

# Articulating Medieval Logic

TERENCE PARSONS

**Q**UOD UIDEREA  
Pauli Veneti logica nō  
qua indigere expositiōe  
tum ppter ipsius disti/  
cultatem tum ppter alio  
rum obscuram diligen/  
tiam modicāq; sollicitudinē cōstitui ne eos  
cōmentarios edere bipitūq; opus facere  
expōnem. s. 7 questione ut cognitio ipsius  
nō in obscurū sed in clarā lucem p̄ducta  
sit. Sic eis putavi hinc tēriculis illic pro  
uectioribus satisfaccere. Nam quid utilit̄  
opa nostra fieri posset qui unctib; p̄hari  
cupiētib; patētes ut auit fores ad ipsam  
p̄hiam reddere cū possimū sine lo<sup>ca</sup> null<sup>o</sup>  
recte p̄hari queat n̄ rerū causas p̄gnoscere  
possit. Neq; medicā ab̄s q; ipsi<sup>o</sup> cogni/  
tione quis tentet. Non eis in manib; illi<sup>o</sup>  
medici me cōfidāz qui logicam ignorat.  
Taceo theologiam que dei essentia p̄tēplaf  
ubi tota nostra beatitudo p̄sistit quam ne  
dum amplecti dicam sed neq; a longe ui/  
dere poteris. Ceteras omitto disciplinas  
que neq; itelligi neq; recte degustari sine  
logica possunt. ad omniū. n. methodorū

torrespondet quart<sup>o</sup> tractatus qui est de  
p̄bationib; terminorū. **Q**uero opaticū  
intellectus que discussus dicitur correspon  
det tertius tractatus qui est de p̄sequētis  
7 quitus qui ē de obligationibus 7 sextus  
qui ē de insolubilibus. **I**n his eis trib<sup>o</sup> fit  
discursus 7 argumētatio ab uno in aliud.  
**E**x quo patet hūc ordinē a magistro fuisse  
puerū cū tractatus de p̄bationib; fioz.  
debeat p̄cedere tractatū p̄sequētariū. quod  
gratia doctrine a magis factū fuisse puto.  
**P**rima pars cōrinet duos tractatus. **I**n  
primo eis determinat de terminis. **I**n 2<sup>o</sup>  
uero de p̄prietatibus terminorū scilz de  
suppositione ampliatione 7 appellatione.  
**P**rimus tractatus decet cōrinet capitula  
**I**n primo determinat de terminis penendo  
diuisionem 7 diffinitiones corū. **I**n 2<sup>o</sup> de  
terminatur de nomine 7 uerbo 7 oratione.  
**I**n 3<sup>o</sup> de p̄positiōe 7 diuisionib; eius. **I**n  
4<sup>o</sup> de figuris p̄pōnum scilz qualiter p̄pō/  
n:is collocēt in figura 7 de nata eorū. **I**n  
5<sup>o</sup> de equipolentiis. **I**n 6<sup>o</sup> de p̄uēzio/  
nibus. **I**n 7<sup>o</sup> de ipotēticiis. **I**n octauo de  
p̄dicabilibus. **I**n 9<sup>o</sup> de p̄ntis. **I**n. io. de fil

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*To Calvin Normore*

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# Contents

<i>List of Tables</i>	xii
<i>Preface</i>	xiii
Introduction	1
1. An Overview of Aristotelian Logic as seen by Medieval Logicians	6
1.1 Categorical propositions	6
1.2 Logical relations among categorical propositions	9
1.3 The square of opposition	10
1.4 Issues concerning empty terms	12
1.4.1 Universal affirmatives	12
1.4.2 Particular negatives	12
1.5 Conversions	14
1.6 Syllogisms	15
1.7 Infinite negation	18
1.8 Formal validity	18
2. Aristotle's Proofs of Conversions and Syllogisms	21
2.1 Formal derivations	21
2.2 Proofs of the conversion principles	25
2.2.1 Conversion of universal negatives	26
2.2.2 Conversion of universal affirmatives ( <i>conversion per accidens</i> )	27
2.2.3 Conversion of particular affirmatives	28
2.3 Reduction of all syllogisms to perfect ones	30
2.3.1 Figure 1 syllogisms	30
2.3.2 Reduction to the first figure	31
2.3.3 Figure 2 syllogisms	32
2.3.4 Barocho	33
2.3.5 Figure 3 syllogisms	34
2.3.6 First figure indirect moods	38
2.4 Proving the first figure syllogisms	39
2.5 Propositions with repeated terms	40
2.6 The indispensability of exposition and expository syllogism	41
2.7 A contemporary assessment of Aristotle's basic theory	43
2.7.1 Generalized quantifiers	43
2.7.2 Axiomatizing generalized quantifiers	48
2.7.3 A qualification concerning existential import	49
2.8 Singular propositions	50
2.9 13th-century texts	51
2.10 Summary of Aristotle's rules of proof	53



3. Quantifying Predicates, Singular Term Predicates, Negative Terms	56
3.1 Expanded notation	56
3.2 Equipollences	60
3.3 Semantics and rules	62
3.4 Singular terms as predicates	67
3.5 Infinitizing negation and conversion	72
3.5.1 Conversion by contraposition and obversion	74
3.6 Completeness of the rules	77
3.6.1 Verbs other than the copula	77
3.7 Summary of the rules of proof used so far	77
4. Linguish	81
4.1 Basics	81
4.2 Categorical propositional logical forms	87
4.3 Rules of inference	90
4.3.1 Some theorems that may be of interest	92
4.3.1.1 Symmetry of 'is'	93
4.4 Signification and supposition	95
4.5 Truth conditions	98
4.5.1 Affirmative and negative propositions and existential import	109
4.6 Validity	110
4.7 Completeness of the rules	113
5. Expanding the Notation	123
5.1 Adjectives	123
5.2 Intransitive verbs	126
5.2.1 Being	128
5.3 Transitive verbs	129
5.4 Additional rules for parasitic terms	133
5.5 Some complex terms	134
5.5.1 Attributive adjectives and participles modifying nouns	134
5.5.2 Participles of transitive verbs with their objects	135
5.5.3 Terms modified by complex terms	138
5.6 Relative clauses	140
5.6.1 Semantics of relative clauses	142
5.6.2 Representing ordinary language Latin (and English)	143
5.7 Genitives	145
5.7.1 What the genitive means	146
5.7.2 Relational common nouns	147
5.7.3 Non-relational uses	148
5.7.4 Non-relational possessives	150
5.7.5 Complex terms with genitives	152
5.8 Demonstratives	153
5.9 Molecular propositions	155
5.9.1 Constructions with free markers	158

6. Some Illustrative Topics	160
6.1 Relational expressions and De Morgan's challenge	160
6.1.1 Dealing with explicit relational expressions	161
6.1.2 Dealing with parasitic terms	162
6.2 Buridan on subjects and predicates	164
6.2.1 Identifying subjects and predicates	164
6.2.2 Subjects and predicates in Linguish	167
6.2.3 Agreeing with Buridan (mostly)	169
6.2.4 The asymmetry of subjects and predicates	173
6.2.4.1 Way 1: Quantifier signs	173
6.2.4.2 Way 2: Negative signs	175
6.3 Simple and complex terms	176
6.3.1 Some interesting constructions	176
6.3.2 Simple and complex terms	180
6.3.2.1 The first type of determinable/determinant pairs	181
6.3.2.2 The second type of determinable/determinant pairs	181
7. Modes of Personal Supposition	184
7.1 Introduction to medieval theory	184
7.2 The 14th-century definitions of the modes	186
7.3 Clarification of the definitions	189
7.3.1 The nature of ascent and descent	190
7.3.2 Occurrences of terms have modes of supposition	190
7.3.3 Repeated occurrences must be ignored	190
7.3.4 Empty terms	192
7.4 Causes of the modes	193
7.5 Restricted descent and parasitic terms	197
7.6 A variant account of merely confused supposition	199
7.7 Useful inferences	202
7.7.1 Superiors and inferiors	202
7.7.2 Monotonicity	205
7.7.3 Parasitic terms	206
7.7.4 Additional useful inferences and non-inferences	207
7.8 Refining the theory: Distinguishing two kinds of distributive supposition	209
7.9 Causes of the refined modes	211
7.9.1 Modes of supposition in Linguish	212
7.10 Useful inferences again	214
7.10.1 Complete induction	214
7.10.2 Switching scopes (thereby switching modes of supposition)	216
7.10.3 Algorithms	217
7.11 Modes of supposition as analyses of quantification	218
7.11.1 Categorical propositions whose terms are non-parasitic and simple	220
7.11.2 Categorical propositions whose terms are simple with one or more parasitic terms	221
7.11.3 Categorical propositions with complex terms	221
7.11.4 Rules from inferior to superior and from superior to inferior	222

7.12	Global quantificational import	223
7.12.1	What are modes of common personal supposition?	223
7.12.2	Causes of the modes and global import	225
7.12.3	Parasitic terms	226
8.	Relatives (Anaphoric Words)	227
8.1	Relatives of identity	227
8.2	Reflexive pronouns	229
8.3	Relatives in Linguish	233
8.3.1	The semantics of relatives that fall within the scope of their antecedents	234
8.3.2	Rules of inference for indexed expressions	236
8.4	Non-reflexive relatives of identity	239
8.5	Applying the singulation theory	244
8.6	An application of relatives to syllogistic	252
8.7	Donkey anaphora	254
8.8	Common term relatives of identity and diversity	256
9.	Comparison of Medieval Logic with Contemporary Logic	259
9.1	The expressive power of medieval logic	259
9.1.1	Medieval logic without anaphoric pronouns	260
9.1.2	Medieval logic with anaphoric expressions	261
9.2	Representing medieval logic within modern predicate logic with identity	262
9.3	Representing modern logic within medieval logic: The problem of existential import	262
9.4	Representing modern logic within medieval logic: Grammatical issues	265
9.5	First-order arithmetic in medieval logic	269
9.5.1	Peano's postulates	269
9.5.2	Definition of addition	271
9.5.3	Multiplication	275
10.	Ampliation and Restriction	276
10.1	Univocation as the source of ampliation and restriction	276
10.2	Ampliation and restriction by tenses	280
10.2.1	Tenses	280
10.2.2	A Complexity: Ambiguity or disjunction?	280
10.2.3	Coordination of times in tensed sentences	281
10.2.3.1	Coordination of times between subject and predicate	281
10.2.3.2	Tenses with relative clauses	282
10.2.3.3	Coordination of times among parts of the subject or predicate	285
10.2.3.4	Subclauses and coordination of tense	285
10.2.4	Buridan's special use of appelliation	286
10.2.5	Tenses in Linguish	288
10.2.5.1	If singular terms are not subject to restriction and ampliation	294
10.2.6	Infinetizing negation	294
10.3	Ampliation by modal terms	295
10.3.1	What are modal propositions?	295
10.3.2	Modal propositions: Semantics	296

10.3.3 Differences between medieval and modern readings	297
10.3.4 Modal propositions in Linguish	298
10.4 Ampliation due to semantic words	299
10.4.1 Looking ahead	299
10.5 Ampliation due to words which pertain to the soul	300
10.6 Promising and owing	302
Appendix: Artificial Quantifiers in Early 16th-Century Logic	306
1. The signs	307
2. What the signs mean	308
3. The signs are, in a sense, logically dispensable	310
4. A doubt: Certain examples do not work as advertised	311
5. Another doubt: The paradigm use of sign 'd' is incoherent	313
6. Some examples from John Major	315
<i>Selected Bibliography</i>	319
<i>Index</i>	328

# List of Tables

1.1.	Logical forms of categorical propositions	8
1.2.	The quality and quantity of propositions	8
1.3.	Aristotle's contributions to the diagram	10
1.4.	The square of opposition	11
1.5.	All valid categorical syllogisms	16
2.1.	Figure 1 from <i>Prior Analytics</i> , chapter 4	30
2.2.	Figure 2 from <i>Prior Analytics</i> , chapter 5	32
2.3.	Figure 3 from <i>Prior Analytics</i> , chapter 6	34
2.4.	Figure 1 indirect moods from <i>Prior Analytics</i> , chapter 7	38

# Preface

I have benefitted greatly from discussion with the participants in several meetings of the Moody Conference over the years, as well as occasional collaborative seminars at UCLA, from participants in the 2001 Copenhagen meeting on Buridan's philosophy, in the 2003 Midwest Conference on Medieval Philosophy in Omaha, and at a pair of talks in 2006 at the University of Barcelona, from the 2007 International Conference on the Square of Opposition in Montreux, from the 17th meeting of the European Symposium for Medieval Logic and Metaphysics in Leiden in 2008, and the 19th meeting in Geneva in 2012, from talks at McGill University in 2008 and again in 2010, from a talk at the University of Nevada at Las Vegas in 2009, and another at the Society for Exact Philosophy in 2010.

I have particularly benefitted in various ways from interactions with E.J. Ashworth, Steve Barney, David Blank, John Carriero, Sten Ebbesen, Elizabeth Karger, Peter King, Gyula Klima, Henrik Lagerlund, Marko Malink, Chris Martin, John Martin, Gary Matthews, Ana Maria Mora, Catarina Dutilh Novaes, Claude Panaccio, Alex Radulescu, Stephen Read, Paul Vincent Spade, Joke Spruyt, Paul Thom, Sara Uckelman, Jack Zupko.

This book has been years in preparation. Throughout this time I have benefitted from occasional sabbaticals granted by the University of California, and I have been nurtured by my family and by my UCLA colleagues. I have been aided by discussions with my various colleagues. I owe special debts of gratitude to Brian Copenhaver and to David Kaplan, and especially to Calvin Normore for his support and insight over the years. And, of course, to my wife Anette, who claims not to understand the work I do, but in some ways understands it most of all.

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# Introduction

Modern logic can be seen as a group of theories and practices clustered around a well-studied and well-understood paradigm theory, namely, first-order predicate logic with relation symbols and identity. This central theory is a formal logic, and formal techniques can be used to validate a vast number of arguments using only a small number of basic principles.

The main theme of this book is that medieval logic can also be seen as a group of theories and practices clustered around a core theory which is a paradigm of logic; this theory consists of a number of widely known principles, all of which can be derived from a very simple core of rules and axioms. Unlike today, however, this was not widely known, and there were only a few attempts to carry out the project of deriving most principles from a basic few. (Buridan's *TC* is a prominent exception.) It has taken me some time to arrive at this view. Medieval writings by logicians can seem to consist of a variety of unsystematic and disparate remarks, and it is not at all obvious whether or how they fit together. That is what this book is about.

There are two striking differences between medieval logic and modern logic. One is the assumptions that are made concerning existential import; in medieval logic '*every A is B*' entails that something is *A*. This needs to be taken seriously, and the details need to be worked out, but from a point of view of general logical principle, this difference is not a great one. The other difference is that medieval logic is formulated entirely within a natural language, Latin. This is a major constraint that needs to be respected and dealt with. It shapes the development of the theory from start to finish. To be sure, medieval Latin is a somewhat artificial natural language. In medieval times it was no longer anybody's native tongue; everyone learned it by schooling, as a second language. However, what is important for my purposes is that it has the grammatical structure of a natural language; in particular, it obeys the "theta-criterion", as discussed in section 4.1.

Medieval logic begins from the logical theory developed by Aristotle. It is well known that Aristotle formulated a system of logic involving conversions (*Some A is a B*; therefore *some B is an A*) and syllogisms. It is also fairly well known that he assumed certain "first figure" syllogisms as axiomatic; these are argument forms such as:

$$\begin{array}{l} \text{Every } B \text{ is a } C \\ \text{Every } A \text{ is a } B \\ \therefore \text{Every } A \text{ is a } C \end{array}$$



and he proved all of the other forms from these and the conversion principles. What is much less well known is that he did not just assume the conversion principles; he proved them. I see the techniques that he used to prove those principles as much more important and interesting than the developed system of logic for which he is known. One of these techniques is well known today: indirect derivation, or, as he called it, *reductio*: to prove a proposition, assume its contradictory and then derive something absurd. Two other principles are often lumped together under the Greek term 'ekthesis'. One of these is that if you are given an existential proposition, you can "choose one of the things that makes it true." In modern logic, this is existential instantiation; given ' $\exists xFx$ ,' pick some name ' $n$ ' that is not used elsewhere in the derivation and write ' $Fn$ ,' and then reason from this rather than from ' $\exists xFx$ .' In Aristotle's notation you are given something like '*some F is a G*' and you pick a name ' $n$ ' not used elsewhere, and write both '*n is an F*' and '*n is a G*.' (In modern logic one would write the conjunction of these formulas, but Aristotle didn't bother with conjunctions.) The third technique is an analogue of our modern existential generalization: given ' $Fn$ ' one can infer ' $\exists xFx$ .' In Aristotle's notation, given both '*n is an F*' and '*n is such and such*' one infers '*some F is such and such*.' Using these three techniques Aristotle proved the conversion principles, and he also made occasional use of those principles in reducing some syllogisms to others. This was a major advance. What was not known then, or throughout the Middle Ages, is that using these three techniques, one may also prove all of the first figure syllogisms,<sup>1</sup> so that those three principles provide a foundation for all of Aristotle's well-known system of logic. This is laid out in Chapters 1 and 2 of this book.

Medieval logicians inherited Aristotle's work together with propositional logic as developed principally by Stoic logicians. Chapter 3 addresses the evolution of this theory in the early 13th century. Much of the additional advances at this time were driven rather straightforwardly by expansions of the logical notation. For example, Aristotle did not allow quantified predicate terms; he argued that one should not write '*Every man is every animal*' because it is not true. But '*No man is every animal*' is true, and it has a quantified predicate, and logicians began using such forms freely. They also constructed sentences with negations sprinkled throughout. Once you can write things like '*Every A isn't a B*' it doesn't take much to notice that this is equivalent to '*No A is a B*,' and logicians formulated principles for interchanging quantifier signs and negations, holding, for example, that '*No A . . .*' is equivalent to '*Not some A . . .*,' and that '*Some A . . .*' is equivalent to '*Not every A not . . .*,' and so on. And once singular terms are allowed to occur anywhere that a quantified common term can, it is clear that singular terms permute with negations, and with other terms, so that '*Socrates no stone is*' is equivalent to '*No stone Socrates is*.' And once instances of the transitivity of identity became formulable it too was recognized as a valid principle. All of this results in a rich system of logic in which a few fundamental principles permit the derivation of the rest.

<sup>1</sup> Thom 1976.

The richness of medieval logic is especially interesting because the entire enterprise is formulated entirely within natural language—at least, within a somewhat regimented version of natural language. So this is a version of logic in which there is no logical form except for grammatical form.<sup>2</sup> Logicians made this work in part by stipulating how Latin is to be understood, holding e.g. that surface order of words determines their semantic scope, so that a sentence having Latin words in this order: ‘*A woman owns each cat*’ is understood to have exactly one reading, meaning that there is a woman such that she owns every cat. To articulate the other reading that is possible in natural English you would have to use a Latin sentence with the word order: ‘*Each cat a woman owns*,’ which is completely grammatical in Latin, and is stipulated to mean, unambiguously, that every cat is such that some woman owns it. This stipulation takes advantage of the relatively free word order of Latin to express quantifier scope. (It is distinctive of medieval logicians that they spend substantial time on matters of scope.) To make clear the logical theory that was developed it is essential to know the exact grammatical forms of the propositions that are employed. Chapter 4 develops a system for encoding and clarifying the grammatical structures of propositions, and there are additional expansions and applications in Chapters 5 and 6.

These expansions of the notation permit the validation of rather complex arguments, such as:

*Some farmer’s every donkey sees every horse which a merchant owns.*  
 ∴ *Not every horse no donkey sees.*

However, the resulting system of logic, because of grammatical constraints, is still limited in its expressive resources. To become adequate one also needs anaphoric pronouns, as in ‘*Some woman owns a donkey which she feeds*.’ This is the task of Chapter 8. If I am right, we are confronted in the texts by two ideas about how anaphoric pronouns work. One of these—the method of *singulation*—is invoked as an analysis of reflexive pronouns. This is roughly the idea that anaphoric pronouns are unaffected by inferences involving their antecedents. For example, given that Socrates is a man and every man loves his mother, we infer that Socrates loves his mother, where the ‘*his*’ remains unchanged while its antecedent changes from ‘*every man*’ to ‘*Socrates*.’ This method works well for reflexives and for a host of other pronouns as well. There is a second method that is much discussed, and that works well in a fairly broad range of cases, but gives clearly wrong results in quite a few. This is roughly the idea that an anaphoric pronoun is an independent term; it stands for the same things as its antecedent, and has the same kind of quantificational status. As Reinhard Hülsen

<sup>2</sup> This needs certain qualification. For example, in Latin all of the following sentences have the same grammatical form: ‘Man is a noun,’ ‘Man is a species,’ ‘A man is a philosopher.’ In the first, the term ‘man’ is said to have “material supposition”; it is taken to stand for itself. In the second, the term is said to have “simple supposition”; it is taken to refer to a form (if there are such things; otherwise it refers to a concept or a word). In the third, the term ‘man’ is said to have “personal supposition”; it is taken to stand for individual men. All of the logic investigated in this book pertains to this last interpretation.

notes,<sup>3</sup> if the first method, the one developed for reflexives, is used in place of the second method, a much better theory results. And at least some later medieval logicians did just that. With the first method, inference principles previously given yield a system of logic that is similar to the predicate calculus in richness and power. At least, this is what I argue in Chapter 9.

In addition to the topics already mentioned, we look in Chapter 7 at what is most distinctive of late medieval logic, and most well known: the useful theory of modes of personal supposition. We also examine the special terms that were used to accommodate sentences containing three or more main quantified phrases. A great deal is now known about these matters, but it is clear that this project will leave far more questions open than it answers.

Chapter 10 and the Appendix touch on further developments of the logical theory.

In focusing on principles of inference that were widely acknowledged I may fail to convey an accurate impression of the diversity of the various writings in medieval logic, and of the various ways in which authors disagreed with one another. These differences and disagreements are widely discussed in the secondary literature. Much of this is fascinating, but it is not my goal to cover it all here. Instead I focus on a few principles that are widely acknowledged and that are rarely debated. It has been suggested that I am trying to impersonate a 15th-century logician who happens to have the skills and habits of a 21st-century graduate student. I discuss known medieval views with some care, and show how far one can go without introducing any logical principles beyond the medieval ones.

I have tried to give enough quotes and citations to ground my assertions in the historical record. However, it is a skewed record. First of all, my work is based entirely on western European texts that have been edited and published, and there are a vast number of yet unedited manuscripts. Further, most of the work I report on here is available in English translation, so untranslated works are not emphasized. Fortunately, the number of first-rate English translations has now reached the point where someone can learn much about medieval logic directly without being able to read Latin. Almost all of the quotations that I give are in English. When quoting from a published translation I use the translator's own words; otherwise I am responsible for them. Most citations to medieval works are given in terms of an abbreviation of the title of the work (e.g. 'SD' for '*Summulae de Dialectica*') followed by a series of numbers, such as '2.3.7'.<sup>4</sup> Page numbers, when given, refer to the English translation, or to the page numbers of the Latin edition if they are included in the English translation, or if there is no published translation. The texts referred to mostly date from 1200 to 1425, after Abelard and up to and including Paul of Venice, with a few later texts also discussed.

<sup>3</sup> Hülsen 1994, 2000.

<sup>4</sup> The meaning of the numbers will vary, depending on how the text in question is demarcated; e.g. it might mean the seventh section of the third chapter of the second book, or the seventh subsection of the third section of the second chapter. Such numbering is usually common to both the Latin edition and to the English translation.

In addition to several anonymous writings, the main logicians referred to are Peter of Spain, William Sherwood, Lambert of Lagny, Roger Bacon, Walter Burley, William Ockham, John Buridan, Albert of Saxony, Marsilius of Inghen, John Wyclif, and Paul of Venice. This group includes both metaphysical realists and nominalists. Although realists and nominalists provide semantic accounts that differ in important details,<sup>5</sup> the logical principles that they endorse are pretty much the same, and so metaphysical differences are mostly not relevant. Likewise for disputes between Thomists and non-Thomists.<sup>6</sup>

It should be apparent that this book does not contain any new historical discoveries; rather it relies on discoveries by others. Over the years I have benefitted enormously from the secondary literature in learning about medieval logic. However, much of the discussion there is not directly relevant to the issues taken up here, and it would be distracting to include it—and it would lengthen this book considerably. I want to hereby acknowledge my indebtedness to the many authors who have published on this topic, and from whom I have learned.

This book is not meant as a general introduction to medieval logic.<sup>7</sup> Further, there are several areas of great logical interest that are not discussed here at all: these include medieval discussion of Aristotle's topics<sup>8</sup> and fallacies,<sup>9</sup> insolubles (semantic and other paradoxes),<sup>10</sup> obligations (rules for debates),<sup>11</sup> syncategoremata (special logical principles of individual words),<sup>12</sup> sophisms (logical and grammatical puzzles),<sup>13</sup> exponible (analyses of special words, such as 'only'),<sup>14</sup> future contingents,<sup>15</sup> consequences (meta-principles of propositional logic),<sup>16</sup> and systems of modal logic<sup>17</sup> (though modal sentences are discussed to some extent). These are all widely treated elsewhere.

<sup>5</sup> For an illustration of some of the contrast between realists and nominalists see Klima 2011.

<sup>6</sup> Aquinas did not discuss general principles of logic much. A full set of views are laid out by his follower John of Saint Thomas (Jean Poinso) in a very competent work from the early 1600s; the logical doctrines laid out there mesh nicely with the ones discussed here. I do not cite passages from this work because it was written considerably later than the period I am discussing.

<sup>7</sup> For a good general introduction see Kretzmann, Kenny, and Pinborg 1982; Marenbon 1987; also Lagerlund 2012, Broadie 2002, Spade 1996. Boehner 2012 is still a good overview.

<sup>8</sup> See Stump 1982.

<sup>9</sup> See the Introduction to Copenhaver, forthcoming, as well as Peter of Spain's own discussions; also the Introduction to Klima 2001, as well as Buridan's own discussions.

<sup>10</sup> For a very brief overview see Spade 1982; also Spade 1988b, Simmons 2008, Yrjönsurri 2008.

<sup>11</sup> For a brief overview see Stump 1982 and Spade 1982b; also Dutilh Novaes 2007.

<sup>12</sup> For a brief overview see Kretzmann 1982.

<sup>13</sup> For a brief overview see Kretzmann 1982; also Read 1993.

<sup>14</sup> For a brief overview see Kretzmann 1982.

<sup>15</sup> For a brief overview see Normore 1982.

<sup>16</sup> For a brief overview see Boh, "Consequences," pp. 300–14 in Kretzmann, Kenny, and Pinborg 1982; also Dutilh Novaes 2007.

<sup>17</sup> For a brief overview see Knuutila 1982 and 2008; also Thom 2003.

# 1

## An Overview of Aristotelian Logic as seen by Medieval Logicians

Medieval Logic is built on a foundation of logical terminology, principles, and methodology contained in the traditional liberal arts, in particular in that part of the Trivium called Logic or Dialectic. This material is mostly from the writings of Aristotle and his commentators, plus the Stoics and others, much of it as interpreted by Boethius.<sup>1</sup> This chapter and the next are devoted to these fundamental parts of logic that medieval logicians accepted as the basis of their work.

I begin with an account of the forms of propositions that constitute the subject matter of Aristotle's symbolic logic, as understood by medieval logicians. In keeping with medieval terminology, I use the term 'proposition' to refer to what we today would call a meaningful sentence. It stands for a sentence with meaning, not for a sentential form, and not for an abstract meaning expressed by a sentence which is named by a that-clause. So '*Snow is white*' and '*Schnee ist weiss*' are different propositions.<sup>2</sup>

### 1.1 Categorical propositions

I begin with an oversimplified account of what Aristotle had to say, mostly in the first seven sections of his short work that is today called *On Interpretation*, and in the first seven sections of his longer work called *Prior Analytics*. This is definitely not the only way to interpret Aristotle's works, but it's a common and straightforward one, and it fits the usual medieval explanations. I also use some standard medieval terminology which Aristotle didn't use.

<sup>1</sup> The parts of Aristotle's symbolic logic that were well known around the year 1000 were his *On Interpretation*, *Categories*, and material (often second-hand) from the first several sections of his *Prior Analytics*. This subject matter was later called the "Old Logic," to distinguish it from the "New Logic" which was based on several additional writings by Aristotle that became available later.

<sup>2</sup> Opinions differed on whether there are also mind-independent entities corresponding to propositions. For many medievals, propositions are tokens, not types, and this is important in certain cases, such as addressing semantic paradoxes, where two tokens of the same type might differ in truth value. But for the most part little would be changed in the theory if propositions were types.

In any system of logic it is essential to be clear about the vocabulary you are using, and about what propositions can be made using it. Aristotle was clear about this. The basic propositions are *categorical* propositions—meaning something like “predicational” propositions. Every such proposition consists of a subject term (perhaps modified) and a copula (perhaps with a negation) and a predicate term. The copula is ‘is.’<sup>3</sup> Every predicate term is a common noun, such as ‘*donkey*’ or ‘*animal*’ or ‘*substance*’ or an adjective such as ‘*just*.’ Every subject term is a proper noun or a common noun. There are eight forms of proposition; four affirmative and four negative. First, there are affirmative and negative “universal” propositions:

Affirmative	Negative
<i>Every A is a B</i>	<i>No A is a B</i>

where ‘A’ and ‘B’ are common nouns or adjectives. Examples are ‘*Every donkey is an animal*’ and ‘*No donkey is an animal*.’ In these examples I have used the indefinite article ‘a’ in order to form grammatical English propositions. There is no indefinite article in Greek, and so the indefinite article appears here as an artifact of English grammar.

There are “particular” propositions:

Affirmative	Negative
<i>Some A is a B</i>	<i>Some A isn’t a B</i> <sup>4</sup>

Examples are: ‘*Some donkey is an animal*’ and ‘*Some donkey isn’t an animal*.’

There are “indefinite” propositions:

Affirmative	Negative
<i>An A is a B</i>	<i>An A isn’t a B</i>

Examples are ‘*A donkey is an animal*’ and ‘*A donkey isn’t an animal*.’ Again, the English examples contain indefinite articles, where Greek and Latin have nothing at all. Finally, there are “singular” propositions:

Affirmative	Negative
<i>n is a B</i>	<i>n isn’t a B</i>

where ‘n’ is a singular term: specifically, ‘n’ is a proper name, such as ‘*Socrates*,’ or a demonstrative term such as ‘*this*’ or ‘*this animal*.’ Examples are ‘*Socrates is an animal*’ and ‘*Socrates isn’t an animal*.’

<sup>3</sup> It may be more true to Aristotle to say that there are two (or more) copulas, an affirmative one, ‘is,’ and a negative one ‘isn’t.’ This will not be important for the applications discussed in this chapter.

<sup>4</sup> In the Latin texts that we will be dealing with the negation naturally precedes the verb, so the word order is ‘*Some A not is B*.’ Generally I use a grammatical English form. I use the contracted form ‘isn’t’ instead of ‘is not’ in order to de-emphasize the English word order, which differs from the Latin.

Table 1.1 shows the logical forms of categorical propositions:

Table 1.1. Logical forms of categorical propositions

	Affirmative	Negative
Universal	<i>Every A is a B</i>	<i>No A is a B</i>
Particular	<i>Some A is a B</i>	<i>Some A isn't a B</i>
Indefinite	<i>An A is a B</i>	<i>An A isn't a B</i>
Singular	<i>n is a B</i>	<i>n isn't a B</i>

Eventually people began describing the status of being affirmative or negative as the *quality* of a proposition, and the other statuses as the *quantity* of a proposition. Table 1.2 shows the eventual classification:<sup>5</sup>

Table 1.2. The quality and quantity of propositions

Quantity	Quality	
	Affirmative	Negative
UNIVERSAL	<i>Every A is a B</i>	<i>No A is a B</i>
PARTICULAR	<i>Some A is a B</i>	<i>Some A isn't a B</i>
INDEFINITE	<i>An A is a B</i>	<i>An A isn't a B</i>
SINGULAR	<i>n is a B</i>	<i>n isn't a B</i>

<sup>5</sup> Many centuries later, Peter of Spain lays out the kinds of non-modal categorical propositions and their ingredients. His terminology was standard:

Of categorical propositions, one is universal, another particular, another indefinite, another singular.

A **universal** proposition is one in which a common term determined by a universal sign is made the subject, like 'every man runs.' Or else a universal proposition is one that signifies that something is in every item or none.

A **common** term is one that is naturally suited to be predicated of many, like 'man' of Sortes, Plato and each one of the other men.

**Universal signs** are these: 'every,' 'none,' 'nothing,' 'any-at-all,' 'either,' 'neither' and the like.

A **particular** proposition is one in which a common term determined by a particular sign is made the subject, like 'some man runs.'

**Particular signs** are these: 'some', 'a-certain', 'the-other', 'the-remaining' and the like.

An **indefinite** proposition is one in which a common term without a sign is made the subject, like 'man runs.'

A **singular** proposition is one in which a singular term, or a common term joined with a demonstrative pronoun, is made the subject, like 'Sortes runs' or 'that man runs.'

A **singular term** is one that is naturally suited to be predicated of just one item.

Also, of categorical propositions, one is affirmative, another negative.

An **affirmative** categorical proposition is one in which the predicate is affirmed of the subject, like 'a man runs.'

A **negative** categorical proposition is one in which the predicate is eliminated from the subject, like 'a man does not run.' (Peter of Spain *LS* I.8–9 (4))

I call these eight forms “standard” categorical propositions, to distinguish them from the extended forms to be discussed later.

#### APPLICATIONS

Classify each of the following as to quantity and quality.

*A wolf is an animal*

*Madonna is a singer*

*No actor is a tycoon*

*Some actor isn't a dancer*

*George Washington isn't an actor*

*Every actor is a dancer*

## 1.2 Logical relations among categorical propositions

Singular propositions: Affirmative and negative singular propositions with the same subject and predicate are contradictories; that is, one of them must be true and the other one false.<sup>6</sup> So given ‘*Socrates is a philosopher*’ and ‘*Socrates isn't a philosopher*’ we know as a matter of logic that one of these is true and the other one false, though we may not know which is which. If Socrates actually exists, then ‘*Socrates is a philosopher*’ is true if and only if that man, Socrates, is actually a philosopher, and ‘*Socrates isn't a philosopher*’ is true if and only if that man, Socrates, is not a philosopher. In case Socrates does not exist, the affirmative form: ‘*Socrates is a philosopher*’ is false, and the negative form: ‘*Socrates isn't a philosopher*’ is true. Since Socrates actually died a long time ago, he does not now exist, and so today ‘*Socrates is a philosopher*’ is false and ‘*Socrates isn't a philosopher*’ is true.<sup>7</sup>

Most medieval theorists assume that every well-formed categorical proposition is either true or false (and not both). (The possible exceptions that were discussed are

<sup>6</sup> In *OI* Aristotle uses the word ‘contradictory’ to mean something like what we would call ‘opposite,’ and he investigates under what conditions pairs of “contradictories” must have one member true and the other false. Sometimes they can be true together, as in ‘a man is white’ and ‘a man isn't white’ (*OI* 17b30). (See Whitaker 1996 for discussion.) But in the ensuing tradition, ‘contradictory’ is taken to mean having opposite truth values. Ammonius, writing as early as 470 CE, says (81.14–16) “the definition of contradiction . . . is a conflict of an affirmation and a negation which always divide the true and the false so that when one of them is false the other is true, and *vice versa*.” Peter of Spain (*LS* 1.14): “The law of contradictories is this, that if one is true, the remaining one is false, and conversely, for they cannot be true or false at the same time with any [subject] matter.”

<sup>7</sup> Aristotle *OI* 17b26 (48): “Of contradictory statements about a universal taken universally it is necessary for one or the other to be true or false; similarly if they are about particulars, e.g. ‘Socrates is white’ and ‘Socrates isn't white.’” (‘particulars’ could as well be translated ‘singulars.’) *Categories* 13b30–34 (37): “For take ‘Socrates is sick’ and ‘Socrates isn't sick’: if he exists it is clear that one or the other of them will be true or false, and equally if he does not; for if he does not exist ‘he is sick’ is false but ‘he isn't sick’ true.”



mostly semantic paradoxes or contingent statements about the future, which are not discussed here.)

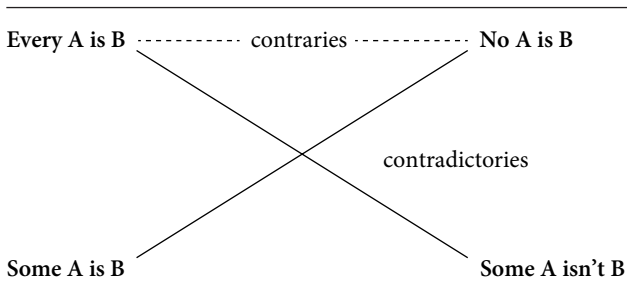
In his essay *On Interpretation* Aristotle states that a universal affirmative proposition is the contradictory of the corresponding particular negative proposition, so ‘*Every man is a philosopher*’ is true if and only if ‘*Some man isn’t a philosopher*’ is false, and vice versa. This is supposed to be apparent, given one’s understanding of what those propositions say. He holds the same for the negative case; the universal negative is the contradictory of the corresponding particular affirmative propositions, so that ‘*No man is a philosopher*’ is true if and only if ‘*Some man is a philosopher*’ is false, and vice versa.

Aristotle also holds that corresponding universal propositions are contraries, meaning that they cannot both be true, although they might both be false. For example, ‘*Every man is a philosopher*’ and ‘*No man is a philosopher*’ cannot both be true, though they can both be false. Most logicians today assume that these propositions are not contraries; they are not contraries because they are both true if there are no humans; they are “vacuously” true. Aristotle did not discuss this possibility, but medieval logicians did, and they concluded that Aristotle was right to take them to be contraries. This is because the universal affirmative proposition ‘*Every man is a philosopher*’ is false if there are no men. We will discuss this further later.

### 1.3 The square of opposition

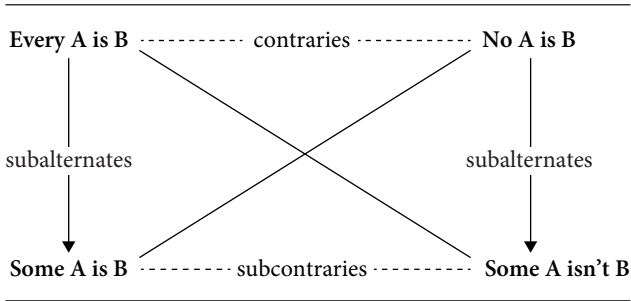
Some commentators on Aristotle found it convenient to use a diagram to keep straight Aristotle’s assumptions about the logic of these propositions. The diagram (see Table 1.3) begins with some of the relationships we have discussed; the universal propositions at the top are contraries, and diagonally opposite propositions are contradictories:

Table 1.3. Aristotle’s contributions to the diagram



The doctrines that Aristotle himself laid out entail others. All of these together were typically encapsulated in a square diagram, today called the “square of opposition” (Table 1.4):

Table 1.4. The square of opposition



Aristotle contributed the top side and the diagonals (and he discussed the bottom at *PA* 2.15, where he calls the bottom propositions “opposites verbally”). The bottom and side relationships follow from the others. In particular, since the propositions at the top are contraries, their contradictories at the bottom are “subcontraries”: they can both be true but their form prevents them from both being false.<sup>8</sup>

Universal and particular affirmative propositions are related by *subalternation*, as are universal and particular negatives: each universal proposition entails the particular directly below it.<sup>9</sup> This is easy to see. For the affirmative case, suppose that ‘*Every A is B*’ is true. Then its contrary, ‘*No A is B*’ must be false. So the contradictory of ‘*No A is B*’, namely, ‘*Some A is B*’ must be true. The reasoning is the same for the negative case: if ‘*No A is B*’ is true, then its contrary, ‘*Every A is B*’, must be false, so the contradictory of ‘*Every A is B*’, which is ‘*Some A isn't B*’, is true.

#### APPLICATIONS

Say how each of the following pairs of propositions are logically related.

*Some wolf is an animal, Some wolf isn't an animal*

*Madonna is a singer, Madonna isn't a singer*

*No actor is a tycoon, Some actor is a tycoon*

*Some actor isn't a dancer, No actor is a dancer*

*George Washington isn't an actor, George Washington is an actor*

*Every actor is a dancer, Some actor isn't a dancer*

<sup>8</sup> “The law of **subcontraries** is this, that if one is false, the remaining one is true, and not conversely, for they can both be true at the same time” (Peter of Spain *LS* 1.14).

