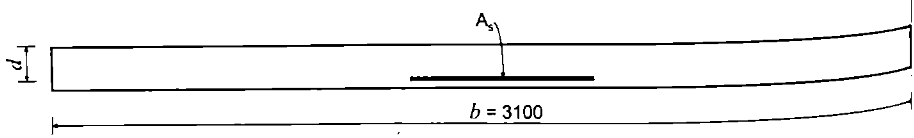
h_s = 160 mm

Effective depth (based on a 25 mm average cover and 15M rebar size):

d = 120 mm



Column strip
cross-section



Middle strip
cross-section

b) Section properties for deflection calculations (see Section 4.3)

- Gross moment of inertia for a rectangular section: [4.2]

$$I_g = \frac{b \cdot h_s^3}{12}$$

- Cracked section properties:

i) Reinforcement ratio: [3.1]

$$\rho = \frac{A_s}{b \cdot d}$$

ii) Neutral axis depth for the cracked section:

$$\bar{y} = d\left(\sqrt{(n\rho)^2 + 2n\rho} - n\rho\right)$$ [4.9]

iii) Moment of inertia for the cracked section:

$$I_{cr} = \frac{b \cdot \bar{y}^3}{3} + n \cdot A_s \cdot (d - \bar{y})^2$$ [4.10]

- The cracking moment (M_{cr}):

$$M_{cr} = \frac{f_r^* \cdot I_g}{y_t}$$ [4.1]

where y_t is the distance from the centroid of the section to the extreme tension fibre. For a rectangular slab section it follows that

$$y_t = \frac{h_s}{2} = \frac{160}{2} = 80 \text{ mm}$$

- The effective moment of inertia (I_e):

A23.3 Eq. 9.1

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$ [4.11]

- Average effective moment of inertia
  i) For an interior span (e.g. column strip for span BC):

$$I_{e,avg} = 0.7I_{e,m} + 0.15(I_{e1} + I_{e2})$$ [4.18]

  ii) For an end span (e.g. middle strip for span 1–2):

$$I_{e,avg} = 0.75I_{e,m} + 0.25I_{ec}$$ [12.39]

c) Deflection calculation equations

The deflections need to be calculated for the dead load and the combined dead plus live load. The maximum deflection for a column strip or a middle strip can be determined from the following equation:

$$\Delta = k\left(\frac{5}{48}\right)\frac{M_m l_n^2}{E_c I_e}$$ [4.12]

where

$$k = 1.2 - 0.2\frac{M_o}{M_m}$$ [4.17]

and

$$M_o = M_m + (M_1 + M_2)/2$$ [12.37]

Note that the factored bending moments for column strips and middle strips were calculated in Step 3. Since the service load moments are required for deflection calculations, it is required to prorate the factored bending moment. In this example, two different load cases are used: the dead load and the dead plus live load. The following scaling factors are used to find service level bending moments:

Dead load deflection: $(DL_w + DL_s)/w_f$ = 5.28 kPa/12.0 kPa = 0.44

Dead plus live load deflection: $[(DL_w + DL_s) + LL_w]/w_f$ = 8.88 kPa/12.0 kPa = 0.74

The deflection calculation procedure is as follows:

1. Calculate the dead load deflection (Δ_D) due to the dead load bending moment (M_D).

2. Calculate the deflection (Δ_{D+L}) due to the combined dead and live load corresponding to the total bending moment (M_{D+L}).
3. Find the immediate live load deflection (Δ_L) :

$$\Delta_L = \Delta_{D+L} - \Delta_D$$

Deflection calculations are presented in the table below (see the sketch showing notation related to the bending moments). Note that the design of flexural reinforcement has been omitted from this example (refer to Example 12.1 for the reinforcement calculation).

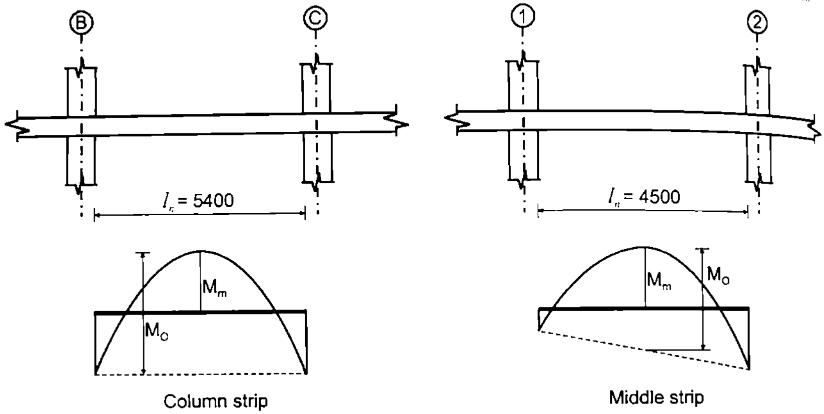


Table 12.23 Immediate deflection calculations for the column strip and the middle strip

		Column Strip			Middle Strip		
		Support B	Midspan	Support C	Support 1	Midspan	Support 2
l_n	(mm)		5400			4500	
M_f	($\times10^6$ Nmm)	124.0	48.0	124.0	0	37.0	33.0
A_s	(mm^2)	3200	1200	3200	3200	1200	3200
ρ		0.011	0.004	0.011	0.009	0.003	0.009
$\bar{y}$	(mm)	41	27	41	37	24	37
I_{cr}	($\times10^6$ mm^4)	217	100	217	231	104	231
I_g	($\times10^6$ mm^4)	819	819	819	1060	1060	1060
M_{cr}	($\times10^6$ Nmm)	16.8	16.8	16.8	21.7	21.7	21.7
M_D	($\times10^6$ Nmm)	54.6	21.1	54.6	0	16.3	14.5
$M_{a,D}$	($\times10^6$ Nmm)	75.7	75.7	75.7	23.5	23.5	23.5
k_D		0.48	0.48	0.48	0.91	0.91	0.91
I_{eD}	($\times10^6$ mm^4)	230	460	230	0	1060	1060
$I_{eD,av}$	($\times10^6$ mm^4)	390	390	390	1060	1060	1060
Δ_D	(mm)		3.2			1.2	
M_{D+L}	($\times10^6$ Nmm)	92.0	36.0	92.0	0	27.0	24.0
$M_{a,D+L}$	($\times10^6$ Nmm)	127.0	127.0	127.0	39.6	39.6	39.6
k_{D+L}		0.48	0.48	0.48	0.91	0.91	0.91
I_{eD+L}	($\times10^6$ mm^4)	220	180	220	0	580	810
$I_{eD+L,av}$	($\times10^6$ mm^4)	190	190	190	640	640	640
Δ_{D+L}	(mm)		11.2			3.4	
Δ_L	(mm)		8.0			2.2	

5. **Check whether immediate deflections are within the CSA A23.3 limits.**

CSA A23.3 limits immediate live load deflections for an occupancy where the roof or floor construction is supporting or is attached to non-structural elements which are not likely to be damaged by large deflections (CSA A23.3 Table 9.3): $l_n/360$.

