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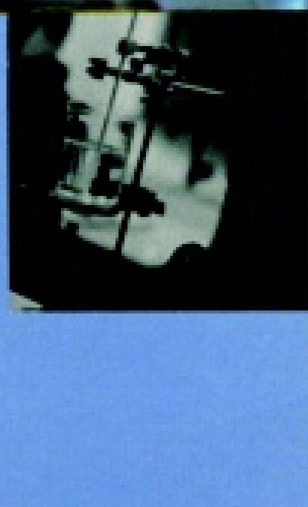
PRACTICAL PHYSICS



R.K. Shukla
Anchal Srivastava

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Preface

The book is an outcome of the experience of the authors, which they have acquired in guiding the under-graduate and post-graduate students. The book is written in a simple and systematic form to enable the students to follow and perform the experiments on their own. Procedure for conducting each experiment is given in detail. Important precautions for performing the experiments have been listed. Viva-voce given at the end of each chapter sets on the thinking process in the mind of the reader.

The book is divided into sixteen chapters. Chapter 1 deals with the experimental errors and the general instructions for the performance of different experiments.

Detailed description of several apparatus are given in chapter 2 which would be helpful in enabling the students to handle the laboratory apparatus and to take measurements.

The experiments in the under-graduate courses prevalent at various Indian Universities are contained in chapters 3 to 15. The theory and the procedure associated with each experiment have been thoroughly described. The precautions to be taken are also enumerated. The determination of experimental error for the quantity to be determined is also given.

Tables of physical constants, log tables etc. are given in chapter 16.

We are also greatful to Prof. T.P. Pandya and Prof. L.M. Bali for their constructive suggestions and constant encouragement. We extend our thanks to Prof. G.P. Gupta, Head of our department for his constant support.

We hope that the book will prove helpful and will meet the needs of the students at under-graduate level of almost all the Indian Universities. We shall welcome suggestions for the improvement of this book.

R.K. Shukla
Anchal Srivastava

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Introductory Concepts

1.1 AIM OF THE EXPERIMENT

Experiments form the foundation of the growth and development of science. The chief aim of experimentation in science is to discover the law which governs a certain phenomenon or to verify a given law which has been derived from a theory. A general scheme of scientific investigation known as Scientific Method involves the following steps:

Observations : Qualitative information about a phenomenon collected by unaided senses.

↓

Experimentation : Quantitative measurements (with the help of instruments) of certain physical quantities which have some bearing on the phenomenon.

↓

Formulation of hypothesis : Analysis of the data to determine how various measured quantities affect the phenomenon and to establish a relationship between them, graphically or otherwise.

↓

Verification : The hypothesis is verified by applying it to other allied phenomena.

↓

Predictions of new phenomena.

↓

New experiments to test the predictions.

↓

Modification of the law if necessary

The above discussion show that experimentation is vital to the development of any kind of science and more so to that of Physics.

1.2 IMPORTANCE OF LABORATORY WORK

Physics is an experimental science and the history of science reveals the fact that most of the notable discoveries in science have been made in the laboratory. Seeing experiments being performed i.e., demonstration experiments are important for understanding the principles of science. However, performing experiments by one's own hands is far more important because it involves learning by doing. It is needless to emphasise that for a systematic and scientific training of a young mind; a genuine laboratory practice is a must. For the progress of science and acceptance of various hypothesis measurements play a key role.

1.3 GENERAL INSTRUCTIONS FOR PERFORMING EXPERIMENTS

1. Before performing an experiment, the student should first thoroughly understand the theory of the experiment. The object of the experiment, the kind of apparatus needed and the procedure to be followed should be clear before actually performing the experiment. The difficulties and doubts if any, should be discussed with the teacher.
2. The student should check up whether the right type of apparatus for the experiment to be performed is given to him or not.
3. All the apparatus should be arranged on the table in proper order. Every apparatus should be handled carefully and cautiously to avoid any damage. Any damage or breaking done to the apparatus accidentally, should be immediately brought to the notice.
4. Precautions meant for the experiment should not only be read and written in the practical file but they are to be actually observed while doing the experiment.
5. All observations should be taken systematically, intelligently and should be honestly recorded on the fair record book. In no case an attempt should be made to cook or change the observations in order to get good results.
6. Repeat every observation, number of times even though their values each time may be exactly the same. The student must bear in mind the proper plan for recording the observations.
7. Calculations should be neatly shown using log tables. The degree of accuracy of the measurement of each quantity should always be kept in mind so that the final result does not show any fictitious accuracy. So the result obtained should be suitably rounded off.
8. Wherever possible, the observations should be represented with the help of graph.
9. Always mention the proper unit with the result.

1.4 HOW TO RECORD AN EXPERIMENT IN THE PRACTICAL FILE

A neat and systematic recording of the experiment in the practical file is very important in achieving the success of the experimental investigations. The students may write the experiment under the following heads in their fair practical note-books.

Date............. Experiment No.............. Page No..............

1. Object/Aim
2. Apparatus used
3. Formula used
4. Theory
5. Procedure/Method
6. Observation
7. Calculations
8. Results
9. % error
10. Sources of error and precaution

Object/Aim: The object of the experiment to be performed should be clearly and precisely stated.

Apparatus used: The main apparatus needed for the experiment are to be given under this head. If any special assembly of apparatus is needed for the experiment, its description should also be given in brief and its diagram should be drawn on the left hand page.

Diagram: A circuit diagram for electricity experiments and a ray diagram for light experiment is a must. Simply principle diagram wherever needed, should be drawn neatly on left hand page.

Theory: The principle underlying the experiment should be mentioned here. The formula used should also be written explaining clearly the symbols involved. Derivation of the formula may not be required.

Procedure: The various steps to be followed in setting the apparatus and taking the measurements should be written in the right order as per the requirement of the experiment.

Observations: The observations and their recording, is the heart of the experiment. As far as possible, the observations should be recorded in the tabular form neatly and without any over-writing. In case of wrong entry, the wrong reading should be scored by drawing a horizontal line over it and the correct reading should be written by its side. On the top of the observation table the least counts and ranges of various measuring instruments used should be clearly given. If the result of the experiment depends upon certain environmental conditions like temperature; pressure, place etc., then the values of these factors should also be mentioned.

Calculations: The observed values of various quantities should be substituted in the formula and the computations should be done systematically and neatly with the help of log tables. Wherever possible, graphical method for obtaining result should be employed.

Estimation of error: Percentage error may also be calculated if the standard value of the result is known.

Result: The conclusion drawn from the experimental observations has to be stated under this heading. If the result is in the form of a numerical value of a physical quantity, it should be expressed in its proper unit. Also mention the physical conditions like temperature, pressure, etc, if the result happens to depend upon them.

Sources of error and precautions: The possible errors which are beyond the control of the experimenter and which affect the result, should be mentioned here.

The precautions which are actually observed during the course of the experiment should be mentioned under this heading.

1.5 ERRORS AND OBSERVATIONS

We come across following errors during the course of an experiment:

1. **Personal or chance error:** Two observers using the same experimental set up, do not obtain exactly the same result. Even the observations of a single experimenter differ when it is repeated several times by him or her. Such errors always occur inspite of the best and honest efforts on the part of the experimenter and are known as personal errors. These errors are also called chance errors as they depend upon chance. The effect of the chance error on the result can be considerably reduced by taking a large number of observations and then taking their mean.

2. **Error due to external causes:** These are the errors which arise due to reasons beyond the control of the experimenter, e.g., change in room temperature, atmospheric pressure etc. A suitable correction can however, be applied for these errors if the factors affecting the result are also recorded.
3. **Instrumental errors:** Every instrument, however cautiously designed or manufactured, possesses imperfection to some extent. As a result of this imperfection, the measurements with the instrument cannot be free from errors. Errors, however small, do occur owing to the inherent manufacturing defects in the measuring instruments. Such errors which arise owing to inherent manufacturing defects in the measuring instruments are called instrumental errors. These errors are of constant magnitude and suitable corrections can be applied for these errors.

1.6 ACCURACY OF OBSERVATIONS

The manner in which an observation is recorded, indicates how accurately the physical quantity has been measured. For example, if a measured quantity is recorded as 50 cm, it implies that it has been measured correct 'to the nearest cm'. It means that the measuring instrument employed for the purpose has the least count (L.C.) = 1 cm.

Since the error in the measured quantity is half of the L.C. of the measuring instrument, therefore, here the error is 0.5 cm in 50 cm. In other words we can say, error is 1 part in 100.

If the observation is recorded in another way i.e., 50.0 cm, it implies that it is correct 'to the nearest mm. As explained above, the error now becomes 0.5 mm or 0.05 cm in 50 cm. So the error is 1 part in 1000.

If the same observation is recorded as 50.00 cm. It implies that reading has been taken with an instrument whose L.C. is 0.01 cm. Hence the error here becomes 0.005 cm in 50 cm, i.e., 1 part in 10,000.

It may be noted that with the decrease in the L.C. of the measuring instrument, the error in measurement decreases, in other words accuracy of measurement increases. When we say the error in measurement of a quantity is 1 part in 1000, we can also say that accuracy of the measurement is 1 part in 1000. Both the statements mean the same thing. Thus from the above discussion it follows that:

(*a*) the accuracy of measurement increases with the decrease in the least count of the measuring instrument; and

(*b*) the manner of recording an observation indicates the accuracy of its measurement.

1.7 ACCURACY OF THE RESULT

The accuracy of the final result is always governed by the accuracy of the least accurate observation involved in the experiment. So after making calculation, the result should be expressed in such a manner that it does not show any superfluous accuracy. Actually the result should be expressed upto that decimal place (after rounding off) which indicates the same accuracy of measurement as that of the least accurate observation made.

1.8 PERMISSIBLE ERROR IN THE RESULT

Even under ideal conditions in which personal errors, instrumental errors and errors due to external causes are some how absent, there is another type of error which creeps into the observations because of the limitation put on the accuracy of the measuring instruments by their least counts. This error is known as the permissible error.

1.9 HOW TO ESTIMATE THE PERMISSIBLE ERROR IN THE RESULT

Case I: When the formula for the quantity to be determined involves the product of only first power of the measured quantities: Suppose in an experiment, there are only two measured quantities say p and q and the resultant quantity s is obtained as the product of p and q, such that

$$s = p \cdot q \qquad \text{...(1)}$$

Let Δp and Δq be the permissible errors in the measurement of p and q respectively. Let Δs be the maximum permissible error in the resultant quantity s. Then

$$s \pm \Delta s = (p \pm \Delta p) \cdot (q \pm \Delta q) \qquad \text{...(2)}$$

From (1) and (2), we have

$$(s \pm \Delta s) - s = p \cdot q \pm p \cdot \Delta q \pm q \cdot \Delta p \pm \Delta p \cdot \Delta q - p \cdot q$$

$$\Delta s = \pm\, p \cdot \Delta q \pm q \cdot \Delta p \pm \Delta p \cdot \Delta q \qquad \text{...(3)}$$

The product of two very small quantitites, i.e. (the product $\Delta p \cdot \Delta q$) is negligibly small as compared to other quantities, so equation (3) can be written as

$$\Delta s = \pm\, p \cdot \Delta q \pm q \cdot \Delta p \qquad \text{...(4)}$$

For getting maximum permissible error in the result, the positive signs with the individual errors should be retained so that the errors get added up to give the maximum effect.

Thus equation (4) becomes

$$\Delta s = p \cdot \Delta q + q \cdot \Delta p \qquad \text{...(5)}$$

Dividing L.H.S. by s and R.H.S. by the product $p \cdot q$, we get

$$\frac{\Delta s}{s} = \frac{p . \Delta q}{p . q} + \frac{q . \Delta p}{p . q}$$

or

$$\left|\frac{\Delta s}{s}\right|_{\text{max.}} = \frac{\Delta q}{q} + \frac{\Delta p}{p} \qquad \text{...(6)}$$

Expressing the maximum permissible error in terms of percentage, we get

$$\frac{\Delta s}{s} = \left(\frac{\Delta q}{q} + \frac{\Delta p}{p}\right) \times 100\% \qquad \text{...(7)}$$

The result expressed by equation (7) can also be obtained by logarithmic differentiation of relation (1). This is done as follows:

On taking log of both the sides of equation (1), one gets

$$\log s = \log p + \log q \qquad \text{...(8)}$$

On differentiating (8), one gets

$$\frac{\Delta s}{s} = \frac{\Delta p}{p} + \frac{\Delta q}{q} \qquad \text{...(9)}$$

$$\because \qquad \Delta (\log x) = \frac{\Delta x}{x}.$$

The result (6) and (9) are essentially the same.

Case II: When the formula for the physical quantity to be determined contains higher powers of various measured quantities.

Let $$s = p^a q^b r^c \qquad ...(10)$$

Then taking log of both sides of (10), we have

$$\log s = a \log p + b \log q + c \log r \qquad ...(11)$$

On differentiating equation (11), we get

$$\frac{\Delta s}{s} = a\frac{\Delta p}{p} + b\frac{\Delta q}{q} + c\frac{\Delta r}{r} \qquad ...(12)$$

Thus $$\left|\frac{\Delta s}{s}\right|_{\text{max.}} = \left(a\frac{\Delta p}{p} + b\frac{\Delta q}{q} + c\frac{\Delta r}{r}\right) \times 100\% \qquad ...(13)$$

Since maximum permissible error can be conveniently estimated by logarithmic differentiation of the formula for the required quantity, so, the maximum permissible error is also called as the Maximum Log Error.

In actual practice the maximum permissible error is computed by logarithmic differentiation method.

1.10 ESTIMATING MAXIMUM PERMISSIBLE ERROR

For determination of resistivity of a material, the formula used is

$$\rho = \frac{R \times \frac{1}{4}\pi d^2}{l},$$

the maximum permissible error in ρ is computed as follows.

Resistivity ρ is a function of three variable R, d and l.

Taking log of both sides, we get

$$\log \rho = \log R + 2 \log d - \log l + \log\left(\frac{\pi}{4}\right)$$

Differentiating with respect to variable itself, we get

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta d}{d} - \frac{\Delta l}{l} + 0 \qquad \left(\because \frac{\pi}{4} \text{ is constant}\right)$$

changing negative sign into positive sign for determining maximum error, we have

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l}$$

The error is maximum due to error in physical quantity occuring with highest power in the working formula.

In an experiment, the various measurements were as follows:

$R = 1.05\Omega$, $\Delta R = 0.01\Omega$

$d = 0.60$ mm, $\Delta d = 0.01$ mm = least count of the screw gauge

$l = 75.3$ cm, $\Delta l = 0.1$ cm = least count of the metre scale

$$\frac{\Delta\rho}{\rho} = \frac{0.01}{1.05} + \frac{2\times 0.01}{0.60} + \frac{0.1}{75.3} = 0.0095 + 0.0334 + 0.0013 = 0.0442 = 0.044$$

The permissible error, in ρ in the above case is

$$\frac{\Delta\rho}{\rho} = 0.044 \times 100$$
$$= 4.4\%, \text{ out of which } 0.0334 \times 100 \text{ i.e. } 3.3\% \text{ is due to error in } d.$$

1.11 PERCENTAGE ERROR

The mean value of the experimentally determined quantity is compared with the standard (correct) value of the quantity. It is done as follows:

$$\text{Percentage Error} = \left(\frac{\text{standard value} - \text{calculated value}}{\text{standard value}}\right) \times 100$$

1.12 SIGNIFICANT FIGURES (PRECISION OF MEASUREMENT)

No measurement of any physical quantity is absolutely correct. The numerical value obtained after measurement is just an approximation. As such it becomes quite important to indicate the degree of accuracy (or precision) in the measurement done in the experiment. Scientists have developed a kind of short hand to communicate the precision of a measurement made in an experiment. The concept of significant figures helps in achieving this objective. To appreciate and understand the meaning of significant figures let us consider that in an experiment, the measured length of an object is recorded as 14.8 cm. The recording of length as 14.8 cm means (by convention) that the length has been measured by an instrument accurate to one-tenth of a centimetre. It means that the measured length lies between 14.75 cm and 14.85 cm. It also indicates that in this way recording of lengths as 14.8 cm, the figures 1 and 4 are absolutely correct where as the figures '8' is reasonably correct. So in this way of recording a reading of a measurement, there are three significant figures. Let us now consider another way of recording a reading. Let the measured length be written as 14.83 cm. This way of writing the value of length, means that the measurement is done with an instrument which is accurate upto one-hundredth of centimetre. It means that the length lies between 14.835 cm and 14.825 cm, which shows that in 14.83 cm, the figures 1, 4 and 8 are absolutely correct and the fourth figure '3' is only reasonably correct. Thus this way of recording length as 14.83 cm contains four significant figures. Thus this significant figure is a measured quantity to indicate the number of digits in which we have confidence. In the above measurement of length, the first measurement (14.8 cm) is good to three significant figures whereas the second one (14.83 cm) is good to four significant figures. From the above discussion, we should clearly understand that the two ways of recording an observation such as 15.8 cm and 15.80 cm represent two different degrees of precision of measurements.

1.13 ROUNDING OFF

When the quantities with different degees of precision are to be added or subtracted, then the quantities should be rounded off in such a way that all of them are accurate upto the same place of decimal.

For rounding off the numerical values of various quantities, the following points are noted.

1. When the digit to be dropped is more than 5, then the next digit to be retained should be increased by 1.
2. When the digit to be dropped is less than 5, then the next digit should be retained as it is, without changing it.
3. When the digit to be dropped happens to be the digit 5 itself, then (*a*) the next digit to be retained is increased by 1, if the digit is an odd number. (*b*) the next digit is retained as it is, if the digit is an even number.

After carrying out the operations of multiplication and division, the final result should be rounded off in such a manner that its accuracy is the same as that of the least accurate quantity involved in the operation.

1.14 LOGARITHMS

Logarithm of number to a given base is the number to which the base must be raised to get the number. For example, we have

$$8 = 2^3 \text{ it means log of 8 to the base 2 is equal to 3}$$

or $$\log_2 8 = 3$$

Napierian Logarithms: In these logarithms, the base used is e, value of e is equal to 2.17828.

Common Logarithms: In these logarithms, the base used is 10. These are used in all arithmetical calculations.

General Relations:

1. The logarithms of 1 to any base is 0.
2. The logarithms of the product of two or more numbers is equal to the sum of their logarithms.

   Thus $$\log_a m \cdot n = \log_a m + \log_a n$$
3. The logarithm of a fraction is equal to the difference between the logarithms of the numerator and that of the denominator.

   Thus $$\log_a \frac{m}{n} = \log_a m - \log_a n$$
4. The logarithm of a number raised to any power (integral or fraction) is equal to the product of the index of power and the logarithm of the number.

   Thus $$\log_a m^n = n \log_a m$$

Characteristic and Mantissa: Logarithms consist of an integral part called the characteristic, and a fractional part called mantissa. For example, 856 lies between 800 and 900, or between 10^2 and 10^3, and so the logarithm of it lies between 2 and 3.

$$\log 856 = 2 + \text{a fraction}$$

Hence, 2 is the characteristic and the unknown fraction (which is always positive) is the mantissa.

The mantissa is determined with the help of logarithm and antilogarithm tables, while reading the mantissa from the tables, the position of the decimal points and zeroes at both ends of the number are ignored. These do not affect the mantissa. For example, the mantissa of 856, 85.6 and 0.00856 is the same.

1.15 HOW TO READ FOUR FIGURE LOGARITHM TABLES

To read the mantissa for 2345, look for 23 in the extreme left vertical column. Move horizontally against it to the number under column marked 4 at the top. This is 3692. Move forward in the same horizontal line and note down the number under mean difference 5. This is 9. On adding the number 9 to 3692, we get 3701. This is the mantissa of 2345.

Therefore,
$$\begin{aligned}\log 2345 &= 3.3701\\ \log 234.5 &= 2.3701\\ \log 23.45 &= 1.3701\\ \log 2.345 &= 0.3701\\ \log 0.2345 &= \bar{1}.3701\\ \log 0.002345 &= \bar{3}.3701\end{aligned}$$

Antilogarithms: The antilogarithm of the logarithm of a number is equal to the number itself. For example, if the number is x, then antilog (log x) = x.

Thus, antilogarithm tables are used for finding out the number from its logarithm. If it is required to find the number whose logarithm is 1.3456 look for 0.34 in the extreme left vertical column of the antilogarithm table. Move horizontally against it to the number under column marked '5' at the top. The number is 2213. Move forward in the same horizontal line and note down the number under mean difference 6. The number is 3. Adding this number to 2213, we get 2216. Since the characteristic in the given logarithm is 1, there are two figures to the left of the decimal point in the required number. Hence the required number is 22.16.

Therefore,
$$\begin{aligned}\text{Antilog } 1.3456 &= 22.16\\ \log 22.16 &= 1.3456\end{aligned}$$

Example of addition: Suppose we have to add $\bar{2}.6495$ and 0.9419

For the mantissa part, we have
$$.6495 + .9419 = 1.5914$$

And for the characteristic part, we have
$$\bar{2}+0+1 = -2 + 0 + 1 = -1 = \bar{1}$$

Thus the sum is $\bar{1}.5914$.

Example of subtraction: Suppose we have to subtract $\bar{2}.9419$ from $\bar{3}.6495$.

For the mantissa part, we have
$$0.6495 - 0.9419 = .7076 \text{ with a borrow } 1.$$

And for the characteristic part, we have $\bar{3}-\bar{2}-1$ (borrow)
$$= -3 + 2 - 1 = -2 = \bar{2}$$

Thus $\quad \bar{3}.6495-\bar{2}.9419 = \bar{2}.7076$

Example: Calculate $\dfrac{(456.2)^{1/2} \times 0.024}{(325)^{1/4}}$

Let x be the required number, then

$$x = \frac{(456.2)^{1/2} \times (0.024)}{(325)^{1/4}}$$

taking log on both sides, we get

$$\begin{aligned}
\log x &= \frac{1}{2}\log(456.2) + \log(0.024) - \frac{1}{4}\log(325) \\
&= \frac{1}{2} \times 2.6592 + \bar{2}.3802 - \frac{1}{4} \times 2.5119 \\
&= 1.3296 + \bar{2}.3802 - 0.6280 = \bar{1}.0818 \\
x &= \text{antilog } \bar{1}.0818 = 0.1207
\end{aligned}$$

1.16 GRAPH

A graph is a pictorial way to show how two physical quantities are related. It is a numerical device dealing not in cm, ohm, time, temperature etc but with the numerical magnitude of these quantities. Two varying quantities called the variables are the essential features of a graph.

Purpose: To show how one quanitity varies with the change in the other. The quantity which is made to change at will, is known as the independent variable and the other quantity which varies as a result of this change is known as dependent variable. The essential features of the experimental observations can be easily seen at a glance if they are represented by a suitable graph. The graph may be a straight line or a curved line.

Advantage of Graph: The most important advantage of a graph is that, the average value of a physical quantity under investigation can be obtained very conveniently from it without resorting to lengthy numerical computations. Another important advantage of graph is that some salient features of a given experimental data can be seen visually. For example, the points of maxima or minima or inflexion can be easily known by simply having a careful look at the graph representing the experimental data. These points cannot so easily be concluded by merely looking at the data. Whenever possible, the results of an experiment should be presented in a graphical form. As far as possible, a straight line graph should be used because a straight line is more conveniently drawn and the deduction from such a graph are more reliable than from a curved line graph.

Each point on a graph is an actual observation. So it should either be encircled or be made as an intersection (i.e. cross) of two small lines. The departure of the point from the graph is a measure of the experimental error in that observation.

How to Plot a Graph? The following points will be found useful for drawing a proper graph:

1. Examine carefully the experimental data and note the range of variations of the two variables to be plotted. Also examine the number of divisions available on the two axes drawn on the graph paper. After doing so, make a suitable choice of scales for the two axes keeping in mind that the resulting graph should practically cover almost the entire portion of the graph paper.

2. Write properly chosen scales for the two axes on the top of the graph paper or at some suitable place. Draw an arrow head along each axis and write the symbol used for the corresponding variable alongwith its unit as headings of observations in the table, namely, d/mm, R/Ω, l/cm, T/S, I/A etc. Also write the values of the respective variables on the divisions marked by dark lines along the axes.
3. After plotting the points encircle them. When the points plotted happen to lie almost on a straight line, the straight line should be drawn using a sharp pencil and a straight edged ruler and care should be taken to ensure that the straight line passes through the maximum number of points and the remaining points are almost evenly distributed on both sides of the line.
4. If the plotted points do not lie on a straight line, draw free hand smooth curve passing through the maximum number of points. Owing to errors occuring in the observations, some of the points may not fall exactly on the free hand curve. So while drawing a smooth curve, care should be exercised to see that such points are more or less evenly distributed on both sides of the curve.
5. When the plotted points do not appear to lie on a straight line, a smooth curve is drawn with the help of a device known as French curve. If French curve is not available, a thin flexible spoke of a broom can also be used for drawing smooth curve. To make the spoke uniformly thin throughout its length, it is peeled off suitably with a knife. This flexible spoke is then held between the two fingers of left hand and placed on the graph paper bending it suitably with the pressure of fingers is such a manner that the spoke in the curved position passes through the maximum number of points. The remaining points should be more or less evenly distributed on both sides of the curved spoke. In this bent position of the spoke, a smooth line along the length of the spoke is drawn using a sharp pencil.
6. A proper title should be given to the graph thus plotted.
7. Preferably a millimetre graph paper should be used to obtain greater accuracy in the result.

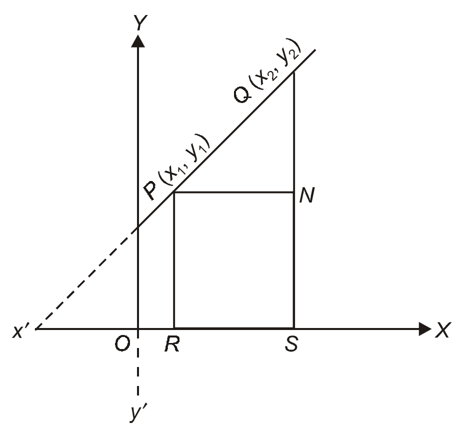
1.17 CALCULATIONS OF SLOPE OF A STRAIGHT LINE

In order to compute the value of slope m of the straight line graph, two points $P\ (x_1,\ y_1,)$ and $Q\ (x_2,\ y_2)$ widely separated on the straight line are chosen. PR and QS are drawn $\perp$ to x-axis and $PN \perp QS$.

The slope of the line

$$m = \frac{QN}{PN} = \frac{y_2 - y_1}{x_2 - x_1}$$

This method of calculating the slope emphasises the important fact that: QN and PN which are $(y_2 - y_1)$ and $(x_2 - x_1)$ must be measured according to the particular scales chosen along y and x-axis respectively, the angle θ must not be measured by a protractor and the values of tan θ should not be read from trigonometric table.



Computing the value of slope m of the line.

1.18 INTERNATIONAL SYSTEM OF UNITS (S.I. UNITS)

S.I. is the abbreviation for Le Systeme International d Unites which is French translation of the International System of Units. International system was accepted in 1960, in the general conference of weights and measures. There are seven fundamental units and two supplimentary units.

Fundamental Units

Physical Quantity	**Unit**	**Symbol**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	Cd
Amount of substance	mole	mol

Suppliment Units

Physical Quantity	*Unit*	*Symbol*
Angle	Radian	rad
Solid angle	Steradian	sr

Rest of the physical quantities can be expressed in terms of the above fundamental quantites.

To express large variations in magnitudes of quantities, their multiples or submultiplies are expressed by prefixes as follows:

Multiples			*Submultiples*		
Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	K	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

1.19 RULES FOR MEASUREMENTS IN THE LABORATORY

Measurements of most of the physical quantities in the laboratory should be done in the most convenient units, e.g., mass of a body in gram, measurements using micrometer screw in mm, small currents in electronic tubes, diode and triode in mA etc.

Calculations: All the measured quanitites must be converted into S.I. units before substituting in the formula for the calculation of the result.

2

Instruments and Accessories

An instrument is a device used to determine the value of a quantity or variable. We discuss below the instruments required to perform the experiments. The procedure for handling the instruments and the methods of taking readings are also outlined, where necessary. Some accessories and devices are additionally considered.

2.1 SOME INSTRUMENTS FOR MEASUREMENT OF LENGTH

(*i*) Metre Scale

To measure a length a metre scale is generally used. The scale is graduated in centimetres and millimetres and is one meter in length. For the measurement of a length with a metre scale, adopt the following procedure.

(*a*) Note the value of one smallest division of the scale.
(*b*) Hold the scale on its side such that the marking of the scale are very close to the points between which the distance is to be measured.
(*c*) Take readings by keeping the eye perpendicular to the scale above the point at which measurement is made.
(*d*) Avoid using zero of the scale as it may be damaged. Measure the distance as a difference of two scale readings. For situation where direct placing of the scale is inconvenient, use a divider. In this case the divider is set to the length to be measured and then transfered to the scale for actual measurement of the length.

A metre scale can be used with an accuracy of 0.1 cm. To measure a small length with an accuracy more than that obtainable from a metre scale, the instruments used are (1) the diagonal scale, (2) the slide callipers, (3) the screw gauge.

Since we shall use mainly the slide callipers and the screw gauge we shall describe these two instruments here. In addition the spherometer which is used to measure the radius of curvature of a spherical surface will be considered.

(*ii*) The Vernier and the Slide Callipers

The vernier consists of an auxilliary scale, called the vernier scale, which is capable of sliding along the edge of a main scale (Fig. 2.1). With the help of the vernier scale, length can be measured with an accuracy greater than that obtainable from the main scale. The graduations on the vernier scale are such that n divisions of this scale are generally made to coincide with $(n - 1)$ divisions of the main scale. Under this condition, lengths can be measured with an accuracy of $\frac{1}{n}$ of the main scale division.

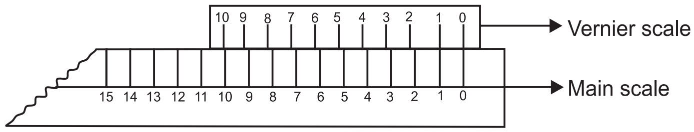


Fig. 2.1

The vernier constant (v.c) is given by

vernier constant (v.c) = (1 main scale div – 1 vernier scale div.) × value of 1 main scale division.

The same principle is employed in the construction of circular vernier used to measure angles.

The slide callipers consists of a steel scale, called the main scale (m.s) with a jaw (A) fixed at one end at right angles to its length as shown in Fig. 2.2. A second jaw (B) carrying a vernier scale and capable of moving along the main scale can be fixed to any position by means of a screw cap *S*. The main scale is graduated in centimetres or inches. The zero of the main scale and vernier scale coincide when the moveable jaw is allowed to touch the fixed jaw. To use the side callipers proceed as follows:

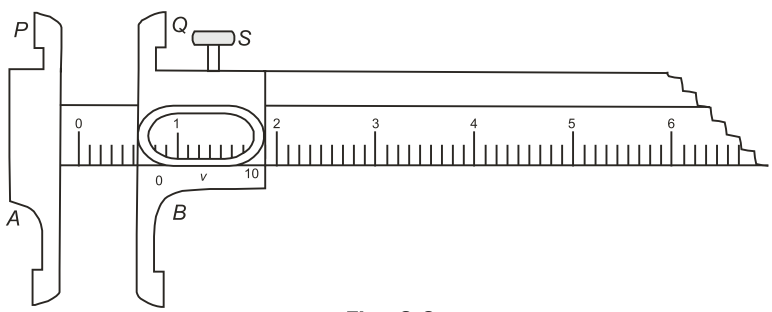


Fig. 2.2

(*i*) Loosen the screw *S* and let the two jaws touch each other. If the zeros or both the vernier and the main scales coincide then the instrumental error is zero. If not, find the instrumental error (below).

(*ii*) Find the vernier constant (v.c) of the slide callipers by recording data in the tabular form as shown in Table 2.1.

(*iii*) Put the movable jaw away from the fixed jaw and hold the object whose length is to be measured against the fixed jaw. Allow the movable jaw to touch the object. Lock the movable jaw by means of the screw *S*.

(*iv*) Find the length L of the object by taking readings (*a*) on the main scale and (*b*) on the vernier scale. Note the readings as follows:

(*a*) If the zero of the vernier stands after *l*th division of the main scale then note the reading of the main scale corresponding to the *l*th division. Let the reading be *l* (cm).

(*b*) If the pth division of the vernier is found to be in line with a main scale graduation then the vernier scale reading is $p \times$ (vernier constant).

Therefore, the length *L* of the object is

$$L = l + p \times \text{v.c. cm},$$

where v.c. is given in cm.

Table 2.1: Determination of the vernier constant
…division (say, *m*) of the vernierscale = …division (say, *n*) of the main scale

Value of 1 smallest main scale division (l_1) *(cm)*	*Value of 1 vernier division* $\left(l_2 = \frac{n}{m} l_1\right)$ *(cm)*	*Vernier Constant* = ($l_1 - l_2$) *(cm)*

A length correct to one-tenth or one-fiftieth part of a millimetre can be measured with slide callipers.

An instrumental error or zero error exists when, with the two jaws touching each other, the zero of the vernier scale is ahead of or behind the zero of the main scale. If x divisions of the vernier scale coincides with a certain mark of the main scale, 'the instrumental error is $y = x \times$ v.c. The error is positive when the vernier zero is on the right and is negative when the vernier zero is cm the left side of the main scale zero. If the instrumental error is positive it is to be subtracted from the measured length to obtain the correct length. If the error in negative is to be added to the measured length.

To measure the internal diameter of a cylinder the jaws P and Q are used.

(*iii*) The Screw Gauge

It consists of a *U*-shaped piece of steel, one arm of which carries a fixed stud B where as the other arm is attached to a cylindrical tube [Fig. 2.3(a)]. A scale S graduated in centimeters or inches is marked on this cylinder. An accurate screw provided with a collar, moves inside the tube. The screw moves axially when it is rotated by the milled head (H). The fixed and the movable studs are provided with plane surfaces.

The beveled end of the collar is generally divided into 50 or 100 equal divisions forming a circualr scale. Depending upon the direction of rotation of the screw the collar covers or uncovers the straight scale divisions. When the movable stud is made to touch the fixed stud, the zero of the linear (straight) scale should coincide with the zero of the circular scale. If they do not coincide then the screw gauge is said to possess an instrumental error.

A ratched wheel (R) which slips over the screw top can be seen in some screw gauges. The purpose of the ratched wheel is to give light pressure on the object during its measurement so that the object does not suffer any deformation.

The principle of the instrument is the conversion of the circular motion of the screw head into the linear motion of the movable stud. For an accurately cut screw, if the screw head is rotated through equal angles then the screw will move axially through equal distances.

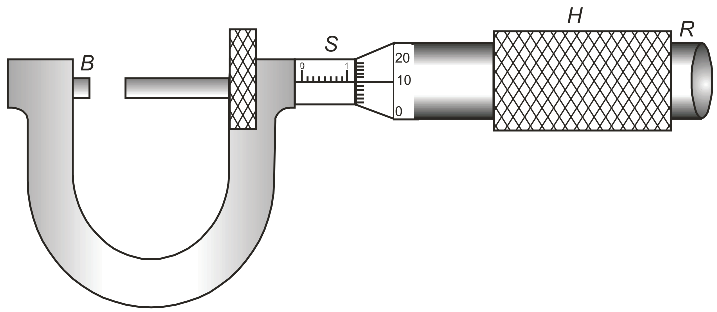


Fig. 2.3(a)

The pitch of the screw is defined as its axial displacement for a complete rotation.

The least count of the screw gauge refers to the axial displacement of the screw for a rotation of one circular division. Thus, if n represents the number of divisions on the circular scale and the pitch of the screw is m scale divisions, then the least count (L.C.) of the screw gauge is given by

$$\text{Least Count (L.C.)} = \frac{m}{n} \text{ scale divisions.}$$

The determination of least count (L.C.) of a screw gauge is conveniently done by completing the following Table 2.2.

Table 2.2: Determination of least count

Pitch of the screw m (cm)	*No of divisions n on the circular scale*	*Least Count = m/n (cm)*

If the pitch of the screw corresponds to one full scale division on the linear scale and the instrumental error is zero then the reading L of an object between the studs is given by

$$L = l + s \times \text{(l.c.)},$$

Where l represents the linear scale reading and s represents the number of circular scale divisions, sth division coinciding with the reference line.

When the studs are in touch with each other and the zero of the circular scale has crossed the reference line on the tube, the instrumental error is considered to be –ve. This error is, therefore, required to be added to the apparent reading. But if the zero on the circular scale fails to reach the reference line, the error is considered to be +ve and is required to be subtracted from the apparent reading. The magnitude of the instrumental error is the axial distance through which the screw should move in order to bring the zero of the circular and the main scale in coincidence when the studs are in contact with each other. If x circular scale division are above or below the reference line, then the magnitude of the zero or the instrumental error is $L = x \times$ least count.

Back-lash error: When a screw moves through a threaded hole there is always some misfit between the two. As a result, when the direction of rotation of the screw is reversed, axial motion of the screw takes place only after the screw head is rotated through a certain angle. This lag between the axial and the circular motion of the screw head is termed the back-lash error. This error is present in micrometer screw gauge. The error is small when the instrumental is new and it gradually increases with the use of the instrument. In order to get rid of this error, the screw head should always be rotated in the same direction while measurement is made.

(*iv*) The Spherometer

This instrument is used to measure (*i*) the radius of curvature of a spherial surface and (*ii*) the thickness of a very thin glass plate. It consists of a micrometer screw S moving through a nut N, the latter being supported on their three legs. A, B and C, as shown in [Fig. 2.3(*b*)]. The points A, B and C form the corners of an equilateral triangle and the tip of the screw S passes through the center of this triangle. Attached to this screw is a circular metallic disc D which is divided into 100 equal divisions. A linear vertical scale S_1, adjacent to the edge of the circular scale give the vertical displacement of the circular scale. When the three legs and the tip of the screw rest on the same plane P, the zero mark of the circular scale coincides with the zero of the linear scale S_1.

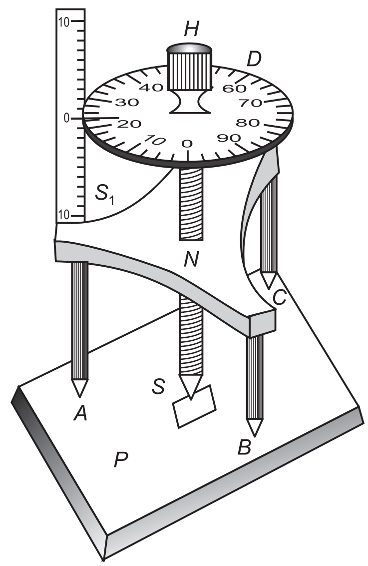


Fig. 2.3(b)

Before using the instrument, the least count (l.c.) is determined. For this, note the shift in position of the circular scale on the linear scale for one complete rotation of the former. This gives the pitch of the instrument. The least count is obtained by dividing the pitch by the total number of circular scale division.

The procedure of using this instrument for the measurement of radius of curvature is as follows:

(*i*) Place the spherometer on the spherical surface whose radius of curvature is to be determined and raise or lower the screw S by means of the head H so that the tip of the screw just touches the curved surface.

(*ii*) Note the reading of the circular scale (c.s.). Let this initial reading be a.

(*iii*) Replace the spherical surface by a plane glass plate.

(*iv*) Lower or raise gradually the screw so that the tip of the screw just touches the glass plate. To obtain the exact point of touching the surface of the plate, look tangentially along the glass plate and move the screw until the gap between the tip of the screw and its images just vanishes. During the movement of the screw, note the number (n) of the complete rotations of the circular scale. Multiply this number by the pitch to get the value of the vertical shift due to those complete rotations of the circular scale. Let it be denoted by M. Note also the final reading (b) on the circular scale when the tip just touches the plate. The difference between the final and the initial circular scale reading ($b \sim a$) gives the additional circular scale readings. Multiply ($b \sim a$) by the least count to get the value of the additional circular scale reading. Call it N. Thus the value of the elevation h is $M + N$.

(v) Measure h for the three different positions on the spherical surface, if possible.

If d represents the average distance between any two legs of the spherometer, then the radius of curvature R of the spherical surface is given by

$$R = \frac{h}{2} + \frac{d^2}{6h}$$

Results:

Table 2.3: To determine the least count of the instrument

Value of 1 smallest linear scale div. (mm)	*Pitch (p) of the screw (mm)*	*No. of circular scale divisions (N)*	*Least count (l.c.) = p/n (mm)*

Table 2.4: To determine the elevation h

No. of obs.	*Initial reading of C.S. when the screw touches the spherical surface (a)*	*Readings when the screw touches the plate*		*Shift for complete rotations M = n × pitch (mm)*	*Additional C.S. reading (b – a)*	*Value of (b – a) C.S. reading N = (b – a) × l.c.*	*Value of 'h' (h = M + N) (mm)*	*Mean (h) (mm)*
		No. of complete circular scale rotation (n)	*Final reading on the C.S. (b)*					
1								
2								
3								

Table 2.5: To determine the radius of curvature R

Distance Between			*Average distance any two legs* $d = \frac{x+y+z}{3}$ *(cm)*	*value of 'h' from Table 2.4 (cm)*	$R = \frac{d^2}{6h} + \frac{h}{2}$ *(cm)*
A and B (x cm)	*B and C (y cm)*	*C and A (z cm)*			

[while using this instrument take care of avoiding the back-lash error by rotating the screw always in the same direction.]

2.2 TRAVELLING MICROSCOPE

The travelling microscope [Fig. 2.4 (*a*)] is an instrument well-suited for the purpose of measuring small vertical or horizontal distances with high accuracy. It consists of a compound microscope that is capable of independent horizontal and vertical movements. The amount of movement in the vertical direction can be obtained from the scale M_1 and the vernier V_1 where as that in the horizontal direction can be obtained the scale M_2 and the vernier V_2. The microscope may be raised or lowered along a vertical pillar *PN* and its axis may be fixed horizontally, vertically, or in between them by the screw S_1. The screw S_2 is used to move the pillar *PN* horizontally. The object is viewed through the eye piece *E* when the objective lens *O* is turned towards the object. The focal length of the objective generally lies between 3 to 4 cm. The focussing of the microscope is accomplished by a screw attached to the body of the microscope. The cross-wire of the eye piece is focussed by moving eye piece in or out. Four screws at the base of instrument are used for its levelling.

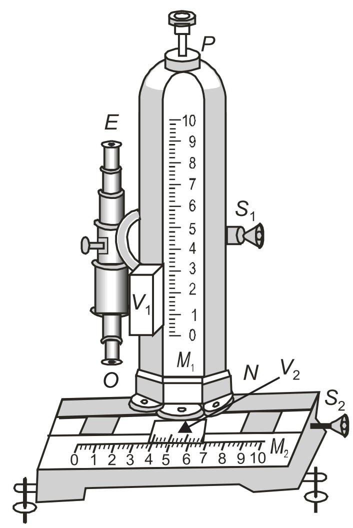


Fig. 2.4(a)

Before using the instrument note the vernier constants of the both the verniers V_1 and V_2. Generally the vernier constants of V_1 and V_2 are the same.

The procedure to measure the horizontal or the vertical distance between two points is as follows:

(*i*) Level the instrument with the help of the base screw and a spirit level.

(*ii*) View one of the points through the microscope and focus the cross-wire with the image of the point. Note the readings of the main scale and the vernier scale (v.s.).

(*iii*) Displace the microscope vertically or horizontally, as required by means of the screw S_1 or S_2 to view the second point and focus the cross wire with the image of the second point. Note again the reading of the main scale and the vernier scale.

(*iv*) Calculate the difference (horizontal or vertical) between these two readings to obtain the distance between the points.

2.3 CATHETOMETER

This instrument is used to measure vertical distances accurately in experiments where the range of a travelling microscope is inadequate. A typical form of the instrument is shown in Fig. 2.4(*b*).

It consists of a graduated rod AA' held vertically by means of a stand provided with three levelling screws (S_1, S_2 and S_3) at the base. The rod AA' carries a slide to which a telescope T is mounted horizontally. The rod along with the telescope can be rotated about a vertical axis. The screw F is used for horizontal adjustment of the telescope. A spirit level L is mounted on the top of the telescope to indicate the horizontal condition of the axis of the telescope.

The slide along with the telescope can be moved vertically up and down and can be fixed at any position by means of a screw S. The carriage holding the telescope has a vernier V. This vernier along with a micro meter screw M is used for measuring the small vertical distance.

To measure a vertical distance with the help of a cathetometer adopt the following procedure:

(*i*) Make the telescope parallel to the line joining any two of the levelling screws of the stand. (Some cathetometer are provided with two adjustement screw and a pin of fixed length is fixed to the third leg. For such cathetometers the telescope is set parallel to a line that passes through the fixed pin and any one of the two screws.) If the bubble of the spirit level is away from the centre of its scale then bring it half way back to the centre by turning the screw F.

(*ii*) Now turn the base screws simultaneously by equal amounts in opposite directions until the bubble reaches the centre of the scale. Note that these screws are in a line which is parallel to the telescope axis. (For those having two adjustable screws and a fixed pin, adjust the screw that in conjunction with the pin forms an imaginary line parallel to the axis of the telescope).

(*iii*) Turn the rod AA' through 180°. If the bubble of the spirit level is found away from the centre, bring it to the centre by first adjusting the screw F and then the two base screws, as discussed in steps (*i*) and (*ii*).

(*iv*) Turn the rod AA' again through 90° so that the telescope is now set at right angles to the previous line joining the two base screws. Adjust the remaining base screw and bring the bubble back to the centre, if necessary.

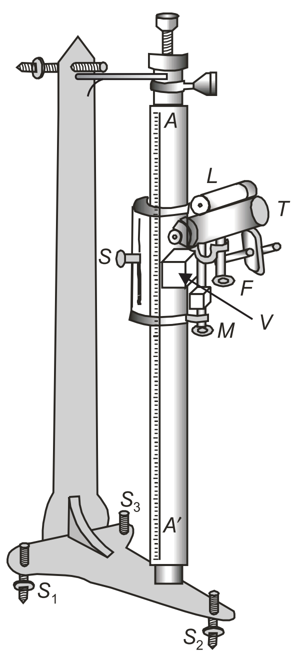


Fig. 2.4(b)

In practice, the above four steps are to be repeated several times in order that the bubble remains at the centre of its scale irrespective of the direction in which the telescope is set.

To measure accurately the vertical distance between two points very close to each other, focus the telescope on the first point until the image of the point coincides with the cross-wires of the telescope. Note the vernier reading. Alter the vertical position of the telescope and focus on the second point so that its image coincides with the cross-wires. Again note the vernier reading. The difference between the first and the second vernier readings gives the vertical distance between the points.

A very familiar application of cathetometer in the laboratory in the measurement of the vertical depression of the mid-point of a bar place horizontally between two knife edges and loaded at the middle.

2.4 THE BALANCE

The mass of a body is determined by weighing the body in a balance. The essential parts of a physical balance are shown in Fig. 2.5.

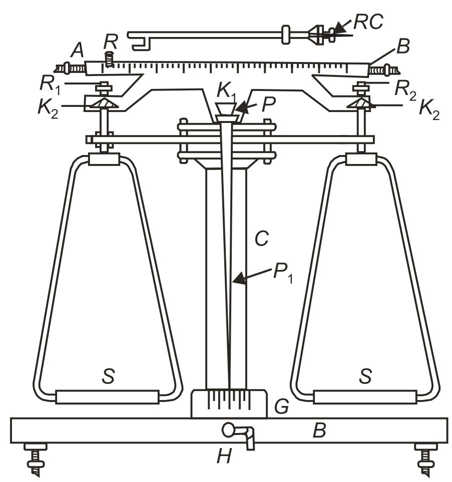


Fig. 2.5

1. **The beam:** It is a horizontal metal rod *AB* mounted at its centre by an agate Knife-edge K_1. The agate Knife-edge, in turn, rests on an agate plate *P* attached to the pillar *C* when the balance is in use. At the two ends of the beam two identical agate Knife-edges (K_2, K_2) are attached with their sharp edges upwards.
2. **The stirrups (R_1, R_2) and the pans (*S*, *S*):** The stirrups are provided with agate pieces which rest on the terminal Knife-edges (K_2, K_2). Each stirrup carries a pan (*S*). The distances of the centres of gravity of the stirrups from the sharp Knife-edges K_1 are called arms and are equal in length.
3. **The pillar (*C*):** This is a vertical rod which can be raised or lowered when required by means of a handle (*H*) fixed at the front of the wooden base of the instrument. At the top of the pillar there is an agate plate upon which the central Knife-edge (K_1) of the beam rests.
4. **The pointer (P_1):** This is attached at its upper end at the middle of the beam where as its lower end can move freely over a graduated scale (*G*) which is fixed at the foot of the pillar. When the beam is horizontal, the tip of the pointer is on the zero mark of the scale.
5. **The base-board (*B*):** The base of the instrument, called the base-board, is provided with levelling screws. The screws are adjusted to make the pillar vertical and the beam horizontal. A plumb line (not shown in the figure) suspended from the top of the pillar is used for correct adjustment of the pillar and the beam.

   When the balance is not in use, the beam is lowered and is allowed to rest on another support. In this case, the botton surfaces of the pans just touch the base board. In the rest position of the beam its central Knife-edge (K_1) is separated from the agate plate. At both ends of the beam there is a screw which can be used to alter the effective weight of each side through a small range.

6. **Rider and its function:** The beams of some physical balances are graduated from the centre outwards. Here weights smaller than 10 mgm are obtained by moving a small piece of wire, called the rider (*R*) along the beam by means of a rider hook and a rider carriage (R.C.). The mass of the rider is usually made 10 mg and each division on the beam corresponds to 0.1 mgm or 0.2 mgm depending upon the construction. Before using the rider it is better to ascertain its correct mass.
7. **Weight box:** The body to be weighed is placed on the left pan and various weights of known masses are placed on the right pan. These weights of known masses are contained in a box called the 'weight box.' Inside the box there are bodies of masses ranging from 100 gm to 10 mgm or less. Masses weighing 1 gm or greater are generally made of brass whereas the lighter masses are made of aluminium. The mass of each is marked on it. These standard weights are carefully handled with a pair of forceps also contained in the box.

   A delicate balance contains various other parts of finer weighing. The balance is always kept within a glass case to prevent external disturbance during weighing.

Sensitivity of a balance: The sensitivity of a balance is defined as the number of scale division through which the pointer moves on account of an excess of weight of 1 mgm on one of the pans.

In Fig. 2.6, A_1, B_1, represents the position of the beam when it is deflected by an angle α from its rest position AB due to an excess weight of W on one of the pans. If l is the length of the pointer and d (= PP_1) is the deflection of the pointer then we have $\alpha = \dfrac{d}{l}$.

Suppose the weight W of the beam and the pointer acts at its centre of gravity G where $OG = h$. Under equilibrium condition the moment due to ω, i.e. $\omega a \cos\alpha$ is balanced by the moment due to W, i.e. $Wh \sin\alpha$, where a (= OB_1) is the length of the arms of the balance. Thus,

$$Wh \sin\alpha = \omega a \cos\alpha$$

or,

$$\tan\alpha = \frac{\omega a}{Wh}$$

When a is small,

$$\tan\alpha, = \alpha = \frac{d}{l}$$

Therefore,

$$\frac{d}{l} = \frac{\omega a}{Wh}$$

Hence the sensitivity of the balance is

$$\frac{d}{\omega} = \frac{al}{Wh}.$$

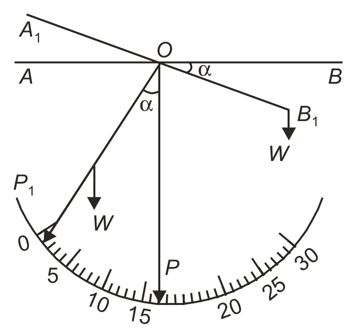


Fig. 2.6

Note that in the above equations ω and W are both expressed in the same units, i.e gms or mgms. If ω is in mgm and W is in gm, the equation will be

$$\frac{d}{\omega} = \frac{al}{1000Wh}.$$

This shows that the sensitivity of a balance can be increased by (*i*) increasing the length of the arm, (*ii*) diminishing the weight of the beam and the pointer, (*iii*) decreasing the distance h of the centre of gravity of the beam and the pointer from the fulcrum and (*iv*) increasing the length of the pointer.

The sensitivity of a balance is a maximum and is independent of the load when the three knife-edges K_1 and (K_2, K_2) are in the same horizontal plane. When the middle Knife-edge is below the plane of the terminal knife-edges and equal leads placed on each pan, the sensitivity increases initially with load since the beam bends and lowers the position of the terminal knife-edges. On the other hand, if the middle knife-edge lies above the plane of the terminal knife-edge then the sensitivity decreases continuously with increasing load. Practical balances are constructed with the central knife-edge lying a little below the plane of the terminal knife-edge. As a result, the sensitivity of a balance initially increases with load, reaches a maximum and then decreases, as shown in Fig. 2.7.

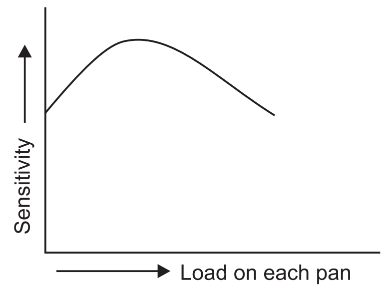


Fig. 2.7

A balance is said to be stable if the arms of the balance quickly return to their equilibrium positions after the balance is disturbed. For a better stability, the arms are made short. In order to make a compromise between stability and sensitivity, generally balances with short arms but with long pointers are made.

Weighing a body with a balance: The weight of a body may be determined either by the method of oscillation or by the method of equal displacements.

(*a*) **Method of oscillation:** To determine the rest position of the pointer: When the beam of a balance oscillates, it executes simple harmonic vibrations with gradually diminishing amplitude and a long time is spent before it comes to rest. To save time the rest position of the pointer is determined from the amplitudes of oscillations. For this, the exterme left of the scale over which the pointer moves is marked zero (as shown in Fig. 2.6). During the oscillation of the pointer, five readings on the turning points of the pointer are taken; three on one side (say, left) and two on the other (say, right) of the rest position. Calculate the mean of the three readings on the left and that of the two readings on the right. The mean of these two means gives the rest position of the pointer.

Suppose P is the rest position of the pointer when no weights are placed on the pans. Let Q be the rest position of the pointer when the body to be weighed is placed on the left pan and a weight W_1, gm is placed on the right pan. Also let R be the rest position of the pointer when an extra load of m mgm is placed over W_1 gm.

Therefore, ($Q - R$) divisions represent the change in the rest position of the pointer due to the extra weight m mgm. Hence ($Q - P$) divisions shift of the rest position of the pointer will correspond to $\frac{Q-P}{Q-R} \times \frac{m}{1000}$ gm. The correct weight of the body, is therefore,

$$\mathrm{W} = W_1 + \frac{Q-P}{Q-R} \times \frac{m}{1000} \text{ gm}$$

Procedure:

(*i*) Level the balance by adjusting the levelling screws. Removes dust particles, if any, on the pans by a camel-hair brush.

(*ii*) Adjust the screw nuts at the ends of the beam until the pointer swings almost equally on both sides of the central line of the scale.

(*iii*) Disturb the balance a little and note the reading of the five successive turning points of the pointer; three to the left and two to the right. Calculate the average of the two sets separately and obtain the mean of these two averages. This is the resting position P of the pointer and is called the zero-point.

(*iv*) **Adjust the balance:** Place the body to be weighed at the centre of the left hand pan and suitable weights W_1 at the centre of the right-hand pan.

(*v*) Determine the resting position of the pointer with the body and the weights by adopting the procedure described in step (iii) Denote the resting position by Q. The weight W_1 should be such that Q lies about 5 divisions to the right of the zero point.

(*vi*) Put an additional 10 mgm weight on the right hand pan and find the view rest position of the pointer following step (iii), let it be R.

Experimental Results:

Table 2.6: Recording of data

Load on the		*Turning Points (divisions)*		*Mean of Turning Points (divisions)*		*Resting positions (divisions)*
Left hand Pan	*Right hand Pan*	*Left*	*Right*	*Left*	*Right*	
zero	zero	(*i*) (*ii*) (*iii*)	(*ii*) (*ii*)			(P)
Body	(W_1 gm)	(*i*) (*ii*) (*iii*)	(*i*) (*ii*)			(Q)
Body	(W_1 gm + 10 mgm)	(*i*) (*ii*) (*iii*)	(*i*) (*ii*)			(R)

Calculation:

$$W = W_1 + \frac{Q-P}{Q-R} \times \frac{10}{1000} \text{ gm}$$

Substitute the experimental values of W_1, P, Q and R and obtain W, the correct weight of the body.

(*b*) **Method of equal displacements:** In this method adopt the following procedure:

(*i*) Level the balance by adjusting the levelling screw. The plumb line will then be vertical.

(*ii*) The dust particles on the pans are then brushed off.

(*iii*) The screw nuts at the ends of the beam are to be shifted in or out till the pointer swings equally on both sides of the central line of the scale.

(*iv*) The body to be weighed is then placed at the centre of the left pan. Suitable standard weights are placed at the centre of the right pan until the pointer swings equally on both sides of the central line of the scale. If the lowest weight in the weight box fails to achieve this, use a rider till the swings of the pointer about the central line are equal.

(*v*) Make a detailed recording of the weights put on the right pan and note the readings of the rider. Sum the weights to obtain the weights of the body.

Precautions in weighing

(*i*) Each time a standard load is placed on the right hand pan, test the balancing of the beam by raising it a little with the handle H after closing the door. Final balance is to be checked by raising the beam in full.

(*ii*) The standard weights should always be handled with a pair of forceps.

(*iii*) The body to be weighed and the standard weights should be placed at the centre of the pans.

(*iv*) While reading the turning points of the pointers, avoid parallax by keeping the eyes perpendicular to the scale and the pointer.

N.B.: Do not weigh a too hot or a too cold body in a balance. The different parts of the balance may expand (or contract) when a too hot (or cold) body is placed on the pan; weighing is therefore not accurate.

2.5 THE OPTICAL BENCH

There are two types of optical bench: (*i*) ordinary and (*ii*) accurate types. We describe below the second type. The first type is a simple one and is used for measurements of optical constants of mirrors and lenses. A student, acquainted with the accurate type of optical bench, will be able to handle the ordinary type easily.

The optical bench employed for biprism work is shown in Fig. 2.8. It consists of two long horizontal steel rails, R_1R_2 and R_3R_4 placed at a fixed distance apart. On one of these rails, a scale is graduated in mm. This is shown in Fig. 2.8 on the rail R_1R_2. Several uprights such as U_1, U_2, U_3, U_4 having verniers attached to their bases can slide over the rails. L_1, L_2, L_3 and L_4 are four levelling screw; U_1 is the upright carrying the slit S, U_2 is the upright carrying the biprism, U_3 is the upright carrying the micrometer eyepiece E, and U_4 is the upright carrying a convex lens L; T_1 and T_2 are two tangent screws by which the slit and the biprism can be rotated in their planes about a horizontal axis, M is a micrometer screw provided with a circular and a linear scale; the screw attached to the upright U_2 can move the upright perpendicular to the bench.

From the scale and the vernier, the shift of any upright from any position can be determined. The height of the uprights from the bed of the bench can be adjusted; the bed can be made horizontal by the levelling screws, L_1, L_2, L_3 and L_4. The screw represented by L_4 is not seen in the figure.

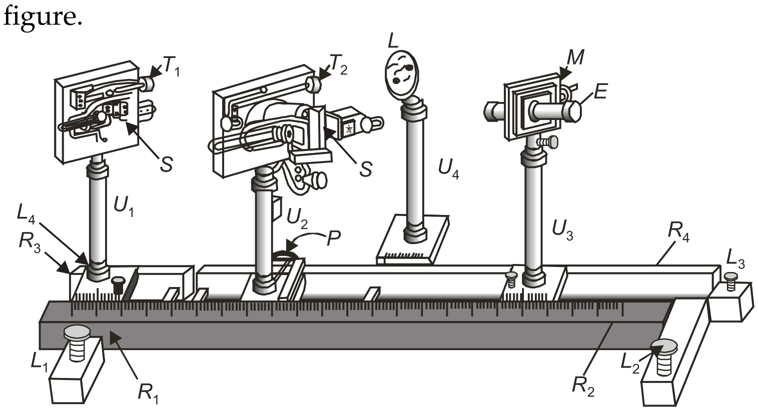


Fig. 2.8

Index error: The distance between any two stands, as obtained from the difference of the bench scale readings, may not be the true distance between them. Thus an error, known as index error, may exist between any two stands. If L be the actual distance between the two stands and d be the distance obtained from the bench scale readings, then the index error is given by $x = L - d$. This is to be added algebraically to the quantity to be corrected for index error. When the distances are measured from the centre of an equi-convex lens of thickness t, the index error becomes $\left(l - d + \dfrac{t}{2}\right)$.

2.6 PARALLAX

Let any eye E be placed in a straight line joining two object points O and O', as shown in Fig. 2.9. Now if the eye is moved in a direction perpendicular to the line EOO', then the more distant object O' will appear to move with the eye. This phenomenon is known as parallax. To eliminate parallax, either the nearer object O should be moved away from the eye E, or the distant object O' should be moved towards the eye until there is no separation between them as the eye is moved in a direction perpendicular to the line joining the two objects.

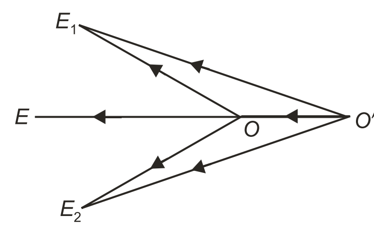


Fig. 2.9

2.7 THE SPECTROMETER

This instrument (Fig. 2.10) is normally used to study spectra and to measure refractive indices. It has the following essential parts:

(*i*) **Collimator (*C*):** It consists of a horizontal tube with a converging achromatic lens O_c at one end of the tube and a vertical slit S (shown separately in the right side of Fig. 2.10) of adjustable width at the other end. The slit can be moved in or out of the tube by a rack and pinion arrangement F_c and its width can be adjusted by turning the screw F. The collimator is rigidly fixed to the main part of the instrument and can be made exactly horizontal by two screws C_1 and C_2 below it. When properly focussed, the slit lies in the focal plane of the lens O_c. Thus the collimator provides a parallel beam of light.

(*ii*) **Prism table (*P*):** It is a small circular table and capable of rotation about a vertical axis. It is provided with three levelling screws, shown separately in Fig. 2.11, as P_1, P_2, and P_3. On the surface of the prism table, a set of parallel, equidistant lines parallel to the line joining two of the levelling screws, is ruled. Also, a series of concentric circles with the

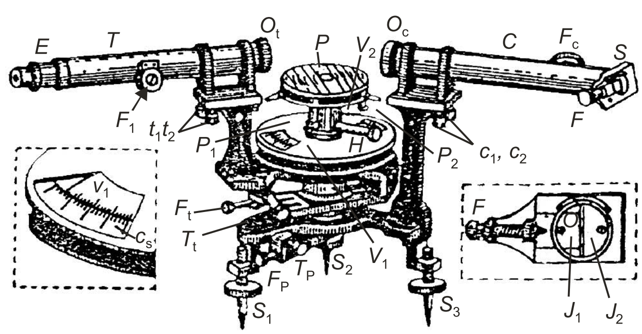


Fig. 2.10

centre of the table as their common centre is ruled on the surface. The screw H (Fig. 2.11) fixes the prism table to the two verniers V_1 and V_2 and also keep it at a given height. These two verniers rotate with the table over a circular scale are graduated in fraction of a degree. The angle of rotation of the prism table can be recorded by these two verniers. The screw F_1 fixes the prism table and the screw T_p is the tangent screw for the prism table by which a smaller rotation can be imparted to it. It should be noted that a tangent screw functions only after the corresponding fixing screw is tightened.

(*iii*) **Telescope (*T*):** It is a small astronomical telescope with an achromatic doublet as the objective O_t and the Ramsden type eye-piece E. The eye-piece is fitted with cross-wires and slides in a tube which carries the cross-wires. The tube carrying the cross wires in turn, slides in another tube which carries the objective. The distance between the objective and the cross-wires can be adjusted by a rack and pinion arrangement F_1. The Telescope can be made exactly horizontal by two screws t_1 and t_2. It can be rotated about the vertical axis of the instrument and may be fixed at a given position by means of the screw F_t slow motion can be imparted to the telescope by the tangent screw T_t.

(*iv*) **Circular Scale (*C.S.*):** This is shown separately in the left hand side of the Fig 2.10. It is graduated in degrees and coaxial with the axis of rotation of the prism table and the telescope. The circular scale is rigidly attached to the telescope and turned with it. A separated circular plate mounted coaxially with the circular scale carries two verniers, V_1 and V_2, 180° apart. When the prism table is clamped to the spindle of this circular plate, the prism table and the verniers turn together.

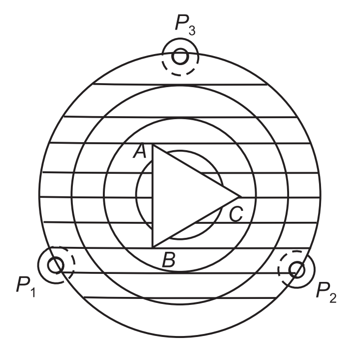


Fig. 2.11

The whole instrument is supported on a base provided with three levelling screws S_1, S_2, and S_3. One of these is situated below the collimator. In Fig. 2.10, this screw has been called S_3.

2.8 ADJUSTMENTS OF THE SPECTROMETER

The following essential adjustments are to be made step by step in a spectrometer experiment:

(*i*) Levelling

Levelling the apparatus means making (*a*) the axis of rotation of the telescope vertical, (*b*) the axis of the telescope and that of the collimator horizontal, and (*c*) the top of the prism table horizontal. The following operations are performed for the purpose.

(*a*) Levelling of telescope: Place a spirit level on the telescope tube T making its axis parallel to that of the telescope. Set the telescope parallel to the line joining the levelling screw S_1 and S_2. Bring the air bubble of the spirit level halfway towards the centre by turning the screw S_1 and S_2 by equal amounts in the opposite direction. Next bring the bubble at centre by turning the levelling screw t_1 and t_2 below the telescope by equal amounts in opposite directions.

Now rotate the telescope through 180° so that it is placed to its first position on the other side. Bring the air bubble at centre as before, i.e. half by the screws S_1 and S_2 and the other half by t_1 and t_2. Repeat the operations several times so that the bubble remains at the centre for both positions of the telescope.

Next place the telescope in the line with the collimator and bring the air bubble of the spirit level at the centre by turning the screw below the collimator, i.e. S_3 check the first adjustment after this second one is made. The axis of the rotation of the telescope has thus become vertical and the axis of the telescope has become horizontal.

(*b*) Levelling of collimator: Remove the spirit level from the telescope. Place it on the collimator along its length. Bring the air bubble of the spirit level at the centre by adjusting the levelling screws C_1 and C_2 below the collimator. This makes the axis of the collimator horizontal.

(*c*) Levelling of the prism table: Place a spirit level at the centre of the prism table and parallel to the line joining two of the levelling screw of the prism table. Bring the air bubble of the spirit level at the centre by turning these two screws in the opposite directions. Now place the spirit level perpendicular to the line joining the two screws and bring the bubble at the centre by adjusting the third screw. This makes the top of the prism table horizontal.

(*ii*) Alignment of the Source

Place the Bunsen burner at a distance of 15 to 20 cms from the slit in such a way that the axis of the collimator passes through the centre of the flame. Soak the asbestos wound round the iron or copper ring in a concentrated solution of sodium chloride. Place the ring round the flame at such a height that the brightest part of the flame lies opposite to the slit.

Now place a screen with an aperture between the source and the slit so that light from the source can reach the slit without obstruction while, at the same time, stray light is prevented from reaching the observer's eyes directly.

(*iii*) Focussing the Cross-wires

Rotate the telescope towards any illuminated background. On looking through the eye-piece, you will probably find the cross-wires appear blurred. Move the eye-piece inwards or outwards until the cross-wire appear distinct.

(*iv*) Adjustment of the Slit

Place the telescope in line with the collimator. Look into the eye-piece without any accommodation in the eyes. The image of the slit may appear blurred. Make the image very sharp by turning the focussing screw of the telescope and of the collimator, if necessary. If the images does not appear vertical, make it vertical by turning the slit in its own plane. Adjust the width of the slit so that its image may have a breadth of about one millimetre.

(*v*) Focussing for Parallel Rays

Schuster's method: This is the best method of focussing the telescope and the collimator for parallel rays within the space available in the dark room. The method is explained below:

Place the prism on the prism table with its centre conciding with the centre of the table and with its refracting edge vertical.* Rotate the prism table so that one of the refracting faces of

* Use optical levelling as described on page 31, if necessary.

the prism AB (Fig. 2.12) is directed towards the collimator and light from the collimator is incident on the refracting face at an angle of about 45° to the face. Look through the other face AC of the prism for the refracted beam which is bent towards the base of the prism. You will see with the naked eye the image of the slit formed by refraction through the prism. Slightly rotating the prism table, first in one direction and then in the other, you will also see that the image moves. Now turn the prism table in the proper direction so that the images of the slit moves towards the direct path of rays from the collimator and reaches the positions of minimum deviation. At this position of the prism the images will move away from the direct path of rays in whatever direction the prism table is turned. Next bring the telescope between the prism and the eye and move it slightly this way or that so that you can see the image of the slit in it. Slightly rotate the prism table so that you find the image through the telescope exactly at the position of minimum deviation. Displace the telescope from this position in a direction away from the direct path of rays. For this position of the telescope, the image of the slit can be brought on the cross-wires for two positions of the prism. In one position, the angle of incidence of the prism is larger than that at minimum deviation and in the other position it is smaller. In the former position, the refracting edge of the prism is nearer to you than at minimum deviation, and in the latter it is more remote from you. The former position of the prism is called the slant position and the latter position is called the normal position.

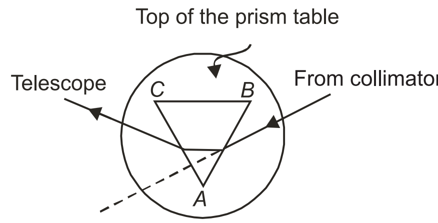


Fig. 2.12

Next perform the following operations:

(*a*) Bring the image on the cross-wires of the telescope by rotating the prism table in such a way that the refracting edge of prism is nearer to you (the prism is at the slant position) than at minimum deviation. Focus the image by rack and pinion arrangement of the telescope. The image now becomes very narrow.

(*b*) Next rotate the prism table in the opposite direction. The refracting edge of the prism will move away from you. Go on rotating the prism table until the image moving towards the position of minimum deviation, turns back and reaches the cross-wires again. Keeping the prism at this position, i.e. at the normal, focus the image by the rack and pinion arrangement of the collimator. The image is now very wide.

Repeat the operations (*a*) and (*b*) several times in succession till the image remains sharp for both the positions of the prism. The telescope and the collimator are then focussed for parallel rays.

Mnemonic: The operations in Schuster's method can be easily remembered as follow: When the refracting edge of the prism is nearer to you, focus the image by the telescope. When the refracting edge of the prism is away from you, focus by the collimator. Or simply, near-near, away-away. Alternatively, the operations can be remembered by noting that when the image is Broad, focussing is done by the collimator. When the image is thin, focus by the Telescope, Simply, $b - c$, $t - t$.

2.9 THEORY OF SCHUSTER'S METHOD

When a narrow beam of light is incident on one face (*AB*) of a prism, the emergent beam from the other face (*AC*) appears to come a point image I [Fig. 2.13 (*a*)].

The relation between the distance of the image from the prism, v and that of the object u is given by

$$v = u\left(\frac{\cos^2 i'}{\cos^2 r'}\right)\Big/\left(\frac{\cos^2 i}{\cos^2 r}\right),$$

where i and r are respectively the angle of incidence and refraction at the first face, and r' and i' are those at the second face.

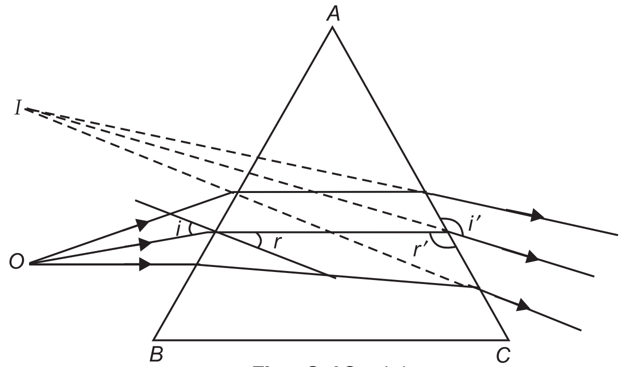


Fig. 2.13 (a)

When the prism is at the position of minimum deviation for the mean ray of the beam, $i = i'$ and $r = r'$. In that case, $v = u$, i.e. the distance of the image from the prism is equal to the distance of the object. This is shown in Fig. 2.13 (*b*).

When the prism is at the normal position, the angle of incidence is smaller than that at minimum deviation. Under this condition, $i < i'$, and $(\cos^2 i/\cos^2 r) > (\cos^2 i'/\cos^2 r')$. Thus $v < u$ i.e. the image is nearer to the prism than the object [Fig. 2.13 (c)].

Similarly it can be shown that for the slant position of the prism when the angle of incidence is larger than that at minimum deviation, $v > u$, i.e. the image of the slit formed by refraction at the prism is at a longer distance than the object [Fig. 2.13 (d)].

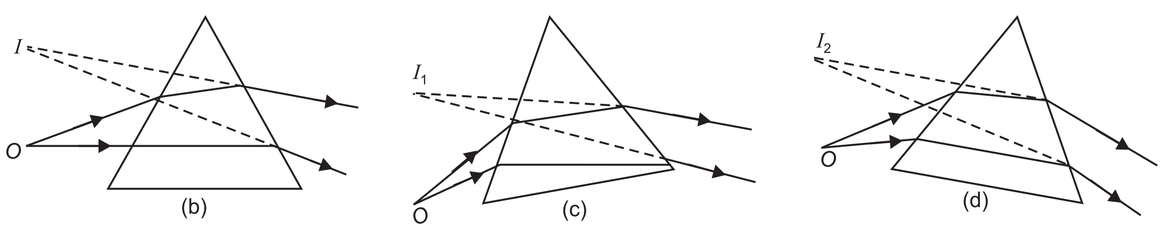


Fig. 2.13

Now in operation (*a*) of Sec 2.8 (*v*), when the prism is in the slant position, the image of the slit is focussed in the telescope by its rack and pinion arrangement. This means that the telescope is focussed on a remote point (since the image is formed at a greater distance from the object.). Next in operation (*b*) when the prism is turned to the normal position, the image moves nearer to the prism and goes out of focus of the telescope. This time, when the image is focussed in the telescope by moving the collimator lens nearer to the slit, the image is pushed to the previous position of focus of the telescope. If the prism is now changed from the normal to the slant position (as is done when the operation (*a*) is again repeated), the image moves further away from the prism. When the image is focussed by the telescope the telescope is focussed on a more remote point.

Thus with every adjustment of the collimator, the image formed by refraction at the prism at the normal position is pushed at the position corresponding to the slant position of the prism and in the latter position the telescope is focussed on the point which moves away to greater and greater distances. Ultimately the images corresponding to the two positions of the prism are formed at very great distances and appear to be in focus for both positions of the prism. The spectrometer is thus focussed for parallel rays.

Optical levelling of a prism: The levelling of a prism table by the method discussed in Sec. 2.8 (*i*)*c* makes the refracting faces of the prism vertical only when the bottom face of the prism, which is placed on the prism table, is perpendicular to its three edges. But if the bottom face is not exactly perpendicular to the edges, which is actually the case, the prism should be levelled by the optical method, as described below:

(*i*) Illuminate the slit by sodium light and place the telescope with its axis making an angle of about 90° with that of the collimator.

(*ii*) Place the prism on the prism table with its centre coinciding with that of the table and with one of its faces (faces *AB* in Fig. 2.11) perpendicular to the line joining the two screw P_1 and P_2 and of the prism table.

(*iii*) Rotate the prism table till the light reflected from this face *AB* of the prism enters the telescope. Look through the telescope and bring the image at the centre of the field of the telescope by turning the screws P_1 and P_2 equally in the opposite directions.

(*iv*) Next rotate the prism table till the light reflected from the other face *AC* of the prism enters the telescope, and bring the image at the centre of the field by turning the third screw P_2 of the prism table.

Care in handling the prism: The reflecting surfaces of the prism should be cleaned with a piece of cloth soaked in alcohol. Do not touch the refracting surfaces by hand. Place the prism on the prism table or remove it from the prism table by holding it with fingers at the top and bottom faces.

2.10 A SLIDING RHEOSTAT

A rheostat offers a resistance that can be altered in a continuous manner.

A sliding rheostat [Fig. 2.14 (*a*)] consists of a coil of bare wire wound uniformly round a porcelain cylinder. Manganin or eureka is generally chosen for the material of the wire. The ends of the coil are connected to two binding screws, *A* and *B*. *C* is a metal contact which moves along a rod *DE* which also contains a binding screw at *E*.

If the terminals *A* and *B* are connected to a circuit, the whole of the coil resistance is included in the circuit and the contact becomes ineffective. But if the terminal *A* and *E* of the rheostat are connected to the circuit, the actual resistance included in the circuit is determined by the portion of the coil between *A* and *C*, and so can be varied by varying the position of the contact *C*, as is evident from Fig. 2.14(*b*). Thus, by sliding the contact *C* the resistance between the terminal *A* and *E* (and also between terminal *B* and *E*) can be changed almost continuously.

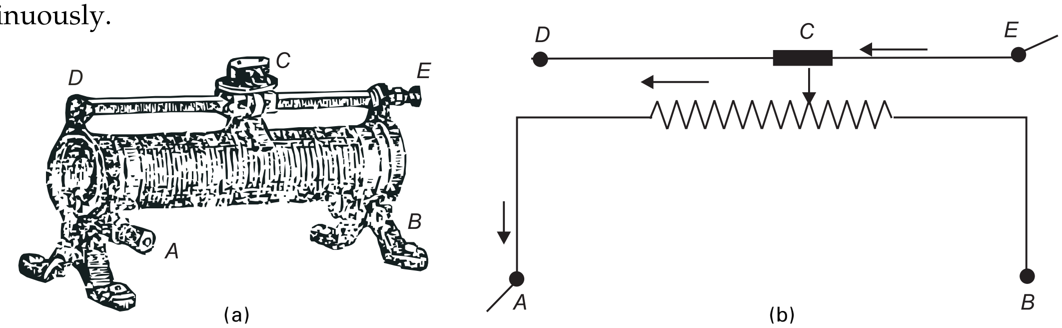


Fig. 2.14

TELESCOPES AND MICROSCOPES

2.11 POWER OF ACCOMMODATION OF THE EYE

Our eye is a natural lens fixed in its place through muscles. This lens has the power to change its focal length and it is because of this that we can see clearly near as well as far objects.

When we see a distant object, the parallel rays of lights falling on our eye are focussed by the lens on the retina. Therefore, the object is seen distinctly. At this moment our eye is in a relaxed state (there is no tension in muscles) and the focal-length of the lens is maximum. When, however, we see a near object, the muscles contract to increase the curvature of the lens. Hence the focal length of the lens decreases and again a clear image of the object is formed at the retina. This power of changing the focal length of the eye is called power of accomodation. As we see more and more near objects, more and more power of accomodation is to be applied. But there is a limit of applying the power of accommodation. The nearest distance up to which eye can see clearly (by applying maximum power of accomodation) is called 'least distance of distinct vision'. For normal eye, this distance is 25 cm. If an object is placed at a distance less than 25 cm from the eye, it will not be seen distinctly.

2.12 VISUAL ANGLE: MAGNIFYING POWER OF OPTICAL INSTRUMENTS

Visual Angle: The angle which an object subtends at our eye is called the 'visual angle.' The apparent size of an object as seen by our eye depends upon the visual angle. Greater the visual angle, larger the apparent size of the object.

In Fig. 2.15, an object *AB* subtends a visual angle α at the eye and its image *CD* is formed at the retina. When the same object is brought nearer the eye in the position A_1, B_1, it subtends a larger visual angle β at the eye and a larger image CD_1, is formed at the retina. Obviously, the same object appears bigger to the eye. Thus in order to see an object larger, the visual angle should be increased. Telescope and Microscopes are the aids to increase the apparent size of object by increasing visual angle.

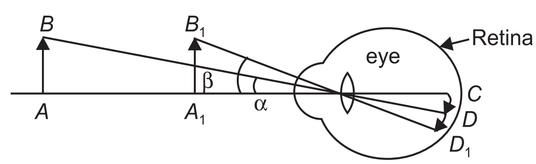


Fig. 2.15

Very distant objects (such as moon), although very big in size, appear very small because they subtend very small visual angle at the eye. To see them bigger, we cannot decrease their distance. But if, with the help of proper lenses, a small angle of the distant object be formed 'close' to the eye then this image will subtend a large visual angle at the eye and the object will appear large. Telescope is based exactly on this principle.

Very small objects subtend small visual angle due to their smallness. We can increase the visual angle by bringing these objects closer to eye, but we cannot do so beyond a certain limit (25 cm) because then the object will not be seen distinct. If, with the help of proper lenses, a 'large' image of small object is formed, then this image will subtend a large visual angle at the eye and the object will appear large. Microscope is based on this principle.

Magnifying Power: The purpose of microscopes and telescope is to increase the visual angle. Therefore, the power of these instruments is measured by their power of increasing the visual angle. This is called the 'magnifying power' of the instrument. The magnifying power of an optical instrument is defined as the ratio of the visual angle subtended by the image formed by the instrument at the eye to the visual angle subtended by the object at the unaided eye.

2.13 ASTRONOMICAL (REFRACTING) TELESCOPE

A telescope is an optical instrument used to see distant objects. The image of the distant object formed by the telescope subtends a large visual angle at the eye, so that the object appears large to the eye.

Construction: It consists of a long cylindrical metallic tube carrying at one end an achromatic convex lens of large focal length and large aperture* which is called the 'objective lens'. At the other end of the tube is fitted a smaller tube which can be moved in and out in the bigger tube by a rack and pinion arrangement. At the other end of the smaller tube is fitted an achromatic convex lens of small focal length and small aperture** which is called the 'eye-piece'. Cross-wires are mounted in the smaller tube at the focus of the eye-piece.

Adjustment: First of all the eye-piece is moved backward and forward in the smaller tube and focussed on the cross-wire. Then the objective-lens is directed toward the object which is to be observed. Now, by rack and pinion arrangement, the smaller tube is moved in the larger tube until the distance of the objective lens from the cross-wire is so adjusted that there is no parallax between the image of the object and the cross-wires. In this position a distinct image of the object will be seen. This image is formed by refraction of the light through the lenses. Hence this telescope is called a 'refracting' telescope.

Formation of Image: In Fig. 2.16 are shown the objective-lens O and the eyepiece E of a telescope. AB is a distant object whose end A is on the axis of the telescope. The lens O forms a small, real and inverted image $A'B'$ at its second focus F_o. This image lies inside the first focus F_e' of the eye-piece E and acts as an object for the eye-piece which forms a virtual, erect and magnified final image $A''\,B''$. To find the position of B'', two dotted rays are taken from B'. One

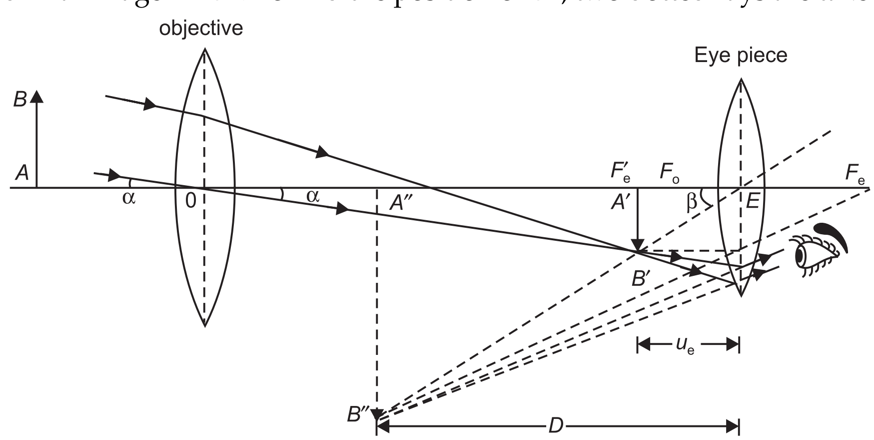


Fig. 2.16

(*) objective of 'large' aperture is taken so that it may collect sufficient light and form bright image of very distant objects (stars, etc).

(**) Eyepiece of small aperture is taken so that the whole light may enter the eye.

ray, which passes through the optical centre E, goes straight and the second ray which is taken parallel to be principal axis goes, after refraction, through the second focus F_e of E. The two refracted rays when produced backward meet at B''.

Magnifying power: The magnifying power (angular magnification) of a telescope is defined by

$$M = \frac{\text{angle subtended by the final image at the eye}}{\text{angle subtended by the object at the eye when the object is in actual position}}$$

Since eye is near the eye-piece E, the angle β subtended by the final image $A''B''$ at the eyepiece may be taken as the angle subtended at the eye. In the same way, since the object AB is very far from the telescope, the angle α subtended by the object at the objective may be taken as the angle subtended at the eye.

Then

$$M = \frac{\beta}{\alpha}$$

Since angles β and α are very small, we can write

$$\beta = \tan \beta = \frac{A'B'}{EA'}$$

$$\alpha = \tan \alpha = \frac{A'B'}{OA'}$$

$$\therefore \qquad M = \frac{A'B' / EA'}{A'B' / OA'} = \frac{OA'}{EA'}$$

If the focal length of the objective O be f_o and the distance of $A'B'$ from the eye-piece E be u_e then, with proper sign, $OA' = + f_o$ and $EA' = -u_e$. Thus by the above equation, we have

$$M = \frac{-f_o}{u_c} \qquad ...(i)$$

This is general formula of magnifying power. Now there are two possibilities:

(*i*) **The final image is formed at the least distance D of distinct vision:** If the distance of the final image $A''B''$ from the eye-piece be D, then in applying the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for the eye-piece, we shall have

$$v = -D,\ u = -u_e \text{ and } f = +f_e$$

where f_e is the focal length of the eyepiece. We get

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right).$$

Substituting this value of $1/u_e$ in eq. (*i*), we have

$$M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right) \qquad ...(ii)$$

We shall substitute only the numerical values of f_o, f_e and D in this formula. In this position the length of the telescope will be $f_o + u_e$.

(*ii*) When the final image is formed at infinity: To see with relaxed eye the final image should be formed at infinity (Fig. 2.17). For this, the distance between the objective and the eye-piece is adjusted so that the images $A'B'$ formed by the objective O is at the focus F_e' of the eye-piece ($u_e = f_e$). This adjustment of the telescope is called normal adjustment. Substituting $u_e = f_e$ in eq (*i*), we get,

$$M = \frac{-f_o}{f_e} \qquad ...(iii)$$

In this position the length of the telescope will be $f_o + f_e$.

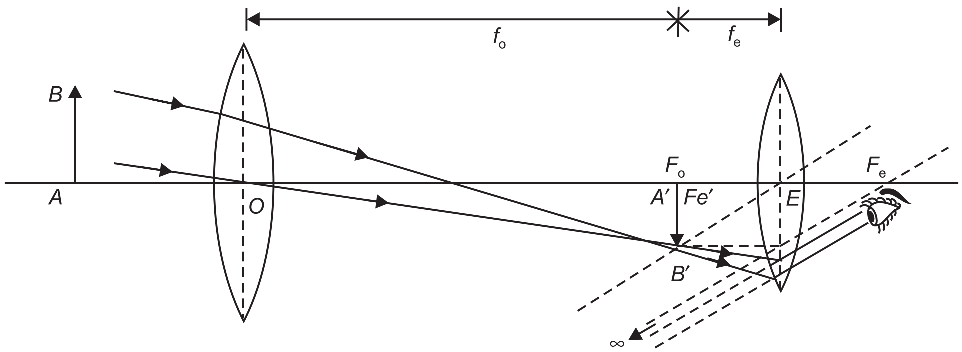


Fig. 2.17

It is clear from eq. (*ii*) and (*iii*) that in order to increase the magnifying power of a telescope the focal length f_o of the objective lens should be large and the focal length f_e of the eye-piece should be small. Negative sign indicates that the final image is inverted.

2.14 REFLECTING TELESCOPE

To obtain a 'bright' image of a distant object by means of a refracting telescope it is essential that the objective is of a large aperture so that it may collect enough light coming from the object. But objectives of very large aperture are difficult to manufacture and are very costly. The same can be achieved by using a concave mirror of large aperture instead of a lens.

Construction: In a reflecting telescope the objective is a concave mirror M_1 (Fig. 2.18) of the large focal-length and large aperture which is fitted at one end of a wide tube. The open end to the tube is directed towards the distant object to be seen. The tube carries a plane mirror M_2 which is

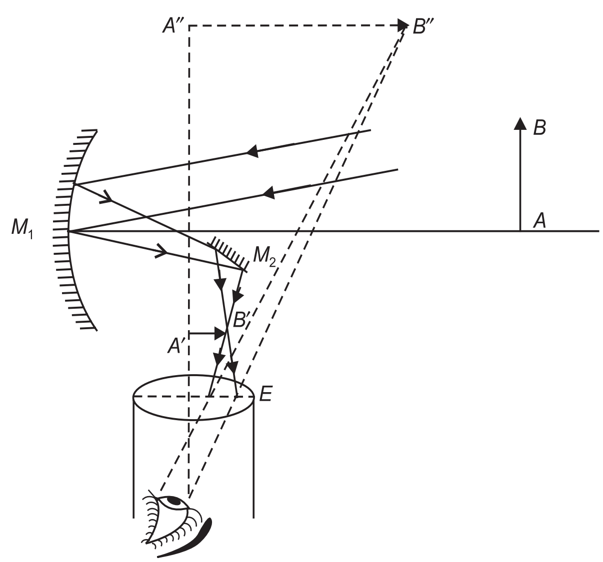


Fig. 2.18

placed between the concave mirror M_1 and its focus, and is inclined at an angle of 45° to the principal axis of M_1. A small side-tube carries a lens E of small focal length and small aperture. E is called the eye-piece.

Formation of Image: The (parallel) rays from a distant object AB fall on the concave mirror M_1. The reflected convergent beam is received by the plane mirror M_2 which reflects this beam to form a small, real image $A'B'$. The image $A'B'$ acts as object for the eye-piece E, which forms a magnified virtual image $A''B''$ between the least distance of distinct vision and infinity. If $A'B'$ is in the focal-plane of E then the final image will be formed at infinity.

Magnifying Power: Magnifying power of the telescope is

$$M = \frac{\text{angle subtending by the final image at the eye}}{\text{angle subtended by the object at the eye}}$$

It can be shown that when the final image is formed at infinity, then the magnifying power is given by

$$M = \frac{f_o}{f_e},$$

Where f_o and f_e are the focal lengths of the concave mirror M_1 and the eye-piece E.