<sup>9</sup> “The law of **subalternates** is this, that if the universal is true, the particular is true, and not conversely, for the universal can be false with its particular being true. And if the particular is false, its universal is false, and not conversely” (Peter of Spain *LS* 1.14).

## 1.4 Issues concerning empty terms

There are two issues concerning the relations encoded in the square having to do with the truth values of propositions with empty subject terms.

### 1.4.1 *Universal affirmatives*

The first issue concerns universal affirmative propositions when the subject term is empty; for example, ‘*Every donkey is a mammal*’ when there are no donkeys. Suppose that the term ‘*A*’ is empty.<sup>10</sup> Then ‘*Some A is B*’ is false.<sup>11</sup> According to the principles embodied in the square, ‘*No A is B*’ is its contradictory, and so it must be true. So, by the law of contraries, the universal proposition ‘*Every A is B*’ must be false. This goes against the modern custom whereby a universal affirmative proposition with an empty subject term is considered to be trivially true. This is because the canonical translation of ‘*Every A is B*’ into symbolic logic is ‘ $\forall x(Ax \rightarrow Bx)$ ’, which is true when ‘*A*’ is empty.

Modern students often balk at the proposal that universal affirmatives are true when their subject terms are empty. In response they may be told:

This is a convention which is useful in logic—it makes for theoretical simplicity. Ordinary usage is unclear regarding such propositions with empty subjects. If you think that universal affirmatives are false when their subjects are empty, then you may simply represent them by adding a condition: symbolize them as ‘ $\exists xAx \ \& \ \forall x(Ax \rightarrow Bx)$ .’

It is apparent that one can also adopt the opposite convention, that universal affirmatives are false when their subject term is empty. This is a convention that is convenient for doing logic—it makes for theoretical simplicity (we return to this shortly). If you want to represent ordinary usage in the contemporary way, as ‘ $\forall x(Ax \rightarrow Bx)$ ’, then just write ‘*Every A is B, or no A is A.*’<sup>12</sup>

### 1.4.2 *Particular negatives*

The other issue with the traditional square of opposition concerns particular negatives. Suppose that the term ‘*A*’ is empty. Then ‘*Some A is B*’ is false. So according to the principles embodied in the square, ‘*No A is B*’ is its contradictory, and is thus true. So, by the principle of subalternation, ‘*Some A isn’t B*’ is also true. But to modern ears, ‘*Some A isn’t B*’ should be false if ‘*A*’ is empty. After all, ‘*some A*,’ has scope over the rest of the proposition. What is going on?

This result is built into the diagram in other ways as well. Again, suppose that ‘*A*’ is empty, so that ‘*Some A is B*’ is false. Then its subcontrary, ‘*Some A isn’t B*’ must be true.

<sup>10</sup> By an empty term I mean one that has no individuals falling under it. Aristotle uses ‘goat-stag’ for an example (*Posterior Analytics* 2.7).

<sup>11</sup> This presupposes an “extensional” interpretation of ‘*Some A is B*’; on this understanding the proposition has exactly the truth conditions assumed in modern logic:  $\exists x(Ax \ \& \ Bx)$ . For an interesting and plausible alternative see Malink 2013.

<sup>12</sup> The issue becomes more complicated when propositions become more complex. See section 9.3 for discussion.

Or suppose that ‘*Some A is B*’ is false; then its superalternate ‘*Every A is B*’ is also false; so the contradictory of that, ‘*Some A isn’t B*’, is again true.

Some authors did not notice this result, but many did. Most who noticed held that this is the right result:<sup>13</sup> if ‘*A*’ is empty, then ‘*Some A isn’t B*’ is indeed true. This may not accord well with ordinary speakers of Latin, but logicians insisted that this was the right way to read the proposition. This is part of their regimentation of the language that will be discussed later. It may be defended in the way that any regimentation is defended, by claiming that it is useful for logical purposes, even if it does not conform well to ordinary usage.<sup>14</sup>

Of course, this proposal will not work unless other parts of logic are formulated with this in mind. For example, we do not want to include the validity of ‘*Some A isn’t B ∴ Some A is A*.’ This inference, of course, is not considered valid.

I said that these proposals make for theoretical simplicity, but I didn’t say how. It is this:

Affirmative categorical propositions are false when any of their main terms are empty.  
 Negative categorical propositions are true when any of their main terms are empty.

These principles hold for singular propositions and for the forms of categorical propositions we have discussed; they will hold without exception, even when categorical propositions are expanded far beyond the forms that Aristotle discussed.

#### APPLICATIONS

Classify each of the following as to truth or falsehood. (Assume that there are now no dodos, that Elvis does not exist, and that Madonna exists.)

<i>Some dodo is an animal</i>	<i>Some dodo isn’t an animal</i>
<i>Madonna is a singer</i>	<i>Elvis isn’t a singer</i>
<i>No dodo is an animal</i>	<i>Every dodo is a dodo</i>

<sup>13</sup> Abelard (*D*, p. 170) thought that this is wrong; he held that the particular negative form should be read ‘*Not every A is B*’ instead of ‘*Some A isn’t B*’. He blamed the latter “misreading” on Boethius, who wrote the latter form instead of the former, which Aristotle had used. But Aristotle (*PA* 27a36) uses both forms interchangeably.

<sup>14</sup> I think that this interpretation of Aristotle’s logic is consistent with his writings. Other writers disagree with this. For example, Robin Smith in the *Stanford Encyclopedia of Philosophy* on Aristotle’s Logic says: “Aristotle in effect supposes that *all* terms in syllogisms are non-empty.” (Smith does not give reasons in that work.) This view has a substantial following; it was reached in Kneale and Kneale’s classic 1962: “In order to justify Aristotle’s doctrine as a whole it is necessary, then, to suppose that he assumed application for *all* the general terms with which he dealt” (60). (I am not certain that I follow their reasoning. I think that it relies on only considering the two options: that universals never have existential import, or that universals, both affirmative and negative, always have existential import.)

## 1.5 Conversions

The square of opposition deals with logical relations among propositions which have the same subject and same predicate. There are also other relations. If a proposition is generated from another by interchanging the original subject and predicate, those propositions are candidates for a logical relation of “conversion”:<sup>15</sup>

**Simple conversion:** A proposition is said to *convert simply* if it entails the result of interchanging its subject and predicate terms. Universal Negative and Particular Affirmative propositions convert simply, resulting in equivalent propositions:

‘No A is a B’ converts simply to ‘No B is an A’

‘Some A is a B’ converts simply to ‘Some B is an A.’

**Conversion per Accidens** (Accidental conversion): Whereas simple conversion produces a proposition equivalent to the original, conversion *per accidens* is not symmetric. A universal proposition may be converted *per accidens*: you interchange its subject and predicate terms and change the universal sign to a particular sign (adding a negation, if necessary, to preserve quality). This inference is not reversible.

‘Every A is a B’ converts per accidens to ‘Some B is an A’

‘No A is a B’ converts per accidens to ‘Some B isn’t an A.’<sup>16</sup>

Since the universal changes to a particular, this form of conversion is sometimes called “conversion by limitation.” Aristotle himself discusses *per accidens* conversion of the universal affirmative form; later writers typically include both forms. (Cf. Roger Bacon, *ASL*, para 279.) We discuss proofs of both forms in the next chapter.<sup>17</sup>

### APPLICATIONS

Say which of the following are valid conversions, which are invalid conversions, and which are not conversions at all.

*Some dodo is an animal* ∴ *Some animal is a dodo*

*No dodo is an animal* ∴ *No animal is a dodo*

*Some dodo isn’t an animal* ∴ *Some animal isn’t a dodo*

*Every dodo is an animal* ∴ *Every animal is a dodo*

*Every dodo is an animal* ∴ *Some animal is a dodo*

*Every dodo is an animal* ∴ *Some dodo is an animal*

<sup>15</sup> The conversion laws were proved by Aristotle in *Prior Analytics* I.2 (2–3). They appear (usually without proof) in every major logic text in the Aristotelian tradition, except that conversion per accidens is sometimes dropped from 20th-century texts.

<sup>16</sup> Cf. Peter of Spain, *LS* I.15, (8). Many authors (including Aristotle) did not mention converting the Universal Negative in this way. This conversion is a consequence of other logical relations assumed in the square. Buridan (*SD* I.6.3) argues: “We should note that in this type of conversion even a universal negative is validly converted into a particular negative: ‘No B is A; therefore, some A isn’t B.’ For by simple conversion the following is valid: ‘No B is A; therefore no A is B;’ whence further ‘No A is B; therefore, some A isn’t B’ for a universal implies its subaltern particular; therefore ‘No B is A; therefore, some A isn’t B’ is valid, since whatever follows from the consequent follows from the antecedent.”

<sup>17</sup> An additional mode of conversion—conversion by contraposition—is discussed in section 3.5.

## 1.6 Syllogisms

Syllogisms form the heart of what is today commonly called Aristotelian logic. A syllogism is a special form of argument containing two premises and a conclusion, each of which is a non-singular standard form categorical proposition. There are three distinct terms;<sup>18</sup> one of them, the “middle” term, occurs once in each premise. Each of the other terms occurs once in a premise and once in the conclusion. An example of a syllogism is any argument having the following pattern:

*Every M is P*  
*Some M is S*  
 ∴ *Some S is P*

The first premise is called the *major* premise and the second is called the *minor*. These individual argument patterns are called “moods,”<sup>19</sup> and the moods in turn are classified into three “figures.” The first figure includes moods in which the middle term occurs as subject in the first premise and predicate in the second; in the second figure the middle term is predicate in both; and in the third figure it is the subject in both. Aristotle discusses some of the first figure moods as well as the second and third figure moods in chapters 4–6 of *Prior Analytics I*, and he discusses some additional first figure moods in chapter 7, for a total of 19 good moods.

There are five additional valid patterns that neither he nor many medieval authors mention; these are forms which conclude with a particular proposition when the super-alternate universal proposition is provable from the same premises.<sup>20</sup> An example is:

*Every A is B*  
*Every C is A*  
 ∴ *Some C is B*

The moods are divided into direct and indirect, where a direct mood is one in which the first premise contains the predicate of the conclusion. In the second and the third figures all of the moods that Aristotle discusses are direct. Interchanging the premises in these figures produces indirect moods that are in the same figure. Since interchanging the premises of a mood cannot affect validity, the results of swapping premises are not listed as additional moods (though in the first figure the direct and indirect moods are separately listed). Sometimes only the direct moods are listed in figure 1, and the

<sup>18</sup> Aristotle also discusses cases in which two terms are identified, so there are four occurrences of one term and two of the other.

<sup>19</sup> The term ‘mood’ might better be translated ‘mode,’ but the former term is standard in the tradition.

<sup>20</sup> Aristotle has only 19 moods because he is examining which combinations of premises can yield a valid syllogism, and there are 19 of these. Five of those yield more than one conclusion, and thus there are 24 all told if the pattern includes the form of the conclusion. The tradition typically discusses only the forms with the stronger conclusion. E.g. Arnauld and Nicole 1662, 142: “people have been satisfied with classifying syllogisms only in terms of the nobler conclusion, which is the general. Accordingly they have not counted as a separate type of syllogism the one in which only a particular conclusion is drawn when a general conclusion is warranted.”

Table 1.5. All valid categorical syllogisms

Figure 1 (ch. 4)	Figure 1: indirect (ch. 7)	Figure 2 (ch. 5)	Figure 3 (ch. 6)
<b>Barbara</b> (26a1) Every M is P Every S is M ∴ Every S is P	<b>Baralippton</b> Every M is S Every P is M ∴ Some S is P	<b>Cesare</b> (27a5) No P is M Every S is M ∴ No S is P	<b>Darapti</b> (28a18) Every M is P Every M is S ∴ Some S is P
Every M is P Every S is M ∴ Some S is P	<b>Celantes</b> (29a26) No M is S Every P is M ∴ No S is P	No P is M Every S is M ∴ Some S isn't P	<b>Felapto</b> (28a27*) No M is P Every M is S ∴ Some S isn't P
<b>Celarent</b> (26a3) No M is P Every S is M ∴ No S is P	No M is S Every P is M ∴ Some S isn't P	<b>Camestres</b> (27a8) Every P is M No S is M ∴ No S is P	<b>Disamis</b> (28b7) Some M is P Every M is S ∴ Some S is P
No M is P Every S is M ∴ Some S isn't P	<b>Dabitis</b> (29a26) Every M is S Some P is M ∴ Some S is P	Every P is M No S is M ∴ Some S isn't P	<b>Datisi</b> (28b12*) Every M is P Some M is S ∴ Some S is P
<b>Darii</b> (26a23) Every M is P Some S is M ∴ Some S is P	<b>Fapesmo</b> (29a22) Every M is S No P is M ∴ Some S isn't P	<b>Festino</b> (27a31) No P is M Some S is M ∴ Some S isn't P	<b>Bocardo</b> (28b18*) Some M isn't P Every M is S ∴ Some S isn't P
<b>Ferio</b> (26a25) No M is P Some S is M ∴ Some S isn't P	<b>Frisesororum</b> (29a22) Some M is S No P is M ∴ Some S isn't P	<b>Barocho</b> (27a36) Every P is M Some S isn't M ∴ Some S isn't P	<b>Ferison</b> (28b33) No M is P Some M is S ∴ Some S isn't P

indirect moods of the first figure are said to constitute a fourth figure, but that was not the custom in medieval times.

All 24 valid moods are included in Table 1.5, using medieval names of the moods with Peter of Spain's spelling (*LS* 4.13).<sup>21</sup> (For the significance of these names, see section 2.9.)

Every valid categorical syllogism either has one of these forms explicitly, or it yields one of these forms by transposing the premises. The unnamed moods in the chart were not mentioned by Aristotle. Chapter numbers indicate the chapter of *Prior Analytics I* in which Aristotle discusses the figure. The specific reference codes indicate where in the chapter he discusses the mood.

<sup>21</sup> Some authors say that Aristotle has only 14 valid moods; these are the moods he discussed explicitly in chapters 4–6 of the *Prior Analytics*. Five more are sketched in chapter 7. Medieval authors included all 19. Aristotle also provides counterexamples for all invalid moods, for a complete case-by-case coverage of all possible syllogisms.

The names of the moods encode logically significant information about the mood. These are discussed in section 2.9. But part of the code is already evident here. In each case the first three vowels of the name of the mood indicate the quality and quantity of the premises and conclusion, in order, using the correlation:

- a: universal affirmative
- e: universal negative
- i: particular affirmative
- o: particular negative

For example, *Baroco* has a universal affirmative proposition ('a') as its first premise, and it has particular negative propositions ('o') for its second premise and conclusion. If you know which figure the mood is, and (in the first figure) if you know whether it is direct or indirect, this completely determines the logical form of the mood.

Some of the valid forms of syllogism, such as *Darapti*, rely on universal affirmatives having existential import. None of the 19 traditional syllogisms rely on particular negatives being true when their subject term is empty, although two of the unmentioned moods rely on this (namely, the unmentioned first figure indirect mood and the second of the unmentioned second figure moods).

#### APPLICATIONS

Identify the mood of each of the following syllogisms. (In answering this it will be efficient to first identify the figure of the syllogism by locating the positions of the middle term.)

- Some dodo is a bird*  
*Every dodo is an animal*  
∴ *Some animal is a bird*
- Every dodo is a bird*  
*Some animal is a dodo*  
∴ *Some bird is an animal*
- Every bird is an animal*  
*Every dodo is a bird*  
∴ *Every dodo is an animal*
- No dodo is a bird*  
*Every animal is a bird*  
∴ *No animal is a dodo*
- No dodo is a bird*  
*Some dodo is an animal*  
∴ *Some animal isn't a bird*



## 1.7 Infinite negation

In addition to terms consisting of common nouns or adjectives, Aristotle included what he called “infinite” or “indefinite” terms. Such a term consists of a common noun with the prefix ‘non,’ such as ‘non-donkey.’ Aristotle insisted that such a term is not a noun, although it occurs in exactly the same place in a proposition as a common noun. These terms require special discussion because the same word occurs both as sentential negation and as term negation in Greek, and this is also true in Latin, which uses ‘non’ for both purposes. Conventions of word spacing did not allow one to distinguish between them. So e.g. the Latin sentence:

*Non homo est animal*

is ambiguous between the false proposition ‘Not: [a] man is [an] animal’ and the true proposition ‘[A] non-man is [an] animal.’ Aristotle took pains to distinguish the two kinds of negation with respect to forming contradictories. For example he pointed out that these are four different propositions:

*A man is just*

*A man is not just*

*A man is non-just*

*A man is not non-just*

Later logicians made much of the presence of indefinite terms, proposing special equivalences governing them. These are discussed in section 3.5.

## 1.8 Formal validity

We have used the term ‘valid’ in assessing Aristotle’s syllogisms. He himself did not use such terminology. He merely defined what a deduction is:

A *deduction* is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so. (*PA* 1.1)

Many medieval logicians interpreted this modally, holding that a purported argument is *good* if and only if it is not possible for its premises all to be true when its conclusion is false. Variants of this idea were common. Notice that this says nothing about an argument’s being formal. And indeed, it was common in the medieval period to take the following as a paradigm of a good argument:

*Socrates is a man*

∴ *Socrates is an animal*

But in Aristotle’s systematic development—his theory of syllogisms—he concentrates on *formally good* arguments. A formally good argument remains good if the non-logical terms are replaced by other terms. The argument just given is not formally good

because it does not remain good when ‘animal’ is changed to ‘stone.’ Aristotle’s endorsed syllogisms however are all formally good. For example, this instance of Barbara is good by Aristotle’s definition:

*Every animal is a substance*  
*Every donkey is an animal*  
 $\therefore$  *Every donkey is a substance*

And it will still be good by his definition even if the terms are changed, as in:

*Every stone is a tree*  
*Every monkey is a stone*  
 $\therefore$  *Every monkey is a tree*

It is clear that Aristotle is interested in formal goodness of an argument, since at *PA* 1.4 he says that no argument of the form:

*No C is B*  
*Some B isn’t A*  
 $\therefore$  *Some A is C*

is good because this argument is no good:

*No raven is a horse*  
*Some horse isn’t white*  
 $\therefore$  *Some white thing is a raven*

This certainly does establish that no argument of the first form is *formally* valid, but it does not address whether any such argument might be non-formally valid.

Medieval logicians held a wide variety of views about the nature of validity, and many articulated notions of formal validity. I cannot survey them all here.<sup>22</sup> For my purposes I need a straightforward notion that will allow me to classify principles that are intended to validate a class of resembling arguments as good arguments. I will use this simple modern notion: an argument is formally valid if and only if no matter how its terms and verbs other than the copula are interpreted one never gets all true premises and a false conclusion. All of Aristotle’s conversions and syllogisms are clearly valid in this sense, and many additional principles will also be valid in this sense. Using this notion we can raise the question of whether the available rules of inference capture all valid arguments.

There is a well-known objection to using this notion of validity in modern logic. The objection holds that it classifies certain intuitively invalid arguments as valid. An example is this argument:

$\forall x x=x$       <everything is self-identical>  
 $\therefore \exists x \exists y \sim x=y$       <there are at least two distinct things>

<sup>22</sup> For a brief review see Dutilh Novaes 2008, section 3.3, 474–7.

In this argument there are no terms at all, and the only verb is the copula. Since there are in fact two distinct things, it is vacuously true that no matter how the terms and verbs other than the copula are interpreted, one never gets a true premise and a false conclusion. But the conclusion—that there are at least two things—does not follow intuitively from the premise, since it *could* be that there are not two things. This is an awkwardness for symbolic logic. But it is not a problem for medieval logic. For from a medieval point of view the source of the awkwardness rests on a defect in the notation of symbolic logic. Consider each of the following phrases:

*some donkey*  
*some tree*  
*some thing*

Each of these consist of a formal logical word, ‘*some*,’ and a common noun. The common noun is not a piece of formal logical notation. And when the two words in the last phrase are put together to form the word ‘*something*,’ this should not be a piece of formal logical notation either. The symbolic argument earlier ought to be worded something like:

*for every thing*  $x, x=x \quad \therefore \quad$  *for some thing*  $x, \text{for some thing } y, \sim x=y$

And this is not valid on the substitutional criterion, because the following has the same form, and its conclusion isn’t true:

*for every moon (of earth)*  $x, x=x$   
 $\therefore$  *for some moon (of earth)*  $x, \text{for some moon of (earth)}$   $y, \sim x=y$

For definiteness, throughout this book I will be employing the modern notion, applied to medieval symbolism. The rules of inference that I discuss throughout are valid in this modern sense (and they are also valid in several of the senses employed by medieval logicians).

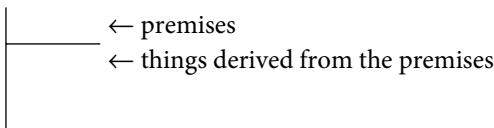
# 2

## Aristotle's Proofs of Conversions and Syllogisms

Early in Aristotle's *Prior Analytics* I 1–7 he proves the conversion principles. Then he assumes that certain syllogistic moods are “perfect,” and he “reduces” all of the remaining valid syllogisms to the perfect ones—that is, he derives the imperfect syllogisms using the conversion principles and the perfect syllogisms. In my opinion, the logical techniques that Aristotle himself uses to prove the conversion principles and derive all syllogisms from the evident ones are of much greater interest than the conversions and syllogisms that he validates. The goal of this chapter is to look in detail at his proofs and to state a very simple system of rules within which the proofs can be carried out formally. These are all rules which were known by medieval logicians.

### 2.1 Formal derivations

Aristotle's proofs are stated informally. To come up with formal analogues I will use a lines-and-bars natural deduction system. In this system a derivation contains a vertical bar with a horizontal bar extending to its right. Any proposition may occur on a line above the main horizontal bar; these are the premises of the derivation. Lines below the bar can only appear if justified by a rule of derivation.



As an illustration, a formal derivation in modern symbolic logic to establish the validity of this argument:

A&B  
B→C  
∴ C

could look like this:

1.	A&B	
2.	B→C	
3.	B	from 1 by Simplification
4.	C	from 2 and 3 by Modus Ponens

Derivations may also contain subderivations. A subderivation is exactly like a derivation except that it contains only one line (the “assumption”) above its horizontal bar.

There are two sorts of rules. One sort lets you infer things from available lines depending on their logical form. (Available lines are previous lines that are not within an already completed subderivation.) Another kind of rule lets you infer things from a subderivation itself. For example, the modern rule of “conditional proof” lets you write ‘A→B’ immediately after any subderivation whose assumption is ‘A’ and whose last line is ‘B.’

A properly constructed derivation shows that an argument is valid if the sentences above the main horizontal line in the derivation are all among the premises of the argument, and the last line of the derivation (1) is the conclusion of the argument and (2) is not within a subderivation and (3) does not contain a name introduced by rule EX discussed shortly.

Aristotle primarily used three principles of proof. One is indirect proof, which he called “reduction to the impossible,” or just “reductio.” The idea is that you can prove a proposition if you assume its contradictory (as an assumption of a subderivation) and then derive something impossible (within that subderivation). For him, you have derived something impossible if you derive some proposition that is the contradictory or the contrary of some other available proposition in the derivation. This rule then lets you immediately follow the subderivation by the contradictory of its assumption.

#### Indirect proof (Reductio):

If there is a subderivation of this form:

P	
A	← a contrary or contradictory of an available line

then immediately following the subderivation one may write

Q

provided that Q is a contradictory of P

For this to work you need a specification of which pairs of propositions are contradictories and which are contraries. These are given in his *On Interpretation* (discussed in section 1.3).

**Contradictories and contraries:**

- ‘Every A is B’ and ‘Some A isn’t B’ are contradictories.
- ‘No A is B’ and ‘Some A is B’ are contradictories.
- ‘Every A is B’ and ‘No A is B’ are contraries.

This technique also permits a trivial version in which the assumption is the only thing in the subderivation. If you assume a proposition, and if *it* is the contradictory or contrary of some available line, then you may follow its subderivation by its contradictory. An example is the following derivation showing the validity of subalternation for universal affirmatives. To show this argument valid:

Every A is B  
 ∴ Some A is B

we can give this very short derivation:

- |           |   |              |              |  |
|-----------|---|--------------|--------------|--|
| 1.        | Every A is B  |              |              |  |
| 2.        | <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">No A is B</td> <td style="padding-left: 10px;">&lt;assumption&gt;</td> </tr> </table> | No A is B    | <assumption> |  |
| No A is B | <assumption>  |              |              |  |
| 3.        | Some A is B   | 1 2 reductio |              |  |

Since the assumption on line 2 is the contrary of the proposition on line 1, you may infer its contradictory, which is the proposition on line 3.

**APPLICATION**

Give a similar proof of the validity of subalternation from any universal negative proposition to the particular negative with the same subject and predicate.

Aristotle also uses a technique that he calls exposition (*ecthesis*); Robin Smith translates this in *PA* 28a24 (9) as “setting-out.” Following Thom 1976, I will call the explicit laying out portion ‘Exposition,’ abbreviated as ‘EX.’ This is a kind of existential instantiation. If there is a proposition of the form ‘some T is P’ then a singular term ‘n’ that has not occurred earlier in the derivation is chosen to “set out” one of the T’s. Two steps are to be produced: one states that n is one of the T’s, and the other says of n that it is P. For example, if ‘some donkey is an animal’ occurs in the derivation, this may be followed by choosing the term ‘d’ to stand for such a donkey; one then enters the lines:

d is a donkey  
 d is an animal

The exposition rule for particular affirmatives is:<sup>1</sup>

**EX (Exposition)**

some T is P  
 $\therefore n$  is T  
 $\therefore n$  is P      where  $n$  is a name that does not already occur in the derivation

A use of exposition is often followed later in the derivation by an analogue of existential generalization. Medieval logicians called this generalization step “Expository Syllogism,” and I will too, abbreviating it as ‘ES.’ The generalization rule has two forms:

**ES (Expository Syllogism)**

$n$ is T	$n$ is T
$n$ is P	$n$ isn't P
$\therefore$ some T is P	$\therefore$ some T isn't P

where  $n$  is any singular term

These two rules will be illustrated in section 2.2. Our task now is to look in detail at Aristotle's own proofs, and to formalize them using no more than the technique of reductio and the two basic rules just stated.

<sup>1</sup> This rule is similar to the well-known “existential instantiation” in modern logic. An example of a use of existential instantiation is this derivation to show the validity of the argument:  $\exists x(Fx \& Gx) \therefore \exists xFx$ :

$\exists x(Fx \& Gx)$	premise
$Fa \& Ga$	existential instantiation
$Fa$	simplification
$\exists xFx$	existential generalization

This rule is variously formulated and it usually comes with several restrictions involving the use of free variables and constraints on the rule of universal generalization due to the presence of existential instantiations. Since we will not be using derivations containing formulas with free variables, and we will not have a rule of universal generalization, we will not need such extra constraints. This is discussed further in section 6.8.

In modern textbooks it is common to use a different rule, often called “existential specification.” With this rule, instead of following an existentially quantified formula with an instance of it, one begins a subproof using that instance as the assumption of the subproof. Then when you have derived what you want in the subproof you end the subproof, immediately following it by a repetition of the sentence on its last line, reclassifying the sentence as now depending logically on the existentially generalized formula. Cf. Mates 1972 (122). I could have used such a technique here; I avoid it primarily in order to avoid the complexity of the additional subproofs.

## 2.2 Proofs of the conversion principles

Here is the part of *Prior Analytics* §2 where Aristotle states and proves the conversion principles. (In this translation a ‘privative’ proposition means a negative one.) Aristotle begins by stating the conversion principles:

[Conversions stated:]

It is necessary for a universal privative premise of belonging to convert with respect to its terms. For instance, if no pleasure is a good, neither will any good be a pleasure.

And the positive premise necessarily converts, though not universally but in part. For instance, if every pleasure is a good, then some good will be a pleasure.

Among the particular premises, the affirmative must convert partially (for if some pleasure is a good, then some good will be a pleasure),

but the privative premise need not (for it is not the case that if man does not belong to some animal, then animal will not belong to some man).

He immediately gives proofs of the principles:

[Conversions proved:]

First, then, let premise AB be universally privative. Now, if A belongs to none of the Bs, then neither will B belong to any of the As. For if it does belong to some (for instance to C), it will not be true that A belongs to none of the Bs, since C is one of the Bs.

And if A belongs to every B, then B will belong to some A. For if it belongs to none, neither will A belong to any B; but it was assumed to belong to every one.

And similarly if the premise is particular: if A belongs to some of the Bs, then necessarily B belongs to some of the As. (For if it belongs to none, then neither will A belong to any of the Bs.)

[Non-conversions:] But if A does not belong to some B, it is not necessary for B also not to belong to some A (for example if B is animal and A man: for man does not belong to every animal, but animal belongs to every man).

Aristotle uses this terminology ‘belongs to’ throughout the *Prior Analytics*, but not much elsewhere. I will follow the general practice of rephrasing this into the more familiar forms, based on the equivalents:

Every A is B	B belongs to every A
No A is B	B belongs to no A
Some A is B	B belongs to some A
Some A isn't B	B does not belong to some A

Filling in some pronominal reference, his text just quoted presents the following three arguments justifying conversions:

*If no B is A, A will not be any B. For if some A (for instance, C) is B, it will not be true that no B is A—since C is one of the B's.*



*If every B is A, then some A will be B. For if no A is B, then no B will be A—but it was assumed that every B is A.*

*If some B is A, necessarily some A is B. For if no A is B, no B will be A.*

We will consider each of these arguments in detail.

### 2.2.1 Conversion of universal negatives

The first argument uses a six-line proof to justify simple conversion of the universal negative form. The argument begins with:

*If no B is A, A will not be any B;*

This identifies the argument to be validated. The proof will be of this form:

1.	No B is A	Premise
6.	No A is B	Conclusion

He goes on: *For if some A . . . is B, it will not be true that no B is A.* This is a part of the argument which leads from the assumption of ‘Some A is B’ to a disproof of ‘No B is A.’ This can be interpreted in different ways, but the most natural one, which fits into the rest of Aristotle’s reasoning, is that this is a use of *reductio ad absurdum*. The technique is to prove ‘No A is B’ by assuming its contradictory, ‘Some A is B,’ and then deriving something absurd from it. The absurdity shows that ‘Some A is B’ is false, so its contradictory, ‘No A is B,’ is true. So the argument has this structure:

1.	No B is A	
2.	Some A is B	← Assumption of the contradictory of ‘No A is B’
5.	???	← Something absurd
6.	No A is B	Reductio from the subproof

The rest of the argument fills in the remaining lines:

*. . . some A (for instance, C) . . . , for C is one of the B’s.*

So we need to fill in the proof with “for instance, C (which is some A)” and “for C is one of the B’s”:

1.	No B is A		
2.	Some A is B		
3.	C is A	“for instance, C (which is some A)”	from 2 by EX
4.	C is B	“C is one of the B’s”	from 2 by EX
5.	???		
6.	No A is B	Reductio from the subproof	

Lines 3 and 4 are an application of Exposition. It is easy to fill in the remainder of the proof, following a pattern that Aristotle uses elsewhere. The point of this subproof is to arrive at some proposition that contradicts or is contrary to something that is already there. In fact, the proposition ‘Some B is A’ follows immediately from the two previous lines by expository syllogism, and it contradicts the original premise:

1.	No B is A		
2.	Some A is B		
3.	C is A	EX from 2	“for instance, C”
4.	C is B	EX from 2	“one of the B’s”
5.	Some B is A	ES from 3,4	
6.	No A is B	Reductio: line 5 contradicts line 1	

This completes the proof. The technique used in steps 3–5 is a common one in modern logic: existentially instantiate line 2 and then existentially generalize 3 and 4 to get step 5.<sup>2</sup>

This particular argument is in fact the most complicated one that Aristotle gives. The remaining proofs are easier because once Aristotle has established a principle he lets himself use it thereafter. So we now have a right to use conversion for universal negatives as a derived principle of inference. The next two arguments are short and easy.

### 2.2.2 Conversion of universal affirmatives (conversion per accidens)

*If every B is A, then some A will be B. For if no A is B, then no B will be A—but it was assumed that every B is A.*

<sup>2</sup> There is some dispute about the interpretation of Aristotle here. I am interpreting Aristotle as introducing a term, C, which is a singular term. This term is then used in a kind of existential generalization. Logically, this makes perfect sense of his reasoning. It is also the interpretation given by various medieval authors, who referred to the generalization step as an “expository syllogism” (cf. Buridan’s QIAPP, Question 6). However, some 20th-century scholars believe that Aristotle did not introduce a singular term at this point; instead, ‘C’ must be a general term. This is not straightforward since there is no quantifier symbol preceding the ‘C’, so lines 3 and 4 appear to be indefinite propositions, and thus equivalent to particulars, in which case line 5 does not follow. So instead, it is sometimes suggested that when Aristotle says ‘for C is one of the Bs’ he actually means ‘for every C is B.’ Then the generalization step is of the form:

Every C is A  
 Every C is B  
 ∴ Some B is A

This interpretation then makes the generalization step have the form of the mood Darapti, which is a form of reasoning that Aristotle takes to be in need of justification. The justification that he gives later for Darapti (we return to this shortly) makes indirect use of the conversion of universal negatives, so his reasoning would be circular. (This argument in favor of interpreting Aristotle as using a singular term here is already spelled out by Alexander of Aphrodisias sometime around 200 CE; see Alexander of Aphrodisias 1991 32,34–34,3, pp. 88–9.) This does not exhaust the options, but exploring others would introduce a lengthy digression. For a discussion of these issues see Smith 1982, especially section 4. For a very interesting alternative view see Malink 2013, which may be a more accurate account of Aristotle’s intent, though one which conflicts with medieval principles.

The initial part again announces what is to be proved:

- |    |              |
|----|--------------|
| 1. | Every B is A |
|    |              |
| 4. | Some A is B  |

The next part is: *for if no A is B, then no B will be A*. Clearly a subproof is here being offered which begins with the assumption ‘No A is B’ and continues on to ‘No B is A.’ In fact, that inference is conversion for universal negatives, which has just been proved. So we have as the argument:

- |    |              |                      |
|----|--------------|----------------------|
| 1. | Every B is A |                      |
| 2. | No A is B    |                      |
| 3. | No B is A    | Conversion of line 2 |
| 4. | Some A is B  |                      |

The next step is

*but it was assumed that every B is A.*

The ‘but’ suggests some kind of opposition of some line to ‘Every B is A,’ which is the initial premise. In fact, it is apparent that line 3 is contrary to the premise on line 1. This short proof is *already* in the form of a completed reductio:

- |    |              |  |
|----|--------------|--|
| 1. | Every B is A |  |
| 2. | No A is B    |  |
| 3. | No B is A    | Conversion of line 2                   |
| 4. | Some A is B  | Reductio; line 3 is contrary to line 1 |

(You might wonder why conversion *per accidens* is so easy to prove, since it is nowadays not considered legitimate to infer ‘Some A is B’ from ‘Every B is A.’ The key assumption that Aristotle makes is that the universal affirmative and universal negative forms are contraries. Whether this is coherent is discussed in section 1.4.)

### 2.2.3 Conversion of particular affirmatives

*If some B is A, necessarily some A is B. For if no A is B, no B will be A.*

As usual, he first indicates the task, which is to validate this argument:

- |    |             |
|----|-------------|
| 1. | Some B is A |
|    |             |
| 4. | Some A is B |

Then we have: *For if no A is B, no B will be A.* The proof is so terse that we expect a simple structure, somehow involving ‘no A is B’ and ‘no B is A’. I suggest that he is here using *reductio*, and the structure is to assume ‘No A is B’ and derive ‘No B is A’:

- |    |             |   |
|----|-------------|---|
| 1. | Some B is A |   |
| 2. | No A is B   |   |
| 3. | No B is A   | Conversion of the previous line                     |
| 4. | Some A is B | Reductio: the previous line contradicts the premise |

This is trivial from the previously proved conversion for universal negatives. This conversion is so salient that it would be natural for Aristotle to use it without needing to restate it.<sup>3</sup>

So Aristotle validates the conversion principles using three simple rules of reasoning. He also shows that conversion is *not* valid for particular negatives, by giving a counter-example. Let B = Animal and A = Man. Then this argument has a true premise and a false conclusion:

*Some animal isn't [a] man / ∴ Some man isn't [an] animal.*

Oddly, Aristotle does not discuss conversion *per accidens* for universal negative propositions, even though this principle can be established as easily as the others. An example of a proof would be:

*If no B is A, then some A will not be B. For if no B is A, no A is B, so it can't be that every A is B.*

- |    |                |  |
|----|----------------|--|
| 1. | No B is A      |  |
| 2. | Every A is B   |  |
| 3. | No A is B      | Conversion of premise                  |
| 4. | Some A isn't B | Reductio: line 3 is contrary to line 2 |

This is a principle that is often unmentioned in the tradition.<sup>4</sup>

We now have five principles that can be appealed to in further proofs; the fundamental ones:

**Reductio**      **Exposition (EX)**      **Expository syllogism (ES)**

and two derived ones:

**Simple conversion (two forms)**      **Conversion *per accidens***

(Subalternation is not included here because Aristotle does not make use of it in this context.)

<sup>3</sup> People sometimes ask: “Didn’t Aristotle already prove conversion for particular affirmatives in the sub-proof in the middle of his proof of conversion for universal negatives?” Yes, I think so. So he did not use an optimal strategy in these proofs; he could have proved conversion for particular affirmatives first, in a simple proof, and then easily proved conversion for universal negatives from that. My job here is to explain Aristotle’s reasoning, not to improve on it.

<sup>4</sup> But it is mentioned in several places, e.g. in Paul of Venice *LM* (123).

## 2.3 Reduction of all syllogisms to perfect ones

Chapter 3 of *Prior Analytics* I concerns modal propositions, which are not discussed here. In chapters 4–7 Aristotle turns to the assertoric syllogisms.

### 2.3.1 Figure 1 syllogisms

Aristotle begins in chapter 4 with the first figure direct moods: moods in which the first premise contains the predicate of the conclusion as its own predicate, and the second premise has the subject of the conclusion as its own subject. He recognizes four good moods within this figure. Table 2.1 states them, using their medieval names.

Table 2.1. Figure 1 from *Prior Analytics*, chapter 4

<b>Barbara</b> (26a1)	Every M is P Every S is M ∴ Every S is P	<b>Darii</b> (26a23)	Every M is P Some S is M ∴ Some S is P
<b>Celarent</b> (26a3)	No M is P Every S is M ∴ No S is P	<b>Ferio</b> (26a25)	No M is P Some S is M ∴ Some S isn't P

Aristotle's argument for the goodness of these moods is simple: no argument is needed because they are *perfect* (sometimes translated “complete”).<sup>5</sup> A perfect syllogism is defined to be one whose goodness is *evident* in itself (*PA* 1.1), and these moods are all evident. (He offers no argument that these particular forms are evident. That, I imagine, is supposed to be evident.)

The perfect syllogisms thereafter are taken to justify other forms of syllogisms. Thus, for Aristotle's purposes the rules stated at the end of the last section may be expanded by adding the four first figure moods.

Aristotle also gives counterexamples to some first figure bad moods. A form of argument that is not a (good) deduction is one that can have true premises and a false conclusion. Aristotle shows that several first figure moods are not deductions; he does this by providing counterexamples to them.

There are two valid first figure moods that Aristotle does not discuss. Since Aristotle's goal seems only to be to find which premise pairs lead to valid syllogisms,

<sup>5</sup> In spite of the fact that no argument is needed, some commentators think that Aristotle did provide an argument, or at least he stated a semantic principle that supports these moods, when he says at *PA* 1.1 24b28–32 (2):

For one thing to be in another as a whole is the same as for one thing to be predicated of every one of another. We use the expression ‘predicated of every’ when none of the subject can be taken of which the other term cannot be said, and we use ‘predicated of none’ likewise.

One way to read this is that it is simply giving the equivalence ‘Every A is B iff no thing which is A isn't B.’ On this interpretation it discusses a relation between two terms, and it's hard to see how it can validate a syllogism involving three terms. But it can also be read ‘Every A is B iff there is no term C which is part of A that B is not said of’ and this can be viewed as a kind of meta-validation of Barbara and Darii. Viewed in this way it is traditionally called (including the negative clause) “the dictum of all and none,” or, in Latin the “*dici de omni et de nullo*.”

if multiple conclusions are possible, he only discusses one of them. The unmentioned moods can easily be validated using the forms that he accepts as perfect. For example, this mood:

Every M is P  
 Every S is M  
 $\therefore$  Some S is P

can be validated as follows:

- |    |              |   |
|----|--------------|---|
| 1. | Every M is P |   |
| 2. | Every S is M |   |
|    |              |   |
| 3. | No S is P    |   |
|    |              |   |
| 4. | Every S is P | from the first two premises by Barbara                    |
| 5. | Some S is P  | Reductio: the previous line is contrary to the assumption |

These forms played little role in the medieval period.