Live load deflections are determined by the following equation (see Step 4):

$$\Delta_L = \Delta_{D+L} - \Delta_D$$

The check is performed for the column strip, the middle strip, and the slab panel. Span length for the slab panel is determined as follows:

$$l_n = \sqrt{(5.4\ \text{m})^2 + (4.5\ \text{m})^2} = 7.0\ \text{m}$$

The deflection calculations are summarized below.

Table 12.24 Immediate deflections: summary calculations

		Column Strip	Middle Strip	Slab Panel
l_n	(mm)	5400	4500	7000
Δ_D	(mm)	3.2	1.2	=3.2+1.2=4.4
Δ_{D+L}	(mm)	11.2	3.4	=11.2+3.4=14.6
Δ_L	(mm)	8.0	2.2	=14.6−4.4=10.2
CSA A.23.3 deflection limit: $l_n/360$	(mm)	5400/360 = 15	4500/360 = 12	7000/360 = 19
Deflection check	(mm)	8.0<15.0 OK	2.2<12.0 OK	10.2<19.0 OK

6. **Check whether long-term deflections are within the CSA A23.3 limits.**

Long-term deflections will be calculated separately for the column strip and the middle strip. It is assumed that 20% of the live load is sustained, and the corresponding deflection is

$$\Delta_{LS} = 0.2\Delta_L$$

The procedure has the three steps, and it is presented in Table 12.25. A sample calculation for the column strip is shown below.

a) Find the sustained deflections after 1 month.

Calculate $s = 0.5$ by interpolation from Table 4.1. Therefore,

$$\zeta_{t1} = 1 + \frac{s}{1+50\rho'} = 1 + \frac{0.5}{1+0} = 1.5 \quad [4.15]$$

where $\rho' = 0$ since there is no compression reinforcement. Finally,

$$\Delta_{t1} = \zeta_{t1} \cdot (\Delta_D + \Delta_{LS}) = 1.5(3.2 + 0.2 \cdot 8.0) = 7.2\ \text{mm}$$

b) Find the sustained deflections after 5 years

Use $s = 2$ from Table 4.1, and calculate $\zeta_{t2} = 3$ from equation [4.15]. Finally,

$$\Delta_{t2} = \zeta_{t2} \cdot (\Delta_D + \Delta_{LS}) = 3.0(3.2 + 0.2 \cdot 8.0) = 14.4\ \text{mm}$$

c) Find the final long-term deflection.

$$\Delta_t = (\Delta_{t2} - \Delta_{t1}) + (\Delta_L - \Delta_{LS}) = (14.4 - 7.2) + (8.0 - 0.2 \cdot 8.0) = 13.6\ \text{mm} \quad [12.41]$$

Once the column strip and middle strip deflections have been calculated, the long-term deflections for the slab panel can be calculated by summing up these two values.

Table 12.25 Long-term deflections: summary calculations

		Column Strip	Middle Strip	Slab Panel
l_c	(mm)	5400	4500	7000
Δ_D	(mm)	3.2	1.2	
Δ_l	(mm)	8.0	2.2	
Sustained deflections after 1 month $s = 0.5$ $\zeta_{t1} = 1.5$ $\Delta_{t1} = \zeta_{t1} \cdot (\Delta_D + \Delta_{LS})$	(mm)	7.2	2.5	
Sustained deflections after 5 years $s = 2$ $\zeta_{t2} = 3$ $\Delta_{t2} = \zeta_{t2} \cdot (\Delta_D + \Delta_{LS})$	(mm)	14.4	4.9	
$\Delta_t = (\Delta_{t2} - \Delta_{t1}) + (\Delta_l + \Delta_{LS})$	(mm)	13.6	4.2	=13.6+4.2=17.8
CSA A23.3 deflection limit: $l_n/240$	(mm)	5400/240 = 22	4500/240 = 19	7000/240 = 29
Deflection check	(mm)	13.6<22 OK	4.2<19 OK	17.8<29 OK

Note that the CSA A23.3 limits for immediate and long-term deflections may be different. In this example, the limit for immediate live load deflection is $l_n/360$, while the long-term deflection limit which includes the effect of sustained and transient loads is $l_n/240$. Both limits apply to roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections.

Example 12.12

Two-Way Flat Plate - Deflection Calculations According to the Crossing Beam Method and the Alternative Procedure to Account for the Effect of Construction Loads

Consider a floor plan of a two-way slab system without beams (flat plate) from Example 12.11.

Use the Crossing Beam Method to calculate immediate and long-term deflections for the slab panel, however use the effective moment of inertia at the construction load level for all deflection components. Assume that the construction load is equal to twice the dead load.

SOLUTION: The approach for solving this problem is similar to that taken in Example 12.11, with the following exceptions:

1) The effective moment of inertia for construction dead load, I_{eD}, was found assuming that the moment due to construction dead load, M_{const}, is twice the actual value (M_D). This calculation is illustrated below, for a column strip (bending moment at support B):

$$M_{const} = 2 \cdot M_D = 2 \cdot (54.6 \cdot 10^6) = 109.2 \cdot 10^6 \text{ Nmm}$$

$$I_{e(constr)} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \qquad [4.11]$$

$$= 217 \cdot 10^6 + \left(819 \cdot 10^6 - 217 \cdot 10^6\right)\left(\frac{16.8 \cdot 10^6}{109.2 \cdot 10^6}\right)^3 \cong 220 \cdot 10^6 \text{ mm}^4$$

Note that the deflections due to dead load are calculated using the actual M_D value (equal to 54.6×10^6 Nmm for the column strip at support B).

2) The effective moment of inertia for the total dead plus live load, $I_{e(D+L)}$, is equal to $I_{e(constr)}$ as discussed above (note that two different values were used in Example 12.11).

The deflection calculation procedure was revised to take into account the effect of construction loads by considering the effective moment of inertia corresponding to the assumed cracked value, as shown in Figure 12.90.

Table 12.26 Immediate deflection calculations for the column strip and the middle strip considering the effect of construction loads

		Column Strip			Middle Strip		
		Support B	Midspan	Support C	Support 1	Midspan	Support 2
l_n	(mm)		5400			4500	
M_u	($\times 10^6$ Nmm)	124.0	48.0	124.0	0	37.0	33.0
A_s	(mm^2)	3200	1200	3200	3200	1200	3200
ρ		0.011	0.004	0.011	0.009	0.003	0.009
$\bar{y}$	(mm)	41	27	41	37	24	37
I_{cr}	($\times 10^6$ mm^4)	217	100	217	231	104	231
I_g	($\times 10^6$ mm^4)	819	819	819	1060	1060	1060
M_{cr}	($\times 10^6$ Nmm)	16.8	16.8	16.8	21.7	21.7	21.7
M_D	($\times 10^6$ Nmm)	54.6	21.1	54.6	0	16.3	14.5
M_{aD}	($\times 10^6$ Nmm)	75.7	75.7	75.7	23.5	23.5	23.5
k_D			0.48			0.91	
$I_{e,constr}$	($\times 10^6$ mm^4)	220	150	220	0	386	576
$I_{e,cracked}$	($\times 10^6$ mm^4)	170	170	170	430	430	430
Δ_D	(mm)		7.5			2.9	
M_{D+L}	($\times 10^6$ Nmm)	92.0	36.0	92.0	0	27.0	24.0
M_{aD+L}	($\times 10^6$ Nmm)	127.0	127.0	127.0	39.6	39.6	39.6
k_{D+L}			0.48			0.91	
$I_{e,constr}$	($\times 10^6$ mm^4)	220	150	220	0	386	576
$I_{e,cracked}$	($\times 10^6$ mm^4)	170	170	170	430	430	430
Δ_{D+L}	(mm)		12.6			4.9	
Δ_L	(mm)		5.1			2.0	

According to the approach presented in this example, the same effective moment of inertia $I_{e(constr)}$ was used to calculate the dead load deflection (Δ_D) and the deflection due to total load (Δ_{D+L}). Therefore, we could have found (Δ_L) directly by finding the live load moment (M_L) and solving for (Δ_L). The deflection calculations are summarized in Tables 12.27 and 12.28.