The modern reflecting telescopes carry paraboloidal mirror which is free from spherical aberration. One of the largest reflecting telescope of the world is kept at Mount Palomer in California. The aperture of its objective mirror is 200 inch (≈ 5 meter). It is used to study distant stars and planets.

Merits:

(*i*) The image formed by a reflecting telescope is brighter than that formed by a refracting telescope.

(*ii*) Further, in reflecting telescope the image is free from chromatic aberration, while this defect persists in the image formed by a refracting telescope.

(*iii*) With the use of paraboloidal mirror the image may also be made free from spherical aberration.

(*iv*) The objective of the telescope should have a larger aperture. It is difficult to construct lenses of large aperture because the glass becomes distorted during the manufacturing process. The image formed by such a lens becomes distorted. On the other hand, the image produced by a mirror is not affected by any distortion in the interior of the glass.

2.15 SIMPLE MICROSCOPE

A microscope is an optical instrument which forms large image of a close and minute object.

In the simplest form a simple microscope or magnifying glass is just a thin, short-focus convex lens carrying a handle. The object to be seen is placed between the lens and its focus and the eye is placed just behind the lens. Then, the eye sees a magnified, erect and virtual image on the same side as the object. The position of the object between the lens and its focus is so adjusted that the image is formed at the least distance of distinct vision (D) from the eye. The image it is then seen most distinctly.

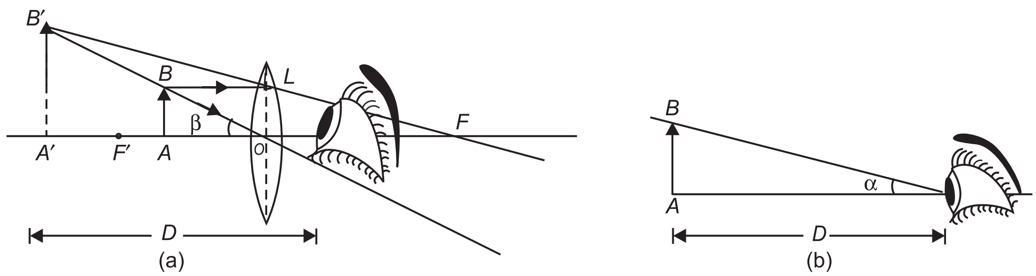


Fig. 2.19

In Fig. 2.19 (*a*), *AB* is a small object placed between a lens *L* and its first focus *F*′. Its magnified virtual image *A*′*B*′ is formed at distance *D* from the lens, the distance of the image *A*′*B*′ from the eye is also *D*.

Magnifying Power: Let β be the angle subtended by the image *A*′*B*′ at the eye [Fig. 2.19 (*a*)] and α the angle subtended by the object *AB* at the eye when placed directly at a distance *D* from the eye [Fig. 2.19 (*b*)]. Then, the magnifying power of the simple microscope is given by

$$M = \frac{\text{angle subtended by the image at the eye}}{\substack{\text{angle subtended by the object at the eye when placed} \\ \text{at least distance of distict vision}}} = \frac{\beta}{\alpha}$$

$$\beta = \tan\beta = AB/OA$$
$$\alpha = \tan\alpha = AB/D.$$

$$\therefore \qquad M = \frac{AB/OA}{AB/D} = \frac{D}{OA}.$$

But $OA = u$ (distance of the object from the lens.)

$$\therefore \qquad \boxed{M = \frac{D}{u}.} \qquad \ldots(i)$$

The image *A*′*B*′ is being formed at a distance *D* from the lens. Hence, in the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we shall put $v = -D$ and $u = -u$(with proper sign). Thus,

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

or
$$\frac{1}{u} = \frac{1}{D} + \frac{1}{f}$$

or
$$\frac{D}{u} = 1 + \frac{D}{f}$$

Putting this value of D/u in eq. (*i*), we get

$$\boxed{M = 1 + \frac{D}{f}.}$$

We shall substitute only the numerical values of D and f. Thus M is positive which means that an erect image is formed. It is also clear, that shorter the focal length of the lens, larger is the magnifying power.

If the eye is kept at the distance d from the lens, then $v = -(D - d)$, and the magnifying power will be

$$\boxed{M = 1 + \frac{D - d}{f}.}$$

Thus, magnifying power is reduced. Hence to obtain maximum magnifying power, the eye must be very close to the lens.

To see with relaxed eye, the image $A'B'$ should be formed at infinity. In this case, the object AB will be at the focus of the lens, that is, $u = f$. Thus, from eq. (1) we have

$$\boxed{M = \frac{D}{f}.}$$

2.16 COMPOUND MICROSCOPE

Construction: It consists of a long cylindrical metallic tube carrying at one end an achromatic convex lens O of small focal length and small aperture (Fig. 2.20). This lens is called the 'objective lens'. At the other end of the tube is fitted a smaller tube. At the outer end of this smaller tube is fitted an achromatic convex lens E whose focal length and aperture are larger than that of the objective lens. The lens E is towards the eye and is called the eye-piece. The entire tube can be moved forward and backward by rack and pinion arrangement.

Adjustment: First of all the eye-piece is moved forward or backward in the tube and brought in a position so that on seeing through it the cross-wire appear distinct. Then the object is placed just below the objective lens and the entire tube is moved by rack and pinion arrangement until the image of the object is formed on the cross-wire and there is no parallax between the image and the cross-wire. In this position the image of the object will be seen distinctly.

Formation of Image: Suppose AB is a small object placed slightly away from the first focus F_o' of the objective O (Fig. 2.20) which form a real inverted and magnified image $A'B'$. This image lies between the eye-piece E and its first focus F_e' and acts as an object for the eye-piece which forms a magnified, virtual final image $A''B''$. To find the position of B'', two dotted rays are taken from B''. One ray; which is parallel to the principal axis passes, after refracting, through the second focus F_e of E. The other ray which passes through the optical centre of E travel straight. Both the refracted rays when produced backward meet at B''. The image $A''B''$ is generally formed at the least distance of distinct vision although it can be formed any where between this position and infinity. The rays by which the eye sees the image are clearly shown in the Fig. 2.20.

Magnifying Power: Suppose the final image $A''B''$ subtends an angle β at the eye-piece E. Since eye is very near to the eye-piece, the angle β can also be taken as subtended by $A''B''$ at the eye. Suppose when the object AB is at the least distance of distinct vision D, then it subtends an angle α at the eye. The magnifying power of the microscope is

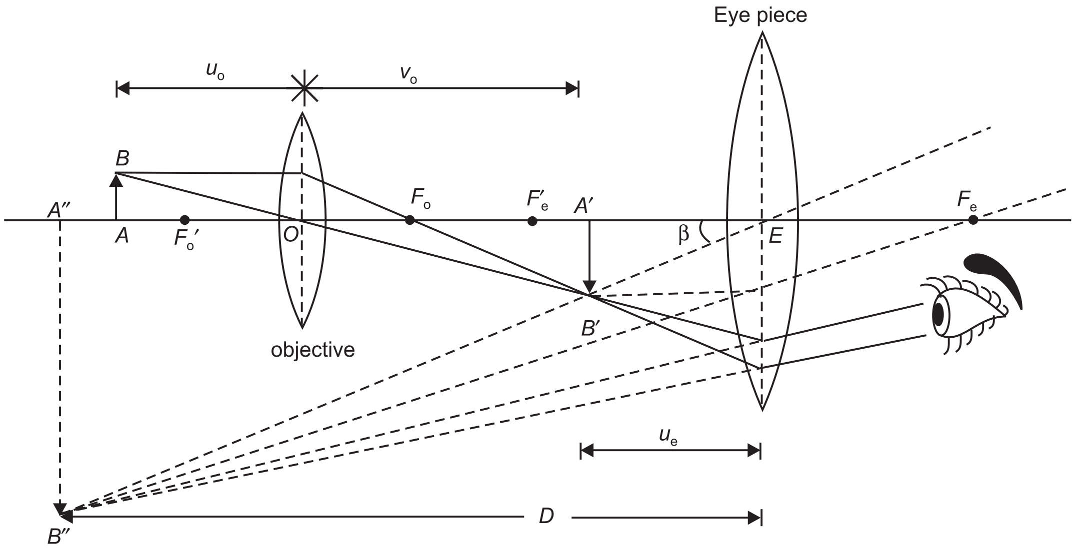


Fig. 2.20

$$M = \frac{\text{angle subtended by the final image at the eye}}{\text{angle subtended by the object, when placed at least distance of distinct vision}}$$

$$= \frac{\beta}{\alpha}$$

Since β and α are very small, we can write

$$\beta = \tan \beta = \frac{A'B'}{EA'}$$

$$\alpha = \tan \alpha = \frac{AB}{D}$$

$$\therefore \qquad M = \frac{\beta}{\alpha} = \frac{A'B'/EA'}{AB/D} = \frac{A'B'}{AB}\left(\frac{D}{EA'}\right).$$

If the distance of the object AB and the image $A'B'$ from the objective O be u_o and v_o respectively, then from the magnification formula we have (taking proper sign)

$$\frac{A'B'}{AB} = \frac{+v_o}{-u_o}.$$

Similarly, if the distance of $A'B'$ from the eye-piece be u_e, then $EA' = -u_e$. Therefore, from the above formula, we have

$$M = \frac{-v_o}{u_o}\left(\frac{-D}{-u_e}\right) = \frac{-v_o}{u_o}\left(\frac{D}{u_e}\right). \qquad \text{...}(i)$$

Now there are two possibilities:

(*i*) *The final image is formed at the least distance D of distinct vision:* If the distance of the final image $A''B''$ from the eye-piece be D, then applying the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for the eye-piece, we shall have

$$v = -D,\ u = -u_e \text{ and } f = +f_e,$$

where f_e is the focal length of the eye-piece. Now, we get

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or
$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e}$$

or
$$\frac{D}{u_e} = 1 + \frac{D}{f_e}$$

substituting this value of D/u_e in eq. (*i*), we get

$$M = \frac{-v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \qquad ...(ii)$$

In this position the length of the microscope will be $v_o + u_e$.

(*ii*) *When the final image is formed at infinity:* To see with relaxed eye, the final image $A''B''$ should be formed at infinity (Fig. 2.21). In this case the image $A'B'$ will be at the focus F_e' of the eye-piece E i.e. $u_e = f_e$. Substituting this value in equation (*i*), we get the magnifying power of the relaxed eye, which is given by

$$M = \frac{-v_o}{u_o}\left(\frac{D}{f_e}\right). \qquad ...(iii)$$

In this position the length of the microscope will be $v_o + f_e$.

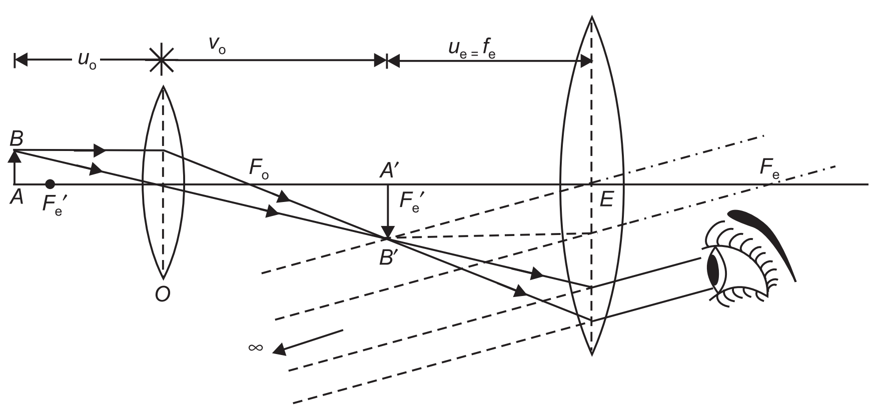


Fig. 2.21

In formula (*ii*) and (*iii*) we shall substitute only the numerical values of v_o, u_o, f_e and D. Negative sign shows that the final image is inverted. It is clear from these formula that in order to increase the magnifying power of microscope:

1. u should be small i.e. the object AB should be placed quite close to the objective O. But to obtain a real and magnified image of the object, the object should be placed beyond the focal length f_o of the objective. Hence, for greater magnifying power of the microscope, the focal length of the objective should be small.
2. The distance v_o of the image $A'B'$ from the objective O should be large. For this, the object should be placed near the first focus of the objective.
3. The focal length f_e of the eye-piece should be small.

Thus it is clear that the magnifying power of the microscope depends upon the focal lengths of both the lenses. Hence by taking proper focal lengths the magnification can be increased.

2.17 RESOLVING POWER OF OPTICAL INSTRUMENTS

Concept of Resolving Power: When light from a point-source passes through an optical instrument, the image of the point-objects is not a sharp point, but a spot is obtained which is called the diffraction pattern. This happens because of the wave-form of light. Hence if two point objects are very close to each other, then their diffraction patterns will also be very close and will overlap each other. If the overlapping in the diffraction patterns is small, then both the objects are seen separate in the optical instrument i.e, the optical instrument is able to resolve the objects. If, however, the overlapping is large, then the objects will not be seen as separate; they will be seen as one. In other words the optical instrument is not resolving them. The power of an optical instrument to produce distinctly separate images of two close objects is called the 'resolving power' of that instrument.

One eye is also an optical instrument. If two small objects are placed very close to each other, then it is not necessary that our eye see them separate. This can be seen by a simple experiment. Suppose there is a wall before us on which is pasted a white paper having a number of back parallel lines drawn at separations of 2 mm. When we are quite near the wall, these lines are seen as separate. But as we move away from the wall, a stage is reached when the lines appear mixing with each other and we can no longer distinguish that the lines are separated from one another.

As we move away from the wall, the angle subtended at our eye by any two lines goes on decreasing. From this we conclude that seeing two close objects as separate depends upon the angle subtended by them at the eye. It is seen by experiment that if this angle is less than $1'$ (1 minute) or $\left(\frac{1}{60}\right)^\circ$ then lines will not be seen as separate. This angle is called the 'resolving limit' of the eye.

Similarly, an optical instrument has a limit to form separate images of two objects placed very close to each other. That minimum distance between two objects when they can be seen as separate by an optical instrument is called the 'limit of resolution' of that instrument. Smaller the limit of resolution of an optical instrument, greater is said to be its resolving power.

Resolving Power of Telescope : Necessity of Large-aperture Objective: The resolving power of a telescope is its ability to show two distant closely-lying objects as just separate. The reciprocal of resolving power is the limit of resolution' of the telescope.

The limit of resolution of telescope is measured by the angle subtended at its objective by those two distant objects which are seen just separate through the telescope. Its value is directly proportional to the wavelength λ of the light used and inversely proportional to the aperture (diameter) *d* of the objective of the telescope.

$$\text{Limit of resolution of telescope} \propto \frac{\lambda}{d} = 1.22\frac{\lambda}{d} \text{ radian.}$$

Telescope is used to see distant objects which are generally seen in sunlight. Therefore, we have no control on λ. Hence, to reduce the limit of resolution of a telescope., we must use objective lens of large aperture (*d*). Larger the aperture of the objective lens, smaller the limit of resolution, or greater the resolving power of the telescope.

There is also an additional advantage of large objective. It sends greater amount of light in the telescope and so intense images are formed. Thus, objects extremely far away (whose luminosity appears feeble because of distance) can also be seen.

Resolving Power of Microscope : Necessity of Light of Small Wavelength: The resolving power of a microscope is its ability to show two nearly closely-lying objects as just separate. The reciprocal of resolving power is the 'limit of resolution' of the microscope.

The limit of resolution of microscope is measured by the minimum distance between those two point-objects which are seen just separate through the microscope. Its value is directly proportional to the wavelength λ of light and inversely proportional to the angle of the cone of lights-rays from any one object entering the microscope:

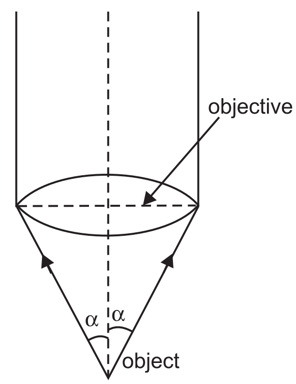


Fig. 2.22

$$\boxed{\text{Limit of resolution of microscope} \propto \frac{\lambda}{\text{cone angle}}}$$

If the angle of the cone of light rays entering the objective of a microscope be 2α (Fig. 2.22), then

$$\boxed{\text{Limit of resolution of microscope} \propto \frac{\lambda}{2\alpha} = \frac{1.22\lambda}{2\sin\alpha}.}$$

If instead of air a liquid of refractive index *n* be filled between the object and the objective of the microscope, then the limit of resolution will become $\dfrac{1.22\lambda}{2n\sin\alpha}$.

Microscope is used to see close objects (such as biological slides, minute particles etc). Those objects are illuminated by a light source. Now, to reduce the limit of resolution of a microscope, we cannot increase the cone-angle (because then the aperture of the lens will have to be increased), but we can decrease λ. For example, we can reduce the limit of resolution by using blue light instead of ordinary light. In visible light the minimum wavelength is 4000 Å. If the distance between two objects is less than this, then we cannot see them separate in the visible light by means of a microscope.

2.18 ELECTRON MICROSCOPE

This microscope is used to see very minute particles distinctly. In this microscope an electron-beam is used instead of light-rays. The electron-beam is focussed by magnetic and electric fields. It behaves as a wave of wavelength of the order of 1 Å; which is 5000 times smaller than the mean wavelength of visible light. Hence an electron microscope can resolve 5000 times compared to an optical microscope.

2.19 EYE-PIECE (OCULARS)

Microscope and telescope suffer from various defects due to the following reasons:

1. Chromatic aberration of the lenses used, and
2. Spherical aberration

The first defect is removed by using achromatic combination of lenses and the second defect is removed by using the more spherical surfaces towards the incoming parallel rays.

As already discussed, the eye cannot be held close to the eye-piece because the rays from the extremities of the image are refracted through the peripheral portion of the eye-piece lens. The best position for the eye is the eye ring. Therefore, to avoid this difficulty a field Lens is used and usually an eye-piece consists of two lenses, the field lens and the eye lens. The angular object field and the angular image field can be increased considerably.

In Fig. 2.23 L_1 is the objective and the extreme ray that can be collected by the eye lens L_3 in the absence of L_2 makes an angular object field α and the angular image field will be β. A is the centre of the exit pupil. When a lens L_2 is placed where the image of the object due to the objective is formed, the dotted ray which was first not collected by the eye lens is now collected by it. The centre of the exit pupil shifts to B. The angular object field is α' where $\alpha' > \alpha$ and the angular image field is β' where $\beta' > \beta$. This combination of the field lens L_2 and the eye lens L_3 forms the eye-piece or the ocular.

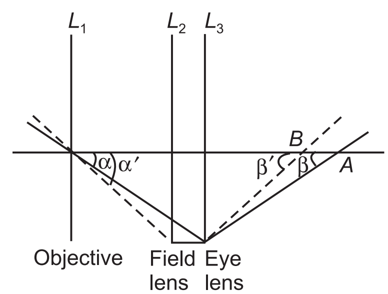


Fig. 2.23

The functions of the field lens are:

1. It increases the angular object field.
2. It brings the centre of the exit pupil nearer the eye lens and increases the angular image field.
3. It helps to minimise spherical aberration and chromatic aberration.

2.20 HUYGENS EYEPIECE

This eye-piece is achromatic and the spherical aberration is also eliminated. It consists of two lenses having focal length in the ratio 3:1 and the distance between them is equal to the difference in their focal lengths. The focal lengths and the positions of the two lenses are such that each lens produces an equal deviation of the ray and the system is achromatic.

Suppose the field lens and the eye lens of focal lengths f_1 and f_2 are placed D distance apart. If F is the focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{D}{f_1 f_2}$$

Differentiating

$$\frac{-dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2} + D \cdot \left(\frac{df_1}{f_1^2 f_2} + \frac{df_2}{f_1 f_2^2}\right)$$

As the dispersive power

$$\omega = \frac{dF}{F} = \frac{-df_1}{f_1} = \frac{-df_2}{f_2}$$

$$\frac{\omega}{F} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - D\left(\frac{\omega + \omega}{f_1 f_2}\right)$$

For achromatism $\dfrac{\omega}{F} = 0$

$$\therefore \qquad \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{2D\omega}{f_1 f_2} = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{2D}{f_1 f_2} = 0$$

$$\therefore \qquad \frac{f_1 + f_2}{f_1 f_2} = \frac{2D}{f_1 f_2}$$

or

$$D = \frac{f_1 + f_2}{2}$$

Also, for equal deviation of a ray by the two lenses, the distance between the two lenses should be equal to $f_1 - f_2$.

Thus, to satisfy both the conditions Huygens constructed an eye-piece consisting of two plano-convex lenses of focal lengths $3f$ and f placed at a distance of $2f$ from each other.

ll, is the image of the distant object formed by the objective in the absence of the field lens. With the field lens, the rays get refracted on passing through it and the image *l′l′*, is formed.

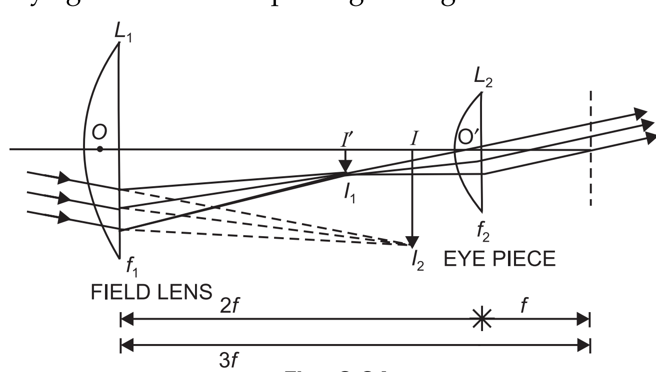


Fig. 2.24

This image lies at the focus of the eye lens so that the final image is seen at infinity The focal length of the combination

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{3f^2}$$

$$\frac{1}{F} = \frac{2}{3f} \quad \therefore F = \frac{3}{2}f$$

The equivalent lens must be placed behind the field lens at a distance

$$= \frac{F \times d}{f_2} = \frac{F \times 2f}{f} = \frac{\frac{3}{2}f \times 2f}{f} = 3f$$

i.e, $3f$ from the field lens or at a distance f behind the eye lens.

In the modern design, the focal length of the lenses are $2f$ and f placed at a distance of 1.5 f from each other.

Huygen eye-piece is known as the negative eye-piece because the real inverted image formed by the objective lies behind the field lens and this image acts as a virtual object for the eye lens. The eye-pieces cannot be used to examine directly an object or a real image formed by the objective. The eye-piece is used in microscopes or other optical instruments using white light only.

Moreover, the cross-wires must be placed (if the measurement of final image is required) between the field lens and the eye lens. But the cross wire are viewed through the eye lens only while the distant object is viewed by rays refracted through both the lenses. Due to this reason relative length of the cross-wires and the image are disproportionate. Hence cross wires cannot be used in a Huygens eye-piece and this is a disadvantage. Hence, Huygens eye-piece cannot be used in telescopes and other optical instruments with which distance and angles are to be measured.

2.21 CARDINAL POINTS OF A HUYGENS EYE-PIECE

The first principal point is at a distance α from the field lens.

$$\alpha = \frac{Fd}{f_2} = \frac{\frac{3}{2}f \times 2f}{f} = 3f$$

The second principal point is at a distance β from eye lens.

$$\beta = \frac{-Fd}{f_1} = \frac{-\frac{3}{2}f \times 2f}{3f} = -f$$

Therefore, the first principal point P_1 is at a distance $3f$ on the right of the field lens and the second principal point P_2 is at a distance f to the left of the eye lens. Since the system is in the air, the nodal points coincide with the principal points Fig. 2.25.

The first point F_1 is at a distance of $\frac{3}{2}f$ from the first principal point and the second focal point F_2 is at a distance of $\frac{3}{2}f$ from the second principal point.

Therefore, F_1 is at a distance of $3f - \frac{3}{2}f = \frac{3}{2}f$ to the right of the field lens L_1 and F_2 is at a distance of $\frac{3}{2}f - f = \frac{1}{2}f$ to the right of the eye lens.

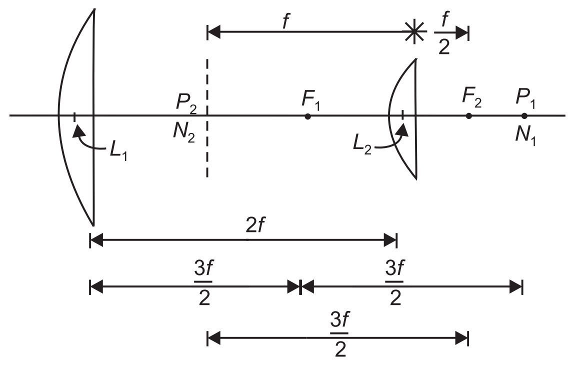


Fig. 2.25

2.22 RAMSDEN EYE-PIECE

It consists of two plano-convex lenses of equal focal length separated by the distance equal to two-thirds the focal length of either. The convex faces are towards each other and the eye-piece is placed beyond the image formed by the objective (Fig. 2.26). In this eye-piece cross wire are provided and it is used in optical instruments where accurate quantitative measurements are made let F be the focal length of the equivalent lens.

Thus
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$\therefore$
$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{\frac{2}{3}f}{f^2}$$

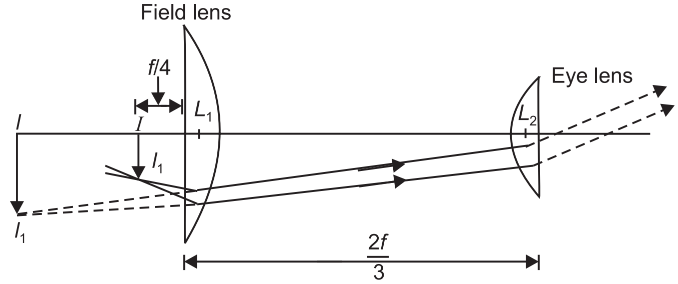


Fig. 2.26

$$\frac{1}{F} = \frac{+2}{f} - \frac{2}{3f} = \frac{+4}{3f}$$

$\therefore$
$$F = \frac{+3}{4}f$$

The equivalent lens must be placed at a distance $\frac{3}{4}f$ behind the field lens at a distance α from it.

$$\alpha = \frac{F \times d}{f_2} = \frac{F}{f} \times \frac{2}{3} f = \frac{\frac{3}{4} f}{f} \times \frac{2}{3} f = \frac{f}{2}$$

Thus, equivalent lens is in between the field lens and the eye lens.

As the focal length of the eye-piece (equivalent lens) is $\frac{3}{4} f$ the image of the object due to the objective must be formed at a distance $\frac{3}{4} f - \frac{1}{2} f = \frac{1}{4} f$ in front of the field lens. This image will act as an object for the eye-piece and the final image will be formed at infinity. The cross wires must be placed at the position where the image due to the objective is formed i.e, at a distance of $\frac{1}{4} f$ in front of the field lens. This is the advantage of Ramsden eye-piece over the Huygens eye-piece.

The chromatic aberration in a Ramsden eye-piece is small. In some cases, both the lenses of the eye-piece are made of a combination of crown and flint glass and chromatic aberration is eliminated. As both the lenses are plano-convex with their convex surface facing each other the spherical aberration produced is small.

2.23 CARDINAL POINTS OF A RAMSDEN EYE-PIECE

The first principal point is at a distance α from the field lens.

$$\alpha = \frac{Fd}{f_2} = \frac{\frac{3}{4} f}{f} \times \frac{2}{3} f = \frac{f}{2}$$

The second principal point is at a distance β from the eye lens.

$$\beta = \frac{-Fd}{f_1} = \frac{-\frac{3}{4} f}{f} \times \frac{2}{3} f = \frac{-f}{2}$$

Therefore, the first principal point P_1 is at a distance of $\frac{f}{2}$ to the right of the field Lens and the second principal point P_2 is at a distance of $\frac{f}{2}$ to the left of the eye lens. Since the system is in air, the nodal points coincide with the principal points (Fig. 2.27). The first focal point F_1 is at a distance of $\frac{3}{4} f$ from the first principal point and the second focal point F_2 is at a distance of $\frac{3}{4} f$ from the second principal point.

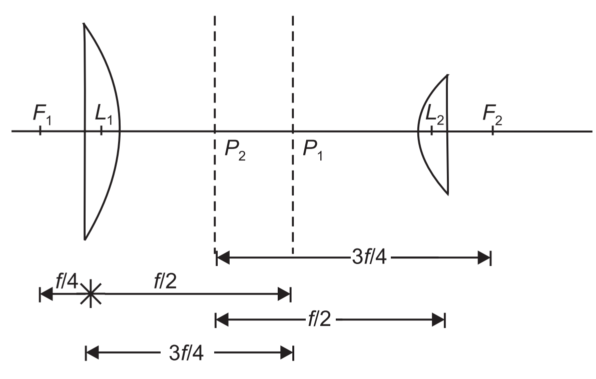
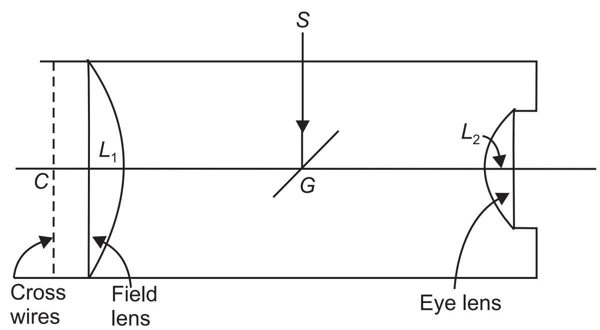


Fig. 2.27

Therefore F_1 is at a distance of $\frac{3}{4}f - \frac{f}{2} = \frac{f}{4}$ to the left of the field lens and F_2 is at a distance of $\frac{3}{4}f - \frac{f}{2} = \frac{f}{4}$ to the right of the eye lens.

2.24 GAUSS EYE-PIECE

Gauss eye-piece is a modification of Ramsden eye-piece. The field lens and the eye lens are two plano-convex lenses of equal focal length and separated by a distance equal to two-third of the focal length of either.



To illuminate the field of view a glass plate G is held at an angle of 45° to the axis of the lens system. S is a source of light. Light reflected from G illuminates the field of view. The cross wire is C kept at a distance $\frac{1}{4}f$ front of the field lens. This eye-piece is used in the telescope of a spectrometer.

2.25 COMPARISON OF EYE-PIECES

	Huygens Eye-piece	*Ramsden Eye-piece*
1.	It is a negative eyepiece. The image formed by the objective lies in between the two lenses. Therefore no cross wires can be used.	It is a positive eye-piece. The image formed by the objective lies in front of the field lens. Therefore cross wire can be used.
2.	The condition for minimum spherical aberration is satisfied.	The condition for minimum spherical aberration is not satisfied. But by spreading the deviations over four lens surfaces, spherical aberration is minimised.
3.	It satisfies the condition for archomatism.	It does not satisfy the condition for archromatism but can be made achromatic by using an achromatic doublet (crown and flint) at the eye lens.
4.	It is achromatic for all colours.	It is achromatic for only two chosen colours.
5.	The other aberration like distortion which is pincushion type is not well removed.	The other aberrations are better removed. There is no coma and distortion is 5% higher.

6.	The eye-clearance is too small and less comfortable.	The eye clearance is 5% higher.
7.	It is used for qualitative purposes in microscope and telescope.	It is used for quantitative purposes in microscopes and telescopes.
8.	Its power is +ve.	Its power is +ve.
9.	The two principal planes are crossed.	The two principal planes are crossed.
10.	It cannot be used as a simple microscope because the first focal plane lies to the right of the field lens and the focal plane is virtual.	It can be used as a simple microscope because the first principal plane lies to the left of the field lens and the focal plane is real.
11.	The nodal points coincide with the principal points.	The nodal points coincide with the principal points.

2.26 SPECTRUM

When the atoms or molecules of a substance are excited to high energies, they jump back to their normal ground state either directly or in several steps. While returning from the excited state to the lower energy state, the difference of energy ΔE between the two states is radiated out as a quantum of frequency ν given by $\Delta E = h\nu$, where h is Plank's constant.

The human eye is sensitive only to the radiations having their wave length in the range 4000 Å to 8000 Å. This is known as the visible region of spectrum and the eye is not able to sense radiation beyond violet region (λ < 4000 Å) and the red region (λ > 8000 Å). In between these two extremes the wave-length are divided into Violet, Indigo, Blue, Green, Yellow, Orange and Red colour sub-regions abbreviated as VIBGYOR. The eye is not equally sensitive in all these regions. It is most sensitive in the yellow region of visible spectrum.

Usually the light emitted by a source contains several wave-lengths. The process of separating these wave-lengths is called 'dispersion'. The term dispersion is popularly employed for 'separation of colours' but truely speaking dispersion means separations of wave-lengths. Two close wave-lengths separated by an instrument are said to be dispersed, yet to eye they appear to have the same colour.

In all instruments producing dispersion, the light from a source is made to illuminate a narrow slit and the instrument produces several images of the slit-one corresponding to each wave-length. It is why, instead of saying than a given source emits so many wave-lengths, we commonly say the source emits so many lines. The whole set of wave-length wise separated images of the slit taken together is called the spectrum and the instrument used for study of spectrum of a source is called a spectrometer, spectroscope or a spectrograph depending upon whether we make direct measurements, just observe the spectrum or take a photograph of it.

Types of Spectrum: The spectrum of a source can be obtained in two ways. In the first method the, atoms or molecules of the substance, which at normal temperature exist in the ground state, are excited to higher energy state by either heating, irradiation, ion-bombardment or applying high electric field etc, where their life time is very small (~10^{-8} sec) and they return back to their ground state either directly or through intermediate energy levels emitting out photons of energy equal to difference of the energies between the two levels. The spectrum obtained by such process is called the emission spectrum. In the other method, white light is allowed to pass through the substance under study. The substance absorbs certain wave lengths from the incident light. These wave-lengths correspond to the diference of energy between various excited states and the ground state. The transmitted light when examined shows these lines missing from the continuous spectrum. Such a spectrum is called the absorption spectrum. Both the emission as well as the absorption spectrum are the characteristic spectrum of the substance under study. Both these spectra can be either of the following three types:

Line Spectrum: A Line spectrum consists of the fine bright lines widely scattered on a dark back ground in case of emission spectrum, or fine dark line on a continuous coloured back-ground i.e, the lines missing in various regions of white light spectrum. The line spectrum is produced by substances in the atomic state i.e., by elements alone. The line spectrum of each element is the characteristic of its own and can be used to identify it. Line spectra are emitted by vaporised elements in flames and by gases in discharge tube. Arcs and sparks give lines characteristic of the electrodes. Absorption line spectrum is obtained by passing light from white light source through the vapours of the element or the gases.

Band Spectrum: A band spectrum consists of a number of groups of lines in different regions of spectrum with dark background. Each band starts with a bright line and then the intensity of lines gradually fades out. In absorption band spectrum such band or group of lines are found missing from a continuous spectrum. The band spectrum is obtained for the substances in the molecular state. The band spectrum of a molecule is characteristic of its own.

Continuous Spectrum: A continuous spectrum consists of all possible wave-lengths in the given region of spectrum. In emission a continuous spectrum appears as a bright patch of light with gradually changing colours from violet to red. Maximum intensity occurs in a particular region. Continuous spectrum is emitted by materials in the incandescent state i.e, at very high temperatures. The spectrum resembles black body spectrum. The maximum of intensity occurs at a wave-length depending upon the temperature. As the temperature increases the maximum of intensity shifts towards the violet end of the spectrum.

2.27 SOURCES OF LIGHT

Discharge Tubes: Elements which either exist normally in the gaseous state or can easily be converted into vapours such as Hg, Na are excited in a discharge tube. These discharge tubes can be classified under two categories (*a*) Cold cathode type, and (*b*) Hot cathode type or vapour lamps.

(*a*) *Cold Cathode Discharge Tubes:* These tubes are used for substances which are in the gaseous form at ordinary temperatures, such as Argon, Neon, Xenon, Oxygen, Hydrogen Helium etc. In a glass tube of the shape shown in Fig. 2.28, the gas is filled at a low pressure, 5 to 1 mm of Hg depending upon the nature of the gas. The electrodes are of aluminium welded to the ends of platinum or tungsten wire fused through the glass tube. The aluminium is used because it sputters less. A high voltage of the order of 3000 volts or more is required for working of these tubes. Therefore, either a transformer or an induction coil is required to run these discharge tubes. The current capacity of these tubes is small, therefore, either the transformer or the induction coil should itself have a suitable high resistance or the same may be introduced externally. When worked, a diffuse glow of the positive column fills the tubes. The middle part of the tube is made narrow so that the intensity is concentrated in a narrower cross-section. The light from this capillary portion is focussed onto the slit of the spectrometer for the study of spectrum.

Fig. 2.28

(*b*) *Hot Cathode Tubes:* For elements which at normal temperature do not exist in the gaseous state, arrangements are required to first convert them into vapours before passing the electric discharge through them. Sodium and mercury vapour lamps employ such arrangement and are usually classified as hot cathode tubes. These are described below:

2.28 SODIUM VAPOUR LAMP

As shown in Fig. 2.29 sodium vapour lamp consists of a U-shape glass tube-filled with neon gas at a pressure of about 10 mm of mercury. Some specks of metallic sodium are introduced in the tube which usually get deposited on its walls. The U-tube is fitted with two electrodes made of tungsten spiral and coated with barium-oxide.

This device provides a copious supply of electrons from the electrodes which are always hot due to discharge. To start the discharge through the tube a high voltage of about 500 volts is required which is supplied by a single wound auto-transformer. When the potential is applied between the electrodes situated at either end of the U-tube, the discharge initially passes through the neon gas which gives light of red colour. This discharge causes an increase

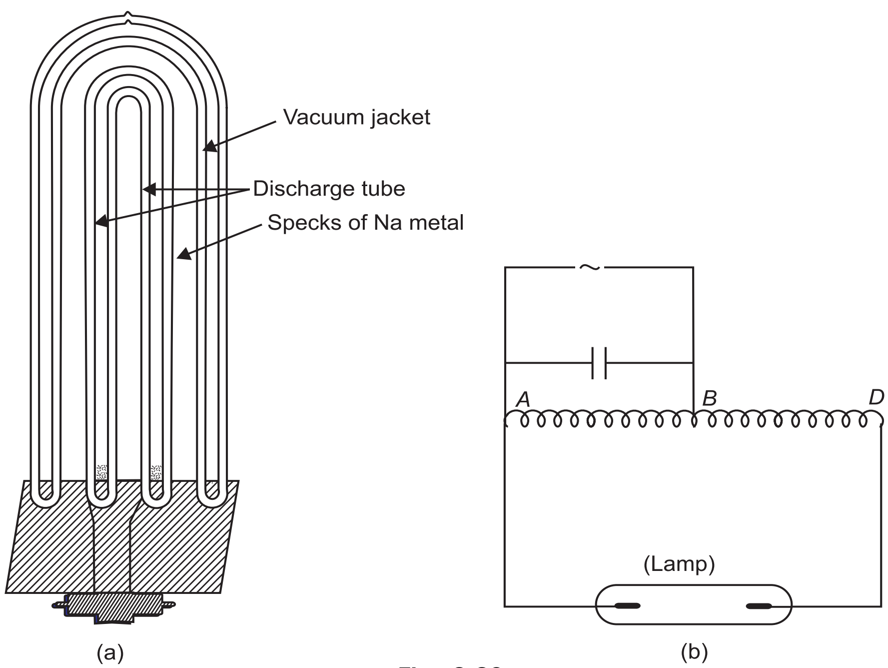


Fig. 2.29

in the temperature and the metallic sodium starts evaporating, the colour of light starts acquiring yellow tinge. When the vapour pressure of sodium rises to about 0.01 mm of Hg the discharge is then maintained by sodium vapours emitting out characteristic golden yellow light. Though in running condition the relative concentrations of neon and sodium are 8000:1, still the discharge passes through the sodium Vapour. This is because of the fact that the energy required to excite Na atoms is much less than that required for the Ne atoms. The neon atoms lose their energy by collision with sodium atoms before acquiring enough energy for their (neon) own excitation. Hence light is predominatly that of sodium. In the beginning since the vapour pressure of sodium is very low it can not support the discharge; neon helps in maintaining it and also provides a path for the start of discharge. The working temperature of the lamp is high, about 250° C, at which the vapour pressure of sodium is high enough to allow the discharge to pass through it and to maintain this temperature with the smallest possible heat loss, the sodium discharge tube is surrounded by a double walled evacuated glass jacket.

The electric circuit of the sodium vapour lamp is shown in Fig. 2.29 (*b*). Since the electrodes are not preheated, and the ionization potential of neon is high (21.5V) a high voltage is necessary to start the discharge. This high voltage is obtained by a step-up auto-leak transformer *ABD*. The *AC* mains is connected across portion *AB* of the single winding *ABD* and the sodium lamp across portion *AD*. When *AC* mains is applied, a high voltage of about 500 volts is developed across *AD* due to self inductance, which is applied to the lamp. At this stage discharge starts in neon. Initially when the resistance of the lamp is high nearly all voltage acts across the lamps. When the discharge sets up the resistance of the lamp falls and the current rises. With this rise of current the leakage reactance of the transformer also increases enormously by the use of magnetic shunts. Thus, with increase of current the effective resistance of coil *AD* also increases and it shares a substantial part of the voltage. In other words, while the lamp gets a large starting voltage this voltage automatically falls as the discharge sets up (no separate choke is needed). The working voltage is usually about 140 volts. The condenser in the circuit is used for suitably adjusting the 'power factor.'

Sodium produces only one doublet line called D-line which consists of two wave-lengths, D_1 line (λ = 5890 Å) and D_2 line (λ_2 = 5896 Å). Since difference between D_1 and D_2 lines is very small (6Å) they ordinarily appear as a single line.

2.29 MERCURY VAPOUR LAMP

It consists of a cylindrical tube containing some mercury and argon at a pressure of about 10 mm of Hg. The main electrodes are *A* and *B*, but an auxiliary electrode *A′* is provided nearer to *B*. This acts to start the discharge through the tube and is connected to the more distant main electrode *A* through a ballast resistance *r* of about 50,000 ohms. The main electrodes are in the form of spirals of tungsten wire, each holding a pipet of barium oxide for electron omission.

When the lamp is connected to *A.C.* mains through a choke-coil, initial discharge is started by argon contained in the tube. The discharge taking place between the auxiliary electrode *A′* and the adjacent main electrode *B*. This produces sufficient ionisation in the tube and the discharge spreads to other main electrode *A*. This is because the ionised argon gas between *A* and *B* becomes more conducting and due to high resistance of the ballast, the discharge across *A′B* is suppressed. Discharge between *AB* through argon produces sufficient amount of heat to vaporise mercury and then the discharge is taken over (from argon) by mercury giving very brilliant light. The amount of mercury in the discharge tube is just enough to give a pressure of 1 atmosphere at the temperature of working of the tube, which is about 600° C. For maintaining this high temperature the discharge tube is enclosed in an evacuated outer glass jacket.

The mercury vapour lamp does not need a higher voltage to start the discharge. If it works at a lower voltage of about 90–100 volts. A suitable choke is connected in series with the lamp (see Fig. 2.30 (*b*)). This shares the voltage as well as controls the current. A capacitor is also connected across the lamp-choke combination to obtain a suitable 'power-factor.'

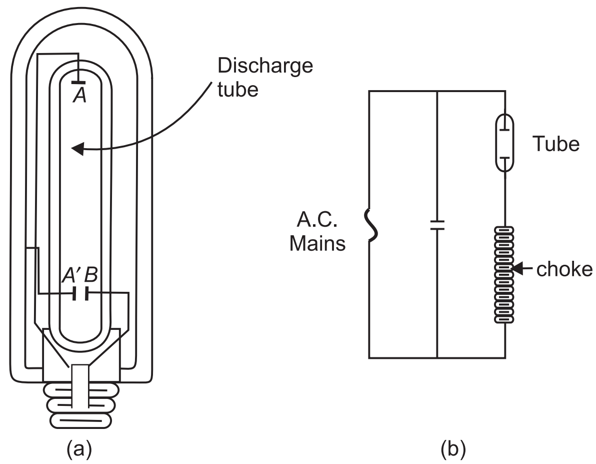


Fig. 2.30

The efficiency of the mercury vapour lamp is very high. It radiates chiefly in green, violet and ultraviolet regions of spectrum.

2.30 ELECTROMAGNETIC SPECTRUM

In the spectrum of sun-light we see all colours from red to violet. This spectrum is called the visible spectrum. The longest wavelength in the red region is nearly 7.8×10^{-7} meter and the smallest wavelength in the violet region is 4.0×10^{-7} meter. Thus the visible spectrum extends from 7.8×10^{-7} meter to 4.0×10^{-7} meter. After Newton, it was discovered that the sun's spectrum is not limited from red to violet colour, but is considerably spread above the red colour and below the violet colour. These parts of the spectrum are not observed by our eye. The part of longer wavelength above the red colour is called the 'infrared spectrum' and that of smaller wavelength below the violet colour is called the 'ultra-violet spectrum.'

Later on were discovered *x*-rays, γ-rays and radio waves. Now it has been established that all these radiations (including visible spectrum) are electro-magnetic waves. The range of the wavelength of these waves is very large and on this basic they can be given an order. This order is called the 'electro-magnetic spectrum.' It ranges from the very small gamma rays to the very long radio waves. The visible spectrum is only a very small part of the electro-magnetic spectrum. The wavelength-ranges, the method of production and the properties of the whole electro-magnetic spectrum are summarised below:

1. **Gamma Rays:**
   Wave length range: From 10^{-13} to 10^{-10} meter.
   Production: Emitted on the disintegration of the nuclei of atoms.
   Properties: Chemical reaction on photographic plates, fluorescence, phosphorescence, ionisation, diffraction, highly-penetrating and uncharged.
2. **X-rays:**
   Wavelength range: From 10^{-10} to 10^{-8} meter.
   Production: Produced by striking high-speed electrons on heavy target.
   Properties: All properties of γ-rays, but less penetrating.

3. **Ultra-violet Radiation:**

Wavelength range: From 10^{-8} to 4×10^{-7} meter.
Production: Produced by sun, arc, spark, hot vacuum spark and ionised gases.
Properties: All properties of γ-rays, but less penetrating, produce photo-electric effect.

4. **Visible Radiation:**

Wavelength range: From 4×10^{-7} to 7.8×10^{-7} meter.
Production: Radiated from ionised gases and incandescent bodies.
Properties: Reflection, refraction, interference, diffraction, polarisation, photo-electric effect, photographic action and sensation of sight.

5. **Infra-red Radiation:**

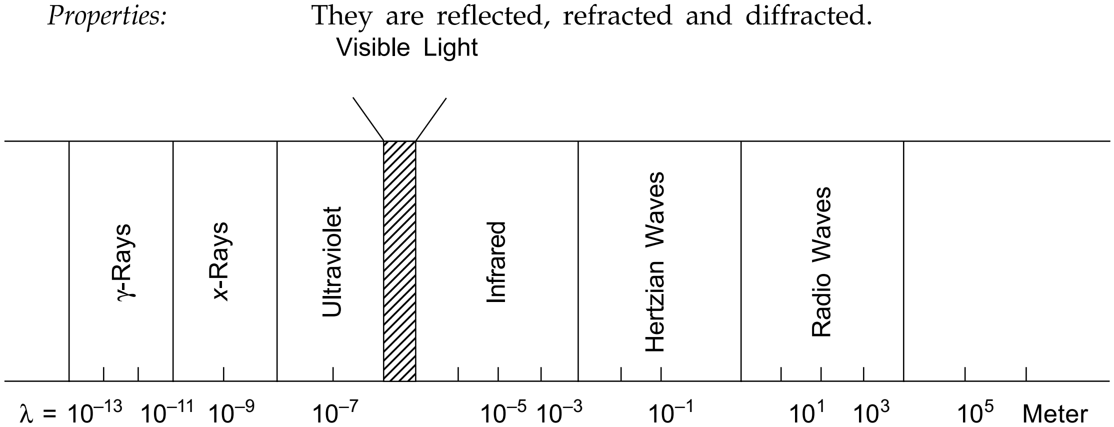
Wavelength range: From 7.8×10^{-7} to 10^{-3} meter.
Production: From hot bodies.
Properties: Heating effect on the thermopiles and bolometer, reflection, refraction, diffraction, photographic action.

6. **Hertzian or Short Radio Waves:**

Wavelength range: From 10^{-3} to meter.
Production: Produced by spark discharge.
Properties: They are reflected, refracted and produce spark in the gaps of receiving circuits. Waves of wavelength from 10^{-3} m to 3×10^{-2} m are also called 'microwaves.'

7. **Long Radio Waves or Wireless:**

Wavelength range: From 1 to 10^4 meter.
Production: From spark gap discharge and oscillating electric circuits.
Properties: They are reflected, refracted and diffracted.



(Electro-magnetic Spectrum)

3

Elasticity

3.1 ELASTICITY

If an external force is applied on a body such that it does not shift the body as a whole, it produces a change in the size or shape or both of the body. In such a case the body is said to be deformed and the applied external force is called the deforming force. This force disturbs the equilibrium of the molecules of the solid bringing into picture the internal forces which resist a change in shape or size of the body. When the deforming force is removed, these internal forces tend to bring the body to its original state. This property of a body by virtue of which it resists and recovers from a change of shape or size or both on removal of deforming force is called its elasticity. If the body is able to regain completely its original shape and size after removal of deforming force it is called perfectly elastic while, if it completely retains its modified size and shape, it is said to be perfectly plastic.

It need hardly be pointed out that there exists no such perfectly elastic or perfectly plastic bodies in nature. The nearest approach to the former is a quartz fibre and to the latter, ordinary putty. All other bodies lie between these two extremes.

3.2 LOAD

Any combination of external forces acting on a body (*e.g.*, its own weight, along with the forces connected with it, like centrifugal force, force of friction etc.) whose net effect is to deform the body, *i.e.,* to change its form or dimensions, is referred to as a load.

3.3 STRESS

A body in equilibrium under the influence of its internal forces is, as we know, in its natural state. But when external or deforming forces are applied to it, there is a relative displacement of its particles ad this gives rise to internal forces of reaction tending to oppose and balance the deforming forces, until the elastic limit is reached and the body gets permanently deformed. The body is then said to be stressed or under stress.

If this opposing or recovering force be uniform, *i.e.,* proportional to area, it is clearly a distributed force like fluid pressure and is measured in the same manner, as force per unit area, and termed stress.

If $\vec{F}$ be the deforming force applied uniformly over an area A, we have stress = $\frac{F}{A}$. If the deforming force be inclined to the surface, its components perpendicular and along the surface are respectively called normal and tangential (or shearing) stress. The stress is, however, always normal in the case of a change of length or volume and tangential in the case of a change of shape of a body.

Its dimensional formula is $ML^{-1}\,T^{-2}$ and its units in M.K.S. and C.G.S. systems are respectively newton/m^2 and dyne/cm^2.

3.4 STRAIN

When the body is deformed on application of external force, it is said to be strained. The deformation of the body is quantitatively measured in terms of strain which is defined as the change in some dimension of the body per unit that dimension. A body can be deformed in three ways *viz.* (*i*) a change in length producing longitudinal strain, (*ii*) a change in volume producing volume strain and, (*iii*) a change in shape without change in volume (*i.e.,* shearing) producing shearing strain. Thus change in length per unit length is called longitudinal strain, change in volume per unit volume is called volume strain, and change in an angle of body is called shearing strain being a ratio the strain is a dimensionless quantity and has no unit.

3.5 HOOKE'S LAW

This law states that when the deforming force is not very large and strain is below a certain upper limit, stress is proportional to strain *i.e.,*

$$\text{stress} \propto \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant}.$$

This ratio of stress and strain, which is constant, is known as the modulus of elasticity and depends upon the material of the body. The limit upto which Hooke's law holds is called the limit of elasticity. Thus Hooke's law may be stated as within elastic limit, stress is proportional to strain.

3.6 ELASTIC LIMIT

In the case of a solid, if the stress be gradually increased, the strain too increases with it in accordance with Hooke's law until a point is reached at which the linear relationship between the two just ceases and beyond which the strain increases much more rapidly than is warranted by the law. This value of the stress for which Hooke's law just ceases to be obeyed is called the elastic limit of the material of the body for the type of stress in question.

The body thus recovers its original state on removal of the stress within this limit but fails to do so when this limit is exceeded, acquiring a permanent residual strain or a permanent set.

3.7 TYPES OF ELASTICITY

Since in a body there can be three types of strain *viz.* , longitudinal strain, volume strain and the shearing strain, correspondingly we have three types of elasticity as described below.

3.8 YOUNG'S MODULUS (OR ELASTICITY OF LENGTH)

When the deforming force is applied to a body in such a manner that its length is changed, longitudinal or linear strain is produced in the body. The internal force of reaction or the restoring force trying to restore its length, restoring force acts along the length of the body and its magnitude per unit cross-sectional area is the normal stress. The ratio of this normal stress and the linear strain is called Young's modulus of elasticity Y.

Thus if a uniform wire of length L and cross-section area A is stretched in length by an amount l by a force F acting along its length, the internal restoring force equals the external force in the equilibrium state, then

$$\text{Longitudinal strain} = \frac{l}{L}$$

and

$$\text{Normal stress} = \frac{F}{A}$$

$$\therefore \qquad Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{\frac{F}{A}}{\frac{l}{L}}$$

The dimensional formula for Young's modulus is ($ML^{-1}T^{-2}$) and its units in M.K.S. and C.G.S. systems are respectively newton/metre^2 and dyne/cm^2.

3.9 BULK MODULUS (OR ELASTICITY OF VOLUME)

When a force is applied normally over the whole surface of the body, its volume changes while its shape remains unchanged. In equilibrium state, internal restoring force equals the external force. The magnitude of normal force per unit area is the normal stress. This may also be appropriately called pressure. The ratio of normal stress and the volume strain is called bulk modulus of elasticity, K.

Thus, if v is the change in volume produced in the original volume V of the body by application of force F normally on surface area A of the body, we have

$$\text{Volume strain} = \frac{v}{V}$$

$$\text{Normal stress} = \frac{F}{A} = p$$

$$\therefore \qquad \text{Bulk modulus} = -\frac{\text{Pressure}}{\text{Volume strain}}$$

or

$$K = -\frac{P}{v/V}$$

The minus sign has been introduced to give K a positive value. This is because an increase in pressure (p-positive) causes a decrease in volume (V-negative) of the body.

K is also sometimes referred to as incompressibility of the material of the body and, therefore, $\frac{1}{K}$ is called its compressibility.

Since liquids and gases can permanently sustain only a hydrostatic pressure, the only elasticity they possess is Bulk modulus (K).

The dimensional formula for bulk modulus of elasticity is $[ML^{-1}\,T^{-2}]$ and its units in MKS and C.G.S. systems are newton/metre^2 and dyne/cm^2 respectively.

3.10 MODULUS OF RIGIDITY (TORSION MODULUS OR ELASTICITY OF SHAPE)

When under application of an external force the shape of the body changes without change in its volume, the body is said to be sheared. This happens when a tangential force is applied to one of the faces of the solid.

Consider a rectangular solid *ABCD abcd* whose lower face *DC cd* is fixed and a tangential force F is applied to its upper face. *ABba* as shown in figure. The layers parallel to the lower face slip through distances proportional to their distance from the fixed face such that finally face *ABba* shifts to *A′B′b′a′* and solid takes the form *A′B′C Da′b′cd* its volume remaining unchanged. Thus one face of solid remains fixed while the other is shifted laterally. The angle $ADA' = \theta = \left(=\frac{l}{L} \text{ where } AD=L \text{ and } AA'=l\right)$ through which the edge AD which was initially perpendicular to the fixed face is turned, is called the shearing strain or simply the shear. Due to this shearing of the solid, tangential restoring force is developed in the solid which is equal and opposite to the external force. The ratio of tangential stress and shearing strain is called modulus of rigidity, η. Thus

$$\text{Tangential stress} = \frac{F}{A}$$

$$\text{shearing strain } \theta = \frac{AA'}{AD} = \frac{l}{L}$$

$$\text{Modulus of rigidity} = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\eta = \frac{\frac{F}{A}}{\theta}$$

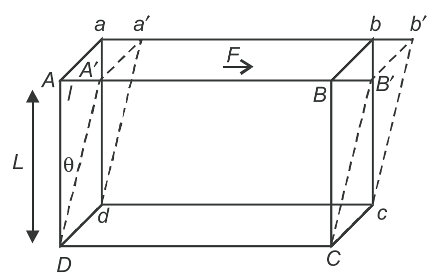


Fig. 3.1

where A is the area of the upper face of the solid.

The dimensional formula for modulus of rigidity is $ML^{-1}T^{-2}$ and its MKS and CGS units are newton/m^2 and dyne/cm^2 respectively.

3.11 AXIAL MODULUS

This is defined as longitudinal stress required to produce unit linear strain, unaccompanied by any lateral strain and is denoted by the Greek letter Z. It is thus similar to Young's modulus, with the all-important difference that here the lateral strain produced (in the form of lateral contraction) is offset by applying two suitable stresses in directions perpendicular to that of the linear stress. So that the total stress is Young's modulus plus these two perpendicular stresses.

3.12 POISSON'S RATIO

When we apply a force on a wire to increase its length, it is found that its size change not only along the length but also in a direction perpendicular to it. If the force produces an extension in its own direction, usually a contraction occurs in the lateral or perpendicular direction and vice-versa. The change in lateral dimension per unit lateral dimension is called lateral strain. Then, within elastic limit, the lateral strain (though opposite in sign) is proportional to the longitudinal strain *i.e.,* the ratio of lateral strain to the longitudinal strain within the limit of elasticity is a constant for the material of a body and is called the Poisson's ratio. It is usually denoted by σ.

Consider a wire of length L and diameter D. Under application of an external longitudinal force F, let l be the increase in the length and d the decrease in the diameter, then

$$\text{Longitudinal strain } \alpha = \frac{l}{L}$$

$$\text{Lateral strain } \beta = \frac{d}{D}$$

$$\therefore \qquad \text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{\beta}{\alpha} = \frac{\frac{d}{D}}{\frac{l}{L}}$$

Being ratio of two strains, Poisson's ratio has no units and dimensions.

3.13 RELATION BETWEEN ELASTIC CONSTANT

In the above discussion we have defined four elastic constants, *viz.* Y, K, η and σ. Out of these only two are independent while other two can be expressed in terms of the two independent constants. Hence if any two of them are determined, the other two can be calculated. The following are the inter-relations between the four elastic constants.

(*i*) $Y = 3K(1 - 2\sigma)$

(*ii*) $Y = 2\eta(1 + \sigma)$

(*iii*) $Y = \dfrac{9K\eta}{3K + \eta}$

(*iv*) $\sigma = \dfrac{3K - 2\eta}{6K + 2\eta}$

3.14 LIMITING VALUES OF POISSON'S RATIO (σ)

From the relation $3k(1 - 2\sigma) = 2\eta\ (1 + \sigma)$, where, as we know, K and η are both positive quantities it follows therefore, that

(*i*) If σ be a positive quantity, the expression on the right hand side in the relation above will be positive. The expression on the left hand side too must therefore be positive. This is, obviously, possible when $2\sigma < 1$ or $\sigma < \frac{1}{2}$ or 0.5; and

(*ii*) If σ be a negative quantity, the left hand expression in the above relation will be positive and hence the expression on the right hand side too must be positive, and this can be so only if σ be not less than –1.

Thus, theoretically, the limiting values of σ are –1 and 0.5, though in actual practice it lies between 0.2 and 0.4 for most of the materials.

3.15 TWISTING COUPLE ON A CYLINDER

(*i*) *Case of a solid cylinder or wire:* Let a solid cylinder (or wire) of length L and radius R be fixed at its upper end and let a couple be applied to its lower end in a plane perpendicular to its length (with its axis coinciding with that of the cylinder) such that it is twisted through an angle θ.

This will naturally bring into play a resisting couple tending to oppose the twisting couple applied, the two balancing each other in the position of equilibrium.

To obtain the value of this couple, let us imagine the cylinder to consist of a large number of hollow, coaxial cylinder, one inside the other and consider one such cylinder of radius x and thickness dx, As will be readily seen, each radius of the base of the cylinder will turn through the same angle θ but the displacement (BB') will be the maximum at the rim, progressively decreasing to zero at the centre (O) indicating that the stress is not uniform all over.

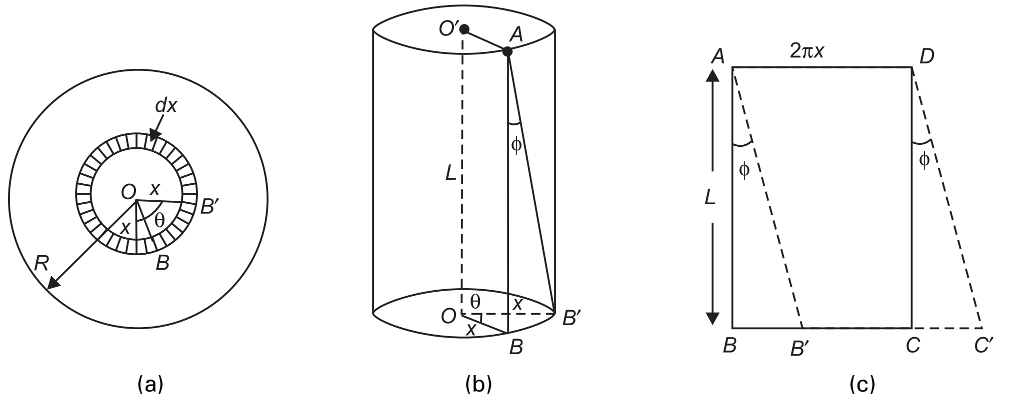


Fig. 3.2

Thus, a straight line AB, initially, parallel to the axis OO' of the cylinder will take up the position AB' or the angle of shear (or shear) = $\angle BAB' = \phi$. This may be easily visualised if we imagine the hollow cylinder to be cut along *AB* and spread out when it will initially have a rectangular shape *ABCD*, and will acquire the shape of a parallelogram *AB′C′D* after it has been twisted, so that angle of shear = $BAB' = \phi$.

Now, $BB' = x\theta = L\phi$, where, the shear $\phi = \dfrac{x\theta}{L}$ and will obviously have the maximum value when $x = R$, *i.e.*, at the outermost part of the cylinder and the least at the innermost.

If η be the coefficient of rigidity of the material of the cylinder, we have

$$\eta = \frac{\text{shearing stress}}{\text{shear}} \text{ or shearing stress} = \eta \times \text{shear}$$

or
$$\text{shearing stress} = \eta\phi = \frac{\eta x\theta}{L}$$

$\therefore$ Shearing force on face area of the hollow cylinder $= \left(\dfrac{\eta x\theta}{L}\right) \times$ face area of the cylinder

$$= \left(\frac{\eta x\theta}{L}\right) \times 2\pi x dx = \left(\frac{2\pi\eta\theta}{L}\right) x^2\, dx$$

And the moment of the force about the axis *OO′* of the cylinder

$$= \left(\frac{2\pi\eta\theta}{L}\right) x^2\, dx \cdot x = \left(\frac{2\pi\eta\theta}{L}\right) x^3\, dx$$

$\therefore$ twisting couple on the whole cylinder

$$= \int_0^R \frac{2\pi\eta\theta}{L} x^3\, dx = \frac{\pi\eta R^4}{2L}\theta$$

or, twisting couple per unit twist of the cylinder or wire, also called torsional rigidity of its material, is given by

$$C = \frac{\pi\eta R^4}{2L}$$

(*ii*) *Case of a hollow cylinder*: If the cylinder be a hollow one, of inner and outer radii R_1 and R_2 respectively we have

$$\text{twisting couple on the cylinder} = \int_{R_1}^{R_2} \frac{2\pi\eta\theta}{L} x^3\, dx = \frac{\pi\eta}{2L}\left(R_2^4 - R_1^4\right)\theta$$

$\therefore$ twisting couple per unit twist, say $C' = \dfrac{\pi\eta}{2L}(R_2^4 - R_1^4)$

Now, if we consider two cylinders of the same material, of density ρ, and of the same mass *M* and length *L*, but one solid, of radius *R* and the other hollow of inner and outer radii R_1 and R_2 respectively.

We have
$$\frac{C'}{C} = \frac{R_2^4 - R_1^4}{R^4} = \frac{(R_2^2 + R_1^2)\left(R_2^2 - R_1^2\right)}{R^4}$$

Since $M = \pi(R_2^2 - R_1^2)\, L\rho = \pi R^2 L\rho$, we have $(R_2^2 - R_1^2) = R^2$

or $$\frac{C'}{C} = \frac{(R_2^2 + R_1^2)R^2}{R^4} = \frac{(R_2^2 + R_1^2)}{R^2}$$

Again, because $R_2^2 - R_1^2 = R^2$, we have $R_2^2 = R^2 + R_1^2$

And, therefore, $R_2^2 + R_1^2 = R^2 + R_1^2 + R_1^2 = R^2 + 2R_1^2$, *i.e.,*

$$(R_2^2 + R_1^2) > R^2.$$

Clearly, therefore, $\frac{C'}{C} > 1$ or $C' > C$

or, the twisting couple per unit twist is greater for a hollow cylinder than for a solid one of the same material, mass and length.

This explains at once the use of hollow shafts, in preference to solid ones, for transmitting large torque in a rotating machinery.

3.16 OBJECT

To determine the value of modulus of rigidity of the material of a wire by statical method using vertical pattern apparatus (Barton's apparatus).

Apparatus: Barton's vertical pattern torsion apparatus, a wire, screw gauge, a vernier callipers, meter scale, set of weights, thread and a meter scale.

Formula Used: The modulus of rigidity η is given by

$$\eta = \frac{2\, mg\, Dl}{\pi\theta r^4}$$

where m = load suspended from each pan
g = acceleration due to gravity
D = Diameter of the heavy cylinder
r = radius of the experimental wire
l = length of the wire
θ = angle of twist in degree

Description of apparatus: The vertical pattern of torsion apparatus (Barton's apparatus) shown in figure is used for specimen available in the form of a long thin rod, whose upper end is fixed securely to a heavy metallic frame and lower end is fixed to a heavy metal cylinder *C*. This heavy cylinder keeps the wire vertical. Flexible cord attached to two diametrically opposite pegs on the cylinder leave it tangentially diametrically opposite points after half a turn. These cords pass over two frictionless pulleys P_1 and P_2, fixed in the heavy frame, and at their free end carry a pan each of equal weight. When equal loads are placed on the pans, couple acts on the cylinder which produces a twist in the rod. The twists are measured by double ended pointer which move over the concentric circular scales graduated in degrees. The three levelling screws are provided at the base of the metallic frame supporting the rod, to make it vertical. In this case the centres of the scale fall on the axis of the rod.

Theory: The modulus of rigidity (η) is defined as the ratio of shearing stress to shearing strain. The shearing stress is the tangential force *F* divided by the area *A* on which it is applied and the shearing strain is the angle of shear ϕ, therefore:

$$\eta = \frac{\frac{F}{A}}{\phi}$$

or $$F = \eta A\phi \qquad ...(1)$$

The twisting of a wire can be related to shearing as follows. Consider a solid cylinderical wire (length l and radius r) as consisting of a collection of thin hollow cylindrical elements. One such element is shown shaded in Fig. 3.5 (*b*). A line PQ of length l is drawn on the surface of the element parallel to its axis OO'. If the wire is clamped at the top and the bottom is twisted through the same angle and the line PQ would become PQ'. The same picture is shown in cross-section in Fig. 3.5(*b*). If we consider this element only, cut it along the line PQ and unroll it, the picture would be seen in Fig. 3.4. This picture is similar to Fig. 3.2(c), with an area A, being given by $2\pi x dx$ and the shearing angle being $QPQ' = \phi$

From Fig. 3.4(*b*) (if ϕ is small), the arc QQ' is equal to ($l \times \phi$) and from Fig. 3.5 (*b*), the arc QQ' is equal ($x \times \theta$). Therefore $l\phi = x\theta$

$$\phi = \frac{x\theta}{l} \qquad ...(2)$$

From Fig. 3.5, we see that the area (A) on which the shearing tangential force is applied to the element $A = 2\pi x dx.$

Substituting for A and ϕ in Eq. (1), we obtain

$$F = \eta\ 2\pi x dx\ \frac{x\theta}{l}$$

or $$F = 2\pi\eta \frac{x^2\theta}{l} dx$$

The moment of this force about the axis of the element (OO') is

$$F \times x = 2\pi\eta \frac{x^3\theta}{l} dx$$

We can calculate the total moment required to twist the entire solid cylinder (wire), by integrating over all the elements. The moment, or couple, C is given by

$$C = \int_0^r 2\pi\eta \frac{\theta}{l} x^3\, dx$$

or $$C = \left[2\pi\eta \frac{\theta}{l}\frac{r^4}{4}\right]$$

or $$C = \frac{\pi\eta\theta r^4}{2l} \qquad ...(3)$$

In this experiment, the twisting or shearing moment is provided by the couple applied to the thick heavy cyclinder of diameter D, fixed at the bottom of the wire. This couple is due to the tension in the threads bearing the weights (Fig. 3.3(*b*). Thus $C = mgD$, where mg is the tension in each thread. Therefore,

$$mgD = \frac{\pi\eta\theta r^4}{2l}$$

or $$\eta = \frac{2mgDl}{\pi\theta r^4} \text{ where } \theta \text{ is measured in degree}$$

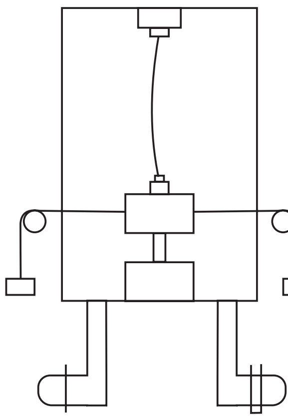


Fig. 3.3(a): Rigid apparatus

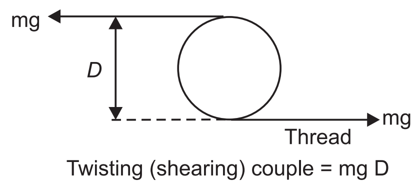


Fig. 3.3(b): Heavy thick solid cylinder

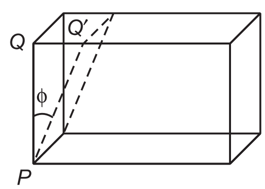


Fig. 3.4: Illustrating shear: shearing angle is $QPQ' = \phi$

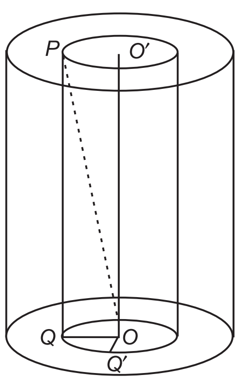


Fig. 3.5(a)

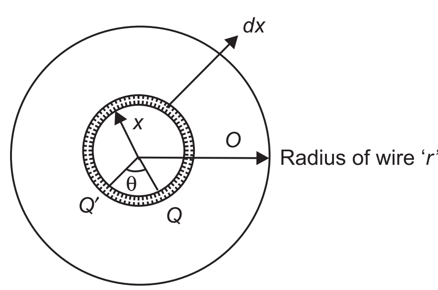


Fig. 3.5(b): Cross section of wire showing element (shaded and the angle of twist θ. $QQ' = x\theta$)

Manipulation

1. Level the base of the Barton's apparatus by the levelling screws at the base using spirit level so that the wire hangs freely vertically and can be twisted without any friction.

2. Wind the thread around the thick cylinder as shown in Fig. 3.3(*b*), pass the two ends over the pulleys and attach the pans to them.
3. Take readings of the pointers on the circular scale with no weights on the pans.
4. Add weights of 10 gm on each pan and take the readings again, repeating this until the total weight on each pan is 50 gms. Take readings again while the weights are reduced to zero.
5. Measure the diameter of the wire using the screw gauge and the diameter of the thick cylinder using a vernier callipers. Take readings of diameters along the entire length in mutually perpendicular directions to correct for any departure from uniform or circular cross-section (some places the area is elliptical hence we measure many places).
6. Measure the length of the wire which is being twisted.
7. Using the vernier callipers measure the diameter of the metallic cylinder.

Observations: (A) Table for the measurement of angle of twist (θ):

S.No.	*Load in each pans (in gm)*	*Pointer readings When load*				*Mean*	*Deflection for 30 gms Load*	*Mean Deflection for 30 gms load*
		Increasing		*Decreasing*				
		Right	*Left*	*Right*	*Left*			
1	0							
2	10							
3	20							
4	30							
5	40							
6	50							

Mean deflection angle for 30 gm load = _______ radian.

(*i*) The wire should not be twisted beyond elastic limits

(*ii*) Table for the measurement of diameter of the given wire.

Least count of screw gauge =

Zero error = ±

S. No.	1	2	3	4	5	6	7	8	9	10	Mean
Diameter in one direction (in cm)											
Diameter perpendicular to above (in cm)											

Diameter of wire corrected for zero error =

Radius r =

(*iii*) Table for the measurement of diameter of the given cylinder

Least count of vernier callipers =

Zero error =

S. No.	1	2	3	4	5	6	7	8	9	10	Mean
Diameter in one direction (in cm)											
Diameter perpendicular to above (in cm)											

Diameter (D) of cylinder corrected for zero error =

(*iv*) Length of the wire l =

Calculations:

$$\eta = \frac{2mg\,Dl}{\pi\theta\, r^4}$$

Result: The value of modulus of rigidity, η of the given iron wire as determined from Barton's apparatus is =

Standard value = η for iron = $7.2 - 8.5 \times 10^{10}$ N/m^2

Percentage error = %

Theoretical error = $\eta = \dfrac{2\,mg\,Dl}{\pi\,\theta\,r^4}$

Taking log, and differentiating, we get

$$\frac{\delta x}{\eta} = \frac{\delta M}{M} + \frac{\delta D}{D} + \frac{\delta l}{l} + \frac{\delta l}{l} + 4\frac{\delta l}{r} + \frac{\delta\theta}{\theta}$$

$$= \_\_\_\_\_\_\_\_\_\_$$

Maximum permissible error =

Precautions:

1. First of all base of the instrument should be levelled using sprit level.
2. The wire must be of uniform circular cross-section, free of links, hanging freely and vertically, firmly clamped at the top.
3. Too much weights must not be put on the pans, else the wire may twist beyond elastic limit.
4. The wire should be trained before the readings are taken.
5. The radius of the wire must be taken carefully since its occurs fourth power is occurring in the formula.
6. The pulleys should be frictionless.
7. Load should be increased of decreased gradually and gently.
8. The chord wound round the cylinder should be thin and strong.
9. Before starting experiment ensure that the upper end of the rod is firmly clamped. If it is not so, the rod may slip at this end on application of load.
10. The length of the wire, it measured between the two clamps.

BENDING OF BEAMS

3.17 BEAM

A rod or a bar of circular or rectangular cross-section, with its length very much greater than its thickness (so that there are no shearing stresses over any section of it) is called a beam.

If the beam be fixed only at one end and loaded at the other, it is called a cantilever.

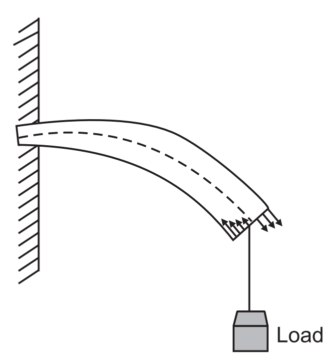


Fig. 3.6

3.18 BENDING OF A BEAM

Suppose we have a beam, of a rectangular cross-section, say, fixed at one end and loaded at the other (within the elastic limit) so as to be bent a little, as shown in figure with its upper surface becoming slightly convex and the lower one concave. All the longitudinal filaments in the upper half of the beam thus get extended or lengthened, and therefore under tension, and all those in the lower half get compressed or shortened and therefore under pressure.

These extensions and compressions increase progressively as we proceed away from the axis on either side so that they are the maximum in the uppermost and the lowermost layers of the beam respectively. There must be a layer between the uppermost and the lowermost layers where the extensions in the upper half change sign to become compressions in the lower half. In this layer or plane, which is perpendicular to the section of the beam containing the axis, the filaments neither get extended nor compressed, *i.e.* retain their original lengths. This layer is therefore called the neutral surface of the beam.

3.19 THEORY OF SIMPLE BENDING

There are the following assumptions:

(*i*) That Hooke's law is valid for both tensile and compressive stresses and that the value of Young's modulus (Y) for the material of the beam remains the same in either case.

(*ii*) That there are no shearing stresses over any section of the beam when it is bent. This is more or less ensured if the length of the beam is sufficiently large compared with its thickness.

(*iii*) That there is no change in cross-section of the beam on bending. The change in the shape of cross-section may result in a change in its area and hence also in its geometrical moment of inertia I_g. Any such change is however always much too small and is, in general, ignored.

(*iv*) That the radius of curvature of the neutral axis of the bent beam is very much greater than its thickness.

(*v*) That the minimum deflection of the beam is small compared with its length.

3.20 BENDING MOMENT

When two equal and opposite couples are applied at the ends of the rod it gets bent. The plane of bending is the same as the plane of the couple. Due to elongation and compression of the filaments above and below the neutral surface, internal restoring forces are developed which constitute a restoring couple. In the position of equilibrium the internal restoring couple is equal and opposite to the external couple producing bending of the beam. Both these couples lie in the plane of bending. The moment of this internal restoring couple is known as bending moment.

Expression for bending moment: Consider the forces acting on a cross-section through CD Fig. 3.7 of a bent beam. The external couple is acting on end B in the clockwise direction. The filaments above the neutral surface are elongated such that change in length is proportional to their distance from neutral axis. Therefore, the filaments to the right of CD and above neutral axis exert a pulling force towards left due to elastic reaction. Similarly, since the filaments below neutral axis are contracted, with change in length proportional to their distance from neutral axis, due to elastic reaction they exert a pushing force towards right as shown in figure.

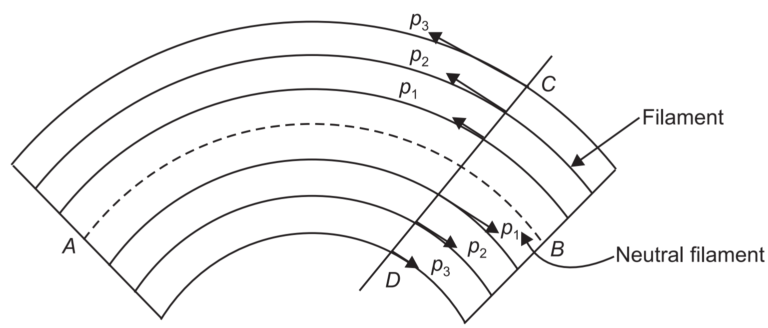


Fig. 3.7

Thus on *CD*, the forces are towards left above neutral axis while they are towards right below it. These forces form a system of anticlockwise couples whose resultant is the internal restoring couple. This couple is equal and opposite to the external couple producing bending in the beam, and keeps the position of beam to the right of *CD* in equilibrium. The moment of this internal restoring couple acting on the cross-section at *CD* is termed as bending moment. Its value is given by

$$G = \frac{YI_g}{R}$$

Where $I_g = \Sigma\delta a.\ z^2$ is called the geometrical moment of inertia of the cross-section of the beam about the neutral filament (This quantity is analogous to the moment of inertia with the difference that mass is replaced by area). For a beam of rectangular cross-section of width *b* and thickness *d*

$$I = \frac{1}{12} bd^3$$

and bending moment $$G = \frac{Ybd^3}{12R}$$

For a beam of circular cross-section of radius *r* its value is

$$I = \frac{1}{4}\pi r^4$$

and Bending moment $$G = \frac{Y\pi r^4}{4R}$$

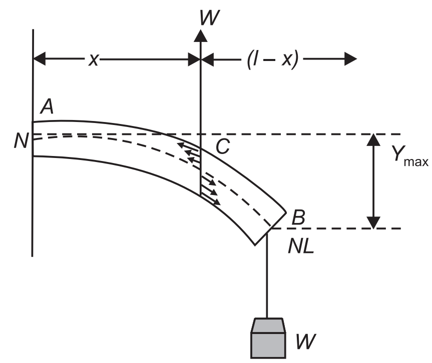
In the position of equilibrium, this bending moment balances the enternal bending couple τ, thus

$$\tau = C = \frac{YI}{R}$$

or $$R = \frac{YI}{\tau}$$

showing that the beam of uniform cross-section is bend into an arc of circle, since *R* is constant for given τ.

3.21 THE CANTILEVER



When a beam of uniform cross-section is clamped horizontally at one end and could be bent by application of a load at or near the free end, the system is called a cantilever.

When the free end of the cantilever is loaded by a weight W (= Mg), the beam bends with curvature changing along its length. The curvature is zero at the fixed end and increases with distance from this end becoming maximum at the free end. This is because of the fact that at distance x from fixed end, for equilibrium of portion CB of the beam the moment of external couple is $W(l - x)$, where l is the length of the cantilever. Thus for equilibrium of portion CB of cantilever, we have

$$G = \frac{YI}{R} = W(l - x)$$

Here it is assumed that the weight of the beam is negligible.

Now the radius of curvature R of the neutral axis at P distant x from fixed end, and having depression y is given by

$$\frac{1}{R} = \frac{d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}}$$

where (dy/dx) is the slope of the tangent at print $P(x, y)$. If the depression of the beam is small $\left(\frac{dy}{dx}\right)$ will be very small quantity in comparison to 1 and is, therefore, negligible. Hence

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Thus $$C = YI\frac{d^2y}{dx^2} = W(l - x)$$

or $$\frac{d^2y}{dx^2} = \frac{W}{YI}(l - x)$$

Integrating twice under the condition that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$, we get

$$y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{\sigma}\right)$$

This gives the depression of the beam at distance x from the fixed end, under the assumption that the weight of the beam itself is negligible.

At the loaded end where $x = l$, the depression is maximum and is given by

$$y_{\max} = \delta = \frac{Wl^3}{3YI}$$

If the beam is of rectangular cross-section (breadth b and thickness d), $I = \frac{1}{12}\ bd^3$, so that

$$\delta = \frac{4\,Wl^3}{ybd^3}$$

and for beam of circular cross-section of radius r, $I = \frac{\pi r^4}{4}$, so that

$$\delta = \frac{4Wl^3}{3\,y\pi r^4}$$

3.22 BEAM SUPPORTED AT BOTH THE ENDS AND LOADED IN THE MIDDLE

The arrangement of a beam supported at its both the ends and loaded in the middle is the most convenient method of measurements. A long beam AB of uniform cross-section is supported symmetrically on two knife edges K_1 and K_2 in the same horizontal plane and parallel to each other at a distance l apart. When the beam is loaded at its middle point c by a weight W, this generates two reactions equal to $\frac{W}{2}$ each, acting vertically upwards at the two knife-edges. The beam is bent in the manner as shown in figure. The maximum depression is produced in the middle of the beam where it is loaded.

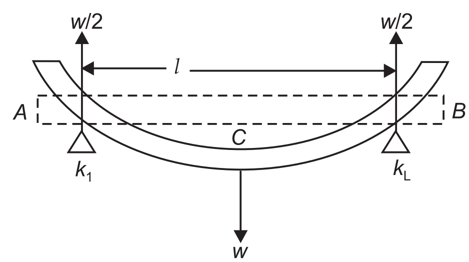


Fig. 3.8

By consideration of symmetry it is clear that the tangent to the beam at C will be horizontal. Hence the beam can be divided into two portions AC and CB by a transverse plane through the middle point of the beam. Each portion can be regarded as a cantilever of length $\frac{l}{2}$, fixed horizontally at point C and carrying a load $\frac{W}{2}$ in the upward direction at the other end (*i.e.,* these are inverted cantilevers). The elevation of the ends A and B above middle point C is equal to the depression of the point C. The depression at the middle point is thus obtained as

$$\delta = \frac{\left(\frac{W}{2}\right)\times\left(\frac{l}{2}\right)^3}{3YI} = \frac{Wl^3}{48YI}$$

$$\delta = \frac{Mgl^3}{48\,YI} \quad \text{since } W = \text{Mg}$$

Hence for a rectangular beam of breadth b and thickness d

$$\delta = \frac{Mgl^3}{4Ybd^3}$$

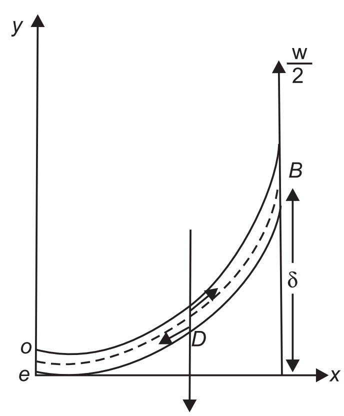


Fig. 3.9

and for a beam of circular cross-section of radius r

$$\delta = \frac{Mgl^3}{12\pi Y r^4}$$

From these expressions, knowing the dimensions of the beam and the depression at the middle point by a known weight, the value of Young's modulus Y, for the material of the beam can be calculated.

Methods of measuring depression: The depression of the beam supported at its ends and loaded in the middle is usually measured by either of the methods *viz.*

(*i*) by micrometer screw or spherometer with some electrical indicating device like a bell, bulb, galvanometer or voltmeter.
(*ii*) by single optical lever method
(*iii*) By double optical lever or Koenig's method

3.23 OBJECT

To determine the Young's modulus of the material of a given beam supported on two knife-edges and loaded at the middle point.

Apparatus used: Two parallel knife edges on which the beam is placed, a hook to suspend weights, a scale attached to the hook, 0.5 kg weights, a cathetometer, a vernier callipers and a meter scale.

Formula used: The Young's modulus (Y) for a beam of rectangular cross-section is given by the relation

$$Y = \frac{Mgl^3}{4bd^3 y} \text{ Newton / meter}^2$$

Where M = load suspended from the beam, g = acceleration due to gravity, l = length of the beam between the two knife edges, b = breadth of the beam, d = thickness of the beam and y = depression of the beam in the middle.

Theory: Let a beam be supported horizontally on two parallel knife edges A and B (Figure 3.8), distance l apart and loaded in the middle C with weight W. The upward reaction at each knife edge being $\frac{W}{2}$ and the middle part of beam being horizontal, it may be taken to be combination of the two inverted cantilevers CA and CB, each of effective length $\frac{l}{2}$ fixed at C and bending upward under a load $\frac{W}{2}$ acting on A and B. Let the elevation of A and B above C or the depression of C below A and B be 'y'.

Consider the section DB of the cantilever CB at a distance x from its fixed end C (Figure 3.9). Bending couple due to load $\frac{W}{2} = \frac{W}{2}\left(\frac{l}{2} - x\right)$. The beam being in equilibrium, this must be just balanced by the bending moment (or the moment of the resistance to bending) $\frac{YI_g}{R}$, where R is the radius of curvature of the section at D and I_g is the geometrical moment of inertia of the cross-section of the beam about an axis passing through the centre of the beam and perpendicular to it. Therefore, we have

$$\frac{YI_g}{R} = YI_g \frac{d^2y}{dx^2} = \frac{W}{2}\left(\frac{l}{2} - x\right) \text{ since } \frac{1}{R} = \frac{d^2y}{dx^2}$$

Which on integration gives $\frac{dy}{dx} = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right] + c$

since at $x = 0$, $\frac{dy}{dx} = 0$, we have $c = 0$ and therefore,

$$\frac{dy}{dx} = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right]$$

or
$$dy = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right]dx$$

Which on further integration between the limits $x = 0$ and $x = \frac{l}{2}$ gives

$$y = \frac{W}{2YI_g}\left[\frac{l^3}{16} - \frac{l^3}{48}\right]$$

$$= \frac{Wl^3}{48YI_g}$$

If the cross-section of the beam be rectangular of breadth 'b' and thickness 'd', we have $I_g = \frac{bd^3}{12}$. Hence above equation can now be written as

$$y = \frac{Wl^3}{4Ybd^3}$$

or
$$y = \frac{Mgl^3}{4Ybd^3}$$

Where M is the mass suspended from the hook. The depression y of the mid-point is noted directly with the help of cathetometer.

Manipulation:

1. Adjust the cathetometer so that
   (*a*) The vertical column that carries the microscope and scale is vertical.
   (*b*) As microscope is moved up and down the column, the axis of the microscope is parallel to some fixed direction in the horizontal plane.

2. Support the experimental beam symmetrically on the knife edges with equal lengths projecting beyond the knife edges.
3. Measure the distance between knife edges with a meter scale. This gives the length l of the beam under flexure.
4. Suspend the hanger with a graduated scale attached to it, on the mid-point of the beam. Focus the microscope horizontal cross-wire of the microscope on a certain division of the scale. Take the reading of the vernier scale of cathetometer (observation table I)
5. Suspend a weight of 1 kg on the hanger. A depression is produced in the beam. Move the microscope downwards till its horizontal cross-wire again coincides with the previous division of the scale attached to the hook (or hanger). Take the readings of the vernier scale of the cathetometer. The difference between (4) and (5) gives the depression in the beam due to 1 kg weight.
6. Repeat the previous step for loads increasing by 1 kg at a time and then for loads decreasing in that order.
7. Measure the distance between knife edges (observation II)
8. Measure the breadth 'b' and thickness 'd' of the beam precisely using vernier callipers and screw gauge respectively (observation table III).
9. A graph is plotted between M and y.

From graph find the slope. calculate Y using the formula and slope of the graph.

Observation: (I) Readings for the depression 'y' in the beam due to the load applied:
Least count of vernier scale of cathetometer =
Zero error of the cathetometer =

S.No.	*Load applied M (in kg)*	*Reading (in cm) of cathetometer with*		*Mean of i and ii (in cm)*	*Depression produced in the beam for 3 kg of load y (in cm)*	*Mean depression produced in the beam for 3 kg of Load y (in cm)*
		Load increasing (i)	*Load decreasing (ii)*			
1	0					
2	1					
3	2					
4	3					
5	4					
6	5					

(II) The distance between two knife edges (l) =
(III) Readings for the breadth (b) and thickness (d) of the beam:
Least count of Vernier callipers =
Zero error of the vernier callipers =

Observation	1	2	3	4	5	6	7	8	9	10	Mean
Breadth b (in cm)											
Thickness d (in cm)											

Calculations: The Young's modulus of the material (Iron) of the beam can be calculated by substituting the values of M, y, L, b, d and g in C.G.S. in Eq.

$$Y = \frac{Mgl^3}{4ybd^3}$$

From graph = $\dfrac{gl^3}{4bd^3}\left(\dfrac{M}{y}\right)$

Result: The value of Young's modulus of the material (Iron) of the beam is found to be =

Standard Value =

Percentage error =

Precautions and Sources of Error

1. The beam must be symmetrically placed on the knife edges with equal lengths projecting out beyond the knife edges.
2. The hanger should be suspended from the centre of gravity of the beam.
3. The loads should be placed or removed from the hanger as gently as possible and the reading should be recorded only after waiting for sometime, so that the thermal effects produced in the specimen, get subsided.
4. Avoid the backlash error in the cathetometer.
5. Since the depth (thickness) of the beam appears as its cube in the formula and is relatively a small quantity, it should be determined by measuring it at several places along length by screw gauge.