#### APPLICATIONS

Give a similar proof of the validity of the other first figure form that Aristotle does not discuss, namely:

No M is P  
 Every S is M  
 $\therefore$  Some S isn't P

Show by giving counterexamples that these first figure moods are not valid.

Some M is P  
 Some S isn't M  
 $\therefore$  Every S is P

No M is P  
 Some S is M  
 $\therefore$  No S is P

### 2.3.2 Reduction to the first figure

In sections 5–6 Aristotle goes on to discuss the second and third figures of syllogism. He thinks that these are not perfect, but they may be “made perfect” (PA 28a5) by *reducing* them to first figure syllogisms. A reduction goes as follows: You state the premises of the mood in question, and then you argue to its conclusion using the logical techniques that we have indicated earlier (the rules and, usually, the first figure direct syllogisms).

It is far from obvious that this can be done, and it is neat that he does it simply and straightforwardly.

What will reduction accomplish? This is not made clear. It does *not* turn an imperfect syllogism into a perfect one. That is impossible; an imperfect syllogism is imperfect precisely because it is not evident *on its own*. No argumentation will change this. The reduction does not make a mood perfect, but it establishes a kind of trust-worthiness for it. In particular, after the reduction it is known that whenever an inference has the form of the imperfect mood in question, one can get from its premises to its conclusion using an argument that makes appeal only to a perfect syllogism (and to the three basic rules of argumentation plus the others that have been previously justified).

### 2.3.3 Figure 2 syllogisms

Table 2.2 shows the good figure 2 syllogisms that Aristotle discusses.

Table 2.2. Figure 2 from *Prior Analytics*, chapter 5

<b>Cesare</b> (27a5)	<b>Festino</b> (27a31)
No P is M	No P is M
Every S is M	Some S is M
∴ No S is P	∴ Some S isn't P
<b>Camestres</b> (27a8)	<b>Baroco</b> (27a36)
Every P is M	Every P is M
No S is M	Some S isn't M
∴ No S is P	∴ Some S isn't P

The proofs that Aristotle gives for the figure 2 and figure 3 syllogisms mostly employ the conversion principles together with first figure forms. I will consider two of the second figure syllogisms as examples. The first is Cesare. Aristotle's argument for Cesare is:

**Cesare:** *Let no N be M, but every X be M. Then, since the privative converts, no M will be N; but every X was assumed to be M, so that no X will be N (this has been proved earlier).*<sup>6</sup>

What is to be shown (Cesare) is:

- |       |              |
|-------|--------------|
| 1.    | No N is M    |
| 2.    | Every X is M |
| <hr/> |              |
| 4.    | No X is N    |

The argument goes: *Then, since the privative converts, no M will be N.* This adds a line by conversion:

<sup>6</sup> As usual, I have reworded this in the standard way. A literal translation goes: *Let M be predicated of no N, but of every X. Then, since the privative converts, N will belong to no M. But M was assumed to belong to every X, so that N belongs to no X (for this has been proved earlier) (27a5–9).*

- |    |              |                                 |
|----|--------------|---------------------------------|
| 1. | No N is M    |                                 |
| 2. | Every X is M |                                 |
|    |              |                                 |
| 3. | No M is N    | Conversion of the first premise |
| 4. | No X is N    |                                 |

The argument continues: *but every X was assumed to be M, so that no X will be N (for this has been proved earlier)*. This fleshes out the reasoning (without adding any lines at all), by pointing out that the perfect mood Celarent justifies the last line:

- |    |              |  |
|----|--------------|--|
| 1. | No N is M    |  |
| 2. | Every X is M |  |
|    |              |  |
| 3. | No M is N    | Conversion of first premise                              |
| 4. | No X is N    | CELARENT: from the previous line plus the second premise |

Several of the moods have proofs that are this brief and straightforward—not using any subproofs at all. Two, however, require the use of reductio. One of these is Barocho:

#### 2.3.4 Barocho

Two arguments are given. The first argument is:

*If every N is M, but some X isn't M, it is necessary for some X not to be N. For if every X is N, and every N is M, every X must be M; but it was assumed that some X isn't M.*

The first sentence states what is to be shown:

- |    |                |
|----|----------------|
| 1. | Every N is M   |
| 2. | Some X isn't M |
|    |                |
| 5. | Some X isn't N |

Then we have: *for if every X is N, and every N is M, every X must be M; but we assumed that some X isn't M*. This must be a reductio within the main argument:

- |              |  |              |
|--------------|--|--------------|
| 1.           | Every N is M   |              |
| 2.           | Some X isn't M   |              |
|              |  |              |
| 3.           | <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Every X is N</td> </tr> </table> | Every X is N |
| Every X is N |  |              |
|              |  |              |
| 5.           | Some X isn't N   |              |

Barbara supplies what is needed:

- |              |  |                                     |                            |
|--------------|--|-------------------------------------|----------------------------|
| 1.           | Every N is M   |                                     |                            |
| 2.           | Some X isn't M   |                                     |                            |
|              |  |                                     |                            |
| 3.           | <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Every X is N</td> </tr> </table> | Every X is N                        |                            |
| Every X is N |  |                                     |                            |
|              |  |                                     |                            |
| 4.           | <table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 10px;">Every X is M</td> </tr> </table> | Every X is M                        | BARBARA from lines 1 and 3 |
| Every X is M |  |                                     |                            |
| 5.           | Some X isn't N   | Reductio; line 4 contradicts line 2 |                            |



Aristotle also gives a second argument: *And if every N is M but not every O is M there will be a deduction that not every O is N: the demonstration is the same.* This is the same argument for *another* form of Baroco, in which the wording 'not every X is Y' is used instead of 'some X is not Y'. This seems to indicate that Aristotle holds these two forms ('Some X isn't Y' and 'Not every X is Y') to be interchangeable.

## APPLICATION

"Reduce" the second figure moods not discussed here, namely:

Every P is M	<Camestres>
No S is M	
∴ No S is P	
No P is M	<Festino>
Some S is M	
∴ Some S isn't P	

## 2.3.5 Figure 3 syllogisms

Table 2.3 shows the good figure 3 syllogisms that Aristotle discusses.

Table 2.3. Figure 3 from *Prior Analytics*, chapter 6

<b>Darapti</b> (28a18)	<b>Datisi</b> (28b12*)
Every M is P	Every M is P
Every M is S	Some M is S
∴ Some S is P	∴ Some S is P
<b>Felapto</b> (28a27*)	<b>Bocardo</b> (28b18*)
No M is P	Some M isn't P
Every M is S	Every M is S
∴ Some S isn't P	∴ Some S isn't P
<b>Disamis</b> (28b7)	<b>Ferison</b> (28b33)
Some M is P	No M is P
Every M is S	Some M is S
∴ Some S is P	∴ Some S isn't P

Various moods of figure 3 can be reduced as before. But new options also arise. For example, Aristotle reduces Darapti by means of a very simple proof (using only conversion *per accidens* and Darii). Then he goes on to remark that Darapti can also be proved by reductio, and by exposition. He gives the following proof with exposition:

*For if both every S is P and every S is R, if some one of the S's be chosen (for instance N), this will be both P and R, consequently some R will be P.*

This seems to produce the following form of proof:

1.	Every S is P	
2.	Every S is R	
3.	N is S	By ??? “for instance”
4.	N is P	By ???
5.	N is R	By ???
6.	Some R is P	ES from the last two lines

Two new forms of argument are used here, without comment. One is an analogue of Exposition but applied to a universal affirmative proposition. This is clearly valid, since exposition is applicable to the corresponding particular affirmative, which can be derived from the universal affirmative by an application of conversion *per accidens* followed by an application of simple conversion:

Every S is P	
Some P is S	from previous line by conversion <i>per accidens</i>
Some S is P	from previous line by simple conversion

I will thus assume that it is a legitimate shortcut to apply exposition directly to a universal proposition, as Aristotle does here.

1.	Every S is P	
2.	Every S is R	
3.	N is S	EX for universal affirmative, from the first premise
4.	N is P	EX for universal affirmative, from the first premise
5.	N is R	By ???
6.	Some R is P	ES from the last two lines

But then how do we derive ‘N is R’? It appears to me that Aristotle is here using that most famous of valid arguments, epitomized by:

*Every man is mortal*  
*Socrates is a man*  
*Therefore, Socrates is mortal.*

It is odd that this famous argument form has no name in either traditional (“Aristotelian”) logic or in modern logic. In modern logic the first premise would be a quantified conditional, and the argument can be justified by universal instantiation followed by modus ponens. In Aristotle’s logic, it is easy to derive from principles we are already using. Let me call the principle we are discussing “universal application”:

**Rule UA (Universal Application):**

Positive form: From ‘n is X’ and ‘Every X is Y’ infer ‘n is Y’  
 Negative form: From ‘n is X’ and ‘No X is Y’ infer ‘n isn’t Y’

(I use small letter 'n' here to indicate that this rule applies to singular terms. There were no small letters in Aristotle's time.)

These forms of Universal Application can be easily derived from our basic rules as follows:

1.	Every X is Y	
2.	n is X	
3.	n isn't Y	
4.	Some X isn't Y	2 3 ES
5.	n is Y	reductio: line 4 contradicts line 1.

1.	No X is Y	
2.	n is X	
3.	n is Y	
4.	Some X is Y	2 3 ES
5.	n isn't Y	reductio: line 4 contradicts line 1.

So Universal Application is another derived rule that we can use. So the argument for **Darapti**, using rule UA, takes the form:

1.	Every S is P	
2.	Every S is R	
3.	N is S	1 EX
4.	N is P	1 EX
5.	N is R	2 3 UA
6.	Some R is P	4 5 ES

Hereafter I will assume UA as an available rule.

Something interesting happens in the proof of **Bocardo**. Aristotle gives a reduction by reductio, but then he says that reductio is unnecessary if you use exposition: "This can also be proved without the leading-away, if some one of the Ss should be chosen to which P does not belong." This alternative proof would have the form:

1.	Every S is R	
2.	Some S isn't P	
3.	X is S	2 EX
4.	X isn't P	2 EX
5.	X is R	1 3 UA
6.	Some R isn't P	4 5 ES

Now this is an interesting move, for it appears to apply exposition to a particular negative proposition. And this is not in general valid. Suppose, for example, that there are no

As. Then, as we saw in section 1.4, the negative proposition ‘Some A isn’t B’ is true. If we could apply exposition to this proposition, then the following derivation would be good:

1.	Some A isn’t B	
2.	n is A	1 EX
3.	n isn’t B	1 EX
4.	Some A is A	2,2 ES (line 2 is used twice)

And if there are no As, the premise is true and the last line is false.

What then is going on? It is possible, of course, that Aristotle just made a mistake, overlooking the problem of existential import. I don’t know of any way to disprove this interpretation. However, other alternatives are worth considering. An obvious option to note is that exposition is indeed valid from a particular negative if there is some independent way to establish that its subject is not empty. And in the example under discussion that is the case. For the argument has two premises, and the first premise is a universal affirmative proposition which has the term in question as subject. So let us consider this more general version of exposition:

**EX (Exposition)**

some T is P	some T isn’t P
∴ n is T	∴ n is T
∴ n is P	∴ n isn’t P

where **n** is a name that does not already occur in the derivation, and where ‘**T**’ occurs affirmatively in an available affirmative proposition.

(For the time being we can say that any main term<sup>7</sup> of an affirmative proposition occurs affirmatively in it.)

In justifying a use of this rule one must cite both the line which is being instantiated and the line on which ‘**T**’ occurs affirmatively.

In the affirmative version of this rule, the premise ‘Some T is P’ itself is already an affirmative proposition in which ‘T’ occurs as a main term, so no additional premise is necessary. When the premise is not affirmative (‘Some T isn’t P’), there must be some available affirmative proposition containing ‘T’ as a main term.

With this revised rule, the proof of Bocardo is correct as Aristotle gives it.

I am not aware of any discussion of this revised version of the rule, ancient or medieval.

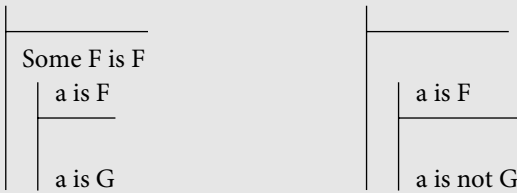
<sup>7</sup> So far, all terms are main terms, so the word ‘main’ is redundant here. In later chapters we consider propositions of more complex forms in which there may be terms which occur within other complex terms. An example is ‘B’ in ‘Some A which is not B is C.’ That complex proposition is affirmative, and it contains ‘B,’ but not as a main term. Its truth does not establish that ‘B’ is not empty.

APPLICATIONS

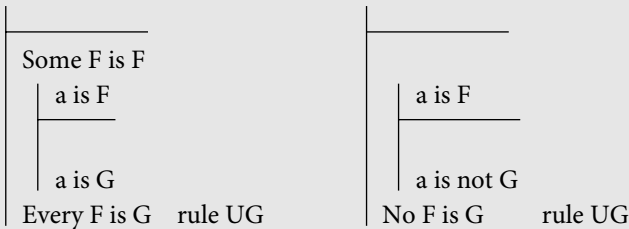
The rule Universal Application is similar to the modern rule of universal instantiation. In modern logic much use is made of another universal quantifier rule: universal generalization. In Aristotle's framework the effect of this rule can be accomplished in a slightly roundabout fashion.

**Universal Generalization (derived rule)**

Given a derivation of either of these forms:



where the name 'a' does not occur in the derivation preceding the subderivation, one may infer the appropriate universal generalization:



Show that one can get the effect of this rule UG using our already existing rules. (The derivation will use reductio and EX and ES, and will rely on the fact that if 'a' does not already occur before the subderivation then any subderivation using a non-occurring name other than 'a' is equally good.)

2.3.6 First figure indirect moods

Table 2.4 shows the good figure 1 indirect moods in chapter 7 of Aristotle's *Prior Analytics*.

Table 2.4. Figure 1 indirect moods from *Prior Analytics*, chapter 7

<p><b>Baralipton</b>                  Every M is S                  Every P is M                  ∴ Some S is P</p>	<p><b>Dabitis</b>                  Every M is S                  Some P is M                  ∴ Some S is P</p>	<p><b>Frisesororum (29a22)</b>                  Some M is S                  No P is M                  ∴ Some S isn't P</p>
<p><b>Celantes</b>                  No M is S                  Every P is M                  ∴ No S is P</p>	<p><b>Fapesmo (29a22)</b>                  Every M is S                  No P is M                  ∴ Some S isn't P</p>	

In chapter 7, Aristotle states Fapesmo and Frisesomorum without proof. He then states that “a deduction always results by means of conversion.” Presumably this means that given any valid mood, if you convert something in it the resulting mood is also valid. By doing this you can generate Fapesmo from Felapto and Frisesomorum from Ferison. Undiscussed consequences also include: that Baralipton comes from Barbara, Celantes from Cesare, and Dabitis from Datisi. This completes the good figure 1 indirect moods.

#### APPLICATIONS

Reduce the following first figure indirect moods to first figure direct moods:

**Fapesmo**  
**Baralipton**  
**Dabitis**

In chapter 7, Aristotle notes that the universal second figure moods have all been reduced to the direct first figure universal moods (the ones we are calling Barbara and Celarent). He then shows that the direct first figure particular moods can be reduced to universal second figure moods. The result of this is that all of the moods are reducible, either directly or indirectly, to the direct universal moods of the first figure.

#### APPLICATIONS

Reduce the first figure particular moods to universal second figure moods:

**Darii**  
**Ferio**

## 2.4 Proving the first figure syllogisms

Aristotle has been criticized for choosing the first figure moods as basic rules; many people find some of the other moods just as evident as the first figure ones. What is surprising is that he needn't have chosen any such rules at all. This is because the same resources that allow Aristotle to prove the conversion rules and to reduce all moods to first figure moods suffice for proving the first figure moods as well. He needn't have chosen any evident syllogisms at all.<sup>8</sup> Proofs to show this are:

<sup>8</sup> This is shown in a somewhat different and elegant form in Thom 1976. He notices that Aristotle gives proofs of Bocardo and of Datisi that do not employ the first figure syllogisms. Then he points out that Barbara may be reduced to Bocardo and Celarent to Datisi.

BARBARA

1.	Every B is A			
2.	Every C is B			
3.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Some C isn't A</td> <td></td> </tr> </table>	Some C isn't A		
Some C isn't A				
4.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c is C</td> <td style="padding-left: 5px;">3 2 EX</td> </tr> </table>	c is C	3 2 EX	
c is C	3 2 EX			
5.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c isn't A</td> <td style="padding-left: 5px;">3 2 EX</td> </tr> </table>	c isn't A	3 2 EX	
c isn't A	3 2 EX			
6.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c is B</td> <td style="padding-left: 5px;">4 2 UA</td> </tr> </table>	c is B	4 2 UA	
c is B	4 2 UA			
7.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Some B isn't A</td> <td style="padding-left: 5px;">5 6 ES</td> </tr> </table>	Some B isn't A	5 6 ES	
Some B isn't A	5 6 ES			
8.	Every C is A	reductio: 7 contradicts 1		

(In steps 4 and 5 line 2 provides the existential import needed to apply EX to line 3.)

CELARENT

1.	No B is A			
2.	Every C is B			
3.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Some C is A</td> <td></td> </tr> </table>	Some C is A		
Some C is A				
4.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c is C</td> <td style="padding-left: 5px;">3 3 EX</td> </tr> </table>	c is C	3 3 EX	
c is C	3 3 EX			
5.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c is A</td> <td style="padding-left: 5px;">3 3 EX</td> </tr> </table>	c is A	3 3 EX	
c is A	3 3 EX			
6.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">c is B</td> <td style="padding-left: 5px;">2 4 UA</td> </tr> </table>	c is B	2 4 UA	
c is B	2 4 UA			
7.	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">Some B is A</td> <td style="padding-left: 5px;">5 6 ES</td> </tr> </table>	Some B is A	5 6 ES	
Some B is A	5 6 ES			
8.	No C is A	reductio: 7 contradicts 1		

Of course, the fact that the first figure moods can be proved in this way might mean little to Aristotle, since he already regards those moods as perfect.

APPLICATIONS

Give similar derivations to directly validate the other two first figure moods, namely:

- Darii**
- Ferio**

## 2.5 Propositions with repeated terms

Aristotle mentions only briefly propositions in which the subject and predicate terms are the same (e.g. at *PA* 2.22, 68a19). If these are allowed, it is well known that a natural additional principle becomes valid:

Some A is A  
 ∴ Every A is A

The proposition ‘Every A is A’ is not provable by itself because it is false when ‘A’ is empty. The particular proposition ‘Some A is A’ rules out emptiness.

This inference is provable by *reductio* using exposition applied to the O form:

1.	Some A is A	
2.	Some A isn’t A	
3.	c is A	1 2 EX
4.	c isn’t A	1 2 EX
5.	Every A is A	reductio: 4 contradicts 3

In this derivation the only role played by the premise is to legitimize the use of Exposition applied to the particular negative form.

## 2.6 The indispensability of exposition and expository syllogism

Typically, “Aristotelian” logic is understood as follows: The first figure direct syllogisms are taken for granted. Then, using the conversion principles, all of the remaining syllogisms are validated. However, as we have seen, Aristotle did not take the conversion principles for granted; he proved them, using *reductio* and exposition and expository syllogism. Since exposition and expository syllogism are often not discussed at all, one naturally asks whether they can be dispensed with. The simple answer is “no.” That is, if exposition and expository syllogism are eliminated from the resources we have with which to validate arguments, and if they are not replaced by some other principles not already present, Aristotle’s project could not be carried out. That is, the project of proving the conversion principles *and* all of the syllogistic moods assuming only the first figure direct syllogisms (and using the *reductio* technique) cannot succeed. This is because the principles of exposition and expository syllogism are vital to proving the principles of simple conversion. This is easy to show. Suppose that we reinterpret the truth conditions for universal negative propositions so as to require existential import for the subject term. That is, a proposition of the form ‘No A is B’ is to be true if and only if something is an A and nothing is both A and B. Make a corresponding change in the truth conditions for particular affirmative propositions so that they remain the contradictories of universal negatives; that is, ‘Some A is B’ is to be true either if there are no As, or at least one thing is both A and B. Leave the truth conditions for universal affirmatives as they are, and likewise for particular negative propositions. Then all of the relations in the square of opposition remain unchanged. Diagonally opposite propositions are still contradictories, universal affirmative and universal negative propositions are still contraries, and each universal proposition still entails the particular proposition directly below it. Accidental conversion remains valid, since that depends only on the universal



propositions being contraries (assuming that diagonally opposite propositions are contradictories).<sup>9</sup> But simple conversion for particular affirmatives and universal negatives fails, for 'Some chimera is a stone' is true but 'Some stone is a chimera' is false; likewise for 'No stone is a chimera' and 'No chimera is a stone.' Without exposition and expository syllogism, the failure of these conversions cannot be disproved. All of the first figure syllogisms remain valid, and could be assumed as Aristotle did. However, some of the other syllogisms would fail; for example, this instance of Celantes:

No substance is a chimera  
 Every stone is a substance.  
 So no chimera is a stone.                      <false because the subject is empty>

So without exposition and expository syllogism Aristotle's project cannot be carried out.

Now this claim flies in the face of an old tradition according to which simple conversion can be proved using only first figure (direct) syllogistic principles. The argument is given by Alexander of Aphrodisias (1991, 90):

suppose that A holds of no B, and—if it does not convert—let B hold of some A. We get, in the first figure, that A does not hold of some A—which is absurd.

The argument is a simple reductio:

1.	No B is A	
2.	Some A is B	hypothesis for reductio
3.	Some A isn't A	1 2 Ferio
4.	No A is B	Reductio; line 3 is absurd

The catch is simple: line 3 is not absurd; it is true when A is empty. This is so on Aristotle's own account, as discussed in section 1.4.2. And this is exactly the case in which 'No A is B' and 'No B is A' can diverge in truth value on the variant model proposed earlier. So this traditional argument designed to avoid assuming exposition and expository syllogism fails.

So without exposition and expository syllogism, nothing forces simple conversion to hold, and without simple conversion nothing forces some of the imperfect syllogisms to hold. So those principles are essential to Aristotle's project. And as we will see throughout this book, they are essential to having a medieval logic that can ground its widely held assumptions in simple basic rules.

<sup>9</sup> Actually, the proof that Aristotle gives also uses simple conversion, which is no longer valid. What is needed is the fact that 'Every A is B' and 'No B is A' are contraries. They are indeed contraries using the reinterpreted truth conditions, but this assumption was not one of Aristotle's original ones. So there would also be a problem *proving* accidental conversion on these conditions, though it would be valid. But this is not relevant to the point under discussion, which is that simple conversion no longer holds.

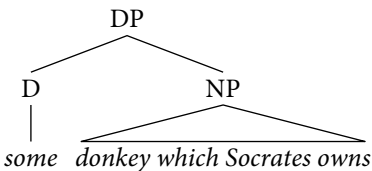
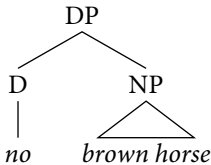
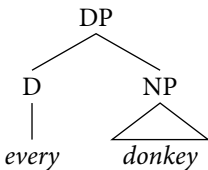
## 2.7 A contemporary assessment of Aristotle's basic theory

(This section may be skipped without loss of comprehension.)

Aristotle has been criticized for focusing on a particularly limited and arbitrary set of sentences with which to develop logical principles: 'Every A is B,' 'No A is B,' 'Some A is B,' 'Some A isn't B.' He has also been criticized for selecting the four forms mentioned previously: Barbara, Celarent, Darii, and Ferio as axiomatic (i.e. for choosing to use them as accepted principles from which to derive the others). I think instead that the sentence forms that he focuses on are central and important, and his choice of axiomatic principles is (mostly) not arbitrary at all. To make this case I turn briefly to a theory from contemporary linguistics, the theory of generalized quantifiers.

### 2.7.1 Generalized quantifiers<sup>10</sup>

Generalized quantifiers are determiner phrases of natural language, phrases like 'every donkey,' 'no brown horse,' 'some donkey which Socrates owns,' and so on. Structurally, they consist of a determiner (D) and a noun phrase (NP):



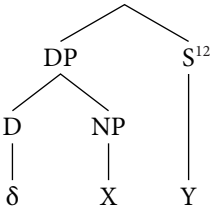
Elsewhere in this book I call these phrases "denoting phrases," using a term from the philosophy of language. Determiner phrases are the same as denoting phrases.<sup>11</sup> In the

<sup>10</sup> For background on generalized quantifiers, see e.g. Keenan 1996.

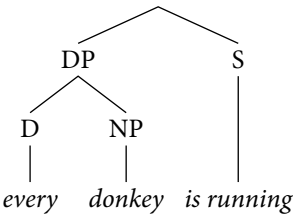
<sup>11</sup> In the philosophical tradition it is unclear whether or not proper names are denoting phrases; likewise, it is unclear in the linguistic tradition whether proper names are determiner phrases. I use both 'denoting phrase' and 'determiner phrase' so as to include proper names.

present section I call them determiner phrases because the theory to be discussed comes from modern linguistics, in which this terminology is used.

According to generalized quantifier theory, a determiner stands for a relation between two classes. A sentence having a DP with scope over the rest of the sentence is true if the relation that the determiner stands for relates the class of things picked out by its NP to the class of things picked out by the rest of the sentence. That is:



is true when the relation that  $\delta$  stands for relates the class of things that X stands for to the class of things that Y stands for. For example, this sentence:



is true when the relation that ‘every’ stands for relates the class of donkeys to the class of things that are running. The relation that ‘every’ stands for is in fact the relation that relates a class X (here, the donkeys) to a class Y (here, the things that run) if and only if X is a subset of Y. So the sentence:

*every donkey is running*

is true when the class of donkeys is a subset of the class of running things. And that seems to be right. (On Aristotle’s view, adopted by the medievals, the sentence is true when the class of donkeys is *non-empty* and is a subset of the class of running things. This difference between the ancient and modern theory is discussed in section 2.7.3.)

The three determiners that Aristotle discusses stand (according to modern theory) for these relations:

<i>every</i>	relates X to Y	when	X is a subset of Y	$X \subseteq Y$
<i>some</i>	relates X to Y	when	X and Y overlap	$X \cap Y \neq \emptyset$
<i>no</i>	relates X to Y	when	X and Y are disjoint	$X \cap Y = \emptyset$

<sup>12</sup> ‘S’ is used here for whatever the category is of the part of the sentence that the DP combines with. In some contemporary theories, DPs are all subject to a principle of “quantifier raising,” and they typically end up with scope over a “sentence,” symbolized by ‘S,’ which contains a free variable (a “trace”) that is bound by the DP.

Natural language determiners have a number of properties that are especially interesting. One is this: it appears that every natural language determiner—in any language—is “conservative,” meaning that the determiner relates  $X$  to  $Y$  if and only if it relates  $X$  to  $X \cap Y$ . For example, every donkey is running if and only if every donkey is a donkey that is running. And some brown horse is an animal if and only if some brown horse is a brown horse that is an animal. And so on. This is a remarkable fact, since it's easy to define relations between classes such that any determiner that stood for one of them would not be conservative. For example, consider the relation that holds between  $X$  and  $Y$  if and only if something that is not  $X$  is  $Y$ . Then the class of donkeys stands in that relation to the class of brown things, since some things that are not donkeys are brown; but the class of donkeys does not stand in that relation to the class of donkeys that are brown things, since nothing that is not a donkey is a donkey that is brown.<sup>13</sup> Naturally, Aristotle's determiners, which are found in natural language, are all conservative.

It is important that the determiners that Aristotle focuses on have other, special, properties that do not hold of natural language determiners in general. In particular, they are “monotonic in both places.” Monotonicity is a logically important property of determiners. A determiner can be monotonic up on the left, or monotonic down on the left, or neither, and it can be monotonic up on the right or monotonic down on the right, or neither. We indicate that a determiner is monotonic by writing arrows to its left or right:

- $\downarrow D$  monotonic down on the left
- $\uparrow D$  monotonic up on the left
- $D \downarrow$  monotonic down on the right
- $D \uparrow$  monotonic up on the right

For a determiner to be monotonic up on the left means that when it relates class  $X$  to class  $Y$ , it also relates any superclass of  $X$  to  $Y$ :

If  $\uparrow D$  relates  $X$  to  $Y$ , and if  $X \subseteq Z$ , then  $\uparrow D$  relates  $Z$  to  $Y$

Likewise for monotonicity up on the right:

If  $D \uparrow$  relates  $X$  to  $Y$ , and if  $Y \subseteq Z$ , then  $D \uparrow$  relates  $X$  to  $Z$

The determiner ‘some’ is monotonic up on the left, and also on the right:  $\uparrow \text{some} \uparrow$ .

Monotonicity-up on the left: If ‘some’ relates the class of donkeys to the class of running things, and if the class of donkeys is a subset of the class of animals, then ‘some’ must relate the class of animals to the class of running things.

<sup>13</sup> Another example: If ‘only’ were a determiner in the context ‘Only As are Bs’ it would fail to be conservative, since ‘Only donkeys are animals’ is false but ‘Only donkeys are donkeys which are animals’ is true. Most modern linguists do not consider ‘only’ a determiner. Nor do medieval logicians; it is typical to take ‘Only As are Bs’ to be an “exponible” proposition, that is, one in need of expounding. For example, Billingham (para 18 in de Rijk 1982) says “An exponible term is what has two, or many, exponents with which it is converted. And in this it differs from a resoluble . . . . But in the exponents of an exponible term it follows well and conversely: as ‘only a man runs; therefore a man runs and nothing other than a man runs’ and conversely.”

That is, if some donkey is running, and every donkey is an animal (donkeys  $\subseteq$  animals), then some animal must be running.

Monotonicity up on the right: If 'some' relates the class of donkeys to the class of running things, and if the class of running things is a subset of the class of moving things, then 'some' must relate the class of donkeys to the class of moving things.

That is, if some donkey is running, and every running thing is moving (running things  $\subseteq$  moving things), then some donkey must be a moving thing.

The determiner 'no' is the opposite of 'some'; it is monotonic down on the left and also monotonic down on the right. Examples to illustrate this are:

Monotonicity down on the left: If 'no' relates the class of animals to the class of running things, and if the class of donkeys is a subset of the class of animals, then 'no' must relate the class of donkeys to the class of running things.

That is, if no animal is running, and every donkey is an animal (donkeys  $\subseteq$  animals), then no donkey can be running.

Monotonicity down on the right: If 'no' relates the class of animals to the class of running things, and if the class of brown things is a subset of the class of running things, 'no' relates the class of animals to the class of brown things.

That is, if no animal is running, and every brown thing is running (brown things  $\subseteq$  running things), then no animal can be brown.

The determiner ' $\downarrow$ every $\uparrow$ ' is monotonic down on the left, and monotonic up on the right.

Monotonicity down on the left: If 'every' relates the class of animals to the class of running things, and if the class of donkeys is a subset of the class of animals, then 'every' must relate the class of donkeys to the class of running things.

That is, if every animal is running, and every donkey is an animal (donkeys  $\subseteq$  animals), then every donkey must be running.

Monotonicity up on the right: If 'every' relates the class of donkeys to the class of running things, and if the class of running things is a subset of the class of brown things, 'every' must relate the class of donkeys to the class of brown things.

That is, if every donkey is running, and every running thing is brown (running things  $\subseteq$  brown things), then every donkey must be brown.

Most determiners are not doubly monotonic in this way. The determiner 'few' is monotonic down on the right—if few X's are Y, then few X's are Z if  $Z \subseteq Y$ —but it is neither monotonic up nor monotonic down on the left. It is not monotonic up on the left because few infants can lift 20 pounds, and infants are a subclass of humans, but it's not true that few humans can lift 20 pounds. And it's not monotonic down on the left because few humans weigh less than five pounds, and preemies are a subclass of humans, but it's not true that few preemies weigh less than five pounds. The determiner

'five' (meaning exactly five) is not monotonic in any way. If five cars are blue it does not follow that five vehicles (a superclass of cars) are blue, or that five convertibles are blue (a subclass of cars) or that five cars are colored things (a superclass of blue things) or that five cars are small blue things (a subclass of blue things).

#### APPLICATIONS

Determine which monotonicity properties the following determiners of English have (if any), and explain why.

*many*

*most*

*at least three*

*at most three*

In addition to conservativity and monotonicity there is another property worth mentioning, which I will call non-triviality. A determiner  $\delta$  is non-trivial if whenever class A is non-empty, there is some class B such that ' $\delta$  A is B' is true, and there is some class C such that ' $\delta$  A is C' is not true.

An example of a trivial determiner is one that relates every class to every class. There is no simple natural language determiner that does so, so let me invent one, call it 'trivial-any'. Its meaning is:

*trivial-any* relates class A to class B if and only if A and B are classes.

Any simple affirmative sentence made using 'trivial-any' is automatically true. So some truths using trivial-any are:

*Trivial-any donkey is an animal*

*Trivial-any donkey is not an animal*

*Trivial-any stone is a donkey*

and so on

All of Aristotle's determiners are non-trivial.

What other determiners have all of the neat properties shared by those that Aristotle uses? We may state here a relevant result of the study of generalized quantifiers: There are only *four possible* determiners of any natural language that are all of:<sup>14</sup>

Conservative

Monotonic on the left

Monotonic on the right

Non-trivial

<sup>14</sup> Cf. van Benthem 2008.

Three of the determiners of this sort already exist in most natural languages; in English, they are 'every,' 'some,' and 'no.' The fourth apparently does not exist in any natural language,<sup>15</sup> but it is easy to see what it would be like if there were one. It would be equivalent to the fourth corner of Aristotle's square of opposition. Such a determiner,  $\delta$ , would hold under these conditions:

$\delta$  A is B  $\approx$  some A is not B

So, as others have noted,<sup>16</sup> Aristotle chose for his logical notation the three (or four) most logically interesting determiners (or quasi-determiners) that can exist in any language. This is why I say that his selection of sentential forms is especially interesting in logic.

### 2.7.2 Axiomatizing generalized quantifiers

Suppose that we want to axiomatize the monotonicity properties of the determiners that are used in Aristotle's logical theory. It would be natural to choose something very close to the syllogistic forms that Aristotle considers as "perfect." For these tell us much of the story about these logical behaviors of the interesting determiners, at least regarding the monotonicity properties of 'every,' 'no,' and 'some.'

Barbara tells us that 'every' is monotonic down on the left and monotonic up on the right. This is because monotonicity down on the left for 'every' means:

If every X is Y, and if every Z is X (if  $Z \subseteq X$ ) then every Z is Y.

And monotonicity up on the right for 'every' means:

If every X is Y, and if every Y is Z (if  $Y \subseteq Z$ ) then every X is Z.

And both of these are correct according to Barbara:

Every M is P  
 Every S is M  
 $\therefore$  Every S is P

The first follows by putting 'X' for M and 'Y' for P and 'Z' for S. The second follows by putting 'X' for S and 'Y' for M and 'Z' for P.

Celarent tells us that 'no' is monotonic down on the left:

If no X is Y and if  $Z \subseteq X$  then no Z is Y.

This follows by putting 'X' for M and 'Y' for P and 'Z' for S in:

No M is P  
 Every S is M  
 $\therefore$  No S is P

<sup>15</sup> Cf. Horn 2001, section 4.5.

<sup>16</sup> Van Benthem 2008.

Monotonicity down on the right follows from this by applying simple conversion to any universal negative proposition.

Darii tells us that 'some' is monotonic up on the right:

If some X is Y and if  $Y \subseteq Z$  then some X is Z.

This follows by putting 'X' for S and 'Y' for M and 'Z' for P in:

Every M is P  
 Some S is M  
 $\therefore$  Some S is P

Monotonicity up on the left follows from this by applying simple conversion to any particular affirmative proposition.

So the choice of the first three moods of the first figure, together with the principles of simple conversion, are ideal forms to take as axiomatic for the symbolism Aristotle uses. This is why I say that Aristotle's choice of axiomatic principles is not arbitrary. (Or not *very* arbitrary; he could have omitted Ferio.)

#### APPLICATIONS

Suppose there is a new determiner added to English, spelled '*smnot*.' It works like this:

'*smnot* A is B' is true if and only if some A is not B

Particular negative propositions that occur in the lower right-hand corner of the square of opposition could then be worded '**Smnot** A is B.'

Say whether '*smnot*' is left monotonic up, or left monotonic down, or neither, and whether it is right monotonic up, or right monotonic down, or neither. In each case cite a syllogistic mood which would validate your claim (assuming that '*smnot* A is B' is equivalent to 'some A isn't B.')

### 2.7.3 A qualification concerning existential import

There remains an issue concerning existential import. We have been discussing parts of Aristotle's theory from a modern perspective. Our applications of monotonicity to 'every,' 'some,' and 'no' as discussed earlier presume that the subjects of universal propositions do not have existential import, and the subjects of particulars do. And we have seen that Aristotle's assumptions rule this out. We have avoided trouble about this by confining our discussion to issues where this fact is not relevant. For example, the differences between the modern and the ancient interpretations of propositions *do not show up* in the first figure moods that Aristotle took to be perfect. But if we insist on giving propositions the ancient readings, then we notice, e.g. that Barbara does not



actually entail standard monotonicity for 'every'. This is because Barbara by itself does not force monotonicity to hold in complete generality. This is because its premises are themselves universal affirmatives, and since such propositions have existential import for their subject terms, Barbara never gets to apply non-vacuously to a case with an empty subject. So all that is shown is that monotonicity holds for all applications in which the subject terms of universal affirmatives are non-empty. This is a coherent, but qualified form of monotonicity.

For a determiner to be *qualifiedly* monotonic<sup>17</sup> down on the left means that when it relates a class X to class Y, it also relates any *non-empty* subclass of X to Y:

If  $\downarrow D$  relates X to Y, and if  $Z \subseteq X$  and  $Z \neq \emptyset$ , then  $\downarrow D$  relates Z to Y

Likewise for qualified monotonicity down on the right. And for a determiner to be *qualifiedly* monotonic up on the left means that when it relates a *non-empty* class X to a class Y, it also relates any superset of X to Y:

If  $\uparrow D$  relates X to Y, and if  $X \neq \emptyset$  and  $X \subseteq Z$ , then  $\uparrow D$  relates Z to Y

Likewise for qualified monotonicity up on the right.

Applications of Barbara, Celarent, and Darii only directly prove that the determiners 'every', 'some', and 'no' are qualifiedly monotonic. In fact, it is easy to extend that result, using only reductio and simple conversion, to show that 'some' and 'no' are indeed fully monotonic both on the left and on the right, and 'every' is fully monotonic up on the right. But it cannot be shown within Aristotle's system that 'every' is fully monotonic down on the left, because that isn't true in the theory as a whole. A counterexample is that 'every' relates the class of animals to the class of living things, and the class of chimeras (which is empty) is a subclass of the class of animals, but 'every' does not relate the class of chimeras to the class of living things.<sup>18</sup>

For many purposes (such as those discussed in section 9.4) the difference between monotonicity and qualified monotonicity is not important. Applications in which it may be important are discussed in section 9.3.

## 2.8 Singular propositions

What happens when we turn to syllogisms containing singular propositions?

We already have established some of these; we have assumed Expository Syllogism, and we have proved affirmative and negative forms of Universal Application. I think

<sup>17</sup> Westerstahl 2012 calls this "weakened monotonicity."

<sup>18</sup> In fact there is no interpretation of 'every' as a relation between two classes that satisfies the full Aristotelian square once the notation is generalized (as medieval logicians generalized it). For example, one may not in general hold that 'every' relates a class A and a class B if and only if A is non-empty and A is a subset of B. For this would make 'Every chimera is no stone' false, since the class of chimeras is empty, and on the medieval view that sentence is true. (It is true since it is negative and any negative proposition with an empty main term is true.)

that we don't need any more rules than we already have. This is partly because the symbolism is so restricted. For example, our current rules do not validate this version of Leibniz's Law:

a is b  
 b is F  
 ∴ a is F

This is not validated for the simple reason that it is not well formed. The first premise is not among Aristotle's categorical propositions since he does not use propositions which have singular terms as predicates. So this inference is not yet expressible. We will consider such propositions in the medieval versions of the theory.

## 2.9 13th-century texts

(This short section may be skipped without affecting understanding of later material.)