Table 12.27 Immediate deflections: summary calculations

		Column Strip	Middle Strip	Slab Panel
l_n	(mm)	5400	4500	7000
Δ_D	(mm)	7.5	2.9	=7.5+2.9=10.4
Δ_{D+L}	(mm)	12.6	4.9	=12.6+4.9=17.5
Δ_L	(mm)	5.1	2.0	=17.5−10.4=7.1
CSA A23.3 deflection limit: $l_n/360$	(mm)	5400/360 = 15	4500/360 = 12	7000/360 = 19
Deflection check	(mm)	5.1<15.0 OK	2.0<12.0 OK	7.1<19.0 OK

Table 12.28 Long-term deflections: summary calculations

		Column Strip	Middle Strip	Slab Panel
l_n	(mm)	5400	4500	7000
Δ_D	(mm)	7.5	2.9	
Δ_L	(mm)	5.1	2.0	
Sustained deflections after 1 month $s = 0.5$ $\xi_{t1} = 1.5$ $\Delta_{t1} = \xi_{t1} \cdot (\Delta_D + \Delta_{LS})$	(mm)	12.8	5.0	
Sustained deflections after 5 years $s = 2$ $\xi_{t2} = 3$ $\Delta_{t2} = \xi_{t2} \cdot (\Delta_D + \Delta_{LS})$	(mm)	25.6	10.0	
$\Delta_t = (\Delta_{t2} - \Delta_{t1}) + (\Delta_L + \Delta_{LS})$	(mm)	16.9	6.6	=16.9+6.6=23.5
CSA A23.3 deflection limit: $l_n/240$	(mm)	5400/240 = 22	4500/240 = 19	7000/240 = 29
Deflection check	(mm)	16.9<22 OK	6.6<19 OK	23.5<29 OK

12.10.4 Deflection Calculations Using the Computer-Aided Iterative Procedure and 2-D Equivalent Frames

The computer-aided iterative procedure uses structural analysis of 2-D equivalent frames to determine deflections in two-way slabs. Each member is divided into several segments characterized by different section properties. Cracked section properties are used where the bending moment exceeds the cracking moment, while gross section properties are used elsewhere along the span. The procedure is described in Section 4.6 as related to reinforced concrete flexural members. An application of this procedure to a two-way flat plate system is illustrated through the following example.

Example 12.13

Two-Way Flat Plate - Deflection Calculations Using The 2-D Computer-Aided Iterative Procedure

Consider the flat plate floor system discussed in Example 12.11.

Compute immediate deflections using the computer-aided iterative procedure.

Given: $f_c' = 30$ MPa
$E_c = 24650$ MPa

SOLUTION: In order to solve this problem, it is required to perform an analysis of two 2-D equivalent frame models: i) an equivalent frame along gridline 2 (between gridlines B and C), and ii) an equivalent frame along gridline B (between gridlines 1 and 2). The iterative analysis will be explained in detail for the equivalent frame along gridline 2.

1. **Find the cross-sectional properties required for the analysis.**

a) Determine cross-sectional dimensions for the slab.

The equivalent frame has the same properties as discussed in Example 12.3. The slab-beam has the width equal to the design strip, that is,

$b = 4800$ mm

and

$h_s = 160$ mm

b) Find the gross moment of inertia for the frame section.

$$I_g = \frac{b \cdot h_s^3}{12} = \frac{4800 \cdot (160)^3}{12} = 16.4 \cdot 10^8 \text{ mm}^4 \qquad [4.2]$$

c) Find the cracking moment (M_{cr}).

$$M_{cr} = \frac{f_r^* \cdot I_g}{y_t} = \frac{1.64 \cdot (16.4 \cdot 10^8)}{80} = 33.5 \text{ kNm} \qquad [4.1]$$

where the reduced modulus of rupture is $f_r^* = 0.5 \cdot f_r = 1.64$ MPa and $y_t = 80$ mm

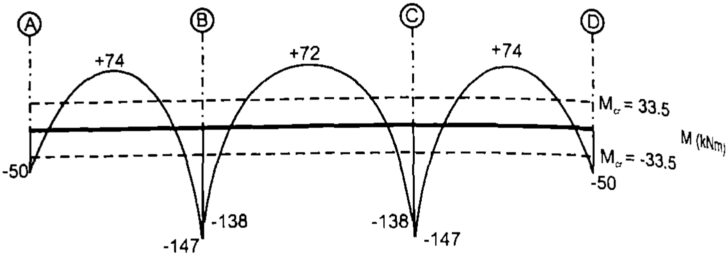
d) Find the cracked moment of inertia:

$$I_{cr} = \frac{b \cdot y^3}{3} + n \cdot A_s \cdot (d - y)^2 \qquad [4.10]$$

The cracked moment of inertia is determined in the same manner as in Example 12.11. Note that I_{cr} values are different along the slab span, depending on the amount of top and bottom reinforcement.

2. **Perform an elastic analysis of the frame using gross cross-sectional properties.**

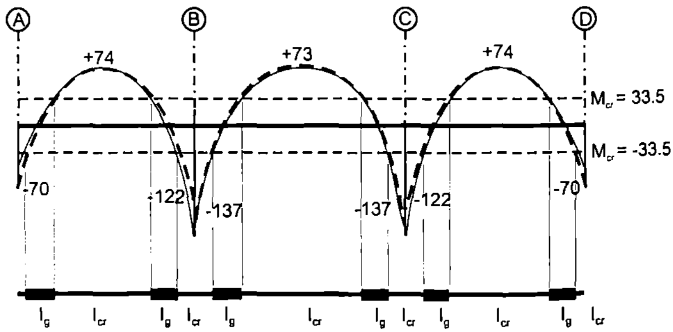
The frame will be modelled using prismatic slab-beam elements. Each span can be divided into 10 or 20 equal rigidly joined segments. Initially, all segments are assigned a gross moment of inertia (I_g) and an elastic analysis is performed. The resulting bending moment diagram for the equivalent frame along gridline 2 is shown on the following sketch. Note that the cracking moment value is superimposed on the diagram to identify cracked regions where bending moment exceeds the cracking moment, that is, $M > M_{cr}$.