Theoretical error: The value of Young's modulus for the material of the beam is given by

$$Y = \frac{Mgl^3}{4bd^3y}$$

Taking log and differentiating above expression, we get

$$\frac{\delta Y}{Y} = \frac{3\delta l}{l} + \frac{\delta b}{b} + \frac{3\delta d}{d} + \frac{\delta y}{y}$$

Maximum possible error = _______%.

3.24 OBJECT

To determine the Poisson's ratio for rubber.

Apparatus used: Rubber tube with metal sleeve and rubber stopper, metre scale, small pointer, slotted weights, hanger, Burette and rubber stopper

Formula used: The Poisson's ratio σ for rubber is given by

$$\sigma = \frac{1}{2}\left[1 - \frac{1}{A}\frac{dV}{dL}\right]$$

Where A = area of cross-section of rubber tube

$= \pi \dfrac{D^2}{4}$ where D is its diameter

dV = Small increase in the volume of the tube when stretched by a small weight

dL = corresponding increase in the length of the tube.

Theory: Let a rubber tube suspended vertically be loaded at its lower end with a small weight. This stretches the rubber tube slightly with a consequent increase in its length and internal volume. Let V be the original volume of the tube, A its area of cross-section, L its length and D its diameter. Then, if for a small increase of volume dV, the corresponding increase in length is dL and the decrease in area is dA, we have

$$V + dV = (A - dA)(L + dL)$$
$$= AL + AdL - LdA - dAdL$$

Putting $AL = V$ and neglecting $dAdL$ a product of two small quantities, we have

$$dV = AdL - LdA \qquad ...(1)$$

Now $$A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$

Differentiating $$dA = \frac{\pi D}{2} dD = \frac{2A}{D} dD$$

Substituting this value of dA in equation (1) we get

$$dV = AdL - \frac{2AL}{D} dD$$

or $$\frac{dV}{dL} = A - \frac{2AL}{D} \cdot \frac{dD}{dL}$$

Whence $$\frac{LdD}{DdL} = \frac{1}{2}\left(1 - \frac{dV}{AdL}\right)$$

Poisson's ratio $$\sigma = \frac{\dfrac{L}{dL}}{\dfrac{D}{dD}} = \frac{1}{2}\left(1 - \frac{1}{A}\frac{dV}{dL}\right)$$

This equation can be employed to calculate σ, if other quantities are determined.

Description of apparatus: A rubber tube about one metre long and 4 cms in diameter is suspended in a vertical position as shown in Fig. 3.10. Its two ends are closed by means of two rubber corks A and B such that both ends are water tight. A burette C about 50 cms long and 1 cm in diameter open at both ends is inserted in the rubber tube through the upper cork A. The tube is held vertical with most of its portion out of the rubber tube.

Water is filled in the rubber tube till it rises in the glass tube from the end A of rubber tube. A pointer P is fixed at lower end B which moves on a scale S when weights are placed on the hanger.

Procedure

1. The apparatus is suspended through a clamp fixed at a convenient height.
2. Pour water in the rubber tube until the water meniscus appears nearly at the top of the burette.
3. Note down the position of the pointer on the scale and the reading of water meniscus in burette. We take this as zero position though the hanger remains suspended from the hook.
4. Measure the diameter of the rubber tube at a number of points with a vernier callipers and find its mean value, and thus calculate the area of cross-section of the tube.
5. Place gently a load, of say 100 gm, on the hanger at the lower end of the tube and wait for about 5 minutes. Note down the readings of pointer and the meniscus of water.
6. The difference between the two readings of the burette gives the change in volume dV for the load on the hanger and the difference between the two readings of the pointer on the scale gives the corresponding change in length.
7. Now increase the load on the hanger in equal steps of, say 100 gm, till maximum permissible load within elastic limit is reached, taking down the reading of the burette and the pointer after addition of each load when the apparatus has settled down.
8. Repeat the above procedure for weights decreasing.
9. Take the mean of the two readings of the burette for the same load on the hanger obtained with increasing and decreasing load, and then subtracting the mean readings for zero load on the hanger from the mean reading for any load, calculate the change in volume dV of the rubber tube for various loads on the hanger.

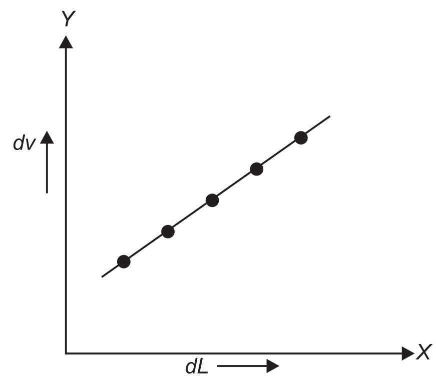


Fig. 3.10

10. Similarly calculate the volume of corresponding change in length dL for the various loads.
11. Calculate the value of $\frac{dV}{dL}$ for each set of observations separately and find its mean value for σ.
12. Plot a graph taking dL along the X-axis and the corresponding value of dV along the Y-axis. This will come out to be a straight line as shown in figure 3.10.

    Its slope will give the average value of $\frac{dV}{dL}$.
13. Also use this value to calculate again the value of σ.

Observation:

Least count of the scale =

Least count of microscope =

(A) Measurement of diameter of the rubber tube.

Vernier constant =

Zero error =

S.No.	*Reading along any diameter α cm*	*Reading along perpendicular diameter β cm*	*Diameter* $\left(\frac{\alpha+\beta}{2}\right)$ cm

Mean uncorrected diameter =

(B) Determination of change in volume dV and the corresponding change in length dL.

S.No.	*Load on the hanger*	*Reading of burette*			*Change in volume dV*	*Reading of pointer P on scale S*			*Change in length dL*	$\frac{dv}{dL}$	*Mean*
		Load increasing x	*Load decreasing* y	*Mean* $\left(\frac{x+y}{2}\right)$		*Load increasing* x'	*Load decreasing* y'	*Mean* $\frac{x'+y'}{2}$			

Calculations: Mean corrected diameter of the rubber tube = ... cm.

$\therefore$ Area of cross-section of the tube A = $\frac{\pi D^2}{4}$ = ... sq. cm

$$\sigma = \frac{1}{2}\left(1 - \frac{1}{A}\frac{dV}{dL}\right)$$

Also from graph calculate the value of slope $\frac{dV}{dL}$ and substitute this value in above equation.

Result: The value of Poisson's ratio for rubber as obtained experimentally

(*i*) by calculations =

(*ii*) by graph =

Standard value of σ =

$\therefore$ Error = %

Sources of error and precautions

1. Microscope should be used to measure internal radius of the rubber as also to measure the radius of the capillary.
2. Hanger should be stationary at the time of taking down the observations.
3. There should be no air bubble inside the rubber tube or the burette.
4. Weights should be placed or removed gently and in equal steps.
5. After each addition or removal of load wait for about 5 minutes before taking observations in order to allow the apparatus to settle down to new conditions of stress and strain.
6. The load suspended at the lower end of the rubber tube should not exceed the maximum load permissible within elastic limit.

3.25 FLY-WHEEL (MOMENT OF INERTIA)

Object: To determine the moment of inertia of a fly-wheel about its axis of rotation.

Apparatus: The flywheel, weight box, thread, stop-watch, meter scale and vernier callipers.

Description of apparatus: A fly-wheel is a heavy wheel or disc, capable of rotating about its axis. This fly-wheel properly supported in bearings may remain at rest in any position, *i.e.,* its centre of gravity lies on the axis of rotation. Its moment of inertia can be determined experimentally by setting it in motion with a known amount of energy.

Theory: The flywheel is mounted in its bearings with its axle horizontal and at a suitable height from the ground, and a string carrying a suitable mass m at its one end and having a length less than the height of the axle from the ground, is wrapped completely and evenly round the axle. When the mass m is released, the string unwinds itself, thus setting the flywheel in rotation. As the mass m descends further and further the rotation of the flywheel goes on increasing till it becomes maximum when the string leaves the axle and the mass drops off.

Let h be the distance fallen through by the mass before the string leaves the axle and the mass drops off, and let v and w be the linear velocity of the mass and angular velocity of the flywheel respectively at the instant the mass drops off. Then, as the mass descends a distance h, it loses potential energy mgh which is used up: (*i*) partly in providing kinetic energy of translation $\frac{1}{2}mv^2$ to the falling mass itself, (*ii*) partly in giving kinetic energy of rotation $\frac{1}{2}I\omega^2$ to the flywheel (where I is the moment of inertia of the flywheel about the axis of rotation) and (*iii*) partly in doing work against friction.

If the work done against friction is steady and F per turn, and, if the number of rotations made by the flywheel till the mass detaches is equal to n_1, the work done against friction is equal to n_1F. Hence by the principle of conservation of energy, we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + n_1F \qquad ...(1)$$

After the mass has detached the flywheel continues to rotate for a considerable time t before it is brought to rest by friction. If it makes n_2 rotations in this time, the work done against friction is equal to n_2F and evidently it is equal to the kinetic energy of the flywheel at the instant the mass drops off. Thus,

$$n_2F = \frac{1}{2}I\omega^2$$

$$F = \frac{1}{2}\frac{I\omega^2}{n_2} \qquad ...(2)$$

Substituting this value of F in Eq. (1), we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2\frac{n_1}{n_2}$$

Whence

$$\text{I} = \frac{2mgh - mv^2}{\omega^2\left(1+\frac{n_1}{n_2}\right)}$$

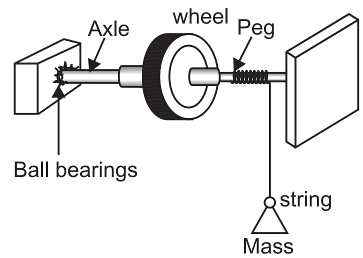


Fig. 3.11

If r be the radius of the flywheel,

$$v = r\omega$$

$\therefore$

$$\text{I} = \frac{2mgh - mr^2\omega^2}{\omega^2\left(1+\frac{n_1}{n_2}\right)} \qquad ...(3a)$$

$$= \frac{m\left(2\frac{gh}{\omega^2} - r^2\right)}{\left(1+\frac{n_1}{n_2}\right)} \qquad ...(3b)$$

After the mass has detached, its angular velocity decreases on account of friction and after some time t, the flywheel finally comes to rest. At the time of detachment of the mass the angular velocity of the wheel is ω and when it comes to rest its angular velocity is zero. Hence, if the force of friction is steady, the motion of the flywheel is uniformly retarded and the average angular velocity during this interval is equal to $\frac{\omega}{2}$. Thus,

$$\frac{\omega}{2} = \frac{2\pi n_2}{t}$$

or

$$\omega = \frac{4\pi n_2}{t} \qquad ...(4)$$

Thus observing the time 't' and counting the rotations n_1 and n_2 made by the flywheel its moment of inertia can be calculated from equation (3) and (4).

For a fly-wheel with large moment of inertia (I), $\frac{1}{2}mv^2$ may be neglected, the equation (3a) becomes

$$I = \frac{2mgh}{\omega^2\left(1+\frac{n_1}{n_2}\right)}$$

Substituting the value of ω from Eq. (4) in above equation, we get

$$\boxed{I = \frac{mgh}{\frac{8\pi^2 n_2^2}{t^2}\left(1+\frac{n_1}{n_2}\right)}} \qquad ..(5)$$

This formula is used to calculate the moment of inertia of a flywheel.

Procedure

1. Attach a mass m to one end of a thin thread and a loop is made at the other end which is fastened to the peg.
2. The thread is wrapped evenly round the axle of the wheel.
3. Allow the mass to descend slowly and count the number of revolutions n_1 during descent.
4. When the thread has unwound itself and detached from the axle after n_1 turns, start the stop watch. Count the number of revolutions n_2 before the flywheel comes to rest and stop the stop watch. Thus n_2 and t are known.
5. Repeat the experiment with three different masses.
6. Calculate the value of I using the given Eq. (5).

Observation: Least count of the stop-watch = ... sec.

Table for determination of n_1, n_2 and t

S.No.	*Total load applied (in kg)*	*No. of revolutions of flywheel before the mass detached n_1*	*No. of revolutions of flywheel to come to rest after mass detached n_2*	*Time for n_2 revolutions t (sec)*
1				
2				
3				

Calculation: $I = \dfrac{mgh}{\frac{8\pi^2 n_2^2}{t^2}\left(1+\frac{n_1}{n_2}\right)}$

Result: The moment of inertia of the flywheel is ..... kg–m^2.

Sources of error and precautions

1. The length of the string should be always less than the height of the axle of the flywheel from the floor so that it may leave the axle before the mass strikes the floor.
2. The loop slipped over the peg should be quite loose so that when the string has unwound itself, it must leave the axle and there may be no tendency for it to rewind in the opposite direction.

3. The string should be evenly wound on the axle, *i.e.,* there should be no overlapping of, or a gap left between, the various coils of the string.
4. To ensure winding to whole number of turns of string on the axle the winding should be stopped, when almost complete at a stage where the projecting peg is horizontal.
5. To determine h measure only the length of the string between the loop and the mark at the other end where the string left the axle before the start of the flywheel.
6. The string used should be of very small diameter compared with the diameter of the axle. If the string is of appreciable thickness half of its thickness should be added to the radius of the axle to get the effective value of r.
7. The friction at the bearings should not be great and the mass tied to the end of the string should be sufficient to be able to overcome the bearing-friction and so to start falling of its own accord.
8. Take extra care to start the stop-watch immediately the string leaves the axle.

Criticism of the method: In this method the exact instant at which the mass drops off cannot be correctly found out and hence the values of n_1, n_2 and t cannot be determined very accurately. The angular velocity ω of the flywheel at the instant the mass drops off has been calculated from the formula $\omega = \dfrac{4\pi n_2}{t}$ on the assumption that the force of friction remains constant while the angular velocity of the flywheel decreases from ω to zero. But as the friction is less at greater velocities, we have no justification for this assumption. Hence for more accurate result, ω should be measured by a method in which no such assumption is made *e.g.,* with a tuning fork.

3.26 TORSION TABLE (ELASTICITY)

Object: To determine the modulus of rigidity of the material of the given wire and moment of inertia of an irregular body with the help of a torsion table.

Apparatus used: Torsion pendulum, a fairly thin and long wire of the material to be tested, clamps and chucks, a stop-watch, an auxillary body (a cylinder), screw guage, vernier callipers, meter scale, spirit level, a balance and a weight box.

Formula used:

$$\eta = \frac{8\pi I_1 l}{(T_1^2 - T_0^2) r^4}$$

where

$$I_1 = \frac{1}{2} MR^2$$

$$I_2 = \frac{T_2^2 - T_0^2}{T_1^2 - T_0^2} I_1$$

T_0 is the time period of oscillations of the torsion table only. T_1 is the time period of oscillations of the torsion table plus a regular body of moment of inertia I_1 placed on the table, its axis coinciding with the axis of the wire. T_2 is the time period for the torsion table plus an irregular body placed on the table. l is the length of the wire between two clamps, r is the radius of the wire, M is the mass of the regular cylinder, R is the radius of the regular cylinder.

Description of apparatus: The torsion table is illustrated in figure 3.12. One end of a fairly thin and long wire is clamped at A to a rigid support and the other end is fixed in the centre of a projection coming out of the central portion of a circular disc B, of aluminium or brass. On the upper face of the disc are described concentric circles and a concentric groove is cut in which three balancing weights can be placed. Beneath the disc is a heavy iron table T provided with three levelling screws. The plumb-line arrangement between the disc and the table serves to test the horizontality of the disc.

Theory: When the disc is rotated in a horizontal plane and then released, it executes torsional vibrations about the wire as the axis. If I_0 be the moment of inertia of the disc with its projection about the wire as the axis, its period of oscillation is, from eq.

$$T = 2\pi\sqrt{\frac{I}{C}} \text{ given by}$$

$$T_0 = 2\pi\sqrt{\frac{I_0}{C}} \qquad ...(1)$$

Where C is the torsional couple per unit radian twist. If an auxiliary body of known moment of inertia I is placed centrally upon the disc, then the period of oscillation T of the combination is given by

$$T = 2\pi\sqrt{\frac{I_0 + I}{C}} \qquad ...(2)$$

Squaring equations (1) and (2) and then subtracting equation (1) from Eq. (2), we get

$$T^2 - T^2_0 = \frac{4\pi^2 I}{C}$$

But from Eq. $\tau = \dfrac{\pi\eta r^4\phi}{2l}$, C is also-equal to $\dfrac{\pi\eta r^4}{2l}$. Hence the above equation yields.

$$\eta = \frac{8\pi I l}{(T^2 - T_0^2) r^4} \qquad ...(3)$$

From Eq. (3) the value of modulus of rigidity η of the wire can be calculated, if its length l and radius r are determined and the periods T_0 and T observed.

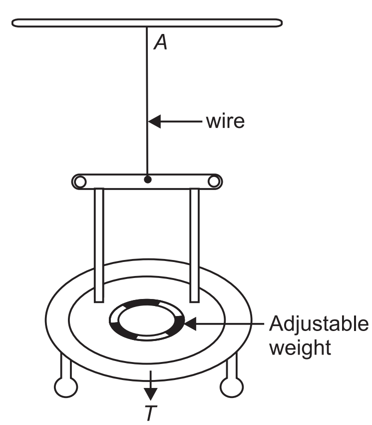


Fig. 3.12

Methods

1. Set up the torsion pendulum as shown in figure.
2. Level the heavy iron table T by the levelling screws and test the levelling with a spirit level.
3. Adjust the positions of the balancing weights in the groove in the disc such that the disc is horizontal as indicated by the plumb-line arrangement between the disc and the table. Place a vertical pointer in front of the disc and just behind it put a mark on the disc when the latter is at rest.

4. Rotate the disc slightly in a horizontal plane and then release it. It will perform torsional oscillations when undesirable motions have subsided, say after two or three oscillations, begin timing the oscillations of the disc with an accurate stop-watch by observing the transits of the mark on the disc past the vertical pointer. Determine twice the time for a large number of oscillations say, 30 and taking at least two sets of observations with different number of oscillations, find the mean period T_0 of the disc.
5. Place a right cylinder centrally upon the disc and determine as before, the period T of the combination. Note T will be greater than T_0.
6. Measure the length of the wire between the clamps with a metre scale and the diameter of the wire at several places with a screw gauge. Also weight the cylinder in a physical balance and find its diameter with vernier callipers.
7. Calculate the moment of inertia of the cylinder from the formula $I = MR^2/2$ and then the value of modulus of rigidity of the wire from Eq. (3).

Observation: (A) Determination of periods T and T_0

Least count of stop-watch = ... sec

S.No.	*No. of oscillation*	*Determination of T_0*			*Determination of T*		
		Time taken		*T_0 sec*	*Time taken*		*T sec*
		min	*sec*		*min*	*sec*	
			Mean T_0			Mean T	

(B) (*i*) Length of the wire = ...

(*ii*) Measurement of diameter of the wire

Least count of the screw gauge =

Zero error =

S.No.	*Reading along any diameter (a) c.m.*	*Reading along perpendicular diameter (b) c.m.*	*$\frac{a+b}{2}$(c.m.)*
		Mean uncorrected diameter	

(C) (*i*) Mass of the cylinder =
(*ii*) Measurement of diameter of the cylinder
Vernier constant of the callipers =
zero error =
Diameter of the cylinder—(*i*) ...... cm
(*ii*) ...... cm
(*iii*) ...... cm
(*iv*) ...... cm
Mean diameter = ... cm

Calculations:

Mean corrected diameter of the wire = ... cm
Radius of the wire = ... cm
Mean corrected diameter of the cylinder = cm
∴ Radius of the cylinder = ... cm

$$I = \frac{MR^2}{2}$$
$$= ......$$
$$\eta = \frac{8\pi Il}{(T^2 - T_0{}^2)\, r^4}$$

Result: The modulus of rigidity of the material of the given wire = ... dynes/cm^2
standard value = ... dynes/cm^2
∴ error = %

Sources of error and precautions:

1. The disc should always remain horizontal so that its moment of inertia I_0 remain unaltered throughout the experiment. Consequently the balancing weights, when once adjusted, should not be disturbed in subsequent observations for T.
2. The motion of the torsion pendulum should be purely rotational in a horizontal plane.
3. The suspension wire should be free from kinks and should be fairly thin and long, say about 70 cm, and 0.1 cm thick, so that torsional rigidity may be small and hence the periods of the pendulum large.
4. The wire should not be twisted beyond elastic limit otherwise the torsional couple will not be proportional to the value of the twist.
5. The auxiliary body (cylinder) should be of uniform density throughout e.g., of brass, and should be placed centrally upon disc so that its axis is coincident with the axis of the suspension wire.
6. As the periods occur raised to the second power in the expression for η, they must be measured very accurately by timing a large number of oscillations with a stop-watch reading upto $\frac{1}{5}$ sec.
7. As the radius of the wire occurs raised to the fourth power in the expression for η and is a small quantity, the diameter of the wire must be measured very accurately with a screw gauge. Reading should be taken at several points along the length of the wire and at each

point two mutually perpendicular diameters should be measured. The diameter of the cylinder should be similarly measured with a vernier callipers.

3.27 OBJECT

To determine the restoring force per unit extension of a spiral spring by statical and dynamical methods and also to determine the mass of the spring.

Apparatus used: A spiral spring, a pointer and a scale-pan, slotted weights and a stop watch.

Formula used: 1. Statical method

The restoring force per unit extensions (k) of the spring is given by

$$k = \frac{Mg}{l} \text{ Newton/meter}$$

where M = mass applied at the lower end of the spring

g = acceleration due to gravity

l = extension produced in the spring

2. Dynamical method

$$k = \frac{4\pi^2 (M_1 - M_2)}{(T_1^2 - T_2^2)} \text{ Newton/meter}$$

where M_1, M_2 = Masses applied at the lower end of the spring successively

T_1, T_2 = Time periods of the spring corresponding to masses M_1 and M_2 respectively.

3. The mass m of the spring is given by

$$m = 3\left[\frac{M_1T_2^2 - M_2T_1^2}{T_1^2 - T_2^2}\right] kg$$

where the symbols have their usual meanings.

Theory: A spiral-spring consists of a uniform wire, shaped permanently to have the form of a regular helix. A flat spiral-spring is one in which the plane of the spiral is perpendicular to axis of the cylinder. We deal here only with flat spiral-spring.

The spring is suspended from a rigid support with a hanger on the other end. A mass M is placed on the hanger so that spiral spring stretched vertically downward. If the extension of the spring is small, the force of elastic reaction F is, from Hooke's law, proportional to the extension l, or

$$F \propto l$$

or

$$F = kl$$

where k is a constant giving a measure of the stiffness of the spring and is called the restoring force per unit extension of the spring.

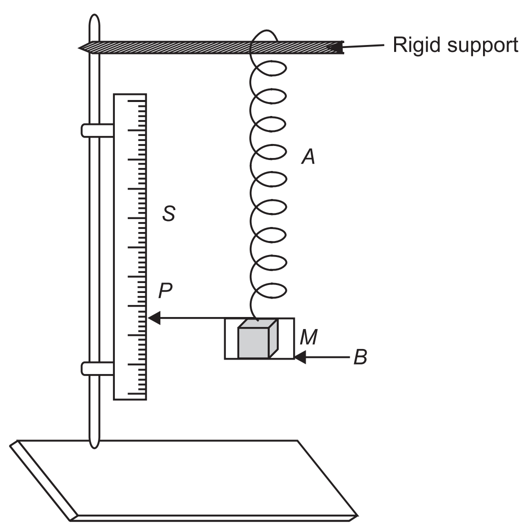


Fig. 3.13

But in the equilibrium state

$$F = Mg$$

$$\therefore \qquad kl = Mg$$

or
$$\boxed{k = \frac{Mg}{l}}$$

from which k may be calculated, if the extension l for a known load M at the end of the spring is determined.

If the mass at the end of the spring is now displaced vertically downward and then released, then for small oscillations, the restoring force at any instant is proportional to the displacement x, *i.e.*, $F = kx$.

If $\frac{d^2x}{dt^2}$ is the acceleration of the mass at that instant, the inertial reaction of the system is $(M+m)\frac{d^2x}{dt^2}$, where m is the effective mass of the spring. Equating the sum of these forces to zero we get from Newton's third law of motion, the equation

$$(M+m)\frac{d^2x}{dt^2} + kx = 0$$

or
$$\frac{d^2x}{dt^2} + \frac{k}{(M+m)}x = 0$$

This equation represents a S.H.M. whose period is given by

$$T = 2\pi\sqrt{\frac{M+m}{k}} \qquad ...(1)$$

If the experiment is performed with two masses M_1 and M_2 suspended successively at the end of the spring and the respective periods are T_1 and T_2, we have

$$T_1 = 2\pi\sqrt{\frac{M_1+m}{k}} \qquad ...(2)$$

and
$$T_2 = 2\pi\sqrt{\frac{M_2+m}{k}} \qquad ...(3)$$

Squaring Eq. (2) and (3) and subtracting, we get

$$T_1^2 - T_2^2 = \frac{4\pi^2}{k}(M_1 - M_2)$$

Whence
$$\boxed{k = \frac{4\pi^2(M_1 - M_2)}{T_1^2 - T_2^2}} \qquad ...(4)$$

From which k may be calculated.

Squaring Eq. (2) and (3) and then dividing Eq. (2) by Eq. (3) we have

$$\frac{T_1^2}{T_2^2} = \frac{M_1 + m}{M_2 + m}$$

Whence on simplification

$$m = \frac{M_1 T_2^2 - M_2 T_1^2}{T_1^2 - T_2^2}$$

It can be shown that the mass of the spring m' is three times the effective mass of the spring.

$$\therefore \text{ Mass of the spring } m' = 3\left[\frac{M_1 T_2^2 - M_2 T_1^2}{T_1^2 - T_2^2}\right] \qquad ...(5)$$

Squaring and rearranging Eq. (1) gives

$$T^2 = \left(\frac{4\pi^2}{k}\right)M + \left(\frac{4\pi^2}{k}\right)m \qquad ...(6)$$

A plot of T^2 against M gives a straight line as shown in figure 3.14. The negative intercept OP on the x-axis equals m and may be used to find m graphically.

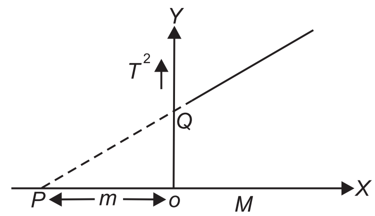


Fig. 3.14

Description of apparatus: A spiral spring A whose restoring force per unit extension is to be determined is suspended from a rigid support as shown in figure 3.13. At the lower end of the spring, a small scale pan is fastened. A small horizontal pointer P is also attached to the scale pan. A millimeter scale S is also set in front of the spring in such a way that when spring vibrates up and down, the pointer freely moves over the scale.

Procedure: (*a*) *Statical Method*

1. Hang a spiral spring A from a rigid support as shown in figure 3.13 and attached a scale pan B.
2. With no load in the scale-pan, note down the zero reading of the pointer on the scale.
3. Place gently in the pan a load of, say 100 gm.
4. Now the spring slightly stretches and the pointer moves down on the scale. In the steady position, note down the reading of the pointer. The difference of the two readings is the extension of the spring for the load in the pan.
5. Increase the lead in the pan in equal steps until maximum permissible load is reached and note down the corresponding pointer readings on the scale.

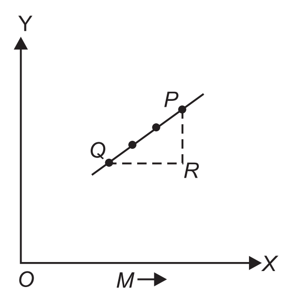


Fig. 3.15

6. The experiment is repeated with decreasing loads.
7. Plot a graph as illustrated in figure 3.15 between the load and the scale readings taking the load on X-axis and corresponding scale readings on Y-axis. The graph will be a straight line. Measure PR and QR and calculate k from the formula $k = \frac{QR}{PR} \times g$ Newton/meter.

Observation: Mass of the pan =

Table for the extension of the spiral-spring

S.No.	*Load in the pan*	*Reading of pointer on the scale in cm*			*Extension for ...... gms (in cm)*
		Load increasing	*Load decreasing*	*Mean*	

Average extension for .... kgs load = ... cm.

Calculations: Statical experiment

The restoring force per unit extension of the spring

$$k = \frac{mg}{l} = \text{...... Newton/meter}$$

A graph is plotted between the load and scale readings described in point (7) in the procedure. From graph

$$PR = \text{.....}$$

$$QR = \text{.....}$$

$$\therefore \qquad k = \frac{QR}{PR} \times g = \text{....... Newton/meter}$$

Result: The restoring force per unit extension of the spring = ... Newton/meter

The mass of the spring = ... gm

Sources of errors and precautions

1. The axis of the spiral spring must be vertical.
2. The scale should be set up vertically and should be arranged to give almost the maximum extension allowed.
3. The pointer should move freely over the scale and should be just not in contact with it.
4. The spiral spring should not be stretched beyond elastic limit.
5. The load in the scale-pan should be placed gently and should be increased in equal steps.
6. While calculating the mean extension of the spring for a certain load, successive difference between consecutive readings of the pointer on the scale should not be taken.

Procedure *(dynamical experiment)*

1. Load the pan. Displace the pan vertically downward through a small distance and release it. The spring performs simple harmonic oscillations.

2. With the help of stop watch, note down the time of a number of oscillation (say 20 or 30). Now divide the total time by the number of oscillations to find the time period (time for one oscillation) T_1.
3. Increase the load in the pan to M_2. As described above, find the time period T_2 for this load.
4. Repeat the experiment with different values of load.
5. Plot a graph between T^2 and M. The slope is $\frac{4\pi^2}{k}$ and the intercept on the negative side of the x-axis is m.

Observations: Measurement of periods T_1 and T_2 for the loads M_1 and M_2. Least count of stop-watch = sec.

S.No.	Load in the pan		No. of Oscillations	Time taken with load		Periods		K Newton/sec	m
	M_1	M_2		M_1	M_2	T1	T_2		
	gms	gms		min sec	min sec				
							Mean		

Calculations: Restoring force per unit extension of the spring

$$k = \frac{4\pi^2 (M_1 - M_2)}{T_1^2 - T_2^2}$$

$$= \text{..... Newton/metre}$$

Mass of the spiral spring m' $= 3\left[\dfrac{M_1T_2^2 - M_2T_1^2}{T_1^2 - T_2^2}\right]$

$$= \text{.... gm}$$

Result: The restoring force per unit extension of the spring as determined by dynamical experiment = ... Newton/m

The mass of spring = ... gm

Sources of error and precautions

1. The spiral spring should oscillate vertically.
2. The amplitude of oscillation should be small.

3. As in the expression for k and m' the periods T_1 and T_2 occur raised to the second power, they should be accurately determined by timing a large number of oscillations correct upto the value measurable with the stop-watch.

3.28 OBJECT

To study the oscillations of a rubber band and a spring.

Apparatus used: Rubber bands (cycle tube), a pan, mounting arrangement, weight of 50 gm, stop watch and spring.

Formula used: (1) For experimental verification of formula

$$\frac{T_1}{T_2} = \sqrt{\left(\frac{m_1 g}{m_2 g}\right)} \times \sqrt{\frac{k_{x'_0}}{k_{x_0}}}$$

where T_1 = Time period of a rubber band when subjected to a load $m_1 g$.

T_2 = Time period of the same rubber when subjected to a load $m_2 g$.

k_{x_0} = force constant of rubber band corresponding to equilibrium extension x_0.

$k_{x'_0}$ = force constant of rubber band corresponding to equilibrium extension x'_0.

Here x_0 and x'_0 are the equilibrium extensions corresponding to loads $m_1 g$ and $m_2 g$.

(2) The entire potential energy U (joule) of the system is given by

$$U = U_b - mg \cdot x$$

where U_b = potential energy of the rubber band or springs.

x = displacement from the equilibrium position due to a load mg.

$-mg\,x$ = gravitational energy of mass m which is commonly taken as negative.

Procedure:

1. Set up the experimental arrangement as shown in figure 3.16 in such a way that when a load is subjected to the rubber band, the pointer moves freely on metre scale. Remove the load and note down the pointer's reading on metre scale when rubber band is stationary.
2. Place a weight of 0.05 kg on the pan. Now the rubber band is stretched. Note down the pointer reading on the meter scale.
3. Continue the process (2) of loading the rubber band in steps of 0.05 kg and noting the extension within the elastic limit.
4. The reading of the pointer is also recorded by removing the weights in steps. If the previous readings are almost repeated then the elastic limit has not exceeded. For a particular weight, the mean of the corresponding readings gives the extension for that weight.
5. Again place 0.05 kg in the pan and wait till the pointer is stationary. Now slightly pull down the pan and release it. The pan oscillates vertically with amplitude decreasing pretty quickly. Record the time of few oscillations with the help of sensitive stop watch. Calculate the time period. Repeat the experiment for other loads to obtain the corresponding time period.
6. Draw a graph between load and corresponding extension. The graph is shown in figure 3.17. Take different points on the curve and draw tangents. Obtain the values of Δm and

Δx for different tangents. Calculate the force constant using the following formula.

$$k_{x_0} = g\left(\frac{\Delta m}{\Delta x}\right)_{x_0}$$

Record the extensions from graph and corresponding force constants in the table.

7. Calculate the time periods by using the formula

$$T_1 = 2\pi\sqrt{\frac{m_1}{k_{x_0}}}$$

and $$T_2 = 2\pi\sqrt{\frac{m_2}{k_{x'_0}}}$$

Compare the experimental time periods with calculated time periods.

8. From load extension graph, consider the area enclosed between the curve and the extension axis for different load increasing in regular steps. The areas are shown in figure 3.18. The area gives U_b corresponding to a particular extension.

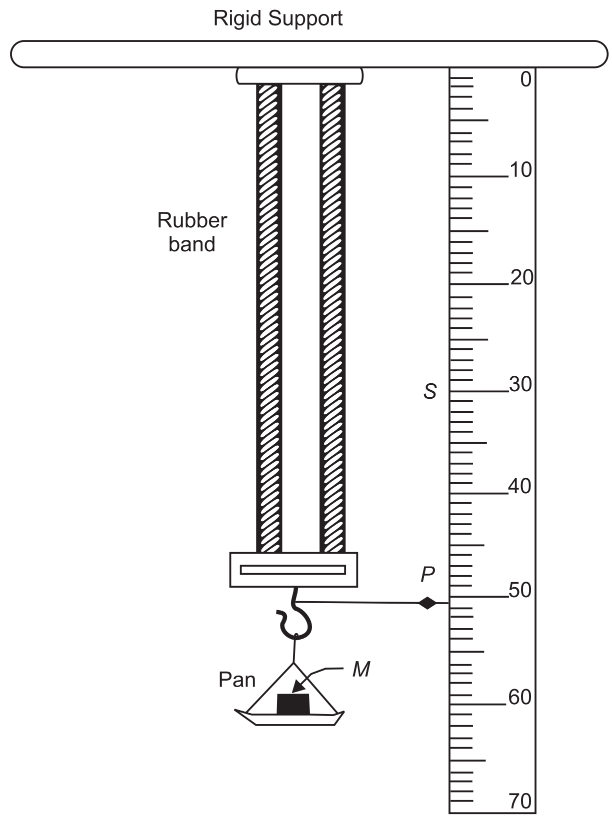


Fig. 3.16

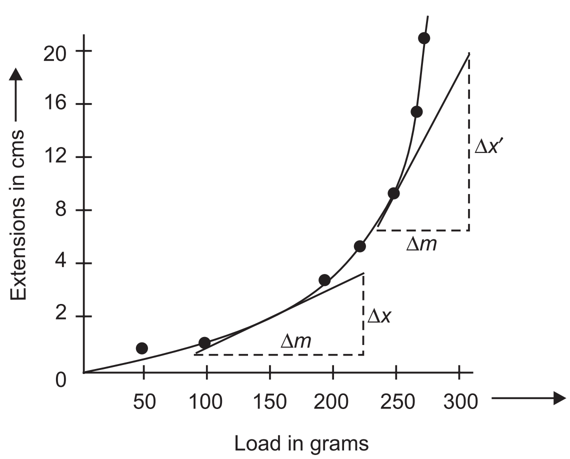


Fig. 3.17

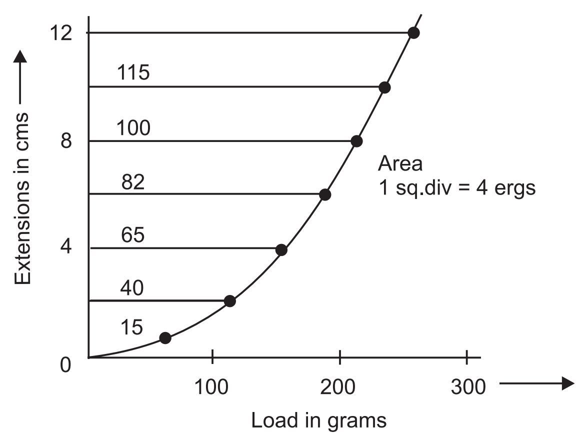


Fig. 3.18

9. Calculate U_m for mass = 100 g and obtain the value of U by the formula

$$= U_b + U_m$$

10. Draw a graph in extension and the corresponding energies i.e. U_b, U_m and U. The graph is shown in Fig. 3.19.
11. Same procedure can be adopted in case of a spring.

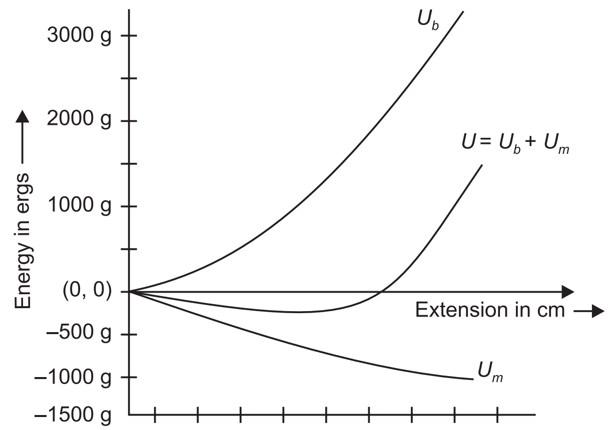


Fig. 3.19

Observations: (I) For load extension graph

S. No.	*Mass suspended in gm*	*Reading of pointer with load*		*Mean (a + b)/2 (meter)*	*Extension in rubber band (meter)*
		Increasing (a) meter	*Decreasing (b) meter*		
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					

Original length of the rubber band = ... cm

(II) For oscillations of the band

S. No.	*Mass Suspended in gm*	*No. of oscillations*	*Time sec*	*Time period (observed) from graphs*	*Eq. Ex. tension*	*k from graph*	*Period (Cal.)*
1.							
2.							
3.							
4.							

Calculations: From Graph

$$k_{x_0} = g\left(\frac{\Delta m}{\Delta x}\right)_{x_0} = ...$$

$$k_{x'_0} = g\left(\frac{\Delta m}{\Delta x'}\right)_{x'_0} = ...$$

$$T_1 = 2\pi\sqrt{\frac{m_1}{k_{x_0}}} = ...\text{ second}$$

$$T_2 = 2\pi\sqrt{\frac{m_2}{k_{x'_0}}} = \text{second.}$$

Results:

1. The force constant of rubber band is a function of extension *a* in elastic limit.
   If the same experiment is performed with spring, then it is observed that the force constant is independent of extension *a* within elastic limit.
2. From table (II), it is observed that the calculated time periods are the same as experimentally observed time periods.
3. U_b, U_m and U versus extension are drawn in the graphs.

Sources of error and precautions:

1. The rubber band should not be loaded beyond 8% of the load required for exceeding the elastic limit.
2. Time period should be recorded with sensitive stop watch.
3. The experiment should also be performed by decreasing loads.
4. The experiment should be performed with a number of rubber bands.
5. Amplitude of oscillations should be small.
6. For graphs, smooth waves should be drawn.

3.29 OBJECT

To determine Young's Modulus, Modulus of rigidity and Poisson's ratio of the material of a given wire by Searle's dynamical method.

Apparatus used: Two identical bars, given wire, stop watch, screw-gauge, vernier callipers, balance, candle and match box.

Formula used: The Young's Modulus (Y), modulus of rigidity (η) and Poisson's ratio (σ) are given by the formula,

$$Y = \frac{8\pi I l}{T_1^2 r^4}\text{ Newton/meter}^2$$

$$\eta = \frac{8\pi I l}{T_2^2 r^4}\text{ Newton/meter}^2$$

$$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

Description of the apparatus: Two identical rods AB and CD of square or circular cross section connected together at their middle points by the specimen wire, are suspended by two silk fibres from a rigid support such that the plane passing through these rods and wire is horizontal as shown in figure 3.20.

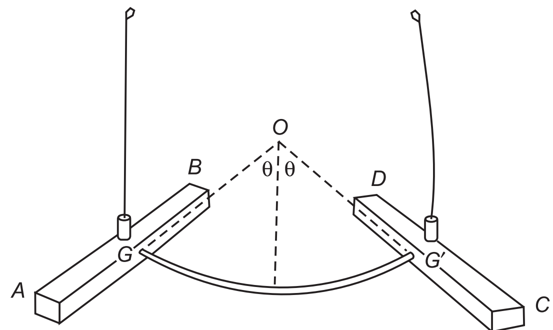


Fig. 3.20

Theory: Two equal inertia bars AB and CD of square section are joined at their centres by a fairly short and moderately thin wire GG' of the material whose elastic constants are to be determined, and the system is suspended by two parallel torsionless threads, so that in the equilibrium position the bars may be parallel to each other with the plane $ABDC$ horizontal. If the two bars be turned through equal angles in opposite directions and be then set free, the bars will execute flexural vibrations in a horizontal plane with the same period about their supporting threads.

When the amplitude of vibration is small, the wire is only slightly bent and the distance GG' between the ends of the wire measured along the straight line will never differ perceptibly from the length of the wire so that the distance between the lower ends of the supporting threads remains practically constant and hence the threads remain vertical during the oscillations of the bars, and there is thus no horizontal component of tensions in the threads acting on the wire.

The mass of the wire is negligible compared with that of the bars so that the motion of G and G' at right angles to GG' may be neglected. Further, since the horizontal displacement of G and G' are very small compared with the lengths of the supporting threads, the vertical motion of G and G' is also negligible. The centres of gravity of the bars, therefore, remain at rest, and hence the action of the wire on either bar and vice versa is simply a couple which, by symmetry must have a vertical axis. The moment of this couple called the "bending moment" is the same at every point of the wire and thus the neutral filament of the wire is bent into a circular arc.

If ρ is the radius of the arc, Y the Young's modulus for the material of the wire and I the geometrical moment of inertia of the area of cross-section of the wire about an axis through the centroid of the area and perpendicular to the plane of bending, the bending moment is from equation $G = \dfrac{YI}{\rho}$. If l is the length of the wire and θ the angle turned through by either bar, $\rho = \dfrac{l}{2\theta}$ and $G = \dfrac{2YI\theta}{l}$; and if $\dfrac{d^2\theta}{dt^2}$ is the angular acceleration of each bar towards its equilibrium position and K the moment of inertia of the bar about a vertical axis through its C.G., the torque due to inertial reaction is $K\dfrac{d^2\theta}{dt^2}$. Hence equating the sum of these two torques to zero, we get, from Newton's third law, the equation

$$K\frac{d^2\theta}{dt^2}+\frac{2YI\theta}{l} = 0$$

or
$$\frac{d^2\theta}{dt^2}+2\frac{YI\theta}{Kl} = 0$$

The motion of the bars is, therefore, simple harmonic and hence the period of the flexural vibrations is given by

$$T_1 = 2\pi\sqrt{\frac{Kl}{2YI}}$$

whence
$$y = \frac{2\pi^2 Kl}{T_1^2 I}$$

If the radius of the wire be r, $I = \frac{1}{4}\pi r^4$ and hence

$$y = \frac{8\pi Kl}{T_1^2 r^4} \qquad ...(1)$$

If the length and breadth (horizontal) of the bar be a and b respectively,

$$K = M\left(\frac{a^2+b^2}{12}\right)$$

where M = mass of the bar.

Now the suspensions of the bars are removed and one of the bars is fixed horizontally on a suitable support, while the other is suspended from a vertical wire. If the wire is twisted through an angle and the bar allowed to execute torsional oscillations, the period of oscillations is given by

$$T_2 = 2\pi\sqrt{\frac{K}{C}}$$

where C is the restoring couple per unit radian twist due to torsional reaction of the wire and is equal to $\pi\eta\, r^4/2l$, where η is the modulus of rigidity for the material of the wire. Thus

$$C = \frac{\pi\eta r^4}{2l} = \frac{4\pi^2 K}{T_2^2}$$

whence
$$\eta = \frac{8\pi Kl}{T_2^2 r^4} \qquad ...(2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{Y}{\eta} = \frac{T_2^2}{T_1^2}$$

Now $Y = 2\eta\ (1 + \sigma)$, where σ is the Poisson's ratio. Hence

$$\sigma = \frac{Y}{2\eta} - 1$$

or

$$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

Procedure:

1. Weigh both the bars and find the Mass M of each bar.
2. The breadth 'b' of the cross bar is measured with the help of vernier callipers. If the rod is of circular cross-section then measure its diameter D with vernier callipers.
3. Measure the length L of the bar with an ordinary meter scale.
4. Attach the experimental wire to the middle points of the bar and suspend the bars from a rigid support with the help of equal threads such that the system is in a horizontal plane Fig. 3.21(*a*).
5. Bring the two bars close together (through a small angle) with the help of a small loop of the thread as shown in Fig. 3.21(*b*).
6. Burn the thread. Note the time period T_1 in this case.
7. Clamp one bar rigidly in a horizontal position so that the other hangs by the wire Fig. 3.21(*c*). Rotate the free bar through a small angle and note the time period T_2 for this case also.

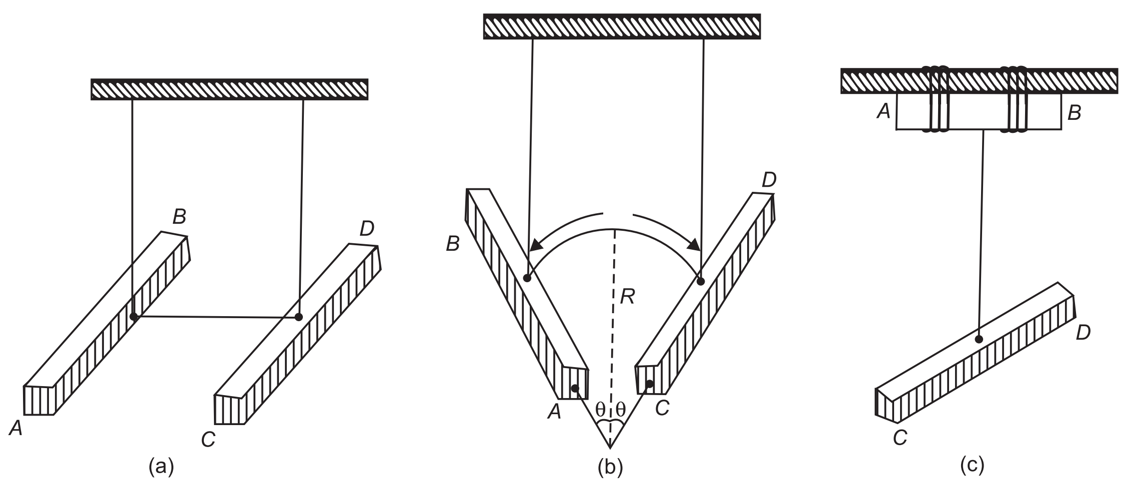


Fig. 3.21

8. Measure the length l of the wire between the two bars with meter scale.
9. Measure the diameter of the experimental wire at a large number of points in a mutually perpendicular directions by a screw gauge. Find r.

Observations: (A) Table for the determination of T_1 and T_2.

Least count of the stop-watch = ... secs.

S.No.	No. of oscillations (n)	Time		T_1	Time period $T_1 = (a/n)$ sec	Mean T_1 secs	Time		T_2	Time period $T_2 = (b/n)$ secs	Mean T_2 secs
		Min	Secs	Total secs (a)			Min	Secs	Total secs (b)		
1.	10										
2.	15										
3.	20										
4.	25										
5.	30										
6.	35										

(B) Mass of either of the *AB* or *CD* rod = ... gms = ... kg.
(C) Length of the either bar (*L*) = ... cms.
(D) Table for the measurement of breadth of the given bar.

$$\text{Least count of vernier callipers} = \frac{\text{Value of one div. of main scale in cm}}{\text{Total no. of divisions on vernier scale}} = \text{...... cm.}$$

Zero error of vernier callipers = ± ...... cms.

S. No.	Reading along any direction (→)			Reading along a perpendicular direction (↑)			Uncorrected breadth $b = \frac{(x+y)}{2}$ cm	Mean corrected breadth b cm
	M.S.	V.S.	Total x–cm	M.S.	V.S.	Total y–cm		
1.								
2.								
3.								
4.								
5.								
6.								

b = ... cm = ... meter

If the bars are of circular cross-section then the above table may be used to determine the diameter *D* of the rod.
(E) Length (*l*) of wire = cms.
(F) Table for the measurement of diameter of the given wire.

$$\text{Least count of screw gauge} = \frac{\text{Pitch}}{\text{total no. of divisions on circular scale}} = \text{...... cm.}$$

Zero error of screw gauge = ± ...... cms.

S. No.	Reading along any Direction (→)			Reading along a perpendicular direction (↑)			Uncorrected diameter (X + Y)/2 cm	Mean uncorrected diameter cm	Mean corrected diameter (d) cm	Mean radius r = d/2 cm
	M.S. reading	V.S. reading	Total X–cm	M.S. reading	V.S. reading	Total Y–cm				
1.										
2.										
3.										
4.										
5.										
6.										

r = ... cms = ... meter

N.B.: Record the mass and dimensions of the second inertia bar also, if the two bars are not exactly identical.

Calculations:

$$I = \frac{M(L^2 + b^2)}{12} = \text{......} \text{ kg} \times m^2 \text{ (for square cross-section bar)}$$

$$I = M\left(\frac{L^2}{12} + \frac{d^2}{16}\right) = \text{......} \text{ kg} \times m^2 \text{ (for circular bar)}$$

(*i*) $Y = \dfrac{8\pi Il}{T_1^2 r^4}$ = ... Newton/meter^2

(*ii*) $\eta = \dfrac{8\pi Il}{T_2^2 r^4}$ = ... Newton/meter^2

(*iii*) $\sigma = \dfrac{T_2^2}{2T_1^2} - 1$ = ...

Results: The values of elastic constants for the material of the wire are

Y = ... Newton/meter^2

η = ... Newton/meter^2

and σ = ...

Standards Results:

Y = ... Newton/meter^2

η = ... Newton/meter^2

and σ = ...

Percentage errors:

Y = ... %

η = ... %

and σ = ... %

Precautions and sources of error:

1. Bars should oscillate in a horizontal plane.
2. The amplitude of oscillations should be small.
3. The two bars should be identical.

4. Length of the two threads should be same.
5. Radius of wire should be measured very accurately.

Theoretical error:

$$Y = \frac{8\pi Il}{T_1^2 r^4} = \frac{8\pi l}{T_1^2 \left(\frac{d}{2}\right)^4} \times \frac{M\left(L^2 + b^2\right)}{12}$$

Taking log and differentiating

$$\frac{\delta Y}{Y} = \frac{\delta l}{l} + \frac{\delta M}{M} + \frac{2L\,\delta L}{\left(\frac{L^2 + b^2}{12}\right)} + \frac{2b\,\delta b}{\left(\frac{L^2 + b^2}{12}\right)} + \frac{2\delta T_1}{T_1} + \frac{4\delta d}{d}$$

Maximum possible error = ... %
Similarly find it for η and σ.

3.30 OBJECT

To determine the value of the modulus of rigidity of the material of a given wire by a dynamical method using Maxwell's needle.

Apparatus used: Maxwell's needle, screw gauge, given wire, meter scale, stop watch, physical balance and weight box.

Formula used: The modulus of rigidity η of the material of the wire is given by

$$\eta = \frac{2\pi l\,(M_S - M_H)\,L^2}{r^4\,(T_2^2 - T_1^2)} \text{ Newton/meter}^2$$

where l = length of the experimental wire
L = length of the brass tube
r = radius of the wire
M_S = mass of each of the solid cylinder
M_H = mass of each of the hollow cylinder
T_1 = time period when solid cylinders are placed in the middle
T_2 = time period when hollow cylinders are placed in the middle

Description of apparatus: Maxwell's needle is shown in figure 3.22. It consists of a hollow cylindrical brass tube of length L, suspended by a wire whose modulus of rigidity is to be determined. The tube is open at both ends. The hollow tube is fitted with four brass cylinders, two solids SS and two hollow HH, each having a length $\frac{L}{4}$ and same radii. These cylinders are inserted in the hollow tube symmetrically so that either solid cylinders SS are inside and hollow HH outside or hollow one inside and solid cylinder outside. A mirror M is attached to the wire for counting vibrations with lamp and scale arrangement.

Theory: Let the two hollow cylinders be placed in the middle and the solid ones at the two ends of the tube and let the combination be slightly rotated in a horizontal plane and then

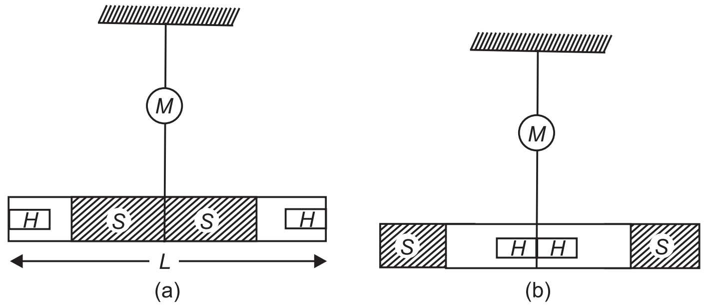


Fig. 3.22

released. The body will then execute S.H.M., about the wire as the axis and the period of oscillation is given by

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}} \qquad ...(1)$$

where I_1 is the moment of inertia of combination about the wire as the axis and C the restoring couple per unit twist due to torsional reaction.

Now let the positions of the hollow and solid cylinders be interchanged so that the solid cylinder are now in the middle. Then, if I_2 is the moment of inertia of the new combination about the axis of rotation, the new period of oscillation is given by

$$T_2 = 2\pi\sqrt{\frac{I_2}{C}} \qquad ...(2)$$

Squaring equation (1) and (2) and subtracting eqn. (2) from eqn. (1), we get

$$T_1^2 - T_2^2 = \frac{4\pi^2}{C}(I_1 - I_2)$$

whence $$C = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2}$$

But from eq. $C = \dfrac{\pi\eta r^4}{2l}$ where r is the radius and l the length of the wire whose modulus of rigidity is η.

Hence $$\frac{\pi\eta r^4}{2l} = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2}$$

or $$\eta = \frac{8\pi l (I_1 - I_2)}{r^4 (T_1^2 - T_2^2)} \qquad ...(3)$$

Now let m_1 and m_2 be the masses of each of the hollow and the solid cylinders respectively and I_0, I' and I'' be the moments of inertia of the hollow tube, the hollow cylinder and the solid cylinder respectively about a vertical axis passing through their middle points. Then, if L is the length of the hollow tube,

$$I_1 = I_0 + 2I' + 2m_1 \left(\frac{L}{8}\right)^2 + 2I'' + 2m_2 \left(\frac{3L}{8}\right)^2$$

and
$$I_2 = I_0 + 2I'' + 2m_2 \left(\frac{L}{8}\right)^2 + 2I' + 2m_1 \left(\frac{3L}{8}\right)^2$$

Hence
$$I_1 - I_2 = 2m_2 \left[\left(\frac{3L}{8}\right)^2 - \left(\frac{L}{8}\right)^2\right] - 2m_1 \left[\left(\frac{3L}{8}\right)^2 - \left(\frac{L}{8}\right)^2\right] = (m_2 - m_1)\,\frac{L^2}{4}$$

Putting this value of $(I_1 - I_2)$ in equation (3) we get

$$\boxed{\eta = \frac{2\pi l\,(m_2 - m_1)\,L^2}{r^4\,(T_1^2 - T_2^2)}}$$

This expression can be used to find the value of modulus of rigidity η of the material of the suspension wire.

Procedure:

1. Suspend the Maxwell's needle from the lower end of a thin and long wire of experimental material and fix the upper end to a rigid support.
2. By slightly rotating the Maxwell's needle about the wire in the horizontal plane, it is allowed to perform torsional oscillations. Keep the solid cylinders inside. Start the stop watch and simultaneously count the number of oscillations. In this way determine the time period of 10 oscillations. Similarly, obtain time periods for 20, 30, 40 and 50 oscillations and evaluate the mean value of time period T_1.
3. Interchange the positions of the cylinders (hollow cylinders inside) and find out the value of the time period T_2 for the system in this case. You will find that $T_2 > T_1$.
4. Measure the diameter of the wire at a large number of points in two mutually perpendicular directions at each point. Measure also the length l of the wire.
5. Using a meter scale, measure length L of the tube. Determine values of $(M_S - M_H)$ with the help of a physical balance.

Observations: (I) Table for the determination of T_1 and T_2.
Least count of the stop watch = ... secs.

S. No.	No. of ocillations	Solid cylinders on the inner side					Solid cylinder on the outer side				
		Time taken			Time period T_1 secs	Mean T_1 secs	Time taken			Time period T_2 secs	Mean T_2 secs
		Min	Secs	Total secs			Min	Secs	Total secs		
1.											
2.											
3.											
4.											
5.											
6.											

(II) Table for the determination of radius 'r' of the wire

$$\text{Least count of screw gauge} = \frac{\text{Pitch}}{\text{Total no. of divisions on circular scale}} = \text{...... cm}$$

Zero error of screw gauge = ± ...... cm

S. No.	*Reading along any direction (→)*			*Reading along a perpendicular direction (↑)*			*Uncorrected diameter (X + Y)/2 cm*	*Mean Uncorrected diameter cms*	*Mean corrected diameter (d) cms*	*Mean radius r = (d/2) cm*
	M.S.	*V.S.*	*Total X–cms*	*M.S.*	*V.S.*	*Total Y–cms*				

r = ... cm = ... meter

(III) Mean mass of a hollow cylinder M_H = ... kg.
(IV) Mean mass of solid cylinder M_S = ... kg.
(V) Length of the Maxwell's needle L = ... meter
(VI) Length of the wire l = ... meter

Calculations: Modulus of rigidity η is calculated by using the following formula:

$$\eta = \frac{2\pi l\,(M_S - M_H)\,L^2}{r^4\,(T_1^2 - T_2^2)} \text{ Newton/meter}^2$$

Result: The modulus of rigidity of the material of the wire (.....) as found experimentally
= ...... Newton/meter^2

Standard value: Standard Value of η for ...... = ... Newton/meter^2
Percentage error = ... %

Precautions and sources of error:

1. The two sets of cylinders should be exactly identical and the hollow tube should be clamped exactly in the middle.
2. The Maxwell's needle should always remain horizontal so that the moment of inertia of the hollow tube about the axis of rotation remains unaltered throughout the whole

experiment. Hence while placing the cylinders inside the tube, no portion of them should be left projecting outside the hollow tube.

3. The motion of the Maxwell's needle should be purely rotational in a horizontal plane. All undesirable motions (up and down, or pendular) should be competely checked.
4. As in the expression for η the periods occur raised to the second power, they must be carefully measured by timing a large number of oscillations with an accurate stop-watch up to an accuracy of say, $\frac{1}{5}$ of a second.
5. The wire should not be twisted beyond elastic limit otherwise the restoring couple due to torsional reaction will not be proportional to value of the twist.
6. There should be no kinks in the wire. The wire should be fairly long and thin particularly when the rigidity is high so that the restoring couple per unit twist due to torsional reaction may be small and hence the period of oscillation of the Maxwell's needle is large.
7. In the expression for η the radius occurs raised to the fourth power and is a very small quantity usually of the order of 0.1 cm. Hence the diameter must be measured very accurately. Readings should be taken at several points equally spaced along the wire and two diameters at right angles to each other should be measured at each point, care being taken not to compress the wire in taking the readings.

3.31 OBJECT

To study the variation of moment of inertia of a system with the variation in the distribution of mass and hence to verify the theorem of parallel axes.

Apparatus used: Maxwell's needle apparatus with solid cylinders only and a stop watch or a light aluminium channel about 1.5 metre in length and 5 cm in breadth fitted with a clamp at the centre to suspend it horizontally by means of wire, two similar weights, stop watch and a metre scale.

Formula used: The time period T of the torsional oscillations of the system is given by

$$T = 2\pi \sqrt{\left[\frac{I_0 + 2I_S + 2m_S x^2}{C}\right]}$$

where I_0 = moment of inertia of hollow tube or suspension system.

I_S = moment of inertia of solid cylinder or added weight about an axis passing through their centre of gravity and perpendicular to their lengths.

m_S = mass of each solid cylinder or each added weight

x = distance of each solid cylinder or each added weight from the axis of suspension.

C = torsional rigidity of suspension wire.

Squaring the above equation

$$T^2 = \frac{4\pi^2}{C}\left[I_0 + 2I_S + 2m_S x^2\right] = \frac{8\pi^2 m_S x^2}{C} + \frac{4\pi^2}{C}(I_0 + 2I_S)$$

This equation is of the form $y = mx + C$. Therefore, if a graph is plotted between T^2 and x^2, it should be a straight line.

Description of the apparatus: The main aim of this experiment is to show that how the moment of inertia varies with the distribution of mass. The basic relation for this is $I = \Sigma mx^2$.

Two equal weights are symmetrically placed on this system. By varying their positions relative to the axis of rotation, the moment of inertia of the system can be changed.

The Maxwell's needle with two solid cylinders can be used for this purpose. The two weights are symmetrically placed in the tube on either side of the axis of rotation and their positions are noted on the scale engraved by the side of the groove on the hollow X tube as shown in figure 3.24. The time period of the torsional oscillations is now determined. Now the positions of these cylinders are changed in regular steps which cause the variation in distribution of mass. By measuring the time periods in each case, the moment of inertia of the system is studied by the variation in the distribution of mass. For the successful performance of the experiment, the moment of inertia of the suspension system should be much smaller than the moment of inertia of the added weights so that a large difference in the time period may be obtained by varying the position of the added weights. For this purpose a light aluminium channel of about 1.5 metre in length and 5 cm in breadth may be used as shown in Figure (2).

Procedure:

1. As shown in Fig. 3.24(a), put the two solid cylinders symmetrically on either side in the hollow tube of Maxwell's needle and note the distance x of their centre of gravity from the axis of rotation or

   As shown in Fig. 3.24(b), put the two equal weights on the aluminium channel symmetrically on either side of axis of rotation and note the distances x of their centre of gravity from the axis of rotation.
2. Rotate the suspension system slightly in the horizontal plane and then release it gently. The system executes torsional oscillations about the suspension wire.

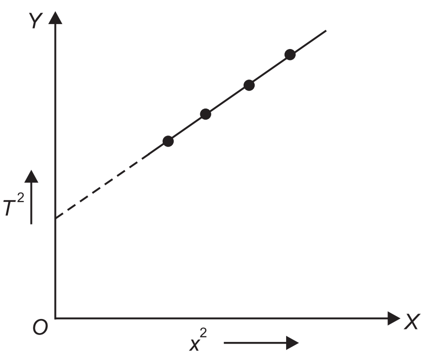


Fig. 3.23

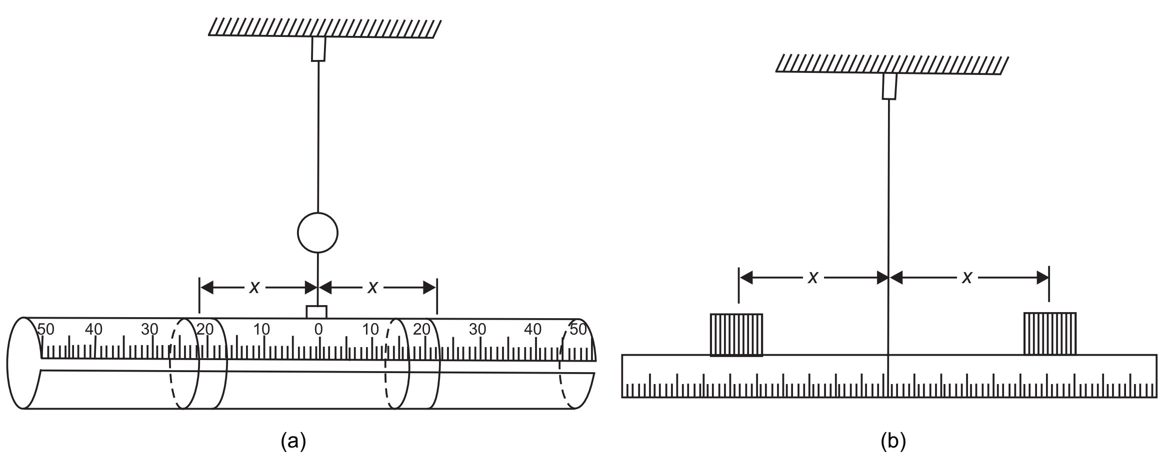


Fig. 3.24

3. Note the time taken by 25–30 oscillations with the help of a stop watch and then divide the total time by the number of oscillations to calculate the time period T.
4. Now displace both the cylinders or added weights by a known distance say 5 cm away from the axis of rotation and determine the time period as discussed above.
5. Take at least 5 or 6 such observations at various values of x by displacing the weights in regular steps of 5 cm.
6. Now plot a graph between x^2 on x-axis and corresponding values of T^2 on y-axis. The graph is shown in figure 3.23.

Observations: Table for time period T and the distance x of the weight:

S. No.	*Distance of the Cylinder of added weight from axis of rotation x meter*	*x^2 (m^2)*	*Time Period*				*T^2 sec*
			No. of oscillations n	*Time taken t sec*	*Time period T = (t/n) sec*	*Mean time period T, sec*	
1.	x_1	x_1^2	20 25 30	....	....	......	......
2.	x_2	x_2^2	20 25 30				
3.	x_3	x_3^2	20 25 30				
4.	x_4	x_4^2	20 25 30				

Result: Since the graph between T^2 and x^2 comes out to be a straight line, it verifies that the basic theorem $I = \Sigma mx^2$ from which theorem of parallel axes follows, is valid.

Sources of error & precautions:
1. The suspension wire should be free from kinks.
2. The suspension system should always be horizontal.
3. The two solid cylinders or added weights should be identical.
4. Oscillations should be purely rotational.
5. The suspension wire should not be twisted beyond elastic limits.
6. Periodic time should be noted carefully.

3.32 VIVA-VOCE

Q. 1. What do you understand by elasticity?

Ans. The property of the body by virtue of which it regains its original size and shape, when the external forces are removed, is known as elasticity.

Q. 2. What are elastic and plastic bodies?

Ans. Bodies which regain their shape or size or both completely as soon as deforming forces are removed are called perfectly elastic while if completely retain their deformed form is known as perfectly plastic.

Q. 3. What is meant by limit of elasticity?

Ans. If the stress be gradually increased, the strain too increases with it in accordance with Hooke's law until a point is reached at which the linear relationship between the two just ceases and beyond which the strain increases much more rapidly than it is warranted by the law. This value of the stress for which Hooke's law just ceases to be obeyed is called the elastic limit of the material of the body.

Q. 4. What do you mean by stress?

Ans. When a force acts on a body, internal forces opposing the former are developed. This internal force tends to restore the body back to its original form, the restoring or recovering force measured per unit area is called stress. Thus, if F be the force applied normally to an area of cross-section a then stress = F/a.

Q. 5. What do you understand by strain?

Ans. Relative change produced in size or shape or both of body which is subjected to stress is called strain. It is of three types: (*i*) Linear strain, (*ii*) volume strain and (*iii*) Shape strain or Shearing strain.

Q. 6. Explain the all three types of strain.

Ans. (*i*) When a wire is subjected to a tension or compression, the resulting deformation is a change in length and the strain is called linear strain.

(*ii*) If the pressure increments is applied to a body in such a way that the resulting deformation is in volume, without change in shape, the strain is called volume strain.

(*iii*) When tangential stresses act on the faces of the body in such a way that the shape is changed, of course volume remaining the same, the strain is called shearing strain.

Q. 7. How do you differentiate between stress or pressure?

Ans. Though both of them are defined as force per unit area, they carry different meanings. By pressure we mean an external force which necessarily acts normal to the surface, while in stress we take the internal restoring force produced in a body due to the elastic reaction, which do not always act normal to the surface.

Q. 8. What is Hooke's law?

Ans. This law states that within elastic limit, the stress is proportional to strain i.e., stress/strain = a constant, called modulus of elasticity.

Q. 9. How many types of moduli of elasticity do you know?

Ans. There are three types of moduli of elasticity: (*i*) Young's modulus (*ii*) Bulk modulus (*iii*) Modulus of rigidity,

Q. 10. What are the units and dimensions of modulus of elasticity?

Ans. The dimensional formula for modulus of elasticity is $ML^{-1}T^{-2}$ and its units in MKS and CGS systems are newton/metre^2 and dyne/cm^2 respectively.

Q. 11. Define Young's modulus?

Ans. It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limits.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

Q. 12. Define Bulk modulus?

Ans. The ratio of normal stress to volume strain within elastic limit is called as Bulk modulus?

$$K = \frac{\text{normal stress}}{\text{volume strain}}$$

Q. 13. Define modulus of rigidity?

Ans. It is defined as the ratio of tangential stress to shearing strain within elastic limits and is denoted by η.

Q. 14. What is compressibility?

Ans. The ratio of volume strain and normal stress is called compressibility $\left[K = \frac{\text{normal stress}}{\text{volume strain}};\ K \text{ is also referred to as incompressibility of the material of the body and } \frac{1}{K} \text{ is called its compressibility}\right]$.

Q. 15. What is the effect of temperature on elastic moduli?

Ans. In general value of elastic moduli decreases with rise in temperature.

Q. 16. Do you know any material whose elasticity increases with decrease of temperature?

Ans. Rubber is such a substance.

Q. 17. Do you know any material whose elasticity is little affected by temperature?

Ans. Some nickel steel alloys *e.g.* elinvar.

Q. 18. How does the elastic limit of a metal change by drawing, hammering and annealing it?

Ans. Drawing and hammering tend to diminish the elastic limit. Annealing tends to increase the elastic limit.

Q. 19. What is the practical use of the knowledge of elastic moduli?

Ans. This enables to calculate the stress and strain that a body of given size can bear. This helps in designing of the body.

Q. 20. How do you explain the meaning of the terms (*i*) limit of proportionality, (*ii*) elastic limit, (*iii*) yield point, (*iv*) breaking stress and (*v*) tensile strength?

Ans.

(*i*) **Limit of proportionality.** Limit upto which extension of wire is proportional to the deforming force. Beyond it, Hooke's law is not obeyed.

(*ii*) **Elastic limit.** The limit beyond which the body does not regain completely its original form even after removal of deforming force. It is very close to limit of proportionality.

(*iii*) **Yield Point.** It is the point beyond which increase in the length is very large even for small increase in the load, and the wire appears to flow.

(*iv*) **Breaking stress.** The maximum stress developed in the wire just before it breaks.

(*v*) **Tensile strength.** Breaking stress for wire of unit cross-section.

Q. 21. What is elastic after effect?

Ans. Some bodies do not regain their original form instantly after the removal of the deforming force, *e.g.*, glass. The delay in recovering the original condition after the deforming force has been withdrawn is known as 'elastic after effect.'

Q. 22. Which substances are exception to above behaviour and to what use are they put?

Ans. Quartz, phosphor-bronze and silver are a few materials which are exception to above behaviour and are extensively used as suspension fibres in many instruments like electrometers, galvanometers, etc.

Q. 23. What precaution is taken in experiments to guard against this error?

Ans. We wait for some time after removal of each load so that body recovers its original conditions completely. For metals not much time is required for this recovery.

Q. 24. What do you understand by elastic fatigue?

Ans. When the wire is vibrating continuously for some days, the rate at which the vibrations die away is much greater than when the wire was fresh. The wire is said to be 'tired' or fatigued' and finds it difficult to vibrate. This is called elastic 'fatigue.'

Q. 25. What is the effect of impurity on elasticity?

Ans. The addition of impurity may increase or decrease the elasticity of material. When a little carbon is added to molten iron, steel is produced which is more elastic than pure iron. While addition of 2% of potassium to gold increases its elasticity many times.

Q. 26. What is elastic hysteresis?

Ans. When a material specimen is subjected to rapid cyclic variations of mechanical strains, the sample is not able to keep pace with the external force. The phenomenon is called elastic hysteresis.

Q. 27. What is Poisson's ratio? What are its units and dimensions?

Ans. Within the elastic limits, the ratio of the lateral strain to the logitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

It has no unit.

Q. 28. Are there any limits to Poissons ratio value?

Ans. Yes, it lies between –1 and $\frac{1}{2}$.

Q. 29. The value of $\boldsymbol{\sigma}$ is $\frac{1}{2}$ for a material? What inference can you draw about this material?

Ans. It is incompressible.

Q. 30. What is the significance of negative value of Poisson's ratio?

Ans. This means that when a stretching force is applied to a body of that material, it will produce increase in length as well as increase in the perpendicular direction. No known substance shows this behaviour.

Q. 31. Can the method used for determination of Poisson's ratio for rubber be used for glass also?

Ans. No, because in case of glass extension will be extremely small.

Q. 32. What do you mean by a beam?

Ans. A bar of uniform cross-section (circular or rectangular) whose length is much greater as compared to thickness is called a beam.

Q. 33. What are longitudinal filament?

Ans. A rectangular beam may be supposed as made up of a number of thin plane layers parallel to each other. Further, each layer may be considered to be a collection of thin fibres lying parallel to the length of beam. These fibres are called longitudinal filaments.

Q. 34. What is a neutral surface?

Ans. There is a plane in the beam in which the filaments remain unchanged in length when equal and opposite couples are applied at the ends of the beam. The plane is called a neutral plane or neutral surface.

Q. 35. In this experiment, the beam is simply bent, not extended. How is the bending of the beam related, with Young's modulus of the beam?

Ans. When the beam is depressed, it becomes curved. There is elongation of fibres of beam on convex side and contraction on concave side. The longitudinal stress and strain come in picture.

Q. 36. How will the value of *Y* change with a change in length, breadth or thickness of the beam?

Ans. Y remains the same because it is constant for a material.

Q. 37. What do you understand by geometrical moment of inertia?

Ans. Geometrical moment of inertia = $\delta A \cdot K^2$ where δA = area of cross section & K = radius of gyration.

Q. 38. What is bending moment?

Ans. The moment of balancing couple (internal couple) formed by the forces of tension and compression at a section of bent beam is called as bending moment. Bending moment = $\frac{YI_g}{R}$. Here Y = Young's modulus, I_g = geometrical moment of inertia and R = radius of curvature of arc.

Q. 39. Do all the filaments other than those lying in the neutral surface suffer equal change in length?

Ans. The extensions and compressions increase progressively as we proceed away from the axis on either side, so that they are the maximum in the uppermost and the lowermost layers of the beam respectively.

Q. 40. What do you mean by Flexural rigidity?

Ans. The quantity *YI* is called the 'flexural rigidity'. It measures the resistance of the beam to bending and is quantitatively defined as the external bending moment required to produce unit radius of curvature of the arc into which the neutral axis is bent.

Q. 41. What is a cantilever.

Ans. If the beam be fixed only at one end and loaded at the other, it is called a cantilever.

Q. 42. How does the curvature change along its length?

Ans. Its upper surface becoming slightly convex and the lower one concave.

Q. 43. In the experiment with the beam supported at ends and loaded in the middle, how is the principle of cantilever involved?

Ans. It behaves like a two cantilever.

Q. 44. Why do you measure the length between the two knife edges instead of its full length?

Ans. Before and after the knife edges it is assumed that the portion is inside the clamp so that we apply the principle of cantilever.

Q. 45. Does the weight of the beam not contribute to the depression at the centre? If yes, why have you not taken it into account?

Ans. Yes, it does. This is ignored because in the experiment we calculate the depression due to different weights. Hence the depression due to the weight of the beam is automatically cancelled.

Q. 46. What precaution do you take in placing the beam on knife edges?

Ans. It should be placed in such a way that the knife edges are perpendicular to its length. The beam should rest symmetrically on the knife edges so that equal portions project outside the knife edges. This ensures equal reaction at each knife edge as has been assumed in the theory.

Q. 47. You have kept the beam on the knife edges. Can you keep it with its breadth vertical.

Ans. In that case depression will be very small, because the breadth will now become the thickness of the beam and the depression $\delta \propto \frac{1}{a^3}$.

Q. 48. What is the practical use of this information?

Ans. It is utilized in the construction of girders and rails. Their depth is made much larger as compared to their breadth. These can, therefore, bear much load without bending too much.

Q. 49. Girders are usually of I shape. Why are they not of uniform cross-section?

Ans. In girders filament of upper half are compressed while those of lower half are extended. Since the compression and extension are maximum near the surface, the stresses on the end filaments are also maximum. Hence the top and bottom are thicker than the central portion. This saves a great deal of material without sacrificing the strength of the girder.

Q. 50. Which of the quantities '*b*' or '*d*' should be measured more accurately and why?

Ans. The depth of the bar should be measured very carefully since its magnitude is small and it occurs in the expression of 'y' in the power of three. An inaccuracy in the measurement of the depth will produce the greatest proportional error in y.

Q. 51. What type of beam will you select for your experiment?

Ans. Moderately long and fairly thin.

Q. 52. Why do you select such a beam?

Ans. For two reasons: (*i*) the depression will be large which can be measured more accurately and (*ii*) in bending of beam shearing stress is also produced in addition to the longitudinal one and it produces its own depression. This depression will be negligible only when the beam is long and thin.

Q. 53. How do you ensure that in your experiment the elastic limit is not exceeded?

Ans. The consistency in the readings of depressions both for increasing load and decreasing load indicates that in the experiment the elastic limit is not exceeded.

Q. 54. Does the weight of the bar have any effect?

Ans. The weight of the beam leads to an 'effective load' different from m. However, since the depression due to a load is calculated by subtracting the zero-load readings the weight of the bar does not affect the result.

Q. 55. How do you produce shearing in a rod or wire?

Ans. The upper end of the experimental rod is clamped while the lower end is subjected to a couple. Now each cross-section of rod is twisted about the rod. The angle of twist for any cross-section of the rod being proportional to its distance from fixed end. Thus the material particles of the rod are relatively displaced with respect to the particles in adjoining layer and the rod is sheared.

Q. 56. What is the angle of twist and how does this angle vary?

Ans. When one end of the rod is clamped and a couple is applied at other end, each circular cross section is rotated about the axis of the rod through certain angle. This angle is called the angle of shear.

Q. 57. What is the difference between the angle of twist and the angle of shear? How are these angles related?

Ans. $\phi = \frac{x\theta}{l}$, where l is the length of the rod and x being the radius of co-axial cylinder under consideration.

Q. 58. What are the values of twisting couple and restoring couple?

Ans. Twisting couple is equal to Mgd, where M = mass placed at each pan and 'd' being the diameter of cylinder.

Restoring couple = $\frac{\pi\eta r^4}{2l}\theta$.

Q. 59. What is the condition of equilibrium in 'η' experiment?

Ans. Mgd = $\frac{\pi\eta r^4}{2l}\theta$

Q. 60. What do you mean by torsional rigidity of a wire?

Ans. This is defined as restoring couple per unit radian twist *i.e.*,

$$C = \frac{\pi\eta r^4}{2l}$$

Q. 61. Do you prefer to use an apparatus provided with a cylinder of larger or smaller radius?

Ans. We shall prefer to take a cylinder of larger radius because twisting couple will be greater.

Q. 62. On what factors does the twist produced in the wire for a given twisting couple depend?

Ans. It depends upon the torsional rigidity of the wire. Smaller the rigidity, greater is the twist and vice-versa. Now $c = \frac{\pi\eta r^4}{2l}$. Hence twist will depend upon length, radius and material of the wire.

Q. 63. Why it is called a statical method?

Ans. This is called a statical method because all observations are taken when all parts of apparatus are stationary.

Q. 64. In vertical apparatus why are levelling screws provided at the base?

Ans. These screws are used to adjust the apparatus such that the experimental wire passes through the centres of the graduated scales to avoid the error due to eccentricity.

Q. 65. Why do you read both the ends of the pointer?

Ans. This eliminates the error due to any eccentricity left even after above adjustment.

Q. 66. Why is the value of η for a thinner wire slightly higher than that for a thicker wire of the same material?

Ans. The wires are drawn by squeezing the molten metal through holes (dies), hence the outer layers are necessarily tougher than the inner ones. Therefore the value of η for a thinner wire is slightly higher than for a thicker wire of the same material.

Q. 67. Why do you measure the radius of wire so accurately?

Ans. Because it occur in fourth power in the formula.

Q. 68. Will the value of modulus of rigidity be the same for thin and thick wires of the same material?

Ans. For a given material, the value should be the same. But we know that wires are made by drawing the metal through a hole. The outer surfaces become harder than inner core. Thus the rigidity of a fine wire will be greater than thick wire.

Q. 69. When the modulus of rigidity can be determined by statical method, what is the necessity of dynamical method.

Ans. The statical method is suitable for thick rods while dynamical method is suitable for thin wires.

Q. 70. What is a torsional pendulum?

Ans. A body suspended from a rigid support by means of a long and thin elastic wire is called torsional pendulum.

Q. 71. Why it is called a torsional pendulum?

Ans. As it performs torsional oscillations, hence it is called a torsional pendulum.

Q. 72. What are various relationship between elastic constants?

Ans. $Y = 2\eta(1+\sigma)$, $Y = 3k\,(1-2\sigma)$

$$\sigma = \frac{3k-2\eta}{6k+2\eta}, \quad Y = \frac{9\eta k}{\eta+3k}$$

Q. 73. What is the unit of Poisson's ratio?

Ans. It has no unit because it is a ratio.

Q. 74. When do you measure the diameter of the tube?

Ans. We measure the diameter of tube at no load position.

Q. 75. Can you not calculate the change in volume by knowing the longitudinal extension?

Ans. No, the contraction in diameter should also be known.

Q. 76. If a graph is plotted between change in volume and change in length, what kind of curve will you get?

Ans. It gives a straight line.

Q. 77. Can you use this method for determining σ for glass?

Ans. No, in case of glass the extension produced is so small that it can not be measured.

Q. 78. Which method do you suggest for glass?

Ans. Cornu's method.

Q. 79. What is a flywheel?

Ans. It is a large size heavy wheel mounted on a long axle supported on ball bearing.

Q. 80. Why the mass of a flywheel is concentrated at rim?

Ans. This increases the radius of gyration and hence the moment of inertia of the flywheel.

Q. 81. What is the practical utility of a flywheel?

Ans. It is used in stationary engines to ensure a uniform motion of the machine coupled to the engine.

Q. 82. Is flywheel used in mobile engines also?

Ans. No, it is not needed in mobile engines, because the heavy body of vehicle itself serves the purpose of flywheel.

Q. 83. How and why does the flywheel start rotating?

Ans. When a weight is hanged near the axle, it has certain amount of potential energy. After realising it, its potential energy is converted into kinetic energy of its own motion, into kinetic energy of rotating of flywheel and in overcoming the force of friction between the axle and ball bearings.

$$\therefore \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 2\pi n_1 F.$$

Q. 84. Why the flywheel continues its revolutions even after the cord has slipped off the axle.

Ans. It continues its revolutions due to its large moment of inertia.

Q. 85. Then, why does it stop after a very short time?

Ans. The energy of the flywheel is dissipated in overcoming the friction at the ball bearings.

Q. 86. What is the purpose of finding n_2?

Ans. The friction offered by the ball bearings is very small but it is not negligible. To account for the work done by weight against friction, the number of revolutions made by flywheel after the weight is detached should be found.

Q. 87. Can you use a thin wire instead of a string?

Ans. No, we cannot use a wire because metals are pliable and so when the wire unwinds itself, some amount of work will also be done in straightening the wire.

Q. 88. Why do you keep the loop slipped over the peg loose?

Ans. We keep the loop slipped over the peg loose so that it may get detached as soon as the string unwinds itself and does not rewind in opposite direction.

Q. 89. What is the harm if the thread overlaps in winding round the axle.

Ans. In this case the couple acting on the wheel will not be uniform and hence the flywheel will not rotate with uniform acceleration.

Q. 90. What is a spiral spring?

Ans. A long metallic wire in the form of a regular helix of given radius is called a spiral spring.

Q. 91. What types of springs do you know?

Ans. There are two types of springs: (*i*) Flat and (*ii*) non-flat. When the plane of the wire is perpendicular to the axis of the cylinder, it is flat and when the plane of wire makes certain angle with the axis of cylinder, it is non flat.

Q. 92. What is the effective mass of a spring?

Ans. In calculations total energy of the spring, we have a quantity $\left(M+\frac{m}{3}\right)$ where M is the mass suspended and m, the mass of the spring. The factor $\left(\frac{m}{3}\right)$ is called the effective mass of the spring.

Q. 93. What do you understand by restoring force per unit extension of a spiral spring?

Ans. This is defined as the elastic reaction produced in the spring per unit extension which tends to restore it back to its initial conditions.

Q. 94. How does the restoring force change with length and radius of spiral spring?

Ans. This is inversely proportional to the total length of wire and inversely proportional to the square of radius of coil.

Q. 95. How the knowledge of restoring force per unit extension is of practical value?

Ans. By the knowledge of restoring force per unit extension, we can calculate the correct mass and size of the spring when it is subjected to a particular force.

Q. 96. How are Y and η involved in this method?

Ans. First of all the wire is placed horizontally between two bars. When the bars are allowed to vibrate, the experimental wire bent into an arc. Thus the outer filaments, are elongated while inner ones are contracted. In this way, Y comes into play. Secondly, when one bar oscillates like a torsional pendulum, the experimental wire is twisted and η comes into play.

Q. 97. Is the nature of vibrations the same in the second part of the experiment as in the first part?

Ans. No, in the second case, the vibrations are torsional vibrations.

Q. 98. Should the moment of inertia of the two bars be exactly equal?

Ans. Yes, if the two bars are of different moment of inertia, then their mean value should be used.

Q. 99. Do you prefer to use heavier or lighter bars in this experiment?

Ans. We shall prefer heavier bars because they have large moment of inertia. This increases the time period.

Q. 100. Can you not use thin wires in place of threads?

Ans. No, because during oscillations of two bars, the wires will also be twisted and their torsional reaction will affect the result.

Q. 101. From which place to which place do you measure the length of wire and why?

Ans. We measure the length of the wire from centre of gravity of one bar to the centre of gravity of the other because it is length of the wire which is bent or twisted.

Q. 102. Is there any restriction on the amplitude of vibration in both part of experiments?

Ans. When the two rods vibrate together, the amplitude of vibration should be small so that the supporting threads remain vertical and there is no horizontal component of tension in the threads. In case of torsional oscillations there is no restriction on the amplitude of oscillations but the wires should not be twisted beyond elastic limits.

Q. 103. Why do the bars begin oscillating when the thread tied to them is burnt? Do they perform S.H.M.?

Ans. When the wire is bent into circular arc and the thread is burnt, the wire tries to come back to its original position due to elastic reaction. In doing so it acquires kinetic energy. Due to this energy the wire overshoots the initial position and becomes curved in another direction. The process is repeated and the bar begins to oscillate. Yes, the rod performs simple harmonic motion.

Q. 104. What do you mean by Poisson's ratio?

Ans. Within the elastic limits, the ratio of the lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Q. 105. What is the principle of Maxwell needle experiment?

Ans. When the Maxwell's needle is given a small angular displacement and released, it starts oscillating. The periodic time of the needle is related with the elasticity of the wire.

Q. 106. On what factors does the periodic time depend?

Ans. It depends upon (*i*) moment of inertia of the needle about wire, and (*ii*) length, radius and material of wire.

Q. 107. What would be the change in periodic time when (*i*) *I* is doubled, (*ii*) *l* is doubled and (*iii*) *r* is doubled.

Ans. We know that $T = 2\pi\sqrt{\frac{I}{C}}$ where $C = \frac{\pi\eta r^4}{2l}$, hence

(*i*) T increases to $T\sqrt{2}$

(*ii*) T increases to $T\sqrt{2}$

(*iii*) T decreases to $T/4$.

Q. 108. Does the time period change, by changing the position of hollow and solid cylinders? Why.

Ans. Yes, the distribution of mass is changed about the axis of rotation and hence the moment of inertia of the needle is changed.

Q. 109. When you change neither the mass of the needle nor the axis of rotation, why does the period change?

Ans. We known that time period depends upon the moment of inertia of oscillating body while the moment of inertia depends upon the distribution of mass besides the total mass and axis of rotation.

Q. 110. Will you prefer to use a long and thin wire or a short and thin wire?

Ans. We shall prefer to use a long and thin wire so that C will be small and periodic time will be greater.

Q. 111. Will the value of modulus of rigidity be the same for thin and thick wires of the same material?

Ans. For a given material, the value should be the same. But we know that wires are made by drawing the metal through a hole. The outer surface becomes harder than inner core. Thus the rigidity of a fine wire will be greater than thick wire.

Q. 112. When the modulus of rigidity can be determined by statical method, what is the necessity of dynamical method?

Ans. The statical method is suitable for thick rods while dynamical method is suitable for thin wires.

Q. 113. How the moment of inertia of a system can be changed?

Ans. The moment of inertia of a system can be changed by varying the distribution of mass.

Q. 114. Which apparatus you are using for this purpose?

Ans. We are using Maxwell's needle for this purpose.

Q. 115. How do you vary the distribution of mass here?

Ans. By changing the positions of two weights symmetrically inside the tube.

Q. 116. What will be the effect on time period of the system by varying the distribution of mass?

Ans. The time period T increases as x increases.

Q. 117. Can you verify the theorem of parallel axes with this experiment?

Ans. Yes, if a graph is plotted between T^2 and x^2, it comes out to be a straight line. This verifies the theorem of parallel axes.

Q. 118. What type of motion is performed by the needle?

Ans. The needle performs the simple harmonic motion.

Q. 119. Should the amplitude of vibration be small here?

Ans. It is not necessary because the couple due to torsional reaction is proportional to the angle of twist. Of course, the amplitude should not be so large that the elastic limit is crossed as the wire is thin and long.