Starting sometime before the year 400, logic was standardly taught as part of the Trivium to students entering higher education. So there was a need for textbooks. By the late medieval period, Aristotle's logic was taught in the following manner. First, the conversion principles were taken without proof. Aristotle's reduction of syllogisms to the first figure moods was often ignored. Eventually it was taught in a form that was susceptible to rote memorization, based on special names of the various moods. The names of the individual moods that we have used in the theory given here are taken from Peter of Spain (Peter of Spain *LS* 4.13). These names are all post-Aristotelian inventions; they are cleverly devised so as to encode useful logical information. Medieval students were expected to memorize a verse consisting of the names. Peter's version of the verse is:

Barbara Celarent Darii Ferio Baralipon  
 Celantes Dabitis Fapesmo Frisesomorum.  
 Cesare Cambestres Festino Barocho Darapti.  
 Felapto Disamis Datisi Bocardo Ferison.

The names in the verse contain codes which show how to mimic Aristotle's reduction of all moods to the first figure direct moods. Here are Peter's instructions (*LS* 4.13) for doing this:

In these four verses are nineteen words representing the nineteen moods of the three figures, so that by the first word we understand the first mood of the first figure, by the second word the second mood, and so on for the others. Hence, the first two verses represent the moods of the first figure, while the third verse—except for its last word—represents the moods of the second figure, so that the first word of the third verse represents the first mood of the second figure, the second word the second mood, and so on for the others. But the last word of the third verse, along with the words remaining in the fourth verse, represent the moods of the third figure in order.

It must be recognized, however, that by the vowels A E I O are understood the four genera of propositions. Thus, by the vowel A we understand the universal affirmative, by E the universal negative, by I the particular affirmative and by O the particular negative. Also, in each word are three syllables, and if anything is left over, it is superfluous—except M, as will be made clear later. And by the first of those three syllables we understand the major proposition of the syllogism, by the second the minor and by the third the conclusion. For example, the first word—BARBARA—has three syllables with an A in each one, and the A put there three times signifies that the first mood of the first figure consists of two universal affirmatives concluding a universal affirmative. And the same understanding applies to the other words regarding the vowels put into them.

Also, it must be recognized that the first four words of the first verse begin with these consonants, B C D F, like all the other words that follow. By this it must be understood that all the moods indicated by a word beginning with B are to be reduced to the first mood of the first figure, and all the moods signified by a word beginning with C to the second mood, those beginning with D to the third and those with F to the fourth. Also, wherever an S is put in these words, it signifies that the proposition understood by the immediately preceding vowel is to be converted simply. And by P it signifies that the proposition is to be converted accidentally. Wherever M is put, it signifies that a transposition in premises is to be done, and a transposition is making a minor out of a major, and the converse. Where C is put, however, it signifies that the mood understood by that word is to be confirmed by impossibility.

Earlier, (LS 4.8) Peter has given an example of what he calls reduction by impossibility:

The fourth [mood of the second figure] consists of a universal affirmative and a particular negative concluding a particular negative, like

Every man is an animal;  
 a-certain stone is not an animal;  
 therefore, a-certain stone is not a man.

And this is reduced to the first mood of the first figure by impossibility.

Peter goes on to explain (LS 4.9):

And to reduce by impossibility is to infer, from the opposite of the conclusion and one of the premises, the opposite of the other premise. The opposite of the conclusion is used—namely,

every stone is a man—

along with the major of the fourth mood just mentioned, and there will be a syllogism in the first mood of the first figure, as follows:

Every man is an animal;  
 every stone is a man;  
 therefore, every stone is an animal.

This conclusion is opposed to the minor premises of the fourth mood. And this is to confirm by impossibility.

These instructions work perfectly provided that conversion by limitation is used in the correct order; from universal to particular in premises, and from particular to universal in conclusions (the verse is written so as to require this).

## APPLICATIONS

Use Peter's instructions to reduce the following syllogistic moods.

*Cesare*

*Cambestres*

*Fapesmo*

*Bocardo*

This little theory allows one to reproduce something like Aristotle's reductions by rote memorization of the verse and by the techniques encoded therein. It is a pity that the material is so easily learnable without much logical comprehension. In particular, it is a pity that Aristotle's proofs of the conversion principles, using exposition and expository syllogism, are not included.

I say that Peter's text contains "something like" Aristotle's reductions because Peter's version of *reductio* is not Aristotle's. In another work, *S* 10.9 (431), Peter distinguishes between a *conversive* syllogism which "is always formed after some other syllogism has previously been formed," and a "syllogism *ad impossibile*," where this is not necessary. He explains the former: "a conversive syllogism is formed after another syllogism has previously been formed by taking the opposite of the <latter's> conclusion in combination with one of its premises in order to destroy the remaining one." What Peter calls reduction *per impossibile* in his rules for reducing syllogisms seems to be the conversive technique: you validate an argument by validating a different argument, one of whose premises is the contradictory of the desired conclusion, and whose conclusion is the contradictory of one of the desired premises. It relies on the principle:

If the argument:  $P$ , *contradictory-of- $R$*   $\therefore$  *contradictory-of- $Q$*  is valid,  
then:  $P$ ,  $Q$   $\therefore$   $R$  is valid.

It is ironic that Peter calls this reduction by impossibility, since no impossibility need be involved in either of the arguments. Although this technique successfully handles the reduction of syllogisms, I believe that it could not be used to derive conversion by limitation, since that *reductio* appeals to the absurdity of contrary propositions, whereas the conversive technique is limited to mimicking *reductios* in which the absurdity involves contradictories.

## 2.10 Summary of Aristotle's rules of proof

It is convenient here to summarize the rules of proof that underlie Aristotle's logic. We still have exactly three basic principles; the others can all be derived from these:

## BASIC PRINCIPLES:

**Reductio**  
**Exposition (EX)**  
**Expository Syllogism (ES)**

## DERIVED PRINCIPLES:

**Simple conversion**  
**Conversion *per accidens***  
**Subalternation**  
**Exposition (EX) for universal affirmatives**  
**Universal Application**  
**All standard syllogisms**

We are understanding these principles in the following ways.

## BASIC PRINCIPLES

## Indirect proof (Reductio):

P	
A	

$\therefore Q$

where **P** is a contradictory of **Q**, and where **A** is a contrary or contradictory of some line that dominates it.

## EX (Exposition)

some T is P	some T isn't P
$\therefore n$ is T	$\therefore n$ is T
$\therefore n$ is P	$\therefore n$ isn't P

where **n** is a name that does not already occur in the derivation, and where 'T' occurs affirmatively in an available affirmative proposition. For the time being we can say that any main term of an affirmative proposition occurs affirmatively in it.

In justifying a use of this rule one must cite both the line which is being instantiated and the line on which 'T' occurs affirmatively.

**ES (Expository Syllogism)**

n is P	n isn't P
n is T	n is T
∴ some T is P	∴ some T isn't P

where n is any singular term

**DERIVED PRINCIPLES:****Simple conversion**

Some A is B	No A is B
∴ Some B is A	∴ No B is A

**Conversion *per accidens***

Every A is B	No A is B
∴ Some B is A	∴ Some B isn't A

**Subalternation**

Every A is B	No A is B
∴ Some A is B	∴ Some A isn't B

**UA (Universal Application):**

n is X	n is X
Every X is Y	No X is Y
∴ n is Y	∴ n isn't Y

**All standard syllogisms discussed earlier (as described)**

# 3

## Quantifying Predicates, Singular Term Predicates, Negative Terms

By the 1200s, medieval authors were using propositions that are more complex than those introduced by Aristotle. This chapter explores the consequences of expanding Aristotle's syntax. Mostly, everything works smoothly, with all of Aristotle's techniques continuing to apply to the new propositions. I have called Aristotle's propositions 'standard categorical propositions'; I will call the expanded forms 'categorical propositions.' The expanded forms of categorical propositions are discussed here in their simplest form. The material will be dealt with in more detail in the next chapter.

### 3.1 Expanded notation

We explore here three main expansions of Aristotle's syntax. The first has to do with quantified predicates. Aristotle rejected propositions with explicitly quantified predicates, such as '*Every man is every animal*' because, he said, they are not true.<sup>1</sup> Medieval writers noticed that this proposition would be true if there were exactly one animal, which was a man.<sup>2</sup> It is also clear that other propositions with quantified predicates are already true, propositions such as '*No man is every animal*'.<sup>3</sup> By the 13th century such propositions were clearly acceptable.

<sup>1</sup> Aristotle *O17*: "It is not true to predicate a universal universally of a subject, for there cannot be an affirmation in which a universal is predicated universally of a subject, for instance 'every man is every animal'."

<sup>2</sup> Ockham *SL* II.4: "it should be noted that every universal proposition in which the predicate is taken universally is false if the subject or predicate is predicated of more than one thing. However, if the predicate were predicated of exactly one thing and if the same held for the subject, then the proposition could be true. Hence, if there were only one animal, say one man, then 'Every man is every animal' would be true, as would 'Every animal is every man.' But if there were more than one man or if there were any number of animals greater than one, then these propositions would be false. Therefore, 'Every phoenix is every animal' is false, even though 'Every phoenix is every phoenix' is true. Still, sometimes an indefinite or particular proposition, in which a universally taken predicate is predicated, can be true, even though the subject has many things contained under it. For example, if there were only one man, then even if there were many animals, 'Some animal is every man' would be true."

<sup>3</sup> Sherwood *S* 1.9 (27–8): "It is asked whether ['every'] can be added to a predicate. It seems that it can, since 'no man is every man' is true. . . . it can be added to a term as such—e.g., to 'man'—and this [resultant] whole can be predicated." See also Bacon *ASL* Part 2, para 238.

Second, logicians permitted negations to occur more widely than just before the copula, so that ‘*Not every donkey is running*’ is recognized as a logically useful proposition whose negation sign is the same word that occurs in ‘*Some donkey isn’t running*’.

Third, by the 14th century logicians regularly used propositions in which singular terms occur as predicates, such as ‘*Some man isn’t Socrates*.’ All of these expanded forms of proposition have a rich logic which will be explored in this chapter.

Some writers (e.g. Buridan *SD* 5.1.8) made use of propositions in which the copula occurs at the end, such as ‘*No man every animal is*.’ In classical Latin the standard position for the verb in a simple sentence is at the end. However, by medieval times, many scholars began life speaking languages in which the verb occurs in the middle, as in English, French, and German. When they used Latin they customarily put the verb in the middle. This was so common that Buridan called propositions with the verb at the end “propositions not in accordance with common usage” (Buridan *SD* 5.1.8). For logical reasons, it is convenient to include such propositions, since putting the verb at the end yields a useful canonical form where the verb does not interfere with relations among negations and quantifiers. I will thus describe a notation of logical forms in which the verb initially occurs at the end, and logical principles will apply directly to such forms. Ordinary propositions, with verbs either in the middle or at the end can be generated from the logical forms, and it will be important to carefully state how to do this.

We will proceed as follows. First we will state rules for generating what we call logical forms. These will appear in bold face, and this custom of having logical forms appear in bold face will be maintained throughout the book. Then we will give rules for converting logical forms into propositions of ordinary language. The propositions of ordinary language will be given in italics, and this too will be a practice followed for the rest of the book.

So first we need to carefully describe how logical forms are constructed. The process will be to begin with a verb and then to add denoting phrases and negations so as to form a completed proposition. (A denoting phrase is either a singular term or a common term preceded by a quantifier sign.) In the present notation, the only verb we have is ‘is,’ so we will begin with that.

One more idiosyncrasy: It will be convenient to use a dot, ‘.’, to make logical forms that underlie indefinite propositions. For English, this dot may be considered to represent the indefinite article. For Latin, which lacks indefinite articles, it represents only a position in the sentence.

**Atomic:** The single word ‘is’ is a partly constructed propositional logical form

**Complex:** If  $\phi$  is a partly constructed propositional logical form with less than two terms, then the following are partly constructed propositional forms (where ‘t’ is any singular term and ‘T’ any common term):



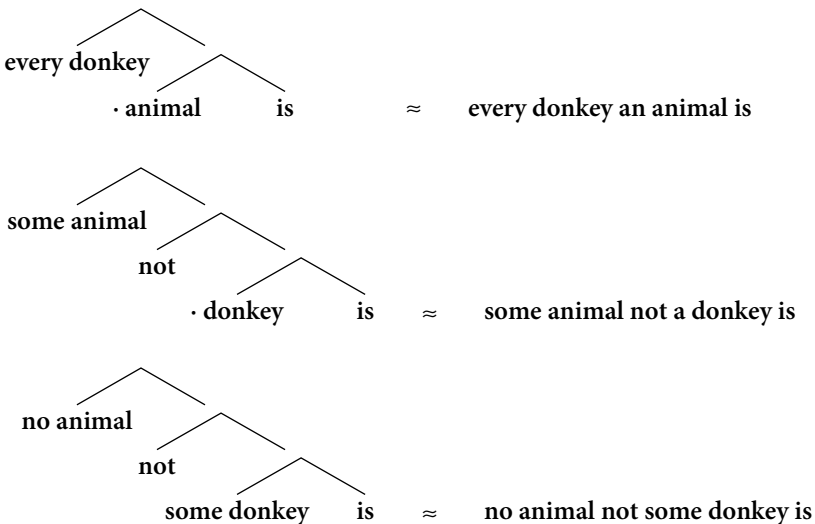
- T  $\phi$
- some T  $\phi$
- no T  $\phi$
- every T  $\phi$
- t  $\phi$

**Negations:** If  $\phi$  is a partly constructed propositional logical form, so is ‘not  $\phi$ ’.<sup>4</sup>

Finally, any partly constructed propositional logical form with two terms is a categorical propositional form (or a “categorical proposition” for short).

As mentioned in the previous chapter, I will use the term ‘denoting phrase’ for any combination of a quantifier word (or ‘.’) and common term, or for any singular term. So the construction rule just given can be summed up by the following: start with ‘is’ and keep putting denoting phrases or negations on the front, but do not put on more than two denoting phrases.

Examples of constructions of categorical propositions are given here:



It is most convenient to develop semantics and logic using the logical forms generated here. We also need to give a rigorous way to generate the actual sentences that medieval logicians use. We can do this as follows:

<sup>4</sup> It is possible to restrict this rule to apply only to partly constructed propositions which do not begin with ‘not,’ so that double negations never occur. However, it is more natural from a modern perspective to admit double (and triple, and . . .) negations, and I will do so. Double negations are always dispensable.

To turn a categorical propositional logical form (as generated previously) into a sentence of natural language:

1. Unbold the logical form and replace the symbol ‘ $\cdot$ ’ by ‘*a(n)*’ for English, and by nothing for Latin.
2. If  $\phi$  is a categorical proposition containing ‘*is*’, where there is a denoting phrase immediately to the left of ‘*is*’, the ‘*is*’ may optionally move to the left of the denoting phrase.
3. For English, replace ‘*not is*’ by ‘*isn’t*’.

The last clause accommodates the fact that in Latin the common position of a negation sign is immediately preceding the verb, whereas in English the ‘*not*’ comes after the verb ‘*is*.’ These rules generate the Latin word orders.

An example using the indefinite article:

**some animal not · donkey is**

unbold and replace the dot by ‘*a*’ for English, and by nothing for Latin:

⇒ *some animal not a donkey is*

This is one of the sentences generated from that logical form. We can also move the verb to the middle position:

⇒ *some animal not is a donkey*

change ‘*not is*’ to ‘*isn’t*’ for English:

⇒ *some animal isn’t a donkey*

(For Latin, omit the ‘*a*.’)

Not every form with its verb at the end can generate a proposition with its verb shifted to the middle like this; the rule applies only if there is nothing between the copula and the denoting phrase to its left. For example, the proposition ‘*Some A every B not is*’ is a well-formed proposition meaning that there is some A such that every B is such that it (the A) is not it (the B). It is grammatical Latin. The ‘*is*’ cannot be moved forward to make ‘*Some A is every B not*’, which is not well formed. (In Latin that is not a form that would be used by a logician.<sup>5</sup>)

Although our logical forms all have the verb at the end, this does not mean that such a word order is privileged in any way. Indeed, most medieval logicians primarily made use of the word order with the verb in middle position. But they would recognize the sentence with either order.

<sup>5</sup> It is not an ordinary grammatical sentence. It could be produced, however, by the principle that in Latin a word may be emphasized by shifting it into an unusual position. The sentence would then be understood as one in which the negation has been moved for emphasis.

## APPLICATIONS

Generate each of the following sentences by first generating a propositional logical form and then converting it to its ordinary language form.

- No animal is every donkey*  
*Not every animal is a donkey*  
*Socrates isn't a donkey*  
*No donkey is Socrates*  
*Not every animal isn't a donkey*

### 3.2 Equipollences<sup>6</sup>

With these expanded forms, lots of logically equivalent forms are possible. One common equivalence pattern resembles our contemporary principles of quantifier exchange, such as the equivalence of ‘ $\forall x$ ’ with ‘ $\sim\exists x\sim$ ’. Writers acknowledged these by formulating a series of equipollences—pairs of logically equivalent propositions that differ only in having one part replaced by an “equipollent” part.<sup>7</sup>

**William Sherwood’s Equipollences:**<sup>8</sup> Replacing a phrase here by another on the same line yields a logically equivalent proposition:

every A	no A not	not some A not
no A	not some A	every A not
some A	not no A	not every A not
some A not	not no A not	not every A

An example of a new result is the equivalence of the form ‘**No A no B is**’ (*No A is no B*) with the form ‘**Every A not no B is**’ (*Every A isn’t no B*), which is in turn equivalent to ‘**Every A some B is**’ (*Every A is some B*). Another is the equivalence of ‘**Some A some**

<sup>6</sup> The term ‘equipollent’ appears as early as the *Introductiones Montane Minores* (pp. 37–9 in De Rijk 1967 part II), which de Rijk dates about 1130 (p. 147 in De Rijk 1967 part I), and the *Introductiones Norimbergenses* (p. 139 in De Rijk 1967), which de Rijk dates to the middle of the 12th century (p. 155 in De Rijk 1967 part I), where it is used to characterize the principle now called obversion; an instance of this is that ‘every A is B’ is said to be equipollent to ‘no A is non-B’. (See section 3.4 for a discussion of the prefix ‘non.’) By sometime in the second half of the 12th century (p. 410 in De Rijk 1967 part I) in the *Dialectica Monacensis* (pp. 474–6 in De Rijk 1967 part II) it is used to characterize the quantifier equivalences discussed here by Sherwood; it is also used to characterize the principle that ‘Socrates runs’ is equipollent to ‘Not Socrates doesn’t run.’ The term ‘equipollent’ is not usually defined, but it seems to just mean the logical equivalence between two propositions. So understood, simple conversion produces equipollent propositions, but I am not aware of anywhere that the term is used to characterize the results of conversions.

<sup>7</sup> Sherwood *IL* I.19 words things this way; he takes equipollence to be a relation that relates the pairs of signs in the chart. Paul of Venice *LP* 1.10 (128) speaks as if it is whole propositions that are equipollent.

<sup>8</sup> These equivalences are from William Sherwood *IL* I.19. They appear widely in other works, then and later; cf. Wyclif *TDL* first tractate, chapter VII (Vol. I, 22–3).

**B is'** (*Some A is some B*) with '**Not every A no B is'** (*Not every A is no B*). Likewise, '**No A every B is'** (*No A is every B*) is equivalent to '**Every A some B not is'** (*Every A some B isn't*).<sup>9</sup>

It is best to see these equipollences as operating directly on propositional forms whose verbs are on the end, and indirectly on sentences generated from these with the verb moved to the middle. If applied directly to a sentence with its verb in the middle the verb can get in the way of direct application of the equivalences. For example, '*Some A not some B is*' can have its '*not some B*' changed directly to '*no B*' to yield '*Some A no B is*' (which can have its verb shifted left to yield '*Some A isn't B*'). But if the verb is in the middle, as in '*Some A not is some B*'; the '*not*' and '*some*' are not contiguous, and the equipollence rules given earlier do not literally directly apply. (Medieval logicians did not fuss over the exact position of the verb when applying these principles.)

Notice that application of any of these equipollences leaves an affirmative proposition affirmative, and a negative proposition negative, where a proposition is affirmative if it contains no negative signs or an even number of negative signs; otherwise it is negative. (This assumes that '*no*' and '*not*' are the only negative signs.)

To the equipollences we should add the equivalence of '*some*' with '*·*', the symbol for the indefinite article:

some A     ≈     ·A

This assumes that indefinites (denoting phrases with '*·*') are exactly equivalent to denoting phrases with '*some*' so far as truth conditions are concerned. Aristotle himself states that indefinite sentences are equivalent to particular sentences.<sup>10</sup> This is probably not accurate regarding the normal Greek that was spoken at his time, and many later logicians took this to be a stipulation.<sup>11</sup> In any event, it was for this reason that Aristotle did not give any additional logical principles governing indefinite sentences (they are absent from his syllogistic forms).

This is a point that merits some discussion, because in English, indefinite sentences seem to have at least two quite different uses. One is probably the sort that Aristotle

<sup>9</sup> It seems that the equipollences were originally proposed in application to subjects only, where they yielded some of the relations in the square of opposition. However, eventually they were applied to expressions inside of propositions.

<sup>10</sup> Aristotle *PA* 1.7 (11): "It is also clear that putting an indeterminate premise in place of a positive particular will produce the same deduction in every figure."

<sup>11</sup> Buridan *SD* 1.4.2: "indefinites have to be judged in the same way with respect to truth and falsity and with respect to oppositions and consequences as particulars." Elsewhere Buridan admits that this is an artificial stipulation; in *SD* 4.2.2 he says "Indeed, by an indefinite proposition [people] quite often mean a universal and not a particular proposition."

Ockham *SL* 2.3: "It should first be noted that if a proposition is not called indefinite or particular except when its subject term supposits personally, then an indefinite and a particular are always interchangeable. For example, the following are interchangeable: '[A] man is running' and 'Some man is running'; '[An] animal is a man' and 'Some animal is a man'; '[An] animal is not a man' and 'Some animal is not a man.'"

Sherwood *IL* 1.15 (31): "Note that it is unnecessary to add the indefinite class [of statements] to this last division, because indefinite judgment is like particular judgment."

had in mind. If I say ‘A dog ate my lunch’ then this is probably the same as ‘Some dog ate my lunch.’ But indefinite propositions are also used to make generic statements: ‘A whale is a mammal,’ ‘A horse is a quadruped,’ ‘A bird has wings.’ It is apparent that these do not mean the same as ‘Some whale is a mammal,’ etc. I think it would be a distortion of the tradition to lump these together with the indefinite propositions that are said to be equivalent to particulars. (There is in fact a substantial body of medieval discussion of generic indefinites, focusing on examples like ‘A man is an animal,’ which is taken by some to be equivalent to a usage from Aristotle’s *Categories* using the form ‘Animal is predicated of man.’) Hereafter we will treat indefinites as equivalent to particulars, with the understanding that we are not discussing propositions with generic readings.

With this in mind, suppose that we take Aristotle’s original list of four (non-singular, non-indefinite) categorical propositions and add the quantifier sign ‘every’ to the predicate. This gives us a list of eight forms:

<i>Every A is [a] B</i>	<i>Every A is every B</i>
<i>No A is [a] B</i>	<i>No A is every B</i>
<i>Some A is [a] B</i>	<i>Some A is every B</i>
<i>Some A isn’t [a] B</i>	<i>Some A isn’t every B</i>

Using Sherwood’s equipollences together with double negation and the equivalence of indefinites with particulars it is straightforward to show that every proposition without singular terms in the expanded notation is equivalent to one of these eight forms.

#### APPLICATIONS

Use Sherwood’s equipollences together with double negation to show that the propositional forms underlying each of the following sentences is equivalent to a proposition with one of the eight forms just given.

*Not every animal is every donkey*  
*Every donkey isn’t an animal*  
*Some animal isn’t every donkey*  
*No donkey some animal isn’t*  
*No animal is no animal*

### 3.3 Semantics and rules

The semantics of these propositions is straightforward, provided that we keep in mind that the surface order of denoting phrases corresponds to their semantic

scope,<sup>12</sup> and that an affirmative proposition is false (and a negative one true) when any of its main terms are empty. In the next chapter we will explore the semantics more deeply, but a rough understanding will suffice for present purposes.

**Validity:** It is easy to define a modern form of validity for this new notation. Let an *interpretation* specify what things each term stands for (stipulating that a singular term stands for at most one thing). Then an argument is formally valid iff there is no interpretation that makes the premises all true and the conclusion false. (Since quantifier signs only occur along with common terms, there is no “domain” or “universe” to be specified.)<sup>13</sup>

**Rules:** We can now state an expanded set of rules of inference for proofs of propositions using the expanded notation. The rules are mostly extended versions of those discussed in Chapter 2.

I begin with a simple one. In the expanded notation, it is now possible for double negations to arise, so we need a rule for this:

#### Double negation

‘not not’ can always be deleted, and it can be inserted wherever it is grammatical to do so

Then we have the quantifier equipollences (recalling that ‘∃’ is being used non-generically):

<sup>12</sup> The question of whether surface order of denoting phrases determines scope was unsettled for a while. A popular early view was that how to read scope is up to the person speaking. In Anonymous [*On the Properties of Discourse*], p. 717: “When it is said [regarding] ‘every man is [an] animal,’ that the supposition of this term ‘animal’ is multiplied, and [that] ‘animal’ does not stand there for one animal, but for many, this is not [the case] except by the intention of the speaker. For I am able thus to talk about one animal only, and according to this it is false, or for many in common, and thus true. Therefore, that it supposits for one, or for many, is not [the case] except by the intention of the speaker.” Also, *On the Properties of Discourse*, p. 719: “It should be said, however, according to the truth, that this: ‘For every man existing some man sees him’ is twofold {ambiguous}, because the commonness of this term ‘some man’ is able to be multiplied or not to be multiplied. However, the cause on account of which it ought simply to be multiplied is not shown from the force of this locution. For it is not necessary in truth that it be multiplied. For it is possible for all to see all, or some [to see] all. Therefore, that it is multiplied will not be [the case] except by the intention of the speaker.”

Later writers tended to ignore this option, and to see scope as determined by word order. In fact, in 1277, Robert Kilwardby prohibited the faculty at Oxford from teaching several propositions, one of which is the proposition that *an animal is every man*. This proposition would be acceptable in ordinary parlance, as a stylistic variant of *every man is an animal*; presumably the word order made this reading unacceptable in teaching logic. (See Uckelman 2009, 20–32.)

<sup>13</sup> It is often suggested that forms of Aristotelian logic can be symbolized in the monadic predicate calculus. The expansions of notation discussed in this chapter require the use of a slightly richer notation; they are equivalent to forms in monadic predicate logic *with identity*. They also require the use of “quantifier overlay,” that is, quantifiers within the scopes of other quantifiers, to symbolize e.g. ‘Some A is every B.’

**Substitution of equipollences:** Any proposition is equivalent to the result of replacing an expression in it by an equipollent expression, with the equipollent pairs given here.

**Quantifier equipollences:**

every T	≈	no T not	≈	not some T not
no T	≈	not some T	≈	every T not
some T	≈	not no T	≈	not every T not
some T not	≈	not no T not	≈	not every T
·T	≈	some T		

Because of double negation and the quantifier equipollences, it becomes possible to formulate a general principle of contradiction that Aristotle did not have: any proposition is contradictory to itself preceded by ‘not.’<sup>14</sup> We also have an expansion of his specification of contraries: that any proposition beginning with ‘every T’ is contrary to the same proposition with ‘not’ inserted after ‘every T’ (which is equivalent to changing ‘every’ to ‘no’).<sup>15</sup> Reductio (indirect proof) thus becomes much more widely applicable.

**Indirect proof (Reductio):**



Q

where P is a contradictory of Q, and where A is a contradictory or contrary of some available line.

**Contradictories:** Any proposition  $\phi$  is a contradictory of ‘not  $\phi$ ’, and vice versa.

**Contraries:** Any proposition of the form ‘every T  $\phi$ ’ is contrary to ‘no T  $\phi$ ’, and vice versa.

<sup>14</sup> Peter of Spain *S* 5.38 (231): “Whenever a negation is placed before some proposition, whether categorical or conditional, it always contradicts the latter.” Also Wyclif *TDL* first tractatus, chapter VII: “‘With negation preposed, contradiction’: This is when a negation is preposed to a universal sign or a particular [sign] in some proposition, then [this] makes a proposition equipollent to that proposition which was its contradictory before the appearance of the negation, as this proposition: ‘Not everyone who says to me, “Lord, Lord,” will enter into the kingdom of heaven’ is equipollent to this: ‘Some man who says to me “Lord, Lord,” will not enter into the kingdom of heaven.’ And this proposition ‘not none are chosen’ is equipollent with this: ‘Some or many are chosen.’”

<sup>15</sup> William Sherwood *IL* 1.19 (37) states the general principle of equipollence: “every universal sign is equivalent to its contrary with a following negation.” This presupposes that ‘every’ and ‘no’ are contrary signs; that is, they produce contrary propositions when preceding the same phrase, as Aristotle says.

These rules improve on Aristotle's in the treatment of contradictories. For we now have a general procedure to make the contradictory of any proposition—adding a 'not' to its front—instead of postulating the diagonals of the square of opposition. Aristotle's own stipulations are now provable, since 'every  $A \cdot B$  is' is equivalent to 'not some  $A$  not  $\cdot B$  is' by quantifier equipollences, and likewise 'some  $A \cdot B$  is' is equivalent to 'not no  $A \cdot B$  is.' Previously in order to prove 'no stone is a man' from some premises by reductio we posited 'some stone is a man' and derived something impossible. Now we posit 'not no stone is a man' and, if we like, we can turn it into 'some stone is a man' by the equipollences. Everything else proceeds as before.

In addition to Aristotle's original principles of simple conversion and conversion *per accidens* it is now possible to derive general principles of subalternation using reductio and the equipollences:

#### Subalternation (DERIVED RULE)

every $T \phi$	no $T \phi$
$\therefore$ some $T \phi$	$\therefore$ some $T$ not $\phi$

where  $\phi$  is any string of symbols such that preceding it by a denoting phrase makes a propositional logical form

#### APPLICATIONS

Using only the equipollence rules and reductio, show that the logical forms of these arguments are valid.

<i>Every A is a B</i>	<i>Some A isn't a B</i>
$\therefore$ <i>Some A is a B</i>	$\therefore$ <i>Not every A is a B</i>
<i>No A is a B</i>	<i>Not some A is a B</i>
$\therefore$ <i>Some A isn't a B</i>	$\therefore$ <i>Not every A is a B</i>

As we use our rule of exposition, it will be increasingly convenient to have a simple way of stating that a term is/is not empty. For simplicity, I will use the term itself followed by 'is non-empty' in angle brackets to mean this. Actually we can take this notation to abbreviate a certain affirmative proposition with that term as the only main term, one that is true whenever the term is non-empty. So if 'T' is a general term, '<T is non-empty>' will be taken as an abbreviation of the proposition 'Some  $T \cdot T$  is' ('some  $T$  is a  $T$ ').<sup>16</sup> And

<sup>16</sup> It would be more faithful to the historical tradition to use the form '*An F is*,' in which there is a subject term and a copula, but no predicate term. A common view was that this has the meaning of '*An F is a being*.' That noun is supposed to be true of everything there is. We could use that construction here, but it would require some way to say that '*being*' is true of everything that there is. One way to do that would be to state the equivalence of '*n is a being*' with '*n is n*,' which is the form used here. There are a number of techniques that would work equally well. The method adopted here has the advantage of not introducing any new primitive vocabulary.



when 't' is a singular term '<t is non-empty>' will abbreviate the proposition 'tt is' ('t is t'). We then have an explicit rule which says that any term that occurs as the subject of any affirmative proposition is not empty:

**Non-emptiness**

some T  $\phi$   
 $\therefore$  <T is non-empty>      if 'some T  $\phi$ ' is affirmative

t  $\phi$   
 $\therefore$  <t is non-empty>      if 't  $\phi$ ' is affirmative

It is convenient to have a broader rule for non-emptiness. The following is a derived rule:

**Non-emptiness (DERIVED RULE)**

$\phi$   
 $\therefore$  <T is non-empty>      if ' $\phi$ ' is affirmative and 'T' is a main term in  $\phi$

$\phi$   
 $\therefore$  <t is non-empty>      if ' $\phi$ ' is affirmative and 't' is a main term in  $\phi$

Exposition is now naturally stated using the defined notation:

**EX (Exposition)**

some T  $\phi$   
 <T is non-empty>  
 $\therefore$  n  $\phi$   
 $\therefore$  n · T is  
 where n is a singular term that does not already occur in the derivation  
 or on its last line

In practice it is convenient to combine uses of Non-Emptiness and Exposition, so instead of explicitly inferring non-emptiness on a line and then citing that line in a use of EX, we perform exposition in a single step, citing both the line from which non-emptiness may be inferred and the line which is being expounded. (This is the pattern that was used in Chapter 2.)

Expository syllogism is unchanged:

**ES (Expository Syllogism)**

n  $\phi$   
 n · T is  
 $\therefore$  some T  $\phi$   
 where n is any singular term

## APPLICATIONS

Show that the logical forms of these arguments are valid by using the basic rules given in this chapter.

- Some A is a B* <simple conversion>  
 $\therefore$  *Some B is an A*
- Every A is a B* <conversion *per accidens*> <using simple conversion>  
 $\therefore$  *Some B is an A*
- Every A is a B* <universal application >  
*n is an A*  
 $\therefore$  *n is a B*
- Every A is a B* <*Darapti*> <using rule UA>  
*Every A is a C*  
 $\therefore$  *Some C is a B*

## 3.4 Singular terms as predicates

Some new rules are needed to handle propositions with singular terms as predicates. A guiding principle is that singular terms permute with whatever they come in contact with.<sup>17</sup> They permute with each other:<sup>18</sup>

**Marcus Tully is**  $\approx$  **Tully Marcus is**

*Marcus is Tully*  $\approx$  *Tully is Marcus*

<sup>17</sup> Buridan (*SD* 1.6.3): “As far as the conversion of a singular proposition is concerned, about which the author [Peter of Spain] does not speak, we should say that it is convertible, be it affirmative or negative; and it is converted into a singular if the predicate is singular, into an indefinite or particular if the predicate was a non-distributed common term, and into a universal if the predicate was a distributed common term; for example, ‘This is Socrates; therefore, Socrates is this,’ or ‘This is not Socrates; therefore Socrates is not this,’ or ‘Socrates is a man; therefore, a man is Socrates’ or ‘Socrates does not run; therefore, no runner is Socrates.’”

Ockham (*SL* II.21): “Similarly, a singular affirmative proposition is converted simply into a particular and an indefinite proposition, or into a singular proposition. For example, this follows: ‘Socrates is a man; therefore a man is Socrates,’ and ‘therefore some man is Socrates’—and conversely. Likewise, this follows: ‘Socrates is Plato; therefore Plato is Socrates’—and conversely. “A singular negative proposition is converted simply into either a universal negative or a singular negative. For example, this follows: ‘Socrates is not white; therefore no white thing is Socrates,’ and conversely. Likewise, this follows: ‘Socrates is not Plato; therefore Plato is not Socrates,’ and conversely.”

<sup>18</sup> This does not mean that the names switch roles as grammatical subject and predicate. In a sentence with a transitive verb, case inflections will indicate which is subject and which is predicate: ‘**Socrates**<sub>nominative</sub> **Plato**<sub>accusative</sub> **sees**’ and ‘**Plato**<sub>accusative</sub> **Socrates**<sub>nominative</sub> **sees**’ illustrate the permutation, where ‘**Socrates**’ is the subject in both sentences and ‘**Plato**’ is direct object in both, and the two sentences are logically equivalent. In actual usage they would be treated as stylistic variants. (Both are normal Latin sentences.) When the verb is ‘is’ our current notation offers no means of distinguishing the grammatical roles of subject and predicate nominative. That will change in the next chapter.

They permute with particular denoting phrases (this is a case of conversion):

**Tully a man is**  $\approx$  **A man Tully is**                      *Tully is a man*  $\approx$  *A man is Tully*

And they permute with negation:<sup>19</sup>

**Not Tully a stone is**  $\approx$  **Tully not a stone is**      *Not Tully is a stone*  $\approx$  *Tully isn't a stone*

Given Sherwood's quantifier equipollences, singular terms also permute with universal denoting phrases. The following are all equivalent:

<b>Tully every stone is</b>	
<b>Tully not some stone not is</b>	Quantifier equipollence
<b>Not Tully some stone not is</b>	Permutation with negation
<b>Not some stone Tully not is</b>	Permutation with particulars
<b>Not some stone not Tully is</b>	Permutation with negation
<b>Every stone Tully is</b>	Quantifier equipollence

So we have the rules (using ' $\approx$ ' to relate forms which can replace each other yielding logical equivalent propositions):

**Permutation for singular terms**

Singular terms permute with other denoting phrases:

**t quant T**  $\approx$  **quant T t**    where **quant** is **every, some, no,** or **.**

Singular terms permute with each other:

**t s**                       $\approx$     **s t**

Singular terms permute with negation:

**t not**                       $\approx$     **not t**

**APPLICATIONS**

Show that the logical forms of these arguments are valid.

*Socrates isn't every animal*

$\therefore$  *Some animal isn't Socrates*

*Socrates isn't Plato*

$\therefore$  *Plato isn't Socrates*

*No stone is an animal*

*Some animal is Socrates*

$\therefore$  *Socrates isn't a ston*

<sup>19</sup> Peter of Spain S2.28 (97): "Note that a negation placed before or after a singular term signifies the same (as in 'Socrates is not running' and 'Not: Socrates is running')."

Now that singular terms can occupy predicate positions, transitivity of identity becomes formulable, and it needs to be included as a rule. I am not aware of any discussion of exactly this principle by medieval logicians. However, there are some 14th-century comments by John Buridan on the foundations of syllogistic which supply principles from which transitivity of identity can be derived. Buridan says:

I declare, therefore, that every affirmative syllogism holds by virtue of the principle ‘whatever things are said to be numerically identical with one and the same thing, are also said to be identical between themselves.’ For example, if a white thing is identical with Socrates and a running thing also is identical with Socrates, then it is necessary that a white thing and a running thing should be identical; since, therefore, it amounts to the same thing to say ‘Socrates is identical with a white thing’ and to say ‘Socrates is white,’ the inference ‘Socrates is white and he runs; therefore a running thing is white’ is valid.<sup>20</sup>

...

At this point we should also remark in connection with negative syllogisms that they all are valid in virtue of that other principle, namely: ‘whatever things are so related that one of them is said to be identical and the other is said to be not identical with one and numerically the same thing, they necessarily have to be said not to be identical with each other.’ (SD 5.1.8, 313, 315)

Buridan’s remarks suggest the following principles, from which the rest all follow:

**Quasi-transitivity of identity**

<b>a b is</b>	<b>a b is</b>
<b>a c is</b>	<b>a c isn’t</b>
<b>/∴ b c is</b>	<b>/∴ b c isn’t</b>

(The negative form is easily provable from the affirmative form using reductio.)

Since singular terms permute, transitivity of identity follows immediately from this principle.

**Transitivity of identity (DERIVED RULE)**

<b>a b is</b>
<b>b c is</b>
<b>/∴ a c is</b>

<sup>20</sup> This inference was widely discussed in its application to the Trinity. The worry is that it would validate this inference: *The father is God, and the son is God, so the father is the son*. The premises are both articles of faith, and the conclusion is heretical. A number of ways were found to escape this conclusion. Buridan himself took an unusual way out; he holds that the inference holds in all cases that do not pertain to divine matter. This seems artificial. A more common way was to deny the premises in their interpretation that makes the inference valid, and to say that the intended reading is ‘*The thing which the father is is God*, and likewise for the son. This is to be understood something like “the substance which the father (in some sense) is is God. This allows us to conclude that the substance which the father is is the substance which the son is, but that is unproblematic. For a fuller discussion see Thom 2012.

We can now derive a version of Leibniz’s Law for identity. We derive the principle only for substitutivity of initial terms; the permutation principles and equipollences extend this to terms occurring in other than initial position.