3. Perform an iterative analysis using cracked and gross cross-sectional properties.

The regions where bending moments exceed the cracking moment need to be identified and the cracked moment of inertia needs to be assigned to those segments. The revised cross-sectional properties are used to perform an analysis and to find bending moments. A subsequent analysis may identify new segments where bending moments have exceeded the cracking moment. As a result, cracked moment of inertia should be assigned to those segments. The analysis continues until there is no change in the status (cracked/uncracked) for slab segments. Note that, once the cracked slab regions have been identified by the analysis, those segments must be assigned cracked sectional properties for all subsequent analyses.

In this example, three iterations were performed before convergence was reached. The final bending moment distribution (shown with dashed lines) superimposed on the original bending moment diagram is shown below.



4. Determine the slab deflections.

Short-term deflections:

a) The equivalent frame along gridline 2, between gridlines B and C:

Dead load: $\Delta_D = 6$ mm

Total dead+live load: $\Delta_{D+L} = 10$ mm

Live load: $\Delta_L = \Delta_{D+L} - \Delta_D = 10 - 6 = 4$ mm

b) The equivalent frame along gridline B, between gridlines 1 and 2:

Dead load: $\Delta_D = 3$ mm

Total dead+live load: $\Delta_{D+L} = 5$ mm

Live load: $\Delta_L = \Delta_{D+L} - \Delta_D = 5 - 3 = 2$ mm

c) Total deflections

Dead load: $\Delta_D = 6 + 3 = 9$ mm

Total dead+live load: $\Delta_{D+L} = 10 + 5 = 15$ mm

Live load: $\Delta_L = 4 + 2 = 6$ mm

12.10.5 Deflection Calculations Using the Computer-Aided Iterative Procedure and 3-D Finite Element Analysis

The underlying concept of this approach is similar to the 2-D iterative analysis, except that the slab system is modelled as a 3-D structure. The slab is modelled as a mesh of finite elements, as described in Section 12.7.3. The analysis is initially performed using gross cross-sectional properties, and the regions where cracking has taken place can be identified by the analysis software, depending on the cracking moment. A few software packages are capable of performing an iterative 3-D analysis and deflection predictions for cracked two-way slabs. Before the solution is obtained, the reinforcement detailing of a slab system must be finalized. This method enables a more accurate prediction of long-term deflections compared to other methods. It is especially suitable for deflection predictions in slabs with a non-rectangular column grid, like the one discussed in 12.7.3. The corresponding deflection contour diagram is shown in Figure 12.91.

Figure 12.91 Deflection contours for an irregular two-way slab.

Example 12.14

Two-Way Flat Plate - Deflection Calculations Using The 3-D Finite Element Analysis

Consider the same flat plate floor system discussed in Example 12.11.

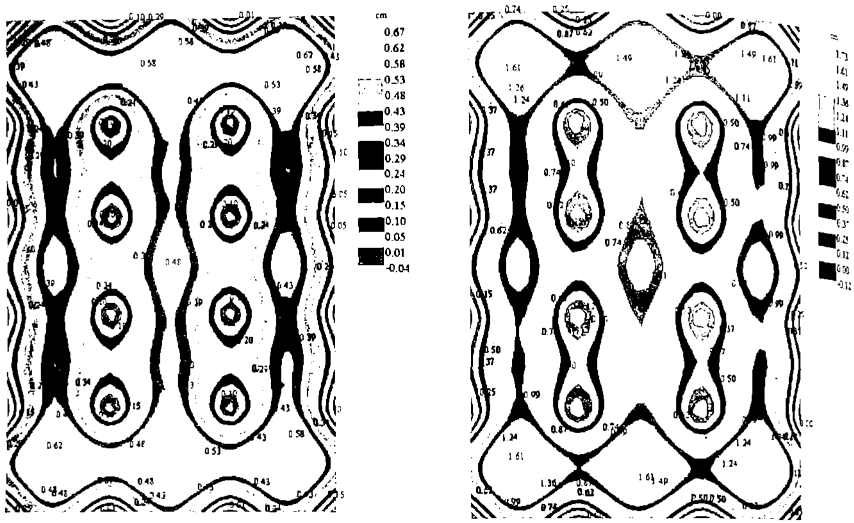
Calculate the immediate deflections using the 3-D Finite Element Analysis procedure.

SOLUTION:

The slab has been modelled as a mesh of finite elements and analyzed using a finite element software package which is able to consider the effect of cracking by performing a numerical iterative structural analysis. The results of the analysis are presented in Figure 12.92.

It can be seen from the figure that the maximum dead load deflection for the slab panel between gridlines B and C and 1 and 2 is $\Delta_D = 6.7$ mm and the dead plus live load deflection is $\Delta_{D+L} = 17.3$ mm. Finally, the immediate live load deflection is equal to:

$$\Delta_L = \Delta_{D+L} - \Delta_D = 17.3 - 6.7 = 10.6 \text{ mm}$$



a) b)

Figure 12.92 Deflection contours for a regular two-way flat slab: a) deflections due to dead load, and b) deflections due to the total load.

Learning from Examples

Previous examples (12.11 to 12.14) were useful to evaluate the results of different deflection calculation procedures. A summary of the results is presented in Table 12.29. A few conclusions are presented below:

1) Both immediate and long-term deflections are within the CSA A23.3 limits according to all calculation procedures, thus the general conclusion is the same irrespective of the method used.
2) Similar immediate live load deflection values are obtained for the Crossing Beam Method (Example 12.11) and the 3-D Finite Element Method (Example 12.14) (10.2 versus 10.6 mm). The value obtained from the Computer-Aided Iterative Procedure (Example 12.13) is the smallest of all (6.0 mm).
3) The results obtained from the Crossing Beam Method depend on whether the effect of construction load has been taken into account. For immediate live load deflections, the standard procedure used in Example 12.11 gives larger values (10.2 mm) compared to the alternative procedure which takes into account the effect of construction load (7.1 mm). However, note that the dead load deflection obtained by the standard procedure (4.4 mm) is significantly less than the value of 10.4 mm obtained from the alternative procedure. As a result, long-term deflections obtained from the alternative procedure (23.5 mm) are significantly higher (more conservative) that those obtained by the standard procedure (17.8 mm).
4) The 3-D FEA takes into account the effect of cracking and it is considered to be the most accurate of all methods. However, it is still appropriate to use an approximate method such as the Crossing Beam Method to estimate deflections in regular slabs.

This comparison illustrates that the margin of error associated with deflection calculation could be large and it depends on the procedure used. The designer must have a good understanding of the governing principles behind various deflection calculation procedures.

Table 12.29 Deflection calculation procedures: a summary of the results for Examples 12.11 to 12.14

		Crossing Beam Method - Standard Procedure (Example 12.11)	Crossing Beam Method - Alternative Procedure (Example 12.12)	Computer - Aided Iterative Procedure (2-D) (Example 12.13)	3-D Finite Element Analysis (Example 12.14)	CSA A23.3 Deflection Limits
			Immediate deflections			
Δ_D	(mm)	4.4	10.4	9.0	6.7	
Δ_{D+L}	(mm)	14.6	17.5	15.0	17.3	$l_n/360 = 19$
Δ_L	(mm)	10.2	7.1	6.0	10.6	
			Long-term deflections			
Δ_t	(mm)	17.8	23.5	*	*	$l_n/240 = 29$

Note

* - The long-term deflections were not calculated for Examples 12.13 and 12.14.