EXERCISE

Q. 1. What do you mean by shearing of body?

Q. 2. How many types of stresses do you know? How will you produce these stresses?

Q. 3. How do you measure the depression at the middle point of the beam?

Q. 4. Why do you load and unload the beam in small steps and gently?

Q. 5. What is meant by modulus of rigidity?

Q. 6. What are shearing stress and strains and what are their units and dimensions?

Q. 7. What is the nature of stress in the case of 'η'.

Q. 8. How and where do you apply tangential stress in this case?

Q. 9. What is the principal of the statical experiment?

Q. 10. What is the value of the restoring couple?

Q. 11. Why do you take readings with increasing and decreasing couples?

Q. 12. What is Poisson's ratio?

Q. 13. What is the value of σ for homogeneous and isotropic materials.

Q. 14. How can you find out the Poisson's ratio for rubber.

Q. 15. Why should you wait for about 5 min, after each addition or removal of a load before taking observation.

Q. 16. Is there any limit to the load placed on the hanger.

Q. 17. Why should there be no air bubble inside the tube?

Q. 18. Why should the spiral spring be suspended exactly vertically?

Q. 19. Why should the extension of the string be small.

Q. 20. Why should the amplitude of oscillation of the spring be small?

Q. 21. Do you get the same value of restoring force per unit extension of the spring from the statical and dynamical experiments. If not, why?

Q. 22. How can you determine the mass of the spring?
Q. 23. What is Maxwell's needle?
Q. 24. How do you measure η with Maxwell's needle?
Q. 25. Which is better: an ordinary pointer or a telescope and scale method of observing oscillations?
Q. 26. Can you improve the pointer method to be nearly as good?
Q. 27. What is parallax and how can you best remove it?
Q. 28. Is it necessary that the oscillations should have small amplitude, if not, why?
Q. 29. Why do you measure the diameter so accurately?
Q. 30. Why should the needle be kept horizontal throughout the experiment?
Q. 31. Why does the needle oscillate when released after twisting the wire?
Q. 32. Which is better, statical or dynamical method?
Q. 33. Do you get the same value for η from the statical and dynamical methods? If not, why?

4

Acceleration Due to Gravity

4.1 ACCELERATION DUE TO GRAVITY

Earth attracts all bodies on or near its surface towards its centre due to its gravitational field. On account of this force of attraction bodies falling freely experience a constant acceleration towards earth's centre. This acceleration does not depend upon the nature, shape, size and mass of the body. It is called the acceleration due to gravity and is represented by 'g'. It is also defined as the force of attraction exerted by the earth on a body of unit mass. Its value changes from place to place on the earth's surface, being minimum at equator and maximum at poles. As we go above or below the earth's surface its value decreases. Its average value is taken as 9.81 $m.s^{-2}$ at sea level and 45° latitude. In M.K.S. system its unit is ms^{-2} and its dimensions are $[LT^{-2}]$.

4.2 PERIODIC MOTION

A motion which repeats itself over and over again after regularly recurring intervals of time, called its time-period, is referred to as a periodic motion.

If a particle, undergoing periodic motion, covers the same path back and forth about a mean position, it is said to be executing on oscillatory (or vibratory) motion or an oscillation (or a vibration). Such a motion is not only periodic but also bounded, *i.e.,* the displacement of the particle on either side of its mean position remains confined within a well-defined limit.

4.3 SIMPLE HARMONIC MOTION

A particle may be said to execute a simple harmonic motion if its acceleration is proportional to its displacement from its equilibrium position, or any other fixed point in its path, and is always directed towards it.

Thus, if F be the force acting on the particle and x, its displacement from its mean or equilibrium position, we have $F = -cx$ where c is a positive constant, called the force constant.

Now, in accordance with Newton's second law of motion, $F = ma$. So that, substituting $-cx$ for F and $\frac{d^2x}{dt^2}$ for acceleration a, we have

$$-cx = m\frac{d^2x}{dt^2} \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{c}{m}x = 0$$

This equation is called the differential equation of motion of a simple harmonic oscillator or a simple harmonic motion, because by solving it we can find out how the displacement of the particle depends upon time and thus know the correct nature of the motion of the particle.

To solve the equation, we may put it in the form $\dfrac{d^2x}{dt^2} = -\left(\dfrac{c}{m}\right)x$, the negative sign, as we know indicating that the acceleration is directed oppositely to displacement x.

Putting $\dfrac{c}{m} = \omega^2$, where ω is the angular velocity of the particle, the equation takes the form

$$\frac{d^2x}{dt^2} = -\omega^2 x = -\mu x \qquad ...(1)$$

where μ is a constant, equal to ω^2. Or, since $\dfrac{d^2x}{dt^2} = -\mu$ if $x = 1$, we may define μ as the acceleration per unit displacement of the particle.

Multiplying both sides of the equation by $2\dfrac{dx}{dt}$, we have

$$2\frac{dx}{dt}\frac{d^2x}{dt^2} = -\omega^2 \cdot 2x\frac{dx}{dt},$$

integrating which with respect to t, we have

$$\left(\frac{dx}{dt}\right)^2 = -\omega^2x^2 + A \qquad ...(2)$$

where A is a constant of integration.

Since at the maximum displacement (or amplitude) a of the oscillator (or the oscillation), the velocity $\dfrac{dx}{dt} = 0$, we have

$$0 = -\omega^2a^2 + A, \text{ hence, } A = \omega^2a^2$$

Substituting this value of A in relation (2), therefore, we have

$$\left(\frac{dx}{dt}\right)^2 = -\omega^2x^2 + \omega^2a^2 = \omega^2(a^2 - x^2),$$

Hence, the velocity of the particle at any instant t, is given by

$$\frac{dx}{dt} = \omega\sqrt{a^2 - x^2} \qquad ...(3)$$

Putting equation (3) as $\dfrac{dx}{\sqrt{a^2 - x^2}} = \omega\, dt$ and integrating again with respect to t, we have

$$\sin^{-1}\frac{x}{a} = \omega t + \phi$$

or

$$\boxed{x = a\sin(\omega t + \phi)} \qquad ...(4)$$

This gives the displacement of the particle at an instant t in terms of its amplitude 'a' and its total phase ($\omega t + \phi$), made up of the phase angle ωt ϕ is called the initial phase, phase constant or the epoch of the particle, usually denoted by the letter e. This initial phase or epoch arises because of our starting to count time, not from the instant that the particle is in some standard position, like its mean position or one of its extreme positions, but from the instant when it is anywhere else in between.

Thus, if we start counting time when the particle is in its mean position, *i.e.*, when $x = 0$ at $t = 0$, we have $\phi = 0$ and, therefore,

$$x = a \sin \omega t$$

And, if we start counting time when the particle is in one of its extreme positions, *i.e.*, when $x = a$ at $t = 0$, we have $a = a \sin(0 + \phi) = a \sin \phi$, *i.e.*, $\sin \phi = 1$ or $\phi = \frac{\pi}{2}$. So that, $x = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos \omega t$. Thus, a simple harmonic motion may be expressed either in terms of a sine or a cosine function. The time-period of the particle,

$$T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Since T is quite independent of both a and ϕ, it is clear that the oscillations of the particle are isochronous, *i.e.*, take the same time irrespective of the values of a and ϕ.

The number of oscillations (or vibrations) made by the particle per second is called its frequency of oscillation or, simply, its frequency, usually denoted by the letter n. Thus, frequency is the reciprocal of the time-period, *i.e.*,

$$n = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$$

Since ω is the angle described by the particle per second, it is also referred to as the angular frequency of the particle.

4.4 ENERGY OF A HARMONIC OSCILLATOR

P.E. at displacement x is given by $U = \frac{1}{2} m\omega x^2 = \frac{1}{2} m\left(\frac{c}{m}\right)x^2 = \frac{1}{2} cx^2$

The maximum value of the potential energy is thus at $x = a$ is $U = \frac{1}{2} ca^2$

K.E. of the particle at displacement $x = \frac{1}{2} m\omega^2 (a^2 - x^2) = \frac{1}{2} c\,(a^2 - x^2)$

The maximum value of K.E. is at $x = 0$ and is also equal to $= \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} ca^2$

Total energy of the particle at displacement x *i.e.*, E = K.E. + P.E.

$$= \frac{1}{2} m\omega^2 (a^2 - x^2) + \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} ca^2$$

Maximum value of K.E. = maximum value of P.E. = total energy E $= \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} ca^2$.

Average K.E. of the particle $= \frac{m\omega^2 a^2}{4} = \frac{1}{4} ca^2$

Average P.E. of the particle $= \frac{1}{4} m\omega^2 a^2 = \frac{1}{4} ca^2$ = half of the total energy.

4.5 THE SIMPLE PENDULUM

The simple pendulum is a heavy point mass suspended by a weightless inextensible and flexible string fixed to a rigid support. But these conditions defines merely an ideal simple pendulum which is difficult to realize in practice. In laboratory instead of a heavy point mass we use a heavy metallic spherical bob tied to a fine thread. The bob is taken spherical in shape because the position of its centre of gravity can be precisely defined. The length (l) of the pendulum being measured from the point of suspension to the centre of mass of the bob. In Fig. 4.1, let S be the point of suspension of the pendulum and O, the mean or equilibrium position of the bob. On taking the bob a little to one side and then gently releasing it, the pendulum starts oscillating about its mean position, as indicated by the dotted lines. At any given instant, let the displacement of the pendulum from its mean position SO into the position SA be θ. Then, the weight mg of the bob, acting vertically downwards, exerts a torque or a moment $-mgl \sin\theta$ about the point of suspension, tending to bring it back to its mean position, the negative sign of the torque indicating that it is oppositely directed to the displacement (θ).

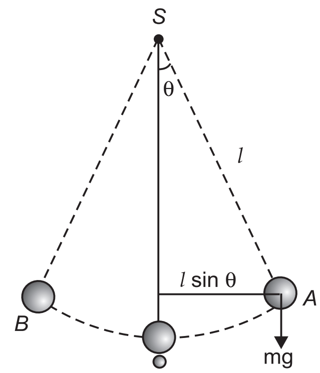


Fig. 4.1

If $\frac{d^2\theta}{dt^2}$ be the acceleration of the bob, towards O, and I, its M.I. about the point of suspension (S), the moment of the force or the torque acting on the bob is also equal to $I\frac{d^2\theta}{dt^2}$. We, therefore, have

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

Now, expanding sin θ into a power series, in accordance with Maclaurin's theorem, we have $\sin\theta = \theta - \frac{\theta^3}{!3} + \frac{\theta^5}{!5}$ .... if, therefore, θ be small, *i.e.*, if the amplitude of oscillation be small, we may neglect all other terms except the first and take sin θ = θ, so that

$$I\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta.$$

or, since M.I. of the bob (or the point mass) about the point of suspension (S) is ml^2, we have

$$\frac{d^2\theta}{dt^2} = \frac{-mgl}{ml^2}\theta = \frac{-g}{l}\theta = -\mu\theta$$

where $\frac{g}{l} = \mu$, the acceleration per unit displacement.

The acceleration of the bob is thus proportional to its angular displacement θ and is directed towards its mean position *O*. The pendulum thus executes a simple harmonic motion and its time-period is, therefore, given by

$$T = 2\pi\sqrt{\frac{1}{\mu}} = 2\pi\sqrt{\frac{1}{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

The displacement here being angular, instead of linear, it is obviously an example of an angular simple harmonic motion. It is also evident from above expression that the graph between l and T^2 will be a straight line with a slope equal to $\frac{4\pi^2}{g}$.

4.6 DRAWBACKS OF A SIMPLE PENDULUM

Though simple pendulum method is the simplest and straightforward method for determination of '*g*', it suffers from several defects:

(*i*) The conditions defining an ideal simple pendulum are never realizable in practice.
(*ii*) The oscillations in practice have a finite amplitude *i.e.,* the angle of swing is not vanishingly small.
(*iii*) The motion of the bob is not purely translational. It also possesses a rotatory motion about the point of suspension.
(*iv*) The suspension thread has a finite mass and hence a definite moment of inertia about point of suspension.
(*v*) The suspension thread is not inextensible and flexible. Hence it slackens when the limits of swing are reached. Thus effective length of the pendulum does not remain constant during the swing.
(*vi*) Finite size of the bob, yielding of the support and the damping due to air drag also need proper corrections.
(*vii*) The bob also has a relative motion with respect to the string at the extremities of its amplitude on either side.

Most of the defects are either absent or much smaller in the case of a rigid or compound pendulum.

4.7 THE COMPOUND PENDULUM

Also called a physical pendulum or a rigid pendulum, a compound pendulum is just a rigid body, of whatever shape, capable of oscillating about a horizontal axis passing through it.

The point in which the vertical plane passing through the *c.g.* of the pendulum meets the axis of rotation is called its point or centre of suspension and the distance between the point of suspension and the *c.g.* of the pendulum measures the length of the pendulum.

Thus figure shows a vertical section of a rigid body or a compound pendulum, free to rotate about a horizontal axis passing through the point or centre of suspension *S*. In its normal position of rest, its *c.g., G*, naturally lies vertically below S, the distance *S* and *G* giving the length l of the pendulum.

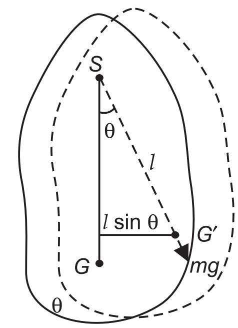


Fig. 4.2

Let the pendulum be given a small angular displacement θ into the dotted position shown, so that its *c.g.* takes up the new position G' where $SG' = l$. The weight of the pendulum, mg, acting vertically downwards at G' and its reaction at the point of suspension S constitute a couple (or a torque), tending to bring the pendulum back into its original position.

Moment of this restoring couple = $-\, mgl \sin \theta$, the negative sign indicating that the couple is oppositely directed to the displacement θ. If I be the moment of inertia of the pendulum about the axis of suspension (through S) and $\frac{d^2\theta}{dt^2}$, its angular acceleration, the couple is also equal to $I\frac{d^2\theta}{dt^2}$. So that, we have

$$I\frac{d^2\theta}{dt^2} = -\, mgl \sin \theta$$

'Again, $\sin \theta = \theta - \frac{\theta^3}{!3} + \frac{\theta^5}{!5}$......., so that, if θ be small, $\sin\theta \approx \theta$ and, therefore, $I\frac{d^2\theta}{dt^2} = -\, mgl\theta$, $\frac{d^2\theta}{dt^2} = -\left(\frac{mgl}{I}\right)\theta = -\,\mu\theta$, where $\frac{mgl}{I} = \mu$, the acceleration per unit displacement.

The pendulum thus executes a simple harmonic motion and its time-period is given by

$$T = 2\pi\sqrt{\frac{1}{\mu}} = 2\pi\sqrt{\frac{1}{\frac{(mgl)}{I}}} = 2\pi\sqrt{\frac{I}{mgl}}$$

Now, if I_0 be the moment of inertia of the pendulum about an axis through its *c.g.*, G, parallel to the axis through S, we have, from the theorem of parallel axes, $I = I_0 + ml^2$. And if k be the radius of gyration of the pendulum about this axis through G, we have $I_0 = mk^2$. So that, $I = mk^2 + ml^2 = m(k^2 + l^2)$.

$$\therefore \qquad T = 2\pi\sqrt{\frac{m(k^2 + l^2)}{mgl}} = 2\pi\sqrt{\frac{k^2 + l^2}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

Thus, the time-period of the pendulum is the same as that of a simple pendulum of length $L = \left(\frac{k^2}{l} + l\right)$ or $\frac{k^2 + l^2}{l}$. This length L is, therefore, called the length of an equivalent simple pendulum or the reduced length of the compound pendulum.

Since k^2 is always greater than zero, the length of the equivalent simple pendulum (L) is always greater than l, the length of the compound pendulum.

4.8 CENTRE OF OSCILLATION

A point O on the other side of the C.G. (G) of the pendulum in a line with SG and at a distance $\frac{k^2}{l}$ from G is called the centre of oscillation of the pendulum and a horizontal axis passing through it, parallel to the axis of suspension (through S) is called the axis of oscillation of the pendulum.

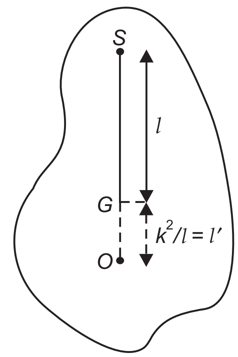


Fig. 4.3

Now, $GO = \frac{k^2}{l}$ and $SG = l$. So that, $SO = SG + GO = l + \frac{k^2}{l} = L$, the length of the equivalent simple pendulum, *i.e.*, the distance between the centres of suspension and oscillation is equal to the length of the equivalent simple pendulum or the reduced length (L) of the pendulum and we, therefore, have

$$T = 2\pi\sqrt{\frac{L}{g}}$$

4.9 INTERCHANGEABILITY OF CENTRES OF SUSPENSION AND OSCILLATION

If we put $\frac{k^2}{l} = l'$, we have $L = l + \frac{k^2}{l} = l + l'$ and, therefore,

$$T = 2\pi\sqrt{\frac{(l+l')}{g}}$$

If how we invert the pendulum, so that it oscillates about the axis of oscillation through O, its time period, T', say, is given by

$$T' = 2\pi\sqrt{\frac{(k^2+l'^2)}{l'g}}$$

Since $\frac{k^2}{l} = l'$, we have $k^2 = ll'$.

Substituting ll' for k^2 in the expression for T', therefore, we have

$$T' = 2\pi\sqrt{\frac{(ll'+l'^2)}{l'g}} = 2\pi\sqrt{\left(\frac{l+l'}{g}\right)} = T,$$

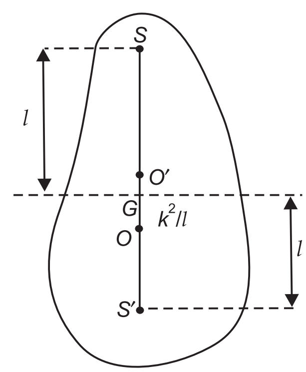


Fig. 4.4

i.e. the same as the time-period about the axis of suspension.

Thus, the centres of suspension and oscillation are interchangeable or reciprocal to each other, *i.e.*, the time-period of the pendulum is the same about either.

There are two other points on either side of G, about which the time-period of the pendulum is the same as about S and O. For, if with G as centre and radii equal to l and $\frac{k^2}{l}$

respectively, we draw two circles so as to cut SG produced in S and O' above, and at O and S' below G, as shown in figure, we have $SG = GS' = l$ and $GO' = GO = \frac{k^2}{l} = l'$

$$\therefore \qquad O'S' = GS' + GO = l + \frac{k^2}{l} = l + l' = SO.$$

Thus, there are four points in all, *viz.* S, O, S' and O', collinear with the *c.g.* of the pendulum (G) about which its time-period is the same.

4.10 MAXIMUM AND MINIMUM TIME-PERIOD OF A COMPOUND PENDULUM

For the time-period of a compound pendulum, we have the relation

$$T = 2\pi\sqrt{\frac{(k^2 + l^2)}{lg}}$$

squaring which, we have

$$T^2 = \frac{4\pi^2(k^2 + l^2)}{lg} = \frac{4\pi^2}{g}\left(\frac{k^2 + l^2}{l}\right) = \frac{4\pi^2}{g}\left(\frac{k^2}{l} + l\right)$$

Differentiating with respect to l, we have

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right)$$

a relation showing the variation of T with length (l) of the pendulum.

Clearly, T will be a maximum or a minimum when $\frac{dT}{dl} = 0$, *i.e.*, when $l^2 = k^2$ or $l = \pm k$ or when $l = k$, because the negative value of k is simply meaningless.

Since $\frac{d^2T}{dl^2}$ comes out to be positive, it is clear that T is a minimum when $l = k$, *i.e.*, the time-period of a compound pendulum is the minimum when its length is equal to its radius of gyration about the axis through its *c.g.*, And the value of this minimum time-period will be

$$T_{\min} = 2\pi\sqrt{\frac{(k^2 + k^2)}{kg}} = 2\pi\sqrt{2k/g}$$

If $l = 0$ or ∞, $T = \infty$ or a maximum. Ignoring $l = \infty$ as absurd, we thus find that the time-period of a compound pendulum is the maximum when length is zero, *i.e.,* when the axis of suspension passes through its *c.g.* or the *c.g.* itself is the point of suspension.

4.11 ADVANTAGES OF A COMPOUND PENDULUM

(*i*) The errors due to finite weight of string and its extensibility are eliminated.
(*ii*) The uncertainty in the motion of the bob is absent.
(*iii*) Due to larger mass of the body, viscous forces due to air have negligible effect.
(*iv*) The errors due to finite amplitude of swing and yielding of the support can be determined and corrections applied for them.
(*v*) The equivalent length of the simple pendulum in this case can be determined more accurately as the position of centre of suspension is known and that of the centre of oscillation is determined graphically.

4.12 DETERMINATION OF THE VALUE OF *G*

From the interchangeability of the points of suspension and oscillation it would appear that the easiest method of determining the value of '*g*' at a place would be to locate two points on either side of the C.G. of the pendulum about which the time-period of the pendulum is the same. These points would then correspond to the centres of suspension and oscillation of the pendulum and the distance between them would give *L*, the length of the equivalent simple pendulum. So that, if *T* be the time-period of the pendulum about either of these, we shall have $T = 2\pi\sqrt{\dfrac{L}{g}}$, and, therefore, $g = \dfrac{4\pi^2 L}{T^2}$.

4.13 OBJECT

To determine the value of '*g*', and the moment of inertia of a bar about C.G. by means of a bar pendulum.

Apparatus: A bar pendulum, steel knife edge, support for the knife edge, a stop watch, telescope and a meter scale.

Description of the apparatus: The bar pendulum consists of a uniform rectangular long metal bar having several holes drilled along its length so that the line of holes passes through the centre of gravity. Any desired hole may be slipped on to a fixed horizontal knife edge and the bar can be made to oscillate about it in a vertical plane. The knife edge is a piece of hard steel grounded to have a sharp edge. The knife edge rests on two glass plates one on each side placed on a rigid support. The knife edge is therefore horizontal and the bar swings regularly without twisting.

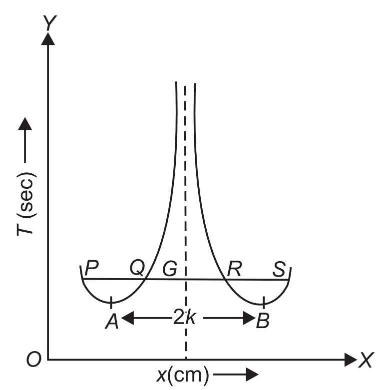


Fig. 4.5

Theory: In this experiment a bar is allowed to oscillate about horizontal knife edge passing through successive holes from one end to the other end. Time period is determined for each case. A graph is then plotted with distance of knife edge from one end of the bar on *x*-axis and the corresponding time period on *y*-axis. A graph of the type shown in figure and consisting of two symmetrical branches is obtained.

The time period is maximum at points A and B. The distance between these points is double the radius of gyration k of the bar about a parallel axis through the C.G. of the bar. These points are symmetrically situated on either side of centre of gravity of the bar. Their middle point gives the position of C.G. of the bar. In this case the equivalent length of simple pendulum is $L = 2k$ and the time period is given by

$$T_{\min} = 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{AB}{g}}$$

Any line drawn parallel to distance axis cuts the graph at four points P, Q, R and S, about which the time periods are equal. Pairs P, S and Q, R are symmetrically situated on either side of the C.G. of the bar. If P(or S) is taken as point of suspension R (or Q) becomes the point of oscillation. The equivalent length of simple pendulum L, then equals the distance PR or QS. If $PG = GS = l_1$ and $QG = GR = l_2$, then time-period about these points is given by

$$T = 2\pi\sqrt{\frac{l_1 + l_2}{g}}$$

Thus knowing $l_1 + l_2 = PR = QS$ and T we can find the value of 'g'.

It may be noted that l_1 and l_2 are related as

$$l_2 = \frac{k^2}{l_1}$$

Thus the radius of gyration of the bar about a parallel axis through its centre of gravity is obtained

$$k = \sqrt{l_1 \times l_2} = \sqrt{PG \times GR} = \sqrt{QG \times GS}$$

Now, instead of calculating the value of g as above, a better method, suggested by Ferguson in the year 1928, is to plot lT^2 along the axis of x and l^2 along the axis of y, which, from the relation $l^2 + k^2 = \left(\frac{lT^2}{4\pi^2}\right)g$ must give a straight line graph, as shown in figure.

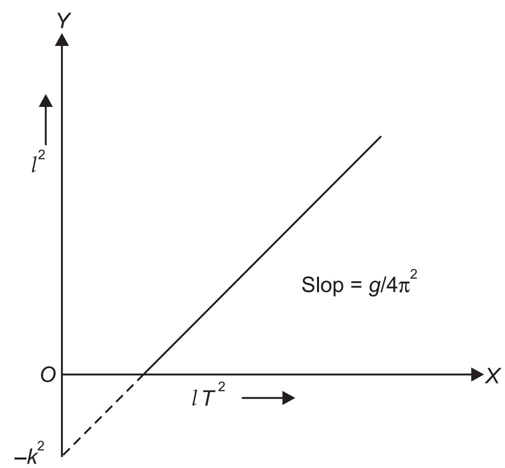


Fig. 4.6

The slope of the curve is $\frac{g}{4\pi^2}$, from where the value of g may be easily obtained. Further, the intercept of the curve on the axis of y-gives $-k^2$ and thus the values of both g and k can be obtained at once.

Procedure

1. Ensure that the knife edge, if fixed, is horizontal otherwise the frame on which movable knife edge is to be rested is horizontal.
2. Find the mass of the bar.
3. Suspend the bar about the knife edge from the hole nearest to one end. Displace the bar slightly to one side in vertical plane and release to put it into oscillations. With the help of

a stop watch find the time for 50 oscillations and hence determine the period of oscillations.

4. Repeat above procedure by suspending bar from successive holes. Beyond the C.G. the bar will turn upside down. Continue till last hole at the other end is reached.
5. Measure the distance from one fixed end to those points in successive holes where the knife edge supports the bar.
6. Plot a graph with distances of knife-edge from one end on *x*-axis and corresponding time period on *y*-axis. A curve of the type shown in Fig. 4.5 is obtained.
7. Measure the value $2k$ = AB from the graph and $T_{\min}$ and hence calculate the value of '*g*'. Alternately choose certain value of T and find $(l_1 + l_2)$ the corresponding mean equivalent length of simple pendulum and hence calculate the value of *g*. This procedure may be repeated to find mean value of '*g*'.
8. Plot the curve lT^2 against l^2 (Fig. 4.6). Calculate '*g*' and '*k*' and compare the results found in above.

Observations: 1. Mass of the bar =

2. Measurement of periodic time and distance of point of support:

S.No.	*Order of Holes*	*Distance of support C.G. (in cm)*	*No. of Oscillations*	*Time (sec)*	*Periodic time (Sec)*
1.					
2.					
3.					
4.					
5.					
		Position of Center of gravity (Turn the bar pendulum)			
6.					
7.					

3. Measurement of T, l_1, l_2 from the graph :

S.No.	*T(in sec)*	l_1 *(in meter)*	l_2 *(in meter)*	$l = l_1 + l_2$ *(in meter)*	$k^2 = l_1 l_2$
1.					
2.					
3.					

Calculation: (*i*) From graph $AB = 2k$ = ... cm

Minimum time period T_{min} = ... sec

Since $$T_{\min} = 2\pi\sqrt{\frac{AB}{g}}$$

$$g = \frac{4\pi^2}{T_{\min}^2} \times AB = \text{cm/sec}^2$$

(*ii*) From graph T = sec.

$$PS = \text{....... cm}, \quad \therefore l_1 = \frac{1}{2}PS = \ldots\text{ cm}$$

$$QR = \text{.....}\ \text{cm}, \quad \therefore l_2 = \frac{1}{2}QR = \ldots\ \text{cm}$$

Thus, mean $\quad L = l_1 + l_2 = \text{.....}\ \text{cm}$

Hence $\quad g = \dfrac{4\pi^2}{T^2}L = \text{.....}\ \text{cm/sec}^2$

Repeat the calculation for two or three different values of T and calculate the mean value of g.

(*iii*) From graph, for a chosen value of T,

$$PS = \text{......}\ \text{cm} \qquad \therefore l_1 = \frac{1}{2}PS = \text{......}\ \text{cm}$$

$$QR = \text{......}\ \text{cm} \qquad \therefore l_2 = \frac{1}{2}QR = \text{.....}\ \text{cm}$$

$\therefore \quad k = \sqrt{l_1 \cdot l_2} = \text{......}\ \text{cm}$

Repeat this calculation for 2 or 3 different values of T and calculate mean value of radius of gyration.

Result: The value of 'g' at ..... is = ... cm/sec^2

Value of 'g' from graph =

Value of k from graph =

Moment of Inertia of the bar =

Standard result: The value of g at .... = ...

Percentage error: ....%

Sources of error and precautions

1. The knife edge is made horizontal.
2. If the knife edge is not perfectly horizontal the bar may be twisted while swinging.
3. The motion of bar should be strictly in a vertical plane.
4. Time period should be noted only after all types of irregular motions subside.
5. The amplitude of swing should be small ($4^\circ - 5^\circ$). So that the condition $\sin\theta = \theta$ assumed in the derivation of formula remains valid.
6. The time period of oscillation should be determined by measuring time for a large number of oscillations with an accurate stop watch.
7. The graph should be drawn smoothly.
8. All distances should be measured and plotted from one end of the rod.

Theoretical error: g is given by the formula

$$g = \frac{4\pi^2 L}{T^2}$$

Taking log and differentiating

$$\frac{\delta g}{g} = \frac{\delta L}{L} + \frac{2\delta T}{T} = \text{.......}$$

Maximum possible error = ... %

Correct way of plotting graph in period of oscillation T and hole distance: The centre of gravity of the bar is to be at the centre of bar, it is necessary that distribution of mass in the bar should be uniform all along its length. A small non-uniformity will, however, shift the position of C.G. from centre of bar. It is, therefore, always preferred to measure the distance of holes from one end of the bar instead of measuring from its centre and then plot a graph as shown in figure.

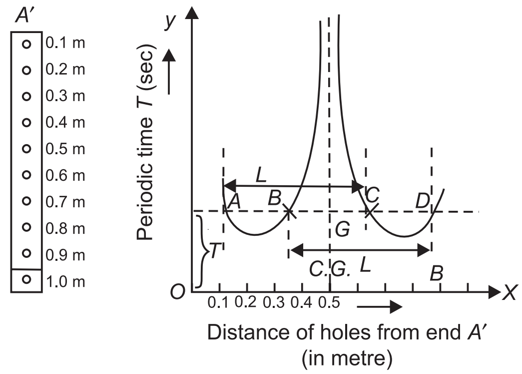


Fig. 4.7

At a particular time period T_1, a horizontal line is drawn giving four points of intersection with the curve about which time period is same.

From graph $AC = BD = l + \dfrac{k^2}{l} = L$.

Centre of AD will point C.G. of bar

we can also find $L = \dfrac{(AC + BD)}{2}$.

Drawback: A drawback of the method is that it being well high impossible to pin-point the position of the c.g. of the bar or the pendulum (as, in fact, of any other body), the distances measured from it are not vary accurate. Any error due to this is, however, eliminated automatically as the graph is smoothed out into the form of a straight line.

Superiority of a compound pendulum over a simple pendulum: The main points of the superiority of a compound pendulum over a simple pendulum are the following:

1. Unlike the ideal simple pendulum, a compound pendulum is easily realisable in actual practice.
2. It oscillates as a whole and there is no lag like that between the bob and the string in the case of a simple pendulum.
3. The length to be measured is clearly defined. In the case of a simple pendulum, the point of suspension and the C.G. of the bob, the distance between which gives the length of the pendulum, are both more or less indefinite points, so that the distance between them, *i.e.*, l, cannot be measured accurately.
4. On account of its large mass, and hence a large moment of inertia, it continues to oscillate for a longer time, thus enabling the time for a large number of oscillations to be noted and its time-period calculated more accurately.

4.14 TIME-PERIOD OF A PENDULUM FOR LARGE AMPLITUDE

A simple or a compound pendulum oscillating with large amplitude is a familiar example of an anharmonic oscillator. The oscillations are simple harmonic only if the angular amplitude θ be infinitely small *i.e.,* when in the expansion of sin θ into a power series all other terms except the first are negligibly small.

If θ be appreciably large, so that the second term $\left(\frac{\theta^3}{\lfloor 3}\right)$ in the power series can not be neglected. Hence, time-period of the simple pendulum, *i.e.,*

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{gl}}\left(1 - \frac{\theta_1^2}{16}\right)$$

Putting $2\pi\sqrt{\frac{l}{g}}$, the time-period when the amplitude is small, equal to T_0, we have

$$T = T_0\left(1 + \frac{\theta_1{}^2}{16}\right)$$

Similarly, for a compound pendulum, we shall obtain

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{\theta_1^2}{16}\right)$$

or

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l^2} + l}{g}}\left(1 + \frac{\theta_1^2}{16}\right)$$

Since $2\pi\sqrt{\frac{\left(\frac{k^2}{l^2}\right) + l}{g}}$ is the time-period T_0 for oscillations of small amplitude, we have

$$T = T_0\left(1 + \frac{\theta_1{}^2}{16}\right),$$

indicating that the time-period increases with amplitude.

Since the amplitude of the pendulum in both cases does not remain constant but goes on progressively decreasing from θ_1 in the beginning to, say, θ_2 at the end, we may take $\theta_1\theta_2$ in place of $\theta_1{}^2$. So that, in either case,

$$T = T_0\left(1 + \frac{\theta_1\theta_2}{16}\right).$$

Now, what we actually observe is T. The correct value of the time period (*i.e.,* if the amplitude be small) is, therefore, given by

$$T_0 = \frac{T}{\left(1 + \frac{\theta_1\theta_2}{16}\right)} = T\left(1 - \frac{\theta_1\theta_2}{16}\right)$$

4.15 OBJECT

To determine the value of acceleration due to gravity at a place, by means of Kater's reversible pendulum.

Apparatus used: A Kater's pendulum, a stop watch, a telescope and Sharp knife edges.

Formula used: The value of 'g' can be calculated with the help of the following formula:

$$g = \frac{8\pi^2}{\left[\dfrac{T_1^2 + T_2^2}{l_1 + l_2} + \dfrac{T_1^2 - T_2^2}{l_1 - l_2}\right]} = \frac{8\pi^2}{\left[\dfrac{T_1^2 + T_2^2}{l_1 + l_2}\right]}$$

$$\text{neglecting } \left[\frac{T_1^2 - T_2^2}{l_1 - l_2}\right]$$

where T_1 = time period about one knife edge
T_2 = time period about the other knife edge
l_1 = distance of one knife edge from centre of gravity of the pendulum
l_2 = distance of the other knife edge from the centre of gravity of the pendulum.
$(l_1 + l_2)$ = distance between two knife edges.

Description of apparatus: A Kater's pendulum is a compound pendulum in the form of a long rod, having two knife edges K_1 and K_2 fixed near the ends facing each other but lying on opposite sides of the centre of gravity. The position of the centre of gravity of the bar can be altered by shifting the weights M and m upwards or downwards, which can be slided and fixed at any point. The smaller weight m, having a micrometer screw arrangement is used for the finer adjustment of the final position of centre of gravity. The centre of gravity lies un-symmetrically between K_1 and K_2 due to the weight w, fixed at one end. Kater's pendulum is shown in Fig. 4.8.

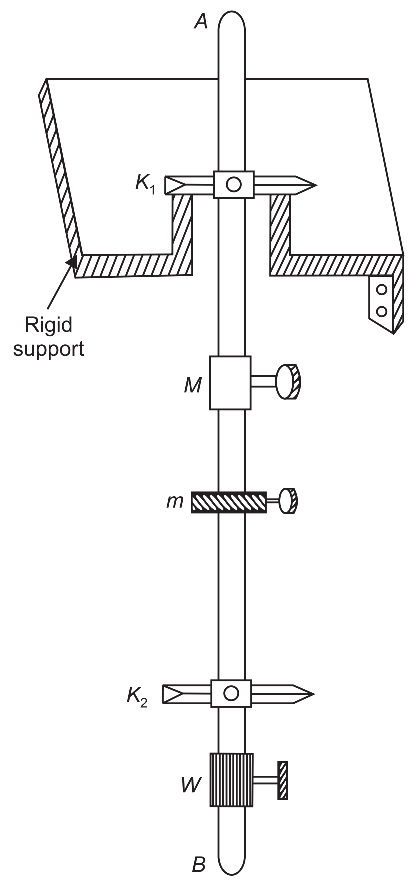


Fig. 4.8

Theory: Kater's pendulm is a compound pendulum of special design. For such a pendulum, if it is possible to get two parallel axes of suspension on either side of the centre of gravity (C.G.) about which the periods of oscillation of the pendulum are the same, then the distance between those axes will be the length L of the equivalent simple pendulum. If T is the equal periodic time then

$$T = 2\pi\sqrt{\frac{L}{g}} \text{ or } g = \frac{4\pi^2 L}{T^2} \qquad \text{...(1)}$$

According to Bessel, an accurate determination of g does not require the tedious process of making the two periods exactly the same. It is sufficient if the two periods are made nearly equal.

If T_1 and T_2 represent two nearly equal periods about the axes of suspension, and l_1 and l_2 represent respectively the distances of the two axes of suspension from the C.G. of the pendulum then

$$T_1 = 2\pi \sqrt{\frac{l_1^2 + k^2}{l_1 g}} \text{ and } T_2 = 2\pi \sqrt{\frac{l_2^2 + k^2}{l_2 g}} \qquad \text{...(2)}$$

where k is the radius of gyration of the pendulum about an axis passing through its C.G.

From Eq. (2), we get

$$l_1 g T_1^2 = 4\pi^2 (l_1^2 + k^2) \text{ and } l_2 g T_2^2 = 4\pi^2 (l_1^2 + k^2)$$

On subtraction we obtain

$$g (l_1 T_1^2 - l_2 T_2^2) = 4\pi^2 (l_1^2 - l_2^2)$$

or,

$$\frac{4\pi^2}{g} = \frac{l_1 T_1^2 - l_2 T_2^2}{l_1^2 - l_2^2} = \frac{1}{2}\left[\frac{T_1^2 + T_2^2}{l_1 + l_2} + \frac{T_1^2 - T_2^2}{l_1 - l_2}\right] \qquad \text{...(3)}$$

Equation (3) is the working formula of the experiment and shows that by measuring l_1 and l_2 and finding the periods T_1 and T_2, the acceleration due to gravity g can be determined.

If T_1 and T_2 are very nearly equal and there is a considerable difference between l_1 and l_2 i.e., the *CG* is not nearly midway between the knife edges, (which is an essential feature of the Kater's pendulum) the second term will be very small compared with the first. Hence an approximate knowledge of $(l_1 - l_2)$ will be sufficient and so it need not be measured very accurately.

Procedure:

1. Place one of the knife edges of the pendulum on a rigid support so that the metallic cylinder is in the downward direction. Draw a vertical sharp mark along the length of the pendulum and focus the sharp mark through a low power telescope by keeping it at a distance in front of the pendulum. Allow the pendulum to oscillate through a very small amplitude and observe the oscillations through the telescope.
2. Measure the time taken for a small number of oscillations (say 20) by means of a precision stop watch.

   Now, place the pendulum on the second knife-edge, and after allowing it to oscillate measure the time taken for the same number of oscillations. The times in the two cases may be widely different.
3. Shift the position of the weight M in one direction and measure again the times for the same number of oscillations as before when the pendulum oscillates first about the knife-edge K_1 and then about the knife-edge K_2. If the shift of the weight M increases the difference in the times of oscillations about the two knife-edges then shift the position of the weight M in the opposite direction. Otherwise shift the weight M in the same direction by a small amount and repeat operation (2).
4. The shifting of the weight M and the repetition of operation (2) are to be continued till the two times are nearly equal.

5. Repeat the process now with more number of oscillations (say, 10, 20, etc). Until the time for 50 oscillations about the two knife-edges are very nearly equal. Note these two very nearly equal times.
   While observing times for more than 20 oscillations, the equality of the two times is to be approached by finer adjustments attached with the weight *m* and shifting it precisely.
6. Measure the time for 50 oscillations about each knife-edge three times and then calculate the mean time for 50 oscillations about each of the knife-edges. From these, determine the time periods T_1 and T_2 about the knife-edges.
7. Now place the pendulum horizontally on a sharp wedge which is mounted on a horizontal table to locate the C.G. of the pendulum. Mark the position of the C.G. and measure the distances of the knife-edges, i.e., l_1 and l_2 from the C.G. by a metre scale.
8. Substituting the values of l_1, l_2, T_1 and T_2 in Eq. (3) calculate g.

Observations:

1. Preliminary record of times of oscillations during adjustment of positions of the weights.

No. of observation	*Adjustment made by shifting the weight*	*No. of oscillations considered*	*Time of oscillations observed about the knife-edge (sec.)*	
			K_1	K_2
1 2 3 4 5 etc.	*M*	5 5 10 10 20 etc.	– – – – –	
1 2 3 4 5 etc.	*m*	20 20 30 30 30 etc.		

2. Determination of final time periods T_1 and T_2

No. of observation	*Oscillations observed about the knife-edge*	*Time for 50 oscillations (sec)*	*Mean time for 50 oscillations (sec)*	*Time periods T_1 & T_2 (sec.)*
1 2 3 4	K_1			(T_1)
1 2 3 4	K_1			(T_2)

3. Determination of the distances l_1 and l_2:

No. of observation	Distance of K_1 from C.G. l_1 (cm)	Mean l_1 (cm)	Distance of K_2 from C.G. l_2 (cm)	Mean l_2 cm
1 2 3				

4. Calculation of g:

T_1 (sec) from Table 2	T_2 (sec) from Table 2	l_1 (cm) from Table 3	l_2 (cm) from Table 3	g (cm/sec^2) from Eq. (3)

Result: The value of acceleration due to gravity at ...... = ... m/sec^2.

Standard Result: The value of g at ...... = ... m/sec^2.

Percentage error: ......

Precautions and sources of error:

1. The amplitude of oscillation must be kept very small so that the motion is truly simple harmonic.
2. The knife-edges must be horizontal and parallel to each other so that the oscillations are confined in a vertical plane and the pendulum remains in a stable position.
3. To save time, preliminary observations of the times of oscillations should be made with a smaller number of oscillations. As the difference between the periods decreases, the number of oscillations observed should be increased.
4. Correction for the finite arc of swing of the pendulum may be included by measuring the half angles of the swing in radians (α_1, α_2) at the start and at the end respectively, and using the formula $T_0 = T\left(1 - \frac{\alpha_1\alpha_2}{16}\right)$, where T_0 is the correct period and T is the observed period.
5. For greater accuracy, measure the time period by the methods of coincidence.
6. A very accurate stop-watch should be used for timing the oscillations and it must give results correct to one-tenth of a second.

4.16 VIVA-VOCE

Q. 1. What do you mean by gravity?

Ans. The property of the earth by virtue of which it attracts the bodies towards its centre, is known as gravity.

Q. 2. What is gravitation?

Ans. Every particle of the universe attracts another particle by a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$F \propto m_1 m_2;\ F \propto \frac{1}{r^2} \quad \therefore F \propto \frac{m_1 m_2}{r^2}$$

Q. 3. What is acceleration due to gravity? How is it defined?

Ans. Acceleration due to gravity is numerically equal to the force of attraction with which a unit mass is attracted by the earth towards its centre.

Q. 4. What are the units and dimensions of '*g*'?

Ans. The units and dimensions of accelerations due to gravity are cm per sec^2 and LT^{-2} respectively.

Q. 5. What is meant by *G*? How are *G* and *g* related?

Ans. *G* is Universal gravitational constant and is equal to the force of attraction between two unit masses placed unit distance apart. $g = G\frac{M}{R^2}$ where *M* is the mass of the earth and *R* is its radius.

Q. 6. Why does the value of '*g*' change at the surface of the earth?

Ans. This is for two reasons:

(*i*) the earth is not a perfect sphere its diameter at poles is about 21 km shorter than that at the equator, which means *R* will change in the relation $g = G\frac{M}{R^2}$ and

(*ii*) earth is spinning about its own axis. Therefore bodies on its surface move on circular path. A part of gravitational force is used up in providing necessary centripetal force and hence effective force of gravity is decreased, decreasing the value of '*g*'.

Q. 7. How does the value of *g* vary at different places and at different distances from the surface of the earth.

Ans. (i) The value of *g* decreases with altitude.

(*ii*) The value of *g* decreases as we go below the surface of earth.

(*iii*) *g* is greatest at poles and least at equator.

Q. 8. What is the value of *g* at the earth's centre?

Ans. The value of *g* at earth's centre is zero.

Q. 9. What is the use of the knowledge of the value of 'g'.

Ans. Its knowledge is quite important in geophysical prospecting of mineral deposits inside the earth. It is also required in many theoretical calculations.

Q. 10. What is a simple pendulum?

Ans. A simple pendulum is just a heavy particle, suspended from one end of an inextensible, weightless string whose other end is fixed to a rigid support.

Q. 11. What is a compound pendulum?

Ans. A compound pendulum is a rigid body, capable of oscillating freely about a horizontal axis passing through it (not through its centre of gravity) in a vertical plane.

Q. 12. Suppose a clear hole is bored through the centre of the earth and a ball is dropped in it, what will happen to the ball?

Ans. The ball will execute simple harmonic motion about centre of the earth.

Q. 13. What are the centres of suspension and oscillation?

Ans. **Centre of suspension.** It is a point where the horizontal axes of rotation intersects the vertical section of the pendulum taken through centre of gravity.

Centre of Oscillation. This is another point, on other side of centre of gravity at a distance $\frac{k^2}{l}$ from it and lying in the plane of oscillation. k being radius of gyration and l, the distance of centre of suspension from centre of gravity.

Q. 14. For how many points in a compound pendulum the time period is the same?

Ans. The time period about centre of suspension and centre of oscillation is the same. By reversing the pendulum, we have two more points (centre of suspension and oscillation) about which the time period is same. In this way, there are four points collinear with C.G. about which time period is same. Two points lie on one side of C.G. and two on another side of C.G.

Q. 15. How does the time period of oscillation of a compound pendulum depend upon the distance of the centre of suspension from the centre of gravity?

Ans. The time period is infinite at centre of gravity. It decreases rapidly and becomes minimum when the distance is equal to radius of gyration. At still greater distances the period again increases.

Q. 16. What do you mean by an equivalent simple pendulum?

Ans. This is a simple pendulum of such a length that its periodic time is same as that of a compound pendulum.

Q. 17. What is the length of equivalent simple pendulum?

Ans. We know that $T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}} = 2\pi\sqrt{\frac{L}{g}}$ where $L = \sqrt{\frac{k^2}{l}+l}$, known as length of equivalent simple pendulum.

Q. 18. What will be the period of oscillation of a compound pendulum if centre of suspension coincides with the centre of gravity?

Ans. No oscillations are possible. Time period may be taken as infinite in this case.

Q. 19. What will be the form of l^2 vs T^2l graph and why?

Ans. It will be a straight line, since $T^2 l = \frac{4\pi^2}{g} l^2 + \frac{4\pi^2}{g} k^2$ which is of the form $y = mx + c$.

Q. 20. How will you find the values of 'g' and k from the graph?

Ans. Slope of this graph will be $\tan\theta = \frac{4\pi^2}{g}$ from which 'g' can be calculated. The -ve intercept on l^2 -axis will give the value of k^2, square root of which will be the value of k.

Q. 21. Why are the knife edges kept horizontal?

Ans. So that bar may oscillate in a vertical plane and may not slip off.

Q. 22. How much should be the amplitude of vibration?

Ans. The amplitude of vibration should not be large because in the deduction of the theory it has been assumed that $\sin\theta = \theta$

Q. 23. There is another method to determine g superior than compound pendulum?

Ans. Yes, Kater's reversible pendulum.

Q. 24. When does the minimum period of a compound pendulum occur?

Ans. The time period of a compound pendulum becomes a minimum when the distance of the centre of suspension from its C.G. equals the radius of gyration of the pendulum about an axis passing through the C.G., the axis being parallel to the axis of rotation.

Q. 25. What is Kater's reversible pendulum?

Ans. This is improved form of compound pendulum. It consists of a long rod having two fixed knife edges, a heavy bob and two weights which can be moved and fixed at desired places.

Q. 26. What is the principle involved in this method?

Ans. If the two points (points of suspension and oscillation) opposite to C.G. are found, then the distance between them will be equal to the length of equivalent simple pendulum. Now the value of g can easily be calculated by using the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Q. 27. Is it necessary to adjust the time periods about the two knife edges to exact equal?

Ans. No, it is a very tedious job. It is sufficient to make them nearly equal but the formula should be modified accordingly.

Q. 28. Which formula will you use when two periods are nearly equal?

Ans.

$$g = \frac{8\pi^2}{\left[\dfrac{T_1^2 + T_2^2}{l_1 + l_2} + \dfrac{T_1^2 - T_2^2}{l_1 - l_2}\right]}$$

Q. 29. What are the functions of weights M and m?

Ans. The two weights are used to make the time periods equal about the two knife edges.

Q. 30. Why is one weight larger than the other?

Ans. The time period is roughly adjusted by larger weight and then for the finer adjustment smaller weight is used.

Q. 31. Does this pendulum give more accurate value of g than bar pendulum?

Ans. Yes.

Q. 32. Is it necessary to determine the position of C.G.?

Ans. If time period about two points is same, then there is no necessity of determining the position of C.G.

Q. 33. How does this experiment give accurate result?

Ans. The working formula of this experiment contains two terms: the denominator of the first term ($l_1 + l_2$) is the distance between the two knife-edges, which can be measured accurately. The denominator of the second term ($l_1 - l_2$) however requires that the position of the C.G. be determined accurately which in practice involves certain inaccuracy. But since the numerator of this term, i.e. $T_1^2 - T_2^2$, is very small the contribution of error due to $l_1 - l_2$ is very small.

EXERCISE

Q. 1. What are the drawbacks of a simple pendulum which have been overcome in a compound pendulum?

Q. 2. What is second's pendulum?

Q. 3. Does the time period of a simple pendulum depend upon the size and material of the bob and its temperature.

Q. 4. How are these points related?

Q. 5. When will it be maximum or minimum.

Q. 6. What do you mean by radius of gyration?

Q. 7. How will you determine the value of radius of gyration of the compound pendulum and the portion of its centre of gravity by plotting a graph between T and l?

Q. 8. What will happen to the time period if a small weight is added to lower end of the bar?

Q. 9. Why is the amplitude of oscillations kept small?

Q. 10. Is it essential that (*i*) the bar be symmetrical (*ii*) uniform (*iii*) the holes be in a line (*iv*) this line should pass through C.G.?

5

Surface Tension

5.1 SURFACE TENSION

When a small quantity of water is poured on a clean glass plate, it spreads in all directions in the form of a thin film. But when a small quantity of mercury is poured on the glass plate, it takes the form of a spherical drop. Similarly, if a small quantity of water is poured on a greasy glass plate, it also takes the form of small globules like mercury. This shows that the behaviour of liquids is controlled not only by gravitational force (weight) but some other force also acts upon it which depends upon the nature of the surfaces in contact. If the weight of the liquid is negligible then its shape is perfectly spherical. For example rain drops and soap bubbles are perfectly spherical. We know that for a given volume, the surface area of a sphere is least. Hence we may say that the free surface of a liquid has a tendency to contract to a minimum possible area.

The free surface of a liquid behaves as if it is in a state of tension and has a natural tendency to contract and occupy minimum surface area. The behaviour is like that of a stretched elastic or rubber membrane with an important difference that whereas the tension in a membrane increases with stretching the tension in a liquid surface is independent of extension in the area. This property of the liquid is known as surface tension. Various experiments suggest that the surface film exerts a force perpendicular to any line drawn on the surface tangential to it. The surface tension of a liquid can be defined in the following way.

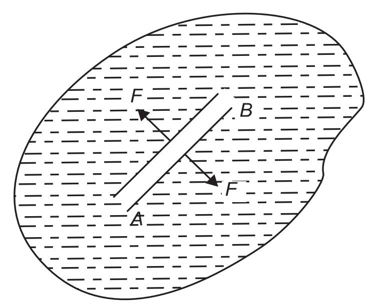


Fig. 5.1

5.2 DEFINITION OF SURFACE TENSION

Let an imaginary line *AB* be drawn in any direction in a liquid surface. The surface on either side of this line exerts a pulling force on the surface on the other side. This force lies in the plane of the surface and is at right angles to the line *AB*. The magnitude of this force per unit length of *AB* is taken as a measure of the surface tension of the liquid. Thus if *F* be the total force acting on either side of the line *AB* of length *l*, than the surface tension is given by

$$T = \frac{F}{l}.$$

If $l = 1$ then $T = F$. Hence, the surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface, acting at right angles on either side of an imaginary line drawn in that surface. Its unit is 'newton/meter' and the dimensions are MT^{-2}.

The value of the surface tension of a liquid depends on the temperature of the liquid, as well as on the medium on the other side of the surface. It decreases with rise in temperature and becomes zero at the critical temperature. For small range of temperature the decrease in surface tension of a liquid is almost linear with rise of temperature.

The value of surface tension for a given liquid also depends upon the medium on outer side of the surface. If the medium is not stated, it is supposed to be air.

5.3 SURFACE ENERGY

Consider a soap film formed in a rectangular framework of wire *PQRS* with a horizontal weightless wire *AB* free to move forward or backward. Due to surface tension the wire *AB* is pulled towards the film. This force acts perpendicular to *AB* and tangential to the film. To keep *AB* in equilibrium a force *F* has to be applied as shown in figure. If *T* is the surface tension of the film, then according to definition.

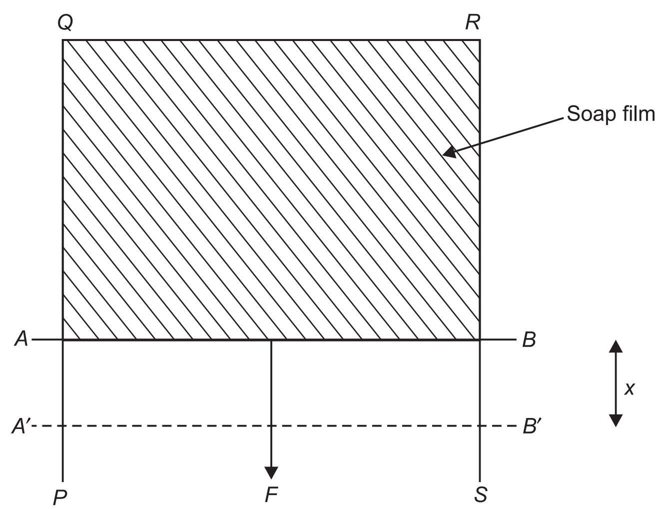


Fig. 5.2

(*i*) the film pulls the wire by a force $2l \times T$ because the film has two surfaces and l is the length of wire *AB.* Thus for equilibrium

$$2l \times T = F$$

or $$T = \frac{F}{2l}$$

if $$2l = 1,\ T = F$$

Now suppose that keeping the temperature constant the wire *AB* is pulled slowly to *A′B′* through a distance x. In this way the film is stretched by area $\Delta A = 2lx$ and work done in this process is

$\therefore$ $$W = Fx$$

Hence $$W = Fx = 2lxT = \Delta A \times T$$

or $$T = \frac{W}{\Delta A}$$

if $\Delta A = 1$, then $$\boxed{T = W}$$

The work done in stretching the surface is stored as the potential energy of the surface so created. When the surface is stretched its temperature falls and it therefore, takes up heat from surrounding to restore its original temperature. If H is the amount of heat energy absorbed per unit area of the new surface, the total or intrinsic energy E per unit area of the surface is given by

$$E = T + H$$

The mechanical part of energy which is numerically equal to surface tension T is also called free surface energy. The surface tension of a liquid is also very sensitive to the presence of even small quantities of impurities on the surface. The surface tension of pure liquid is greater than that of solution but there is no simple law for its variation with concentration.

5.4 MOLECULAR THEORY OF SURFACE TENSION

Laplace explained the surface tension on the basis of molecular theory. The molecules do not attract or repel each other when at large distances. But they attract when at short distances. The force of attraction is said to be cohesive when it is effective between molecules of the same type. But the force of attraction between molecules of different types is called adhesive force. The greatest distance upto which molecules can attract each other is called the molecular range or the range of molecular attraction. It is of the order of 10^{-7} cm. If we draw a sphere of radius equal to molecular range with a molecule as centre, then this molecule attracts only all those molecules which fall inside this sphere. This sphere is called the sphere of influence or the sphere of molecular attraction.

When a molecule is well inside the liquid it is attracted in all directions with equal force and resultant force, on it is zero and it behaves like a free molecule. If the molecule is very close to the surface of liquid, its sphere of influence is partly inside the liquid and partly outside. Therefore the number of liquid molecules pulling it below is greater than the number of those in the vapour attracting it up. Such molecules experience a net force towards the interior of the liquid and perpendicular to the surface. The molecules which are situated at the surface experience a maximum downward force due to molecular attraction. The liquid layer between two planes FF' and SS' having thickness equal to molecular range is called the surface film. When the surface area of a liquid is increased, more molecules of the liquid are to be brought to surface. While passing through surface film each molecule experiences a net inward force due to cohesion. Mechanical work has to be done to overcome this force. This work is stored in the surface molecules in the form of potential energy. Thus potential energy of molecules in surface film is greater than that in the interior of the liquid. It is a fundamental property of every mechanical system to acquire a stable equilibrium in a state in which its potential energy is minimum. Therefore, to

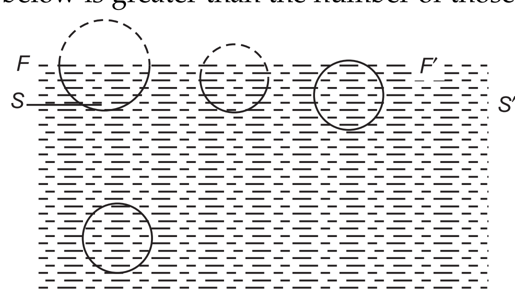


Fig. 5.3

minimize their potential energy the molecules have a tendency to go into the interior of the liquid as a result of which the surface has a tendency to contract and acquire a minimum area. This tendency is exhibited as surface tension.

The potential energy of the molecules in the unit area of the surface film equals the surface energy.

5.5 SHAPE OF LIQUID MENISCUS IN A GLASS TUBE

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The nature of the curvature (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and those of the solid.

In Fig. 5.4(*a*), water is shown to be in contact with the wall of a glass tube. Let us consider a molecule *A* on the water surface near the glass. This molecule is acted upon by two forces of attraction:

(*i*) The resultant adhesive force *P*, which acts on *A* due to the attraction of glass molecules near *A*. Its direction is perpendicular to the surface of the glass.

(*ii*) The resultant cohesive force *Q*, which acts on *A* due to the attraction of neighbouring water molecules. It acts towards the interior of water.

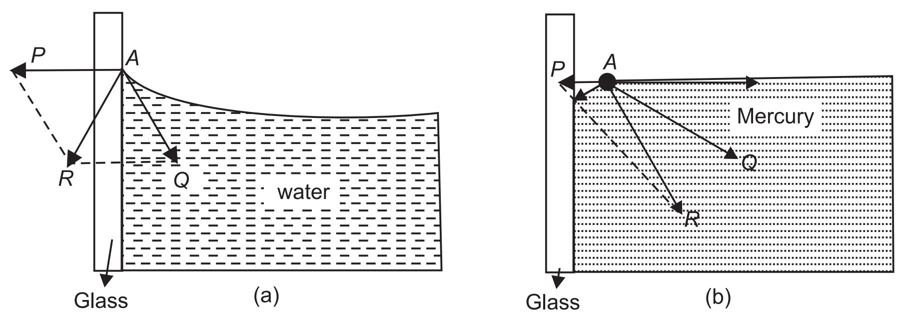


Fig. 5.4

The adhesive force between water molecules and glass molecules is greater than the cohesive force between the molecules of water. Hence, the force *P* is greater than the force *Q*. Their resultant *R* will be directed outward from water. In Fig. 5.4(*b*), mercury is shown to be in contact with the wall of a glass tube. The cohesive force between the molecules of mercury is far greater than the adhesive force between the mercury molecules and the glass molecules. Hence, in this case, the force *Q* will be much greater than the force *P* and their resultant will be directed towards the interior of mercury.

5.6 ANGLE OF CONTACT

When the free surface of a liquid comes in contact of a solid, it becomes curved near the place of contact. The angle inside the liquid between the tangent to the solid surface and the tangent to the liquid surface at the point of contact is called the angle of contact for that pair of solid and liquid.

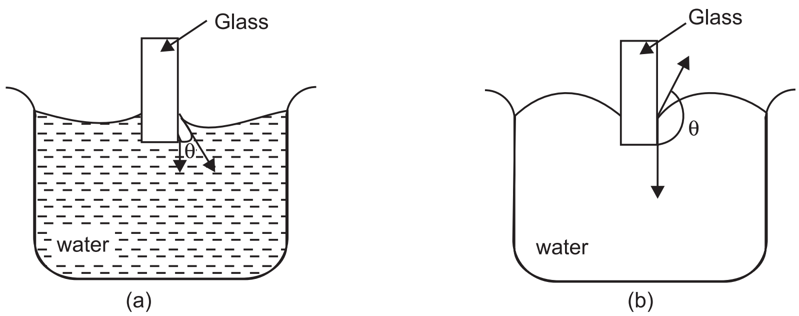


Fig. 5.5

The angle of contact for those liquids which wet the solid is acute. For ordinary water and glass it is about 8°. The liquids which do not wet the solid have obtuse angle of contact. For mercury and glass the angle of contact is 135°. In Fig. 5.5(*a*) and (*b*) are shown the angles of contact θ for water-glass and mercury-glass.

The angle of contact for water and silver is 90°. Hence in a silver vessel the surface of water at the edges also remains horizontal.

5.7 EXCESS OF PRESSURE ON CURVED SURFACE OF LIQUID

The free surface of a liquid is always a horizontal plane. If we consider any molecule on such a surface the resultant force due to surface tension is zero as in Fig. 5.6(*a*). On the other hand if the surface tension acts normally to the surface towards the concave side. Thus for convex meniscus the resultant is directed normally inwards towards the interior of the liquid while for concave meniscus this resultant force is directed normally outwards. As a result of these forces the curved surface has a tendency to contract and become plane. Consequently to maintain a curved liquid surface in equilibrium, there must exist an excess of pressure on the concave side compared to the convex side, which of itself would produce an expansion of the surface.

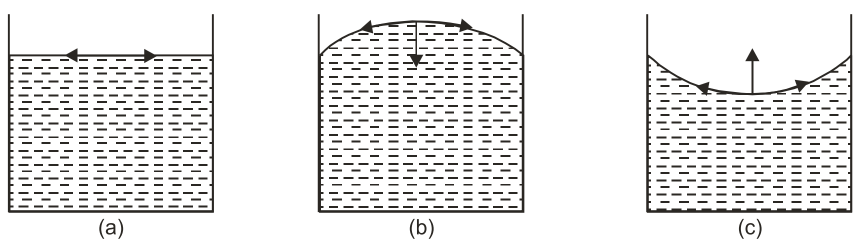


Fig. 5.6

Hence we conclude that there exists an excess of pressure on the concave side of the surface. If the principal radii of curvature of the surface are R_1 and R_2 respectively the magnitude of this excess of pressure on concave side is given by

$$p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(*i*) **Spherical surface:** For spherical surfaces like a liquid drop or air bubble in a liquid there is only one surface and the two principal radii of curvature are equal ($R_1 = R_2 = R$) and we have,

$$p = \frac{2T}{R}$$

But in case of a soap bubble or other spherical films we have two surfaces hence

$$p = \frac{4T}{R}$$

(*ii*) **Cylindrical surface:** In this case one principal radius of curvature is infinite. Therefore

$$p = \frac{T}{R}$$

For cylindrical film since there are two surfaces hence

$$p = \frac{2T}{R}$$

5.8 CAPILLARITY RISE OF LIQUID

When a glass capillary tube open at both ends is dipped vertically in water, the water rises up in the tube to a certain height above the water level outside the tube. The narrower the tube, the higher is the rise of water Fig. 5.7(*a*). On the other hand, if the tube is dipped in mercury, the mercury is depressed below the outside level Fig. 5.7(*b*). The phenomenon of rise or depression of liquid in a capillary tube is called capillarity. The liquids which wet glass (for which the angle of contact is acute) rise up in capillary tube, while those which do not wet glass (for which the angle of contact is obtuse) are depressed down in the capillary.

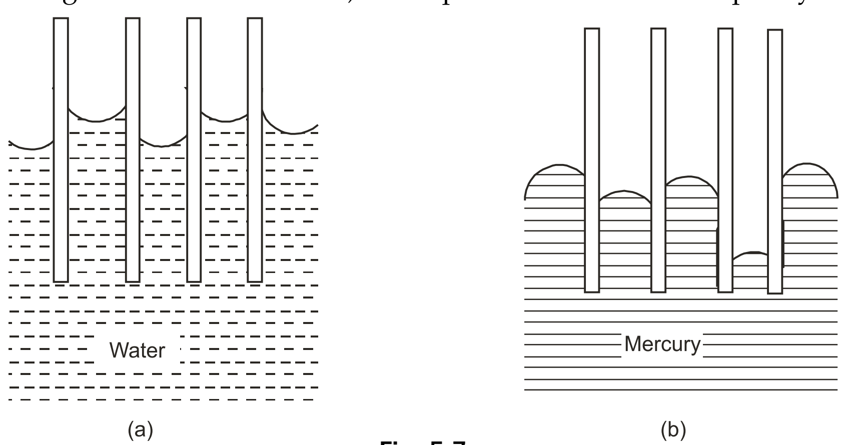


Fig. 5.7

Explanation: The phenomenon of capillarity arises due to the surface tension of liquids. When a capillary tube is dipped in water, the water meniscus inside the tube is concave. The pressure just below the meniscus is less than the pressure just above it by $\frac{2T}{R}$, where T is the surface tension of water and R is the radius of curvature of the meniscus. The pressure on the surface of water is atmospheric pressure P. The pressure just below the 'plane' surface of water outside the tube is also P, but that just below the meniscus inside the tube is $P - \frac{2T}{R}$ Fig. 5.8(*a*). We know that pressure at all points in the same level of water must be the same. Therefore, to make up the deficiency of pressure, $\frac{2T}{R}$, below the meniscus, water begins to flow from outside into the tube. The rising of water in the capillary stops at a certain height '*h*' Fig. 5.8(*b*). In this position the pressure of the water-column of height '*h*' becomes equal to $\frac{2T}{R}$, that is $h\rho g = \frac{2T}{R}$ where ρ is the density of water and '*g*' is the acceleration due to gravity. If *r* be the radius of the capillary tube and θ the angle of contact of water-glass, then the radius of curvature *R* of the meniscus is given by $R = \frac{r}{\cos\theta}$ Fig. 5.8(*c*).

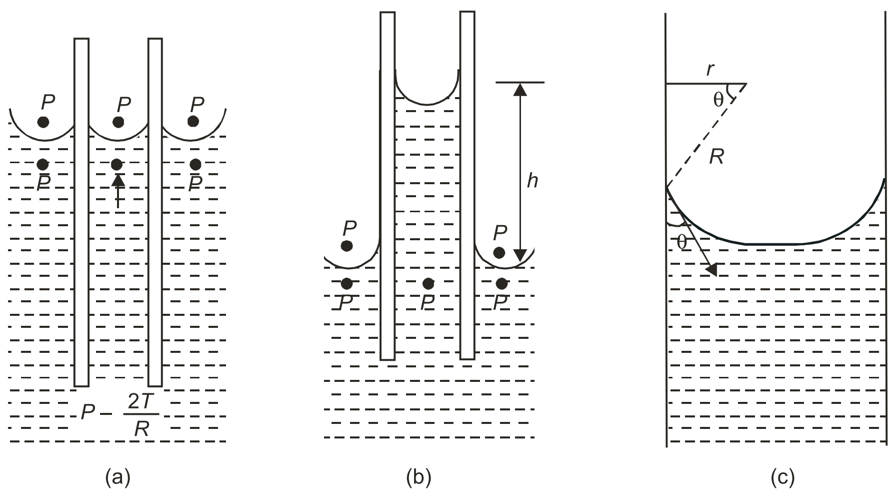


Fig. 5.8

$$\therefore \qquad h\rho g = \frac{2T}{\frac{r}{\cos\theta}}$$

$$h = \frac{2T\cos\theta}{r\rho g}$$

This shows that as *r* decreases, *h* increases, that is, narrower the tube, greater is the height to which the liquid rises in the tube.

Rising of liquid in a capillary tube of insufficient length: Suppose a liquid of density ρ and surface tension *T* rises in a capillary tube to a height '*h*' then

$$h\rho g = \frac{2T}{R}$$

Where *R* is the radius of curvature of the liquid meniscus in the tube. From this we may write

$$hR = \frac{2T}{\rho g} = \text{constant (for a given liquid)}$$

When the length of the tube is greater than h, the liquid rises in the tube to a height so as to satisfy the above equation. But if the length of the tube is less than h, then the liquid rises up to the top of the tube and then spreads out until its radius of curvature R increases to R', such that

$$h'R' = hR = \frac{2T\cos\theta}{\rho g} = \text{const.}$$

It is clear that liquid cannot emerge in the form of a fountain form the upper end of a short capillary tube.

On the other hand if the capillary is not vertical, the liquid rises in the tube to occupy length l such that the vertical height h, of the liquid is still the same as demanded by the formula.

If a tube of non-uniform bore or of any cross-section may be used the rise depends upon the cross-section at the position of the meniscus. If the bore is not circular, the formula for capillary rise is not as simple as Eq. $\left(h = \frac{2T\cos\theta}{r\rho g}\right)$.

5.9 OBJECT

To find the SURFACE TENSION of a liquid (water) by the method of CAPILLARY RISE.

Apparatus used: A capillary tube, petridish with stand, a plane glass strip, a pin, a clamp stand, traveling microscope, reading lens and some plasticine.

Formula used: The surface tension of a liquid is given by the formula.

$$T = r\left(h + \frac{r}{3}\right)\rho g/2 \text{ Newton/meter}$$

Where r = radius of the capillary tube at the liquid meniscus

h = height of the liquid in the capillary tube above the free surface of liquid in the petridish

ρ = density of water ($\rho = 1.00 \times 10^3$ Kg/m^3 for water)

Theory: When glass is dipped into a liquid like water, it becomes wet. When a clean fine bore glass capillary is dipped into such a liquid it is found to rise in it, until the top of the column of water is at a vertical height 'h' above the free surface of the liquid outside the capillary. The reason for this rise is the surface tension, which is due to the attractive forces between the molecules of the liquid. Such forces called cohesive forces try to make the surface of the liquid as small as possible. This is why a drop of liquid is of spherical shape.

Since the surface tension tries to reduce the surface of a liquid we can define it as follows. If an imaginary line of unit length is drawn on the surface of a liquid, then the force on one side of the line in a direction, which is perpendicular to the line and tangential to the surface, is called SURFACE TENSION.

When the liquid is in contact with the glass then on the line of contact the cohesive forces (or surface tension) try to pull the liquid molecules towards the liquid surface and the adhesive forces *i.e.* the forces between the molecules of the glass and the liquid try to pull the liquid molecules towards the glass surface. Equilibrium results when the two forces balance each other. Such equilibrium arises after the water has risen in the capillary to a height of 'h'. This column has weight equal to mg where m is the mass of the water in the column. This balances the upward force due to the surface tension which can be calculated as follows:

The length of the line of contact in the capillary between the surface of the water and the glass is $2\pi r$ where 'r' is the radius of the capillary. As seen from Fig. 5.9, the surface tension T acts in the direction shown and θ is called the angle of contact. The upward component of T is given by $T \cos \theta$ and therefore, recalling that T is the force per unit length, we get the total upward force equal to $2\pi rT\cos\theta$. This must balance the weight mg and we have

$$mg = 2\pi rT \cos \theta$$

For water-glass contact, $\theta = 0°$ and so $\cos \theta = 1$

Therefore, $$mg = 2\pi rT \qquad \text{...(1)}$$

Now $$m = \rho V \qquad \text{...(2)}$$

Where ρ is the density of the liquid and V is the Volume of the column of water

Since radius of the capillary is 'r'

$$V = \pi r^2 h + \text{volume in meniscus (See Fig. 5.9).}$$

and the volume of liquid in the meniscus = volume of cylinder radius 'r' & height 'r' – volume of hemisphere of radius 'r'

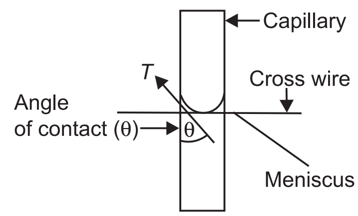


Fig. 5.9: Magnified view of the meniscus in the capillary. The position of the horizonal cross wire is also shown.

i.e. $$\text{volume in meniscus} = \pi r^3 - \left(\frac{2}{3}\right)\pi r^3$$

$$= \left(\frac{1}{3}\right)\pi r^3$$

Therefore, $$V = \pi r^2 h + \left(\frac{1}{3}\right)\pi r^3$$

$$= \pi r^2\left(h + \frac{r}{3}\right) \qquad \text{...(3)}$$

Substituting the value of V from Eq. (3) in Eq. (2) we obtain

$$m = \rho\pi r^2\left(h + \frac{r}{3}\right)$$

and so, Eq. (1) becomes

$$2\pi rT = \rho\pi r^2\left(h + \frac{r}{3}\right)g$$

or $$\boxed{T = \frac{1}{2}\rho gr\left(h + \frac{r}{3}\right)}$$

For water in C.G.S. units, $\rho = 1$ and we finally obtain the formula for a glass capillary dipping in water to be

$$T = \frac{1}{2}gr\left(h + \frac{r}{3}\right) \text{ in C.G.S. units, } i.e. \text{ dynes/c.m. at......°c}$$

Where r = radius of the capillary tube at the liquid meniscus, h = height of the liquid in the capillary tube above the free surface of liquid in the beaker.

Procedure:

1. Mount the capillary on the glass strip using the plasticine and clamp the glass strip so that the capillary is vertical. Pass the pin through the hole in the clamp and secure it so that the tip of the pin is close to the capillary and slightly above its lower end.
2. Fill the petridish with water and place it on the adustable stand just below the capillary and the tip of the pin. Then raise the stand till the capillary dips into the water and the surface of the water just touches the tip of the pin. Observing can ensure that the tip of the pin and its image in the water surface just touch one another. The apparatus will now be in the position as shown in Fig. 5.10.
3. Level the base of the traveling microscope, so that the upright is vertical. Find the least count of the traveling microscope and the room temperature.
4. Place the microscope in a horizontal position so that its objective is close to the capillary. Focus the crosswire and then move the entire microscope until the capillary is in focus. Now raise (or lower) the microscope until the meniscus is seen. The inverted image (with the curved position above) will be seen in the microscope.
5. Move the microscope and ensure that both the meniscus and the tip of the pin can be seen within the range of the vertical scale.

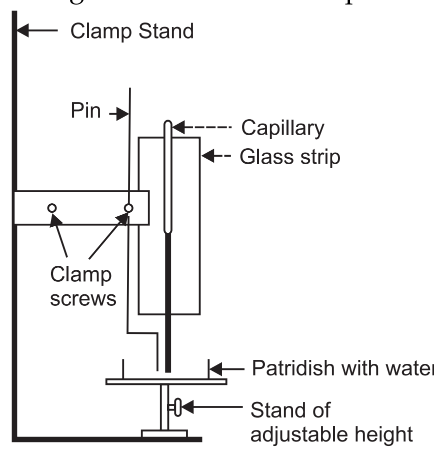


Fig. 5.10: Setup for observing the capillary rise.

6. Place the horizontal crosswire so that it is tangential to the meniscus and take the reading on the vertical scale. Repeat it four times.
7. Carefully lower and remove the petridish without disturbing the capillary or the pin. Lower the microscope until it is in front of the tip of the pin. Refocus if necessary, Now take five readings for the tip of the needle, which gives the position where the surface of the water had been earlier. Thus the difference between the readings for the meniscus and the tip of the needle gives the height of the column of water 'h'.
8. The temperature of the liquid is measured and its density ρ at this temperature is noted from the table of constants.
9. To find the radius of the capillary 'r' remove the capillary and the glass strip from the clamp and cut at the point where the meniscus. Place then horizontally so that the tip of the capillary points into the objective of the microscope. Now find the capillary internal diameter by placing the crosswire tangent in turn to opposite sides of the capillary. The position is as shown in Fig. 5.11. The horizontal and vertical diameters must each be determined ten times. This reduces error due to the small value of the diameter and the possibility of an elliptic cross section.

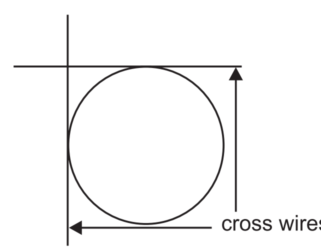


Fig. 5.11: Measurement of capillary radius.

10. The experiment is repeated for different capillary tubes.

Observations: Room temperature = _ _ _ _°C

1. Least count of the traveling microscope = =

2. Reading for the height of the column of water *'h'*

Sl. No	*Reading for Meniscus A (in c.m.)*	*Reading for Tip of needle B (in c.m.)*	*Difference 'h' (in c.m.) h = A – B*
1.			
2.			
3.			
4.			
5.			

Mean value of *'h'* = *c.m.*

3. Reading for diameter of capillary

Sl. No.	*Reading for Horizontal Diameter*			*Reading for Vertical Diameter*			*Diameter (x + y)/2 = D c.m.*	*Radius r = D/2 cm*	*Mean Radius c.m.*
	Left (c.m.)	*Right (c.m.)*	*Difference x (c.m.)*	*Upper (c.m.)*	*Lower (c.m.)*	*Difference y (c.m.)*			
1.									
2.									
3.									
4.									
5.									
6.									
7.									
8.									
9.									
10.									

Radius of the capillary *'r'* = *c.m.*

Result:

Surface tension of water at temperature _ _ _ _ °C

= dynes/cm = × N/m

Precautions:

1. The water surface and the capillary must be clean since the surface tension is affected by contamination.
2. Capillary tube must be vertical.
3. Capillary tube should be clean and dry.
4. Capillary tube should be of uniform bore.
5. Since the capillary may be conical in shape, so it would be better to break the capillary at the site of the meniscus and find the diameter at that point. However, this is not permitted in our laboratory.
6. The diameter of the tube must be measured very accurately in two perpendicular directions at the point upto which liquid rises.

Theoretical error:

$$T = r\left(h+\frac{r}{3}\right)\frac{g}{2} = \frac{gD}{4}\left(h+\frac{D}{6}\right)$$

Where D is the diameter of capillary tube.
Taking log and differentiating,

$$\frac{\delta T}{T} = \frac{\delta D}{D} + \frac{\delta h}{\left(h+\frac{D}{6}\right)} + \frac{\delta\left(\frac{D}{6}\right)}{\left(h+\frac{D}{6}\right)}$$

Maximum permissible error = %

5.10 OBJECT

To determine the surface tension of a liquid (water) by Jaeger's method.

Apparatus used: Jaeger's apparatus, a glass tube of about 5 mm diameter, a microscope, a scale, beaker, thermometer.

Formula used: The surface tension T of the liquid is given by the formula

$$T = \frac{rg}{2}(H\rho_1 - h\rho), \text{ Newton/meter}$$

where r = radius of the orifice of the capillary tube
g = acceleration due to gravity
H = maximum reading of the manometer just before the air bubble breaks away.
ρ_1 = density of the liquid in the manometer
h = depth of the tip of the capillary tube below the surface of the experimental liquid.
ρ = density of the experimental liquid.
(For water $\rho = 1.00 \times 10^3$ Kg/m^3).

Description of apparatus: The apparatus consists of a Woulf's bottle W fitted in one mouth with a bottle B containing water through a stop-cock K. The other mouth is joined to a manometer M and vertical tube BC as shown in Fig. 5.13. The end C of the tube is drawn into a fine capillary with its cut smooth. For practical purpose, a separate tube C drawn out into a fine jet is connected to the manometer by means of a short piece of Indian rubber J. The end C is kept in the experimental liquid at a known depth of a few cms. The liquid contained in the manometer is of low density in order to keep the difference in the level of liquid in the two limbs of M quite large for a given pressure difference.

Theory: Let the capillary tube be dipped in the liquid, the latter will rise in it and the shape of the meniscus will be approximately spherical. If air is forced into the glass tube by dropping water from the funnel (or burette) into the Woulf's bottle, the surface of the liquid in the capillary tube is pressed downwards and as the pressure of air inside the tube increases the liquid surface goes on sinking lower and lower until finally a hemispherical bubble of radius r equal to that of the orifice of the capillary tube protrudes into the liquid below, the pressure inside the bubble being greater than that outside by

$$p = \frac{2T}{r} \qquad ...(1)$$

where T is the surface tension of the liquid.

Suppose that the bubble is formed at the end of a narrow tube of radius r at a depth h_1 in the liquid and that the bubble breaks away when its radius is equal to the radius of the tube. The pressure outside the bubble is $P + h_1\rho_1 g$, where P is the atmospheric pressure and g is the acceleration due to gravity. If the pressure inside the bubble is measured by an open-tube manometer containing a liquid of density ρ_2 then the inside pressure is given by $P + h_2\rho_2 g$, where h_2 represents the difference in heights of the liquid in the two arms of the manometer at the moment the bubble breaks away. Therefore the excess pressure p inside the bubble is $p = (P + h_2\rho_2 g) - (P + h_1\rho_1 g) = g\,(h_2\rho_2 - h_1\rho_1)$.

Hence the surface tension T of the liquid is

$$T = \frac{gr}{2}(h_2\rho_2 - h_1\rho_1) \qquad ...(2)$$

In deriving the expression (2) we have assumed that the bubble breaks away when its radius becomes equal to that of the tube. This assumption is not quite correct and so the above expression for T is inaccurate. The correct expression for T is given by

$$T = \frac{1}{2} f(r)\ g\,(h_2\rho_2 - h_1\rho_1) \qquad ...(3)$$

where $f(r)$ is an unknown quantity having the same dimension as r. This is Jaeger's formula and is also the working formula of the present experiment. Here $f(r)$ is first obtained from the given value of the surface tension of water at room temperature using Eq. (3). With this value of $f(r)$, the surface tension of water at other temperatures are determined.

If the quantities on the right-hand side of Eq. (3) are expressed in the CGS System of Units, T is obtained in dyne/cm.

Procedure

1. Take a clean glass tube having one end into a fine jet. Hang this tube vertically inside the experimental liquid (water) from the tube B with the help of Indian rubber joint J.
2. Before proceeding with experiment the apparatus is made air tight.
3. Now the liquid in the tube BC stands at a certain level higher than that in the outer vessel. The stop cock K is opened slightly so that water slowly falls into the bottle W and forces an equal volume of air into the tube BC. The liquid in the tube BC, therefore, slowly goes down and forms a bubble at the end C. A difference of pressure between inside and outside of apparatus is at once set up which is shown by the manometer.
4. The radius of bubble gradually decreases as the inside pressure increases till it reaches the minimum value. At this stage the shape of the bubble is nearly that of a hemisphere of the radius r equal to the radius of the opening at C.
5. The difference in the level of the liquid in the two limbs of the manometer is now maximum, say, H and is noted. The bubble now becomes unstable since a small increase in the radius decreases the internal pressure necessary to produce equilibrium. As the external pressure is constant, there can be no more equilibrium state and hence the bubble breaks away.

6. With the help of scale, measure the depth of the orifice below the level of water.
7. Repeat the whole process, fixing the orifice C at different depths below the surface of water in the beaker.
8. The radius of end C is found by a microscope.

Observations: (I) Table for the measurement of H and h

S. No.	*Depth of the orifice below the level of liquid h meter*	*Manometer reading*		*H = (x ~ y) meter*	*Mean H meter*
		one limb x meter	*other limb y meter*		

(II) Temperature of water = ... C°
(III) Table for the measurement of diameter of orifice.

$$\text{Least count of the microscope} = \frac{\text{Value of one div. on main scale in cm}}{\text{Total no. of divisions on vernier scale}} = \text{...... cm}$$

S. No.	*Microscope reading along any direction (↑)*						*Difference (b ~ a) X – cms*	*Microscope reading along a perpendicular direction (→)*						*Difference (d – c) Y – cm*	*Mean diameter (x + y)/2 = D*	*Mean radius r = D/2 cm*
	Left edge of Capillary			*Right edge of Capillary*				*Lower edge of Capillary*			*Upper edge of Capillary*					
	M.S.	*V.S.*	*Total a cm*	*M.S.*	*V.S.*	*Total b cm*		*M.S.*	*V.S.*	*Total c cm*	*M.S.*	*V.S.*	*Total d cm*			
1																
2																
3																
4																

Mean r = ... cm = ... meter

Calculations: $T = \dfrac{rg}{2}(H-h)$ $\rho_1 = \rho = 100 \times 10^3$ kg/m^3, for water being in beaker and manometer

= ...... Newton/meter

Result: The surface tension of water at ...... °C = ... dyne/cm.
Standard value ...... °C = ...
Error = ...

Graphical method: Draw a graph as shown in Fig. 5.12.

The expression $T = \dfrac{rg}{2}(H-h)$ can be put in the form $H = h + \dfrac{2T}{rg}$ which represents a straight line $y = mx + C$ and hence on plotting H on y–axis and h on x-axis a straight line will be obtained, the intersection of which with y-axis will be $\frac{2T}{rg}$.

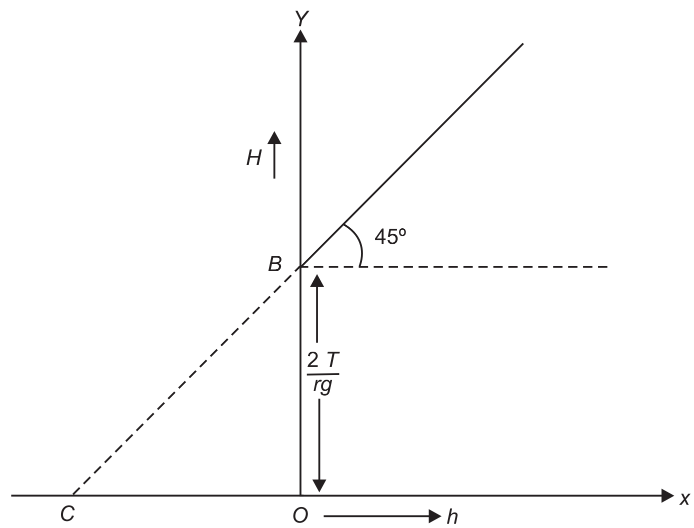


Fig. 5.12

From graph $\frac{2T}{rg} = OB$

or $T = \frac{rg}{2} \times OB$

$= ......$ Newton/meter

Sources of error and precautions

1. There should be no leakage in the apparatus. It is distinctly advantageous to have the apparatus in one piece avoiding the use of rubber joints.
2. The manometer should contain xylol so that the height H may be large, the density of xylol is less than that of water.
3. To damp the oscillations of the liquid in the manometer, its open end may be drawn out to a capillary tube.
4. Diameter of the capillary tube must be measured only at the orifice dipping in the liquid.
5. The orifice of the capillary tube should be circular and very small, about 0.3 mm in diameter, so that the maximum pressure in a bubble may occur when it is hemispherical.
6. While measuring the diameter of the orifice of the capillary tube, several observations of mutually perpendicular diameters should be taken. This reduces the error due to ellipticity of the cross-section to minimum.

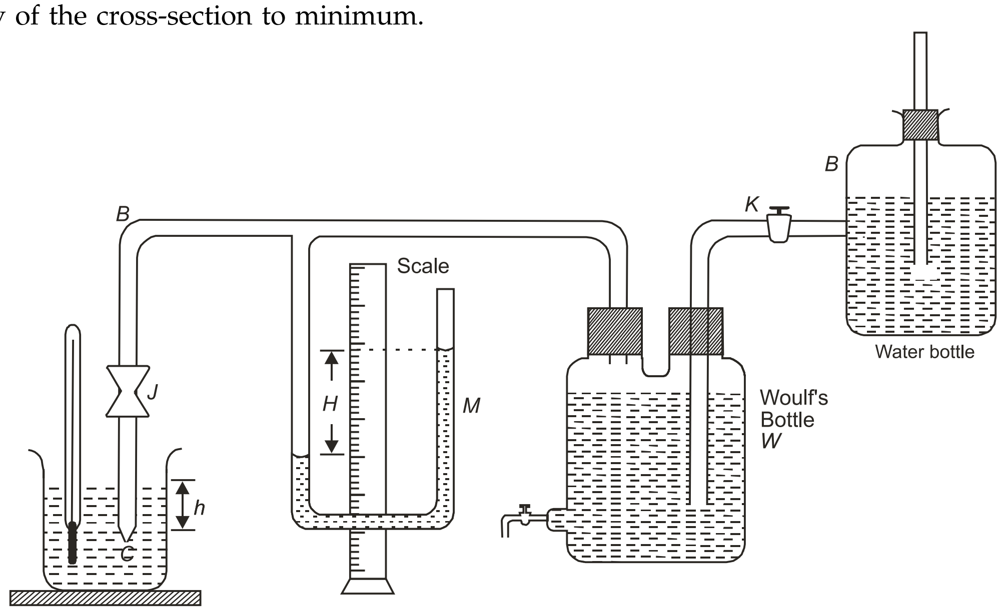


Fig. 5.13

7. In the case of a micrometer microscope error due to backlash should be avoided while taking observations for diameter.
8. The capillary tube should be clean. All traces of grease must be carefully removed as they are fatal to surface tension experiments.
9. As the surface tension of a liquid depends upon its temperature, the temperature of the experimental liquid should be recorded.
10. The bubbles should be formed singly and slowly, say at the rate of one in about ten seconds.

5.11 VIVA-VOCE

Q. 1. What do you understand by the phenomenon of surface tension?

Ans. The free surface of a liquid behaves as if it were in a state of tension having a natural tendency to contract like a stretched rubber membrane. This tension or pull in the surface of a liquid is called surface tension.

Q. 2. How do you define surface tension?

Ans. When a straight line of length unity is drawn on the liquid surface, the portions of the surface on both sides of the line tend to draw away from each other with a force which is perpendicular to the line and tangential to the liquid surface. This force is called surface tension.

Q. 3. What are the units and dimensions of surface tension?

Ans. The C.G.S units of surface tension is dyne/cm and its S.I. unit is Newton/m.

Q. 4. What do you mean by 'cohesive' and 'adhesive forces'?

Ans. The force of attraction between the molecules of the same substance is known as cohesive force while the force of attraction between the molecules of different substances is known as adhesive force.

Q. 5. What do you mean by surface energy of a liquid?

Ans. Whenever a liquid surface is enlarged isothermally, certain amount of work is done which is stored in the form of potential energy of the surface molecule. The excess of potential energy per unit area of the molecules in the surface is called surface energy.

Q. 6. What do you mean by angle of contact?

Ans. The angle between the tangent of the liquid surface at the point of contact and solid surface inside the liquid is known as angle of contact between the solid and the liquid.