**Substitutivity of identity (DERIVED RULE)**

**n m is**  
**n φ**  
 ∴ **m φ**

Wyclif states this pattern, including it among expository syllogisms.<sup>21</sup>

This may be proved as follows. It will suffice to prove two instances of the principle, namely:

<b>n m is</b>	<b>n m is</b>
<b>n some T is</b>	<b>n some T not is</b>
∴ <b>m some T is</b>	∴ <b>m some T not is</b>

Proof of the first instance:

- |       |                    |                        |
|-------|--------------------|------------------------|
| 1.    | <b>n m is</b>      |                        |
| 2.    | <b>n some T is</b> |                        |
| <hr/> |                    |                        |
| 3.    | <b>some T n is</b> | 2 Permutation          |
| 4.    | <b>t some T is</b> | 3 3 EX                 |
| 5.    | <b>t n is</b>      | 3 3 EX                 |
| 6.    | <b>n t is</b>      | 5 Permutation          |
| 7.    | <b>m t is</b>      | 1 6 Quasi-Transitivity |
| 8.    | <b>t m is</b>      | 7 Permutation          |
| 9.    | <b>some T m is</b> | 4 8 ES                 |
| 10.   | <b>m some T is</b> | 9 Permutation          |

This proof is complex because we are being fastidious, justifying every change of order. If we suppress reference to the changes of order of the elements of the propositions, an informal rendition would look much simpler and more natural:

- |       |                               |                        |
|-------|-------------------------------|------------------------|
| 1.    | <i>n is m</i>                 |                        |
| 2.    | <i>n is some T</i>            |                        |
| <hr/> |                               |                        |
| 3.    | <i>Let t be a T that n is</i> | 2 EX                   |
| 4.    | <i>n is t</i>                 | 2 EX                   |
| 5.    | <i>m is t</i>                 | 1 4 Quasi-transitivity |
| 6.    | <i>m is some T</i>            | 3 5 ES                 |

<sup>21</sup> Wyclif *TDL* first tractatus, chapter XI: “And it should be noted that in every figure it is possible to have an expository syllogism. In the first figure thus: ‘*This is a man*, and *Socrates is this*: therefore, *Socrates is a man*.”

Proof of the other instance:

1.	<b>n m is</b>	
2.	<b>n some T not is</b>	
3.	<b>not m some T not is</b>	
4.	<b>some T n not is</b>	2 Permutation
5.	<b>t some T is</b>	3 4 EX
6.	<b>t n not is</b>	3 4 EX
7.	<b>n t not is</b>	6 Permutation
8.	<b>m t not is</b>	1 7 Quasi-Transitivity
9.	<b>t m not is</b>	8 Permutation
10.	<b>some T m not is</b>	5 9 ES
11.	<b>m some T not is</b>	10 Permutation
12.	<b>m some T not is</b>	3 11 Reductio

By symmetry of **n** and **m**, the two principles just proved along with a simple use of indirect derivation entail the following forms:

<b>n m is</b>	<b>n m is</b>
<b>not n some T is</b>	<b>not n some T not is</b>
<b>∴ not m some T is</b>	<b>∴ not m some T not is</b>

It is then straightforward to show that every proposition expressible in our notation that contains exactly one singular term is equivalent to one of these four forms:

<b>n some T is</b>	<b>n some T not is</b>
<b>not n some T is</b>	<b>not n some T not is</b>

So Substitutivity of Identity is established for propositions containing exactly one singular term. It is not difficult to establish the principle for propositions with two singular terms.

#### APPLICATIONS

Show that the logical forms of these arguments are valid.

<i>Marcus is Tully</i>	<i>Not every orator is Cicero</i>
<i>Tully is an orator</i>	<i>Marcus is Cicero</i>
<i>∴ Marcus is an orator</i>	<i>∴ Some orator isn't Marcus</i>
<i>Cicero is Tully</i>	<i>Not every orator is Cicero</i>
<i>Marcus is Tully</i>	<i>Marcus is Cicero</i>
<i>∴ Not Marcus isn't Cicero</i>	<i>∴ Marcus some orator isn't</i>

As noted earlier, self-identity does not generally hold, but it holds for non-empty names:

**Self-identity for non-empty terms**

<t is non-empty>

∴ t t is

(Of course, if ‘<t is non-empty>’ is taken to abbreviate the particular proposition ‘t t is,’ as we have done, this rule is completely trivial.)

### 3.5 Infnitizing negation and conversion

Recall from Chapter 1 the theory of “conversion” inherited from Aristotle: conversion means the interchange of the subject and predicate terms of a categorical proposition. “Simple conversion” was universally recognized as being valid for the universal negative and particular affirmative forms, and conversion by limitation held for the universal forms.

Conversions become more interesting when term negation is allowed. We do this in English with the prefix ‘non-,’ writing complex terms like ‘non-donkey.’ The Latin version of the negative prefix is called “infnitizing negation” by medieval authors; it is distinguished from ordinary propositional negation, which is called “negating negation.” The title “infnitizing” comes from Aristotle’s calling terms with a negative prefix “infnite names.”<sup>22</sup> There was a special reason to be sensitive to this distinction, because in Latin negating negation and infnitizing negation are spelled the same (‘non’) and there are no hyphens to tell you whether or not to attach ‘non’ to a term. Thus a proposition containing a negation is often ambiguous between the two. (This is true in Greek as well, so the ambiguity was present when Aristotle made the distinction.) An example of such a sentence is:

*Non homo est animal*

which is ambiguous between the false proposition ‘Not: [a] man is [an] animal’ and the true proposition ‘[A] non-man is [an] animal.’ Examples of propositions with infnitizing negations:

**Some non-donkey some stone is**

*Some non-donkey is some stone*

**Socrates · non-donkey is**

*Socrates is [a] non-donkey*

<sup>22</sup> *Of* §2. Although ‘infnite name’ is the commonest translation, the expression can also be translated as ‘indefinite name’ or ‘unlimited name.’ It does not seem to mean that the term supposits for an infinite number of things (though that is often so).

When deciding whether a proposition is affirmative or negative, infinitizing negation does not have the effect that negating negation has in reversing quality. Although this proposition is negative: ‘*Some donkey isn’t an animal*’, this one is affirmative: ‘*Some donkey is a non-animal*’.

Typically infinitizing negation combines with a simple term, not a complex term such as ‘*grey donkey*’; Sherwood and others argue for this,<sup>23</sup> and other writers seem to assume it. Tentatively, I adopt that option.<sup>24</sup> We can accommodate infinite terms by stipulating:

### Infinitizing negation

If ‘**T**’ is a simple common term, ‘**non-T**’ is a common term.

If ‘**t**’ is a simple singular term, ‘**non-t**’ is a common term.

The term ‘**non-T**’ stands for every existing thing that ‘**T**’ does not stand for.

The term ‘**non-t**’ stands for every existing thing that ‘**t**’ does not stand for.

We might want to modify this by dropping the condition ‘existing.’ With such a change, in a present tense sentence ‘**non-donkey**’ would stand for past, present, and future fish, planets, dodos, and merely possible gold mountains. As we have worded the condition, the sentence ‘*Every non-donkey is a being*’ turns out true; on the changed version it might be false. (This would depend on whether the ‘*every*’ is restricted to presently existing things. See Chapter 10 for discussion.) As we have worded the condition it obeys the common constraint that all common terms in an ordinary present tense sentence are restricted to standing for presently existing things. But I am not sure that there is unanimity on the question.<sup>25</sup> Other issues like this arise when past tense sentences and modal sentences are involved. We return to discussing this issue in

<sup>23</sup> Sherwood S XI.8: “suppose that Socrates is black. Then ‘Socrates is a non-man [colored] white’ is false, even though ‘Socrates is not a man [colored] white’ is true. The cause of this is that when the infinitizing negation cuts into one and the same word with that which is infinitized, it is necessary that it fall over one word alone.” Burley *Shorter Treatise* para 198: “the term to be infinitized should be simple and not composed of a substantive and an adjective, or of adjectives. For what is to be infinitized should be one. But an aggregate term like that is not absolutely one. Thus the term ‘white wood’ {word-order in Latin: “wood white”} cannot be infinitized. And if a negation is added, by saying ‘non white wood’ {“non wood white” in Latin}, nothing is infinitized but the term ‘wood’.”

<sup>24</sup> Since ‘*non-*’ can only combine with a simple term, “double negations” such as ‘*non-non-donkey*’ are not permitted. Albert of Saxony *QCL* para. 108 (76) says “Infinite terms cannot be infinitized. This is shown, because infinite terms do not have finite signification, which is required if they could be infinitized. Hence, the term ‘non-man’ cannot be infinitized, nor the term ‘non-horse’.”

<sup>25</sup> Peter of Spain S 2.9–15 has both options. He argues that an infinitized term is ambiguous, for the *non* can either make a “negative” term ‘*non-man*’ that is said of absolutely everything that is not a man, including non-beings, or it can make a “privative” term ‘*non-man*’ which is true only of (existing) beings that are not men. Peter also seems to think that on the first reading, infinitizing negation makes a proposition containing it negative (if it is affirmative without the negative term). His first reading then resembles the result of treating the negation as negating rather than infinitizing. Before and during Peter’s time there were a variety of other views; cf. Kneepkens 1993 for others.



sections 10.2 and 10.4 of the chapter on ampliation and restriction. In the meantime we will use the version as formulated previously.

Infinitizing negation is sometimes explained in terms of relative clauses: something is a non-*P* if and only if it is *a thing which isn't a P*.<sup>26</sup> We will return to this issue when we get to relative clauses. In the meantime, we can introduce some adequate rules that do not involve relative clauses:

Rule Non:

$n \cdot \text{non-}P$ is	$\langle n \text{ is non-empty} \rangle$
$\therefore \langle n \text{ is non-empty} \rangle$ <sup>27</sup>	not $n \cdot P$ is
$\therefore$ not $n \cdot P$ is	$\therefore n \cdot \text{non-}P$ is

### 3.5.1 Conversion by contraposition and obversion

Along with simple conversion and conversion by limitation, many authors writing in the Old Logic tradition discussed **conversion by contraposition**. To convert by contraposition you interchange subject and predicate, changing each from finite to infinite, or vice versa. That is, if a term lacks an infinitizing negation, you put one on, and if it has an infinitizing negation you remove it. Universal affirmatives and particular negatives are said to convert by contraposition as follows:

<i>Every A is B</i>	is said to be equivalent to	<i>Every non-B is non-A</i>
<i>Some A isn't B</i>	is said to be equivalent to	<i>Some non-B isn't non-A</i>

Several early authors endorse conversion by contraposition for universal affirmatives and particular negatives.<sup>28</sup> Others point out that this is not quite correct, since the affirmative form has existential import for the subject and predicate terms, and the contrapositives of such forms have existential import for the negatives of those terms.<sup>29</sup> Examples of nonequivalent pairs are:

*Every chimera is an animal* [false]

<sup>26</sup> Albert of Saxony *SL* 1.6: “this proposition, ‘Not-man runs,’ is equivalent to this proposition ‘something which is not a man is running.’ And this proposition ‘Every not-man runs’ is equivalent to this sentence ‘Everything which is not a man is running.’” (Moody’s translation). Notice that this explanation is only obviously correct for present tense sentences.

<sup>27</sup> The inference of non-emptiness is already guaranteed by the fact that the premise is affirmative. It is included here for comparison with the other inference.

<sup>28</sup> Contraposition is discussed in several 12th- and early 13th-century anonymous texts edited in De Rijk 1967, where it is seen as a type of *equipollence*. It is endorsed in several of these: *Excerpta Norimbergenses* (138–9), *Ars Burana* (190), *Tractatus Anagnini* (238), *Introductiones Parisienses* (362), *Logica: Ut dicit* (385), *Logica: Cum sit nostra* (426), *Dialectica Monacensis* (478). Another text, *Ars Emmerana* (157) endorses contraposition, but then says that it does not hold for the particular negative unless understood with “constancy,” a term that had to do with assuming instances of the terms in question. Sherwood *IL* 3.3 (59) also endorses it.

<sup>29</sup> For example, Buridan *TC* 1.8.100 (224).

versus

*Every non-animal is a non-chimera* [true]

Likewise, a particular negative is vacuously true if its subject term is empty, but its contrapositive is not vacuously true if that very same term (which now occurs within the predicate) is empty.

*Some donkey isn't a being* [false]

versus

*Some nonbeing isn't a non-donkey* [true]

(These invalid transitions are not yielded by our rules of inference. The problematic inferences become OK when a premise is added making the term in question non-empty.)

A similar thing happens with the principle of **obversion**.<sup>30</sup> This is the principle that states that you can change a proposition to an equivalent form if you change it from affirmative to negative (or vice versa) and change the predicate term from finite to infinite (or infinite to finite). Some examples are:

<i>Every A is B</i>	is said to be equivalent to	<i>No A is non-B</i>
<i>No A is B</i>	is said to be equivalent to	<i>Every A is non-B</i>
<i>Some A is B</i>	is said to be equivalent to	<i>Some A isn't non-B</i>
<i>Some A isn't B</i>	is said to be equivalent to	<i>Some A is non-B</i>

It is apparent that these inferences are valid when moving from affirmative to negative, but not in the reverse direction, because the terms may be empty. Buridan (*TC* I.8.107 (226)) makes this clear, stating:

From any affirmative there follows a negative by changing the predicate according to finite and infinite, keeping the rest the same, but there is no formal consequence from a negative to an affirmative, although there is a consequence under the assumption that all of the terms supposit for something.

The good direction he gives as:

*Every A is B; therefore no A is non-B.*

The fallacious direction is illustrated by

*A chimera isn't a woman; therefore a chimera is a non-woman.*

<sup>30</sup> Although 'contraposition' is a medieval term, 'obversion' is not. I don't think medieval authors had terminology for this pattern of reasoning, though they discussed it.

Some medieval writers before Buridan accepted the fallacious versions, and some did not.<sup>31</sup> These principles are already taken care of by our rules. Proofs of the valid directions for two of the four forms are:

1.	<b>every A · B is</b>	
2.	<b>not no A · non-B is</b>	
3.	<b>some A · non-B is</b>	2 equipollences
4.	<b>a · A is</b>	3 EX
5.	<b>a · non-B is</b>	3 EX
6.	<b>a · B is</b>	1 4 UA
7.	<b>not a · B is</b>	5 Non
8.	<b>no A · non-B is</b>	6 7 reductio

1.	<b>some A · non-B is</b>	
2.	<b>not some A not · B is</b>	
3.	<b>every A · B is</b>	2 equipollences
4.	<b>a · non-B is</b>	1 EX
5.	<b>a · A is</b>	1 EX
6.	<b>a · B is</b>	3 5 UA
7.	<b>not a · B is</b>	4 Non
8.	<b>some A not · B is</b>	6 7 reductio

The last inference can be reversed if you are given the non-emptiness of the subject term:<sup>32</sup>

1.	<b>some A not · B is</b>	
2.	<b>&lt;A is non-empty&gt;</b>	
3.	<b>a not · B is</b>	1 2 EX
4.	<b>a · A is</b>	1 2 EX
5.	<b>&lt;a is non-empty&gt;</b>	4 non-emptiness
6.	<b>not a · B is</b>	3 Permute
7.	<b>a · non-B is</b>	5 6 Non
8.	<b>some A · non-B is</b>	4 7 ES

<sup>31</sup> Obversion is discussed in several 12th and early 13th-century anonymous texts edited in De Rijk 1967, where it is seen as a type of *equipollence*. It is endorsed in four of these: *Excerpta Norimbergenses* (138–9), *Logica: Ut dicit* (385), *Logica: Cum sit nostra* (426), *Dialectica Monacensis* (478). Three texts, *Ars Burana*, *Introductiones Parisienses*, *Tractatus Anagnini* omit it (while including a discussion of contraposition). One text, *Introductiones Montane Minores* (37–8) objects to it because it mishandles empty terms. Roger Bacon ASL para 261 is clear in rejecting obversion when going from negative to affirmative: “Infinitezation does not follow for any other reason than because a proposition with an infinite predicate is affirmative, and so the being [of the predicate] is affirmed of the subject. In a negative, however, i.e., in a proposition with a denied predicate, the being [of the predicate] is denied of the subject, and from the negation of something there does not follow the affirmation of the same.”

<sup>32</sup> Buridan makes a similar point, assuming that all of the terms stand for something. It is sufficient to assume that the subject term stands for something.

## APPLICATIONS

Consider the remaining alleged equivalents, namely:

<i>No A is B</i>	equivalent to	<i>Every A is non-B</i>
<i>Some A is B</i>	equivalent to	<i>Some A isn't non-B</i>

Give derivations to validate the affirmative to negative directions of each.

Give a derivation to validate the negative to affirmative direction of one of the equivalences when given the appropriate non-emptiness condition.

### 3.6 Completeness of the rules

Completeness is the condition that

**Completeness:** For any argument from a set of propositions  $\Gamma$  to a proposition  $\phi$ , if the argument is valid then  $\phi$  is derivable from  $\Gamma$  by the stated rules.

In the next chapter a completeness proof is given for a system of rules similar to the one given here. The proof given there can be turned into a completeness proof for the present notation by erasing all of the markers and parentheses in the formulas discussed there.

#### 3.6.1 Verbs other than the copula

Another innovation was widely practiced, though usually without comment. This is the use of verbs other than the copula, as in '*Socrates sees Brownie*.' Although we have confined ourselves to propositions containing the copula as their only verb, a survey of the rules given earlier shows that except for the rules that explicitly mention identity, none of them depend on the nature of the verb itself. As we move forward we will allow for other verbs, with details to be given in the next chapter.

Some expansions of the ancient logic have been discussed here using pretty much the forms of expression that medieval authors themselves used. But things will become more complex. The next chapter enhances the notation of the present chapter so that more complex forms can be dealt with.

### 3.7 Summary of the rules of proof used so far

Here is a summary of the basic rules for derivations discussed in this chapter.

**Double negation**

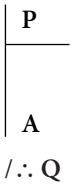
**not not** can always be deleted, and it can be inserted wherever it is grammatical to do so

**Substitution of equipollences:** If  $\phi$  differs from  $\phi^*$  only in containing  $\psi$  where  $\phi^*$  contains  $\psi^*$ , and if  $\psi$  and  $\psi^*$  are equipollent ( $\psi \approx \psi^*$ ), then  $\phi / \therefore \phi^*$

**Quantifier equipollences:**

every T	$\approx$	no T not	$\approx$	not some T not
no T	$\approx$	not some T	$\approx$	every T not
some T	$\approx$	not no T	$\approx$	not every T not
some T not	$\approx$	not no T not	$\approx$	not every T
·T	$\approx$	some T		

**Indirect proof (Reductio):**



where P is a contradictory of Q, and where A is a contradictory or contrary of some other line that dominates it.

**Contradictories:** Any proposition  $\phi$  is a contradictory of ‘not  $\phi$ ’, and vice versa.

**Contraries:** Any proposition of the form ‘every T  $\phi$ ’ is contrary to ‘no T  $\phi$ ’, and vice versa.

**Non-emptiness**

some T  $\phi$

$\therefore$  <T is non-empty> if ‘some T  $\phi$ ’ is affirmative

t  $\phi$

$\therefore$  <t is non-empty> where ‘t  $\phi$ ’ is affirmative

**EX (Exposition)**

some  $T \phi$   
 $\langle T \text{ is non-empty} \rangle$   
 $\therefore n \phi$   
 $\therefore n \cdot T \text{ is}$   
 where  $n$  is a singular term that does not already occur in the derivation

As noted earlier, it is often convenient to collapse Non-Emptiness and EX into a single step.

**ES (Expository Syllogism)**

$n \phi$   
 $n \cdot T \text{ is}$   
 $\therefore \text{some } T \phi$   
 where  $n$  is any singular term

**Permutation for singular terms**

Singular terms permute with other denoting phrases:

$t \text{ quant } T \approx \text{quant } T t$ , where **quant** is every, some, no, or .

Singular terms permute with each other:

$t s \approx s t$

Singular terms permute with negation:

$t \text{ not} \approx \text{not } t$

**Quasi-transitivity of identity**

$a b \text{ is} \quad a b \text{ is}$   
 $a c \text{ is} \quad a c \text{ not is}$   
 $\therefore b c \text{ is} \quad \therefore b c \text{ not is}$

**Self-identity for non-empty terms**

$\langle t \text{ is non-empty} \rangle$   
 $\therefore t t \text{ is}$

Rule Non:

$n \cdot \text{non-P is}$	$\langle n \text{ is non-empty} \rangle$
$\therefore \langle n \text{ is non-empty} \rangle$	$\text{not } n \cdot P \text{ is}$
$\therefore \text{not } n \cdot P \text{ is}$	$\therefore n \cdot \text{non-P is}$

# 4

## Linguish

### 4.1 Basics

The propositions that we have discussed so far have very simple forms. There is always a single verb, which is ‘is,’ and two terms, plus a quantifier symbol for each common term. There may also be some negation signs. By the 14th century medieval logicians regularly dealt with propositions which are much more complex: there are verbs other than the copula, there are relative clauses, there are terms which are related by the grammatical “possessive,” as in ‘*Every woman’s donkey is running*,’ and there are various kinds of complex terms, as in ‘*A donkey seeing some horse is running*.’ When things get complicated we will need a way to keep track of how the parts of sentences are grammatically and logically related to one another. The job of this chapter is to devise a notation that will permit this. For the sake of continuity, our new notation will be an enhanced version of that of the last chapter.

An example: suppose we start with a verb, the transitive verb ‘sees.’ It takes a subject and a direct object. We can precede the verb with a term that is to be its subject, say ‘Socrates.’ This gives us ‘Socrates sees,’ in which the subject role is taken by ‘Socrates,’ and the direct object role is not yet taken. We can put another term in front of this, with the understanding that it takes the role of the direct object of ‘sees’; an example would be ‘Some horse Socrates sees’ (where the term ‘horse’ has brought a quantifier word with it). We cannot now add any more terms, because there are no grammatical roles for them to fill. So we have a complete sentence, one that is grammatical in Latin, where it means that Socrates sees some horse.

We could have put the same terms in front of the same verb in the same order, but with the understanding that ‘Socrates’ takes the direct object role and ‘some horse’ takes the subject role. The sentence would then mean that there is some horse that sees Socrates.

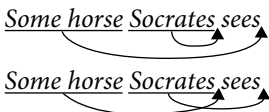
In order to have a notation that makes clear which term is the subject and which the direct object, we need to have some device to encode that information. In Latin, this job is done partly by case inflections on the terms. In our first example, ‘Socrates’ would receive a nominative case inflection and ‘horse’ would receive an accusative inflection; in the second example, these inflections would be reversed. However, case



inflections fall short of doing the job of determining grammatical relations in two ways. One is that sometimes a word with a nominative case inflection is spelled the same as the same word with an accusative case inflection, so you can't tell from the spelling which inflection the word has. We could treat this as a simple case of ambiguity, and add subscripts to words indicating which case inflection they bear. But this would not address a second problem, which is that when there is more than one verb in a sentence, case inflections will not tell you which verb a term is related to. For example, in 'A *donkey which sees a horse sees a man*,' both 'horse' and 'man' are accusative, and there are two verbs which take direct objects. To understand the sentence you have to know that 'horse' is the direct object of the first 'sees,' and 'man' is the direct object of the second 'sees.' This is easy to determine in English by the English word order. But Latin word order is freer than English, and in complicated sentences there may be several grammatical relationships to keep track of.

Our approach will utilize a version of what linguists call the "theta-criterion." The idea behind the theta-criterion is that certain words, such as verbs, provide roles, called theta-roles, and each of these roles must be filled by exactly one denoting phrase. (Cf. Fromkin et al. 2000.) Further, it is only by filling a theta-role that a denoting phrase can get into a sentence. As an example, the verb 'see' provides two theta-roles, that of the seer and that of the thing seen. If the names 'Socrates' and 'Plato' fill these theta-roles to make 'Socrates sees Plato' then no additional denoting phrase may be put into the sentence, so that 'Socrates Cicero sees Plato' for example is not well formed.

One way to implement this idea would be to draw arrows to indicate which theta-role is being filled by which denoting phrase. So we could distinguish the two ways to read 'Some horse Socrates sees' as follows:



where the first indicates that 'Socrates' fills the subject role and 'Some horse' fills the direct object role, and the second indicates the opposite. These arrows work fine in the simplest cases, but they make an unreadable mess in complicated ones. So I will resort instead to the use of letters as grammatical role markers for the theta-roles, adopting the letters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ ,  $\eta$ , for this purpose (adding subscripts if more are needed). Each verb will be accompanied by such markers indicating the grammatical roles that the verb provides for filling by terms. In English the theta-roles often correspond to places in the sentence, so that the denoting phrase filling the seer role occupies subject position, preceding the verb, and the denoting phrase filling the seen role occupies direct object position, following the verb. I will put the role markers in those positions so that it is easy for English speakers to keep track of them without additional explanation. For example, we will have forms like:

$\alpha$  **sees**  $\beta$   
 $\gamma$  **runs**  
 $\delta$  **is**  $\epsilon$

In all three cases the marker before the verb indicates the theta-role filled by the subject of the verb. The ' $\beta$ ' coming after 'sees' indicates the theta-role filled by the direct object. The verb 'run' provides only the subject role. The ' $\epsilon$ ' after 'is' indicates the predicate role. (I am not quite sure what to call this role. It is the role that is filled by a term occupying what is usually called a "predicate nominative" position.) When a denoting phrase is added to the verb to build up a sentence the term is to be accompanied by a marker that matches some available marker provided by the verb; this indicates that the term bears to that verb whatever grammatical relation the marker indicates. Thus in the examples discussed previously we had two structures whose meanings will be encoded as:

(some horse  $\delta$ )(Socrates  $\epsilon$ )  $\epsilon$  sees  $\delta$   
 (some horse  $\epsilon$ )(Socrates  $\delta$ )  $\epsilon$  sees  $\delta$

The first indicates that 'Socrates' is the seer and 'horse' is the thing seen; the second indicates the reverse. I have used parentheses to group together terms with their markers, including any quantifier sign that precedes the term. I am pretty sure that no harm at all is done if these parentheses enclosing denoting phrases are omitted; the structures are sufficiently clear without them. But the parentheses make it easy visually to keep track of the relationships.

It is important that each role gets a unique grammatical marker. Thus, not only must the subject and the object of 'sees' have different markers, the subject and object of 'owns' must not only be different from one another, they must also be different from the markers for the subject and object of 'sees.' This is because as we conceive of roles, the subjects of different verbs are different items. A role is not just a status like "subject," it is a status of being the subject of a particular occurrence of a particular verb in the sentence. No denoting phrase may be the subject simultaneously of two different verbs. At least, not in natural language.<sup>1</sup>

I call sentential structures with the roles indicated as earlier *logical forms*. These logical forms are meant to underlie actual sentences of Latin with the indicated grammatical structure. The sentences that they represent are the ones that are generated by this simple procedure, similar to that from the last chapter:

<sup>1</sup> Note that in a sentence like 'Some horse prances and runs' the 'horse' is not the subject of both verbs. It is the subject of a unique conjunctive verb phrase 'prances and runs.' (Such constructions are not implemented in this text.) Also in the sentence 'Some horse prances and it runs' the subject is still not the subject of the second verb; the subject of the second verb is 'it.' If 'it' is understood to have 'horse' as its grammatical antecedent, then this is still not a conjunction in which 'horse' is subject of both verbs. These constructions (where pronouns have antecedents) are discussed in Chapter 8.

**To generate sentences from logical forms:**

- *In a logical form, erase the parentheses and the theta-role-markers.*
- *Replace ‘.’ by the indefinite article in English, and by nothing in Latin.*
- *Mark each term to indicate its grammatical case (null marking indicates nominative).*
- *Optionally move any verb which is immediately to the right of a denoting phrase to the left of that denoting phrase.*

For example, from the logical form ‘(some horse  $\epsilon$ )(Socrates  $\delta$ )  $\epsilon$  sees  $\delta$ ’ we erase the parentheses and role markers to get ‘Some horse Socrates<sub>acc</sub> sees,’ and we can optionally move the verb to get: ‘Some horse sees Socrates<sub>acc</sub>.’

The sentences generated from logical forms are the things that medieval logicians took to be their subject matter. They are meaningful sentences of Latin. The theta-role markers in the logical forms explicitly indicate grammatical (syntactic) relationships present in the sentences that are generated by those forms. This encapsulates the idea that the subject matter of medieval logic consists of meaningful sentences *together with* their grammatical (syntactic) structure.

If two different logical forms produce the same sentence upon erasure of the markers and parentheses, then that sentence is ambiguous. (When things get complicated there will be ample cases of such ambiguity.) Medieval logicians are sensitive to such ambiguities. When logicians describe logical relationships among ambiguous sentences, they always have in mind certain grammatical forms that they take the sentences to have.

I assume that it is appropriate to apply medieval methods to the structures with markers, since these are just sentences with their grammatical structure made explicit. I assume that this grammatical structure was something clearly known to logicians. (There won’t be anything subtle about it.)

I use the title “Linguish” to stand for the system of logical forms together with the algorithms we use to generate (transliterations of) actual sentences of Latin. It is essential to my enterprise that a logical form of Linguish is transformable into a unique actual sentence of Latin of the sort that medieval logicians discuss. I write the logical forms using English vocabulary, but this is only for the convenience of English readers.

Some examples may be helpful. The logical form:

(some horse  $\delta$ )(Socrates  $\epsilon$ )  $\delta$  sees  $\epsilon$

directly generates the English sentence:

*Some horse Socrates<sub>acc</sub> sees*

and the Latin sentence:

*Aliquis equus Sortem<sup>2</sup> videt*

The Latin sentence, unlike the English sentence, has case endings on the words to clarify their cases. The ‘*uis*’ of ‘*aliquis*’ and the ‘*us*’ of ‘*equus*’ tell us that ‘*some horse*’ is nominative (and so it must be the subject of ‘*sees*’), and the ‘*em*’ on ‘*Sortem*’ tells us that ‘*Socrates*’ is accusative (and so it must be the direct object of ‘*sees*’). So in this case the Latin sentence is unambiguous whereas the English version is not. To disambiguate the English sentence I have subscripted the term ‘*Socrates*’ with the sign ‘*acc*’ to indicate that it has accusative case. To avoid clutter, I take a lack of marking on a term to indicate that it has nominative case.

Alternatively, the logical form:

(some horse<sub>ε</sub>)(Socrates<sub>δ</sub>)<sub>δ</sub> sees<sub>ε</sub>

directly generates the same English sentence:

*Some horse<sub>acc</sub> Socrates sees*

and the Latin sentence:

*Aliquem equum Sortes videt*

The Latin spellings are different from those in the first example; this Latin sentence requires that ‘*Socrates*’ is the subject of ‘*sees*’ and ‘*some horse*’ is the direct object.

I have spoken cavalierly of the “English” sentences generated previously, even though they are pretty unusual. I assume that they can be understood as intended, using the case subscripts as a guide.

The logical forms directly generate sentences whose verbs occur at the end, but there is nothing special about that order. As in the last chapter, we allow those verbs to migrate to the left under certain circumstances. E.g. the logical form:

(some horse<sub>δ</sub>)(Socrates<sub>ε</sub>)<sub>δ</sub> sees<sub>ε</sub>

means that there is a horse that sees Socrates. Using the verb-movement option, it generates the quasi-English sentences:

*Some horse Socrates<sub>acc</sub> sees* <no movement yet>

*Some horse sees Socrates<sub>acc</sub>*

*Sees some horse Socrates<sub>acc</sub>*

and generates the Latin sentences:

*Aliquis equus Sortem videt*

*Aliquis equus videt Sortem*

*Videt aliquis equus Sortem*

<sup>2</sup> It is common in medieval manuscripts for Socrates’ name to be spelled ‘Sortes’ in the nominative and ‘Sortem’ in the accusative.

The first English sentence is grammatical only in poetry or unusual writing. The second is perfectly grammatical. The third is completely ungrammatical. The corresponding Latin sentences are all grammatical. They all say that some horse sees Socrates. For a medieval logician the center form would be most natural, and the first form would be clearly understandable. The bottom form is not uncommon in classical Latin; it would be used to emphasize the verb for some reason. I'll follow the practice of most medieval logicians in mostly ignoring these forms, as stylistically unusual.

Consider a related logical form, one like the previous one with its markers changed:

(some horse  $\epsilon$ )(Socrates  $\delta$ )  $\delta$  sees  $\epsilon$

This means that Socrates sees some horse. It generates almost the same surface forms as before:

*Some horse<sub>acc</sub> Socrates sees*

*Some horse<sub>acc</sub> sees Socrates*

*Sees some horse<sub>acc</sub> Socrates*

and generates the Latin sentences:

*Aliquem equum Sortes videt*

*Aliquem equum videt Sortes*

*Videt aliquem equum Sortes*

In this case the first English sentence could be used in poetry with the right meaning. Likewise the second sentence, although its normal interpretation has the horse seeing Socrates, which is not what the logical form says. The third is still ungrammatical. As before the Latin sentences are all grammatical and they mean exactly what the logical form will be taken to mean: that there is a horse that Socrates sees. Notice that the Latin case endings have changed, so as to require that 'Socrates' is the subject and 'some horse' is the direct object.

As a general policy from now on, when I give a logical form of Linguish I will usually also give the sentence or sentences that it generates, with an explanation of unusual word order if necessary.

Notice that when a natural language form is generated from a logical form, the order of the denoting phrases is the same in the native language sentence as in the logical form. This is wrong for English, which generally requires that its subject precede the verb and its direct object follow the verb, even if the sentence may have a meaning in which the direct object quantifier takes scope over that of the subject, as in one natural reading of 'A boy dated each girl.' As mentioned earlier, by the late medieval period logicians were reading sentences as if the surface order of the quantifier expressions corresponds exactly with the semantic scope of those expressions. They thus used a regimented version of Latin, one that was very convenient for use in logic. Since it is their usage that is under investigation, I will take this regimentation for granted, including the assumption that the surface order of terms corresponds to their semantic scope.

## 4.2 Categorical propositional logical forms

We will work up to the full complexity of Linguish slowly. We begin with a stock of verbs, with markers indicating their theta-roles:

- $\alpha$  **exists**
- $\alpha$  **runs**
- $\alpha$  **is**  $\beta$
- $\alpha$  **sees**  $\beta$
- $\alpha$  **owns**  $\beta$

In addition to these we will focus entirely on analogues of the logical forms of the last chapter. We will then give analogues of the rules of inference from the last chapter; these will be mostly the same as before but with grammatical markers added. Then using the markers in the logical forms we will formulate a precise semantics for each form—and thus indirectly a semantics for the actual sentences generated from that form. We can then define formal validity and prove a completeness theorem. The techniques of this chapter will continue to be used later when we complicate the forms to accommodate more complex sentences.

We begin with a definition of categorical propositional logical forms. In the following it is convenient to borrow some modern terminology. When a denoting phrase is added to a formula I will say that its grammatical marker *binds* the marker in the formula that uses the same Greek letter. A marker is *bound* in a formula if some denoting phrase binds it; otherwise it is *free*.

- Any verb accompanied by one or more distinct markers to indicate grammatical roles is a categorical formula. Examples were given earlier.
- If ‘ $\phi$ ’ is a categorical formula, ‘P’ a common term, ‘t’ a singular term, these are categorical formulas:

**not**  $\phi$

(**every**  $P \delta$ )  $\phi$     where the marker ‘ $\delta$ ’ appears free in  $\phi$

(**some**  $P \delta$ )  $\phi$     where the marker ‘ $\delta$ ’ appears free in  $\phi$

(**no**  $P \delta$ )  $\phi$     where the marker ‘ $\delta$ ’ appears free in  $\phi$

( $\cdot P \delta$ )  $\phi$     where the marker ‘ $\delta$ ’ appears free in  $\phi$

( $t \delta$ )  $\phi$     where the marker ‘ $\delta$ ’ appears free in ‘ $\phi$ ’

- A categorical propositional logical form is a categorical formula with no free markers.

Notice that there is no “vacuous quantification.” This is because when a quantificational phrase is put on the front of the formula, the marker indicates its grammatical

role in the resulting sentence. Vacuous quantification would be a case in which a denoting phrase occurs in a sentence without having any grammatical role in that sentence. That would not be grammatically well formed.

Examples of the construction of some logical forms:

$\alpha \text{ is } \beta \Rightarrow (\text{some animal } \beta) \alpha \text{ is } \beta \Rightarrow (\text{every donkey } \alpha) (\text{some animal } \beta) \alpha \text{ is } \beta$   
*Every donkey some animal is*  
*Every donkey is some animal*

$\alpha \text{ is } \beta \Rightarrow (\cdot \text{ donkey } \beta) \alpha \text{ is } \beta \Rightarrow \text{not } (\cdot \text{ donkey } \beta) \alpha \text{ is } \beta \Rightarrow$   
**(some animal  $\alpha$ ) not ( $\cdot$  donkey  $\beta$ )  $\alpha$  is  $\beta$**   
*Some animal not a donkey is*  
*Some animal isn't a donkey*

$\alpha \text{ is } \beta \Rightarrow (\cdot \text{ donkey } \beta) \alpha \text{ is } \beta \Rightarrow (\text{no stone } \alpha)(\cdot \text{ donkey } \beta) \alpha \text{ is } \beta$   
*No stone a donkey is*  
*No stone is a donkey*

$\alpha \text{ is } \beta \Rightarrow \text{not } \alpha \text{ is } \beta \Rightarrow (\cdot \text{ donkey } \beta) \text{not } \alpha \text{ is } \beta \Rightarrow$   
**( $\cdot$  stone  $\alpha$ )( $\cdot$  donkey  $\beta$ ) not  $\alpha$  is  $\beta$**   
*A stone a donkey not is*  
*A stone a donkey isn't*

In this last example, the verb cannot move to a middle position since it is not contiguous to a denoting phrase. (Recall that 'isn't' is an English abbreviation of the Latin word order 'not is'.)

These examples all use the verb 'is.' Some examples with other verbs are:

$\alpha \text{ exists } \Rightarrow (\text{some animal } \alpha) \alpha \text{ exists}$   
*Some animal exists*

$\alpha \text{ sees } \beta \Rightarrow (\cdot \text{ donkey } \beta) \alpha \text{ sees } \beta \Rightarrow (\text{no stone } \alpha)(\cdot \text{ donkey } \beta) \alpha \text{ sees } \beta$   
*No stone a donkey<sub>acc</sub> sees*  
*No stone sees a donkey<sub>acc</sub>*

$\alpha \text{ sees } \beta \Rightarrow (\cdot \text{ donkey } \beta) \alpha \text{ sees } \beta \Rightarrow \text{not } (\cdot \text{ donkey } \beta) \alpha \text{ sees } \beta \Rightarrow$   
**(some animal  $\alpha$ ) not ( $\cdot$  donkey  $\beta$ )  $\alpha$  sees  $\beta$**   
*Some animal not a donkey<sub>acc</sub> sees*  
*Some animal doesn't see a donkey<sub>acc</sub>*

This last example illustrates one additional complication in generating sentences. When the word 'not' occurs directly in front of a verb in Latin, that word order is normal. In English the helping verb 'do' is needed for verbs other than the copula. Here is an expanded statement of the principles for generating sentences from logical forms:

**To generate sentences from logical forms:**

- In a logical form, erase the parentheses and the theta-markers.
- Replace ‘.’ by the indefinite article in English, and by nothing in Latin.
- Mark each term for grammatical case, depending on its grammatical marker.
- Optionally move any verb which is immediately to the right of a denoting phrase to the left of that denoting phrase.<sup>3</sup>
- For English, replace ‘not is’ by ‘isn’t’ and replace ‘not VERBs’ by ‘doesn’t VERB’ for verbs other than ‘is.’

$\alpha$  owns  $\beta \Rightarrow (\cdot \text{ donkey } \alpha) \alpha$  owns  $\beta \Rightarrow (\text{no stone } \beta) (\cdot \text{ donkey } \alpha) \alpha$  owns  $\beta$

*No stone<sub>acc</sub> a donkey owns*

*No stone<sub>acc</sub> owns a donkey*

*<i.e. no stone is owned by a donkey>*

$\alpha$  sees  $\beta \Rightarrow \text{not } \alpha$  sees  $\beta \Rightarrow (\cdot \text{ donkey } \beta) \text{not } \alpha$  sees  $\beta \Rightarrow$

$(\cdot \text{ stone } \alpha) (\cdot \text{ donkey } \beta) \text{not } \alpha$  sees  $\beta$

*A stone a donkey<sub>acc</sub> not sees*

*A stone a donkey<sub>acc</sub> doesn’t see*

**APPLICATIONS**

Generate the following sentences by first generating a propositional logical form and then converting it to its ordinary language form. (Interpret the sentence using ordinary English word order.) If the form cannot be generated, explain why.