12.10.6 Practical Guidelines for Deflection and Cracking Control

Deflection control is one of the key design considerations for two-way slabs. Several construction- or design-related options are available to ensure that deflections are within the CSA A23.3 allowable limits, and that the cracking is not excessive. The designer can control deflections in two-way slabs by means of some of the following construction procedures:

1. Specify additional construction procedures, such as delay strips and control joints, to allow the slab to shrink more freely during construction.
2. Specify a delayed formwork stripping and reshoring procedure to reduce the effect of creep due to cracking of early age concrete.
3. Perform cambering of the slab by adjusting the formwork to have a camber or a "crown" at midspan regions. The purpose of cambering is to reduce the appearance of sag in the slab. The amount of camber is somewhat subjective, and can be as high as the total anticipated long-term deflection due to sustained loads. This strategy can be effective for reducing visual and functional effects of deflections in thin slabs, and particularly in slabs with a large dead/live load ratio.
4. Where long-term deflections are likely to induce significant strains in architectural building components, the problem can be mitigated by providing vertical slip joints in partitions and window walls. This measure does not restrict the deflections in the slab, but it prevents damage in non-structural elements.

Apart from design-related errors, some of the most common causes of excessive slab deflections are due to poor construction practices. Relevant construction considerations for two-way slabs are discussed in the next section.

12.11 CONSTRUCTION CONSIDERATIONS AND DRAWINGS

Efficiently designed two-way slabs are considered to be among the most cost-effective structural systems. The overall construction cost associated with two-way slabs is influenced by the amount of concrete and steel, labour and material costs associated with the formwork and shoring and reshoring. The labour costs are a significant component of two-way slab construction, thus a design that minimizes labour usage by taking into consideration simplicity and speed of the formwork erection and rebar placement often results in the most cost-efficient solution. An optimal design strikes the balance between material usage and labour efficiency. To produce a safe and a cost-effective design solution, the designer needs to consider a few important constructability issues, such as i) simplify formwork requirements to speed up the construction process, ii) consider the shoring and reshoring in design, and ii) produce a simple reinforcement layout. These issues are discussed next.

Formwork When the design results in a labour-intensive formwork, the construction process is less efficient and the formwork cost is higher. Flat plates are easy to form, however forming the slabs with beams or drop panels/capitals is more difficult and time-consuming. When construction speed is important, the designer could attempt to simplify the formwork by designing a flat plate instead of a flat slab or a slab with beams. This might be done at the expense of a thicker slab and a larger amount of steel.

Shoring and reshoring After the concrete has been placed in the forms, formwork and shoring are required to support the slab before the concrete gains sufficient strength to be able to support the slab self-weight. Shoring must be adequately designed by considering the deflection and/or settlement tolerances in order to avoid additional stresses in the slab.

In multi-storey high-rise construction, contractors usually wish to pour one floor every few days, thus the formwork and the shoring need to be stripped once the concrete at a specific floor level has gained sufficient strength. The same process is repeated at each floor level. As the building construction progresses, a temporary construction load on each floor slab may exceed the permanent service (specified) design load. The problem is often compounded by the fact that the temporary construction load is placed on a partially cured slab.

The most vulnerable stage in the slab construction is after the shoring is removed and before the reshoring is installed, because young concrete hasn't reached sufficient design strength while being subjected to loads sometimes beyond the design service loads. Early age construction loading can lead to excessive immediate and long-term deflections in flat plates and flat slabs.

Construction sequencing must be considered in the design of flat slabs for multi-storey buildings. The designer may need to specify the maximum construction loads and work with the contractor to ensure that a proper amount of shoring and reshoring is in place in order to avoid overloading of slab during construction. An example of a concrete building under construction is shown in Figure 12.93. The concrete is being placed onto the flyform on the top floor, while partially cured slabs below have reshoring to help support the load from the young (wet) concrete.

Reinforcement placement and detailing Two-way slabs, especially slabs with complex geometry, are often designed using sophisticated computer-based analysis tools. Regardless of the analysis method used, the designer needs to exercise judgment while using the analysis results to prepare design drawings and specifications. The designer must use good judgment by taking into account the underlying assumptions, and also recognize the approximations associated with the structural model. The designer can take advantage of moment redistribution, discussed earlier in this chapter, to help even out negative bending moments in different spans. This could help produce similar bending moments and reinforcement for different spans. The resulting design solution and reinforcement details should be simple, repetitive, and uniform as much as possible. One of the key objectives is to ensure that the construction crew can easily understand the design drawings without confusion. For example, it is a good idea to specify reinforcement mats that are symmetrical over the columns. A repeated use of similar mats in both orthogonal directions is a good strategy for minimizing placement errors. In many cases, an optimal design solution includes consideration of practical construction constraints.

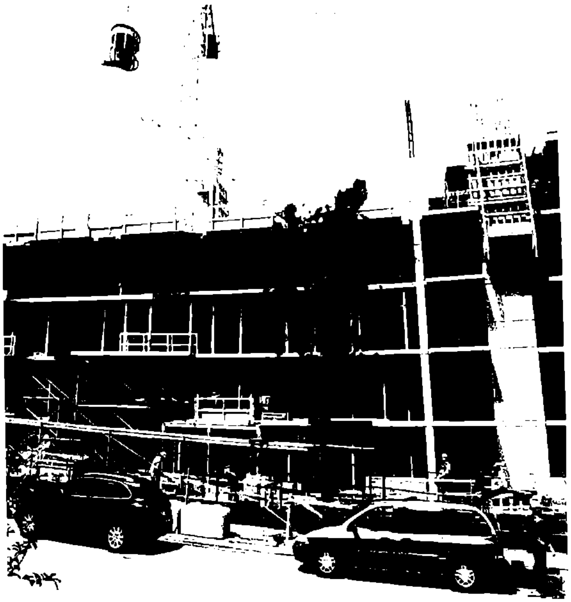


Figure 12.93 A concrete building under construction showing shoring and reshoring of flat slab floors (John Pao).

An example of a reinforcement plan for a flat plate designed in Examples 12.1 and 12.3 is presented in Figure 12.94.

It can be seen from the drawing that four different mats have been specified (A to D). Note that Mat A has been specified over all of the interior columns. The mat is symmetrical with regards to a column.

Additional relevant information concerning the reinforcement placement is provided on the notes and diagrams, such as the one shown in Figure 12.95. Note that the information related to anchorage is contained in the notes. For example, Note 6 specifies details of top reinforcement at end spans, which is usually hooked at one end (H1E).

A flat plate floor system under construction is shown in Figure 12.96.

It is important to ensure a reasonably accurate placement of reinforcement in compliance with the specified tolerances. Misplacement of reinforcement might have significant consequences upon the structural performance. For example, the placement of top reinforcement over columns at a lower level than specified by the design may lead to excessive cracking and rotation at the supports due to high bending moments, thereby resulting in increased midspan deflections.

The designer also needs to be familiar with the requirements related to the curing of concrete. Conformance to specified field curing as defined in the CSA A23.1 Tables 2 and 20 is essential to assure an adequate early strength gain. An insufficient early strength gain prior to stripping formwork might result in a significant reduction in the slab flexural stiffness and increased deflections.