Q. 7. What is the value of angle of contact of (*i*) water and glass, and (*ii*) mercury and glass?

Ans. (*i*) 8° and (*ii*) 135°

Q. 8. What is the effect of temperature on surface tension?

Ans. It decreases with rise in temperature and becomes zero at the boiling point of the liquid.

Q. 9. What is the effect of impurities on surface tension?

Ans. The soluble impurities generally increases the surface tension of a liquid while contamination of a liquid surface by impurities (dust, grease etc) decreases the surface tension.

Q. 10. Do you think that surface tension depends only upon the nature of the liquid?

Ans. No, this also depends upon the nature of surrounding medium.

Q. 11. Why does water rise in the capillary tube?

Ans. The vertical component of the surface tension acting all along the circle of contact with the tube ($2\pi rT\cos\theta$) pulls the water up.

Q. 12. How far does the water rise?

Ans. The water in the capillary tube rise only upto a height till the vertical component of surface tension is balanced by weight of water column in capillary.

Q. 13. On what factors the rise of water depend?

Ans. It depends upon: (*i*) surface tension of water (*ii*) Angle of contact and (*iii*) Radius of the tube.

Q. 14. Is it that the liquid always rises in a capillary?

Ans. No, the level of mercury in a capillary tube will be depressed.

Q. 15. How will the rise be affected by (*a*) using capillary of non-uniform bore, (*b*) changing radius of the capillary (*c*) change in shape of the bore.

Ans. (*a*) No effect since the rise of liquid in the capillary depends upon the radius of the tube at the position of the meniscus. Theoretically there is no difficulty if we use a conical capillary.

(*b*) The capillary exent is inversely proportional to the radius, hence change in radius will affect the height of liquid column.

(*c*) No effect.

Q. 16. In the experiment of capillary rise where do you measure the radius of the tube and why?

Ans. The radius is measured at the position of the meniscus because the force of surface tension balancing the water column corresponds to this radius. If the capillary is of uniform bore, then r can be measured at the end.

Q. 17. Will water rise to the same height if we use capillaries of the same radius but of different materials?

Ans. Since surface tension of a liquid depends upon the surface in contact, therefore, in capillaries of different materials, the rise of water will be different.

Q. 18. In your formula for calculating the value of surface tension you have added $\frac{r}{3}$ to the observed height '*h*'. Why have you done so?

Ans. In the experiment '*h*' is measured from free surface of water to the lowest point of the meniscus. To take into account the weight of the liquid above this point. We add this correcting factor $\frac{r}{3}$.

Q. 19. What will be the effect of inclining the tube?

Ans. The vertical height of column will be the same. If l is the length of column. α the angle that capillary makes with vertical and h the vertical height, $l\cos\alpha = h$.

Q. 20. What happens when a tube of insufficient height is dipped in a liquid? Does the liquid over-flow, How is the equilibrium established in this case?

Ans. No, the liquid will not over-flow. The liquid rises to the top of the tube, slightly spreads itself there and adjusts its radius of curvature to a new value R' such that $R'l = Rh$ where l is the length of the tube of insufficient height, R its radius and h the height of column as demanded by formula. In this case $R' > R$ and meniscus becomes less concave.

Q. 21. How will the rise of liquid in a capillary tube be affected if its diameter is halved?

Ans. The height to which the liquid rises, will be doubled.

Q. 22. Why do you measure the diameter in two mutually perpendicular directions?

Ans. This minimises the error due to ellipticity of the bore of the capillary.

Q. 23. Why should the capillary be kept vertical while measuring capillary ascent?

Ans. If the tube is not kept vertical the meniscus will become elliptical and the present formula will not hold.

Q. 24. Should the top of the capillary tube be open or closed?

Ans. It should be open.

Q. 25. Is there any harm if the top of the capillary is closed?

Ans. The rise of water will not be completed as the air above water presses it.

Q. 26. What may be the possible reason if quite low value of surface tension is obtained by this method?

Ans. It may be due to contamination of the capillary with grease or oil.

Q. 27. How can you test that the tube and the water are not contaminated?

Ans. The lower end of the capillary is dipped to a sufficient depth inside water. The water rises in the capillary. The capillary is then raised up. If water falls rapidly as the tube is raised, this shows that water and capillary tube are not contaminated.

Q. 28. For what type of liquids is this method suitable?

Ans. Which wet the walls of the tube and for which the angle of contact is zero.

Q. 29. Can you study the variation of surface tension with temperature by this method?

Ans. No, as the temperature of liquid at the meniscus can not be determined with the required accuracy.

Q. 30. What type of capillary tube will you choose for this experiment?

Ans. A capillary of small circular bore should be chosen.

Q. 31. Water wets glass but mercury does not why?

Ans. The cohesive forces between the water molecules are less than the adhesive forces between the molecules of water and the glass. On the other hand, the cohesive forces between the mercury molecules are larger than the adhesive forces existing between the molecules of the mercury and the glass. That is why water wets glass but mercury does not.

Q. 32. What is the principle underlying Jaeger's method?

Ans. This method is based on the fact that there is always an excess of pressure inside a spherical air bubble formed in a liquid. The excess of pressure $P = \frac{2T}{r}$ where T is surface tension of liquid and r is the radius of the bubble.

Q. 33. Does the excess of pressure depend upon the depth of the orifice, below the surface of experimental liquid?

Ans. No, it depends upon the surface tension of the liquid and radius of bubble.

Q. 34. After breaking, does the size of the bubble remain same as it is near the surface of the liquid.

Ans. No, the size goes on increasing because hydrostatic pressure goes on decreasing as bubble comes near the surface.

Q. 35. Do you think that the radius of the bubble, as considered here, is always equal to the radius of the orifice?

Ans. No, this is only true when the radius of the orifice of capillary tube is very small.

Q. 36. What should be the rate of formation of bubbles?

Ans. The rate of formation of bubble should be very slow, i.e., one bubble per ten second.

Q. 37. What type of capillary will you choose for this experiment?

Ans. We should use a capillary of small circular bore.

Q. 38. What is the harm if the tube is of larger radius?

Ans. When the tube is of large radius the excess of pressure inside the bubble will be small and cannot be measured with same degree of accuracy.

Q. 39. Will there be any change in the manometer reading if the depth of the capillary tube below the surface of liquid is increased?

Ans. Yes, it will increase.

Q. 40. At what position of the levels should the reading be noted?

Ans. The reading should be taken when the manometer shows the greatest difference of pressure because at this moment the bubble breaks.

Q. 41. Can you study the variation of surface tension with temperature with this apparatus?

Ans. Yes, the experimental liquid is first heated and filled in bottle. Now the manometer readings are taken at different temperatures.

Q. 42. Mention some phenomena based on 'surface tension'.

Ans. (*i*) Calming of waves of oil, (*ii*) floating of thin iron needle on water, (*iii*) gyration of camphor on water, etc.

Q. 43. Why is Xylol used as a manometric liquid in preference to water?

Ans. The density of Xylol is lower than that of water. This gives a larger difference of levels in the two limbs of the manometer tube.

Q. 44. Why is the open end of the manometer drawn out into a capillary?

Ans. To minimise the oscillations of the manometric liquid due to surface tension.

Q. 45. At what temperature surface tension is zero.

Ans. At critical temperature.

EXERCISE

Q. 1. Why does liquid surface behave like a stretched rubber membrane?
Q. 2. Do you know any other form of the definition of surface tension?
Q. 3. How do you say that work is done in enlarging the surface area of a film?
Q. 4. Distinguish between surface tension and surface energy?
Q. 5. Mercury does not stick to the finger but water does, explain it?
Q. 6. What is the shape of a liquid surface when it is in contact with a solid?
Q. 7. When will the meniscus be convex or concave and why?
Q. 8. Is there any practical use of the property of surface tension?
Q. 9. Can't you use a wider glass tube instead of a capillary tube? if not why?
Q. 10. How do you explain the shape of meniscus of a liquid in the capillary tube?
Q. 11. Why is it difficult to introduce mercury in a capillary tube?
Q. 12. How do you measure the radius at the meniscus?
Q. 13. How will you clean the capillary tube?
Q. 14. Why is the pressure inside an air-bubble greater than that outside it.
Q. 15. Why does the apparatus be in one piece.

Q. 16. Why does the liquid in the manometer rise and fall during the formation and breaking away of the bubble?

Q. 17. How can you damp the oscillations of liquid in the manometer?

Q. 18. How will you make sure that the bubbles are being formed at the same depth?

Q. 19. Why should the bubbles be formed singly and slowly? How will you accomplish this adjustment?

Q. 20. In measuring the diameter of the tube, why do you take readings along two mutually perpendicular diameters?

Q. 21. What are the advantages of this method?

Q. 22. How can variation of surface tension with temperature be studied with the help of this apparatus?

6
Viscosity

6.1 IDEAL LIQUID

An ideal liquid is one which has the following two properties:

(*i*) **Zero compressibility:** An ideal liquid is incompressible, that is, on pressing the liquid there is no change in its volume (or density). Most of the liquids may be considered approximately incompressible, because on pressing them the change in their volume is negligible. For example, on pressing water by one atmospheric pressure its volume changes only by a fraction of 0.000048.

(*ii*) **Zero viscosity:** An ideal liquid is non-viscous, that is, when there is a relative motion between different layers of the liquid then there is no tangential frictional-force in between the layers. In actual practice, however, there is some viscosity in all liquids (and gases). It is less in gases; but larger in liquids.

6.2 STREAM-LINED FLOW

When a liquid flows such that each particle passing a certain point follows the same path as the preceding particles which passed the same point, the flow is said to be 'stream-lined' and the path is called a 'stream-line'. The tangent drawn at any point on the stream-line gives the direction of the velocity of the liquid at that point. Clearly, two stream-lines cannot intersect each other, if they would do so then there will be two directions of the velocity of the liquid at the point of intersection which is impossible.

The important property of a stream-line is that the velocity of the liquid can be different at different points of the stream line, but at a particular point the velocity remains constant. In Figure 6.1; A, B and C are three points on the same stream-line of a flowing liquid. All those particles of the liquid which pass through A, also pass through B and C. The velocities of the liquid at these points are v_1, v_2 and v_3, that is, different. But at one and the same point, such as A, the velocity is always v_1 whichever be the particle of the liquid at this point. The same is true for B and C also.

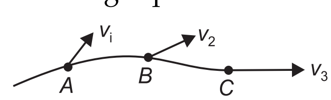


Fig. 6.1

6.3 PRINCIPLE OF CONTINUITY

It states that, when an incompressible and non viscous liquid flows in stream-lined motion through a tube of non-uniform cross-section, then the product of the area of cross-section and the velocity of flow is same at every point in the tube.

Let us consider a liquid flowing in stream-lined motion through a non-uniform tube *XY*. Let A_1 and A_2 be the cross-sectional areas of the tube and v_1 and v_2 the velocities of flow at *X* and *Y* respectively. Let ρ be the density of the liquid.

The liquid entering the tube at *X*-covers a distance v_1 in 1 second. Thus the volume of the liquid entering at the end *X* in 1 second

$$= A_1 \times v_1$$

∴ mass of the liquid entering at the end *X* in 1 second = $\rho A_1 v_1$

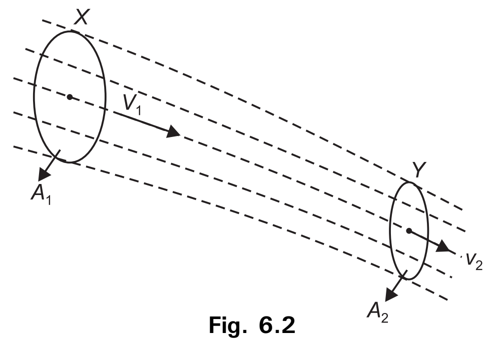


Fig. 6.2

Similarly, mass of the liquid coming out from the end *Y* in 1 second = $\rho A_2 v_2$

But the liquid which enters at one end must leave at the other. Hence both these masses are equal, that is,

$$\rho A_1 v_1 = \rho A_2 v_2$$
$$A_1 v_1 = A_2 v_2$$
$$\boxed{Av = \text{constant}}$$

Thus, at every place in the tube the product of the area of cross-section of the tube and the velocity of flow of the liquid is a constant. Therefore, the velocity of the liquid is smaller in the wider parts of the tube and larger in the narrower parts figure 6.3.

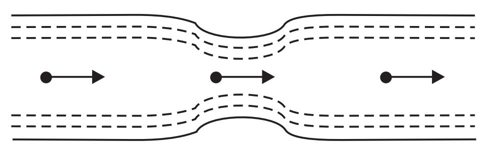


Fig. 6.3

6.4 ENERGY OF A FLOWING LIQUID

There are following three types of energies in a flowing liquid.

(*i*) **Pressure Energy.** If P is the pressure on an area A of a liquid, and the liquid moves through a distance l due to this pressure, then

pressure energy of liquid = work done = force × distance

= pressure × area × distance = $P \times A \times l$

The volume of the liquid is $A \times l$ (area × distance).

∴ pressure energy per unit volume of the liquid = $\dfrac{P \times A \times l}{A \times l} = P$

(*ii*) **Kinetic Energy.** If a liquid of mass m and volume V is flowing with velocity v, then its kinetic energy is $\dfrac{1}{2} mv^2$.

∴ Kinetic energy per unit volume of the liquid = $\dfrac{1}{2}\left(\dfrac{m}{V}\right)v^2 = \dfrac{1}{2}\rho v^2$

where ρ is the density of the liquid.

(*iii*) **Potential Energy.** If a liquid of mass m is at a height h from the surface of the earth, then its potential energy is mgh.

$\therefore$ potential energy per unit volume of the liquid = $\left(\frac{m}{V}\right)gh = \rho gh$

6.5 BERNOULLI'S THEOREM

When an incompressible and non-viscous liquid (or gas) flows in stream-lined motion from one place to another, then at every point of its path the total energy per unit volume (pressure energy + kinetic energy + potential energy) is constant. That is

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Thus, Bernoulli's theorem is in one way the principle of conservation of energy for a flowing liquid (or gas). In this equation the dimensions of each term are the same as of pressure.

P + rgh is called static pressure and $\frac{1}{2}\rho v^2$ is called 'dynamic pressure'.

6.6 VELOCITY OF EFFLUX

Let a vessel be filled with a liquid upto a height H and let there be an orifice at a depth h below the free surface of the liquid. The pressure at the free surface of the liquid and also at the orifice is atmospheric, and so there will be no effect of atmospheric pressure on the flow of liquid from the orifice. The liquid on the free surface has no kinetic energy, but only potential energy, while the liquid coming out of the orifice has both the kinetic and potential energies.

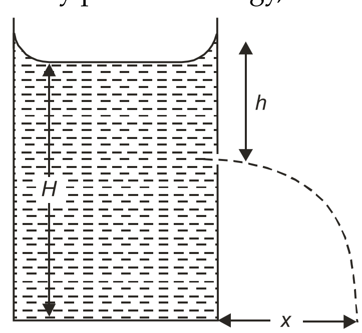


Fig. 6.4

Let P be the atmospheric pressure, ρ the density of the liquid and v the velocity of efflux of the liquid coming out from the orifice. According to Bernoulli's theorem, the sum of the pressure and the total energy per unit volume of the liquid must be the same at the surface of the liquid and at every point of the orifice. Thus

$$P + 0 + \rho gH = P + \frac{1}{2}\rho v^2 + \rho g(H - h)$$

$$\frac{1}{2}\rho v^2 = \rho gh$$

$$v = \sqrt{2gh}$$

This formula was first established in 1644 by Torricelli and is called '*Torricelli's theorem*'.

If a body is dropped freely ($u = 0$) from a height h, then from the third equation of motion, $v^2 = 2gh$, we have

$$v = \sqrt{2gh}$$

Clearly, the velocity of the liquid falling from a height h is $\sqrt{2gh}$. Hence the velocity of efflux of a liquid from an orifice is equal to that velocity which the liquid acquires in falling freely from the free surface of the liquid upto the orifice.

After emerging from the orifice the liquid adopts parabolic path. If it takes t second in falling through a vertical distance ($H - h$), then according to equation $s = \frac{1}{2}at^2$, we have

$$(H - h) = \frac{1}{2}gt^2$$

$$\therefore \qquad t = \sqrt{2(H-h)/g}$$

Since there is no acceleration in the horizontal direction, the horizontal velocity remains constant, the horizontal distance covered by the liquid is

$$\begin{aligned} x &= \text{horizontal velocity} \times \text{time} \\ &= v \times t \\ &= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} \\ &= 2\sqrt{h(H-h)} \end{aligned}$$

This formula shows that whether the orifice in the vessel is at a depth 'h' or at a depth ($H - h$) from the free surface of the liquid, the emerging liquid will fall at the same distance *i.e.* the range x of the liquid will remain the same.

Now, h ($H - h$) will be maximum when $h = H - h$ *i.e.* $h = \frac{H}{2}$

Hence the maximum range of the liquid is given by

$$x_{\max} = 2\sqrt{\left(\frac{H}{2}\right) \times \left(H - \frac{H}{2}\right)} = H$$

Therefore, when the orifice is exactly in the middle of the wall of the vessel, the stream of the liquid will fall at a maximum distance (equal to the height of the liquid in the vessel).

6.7 VISCOSITY

When a solid body slides over another solid body, a frictional force begins to act between them. The force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. The force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB. The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the figure, the lengths of the arrow represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c. Thus each layer tends to decrease the velocity of the layer above it. Similarly , each layer tends to increase the velocity of the layer below it. This means that in

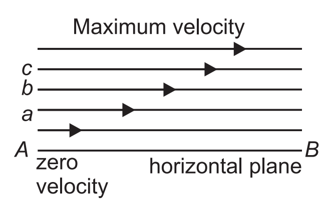


Fig. 6.5

between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called viscous forces. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.

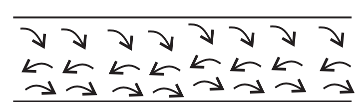
The property of viscosity is seen in the following examples.

(*i*) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerine, etc have a larger viscosity than thinner ones like water. If we pour coaltar and water on to a table, the coaltar will stop soon while the water will flow upto quite a larger distance.
(*ii*) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.
(*iii*) We can walk fast in air, but not in water.
(*iv*) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky. Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

6.8 CRITICAL VELOCITY

When a liquid flows in a tube, the viscous forces oppose the flow of the liquid. Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes *zig-zag* and eddy-currents are developed in it.

Reynold proved that the critical velocity for a liquid flowing in a tube is $v_c = \dfrac{k\eta}{\rho a}$ where ρ is density, η is viscosity of the liquid, a is radius of the tube and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity, the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedly inspite of being very thick (of large viscosity).



6.9 VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

Suppose a liquid is flowing in stream-lined motion on a horizontal surface *OX*. The liquid layer in contact with the surface is at rest while the velocity of other layers increases with increasing distance from the surface *OX*. The highest layer flows with maximum velocity. Let us consider two parallel layers *PQ* and *RS* at distances z and $z + dz$ from *OX*. Let v_x and $v_x + dv_x$ be their velocities in the direction *OX*. Thus the change in velocity in a perpendicular distance dz is dv_x. That is, the rate of change of velocity with distance perpendicular to the direction of flow is $\frac{dv_x}{dz}$. This is called 'velocity gradient.'

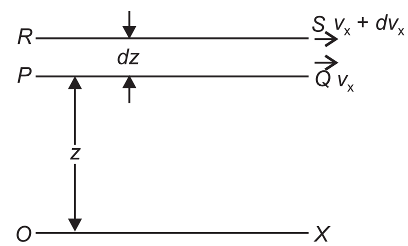


Fig. 6.6

Now let us consider a liquid layer of Area A at a height z above *OX*. The layer of the liquid immediately above it tends to accelerate it with a tangential viscous force F, while the layer immediately below it tends to retard it backward with the same tangential viscous force F. According to Newton, the viscous force F acting between two layers of a liquid flowing in stream-lined motion depends upon two factors:

(*i*) It is directly proportional to the contact-area A of the layer ($F \propto A$).

(*ii*) It is directly proportional to the velocity-gradient $\frac{dv_x}{dz}$ between the layers $\left(F \propto \frac{dv_x}{dz}\right)$.

Combining both these laws, we have,

$$F \propto A\frac{dv_x}{dz}$$

$$F = \pm\,\eta A\frac{dv_x}{dz}$$

Where η is a constant called 'coefficient of viscosity' of the liquid. In this formula if $A = 1$ and $\frac{dv_x}{dz} = 1$, the $\eta = \pm F$. Thus, the coefficient of viscosity of a liquid is defined as the viscous force per unit area of contact between two layers having a unit velocity gradient between them.

In the above formula, ± sign indicate that the force F between two layers is a mutual-interaction force. On the layer A, the layer above it exerts a force in the forward direction while the layer below it exerts an equal force in the backward direction.

Dimensions and unit of coefficient of viscosity: From the above formula, we have

$$\eta = \frac{F}{A\left(\frac{dv_x}{dz}\right)}$$

$$\therefore \qquad \text{dimensions of } \eta = \frac{[MLT^{-2}]}{[L^2]\left[\frac{LT^{-1}}{L}\right]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second)

6.10 EFFECT OF TEMPERATURE ON VISCOSITY

The viscosity of liquids decreases with rise in temperature. On the other hand, the viscosity of gases increases with rise in temperature.

6.11 POISEUILLE'S FORMULA

Poiseuille obtained a formula for the rate of flow of a liquid through a narrow horizontal tube of uniform cross-section under a constant pressure difference between the ends of the tube under the following assumptions:

(*i*) The flow of liquid is stream-lined and the stream-lines are parallel to the axis of the tube.
(*ii*) There is no radial flow of the liquid i.e. the pressure over any cross-section of the tube is constant.
(*iii*) The liquid in contact with the walls of the tube is at rest (no slip).

These conditions require that the pressure difference across the tube should be small and the tube must be of narrow bore (so that vertical hydrostatic pressure may be neglected). These assumptions also ensure that at the same distance from the axis the velocity of flow is the same.

Let the length of the tube be L and the radius of bore be R. Now, consider a cylindrical layer of liquid coaxial with the tube having internal and external radii r and $r + dr$ respectively (Fig. 6.7).

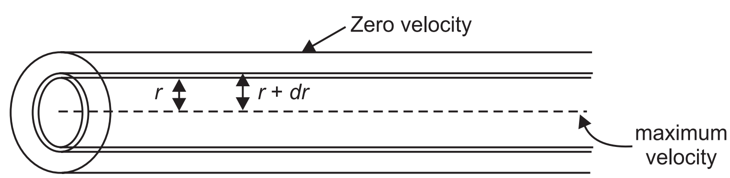


Fig. 6.7

Let v be the velocity of flow at this layer and $\frac{dv}{dr}$ the velocity gradient here. The area of this layer is $A = 2\pi rL$. Now the liquid inside this cylindrical layer is moving faster than the liquid outside it, hence according to Newton's law of viscous drag, the viscous drag on this layer of liquid will be

$$F = \eta A \frac{dv}{dr} = -2\pi rL\eta \frac{dv}{dr}$$

The viscous drag on a layer at distance $r + dr$ from the axis will be

$$F + \frac{dF}{dr} \cdot dr$$

Therefore, if we consider the cylindrical layer of liquid between radii r and $r + dr$ its inner surface is pulled forward by a force F while the outer surface is dragged backward by a force $F + \frac{dF}{dr}\,dr$ the resultant dragging force on this cylindrical shell is $\left(F + \frac{dF}{dr}\,dr\right) - F = \frac{dF}{dr} \cdot dr$

In steady flow the velocity of the shell is constant, this means that the viscous drag must be balanced by the driving force due to pressure P across the ends of the tube. This driving force on the shell will be

$$= \text{Area of cross-section of shell} \times \text{pressure}$$
$$= 2\pi r\, dr\, P$$

Hence for steady flow $\frac{dF}{dr} \cdot dr = 2\pi r\, dr\, P$

or $$\frac{d}{dr}\left(-2\pi r\, L\eta \frac{dv}{dr}\right) dr = 2\pi r\, dr\, P$$

Since pressure P is constant, independent of r, integrating above equation under the condition that at $r = 0$, $\frac{dv}{dr} = 0,$ we get

$$2\pi r L\eta \frac{dv}{dr} = -\pi r^2 P$$

or $$dv = -\frac{P}{2L\eta} r\, dr$$

Integrating this equation we get

$$v = -\frac{P}{2L\eta}\frac{r^2}{2} + c$$

Where c is constant of integration, the initial condition to determine it is that at r = R, $v = 0$, so that

$$0 = -\frac{P}{2L\eta} \cdot \frac{R^2}{2} + c$$

or $$c = \frac{P}{4L\eta} R^2$$

Hence $$v = \frac{P}{4L\eta}\left(R^2 - r^2\right)$$

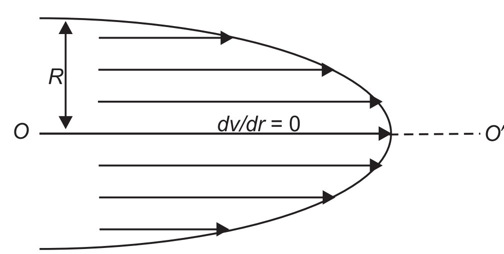


Fig. 6.8

This gives the distribution of velocity of flow of liquid in steady state at any distance r from the axis of the tube. The profile of velocity distribution is parabola Fig. 6.8 with vertex on the axis of the tube, which advances onward.

The velocity of flow cannot be measured conveniently, hence we determine the rate of flow of liquid through entire cross-section of the tube. The volume of liquid flowing per second through the cylindrical shell of radii r and $r + dr$ considered above is

$$dQ = 2\pi r\, dr\, v$$
$$= 2\pi r dr \times \frac{P}{4L\eta}\left(R^2 - r^2\right)$$

The rate of flow of liquid through entire cross-section of the tube is obtained by integrating this expression between the limits $r = 0$ to $r = R$. Thus.

$$Q = \int_0^R dQ = \int \frac{2\pi P}{4L\eta}\, r\,(R^2 - r^2)\, dr = \frac{2\pi P}{4L\eta}\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{2\pi P}{4L\eta}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi P R^4}{8L\eta}$$

Thus knowing the rate of flow of liquid through the capillary tube under pressure P and all the other quantities we can calculate the value of η, the coefficient of viscosity of the liquid.

The Poiseuille's formula determined above has the following limitations:

(*i*) It is true for stream-line flows only. Therefore, the velocity of flow should be below the critical velocity which is inversely proportional to the radius of the tube. Thus the formula is valid only for capillary tubes and not for the tubes of wider bore.

(*ii*) The formula holds only for small pressure differences applied across the tube. In this case the flow of liquid is slow and kinetic energy is small. Hence the force due to pressure difference is almost completely used up in overcoming the viscous drag. Usually a portion $\frac{Q^2 d}{\pi^2 R^4}$, where d is the density of liquid, is used up in providing kinetic energy to the following liquid. Thus actual driving pressure is only $P \simeq \frac{Q^2 d}{\pi^2 R^2}$ and the liquid emerges out at the outlet end with appreciable velocity and does not just trickle out.

(*iii*) The flow is accelerated near the entrance and becomes steady only after a certain distance from the inlet end. This error can be corrected by taking the length of tube as $(L + 1.64R)$ instead of L. Hence tubes of longer length will give better results.

(*iv*) The formula does not hold for gases.

6.12 STOKE'S LAW FOR VISCOUS DRAG ON MOVING BODIES

When a small body is allowed to move through a viscous fluid, the layers of the fluid in contact with the body move with the velocity of the moving body, while those at large distance from it are at rest. Thus the motion of a body through the fluid creates relative motion between different layers of the fluid near it. Viscous forces are developed which oppose the relative motion between different layers of the fluid and hence the medium opposes the motion of the body. This opposing force increases with the velocity of the body. Stoke has shown that for a small sphere of radius r moving slowly with velocity v through a homogeneous fluid of infinite extension, the viscous retarding force is given by

$$F = 6\pi\eta r v$$

Where η is the coefficient of viscosity of the fluid.

It is to be noted that an equilibrium is established when this viscous drag on the body is balanced by driving force. In that case net force on the body becomes zero and it starts moving with constant velocity called its terminal velocity.

Calculation of terminal velocity: Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v, it is subjected to two forces.

(*i*) effective force acting downward = $V(\rho - \sigma)g = \frac{4}{3}\pi r^3(\rho-\sigma)g$

(*ii*) viscous force acting upward = $6\pi\eta\, rv$

$\therefore$ Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

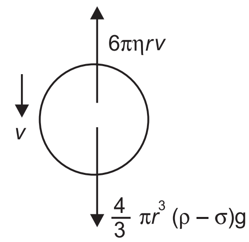


Fig. 6.9

$$6\pi\eta\, rv = \frac{4}{3}\pi r^3(\rho-\sigma)g$$

$$v = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$$

Thus terminal velocity of the ball is directly proportional to the square of its radius.

6.13 EFFECT OF VARIOUS FACTORS ON VISCOSITY OF FLUIDS

(*i*) **Temperature:** The coefficient of viscosity of liquids decreases rapidly with rise in temperature. The effect is so marked that it would be practically meaningless to state the value of viscosity of a liquid without mentioning the temperature.

On the other hand, in case of gases, the viscosity increases with the rise in temperature. This can be explained on the basis of the kinetic theory of gases.

(*ii*) **Pressure:** The coefficient of viscosity of liquids, in general, increases with increase of pressure. However, in case of water there is a decrease in viscosity for the first two hundred atmospheric pressure. The increase of viscosity with pressure is much more in case of very viscous liquids than in case of fairly mobile liquids. The coefficient of viscosity of all gases increases with increase of pressure. At moderate pressures the coefficient of viscosity is independent of pressure. At low pressure it is proportional to pressure. At very high pressure, the coefficient of viscosity of gases increases with increase with pressure.

(*iii*) **Impurity:** The viscosity of a liquid is also sensitive to impurities. For solutions in some cases the coefficient of viscosity is less than that of the pure solvent while in other cases it is greater. There is no set rule for this change. In case of mixtures the coefficient of viscosity is generally less than the arithmetic mean of the coefficients of viscosity of the components of the mixture.

6.14 OBJECT

Determination of the viscosity of water by method of capillary flow. [Poiseuilles method]

Apparatus used: Capillary tube fitted on a board with a manometer and side tubes, constant level reservoir, measuring cylinder, a stopwatch traveling microscope.

Formula used: The coefficient of viscosity η of a liquid is given by the formula.

$$\eta = \frac{\pi PR^4}{8\theta\ell} = \frac{\pi(h\rho g)}{8Q}R^4 \text{ kg/(m - sec) or poise}$$

Where R = radius of the capillary tube

Q = volume of water collected per second
= length of the capillary tube
ρ = density of liquid ($\rho = 1 \times 10^3$ kg / m^3 for water)
h = difference of levels in manometer

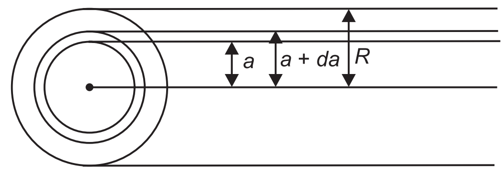


Fig. 6.10

Description of the apparatus: The apparatus used is shown in the Fig. 6.11. Water from the constant level reservoir flows to the union *X*; thence through a capillary tube of known length of a graduated jar. From the unions *X* and *Y* two pieces of rubber tubing make connections to the manometer. The difference of the levels *E* and *F* gives the value of the pressure difference between the ends of the capillary tube *K*.

Manipulations:

1. Arrange the flow of water in such a way that the emergent water is a slow trickle or a succession of drops. This is done to ensure streamlined motion.

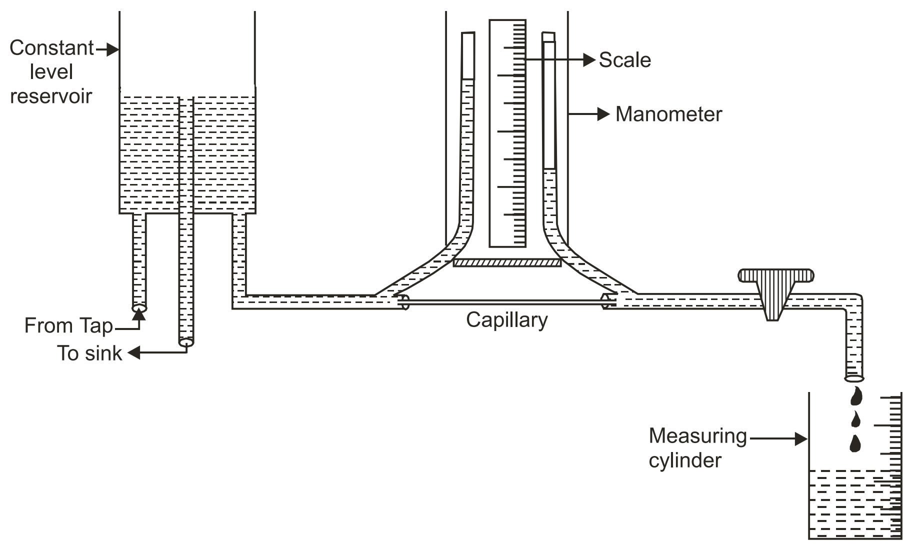


Fig. 6.11

2. When every thing is steady, collect water for two minutes in the graduated jar. Note the quantity collected. From this find Q, the amount of water passing per second.
3. Find the difference in the level of the water in the manometer and from this calculate 'P'.
4. Vary the flow of the water slightly by raising or lowering the reservoir and when every thing is steady, repeat (2) and (3). Thus make 5 sets of Q and P. Take the mean of $\frac{P}{Q}$.
5. Measure the length 'l' of the tube K and also internal diameter of the sample provided.
6. Draw a graph between h and Q and find the value of $\left(\frac{h}{Q}\right)$ from the graph.

Observation: *Length of the tube =*

Sl. No.	Time (in sec.)	Volume of water collected (in cc)	Q (cc/sec.)	Manometer Reading		Pressure Difference h (in cm)	h/Q c.m.$^{-2}$ Sec.
				Left	Right		
1.							
2.							
3.							
4.							
5.							

Mean of $\frac{h}{Q}$ =

Diameter of capillary: Take mean of the ten different sets of readings (each set consists of diameters perpendicular to each other).

Sl. No.	Horizontal Diameter			Vertical Diameter					
	One end reading a	Second end reading b	difference X = a ~ b (in cm)	One end reading c	Second end reading d	Difference Y = C ~ d (in cm)	Diameter d = (X + Y)/2 (in cm)	Mean diameter d (in cm)	Mean radius R = d/2 (in cm)
1.									
2.									
3.									
4.									
5.									
6.									

Radius of capillary =

Result: The coefficient of viscosity of water at = °C =
Standard value η = poise
Percentage error = %

Precautions:
1. The tube should be placed horizontally to avoid the effect of gravity.
2. The diameter should be measure very accurately.
3. The difference of the pressure should be kept constant during the time of one set of the observation.
4. The ratio of flow through the capillary should be very small.
5. There should be no air bubble in the apparatus.
6. The pressure difference should be kept small to obtain stream line motion.

Theoretical error:

$$\eta = \frac{\pi\rho g}{8}\,\frac{\left(\frac{d}{2}\right)^4 h t'}{Q'}$$

Where Q' in the volume of water collected in t sec, and d is diameter of the capillary Taking log and differentiating.

$$\frac{\delta\eta}{\eta} = \frac{4\delta d}{d} + \frac{\delta h}{h} + \frac{\delta t}{t} + \frac{\delta l}{l} + \frac{\delta Q'}{Q'}$$

Maximum permissible error = %
From graph $\eta =$

6.15 ROTATING CYLINDER METHOD

The value of the coefficient of viscosity of liquids like glycerine, water and of air is conveniently determined by means of a rotating cylinder. The apparatus consists of two coaxial metal cylinders. The outer cylinder is clamped on a turn-table which may be rotated with constant angular velocity by means of an electric motor. The inner cylinder is suspended coaxially with the outer cylinder by a phosphor-bronze suspension which carries a mirror to measure the angle of rotation of the inner cylinder by means of telescope and scale arrangement. The space between the inner and outer cylinders is filled with the experimental liquid. When the outer cylinder is set into rotation with constant angular velocity, a couple G is transmitted to the inner cylinder due to the intervening liquid. The couple rotates the inner cylinder through such an angle that the torsional restoring couple just balances the turning moment G due to the liquid.

Let R_1 and R_2 be the radii of the inner and the outer cylinder respectively and the inner cylinder be submerged to a length l. If ω_0 be the angular velocity of the outer cylinder, then the layer in contact with it moves with the same velocity whereas the layer in contact with the inner cylinder is at rest. Thus a relative motion is created between the different layers of the liquid and hence of forces of viscosity are called into play.

Consider a cylindrical layer at a distance r from the axis of rotation, moving with an angular velocity ω, then its linear velocity is

$$v = r\omega$$

Let the angular velocity of rotation increase by $d\omega$ across the cylindrical shell between radii r and $r + dr$ as shown in the Fig. 6.12. Then across the cylindrical shell.

$$\text{Velocity gradient} = \frac{dv}{dr} = \frac{d}{dr}(r\omega) = \omega + r\frac{d\omega}{dr}$$

Since the first term ω on the right-hand side of this equation represents the angular velocity which the layer would have if the fluid were rotating like a rigid body, it is evident that the velocity gradient responsible for producing viscous drag on the layer under reference is $r\dfrac{d\omega}{dr}$.

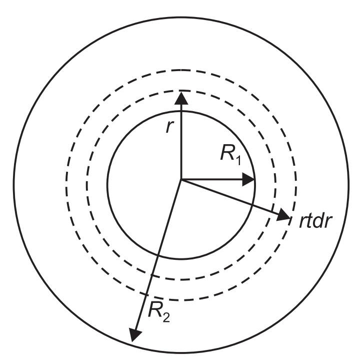


Fig. 6.12

According to Newtons law the viscous forces acting tangentially at the surface of the cylindrical shell of liquid is

$$F = -\eta 2\pi rl \cdot r \cdot \frac{d\omega}{dr} = -2\pi\eta l \cdot r^2 \cdot \frac{d\omega}{dr}$$

Taking the moment of this force about the central axis we get

$$G = F \cdot r = -2\pi\eta l \cdot r^2 \cdot \frac{d\omega}{dr} \cdot r$$

$$= -2\pi\eta l \cdot r^3 \cdot \frac{d\omega}{dr}$$

When the steady state is reached this couple G is constant throughout the liquid and must be equal in magnitude and opposite in sense to the steady external couple acting on the inner cylinder due to twist in the suspension wire.

The above equation can be written as

$$G\frac{dr}{r^3} = 2\pi\eta l d\omega$$

Integrating the above equation we have

$$\int G\frac{dr}{r^3} = 2\pi\eta l d\omega$$

or

$$-\frac{G}{2r^2} + B = 2\pi\eta l.\omega$$

where B is a constant of integration. To evaluate B the condition of motion $r = R_1$, $\omega = 0$ may be applied. This gives

$$B = \frac{G}{2R_1}$$

Whence

$$-\frac{G}{2r} + \frac{G}{2R_1^2} = 2\pi l\eta \cdot \omega$$

We know also that $r = R_2$ and $\omega = \omega_0$, then

$$\frac{G}{2R_1^2} - \frac{G}{2R_2^2} = 2\pi\eta l \cdot \omega_0$$

$$\therefore \qquad G = 4\pi\eta\omega_0 \cdot \frac{R_1^2 R_2^2}{\left(R_2^2 R_1^2\right)} \cdot l$$

The restoring couple offered by the suspension fibre = θC

where C is the couple per unit radian twist.

In the equilibrium position the two couples balance each other hence

$$G = C \cdot \theta$$

or $$\frac{4\pi\eta l\omega_0 R_1^2 R_2^2}{\left(R_2^2 - R_1^2\right)} = C \cdot \theta$$

In the above discussion we have not taken into account the torque on the base of the inner cylinder. To correct for this couple, the cylinder is submerged in the liquid for two different lengths. If $f(\beta)$ be the unknown couple over the base of the inner cylinder, then

$$G = \frac{4\pi\eta l\omega_0 R_1^2 R_2^2}{\left(R_2^2 - R_1^2\right)} + f(\beta)$$

Let l_1 and l_2 be the two lengths of the inner cylinder in the liquid and θ_1 and θ_2 the steady deflections produced. Then

$$\frac{4\pi\eta l_1\,\omega_0\,R_1^2\,R_2^2}{\left(R_2^2 - R_1^2\right)} + f(\beta) = C\theta_1 \qquad ...(1)$$

and $$\frac{4\pi\eta l_2\,\omega_0\,R_1^2\,R_2^2}{\left(R_2^2 - R_1^2\right)} + f(\beta) = C\theta_2 \qquad ...(2)$$

Subtracting (2) from (1) we have

$$\frac{4\pi\eta\omega_0\,R_1^2\,R_2^2\,(l_1 - l_2)}{\left(R_2^2 - R_1^2\right)} = C \cdot (\theta_1 - \theta_2)$$

whence $$\eta = \frac{C\left(R_2^2 - R_1^2\right)(\theta_1 - \theta_2)}{4\pi\omega_0\,R_1^2 R_2^2\,(l_1 - l_2)} \qquad ...(3)$$

To determine C, the torque per unit radian twist due to torsional reaction in the suspension wire, a hollow metal disc D is provided whose moment of inertia r_1 about the axis of rotation can be calculated from a knowledge of its mass and dimensions. This inner cylinder is allowed to oscillate torsionally in air first alone and then with the disc placed centrally upon it. If T_0 and T_1 be the periods of oscillation in the two cases respectively and I_0 the moment of inertia of the cylinder about the axis of rotation, we have

$$T_0 = 2\pi\sqrt{\frac{I_0}{C}}$$

and $$T_1 = 2\pi\sqrt{\frac{I_0 + I_1}{C}}$$

From the above two equations we get

$$C = \frac{4\pi^2 I_1}{T_1^2 - T_0^2} \qquad ...(4)$$

Substituting this value of C in equation (3) we have

$$\eta = \frac{4\pi^2 I_1}{T_1^2 - T_0^2} \times \frac{\left(R_2^2 - R_1^2\right)(\theta_1 - \theta_2)}{4\pi\omega_0 \; R_1^2 \; R_2^2 \left(l_1 - l_2\right)} \qquad \text{...(5)}$$

$$= \frac{\pi I_1}{T_1^2 - T_0^2} \cdot \frac{(R_2^2 - R_1^2)\,(\theta_1 - \theta_2)}{\omega_0 \; R_1^2 \; R_2^2 \; (l_1 - l_2)}$$

If the mass of the disc placed over the cylinder be m and a and b its internal and external radii,

$$I_1 = \frac{m(a^2 + b^2)}{2}$$

Substituting this value of I_1 in above equation, we get

$$\boxed{\eta = \frac{m\left(a^2 + b^2\right)\pi}{2\left(T_1^2 - T_0^2\right)} \times \frac{\left(R_2^2 - R_1^2\right)(\theta_1 - \theta_2)}{\omega_0 R_1^2 R_2^2 \left(l_1 - l_2\right)}}$$

Knowing all the factors on the right hand side of this equation, η can be calculated.

6.16 OBJECT

To determine the coefficient of viscosity of water by rotating cylinder method.

Apparatus required: Rotating cylinder arrangement, stop watch, lamp and scale arrangement, meter scale and water, vernier callipers.

Formula used: The coefficient of viscosity η is given by

$$\eta = \frac{I_2\left(\phi_1 - \phi_2\right)\left(b^2 - a^2\right)\pi}{\left(T_2^2 - T_1^2\right)\omega_0 \, a^2 \, b^2 \left(l_1 - l_2\right)} \text{ Poise}$$

where

I_2 = moment of inertia of an annular metallic disc.
ϕ_1 = deflection produced due to a length l_1 of water.
ϕ_2 = deflection produced due to a length l_2 of water.
a = radius of inner cylinder.
b = radius of outer cylinder.
T_1 = time period of inner cylinder alone
T_2 = time period of inner cylinder with an annular disc placed on it.
ω_0 = angular velocity of the cylinder.

Description of apparatus: The apparatus consists mainly of a revolving table T, upon which is carried the outer cylinder B which can be easily clamped in a position coaxial with the spindle. The spindle can be rotated from 20 to 60 r.p.m by a small motor. The inner cylinder A is suspended in the fluid by means of a long and thin wire and carries a small plane mirror m. The steady twist produced in the wire when the outer cylinder is rotated at a constant speed is read by means of a telescope and scale arrangement. A hollow metal disc D usually of the same external diameter as that of the inner cylinder is also provided with the apparatus. An additional inner cylinder of the same diameter but of a different length is also supplied with the apparatus.

Theory: The viscosity is given by

$$\eta = \frac{m\left(a^2+b^2\right)\pi}{2\left(T_1^2-T_0^2\right)} \times \frac{\left(R_2^2-R_1^2\right)(\theta_1-\theta_2)}{\omega_0 R_1^2 R_2^2\,(l_1-l_2)}$$

Procedure:

1. Set up the lamp and scale arrangement and adjust the position of the spot on the zero of the scale.
2. Fill up the space between the outer cylinder and inner cylinder with a length l_1 of water column with the help of a burette.
3. Take the initial reading of the counter. Start the electric motor and simultaneously a stop watch. For a certain time, note down the total number of revolutions with the help of counter and thus calculate the angular velocity. After some time when the spot becomes stationary, note down its position on the scale.
4. Change the length of water say to a height l_2 and repeat procedure (3). Note down the deflected position of the spot in this case also.
5. Find out the time period of the suspended system alone as well as by putting a metal disc of same radius on it.
6. Note down the distance of lamp and scale arrangement from the mirror m.
7. Find out the diameter of inner as well as outer cylinder.
8. Note down the mass and radius of metal disc.

Observation: Diameter of the inner cylinder = ... meter

$\therefore$ radius of the inner cylinder a = ... meter

Diameter of the outer cylinder = ... meter

$\therefore$ radius of the outer cylinder b = ... meter

Mass of the disc M = ... Kg

Radius of the disc R = ... meter

Distance of the scale from mirror = ... meter

Determination of time periods T_1 and T_2:

S.No.	No. of oscillations	Time of the inner cylinder alone			T_1 sec	Mean T_1	Time of inner cylinder + disc			T_2 Secs	Mean T_2
		Min	Secs	Total secs			Min	Secs	Total		
1.	10										
2.	15										
3.	20										
4.	25										

Determination of angular velocity ω_0:

S. No.	No. of revolution N	Time taken			$\omega_0 = 2\pi N/T$	Mean ω_0
		Min	Secs	Total T secs		
1. 2. 3. 4.	50 60 70 80					

Table for ϕ_1 and ϕ_2:

Distance of scale from mirror D = ... meter
water length l_1 = ... meter
water length l_2 = ... meter

S. No.	Zero position of spot	Final position of spot with		Deflection of the spot with		$\phi_1 = x/2D$ spot	$\phi_2 = x'/2D$
		l_1 of water	l_2 of water	with l_1 x	with l_2 x'		
1 2 3 4							

Calculations:

$$I_2 = MR^2 = \text{-------kg}xm^2$$

$$\eta = \frac{I_2\,(\phi_1 - \phi_2)(b^2 - a^2)\,\pi}{(l_1 - l_2)(T_2^2 - T_1^2)\,\omega_0\,(a^2 b^2)}$$

= .... Poise

Result: The Coefficient of viscosity of water as determined by rotating cylinder apparatus at .... ºC = ... Poise

Standard value at .... ºC = ... Poise
Percentage error = ...

Precautions and sources of error:

1. At one cross-section measure the diameter of the two cylinders along two perpendicular directions. Repeat the measurement in the same cross-section and at different cross-sections. Then find the mean value of R_1 and R_2. The inner and the outer diameters of the annular ring should be similarly measured.
2. The axis of the outer cylinder should be made vertical with the help of the plumb line and levelling screens.
3. If the axis of the outer cylinder does not coincide with the axis of the inner cylinder, the distance between the two cylinders will not remain the same and as such as error is liable to be introduced. Whether the two axes are coincident or not, can be tested in the following manner. A thin rod smeared with powder is laid horizontally attached to the suspension of the inner cylinder such that it protrudes beyond the rim of the outer cylinder and is in

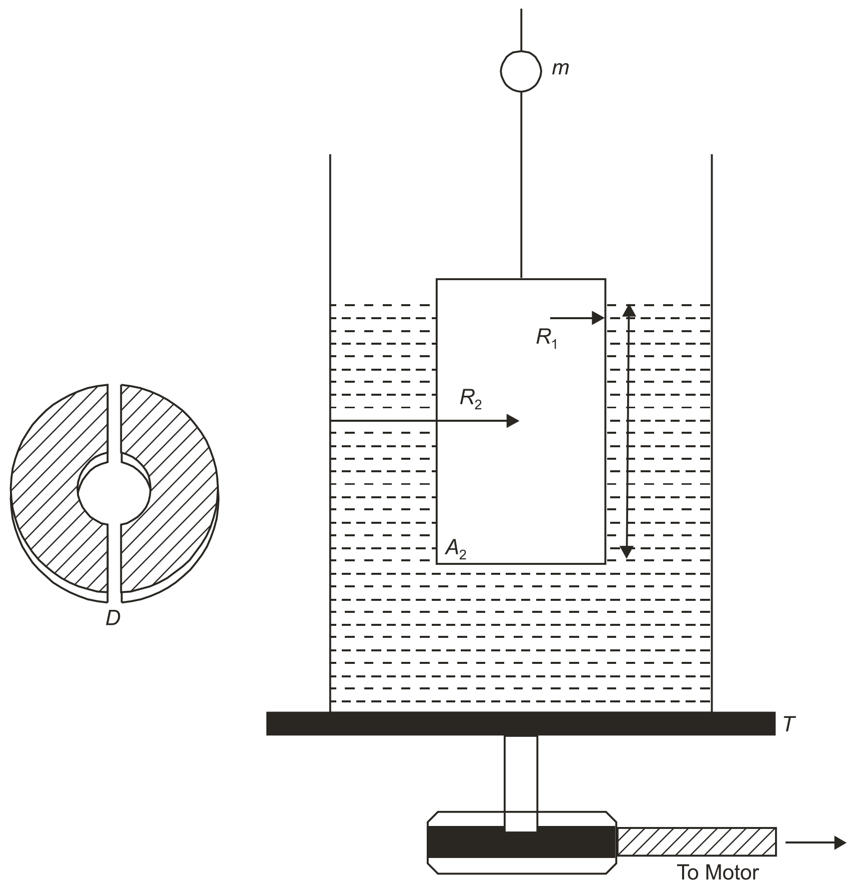


Fig. 6.13

contact with the rim. The outer cylinder is then rotated by hand. If the thin rod is touched by the outer cylinder only at one mark, the axes of the two cylinders are coincident. If the thin rod is touched by the outer cylinder at a number of points as examined with the help of marks made by it on the powdered thin rod, the axes of the two cylinders are not coincident. For making the two axes coincident, move the whole arrangement which carries the outer cylinder suitably and test again with the help of the thin rod.

4. The inner cylinder should be suspended by a long and thin wire so that the value of θ is appreciable.
5. It must be very carefully noted that when water is poured in between the two cylinders, the inner cylinder does not float in it, if it does, it may be necessary to use a heavy annular ring placed over the inner cylinder to submerge it into water. T_0 should be determined with this composite arrangement, of course with air between the cylinders. T_1 is then determined by placing another annular ring of know mass and dimensions.
6. T_0 and T_1 should be carefully determined by timing a large number of oscillations with a stop watch reading upto 1/5th of a second .
7. The speed of rotation should be constant to obtain steady deflection of the inner cylinder and should be such that the motion of the liquid (water) between the two cylinders is stream-line.

6.17 VIVA-VOCE

Q. 1. What do you mean by viscosity?

Ans. The property of a liquid by virtue of which it opposes the relative motion between its different layers is known as viscosity.

Q. 2. Is there any effect of temperature on the coefficient of viscosity of liquids?

Ans. The coefficient of viscosity decreases with rise in temperature.

Q. 3. What is the effect of pressure on coefficient of viscosity?

Ans. The coefficient of viscosity increases with rise of pressure.

Q. 4. What is meant by coefficient of viscosity?

Ans. The coefficient of viscosity is defined as the viscous force acting per unit area between two adjacent layers moving with unit velocity gradient.

Q. 5. What is stream line motion?

Ans. When a liquid flows through a tube in such a manner that each molecule of the fluid travels regularly along the same path as its preceeding molecule, the motion is said to be stream line.

Q. 6. What is turbulent flow?

Ans. Beyond critical velocity, the paths and velocities of the liquid change continuously and haphazardly then the flow is called turbulent flow.

Q. 7. Does the flow of a liquid depend only on its viscosity?

Ans. For velocities well below the critical velocity, the rate of flow is governed by the viscosity and is independent of density. For higher velocities, however, it depends to a far greater extent on the density than on the viscosity.

Q. 8. On what factors does the rate of flow of a liquid through a capillary tube depend?

Ans. It depends upon (*i*) pressure difference p, (*ii*) radius of capillary tube 'r', (*iii*) length of capillary tube 'l', and viscosity of the liquid.

Q. 9. In Poiseuille's method what overcomes the viscous force?

Ans. The constant pressure difference at the two ends of the tube.

Q. 10. Why should the pressure difference across the tube be constant?

Ans. Otherwise rate of flow will change while taking observations.

Q. 11. Should the liquid leave the capillary in a trickle? If so, why?

Ans. Yes, in that case the velocity of flow will be small and the liquid will flow in stream-line. For this pressure difference across the tube should be small.

Q. 12. Why do you not connect the capillary tube directly to the tap?

Ans. Because (*i*) the pressure difference will be large (*ii*) the pressure difference will not remain constant and (*iii*) it will not be possible to change the pressure difference for different sets of observation.

Q. 13. Why do you take a narrow and long capillary tube in the Poiseuille's method?

Ans. Because, the critical velocity of liquid for stream-line flow is inversely proportional to the radius of tube for a narrow tube the critical velocity will be large and hence the flow can be kept stream-line even for larger pressure differences. Further, in Poiseuille's formula $Q \alpha \frac{1}{l}$ *i.e.*, for given pressure difference rate of flow is inversely proportional to length of the tube. Hence for longer tubes rate of flow will be small and within stream line limits.

Q. 14. What is the harm if we take a short and wide tube and keep the rate of flow quite small?

Ans. The rate of flow of water will be quite large even for a small pressure difference and the motion will not be stream line.

Q. 15. Why do you keep the capillary tube horizontal?

Ans. So that the flow of water is not affected by gravity.

Q. 16. Why should the capillary tube be of uniform bore?

Ans. If it is not so the flow will not be stream line due to introduction of radial component of velocity at change in cross-sectional area.

Q. 17. Does the flow of liquid depend only on its viscosity?

Ans. For velocities below critical velocity, the rate of flow is governed by viscosity and is independent of density. However, for higher velocities, the rate of flow depend to a greater extent on density rather than viscosity.

Q. 18. How can the uniformity of tube be tested?

Ans. By introducing a column of mercury in the tube and measuring the length which it occupies in various parts of the tube. If the bore is uniform, the length of mercury thread will be the same throughout.

Q. 19. Is the velocity of the liquids through the tube same everywhere?

Ans. No, it is maximum along axis and decreases towards the wall of the tube. The velocity profile is parabolic.

Q. 20. Can you use this apparatus to determine the viscosity of a gas?

Ans. No.

Q. 21. Can Poiseuille's method be used for determining the viscosity of glycerine?

Ans. No, glycerine is very viscous. This method is suitable only for mobile liquids.

Q. 22. What is Stoke's law and what are its limitations?

Ans. According to Stoke's law, for a body of radius a, moving through a fluid of viscosity η with a velocity v, the viscous drag is given by

$$F = 6\,\pi\eta a v$$

This law holds for an infinite extent of a viscous fluid. In general, in experiments this condition is not satisfied due to finite dimensions of the container. In that case the following two corrections are applied to the terminal velocity.

(*i*) **Ladenburg correction:** This is for wall-effect and the corrected terminal velocity is given by

$$v_\infty = V\left(1 + \frac{2.4a}{R}\right)$$

Where R is the radius of the container and V the observed terminal velocity.

(*ii*) **Correction for end-effect:** Due to finite height of liquid column.

$$v_\infty = V\left(1 + \frac{1.33a}{h}\right)$$

Where h is the total height of the liquid column.

Q. 23. What is the value of coefficient of viscosity of air?

Ans. The value of coefficient of viscosity of air at 20°C is 18.1×10^5 poise.

Q. 24. Why the tiny rain drops appear to us to be floating about as clouds?

Ans. The tiny drops of water have a radius as small as 0.001 cm and their terminal velocity, as they fall through air (η = 0.00018) comes to about 1.2 cm/sec. Hence they appear to us to be floating about as clouds.

Q. 25. For what substances rotating cylinder method can be used?

Ans. This method can be used for liquids and air.

Q. 26. What is the material of suspension wire?

Ans. The material of suspension wire is phosphor bronze. A fine brass or constantan wire may also be used.

Q. 27. Why is the inner cylinder rotated when only the outer one is rotated by electric motor?

Ans. The inner cylinder experiences a couple due to viscous drag of the substance filled between them.

Q. 28. How the inner cylinder comes to rest?

Ans. When the couple due to viscous drag is equal to the torsional restoring couple produced in suspension wire, the inner cylinder comes to rest.

Q. 29. Is there any affect of couple on the base of inner cylinder?

Ans. The base of the inner cylinder is affected by the viscous couple but the affect is eliminated by taking another set with different lengths.

Q. 30. How does the viscous drag come into play?

Ans. The layers of the fluid which are in contact with the outer cylinder move with greater angular velocity than the layers towards the inner cylinder. So the viscous drag comes into play.

Q. 31. How do you find the angular velocity of outer cylinder?

Ans. $\omega = \dfrac{2\pi N}{T}$, where N = total number of revolutions made by outer cylinder in time T.

Q. 32. What is value of coefficient of viscosity of air?

Ans. The value of coefficient of viscosity of air at 20° C is 18.1×10^{-5} poise.

EXERCISE

Q. 1. What is this properly due to?

Q. 2. What are units and dimensions of coefficient of viscosity?

Q. 3. What is the distinction between viscosity and coefficient of viscosity?

Q. 4. What do you mean by 'velocity gradient'? What are its units and dimensions?

Q. 5. What do you mean by critical velocity and on what factors does it depend?

Q. 6. What is constant level tank? Explain its working?

Q. 7. What are the conditions on which the validity of Poiseuille's formula rests and how can they be fulfilled in practice?

Q. 8. Which is the most important quantity to be measured in the experiment and why?

7

Sound

7.1 SPEED OF TRANSVERSE WAVE IN STRETCHED STRING

A string means a wire or a fibre which has a uniform diameter and is perfectly flexible *i.e.* which has no rigidity. In practice, a thin wire fulfills these requirements approximately.

The speed of transverse wave in a flexible stretched string depends upon the tension in the string and the mass per unit length of the string. Mathematically, the speed v is given by

$$v = \sqrt{\frac{T}{m}}$$

Where T is the tension in the string and m is the mass per unit length of the string (not the mass of the whole string).

If r be the radius of the string and d the density of the material of the string, then

$$m = \text{volume per unit length} \times \text{density} = \pi r^2 \times 1 \times d = \pi r^2 d$$

Then the speed of transverse wave is

$$v = \sqrt{\frac{T}{\pi r^2 d}}$$

7.2 VIBRATIONS OF STRETCHED STRING

When a wire clamped to rigid supports at its ends is plucked in the middle, transverse progressive waves travel towards each end of the wire. The speed of these waves is

$$v = \sqrt{\frac{T}{m}} \qquad \text{...(i)}$$

Where T is the tension in the wire and m is the mass per unit length of the wire. These waves are reflected at the ends of the wire. By the superposition of the incident and the reflected waves, transverse stationary waves are set up in the wire. Since the ends of the wire are clamped, there is a node N at each end and a antinode A in the middle (Fig. 7.1).

We know that the distance between two consecutive nodes is $\frac{\lambda}{2}$, where λ is wavelength. Hence if l be the length of the wire between the clamped ends, then

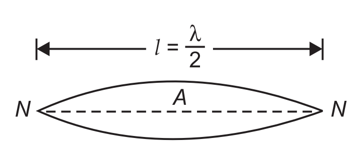


Fig. 7.1

$$l = \frac{\lambda}{2}$$

or $$\lambda = 2l$$

If n be the frequency of vibration of the wire, then $n = \frac{v}{\lambda} = \frac{v}{2l}$

Substituting the value of v from Eq. (i) we have $n = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$ ...(ii)

This is the frequency of the note emitted by the wire.

It is seen from Eq. (ii) that the frequency of the sound emitted from a stretched string can be changed in two ways by changing the length of the string or by changing the tension in the string. In sitar and violin the frequencies of the notes are adjusted by tightening or loosening the pegs of the wires.

7.3 FUNDAMENTAL AND OVERTONES OF A STRING

When a stretched wire is plucked in the middle, the wire usually vibrates in a single segment Fig. 7.2(a). At the ends of the wire are nodes (N) and is the middle an antinode (A). In this condition, the node emitted from the wire is called the "fundamental tone". If l be the length of the wire, and λ_1 be the wavelength in this case, then

$$l = \frac{\lambda_1}{2}$$

or $$\lambda_1 = 2l$$

If n_1 be the frequency of vibration of the wire and v the speed of the wave in the wire, then

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This is the fundamental frequency of the wire.

We can make the wire vibrate in more than one segment. If we touch the middle-point of the wire by a feather, and pluck it at one-fourth of its length from an end, then the wire vibrates in two segments Fig. 7.2(b). In this case, in addition at the ends of the wire, there will be a node (N) at the middle-point also, and in between these three nodes there will be two antinodes (A). Therefore, if λ_2 be the wavelength in this case, then

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

or $$\lambda_2 = \frac{2l}{2}$$

If the frequency of the wire be now n_2, then

$$n_2 = \frac{v}{\lambda_2} = \frac{2v}{2l} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2n_1$$

that is, in this case the frequency of the tone emitted from wire is twice the frequency of the fundamental tone. This tone is called the 'first overtone'.

Similarly, if the wire vibrates in three segments Fig. 7.2(c) and the wavelength in this case be λ_3, then

$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = \frac{3\lambda_3}{2}$$

or $$l_3 = \frac{2l}{3}$$

If the frequency of the wire be n_3, then

$$n_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n_1$$

that is, in this case the frequency of the emitted tone is three times the frequency of the fundamental tone. This tone is called 'second overtone'.

Similarly, if the wire is made to vibrate in four, five .... segments then still higher overtones can be produced. If the wire vibrates in p segments, then its frequency is given by

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

Thus, the frequencies of the fundamental tone and the overtones of a stretched string have the following relationship:

$$n_1 : n_2 : n_3 : ...... = 1 : 2 : 3 : ......$$

These frequencies are in a harmonic series. Hence these tones are also called 'harmonics'. The fundamental tone (n_1) is the first harmonic, the first overtone (n_2) is the second harmonic, the second overtone (n_3) is the third harmonic etc. The tones of frequencies n_1, n_3, n_5 ..... are the odd harmonics and the tones of frequencies n_2, n_4, n_6....... are the 'even harmonics'. Clearly, a stretched string gives both even and odd harmonics.

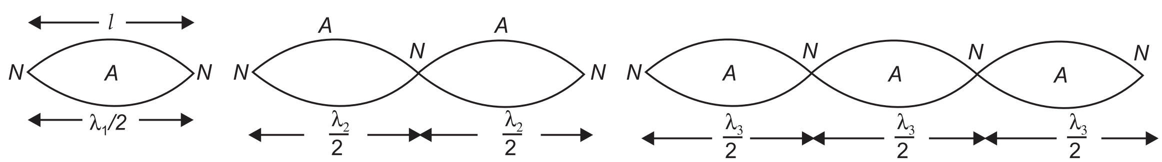


Fig. 7.2

7.4 SONOMETER

It is the simplest apparatus for demonstrating the vibrations of a stretched string. It consists of a hollow wooden box about 1 meter long which is called the 'sound board'. A thin wire is stretched over the sound-board. One end of the wire is fastened to a peg A at the edge of the sound-board and the other end passes over a frictionless pulley P and carries a hanger upon which weights can be placed. These weights produce tension in the wire and press it against two bridges B_1 and B_2. One of these bridges is fixed and the other is movable. The vibrating length of the wire can be changed by changing the position of the movable bridges. The wall of the sound-board contains holes so that the air inside the sound-board remains in contact with the air outside. When the wire vibrates, then these vibrations reach (through the bridges) the upper surface of the sound-board and the air inside it. Along with it, the air outside the sound-board also begins to vibrate and a loud sound is heard.

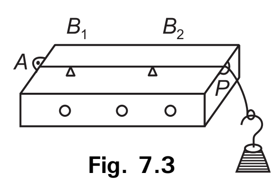


Fig. 7.3

When the sonometer wire is plucked at its middle-point, it vibrates in its fundamental mode with a natural frequency 'n' is given by $n = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$.

Here l is the length of the wire between the bridges, T is the tension in the wire and m is the mass per unit length of the wire. If r be the radius of the wire, d the density of the material of the wire and M the mass of the weights suspended from the wire, then

$$m = \pi r^2 d \text{ and } T = Mg$$

$$\therefore \qquad n = \frac{1}{2l}\sqrt{\frac{Mg}{\pi r^2 d}}$$

7.5 OBJECT

To determine the frequency of A.C. mains by using a sonometer and a horse-shoe magnet.

Apparatus: A sonometer, a step-down transformer, a choke, weights, a meter scale and a scale pan, a horse-shoe magnet, a wire of non-magnetic material (brass or copper wire), a physical balance, a weight-box.

Formula used: The frequency of A.C. mains is given by the following formula.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Where l = length of the sonometer wire between the two bridges when it is thrown into resonant vibrations.

T = tension applied to the wire.

m = mass per unit length of the wire.

Description of apparatus: A sonometer consists of a wooden box AB about 1 metre long. It also carries a wire of uniform cross-section and made of non-magnetic material usually brass. One end of this wire is fixed to a peg at one end of the box. This wire after passing over a pulley at the other end of the box carries a hanger at the other end. Tension is produced in the wire by placing suitable load on this hanger. There are three knife-edge-bridges over the box. Two of them are fixed near the ends of the box while the third one, can be slided along the length of the wire supporting it (Some times only two knife-edge-bridges is provided in this case, one is fixed and other is slide to get maximum vibration). Its position can be read on a scale fitted along the length of the wire. The vibrations of the wire alone can not produce audible sound. But the sound box helps in making this sound louder. When wire vibrates, these vibrations are communicated to the box and the enclosed air in it. Since the box has a large surface and volume it produces sufficient vibrations in air to make it audible.

A permanent horse-shoe magnet is mounted vertically in the middle of the wire with wire passing between its poles. The magnet produces a magnetic field in the horizontal plane and perpendicular to the length of the wire. When the alternating current from mains after being stepped down to 6 or 9 volt is passed through the wire, it begins to vibrate in vertical plane. By adjusting the position of the bridge resonance can be obtained.

Theory: When transverse waves are excited in a stretched wire the bridges act as rigid reflectors of these waves. As a result of this the length of the wire between two bridges becomes a bound medium with waves reflected at both ends. Thus stationary waves are formed with bridges as nodes. Therefore in the fundamental mode, when wire vibrates in one loop, we have

$$\frac{\lambda}{2} = l$$

Where l is the distance between bridges and λ is the wave-length of transverse waves through the string. We know that if the, elastic forces are negligible compared to tension, the velocity of transverse waves in the string is given by

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension and m is mass per unit length of the wire. Therefore, the natural frequency (fundamental mode) of the wire is given by

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

The frequency of the wire can be changed by varying tension T, or length l.

Now when the wire carrying current is placed in a magnetic field perpendicular to its length, the wire experiences a magnetic force whose direction is perpendicular to both the wire as well as the direction of the magnetic field. Thus due to orientations of field and wire, the wire in this case experiences a force in the vertical direction with the sense given by Fleming's left hand rule. Since in the experiment alternating current is being passed through the wire, it will experience an upward force in one half cycle and downward force in next half cycle. Thus the wire gets impulses alternately in opposite directions at the frequency of the current, and consequently it begins to execute forced transverse vibrations with the frequency f of the alternating current. Now if the distance between bridges is so adjusted that the natural frequency of vibrations 'n' of the wire becomes equal to that of the alternating current, resonance will take place, and the wire will begin to vibrate with large amplitude. In this case $f = n$. Hence

$$f = n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

From this the frequency of A.C. mains can be calculated.

Procedure:

1. Arrange the apparatus as shown in Fig. 7.3.
2. Put some weights on the pan and the magnet on the board between the bridges in such a position as to produce magnetic field at right angles to the wire.
3. Connect the primary of the step-down transformer to A.C. mains.
4. Now vary the position of the bridges slowly and symmetrically with respect to the magnet till a stage is reached when the wire vibrates with maximum amplitude. This is the position of resonance. Measure the distance between bridges. Repeat this step 3 or 4 times to find mean value of l.
5. Repeat above steps with load on the hanger increasing in steps of 100 gm till maximum allowable limit is reached. Corresponding to each load find mean l.
6. Repeat the experiment with load decreased in the same steps in which it was increased.
7. From readings with increasing and decreasing load find mean value of l corresponding to each load.
8. Weigh the specimen wire, measure its length and hence calculate its linear density.

Observations:

1. Measurement of 'T' and 'l'

   Mass of the pan =

S.No.	*Total load (including hanger) M gms.*	*Length in resonance l 'cm'*		*Mean length*
		Load increasing	*Load decreasing*	
1. 2. 3. 4. 5. 6.				

2. Measurement of 'm'—

   (i) Mass of the specimen wire = gm

   (ii) Length of the specimen wire = cm

Calculations: Linear density of the wire m = gm/cm

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Results: Frequency of the AC mains is found to be = cycles/sec or Hertz.

Standard value = 50 Hz

Percentage error = ... %

Precautions and sources of error:

1. The wire from which the pan is suspended should not be in contact with any surface.
2. Use choke to limit the current or wire may burn out.
3. The wire may be uniform and free from kinks and joints.

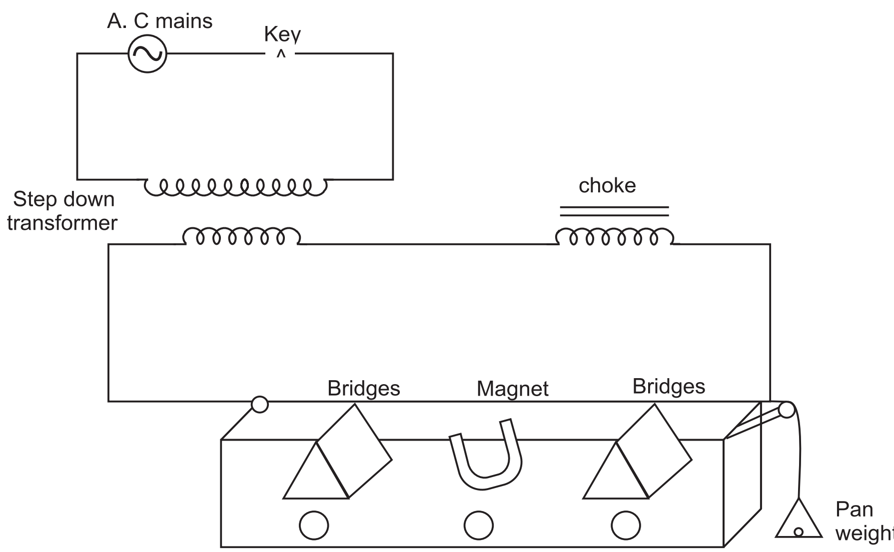


Fig. 7.4

4. The magnetic field should be at center of vibrating loop and must be perpendicular to the length of the wire.
5. The material of the sonometer wire should be non magnetic.
6. The bridges used should give sharp edges to get the well defined nodes.
7. The weights should be removed from the wire otherwise the wire may develop elastic fatigue.
8. In order that the tension in the cord may be exactly equal to the weight suspended, there should be no friction at the pulley.

7.6 OBJECT

To determine the frequency of A.C. mains or of an electric vibrator, by Melde's experiment, using:

(*i*) Transverse arrangement
(*ii*) Longitudinal arrangement

Apparatus used: Electric vibrator, thread and pulley, chemical balance and metre scale.

Formula used:

(*i*) For the transverse arrangement, the frequency n of the fork is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

Where l = length of the thread in the fundamental vibration.
T = tension applied to thread.
M = total mass suspended.
m = mass per unit length of thread.

(*ii*) For the longitudinal arrangement, the frequency of electric vibrator is given by

$$n = \frac{1}{l}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{Mg}{m}}$$

Where the symbols have usual meaning.

Description of the apparatus: An electric vibrator consists of a solenoid whose coil is connected to A.C. mains. The circuit includes a high resistance in the form of an electric bulb as shown in Fig. 7.5. A soft iron rod AB is placed along the axis of the solenoid, clamped near the end A with two screws X and Y while the end B is free to move. The rod is placed between the pole pieces of a permanent magnet NS. One end of the thread is attached to the end B and the other passes over a frictionless pulley and carries a weight.

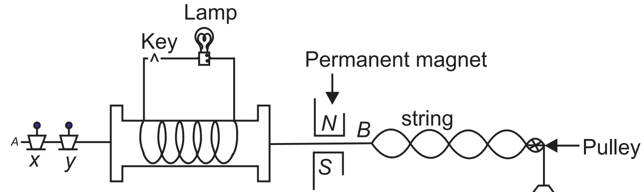


Fig. 7.5

When an alternating current is passed in the coil of the solenoid, it produces an alternating magnetic field along the axis. The rod AB gets magnetised with its polarity changing with the same frequency as that of the alternating current. The rod AB vibrates n times per second due to interaction of the magnetised rod with the permanent magnet.

Procedure: Transverse arrangement

1. Take a uniform thread and attach its one end to the point B of the rod and the other to a light pan by passing it over a frictionless pulley.
2. Connect the A.C. mains as shown in Fig. 7.4.
3. Place the vibrator in the transverse position.
4. The vibrations of maximum amplitude are obtained either by adding weights to the pan slowly in steps or by puting some mass M in the pan and adjusting the length of the thread by moving the vibrator.
5. Note the number of loops p formed in the length L of the thread. This gives the value of l as $l = \frac{L}{p}$.
6. Repeat the above procedure for different loops.

Longitudinal arrangement: In this case the vibrator is adjusted such that the motion of the rod is in the same direction as the length of the thread. The procedure remaining the same as described in case of transverse arrangement.

Observations:

1. Mass of the pan =
   Mass of the thread =
   Length of the thread =
2. Transverse arrangement.
   Table for the determination of T and l.

S.No.	*Tension T applied*			*No. of loops (p)*	*corresponding length of the thread L meter*	*Length l for one loop (L/p)*	*Mean l meter*
	weight placed in pan	*weight of pan*	*Total tension*				

3. Longitudinal arrangement.
   Table for the determination of T and l.

S.No.	*Tension T applied*			*No. of loops (p)*	*corresponding length of the thread L meter*	*Length l for one loop (L/p)*	*Mean l meter*
	weight placed in pan	*weight of pan*	*Total tension*				

Calculations: Mass per unit length of thread $m = \frac{\text{Mass}}{\text{Length}}$ = ...

In transverse arrangement.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}} = \text{...... cycles/sec.}$$

Similarly calculate n from the other sets of observations.

Mean n = ... cycles/sec.

In longitudinal arrangement

$$n = \frac{1}{l}\sqrt{\frac{Mg}{m}}$$

$$= \text{...... cycles/sec.}$$

Similarly calculate n from the other set of observations.

Mean n = ... cycles/sec.

Result: The frequency of A.C. mains, using.

1. Transverse arrangement = ... cycles/sec.
2. Longitudinal arrangement = ... cycles/sec.

Standard result: Frequency of A.C. mains = ... cycles/sec.

Percentage error = ... %

Sources of error and precautions:

1. Pulley should be frictionless.
2. The thread should be then, uniform and inextensible.
3. Weight of the scale pan should be added.
4. The loops formed in the thread should appear stationary.
5. Do not put too much load in the pan.

7.7 VIVA-VOCE

Q. 1. What type of vibrations are produced in the sonometer wire and the surrounding air?

Ans. In the wire transverse vibrations are produced and in the surrounding air longitudinal progressive waves are produced.

Q. 2. How are stationary waves produced in the wire?

Ans. The transverse waves produced in sonometer wire are reflected from the bridges. The two waves superpose over each other and stationary waves are produced.

Q. 3. What do you understand by resonance?

Ans. In case of forced or maintained vibrations, when the frequencies of driver and driven are same then amplitude of vibration of driven becomes large. This phenomenon is called resonance.

Q. 4. Is there any difference between frequency and pitch?

Ans. Frequency is the number of vibrations made by the source in one second while pitch is the physical characteristics of sound which depends upon its frequency.

Q. 5. On what factors does the sharpness or flatness of resonance depend?

Ans. It depends only on natural frequency.

Q. 6. What are the positions of nodes and antinodes on sonometer wire?

Ans. Nodes at the bridges and antinodes at the middle between the two knife bridges.

Q. 7. On what factor does the frequency of vibration of a sonometer wire depend?

Ans. The frequency of the wire can be changed by varying tension T, or length l.

Q. 8. What are the laws of vibrations of strings?

Ans. Strings vibrates according to $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Q. 9. What are the requisites of sonometer wire?

Ans. The wire (*i*) should have uniform linear density m, (*ii*) should not change in length during vibration and (*iii*) should be flexible. A steel or brass wire serves the purpose best.

Q. 10. What is the function of the sonometer board?

Ans. The board is hollow and contains air inside. When the vibrations of wire take place, the energy of vibrations is communicated to the board and from there to the enclosed air. Due to the forced vibrations of this large mass of air loudness of sound is increased. The holes drilled on the sides of the board establish the communication of inside air with external air.

Q. 11. Why are bridges provided on the board?

Ans. The bridges limit the length of vibrating wire. Reflection of transverse waves on the string takes place from these bridges and the stationary waves are formed.

Q. 12. How does the friction affect the result?

Ans. Friction reduces the tension applied to the wire i.e. the tension in wire becomes less than the load suspended from the hanger as a result the calculated values are higher than the actual frequency.

Q. 13. In sonometer experiment is the resonance sharp or flat?

Ans. It is sharp. A slight displacement of bridges causes a considerable fall in amplitude of vibration.

Q. 14. What do you mean by A.C. mains?

Ans. An electric current that reverses its direction with a constant frequency (f). If a graph of the current against time has the form of a sine wave, the current is said to be sinusoidal.

Q. 15. What do you mean by frequency of A.C. mains?

Ans. A current which changes its direction of flow *i.e.* continuously varying from zero to a maximum value and then again to zero and also reversing its direction at fixed interval of time.

Q. 16. What is the frequency of your A.C. mains? What does it represent?

Ans. The number of times the current changes its direction in each second is called the frequency of A.C. mains. It's value is 50 cycles per second.

Q. 17. Does direct current also have any frequency?

Ans. No, it does not change its direction.

Q. 18. In case of non-magnetic wire why does it vibrate? When does the wire resonate?

Ans. It vibrates according to Fleming left hand rule.

Q. 19. Can a rubber string be used in place of wire?

Ans. No, the rubber string will not continue vibrating long because it is not sufficiently rigid.

Q. 20. Why should the magnet be placed with its poles in a line perpendicular to the length of wire ?

Ans. To fulfil the condition for vibration according to Fleming left hand rule.

Q. 21. Why do you use a transformer here? Can't you apply the A.C. directly.

Ans. The transformer is used to step down the A.C. voltage to a small value of about 6-9 volts. This ensures that no high current flows through the sonometer wire and heats it up. The A.C. mains is not directly connected to the wire as it is dangerous for human body, and may also cause a high current to flow through the wire.

Q. 22. What is the construction of your transformer?

Ans. A device for transferring electrical energy from one alternating current circuit to another with a change of voltage, current, phase, or impedance. It consists of a primary winding of N_p turns magnetically linked by a ferromagnetic core or by proximity to the secondary winding of N_s turns. The turns ratio $\frac{N_s}{N_p}$ is approximately equal to $\frac{V_s}{V_p}$ and to $\frac{I_p}{I_s}$, where V_p and I_p are the voltage and current fed to the primary winding and V_s and I_s are the voltage and current induced in the secondary winding assuming that there are no power losses in the core.

Q. 23. In above experiment can't we use an iron wire?

Ans. We can't use an iron wire because in this case wire is attracted by magnet and hence wire does not vibrate.

Q. 24. In this experiment will the frequency of sonometer wire change by changing the distance between the bridges.

Ans. No, the frequency of vibration of the sonomater wire will not change by changing the distance between the bridges because the wire is executing forced vibrations with the frequency of the mains.

Q. 25. Then, what is actually changing when the distance between bridges is changed.

Ans. The natural frequency of the wire.

Q. 26. What is the principle, according to which the wire begins to vibrate, when the alternating current is passed through it?

Ans. When a current carrying wire is placed in a magnetic field, it experiences a mechanical force (given by Fleming's left hand rule) which is perpendicular to the direction of current and magnetic field both.

Q. 27. What is Fleming's left hand rule?

Ans. According to this rule if the thumb and the first two fingers of the left hand are arranged mutually perpendicular to each other, and the first finger points in the direction of magnetic field, the second in the direction of current then the thumb indicates the direction of mechanical force.

Q. 28. What do you understand by linear density of wire?

Ans. Mass per unit length is called the linear density of wire.

Q. 29. Why a choke is used?

Ans. To avoid the heating of the wire.

Q. 30. What are the losses in the transformer.

Ans. Eddy current loss, hysteresis losses in the wire, heating losses in the coils themselves.

Q. 31. Why a special type (horse-shoe type) of magnet is used?

Ans. In this type of magnet the magnetic field is radial.

Q. 32. How does the rod vibrate?

Ans. When alternating current is passed through the solenoid, the iron rod is magnetised such that one end is north pole while other end is south pole. When the direction of current is changed, the polarity of rod is also changed. Due to the interaction of this rod with magnetic field of permanent horse-shoe magnet, the rod is alternately pulled to right or left and thus begins to vibrate with frequency of A.C. mains.

Q. 33. What type of vibrations does the rod execute?

Ans. The vibrations are forced vibrations. The rod execute transverse stationary vibrations of the same frequency as that of A.C.

Q. 34. Can you use a brass rod instead of soft iron rod?

Ans. No, because it is non-magnetic.

Q. 35. How is it that by determining the frequency of the rod, you come to know the frequency of A.C. mains?

Ans. Here the rod vibrates with the frequency of A.C. mains.

Q. 36. What is the construction of an electric vibrator?

Ans. It consists of a solenoid in which alternating current is passed. To avoid the heating effect in the coil of solenoid, an electric bulb is connected in series. A rod passes through the solenoid whose one end is fixed while the other is placed in pole pieces of permanent horse shoe magnet.

Q. 37. What are resonant vibrations?

Ans. If the natural frequency of a body coincides with the frequency of the driving force, the former vibrates with a large amplitude. Now the vibrations are called as resonant vibrations.

Q. 38. When does resonance occur?

Ans. When the natural frequency of the rod becomes equal to the frequency of AC mains, resonance occurs.

Q. 39. How do you change the frequency of the rod?

Ans. We can change the frequency of the rod by changing the vibrating lengths of rod outside the clamps.

Q. 40. Can't we send direct current through solenoid?

Ans. No. In this case the ends of the rod will become permanently either N or S pole and will be pulled to one side.

EXERCISE

Q. 1. What is the cause of variation of current in case of A.C.?

Q. 2. For securing resonance, where do you put the magnet and why?

Q. 3. Does a transformer also change the frequency of A.C.? If not, why?

Q. 4. What is the chief source of error in this experiment?

Q. 5. In case of iron wire which arrangement do you use?

Q. 6. Why do you halve the frequency of the wire to obtain the frequency of A.C. mains in this case?

Q. 7. What is the elastic fatigue and elastic limit and how they are related in the experiment.

8

The Mechanical Equivalent of Heat

8.1 DESCRIPTION OF THE CALLENDER-AND-BARNES CALORIMETER

A heating coil is mounted axially along a horizontal glass tube. This glass tube is further surrounded by a glass jacket to minimise convection of heat. The coil is made of manganin or nichrome. Small length brass tubes are jointed to the two ends of the glass tube by sealing wax. The ends of the heating coil are brought out for external electrical connection by means of two screws. The free ends of the brass tubes are connected with hollow iron bases which have three extra openings i.e. two vertical and one horizontal. The horizontal openings are used for inlet and outlet of water. In one of the vertical openings on both ends of the glass tube a thermometer is inserted through rubber stopper. The other vertical opening on both sides are used to remove any air bubble which might have crept in while flowing water from the tank.

The water reservoir is a small metal vessel having three openings at the bottom. One of the openings is connected to the tap, the middle one to the sink, and the other to the inlet end of the Callender-and-Barnes calorimeter. The height of the reservoir is adjusted and water is allowed to flow through the tube at a constant pressure.

The outlet end of the calorimeter is connected to a small glass tube having a nozzel at the free end by means of a rubber tube. The rate of flow of water from the nozzle is controlled by means of the reservoir attached to the input end. The temperature of the inlet and outlet water are given by the respective thermometers. T_1 and T_2.

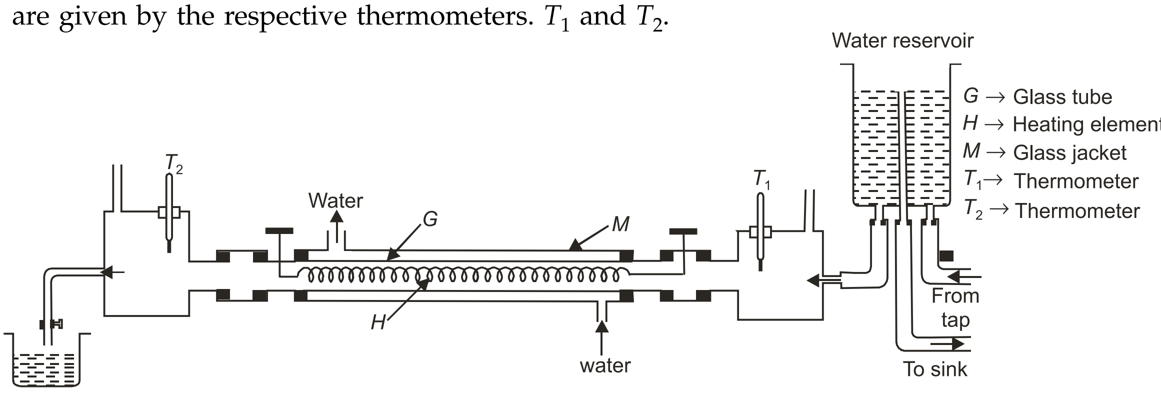


Fig. 8.1

Theory: When a steady electric current flows through the heating coil and a steady flow of water is maintained through the tube, the temperatures at all parts of the apparatus become steady. Under such steady-state conditions, the amount of electrical energy supplied during a known time interval is consumed in heating the amount of water which flows through the tube during the same interval and a small amount of heat is lost by radiation etc., to the surroundings during that interval.

Let the current flowing through the heating coil $= I_1$ amps
the potential difference between the ends of the coil $= V_1$ volts
the rate of flow of water through the tube $= m_1$ gm/sec
the temperature of the inlet water $= \theta_1$°C
the temperature of the outlet water $= \theta_2$°C
and the mean specific heat of water between the temperatures t_1 and t_2 $= s$

Therefore, we can write

$$\frac{V_1 I_1}{J} = m_1 s\,(\theta_2 - \theta_1) + h_1 \qquad ...(1)$$

Where J is the mechanical equivalent of heat (also called Joule's equivalent) and h_1 is the amount of thermal leakage per second from the surface of the tube due to radiation etc.

If V_1, I_1, and m_1 are changed to V_2, I_2, and m_2 while keeping the temperature rise unaltered, then for the same surrounding temperature we can write

$$\frac{V_2 I_2}{J} = m_2 s\,(\theta_2 - \theta_1) + h_2 \qquad ...(2)$$

Subtracting Eq. (1) from Eq.(2), we obtain

$$\frac{V_2\, I_2 - V_1\, I_1}{J} = s\,(m_2 - m_1)\,(\theta_2 - \theta_1) + (h_2 - h_1)$$

For all practical purposes, we may consider $h_1 = h_2$

$$\boxed{J = \frac{V_2\, I_2 - V_1\, I_1}{s\,(m_2 - m_1)\,(\theta_2 - \theta_1)}}$$

Thus, by measuring V_1, V_2, I_1, I_2, m_1, m_2, θ_1 and θ_2, and knowing s, J can be determined in Joules/Calorie.

8.2 OBJECT

To determine the Mechanical Equivalent of heat (J) by the Callender and Barnes method.

Apparatus used: A Callender and Barne's calorimeter, AC mains with a step down transformer, an AC Ammeter and an AC Voltmeter, switch, a rheostat, a stop watch, a measuring jar and 2 thermometers.

Formula used: $J = (E_2C_2 - E_1\, C_1)/(m_1 - m_2)\,(\theta_2 - \theta_1)\, s$ for water $S = 1.0$ Cal/gm °C.

Procedure:

1. Connect the apparatus as shown in the Fig. 8.2.
2. Adjust the tap and the water reservoir till the rate of flow of water through the tube is about (one) c.c per second.

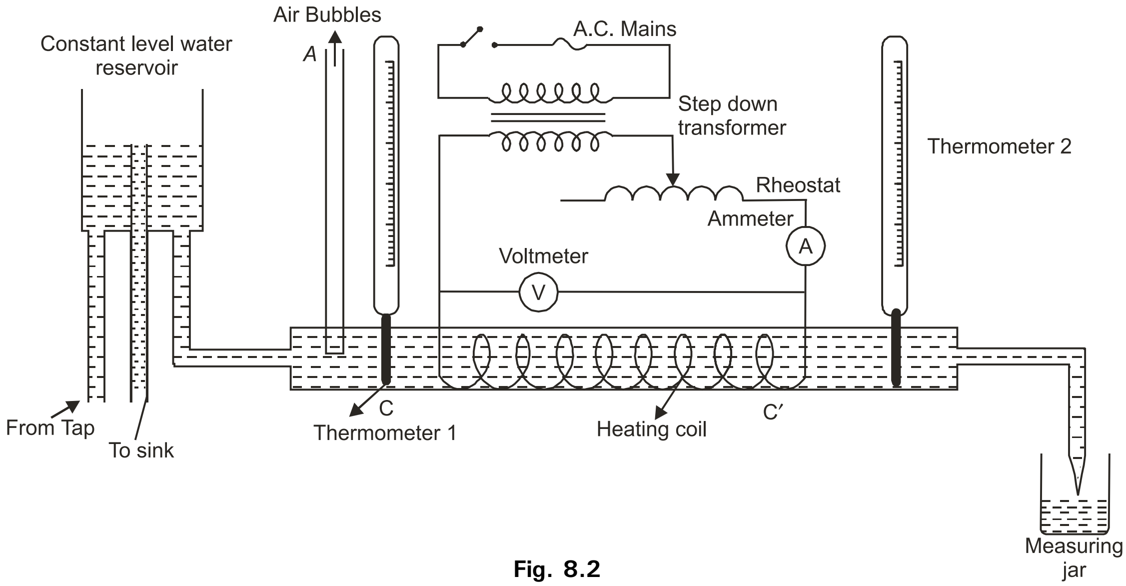


Fig. 8.2

3. Switch on the current and regulate the rheostat so that the current passing is about 2 amperes.
4. As soon as the temperature of the heated water going out becomes steady. Note the temperature of the two thermometers. Note the ammeter and the voltmeter readings.
5. Measure the rate of flow of water at this moment with the help of measuring Jar.
6. Change the rate of flow of water by varying the height of the reservoir and vary the electric current until the two thermometers again indicate their previous readings. Note the new readings of the ammeter and the voltmeter and measure the new rate of flow of water.