*No animal is every donkey*

*Not every animal sees a donkey*

*Socrates isn’t a donkey*

*Socrates doesn’t see a donkey*

*No donkey exists*

*Not every animal isn’t a donkey*

*Socrates sees Plato a stone*

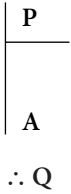
<sup>3</sup> It might be advisable to liberalize the constraint on verb movement so as to allow a verb to move past a ‘not’ when that ‘not’ did not start out directly in front of the verb. This would allow for the generation of the surface proposition ‘Some horse sees not every donkey’ from ‘(Some horse  $\alpha$ ) not (every donkey  $\beta$ )  $\alpha$  sees  $\beta$ ’. I find such wordings awkward, and I do not recall seeing them in any examples discussed by medieval logicians. But they seem to have straightforward meanings. This requires further thought.



### 4.3 Rules of inference

These rules are essentially those of Chapter 3 with parentheses and markers added. For comments on the rules see the previous chapter.

**Indirect proof (Reductio):**



where P is a contradictory of Q, and where A is a contradictory or contrary of the proposition on some line that dominates A.

**Contradictories:** One sentence is a contradictory of another if one consists of the other preceded by ‘not,’ or is equipollent to the other preceded by ‘not.’

**Contraries:** Any sentence of the form ‘(every T<sub>α</sub>) φ’ is contrary to ‘(no T<sub>α</sub>) φ.’

**Substitution of equipollences:** Any proposition is equivalent to the result of replacing an expression in it by an equipollent expression, with the equipollent pairs given here.

**Quantifier equipollences:**

- |                            |   |                              |   |                                 |
|----------------------------|---|------------------------------|---|---------------------------------|
| (every T <sub>α</sub> )    | ≈ | (no T <sub>α</sub> ) not     | ≈ | not (some T <sub>α</sub> ) not  |
| (no T <sub>α</sub> )       | ≈ | not (some T <sub>α</sub> )   | ≈ | (every T <sub>α</sub> ) not     |
| (some T <sub>α</sub> )     | ≈ | not (no T <sub>α</sub> )     | ≈ | not (every T <sub>α</sub> ) not |
| (some T <sub>α</sub> ) not | ≈ | not (no T <sub>α</sub> ) not | ≈ | not (every T <sub>α</sub> )     |
| (· T <sub>α</sub> )        | ≈ | (some T <sub>α</sub> )       |   |                                 |

**Double negation:** not not ≈ -  
 (double not’s are equipollent to the null string)

**Singular terms permute:**

- |  |   |   |
|--|---|---|
| (t <sub>ε</sub> )(quant T <sub>δ</sub> ) | ≈ | (quant T <sub>δ</sub> )(t <sub>ε</sub> ), where quant is ‘every,’ ‘some,’ ‘·’ or no |
| (t <sub>ε</sub> )(r <sub>δ</sub> )       | ≈ | (r <sub>δ</sub> )(t <sub>ε</sub> )  |
| (t <sub>ε</sub> ) not                    | ≈ | not (t <sub>ε</sub> )   |

Abbreviations:

' $\langle T \text{ is non-empty} \rangle$ ' is to abbreviate the proposition ' $(\text{some } T_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$ .'

' $\langle t \text{ is non-empty} \rangle$ ' is to abbreviate the proposition ' $(t_\alpha)(t_\beta)_\alpha \text{ is } \beta$ .'

**Non-emptiness:**

$(\text{some } T_\alpha) \phi$   
 $\therefore \langle T \text{ is non-empty} \rangle$       if ' $(\text{some } T_\alpha) \phi$ ' is affirmative

$(t_\alpha) \phi$   
 $\therefore \langle t \text{ is non-empty} \rangle$       if ' $(t_\alpha) \phi$ ' is affirmative

**EX**       $(\text{some } T_\alpha) \phi$   
              $\langle T \text{ is non-empty} \rangle$   
 $\therefore (n_\alpha) \phi$   
 $\therefore (n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$   
 where **n** is a name that does not already occur in the derivation

**ES**       $(n_\alpha) \phi$   
              $(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$   
 $\therefore (\text{some } T_\alpha) \phi$       where 'n' is any singular term

**Quasi-transitivity of identity:**

$(n_\alpha)(m_\beta)_\alpha \text{ is } \beta$   
 $(n_\alpha)(p_\beta)_\alpha \text{ is } \beta$   
 $\therefore (m_\alpha)(p_\beta)_\alpha \text{ is } \beta$

**Self-identity:**

$\langle n \text{ is non-empty} \rangle$   
 $\therefore (n_\alpha)(n_\beta)_\alpha \text{ is } \beta$

Finally, we need a housekeeping rule, indicating that the choice of which letters to use for grammatical markers is not logically significant:

**Interchange of bound markers:** Any proposition is equipollent to the result of replacing any bound marker in it (in all its occurrences) by a marker that does not occur in it.

In order to avoid lengthy derivations, I will occasionally reorder the bound markers within a formula without mention. I will also assume that all rules that specifically mention the quantifier sign ‘some’ have parallel versions using the indefinite sign ‘.’

4.3.1 *Some theorems that may be of interest*

**Boethius’s principle** (“*nothing is truer than when something is said of itself*”)

<A is non-empty>  
 $\therefore (\text{Every } A_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta$

Proof:

- |    |   |                                       |
|----|---|---------------------------------------|
| 1. | <u>&lt;A is non-empty&gt;</u>   |                                       |
| 2. | <u>not (every <math>A_\alpha</math>)(some <math>A_\beta</math>) <math>\alpha</math> is <math>\beta</math></u> |                                       |
| 3. | (some $A_\alpha$ ) not (some $A_\beta$ ) $\alpha$ is $\beta$  | 2 equipollence                        |
| 4. | ( $a_\alpha$ )(some $A_\beta$ ) $\alpha$ is $\beta$   | 1 3 EX                                |
| 5. | ( $a_\alpha$ ) not (some $A_\beta$ ) $\alpha$ is $\beta$  | 1 3 EX                                |
| 6. | not ( $a_\alpha$ )(some $A_\beta$ ) $\alpha$ is $\beta$   | 5 permutation                         |
| 7. | (every $A_\alpha$ )(some $A_\beta$ ) $\alpha$ is $\beta$  | Reductio: 4 and 6 are contradictories |

Recall that ‘(Every  $A_\alpha$ )( $\cdot A_\beta$ )  $\alpha$  is  $\beta$ ’ does not hold if ‘A’ is empty.<sup>4</sup>

**Generalized subalternation (DERIVED RULE)**

(every $T_\alpha$ ) $\phi$	(no $T_\alpha$ ) $\phi$
$\therefore$ (some $T_\alpha$ ) $\phi$	$\therefore$ (some $T_\alpha$ ) not $\phi$

<sup>4</sup> Buridan SD 1.8.4: “no affirmative is true in which the subject supposits for nothing.”

Ockham SL 2.14: “Now someone might ask: isn’t ‘A chimera is a chimera’ true? It seems that it is true, since the same thing is predicated of itself. And Boethius claims that no proposition is more true than one in which the same thing is predicated of itself. . . . Boethius meant that no proposition in which something is predicated of something is more true than one in which the same thing is predicated of itself. But since his point is a negative one, it is consistent with its being the case that neither proposition is true—neither the one in which the same thing is predicated of itself nor the one in which something else is predicated of it.”

**Substitutivity of identity (DERIVED RULE)**

$(n_\alpha)(m_\beta)_\alpha \text{ is } \beta$   
 $(n_\alpha) \phi$   
 $\therefore (m_\alpha) \phi$

**UA (DERIVED RULE)**

$(\text{every } T_\alpha) \phi$	$(\text{no } T_\alpha) \phi$
$(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$	$(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$
$\therefore (n_\alpha) \phi$	$\therefore \text{not } (n_\alpha) \phi$

A form of Universal Generalization is derivable, but it's just as easy to derive generalizations using reductio, so we won't bother to formulate it.

**Affirmative import (DERIVED RULE)**

$\phi$   
 $\therefore \langle T \text{ is non-empty} \rangle$  where  $\phi$  is affirmative and 'T' occurs as a main term in  $\phi$   
 $\phi$   
 $\therefore \langle t \text{ is non-empty} \rangle$  where  $\phi$  is affirmative and 't' occurs as a main term in  $\phi$

Proof of the first part (for common terms): Take  $\phi$  and replace '(no P<sub>γ</sub>)' by '(every P<sub>γ</sub>) not', and '(· P<sub>γ</sub>)' by '(some P<sub>γ</sub>)', and move all negations and singular terms as far to the right as possible, using equipollences when necessary to change 'not (some P<sub>γ</sub>)' to '(every P<sub>γ</sub>) not' and 'not (some P<sub>γ</sub>)' to '(every P<sub>γ</sub>) not.' If the initial denoting phrase is '(some T<sub>α</sub>)', we are done, because of the Non-Emptiness rule. If it is '(every T<sub>α</sub>)' then use subalternation to get '(some T<sub>α</sub>)' and we are again done. Otherwise, some other denoting phrase is on the front. If that term uses 'every', apply subalternation. Then use EX to infer the result of replacing the initial denoting phrase with a new singular denoting phrase. Now permute the new singular term to the right, and start over.

Proof of the second part (for singular terms): permute the singular term denoting phrases to the front and apply the Non-Emptiness rule.

4.3.1.1 SYMMETRY OF 'IS'

In the notation of Chapter 3 there were two propositions that you could make using two names and the copula; they were:

*Marcus is Tully*  
*Tully is Marcus*

(I am here ignoring the position of the copula itself, which is a surface issue.) In our new notation there are four. The differences cannot be illustrated using only third-person singular forms, but with other persons they are:

*Marcus I am*  
*I am Marcus*  
*Marcus is me* <or *Marcus is I*>  
*Me is Marcus* <or *I is Marcus*>

In the first two examples the subject of the verb is ‘I’ and in the final two the subject is ‘Marcus.’ In the Linguish notation, using only third-person singular terms, the forms are:

(Marcus<sub>α</sub>)(Tully<sub>β</sub>)<sub>α</sub> is<sub>β</sub>  
 (Tully<sub>β</sub>)(Marcus<sub>α</sub>)<sub>α</sub> is<sub>β</sub>  
 (Marcus<sub>α</sub>)(Tully<sub>β</sub>)<sub>β</sub> is<sub>α</sub>  
 (Tully<sub>β</sub>)(Marcus<sub>α</sub>)<sub>β</sub> is<sub>α</sub>

In the first two forms the term ‘Marcus’ is the subject; in the latter two the subject is ‘Tully.’ The first two forms are equivalent to one another by permutation of singular terms; likewise for the final two forms. But what about the first two versus the final two? In fact, we can prove they are equivalent. I will show one particular case of this, the inference:

(Marcus<sub>α</sub>)(Tully<sub>β</sub>)<sub>α</sub> is<sub>β</sub>  
 ∴ (Marcus<sub>α</sub>)(Tully<sub>β</sub>)<sub>β</sub> is<sub>α</sub>

1.	(m <sub>α</sub> )(t <sub>β</sub> ) <sub>α</sub> is <sub>β</sub>	
2.	<m is non-empty>	1 non-emptiness
3.	(m <sub>α</sub> )(m <sub>β</sub> ) <sub>α</sub> is <sub>β</sub>	2 self-identity
4.	(t <sub>α</sub> )(m <sub>β</sub> ) <sub>α</sub> is <sub>β</sub>	1 3 quasi-transitivity of identity
5.	(m <sub>β</sub> )(t <sub>α</sub> ) <sub>α</sub> is <sub>β</sub>	4 permutation
6.	(m <sub>γ</sub> )(t <sub>δ</sub> ) <sub>δ</sub> is <sub>γ</sub>	5 interchange of bound markers
7.	(m <sub>α</sub> )(t <sub>β</sub> ) <sub>β</sub> is <sub>α</sub>	6 interchange of bound markers

I believe that the following derived rule holds in general.

**Symmetry of ‘is’:** (DERIVED RULE)

If ‘α’ and ‘β’ are any two distinct markers, any proposition containing ‘α is β’ is equivalent to the same proposition with ‘β is α.’

(If this is not derivable, it should be posited as a basic rule.)

## APPLICATIONS

Provide derivations to validate the following derived rules.

Generalized Subalternation

Universal Application

Provide derivations to show that empty denoting phrases may be intersubstituted. For example:

<b>not</b> ( $\mathbf{n}_\alpha$ )( $\mathbf{n}_\beta$ ) $\alpha$ <b>is</b> $\beta$	'n' is empty
<b>not</b> ( <b>some</b> $\mathbf{P}_\alpha$ )( <b>some</b> $\mathbf{P}_\beta$ ) $\alpha$ <b>is</b> $\beta$	'P' is empty
( $\mathbf{n}_\alpha$ ) $\phi$	
$\therefore$ ( <b>some</b> $\mathbf{P}_\alpha$ ) $\phi$	

<b>not</b> ( $\mathbf{n}_\alpha$ )( $\mathbf{n}_\beta$ ) $\alpha$ <b>is</b> $\beta$	'n' is empty
<b>not</b> ( <b>some</b> $\mathbf{P}_\alpha$ )( <b>some</b> $\mathbf{P}_\beta$ ) $\alpha$ <b>is</b> $\beta$	'P' is empty
( <b>some</b> $\mathbf{P}_\alpha$ ) $\phi$	
$\therefore$ ( $\mathbf{n}_\alpha$ ) $\phi$	

## 4.4 Signification and supposition

To give the semantics of Linguish, we will proceed in the usual way, by first linking the simple expressions of the language—terms and verbs—to things in the world, and then characterizing truth conditions for sentences based on those links. Our semantics is stated for propositions in logical form; the results are to automatically apply to the real surface propositions that are generated from them, where the logical form indicates how the surface proposition is read.

Two key semantic relations that were used by philosophers during the 12th–14th centuries are *signification* and *supposition*. First, signification.

The word '*signification*' occurs over the centuries in a wide variety of uses. It is used here in one of its common 14th-century meanings. A simple account given by both Ockham and Buridan is that words are signs of (or subordinated to) mental concepts, and the words thereby signify the things those concepts are concepts of. The things in question are things such as particular donkeys, persons, things. All that is needed for the logical theory is that signification is a relation between terms and things.<sup>5</sup>

<sup>5</sup> Realist accounts hold that words are signs of Forms, and words signify the Forms. These will be Forms of particular donkeys, persons, things. In the nominalist tradition, words will typically stand not for Forms, but for the particular things that the Forms are of. For simplicity of exposition, we will not give the details of realist accounts. (They would not affect the logical forms studied here.)

### Signification

Each common term is a sign of a concept which is naturally a concept of some (or no) things. Each common term signifies at time *t* each thing the concept is a concept of at time *t*.

Each singular term is a sign of a concept which is naturally a concept of at most one thing. Each singular term signifies at time *t* the thing (if any) that the concept is a concept of at time *t*.

The concept associated with a written or spoken term is a stable thing; it is assigned to a term independent of its use in a sentence. The written word ‘*young*’ is subordinated to a concept that at any given time is a concept of certain things, and the word signifies those things then; at a later time that concept is no longer a concept of some of those things, and the word no longer signifies them then. This is an idealized account; variants are discussed in Chapter 10 (including an account by Buridan in which signification does not vary with time).

For purposes of discussing truth conditions, the most important semantic relation is what medieval theorists came to call “supposition.”<sup>6</sup> Supposition presupposes signification; that is, only a term that already has signification can have supposition.<sup>7</sup> Supposition in general is usually defined in terms of the accepting or taking of a term for something, or a term’s taking the place of something, when that term occurs in a proposition. An occurrence of a term in a proposition supposits for (stands for) a thing iff it is taken for, or is accepted for, or takes the place of, that thing.<sup>8</sup> “Suppositing for” is

<sup>6</sup> ‘*Supposit*’ was originally a word from the syntactic part of grammar; for a word to supposit originally meant for it to occur as the subject of a verb. It gradually began to be used for a semantic relation between a subject term and what that term stands for in that position. Eventually it was extended to a relation between any occurrence of a term and what that particular occurrence of the term stands for.

<sup>7</sup> Lambert 2c (105) [in Kretzmann and Stump 1988]: “For the signification is the concept of the thing represented by means of the utterance, and before the union of it with the utterance there is no term; rather, a term is constituted in the union of that concept of a thing with an utterance. Supposition, on the other hand, is a certain property of a term that has been constituted in that way.” Also Roger Bacon *ASL* para 212 (107–8): “supposition is not a property except of a term actually ordered within an expression, and not outside it. Signification, however, is a condition of an utterance and a term in and of itself, . . . So it signifies both outside an expression and within an expression, although it supposits only within.”

There is one acknowledged exception to this: a nonsense term can be used to supposit for itself. So in the sentence ‘Bu has no signification’ the sentence is true because ‘Bu’ does indeed supposit for itself in this special context.

<sup>8</sup> Some other explications are: Sherwood *IL* V.1, p. 105: “Signification, then, is a presentation of the form of something to the understanding. Supposition, however, is an ordering of the understanding of something under something else.” Albert of Saxony, *Summa Logica*, Part Two, Chapter One: “Supposition, in the sense here intended, is the interpretation, or usage, of a categorematic term, for some thing or things in a proposition.” Marsilius of Inghen. <1. Suppositions>, page 53: “supposition is the acceptance of a term in a proposition for something, or things” Paul of Venice II.1 (30): “Supposition is the acceptance of a term in a proposition for some thing or things.” Peter of Spain *LS* VI 2–3: “Supposition is the acceptance of a substantival term for something.” Walter Burley *LTPAL* 1.1.1.6 (80): “Speaking generally, supposition is the taking of a term for something.” William Ockham *SL* 1.63 (189): “Supposition is said to be a sort of taking the place of another.”

pretty much what we would express today by “standing for” or perhaps by “referring to” in some of the ways that ‘*refer*’ is used. So ‘*x* supposit for  $\beta$ ’ means essentially ‘*x* stands for  $\beta$ ’. I’ll stick with ‘supposit’ to clarify that it is the medieval theory that is under discussion.

Like signification, “supposition” relates linguistic expressions with extra-linguistic things, but which things a term supposit for depends on its use in a proposition. For purposes of this chapter we will only consider terms used in present tense non-modal propositions without special verbs such as ‘*believe*’ or ‘*signify*’. (Other propositions are discussed in Chapter 10.) We also confine ourselves to occurrences of terms which are not used to stand for themselves (as in ‘Donkey has two syllables’) or for the associated concept or form (as in ‘Donkey is a species’). In the propositions discussed here, supposition can be characterized in terms of signification as follows:

### Supposition

Each common term supposit at time *t* for all of the things that it signifies at *t* and which exist at *t*.<sup>9</sup>

Each proper name supposit at time *t* for the thing (if any) that it signifies at *t*, provided that that thing exists at *t*;<sup>10</sup> otherwise it supposit at *t* for nothing.

Truth conditions for present tense sentences are all given in terms of suppositing with respect to the present time. (That is, the time of utterance of the sentence, or the feigned time.) Until we reach Chapter 10 on tenses, we will suppress explicit mention of the relativity to the present time. That is, instead of saying ‘supposit at the present time’ we will just say ‘supposit.’ (This is for simplicity of expression only.) So we have:

‘*bishop*’ supposit for *x* iff *x* is a (presently existing) bishop

‘*dodo*’ supposit for *x* iff *x* is a presently existing dodo

‘*Magic Johnson*’ supposit for presently existing Magic Johnson

‘*Socrates*’ supposit for presently existing Socrates

<sup>9</sup> I am ignoring a view that is endorsed e.g. by Peter of Spain *SL* (11.16) according to which certain present things do not exist. For example, false propositions do not exist even if they are uttered: “All statables that are false in the present are present but not existent since nothing false exists.” For Peter’s version of this view ‘which exist at *t*’ should be replaced by ‘which are present at *t*’.

<sup>10</sup> The requirement that a term used in a present tense sentence supposit only for presently existing things is called “restriction,” a phenomenon that is discussed in Chapter 10. Before about 1400 restriction for proper names is not generally part of the theory. Certain authors (Anonymous, *Treatise on Univocation* 338; Lambert of Auxerre/Lagny *PT* 5b; Peter of Spain *LS* 9.2) explicitly say that singular terms are not restricted; others state restriction conditions only for common terms, implying thereby that singular terms are not restricted (Anonymous *Cum sit nostra* 450–1; Anonymous *Properties of Discourse* 723–5; William Sherwood *IL* 5.16.1; Walter Burley, *Longer Treatise* paras (206)–(208)). But some later authors (Marsilius of Inghen *Ampliations* 121; Paul of Venice, *Parva Logica* II.8 and passages in Ashworth 1974a II.6 pp 89ff.) give examples in which proper names are said to be amplified or restricted. Ockham *SL* II.22 seems to imply that only common terms are subject to ampliation. Buridan’s views are discussed in Chapter 10.



Since no *dodos* presently exist, '*dodo*' supposits for nothing at all. Likewise, since *Socrates* does not presently exist, '*Socrates*' supposits for nothing.

## 4.5 Truth conditions

In this section we give truth conditions for logical forms of Linguish. We have already explained in an informal way how the symbolism is to be understood, suggesting that the sentences be understood as they are pronounced in English, with the proviso that terms in affirmative sentences have existential import, and terms in negative sentences the reverse. It is not clear how this is to work when things get complicated, and so we will formulate the semantics in a more detailed and rigorous way. Generally, when students learn modern logic they are given an informal explanation of how to read the symbolism. Eventually they catch on, and are able to use the notation well. However, when one wishes to prove something about the logical system, such as a completeness theorem to the effect that any valid argument may be proved to be valid by means of a formal derivation using specified rules of inference, then one needs a rigorous statement of when a sentence in the notation is true. This will be especially important here since the notation is slightly different from the notation we have all learned in logic courses, and some of the sentences are given a slightly different interpretation than we are used to.

The techniques detailed here are modern, not medieval. Medieval writers occasionally discuss truth conditions, and these remarks will be useful as a guide. But their remarks are usually limited to a particular construction, such as the standard categorical propositions that Aristotle used, or molecular sentences that are conjunctions or disjunctions, or conditionals. The idea of giving a single unified theory was not pursued. We will pursue it here, using 20th-century techniques. Our goal is to formulate a precise semantics that agrees with the truth conditions that were discussed by medieval authors, and that permits a general definition of formal validity that will allow us to pose the questions of how adequate are the rules of inference that were discussed in medieval times.

The goal is this: given a specification of what the terms supposit for, and what things the verbs hold of, to state explicitly when an arbitrary logical form in Linguish notation is true.

We begin with what is usually called the "intended interpretation," where what the terms supposit for and what the verbs hold of is determined by the normal meanings of the words used.

### TERMS

'*donkey*' supposits for all presently existing donkeys

'*dodo*' supposits for all presently existing dodos (and thus for nothing)

'*Madonna*' supposits for presently existing Madonna

'*Socrates*' supposits for presently existing Socrates (and thus for nothing)

etc.

## VERBS

‘runs’ holds of all things that presently run

‘sings’ holds of all things that presently sing

‘is’ holds of every pair of things whose first and second members are the same

‘sees’ holds of every pair of things whose first member presently sees the second member

etc.

We will need to specify under what conditions a logical form is true. Usually this is done by making assignments to the variables in the formulas, and giving truth conditions relative to assignments to the variables. There are no variables in Linguish, so we cannot do exactly the same. But there is an equivalent technique, which is to introduce temporary names, and to give truth conditions relative to assignments to the temporary names. These names will be written ‘ $\$k$ ’ where ‘ $k$ ’ is a positive numeral. So that the formula ‘ $(\$3 \alpha) \alpha F$ ’ will be true relative to an assignment to the temporary names if and only if the thing assigned to the temporary name ‘ $\$3$ ’ is in the extension of the predicate ‘ $F$ ’. For a regular verb, the sentence ‘ $(\$5 \alpha) \alpha$  runs’ is true relative to an assignment to the temporary names if and only if the verb ‘runs’ holds of the thing assigned to the temporary name ‘ $\$5$ ’.

Is there anything like this in the medieval tradition? Perhaps. There are some writings on the topic of “proving terms,” which seem to give partial theories of truth conditions for propositions. An example is this passage from Richard Billingham’s *Speculum Puerorum* {“A Boys’ Mirror”}, in De Rijk 1982 (84):

8 First example: ‘*a man runs*.’ This proposition is mediate. And it is proved thus: A MAN RUNS: ‘*this runs*; and *this is a man*; therefore *a man runs*.’ And consequently through inferiors of it, because an inferior is that from which something follows, and not conversely.

Billingham might here be discussing the rule of inference, expository syllogism, in which case this passage may not be relevant. But in context “proving the terms” in ‘*A man runs*’ seems less like giving a proof than giving an explanation of truth conditions for that proposition, using ‘*this*’ somewhat as we will use temporary names. For example, ‘*a man runs*’ is true iff a thing—call it ‘*this*’—which is a man is such that ‘*this runs*’ is true. And ‘*this runs*’ is true relative to an assignment of a thing to the “temporary name” ‘*this*’.

This is the idea we shall pursue. We will introduce a series of temporary names:  $\$1$ ,  $\$2$ ,  $\$3$ , . . . Because they are temporary, they do not come already suppositing for things; instead we will assign them things to supposit for, relative to that assignment. We will use  $\sigma$  to stand for an arbitrary assignment of things to the temporary names.

An assignment,  $\sigma$ , to the temporary names of the language is any way of associating with each temporary name a single thing (or nothing at all).

The notation ' $\sigma(\$1)$ ' will stand for the thing, whatever it is, that  $\sigma$  assigns to the temporary name ' $\$1$ '.

#### DEFINITION OF NOTATION

If  $\sigma$  is an assignment of things to the temporary names, and if ' $\$k$ ' is a temporary name, then  $\sigma(\$k)$  is the thing, if any, that  $\sigma$  assigns to ' $\$k$ '.

If  $\sigma$  assigns nothing at all to ' $\$k$ ' then we say that  $\sigma(\$k)$  is undefined.

For the simplest type of sentence with a two-place verb, and containing no terms except for temporary names, we will want to say:

' $(\$j)_\alpha (\$k)_\beta \alpha V_\beta$ ' is true <sub>$\sigma$</sub>  iff ' $V$ ' holds of the pair  $\langle \sigma(\$j), \sigma(\$k) \rangle$ .

This says e.g. that the logical form underlying ' $\$1$  sees  $\$2$ ' is true on the assignment  $\sigma$  if and only if '*sees*' holds of the pair of things that  $\sigma$  assigns to the names  $\$1$  and  $\$2$ . So we will define truth conditions for whole propositions based on an assignment of things to the temporary names.

One-place "verbs" such as ' $\alpha$  is brown' should be treated similarly.

So let us talk about assignments of things to temporary names. We will use the Greek letters ' $\sigma$ ', ' $\sigma'$ ', ' $\sigma''$ ', and so on for assignments of things to the temporary names of the language.

Here are some assignments:

$\sigma'$  assigns Madonna to ' $\$1$ ' and assigns Socrates to ' $\$2$ ' and assigns Plato to ' $\$3$ '

$\sigma''$  assigns Madonna to both ' $\$1$ ' and ' $\$2$ ' and assigns Bono to ' $\$3$ '

$\sigma'''$  assigns Socrates to ' $\$1$ ' and assigns Bono to ' $\$2$ ' and assigns nothing at all to ' $\$3$ '

These assignments determine these identities:

$\sigma'(\$1) = \text{Madonna}$

$\sigma'(\$2) = \text{Socrates}$

$\sigma'(\$3) = \text{Plato}$

$\sigma''(\$1) = \text{Madonna}$

$\sigma''(\$2) = \text{Madonna}$

$\sigma''(\$3) = \text{Bono}$

$\sigma'''(\$1) = \text{Socrates}$

$\sigma'''(\$2) = \text{Bono}$

$\sigma'''(\$3) = \text{nothing at all}$

Once we have completed our semantic theory, it will turn out that relative to assignment  $\sigma'$ , the formula ' $(\$1 \alpha) \alpha$  sings' is true (because Madonna currently sings) and the formula ' $(\$2 \alpha) \alpha$  sings' is false (because Socrates does not currently sing) and the formula ' $(\$3 \alpha) \alpha$  sings' is false (because Plato does not currently sing). Also, relative to the assignment  $\sigma''$ , ' $(\$3 \alpha) \alpha$  sings' will be false because ' $\$3$ ' is assigned nothing at all.

For a more compact notation we will use 'true $_{\sigma}$ ' to abbreviate 'true relative to the assignment  $\sigma$ '.

'true $_{\sigma}$ ' abbreviates 'true relative to the assignment  $\sigma$ '

So we can also say:

' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma'}$  and ' $(\$2 \alpha) \alpha$  sings' is false $_{\sigma'}$  and ' $(\$3 \alpha) \alpha$  sings' is false $_{\sigma'}$

And for the other two assignments:

' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma''}$  and ' $(\$2 \alpha) \alpha$  sings' is true $_{\sigma''}$  and ' $(\$3 \alpha) \alpha$  sings' is true $_{\sigma''}$   
 ' $(\$1 \alpha) \alpha$  sings' is false $_{\sigma''}$  and ' $(\$2 \alpha) \alpha$  sings' is true $_{\sigma''}$  and ' $(\$3 \alpha) \alpha$  sings' is false $_{\sigma''}$

With this in mind, we will give a recursive description of when any formula is true on any given assignment  $\sigma$  to the temporary names. We begin our account of truth $_{\sigma}$  with atomic formulas, making use of the VERBS condition given earlier (which states which verbs hold of which things or of which pairs of things):

**ATOMIC:**

An atomic formula of the form ' $(\$1 \alpha) \alpha$  V' is true $_{\sigma}$  iff 'V' holds of  $\sigma(\$1)$ .  
 (The formula is not true $_{\sigma}$  if  $\sigma(\$1)$  is undefined.)

An atomic formula of the form ' $(\$1 \alpha)(\$2 \beta) \alpha$  V  $\beta$ ' is true $_{\sigma}$  iff 'V' holds of the pair whose first member is  $\sigma(\$1)$  and whose second member is  $\sigma(\$2)$ .  
 (The formula is not true $_{\sigma}$  if either  $\sigma(\$1)$  or  $\sigma(\$2)$  or both is undefined.)

We can now show that our theory yields the results mentioned previously. For example, ' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma'}$  because ' $(\$1 \alpha) \alpha$  sings' is an atomic formula, and so by the ATOMIC principle just given, ' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma'}$  iff 'sings' holds of  $\sigma'(\$1)$ . But  $\sigma'(\$1)$  = Madonna (given a moment ago) and by the VERBS principle 'sings' holds of a thing iff it sings. So ' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma'}$  iff Madonna sings. And since Madonna does sing, ' $(\$1 \alpha) \alpha$  sings' is true $_{\sigma'}$ . In an orderly form, this reasoning is:

'(\$1 a) a **sings**' is true <sub>$\sigma'$</sub>   
 iff <ATOMIC>  
 'sings' holds of  $\sigma''$ (\$1)  
 iff <Definition of  $\sigma''$ >  
 'sings' holds of Madonna  
 iff <VERBS>  
 Madonna sings

So the theory gives us that '(\$1 a) a **sings**' is true <sub>$\sigma'$</sub>  iff Madonna sings. The theory, of course, doesn't tell us that Madonna actually sings; it only states the conditions under which '(\$1 a) a **sings**' is true <sub>$\sigma'$</sub>

Negations work as expected:

**NEGATION:**

A formula of the form '**not**  $\phi$ ' is true <sub>$\sigma$</sub>  iff it is not the case that  $\phi$  is true <sub>$\sigma$</sub> .

(Also, we freely permute negation with singular terms when applying the semantics.)

So we can show that '**not** (\$1 a) a **sings**' is true <sub>$\sigma'$</sub>  iff it's not the case that Madonna sings:

'**not** (\$1 a) a **sings**' is true <sub>$\sigma'$</sub>   
 iff <NEGATION>  
 it is not the case that '(\$1 a) a **sings**' is true <sub>$\sigma'$</sub>   
 iff <ATOMIC>  
 it is not the case that 'sings' holds of  $\sigma''$ (\$1)  
 iff <Definition of  $\sigma''$ >  
 it is not the case that 'sings' holds of Madonna  
 iff <VERBS>  
 it is not the case that Madonna sings

Modified assignments: Shortly, we will want to talk about what happens when we take an assignment  $\sigma$  and modify it in certain ways. The following terminology will be handy:

When  $\$k$  is a temporary name, and  $t$  is a thing, ' $\sigma[\$k/t]$ ' stands for the assignment that is just like  $\sigma$  except that it assigns the thing  $t$  to the temporary name  $\$k$ .

Similarly, ' $\sigma[\$k/-]$ ' stands for the assignment that is just like  $\sigma$  except that it assigns nothing at all to  $\$k$ .

In case  $\sigma$  already assigns  $t$  to  $\$k$ ,  $\sigma[\$k/t]$  will be the same assignment as  $\sigma$ . Recall the assignment  $\sigma'$  discussed earlier:

$\sigma'$  assigns Madonna to '\$1' and assigns Socrates to '\$2' and assigns Plato to '\$3'

Consider these modifications to  $\sigma'$ :

$\sigma'[\$1/\text{Madonna}]$  is the same assignment as  $\sigma'$

$\sigma'[\$2/\text{Socrates}]$  is the same assignment as  $\sigma'$

$\sigma'[\$1/\text{Socrates}]$  is not the same assignment as  $\sigma'$

$\sigma'[\$1/-]$  is not the same assignment as  $\sigma'$

If you take an assignment and change what it assigns to a t-name, and then apply that assignment to that same t-name, you get what you changed it to. For example, for any assignment  $\sigma$ :

$$\sigma[\$1/\text{Plato}](\$1) = \text{Plato}$$

An assignment that has been modified can be modified again. For example,  $\sigma[\$1/\text{Socrates}][\$2/\text{Bono}]$  is the assignment that you get by starting with  $\sigma$  and changing what it assigns to '\$1' and then taking that changed assignment and changing what it assigns to '\$2'. It is an assignment that assigns Socrates to '\$1' and assigns Bono to '\$2' and assigns Plato to '\$3'.

#### APPLICATIONS

Using the examples given earlier, namely:

$\sigma'$  assigns Madonna to '\$1' and assigns Socrates to '\$2' and assigns Plato to '\$3'

$\sigma''$  assigns Madonna to both '\$1' and '\$2' and assigns Bono to '\$3'

$\sigma'''$  assigns Socrates to '\$1' and assigns Bono to '\$2' and assigns nothing to '\$3'

give the values of each of the following:

$\sigma''(\$3)$

$\sigma'''(\$3)$

$\sigma'''(\$2)$

$\sigma'[\$3/\text{Madonna}](\$3)$

$\sigma'[\$3/\text{Madonna}](\$1)$

$\sigma'[\$3/\text{Madonna}][\$2/\text{Bono}](\$3)$

$\sigma'[\$3/\text{Madonna}][\$2/\text{Bono}](\$1)$

$\sigma''[\$3/\text{Madonna}](\$1)$

$\sigma''[\$3/\text{Madonna}][\$2/\text{Bono}](\$3)$

$\sigma''[\$3/\text{Madonna}][\$2/-](\$1)$

$\sigma'''[\$3/\text{Madonna}][\$1/\text{Bono}](\$2)$

$\sigma'''[\$3/\text{Madonna}][\$2/\text{Bono}](\$1)$

$\sigma'''[\$3/\text{Madonna}][\$2/-](\$2)$

Next, we give the truth conditions for propositional formulas that contain ordinary denoting phrases. Applying one of our clauses for denoting phrases in what follows, we replace the outermost denoting phrase with one containing a temporary name. Then we move to the next denoting phrase, always working with the denoting phrase which has only temporary name denoting phrases to its left. Eventually we get a sentence whose denoting phrases all contain temporary names, at which point we apply the basic conditions for TERMS and for VERBS given earlier.

We will need to talk about sentences that optionally begin with denoting phrases containing temporary names. We will use ‘ $\tau_i$ ’ to stand for any string of zero or more denoting phrases with temporary names.

(SINGULAR) Denoting Phrases:

If  $\rho$  is a non-empty singular term, then ‘ $\tau_i(\rho \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/a]}$ , where  $a$  is the thing that  $\rho$  supposits for (and where ‘ $\$k$ ’ is a temporary name not already occurring in ‘ $\tau_i(\rho \alpha)\phi$ ’).

If  $\rho$  does not supposit for anything, then ‘ $\tau_i(\rho \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/-]}$ .

For example, the sentence ‘(Socrates) $_\alpha$  runs’ has no temporary names, so ‘ $\tau_i$ ’ is empty. Applying the condition SINGULAR we have

If ‘Socrates’ is a singular term, then ‘(Socrates) $_\alpha$  runs’ is  $\text{true}_\sigma$  iff ‘( $\$k \alpha$ ) runs’ is  $\text{true}_{\sigma[\$k/a]}$ , where  $a$  is the thing that ‘Socrates’ supposits for.

If ‘Socrates’ supposits for Socrates, this says that ‘(Socrates) $_\alpha$  runs’ is  $\text{true}_\sigma$  iff ‘( $\$k \alpha$ ) runs’ is  $\text{true}_{\sigma[\$k/\text{Socrates}]}$ . Then our condition ATOMIC says that this is equivalent to saying that ‘(Socrates) $_\alpha$  runs’ is  $\text{true}_\sigma$  iff ‘runs’ holds of Socrates.

For denoting phrases with common terms the conditions are:

(COMMON) Denoting Phrases:

If  $T$  is a common term that supposits for something, then ‘ $\tau_i(\text{every } T \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff for every thing  $a$  that  $T$  supposits for,  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/a]}$ .

If  $T$  does not supposit for anything, ‘ $\tau_i(\text{every } T \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff ‘ $\tau_i(\$k \alpha)\phi$ ’ is  $\text{true}_{\sigma[\$k/-]}$ .

If  $T$  is a common term that supposits for something, then ‘ $\tau_i(\text{some } T \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff for some thing  $a$  that  $T$  supposits for,  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/a]}$ .

If  $T$  does not supposit for anything, ‘ $\tau_i(\text{some } T \alpha)\phi$ ’ is  $\text{true}_\sigma$  iff ‘ $\tau_i(\$k \alpha)\phi$ ’ is  $\text{true}_{\sigma[\$k/-]}$ .

If  $T$  is a common term that supposits for something, then ' $\tau_i(\cdot T \alpha)\phi$ ' is  $\text{true}_\sigma$  iff for some thing  $a$  that  $T$  supposits for,  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/a]}$ .

If  $T$  does not supposit for anything, ' $\tau_i(\cdot T \alpha)\phi$ ' is  $\text{true}_\sigma$  iff ' $\tau_i(\$k \alpha)\phi$ ' is  $\text{true}_{\sigma[\$k/-]}$ .

If  $T$  is a common term that supposits for something, then ' $\tau_i(\text{no } T \alpha)\phi$ ' is  $\text{true}_\sigma$  iff for every thing  $a$  that  $T$  supposits for, it is not the case that  $\tau_i(\$k \alpha)\phi$  is  $\text{true}_{\sigma[\$k/a]}$ .

If  $T$  does not supposit for anything, ' $\tau_i(\text{no } T \alpha)\phi$ ' is  $\text{true}_\sigma$  iff it is not the case that ' $\tau_i(\$k \alpha)\phi$ ' is  $\text{true}_{\sigma[\$k/-]}$ .

This method of handling common terms so as to get the right truth conditions with respect to existential import is equivalent to a method first proposed (so far as I know) in Klima 1988: 24, 50.

Finally we want to talk about a sentence being simply true. If there are no temporary names in a sentence, then it will turn out that it is either true on every assignment whatsoever to the temporary names, or false on every assignment. So it is customary to say that a sentence is true iff it's true on every assignment to the temporary names:

TRUTH: A formula  $\phi$  is true iff for every assignment  $\sigma$ ,  $\phi$  is  $\text{true}_\sigma$ .

This completes the semantic theory.

To illustrate the semantics, let us show first that, assuming that Madonna exists, then according to the theory we have just given, ' $(\text{Madonna } \alpha) \alpha \text{ sings}$ ' is true iff Madonna sings. Here is a proof with each transition in the proof justified by a provision in the theory in angle brackets.