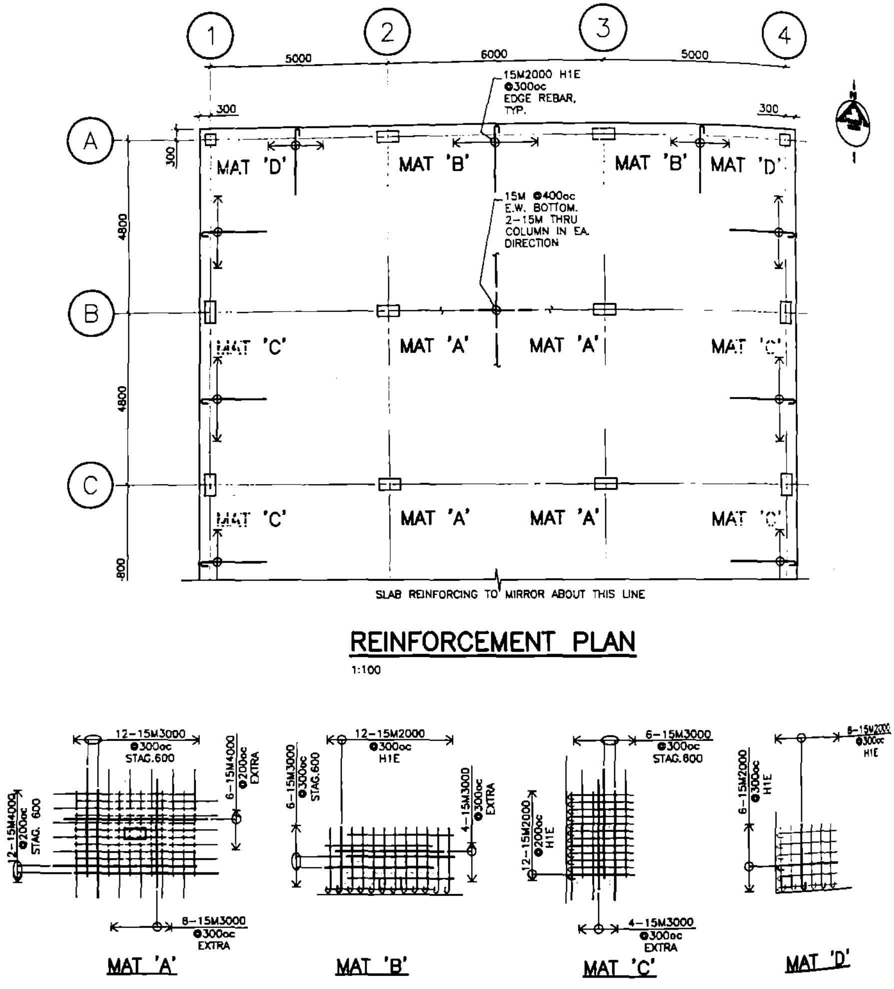


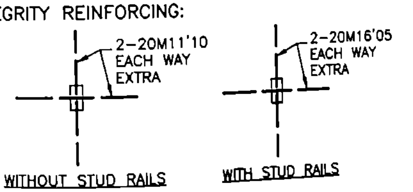
Figure 12.94 Reinforcement plan for a regular flat plate system.

NOTES:

1. THIS DRAWING TO BE READ IN CONJUNCTION WITH GENERAL NOTES ON DWG. S1.1 AND TYPICAL DETAILS ON DWG. S4.1, S4.2 AND S4.3. SEE ARCHITECTURAL DRAWINGS FOR ALL LAYOUT INFORMATION INCLUDING ELEVATIONS, COLUMN LOCATIONS, AND SLAB EDGE LOCATIONS.
2. REFER TO DWG. S3.1 FOR FOOTING, WALL AND COLUMN SCHEDULES.
   REFER TO DWG. S3.2 FOR ZONE DETAILS.
   REFER TO DWG. S3.3 FOR MAT DETAILS.
3. SLAB 7" THICK R/W 15M @12"oc E.W. BOTTOM U.N.O.
4. SPLICE BOTTOM REINF. WITH 3'-0" SPLICE AT COLUMN LINES ONLY. EXTEND BARS 18" PAST CENTRE OF SUPPORTING COLUMN.
5. BAR STAGGERS SHOWN ON PLAN ARE 24" U.N.O.
6. EDGE TOP HOOKED REINF. SHOWN ON PLAN 15M07'04 @12"oc H1E U.N.O.
7. DO NOT USE MAIN SLAB REINF. AS DROPPED CARRY BARS. ADD EXTRA REINF. FOR DROPPED CARRY BARS.
8. CRANK BARS SHOWN ON PLAN AS NEEDED TO SUIT SLAB SLOPES.
9. (A) DENOTES PUNCHING SHEAR REINFORCING SEE DWG. S3.3A

SLAB BAR PLACEMENT ORDER	
UPPER LAYER TOP (NORTH–SOUTH DIRECTION)	UPPER LAYER BOTTOM (EAST–WEST DIRECTION)
LOWER LAYER TOP (EAST–WEST DIRECTION)	LOWER LAYER BOTTOM (NORTH–SOUTH DIRECTION)

INTEGRITY REINFORCING:

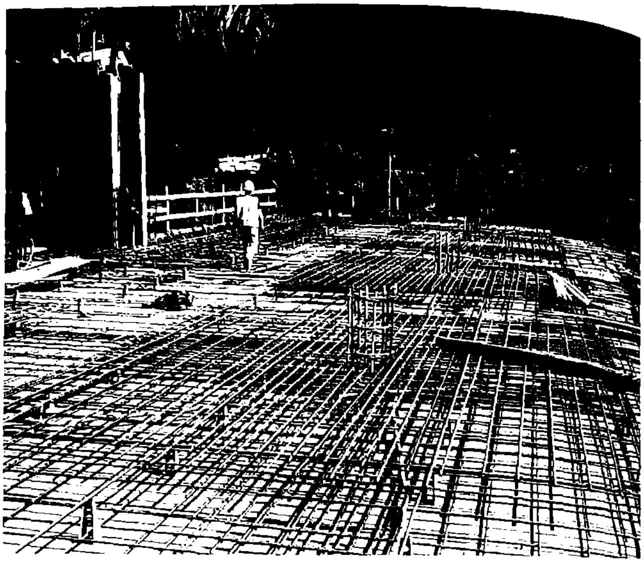


NOTES:

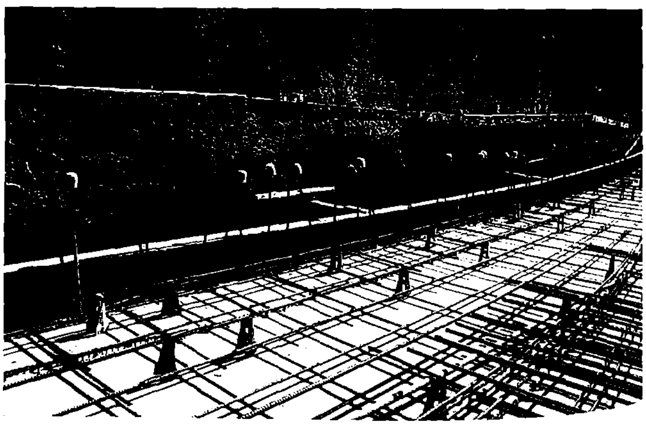
1. INTEGRITY REINFORCING IS REQUIRED FOR ALL COLUMNS.
2. THE TWO (2) INTEGRITY REINFORCING BARS IN EACH DIRECTION MUST PASS THROUGH THE COLUMN.