Observation:

Temperature of the cold water (inlet end) = θ_1 _ _ _ _ _ºC
Temperature of the hot water (exit end) = θ_2 = _ _ _ _ _ ºC

	E (in volts)	*C (in amps)*	*Amount of flow of water per minute unit*			
			I	II	III	Mean
I Case						
II Case						

Result: The value of *J* is found to be = ergs/cal. (C.G.S. units)
= Joule/cal. (M.K. S. units)

Precautions:

1. The rate of flow of water in the tube should be uniform. To ensure this a number of measurements for the rate of out flow of water should be made.
2. Heating of the water should be uniform throughout tube.
3. Thermometers should be very sensitive.

8.3 VIVA-VOCE

Q. 1. Define mechanical equivalent of heat?

Ans. The mechanical equivalent of heat is defined as the amount of work done in order to produce a unit calorie of heat.

Q. 2. Why do you call it by the letter *J*?

Ans. It is represented by *J*, which is the first letter in the name Joule. James Prescot Joule was the first to determine the value of the ratio of work done to the heat produced.

Q. 3. What are the units of *J*?

Ans. In the C.G.S system, the units of *J* are ergs/calorie.

Q. 4. What is standard value of J?

Ans. It is 4.1852×10^7 erg per calorie or 4.1852 *J*/cal.

Q. 5. What is meant by mechanical equivalent of heat?

Ans. The mechanical equivalent of heat *J* is defined as the constant ratio between mechanical work done '*W*' and corresponding heat produced '*H*' i.e., $J = W/H$.

Q. 6. Why is the heating coil taken in the form of helical form?

Ans. Because greater surface area is exposed to water and it keeps water stirred.

Q. 7. Why should a constant level water tank be used?

Ans. If a constant level water tank is not used for steady flow of water, a steady difference of temperature between the thermometers will not be obtained.

Q. 8. What is Joule's law on Heating Effects of Currents?

Ans. The quantity of heat *H* produced due to a current *C* in a conductor is directly proportional to (i) the square of the current (ii) the resistance *R* and (iii) the time t sec. Thus

$$H = \frac{VCt}{J} = \frac{C^2Rt}{J}$$

Q. 9. In what units are current and voltage used in it?

Ans. The current and voltage have been taken in electromagnetic units.

Q. 10. How do you convert these into practical units?

Ans. The practical unit of voltage is a volt such that 1 volt = 10^8 e.m.u. of potential difference. The practical unit of current is an ampere such that 1 ampere = $\frac{1}{10}$ e.m.u. of current. Thus the heat produced, $H = \frac{Vct}{J} \times 10^7 = \frac{C^2Rt}{4.18} = .24C^2\,Rt$

Q. 11. How much work is done in heating by means of currents?

Ans. If a current of *C* amperes passes in a conductor whose ends are maintained at a potential difference of *V* volts, then in t secs the amount of work done.

$$W = VCt \times 10^7 \text{ ergs}$$

Q. 12. Why is this work done when current passes through a conductor?

Ans. When a potential difference is applied across a conductor, than the electric current is due to a flow of electrons in the interatomic space of the conductor. These electrons experience resistance to their motion in that space and so work has to be done against this resistance. This work done appears as heat.

Q. 13. Upon what factors does the work done depend?

Ans. The work done depends upon the following factors:

(*i*) The number of electrons flowing, i.e., the strength of the current in the conductor.

(*ii*) The resistance of the conductor.

(*iii*) The time for which the current flows in the conductor.

Q. 14. Does the heating effect depend upon the direction of current?

Ans. The heating effect of current does not depend upon the direction of current because it is proportional to the square of the current.

Q. 15. Will the heating effect be different for direct and alternating currents?

Ans. No, the effect is the same with both types of currents because it does not depend upon the direction of current.

Q. 16. What is the heating coil made of?

Ans. The heating coil is made of some resistance wire such as nichrome, constantan or manganin.

Q. 17. Can you give an idea of the resistance of the heater coil?

Ans. The resistance of the heater coil can be calculated by ohm's law by dividing any voltmeter reading with the ammeter reading.

Q. 18. Should the resistance be high or low?

Ans. The resistance of heater coil should be low so that it may take up a large current and the heating may be large, because $H \propto C^2$.

Q. 19. Why is the water not electrolyzed if the naked heating coil is placed in the water?

Ans. Water is a poor conductor of electricity and so all the current passes through the coil and not through water. Thus the water is not electrolyzed.

If ordinary water be used and the potential difference used be more than eight volts, some electrolysis may take place.

Q. 20. Why do you use a step down transformer while using AC?

Ans. A step down transformer is used in order that the apparatus may be used safely without any danger of getting a shock.

Q. 21. Can this apparatus be used for any other purpose?

Ans. Originally this was designed for the study of variation of specific heat with temperature. It can also be used for determining the specific heat of air at constant pressure and of mercury or any other liquid.

Q. 22. What important precaution are taken by you?

Ans.
1. Water should flow at a constant pressure and its motion should be slow and continuous.
2. The rate of flow of water and the two temperatures should be noted only when the steady state has been reached.
3. The current is switched on after the tube is filled up with water.
4. The difference of temperature should be maintained at about 5ºC and sensitive thermometers should be used.
5. The sets of observations are taken for different currents for the same difference of temperature.

Q. 23. What other methods of determining *J* do you know? Discuss their relative merits and demerits?

Ans. '*J*' by Searle's Friction Cone method.

Q. 24. What would happen if the flow of water is not steady?

Ans. The difference of temperature will not remain steady.

9

Thermoelectric Effect

9.1 THERMOELECTRIC EFFECT

Seebeck discovered the thermoelectric effect. To study this effect, two wires of different materials say copper and iron are joined at their ends so as to form two junctions. A sensitive galvanometer is included in the circuit as shown in Figure 9.1. This arrangement is called a Cu-Fe thermocouple. When one junction of the thermocouple is kept hot and the other cold, the galvanometer gives deflection indicating the production of current in the arrangement. The current so produced is called thermoelectric currents.

The continuous flow of current in the thermocouple indicates that there must be a source of e.m.f. in the circuit, which is causing the flow of current. This e.m.f is called thermoelectric e.m.f. It is found that for a temperature difference of 100ºC between the hot and cold junction, thermo e.m.f. produced in Cu-Fe thermocouple is 0.0013V and in case of Sb-Bi thermocouple, the thermo e.m.f. produced is 0.008V.

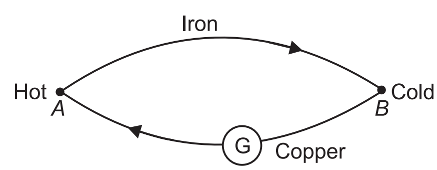


Fig. 9.1

This phenomenon of production of electricity with the help of heat is called thermoelectricity and this effect is called thermoelectric effect or Seebeck effect.

Thus, the phenomenon of production of e.m.f. causing an electric current to flow in a thermocouple when its two junctions are kept at different temperature, is known as Seebeck effect.

9.2 ORIGIN OF THERMO E.M.F.

In a conductor, there are always free electrons. In any conductor, the number of free electrons per unit volume (*electron* density) depends upon its nature. In general, the electron density increases with rise in temperature.

When two metallic wires of different materials are joined at their ends to form a thermocouple, electrons from a metal having greater electron density diffuse into the other with lower value of electron density. Due to this, a small potential difference is established across the junction of the two metals. The potential difference so established is called contact potential and its value depends upon the temperature of the junction for the two given metal obviously, if the two junctions are at the same temperature, the contact potentials at the two junctions will be equal. As the contact potentials at the two junctions tend to send current in opposite directions, no current flows through the thermocouple.

However, if one of the two junctions is heated, more diffusion of electrons takes place at the hot junction and the contact potential becomes more than that at the cold junction. Hence, when the two junctions of a thermocouple are at different temperature, a net e.m.f. called thermo e.m.f. is produced.

It may be pointed out that Seebeck effect is reversible. If we connect a cell in the circuit so as that it sends current in a direction opposite to that due to Seebeck effect Figure 9.2, then it is observed that heat is rejected at the hot junction and absorbed at the cold junction i.e. the hot junction will start becoming hotter, while the cold junction still colder.

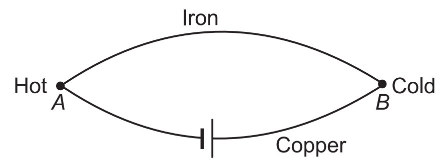


Fig. 9.2

A thermo couple also acts like a heat engine. It constantly absorbs heat at the hot junction, rejects a part at the cold junction and the remaining part is converted into electrical energy, which sends current through that thermocouple.

9.3 MAGNITUDE AND DIRECTION OF THERMO E.M.F.

The magnitude and direction of the thermo e.m.f. developed in a thermocouple depends upon the following two factors.

(*i*) **Nature of the metals forming the thermocouple:** For the experimental investigations, Seebeck arranged a number of metals in the form of a series called thermoelectric series. Some of the metals forming this series are as below:

Sb, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Co, Ni and Bi.

If a thermocouple is formed with wires of any two metals from this series, the direction of current will be from a metal occurring earlier in this series to a metal occurring later in the series through the cold junction. Therefore, in copper-iron (Cu-Fe) thermocouple, the current will flow from iron to copper through cold junction or Copper to Iron through the hot junction. In antimony-bismuth (Sb-Bi) thermocouple, the current flows from antimony to bismuth through the cold junction.

The thermo e.m.f. for a difference of temperature equal to 100ºC is about 0.0013V for Cu-Fe thermocouple and about 0.008V for Sb-Bi thermocouple. As a rule, more the metals are separated in the series, the greater will be the thermo e.m.f.

(*ii*) **The temperature difference between the two junctions of the thermocouple:** To study the effect of difference of temperature between the two junctions, consider a Cu-Fe thermocouple. Its one junction is kept hot by immersing in oil bath and heated with burner.

The other junction is kept cold by immersing it in pounded ice. The temperature of the hot junction can be measured by the thermometer *T* placed in the hot oil bath.

As the temperature of the hot junction is increased by keeping the temperature of the cold junction constant at 0ºC, the deflection in the galvanometer goes on increasing. The deflection in the galvanometer is directly proportional to the thermoelectric current and hence the thermo e.m.f. The graph between thermo e.m.f. and the temperature of

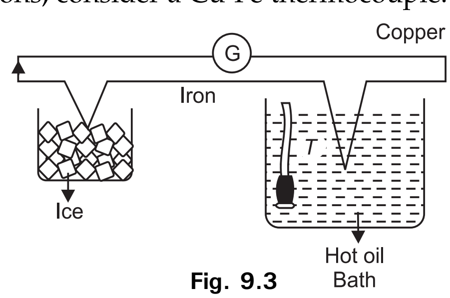


Fig. 9.3

hot junction is found to be parabolic in shape as shown in Figure.

As the temperature of hot junction is further increased, a stage comes, when the thermo e.m.f. becomes maximum.

The temperature of hot junction at which the thermo e.m.f. produced in the thermocouple becomes maximum, is called neutral temperature. For a given thermocouple, neutral temperature has a fixed value. It does not depend upon the temperature of cold junction of the thermocouple. It is denoted by θ_n. For copper-iron thermocouple, neutral temperature is 270ºC.

The temperature of the hot junction, at which the direction of the thermo e.m.f. reverses, is called the temperature of inversion. It is denoted by θ_i.

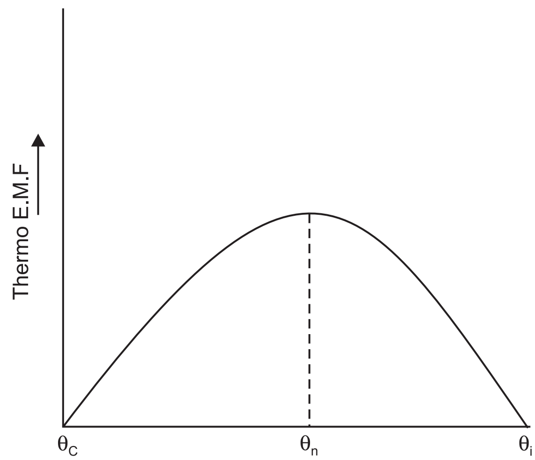


Fig. 9.4

The temperature of inversion is as much above the neutral temperature as the neutral temperature is above the temperature of the cold junction. Then, if θ_c is temperature of the cold junction, then

$$\theta_n - \theta_c = \theta_i - \theta_n$$

or
$$\theta_n = \frac{\theta_i + \theta_c}{2} \quad \text{and} \quad \theta_i = 2\,\theta_n - \theta_c$$

Thus, the netural temperature is the mean of the temperature of inversion θ_i and temperature of the cold junction θ_c, but is independent of θ_i and θ_c. For Cu-Fe thermocouple θ_n = 270ºC. If cold junction is at 0°C, then it follows that θ_i = 540ºC.

If the temperature of cold junction is 0ºC, the graph between the temperature of hot junction and thermo e.m.f. is found to satisfy the equation of the parabola.

$$E = \alpha\theta + \beta\theta^2$$

Where α and β are constants called thermoelectric constants. θ represents the temperature difference between the hot and cold junctions.

9.4 PELTIER EFFECT

Let us consider a bismuth-copper (Bi-Cu) thermocouple. Due to Seebeck effect, in such a thermocouple, thermo electric current flows from copper to bismuth through cold junction. Heat is absorbed at hot junction and is evolved at cold junction.

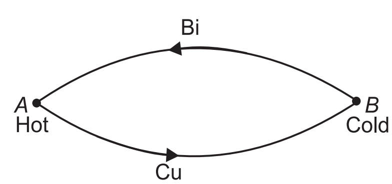


Fig. 9.5

Peltier discovered that whenever two dissimilar metals are connected at a point, an electromotive force (e.m.f.) exists across the junction. Thus, out of the two metals, one is at higher potential than the other. This e.m.f. is found to vary with the change in temperature of the junction.

In the Bi-Cu thermocouple, copper is at higher potential as compared to bismuth. Thus, the Bi-Cu thermocouple appears as if two cells are connected across the two junctions, with their positive terminals to the ends of copper wires. Further, this contact e.m.f. of the cell across the hot junction is greater than that across the cold junction. The net e.m.f. causes the current to flow from copper to bismuth through the hot junction. At the hot junction, as the current flows from Bi to Cu i.e. from lower potential to higher potential, energy will be needed for this purpose. For this reason, in a thermocouple, heat is absorbed at the hot end. On the other hand, energy is given out in the form of heat at the cold junction, as here the current flows from higher to lower potential.

This absorption or evolution of heat at a junction of two dissimilar metals, when current is passed, is known as Peltier effect.

It is also a reversible phenomenon. If the direction of flow of current is reversed, then at a junction heat will be evolved, if earlier heat was absorbed there and vice-versa.

9.5 PELTIER COEFFICIENT(π)

The amount of heat energy absorbed or evolved per second at a junction, when a unit current is passed through it, is known as Peltier Coefficient. It is denoted by π.

Suppose cold junction is at temperature T and hot junction at $T + dT$. If dE is the thermo e.m.f. produced, then it is found that

$$\frac{\pi}{T} = \frac{dE}{dT}$$

Here, $\frac{dE}{dT}$ is rate of change of thermo e.m.f. with temperature. It is called thermoelectric power. It is also known as Seebeck coefficient. If S is Seebeck Coefficient, then

$$S = \frac{dE}{dT}$$

Thus

$$\boxed{\frac{\pi}{T} = \frac{dE}{dT} = S}$$

9.6 THOMSON'S EFFECT

Thomson found that in a Copper wire whose one end is hot and the other kept cold, if current is passed from hotter end to colder end, then heat is evolved along the length of the copper wire. In case, current is passed from colder end to the hotter end, then heat is absorbed along the length of the Copper wire, The explanation lies in the fact that in case of Copper wire, the hot end is at higher potential and the cold end is at lower potential. When current flows from hotter to colder end i.e. from higher to lower potential, the energy is given out in the form of heat. On the other hand, when current is passed from colder to hotter end i.e. from lower to higher potential, the energy is required and it leads to absorption of the heat energy.

In case of bismuth, the effect is just reverse i.e. heat is evolved along the length of bismuth wire, when current is passed from colder to hotter end and heat is absorbed, when current is passed from hotter to colder end. It is because, in case of bismuth, the hot end is at lower potential and cold end is at higher potential.

This absorption or evolution of heat along the length of a wire, when current is passed through a wire whose one end is hot and other is kept cold, is known as Thomson effect. Thomson effect is also a reversible phenomenon. The substances which behave like Copper are said to have positive Thomson effect. Such other substances are antimony, silver, zinc, etc. On the other hand, substances such as cobalt, iron, platinum, etc. which behave like bismuth are said to have negative Thomson effect.

9.7 THOMSON COEFFICIENT

The amount of heat energy absorbed or evolved per second between two points of a conductor having a unit temperature difference, when a unit current is passed, is known as Thomson Coefficient for the material of a conductor. It is denoted by σ.

Thomson coefficient of the material of a conductor is found by forming its thermocouples with a lead wire (Thomson Coefficient of lead is zero). It can be proved that Thomson Coefficient of the material of conductor is given by

$$\sigma = -T\frac{d^2E}{dT^2}$$

Now, Seebeck coefficient is given by

$$S = \frac{dE}{dT}$$

$$\therefore \qquad \frac{dS}{dt} = \frac{d^2E}{dT^2}$$

Thus

$$\sigma = -T\,\frac{d^2E}{dT^2} = -T\left(\frac{dS}{dT}\right)$$

9.8 THERMOPILE

Principle: For a given small difference in temperature of two junctions of a thermocouple, Bi-Sb thermocouple produces a comparatively large e.m.f. and it can be used to detect the heat radiation. When, a number of such thermocouples are connected in series, the arrangement becomes very much sensitive to detect heat radiation as the thermo e.m.f.'s of the thermocouples get added.

A series combination of a large number of Bi-Sb thermocouples is enclosed in a funnel or horn-shaped vessel.

The junction A of each thermocouple is coated with lamp black while the junction B of each thermocouple is well polished and covered with insulating material. The extreme ends of the arrangement are connected to the terminals T_1 and T_2, across which a sensitive galvanometer is connected.

Such an arrangement known as thermopile is shown in Fig. 9.6.

When heat radiations fall on the funnel shaped end of the thermopiles, the set of junction B coated with lamp black absorbs the heat radiation. As a result, the temperature of set of junction B relative to junctions A get raised and thermo e.m.f. is developed in each thermocouple.

The thermoelectric current flows in the same direction (from Sb to Bi through cold junction) in all the thermocouples. Therefore, a large current flowing through the circuit produces deflection in the galvanometer, which indicates the existence of heat radiation.

The thermoelectric effect has the following important applications:

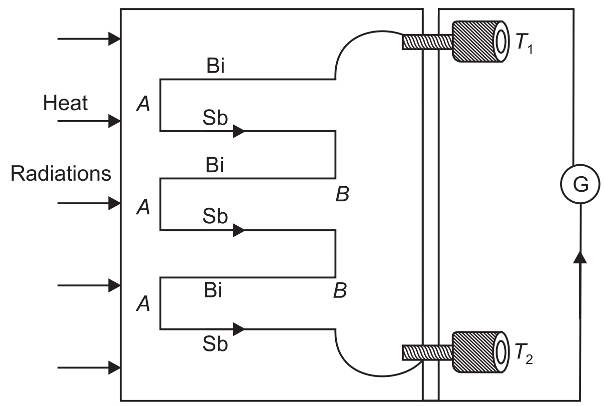


Fig. 9.6

1. A thermocouple is preferred and used to measure temperatures in industries and laboratories. One junction is kept cold at known temperature and other junction is placed in contact with the object, whose temperature is to be measured. The temperature is calculated from the measured value of the thermo e.m.f. The thermocouple is preferred to measure temperature for the following reasons:

(*a*) Since the junction is very small, it absorbs only a very small heat and therefore it does not change the temperature of the object.
(*b*) It quickly attains the temperature of the object.
(*c*) The accuracy in the measurement of temperature is very high. It is because the measurements are made of electrical quantities.

The type of the thermocouple to be used is determined by the range of measurement of the temperature. Different types of thermocouples used in different ranges of temperature are given below:

Thermocouple	Temperature range
Copper-gold and Iron alloy	1 K to 50 K
Copper-constantan	50 K to 400 K
Platinum-platinum rhodium alloy	1500 K to 2000 K.

2. **To detect heat radiation:** A thermopile is a combination of large number of thermocouples in series. It can be used to detect the heat radiation and to note the small difference or the variation in temperatures.

3. **Thermoelectric refrigerator:** If a current is passed through a thermocouple, then due to Peltier effect, heat is removed at one junction and is absorbed at other junction. In case, if on the whole, heat is removed, then the thermocouple acts as a thermoelectric refrigerator. The advantage is that it has, no compressor. No doubt, the cooling effect produced is much low as compared to that in the case of conventional refrigerators. A thermoelectric refrigerator is used, when the region to be cooled is very small and the noise is not acceptable. The thermocouple used as a thermoelectric refrigerator should have following three characteristics:

(*a*) It should have low resistivity, otherwise, loss of energy in the form of heat will be large.
(*b*) It should have low thermal conductivity. It will help in maintaining large temperature difference between the two junctions.
(*c*) It should produce high thermopower.

4. **Thermoelectric generator:** Thermocouples can be used to generate thermoelectrical power in remote areas. It may be done by heating one junction in a flame and exposing the other junction to air. The thermo e.m.f. developed has been used to power radio receivers, etc.

9.9 OBJECT

To calibrate a thermocouple and to find out the melting point of naphthalene.

Apparatus used: One thermocouple apparatus as shown in the Fig. 9.7, a galvanometer, two thermometers, key, Naphthalene, stop watch and heater or gas burner, glass test tube, stand and clamp.

Description of apparatus and theory: The apparatus as shown in the diagram consists of two junctions A and B of copper and constantan. These are placed in the test tubes which themselves are placed in the baths B_1 and B_2. One of them is kept at room temperature are measured by thermometers T_1 and T_2.

When a difference of temperature is produced between two junctions an E.M.F. is set up which produces a deflection in the galvanometer. This deflection is proportional to the difference of temperature between the two junctions. Thus by plotting a graph with known difference of temperature and known deflections we can find out an unknown temperature by noting its deflection and finding out the corresponding temperature from the graph.

Manipulations:

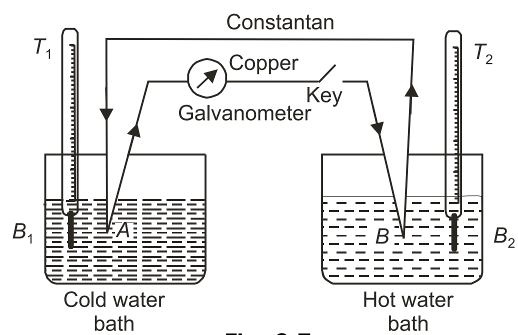


Fig. 9.7

1. Set the apparatus as shown in the diagram.
2. Put junction A in cold water and note temperature T_1.
3. Put junction B in cold water and heat the water.
4. Start observations from the room temperature and take galvanometer readings at intervals of 4ºC – 5ºC and go up to the boiling point of water. To keep the temperature constant for some time, remove the flame at the time of taking observation and keep on a stirring the water.
5. Plot a graph with deflection as ordinate and temperature as abscissa. The graph, in general, should be a parabola (Fig. 9.8(a)) but within a short range of temperature as in the present case (0ºC – 90ºC or 100ºC), the graph will be straight line Fig. 9.8(b). The straight line actually is the straight portion of the parabola. This graph can be used to determine any unknown temperature (within this range).
6. Pour the naphthalene bits into the test tube and immerse the end B of the thermocouple in it keeping the test tube immersed in hot water in a water bath.

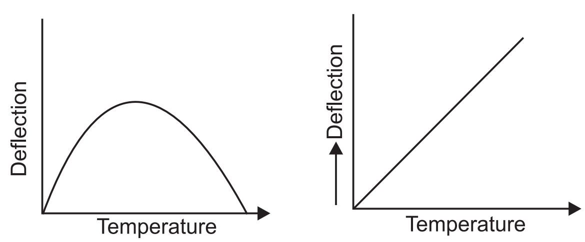


Fig. 9.8

7. Find out the deflection when the naphthalene melts and also when it solidifies again. The deflection should be read at the intervals of 30 seconds. When the naphthalene is about to melt or solidify, the deflection will remain constant during melting and solidification. Note the value of constant deflection.
8. Find out from the previous graph, the value of temperature corresponding to the deflection. This is the melting point of naphthalene.

Observation:

Temperature of the cold junction = _ _ _ _ºC

Initial reading of galvanometer =

(1) With water in B

Sl No.	*Temperature of the hot junction*	*Reading for deflections*		*Mean Deflection*	*Net Deflection*
		While heating	*While cooling*		
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					

(2) With Naphthalene in B

Sl No.	*Reading for deflection while melting*	*Reading for deflection while solidify*	*Mean of constant readings*	*Net deflection*
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				

9.				
10.				
11.				
12.				
13.				
14.				
15.				
16.				

The temperature of melting naphthalene corresponding to this deflection from the graph =

Result: The melting point of the Naphthalene =

Standard value: The melting point of Naphthalene =
Percentage error =

Precautions:
1. The reading of galvanometer should be taken carefully.
2. The hot junction should be carefully dipped in naphthalene.
3. Do not inhale naphthalene vapor, as it may be harmful.
4. The ends of connecting wires should be properly cleaned.
5. The wires forming thermocouple should be in contact with each other at the junction only.

9.10 VIVA-VOCE

Q. 1. On what factors does the direction of thermo electric current depends?
Ans. It depends upon the nature of metals in contact.
Q. 2. What is the direction of current in case of copper-iron and antimony-bismuth couple?
Ans. For copper-iron couple-current flows from copper to iron at hot junction. For antimony-bismuth couple the current flow at cold junction from antimony to bismuth.
Q. 3. What is thermo-couple?
Ans. A thermo couple is a circuit formed by joining two dissimilar metals. Its junctions are kept at different temperature.
Q. 4. What is neutral temperature?
Ans. The neutral temperature is the temperature at which thermoelectric e.m.f. is maximum.
Q. 5. How does the temperature of inversion vary?
Ans. The temperature of inversion for a given couple at hot junction is as much above the neutral temperature as the temperature of cold junction is below it.
Q. 6. What is thermoelectric effect?
Ans. When two junctions of different metals are kept at different temperatures, then an emf produced in the circuit gives rise to a current in the circuit. It is called Seebeck effect.

Q. 7. Is the neutral temperature same for all thermocouples?

Ans. No, it is different for different thermocouples.

Q. 8. What is temperature of inversion?

Ans. The temperature at which thermoelectric emf changes its sign, is called temperature of inversion.

Q. 9. How temperature of inversion θ_i is related to neutral temperature θ_n?

Ans. $\theta_n - \theta_c = \theta_i - \theta_n$

Q. 10. What is calibration curve?

Ans. A curve showing the variation of thermoelectric emf with temperature, is known as calibration curve.

Q. 11. What is Peltier effect?

Ans. When current is passed through the junction of two different metals, one junction is heated while other is cooled.

Q. 12. How thermo emf is generated?

Ans. The concentration of electrons at the interface of two metals is different. The electrons from higher concentration interface, are transferred to lower concentration interface. Thus a constant potential difference is developed when one junction is hot and other is cold. The contact potential is higher at hot junction than that of cold junction, and so thermo emf is generated.

Q. 13. What is Thomson effect?

Ans. Whenever the different parts of the same metal are at different temperature an emf is developed in it. This is called Thomson effect.

Q. 14. What is Seebeck effect?

Ans. When two wires of different metals are joined at their ends and a temperature difference is maintained between the junctions, a current will flow in the circuit. This is known as Seebeck effect.

Q. 15. What is Peltier effect?

Ans. When a battery is inserted in a thermocouple circuit whose two junctions are initially at the same temperature, one of the junctions will became hot and the other cold. This phenomenon is known as Peltier effect.

Q. 16. Explain the existence of an emf at the junction of two metals.

Ans. When two different metals are joined at their ends the free electrons of one metal will flow to the other because the electron density of the two metals is different. As a result of the electron flow, one metal becomes positive with respect to the other and a potential difference is created at the junction. The magnitude of this potential difference, referred to as the contact potential difference, increases with temperature.

Q. 17. Explain the difference between Joule effect and Peltier effect.

Ans. In the case of Joule effect, the generated heat is proportional to the square of the current and is, therefore, independent of the direction of the current. Peltier effect, on the other hand, produces heating or cooling at a junction that is proportional to the current. Thus, a junction which is heated by a current will be cooled when the direction of the current is reversed.

Q. 18. How is a thermocouple constructed? Name some pairs of metals that are generally used for the construction of thermocouples.

Ans. To construct a thermocouple, say copper-constantan, one piece of constantan wire and two pieces of copper wires are taken. After cleaning the ends with emery paper, one end of each of the copper wires are spot-welded with the ends of the constantan wire forming two junctions of the thermocouple. Copper-constantan, copper-iron, platinum-rhodium etc. are the pairs of metals which are used for thermocouples.

Q. 19. Practical applications of thermocouples.

Ans. (*i*) To measure the temperature at a point.
(*ii*) To measure radiant heat.

Q. 20. Can you use an ordinary voltmeter to measure the thermo-emf?

Ans. No, because the emf is in the millivolt range.

Q. 21. What is the general nature of the thermo e.m.f. vs temperature curve? What is the nature of the curve that you have obtained?

Ans. Parabola. We obtain a straight line, because the temperature of the hot junction is much removed from the neutral temperature of the couple. That is, we obtain the straight portion of the parabola.

Q. 22. What is thermo-electric power? What is its value at 60°C for the copper-constantan thermocouple?

Ans. It is defined as the increase in thermo emf. of a thermocouple at a particular temperature of the hot junction per unit degree rise in temperature of the hot junction. For a copper-constantan thermocouple $\frac{dE}{dT}$ at 60ºC is about 40 μv per ºC.

Q. 23. Why does the null point remain constant during melting or freezing of the solid?

Ans. During melting or freezing the temperature does not change. This gives a constant null point reading.

Q. 24. What is the value of the thermo-electric power at the neutral temperature?

Ans. Zero.

Q. 25. How will the emf change if the temperature of the hot junction be increased beyond the neutral temperature?

Ans. The emf will decrease with increase of temperature and at a temperature, called the temperature of inversion, the e.m.f. will be reduced to zero. After the inversion temperature the polarity of thermo-emf will reverse.

Q. 26. What is the difference between heat and temperature?

Ans. The quantity of thermal energy present in a body is called 'heat' whereas the degree of hotness of a body is given by temperature.

Q. 27. Can you explain it on the kinetic theory?

Ans. The total kinetic energy possessed by all molecules of a body gives us the idea of 'heat' while the average kinetic energy possessed by a molecule gives us the 'temperature' of the body. It is given by $\frac{1}{2}m\overline{C}^2$ where m is the mass and $\overline{C}$ is the average velocity of the molecule.

Q. 28. How can you measure heat and temperature?

Ans. Heat is measured by the product of mass, specific heat and the temperature of the body. Temperature is measured by instruments called thermometers.

Q. 29. What types of thermometers do you know?

Ans. *Mecury thermometer:* These can be used for temperatures from –40ºC to 359ºC the freezing and boiling points of mercury.

Alcohol thermometers: These gives the maximum and minimum temperatures of the day and are used in meteorological departments.

Gas thermometers: They can be used from –260ºC to 1600ºC and are used to standardize the mercury thermometers.

Platinum resistance thermometers have the range from –200ºC to 1200ºC approximately.

Thermo-couples are also used to measure temperatures in the range –200ºC to 1700ºC.

Optical pyrometers are used to measure high temperatures from about 600ºC to 6000ºC.

Q. 30. Define specific heat?

Ans. It is defined as the ratio of the quantity of heat required to raise the temperature of a given mass of a substance to the quantity of heat required by the same amount of water to raise its temperature by the same amount.

Q. 31. What is a calorie?

Ans. It is the amount of heat in C.G.S. units required to raise the temperature of one gram of water from 14.5ºC to 15.5ºC.

Q. 32. Name the metals forming the thermoelectric series.

Ans. The following metals form the thermoelectric series:

Sb, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Co, Ni and Bi.

Q. 33. How does the thermoelectric series enable us to know the direction of flow of current in a thermocouple?

Ans. In the thermocouple formed of the two metals from the thermoelectric series, the current flows from the metal occurring earlier in the series to the metal occurring latter in the series through the cold junction.

Q. 34. Give the direction of thermo electric current: (*i*) at the cold junction of Cu-Bi (*ii*) at the hot junction of Fe-Cu (*iii*) at the cold junction of platinum-lead thermocouple.

Ans. (*i*) From Cu to Bi, (*ii*) from Cu to Fe, (*ii*) From lead to platinum.

Q. 35. How does the thermo e.m.f. vary with the temperature of the hot junction?

Ans. The thermo e.m.f. increases with increase in temperature of hot junction, till the temperature becomes equal to be neutral temperature of the hot junction. As the temperature is increased beyond the neutral temperature, thermo e.m.f. starts decreasing.

Q. 36. Write the expression connecting the thermoelectric e.m.f. of a thermocouple with the temperature difference of its cold and hot junctions.

Ans. The relation between thermo e.m.f. (E) and temperature difference (θ) between the cold and the hot junction is given as

$$E = \alpha\theta + \beta\theta^2$$

Q. 37. Name a few metals, which have (i) positive Thomson coefficient and (ii) negative Thomson coefficient.

Ans. (*i*) For copper, antimony, silver and Zinc, Thomson coefficient is positive

(*ii*) For iron, cobalt, bismuth and platinum, Thomson coefficient is negative.

Q. 38. What is the cause of production of thermo e.m.f. in the thermocouple?

Ans. In a thermocouple, heat is absorbed at the hot junction, while it is rejected at the cold junction. The production of thermo e.m.f. in a thermocouple is the result of conservation of the net heat absorbed in the thermocouple into electric energy. In other words, the thermoelectric effect obeys the law of conservation of energy.

Q. 39. Heat is produced at a junction of two metals, when a current passes through. When the direction of current is reversed, heat is absorbed at the junction (i.e. the junction

gets cooler) Is the usual formula (I^2R = Power dissipated as heat) applicable for this situation. If not why not?

Ans. No, the formula is not applicable to the situation, when on reversing the direction of current, the heat is absorbed at the hot junction. It is because Joule's heating effect of current and reversible Seebeck effect are different from each other.

Q. 40. How does thermoelectric series help to predict the direction of flow of current in a thermocouple?

Ans. It helps to know the direction in which current will flow, when a thermocouple is formed with the wires of any two metals in the series. The direction of current will be from a metal occurring earlier in this series to the metal occurring latter in the series through the cold junction.

Q. 41. Why do we generally prefer Sb-Bi thermocouple?

Ans. The metals Sb and Bi are at the two extreme ends of the thermoelectric series and hence for the given temperature of cold and hot junctions, the thermo e.m.f. produced is maximum. It is because more the metals are separated in the series, the greater will be the thermo-e.m.f. produced.

Q. 42. What is a thermopile?

Ans. It is a combination of a large number of thermocouples in series. As such, it is able to detect the heat radiation and to note the small variation or difference in temperature.

10

Refraction and Dispersion of Light

10.1 REFRACTION OF LIGHT

Light rays travel in straight lines in a homogeneous medium. But whenever a light ray passes from one transparent medium to another, it deviates from its original path at the interface of the two media. In the second medium the ray either bends towards the normal to the interface or away from the normal. The bending of the light-ray from its path in passing from one medium to the other medium is called 'refraction' of light. If the refracted ray bends towards the normal relative to the incident ray, then the second medium is said to be 'denser' than the first medium Fig. 10.1(*a*). But if the refracted ray bends away from the normal, then the second medium is said to be 'rarer' than the first medium Fig. 10.1(*b*).

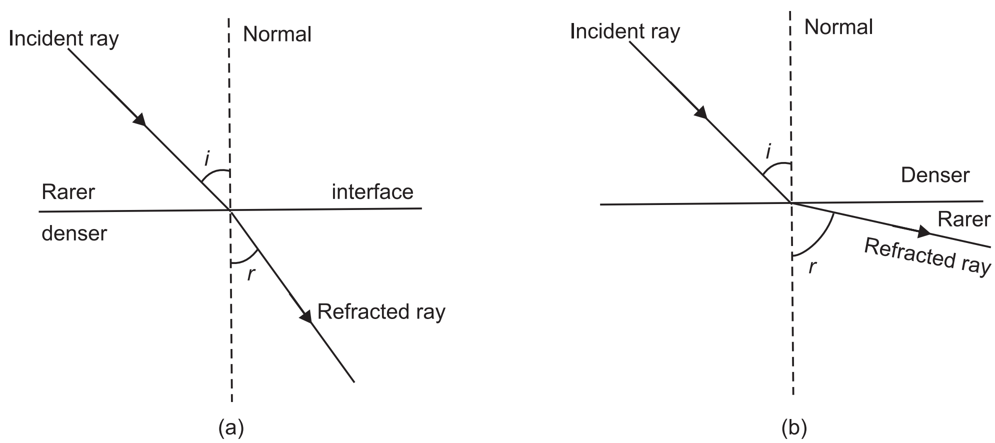


Fig. 10.1

The refraction of light takes place according to the following two laws known as the 'laws of refraction':

1. The incident ray, the refracted ray and the normal to the interface at the incident point all lie in the same plane.
2. For any two media and for light of a given colour (wavelength), the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

If the angle of incidence is i and the angle of refraction is r, then

$$\frac{\sin i}{\sin r} = \text{constant}$$

This law is called the 'Snell's law' and the constant is called the refractive index of the second medium with respect to the first medium.

If the first medium be represented by 1 and second by 2, then the refractive index is represented by ${}_1n_2$. Thus

$$\frac{\sin i}{\sin r} = {}_1n_2$$

If the path of light be reversed, then by the principle of reversibility of light, we have

$$\frac{\sin r}{\sin i} = {}_2n_1$$

Where ${}_2n_1$ is the refractive index of medium 1 with respect to medium 2. From the above two expressions, we have

$${}_1n_2 \times {}_2n_1 = 1$$

or

$${}_1n_2 = \frac{1}{{}_2n_1}$$

If there are three media 1, 2, 3, then

$${}_1n_2 \times {}_2n_3 \times {}_3n_1 = 1$$

If medium 1 is air, medium 2 is water and medium 3 is glass, then

$${}_an_w \times {}_wn_g \times {}_gn_a = 1$$

or

$${}_wn_g = \frac{1}{{}_an_w \times {}_gn_a} = \frac{{}_an_g}{{}_an_w}$$

Refraction of light occurs because the speed of light is different in different media.

According to Huygen's principle, when a wavefront passes from one medium into another, the speed of the secondary wavelets orginating from the wavefront changes in the second medium. As result, the (refracted) wavefront in the second medium bends with respect to the (incident) wavefront in the first medium. If the incident and the refracted wavefronts are inclined at angles i and r respectively with the interfrace of the two media, then

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

Where v_1 and v_2 are the speeds of the wavelets in the first and second media respectively.

But $\sin i/\sin r = {}_1n_2$ (snell's law)

$$\therefore \qquad {}_1n_2 = \frac{v_1}{v_2} = \frac{\text{speed of light in the first medium}}{\text{speed of light in the second medium}}$$

If these speeds are equal then both the wavefronts would have been mutually parallel ($i = r$) and light would have not been bent.

If the incident wavefront be parallel to the interface of the two media ($i = 0$), then the refracted wavefront will also be parallel to the interface ($r = 0$).

In the process of refraction the speed, the wavelength and the intensity of light change, while the frequency of light remains unchanged. The intensity changes because alongwith refraction there is also a partial reflection and an absorption of light.

10.2 REFRACTION THROUGH A PRISM

A prism is a homogeneous, transparent medium such as glass enclosed by two plane surfaces inclined at an angle. These surfaces are called the 'refracting surfaces' and the angle between them is called the refracting angle' or the 'angle of prism'. The section cut by a plane perpendicular to the refracting surfaces is called the 'principal section' of the prism. Let *ABC* be the principal section of a glass prism. The angle *A* is the refracting angle of the prism. Let a monochromatic ray of light *PQ* be incident on the face *AB*. This ray is refracted towards the normal *NQE* and travels in the prism along *QR*. The refracted ray *QR* bends away from the normal *MRE* at *AC* and emerges along *RS* into the air. Thus *PQRS* is the path of the light ray passing through the prism. Let i be the angle of incidence and r the angle of refraction at *AB*; and r' the angle of incidence and i' the angle of emergence at *AC*. Let δ be the angle between the incident ray *PQ* produced forward and the emergent ray *RS* produced backward. δ is called the 'angle of deviation'.

10.3 MINIMUM DEVIATION

For a given prism, the angle of deviation depends upon the angle of incidence of the light-ray falling on the prism. If a light-ray is allowed to fall on the prism at different angles of incidence (but not less than 30°) then for each angle of incidence the angle of deviation will be different. If we determine experimentally the angles of deviation corresponding to different angles of incidence and then plot i against δ, then we shall get a curve as shown in Fig. 10.3. It is seen from the curve that as the angle of incidence i increases, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then again increases. Thus, for one, and only one particular angle of incidence the prism produces minimum deviation. The minimum angle of deviation is represented by δ_m. In the position of minimum deviation, the angle of incidence i and the angle of emergence i' are equal.

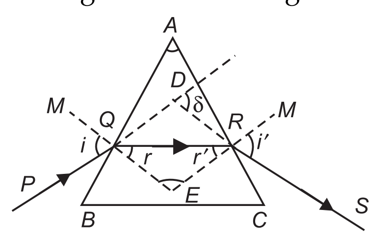


Fig. 10.2

In Fig. 10.2, for the ray *PQRS*, the angle of incidence is i and the angle of emergence is i'. Let the angle of deviation δ be minimum, i.e., $\delta = \delta_m$. If the path of the ray is reversed (*SRQP*), then the angle of incidence will be i' and the angle of emergence will i and δ will still be minimum. Thus δ is minimum for two angles of incidence i' and i. But δ can be minimum only for one angle of incidence. Therefore, it is clear that

$$i = i'$$

Let n be the refractive index of glass with respect to air. Applying snell's law for the refraction of light at the points Q and R, we have

$$n = \frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'}$$

Therefore, if $i' = i$, then $r' = r$. Thus in the position of minimum deviation $\angle AQR$ and $\angle ARQ$ are equal. Hence if the angles of the base of the prism are equal, then in the position of minimum deviation, the light passes in the prism parallel to the base.

10.4 FORMULA FOR THE REFRACTIVE INDEX OF THE PRISM

In Δ QDR, we have

$$\begin{aligned}\delta &= \angle DQR + \angle DRQ \\ &= (i - r) + (i' - r') \\ &= (i + i') - (r + r') \qquad \text{...(1)}\end{aligned}$$

In the quadrilateral $AQER$, $\angle AQE$ and $\angle ARE$ are right angles. Hence the sum of the angles A and E is 180°.

$$A + E = 180^\circ$$

In Δ QER $\quad r + r' + E = 180^\circ$

From these two equations, we have

$$\boxed{r + r' = A} \qquad \text{...(2)}$$

Substituting this value of $r + r'$ in Eq (1), we have

$$\delta = i + i' - A \qquad \text{...(3)}$$

If the prism is in the position of minimum deviation, then

$$i' = i,\ r' = r,\ \delta = \delta_m$$

Hence from the equations (2) and (3), we have

$$2r = A \text{ or } r = A/2$$

and $\quad \delta_m = 2i - A$ or $i = (A + \delta_m)/2$

By Snell's law, $\quad n = \dfrac{\sin i}{\sin r}$

Substituting the value of i and r, we get

$$n = \sin\left(\frac{A + \delta_m}{2}\right) \Big/ \sin\frac{A}{2}$$

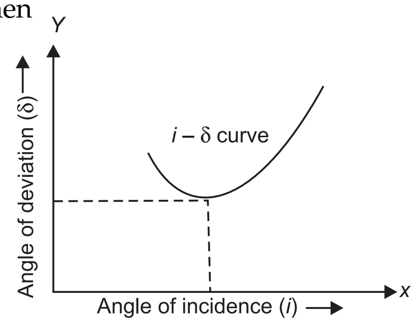


Fig. 10.3

Thus, knowing the angle of minimum deviation and the angle of prism, the refractive index of the material of the prism can be calculated.

10.5 DEVIATION PRODUCED BY A THIN PRISM

If the prism is thin (i.e. its angle A is nearly 5° or less), δ_m will also be small and we can put.

$$\sin\frac{A + \delta_m}{2} = \frac{A + \delta_m}{2} \text{ and } \sin\frac{A}{2} = \frac{A}{2}$$

$\therefore \quad n = \dfrac{(A + \delta_m)/2}{A/2}$

or $\quad \boxed{\delta_m = (n - 1) A}$

It is clear from this expression that the deviation produced by a thin prism depends only upon the refractive index n of the material of the prism and the angle A of the prism. It does not depend upon the angle of incidence.

10.6 CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

When a ray of light passes from a denser medium to a rarer medium (from glass to air), it bends away from the normal at the interface of the two media, that is, the angle of refraction is greater than the angle of incidence. On increasing the angle of incidence, the angle of refraction increases (Snell's law) and for a particular angle of incidence the angle of refraction becomes 90º, that is, the refracted ray grazes along the interface. This angle of incidence is called the 'critical angle' for the interface. Thus, the critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90º. Its value depends upon the two media and the colour of light. For glass-air interface, the critical angle for the visible mean light is about 42º.

If the rarer medium be represented by 1 and the denser medium by 2, then by Snell's law, the refractive index of the rarer medium with respect to the denser medium is given by

$$ {}_2n_1 = \frac{\sin C}{\sin 90^\circ} = \sin C $$

But ${}_2n_1 = \dfrac{1}{{}_1n_2}$, where ${}_1n_2$ is the refractive index of the denser medium with respect to the rarer medium.

Therefore, $$ \frac{1}{{}_1n_2} = \sin C $$

or $$ \boxed{{}_1n_2 = \frac{1}{\sin C}} $$

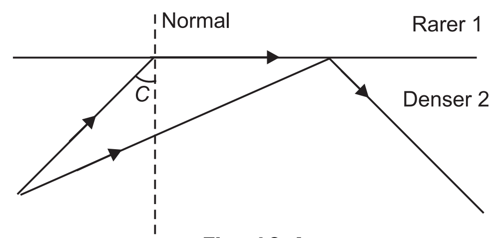


Fig. 10.4

Total internal reflection: When the angle of incidence in the denser medium is increased even very slightly beyond the critical angle, then the ray of light is reflected back completely into the denser medium in accordance to the laws of reflections. This phenomenon is called the 'total internal reflection', total because the whole of the light is reflected back into the denser medium.

10.7 DISPERSION OF LIGHT BY A PRISM

White light is a mixture of lights of different colours. When a beam of white light falls on a prism, it splits into the rays of its constituents colours. This phenomenon is called the 'dispersion' of light. The reason for the dispersion is that in a material medium the light rays of different colours travel with different speeds although in vacuum (or air) rays of all colours travel with the same speed (3×10^8 m/sec). Hence the refractive index n of a material is different for different colour of light. In glass, the speed of violet light is minimum while that of red light is maximum. Therefore, the refractive index of glass is maximum for the violet light and minimum for the red light ($n_V > n_R$). Hence according to the formula $\delta_m = (n - 1)A$, the angle of deviation for the violet light will be greater than the angle of deviation for the red light. When white light enters a prism, then rays of different colours emerge in different directions. The ray of violet colour bends maximum towards the base of the prism, while the ray of red colour bends least (Fig. 10.5). Thus, white light splits into its constituent colours. This is 'dispersion'.

The angle between the emergent rays of any two colours is called 'angular dispersion' between those colours. For example, the angle θ in Fig. 10.5 is the angular dispersion between red and violet rays. If δ_R and δ_V be the angles of (minimum) deviation for the red and the violet rays respectively, then the angular dispersion between them is

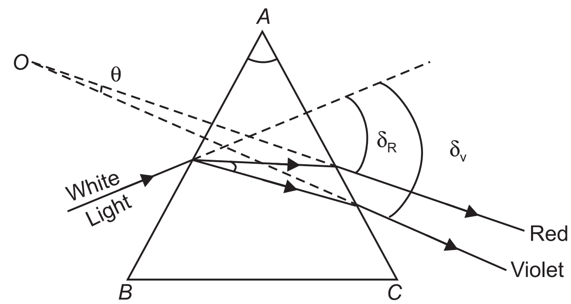


Fig. 10.5

$$\theta = \delta_V - \delta_R$$

Let n_R and n_V be the refractive indices of the material of the prism for the red and the violet rays respectively and A the angle of the prism. Then for a thin prism, we have

$$\delta_R = (n_R - 1)\, A \text{ and } \delta_V = (n_V - 1)\, A.$$

$\therefore$ angular dispersion

$$\begin{aligned}\theta &= (\delta_V - \delta_R)\\ &= (n_V - 1)\, A - (n_R - 1)\, A = (n_V - n_R)\, A.\end{aligned}$$

10.8 DISPERSIVE POWER OF AN OPTICAL MEDIUM

When white light passes through a thin prism, the ratio of the angular dispersion between the violet and red emergent rays and the deviation suffered by a mean ray (ray of yellow colour) is called the 'dispersive power' of the material of the prism. It is denoted by ω.

Let n_V, n_R and n_Y be the refractive indices of the material of the prism for the violet, red and yellow lights respectively and A the angle of the prism. Then the angular dispersion between the violet and the red rays is given by

$$\delta_V - \delta_R = (n_V - n_R)A,$$

and the angle of deviation for the yellow ray is given by

$$\delta_Y = (n_Y - 1)\, A.$$

$\therefore$ dispersive power $\quad \omega = \dfrac{\delta_v - \delta_R}{\delta_Y} = \dfrac{(n_v - n_R)\,A}{(n_Y - 1)\,A}$

$$\boxed{\omega = \frac{n_V - n_R}{n_Y - 1}}$$

Our eye is most sensitive to that part of the spectrum which lies between the F line (sky-green) and the C line (red) of hydrogen, the mean refractive index for this part is nearly equal to the refractive index for the D line (yellow) of sodium. Hence, for the dispersive power, the following formula is internationally accepted:

$$\omega = \frac{n_F - n_c}{n_D - 1}$$

Here F and C are respectively the sky-green and the red lines of hydrogen spectrum and D is the yellow line of sodium spectrum. The wavelengths of these lines are respectively 1861A°, 6563A° and 5893A°. (1A° = 10^{-10} meter)

Thus, the dispersive power depends only upon the material of the prism, not upon refracting angle of the prism. Greater is its value for a material, larger is the span of the spectrum formed by the prism made of that material. Dispersive power flint-glass is more than that of crown-glass.

10.9 PRODUCTION OF PURE SPECTRUM

When a beam of white light coming from a slit S passes through a prism, it splits up into its constituent colours and form a colour band from red to violet on a screen. This colour band is called 'spectrum'. In this spectrum, the different colours are not distinctly separated, but mutually overlap. Such a spectrum is an 'impure spectrum'. The reason for the impure spectrum is that the beam of light contains a large number of rays and each ray produces it own spectrum.

In Fig. 10.6, the rays 1 and 2 form their spectra R_1V_1 and R_2V_2 respectively which overlap, as shown. Clearly, the upper and lower edges of the composite spectrum are red and violet respectively, but in the middle part the colours are mixed.

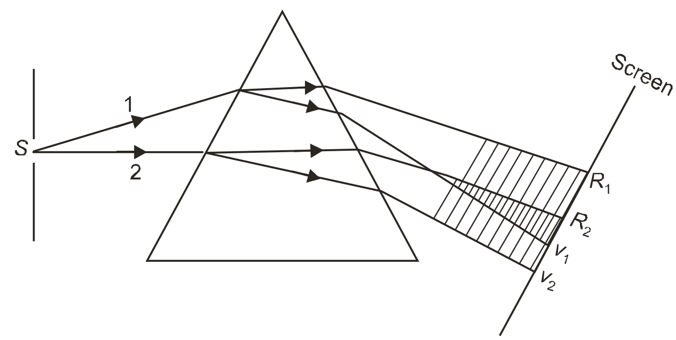


Fig. 10.6

When all the colours in a spectrum are distinctly separated and there is no-overlapping of colours anywhere, then the spectrum is a 'pure spectrum'. In practice, following conditions should be satisfied to obtain a pure spectrum.

(*i*) *The slit should be narrow:* Then only a few rays will fall on the prism and overlapping of colours will be reduced.

(*ii*) The rays falling on the prism should be parallel. Then all the rays will be incident on the prism at the same angle and rays of the same colour emerging from the prism will be parallel to one another which may be focussed at one point.

(*iii*) The rays emerging from the prism should be focussed on the screen by an achromatic convex lens. Then the rays of different colours will be focussed on the screen at different points.

(*iv*) The prism should be placed in minimum-deviation position with respect to the mean ray and the refracting edge of the prism should be parallel to the slit. Then the focussing of different colours at different points will be sharpest.

An arrangement for obtaining a pure spectrum is shown in Fig. 10.7.

S is a narrow slit illuminated by a white-light source placed behind it. The slit is placed at the focus of an achromatic convex lens L_1. Thus, the rays diverging from the slit are rendered parallel by the lens L_1 which is called the 'Collimating lens'. These parallel rays fall on a prism.

(placed in the position of minimum deviation) at the same angle of incidence. The prism splits these rays into their constituent colours. The rays of the same colour are deviated through the same angle and emerge parallel to one another. All these rays are received by another achromatic convex lens L_2 which focuses rays of different colours at different point on a screen placed at the focus of L_2. For example, all red rays are focussed at point R and all violet rays at point V. The rays of intermediate colours are focussed between R and V. Thus a pure spectrum is obtained on the screen.

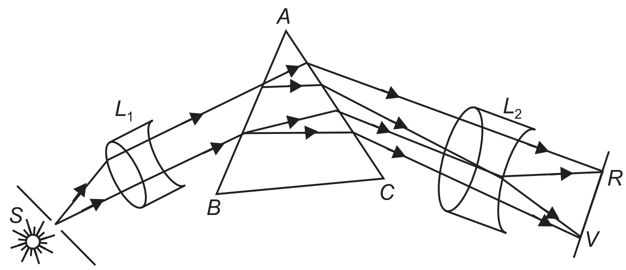


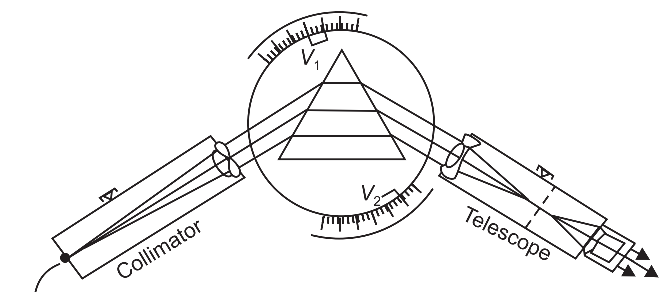
Fig. 10.7

All requirements for obtaining a pure spectrum are fulfilled in spectrometer.

10.10 OBJECT

Determination of the dispersive power of a prism.

Apparatus: A spectrometer, a glass prism, a neon lamp, reading lamp and a magnifying lens.



Schematic diagram of the spectrometer

Fig. 10.8

Formula used: The despersive power of the medium of the prism is given by

$$\omega = \frac{\mu_b - \mu_r}{\mu_y - 1}$$

Where μ_b and μ_r are the refractive indices of the medium for blue and red lines respectively and μ_y refers to the refractive index for the D yellow line of sodium and may be written as:

$$\mu_y = \frac{\mu_b + \mu_r}{2}$$

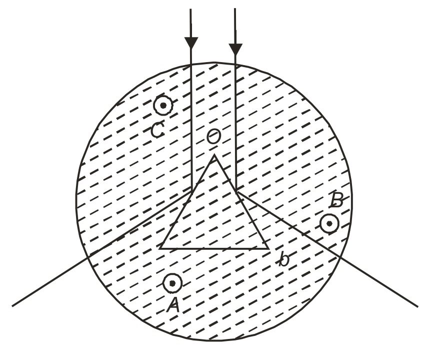
The refractive indices μ_b and μ_r can be determined by using the formulae:

$$\mu_r = \frac{\sin (A + \delta_r)/2}{\sin A/2}, \quad \mu_b = \frac{\sin(A + \delta_b)/2}{\sin A / 2}$$

Where A is the angle of the prism and δ_b and δ_r are the angles of minimum deviation for the blue and red respectively.

Manipulations:

1. Determine the vernier constant of the spectrometer.
2. Turn the telescope towards some brightly illuminated white background and move the eyepiece in or out till the cross-wire is sharply focused.
3. Switch on the neon lamp.
4. Bring the telescope and collimator in the same straight line and move the lamp right and left and up and down and fix its position when the illumination of the slit is maximum.
5. If the image of the slit is not bisected by the horizontal cross-wire in the telescope, adjust the leveling screws of the telescope or collimator till the slit is bisected.
6. Place the prism in the centre of the small prism table in such a way that one of its refracting face is at right angle to the line joining two of the levelling screws on the small prism table.



The Mechanical levelling

Fig. 10.9

Optical Leveling:

7. Turn the table till the edge of the prism is opposite to the middle of the collimator lens. The image of slit will now be reflected from each of the two faces.
8. First get the image of the slit from that face of the prism, which has been kept at right angles to the line joining the two leveling screws on the prism table. If it is not bisected by the horizontal cross-wire it should be made to do so by adjusting either of the two screws. This is done to ensure that the faces of the prism are vertical.
9. Now view the slit through the telescope, as it is reflected from the other face of the prism. If it is not bisected, adjust the third screw. This operation makes the edge of the prism vertical and parallel to the slit.
10. The prism table is thus leveled and the two faces of the prism are made vertical.
11. Turn the prism table till the beam of parallel light from the collimator enters the prism at one face and emerges from the other. Now the refractive image of the slit will be seen. The prism table is moved in a direction to increase the angle of incidence. As we increase the angle of incidence, the refractive ray will move in a particular direction. At one particular angle of incidence, the refractive ray will cease to move. This gives the position of minimum deviation for the prism. If the prism is moved still further, the refractive ray will begin to move in opposite direction. Turn the telescope a little to one side of the image and fix it. It is evident that there are now two positions of the prism, one on each side of that of minimum deviation, which will bring the image of the line again into view in the center of the field of the telescope.
12. The prism is first turned to the position where the angle of incidence is greater than that corresponding to minimum deviation. The telescope is now focused while looking at the spectrum through the telescope.
13. Now rotate the prism table in the opposite direction till the image is again visible through the telescope.
14. Focus the collimator.
15. Turn the prism table again so as to increase the angle of incidence till the refracted rays after going out of the field of view are again visible.

16. Focus the telescope.
17. Again rotate the prism table so as to decrease the angle of incidence and when the image reappears focus the collimator.
18. If all the above operations have been performed correctly, you will find that the refracted image will always be in sharp focus, no matter in which direction the prism is turned. This is known as Schuster's method of focusing the telescope and collimator.
19. Find the angle of the prism.
20. Find the angle of minimum deviation for bright red and greenish blue line of the neon spectrum.

Observations:

1. Vernier constant of the spectrometer =
2. Readings for the angle of the prism 'A'

Sl No.	*Readings for the Image Reflected*				*2A*	*A*
	From Right Face		*From Left Face*		*c – a d – b*	
	Venier A *a*	*Venier B* *b*	*Venier A* *c*	*Venier B* *d*		
1.						
2.						
3.						

Mean A =

3. Readings for the angle of minimum deviation

Sl. no.	*Colour of Light*	*Direct Reading*		*Reading for the Position of Minimum Deviation*		δ_m
		A	*B*	*A*	*B*	
1.	Red					
2.	Red					
3.	Red					
1.	Blue					
2.	Blue					
3.	Blue					

Mean: δ_m (red) = δ_r =, δ_m (Blue) = δ_b =

Calculation:

$$\mu_r = \frac{\sin (A + \delta_r)/2}{\sin A/2} =$$

$$\mu_b = \frac{\sin (A + \delta_b)/2}{\sin A/2}$$

$$\mu_y = \frac{\mu_r + \mu_b}{2} =$$

$$\omega = \frac{\mu_b - \mu_r}{\mu_y - 1}$$

Result: The dispersive power of the medium of the prism is found to be =

Precaution:
1. Use the neon tube sparingly.
2. The axis of the telescope and collimator must be perpendicular to the axis of rotation of the prism table. The three axis should meet at a point.
3. The optical surfaces of the prism should not be touched by hand and should be cleaned by tissue paper only.

10.11 VIVA-VOCE

Q. 1. What do you mean by interference of light?

Ans. When the two waves superimpose over each other, resultant intensity is modified. The modification in the distribution of intensity in the region of superposition is called interference.

Q. 2. What is refractive index?

Ans. The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant of any two media, i.e.,

$$\frac{\sin i}{\sin r} = \mu$$

a constant known as refractive index.

Q. 3. Is it essential in your experiment to place the prism in the minimum deviation position? If so, why?

Ans. Yes, it is essential because we obtain a bright and distinct spectrum and magnification is unity i.e. the distance of the object and image from the prism is same. The rays of different colours after refraction diverge from the same points for various colours.

Q. 4. Will the angle of minimum deviation change, if the prism is immersed in water?

Ans. Yes, the refractive index of glass in water is less than air hence angle of minimum deviation becomes less.

Q. 5. Does the angle of minimum deviation vary with the colour of light?

Ans. Yes, it is minimum for red and maximum for violet colour.

Q. 6. Does the deviation not depend upon the length of the base of the prism?

Ans. No, it is independent of the length of the base. By increasing the length of base, resolving power is increased.

Q. 7. What do you mean by pure spectrum?

Ans. A spectrum in which there is no overlaping of colours is known as pure spectrum. Each colour occupies a separate and distinct position.

Q. 8. Can you determine the refractive index of a liquid by this method?

Ans. Yes, the experimental liquid is filled in a hollow glass prism.

Q. 9. How refractive index vary with wavelengths?

Ans. Higher is the wavelength, smaller is the refractive index.

Q. 10. What is the relationship between deviation and wavelength?

Ans. Higher is deviation, smaller is wavelength i.e. deviation for violet colour is most but wavelength is least.

Q. 11. Which source of light are you using? Is it a monochromatic source of light?

Ans. Neon lamp or mercury lamp. It is not a monochromatic source of light. The monochromatic source contains only one wavelength.

Q. 12. Can you not use a monochromatic source (sodium lamp)?

Ans. Yes, we can use a sodium lamp but it will give only yellow lines and not the full specturm.

Q. 13. What is an eyepiece?

Ans. Eyepiece is a magnifier designed to give more perfect image than obtained by a single lens.

Q. 14. Which eyepiece is used in the telescope of a spectrometer?

Ans. Ramsden's eyepiece.

Q. 15. What is the construction of Ramsden's eyepiece?

Ans. It consists of two plano-convex lenses each of focal length f separated by a distance equal to $2f/3$.

Q. 16. What is the construction of Huygen's eyepiece?

Ans. It consists of two plano-convex lenses one having focal length $3f$ and other with focal length f and separated at distance $2f$.

Q. 17. What are chromatic and spherical abberration?

Ans. The image of white object formed by a lens is coloured and blurred. This defect is known as chromatic abberration. The failure or inability of the lens to form a point image of a axial point object is called spherical abberration.

Q. 18. How these two defects can be minimised?

Ans. The chromatic aberration can be minimised by taking the separation between two lenses $\left[d = (f_1 + f_2)/2\right]$.

The spherical aberration can be minimised by taking the separation as the difference of two focal lengths $d = (f_1 - f_2)/2$.

Q. 19. What is the main reason for which Ramsden's eyepiece is used with a spectrometer?

Ans. In this eyepiece, the cross wire is outside the eyepiece and hence mechanical adjustment and measurements are possible.

Q. 20. What is a telescope? What is its construction?

Ans. It is an instrument designed to produce a magnified and distinct image of very distinct object. It consists of a convex lens and eyepiece placed coaxially in a brass tube. The lens towards the object is called objective. This is of wide aperture and long focal-length. Observations are made by eyepiece. This is fitted in a separate tube which can slide in main tube.

Q. 21. What do you mean by dispersive power? Define it.

Ans. The dispersive power of a material is its ability to disperse the various components of the incident light. For any two colours, it is defined as the ratio of angular dispersion to the mean deviation, i.e.

$$\omega = \frac{\delta_v - \delta_r}{\delta_y}$$

Q. 22. On what factors, the dispersive power depends?

Ans. It depends upon (*i*) material and (*ii*) wavelengths of colours.

Q. 23. Out of the prism of flint and crown glasses, which one will you prefer to use?

Ans. We shall prefer a prism of flint glass because it gives greater dispersion.

Q. 24. What is a normal spectrum?

Ans. A spectrum in which angular separation between two wavelengths is directly proportional to difference of the wavelengths is called a normal spectrum.

Q. 25. Do you think that a prismatic spectrum a normal one?

Ans. No.

Q. 26. Can you find out the dispersive power of a prism with sodium light?

Ans. No, this is a monochromatic source of light.

Q. 27. How many types of spectra you know?

Ans. There are two main types of spectra: (*i*) emission spectra and (*ii*) absorption spectra.

Q. 28. What type of spectra do you expect to get from (*i*) an incandescent filament lamp (*ii*) sun light (*iii*) mecury lamp?

Ans. (*i*) continuous spectrum, (*ii*) band spectrum and (*iii*) Line spectrum.

Q. 29. What is difference between a telescope and microscope?

Ans. Telescope is used to see the magnified image of a distinct object. Its objective has large aperture and large focal-length. The microscope is used to see the magnified image of very near object. Its objective has small focal-length and aperture.

Q. 30. Without touching can you differentiate between microscope and telescope?

Ans. The objective of microscope has small aperture while the telescope has a large aperture.

Q. 31. What is that which you are adjusting in focussing the collimator and telescope for parallel rays?

Ans. In case of collimator, we adjust the distance between collimating lens and slit while in case of telescope the distance between cross wires from the objective lens is adjusted.

Q. 32. What are these distances equal to when both the adjustments are complete.

Ans. The slit becomes at the focus of collimating lens in collimator and cross wires become at the focus of objective lens in telescope.

Q. 33. How can telescope and collimator be adjusted together?

Ans.
(*i*) the prism is set in minimum deviation for yellow colour.
(*ii*) Prism is rotated towards telescope and telescope is adjusted to get a well defined spectrum.
(*iii*) Now the prism is rotated towards collimator and the collimator is adjusted to get well defined spectrum.
(*iv*) The process is repeated till the spectrum is well focussed. This is known as Schuster's method.

Q. 34. Why do you, often, use sodium lamp in the laboratory?

Ans. Sodium lamp is a convenient source of monochromatic light.

Q. 35. Do you know any other monochromatic source of light?

Ans. Red line of cadmium is also monochromatic source.

Q. 36. Why are two verniers provided with it?

Ans. Because one vernier will not give the correct value of the angle of rotation due to eccentricity of the divided circles with respect to the axis of the instruments. Two verniers eliminates this error.

Q. 37. Why are the lines drawn on the prism table?

Ans. With the help of these lines we can place the prism on the table in any particular manner. For example, when we measure the angle of the prism, we keep the prism such that it is at the centre of the table and one of its faces perpendicular to the line joining two of the levelling screws.

Q. 38. Why are the concentric circles drawn on the prism table?

Ans. These help us in placing the prism on the table such that axis of rotation of the table passes through the centre of the circumscribing circle of the prism.

Q. 39. Why is it necessary to place the prism on the table with the help of lines or circles?

Ans. Because, this minimises the error due to lack of parallism of the incident light.

Q. 40. What conclusion will you draw if the spectrum becomes rapidly worse in this process?

Ans. This means that the adjustments of the collimator and the telescope are being done in the wrong order.

EXERCISE

Q. 1. What is critical angle?
Q. 2. Why do you measure 2θ and not θ?
Q. 3. What is a spectrometer?
Q. 4. What are the requisites of a good spectrometer?
Q. 5. Explain the working of its essential mechanical and optical parts.
Q. 6. Why is the slit not permanently fixed at the focus of the collimating lens?
Q. 7. Has the aperture of the collimating lens anything to do with the brightness of the spectrum?
Q. 8. Explain the working of the slit.
Q. 9. Why are lines drawn on the prism table?
Q. 10. Why is it provided with levelling screws and why is the spectrometer not provided with these?
Q. 11. Explain the use of the three levelling screws in the adjustment of the prism table.
Q. 12. What is the function of the clamping and the tangent screw attached to the prism table?
Q. 13. How do you focus the telescope for parallel rays?
Q. 14. Explain the various methods used for this adjustment.
Q. 15. Why is the telescope not permanently focussed for infinity?
Q. 16. How do you level the prism table?
Q. 17. Can a spirit level be used for this adjustment?
Q. 18. What mechanical adjustments should the spectrometer be tested for before making any optical adjustments?
Q. 19. What is the difference between the readings of the two verniers?
Q. 20. Explain the difference between a spectrometer, a spectroscope and a spectrograph.
Q. 21. Why does light undergo dispersion when passed through a prism?
Q. 22. Explain Schuster's method.
Q. 23. What is the theory of Schuster's method?

11

Interference of Light

11.1 INTERFERENCE

The phenomenon of interference of light has proved the validity of the wave theory of light. Thomas Young successfully demonstrated his experiment on interference of light in 1802. When two or more wave trains act simultaneously on any particle in a medium, the displacement of the particle at any instant is due to the superposition of all the wave trains. Also, after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent. This principle was explained by Huygens in 1678.

From the principle of superposition of waves we know that when two wave trains arrive simultaneously at a point, the resultant vibrations have an amplitude different from the sum of the contributions by the two waves acting separately. This modification of amplitude obtained by superposition of two waves in known as interference. It is a characteristic of the wave motion. If the two waves arrive at the point in the same phase, the resulting amplitude is large, and if they arrive in opposite phase resulting amplitude is very small, zero if the two amplitude are equal. The former is called the constructive interference while the latter, the destructive interference. Light also exhibits the phenomenon of interference.

11.2 CONDITION OF INTERFERENCE OF LIGHT

The essential conditions for observing a sustained and good interference pattern are:

1. The two sources of light must be coherent, i.e., they should have a constant phase difference not changing with time.
2. The two wave-trains must have the same frequency i.e. the two sources must be monochromatic.
3. The two-light wave-trains must be transmitting in the same direction or make a very small angle with each other.
4. For good contrast the amplitudes should be equal or nearly equal.
5. For interference of polarised waves, they must be in the same state of polarisation.

11.3 COHERENT SOURCES

The intensity at any point depends upon the phase difference between the two waves arriving at that point, which in turn depends upon

(a) The phase difference between the sources themselves, and
(b) The geometrical arrangement which determines the path difference Δ between the two sources and the point under consideration.

It is clear that the phase difference at a point due to path difference does not change with time as it depends upon the geometrical arrangement of the experiment. Hence to achieve the condition that the total phase difference at the point under consideration may not change with time, the phase difference between the sources themselves should not change with time. Two such sources, whose phase difference does not change with time are known as coherent sources. The condition of coherent sources is the basic condition for obtaining a sustained (i.e., not changing with time) pattern of interference.

If the phase difference between the sources does not vary from time to time, then the brightness or darkness at different points in space is determined by the geometry of the experimental arrangement alone and does not change with time. If, however, the phase difference between the two sources varies with time, than intensity at each observed point will also vary with time. We shall than observe only time average of intensity and this will be the same at all points of observation. It may be noted that at any given instant fringes are still formed but their positions change rapidly with time due to change of phase relation between the sources; and hence the fringes are blurred out to uniform intensity. Therefore, it is important for observing a sustained interference pattern with light that the phase relation between the two sources does not change with time i.e., the sources are coherent.

In the emission of light a huge number of atoms are participating and the emission from each atom is also randomly changing phase from time to time. It is, therefore, impossible to have two independent sources having fixed phase relation. Even two different portions of the same source will not have a fixed phase relation. The only alternative is to get 'photo copies' from the single source. These can be obtained by various methods. In that case whatever change of phase takes place in the source, also take place in these 'copies', so that the difference of phase does not change with time.

In practice two coherent source are realized from a single source by the following methods.

1. A real narrow source and its virtual image produced by reflection as in Lloyd's single mirror.
2. Two virtual images of the same source produced by reflection as in Fresnel's double mirror.
3. Two virtual images of the same source produced by refraction as in Fresnel's biprism.
4. Two real images of the same source produced by refraction as in Billet's split lens.
5. By dividing the amplitude of a portion of the wave-front into two parts by reflection or refraction or both. This type of interference is produced in Newton's rings, Michelson's interferometer etc.

11.4 PHASE DIFFERENCE AND PATH DIFFERENCE

If the path difference between the two waves is λ, the phase difference = 2π

Suppose for a path difference x, the phase difference is δ.

For a path difference λ, the phase difference = 2π

$\therefore$ For path difference x, the phase difference $= \dfrac{2\pi x}{\lambda}$

Phase difference $\delta = \dfrac{2\pi x}{\lambda} = \dfrac{2\pi}{\lambda} \times$ path difference

Analytical Treatment of Interference: Consider a narrow slit S illuminated with monochromatic source emitting waves of wave length λ. Let A and B be two parallel t slits lying very close to each other and equidistant from the source S. The waves arriving at A and B will be in phase. On emergence from the slits A and B, the waves proceed as if they have started from A and B. To find the intensity at any point P on the screen XY placed parallel to A and B, let the general equation of the wave reaching P from A be

$$y_1 = a \sin \frac{2\pi}{\lambda} vt$$

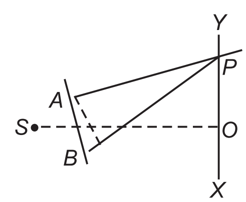


Fig. 11.1

where y_1 is the displacement of particle from its mean position at any time t, v the velocity of propagation of the wave of wave length λ and a the amplitude.

If y_2 be the displacement of the wave reaching P from B and x the path-difference with respect to the first wave from A, then general equation of the second wave is given by

$$y_2 = a \sin \frac{2\pi}{\lambda} (vt + x)$$

When the two waves superimpose, the resultant displacement Y is given by

$$Y = y_1 + y_2 = a \sin \frac{2\pi\, vt}{\lambda} + a \sin \frac{2\pi}{\lambda} (vt + x)$$

Using the relation

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \text{ to simplify, we get}$$

$$Y = 2a \cos \left(\frac{2\pi}{\lambda} \frac{x}{2}\right) \sin \frac{2\pi}{\lambda} \left(vt + \frac{x}{2}\right)$$

This is the equation of a simple harmonic vibration of amplitude

$$A = 2a \cos \left(\frac{2\pi}{\lambda} \frac{x}{2}\right) = 2a \cos \left(\frac{\pi x}{\lambda}\right)$$

For the amplitude to be minimum

$$\frac{\cos \pi x}{\lambda} = 0 \text{ or } \frac{\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2} \ldots (2n+1) \frac{\pi}{2}$$

or $$x = (2n+1) \frac{\lambda}{2}$$

This shows that the intensity is minimum when the path difference between the two wave trains is equal to an odd multiple of half a wave length.

For the amplitude to be maximum

$$\frac{\cos \pi x}{\lambda} = 1 \text{ or } \frac{\pi x}{\lambda} = 0, \pi, 2\pi, \ldots \pi n$$

or $$x = n\lambda$$

Thus the intensity is maximum if the path-difference between the two wave trains is an even multiple of half a wave length.

The amplitude of the resultant wave can be written as

$$A = 2a \cos \frac{\delta}{2}$$

where δ is the phase-difference between the two waves reaching P at any instant, as the phase-difference for a path-difference λ is 2π and hence the phase-difference for a path difference x is

$$\delta = \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda} \times \text{path–difference}$$

$$\therefore \qquad \text{Intensity, } I = A^2 = 4a^2 \cos^2 \frac{\delta}{2}$$

Hence the intensity is proportional to $\cos^2 \frac{\delta}{2}$.

Special Cases:

(*i*) When the phase difference $\delta = 0, 2\pi, 2(2\pi), \ldots n(2\pi)$, or the path difference $x = 0, \lambda, 2\lambda, \ldots n\lambda$

$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(*ii*) When the phase difference, $\delta = \pi, 3\pi, \ldots (2n + 1)\pi$, or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (2n+1)\frac{\lambda}{2}$.

$I = 0$

Intensity is minimum when the path difference is an odd number multiple of half wave-length.

Energy Distribution: It is found that the intensity at bright points is $4a^2$ and at dark points it is zero. According to the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. For, at bright points, the intensity due to the two waves should be $2a^2$ but actually it is $4a^2$. As shown in Fig. 11.2 the intensity varies from 0 to $4a^2$, and the average is still $2a^2$. It is equal to the uniform intensity $2a^2$ which will be present in the absence of the interference phenomenon due to the two waves.

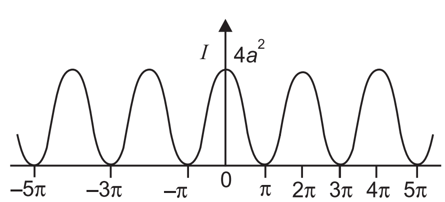


Fig. 11.2

Therefore, the formation of interference fringes is in accordance with the law of conservation of energy.

11.6 THEORY OF INTERFERENCE FRINGES

Consider a narrow monochromatic source S and two pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity. Consider a point P at a distance x from C, The waves reach at the point P from A and B.

Here $$PQ = x - \frac{d}{2},\ PR = x + \frac{d}{2}$$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$= 2xd$$

$$BP - AP = \frac{2xd}{BP + AP}$$

But $$BP = AP = D \text{ (approximately)}$$

$\therefore$ $$\text{Path difference} = BP - AP = \frac{2xd}{2D} = \frac{xd}{D}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$$

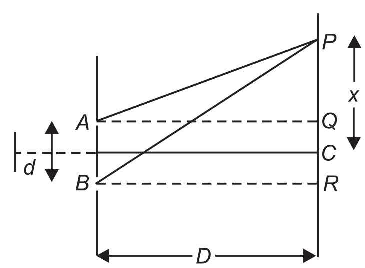


Fig. 11.3

(*i*) **Bright fringes:** If the path difference is a whole number multiple of wavelength λ, the point P is bright

$\therefore$ $$\frac{xd}{D} = n\lambda$$

where $$n = 0, 1, 2, 3, \ldots$$

or $$x = \frac{n\lambda D}{d}$$

This equation gives the distances of the bright fringes from the point C. At C, the path difference is zero and a bright fringe is formed

when $$n = 1, \quad x_1 = \frac{\lambda D}{d}$$

$$n = 2, \quad x_2 = \frac{2\lambda D}{d}$$

$$\ldots\ldots\ldots \qquad \ldots\ldots\ldots$$

$$x_n = \frac{n\lambda D}{d}$$

Therefore the distance between any two consecutive bright fringes.

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$$

(*ii*) **Dark fringes:** If the path difference is an odd number multiple of half wavelength, the point P is dark.

$$\frac{xd}{D} = (2n + 1)\frac{\lambda}{2} \text{ where } n = 0, 1, 2, 3, \ldots$$

or
$$x = \frac{(2n+1)\lambda D}{2d}$$

This equation gives the distances of the dark fringes from the point C.

when
$$n = 0, \quad x_0 = \frac{\lambda D}{2d}$$

$$n = 1, \quad x_1 = \frac{3\lambda D}{2d}$$

....

and
$$x_n = \frac{(2n+1)\lambda D}{2d}$$

The distance between any two consecutive dark fringes.

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d}$$

This shows that the distance between two consecutive dark or consecutive bright fringes is equal. This distance is known as the fringe-width.

If D and d are constants, then the fringe-width $\beta \propto \lambda$.

Hence fringes produced by light of shorter wavelengths will be narrow as compared to those produced by longer wavelengths.

11.7 STOKE'S TREATMENT TO EXPLAIN CHANGE OF PHASE ON REFLECTION

When a ray of light is reflected at the surface of a medium which is optically denser than the medium through which the ray is travelling, a change of phase equal to π or a path difference $\frac{\lambda}{2}$ is introduced. When reflection takes place at the surface of a rarer medium, no change in phase or path-difference takes place.

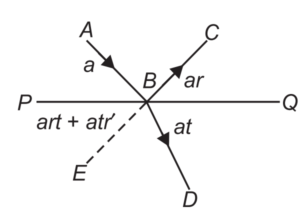


Fig. 11.4

Let PQ be the surface separating the denser medium below it from the rarer medium above it as shown in Fig. 11.4. A ray of light AB of amplitude a incident on this surface is partly reflected along BC and partly refracted into the denser medium along BD. If r is the coefficient of reflection at the surface of a denser medium, i.e., the fraction of the incident light which is reflected, then

Amplitude of the ray $BC = ar$

If 't' is the coefficient of transmission from the rarer into the denser medium i.e., the fraction of the incident light transmitted, then

Amplitude of the refracted ray $BD = at$ if there is no absorption of light, then

$$ar + at = a$$

or

$$r + t = 1$$

If the reflected and the refracted rays are reversed the resultant should have the same amplitude 'a' as that of the incident ray.

When CB is reversed it is partly reflected along BA and partly refracted along BE.

The amplitude of the refracted ray along $BE = art$

Similarly when the ray DB is reversed it is partly refracted along BA and partly reflected along BE. IF r' is the coefficient of reflection at the surface of a rarer medium, then

Amplitude of the reflected ray along $BE = atr'$

The two amplitudes along BA will combine together to produce the original amplitude, only if the total amplitude along BE is zero.

$$\therefore \qquad art + ar't = 0$$

or

$$\boxed{r = -r'}$$

The negative sign shows that when one ray has a positive displacement the other has a negative displacement. Hence the two rays, one reflected on reaching a denser medium and the other reflected on reaching a rarer medium, differ in phase by π from each other.

This explains the presence of a central dark spot in Newton's rings and is also responsible for the reversal of the condition of darkness and brightness produced in the reflected and transmitted systems in colours of thin films and in the fringes produced by Lloyd's single mirror.

11.8 INTERFERENCE IN THIN FILMS

Newton and Hooke observed and developed the interference phenomenon due to multiple reflections from the surface of thin transparent materials. Everyone is familiar with the beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of a soap bubble. Hooke observed such colours in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings when a convex lens was placed on a plane glass-plate. Youngs was able to explain the phenomenon on the basis of interference between light reflected from the top and the bottom surface of a thin film. It has been observed that interference in the case of thin films takes place due to (1) reflected light and (2) transmitted light.

11.9 INTERFERENCE DUE TO REFLECTED LIGHT (THIN FILMS)

Consider a transparent film of thickness 't' and refractive index μ. A ray SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along CQ. The difference in path between the two rays AT and CQ can be calculated. Draw CN normal to AT and AM normal to BC. The angle of incidence is i and the angle of refraction is r. Also produces CB to meet AE produced at P. Here $\angle APC = r$.

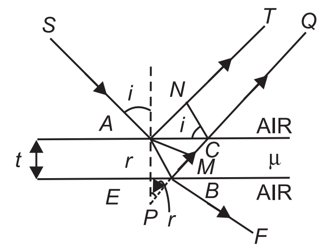


Fig. 11.5

The optical path difference

$$x = \mu\,(AB + BC) - AN$$

Here $\mu = \dfrac{\sin i}{\sin r} = \dfrac{AN}{CM}$

$\therefore$ $AN = \mu \cdot CM$

$$x = \mu\,(AB + BC) - \mu \cdot CM$$
$$= \mu\,(AB + BC - CM) = \mu\,(PC - CM) = \mu PM$$

In the ΔAPM, $\cos r = \dfrac{PM}{AP}$

or $PM = AP \cdot \cos r = (AE + EP) \cos r = 2t \cos r.$

$\because$ $AE = EP = t$

$\therefore$ $x = \mu \cdot PM = 2\mu t \cos r$ ...(*i*)

This equation (*i*), in the case of reflected light does not represent the correct path difference but only the apparent. It has been established on the basis of electromagnetic theory that, when light is reflected from the surface of an optically denser medium (air-medium interface) a phase change π equivalent to a path difference $\dfrac{\lambda}{2}$ occurs. Therefore, the correct path difference in this case,

$$x = 2\mu t \cos r - \frac{\lambda}{2} \quad \text{...(ii)}$$

1. If the path difference $x = n\lambda$ where $n = 0, 1, 2, 3, 4$ etc, constructive interference takes place and the film appears bright.

$\therefore$ $2\,\mu t \cos r - \dfrac{\lambda}{2} = n\lambda$

or $2\,\mu \text{t} \cos r = (2n + 1)\,\dfrac{\lambda}{2}$ ...(*iii*)

2. If the path difference $x = (2n + 1)\,\dfrac{\lambda}{2}$ where $n = 0, 1, 2, \ldots$ etc., destructive interference takes place and the film appears dark.

$\therefore$ $2\mu t \cos r - \dfrac{\lambda}{2} = (2n + 1)\,\dfrac{\lambda}{2}$

or $2\mu t \cos r = (n + 1)\,\lambda$

Here n is an integer only, therefore $(n + 1)$ can also be taken as n.

$\therefore$ $2\mu t \cos r = n\lambda$

where $n = 0, 1, 2, 3, 4, \ldots$ etc

It should be remembered that the interference patterns will not be perfect because the intensities of the rays AT and CQ will not be the same and their amplitude are different. The amplitudes will depend on the amount of light reflected and transmitted through the films. It has been found that for normal incidence, about 4% of the incident light is reflected and 96%

is transmitted. Therefore, the intensity never vanishes completely and perfectly dark fringes will not be observed for the rays *AT* and *CQ* alone. But in the case of multiple reflection, the intensity of the minima will be zero.

Consider reflected rays 1, 2, 3 etc. as shown in Fig. 11.6. The amplitude of the incident ray is a. Let r be the reflection coefficient, t the transmission coefficient from rarer to denser medium and t' the transmission coefficient from denser to rarer medium.

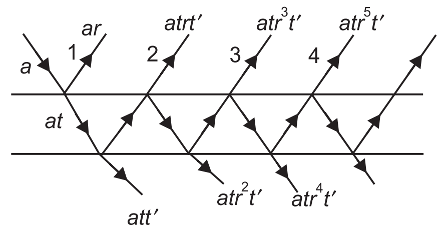


Fig. 11.6

The amplitudes of the reflected rays are: ar, $a{\cdot}tr\,t'$, $atr^3\,t'$, $atr^5\,t'$, ... etc. The ray 1 is reflected at the surface of a denser medium. It undergoes a phase change π. The rays 2, 3, 4 etc. are all in phase but out of phase with ray 1 by π.

The resultant amplitude of 2, 3, 4 etc. is given by

$$A = atrt' + atr^3 t' + atr^5 t' + \dots$$
$$= att' r\,[1 + r^2 + r^4 + \dots]$$

As r is less than 1, the terms inside the brackets form a geometric series.

$$A = att'r\left[\frac{1}{1-r^2}\right] = \left[\frac{att'r}{1-r^2}\right]$$

According to the principle of reversibility

$$tt' = 1 - r^2$$

$$\therefore \qquad A = \frac{a(1-r^2)r}{(1-r^2)} = ar$$

Thus the resultant amplitude of 2, 3, 4, ... etc is equal in magnitude of the amplitude of ray 1 but out of phase with it. Therefore the minima of the reflected system will be of zero intensity.

11.10 INTERFERENCE DUE TO TRANSMITTED LIGHT (THIN FILMS)

Consider a thin transparent film of thickness t and refractive index μ. A ray *SA* after refraction goes along *AB*. At *B* it is partly reflected along *BC* and partly refracted along *BR*. The ray *BC* after reflection at *C*, finally emerges along *DQ*. Here at *B* and *C* reflection takes place at the rarer medium (medium—air interface). Therefore, no phase change occurs. Draw *BM* normal to *CD* and *DN* normal to *BR*. The optical path difference between *DQ* and *BR* is given by,

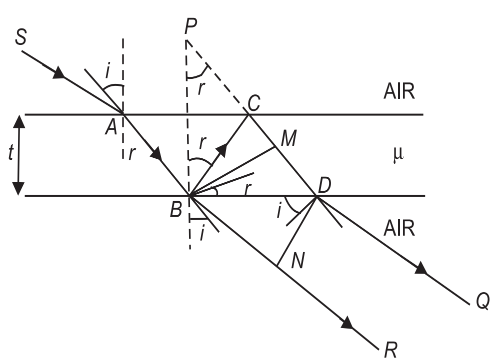


Fig. 11.7

$$x = \mu\,(BC + CD) - BN$$

Also $\qquad \mu = \dfrac{\sin i}{\sin r} = \dfrac{BN}{MD}$ or $BN = \mu{\cdot}MD$

In Fig. 11.7 $\qquad \angle BPC = r$ and $CP = BC = CD$

$\therefore \qquad BC + CD = PD$

$\therefore \qquad x = \mu\,(PD) - \mu\,(MD) = \mu\,(PD - MD) = \mu PM$

In the $\Delta\, BPM$, $\qquad \cos r = \dfrac{PM}{BP}$ or $PM = BP \cdot \cos r$

But, $\qquad BP = 2t$

$\therefore \qquad PM = 2t \cos r$

$\therefore \qquad x = \mu \cdot PM = 2\mu t \cos r$

(*i*) for bright fringes, the path difference $x = n\lambda$

$\therefore \qquad 2\mu t \cos r = n\lambda$

where $\qquad n = 0, 1, 2, 3, \ldots$ etc.

(*ii*) For dark fringes, the path difference $x = (2n + 1)\,\dfrac{\lambda}{2}$

$\therefore \qquad 2\mu t \cos r = \dfrac{(2n+1)\,\lambda}{2}$

where $\qquad n = 0, 1, 2, \ldots$ etc.

In the case of transmitted light, the interference fringes obtained are less distinct because the difference in amplitude between *BR* and *DQ* is very large. However, when the angle of incidence is nearly 45°, the fringes are more distinct.