' $(\text{Madonna } \alpha) \alpha \text{ sings}$ ' is true

iff <TRUTH>

for every  $\sigma$ , ' $(\text{Madonna } \alpha) \alpha \text{ sings}$ ' is  $\text{true}_\sigma$

iff <SINGULAR DP>

for every  $\sigma$ , ' $(\$1 \alpha) \alpha \text{ sings}$ ' is  $\text{true}_{\sigma[\$1/a]}$  where  $a$  is the thing that ' $\text{Madonna}$ ' supposits for

iff <TERMS>

for every  $\sigma$ , ' $(\$1 \alpha) \alpha \text{ sings}$ ' is  $\text{true}_{\sigma[\$1/\text{Madonna}]}$

iff <ATOMIC>

for every  $\sigma$ , ' $\text{sings}$ ' holds of  $\sigma[\$1/\text{Madonna}](\$1)$

iff <DEFINITION OF NOTATION>

for every  $\sigma$ , ' $\text{sings}$ ' holds of Madonna

iff <VERBS>

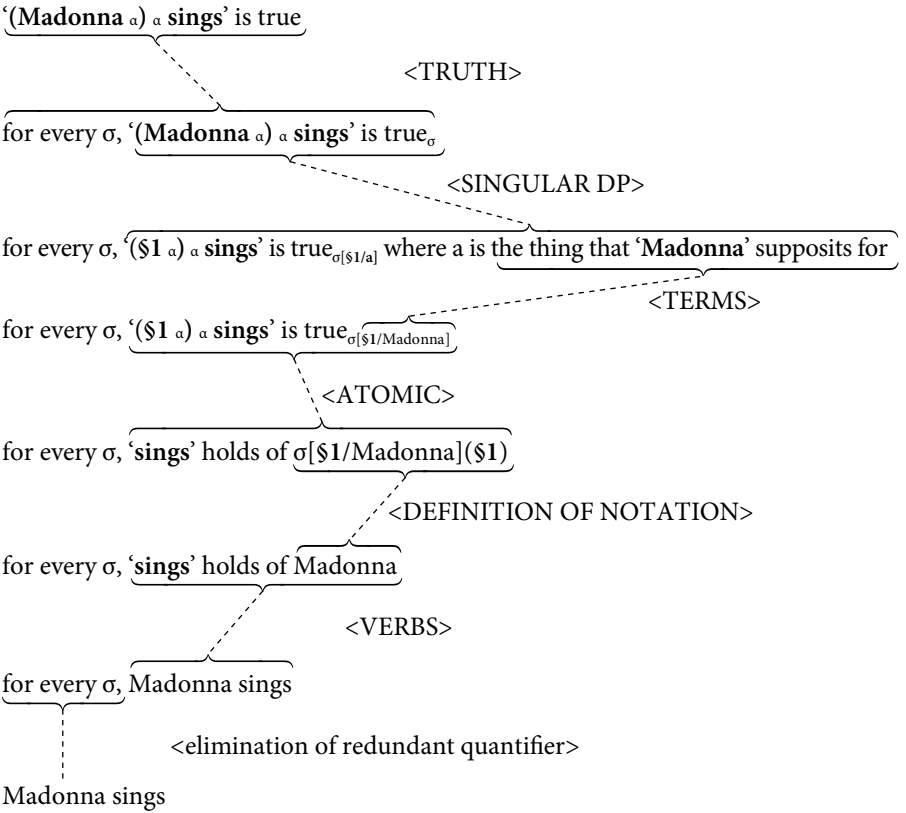
for every  $\sigma$ , Madonna sings

iff <elimination of redundant quantifier>

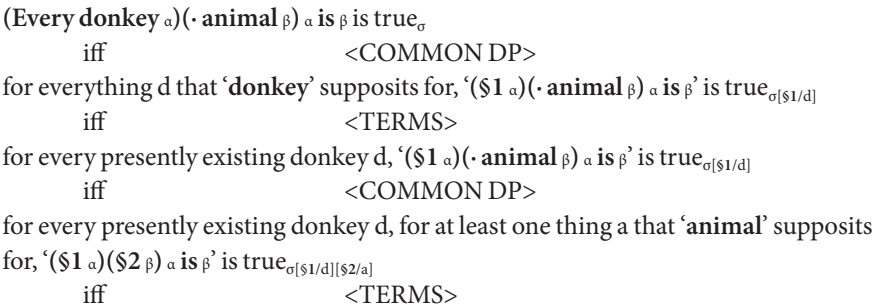
Madonna sings



This proof may be easier to follow if we identify exactly which parts of the conditions are being changed from one step to the next. Here is the same proof with those parts identified:



Here is an example with two denoting phrases. We show that, assuming there is at least one donkey, and at least one animal, then  $\text{'(Every donkey } \alpha \text{)(}\cdot \text{animal } \beta \text{)} \alpha \text{ is } \beta \text{'}$  is  $\text{true}_\sigma$  iff for every presently existing donkey  $d$ , for at least one presently existing animal  $a$ ,  $d=a$ :



for every presently existing donkey  $d$ , for at least one presently existing animal  $a$ ,

' $(\$1_\alpha)(\$2_\beta)_\alpha$  is  $\beta$ ' is  $\text{true}_{\sigma[\$1/d][\$2/a]}$   
iff <ATOMIC>

for every presently existing donkey  $d$ , for at least one presently existing animal  $a$ ,  
'is' holds of the pair whose first member is  $\sigma[\$1/d][\$2/a](\$1)$  and whose second  
member is  $\sigma[\$1/d][\$2/a](\$2)$

iff <VERBS>

for every presently existing donkey  $d$ , for at least one presently existing animal  $a$ ,  
 $\sigma[\$1/d][\$2/a](\$1) = \sigma[\$1/d][\$2/a](\$2)$

iff <DEFINITION OF NOTATION>

for every presently existing donkey  $d$ , for at least one presently existing animal  $a$ ,  
 $d=a$

If no donkeys presently exist, then '**(Every donkey  $\alpha$ )(· animal  $\beta$ )  $\alpha$  is  $\beta$** ' is not  $\text{true}_\sigma$  for any  $\sigma$ . For if there are no donkeys, '**donkey**' supposits for nothing. Then the semantics goes as follows:

'**(Every donkey  $\alpha$ )(· animal  $\beta$ )  $\alpha$  is  $\beta$** ' is  $\text{true}_\sigma$   
iff <COMMON DP>

' $(\$1_\alpha)(\cdot \text{ animal } \beta)_\alpha$  is  $\beta$ ' is  $\text{true}_{\sigma[\$1/-]}$   
iff <COMMON DP>

' $(\$1_\alpha)(\$2_\beta)_\alpha$  is  $\beta$ ' is  $\text{true}_{\sigma[\$1/-][\$2/a]}$  for at least one thing  $a$  that '**animal**' supposits for.

' $(\$1_\alpha)(\$2_\beta)_\alpha$  is  $\beta$ ' is  $\text{true}_{\sigma[\$1/-][\$2/a]}$  for at least one animal,  $a$

But  $\sigma[\$1/-][\$2/a]$  assigns nothing to ' $\$1$ ,' and ' $(\$1_\alpha)(\$2_\beta)_\alpha$  is  $\beta$ ' is never  $\text{true}_\sigma$  for any assignment  $\sigma$  that assigns nothing to one of its temporary names.

An example to show that if some donkeys presently exist, and also some stones, then '**(Some donkey  $\alpha$ ) not (· stone  $\beta$ )  $\alpha$  is  $\beta$** ' is  $\text{true}_\sigma$  iff for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ ,  $d=s$ :

**(Some donkey  $\alpha$ ) not (· stone  $\beta$ )  $\alpha$  is  $\beta$**  is  $\text{true}_\sigma$   
iff

for something  $d$  that '**donkey**' supposits for, ' $(\$1_\alpha)$  **not (· stone  $\alpha$ )  $\alpha$  is  $\beta$** ' is  $\text{true}_{\sigma[\$1/d]}$   
iff

for some presently existing donkey  $d$ , '**not  $(\$1_\alpha)(\cdot \text{ stone } \beta)_\alpha$  is  $\beta$** ' is  $\text{true}_{\sigma[\$1/d]}$   
iff

for some presently existing donkey  $d$ , it is not the case that ' $(\$1_\alpha)(\cdot \text{ stone } \beta)_\alpha$  is  $\beta$ ' is  
 $\text{true}_{\sigma[\$1/d]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently  
existing thing  $s$  that '**stone**' supposits for, ' $(\$1_\alpha)(\$2_\beta)_\alpha$  is  $\beta$ ' is  $\text{true}_{\sigma[\$1/d][\$2/s]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ , ‘ $(\S 1 \alpha)(\S 2 \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/d][\S 2/s]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ ,  $\sigma[\S 1/d][\S 2/s](\S 1) = \sigma[\S 1/d][\S 2/s](\S 2)$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ ,  $d=s$

Recall that a particular negative proposition was said to be true if its subject term is empty, even though its subject term has scope over the rest of the proposition. As an example, we can show that if no dodos presently exist (though there are stones), then ‘(Some dodo  $\alpha$ ) not ( $\cdot$  stone  $\beta$ )  $\alpha$  is  $\beta$ ’ is true.

(Some dodo  $\alpha$ ) not ( $\cdot$  stone  $\beta$ )  $\alpha$  is  $\beta$  is true $_{\sigma}$   
iff

‘ $(\S 1 \alpha)$  not ( $\cdot$  stone  $\beta$ )  $\alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/-]}$   
iff

it is not the case that ‘ $(\S 1 \alpha)(\cdot$  stone  $\beta)$   $\alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/-]}$   
iff

it is not the case that for at least one presently existing thing  $s$  that ‘stone’ supposits for, ‘ $(\S 1 \alpha)(\S 2 \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/-][\S 2/s]}$

But ‘ $(\S 1 \alpha)(\S 2 \beta) \alpha$  is  $\beta$ ’ cannot be true $_{\sigma[\S 1/-][\S 2/s]}$ , since one of its temporary names is assigned nothing. So it is indeed not the case that for at least one presently existing thing  $s$  that ‘stone’ supposits for, ‘ $(\S 1 \alpha)(\S 2 \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/-][\S 2/s]}$

(It is easy to show that the proposition is also true if there are no stones.) This example illustrates the fact that a particular negative proposition is true when its subject term is empty, even though its subject term has wide scope. This fleshes out the discussion of particular negatives in section 1.4, showing that a systematic semantics is possible which makes them true when their subject terms are empty, even when they are read with the scopes that they intuitively seem to have.

#### APPLICATIONS

Using the pattern given earlier, identify which provisions of the semantic theory justify each step:

Assuming that there are both donkeys and stones:

(Some donkey  $\alpha$ ) not ( $\cdot$  stone  $\beta$ )  $\alpha$  is  $\beta$  is true $_{\sigma}$   
iff

for something  $d$  that ‘donkey’ supposits for, ‘ $(\S 1 \alpha)$  not ( $\cdot$  stone  $\beta)$   $\alpha$  is  $\beta$ ’ is true $_{\sigma[\S 1/d]}$   
iff

for some presently existing donkey  $d$ , ‘not  $(\exists 1 \alpha)(\cdot \text{stone } \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\$1/d]}$   
iff

for some presently existing donkey  $d$ , it is not the case that  $(\exists 1 \alpha)(\cdot \text{stone } \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\$1/d]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing thing  $s$  that ‘stone’ supposits for, ‘ $(\exists 1 \alpha)(\exists 2 \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\$1/d][\$2/s]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ , ‘ $(\exists 1 \alpha)(\exists 2 \beta) \alpha$  is  $\beta$ ’ is true $_{\sigma[\$1/d][\$2/s]}$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ , ‘is’ holds of the pair  $\langle \sigma[\$1/d][\$2/s](\exists 1), \sigma[\$1/d][\$2/s](\exists 2) \rangle$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ , ‘is’ holds of the pair  $\langle d, s \rangle$   
iff

for some presently existing donkey  $d$ , it is not the case that for at least one presently existing stone  $s$ ,  $d=s$

Supposing that there are donkeys, show that ‘(No donkey  $\alpha$ )  $\alpha$  runs’ is true if and only if for every donkey  $d$  it is not the case that  $d$  runs.

Supposing that there are donkeys, and there are horses, show that ‘(Every donkey  $\alpha)(\cdot \text{horse } \beta) \alpha$  sees  $\beta$ ’ is true if and only if for every donkey  $d$  there is at least one horse  $h$  such that  $d$  sees  $h$ .

#### 4.5.1 Affirmative and negative propositions and existential import

For Aristotle, categorical propositions are either affirmative or negative. As discussed in Chapter 1, affirmative standard categorical propositions are false when their subject terms are empty, and negative standard categorical propositions are true when their subject terms are empty. This held for medieval philosophers as well. In addition, some of them expanded the scope of this principle, so that the following became widely adopted:

##### Existential import:

All main terms in an affirmative proposition have existential import; that is, if any main term of an affirmative proposition supposits for nothing, then the proposition is false. The reverse is true for negative propositions: if any main term of a negative proposition supposits for nothing, then the proposition is true.

A “main term” of a proposition is a term in that proposition that is not part of another term in the proposition. In the simple structures studied in this chapter, terms occur only as main terms.

With expanded notions of “affirmative” and “negative,” this principle of existential import is validated by the semantics given earlier. To establish these results we define when a proposition is affirmative or negative. We do this by means of a recursive characterization.

#### Affirmative and negative formulas:

Any atomic categorical formula is affirmative.

If  $\phi$  is affirmative (negative) then **not**  $\phi$  is negative (affirmative).

If  $\phi$  is affirmative (negative) then so are the following:

(every  $T_\alpha$ )  $\phi$

(some  $T_\alpha$ )  $\phi$

(·  $T_\alpha$ )  $\phi$

( $t_\alpha$ )  $\phi$

If  $\phi$  is affirmative (negative) then the following is negative (affirmative):

(**no**  $T_\alpha$ )  $\phi$

There is an easy algorithm for determining whether a categorical proposition is affirmative or negative: count the number of negative signs in it, where the negative signs are ‘not’ and ‘no.’ If the sum is odd, the proposition is negative; if it is even, the proposition is affirmative.<sup>11</sup>

Example: The proposition ‘**not** (every animal  $\alpha$ )(no bishop  $\beta$ )  $\alpha$  is  $\beta$ ’ is affirmative, so if there are no bishops it is false. And ‘(some donkey  $\alpha$ ) **not** (· stone  $\beta$ )  $\alpha$  is  $\beta$ ’ is negative, so if there are no donkeys, it is true.

## 4.6 Validity

As mentioned in Chapter 1, Aristotle’s explanation of a good deduction includes both what we would call formally valid arguments and non-formally valid ones. Medieval authors addressed this dichotomy in a variety of ways, which are not summarized here. Instead, in this section we will study formal validity in one modern-day sense. An argument is formally valid in one modern sense if and only if no way of interpreting the terms and verbs in that argument (other than the copula) produces an argument whose

<sup>11</sup> Buridan *SD* 1.5.4 (47), argues that two negations cancel each other out so far as the quality (affirmativity or negativity) is concerned.

premises are true and whose conclusion is false. For example, suppose that you want to show that this argument is not valid:

Every man is an animal  
Socrates is an animal  
∴ Socrates is a man

The argument itself has true premises and a true conclusion, so its actual truth values do not establish that it is invalid. But you could ask “What if ‘Socrates’ referred to a donkey?” Interpreted in that way, the original argument would have true premises and a false conclusion, and this will show that it is invalid. This is the idea that we will explore here. To make sense of this account, we need to say what interpreting is.

We will say that an interpretation, *I*, is any way of specifying what the terms of the language supposit for and what things the verbs hold of. For example, we will say that a name ‘*I*-supposits’ for certain things, and a verb ‘*I*-holds’ of certain things. We have already seen this in the previous section where we talked about the *intended* interpretation of the symbolism. To say e.g. that ‘donkey’ supposits for donkeys *on the intended interpretation*, we just used the unmodified term ‘supposit.’ From now on we will talk about what a term *I*-supposits for, so as to make clear which interpretation we have in mind. For example, we might want to consider an interpretation *I\** which interprets ‘donkey’ as suppositing for trees, and interprets ‘Madonna’ as suppositing for Bono, and so on. Spelling this out we might have e.g.

‘donkey’ *I\**-supposits for all presently existing trees  
 ‘dodo’ *I\**-supposits for all presently existing tamales  
 ‘Madonna’ *I\**-supposits for Bono  
 ‘Socrates’ *I\**-supposits for Michelle Obama  
  
 ‘runs’ *I\**-holds of all things that are presently eating  
 ‘sings’ *I\**-holds of all things that are presently sleeping  
 ‘is’ *I\**-holds of every pair whose first and second members are the same  
 ‘sees’ *I\**-holds of every pair whose first member owns the second member  
 etc.

Notice that ‘is’ is still given as *I\**-holding of pairs whose members are identical. This carries out the medieval idea that ‘is’ is a unique verb with a special meaning. Every other verb can vary in its interpretation.

We will need to say what it means for a logical form to be true on the interpretation *I*, that is, for it to be “*I*-true.” As before, we will need to talk about formulas being *I*-true with respect to an assignment to the temporary names; that is, a formula’s being *I*-true<sub>σ</sub>. The provisions in what follows differ hardly at all from those given previously for the intended interpretation.

TRUTH: A formula  $\phi$  is *I*-true iff for every assignment  $\sigma$ ,  $\phi$  is *I*-true<sub>σ</sub>.

The remaining provisions are the same as before, with truth replaced by I-truth.

ATOMIC:

An atomic sentence of the form ' $(\S 1 \alpha) \alpha V$ ' is I-true <sub>$\sigma$</sub>  iff ' $V$ ' I-holds of  $\sigma(\S 1)$ .  
(The sentence is not I-true <sub>$\sigma$</sub>  if  $\sigma(\S 1)$  is undefined.)

An atomic sentence of the form ' $(\S 1 \alpha)(\S 2 \beta) \alpha V \beta$ ' is I-true <sub>$\sigma$</sub>  iff ' $V$ ' I-holds of the pair whose first member is  $\sigma(\S 1)$  and whose second member is  $\sigma(\S 2)$ .  
(The sentence is not I-true <sub>$\sigma$</sub>  if either  $\sigma(\S 1)$  or  $\sigma(\S 2)$  or both is undefined.)

Negations work as expected:

NEGATION:

A sentence of the form '**not**  $\phi$ ' is I-true <sub>$\sigma$</sub>  iff it is not the case that  $\phi$  is I-true <sub>$\sigma$</sub> .

Also, we freely permute negation with singular terms when applying the semantics.

For non-temporary denoting phrases we have: (As before  $\tau_i$  is any string of zero or more denoting phrases with temporary names.)

(SINGULAR) Denoting Phrases:

If  $\rho$  is a singular term, then ' $\tau_i(\rho \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff  $\tau_i(\S k \alpha)\phi$  is I-true <sub>$\sigma[\S k/a]$</sub> , where  $a$  is the thing that  $\rho$  I-supposits for.

If  $\rho$  does not I-supposit for anything, then ' $\tau_i(\rho \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff  $\tau_i(\S k \alpha)\phi$  is I-true <sub>$\sigma[\S k/-]$</sub> .

(COMMON) Denoting Phrases:

If  $T$  is a common term that I-supposits for something, then ' $\tau_i(\text{every } T \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff for every thing  $a$  that  $T$  I-supposits for, ' $\tau_i(\S k \alpha)\phi$ ' is I-true <sub>$\sigma[\S k/a]$</sub> .

If  $T$  does not I-supposit for anything, ' $\tau_i(\text{every } T \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff ' $\tau_i(\S k \alpha)\phi$ ' is I-true <sub>$\sigma[\S k/-]$</sub> .

If  $T$  is a common term that I-supposits for something, then ' $\tau_i(\text{some } T \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff for some thing  $a$  that  $T$  I-supposits for, ' $\tau_i(\S k \alpha)\phi$ ' is I-true <sub>$\sigma[\S k/a]$</sub> .

If  $T$  does not I-supposit for anything, ' $\tau_i(\text{some } T \alpha)\phi$ ' is I-true <sub>$\sigma$</sub>  iff ' $\tau_i(\S k \alpha)\phi$ ' is I-true <sub>$\sigma[\S k/-]$</sub> .

If  $T$  is a common term that I-supposits for something, then ' $\tau_i(\cdot T \alpha) \phi$ ' is I-true <sub>$\sigma$</sub>  iff for some thing  $a$  that  $T$  I-supposits for, ' $\tau_i(\$k \alpha) \phi$ ' is I-true <sub>$\sigma[\$k/a]$</sub> .

If  $T$  does not I-supposit for anything, ' $\tau_i(\cdot T \alpha) \phi$ ' is I-true <sub>$\sigma$</sub>  iff ' $\tau_i(\$k \alpha) \phi$ ' is I-true <sub>$\sigma[\$k/\cdot]$</sub> .

If  $T$  is a common term that I-supposits for something, then ' $\tau_i(\mathbf{no} T \alpha) \phi$ ' is I-true <sub>$\sigma$</sub>  iff for every thing  $a$  that  $T$  I-supposits for, it is not the case that ' $\tau_i(\$k \alpha) \phi$ ' is I-true <sub>$\sigma[\$k/a]$</sub> .

If  $T$  does not I-supposit for anything, ' $\tau_i(\mathbf{no} T \alpha) \phi$ ' is I-true <sub>$\sigma$</sub>  iff it is not the case that ' $\tau_i(\$k \alpha) \phi$ ' is I-true <sub>$\sigma[\$k/\cdot]$</sub> .

#### APPLICATIONS

Interpretations treat equipollent expressions equivalently. Show that the following holds for any interpretation  $I$ :

'**not not**  $\phi$ ' is I-true if and only if ' $\phi$ ' is I-true

We can now define formal validity for arguments consisting of propositions, that is, of sentences with no temporary names.

An argument is formally valid iff for every interpretation,  $I$ , if the premises of the argument are all I-true then the conclusion is also I-true.

(The following two sections are technical in nature; they may be skipped without loss of continuity. They do not contain any innovative techniques.)

## 4.7 Completeness of the rules

Now that we have an explicit definition of validity, the question arises as to whether the rules we have proposed are adequate to provide derivations for all valid arguments. Completeness is the condition that all (formally) valid arguments can be shown valid by derivations using the rules given earlier.

**Completeness:** For any argument from a set of propositions  $\Gamma$  to a proposition  $\phi$ , if the argument is valid then there is a derivation leading from  $\Gamma$  to  $\phi$  (where no name in  $\phi$  is introduced by rule EX).



When we say that there is a derivation leading from  $\Gamma$  to  $\phi$  we mean that there exists an actual derivation whose premises are members of  $\Gamma$  and which ends with the proposition  $\phi$  (which is not within a subderivation). In this section we will use well-known contemporary techniques to prove that the logical system described in earlier sections is complete. The technique we will use for proving completeness was developed by the logician Leon Henkin in the late 1940s.

It will be useful to have some terminology. We will say that a set of sentences  $\Gamma$  is *d-consistent* (consistent with respect to derivability) iff there is no proposition  $\phi$  such that ' $\phi$ ' is derivable from  $\Gamma$  and '**not**  $\phi$ ' is also derivable from  $\Gamma$ . We will then prove a theorem from which completeness will follow:

**Theorem HMT: The Henkin Modeling Theorem:** If  $\Gamma$  is d-consistent then there is an interpretation  $I$  such that every sentence in  $\Gamma$  is  $I$ -true.

We will prove the HMT theorem, and then we will use this theorem to prove completeness.

Since the quantifier equipollences are both valid and derivable, it will suffice to show that the theorem holds for a stripped-down version of the language. In fact, using the equipollences we may eliminate every use of '**'**' or '**every**' or '**no**' by a combination of '**not**' and '**some**'. So we may concentrate on sentences where common terms  $T$  occur only within denoting phrases of the form '**some**  $T$ '.

The theorem, HMT, will be proved shortly from three lemmas. The first lemma says that any d-consistent set of propositions  $\Gamma_0$  may be embedded in a maximal d-consistent set  $\Gamma$  with witnesses.

**Lemma One:** For any d-consistent set of sentences  $\Gamma_0$  there is a set of sentences  $\Gamma$  such that:

1.  $\Gamma_0 \subseteq \Gamma$
2.  $\Gamma$  is d-consistent
3.  $\Gamma$  is maximal, in the sense that for every proposition  $\phi$ , either ' $\phi$ '  $\in \Gamma$  or '**not**  $\phi$ '  $\in \Gamma$
4. Particular sentences in  $\Gamma$  have "witnesses", i.e. for every sentence in  $\Gamma$  of the form '**(Some**  $T_\alpha$ )  $\phi$ ', if '<**T** is non-empty>' is also in  $\Gamma$ , there are sentences of the form '**(t <sub>$\alpha$</sub> )**(**some**  $T_\beta$ )  $\alpha$  is  $\beta$ ' and '**(t <sub>$\alpha$</sub> )**  $\phi$ ' in  $\Gamma$ . ('**t**' is called the witness for '**(Some**  $T_\alpha$ )  $\phi$ '.)


### Proof of Lemma One

We will show how, starting with a d-consistent set of sentences  $\Gamma_0$ , we can produce a set  $\Gamma$  containing every sentence in  $\Gamma_0$  such that  $\Gamma$  satisfies the provisions of Lemma One.

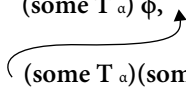
To begin, we choose an infinite set of new names  $n_0, n_1, n_2, \dots$  which do not occur in any sentence in  $\Gamma_0$ . (These names will be used to provide the witnesses just mentioned.) Then we make a sequence of all sentences in the language made from the vocabulary of the sentences in  $\Gamma_0$  together with the new names  $n_0, n_1, n_2, \dots$ . The sequence of sentences will be laid out as:

□, □, □, □, □, □, □, □, □, □, ...

Next, we insert a sentence in front of the sequence that says that  $n_0$  is empty:


 □, □, □, □, □, □, □, □, □, □, ...  
**not  $(n_0 \alpha)(n_0 \beta) \alpha$  is  $\beta$**

Then for each sentence in the sequence of the form ‘(some T  $\alpha$ )  $\phi$ ’ we insert three sentences right after it:

□, □, (some T  $\alpha$ )  $\phi$ ,  □, □, □, □, ...  
 (some T  $\alpha$ )(some T  $\beta$ )  $\alpha$  is  $\beta$ ,  $(n_j \alpha)$ (some T  $\beta$ )  $\alpha$  is  $\beta$ ,  $(n_j \alpha) \phi$

where  $n_j$  is the first of the newly introduced names which does not occur in any sentence in the sequence so far.

Call the sequence that results from making these changes sentences  $S_1, S_2, S_3, \dots$

$S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad \dots$

We then make a corresponding sequence of sets of sentences  $\Gamma_k$ :

$S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad \dots$   
 $\Gamma_0 \quad \Gamma_1 \quad \Gamma_2 \quad \Gamma_3 \quad \Gamma_4 \quad \Gamma_5 \quad \dots$

These sets of sentence are constructed as follows. The first member is our original set  $\Gamma_0$ . Then each set  $\Gamma_{n+1}$  is formed as follows:

If  $\Gamma_n \cup \{S_{n+1}\}$  is d-consistent,  $\Gamma_{n+1} = \Gamma_n \cup \{S_{n+1}\}$   
 If  $\Gamma_n \cup \{S_{n+1}\}$  isn't d-consistent,  $\Gamma_{n+1} = \Gamma_n$

that is, if  $S_{n+1}$  is consistent with what we have so far, we add it to what we have; otherwise we ignore it and go on.

APPLICATIONS

1. Show that for each  $n \geq 0$ ,  $\Gamma_n$  is d-consistent.
2. Show that for each  $n \geq 0$ ,  $\Gamma_n \subseteq \Gamma_{n+1}$ .
3. Show that for each  $n \geq 1$ , if  $S_n$  is not a member of  $\Gamma_n$ , then the negation of  $S_n$  is derivable from  $\Gamma_n$ .
4. Show that if  $S_n \in \Gamma_m$  for some  $m$ , then  $S_n \in \Gamma_n$ .
5. Show that if  $S_n$  is derivable from  $\Gamma_{n-1}$ , then  $S_n \in \Gamma_n$ .
6. Show that ‘**not  $(n_0 \alpha)(n_0 \beta) \alpha$  is  $\beta$** ’ is in  $\Gamma_1$ .

Finally, let  $\Gamma$  be the set containing every sentence in any one of the  $\Gamma_n$ 's. This set of sentences,  $\Gamma$ , will satisfy the conditions of Lemma One.

**Proof that  $\Gamma$  has the properties 1–4 listed in Lemma One**

1.  $\Gamma_0 \subseteq \Gamma$ :

Proof: The sequence  $\Gamma_0, \Gamma_1, \dots, \Gamma_n, \dots$  was constructed by starting with  $\Gamma_0$ . And  $\Gamma$  contains every sentence in any set in the sequence. So it contains all of the sentences in  $\Gamma_0$ .

2.  $\Gamma$  is d-consistent:

Proof: Notice first that each of the sets  $\Gamma_0, \Gamma_1, \dots, \Gamma_n, \dots$  is d-consistent, because the initial set  $\Gamma_0$  is d-consistent by hypothesis, and each further set is either the same as its predecessor, or it differs from its predecessor by containing a sentence which is d-consistent with the others. If the final set  $\Gamma$  were not d-consistent then for some sentence  $S$  you could derive both ‘ $S$ ’ and ‘**not**  $S$ ’ from  $\Gamma$ . The derivation itself would be finite, so it would use only a finite number of sentences from  $\Gamma$  as premises. So there would be some set  $\Gamma_n$  that contains all of the premises. But that  $\Gamma_n$  is d-consistent, so there isn’t any such derivation.

3.  $\Gamma$  is maximal, in the sense that for every proposition ‘ $\phi$ ’, either ‘ $\phi$ ’  $\in \Gamma$  or ‘**not**  $\phi$ ’  $\in \Gamma$ : Proof: Suppose not. Then for some sentence ‘ $\phi$ ’, neither ‘ $\phi$ ’ nor ‘**not**  $\phi$ ’ is in any  $\Gamma_n$ . Now for some integers  $n$  and  $m$ , ‘ $\phi$ ’ is  $S_n$  and ‘**not**  $\phi$ ’ is  $S_m$ . Since ‘ $\phi$ ’ is not in  $\Gamma$ , that means that  $S_n$  was not added to  $\Gamma_{n-1}$  when forming  $\Gamma_n$ . So  $S_n$  is not d-consistent with  $\Gamma_{n-1}$ . But then its negation, namely ‘**not**  $\phi$ ’ is derivable from  $\Gamma_{n-1}$ , and thus from  $\Gamma$ .

Similarly, the negation of  $S_m$ , namely ‘ $\phi$ ’, is derivable from  $\Gamma_{m-1}$ , and thus from  $\Gamma$ . Since both ‘**not**  $\phi$ ’ and ‘ $\phi$ ’ are derivable from  $\Gamma$ ,  $\Gamma$  isn’t d-consistent, contrary to what was proved earlier.

4. Particular sentences in  $\Gamma$  have “witnesses”; that is, for each sentence in  $\Gamma$  of the form ‘(Some  $T_\alpha$ )  $\phi$ ’, if ‘ $\langle T \text{ is non-empty} \rangle$ ’ is also in  $\Gamma$ , there are sentences of the form ‘( $n_j \alpha$ ) (some  $T_\beta$ )  $\alpha$  is  $\beta$ ’ and ‘( $n_j \alpha$ )  $\phi$ ’ is in  $\Gamma$ .

Proof: Suppose that ‘(Some  $T_\alpha$ )  $\phi$ ’ is in  $\Gamma$  and that ‘ $\langle T \text{ is non-empty} \rangle$ ’ is also in  $\Gamma$ . Let  $S_n$  be the earliest occurrence of ‘(Some  $T_\alpha$ )  $\phi$ ’ in the sequence  $S_1, S_2, \dots$ . Then ‘(Some  $T_\alpha$ )  $\phi$ ’  $\in \Gamma_n$ . By construction of the sequence, ‘ $\langle T \text{ is non-empty} \rangle$ ’ is  $S_{n+1}$  and so ‘ $\langle T \text{ is non-empty} \rangle$ ’  $\in \Gamma_{n+1}$ . By rule EX, both ‘( $n_j \alpha$ ) (some  $T_\beta$ )  $\alpha$  is  $\beta$ ’ and ‘( $n_j \alpha$ )  $\phi$ ’ are derivable from  $\Gamma_{n+1}$ , and so they will be members of  $\Gamma_{n+2}$  and  $\Gamma_{n+3}$ , which means that they will both be members of  $\Gamma$ .

This concludes Lemma One.

**APPLICATIONS**

7. Show that for any subset  $\Delta$  of  $\Gamma$ , if  $\phi$  is derivable from  $\Delta$  then  $\phi \in \Gamma$ .

**Lemma Two:** We now construct a certain useful interpretation  $I$  for the language. To assist us in this, we first define what we mean by a name-set.

For each name 'n' let '[n]' be the set of all names 'm' in the language which appear in a sentence in  $\Gamma$  of the form ' $(m_\alpha)(n_\beta) \alpha$  is  $\beta$ '. Call any such set a name-set. If a name 'n' does not appear in a sentence in  $\Gamma$  of the form ' $(m_\alpha)(n_\beta) \alpha$  is  $\beta$ ', then [n] will be empty. Otherwise [n] will contain 'n' itself, typically together with other names.

#### APPLICATIONS

8. Show that if [n] is not empty, then n is a member of [n].
9. Show that these are all equivalent:

' $(m_\alpha)(n_\beta) \alpha$  is  $\beta$ ' is in  $\Gamma$

' $(m_\alpha)(n_\beta) \beta$  is  $\alpha$ ' is in  $\Gamma$

' $(n_\alpha)(m_\beta) \alpha$  is  $\beta$ ' is in  $\Gamma$

' $(n_\alpha)(m_\beta) \beta$  is  $\alpha$ ' is in  $\Gamma$

To say what I is, we need only say how it interprets the terms and how it interprets the verbs.

#### *Interpretation of the terms:*

Singular terms: Each singular term 'n' is to I-supposit for its own name-set [n] if [n] is non-empty; otherwise 'n' is to I-supposit for nothing.

Common terms: Any common term 'T' is to I-supposit for each name-set u such that for some name 'n'  $\in$  u, the sentence ' $(n_\alpha)(\text{some } T_\beta) \alpha$  is  $\beta$ ' is a member of  $\Gamma$ .

#### *Interpretation of the verbs:*

Each verb 'V' that takes a single grammatical marker I-holds of a non-empty name-set u if and only if there is some name 'n' in u such that ' $(n_\alpha) \alpha$  V' is a member of  $\Gamma$ .

Each verb 'V' that takes two grammatical markers I-holds of a pair of non-empty name-sets  $\langle u, v \rangle$  if and only if there are names 'n'  $\in$  u and 'm'  $\in$  v such that ' $(n_\alpha)(m_\beta) \alpha$  V  $\beta$ ' is a member of  $\Gamma$ .

#### APPLICATIONS

10. Show that ' $(n_\alpha)(m_\beta) \alpha$  is  $\beta$ ' is I-true in this interpretation if and only if 'n' and 'm' have the same non-empty name-set.
11. Show that ' $(n_\alpha)(m_\beta) \alpha$  V  $\beta$ ' is I-true if and only if 'V' holds of  $\langle [m], [n] \rangle$ .

This completes the specification of I, and thus of Lemma Two.

**Lemma Three:** A sentence S is I-true iff S is a member of  $\Gamma$ .

Note that the permutations of singular terms, and double negation hold both semantically, and by means of rules of inference. These will be taken for granted. Also, because of consistency and maximality, if Lemma Three holds for 'S' it also holds for 'not S.' This will be taken for granted later.

APPLICATIONS

12. Show that if a sentence S is I-true iff S is a member of  $\Gamma$ , then not S is I-true iff not S is a member of  $\Gamma$ .

The proof of the lemma proceeds by induction on the number of common terms in S. First we show (the "basis step") that the lemma holds for all sentences that contain no common terms. Then we show (the "induction step") that if the lemma holds for all sentences with k or fewer common terms, then it holds for all sentences with k+1 common terms.

**Basis Step: There are no common terms in S.**

*Form with no negations:* Suppose that S is of the form '(n  $\alpha$ )(m  $\beta$ )  $\alpha$  V  $\beta$ '

Case 1: Suppose that 'n' and 'm' both I-supposit for something. Then:

'(n  $\alpha$ )(m  $\beta$ )  $\alpha$  V  $\beta$ ' is I-true  
 iff <TRUTH>  
 for any  $\sigma$ , '(n  $\alpha$ )(m  $\beta$ )  $\alpha$  V  $\beta$ ' is I-true $_{\sigma}$   
 iff <SINGULAR DP>  
 for any  $\sigma$ , '(\$1  $\alpha$ )(\$2  $\beta$ )  $\alpha$  V  $\beta$ ' is I-true $_{\sigma[1/a] [2/b]}$  where a and b are what 'n' and 'm'  
 I-supposit for  
 iff <TERMS>  
 for any  $\sigma$ , '(\$1  $\alpha$ )(\$2  $\beta$ )  $\alpha$  V  $\beta$ ' is I-true $_{\sigma[1/[n]] [2/[m]]}$   
 iff <ATOMIC>  
 for any  $\sigma$ , 'V' I-holds of < $\sigma[1/[n]] [2/[m]]$ (\$1),  $\sigma[1/[n]] [2/[m]]$ (\$2)>  
 iff <DEFINITION OF NOTATION>  
 for any  $\sigma$ , 'V' I-holds of <[n], [m]>  
 iff <REDUNDANT QUANTIFIER>  
 'V' I-holds of <[n], [m]>  
 iff <CONSTRUCTION OF I – application 11 above>  
 '(n  $\alpha$ )(m  $\beta$ )  $\alpha$  V  $\beta$ ' is in  $\Gamma$

(Recall that when I was constructed we made any two-place verb I-true of the pairs of name-sets of names that occur with the verb in a sentence in  $\Gamma$ .)

Case 2: Suppose that 'n' or 'm' I-supposit for nothing.

Then  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  is not I-true.

To show that  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  isn't a member of  $\Gamma$ :

For reductio, suppose that it is a member. Since it entails  $\langle n \text{ is non-empty} \rangle$  and  $\langle m \text{ is non-empty} \rangle$  they too are in  $\Gamma$ . But then the name-sets for 'n' and 'm' aren't empty, and so neither 'n' nor 'm' I-supposit for nothing, as was assumed.

In summary, if 'n' and 'm' both I-supposit for something, then  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  is I-true iff  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  is in  $\Gamma$ .

And if either 'n' or 'm' I-supposit for nothing then  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  isn't I-true and also it isn't in  $\Gamma$ , so again  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  is I-true iff  $\langle (n_\alpha)(m_\beta)_\alpha V_\beta \rangle$  is in  $\Gamma$ .

*Negative form:* As noted previously, if the lemma holds for S, it also holds for **not-S**.

Because of double negation and the permutations, any logical form that contains exactly two singular terms is equivalent to another form that either contains no negations, or contains a single negation on the front. So the lemma holds of every formula which contains no common terms and contains two singular terms.

It is easy to also prove the lemma for the case in which the formula contains exactly one singular term and no common terms.

#### APPLICATIONS

13. Show that the lemma holds when S is of the form  $\langle (n_\alpha)_\alpha V \rangle$ .

**Induction Step:** Assume that Lemma Three is true for sentences containing  $\leq k$  common terms. (This is the "inductive hypothesis.") Show that it is true for any sentence with  $k+1$  terms.

(So far in this text we have only discussed sentences with at most two common terms. But later on we will extend the notation so that a sentence may contain any number of common terms. We give an argument here that can be extended later when additional vocabulary and syntactic structures are added.)

We may assume that there is a common denoting phrase to the left of all other denoting phrases, since the only other alternative is for a singular denoting phrase to come first, and it may be permuted to the right. So S is of the form  $\langle (\text{some } P_\alpha) \phi \rangle$  or  $\langle \text{not } (\text{some } P_\alpha) \phi \rangle$ . If we prove the lemma for the first form, then by consistency and maximality it will hold for the second form as well. We will first show

I. if  $\langle (\text{some } P_\alpha) \phi \rangle$  is in  $\Gamma$ , it is I-true,

and then we will show

II. if  $\langle (\text{some } P_\alpha) \phi \rangle$  is I-true, then it is in  $\Gamma$ .

I. Suppose that  $\langle (\text{some } P_\alpha) \phi \rangle$  is in  $\Gamma$ . To show that  $\langle (\text{some } P_\alpha) \phi \rangle$  is I-true.