Figure 12.95 Notes related to reinforcement in two-way slabs.

Figure 12.96 A flat plate floor under construction: a) reinforcement mats (Ron Hopen), and b) hooked top reinforcement (Svetlana Brzev).



a)



b)

Most two-way slab systems in reinforced concrete buildings are floor and roof slabs supported by columns and/or walls. These systems can be divided into one-way and two-way slabs depending on the manner in which they transfer gravity loads. One-way slabs transfer gravity loads in one direction, while two-way slabs are continuous structures spanning in two directions. Two-way slabs are easy to form and construct, and they are the most popular floor system for multi-storey building construction in Canada.

Types of two-way slabs

The four main types of two-way slab systems are as follows:

- *Flat plate* consists of solid slabs reinforced in two directions and supported directly by columns or walls. This system is economical for short and medium spans. The slab thickness is usually controlled by long-term deflections and punching shear.
- *Flat slab* is a column- or a wall-supported slab system with drop panels, that is, thickened regions of the slab centered at the columns. Drop panels are provided to increase the punching shear capacity of the slab and help to control deflections in midspan regions.
- *Waffle slab* consists of evenly spaced concrete joists spanning in two directions — this system is also known as a two-way joist system. The joists are formed by using standard pans or domes installed in the forms to produce a coffered soffit in the slab. Waffle slabs offer an economical design solution for longer spans, and are particularly advantageous when the use of heavy loads is desired without the use of deepened drop panels, capitals, or support beams.
- *Slab with beams* consists of solid slab panels supported by beams on all four sides. When the ratio of the span lengths for a panel approaches 2.0, load is predominately transferred by bending in the short direction and the panel essentially acts as a one-way slab. As the panel approaches a square shape, a significant load is transferred by bending in both orthogonal directions, and the panel should be treated as a two-way slab.

Design of two-way slabs for flexure

The following four procedures can be used for flexural design of two-way slabs according to CSA A23.3:

1. Direct Design Method (DDM) is a statics-based method which can be used to design regular two-way slabs for flexure (Cl.13.9). The method treats a slab as a wide beam with a width equal to the tributary portion between column centrelines. Prescribed moment coefficient values can be used to determine bending moments at critical locations within a slab span (supports and midspan). The moment at each critical location is distributed transversely between column and middle strips in flat slabs and flat plates, or between beam and slab strips in slabs with beams.
2. Equivalent Frame Method (EFM) idealizes a 3-D building structure consisting of slabs and columns as a series of parallel 2-D equivalent frames in each principal direction of the building. Each equivalent frame is analyzed separately, and the results are combined to create a design solution for an entire slab at the floor level. CSA A23.3 Cl.13.8 refers to this method to as "slab systems as elastic frames".
3. Three-Dimensional Elastic Analysis idealizes a structure as a 3-D model, where slabs are modelled as 2-D (plate) finite elements, and columns are modelled as linear elements. This is a computer-based numerical analysis procedure based on the Elastic Plate Theory (Cl.13.6). This method is appropriate for design of complex slabs with irregular (non-rectangular) plan shapes, or slabs with regular plans characterized by large column and wall offsets relative to a rectangular grid.
4. Theorems of Plasticity (Cl.13.7) are able to predict the ultimate load capacity of a slab: the Yield Line Method (YLM) and Hillerborg's strip method give an upper- and a lower-bound estimate respectively. The YLM can predict the ultimate load (load capacity) for a slab panel with flexure-controlled behaviour. The analysis is performed by applying the Virtual Work Method to compare possible yield patterns (failure scenarios) for a particular slab. The governing yield pattern is the one that gives the least load capacity at the ultimate stage. When the ultimate load capacity in the slab has been reached, plastic

rotation occurs along yield lines, straight cracks across which the reinforcing bars have yielded, while regions between these yield lines act like rigid bodies.

Design of two-way slabs for shear

Shear failure in two-way slabs is sudden and must be carefully considered in the design. The two main CSA A23.3 shear design considerations are related to

- one-way shear (Cl.13.3.6) and
- two way shear (Cl.13.3.3 to 13.3.5).

The two-way shear (or punching shear) resistance is particularly critical for flat plates and flat slabs, due to chances of potentially catastrophic collapse associated with this failure mechanism. Shear stresses in two-way slabs are due to the combined effect of gravity loads and unbalanced bending moments transferred through slab-column connections. A fraction of the bending moment at the slab-column connection is transferred through flexure and resisted by the flexural reinforcement, while the remainder is transferred through shear stresses at the critical perimeter. Shear resistance in two-way slabs is provided by concrete and reinforcement. Shear stresses in a slab with beams may be resisted jointly by the slab and the supporting beams.

Deflections in two-way slabs

Deflection control is one of the key design considerations for two-way slabs. Excessive deflections can cause damage to non-structural elements such as partitions and glazing, and noticeable deflections appear unsafe. Immediate and long-term deflections due to service loads must remain within the limits prescribed by CSA A23.3 Table 9.3.

CSA A23.3 prescribes the following two approaches for deflection control:

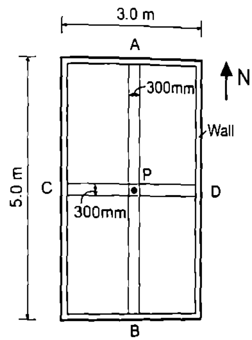
1. Indirect approach permits the use of minimum slab thickness which results in a robust design, thus detailed deflection calculations are not required (Cl.13.2.2). This approach can be applied only to regular two-way slabs.
2. Detailed deflection calculations must be performed for slabs with span-to-thickness ratio below the CSA A23.3 limit.

Deflections in two-way slabs can be estimated by applying one the following three methods:

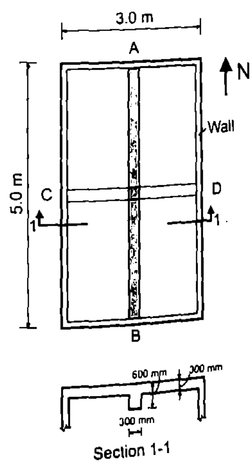
1. The Crossing Beam Method is based on treating a two-way slab as an orthogonal one-way system, thus allowing the deflection calculations by beam analogy.
2. The Equivalent Frame Method uses a linear elastic analysis of 2-D frames. An effective moment of inertia is calculated across the full width to account for cracking.
3. The Finite Element Method is a computer-based method which can be used to obtain both internal forces and deflections in two-way slabs. Depending on the software capabilities, it can be used either for linear elastic or nonlinear analysis which takes into account the effect of cracking.

Unless noted otherwise, the following material properties should be used: concrete $f_c' = 30$ MPa (normal density concrete) and steel $f_y = 400$ MPa.

12.1. Consider a 300 mm thick flat plate panel supported by walls on all sides shown in the figure. The slab is subjected to a 1000 kN point load at midspan (point P). Estimate the reactions at points A and C. Assume that imaginary 300 mm wide strips AB and CD carry the load simultaneously.