11.11 COLOURS OF THIN FILMS

When white light is incident on a thin film, the light which comes from any point from it will not include the colour whose wavelength satisfies the equation $2\mu t \cos r = n\lambda$, in the reflected system. Therefore, the film will appear coloured and the colour will depend upon the thickness and the angle of inclination. If r and t are constant, the colour will be uniform. In the case of oil on water, different colours are seen because r and t vary.

1. If t and r are constant, the path-difference varies with μ or the wavelength of light. White light is composed of various colours, therefore, these colours will appear in the order violet, blue etc, as the wave-length λ increases.
2. If the angle of incidence changes, r also changes and hence the path-difference also changes. If, therefore, we view the film in various directions, different colours will be seen with white light.
3. When the thickness of the film varies, the film passes through various colours for the same angle of incidence.

From what has been said it is clear that colours in the transmitted and the reflected systems are complementary.

11.12 NON-REFLECTING FILMS

Non-reflecting glass surfaces can be prepared by depositing a thin layer or film of a transparent material. The refractive index of the material is so chosen that it has an intermediate value between glass and air. The thickness of the film is so chosen that it introduces a path-difference of $\frac{\lambda}{2}$. For example the refractive index of magnesium fluoride is 1.38. This value is greater than the refractive index of air and smaller than the refractive index of glass (μ = 1.5). The ray *AB* suffers reflection at *B* on the surface of a denser medium and proceeds along *BC*. A part of it moves along *BD* and after suffering reflection again at the surface of a denser medium (glass) emerges out along *EF*. Hence same phase-change occurs in both the rays at each reflection.

The optical path-difference for normal incidence will be $2\mu t$, where μ is the refractive index of the material of film and t its thickness. There will be destructive interference if

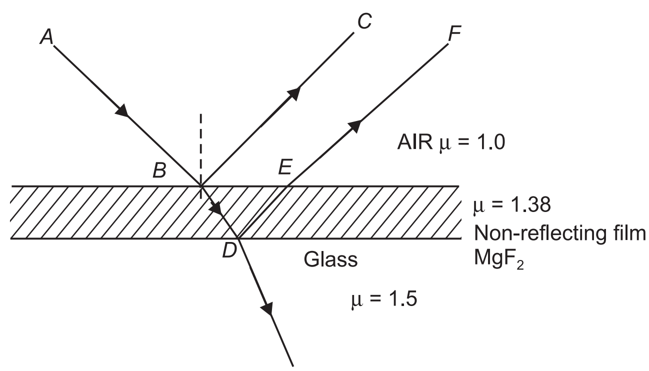


Fig. 11.8

$$2\mu t = (2n - 1)\frac{\lambda}{2}$$

where $\quad n = 1, 2, 3, \ldots$

For $\quad n = 1, \; 2\mu t = \frac{\lambda}{2}$

Hence the minimum thickness of the coating required for reflection at the centre of visible spectrum ($\lambda = 5.5 \times 10^{-5}$cm) for Mg F_2 is given by

$$t = \frac{\lambda}{4\mu} = \frac{5.5 \times 10^{-5}}{4 \times 1.38} = .996 \times 10^{-5}\text{cm}$$

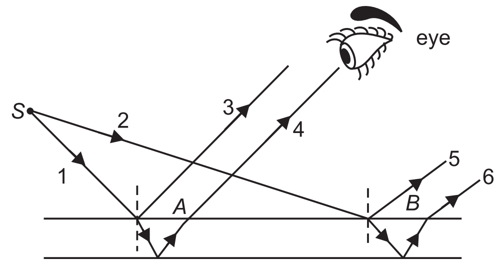
It may be noted that some reflection does take place on both the longer and shorter wavelength and reflected light has a purple colour. By coating the surface of a lens or prism the over all reflection can be reduced from 4 to 5 per cent to a fraction less than one-per cent.

This method is highly useful in reducing loss of light by reflection in instruments like periscope which has a number of air-glass surface.

11.13 NECESSITY OF A BROAD SOURCE

Interference fringes obtained in the case of Fresnel's biprism inclined mirrors and Lloyd's single mirror were produced by two coherent sources. The source used is narrow. These fringes can be obtained on the screen or can be viewed with an eyepiece. In the case of interference in thin films, the narrow source limits the visibility of the film.

Consider a thin film and a narrow source of light at S. The ray 1 produces interference fringes because 3 and 4 reach the eye whereas the ray 2 meets the surface at some different angle and is reflected along 5 and 6. Here, 5 and 6 do not reach the eye. Similarly we can take other rays incident at different angles on the film surface which do not reach the eye. Therefore, the portion A of the film is visible and not the rest.



If an extended source of light is used, the ray 1 after reflection from the upper and the lower surface of the film emerges as 3 and 4 which reach the eye. Also ray 2 from some other point of the source after reflection from the upper and the lower surfaces of the film emerges as 5 and 6 which also reach the eye. Therefore, in the case of such a source of light, the rays incident at different angles on the film are accommodated by the eye and the field of view is large. Due to this reason, to observe interference phenomenon in thin film, a broad source of light is required. With a broad source of light, rays of light are incident at different angles and the reflected parallel beams reach the eye or the microscope objective. Each such ray of light has its origin at a different point on the source.

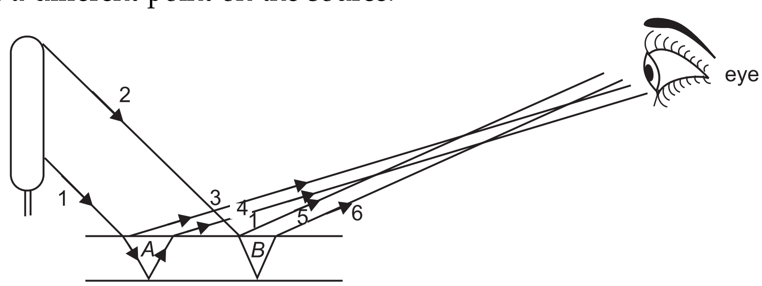


Fig. 11.9

11.14 FRINGES PRODUCED BY A WEDGE SHAPED THIN FILM

Let ABC be a wedge-shaped film of refractive index μ, having a very small angle at A, as shown in Fig. 11.10. If a parallel beam of monochromatic light is allowed to fall on the upper surface and the surface is viewed by reflected light, then alternate dark and bright fringes become visible.

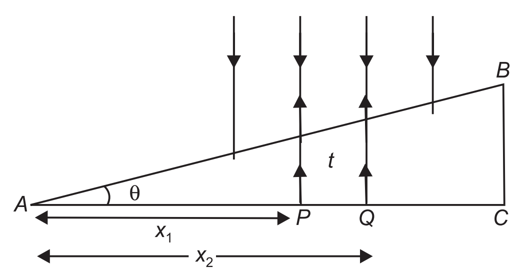


Fig. 11.10

Consider a point P at a distance x_1 from A where the thickness of the film is t. When light is incident normally the total path-difference between the light reflected at R from the upper face AB and that reflected at P from the lower face AC is $2\mu t + \dfrac{\lambda}{2}$ as an additional path-difference of $\dfrac{\lambda}{2}$ is produced in the beam reflected from the upper face AB at R where reflection takes place at the surface of a denser medium. The point P will appear dark and a dark band will be observed across the wedge, if

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = n\lambda$$

The point P will appear bright and a bright band will be observed across the wedge, if

$$2\mu t + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2\mu t = (2n-1)\frac{\lambda}{2}$$

If the nth dark fringe is formed at P, then

$$2\mu t = n\lambda$$

But $$\frac{t}{x_1} = \theta$$

or $$t = x_1\theta$$

$\therefore$ $$2\mu \cdot x_1\theta = n\lambda \qquad \text{...(i)}$$

Similarly for the $(n + 1)$ the dark band, which is formed at Q at a distance x_2 from A, we have

$$2\mu x_2\theta = (n+1)\lambda \qquad \text{...(ii)}$$

Subtracting (i) from (ii), we have

$$2\mu\theta\,(x_2 - x_1) = \lambda$$

or $$\text{Fringe-width } \beta = x_2 - x_1 = \frac{\lambda}{2\mu\theta}$$

Similarly if we consider two consecutive bright fringes the fringe width β will be the same.

A wedge-shaped air film can be obtained by inserting a thin piece of paper or hair between two plane parallel plates.

For air film $\mu = 1$, and $\theta = \frac{t}{x}$

Where t is the thickness of the hair and x its distance from the edge where the two plates touch each other.

11.15 TESTING THE PLANENESS OF SURFACES

If the two surfaces OA and OB are perfectly plane, the air-film gradually varies in thickness from O to A. The fringes are of equal thickness because each fringe is the locus of the points at which the thickness of the film has a constant value. This is an important application of the phenomenon of interference. If the fringes are not of equal thickness it means the surfaces are not plane. The standard method is to take an optically plane surface OA and the surface to be tested OB. The fringes are observed in the field of view and if they are of equal thickness the surface OB is plane. If not, the surface OB is not plane. The surface OB is polished and the process is repeated. When the fringes observed are of equal width, it means that the surface OB is plane.

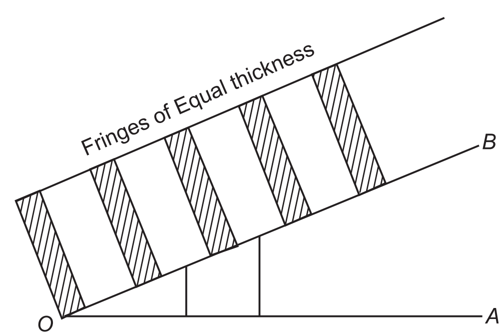


Fig. 11.11

11.16 NEWTON'S RINGS

Circular interference fringes can be produced by enclosing a very thin film of air or any other transparent medium of varying thickness between a plane glass plate and a convex lens of a large radius of curvature. Such fringes were first obtained by Newton and are known as Newton's rings.

When a plane-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are coloured. With monochromatic light, bright and dark circular fringes are produced in the air film.

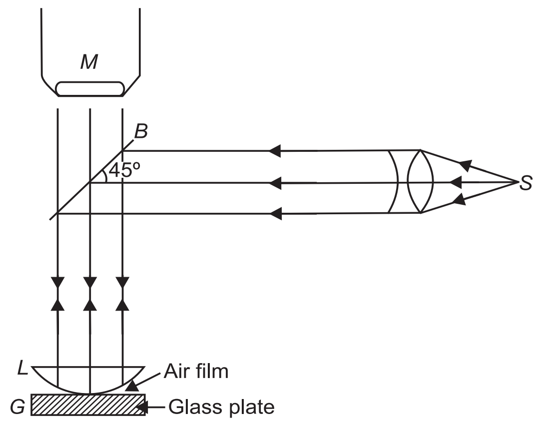


Fig. 11.12

S is a source of monochromatic light as shown in Fig. 11.12. A horizontal beam of light falls on the glass plate *B* at 45°. The glass plate *B* reflects a part of the incident light towards the air film enclosed by the lens *L* and the plane glass plate *G*. The reflected beam from the air film is viewed with a microscope, Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate *G*.

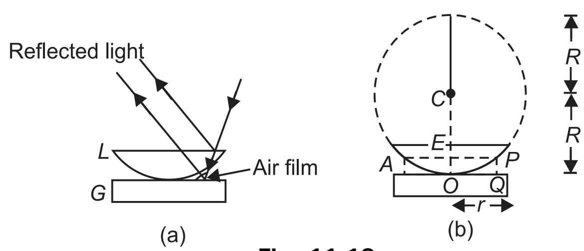


Fig. 11.13

Theory:

(*i*) *Newton's rings by reflected light:* Suppose the radius of curvature of the lens is *R* and the air film is of thickness *t* at a distance of *OQ* = *r* from the point of contact *O*.

Here, interference is due to reflected light. Therefore, for the bright rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \qquad \text{...(i)}$$

where $n = 1, 2, 3, \ldots$ etc.

Here, θ is small, therefore

$$\cos\theta = 1$$

For air, $\mu = 1$

$$2t = (2n-1)\frac{\lambda}{2} \qquad \text{...(ii)}$$

For the dark rings

$$2\mu t \cos\theta = n\lambda$$

or $$2t = n\lambda \qquad ...(iii)$$

where $n = 0, 1, 2, 3, ...$ etc.

In Fig. 11.13(b) $EP \times HE = OE \times (2R - OE)$

But $EP = HE = r,\ OE = PQ = t$

and $2R - t = 2R$ (Approximately)

$$r^2 = 2R \cdot t$$

or $$t = \frac{r^2}{2R}$$

Substituting the value of t in equations (ii) and (iii).

For bright rings $$r^2 = \frac{(2n-1)\lambda R}{2}$$

$$\boxed{r = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$

Fig. 11.14

For dark rings

$$r^2 = n\lambda R$$

$$\boxed{r = \sqrt{n\lambda R}}$$

when $n = 0$, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately, dark and bright rings are produced.

Result: The radius of the dark ring is proportional to

(*i*) $\sqrt{n}$, (*ii*) $\sqrt{\lambda}$ and (*iii*) $\sqrt{R}$.

Similarly the radius of the bright ring is proportional to

(*i*) $\sqrt{\frac{(2n-1)}{2}}$, (*ii*) $\sqrt{\lambda}$ and (*iii*) $\sqrt{R}$.

If D is the diameter of the dark ring

$$D = 2r = 2\sqrt{n\lambda R}$$

For the central dark ring

$$n = 0$$

$$D = 2\sqrt{n\lambda R} = 0$$

This corresponds to the centre of the Newton's rings.

While counting the order of the dark rings 1, 2, 3 etc, the central ring is not counted.

Therefore for the first dark ring

$$n = 1$$

$$D_1 = 2\sqrt{\lambda R}$$

For the second dark ring

$$n = 2$$

$$D_2 = 2\sqrt{2\lambda R}$$

and for the n^{th} dark ring

$$D_n = 2\sqrt{n\lambda R}$$

Take the case of 16th and 9th rings

$$D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R}$$

$$D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$$

The difference in diameters between the 16th and the 9th rings,

$$D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Similarly the difference in the diameters between the fourth and first rings,

$$D_4 - D_1 = 2\sqrt{4\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Therefore, the fringe width decreases with the order of the fringe and the fringes get closer with increase in their order.

For bright rings,

$$r_n^2 = \frac{(2n-1)\lambda R}{2}$$

$$D_n^2 = \frac{2\,(2n-1)\,\lambda R}{2}$$

$$r_n = \sqrt{\frac{(2n-1)\,\lambda R}{2}}$$

In above equation, substituting $n = 1, 2, 3$ (number of the ring) the radii of the first, second, third etc., bright rings can be obtained directly.

11.17 NEWTON'S RINGS BY TRANSMITTED LIGHT

In the case of transmitted light, the interference fringes are produced such that for bright rings,

$$2\mu t \cos\theta = n\lambda$$

and for dark rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2}$$

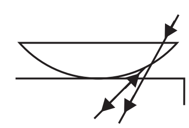


Fig. 11.15

Here, for air $\mu = 1$, and $\cos\theta = 1$

For bright rings $2t = n\lambda$

and for dark rings $2t = (2n-1)\frac{\lambda}{2}$

Taking the value of $t = \frac{r^2}{2R}$, where r is the radius of the ring and R the radius of curvature of the lower surface of the lens, the radius for the bright and dark rings can be calculated.

For bright rings, $r^2 = n\lambda R$

For dark rings, $r^2 = \frac{(2n-1)\,\lambda R}{2}$

where $n = 1, 2, 3, \ldots$, etc.

When $n = 0$, for bright rings $r = 0$

Therefore, in the case of Newton's rings due to transmitted light, the central ring is bright i.e. just opposite to the ring pattern due to reflected light.

11.18 DETERMINATION OF THE WAVELENGTH OF SODIUM LIGHT USING NEWTON'S RINGS

The arrangement used is shown earlier. In Figure S is a source of sodium light. A parallel beam of light from the lens L_1 is reflected by the glass plate B inclined at an angle of 45° to the horizontal. L is a plano-convex lens of large focal length. Newton's rings are viewed through B by the travelling microscope M focussed on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the n^{th} dark ring.

Suppose, the diameter of the n^{th} ring = D_n

$$r_n^2 = n\lambda R$$

But

$$r_n = \frac{D_n}{2}$$

$$\therefore \quad \frac{(D_n)^2}{4} = n\lambda R$$

or

$$D_n^2 = 4n\lambda R \qquad \text{...(i)}$$

Measure the diameter of the $(n + m)$th dark ring.

Let it be D_{n+m} $\therefore$ $\dfrac{(D_{n+m})^2}{4} = (n + m)\,\lambda R$

or

$$(D_{n+m})^2 = 4\,(n + m)\,\lambda R \qquad \text{...(ii)}$$

Subtracting (i) from (ii)

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

$$\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR}$$

Hence, λ can be calculated. Suppose the diameters of the 5^{th} ring and the 15^{th} ring are determined. Then $m = 15 - 5 = 10$.

$$\therefore \quad \lambda = \frac{(D_{15})^2 - (D_5)^2}{4 \times 10\,R}$$

The radius of curvature of the lower surface of the lens is determined with the help of a spherometer but more accurately it is determined by Boy's method.

Hence the wavelength of a given monochromatic source of light can be determined.

11.19 REFRACTIVE INDEX OF A LIQUID USING NEWTON'S RINGS

The experiment is performed when there is an air film between the plano-convex lens and the optically plane glass plate. These are kept in a metal container C. The diameter of the n^{th} and the $(n+m)^{th}$ dark rings are determined with the help of a travelling microscope.

For air $\quad (D_{n+m})^2 = 4\,(n + m)\,\lambda R,\; D_n^2 = 4n\lambda R$

$$D^2_{n+m} - D_n^2 = 4\,m\lambda R \qquad \text{...(i)}$$

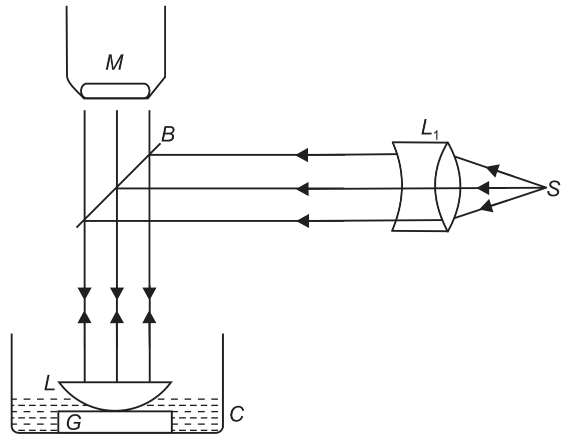


Fig. 11.16

The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the n^{th} ring and the $(n+m)^{th}$ ring are determined.

For the liquid, $2\mu t \cos\theta = n\lambda$ for dark rings

$$2\mu t = n\lambda, \text{ But } t = \frac{r^2}{2R}$$

or
$$\frac{2\mu r^2}{2R} = n\lambda$$

or
$$r^2 = \frac{n\lambda R}{\mu} \quad \text{But } r = \frac{D}{2};$$

$$D^2 = \frac{4n\lambda R}{\mu}$$

If D'_n is the diameter of the n^{th} ring and D'_{n+m} is the diameter of the $(n + m)^{th}$ ring

then
$$(D'_{n+m})^2 = \frac{4\,(n+m)\,\lambda R}{\mu}; \quad (D'_n)^2 = \frac{4n\lambda R}{\mu} \quad ...(i)$$

or
$$(D'_{n+m})^2 - (D'_n)^2 = \frac{4m\lambda R}{\mu} \quad ...(ii)$$

or
$$\mu = \frac{4m\lambda R}{(D^1_{n+m}) - (D^1_n)^2} \quad ...(iii)$$

If m, λ, R, D'_{n+m} and D'_n are known μ can be calculated.

If λ is not known then divide (iii) by (i) we get

$$\mu = \frac{(D_{n+m})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2}$$

11.20 NEWTON'S RINGS WITH BRIGHT CENTRE DUE TO REFLECTED LIGHT

The rings formed by reflected light have a dark centre when there is an air film between the lens and the plane glass plate. At the centre, the two surfaces are just in contact but the two interfering rays are reflected under different conditions due to which a path difference of half a wavelength occurs (since one of the rays undergoes a phase change of π, when reflected from the glass plate).

Consider a transparent liquid of refractive index μ trapped between the two surfaces in contact. The refractive index of the material of the lens is μ_1 and that of the glass plate is μ_2 such that $\mu_1 > \mu > \mu_2$. This is possible if a little oil of sassafaras is placed between a convex lens of crown glass and a plate of flint glass. The reflections in both the cases will be from denser to rarer medium and the two interfering rays are reflected under the same conditions. Therefore, in this case the central spot will be bright.

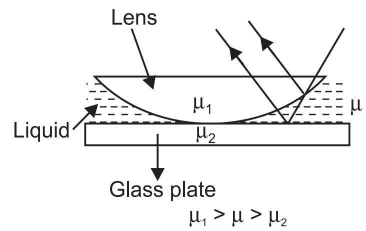


Fig. 11.17

The diameter of the n^{th} bright ring.

$$D_n = 2\sqrt{\frac{n\lambda R}{\mu}}$$

The central spot will also be bright, if $\mu_1 < \mu < \mu_2$, because a path difference of $\frac{\lambda}{2}$ takes place at both the upper and the lower glass-liquid surfaces. Here again the two interfering beams are reflected under similar conditions. In this case also the central spot is bright due to reflected light.

11.21 NEWTON'S RINGS WITH WHITE LIGHT

With monochromatic light, Newton's rings are alternately dark and bright. The diameter of the ring depends upon the wavelength of light used. When white light is used, the diameter of the rings of the different colours will be different and coloured rings are observed. Only the first few rings are clear and after that due to overlapping of the rings of different colours, the rings cannot be viewed.

11.22 INTERFERENCE FILTER

An interference filter is based on the principle of Fabry-Perot interferometer. It consists of an optical system that will transmit nearly a monochromatic beam of light (covering a small range of 50Å).

An interference filter consists of a thin transparent dielectric e.g. magnesium fluoride. There are two glass plates on whose surfaces semi-transparent silver films are deposited by evaporation method. The dielectric is placed between the two glass plates.

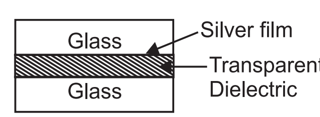


Fig. 11.18

When a beam of light is incident normally on the filter, multiple reflections take place within the film. The interference maxima for the transmitted beam will be governed by

$$2\mu t = n\lambda$$

Here μ is the refractive index of the dielectric and t is its thickness, and n is a whole number. If $\mu t = \lambda$, n will be equal to 2. For the value of $n = 1$, the maximum occurs for a wavelength of 2λ. Here λ and 2λ represent a wide separation in the visible region.

In the case of an interference filter, when the thickness of the dielectric is reduced, the transmitted wavelengths are more widely spaced. For an optical thickness (μt) of the dielectric film of 5000 Å, the transmitted wavelengths for $n = 1, 2, 3$, etc are 10,000 Å, 5000 Å, 3333 Å. These three wavelengths are widely spaced. Only 5000 Å is in the visible region. If there are two maxima in the visible region one of them can be eliminated by using a coloured glass filter. This may be the protecting glass of the dielectric itself.

Interference filters are better as compared to the coloured glass filters because in the case of interference filters light is not absorbed and hence there is no overheating. Interference filters are used in spectroscopic work for studying the spectra in a narrow range of wavelengths.

11.23 OBJECT

Measurement of wave length of sodium light by Newton's Ring.

Apparatus used: A small wooden box open on the top and in front, fitted with a glass plate C and lens D are placed at the bottom, a sodium lamp, a plano-convex lens and a travelling microscope M, sodium lamp.

Formula used: The wave length λ of light given by the formula.

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

where D_{n+p} = diameter of $(n + p)^{th}$ ring

D_n = diameter of n^{th} ring

P = an integer number (of the rings)

R = radius of curvature of the curved face of the plano-convex lens.

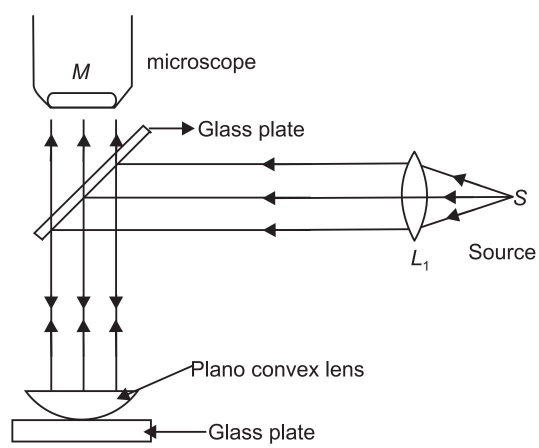


Fig. 11.19

Description of apparatus: The optical arrangement for Newton's ring is shown in Fig. 11.19. Light from a monochromatic source (sodium light) is allowed to fall on a convex lens through a broad slit which renders it into a nearly parallel beam. Now it falls on a glass plate inclined at an angle 45° to the vertical, thus the parallel beam is reflected from the lower surface. Due to the air film formed by a glass plate and a plane-convex lens of large radius of curvature, interference fringes are formed which are observed directly through a travelling microscope. The rings are concentric circles.

Theory: If a convex lens and a plane plate of glass are placed in contact with each other, Newton's ring are formed as a result of interference between two reflected rays from the bottom and top of air film between glass plates and lens. Near the point of contact the thickness of the air film will be very small in comparison with the wave length of light. Consequently, the point of contact of lens and the plate will be surrounded by a circular black spot, when viewed by a transmitted light. As the surface of the lens is a portion of a sphere the thickness of the air film will be increasing from the point of contact towards the periphery of the lens and will be uniform for all points on a circle, concentric with the point of contact. Thus the central black spot shown by reflected light will be surrounded by a concentric bright rings separated by dark rings.

In Fig. 11.20 *COD* represents a lens the convex surface of which is in contact with the plane glass *AB* at *O*, Fig. 11.20 when viewed normally by reflected light the points *C* and *D* equidistant from *O* will lie on a bright or a dark ring according as twice the distance *DB* (or *CA*) is equal to an odd or even number of half wave-length of incident light. From *O* draw the diameter *OF* of the circle of which the curved section of the lens is *COD*. Join *CD* cutting *OF* in *E* let $DB = CA = x_1$ and let D_n denote the diameter *CD* of the n^{th} ring under observation. If *R* is the radius of curvature of the lower surface of the lens, then we have

$$OE \cdot EF = CF \cdot ED$$

or, $$x_1 (2R - x_1) = (D_n/2)^2$$

or, $$2Rx_1 - x_1^2 = D_n^2/4$$

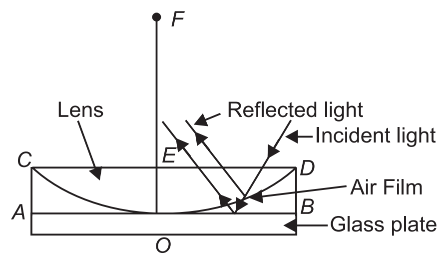


Fig. 11.20

Since x_1 is very small as compared to *R*, we have

$$x_1^2 << 2Rx_1$$

or, $$2Rx_1 = D_n^2/4$$

or, $$x_1 = D_n^2/8R$$

For *C* and *D* to be situated on a bright ring, we have

$$2\mu x_1 = (n + 1/2)\ \lambda$$

or, $$2D_n^2/8R = (n + 1/2)\ \lambda \qquad [\mu = 1 \text{ for air}]$$

or, $$D_n^2/4R = (n + 1/2)\ \lambda \qquad ...(1)$$

When *n* has values 1, 2, 3, ... etc, for the first, second, third, fourth etc rings respectively. The above formula is sufficient to give the value of λ, if *D* is measured by a traveling microscope and *R* is given.

From the formula (1) we see that *R* and λ (For a particular set up) are constant, thus

$$D_n^2 = 4R\ (n + 1/2)\ \lambda$$

$$D_m^2 = 4R\ (m + 1/2)\ \lambda$$

Hence if we draw curve with the square of the diameter as ordinate and the number of rings as abscissa, the graph will be a straight line.

$$D_m^2 - D_n^2 = 4R\lambda\ (m - n)$$

$$\lambda = (D_m^2 - D_n^{2)}/4R\ (m - n)$$

Thus the diameter of the *m*th and *n*th rings are to be found and substituted in the above formula to get λ.

Procedure:

1. Clean the surface of plano-convex lens and glass plate thoroughly.

2. Place the plano-convex lens on the glass plate with its curved surface touching the plate and put the combination in a wooden box.
3. Arrange a plane glass plate *G* at 45° to vertical and allow the parallel rays of sodium light to fall on the thin glass plate lens combination.
4. Focus the eye-piece on the cross wire and adjust the position of microscope by rack and pinion arrangement so that concentric rings are clearly seen in the field of view and crosswire lies on the centre of central dark ring. Clamp the microscope.
5. Adjust one of the cross wires tangential to the rings and then move the microscope in the horizontal direction with the help of slow motion screw towards one side (say left) of the centre till the cross wire coincides tangentially in the middle of the 30^{th} bright or dark ring. Note down the reading of the microscope knowing least count of it.
6. Now move the microscope to the right, set it on every alternate second bright/dark ring and note down the corresponding readings of the microscope till it reaches to the right side (30^{th} ring).
7. Determine the difference in the reading of microscope of the corresponding rings which gives the diameter of the various rings.
8. Remove the plano-convex lens and determine the radius of curvature (if it is unknown).
9. Plot a graph (Fig. 11.21) between the square of diameter of ring on *Y*-axis and number of rings on *X*-axis. A straight line is obtained. Take two points *A* and *B* on *X*-axis for any value of *p* and find the corresponding value of D^2_n and D^2_{n+p} on *Y*-axis and hence determine the difference.
   From graph $D^2_{n+p} - D^2_n = CD$ and $p = AB$

Observations:
Venire constant of the microscope =

No. of Rings	Micrometer Readings (in cm)		D_n (in cm)	D_n^2 (in cm)	$D_m^2 - D_n^2$ (in cm)	Mean $D_m^2 - D_n^2$ (in cm^2)
	Left	Right				
30						
25						
20						
15						
10						
5						

Radius of curvature of given lens = 100 cm (say)

Result: The wave-length of sodium light =
Standard mean wavelength λ = 5893 A.U.
Percentage error = %

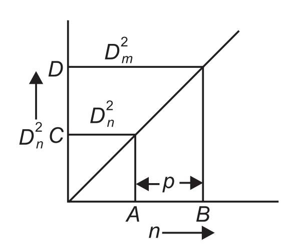


Fig. 11.21

Theoretical Error: In our case $\lambda = (D^2_m - D^2_n)/4\,(m - n)\,R$

Taking logarithm of both sides and differentiating

$$\delta\lambda/\lambda = \delta\,(D^2_m - D^2_n)/D^2_m - D^2_n + \delta R/R$$

$$= \frac{2\{D_m(\delta D_m) + D_n(\delta D_n)\}}{D_m^2 - D_n^2} + \delta R/R$$

$= ......$

Maximum permissible error = %

Precautions:

1. The glass plate and the lens should be very clean before setting up the apparatus.
2. To avoid the error of backlash the readings of microscope should be taken while the microscope is traveling in one direction only.
3. The lens used should be of large radius of curvature.
4. The source of light used should be an extended one.
5. Crosswire should be focussed on a bright ring tangentially.
6. Before measuring the diameters of rings, the range of the microscope should be properly adjusted.

11.24 VIVA-VOCE

Q. 1. What do you mean by interference of light?

Ans. When the two waves superimpose over each other, resultant intensity is modified. The modification in the distribution of intensity in the region of superposition is called interference.

Q. 2. What are interference fringes?

Ans. They are alternately bright and dark patches of light obtained in the region of superposition of two wave trains of light.

Q. 3. Is there any loss of energy in interference phenomenon?

Ans. No, there is no loss of energy in interference phenomenon. Only redistribution of energy takes place. The energy absent at dark places is actually present in bright regions.

Q. 4. What is the physical significance of this phenomenon?

Ans. The phenomenon of interference of light has proved the validity of the wave theory of light.

Q. 5. What are the essential conditions for observing the interference phenomenon in the laboratory?

Ans. (*i*) The two sources should be coherent.

(*ii*) The two sources must emit waves of same wavelength and time period.

(*iii*) The sources should be monochromatic.

(*iv*) The amplitudes of the interfering waves should be equal or nearly equal.

Q. 6. What are the different classes of interference?

Ans. (*i*) Division of wavefront, the incident wavefront is divided into two parts by utilising the phenomenon of reflection, refraction or diffraction.

(*ii*) Division of amplitude, the amplitude of incoming beam is divided into two parts either by partial reflection or refraction.

Q. 7. What is the construction of sodium lamp?

Ans. It consists of a U-shaped glass tube with two electrodes of tungsten coated with barium oxide. The tube is filled with neon gas at a pressure of 10 mm of mercury and some sodium pieces. This tube is enclosed in a vacuum jacket to avoid heat losses.

Q. 8. Why does the sodium lamp give out red light in the beginning?

Ans. First of all discharge passes through neon gas.

Q. 9. Why is the neon gas filled in it at all?

Ans. Initially, no discharge passes through sodium as its vapour pressure is low. First, the discharge passes through neon. Now the temperature rises and sodium vaporises. Now sodium gives its own characteristic yellow colour.

Q. 10. How are these rings formed?

Ans. When a plano-convex surface is placed on a glass plate, an air film of gradually increasing thickness is formed between the two and monochromatic light is allowed to fall normally on film and viewed in reflected light, alternate dark and bright rings are observed. These are known as Newton's ring.

Q. 11. Why are the rings circular?

Ans. These rings are loci of constant thickness of the air film and these loci being concentric circle hence fringes are circular.

Q. 12. Why do you use in extended source of light here?

Ans. To view the whole air film, an extended source is necessary.

Q. 13. What may be the reason if the rings are not perfectly circular?

Ans. (*i*) The plate may not be optically flat.

(*ii*) The surface of the lens may not be the part of a perfect sphere and

(*iii*) The plate and the lens may not be perfectly clean.

Q. 14. In the Newton's rings system, the fringes at the centre are quite broad, but they get closer as we move outward why is it so?

Ans. This is due to the fact that the radii of dark rings are proportional to square root of natural numbers while those of bright rings are proportional to square root of odd natural numbers.

Q. 15. What are the factors which govern the radius of a ring?

Ans. The radius depends upon

(*i*) wavelength of light used.

(*ii*) refractive index 'μ' of enclosed film.

(*iii*) radius of curvature R of convex lens.

Q. 16. What would be your observation in transmitted light?

Ans. Where we have bright fringe in the reflected light, we shall have a dark fringe in the transmitted light and vice-versa. These two systems of fringes are complementary.

Q. 17. Do you get rings in the transmitted light?

Ans. Yes, in this case the colour of rings is complimentary of the reflected light.

Q. 18. Why is the centre of the ring dark?

Ans. Although at centre, the thickness of air film is zero but at the point of contact the two interfering rays are opposite in phase and produce zero intensity.

Q. 19. Sometimes the centre is bright, why?

Ans. This happens when a dust particle comes between the two surfaces at the point of contact.

Q. 20. What will happen if the glass plate is silvered on its front surface?

Ans. The transmitted system of fringes will also be reflected and due to the superposition of the reflected and transmitted (which is also reflected now) systems the uniform illumination will result.

Q. 21. If by chance, you get a bright central spot in your experiment, will you proceed with the experiment with the same system of fringes or will you reject them?

Ans. The system will not be rejected, but we will proceed on with measurement, because the formula employed for the evaluation of λ involves the difference of the squares of the diameters of two rings and the order of fringe at the centre is immaterial.

Q. 22. What will happen when the sodium lamp is replaced by a white light source?

Ans. A few coloured fringes are observed near the centre. The violet colour will come first as we proceed away from the centre.

Q. 23. What will happen if a few drops of a transparent liquid are introduced between the lens and the plate?

Ans. The fringes will contract, with diameter reduced by a factor of $\sqrt{\mu}$.

Q. 24. Can you utilise this procedure for determining the refractive index of a liquid?

Ans. Yes.

Q. 25. Will there be any change in rings if light is obliquely incident?

Ans. The diameter of the rings will increase.

Q. 26. Why do you make the light fall on the convex lens normally?

Ans. The light is allowed to fall normally so that angles of incidence and reflection may be zero so that cos θ may be taken as unity.

Q. 27. In this experiment the rings are observed through the lens. Does it affect the observation of diameter?

Ans. Due to refraction through lens, the observed diameters will be different from their actual values. To avoid this thin lens should be used.

Q. 28. How can you determine R?

Ans. This can be determined either by spherometer or by Boy's method.

Q. 29. Can Newton's rings be formed with the combination of convex and concave lens?

Ans. The plane glass plate is replaced by concave lens i.e. the convex lens is placed over the concave lens.

Q. 30. What are the uses of Newton's rings?

Ans. (*i*) To determine λ of light (monochromatic).

(*ii*) To determine μ of a liquid.

(*iii*) To measure the radius of spherical surface.

(*iv*) To measure expansion coefficient of crystal.

EXERCISE

Q. 1. How do you explain their formation?

Q. 2. Why is it not possible to observe interference of light with two independent sources of light?

Q. 3. What do you mean by coherent sources?

Q. 4. How are coherent sources produced?

Q. 5. Why should the two sources be monochromatic?
Q. 6. Should the two sources be rigorously monochromatic or nearly so?
Q. 7. Should the two sources have exactly equal or nearly equal amplitudes?
Q. 8. Why is the experiment on 'Newton's rings' so called?
Q. 9. Why are the Newton's rings called the curves of equal thickness? Explain.
Q. 10. What do you mean by optically flat surface?
Q. 11. Why should the radius of the lens used in this experiment be large?
Q. 12. Are these fringe's localised? If so where are they formed?
Q. 13. How can you get a bright spot at centre?
Q. 14. To what class of interference does the Newton's rings experiment belong?

12

Diffraction of Light

12.1 DIFFRACTION

Light is known to travel in straight lines. This is a direct inference from the formation of shadows of opaque obstacle. However, it was discovered that with small sources, the shadow of a small object is much larger than that given by geometrical construction and is surrounded by fringes. This can be explained only if one assumes that light travels in the form of waves and bends round the corners of an obstacle. This phenomenon of deviation of light from rectilinear propagation and bending round the corners of an obstacle in known as diffraction. This is an important phenomenon exhibited by waves. In the case of sound waves for which wave-length is sufficiently large diffraction phenomenon can be easily observed. But in the case of light, the wavelength is extremely small and a very careful setting and closer observation is required. The size of the obstacle should be of dimensions comparable with the wave length of light. Careful experiments reveal that there is encroachment of light in the geometrical shadow region of opaque obstacles. Further the intensity of illumination outside the geometrical shadow region is not uniform but shows variation.

The essential difference between interference and diffraction of light is that, in interference the resultant intensity at a point is the resultant of superposition of two wavefronts coming from two coherent sources, whereas in the diffraction phenomenon the resultant intensity at a point is due to superposition of wavelets from two parts of a single wavefront.

12.2 CLASSIFICATION OF DIFFRACTION

The phenomena of diffraction of light is divided into the following two classes depending upon the position of source and the place of observation with respect to the diffracting obstacle.

12.3 FRESNEL'S CLASS OF DIFFRACTION

In this class of diffraction the source of light, or the screen or both are usually at finite distance from the obstacle. The wavefronts employed are spherical or cylindrical. They are treated by construction of half-period zones. The diffraction patterns obtained under this class are very faint, because in these the wavelets reaching any point of the screen from different parts of the exposed wavefront are all in different phases and produce only a feeble resultant. On the screen we get a pattern which is of the shape of the obstacle with some modifications due to diffraction. Further the pattern is formed in a plane which is not focally conjugate to the plane in which the source lies.

12.4 FRAUNHOFFER CLASS OF DIFFRACTION

In this class of diffraction the source and the screen are effectively at infinite distance from the obstacle. The incident light is diffracted in various directions and that diffracted in a particular direction is focussed on a screen by means of a convex lens. The illumination at the screen is greater if the phases of these parallel rays happen to agree. It is not necessary to employ a plane wavefront, we may even employ spherical or cylindrical wavefronts to obtain this class of diffraction. In that case the essential condition is that the pattern must be observed in a plane which is conjugate to the plane in which the source lies. In this class of diffraction the shape of the source is reproduced in the pattern as modified by the diffracting aperture. The diffracting aperture or obstacle do not come in the diffraction pattern.

12.5 FRESNEL'S ASSUMPTIONS

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below:

In Fig. 12.1, *S* is a point source of monochromatic light and *MN* is a small aperture, *XY* is the screen and *SO* is perpendicular to *XY*. *MCN* is the incident spherical wavefront due to the point source *S*. To obtain the resultant effect at a point *P* on the screen, Fresnel assumed that (1) a wavefront can be divided into a large number of strips or zones called Fresnel's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones; (2) the effect at a point due to any particular zone will depend on the distance of the point from the zone; (3) the effect at P will also depend on the obliquity of the point with reference to the zone under consideration, e.g., due to the part of the wavefront at *C*, the effect will be maximum at *O* and decreases with increasing obliquity. It is maximum in a direction radially outwards from *C* and it decreases in the opposite direction. The effect at a point due to obliquity factor is proportional to $(1 + \cos\theta)$ where $\angle PCO = \theta$. Considering an elementary wavefront at *C*, the effect is maximum at *O* because $\theta = 0$ and $\cos\theta = 1$, Similarly, in a direction tangential to the primary wavefront at *C* (along *CQ*) the resultant effect is one half of that along *CO* because $\theta = 90°$ and $\cos 90° = 0$. In this direction *CS*, the resultant effect is zero since $\theta = 180°$ and $\cos 180° = -1$ and $1 + \cos 180° = 1 - 1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz., that if the secondary waves spread out in all directions from each point on the primary wavefront, they should give a wave travelling forward as well as backward as the amplitude at the rear of the wave is zero there will evidently be no back wave.

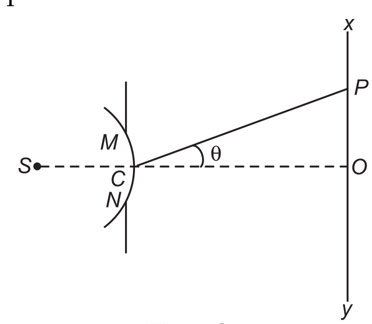


Fig. 12.1

12.6 RECTILINEAR PROPAGATION OF LIGHT

ABCD is a plane wavefront perpendicular to the plane of the paper Fig. 12.2(*a*) and *P* is an external point at a distance *b* perpendicular to *ABCD*. To find the resultant intensity at *P* due to the wavefront *ABCD*, Fresnel's method consists in dividing the wavefront into a number of

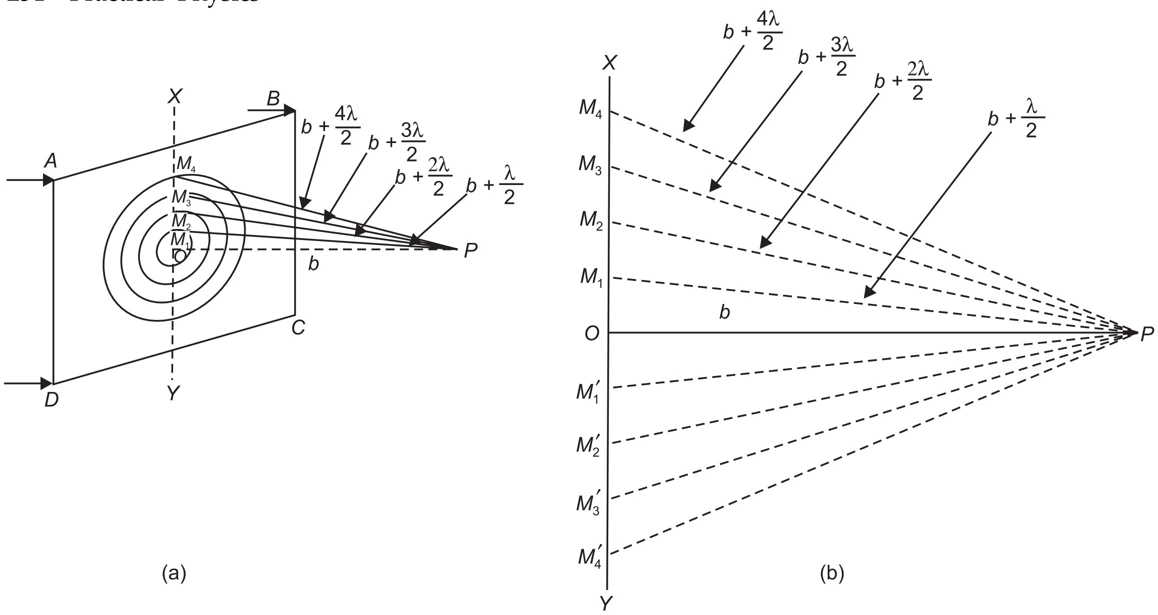
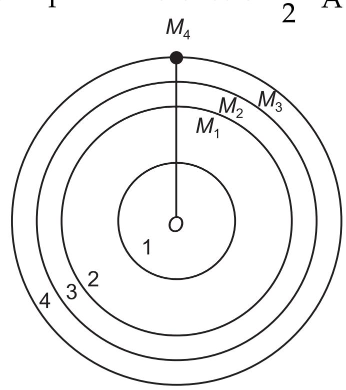


Fig. 12.2

half period elements or zones called Fresnel's zones and to find the effect of all the zones at the point *P*.

With *P* as centre and radii equal to $b+\frac{\lambda}{2}$, $b+\frac{2\lambda}{2}$, $b+\frac{3\lambda}{2}$ etc. construct spheres which will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wavefront. These circular zones are called half-period zones or half period elements. Each zone differs from its neighbour by a phase difference of π or a path difference of $\frac{\lambda}{2}$. Thus the secondary waves starting from the point *O* and M_1 and reaching *P* will have a phase difference of π or a path difference of $\frac{\lambda}{2}$. A Fresnel half period zone with respect to an actual point *P* is a thin annular zone (or a thin rectangular strip) of the primary wavefront in which the secondary waves from any two corresponding points of neighbouring zones differ in path by $\frac{\lambda}{2}$.



(c)

Fig. 12.2

In Fig. 12.2(*c*), *O* is the pole of the wavefront *XY* with reference to the extrnal point *P*. *OP* is perpendicular to *XY*. In Fig. 12.2(*c*) 1, 2, 3 etc. are the half period zones constructed on the primary wavefront *XY*. OM_1 is the radius of the first zone. OM_2 is the radius of the second zone and so on *P* is the point at which the resultant intensity has to be calculated.

$$OP = b,\ OM_1 = r_1\ OM_2 = r_2,\ OM_3 = r_3 \text{ etc.}$$

and $\quad M_1P = b + \frac{\lambda}{2},\ M_2B = b + \frac{2\lambda}{2},\ M_3P = \frac{3\lambda}{2}$ etc.

area of the first half period zone is

$$\pi OM_1^{\,2} = \pi\,[M_1P^2 - OP^2]$$

$$= \left[\left(b+\frac{\lambda}{2}\right)^2 - b^2\right] = \pi\left[b\lambda + \frac{\lambda^2}{2}\right] = \pi b\lambda \text{ approximately} \qquad ...(i)$$

(As λ is small, λ^2 term is negligible).

The radius of the first half period zone is

$$r_1 = OM_1 = \sqrt{b\lambda} \qquad ...(ii)$$

The radium of the second half period zone is

$$OM_2 = \left[M_2P^2 - OP^2\right]^{\frac{1}{2}} = \left[(b+\lambda)^2 - b^2\right]^{\frac{1}{2}} = \sqrt{2b\lambda} \text{ approximately}$$

The area of the second half period zone is

$$= \pi\left[OM_2^2 - OM_1^2\right] = \pi\left[2b\lambda - b\lambda\right] = \pi b\lambda$$

Thus, the area of each half period zone is equal to $\pi b\lambda$. Also the radii of the 1st, 2nd, 3rd etc. half period zones are $\sqrt{1b\lambda}, \sqrt{2b\lambda}, \sqrt{3b\lambda}$ etc. Therefore, the radii are proportional to the square roots of the natural numbers. However, it should be remembered that the area of the zones are not constant but are dependent on (i) λ the wave length of light and (ii) b, the distance of the point from the wavefront. The area of the zone increases with increase in the wavelength of light and with increase in the distance of the point P from the wavefront.

The effect at a point P will depend on (i) the distance of P from the wavefront, (ii) the area of the zone, and (iii) the obliquity factor.

Here, the area of each zone is the same. The secondary waves reaching the point P are continuously out of phase and is phase with reference to the central or the first half period zone. Let m_1, m_2, m_3 etc. represent the amplitudes of vibration of the ether particles at P due to secondary waves from the 1st, 2nd, 3rd etc. half period zones. As we consider the zones outwards from O, the obliquity increases and hence the quantities m_1, m_2, m_3 etc. are of continuously decreasing order. Thus, m_1 is slightly greater than m_2; m_2 is slightly greater than m_3 and so on. Due to the phase difference of π between any two consecutive zones, if the displacement of the ether particles due to odd numbered zones is in the positive direction, then due to the even numbered zones the displacement will be in the negative direction at the same instant. As the amplitude are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it. e.g.

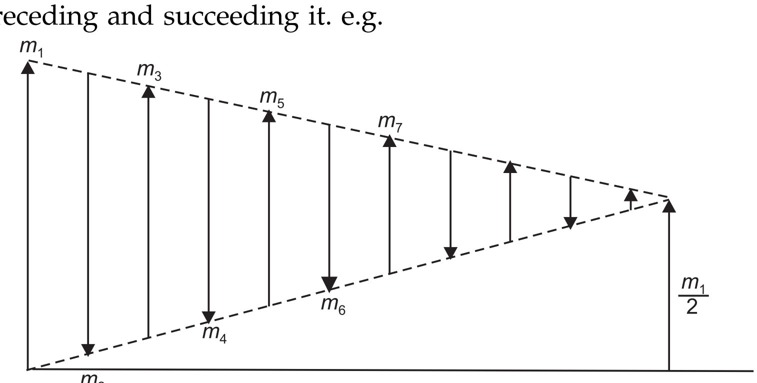


Fig. 12.3

$$m_2 = \frac{m_1 + m_3}{2}$$

The resultant amplitude at P *at* any instant is given by

$$A = m_1 - m_2 + m_3 - m_4 ... + m_n \text{ if } n \text{ is odd.}$$

(If n is even, the last quantity is $-m_n$)

$$\therefore \qquad A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + \cdots$$

But $\qquad m_2 = \frac{m_1}{2} + \frac{m_3}{2}$ and $m_4 = \frac{m_3}{2} + \frac{m_5}{2}$

$$\therefore \qquad A = \frac{m_1}{2} + \frac{m_n}{2} \cdots \text{ if } n \text{ is odd}$$

and

$$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n \cdots \text{ if } n \text{ is even.}$$

If the whole wavefront *ABCD* is unobstructed the number of half period zones that can be constructed with reference to the point P is infinite i.e., $n \to \infty$. As the amplitudes are of gradually diminishing order, m_n and m_{n-1} tend to be zero.

Therefore, the resultant amplitude at P due to the whole wavefront $= A = \frac{m_1}{2}$. The intensity at a point is proportional to the square of the amplitude

$$\therefore \qquad I \propto \frac{m_1^2}{4}$$

Thus, the intensity at P is only one-fourth of that due to the first half period zone alone Here, only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the area of the first half period zone placed at O will screen the effect of the whole wavefront and the intensity at P due to the rest of the wavefront will be zero. While considering the rectilinear propagation of light the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round corners (diffraction effects) cannot be noticed. In the case of sound waves, the wavelengths are far greater than the wavelength of light and hence the area of the first half period zone for a plane wavefront in sound is very large. If the effect of sound at a point beyond as obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect. If the size of the obstacle placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. Thus, rectilinear propagation of light is only approximately true.

12.7 ZONE PLATE

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method is dividing a wavefront into half period zones can be verified with its help.

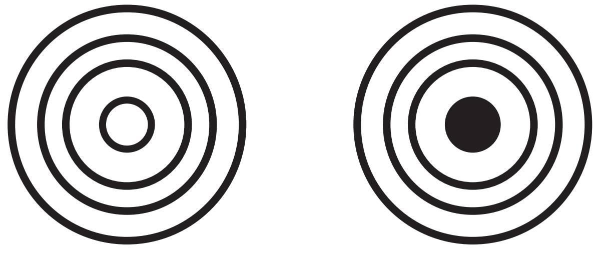


Fig. 12.4

To Construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers. The odd numbered zones (i.e., 1st, 3rd, 5th etc) are covered with black ink and a reduced photograph is taken. The drawing appears as shown in Fig. 12.4(b) the negative of the photograph will be as shown in Fig 12.4(a). In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say b cm from the zone plate. XO is the upper half of the incident plane wavefront. P is the point at which the light intensity is to be considered. The distance of the point P from the wavefront is b. OM_1 (= r_1), OM_2 (= r_2) etc. are the radii of the zones,

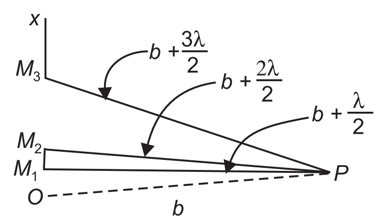


Fig. 12.5

$$r_1 = \sqrt{b\lambda} \text{ and } r_2 = \sqrt{2b\lambda}$$

Where λ is the wavelength of light

$$r_n = \sqrt{n\,b\lambda} \text{ or } b = \frac{r_n^2}{n\lambda}$$

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wavefront can be taken as a plane one with respect to the small area of the zone plate. The even numbered zones cut off the light and hence the resultant amplitude at $P = A = m_1 + m_3 + \ldots$ etc.

In this case the focal length of the zone plate f_n is given by

$$f_n = b = \frac{r_n^2}{n\lambda} \qquad \because r_n^2 = bn\lambda$$

Thus, a zone plate has different foci for different wavelengths, the radius of the n^{th} zone increases with increasing value of λ. It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than that when the whole wavefront is exposed to the point P.

In the first case the resultant amplitude is given by

$$A = m_1 + m_3 + m_5 + \cdots m_n \ (n \text{ is odd})$$

When the whole wavefront is unobstructed the amplitude is given by

$$A = m_1 - m_2 + m_3 - m_4 + \cdots + m_n$$

$$= \frac{m_1}{2}.\ \text{(if } n \text{ is very large and } n \text{ is odd).}$$

If a parallel beam of white light is incident on the zone plate, different colours came to focus at different points along the line OP. Thus, the function of a zone plate is similar to that of a convex (converging) lens and a formula connecting the distance of the object and image paints can be obtained for a zone plate also.

12.8 ACTION OF A ZONE PLATE FOR AN INCIDENT SPHERICAL WAVEFRONT

Let XY represent the section of the zone plate perpendicular to the plane of the paper. S is a point source of light, P is the position of the screen for a bright image, a is the distance of the source from the zone plate and b is the distance of the screen from the plate. OM_1, OM_2, OM_3, (r_1, r_2, r_3) etc. are the radii of the 1st, 2nd, 3rd etc half period zones. The position of the screen is such that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

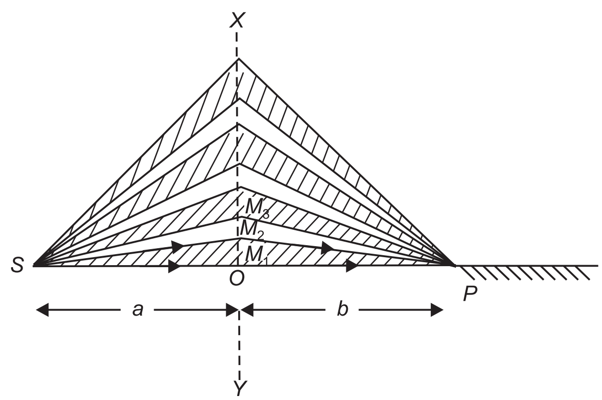


Fig. 12.6

Thus, from Fig. 12.6,

$$SO + OP = a + b$$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad \text{...(i)}$$

$$SM_2 + M_2P = a + b + \frac{2\lambda}{2} \text{ and so on}$$

From the $\Delta\, SM_1O$,

$$SM_1 = \left(SO^2 + OM_1^2\right)^{\frac{1}{2}} = \left(a^2 + r_1^2\right)^{\frac{1}{2}}$$

Similarly from the $\Delta\, OM_1P$

$$M_1P = \left(OP^2 + OM_1^2\right)^{\frac{1}{2}}$$

$$= \left(b^2 + r_1^2\right)^{\frac{1}{2}}$$

Substituting the values of SM_1 and M_1P in eqn. (i)

$$\left(a^2 + r_1^2\right)^{\frac{1}{2}} + \left(b^2 + r_1^2\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a\left(1 + \frac{r_1^2}{a^2}\right)^{\frac{1}{2}} + b\left(1 + \frac{r_1^2}{b^2}\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2}$$

$$r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$$

Similarly for r_n, i.e., the radius of the nth zone, the relation can be written as

$$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

Applying the sign convention

or
$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n} \qquad \text{...(ii)}$$

$$f_n = \frac{r_n^2}{n\lambda}$$

Equation (ii) is similar to the equation $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ in the case of lenses with a and b as the object and image distances and f_n the focal length. Thus, a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

12.9 OBJECT

Determination of the diameter of a wire by diffraction.

Apparatus used: Optical bench with accessories, sodium lamp, and Ramsden's eye- piece with micrometer screw.

Formula used: $d = \frac{D\lambda}{W}$ where W is fringe width, D is the diatance between the pin and the screen, d is diameter of pin.

Theory: Consider a cylindrical-wave front WW' Fig. 12.7 of wavelength coming from a slit S, normal to the plane of paper, falling on a narrow pin having a finite width or diameter $AB = d$, the sides of the pin being parallel to the length of slit. On the screen MM', the geometrical shadow region will be represented by PP'.

The effect of inserting AB in the path of light is to screen of a few of the half period elements. At a point K outside the geometrical shadow, the effect is given by the sum of resultant effects of the half period elements in the upper half of the wave front above the pole O', with respect to point K and those elements which lie between O' and edge of obstacle. If the obstacle is large the effect of half period elements of the lower half of the wave front WW' which are not obstructed by wire, may be neglected. This is because of the fact that these half period elements will be of higher order.

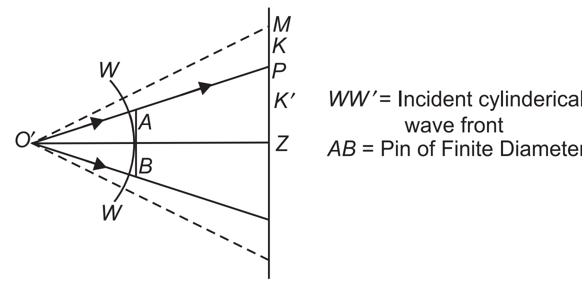


Fig. 12.7

If $O'A$ contains as even number of half period strips the point K will be comparatively dark. If $O'A$ contains odd number of half period strips, the point K will be bright.

The diffraction patterns of the 2 sides of the geometrical shadow are thus similar to the diffraction pattern outside the geometrical shadow of straight edge.

Next consider point K' inside the geometrical shadow region. At this point the displacement is due to the two unobstructed halves of wave front.

The half-period elements in the upper half will combine into a resultant whose phase will be in arrangement with the resultant phase of the half-period elements, lying near the edge A. Similarly the half period elements or zones in lower half will combine into resultant phase. These will be the same as the resultant phase of the half period elements lying near B.

The phase difference between the two resultants to Z is $\frac{dx}{D}$ where D is the distance between the pin and the screen and $x = cz$. Thus the point Z will be bright, if

$$\frac{dx}{D} = 2n\,\frac{\lambda}{2} \qquad \text{...(1)}$$

and dark, if

$$\frac{dx}{D} = (2n + 1)\,\frac{\lambda}{2} \qquad \text{...(2)}$$

where n is the integer.

These maxima and minima obtained inside the geometrical shadow region resemble the fringes observed in an interference experiment. They are not due to diffraction. The fact they may be regarded as due to interference between the two narrow sources estimated at the edges of the obstacle. The distance between two consecutive dark and bright fringes, are called the fringe width W. It can be obtained with the help of eqn. (1) and eqn. (2).

Thus for $n = 1$ and $n = 2$, eqn. (1) gives bright fringes, as:

$$\frac{dx_1}{D} = \lambda \qquad \text{...(3)}$$

$$\frac{dx_2}{D} = 2\lambda \qquad \text{...(4)}$$

$$W = x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

or,

$$W = \frac{D\lambda}{d}$$

This eqn. can be used for determination of wavelength of light.

Manipulations:

1. Level the optical bench with the help of spirit level and leveling screw.
2. Make the slit vertical with the help of plumb line.
3. Bring the slit, the eyepiece at the same height and in the same line. See that the planes of the slit, the wire and eyepiece are transversely normal to the bench.
4. Focus the light by a lens on the slit.
5. Move the pin wide way with the help of tangential screws and vary the slit width till fringes are obtained and are seen through the eyepiece. The visibility will be best when the slit and the pin are parallel to each other.
6. Adjust the line joining the pin and the cross wires parallel to the bed of optical bench. This is accomplished when on moving the eyepiece along the bed of the optical bench, no lateral shift is obtained.
7. In order to adjust the system for no lateral shift, the eyepiece is moved away from the straight edge (pin). In this case the fringes will move to the right or left, but with the help of the base screw provided with pin (wire) it is moved at right angle to the bench in a direction to bring the fringes back to their original position,

   Now move the eyepiece towards the wire and same adjustment is made with the help of eyepiece.

   Using the process again and again the lateral shift is removed.
8. Measure the fringe width and the diameter of the pin.
9. Repeat the experiment with 2 more values of *D*.

Observations:

1. For diameter of pin.

   Least count of screw gauge =

   Zero error of screw gauge =

Readings	1	2	3	4	5	6	7	8	9	10
Along one direction (in cm.)										
$\perp$ to above (in cm.)										

   Mean diameter of a wire =

   Diameter corrected from zero error =

2. **Bench Error:** The distance between cross wire and pin:-

   (a) As measured by bench rod =

   (b) as measured by the bench scale =

   The bench error =

3. **Fringe Width:**

   Least count of Micrometer =

   Observed distance between the cross wire and the pin =

   Corrected distance between the cross wire and the pin =

No. of fringes	*Reading of Micrometer (in cm)*	*Width of 3 Fringes (in cm)*	*Mean (in cm)*	*Fringe width W (in cm)*
1				
2				
3				
4				
5				
6				

Fringe width W =

Result: For sodium light (λ = 5893 Å) diameter 'd' of a pin =

Actual diameter as measured by screw gauge =

Percentage error = ...%

Precautions:

1. The bench error is necessary therefore it should be found.
2. The straight pin should be parallel to the slit.
3. Make the slit as narrow as possible until the fringes are most clear.
4. The cross wire of the microscope should be well focused on the fringes.

13

Polarisation of Light

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e., whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon of polarization has helped to establish that light waves are transverse waves.

13.1 POLARIZATION OF TRANSVERSE WAVES

Let a rope AB be passed through two parallel slits S_1 and S_2. The rope is attached to a fixed point at B. Hold the end A and move the rope up and down perpendicular to AB. A wave emerges along CD and it is due to transverse vibrations parallel to the slit S_1. The slit S_2 allows the wave to pass through it when it is parallel to S_1. It is observed that the slit S_2 does not allow the wave to pass through it when it is at right angles to the slit S_1.

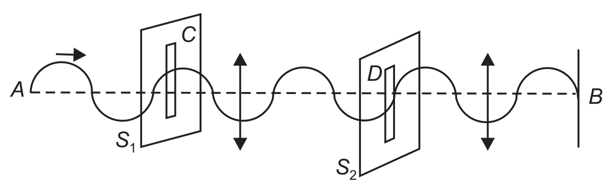


Fig. 13.1(a)

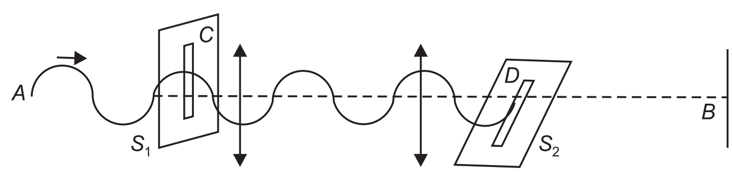


Fig. 13.1(b)

If the end A is moved in a circular manner, the rope will show circular motion up to the slit S_1. Beyond S_1, it will show only linear vibrations parallel to the slit S_1, because the slit S_1 will stop the other components. If S_1 and S_2 are at right angles to each other the rope will not show any vibration beyond S_2.

If longitudinal waves are set up by moving the rope forward and backward along the string, the waves will pass through S_1 and S_2 irrespective of their position.

A similar phenomenon has been observed in light when it passes through a tourmaline crystal. Let light from a source S fall on a tourmaline crystal A which is cut parallel to its axis. The crystal A will act as the slit S_1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal A, no remarkable change is noticed. Now place the crystal B parallel to A.

1. Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of B Fig. 13.2(*a*).

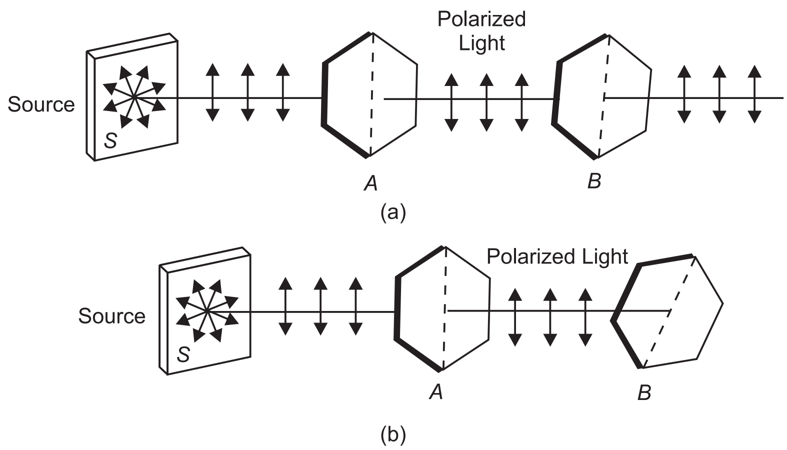


Fig. 13.2

2. Keep the crystal *A* fixed and rotate the crystal *B*. The light transmitted through *B* becomes dimmer and dimmer. When *B* is at right angles to *A*, no light emerges out of *B* Fig. 13.2 (*b*).

If the crystal *B* is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel.

This experiment shows conclusively that light is not propagated as longitudinal or compressional waves. If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal *B* is rotated.

It is clear that after passing through the crystal *A*, the light waves vibrate only in one direction. Therefore light coming out of the crystal *A* is said to be polarized because it has acquired the property of one sidedness with regard to the direction of the rays.

This experiment proves that light waves are transverse waves, otherwise light coming out of *B* could never be extinguished by simply rotating the crystal *B*.

13.2 PLANE OF POLARIZATION

When ordinary light is passed through a tourmaline crystal, the light is polarized and vibrations are confined to only one direction perpendicular to the direction of propagation of light. This is plane polarized light and it has acquired the property of one sidedness. The plane of polarization is that plane in which no vibrations occur. The plane *ABCD* in Fig. 13.3 is the plane of polarization. The vibrations occur at right angles to the plane of polarization and the plane in which vibrations occur is known as plane of vibration. The plane *EFGH* in Fig. 13.3 is the plane of vibration.

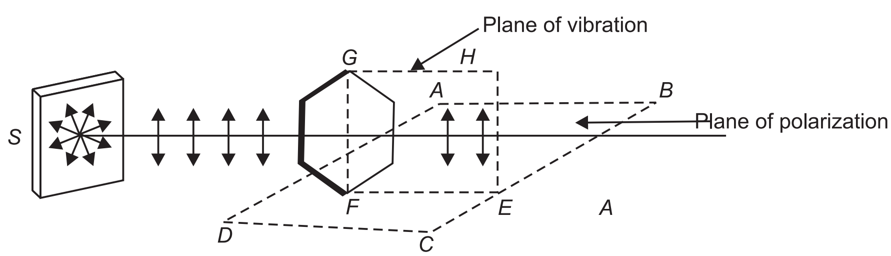


Fig. 13.3

In Fig. 13.4(*a*), the vibrations of the particles are represented parallel (arrow heads) and perpendicular to the plane of the paper (dots).

In Fig. 13.4(*b*), the vibrations are shown only parallel to the plane of the paper.

In Fig. 13.4(*c*), the vibrations are represented only perpendicular to the plane of the paper.

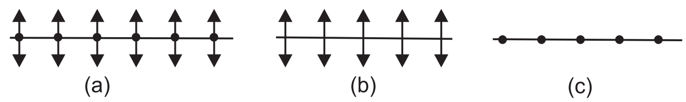


Fig. 13.4

13.3 POLARIZATION BY REFLECTION

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB* on the glass surface Fig. 13.5. Light is reflected along *BC*. In the path of *BC*, place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle. Similarly polarized light by reflection can be produced from water surface also.

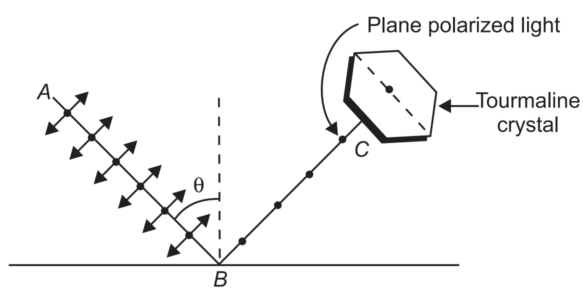


Fig. 13.5

The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface. Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

13.4 BREWSTER'S LAW

In 1811, Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the angle of polarization.

He proved that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and the refracted rays are perpendicular to each other.

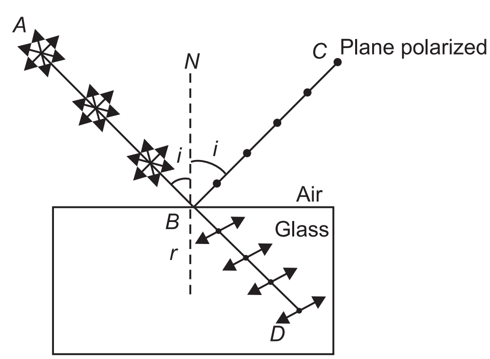


Fig. 13.6

Suppose, unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD Fig. 13.6.

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \quad \text{...(i)}$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad \text{...(ii)}$$

Comparing (i) and (ii)

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \qquad i = \frac{\pi}{2} - r \ \text{ or } \ i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and the refracted rays are at right angles to each other.

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by

$$i = \tan^{-1}(1.52) \ \text{ or } \ i = 56.7°$$

However, 57° is an approximate value for the polarizing angle for ordinary glass. For a refractive index of 1.7 the polarising angle is about 59.5° i.e., the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelengths. Therefore, polarization will be complete only for light of a particular wavelength at a time i.e. for monochromatic light. It is clear that the light vibrating in the plane of incidence is not reflected along BC. In the reflected beam the vibrations along BC cannot be observed, whereas vibrations at right angles to the plane of incidence can contribute for the resultant intensity. Thus, we get plane polarized light along BC. The refracted ray will have both the vibrations: (*i*) in the plane of incidence and (*ii*) at right angles to the plane of incidence. But it is richer in vibrations in the plane of incidence. Hence it is partially plane-polarized.

13.5 BREWSTER WINDOW

One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers and it is called a Brewster window. When an ordinary beam of light is incident normally on a glass window, about 8% of light is lost by reflection on its two surfaces and about 92% intensity is transmitted. In the case of a gas laser filled with mirrors outside the windows, light travels through the window about a hundred times. In this way, the intensity of the final

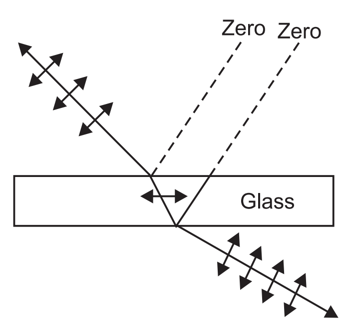


Fig. 13.7

beam is about 3×10^{-4} because $(0.92)^{100} \approx 3 \times 10^{-4}$. It means the transmitted beam has practically no intensity.

To overcome this difficulty, the window is tilted so that the light beam is incident at Brewster's angle. After about hundred transmissions, the final beam will be plane polarized. The light component vibrating at right angles to the plane of incidence is reflected. After about 100 reflections at the Brewster window, the transmitted beam will have 50% of the intensity of the incident beam and it will be completely plane polarized. The net effect of this type of arrangement in that half the amount of light intensity has been discarded and the other half is completely retained Brewster's windows are used in gas lasers.

13.6 POLARIZATION BY REFRACTION

It is found that at a single glass surface or any similar transparent medium, only a small fraction of the incident light is reflected.

For glass ($\mu = 1.5$) at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular vibrations only 85% is transmitted and 15% is reflected. Therefore, if we use a pile of plates and the beam of ordinary light is incident at the polarizing angle on the pile of plates, some of the vibrations perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted through it. When this beam of light is reflected by the second plate, again some of the vibrations perpendicular to the plane of incidence are reflected by it and the rest are transmitted. The process continues and when the beam has traversed about 15 or 20 plates, the transmitted light is completely free from the vibrations at right angles to the plane of incidence and is having vibrations only in the plane of incidence. Thus, we get plane-polarized light by refraction with the help of a pile of plates, the vibrations being in the plane of incidence as shown in Fig. 13.8.

The pile of plates consists of number of glass plates (microscope cover slips) and are supported in a tube of suitable size and are inclined at an angle of 32.5° to the axis of the tube. A beam of monochromatic light is allowed to fall on the pile of plates at the polarizing angle. The transmitted light is polarized perpendicular to the plane of incidence and can be examined by a similar pile of plates which works as an analyser.

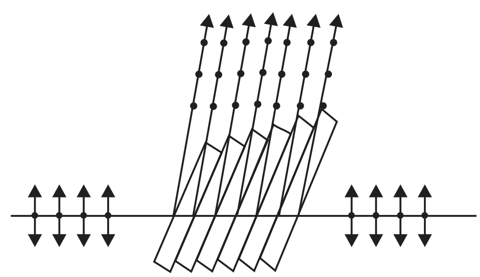


Fig. 13.8

13.7 MALUS LAW

When a beam of light, polarized by reflection at one plane surface is allowed to fall on the second plane surface at the polarizing angle the intensity of the twice reflected beam varies with the angle between the planes of the two surfaces. In the Biot's polariscope it was found that the intensity of the twice reflected beam is maximum when the two planes are parallel and zero when the two planes are at right angles to each other. The same is also true for the twice transmitted beam from the polarizer and analyser. The law of Malus states that the intensity of the polarized light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polarizer.

The intensity I_1 of the polarised light transmitted through the analyser is given by the Malus Law

$$I_1 = I \cos^2 \theta$$

where I is the original intensity and θ is the angle between the planes of the polariser and the analyser.

13.8 OBJECT

To determine the polarizing angle for the glass prism surface and to determine the refractive index of the material using Brewster's Law.

Apparatus: Spectrometer, sodium lamp, glass prism, a Polaroid with attachment etc.

Formula used: According to Brewster's law

$$\mu = \tan \phi$$

Where μ = refractive index of the material of prism.

ϕ = angle of polarization

Theory: Whenever light falls on a smooth surface a portion is reflected and the other is refracted. The fraction reflected and refracted depend upon the surface material of the medium and the angle of incidence. However, if the incident light is unpolarised both reflected and refracted rays are partially polarized. Brewster in 1811 carried out a series of experiments and concluded that for a particular angle of incidence, reflected light is completely polarized in the plane of incidence and the refracted light is partial polarized with predominant vibration of electric vectors in the plane of incidence. This angle of incidence is known angle of polarization or Brewster's angle.

Further, when the angle of incidence is equal to the angle of polarization, the reflected and refractive rays are mutually perpendicular to each other and therefore:

$${}_a\mu_g = \sin i/\sin r = \sin \phi/\sin (90° - \phi) = \sin \phi/\cos \phi = \tan \phi$$

Where ϕ is the angle of polarization. The relation $\mu = \tan \phi$ is known as Brewster's Law.

Procedure:

1. Make the mechanical and optical adjustment of the spectrometer as mentioned in the experiment of dispersive power. Attach the Polaroid attachment to the telescope objective.
2. Place the prism on the prism table such that one of the reflecting surface of prism passes through the prism table's center.
3. The prism table is rotated so that the light coming from the collimator is incident on the face of the prism passing through the center. The telescope is adjusted to get the reflected light on the cross wire. Polaroid attached with the telescope is then slowly rotated and the variation of the intensity of the field of view is observed.
4. The angle of incidence is increased by slightly rotating the prism table. The position of the telescope and Polaroid both are adjusted each time to get minimum intensity. The process is repeated till the reflected light completely disappears for one particular adjustment.
5. The position of the telescope is noted on both the verniers of the circular scale.
6. Now remove the prism and set the telescope for the direct image of the slit and again note the readings of the verniers.

The full procedure 3rd, 4th and 5th is repeated in order to determine the position of the telescope more accurately.

7. Calculate the angle.

Observations:

1. Vernier Constant of the spectrometer =
2. Readings for the determination of the angle of polarization:

Sl.No.	*Vernier*	*Position of telescope for extinction of image (a)*	*Position of telescope for direct image (b)*	*Difference (a – b)*	*Mean*	*Mean θ*
1	A					
	B					
2	A					
	B					
3	A					
	B					

Mean θ =

Calculation: Angle of polarization $\phi = (90° - \theta/2)$ =

$$\mu = \tan\phi =$$

Result:

1. The angle of polarization for air-glass interface is found to be =
2. Refractive index of the material is found to be =

Precautions:

1. The width of the slit should be narrow.
2. If it is not possible to obtain zero intensity position, then it should be adjusted for minimum possible intensity.

13.9 DOUBLE REFRACTION

Certain crystalline materials have the property that a beam of light incident on them: (*i*) breaks into two plane polarised beams with their planes of polarisation mutually perpendicuar, and (*ii*) these two beams in general have different velocities in medium. The phenomenon is called double refraction.

In Fig. 13.9 the incident ray *AB* of light breaks up into two refracted rays *BO* and *BE*. BO is plane polarised in one plane, *BE* is also plane polarised but in a perpendicular plane. There are obviously two refractive indices,

$$\mu_1 = \frac{\sin i}{\sin r_1},\ \ \mu_2 = \frac{\sin i}{\sin r_2}$$

If the angle of incidence i is varies, snell's law $\frac{\sin i}{\sin r}$ = constant holds for one of the rays e.g., *BO* only. For the other ray this law does not generally hold. The ray *BO* which follows the laws of refraction is called ordinary (O) and for it refractive index is constant, say equal to μ_0. The other ray BE is called extra ordinary (E) and for it refractive index μ_e varies with direction.

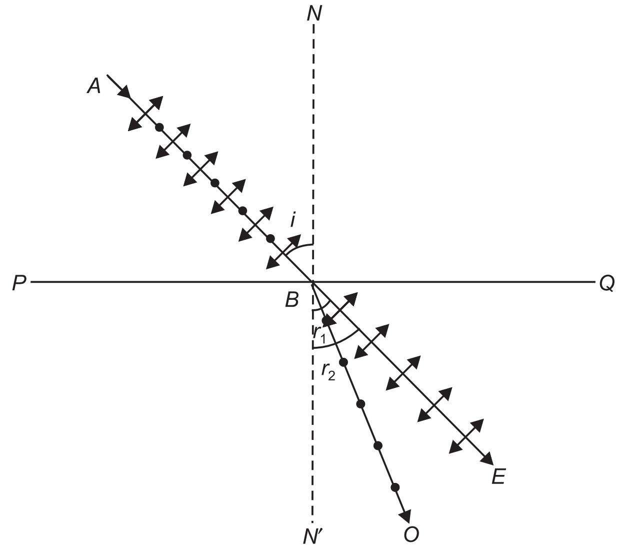


Fig. 13.9

In doubly refracting crystals there is one particular direction in which the ordinary (O) and the extra ordinary (E) travel with equal velocities. This direction is known as optic axis. In the direction of optic axis the refractive index of the crystal for both the ordinary and the extra ordinary is the same (each equal to μ_0). In this direction the crystal will not exhibit double refraction. The crystals in which there is only one such direction are known as uniaxial crystals.

In directions more and more inclined to the optic axis, the difference of velocities for the ordinary and the extra-ordinary becomes larger and larger. This difference is thus maximum in a plane perpendicular to the optic axis. Crystals in which the extra-ordinary ray travels faster than the ordinary are called negative crystal, reverse is the case for positive crystal. Calcite is negative crystal while quartz is positive crystal. The extreme value of refractive index for E-ray (when it travels in the direction of optic axis) is called μ_e. It is to be noted carefully that the refractive index of the crystal for E-ray has all values between μ_0 and μ_e. The symbol μ_e is reserved for the extreme (maximum or minimum) value of refractive index for the extra ordinary (E) ray.

In a doubly refracting crystal a plane containing the optic axis and perpendicular to its opposite faces is called its principal section. In case of a calcite crystal the principal section is a parallelogram whose angles are 71° and 109°, as shown in Fig. 13.10(a). In Fig. 13.10(b) end view of the principal section is shown. It is known that the vibrations in the ordinary are perpendicular to the principal section and in the extra-ordinary they are in the principal section of the crystal. The ordinary ray is, therefore, polarised in the principal section and the extra-ordinary is polarised perpendicular to the principal section. If the principal section of the crystal is also the plane of incidence, the ordinary vibrations can be represented by dots (.) and the extra-ordinary vibrations by dashes (—).

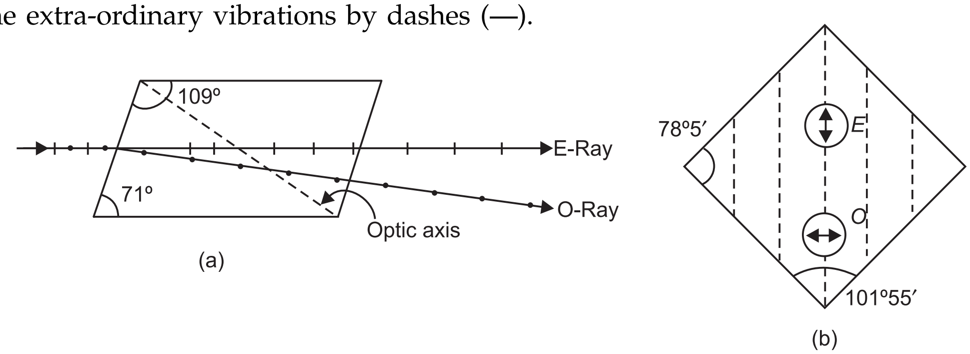


Fig. 13.10

13.10 NICOL PRISM

It is an optical device made from a calcite crystal and is used in many instruments to produce and analyse the plane polarised light.

When light is passed through a doubly refracting crystal it is split up into the *O*-ray and the *E*-ray. Both these rays are plane polarised.One of these rays is cut off by total internal reflection. This prism was designed by William Nicol and is known as Nicol's prism after his name.

Constructions: The nicol prism is constructed from a calcite crystal whose length is nearly three times its width. The end faces of the crystal are cut down so as to reduce the angles at *B* and *D* from 71° in the principal section to a more acute angle of 68°. The crystal is then cut along the plane. A′bc′d perpendicular both to the principal section A′B′C′D and the end faces such that A′C′ makes an angle of 90° with the end faces A′B and C′D as shown in Figure.

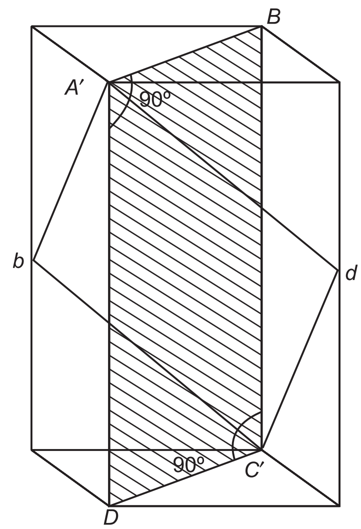


Fig. 13.11(a)

The two cut surfaces are ground, polished optically flat and then cemented together with Canada balsam, a transparent cement so that the crystal is just as transparent as it was previously to its having been sliced. The refractive index of Canada blasam μ_b has a value which lies midway between the refractive index of calcite μ_o for the *O*-ray and μ_e for the *E*-ray. The values of these for the sodium light of mean wavelength λ = 5893 Å are μ_o = 1.658; μ_b = 1.55 and μ_e = 1.486.

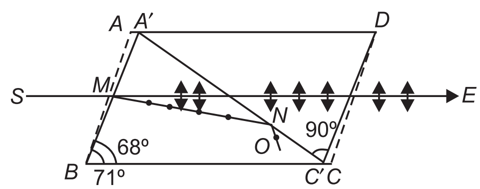


Fig. 13.11(b)

The sides of the prism are blackened to absorb the totally reflected rays.

Action: If a ray of light *SM* is incident nearly parallel to BC′ in the plane of the paper on the face A′B, it suffers double refraction and gives rise to

(*i*) the extraordinary beam *ME*, and
(*ii*) the ordinary beam *MN*.

The *E*-ray passes through along *ME* which is plane polarised and has vibrations in the plane of the paper.

The *O*-ray which is also plane polarised suffers total internal reflection at the Canada balsam layer for nearly normal incidence.

It is because canada balsam is optically more dense than calcite for the *E*-ray and less dense for the *O*-ray. The *E*-ray is refracted through canada balsam and is transmitted but the O-ray moving from a denser calcite medium to the rarer canada balsam medium is totally reflected for angles of incidence greater than the critical angle. The value of critical angle for the ordinary ray for calcite to Canada balsam

$$= \sin^{-1}\left(\frac{1.550}{1.658}\right) = 69.2°$$

If the incident ray makes an angle much smaller than *BMS* with the surface *A′B* the ordinary ray will strike the balsam layer at an angle less than the critical angle and hence will be transmitted.

If the incident ray makes an angle greater than *BMS* the extraordinary ray will become more and more parallel to the optic axis *A′Y* and hence its refractive index will become nearly equal to that of calcite for the ordinary ray. This will then also suffer total internal reflection like the ordinary ray. Hence no light will emerge out of the Nicol's prism. A Nicol's prism, therefore, cannot be used for highly convergent or divergent beams. The angle between the extreme rays of the incoming beam is limited to about 28.

13.11 USES OF NICOL PRISM AS AN POLARISER AND AN ANALYSER

The Nicol's prism can be used both as a polariser and an analyser.

When unpolarised beam of light is incident on a Nicol prism N_1, the light emerging out of it is plane polarised and has vibrations parallel to its principal section. If now this light is made to pass through a second Nicol N_2 the principal section of which is parallel to the principal section of N_1, the light vibrations in N_2 are parallel to its principal section and hence are completely transmitted as shown in Fig. 13.12(*a*). The intensity of the emergent beam is maximum.

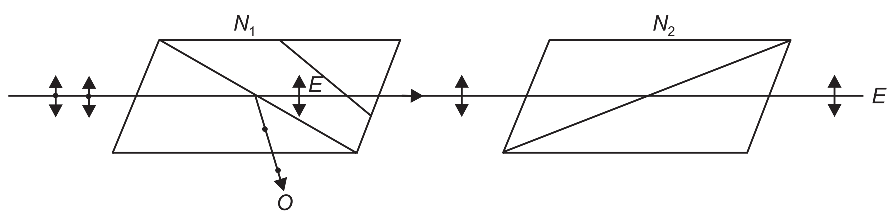


Fig. 13.12(a)

Now if the Nicol N_2 is rotated such that its principal section becomes perpendicular to that of N_1 as shown in Fig. 13.12(*b*), then the vibrations of incident light in N_2 will be perpendicular to the principal section of N_2. These behave as *O*-vibrations for N_2 and are thus totally reflected and hence no light emerges from the second Nicol N_2. In this position the two Nicols are said to be crossed.

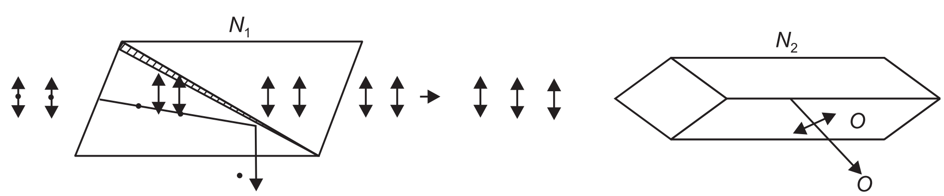


Fig. 13.12(b)

When the Nicol N_2 (analyser) is further rotated the two Nicols are again in parallel position then in this position the *E*-ray is again transmitted through the Nicol N_2.

The first Nicol polarises the light and is called the polariser. The second Nicol analyses the polarised light and is called the analyser.

13.12 PRINCIPAL REFRACTIVE INDEX FOR EXTRAORDINARY RAY

The velocity of the extraordinary ray through a uniaxial crystal depends upon the direction of the ray. Therefore, the refractive index for the extraordinary ray is different along different directions. In the case of a negative crystal, the velocity of the extraordinary ray travelling perpendicular to the direction of the optic axis is maximum and the refractive index is minimum. This refractive index for the extraordinary ray is known as the principal refractive index (μ_e) and is defined as the ratio of the sine of the angle of incidence to the sine of the angle of refraction when the refracted ray travels perpendicular to the direction of the optic axis. This is also defined as the ratio of the velocity in vacuum to the maximum velocity of the extraordinary ray.

$$\mu_e = \frac{\text{velocity of light in vacuum}}{\begin{array}{c}\text{velocity of the extraordinary ray in a direction}\\ \text{perpendicular to the optic axis}\end{array}}$$

For a positive uniaxial crystal, the velocity of the extraordinary ray travelling perpendicular to the optic axis is minimum and the refractive index is maximum. Therefore, the principal refractive index for the positive uniaxial crystal is the ratio of the velocity of light in vacuum to the minimum velocity of the extraordinary ray.

13.13 ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT

Let monochromatic light be incident on the Nicol prism N_1. After passing through the nicol prism N_2, it is plane-polarized and is incident normally on a uniaxial doubly refracting crystal P (calcite or quartz) whose faces have been cut parallel to the optic axis. The vibrations of the plane-polarized light incident on the crystal are shown in Fig. 13.13(b).