12.2. The slab of Problem 12.1 has been modified to include beam AB in the longitudinal direction, as shown in the figure. The slab is subjected to the same point load (1000 kN) as in Problem 12.1. Find the reactions at points A and C, by using the same strips AB and CD of Problem 12.1.

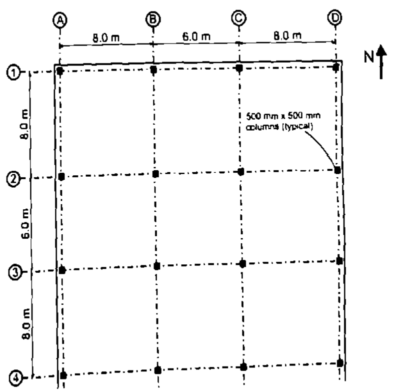


12.3. Consider the flat plate system from Example 12.1. Use the Direct Design Method to determine design bending moments and reinforcement for an interior frame along gridline B. Use the same slab thickness (180 mm) as in Example 12.1. Consider unbalanced moments and moment transfer through slab-column connections in the design.

12.4. Redesign the slab of Problem 12.3 using the Equivalent Frame Method. Compare the bending moments and reinforcement obtained using the Equivalent Frame Method and the Direct Design Method.

12.5. A typical floor plan of a hospital building is shown in the figure. The floor system is a flat plate and it is subjected to specified live load (LL) of 3.6 kPa, and superimposed dead load (DL_s) of 1.5 kPa, in addition to its self-weight. Design a typical interior panel for flexure. Consider unbalanced moments and moment transfer through slab-column connections in the design. Select the slab thickness such that a deflection check is not required according to CSA A23.3. Assume that edge beams are not provided.

a) Use the Direct Design Method.
b) Use the Equivalent Frame Method.
c) Compare the results (bending moments and reinforcement).

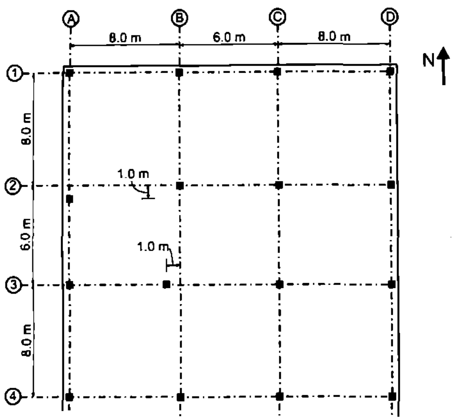


12.6. Redesign the slab from Problem 12.5 by considering drop panels at interior column locations only. Use minimum drop panel dimensions permitted by CSA A23.3. Revise the slab thickness based on the CSA A23.3 requirements for flat slabs. The slab plan dimensions and loading are the same as in Problem 12.5. Design a typical interior panel for flexure.

a) Use the Direct Design Method.
b) Use the Equivalent Frame Method.
c) Compare the results (bending moments and reinforcement).

12.7. Design of the floor system from Problem 12.5 needs to be modified due to architectural constraints. Columns 2-A and 3-B need to be moved and are offset by 1 m relative to the gridlines, as shown in the figure. The slab dimensions and loading are the same as those in Problem 12.5. Design an interior panel for flexure.

a) Use the Direct Design Method.
b) Use the Equivalent Frame Method.
c) Compare the results (bending moments and reinforcement).



12.8. Consider the floor plan from Problem 12.5. The design needs to be modified by providing beams on all sides. Assume 400 mm square columns and beam webs matching the column width. The beam depth should be selected such that the reinforcement ratio is approximately equal to 40% of the balanced ratio (0.4 ρ_b). Revise slab thickness based on the CSA A23.3 requirements for slabs without beams. The plan dimensions and loading are the same as in Problem 12.5. Design a typical interior panel for flexure.

a) Use the Direct Design Method.
b) Use the Equivalent Frame Method.
c) Compare the results (bending moments and reinforcement).

12.9. Consider a two-way flat plate floor system designed in Problem 12.5. Use the slab thickness of 200 mm and an effective depth of 160 mm. The slab loading is the same as in Problem 12.5, resulting in the factored area load $w_f = 12.6kPa$. Design the slab for shear according to the CSA A23.3 requirements. Consider only an interior column at the intersection of gridlines 2 and C.

a) Perform the design by disregarding the effect of unbalanced moments.
b) Consider the effect of unbalanced moments, and also shear and moment transfer at the slab-column connection. Use unbalanced moments calculated in Problem 12.5.

12.10. Consider the slab-column connection of Problem 12.9. Design shear stud reinforcement assuming that the total factored load had to be increased by 30%, but the slab thickness needs to remain unchanged.

12.11. Consider the flat plate system shown in Problem 12.5. A change in the building function took place after the design was performed, and the building is going to have a residential occupancy. The floor is subjected to specified live load (LL) of 1.9 kPa and superimposed dead load (DL_s) of 1.5 kPa, in addition to its self-weight. The owner requires 200 mm slab thickness for this design.

a) Calculate deflection for an interior slab panel according to the Crossing Beam Method. Use the reinforcement designed in Problem 12.5. Consider both immediate and long-term deflections. Consider 20% of the specified live load to be sustained for long-term deflection calculations.
b) Check whether deflections are within the CSA A23.3 limits. Note that, according to the design requirements, the underside of each concrete floor slab serves as the ceiling for the floor below. Consequently, non-structural elements are likely to be damaged by large deflections and that should be taken into account when considering CSA A23.3 deflection limits.

Walls

LEARNING OUTCOMES

After reading this chapter, you should be able to

- identify the eight main types of reinforced concrete walls
- apply the CSA A23.3 requirements for detailing of wall reinforcement
- design bearing walls for gravity load effects
- design basement walls subjected to lateral earth pressure
- design shear walls for flexure and shear effects

13.1 INTRODUCTION

Walls are vertical structural members used to enclose or separate spaces. In addition, walls may be used to retain earth and liquids or resist wind pressures or to contain bulk materials in storage containers. Bearing walls can be designed like columns to support gravity loads or like beams to carry concentrated and uniformly distributed gravity loads and transfer them down to the foundations. Walls also have an important role in resisting lateral loads due to winds and earthquakes.

This chapter is focused on the conceptual design of bearing walls, basement walls, and shear walls. Different types of walls are outlined in Section 13.2. General CSA A23.3 design and detailing requirements are discussed in Section 13.3. The design of bearing walls is discussed in Section 13.4, whereas the design of basement walls is discussed in Section 13.5. Basic concepts related to the design of shear walls are discussed in Section 13.6. Structural drawings and details for reinforced concrete walls are discussed in Section 13.7. Brief coverage of joints in reinforced concrete walls is included in Section 13.8.

Advanced topics related to wall design, such as seismic design and detailing, are beyond the scope of this book. For more details on the seismic design of walls, the reader is referred to Paulay and Priestley (1992).

13.2 TYPES OF WALLS

Based on their function, reinforced concrete walls can be classified as follows:

- retaining walls
- basement walls
- grade beams
- bearing walls
- shear walls
- wall panels
- fire walls
- tilt-up walls