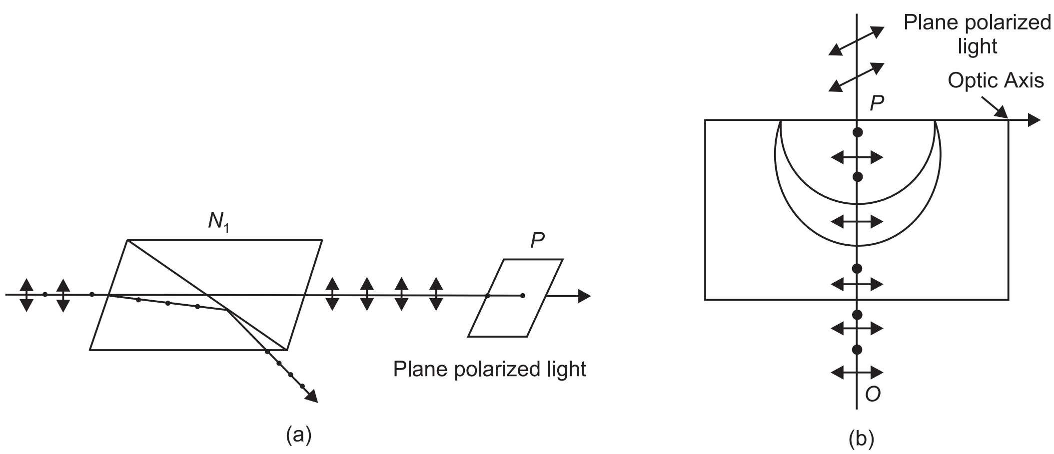


Fig. 13.13

The plane polarized light on entering the crystal is split up into two components, ordinary and extraordinary. Both the rays, in this case, travel along the same direction but, with different velocities. When the rays have travelled through the thickness d in the crystal, a phase difference δ is introduced between them.

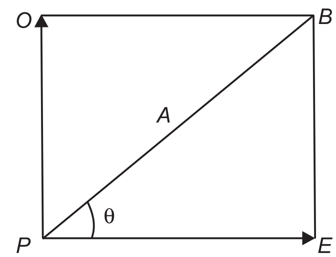


Fig. 13.14

Theory: Suppose the amplitude of the incident plane polarized light on the crystal is A and it makes an angle θ with the optic axis. Therefore, the amplitude of the ordinary ray vibrating along PO is $A \sin\theta$ and the amplitude of the extraordinary ray vibrating along PE is $A \cos\theta$. Since a phase difference δ is introduced between the two rays, after passing through a thickness d of the crystal, the rays after coming out of the crystal can be represented in terms of two simple harmonic motions, at right angles to each other and having a phase difference.

∴ For the extraordinary ray,

$$x = A\cos\theta \cdot \sin(\omega t + \delta)$$

For the ordinary ray,

$$y = A\sin\theta \cdot \sin\omega t$$

Take $A\cos\theta = a$, and $A\sin\theta = b$

$$x = a\sin(\omega t + \delta) \quad \text{...(i)}$$

$$y = b\sin\omega t \quad \text{...(ii)}$$

From Eq. (ii)

$$\frac{y}{b} = \sin\omega t$$

and

$$\cos\omega t = \sqrt{1-\frac{y^2}{b^2}}$$

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$$= \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\cdot\sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1-\frac{y^2}{b^2}}\cdot\sin\delta$$

Squaring and rearranging

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta \quad \text{...(iii)}$$

This is the general equation of an ellipse.

Special cases:

1. When $\delta = 0$, $\sin\delta = 0$ and $\cos\delta = 1$.
   From Equation (iii)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$y = \frac{bx}{a}$$

This is the equation of a straight line. Therefore, the emergent light will be plane polarized.

2. when $\delta = \dfrac{\pi}{2}$, $\cos\delta = 0$, $\sin\delta = 1$

   From Eq. (iii) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

   This represents the equation of a symmetrical ellipse. The emergent light in this case will be elliptically polarized provided $a \neq b$.

3. when $\delta = \dfrac{\pi}{2}$ and $a = b$

   From Eq. (iii), $x^2 + y^2 = a^2$

   This represents the equation of circle of radius a. The emergent light will be circularly polarized. Here the vibrations of the incident plane polarized light on the crystal make an angle of 45° with the direction of the optic axis.

4. For $\delta = \dfrac{\pi}{4}$ or $\dfrac{7\pi}{4}$, the shape of the ellipse will be as shown in Fig. 13.15.
5. For all other values of δ, the nature of vibrations will be as shown in Fig. 13.15.

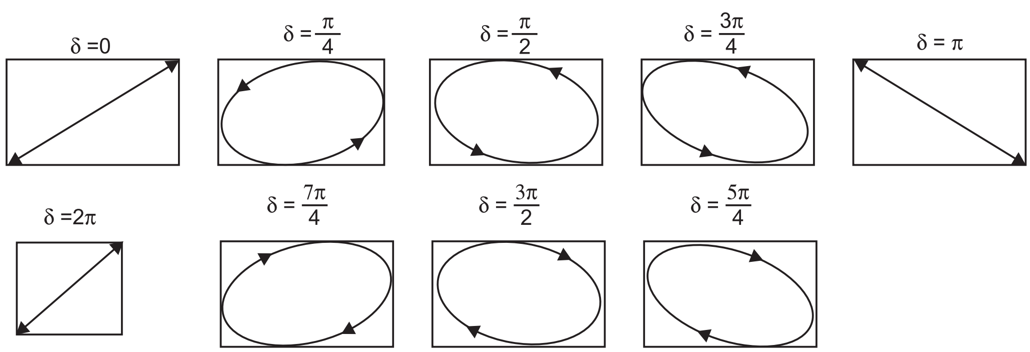


Fig. 13.15

13.14 QUARTER WAVE PLATE

It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane-polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_e respectively, then the path difference introduced between the two rays is given by:

For negative crystals, path difference = $(\mu_o - \mu_e)\, t$
For positive crystals, path difference = $(\mu_e - \mu_o)\, t$

To produce a path difference of $\dfrac{\lambda}{4}$, in calcite

$$(\mu_o - \mu_e)\, t = \frac{\lambda}{4}$$

or $$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

and in the case of quartz $$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

If the plane-polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized.

13.15 HALF WAVE PLATE

This plate is also made from a doubly refracting uniaxial crystal of quartz or calcite with its refracting faces cut parallel to the optic axis. The thickness of the plate is such that the ordinary and the extraordinary rays have a path difference = $\frac{\lambda}{2}$ after passing through the crystal.

For negative crystals, path difference = $(\mu_o - \mu_e)\, t$
For positive crystals, path difference = $(\mu_e - \mu_o)\, t$

To produce a path difference of $\frac{\lambda}{2}$ in calcite

$$(\mu_o - \mu_e)\, t = \frac{\lambda}{2}$$

or $$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

and in the case of quartz

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

When plane-polarized light is incident on a half-wave plate such that it makes an angle of 45° with the optic axis, a path difference of $\frac{\lambda}{2}$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90º. Thus, a half wave plate rotates the azimuth of a beam of plane polarized light by 90º, provided the incident light makes an angle of 45º with the optic axis of the half wave plate.

13.16 PRODUCTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHT

1. **Plane Polarized Light:** A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is split up into extraordinary ray and ordinary ray. The ordinary ray is totally internally reflected back at the Canada balsam layer, while the extraordinary ray passes through the nicol prism. The emergent beam is plane polarized.

2. **Circularly Polarized Light:** To produce circularly polarized light, the two waves vibrating at right angles to each other and having the same amplitude and time period should have a phase difference of $\frac{\pi}{2}$ or a path difference of $\frac{\lambda}{4}$. For this purpose, a parallel beam of monochromatic light is allowed to fall on a nicol prism N_1 (Fig. 13.16). The beam after passing through the prism N_1, is plane polarized. The nicol prism N_2 is placed at some distance from N_1 so that N_1 and N_2 are crossed. The field of view will be dark as viewed by the eye in this position. A quarter wave plate P is mounted on a tube A. The tube A can rotate about on the outer fixed tube B introduced between the nicol prism N_1 and N_2. The plane polarized light form N_1 falls normally on P and the field of view may be bright. The quarter wave plate is rotated until the field of view is dark. Keeping P fixed, A is rotated such that the mark S on P coincides with zero mark on A. Afterwards, by rotating the quarter wave plate P, the mark S is made to coincide with 45° mark on A.

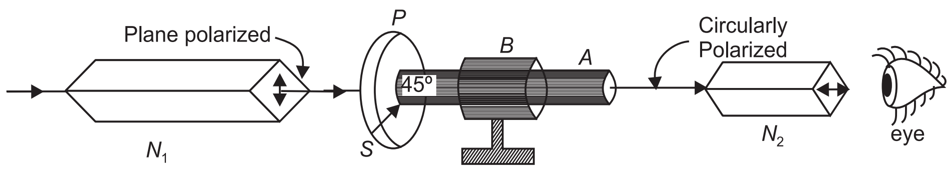
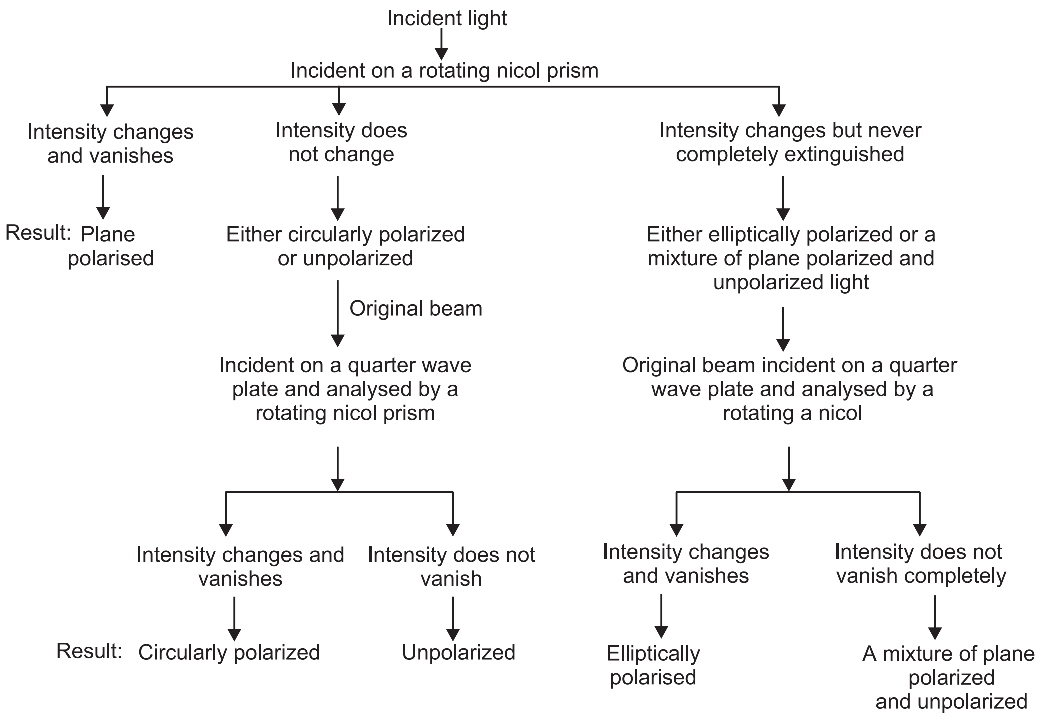


Fig. 13.16

The quarter wave plate is in the desired position. In this case, the vibrations of the plane-polarized light falling on the quarter wave plate make an angle of 45° with the direction of the optic axis of the quarter wave plate. The polarized light is split up into two rectangular components (ordinary and extraordinary) having equal amplitude and time period and on coming out of the quarter wave plate, the beam is circularly polarized. If the nicol prism N_2 is rotated at this stage, the field of view is uniform in intensity similar to the ordinary light passing through the nicol prism.

3. **Ellipticall Polarized Light:** To produce elliptically polarized light, the two waves vibrating at right angles to each other and having unequal amplitudes should have a phase difference of $\frac{\pi}{2}$, or a path difference of $\frac{\lambda}{4}$. The arrangement of Fig. 13.16 can be used for this purpose. A parallel beam of monochromatic light is allowed to fall on the nicol prism N_1. The prism N_1 and N_2 are crossed and the field of view is dark. A quarter wave plate is introduced between N_1 and N_2. The plane polarized light from the nicol prism N_1 falls normally on the quarter wave plate. The field of view is illuminated and the light coming out of the quarter wave plate is elliptically polarized. (The only precaution in this case is that the vibrations of the plane-polarized light falling on the quarter wave plate should not make an angle of 45° with the optic axis, in which case, the light will be circularly polarized). When the nicol N_2 is rotated, it is observed that the intensity of illumination of the field of view varies between a maximum and minimum. This is just similar to the case when a beam consisting of a mixture of plane-polarized light and ordinary light is examined by a nicol prism.

13.17 DETECTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHT



13.18 OPTICAL ACTIVITY

When a polarizer and an analyser are crossed, no light emerges out of the analyser. When a quartz plate cut with its faces parallel to the optic axis is introduced between N_1 and N_2 such that light falls normally upon the quartz plate, the light emerges out of N_2.

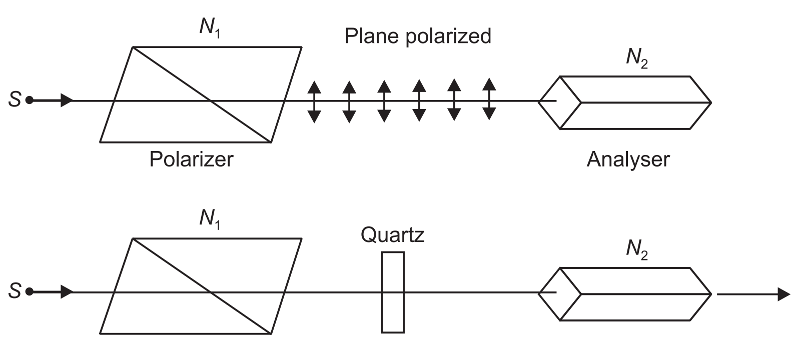


Fig. 13.17

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration is gradually rotated as shown in Fig. 13.18. The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light. The action of turning the plane of vibration occurs

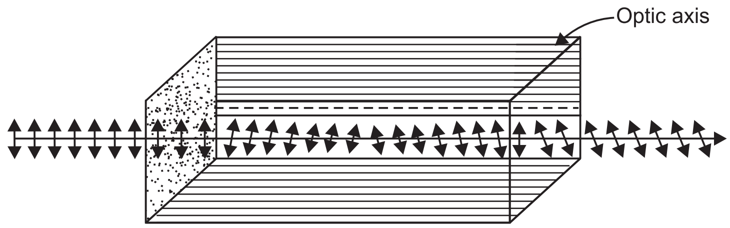


Fig. 13.18

inside the body of the plate and not on its surface. This phenomenon or the property of rotating the plane of vibration by certain crystals or substances is known as optical activity and the substance is known as an optically active substance. It has been found that calcite does not produce any change in the plane of vibration of the plane polarised light. Therefore, it is not optically active.

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate and cinnabar are optically active. Some of the substances rotate the plane of vibration to the right and they are called dextro-rotatory or right handed. Right handed rotation means that when the observer is looking towards light travelling 'towards him, the plane of vibration is rotated in a clockwise direction. The substances that rotate the plane of vibration to the left (anti-clockwise from the point of view of the observer) are known as laevo-rotatory or left-handed.

It has been found that some quartz crystals are dextro-rotatory while others are laevo-rotatory. One is the mirror image of the other in their orientation. The rotation of the plane of vibration in a solution depends upon the concentration of the optically active substance in the solution. This helps in finding the amount of cane sugar present in a sample of sugar solution.

13.19 SPECIFIC ROTATION

Liquid containing an optically active substance e.g., sugar solution, camphor in alcohol etc. rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the solution or density of the active substance in the solvent. (3) Wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimeter (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution. Therefore

$$S_\lambda^t = \frac{10\,\theta}{lc}$$

where S_λ^t, represents the specific rotation at temperature t°C for a wavelength λ, θ is the angle of rotation, l is the length of the solution in cm. through which the plane polarised light passes and c is the concentration of the active substance in g/cc in the solution. The angle through which the plane of polarization is rotated by the optically active substance is determined with the help of a polarimeter, when this instrument is used to determine the quantity of sugar in a solution, it is known as a saccharimeter.

13.20 LAURENT'S HALF SHADE POLARIMETER

It consists of two nicol prism N_1 and N_2. Figure 13.19 N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half wave plate of quartz Q which covers one half of the field of view,

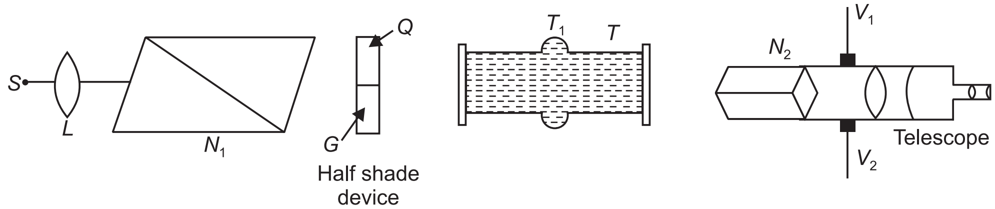


Fig. 13.19

while the other half G is a glass plate. The glass plate G absorbs the same amount of light as the quartz plate Q. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube, light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB. Here AB makes an angle θ with YY′ (Fig. 13.20). On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced between them. The vibration of the beam emerging out of quartz will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser N_2 has its principal plane or section along YY′ i.e., along the direction which bisects the angle AOC, the amplitudes of light incident on the analyzer N_2 from both the halves (i.e., quartz half and glass half) will be equal. Therefore, the field of view will be equally bright (Fig. 13.21(*i*)).

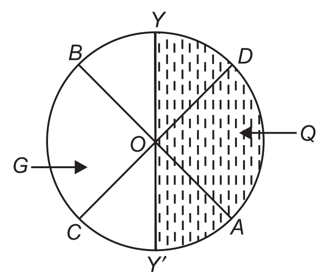


Fig. 13.20

If the analyser N_2 is rotated to the right of YY′ , then the right half will be brighter as compared to the left half (Figure *ii*) on the other hand, if the analyser N_2 is rotated to the left of YY′ , the left half is brighter as compared to the right half (Figure *iii*).

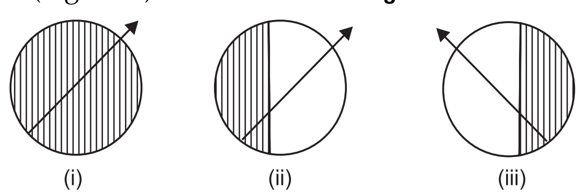


Fig. 13.21

Therefore, to find the specific rotation of an optically active substance (*say, sugar solution*), the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube T. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position so that the whole field of view is equally bright. The new portions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentration of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation θ. The graph is a straight line.

Then from the relation

$$S_\lambda^t = \frac{10\,\theta}{\lambda c}$$

the specific rotation of the optically active substance is calculated.

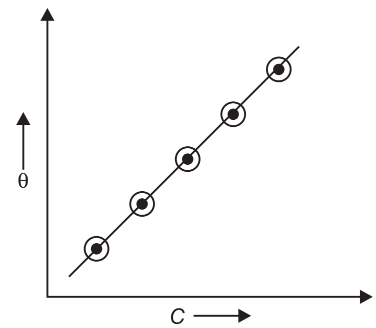


Fig. 13.22

13.21 BIQUARTZ

Instead of half shade plate, a biquartz plate is also used in polarimeters. It consists of two semi-circular plates of quartz each of thickness 3.75 mm. One half consists of right-handed optically active quartz, while the other is left-handed optically active quartz. If white light is used, yellow light is quenched by the biquartz plate and both the halves will have the tint of passage. This can be adjusted by rotating the analyser N_2 to a particular position. When the analyser is rotated to one side from this position, one half of the field of view appears blue, while the other half appears red. If the analyser is rotated in the opposite direction the first half which was blue earlier appear red and the second half which was red earlier appears blue. Therefore, by adjusting the particular position of the analyser, the field of view appears equally bright with tint of passage.

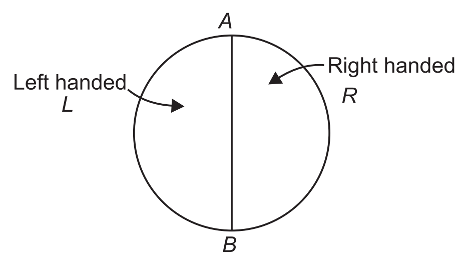


Fig. 13.23

13.22 LIPPICH POLARIMETER

Laurent's polarimeter suffers from the defect that it can be used only for light of a particular wavelength for which the half wave plate has been constructed. To overcome this difficulty, Lippich constructed a polarimeter that can be used for light of any wavelength. The diagram is shown in figure. It consists of two nicol prisms N_1 and N_2 (Fig. 13.24). Behind N_1, there is a nicol prism N_3, that covers half the field of view. The nicols N_1 and N_3 have their planes of vibration inclined at a small angle. Suppose the plane of vibration of N_1 is along AB (Fig. 13.25) and that of N_3 is along CD. The angle between the two planes is θ. When the analyser N_2 is

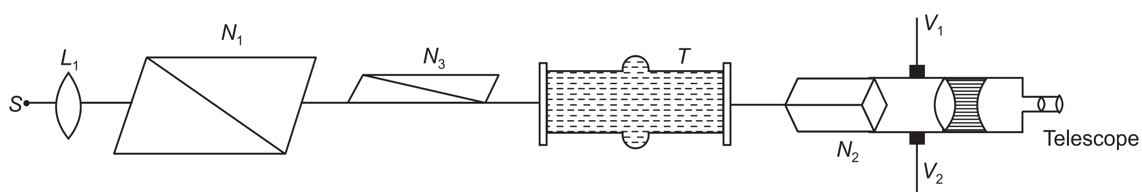


Fig. 13.24

rotated such that the plane of vibration of N_2 is along *AB*, the left half will be more bright as compared to the right half. If the analyser N_2 has its plane of vibration along *CD*, the right half will be more bright as compared to the left half. YY′ is the bisector of the angle *AOC*. Therefore, when the plane of vibration of analyser N_2 is along YY′, the field of view is equally illuminated. For a slight rotation of the analyser, either to the right or to the left, the field of view appears to be of unequal brightness. Therefore, by rotating N_2, the position for equal brightness of the field of view is obtained. To determine the specific rotations of the optically active substance, the procedure is the same as discussed in spectic rotation.

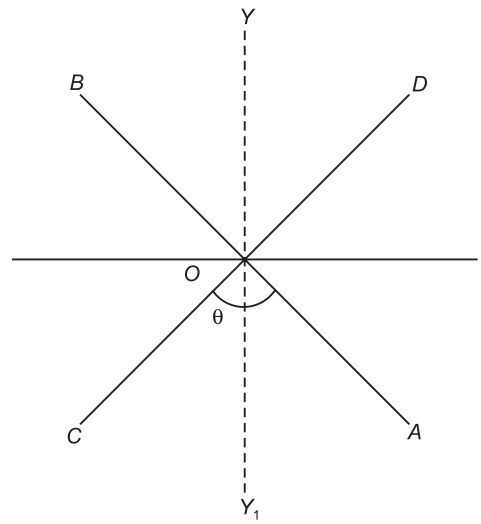


Fig. 13.25

Three-field System: In the improved form, Lippich polarimeter has a three-field system. The defect in the two - field system is that if the eye is off the axis, even for the position of equal brightness of the field of view, one side appears more bright as compared to the other. Just behind N_1, there are two nicol prisms N_3 and N_4 as shown in Fig. 13.26. The planes of vibration of N_3 and N_4 are parallel to each other and make a small angle with the plane of vibration of N_1. For a particular position of the analyser N_2, the field has three parts. The central portion is illuminated by light which has passed through N_1 and N_2, while the other two portion, which are equally bright, are illuminated by light passing through N_1, N_2, and one of the prisms N_3 and N_4.

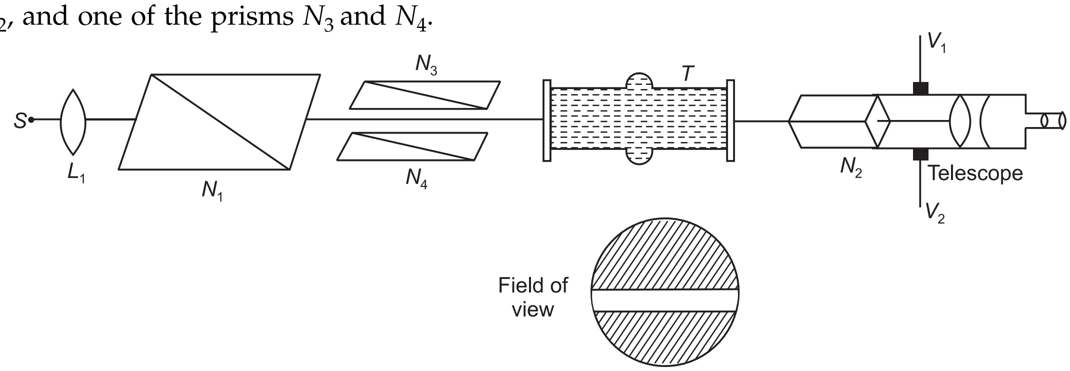


Fig. 13.26

13.23 OBJECT

To find the specific rotation of sugar solution by polarimeter.

Apparatus Used: Polarimeter, white light source, sugar, beakers, graduated jar, disc, weight box, balance.

Formula Used: The specific rotation of the plane of polarization of sugar dissolved in water can be determined by the following formula.

$$S = \theta/lc = \theta v/lm$$

Where θ = rotation produced in degrees
ℓ = length of the tube in decimeter
m = mass of sugar in gms dissolved in water
v = volume of sugar solution

Description of the Apparatus and Theory: Polarimeter in general consists of a source of light a polarimeter and an analyzer provided with a graduated circular scale. Figure 13.27 represents the general optical arrangement of most polarimeters.

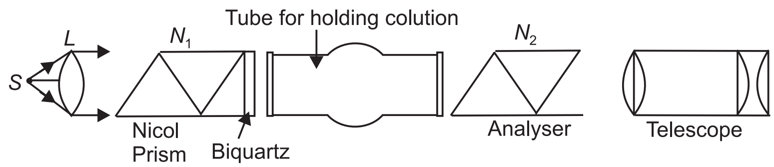


Fig. 13.27

S is a source of light, so placed that it is nearly at a focus of the lens *L* so that parallel pencil of rays enters the Nicol Prism N_1 which serves to polarize the beam of light passing through it. The polarizing nicol is immediately followed by a Laurent half shade plate or a biquartz. The other Nicol prism N_2 analyses the transmitted beam and detects its plane of polarization and is placed in front of a low power telescope. In between N_1 and N_2 is placed the tube *T* containing the liquid under investigation. The tube is closed on both sides with metal caps. When this tube is filled with solution containing an optically active substance, the air bubbles if any will appear at the upper side of the wide portion of the tube. The light from N_1 can pass through N_2 only if N_2 is placed in exactly the same way as N_1. In this case the Nicols are said to be parallel. If however, N_2 is turned from this position by a right angle no light from N_1 can pass through N_2. In this position the Nicols are said to be crossed.

Certain substances like quartz, solution of sugar etc.possess the property of rotating the plane of polarized light. When it passes through them. On inserting the active substance on account of the rotation of plane of polarization, some light will pass through N_2 even when it is set in crossed position. It is found that rotation of N_2 in one direction or the other will again bring N_2 into a plane in which light is once more stopped.

Thus we can get the amount of rotation by measuring the angle through which N_2 has turned.

Specific rotation is defined as the amount of rotation produced by one decimeter of the solution divided by the weight of the dissolved substance in unit volume. Let *W* grams be dissolved in 100 c.c. and suppose a length ℓ cm. of liquid produces a rotation θ.

$$[S]^{D}{}_{t} = \theta \Big/ \left(\frac{\ell}{10}\right) \div \left(\frac{W}{100} V\right)$$

$$= \left(\frac{10\theta}{\ell}\right) \div \left(\frac{W}{100}\right)$$

$$= 1000\ \theta / lw$$

Method:

1. Weigh sugar in a watch glass and dissolve the sugar in 100 c.c. Have distilled water.

2. Clean the polarimeter tube and fill it with distilled water. See that there is no air bubble in the tube when the end caps have been screwed.
   Place the tube in its position inside the polarimeter.
3. Look through the analyser when it will be observed that two portions of the field of view of the sensitive biquartz device are of different colors red and blue.
4. Rotate the analyser till the two portions of the field of view are of same intensity or acquire tint of passage.
5. Take the reading of the analyzer on the circular scale. The settings of the analyzer should be done by rotating the analyzer in the clock-wise as well as by rotating in the anti-clockwise directions.
6. Remove the distilled water from the tube and fill it completely with the sugar solution and again place it in the polarimeter. On looking through the analyzer the previous setting would be disturbed. Adjust the analyzer again till the two portions of the field of views acquire the gray tint shade. Take the reading of the analyzer.
7. Difference between the two settings of the analyzer (6) - (5) gives the value of the angle of rotation.
8. Repeat the experiment with sugar solution of different concentrations.
9. Measure the length of the tube ℓ and also note the room temperature.

Observation:

Room temperature = °C
Weight of the empty watch glass =
Weight of the watch glass + sugar =
Weight of the sugar employed =
Volume of the water taken =
Least count of the analyzer =
Length of the polarimeter tube =

Table for the Angle of Rotation: (1) For 1st solution.

Sl No.	*Position of analyzer with distilled water*				*Position of analyzer with sugar solution*				*Mean θ in degree*	*θ (in degree)*
	Clock wise rotation		*Anti clock wise rotation*		*Clock wise rotation*		*Anti clock wise rotation*		$\frac{1}{4}[(\theta_1-\theta'_1)+(\theta_2-\theta'_2)+(\theta_3-\theta'_3)+(\theta_4-\theta'_4)]$	
	One side vernier θ_1	*Other vernier* θ_2	*One side vernier* θ_3	*Other vernier* θ_4	*One side vernier* θ_1'	*Other vernier* θ_2'	*One side vernier* θ_3'	*Other vernier* θ_4'		
1.										
2.										
3.										

(2) *For 2nd Solution:* Same table as above

Result: The specific rotation of sugar solution at =
Standard value =
Percentage = ...%

Theoretical Error:

$$S = \theta V/ l\mathrm{m}$$

Taking log and differentiating, we get

$$\delta S/S = \delta\theta/\theta + \delta V/V + \delta l/l + \delta \mathrm{m}/\mathrm{m}$$

Maximum theoretical error =

Precaution:

1. The polarimeter tube should be well cleaned.
2. Care should be taken that there is no air bubble when the tube is filled with liquid.
3. Care should be taken in weighing sugar and measuring the quantity of water.
4. Note the temperature of the room and also the wavelength of the light used.
5. Start with a concentrated solution and then go on diluting by adding water to it.

13.24 VIVA-VOCE

Q. 1. What do you mean by polarisation?
Ans. The light which has acquired the property of one sideness is called a polarised light.
Q. 2. What information does it provide about light waves?
Ans. This gives that the light waves are transverse in nature.
Q. 3. How will you distinguish between unpolarised and plane polarised light?
Ans. The unpolarised light is symmetrical about the direction of propagation while in case of plane polarised light, there is lack of symmetry about the direction of propagation.
Q. 4. For what kind of light does this law hold.
Ans. It holds for completely plane polarised light.
Q. 5. In using a Nicol prism, light is made incident almost parallel to its oblong side. What may happen if incident light is too much convergent or divergent?
Ans. Two things may happen in this case: (i) ordinary ray may be incident at Canada Balsam layer at an angle less than the critical angle resulting in its transmission. (ii) the total internal reflection of the extra-ordinary is also possible.
Q. 6. What is the plane of polarisation of plane polarised light obtained from a Nicol?
Ans. Light emerging from a Nicol is polarised at right angles to its principal section.
Q. 7. In which direction is the difference of refractive index for *O* and *E*-rays: (i) least, (ii) greatest?
Ans. Their difference is least (zero) in the direction of optic axis and is greatest in a direction perpendicular to it.
Q. 8. Is there any physical significance in taking μ_0 in a direction perpendicular to optic axis?
Ans. In a plane perpendicular to optic axis, the extraordinary ray also obeys snell's law of refraction i.e., $\frac{\sin i}{\sin r} = \mu_e$, a constant for *E*-ray also.
Q. 9. What do you mean by polarised light?
Ans. The light which has acquired the property of one sideness is called a polarised light.
Q. 10. How does polarised light differ from ordinary light?

Ans. The ordinary light is symmetrical about the direction of propagation while in case of polarised light, there is lack of symmetry about the direction of propagation.

Q. 11. What does polarisation of light tell about the nature of light?

Ans. Light waves are transverse in nature.

Q. 12. Define plane of vibration and plane of polarisation.

Ans. The plane containing the direction of vibration as well as the direction of the propagation of light is called plane of vibration. On the other hand, the plane passing through the direction of propagation and containing no vibration is called plane of polarisation.

Q. 13. What is phenomenon of double refraction?

Ans. When ordinary light is incident on a calcite or quartz crystal, it splits in two refracted rays and this phenomenon is known as double refraction.

Q. 14. Define optic axis and principal section.

Ans. A line passing through any one of the blunt corners and making equal angles with three faces which meet there is the direction of optic axis. A plane containing the optic axis and perpendicular to two opposite faces is called the principal section.

Q. 15. What are uniaxial and biaxial crystals?

Ans. The crystals having one direction (optic axis) along which the two refracted rays travel with the same velocity are called as uniaxial crystal. In biaxial crystals, there are two optic axes.

Q. 16. What do you mean by optical activity, optical rotation and angle of rotation?

Ans. The property of rotating the plane of vibration of plane polarised light about its direction of travel by some crystal is known as optical activity. This phenomenon is known as optical rotation and the angle through which the plane of polarisation is rotated is known as angle of rotation.

Q. 17. What is specific rotation?

Ans. The specific rotation of a substance at a particular temperature and for given wavelength of light may be defined as the rotation produced by one decimeter length of its solution when concentration is 1 gm per cc. Thus

$$\text{specific rotation} = \frac{\theta}{lc}$$

where θ is angle of rotation in degrees, l the length of solution in decimeter and C the concentration of solution in gm per c.c.

Q. 18. On which factors specific rotation depend?

Ans. On temperature and wavelength of light used.

Q. 19. Does angle of rotation and specific rotation depend on strength of sugar solution and length of the tube?

Ans. Angle of rotation is proportional to length and concentration but specific rotation is independent of these factors.

Q. 20. What is a polarimeter?

Ans. It is an instrument used for measuring the angle of rotation of the plane of polarisation by an optically active substance.

Q. 21. What is the unit of specific rotation?

Ans. The unit of specific rotation is Degree/decimeter–gm–cc.

Q. 22. What do you mean by dextro and laevo-rotary substances?

Ans. When the optically active substance rotates the plane of polarisation of light towards right, it is called as right handed or dextro- rotatory. If the substance rotates the plane of polarisation towards left, it is called as left handed or laevorotatory.

Q. 23. Name two different devices used with a polarimeter? What is the difference in their construction?

Ans. (*i*) Half shade, (*ii*) Bi-quartz polarimeter. In half shade, one semi circular plate made is of ordinary glass while the other is of calcite working as half wave plate In Bi-quartz device, the two semi circular plates are made of right handed and left handed quartz with thickness for which angle of rotation for yellow colour is 90º..

Q. 24. Which is better out of these two?

Ans. Bi-quartz because (*i*) It is convenient to arrange white light rather than monochromatic light (*ii*) It is easy to judge accurately the contrast of colours rather than contrast of intensity.

Q. 25. What is main difference in the working of two?

Ans. Half shade polarimeter is used with monochromatic light and in it two halves in eye piece, appear of different intensity. Bi-quartz is used with white light and in it two halves appear of different colours.

Q. 26. What are the main parts of polarimeter?

Ans. Two nicol prisms working as polariser and analyser and a glass tube between them.

Q. 27. Explain the construction and working of a Bi-quartz and half shade device.

Ans. In Bi-quartz, the two semi circular plates are of left handed and right handed quartz. The thickness is taken for which angle of rotation for yellow colour in 90º. White light on entering through it is dispersed in different direction as θ is different for different colours. The observation is taken when two halves are of same colour.

In half shade device, one semi circular plate is of ordinary glass. While the other is of calcite, working as half plate. The observation is taken when two halves are of equal intensity.

Q. 28. What do you understand by dextro and laevo rotatory substances?

Ans. Some optical active substance rotate the plane of polarisation towards right (clockwise) while some towards left (anticlockwise). They are called dextro and laevo rotatory substances respectively.

Q. 29. What does polarisation ascertain about the nature of light?

Ans. Light waves are transverse in nature.

Q. 30. Can you find unknown concentration of sugar solution by polarimeter?

Ans. Yes

Q. 31. What is meant by Saccharimeter?

Ans. It is an apparatus used to find out unknown concentration of sugar solutions using standard value of specific rotation.

Q. 32. Where is the half-shade plate fitted in the polarimeter?

Ans. This is fitted between the polarising Nicol and the polarimeter tube containing the solution.

Q. 33. Is there any arrangement which can work with white light?

Ans. Yes, bi-quartz arrangement.

Q. 34. Can you find from your experiment, the direction of rotation of polarisation?

Ans. No

Q. 35. How can you modify your present experiment to find the direction of rotation?

Ans. The apparatus is modified in such a way that we can study the rotation produced by two different lengths of the solution. When θ is larger for longer lengths, then the direction of rotation gives the direction of the plane of polarisation.

Q. 36. What will be the resultant if plane polarised light is passed through a number of optically active solutions?

Ans. The resultant rotation will be algebraic sum of individual rotations produced by each solution separately.

Q. 37. What is a polaroid?

Ans. This is a device to produce plane polarised light. It consists of ultra - microscopic crystals of quinine iodo sulphate which are embedded in nitro - cellulose films in such a way that their optic axes are parallel to each other.

Q. 38. What is the characteristic of a polaroid?

Ans. It absorbs the ordinary ray while allow the extraordinary rays pass through them.

EXERCISE

Q. 1. How will you produce plane polarised light by reflection?
Q. 2. What is Brewster's angle or polarising angle?
Q. 3. How is it related to μ?
Q. 4. Which are the vibrations totally suppressed in reflection at Brewster's angle?
Q. 5. Are the other vibrations totally suppressed in the refracted beam?
Q. 6. What is Malus law?
Q. 7. What is double refraction?
Q. 8. What are the main characteristics of double refraction?
Q. 9. What are the various methods for obtaining one plane polarised beam out of the two produced by double refraction?
Q. 10. What is dichroism?
Q. 11. What is a polaroid? What does it give you?
Q. 12. How can you use a polaroid as a polariser and as an analyser?
Q. 13. What is a Nicol prism?
Q. 14. What is it made of?
Q. 15. What is a calcite crystal?
Q. 16. What do you mean by principal section of a crystal?
Q. 17. What is optic axis?
Q. 18. Does double refraction take place in the direction of optic axis?
Q. 19. Is there only such direction as optic axis in all the doubly refracting crystals?
Q. 20. What do you mean by negative and positive crystals? Give example of each.
Q. 21. What is the basic principle of working of a Nicol?
Q. 22. Explain how a Nicol prism produces plane polarised light?
Q. 23. How can total internal reflection of the extra-ordinary take place?
Q. 24. When are two Nicols 'parallel'?
Q. 25. When are they crossed?
Q. 26. How will you distinguish between unpolarised, partially plane polarised and completely plane polarised light using a Nicol prism?

Q. 27. Why are there two refractive incides for a doubly refracting crystal?

Q. 28. When the velocity of E-ray inside the crystal varies with its direction, then what you mean by refractive index (μ_e) for the extra-ordinary ray?

Q. 29. What is a quarter wave and half wave plate?

Q. 30. What does a quarter wave plate do?

Q. 31. How can you produce circularly and elliptically polarised light with a Nicol Prism and a quarter wave plate?

Q. 32. What is the function of a half-wave plate?

14

Resolving Power

14.1 RESOLVING POWER

Due to diffraction, the image of a point object formed by an optical instrument has finite dimensions. It consists of a diffraction pattern, a central maximum surrounded by alternate dark and bright rings. Two point objects, are resolvable by an optical instrument if their diffraction pattern are sufficiently small or are far enough apart so that they can be distinguished as separate image patterns. The resolving power of an optical instrument is defined as its ability to produce separate and distinguishable images of two objects lying very close together. The diffraction effects set a theoretical limit to the resolving power of any optical instrument. The term resolving is used in two contexts.

14.2 GEOMETRICAL RESOLVING POWER

When the purpose is to see as separate two objects close together or when fine structure is seen through a telescope or microscope. In the case of telescope (or eye), the resolving power is defined as the smallest angle subtended at the objective of the telescope (or the eye) by two point objects which can be seen just separate and distinguishable. Smaller is this angle, the greater will be the resolving power of the instrument. For a microscope the resolving power is defined as the linear separation which the two neighbouring point objects can have and yet be observed as just separate and distinguishable when seen through the microscope.

14.3 CHROMATIC RESOLVING POWER

This term is used when the instrument such as prism or grating spectrometers are employed for spectroscopic studies. The purpose of these instruments is to disperse light emitted by a source and to produce its spectrum. The chromatic resolving power of an instrument is its ability to separate and distinguish between two spectral lines whose wavelengths are very close. Smaller the wavelength interval at a particular wavelength that can be separated, the greater is the resolving power. If a source emits two close wavelengths λ and $(\lambda + d\lambda)$, the resolving power is mathematically defined as the ratio $\frac{\lambda}{d\lambda}$ provided the wavelength interval $d\lambda$ can just be separated at the wavelength λ.

14.4 CRITERION FOR RESOLUTION ACCORDING TO LORD RAYLEIGH

Lord Rayleigh has set a criterion to decide as to how close the two diffraction patterns can be brought together such that the two images can just be recognised as separate and distinguished from each other. The criterion is applicable to both the geometrical as well as spectroscopic resolving powers. According to Rayleigh's criterion the two point sources are just resolvable by an optical instrument when their distance apart is such that the central maximum of the diffraction pattern of one source coincides in position with the first diffraction maximum of the diffraction pattern of the other source. When applied to the resolution of spectral lines, this principle is equivalent to the condition that for just resolution the angular separation between the principle maxima of the two spectral lines in a given order should be equal to half angular width of either of the principal maximum. In this latter case, it is assumed that the two spectral lines have equal intensities.

In Fig. 14.1(a), *A* and *B* are the central maxima of the diffraction patterns of two spectral lines of wavelengths λ_1 and λ_2. The difference in the angle of diffraction is large and the two images can be seen as separate ones. The angle of diffraction corresponding to the central maximum of the image *B* is greater than the angle of diffraction corresponding to the first minimum at the right of *A*. Hence the two spectral lines will appear well resolved. In Fig. 14.1(b) the central maxima corresponding to the wavelength λ and $\lambda + d\lambda$ are very close. The angle of diffraction corresponding to the first minimum of *A* is greater than the angle of diffraction corresponding to the central maximum of *B*. Thus, the two images overlap and they cannot be distinguished as separate images. The resultant intensity curve gives a maximum as at *C* and the intensity of this maximum is higher than the individual intensities of *A* and *B*. Thus when the spectrograph is turned from *A* to *B*, the intensity increases, becomes maximum at C and then decreases. In this case, the two spectral lines are not resolved.

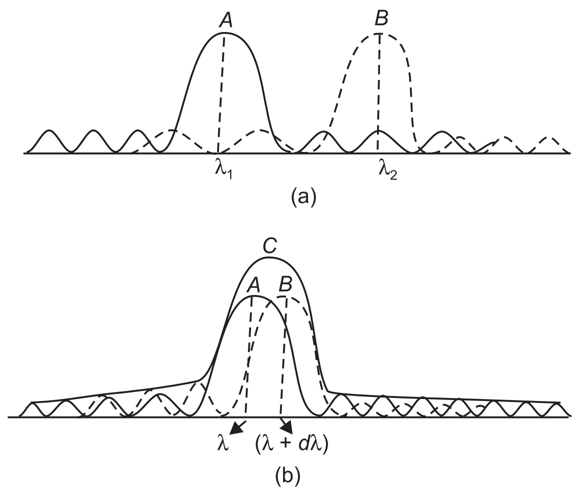


Fig. 14.1

In Fig. 14.1(c), the position of the central maximum of *A* (wavelength λ) coincides with the position of the first minimum of *B*. (wavelength $\lambda + d\lambda$). Similarly, the position of the central maximum of *B* coincides with the position of the first minimum of *A*. Further, the resultant intensity intensity curve shows a dip at *C* i.e., in the middle of the central maxima of *A* and *B* (Here, it is assumed that the two spectral lines are of the same intensity). The intensity at *C* is approximately 20% less than that at *A* or *B*. If a spectrograph is turned from the position corresponding to the central image of *A* to the one corresponding to the image of *B*, there is noticeable decrease in intensity between the two central maxima. The spectral lines can be distinguished from one another and according to Rayleigh they are said to be just resolved Rayleigh's condition can also be stated as follows. Two images are said to be just resolved if the radius of the central disc of either pattern is equal to the distance between the centers of the two patterns.

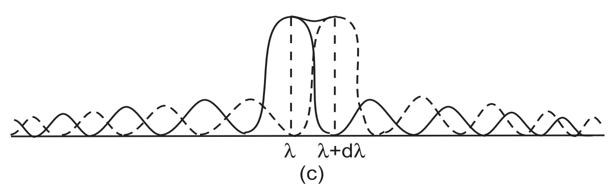


Fig. 14.1

14.5 RESOLVING POWER OF A TELESCOPE

Let a be the diameter of the objective of the telescope. Consider the incident ray of light from two neighbouring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern. Let P_1 and P_2 be the positions of the central maxima of the two images. According to Rayleigh, these two images are said to be resolved if the position of the central maximum of the second image coincides with the first minimum of the first image and vice versa. The path difference between the secondary waves travelling in the directions AP_1 and BP_1 is zero and hence they reinforce one another at P_1. Similarly, all the secondary waves from the corresponding points between A and B will have zero path difference. Thus, P_1 corresponds to the position of the central maximum of the first image.

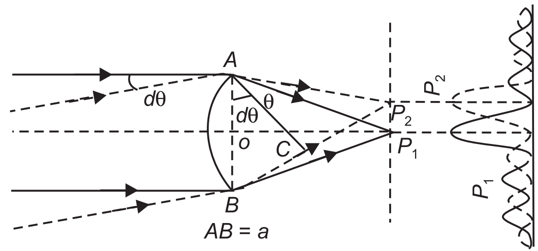


Fig. 14.2

The secondary waves travelling in the directions AP_2 and BP_2 will meet at P_2 on the screen. Let the angle P_2AP_1 be $d\theta$. The path difference equal to BC.

From the $\Delta\, ABC$

$$BC = AB \sin d\theta$$

for small angles

$$BC = AB \cdot d\theta = a \cdot d\theta$$

If this path difference $a \cdot d\theta = \lambda$, the position of P_2 corresponds to the first minimum of the first image. But P_2 is also the position of the central maximum of the second image. Thus, Rayleigh's condition of resolution is satisfied if

$$a \cdot d\theta = \lambda$$

or

$$d\theta = \frac{\lambda}{a} \qquad ...(i)$$

The whole aperture AB can be considered to be made up of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\frac{\lambda}{2}$. All the secondary waves destructively interfere with one another and hence P_2 will be the first minimum of the first image. The equation $d\theta = \frac{\lambda}{a}$ holds good for rectangular aperture. For circular aperture, this equation, according to Airy, can be written as

$$d\theta = \frac{1.22\,\lambda}{a} \qquad ...(ii)$$

where λ is the wavelength of light and a is the aperture of the telescope objective. The aperture is equal to the diameter of the metal ring in which the objective lens is mounted. Here $d\theta$ refers to the limit of resolution of the telescope. The reciprocal of $d\theta$ measures the resolving power of the telescope.

$$\therefore \qquad \frac{1}{d\theta} = \frac{a}{1.22\,\lambda} \qquad ...(iii)$$

From Eq. (iii), it is clear that a telescope with large diameter of the objective has higher resolving power, $d\theta$ is equal to the angle subtended by the two distant object points at the objective.

Thus resolving power of a telescope can be defined as the reciprocal of the angular separation that two distant object points must have, so that their images will appear just resolved according to Rayleigh's criterion.

If f is the focal length of the telescope objective, then

$$d\theta = \frac{r}{f} = \frac{1.22\,\lambda}{a}$$

or

$$r = \frac{1.22\,f\lambda}{a} \qquad \text{...}(iv)$$

where r is the radius of the central bright image.

The diameter of the first dark ring is equal to the diameter of the central image. The central bright disc is called the Airy's disc.

From equation (iv), if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction patterns will appear sharper and the angular separation between two just resolvable point objects will be smaller. Correspondingly, the resolving power of the telescope will be higher.

Let two distant stars subtend an angle of one second of an arc at the objective of the telescope.

1 second of an arc = 4.85×10^{-6} radian. Let the wavelength of light be 5500 Å. Then, the diameter of the objective required for just resolution can be calculated from the equation

$$d\theta = \frac{1.22\lambda}{a}$$

or

$$a = \frac{1.22\lambda}{d\theta} = \frac{1.22 \times 5500 \times 10^{-8}}{4.85 \times 10^{-8}} = 13.9 \text{ cm. (approximately)}$$

The resolving power of a telescope increases with increase in the diameter of the objective. With the increase in the diameter of the objective, the effect of spherical aberration becames appreciable. So, in the case of large telescope objectives, the central portion of the objective is covered with a stop so as to minimize the effect of spherical aberration. This, however, does not affect the resolving power of the telescope.

14.6 OBJECT

To determine the resolving power of telescope.

Apparatus Used: A telescope fitted with a variable width rectangular aperture to its objective, a sodium lamp, three pair of slits a focusing lens, two mountings for the slit pair and the lens.

Formula Used: The theoretical and practical resolving power are given by theoretical resolving power = λ/D and practical resolving power = u/x

Where λ = mean wavelength of light employed.

D = width of the rectangular slit for just resolution of two objects

x = separation between two objects.
u = distance of the objects from the objective of the telescope.

Theory: The resolving power of a telescope is defined as the inverse of the least angle subtended at the objective by two distant point objectives (of equal brightness) which can just be distinguished as separate in its focal plane.

Let the parallel ray of light from two distant objects subtend an angle (θ) at the telescope objective *AOB*. The image of each point object is a fraunhoffer diffraction pattern consisting of a central bright disc surrounded by concentric dark and bright rings due to circular aperture of the objective. The diffraction patterns overlap each other and the two images will just be resolved. According to Rayleigh's criterion, on the first minimum of one image coincide with the central maximum of the other and vice versa.

According to theory the least resolvable angle θ is given by λ/D and the resolving power of the telescope.

$$1/\theta = D/\lambda$$

The resolving power (experimentally) is also given by

$$1/\theta = 1/x/u = u/x$$

Where 'x' is the distance between the two line objects and 'u' is the distance from telescope objective, if θ is the angle when the two images are just resolved.

Method:

1. The focusing lens and a pair of slits are mounted on their respective stands. The slits are made vertical with the help of a plumb line by using the screw attached to the stand.
2. Light from the sodium lamp is focused on the slits by means of the lens.
3. The axis of the telescope is made horizontal by means of spirit level and its height is so adjusted that the images of the pair of slits are symmetrical with respect to the cross point of the cross wires. The inter adjustment can also be obtained by keeping the variable aperture wide open and adjusting the telescope.
4. The images are brought into sharp focus by adjusting the telescope while keeping the variable aperture wide open.
5. The width of the aperture is gradually reduced so that at first the two images appear out and ultimately their separation vanishes. The width of the aperture at this critical position may be measured by means of a micrometer screw or the readings are noted directly on the vernier scale attached to the aperture. Reducing aperture further we note the reading when the illumination (light) just disappears altogether. The difference of these two readings gives width of the aperture required.
6. Now we begin with a closed aperture gradually increasing the width. We take the first reading when the illumination just appears and then when the two images just appear to be separated. The difference giving the width of the aperture is noted.
7. The operation 5 and 6 are repeated.
8. The operation 1 to 7 is repeated for the other two pairs of slits.
9. The distance between the slits (sources) and the objective of the telescope is measured by means of measuring tape.

Observation:

1. Least count of the micrometer attached to the variable width aperture =
2. Minimum width of aperture for resolution D =

3. Distance between the objective of the telescope and the slit sources (u) =
4. Wave length of the light employed =

Pair of Slits	*Distance between the slits (in cm.) x*	*Micrometer reading while aperture width decreasing (cm.)*	*Micrometer reading while aperture width increasing (cm.)*	*Mean (in cm.) D*
A.				
B.				
C.				

Calculations:

Pair of slits	*Theoretical resolving power T = D/λ*	*Experimental resolving Power E = u/x*	*% Difference*
A.			
B.			
C.			

Result: The resolving power of telescope as measured is nearly equal to the theoretical value.

Precautions:

1. The axis of telescope should be horizontal.
2. The screw attached to the variable width aperture should be handled gently. While decreasing the width of the aperture we should stop at the point when illumination just disappears. We should not tight the screw beyond this point. This should be the starting point taking for readings for the increasing aperture.
3. Backlash error in the micrometer screw should be avoided.
4. The axis of the telescope should be at right angles to the plane containing the slits. The two slits should appear equal in height.
5. Care should be taken that distance between the lens and the slit source is more than focal length of lens (about 30 cm).

14.7 VIVA-VOCE

Q. 1. What do you mean by resolving power of a telescope?

Ans. The resolving power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective by two distinct points which can be just seen as separate through the telescope.

Q. 2. On what factors does the resolving power of a telescope depend?

Ans. The resolving power of a telescope is given by

$$\frac{1}{d\theta} = \frac{d}{1.22\,\lambda}$$

Resolving power is directly proportional to d i.e., a telescope with large diameter of objective has higher resolving power and inversely proportional to λ.

Q. 3. Why are the telescopes fitted with objectives of large diameter ?

Ans. To increase the resolving power of telescope.

Q. 4. Does the resolving power of a telescope depend upon the focal length of its objective?

Ans. No.

Q. 5. Does any thing depend on *f*?

Ans. Yes, Magnifying power increases with *f*.

Q. 6. Sometimes an observer gets a higher than the theoretically expected value of resolving power. How do you explain it?

Ans. It is because that Rayleigh criterion is itself quite arbitrary and skilful experimenters can exceed the Rayleigh limit.

Q. 7. Define the magnifying power of the telescope.

Ans. The magnifying power of a telescope is defined as the ratio of angle subtended at the eye by the final image and the angle subtended at the eye by object when viewed at its actual distance.

Q. 8. What is Rayleigh criterion of resolution?

Ans. According to Rayleigh criterion, two point sources are resolvable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice-versa.

Q. 9. What does the term 200 inch written on a telescope indicate?

Ans. This indicate that the diameter of the objective of the telescope is 200 inches.

EXERCISE

Q. 1. What do you mean by resolving limit of a telescope?

Q. 2. What is the resolving power of the eye?

Q. 3. How does the minimum angle of resolution change by putting variable aperture before the telescope objective?

Q. 4. Does the resolving power of a telescope depend upon the distance between the telescope and the objects to be resolved?

Q. 5. Is it possible to attain the theoretical resolving power?

Q. 6. What will be the resolving power of this telescope?

15

Sextant

15.1 OBJECT

To determine the height of a tower by a Sextant.

Apparatus Used: A sextant and a measuring tape.

Formula Used: The height h of a tower is given by the following formula:

$$h = d /(\cot \theta_2 - \cot \theta_1)$$

Where d = distance between the two points of observation.

θ_2 = angular elevation of the tower from one point of observation.

θ_1 = angular elevation at a point distant 'd' from the previous point towards the tower.

15.2 DESCRIPTION OF SEXTANT

Sextant is an optical instrument as shown in Fig. 15.1 and is meant to measure angles. It consists of a graduated circular arc about 60° having two radial fixed arms A and B. There is another arm known as third moving arm C (index arm) that moves over the circular graduated scale. It carries a vernier scale V on one side and plane mirror M_1 (index glass) on the another side. This arm is fitted with clamp and tangent screw, so that it can be adjusted in any desired direction. The plane of the mirror M_1 is perpendicular to the plane of arc. A second mirror M_2 called the horizon glass is fixed to the arm A whose lower half is silvered while upper half is transparent. The plane of this mirror is also perpendicular to the circular arc. A telescope T is fitted to the arm B with its axis perpendicular to the horizon glass. The telescope receives the direct rays through the transparent portion of M_2 and twice reflected rays from M_1 and M_2.

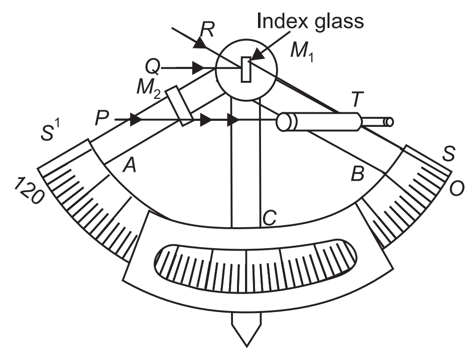


Fig. 15.1

15.3 PRINCIPLE OF WORKING

The distant object is viewed directly through the clean parts of mirrors M_2 and then the movable arm is so rotated that the mirror M_1 and M_2 become parallel. In this position the telescope receives the rays from distant object in two paths as shown in Fig. 15.2. One set of rays PM_2 T through clear part of M_2 and other set of rays starting from R reflected from mirror M_1 and then from the silvered portion of mirror M_2 enter the telescope. Now the zero of the main scale should coincide with the zero of the vernier scale and if it is not so then there is a zero error in the instrument, which should be noted with proper sign.

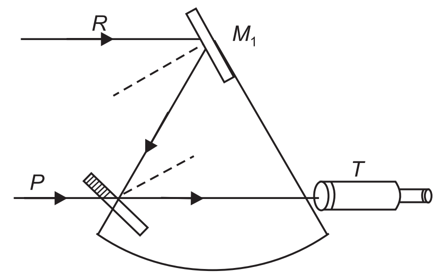


Fig. 15.2

In order to calculate the angle between two objects situated in the direction M_1R and M_2P as shown in Fig. 15.3, the movable arm containing mirror M_1 is moved such that the rays coming directly from P towards telescope and rays coming through the paths RM_1, M_1 M_2 and M_2 T concide with each other. The angle RM_1Q is the angle between the directions of the two objects, which is twice the angle BM_1 C. To facilitate this the circular scale is directly marked as twice the actual degrees.

Theory: Suppose AB is the tower, the height of which is to be measured. Make a mark B' on the tower at the height of your eyes from the ground.

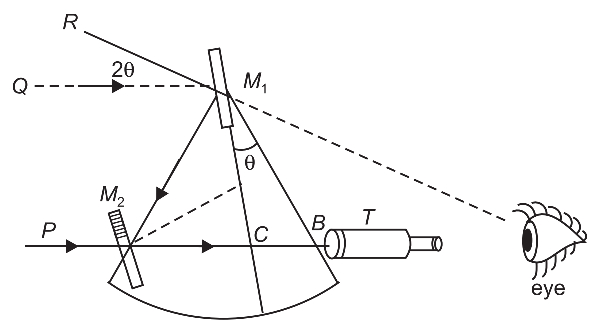


Fig. 15.3

Suppose at a point C (Fig. 15.4) the length AB of the tower subtends an angle θ at your eyes and at some other point D at a distance d from point C in the direction of BC, the angle subtended at your eye is θ_2.

Since $B'D' = AB' \cot \theta_2$

and $B'C' = AB' \cot \theta_1$

$B'D' - B'C' = d$

$AB' (\cot \theta_2 - \cot \theta_1) = d$

or $AB' = d/(\cot \theta_2 - \cot \theta_1)$

Hence the total height of the tower = $AB' + BB'$

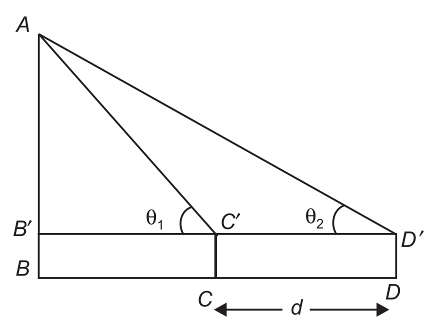


Fig. 15.4

Adjustments:

1. The plane of the index glass M_1 must be perpendicular to the plane of the arc. To test whether this adjustment is complete set the radius bar to about the middle of the arc. Then with the eyes near the index glass look obliquely into the glass so as to set at the same time part of the arc direct and part after reflection into the mirror. If the two positions of the arc appear in the same plane the mirror is in adjustment. If not, then the screws at the back of the mirror must be move till the adjustment is complete.
2. The plane of the horizon glass must be perpendicular to the plane of the instrument. Direct the instrument so as to view some small well-defined distant object and move the radius arm till two images of the distant object appear in the field of view. If by adjusting

the radius arm the two images are made to exactly coincide then the mirror M_2 is parallel to the mirror M_1 and hence perpendicular to the plane of the instrument. If turning the radius arm will not bring the two images in to exact coincidence the screws at the back of the mirror M_2 must be adjusted till complete coincidence of the images can be secured.

3. The line of collimation of the telescope must be parallel to the plane of the instrument.
4. The zero of the scale is said to coincide with the position of the index arm when the two mirrors are parallel. This adjustment is seldom exactly correct and hence the correction (called the index correction), which has to be applied must be determined.

Manipulations:

1. Determine the least count of SEXTANT.
2. Make a mark B' on the tower AB at the height of your eye level from the ground.
3. Spread the tape towards the tower at some distance from it.
4. Determine the angle θ_1, standing at one end of the tape. Take the readings and find the index correction.
5. Step back 20 feet along the tape and measure θ_2 and index correction.
6. Step back another 20 feet and measure θ_3 and index correction.
7. Measure the height of the mark B' on the tower from the ground.

Observations:

L.C. of Sextant =

Sl. no.	*Distance from tower (in meter)*	*Index Correction (a)*	*Angle at the end point of a tower (b)*	*Angular elevation (b – a)*
1				
2				
3				

Height of the mark B' above the ground BB' = meter

Calculations:

$$AB' = d/(\cot \theta_2 - \cot \theta_1)$$

Also $$AB' = d/(\cot \theta_3 - \cot \theta_2)$$

Result: The height of the tower is found to be = meter.

Precautions and Sources of Error:

1. Before performing the experiment the adjustments should be made carefully.
2. Zero reading (index correction) must be found separately at different places.
3. The foot of the tower and two points of observations should be in a straight line.
4. To find out the actual height of the tower, the height of the chalk mark from the foot of tower should be added.

15.4 VIVA-VOCE

Q. 1. Why this instrument is called a Sextant?

Ans. The circular scale of the instrument is one sixth of a circle i.e., an arc of 60°.

Q. 2. Upon what principle does a sextant work?

Ans. This is based on the principle that when a plane mirror is rotated through an angle θ, the reflected ray is turned though 2θ.

Q. 3. Is the incident ray fixed here as mirror is rotated? then?

Ans. No, the incident ray is not fixed, but the reflected ray is fixed in this experiment. On account of the reversibility of light path, the incident and reflected rays are interchangeable.

Q. 4. What is the relation between markings on the scale and the angle turned by the index arm of the sextant?

Ans. The angle subtended by the object is twice the angle of rotation of the index glass which is measured by the vernier. To obtain the angle of elevation of the object, the value of angle obtained from arc scale should be doubled. To avoid the necessity of doubling the reading each time, the instrument is made direct reading by marking the graduations of the arc scale by double the actual numbers, thus the 60° arc scale is marked as 130° scale.

Q. 5. Why are the two images formed when a sextant is directed towards some object?

Ans. One image is formed by the rays directly entering the telescope through the transparent portion of horizon glass and the second by those rays which enter the telescope after reflections from index-glass and silvered portion of horizon glass.

Q. 6. What is the relative setting of M_1 and M_2 when the scale reads zero?

Ans. They are parallel to each other and perpendicular to the bed of the instrument.

Q. 7. What do you mean by zero error of sextant?

Ans. When direct image of a distant object seen through transparent portion of the horizon glass is made to coincide with the image formed by reflection at index and horizon glass the two glasses are parallel. The reading of the index arm on the scale should be zero, if it is not zero, then there is zero error.

Q. 8. What are these coloured glasses meant for?

Ans. These are used when measurement are made with sun or any other bright object.

Q. 9. What is the relative setting of M_1 and M_2 when the scale reads zero?

Ans. The two mirrors are parallel to each other and perpendicular to the bed of the apparatus.

Q. 10. What are other uses of sextant?

Ans. This is used by mariners to find latitude and longitude at a particular place during their voyage.

Q. 11. What is meant by angular diameter θ of the sun?

Ans. The angle subtended by sun's disc at the earth is called angular diameter θ of the sun.

Q. 12. How the angular diameter of the sun is related to the actual diameter?

Ans. The actual diameter D of the sun is related to angular diameter θ by the relation $D = x\theta$, where x is the distance between earth and sun.

EXERCISE

Q. 1. What mechanical adjustments must be secured in a sextant?

16

Tables of Physical Constants

16.1 SPECIFIC RESISTANCE AND TEMPERATURE COEFFICIENT

Substance	*Specific Resistance.* ρ *(ohm-m)* $\times 10^{-8}$	*Temp. Coeff. of resistance* (α), $\times 10^{-4}$ °C^{-1}
Aluminium	2.6	39
Brass	6.6	10
Constantan	49.1	(–4 to + 0.1)
Copper	1.78	42.8
German silver	26.6	(2.3–6)
Gold	2.42	40
Iron	12.0	62
Lead	20.8	43.0
Manganin	44.5	0.02–5
Mercury	95.8	8.9
Nichrome	110	3.5
Nickel	11.8	27
Phosphor Bronze	5–10	35
Platinoid	34.4	2.5
Platinum	11.0	37
Silver	1.65	40

16.2 E.M.F. OF CELLS: VOLTS

Cell	*E.M.F.*	*Cell*	*E.M.F.*
Daniell	1.08–1.09	Cadmium at 20^0C	1.018 54
Grove	1.8–1.9		
Lechlanche	1.45	Lead accumulator	1.9–2.2
		Edison cell	1.45
Voltaic	1.01	Clarke	1.43
Bunsen	1.95	Ni-Fe	1.20

16.3 ELECTRO-CHEMICAL EQUIVALENT OF ELEMENTS

Element	*Atomic weight*	*Valency*	*E.C.E. (g/coulomb)*
Copper	63.57	2	0.000 329 5
Gold	197.2	3	0.006 812
Hydrogen	1.0080	1	0.000 010 45
Lead	207.21	2	0.001 073 6
Nickel	58.69	3	0.000 202 7
Oxygen	16.00	2	0.000 082 9
Silver	107.88	1	0.001 1180
Aluminium	26.97	3	0.0000 935

16.4 REFRACTIVE INDEX OF SUBSTANCES

For sodium light $\lambda = 5896 \times 10^{-10}$m

Solid	*Refractive*	*Liquid*	*Refractive Index*
Diamond	2.417	Canada balsam	1.53
Glass (crown)	1.48 – 1.61	Water	1.333
Glass (flint)	1.53 – 1.96	Alcohol (ethyl)	1.362
Glass (soda)	1.50	Aniline	1.595
		Benzene	1.501
Ice	1.31	Cedar oil	1.516
Mica	1.56 – 1.60	Chloroform	1.450
Rock-salt	1.54	Ether	1.350
		Glycerine	1.47
Quartz (O – Ray)	1.5443	Olive oil	1.46
Quartz (E – Ray)	1.5534	Paraffin oil	1.44
Quartz (fused)	1.458	Kerosene oil	1.39
		Turpentine oil	1.44

16.5 WAVELENGTH OF SPECTRAL LINES: (in Å, 1 Å = 10^{-10} m)

[The visible spectrum colours are indicated—*red, orange, yellow, green, blue, indigo, violet*]

Hydrogen	*Helium*	*Mercury*	*Neon*	*Sodium*
3970 *v*	3889 *v*	4047 *v*	5765 *y*	(D_2) 5890 *0*
4102 *v*	4026 *v*	4078 *v*	5853 *y*	(D_1) 5896 *0*
4340 *b*	4471 *b*	*4358* v	5882 *0*	
4861 *gb*	5876 *y*	4916 *bg*	6507 *r*	
6563 *r*	6678 *r*	4960 *g*	7245 *r*	
	7065 *r*	5461 g		
		5770 *y*		
		5791 *y*		
		6152 *0*		
		6322 *0*		

16.6 ELECTROMAGNETIC SPECTRUM (WAVELENGTHS)

Wireless Wave			*5 metres and above*
Infra Red	3.0×10^{-4} m	to	7.5×10^{-7} m
Visible Red	7.5×10^{-7} m	to	6.5×10^{-7} m
Visible Orange	6.5×10^{-7} m	to	5.9×10^{-7} m
Visible Yellow	5.9×10^{-7} m	to	5.3×10^{-7} m
Visible Green	5.3×10^{-7} m	to	4.9×10^{-7} m
Visible Blue	4.9×10^{-7} m	to	4.2×10^{-7} m
Visible Indigo	4.2×10^{-7} m	to	3.9×10^{-7} m
Ultra Violet	3.9×10^{-7} m	to	1.8×10^{-7} m
Soft X-Rays	2.0×10^{-7} m	to	1.0×10^{-7} m
Hard X-Rays	1.0×10^{-10} m	to	$1.0 \text{ x } 10^{-11}$ m
Gamma Rays	5.0×10^{-11} m	to	$5.0 \text{ x } 10^{-12}$ m
Cosmic Rays	5.0×10^{-14} m		

16.7 MAGNETIC ELEMENTS

Place	*Angle of Declination*	*Angle of Dip*	*Horizontal component (B_H)* Tesla (10^{-4})
Agra	0° 10′ E	40° – 40′	0.348
Ajmer	....	39° – 46′	0.358
Aligarh	0° 20′ E	41° – 50′	0.346
Allahabad	0° 20′ W	37° – 10′	0.353
Amritsar	....		0.300
Bangalore	....	9° – 222′	0.405
Bareilly	0° 20′ E	42° – 20′	0.344
Calcutta	0° 00′	30° – 31′	0.382
Chandausi	0° 30′ E	42° – 40′	0.343
Dehra Dun	0° 50′ E	45° – 50′	0.332
Delhi	0° 40′ E	42° – 52′	0.345
Hyderabad (Deccan)	....	19° – 39.2′	0.397
Gorakhpur	0° 20′ W	39° – 40′	0.358
Gwalior	0° 20′ E	39° – 00′	0.353
Jaipur	0° 30′ E	40° – 30′	0.347
Kanpur	0° 00′	38° – 39′	0.363
Khurja	0° 30′ E	42° – 10′	0.343
Lucknow	0° 10′ W	40° – 00′	0.354
Ludhiana	....	45° – 13.3′	0.335
Meerut	0° 40′ E	43° – 30′	0.339
Mumbai	0° 20′ W	25° – 30′	0.376
Nagpur	....	32° – 09′	0.385
Patna	....	36° – 24′	0.373
Varanasi	0° 30′ W	37° – 10′	0.364

16.8 WIRE RESISTANCE

S.W.G. No.	Diameter (mm)	Resistance (ohm/metre)		
		Copper	Constantan (60 Cu, 40 Ni)	Manganin (84 Cu, 4 Nil, 12 Mn)
10	3.25	0.0021	0.057	0.051
12	2.64	0.0032	0.086	0.077
14	2.03	0.0054	0.146	0.131
16	1.63	0.0083	0.228	0.204
18	1.22	0.0148	0.405	0.361
20	0.914	0.0260	0.722	0.645
22	0.711	0.0435	1.20	1.07
24	0.559	0.070	1.93	1.73
26	0.457	0.105	2.89	2.58
28	0.374	0.155	4.27	3.82
30	0.315	0.222	6.08	5.45
32	0.274	0.293	8.02	7.18
34	0.234	0.404	11.1	9.9
36	0.193	0.590	16.2	14.5
38	0.152	0.950	26.0	23.2
40	0.122	1.48	40.6	36.3
42	0.102	2.10	58.5	53.4
44	0.081	3.30	91.4	81.7
46	0.061	5.90	162.5	145.5

Nichrome (68% Ni, 15% Cr 15.5% Fe, 1.5% Mn)	Resistivity, at 20^0C $(100–110) \times 10^{-8} \Omega$.m

16.9 VISCOSITY'S LIQUID (IN POISE) (1 Pa.s = 10 POISE)

Substance	*Viscosity Pa.s*	*Substance*	*Viscosity Pa.s*
Alcohol (methyl)	0.0119	Olive oil	0.98
Alcohol (methyl)	0.005 91	Turpentine oil	0.014 9
Benzene	0.006 49	Water at 0ºC	0.017 93
Caster oil	9.86	Water at 20ºC	0.010 06
Chloroform	0.005 64	Water at 25ºC	0.008 93
Ether	0.002 34	Water at 30ºC	0.008 00
Glycerine	8.5	Water at 40ºC	0.006 57
Mercury	0.0156	Water at 60ºC	0.00469
		Water at 100ºC	0.002 84

S.I. unit of viscosity Pa.s. (Pascal second)

16.10 DIELECTRIC CONSTANTS OF SOME COMMON MATERIALS (AT 20°C)

Material	*Frequency (Hz)*	*Dielectric constant*
Amber	10^6	2.8
Amber	3×10^9	2.6
Soda glass	10^6	7.5
Fused quartz	10^3 to 10^8	3.8
Liquid paraffin (Medical Grade)	10^3	2.2
Transformer oil (Class B)	10^3	2.2
Marble	10^6	8
Sand (dry)	10^6	3
Sandstone	10^6	10
Paper (Oil impregnated condenser tissue)	10^3	2.3
Mica	10^3 to 10^8	5.4 to 7
Epoxy resin (*e.g.*, Araldite)	10^6	3.3
Cellulose Acetate	10^6	3.5
Vinyl Acetate (Plasticised)	10^6	4
Vinyl Acetate (P.V.C.)	10^6	4
Ebonite (Pure)	10^6	3
Rubber (Vulcanized soft)	10^6	3.2
Rubber, Synthetic	10^6	2.5
Paraffin wax	10^6	2.2
Sulphur	3×10^9	3.4
Walnut wood (dry)	10^7	2.0
Walnut (17% moisture)	10^7	5
Vacuum	any	1.000 00
Air	Upto 3×10^9	1.000 54
Porcelain	10^6	5.5
Barium titanate	10^6	1200
Water	10^9	80
Water	10^{10}	64

16.11 PROPERTIES OF LIQUID

Substance	*Freezing point, °C*	*Boiling point, °C*	*Density at 0ºC kg m^{-3} × 10^3*	*Specific heat*		*Latent heat*		*Coefficient of cubical expansion K^{-1} × 10^{-1}*	*Surface tension at 20°C Nm^{-1} × 10^{-3}*	*Viscosity at 20ºC Poiseulle × 10^{-6}*
				cal gm^{-1} K^{-1}	*J kg^{-1} K^{-1} × 10^3*	*cal gm^{-1}*	*kJ kg^{-1}*			
Alcohol (ethyl)	– 115	78	0.79	0.58	2.436	205	861	11.2	22.3	192
Alcohol (methyl)	– 97	64	0.81	0.61	2.562	267	1120	11.2	–	590
Benzene	–	80.2	0.92	0.40	1.680	93	391	12.4	28.9	649
Ether	– 12	34	0.74	0.56	2.352	89	373	16.3	18.5	264
Glycerine	17	290	1.30	0.59	2.478	–	–	5.3	62.5	149 0000
Paraffin oil	–	–	0.81	0.52	2.184	–	–	9.0	26.0	–
Mercury	– 38.9	357	13.56	0.033	0.140	68	286	1.8	52.0	1560
Turpentine oil	– 10	159	0.87	0.42	1.800	70	294	9.4	27.3	1991
Water	0	100	0.999	1.00	4.200	536	2251	1.5	72	1006

16.12 PROPERTIES OF SOLIDS

Substance	*Density kg m^{-3} × 10^{+3}*	*Relative density (a pure number)*	*Melting point ºC*	*Specific heat*		*Latent heat of fusion*		*Coefficient of linear expansion α per ºC × 10^{-6}*	*Thermal conductivity cal cm^{-1} s^{-1} K^{-1}*
				J kg^{-1} K^{-1}	*cal g^{-1} kg^{-1}*	*kJ Kg^{-1}*	*calg^{-1}*		
Aluminium	2.74	2.74	657	875	0.29	389	96	14	0.503
Copper	8.93	8.93	1089	380	0.091	206	49	17	0.920
Ice	0.92	0.92	0.00	2100	0.502	336	80	51	0.005
Iron (cast)	7.60	7.60	1100	500	0.119	118	28	10	0.114
Iron (wrought)	7.85	7.85	1530	483	0.115	206	49	12	0.160
Steel	7.71	7.71	1400	470	0.112	206	49	11	0.110
Lead	11.37	11.37	327	130	0.031	26	6.2	23	0.083
Brass	8.4 – 8.7	8.4 – 8.7	900	396	0.092	114	27	26	0.260
Constantan	8.88	8.88	1300	412	0.098	–	–	18	0.053
Zinc	7.1	7.1	419	378	0.091	110	24	0	0.265
Glass	2.4 – 2.7	2.4 –2.7	1000	412	0.160	–	–	–	0.0025
Sand	2.63	2.63	–	966	0.230	–	–	–	

16.13 ELASTIC CONSTANTS

Substance	*Young's modulus Y Nm^{-2} $\times 10^{10}$*	*Rigidity modulus, η Nm^{-2} $\times 10^{10}$*	*Bulk modulus, K Nm^{-2} $\times 10^{10}$*	*Poisson's ratio(σ)*	*Breaking stress (kg/mm^2)*
Copper	11.0 – 12.55	3.4 – 3.61	13.0 – 14.33	0.34	40 to 45
Brass	9.0 – 10.2	3.5 – 3.37	6.0 – 6.2	0.34 – 0.38	30 to 50
Iron (cast)	10 – 12.8	5.5 – 5.1	9.5 – 9.7	0.23 – 0.30	–
Iron (wrought)	19 – 21.2	7.7 – 8.5	14.6 – 16.1	0.27	–
Steel	19 – 21.0	7.4 – 7.6	16.5 – 17.8	0.28	40 to 45
Lead	1.6	0.6	4.5	0.44	–
Zinc	8 – 10.7	3.6 – 3.76	3.0 – 6.2	0.30	–
Aluminium	7.0	2.5	7.5	0.34	20 to 25
Constantan	16.32	6.15	15.57	0.32	–
German silver	11.0	4.53	15.03	0.37	–
Manganin	12.5	4.65	12.17	0.33	–
Quartz	5.4	3.0	–	–	–
Rubber	0.05	–	–	0.05	–
Silver	7.1 to 7.4	2.5 – 3.0	–	0.37	40 to 45

16.14 SURFACE TENSION AND VISCOSITY OF WATER: (FROM 0°C TO 100°C)

Tem. in °C	*Surface Tension in $Nm^{-1} \times 10^{-3}$*	*Viscosity in Pa.s. $\times 10^{-6}$*	*Temp. in °C*	*Surface Tension in $Nm^{-1} \times 10^{-3}$*	*Viscosity in Pa.s $\times 10^{-6}$*
0	75.2	1793	–	–	–
10	73.5	1311	60	65.6	469
20	72.0	1006	70	63.8	406
30	70.6	800	80	62.0	356
40	68.9	657	90	60.0	316
50	67.3	550	100	58.0	284

16.15 ACCELERATION DUE TO GRAVITY

Place	*g (m/s^2)*	*Latitude (N)*	*Longitude (E)*	*Elevation (m)*
Agra	9.7905	27° 12′	78° 02′	150
Aligarh	9.7908	27° 54′	78° 05′	187
Allahabad	9.7894	25° 27′	81° 51′	94
Banaras	9.7893	25° 20′	83° 00′	81
Calcutta	9.7880	22° 35′	88° 20′	6
Chennai	9.7828	13° 04′	80° 15′	6
Delhi	9.7914	28° 40′	77° 14′	216
Equator	9.7805	00° 00′	n.a.	0
Jaipur	9.7900	26° 55′	75° 47′	433
Mumbai	9.7863	18° 54′	72° 49′	10
Udaipur	9.7881	24° 35′	3° 44′	563
Srinagar	9.7909	34° 05′	74° 50′	159

Pole	9.8322	90º 00′	n.a.	0
Tiruvananthpuram	9.7812	8º 28′	76º 58′	27
Tirupati	9.7822	13º 38′	79º 24′	169
Madurai	9.7810	9º 55′	78º 07′	133
Bangalore	9.7803	12º 57′	77º 37′	915
Guwahati	9.7899	26º 22′	91º 45′	52
Bhubaneswar	9.7866	20º 28′	85º 54′	23

16.16 THERMOCOUPLE

Thermocouple	*Thermo. e.m.f. when cold junction at 0ºC and hot junction at 100ºC*	*Range of Temperature*
Copper—Constantan	4160 μ V	upto 300ºC
Iron—Nickel	3460 μ V	from 300ºC to 600ºC
Copper—Iron	1220 μ V	
Copper—Nickel	2240 μ V	
Iron—Constantan	5380 μ V	
Aluminium—Constantan	3820 μ V	
Aluminium—Nickel	3460 μ V	
Copper—Aluminium	340 μ V	
Nickel—Chromium		from 600ºC to 1000ºC
Platinum—Platinum and Rhodium alloy		from 1000ºC to 1600ºC
Iridium—Iridium and Rubidium alloy		from 1600ºC to 2000ºC
Tungsten—Molybdenum		from 2000ºC to 3000ºC

16.17 TRANSISTORS AND CRYSTAL DIODES (MANUFACTURED BY BEL)

Transistor		*Diode*	
Type	*No.*	*Type*	*No.*
AC 125	PNP	OA	70
AC 126	PNP	OA	73
AC 127	PNP	OA	72
AC 128	PNP	OA	79
AC 134	PNP	OA	81
AF 114	PNP	OA	85
AF 115	PNP	OA	91
AF 116	PNP	OA	95
AF 117	PNP	DR	25
BC 1488	NPN	DR	100
BF 115	NPN		
2 N 3055	NPN		
2 N 109	NPN		
2 N 175	NPN		

16.18 DATA FOR INTRINSIC AND EXTRINSIC SEMI-CONDUCTORS

Material	*Energy Gap (ev)*	*No. density per m^3* electron-hole pairs at 300 K*	*Mobility Electrons ($m^2V^{-1}s^{-1}$)*	*Mobility holes ($m^2V^{-1}s^{-1}$)*	*Conductivity (Sm^{-1})*	*Density (kg. m^3)*
Semi-conductors						
Germanium	1.12	6×10^{19}	0.39	......	2.18	5320
Silicon	0.76	7×10^{15}	0.135	......	4.4×10^{-4}	2300
Doped Si with P	0.045	2.5×10^{21}	0.135	......	2.5×10^{0}	2300
Insulators						
Diamond	6 to 12	$\approx 10^{7}$				

16.19 DENSITY OF COMMON SUBSTANCES

Substance	*Density (gm/cc)*	*Substance*	*Density (gm/cc)*	*Substance*	*Density (gm/cc)*
a. Solids		Tungeston	19.3	Benzene	0.88
Aluminium	2.7	Tin	7.3	Glycerine	1.26
Iron (pure)	7.88	Lead	11.34	Kerosene	0.799
Iron (wrought)	7.85	Magnesium	1.74	Mercury	13.6
Iron (cast)	7.6	Nickel	8.8	Spirit	0.83
Steel	7.7–7.9	Bronze	8.8–8.9	Milk	1.03
Brass	8.4–8.7	Constantan	8.88	Turpentine oil	0.87
Chromium	6.93	Magnanin	8.50	Water	9999
Copper	8.89	Asbestos	2.0–2.8	***c. Gases***	
Gold	19.3	Cork	0.22–0.26	Air	0.001293
Antimony	6.62	Glass (crown)	2.5–2.7	Carbon dioxide	0.001919
Bismuth	9.78	Glass (flint)	2.9–4.5	Helium	0.000178
Silver	10.5	Zinc	7.1	Hydrogen	0.000089
Mica	2.6–3.2	***b. Liquids***		Nitrogen	0.001251
Platinum	21.45	Alcohol	0.80	Oxygen	0.001429
Selenium	4.8			Steam (at 100ºC)	0.000581

16.20 UNIVERSAL PHYSICAL CONSTANTS

Gravitational constant, G	.....	.....	6.67×10^{-11} newton-m^2/kg^2
Boltzmann constant, k	.....	.....	1.38×10^{-23} joule/K
Stefan-Boltzman constant, s	.....	.....	5.67051×10^{-8} W/m^2-K^4
Wien's displacement law constant, b	.....	.....	2.897756×10^{-3} m-K
Planck's constant, h	.....	.....	6.63×10^{-34} J-s
Universal gas constant, R	.....	.....	8.314 J.mol^{-1}.K^{-1}
Avogadro's constant, N_A	.....	.....	6.023×10^{23} mol^{-1}
Speed of light in vacuum, c	.....	.....	3×10^{8} m/s
Charge of porton, e	.....	.....	1.60×10^{-19} C
Atomic mass unit, a.m.u.	.....	.....	1.6606×10^{-27} kg
Mass of electron, m_e	.....	.....	9.11×10^{-31} kg
Mass of proton, m_p	.....	.....	$1.6726231 \times 10^{-27}$ kg
Mass of neutron, m_n	.....	.....	1.67×10^{-27} kg

Permeability of vaccum, μ_o	.....	.....	$4\pi \times 10^{-7}$ N/A^2
Permittivity of vaccum, ε_o	.....	.....	8.85×10^{-12} C^2/N-m^2
Faraday constant, *F*	.....	.....	9.6×10^4 C/mol
Rydberg constant, *R*	.....	.....	1.10×10^7 m^{-1}

16.21 CRITICAL ANGLE

Substance	*Critical angle*	*Substance*	*Critical angle*
Diamond	25°	Ruby	50°
Glass (crown)	45°	Glass (flint)	37°
Turpentine oil	15°	Water	41°30′

16.22 SPECIFIC ROTATION

Optically active substance	*Solvent*	*Specific Rotation*
Water	Cane sugar	+ 66.5°
Water	Glucose	+ 52°
Water	Fructose	– 91°
Alcohol	Camphor	+ 41°
Pure	Turpentine	– 37°
Pure	Nicotine	– 122°

16.23 CONVERSION FACTORS

Mass:
1 kg = 1000 g
1 tonne = 1000 kg
1 a.m.u. = 1.6606×10^{-27} kg
1 kg = 6.022×10^{23} a.m.u.
1 slug = 14.59 kg
1 kg = 6.852×10^{-2} slug
1 a.m.u. = 931.50 Me V/c^2

Length:
1 km = 0.6215 mile
1 mile = 1.609 km
1 m = 1.0936 yd = 3.281 ft = 39.37 in
1 in = 2.54 cm
1 ft = 12 in = 30.48 cm
1 yd = 3 ft = 91.44 cm
1 Å = 0.1 nm

Speed:
1 km/h = 0.2778 m/s = 0.6215 mi/h
1 mi/h = 0.4470 m/s = 1.609 km/h
1 mi/h = 1.467 ft/s

Density:
1 g/cm^3 = 1000 kg/m^3 = 1 kg/L
(1 g/cm^3) g = 62.4 lb/ft^3

Area:
1m^2 = 10^4 cm^2
1 km^2 = 0.3861 mi^2 = 247.1 acres
1 in^2 = 6.4516 cm^2
1 ft^2 = 9.29×10^{-2} m^2
1 m^2 = 10.76 ft^2
1 mi^2 = 460 acres = 2.590 km^2

Angle:
π rad = 180°
1 rad = 57.30°
1° = 1.745×10^{-2} rad

Force:
1 N = 0.2248 lb = 10^5 dyn
1 lb = 4.4482 N

Volume:
1 $m^3 = 10^6\ cm^3$
1 L = 1000 $cm^3 = 10^{-3}\ m^3$
1 gal = 3.786 L
1 gal = 4 qt = 8 pt = 128 oz = 231 in^3
1 in^3 = 16.39 cm^3
1ft^3 = 1728 in^3 = 28.32 L = 2.832 × $10^4\ cm^3$

Magnetic field:
1 Gauss = 10^{-4} T
1 Tesla = 10^4 G

Energy:
1 kW.h = 3.6 MJ
1 cal = 4.1840 J
1 ft.lb = 1.356 J = 1.286 × 10^{-3} Btu
1 Btu = 778 ft.lb = 252 cal = 1054.35 J
1 eV = 1.602 × 10^{-19} J
1 u.c^2 = 931.50 MeV
1 erg = 10^{-7} J

Pressure:
1 pA = 1 N/m^2
1 atm = 101.325 kPa = 1.01325 bars
1 atm = 14.7 lb/im^2 = 760 mmHg
= 29.9 inHg = 6.895 kPa
1 torr = 1 mmHg = 133.32 Pa
1 bar = 100 kPa

Thermal Conductivity:
1 W/m.K = 6.938 Btu.in/h.ft^2·F°
1 Btu.in / h.ft^2.F° = 0.1441 W/m.K

Power:
1 horsepower = 550 ft.lb/s = 746 W
1 Btu/min = 17.58 W
1 W = 1.341 × 10^{-3} horsepower = 0.7376 ft.lb/s

16.24 COLOUR CODE FOR RADIO—CARBON RESISTANCES

Generally, carbon resistances used in radio network are provided in the laboratory. The value of these resistances is read by means of a colour code printed on the outer casting as shown in fig. 1. The numerical value associated with each colour is indicated in following table. The colour bands are always read left to right from the end that has the bands closest to it, as shown in fig. 1. The first and second bands represents the first and second significant digits, respectively, of the resistance value and the third band is for the number of zeros that follow the second digits. The fourth band represents the tolerance.

Colour coding

Colour	*Digit*	*Multiplier*	*Tolerance*
Black	0	1	..
Brown	1	10^1	..
Red	2	10^2	..
Orange	3	10^3	..
Yellow	4	10^4	..
Green	5	10^5	..
Blue	6	10^6	..
Violet	7	10^7	..
Grey	8	10^8	..
White	9	10^9	..
Gold	..	..	± 5%
Silver	..	..	± 10%
No Colour	..	..	± 20%

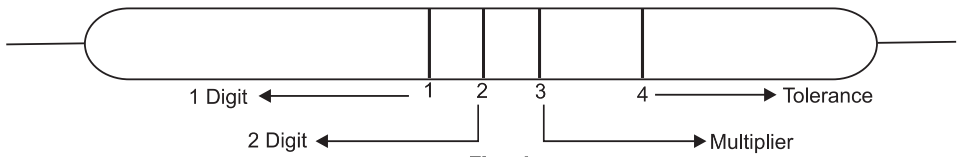


Fig. 1

For example, if a resistor as a colour band sequence : yellow, violet, orange and gold then the resistance will be

1st band	2nd band	3rd band	4th band
Yellow	Violet	Orange	Gold
4	7	10^3	± 5%

$$= 47 \times 10^3\ \Omega \pm 5\%$$

Now 5% of $47 \times 10^3\ \Omega = 2.35 \times 10^3 \Omega$. Therefore, the resistance should be within the range $47 \times 10^3\ \Omega \pm 2.35 \times 10^3\ \Omega$, or between $44.65 \times 10^3\ \Omega$ and $49.35 \times 10^3\ \Omega$

16.25 LOGARITHMS TABLES COMMON LOGARITHMS

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Mean Differences*								
											1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10

	0	1	2	3	4	5	6	7	8	9	*Mean Differences*								
											1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9217	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9360	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9916	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

16.26 ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1418	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1798	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4