By the construction of  $\Gamma$ , ' $(\text{some } P \alpha) \phi$ ' is  $S_n$  for some  $n$ , and ' $\langle P \text{ is non-empty} \rangle$ ' is  $S_{n+1}$ , followed by ' $(n_j \alpha)(\text{some } P \beta) \alpha \text{ is } \beta$ ' and then by ' $(n_j \alpha) \phi$ '. There are two cases to consider:

Case 1: ' $\langle P \text{ is non-empty} \rangle$ ' is added to  $\Gamma_{n+1}$  and is in  $\Gamma$ . By section 4 of the proof of Lemma 1, ' $(n_j \alpha)(\text{some } P \beta) \alpha \text{ is } \beta$ ' and ' $(n_j \alpha) \phi$ ' are also in  $\Gamma$ . By the construction in Lemma 2, since ' $(n_j \alpha)(\text{some } P \beta) \alpha \text{ is } \beta$ ' is in  $\Gamma$ , ' $P$ ' I-supposits for  $[n_j]$ , and thus ' $(n_j \alpha)(\text{some } P \beta) \alpha \text{ is } \beta$ ' is I-true. Since ' $(n_j \alpha) \phi$ ' is in  $\Gamma$  and ' $(n_j \alpha) \phi$ ' has fewer common terms than ' $(\text{some } P \alpha) \phi$ ', by the inductive hypothesis, ' $(n_j \alpha) \phi$ ' is I-true. Since ' $(n_j \alpha)(\text{some } P \beta) \alpha \text{ is } \beta$ ' and ' $(n_j \alpha) \phi$ ' are both I-true, so is ' $(\text{some } P \alpha) \phi$ '.

Case 2: ' $\langle P \text{ is non-empty} \rangle$ ' is not added to  $\Gamma_{n+1}$  and thus is not in  $\Gamma$ . Then by maximality ' $\text{not } \langle P \text{ is non-empty} \rangle$ ' is in  $\Gamma$ . Notice that ' $\text{not } \langle n_0 \text{ is non-empty} \rangle$ ' is also in  $\Gamma$ . By the principle of substitution of empties, ' $(\text{some } P \alpha) \phi$ ' together with the emptiness of both ' $P$ ' and ' $n_0$ ' entails ' $(n_0 \alpha) \phi$ ', which is thus in  $\Gamma$ . This has fewer common terms than ' $(\text{some } P \alpha) \phi$ ', and so by the inductive hypothesis ' $(n_0 \alpha) \phi$ ' is I-true.

Now ' $\text{not } \langle P \text{ is non-empty} \rangle$ ' must also be I-true, because otherwise ' $P$ ' would I-supposit for something of the form  $[u]$ ; so by construction of I, a sentence of the form ' $(\text{some } P \alpha)(u \beta) \alpha \text{ is } \beta$ ' would be in  $\Gamma$ , which would entail ' $\langle P \text{ is non-empty} \rangle$ ', contradicting the fact that ' $\langle P \text{ is non-empty} \rangle$ ' is not in  $\Gamma$ .

Since ' $\text{not } \langle n_0 \text{ is non-empty} \rangle$ ' has no common terms, by the inductive hypothesis, it is also I-true. But then these are all I-true:

**not**  $\langle P \text{ is non-empty} \rangle$   
**not**  $\langle n_0 \text{ is non-empty} \rangle$   
 $(n_j \alpha) \phi$

and so by the substitutivity of empties, ' $(\text{some } P \alpha) \phi$ ' is I-true.

II. Suppose that ' $(\text{some } P \alpha) \phi$ ' is I-true. To show that ' $(\text{some } P \alpha) \phi$ ' is in  $\Gamma$ .

Now ' $P$ ' may or may not I-supposit for something. There are two cases to consider:

Case 1: ' $P$ ' I-supposits for something.

By our semantic theory we have:

' $(\text{some } P \alpha) \phi$ ' is I-true

iff  $\langle \text{TRUTH} \rangle$

For every  $\sigma$ , ' $(\text{some } P \alpha) \phi$ ' is I-true $_\sigma$

iff  $\langle \text{COMMON DP } \langle \text{non-empty case} \rangle \rangle$

For every  $\sigma$ , for some name-set  $[v]$  for which ' $P$ ' I-supposits, ' $(\$1 \alpha) \phi$ ' is I-true $_{\sigma[\$1/[v]]}$

iff  $\langle \text{since 'v' I-supposits for } [v] \rangle$

For every  $\sigma$ , for the name-set  $[v]$  for which ' $v$ ' I-supposits, ' $(\$1 \alpha) \phi$ ' is I-true $_{\sigma[\$1/[v]]}$

iff  $\langle \text{SINGULAR DP } \langle \text{non-empty case} \rangle \rangle$

For every  $\sigma$ , ' $(v \alpha) \phi$ ' is I-true $_\sigma$

iff  $\langle \text{TRUTH} \rangle$

' $(v \alpha) \phi$ ' is I-true

So ' $(v_\alpha) \phi$ ' is I-true, where ' $P$ ' I-supposits for ' $[v]$ '

By the inductive hypothesis, ' $(v_\alpha) \phi$ ' is in  $\Gamma$ .

Since ' $P$ ' I-supposits for ' $[v]$ ', by the construction of I, ' $(\text{some } P_\alpha)(v_\beta)_\alpha \text{ is } \beta$ ' is in  $\Gamma$ .

By maximality and consistency, ' $(v_\beta)(\text{some } P_\alpha)_\alpha \text{ is } \beta$ ' is also in  $\Gamma$ .

Then by the semantic version of rule ES, ' $(\text{some } P_\alpha) \phi$ ' is in  $\Gamma$ .

Case 2: ' $P$ ' does not I-supposit for anything.

Notice first that ' $\text{not } \langle P \text{ is non-empty} \rangle$ ' is in  $\Gamma$ , since other wise ' $\langle P \text{ is non-empty} \rangle$ ' would be in  $\Gamma$ , and since this is an existentially quantified sentence, ' $(\text{some } P_\alpha) (\text{some } P_\beta)_\alpha \text{ is } \beta$ ', it would have a witness in  $\Gamma$  of the form ' $(n_j)_\alpha (\text{some } P_\beta)_\alpha \text{ is } \beta$ ', and then by the construction of I, ' $P$ ' would I-supposit for ' $[n_j]$ ', contrary to the assumption of Case 2.

Now by our semantic theory we have:

' $(\text{some } P_\alpha) \phi$ ' is I-true

iff  $\langle \text{TRUTH} \rangle$

For every  $\sigma$ , ' $(\text{some } P_\alpha) \phi$ ' is I-true $_\sigma$

iff  $\langle \text{COMMON DP } \langle \text{empty case} \rangle \rangle$

For every  $\sigma$ , ' $(S1_\alpha) \phi$ ' is I-true $_{\sigma[S1/[-]]}$

iff  $\langle \text{SINGULAR DP } \langle \text{empty case} \rangle \rangle$

For every  $\sigma$ , ' $(n_0)_\alpha \phi$ ' is I-true $_\sigma$

iff  $\langle \text{TRUTH} \rangle$

' $(n_0)_\alpha \phi$ ' is I-true

So ' $(n_0)_\alpha \phi$ ' is I-true.

By the inductive hypothesis, ' $(n_0)_\alpha \phi$ ' is in  $\Gamma$ .

So then these are all in  $\Gamma$ :

$\text{not } \langle P \text{ is non-empty} \rangle$

$\text{not } \langle n_0 \text{ is non-empty} \rangle$

$(n_0)_\alpha \phi$

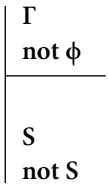
and so by the rule of substitutivity of empties, ' $(\text{some } P_\alpha) \phi$ ' is also in  $\Gamma$ .

This completes the proof of HMT. We may now proceed to prove the Completeness Theorem.

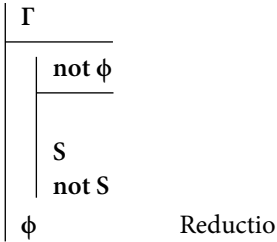
**Completeness Theorem:** For any argument from a set of propositions  $\Gamma$  to a proposition  $\phi$ , if the argument is valid then there is a derivation leading from  $\Gamma$  to  $\phi$  (where no name in  $\phi$  is introduced by rule EX).

**Proof:** We will show this using a reductio argument. Suppose that completeness does not hold, that is, there is an argument from some set of sentences  $\Gamma$  to a sentence  $\phi$  that is valid, but there is no derivation of  $\phi$  from  $\Gamma$ . Then we can show that  $\Gamma \cup \{\text{not } \phi\}$  must be d-consistent. ( $\Gamma \cup \{\text{not } \phi\}$  is the set containing every sentence in  $\Gamma$  and also containing ' $\text{not } \phi$ '.) This is so because if  $\Gamma \cup \{\text{not } \phi\}$  were not d-consistent, one could derive contradictory propositions from  $\Gamma \cup \{\text{not } \phi\}$ . That is, there would be a derivation of the form:





But then one could rearrange the parts of that derivation as follows:



The subderivation is to consist of exactly the same sentences as the whole derivation earlier. This reasoning is justified because when applying our rules of inference, it doesn't matter whether '**not  $\phi$** ' is an original premise or the assumption of a subderivation. The result then is a subderivation leading from '**not  $\phi$** ' to contradictory propositions, and so our reductio rule lets us add ' $\phi$ ' following the subderivation. The result is a derivation leading from  $\Gamma$  to ' $\phi$ ', contradicting our assumption that there is no such derivation.

So  $\Gamma \cup \{\text{not } \phi\}$  is d-consistent. But then by the HMT theorem there is an interpretation  $I$  such that every sentence in  $\Gamma \cup \{\text{not } \phi\}$  is  $I$ -true. But this is an interpretation  $I$  in which every sentence in  $\Gamma$  is  $I$ -true and  $\phi$  is not  $I$ -true. So by the definition of validity, the argument from  $\Gamma$  to  $\phi$  is not valid, contradicting the assumption we made at the beginning of the original reductio argument.

So the Completeness Theorem is proved.

# 5

## Expanding the Notation

In the previous chapter a notation, *Linguish*, was devised. It is intended to be a perspicuous presentation of the grammatical structures of the sentences that medieval logicians dealt with. It encodes only information clearly available to medieval writers. The fundamental rules of inference that were included are confined to rules that were explicitly stated in the literature, or were clearly expressed (with the possible exception of Aristotle's use of exposition applied to a negative proposition). Truth conditions were stated using modern techniques, but the inputs to the provisions used (in terms of the significations and suppositions of terms) are those of the medievals, and the outputs are intended to match the truth conditions of those propositions that were explicitly discussed.

Medieval logicians make free use of forms of language that go far beyond those generated in the previous chapter. The goal of this chapter is to expand the notation to include some of these rich forms, and to add rules of inference to govern them. The notation continues to encode only grammatical information available to medieval logicians. The rules will sometimes go beyond those that were stated by the authors, and this will need discussion. The truth conditions, though expressed using modern notation, are still intended to agree with those attributed to propositions by the authors.

In this chapter we continue to focus on non-modal present tense propositions, leaving additional complications to Chapter 10.

### 5.1 Adjectives

Aristotle's logic is problematic when applied to propositions that have adjectives as predicates. If you apply simple conversion to:

*Some donkey is grey*

you get

*Some grey is [a] donkey*

which, according to some authors, is ungrammatical; a quantifier word like 'some' must combine with a noun, not with an adjective like 'grey'.<sup>1</sup>

<sup>1</sup> Buridan *SD* 4.2.6: "in a conversion the subject should become the predicate and the predicate should become the subject, and an adjective cannot become the subject in itself, unless it gets substantivated."

This problem has a straightforward solution. In Latin it is possible to “substantivate” an adjective, which is to use it as a common noun.<sup>2</sup> (Some authors hold that when this is done, the new noun ends up with a neuter gender, but this idea does not seem to be universal.) For example, the earlier sentence:

*Some donkey is grey*

contains the adjective ‘grey’ which, in this sentence, would bear a singular nominative masculine inflection to agree with ‘donkey.’ But in this sentence:

*Every grey is tired*            (Meaning ‘Every grey thing is tired.’)

it is substantivated; it bears a neuter inflection of its own, and it is essentially a noun. The adjective ‘tired’ is still an adjective; it has an adjectival inflection that agrees with the case and number of ‘grey.’

Since substantivated adjectives literally are nouns, they are already automatically included in the language we have developed. To make this clear, I will append ‘-thing’ to each adjective used substantivally in the logical notation, with the understanding that the result is a common term.<sup>3</sup> In Latin the appended ‘-thing’ is invisible; the status of a term as a noun is understood by means of the fact that the word takes the inflections of a noun. In English the ‘thing’ is needed to produce a (complex) noun. The logical form:

(Every grey-thing<sub>α</sub>)(·donkey<sub>β</sub>)<sub>α</sub> is β

yields the proposition:

*Every grey-thing a donkey is*            or            *Every grey-thing is a donkey*

What about adjectives that occur alone in predicate position? A proposition with an adjectival predicate is logically equivalent to the same proposition with the adjective substantivated. This is forced on us by the fact that particular propositions with adjectives as predicates are said to undergo simple conversion, with the understanding that the adjectives are converted to their substantivated forms. Thus ‘*Some donkey is grey*’ converts to ‘*Some grey-thing is [a] donkey*.’ But ‘*Some grey-thing is [a] donkey*’ is itself a particular affirmative proposition, which in turn automatically converts to ‘*Some donkey is [a] grey-thing*.’ So the form with the adjective in the predicate is logically equivalent to that with the substantivated adjective in the subject, and the form with the

<sup>2</sup> This is not to be confused with the common(er) practice in Latin of using adjectives (with adjectival inflections) without an accompanying noun when the missing noun is “understood” from context. For logical purposes understood nouns need to occur overtly in logical forms.

<sup>3</sup> Regarding the use of ‘thing,’ Buridan *SD* 4.2.2 says: “an adjective substantivated [by being put] in the neuter gender can be a subject, for it is resolved into an adjective and a substantive, e.g., ‘white’ [*album*] means the same as ‘white thing’ [*res alba*].”

substantivated adjective in the subject is equivalent to the form with the substantivated adjective in the predicate. Thus the form with the adjective in the predicate must be equivalent to the form with that adjective substantivated in the predicate. (The use of the substantivated form of the adjective in predicate position would be unusual in Latin, but not ungrammatical.)

We can exploit this equivalence to generate un-substantivated adjectives that occur with ‘is’ by allowing Linguish logical forms to generate both ‘*is ADJ*’ and ‘*[an] ADJ-thing is*’, where the second ‘is’ is the same copula that occurs in all of our previous forms. This equivalence will hold in both Latin and English. Thus, these are equivalent:

(Socrates<sub>α</sub>)<sub>α</sub> is healthy

*Socrates is healthy*

(Socrates<sub>α</sub>)(· healthy-thing<sub>β</sub>)<sub>α</sub> is β

*Socrates a healthy-thing is*

*Socrates is a healthy-thing*

(Later we will discuss how to treat adjectives that appear in attributive position modifying nouns.)

### Adjectival common terms

For each ordinary<sup>4</sup> adjective  $\alpha$ ,  $\alpha$ -**thing** is a common term. It signifies (at a time) whatever is  $\alpha$  at that time.<sup>5</sup>

In transition to natural language, the ‘*thing*’ is pronounced in English, but omitted in Latin; instead, the adjective is given nominal inflections.

We should also have an explicit rule of inference connecting the adjectival form with the associated common term:

### Rule Adj

The forms:  $\alpha$  is Adj and (· Adj-thing<sub>β</sub>)<sub>α</sub> is β are interchangeable everywhere

E.g. Socrates is clever if and only if Socrates is a clever-thing.

<sup>4</sup> As a rough guide, an “ordinary” adjective is one whose meaning can be analyzed along the patterns of Aristotle’s *Categories*, such as ‘x is yellow’  $\approx$  ‘a yellowness is in x.’ These are all “intersective” in the current linguistic sense. The adjective ‘former’ is an example of an adjective that does not pass this test.

<sup>5</sup> This condition on signification is probably inadequate for the semantics of modal contexts. See Chapter 10 for a discussion of problems.

## APPLICATIONS

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

*Socrates is tall*

∴ *A tall thing is Socrates*

*Some donkey is tall*

∴ *Some donkey is a tall thing*

*Some tall thing is grey*

∴ *Some grey thing is tall*

*Every donkey is grey*

∴ *Every donkey is a grey thing*

## 5.2 Intransitive verbs

Intransitive verbs appear in sentences in two ways. We have already seen that logicians were comfortable using them, as in ‘*Brownie runs*.’ But they also appear as common terms. This stems from the common view that there is really only one true verb, and that is the copula. So sentences that have other verbs must be analyzed in such a way that the apparent verb is really a combination of the copula with a common term. In particular, several medieval logicians hold that the use of an intransitive verb is equivalent to the use of the copula along with its present participle. For example, they say that ‘*Brownie runs*’ is equivalent to ‘*Brownie is running*.’ At least, this is how the proposal sometimes appears in English, because people sometimes translate the Latin examples word for word. But the normal way to understand the English sentence ‘*Brownie is running*’ is that it contains the verb ‘*run*’ in its present progressive form: ‘*run*’ ⇒ ‘*is running*’, meaning that Brownie is engaged in the process of running. But that is not what the medieval authors intend. There is no way to understand in Latin the combination of the copula with a participle of a verb as a progressive form of the verb because verbs do not have progressive forms in Latin. Instead, what the authors intend is a form of the copula followed by a common term in participle form. So a better English translation is ‘*Brownie is a running-thing*.’ This seems not to be common Latin usage, but it is what many logicians insisted on, and I will go along with this. As a result, if you want to treat a sentence with an intransitive verb, you can take its form to be that of the copula followed by the verb’s

present participle used as a common term.<sup>6</sup> Since present participles are adjectives in Latin, they can be substantivated and used as nouns. That is what happens here.<sup>7</sup> So we again have a procedure for applying the existing formal theory to a new class of items. We treat these uses of Latin participles as nouns (*'running-thing'*):

(**Brownie**<sub>α</sub>)(· **running-thing**<sub>β</sub>)<sub>α</sub> is β  
*Brownie [a] running-thing is*  
*Brownie is a running thing*

### Participles of intransitive verbs

Any present participle of an intransitive verb followed by *'-thing'* is a common term. It supposits for whatever the verb itself is true of.

In transition to natural language, the *'thing'* is pronounced in English, but omitted in Latin; instead, the participle is substantivated.

Other examples:

(**no stone**<sub>α</sub>)(· **speaking-thing**<sub>β</sub>)<sub>α</sub> is β  
*no stone is a speaking thing*

(**no creature**<sub>α</sub>)(· **creating-thing**<sub>β</sub>)<sub>α</sub> is β  
*no creature is a creating thing*

In order to make a logical transition between the two ways in which an intransitive verb is used, we need to add a rule of inference for each intransitive verb that is used in this way.

<sup>6</sup> Paul of Venice *LP* 1.6 (5): "A categorial proposition is one which has a subject, a predicate and a copula as its principle parts, e.g., 'a man is an animal,' 'man' is the subject; 'animal' is the predicate, and the copula is always that verb 'is' because it conjoins the subject with the predicate. And if someone says 'a man runs' (*homo currit*) is a categorial proposition but it does not have a predicate, it can be replied that it has an implicit predicate, viz., 'running.' This is clear by analyzing that verb 'runs' into 'I am,' 'you are,' 'it is' and its participle."

Buridan *S.D.* 1.3.2 (23): "to make the subject, predicate and copula explicit, such a verb has to be analyzed into the verb 'is' as third adjacent, provided that the proposition is assertoric and in the present tense, and into the participle of that verb, as for example, 'A man runs' is to be analyzed into 'A man is running,' and similarly, 'A man is' into 'A man is a being.'"

Albert of Saxony *SL* 3.1: "a categorial proposition is one which has a subject and a predicate and copula as its principal components. Against this it might be objected that there are many propositions which have only a subject and a predicate, and no copula, such as the sentence 'Man runs'. . . . To this we reply that although they do not have an explicit copula, they do have it implicitly. Thus the verb 'runs,' and in general any active verb, includes in it a participle of present time along with a copula—as is seen when we analyze this sentence 'Man runs' into this 'Man is running.'"

<sup>7</sup> Albert of Saxony *SL* 1.5 (5): "And the participle is also treated as a noun, and not as a verb, because the participle is never the copula in the logical sense, but it can very well be a subject or a predicate."

### Special rule of inference for intransitive verbs

The forms:  $\alpha$  VERBs and  $(\cdot \text{VERBing-thing } \beta) \alpha$  is  $\beta$  are interchangeable.

This lets us prove the equivalence of the logical forms underlying:

*no stone is a speaking thing*  $\approx$  *no stone speaks*  
*no creature is a creating thing*  $\approx$  *no creature creates*

Once participles appear as common terms, they can be used anywhere in a sentence that a common noun can be used. So there are sentences such as these:

**(no speaking-thing  $\alpha$ )  $\alpha$  runs**

*No speaking thing runs*

**(every speaking-thing  $\alpha$ )  $\alpha$  moves**

*Every speaking thing moves*

#### APPLICATIONS

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

- Socrates doesn't speak*  
 $\therefore$  *Socrates isn't a speaking thing*
- Some donkey speaks*  
 $\therefore$  *Some speaking thing is a donkey*
- Some speaking thing runs*  
 $\therefore$  *Some running thing speaks*
- Every donkey speaks*  
 $\therefore$  *Every donkey is a speaking thing*

### 5.2.1 Being

Some authors extended this idea to analyze propositions of the form 'Socrates is' or 'a donkey is.' They took this use of 'is' to be like the use of an intransitive verb, to be analyzed in terms of a copula together with the present participle of the ("intransitive") verb 'is,' namely 'being.' Since that participle already occurs as a noun, the natural analysis of 'Socrates is' is 'Socrates is [a] being.'

(Instead of the view just mentioned, some authors held that 'Socrates is' means 'Socrates is a thing.' Still others were said to hold the view that 'Socrates is' means

‘*Socrates is Socrates*’.<sup>8</sup> It would be neat to appeal to this last view to replace the artificial notation ‘<t is non-empty>’ by ‘(t α) α is.’ However, that would be to take sides on a controversial issue.)

Independent of how to analyze ‘α is,’ the noun ‘*being*’ was widely discussed. It seems to be taken to be the most unrestricted noun which signifies everything, including both God and creatures. Given its use, it is apparent that if ‘*donkey*’ is non-empty then ‘*every donkey is a being*’ is true. I will take this for granted.<sup>9</sup> However, we will not make any use of it before Chapter 10.

### 5.3 Transitive verbs

So far, we have been able to include adjectives and intransitive verbs without putting anything new into the logical notation (except for the ‘-thing’ on the end of adjectives, which has no logical effect at all). One naturally wonders if the same kind of paraphrase is available with transitive verbs. Transitive verbs have present participles just as intransitive verbs do—so one could replace a transitive verb with the copula and its participle and proceed as previously. And indeed, some medieval logicians do so. For example, Buridan (*SD* 1.3.3) discusses this example:

*Every horse [a] man is seeing*

where we have the participle of a transitive verb. If we treat the participles of transitive verbs just like those of intransitives, then a complication needs to be addressed: the verb is ‘*is*,’ and it provides grammatical roles for two terms, as usual. However, there are three terms in this proposition: ‘*horse*,’ ‘*man*,’ and ‘*seeing*.’ The solution is to suppose that the participle of a transitive verb itself provides a grammatical role: the role of direct object of the participle. The participle ‘*seeing*’ has a direct object just as much as ‘*see*’ does. (This is clear in the example from Buridan, for ‘*horse*’ has accusative case there, because of its being the object of the participle.) And something like this seems to be what Buridan has in mind, because he says (*S.D.* 1.3.3, 28) “‘*horse*’ is construed with ‘*seeing*’ to make one extreme of the proposition.” In order to accommodate this idea with our present technique of using markers to indicate grammatical relations, we need to include a grammatical marker along with each participle of a transitive verb. (Intuitively, this is the “same” grammatical marker that would occur after the verb itself if it appeared in its verbal form.) It is heuristic to put that marker immediately after the participle. So take the sentence ‘*Cicero sees a donkey*,’ and represent it as suggested, to get:

<sup>8</sup> See Andrews 1993, 8.

<sup>9</sup> How singular terms interact with ‘*being*’ depends on the decision, discussed in section 4.3, of whether a singular term that signifies something that does not presently exist supposits for that thing in a present tense sentence, or not. If so, then if a singular term such as ‘*Brownie*’ is non-empty, then ‘*Brownie is a being*’ will be true or false depending on whether Brownie currently exists; if not, ‘*Brownie is a being*’ will be true exactly when ‘*Brownie*’ is non-empty. These will usually be equivalent.



(Cicero<sub>α</sub>)(· donkey<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α</sub> is<sub>γ</sub>

*Cicero of-a donkey<sub>acc</sub> a seeing thing is*

*Cicero of-a donkey<sub>acc</sub> is a seeing thing*

*Cicero is of-a donkey<sub>acc</sub> a seeing thing*

The term whose denoting phrase binds a marker in a transitive-verb participle gets an accusative ending in Latin because its grammatical role is direct object of the participle. I have also introduced the preposition ‘of’ in front of the denoting phrase which is the object of the participle because that preposition is often used in English for the direct object of a verb when the verb itself is present in a non-verb form, for example in ‘*the destruction of the city*’, where ‘*the city*’ is understood to be the direct object of ‘*destroy*’. This is for readability only, and it may be ignored if it is not helpful.

Call a term that incorporates a variable as just described a “parasitic” term, since what it supposits for depends on the denoting phrase occupying the role that it provides.

#### Parasitic terms (participles of transitive verbs)

If ‘V’ is a transitive verb, then ‘Ving-<sub>α</sub>-thing’ is a common term.

It goes into propositions where common terms go (understanding that when a denoting phrase containing it is added to a formula, the role marker ‘α’ does not already occur in that formula).

In context, when we specify what a parasitic term supposits for we must do so relative to something else. For example, we will say that ‘seeing-<sub>α</sub>-thing’ supposits for a thing *o* relative to a thing *o’* if and only if *o* sees *o’*.

#### Supposition for parasitic terms: Participles of transitive verbs

The term ‘Ving-<sub>α</sub>-thing’ supposits for a thing *o* relative to *o’* iff ‘V’ holds of  $\langle o, o' \rangle$

Then we need to see how this fits into the recursive semantics. It will be relevant in a case in which it is the main term of the first denoting phrase not containing a temporary name, but where some preceding temporary name binds the role marker provided with the participle. Here is an example when the quantifier sign is ‘some’.

‘ $\tau_j(\$n_\alpha)\tau_k$  (some Ving-<sub>α</sub>-thing<sub>β</sub>)  $\varphi$ ’ is true<sub>σ</sub> iff for some thing *o* such that ‘Ving-<sub>α</sub>-thing’ supposits for *o* relative to  $\sigma$ (‘\$i’), ‘ $\tau_j \tau_k (\$i_\beta) \varphi$ ’ is true<sub>σ[‘\$i’/o]</sub>, where ‘\$i’ is the first temporary name not occurring in ‘ $\tau_j(\$n_\alpha)\tau_k$ ’

If ‘V’ supposits for nothing relative to  $\sigma$ (‘\$i’), replace ‘true<sub>σ[‘\$i’/o]</sub>’ by ‘true<sub>σ[‘\$i’/-]</sub>’.

The earlier example is:

(Cicero<sub>α</sub>)(· donkey<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α is γ</sub>  
*Cicero of-a donkey<sub>acc</sub> a seeing thing is*

This logical form is built up in stages, where each addition of a denoting phrase binds a grammatical marker that is free in the formula it is added to. The formula then says:

Cicero is a thing x such that a donkey is a thing y such that a thing z which sees y is x

For another example, consider the proposition:

(Every woman<sub>α</sub>)(Brownie<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α is γ</sub>  
*Every woman is of-Brownie<sub>acc</sub> a seeing thing*

This has the following truth conditions:

‘(Every woman<sub>α</sub>)(Brownie<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ</sub>

iff (supposing that ‘woman’ is not empty) for any thing w such that w is a woman,

‘(\$1<sub>α</sub>)(Brownie<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ[\$1/w]</sub>

iff for any thing w such that w is a woman, for the thing b which is Brownie:

‘(\$1<sub>α</sub>)(\$2<sub>β</sub>)(· seeing-β-thing<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ[\$1/w][\$2/b]</sub>

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that ‘seeing-β-thing’ supposits<sub>σ[\$1/w][\$2/b]</sub> for s relative to σ[\$1/w][\$2/b] (‘\$2’):

‘(\$1<sub>α</sub>)(\$3<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ[\$1/w][\$3/s]</sub>

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that ‘sees’ holds of <s, σ[\$1/w][\$2/b] (‘\$2’)>:

‘(\$1<sub>α</sub>)(\$3<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ[\$1/w][3/s]</sub>

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that s sees b:

‘(\$1<sub>α</sub>)(\$3<sub>γ</sub>)<sub>α is γ</sub>’ is true<sub>σ[\$1/w][3/s]</sub>

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that s sees b, ‘is’ holds of the pair of things:

<σ[\$1/w][3/s] (‘\$1’), σ[\$1/w][3/s] (‘\$3’)>.

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that s sees b, ‘is’ holds of the pair of things:

<w, s>

iff for any thing w such that w is a woman, for the thing b which is Brownie, for a thing s such that s sees b: w = s.

This then is logically equivalent to:

iff for any thing  $w$  such that  $w$  is a woman, for the thing  $b$  which is Brownie,  $w$  sees  $b$ .

or

iff for any thing  $w$  such that  $w$  is a woman,  $w$  sees Brownie.

This seems to be the right truth conditions for the sentence ‘*Every woman is of-Brownie<sub>acc</sub> a seeing thing.*’

As with participles of intransitive verbs, we need to include a rule of inference to link uses of the verb with uses of its participle. The obvious condition is:

**Participles of transitive verbs: Interchange rule**

The forms:  $(\cdot \text{Ving}_{-\beta}\text{-thing}_{\gamma})_{\alpha}$  is  $\gamma$  and  $_{\alpha} \text{V}_{\beta}$  are interchangeable everywhere.

As with participles of intransitive verbs, participles of transitive verbs can occur with other quantifiers, as in:

*Of-every donkey<sub>acc</sub> a seeing thing is a horse*

meaning that for each donkey, something seeing it is a horse. Or:

*A donkey<sub>acc</sub> every seeing thing sees a horse*

meaning that there is a donkey such that everything seeing it sees a horse.

Notice that we continue with a logical development that is consistent with, though not committed to, the view that there is only one “real” verb, namely ‘*is.*’

**APPLICATIONS**

Take each of the following sentences and generate its equivalent form that contains no verb other than the copula.

*No animal sees Socrates<sub>acc</sub>*

*Some donkey sees every horse<sub>acc</sub>*

*Every bird sees every bird<sub>acc</sub>*

Generate these sentences from their logical forms.

*Of Socrates<sub>acc</sub> no seeing thing runs*

*Of every donkey<sub>acc</sub> some seeing thing is an orating thing*

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

*Plato sees Socrates<sub>acc</sub>*  
 $\therefore$  *Plato of Socrates<sub>acc</sub> is a seeing thing*

*No animal sees Socrates<sub>acc</sub>*  
 $\therefore$  *No animal of Socrates<sub>acc</sub> is a seeing thing*

### 5.4 Additional rules for parasitic terms

With parasitic terms added, we need our rules of inference to accommodate them. Some rules require no changes at all; for example, the quantifier equipollence rules already apply to sentences involving parasitic terms, because parasitic terms *are* common terms. The changes that are needed are due to the fact that a parasitic term carries a free marker along with it, and thus no denoting phrase with a parasitic term can be the initial denoting phrase in a sentence. But several of our rules are explicitly formulated in terms of denoting phrases that appear on the front of a proposition. These rules must be adapted so as to apply somehow to a proposition which has a parasitic term occurring second, preceded by a singular term. For example, we should be able to validate this inference, which resembles simple conversion:

*An animal is of-Socrates<sub>acc</sub> a seeing thing* <An animal is a Socrates seer>  
 $\therefore$  *of-Socrates<sub>acc</sub> a seeing thing is an animal* <A Socrates seer is an animal>

The obvious way to do this is with a proof like this:

1.	( <b>n</b> $\alpha$ )( <b>Socrates</b> $\beta$ )( <b>· seeing-<math>\beta</math>-thing</b> $\gamma$ ) $\alpha$ is $\gamma$	
2.	( <b>n</b> $\alpha$ )( <b>· animal</b> $\beta$ ) $\alpha$ is $\beta$	1 EX
3.	( <b>n</b> $\alpha$ )( <b>Socrates</b> $\beta$ )( <b>· seeing-<math>\beta</math>-thing</b> $\gamma$ ) $\alpha$ is $\gamma$	1 EX
4.	( <b>Socrates</b> $\beta$ )( <b>· seeing-<math>\beta</math>-thing</b> $\gamma$ )( <b>· animal</b> $\alpha$ ) $\alpha$ is $\gamma$	2 3 ES ?????

This would be straightforward if the combination of the singular term, ‘**Socrates**’ with the denoting phrase ‘**a seeing-thing**’ were itself a common term. Is it? The answer seems to be that given the freedom of Latin word order, and given the practices of medieval logicians, it is possible to treat ‘*Socrates<sub>acc</sub> seeing thing*’ as a single complex common term, and it is also possible to treat it as a string of two grammatically related terms. Based on the logical form, it consists of two separate terms. In the next section we will show how this form is equivalent to one consisting of a single complex term. (This will be the rule Complex Term 2 in section 5.5.2.) That equivalence will allow us to complete the derivation just given. That will have to await the construction of complex terms in the next section.

With the introduction of parasitic terms we have expanded on medieval practice in the new rules just given. When medieval writers introduced, say, substantivated versions of participles of transitive verbs, there is no evidence that they recognized the

significance of what they were doing. In context, it appears as if participles of intransitive and transitive verbs were taken to be alike, with no comment made of their important logical and semantic differences. Introducing parasitic terms is straightforward medieval practice, but identifying additional logical rules that pertain to them is, I think, unknown.<sup>10</sup> I am confident that medieval writers would recognize instances of the expanded rules to be given shortly as valid, but they did not give the rules themselves.

## 5.5 Some complex terms

Generally a complex term is made by combining a common term with something that modifies it. This section is devoted to some ways in which terms are combined with modifiers. (Modification of a term by relative clauses is discussed in the following section.) This topic might seem an unnecessary complication, but medieval logicians took the Latin language as is, including many of its complications, and so it would be ahistorical to ignore them.

Medieval authors discussed such constructions, but they rarely gave any explicit rules for handling them. The discussion in this section is meant to be faithful to their discussions (this will be covered in more detail in Chapter 6); the explicit rules given here go beyond their discussion.

### 5.5.1 *Attributive adjectives and participles modifying nouns*

The simplest type of modifying is when an adjective modifies a noun, as in ‘*grey donkey*.’ In Latin an adjective usually follows the noun that it modifies, though it may precede it or even be widely separated from it. As usual, I will generate only the basic form that the logicians themselves used, where the adjective immediately follows the expression it is modifying (though I’ll display natural language versions for English speakers with the adjective preceding the noun).<sup>11</sup> This construction is to include the case in which the adjective is the participle of an intransitive verb, as in ‘*running donkey*.’

#### Attributive adjectives and participle constructions<sup>12</sup>

If ‘P’ is a common term and ‘X’ a term representing an adjective or participle of an intransitive verb, then {PX} is a common term.

{PX}’ suppositis (with respect to time t) for whatever ‘P’ and ‘X’ both supposit for (with respect to t).

<sup>10</sup> Buridan does note the special behavior of certain of these constructions in his discussion of “restricted descent”; but his comments are limited. See section 7.4.

<sup>11</sup> In Latin most adjectives follow the noun that they modify, though certain classes of adjectives typically precede the verb. I’ll ignore this variation.

<sup>12</sup> This provision is aimed at what are usually called “intersective” adjectives, which are the simplest cases. Treating others would take us far afield.

In modern logic these constructions are not present in official notation. When students symbolize sentences of natural language they are expected to apply something like the rules given here in their heads, so that ‘grey donkey’ gets symbolized as a conjunction using predicates for ‘grey’ and ‘donkey’ separately. Given ‘Every grey donkey is running,’ the students are expected to analyze ‘grey donkey’ in their heads, writing down the result of such an analysis: ‘ $\forall x(\text{grey } x \ \& \ \text{donkey } x \rightarrow x \text{ is running})$ .’ Medieval logicians took the complex forms seriously as logical units. If this is done a special rule of inference is needed to get the results that modern students calculate in their heads:

**The Rule Modifier:**

$$\begin{aligned} & (\mathbf{n}_\alpha)(\cdot \{\mathbf{PX}\}_\beta) \alpha \text{ is } \beta \\ \therefore & (\mathbf{n}_\alpha)(\cdot \mathbf{P}_\beta) \alpha \text{ is } \beta \\ \therefore & (\mathbf{n}_\alpha)(\cdot \mathbf{X}\text{-thing}_\beta) \alpha \text{ is } \beta \\ & (\mathbf{n}_\alpha)(\cdot \mathbf{P}_\beta) \alpha \text{ is } \beta \\ & (\mathbf{n}_\alpha)(\cdot \mathbf{X}\text{-thing}_\beta) \alpha \text{ is } \beta \\ \therefore & (\mathbf{n}_\alpha)(\cdot \{\mathbf{PX}\}_\beta) \alpha \text{ is } \beta \end{aligned}$$

This rule may be applied recursively. Some examples are:

{donkey grey}	<grey donkey>
{donkey running}	<running donkey>
{{donkey running} grey}	<grey running donkey>

**APPLICATIONS**

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

- Some grey donkey is a running thing*  
 $\therefore$  *Some running donkey is grey*
- Every donkey is grey*  
 $\therefore$  *Every running donkey is grey*
- Socrates sees a grey donkey<sub>acc</sub>*  
*Every donkey is an animal*  
 $\therefore$  *Socrates sees a grey animal<sub>acc</sub>*

### 5.5.2 Participles of transitive verbs with their objects

The participle of a transitive verb may combine with its direct object (and any quantifier that its direct object might have) to form a complex term. Its object may either precede

or follow the noun being modified. (This is based on observation of examples discussed by logicians.) Examples are:

<i>of-Socrates<sub>acc</sub> seeing thing</i>	i.e. thing seeing Socrates
<i>of-every horse<sub>acc</sub> seeing thing</i>	i.e. thing seeing every horse

These complex common terms occur in sentences such as:

*Every seeing-thing of-some horse<sub>acc</sub> is running*  
*Every of-some horse<sub>acc</sub> seeing-thing is running*

each meaning that everything that sees some horse is running. These constructions are quite common. They may be included as follows:

### Participle of transitive verb with its direct object

If 'P' is the participle of a transitive verb, 'T' a common term, 't' a singular term, and 'Q' a quantifier sign, then these are all common terms:

$\{(Q T \gamma)P\text{-}\gamma\text{-thing}\}$   
 $\{P\text{-}\gamma\text{-thing} (Q T \gamma)\}$   
 $\{(t \gamma)P\text{-}\gamma\text{-thing}\}$   
 $\{P\text{-}\gamma\text{-thing} (t \gamma)\}$

In this type of construction it is a merely stylistic question whether the direct object precedes or follows the participle. To accommodate this it is convenient to have a rule saying that the different forms are equivalent:

### Rule Modifier-Permute:<sup>13</sup>

Any proposition containing ' $\{(Q T \gamma)P\text{-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-}\gamma\text{-thing} (Q T \gamma)\}$ '

Any proposition containing ' $\{(t \gamma)P\text{-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-}\gamma\text{-thing} (t \gamma)\}$ '

Supposition conditions need to be stated by cases, with one case for each quantifier. For example, ' $\{(\text{every } T \gamma)P\text{-}\gamma\text{-thing}\}$ ' supposits for something if and only if that thing is related by the P relation to everything for which 'T' supposits.

We have two forms of a rule for dealing with such complex terms. The first form is confined to instances in which the embedded denoting phrase is singular; it lets us

<sup>13</sup> In some of these constructions a marker precedes the denoting phrase which binds it. This is not problematic so long as the semantics of the whole construction is clear.

replace any such term anywhere. In general, the term is eliminable when it is one term of a simple identity.

**Rule Complex Term 2:**

Form A:  $(Q\{(t_\gamma)P_{-\gamma}\text{-thing}\}_\beta)$  is equipollent to  $(t_\gamma)(Q P_{-\gamma}\text{-thing}_\beta)$

Form B:  $(t_\alpha)(\cdot\{(Q T_\gamma)P_{-\gamma}\text{-thing}\}_\beta)$   $\alpha$  is  $\beta$   
 is logically equivalent to  
 $(t_\alpha)(Q T_\gamma)(\cdot P_{-\gamma}\text{-thing}_\beta)$   $\alpha$  is  $\beta$

Example A: ‘*every Socrates-seeing-thing*’ is equipollent to ‘*of-Socrates every seeing-thing*’, where the former is a single complex denoting phrase and the latter is a string of two denoting phrases.

Example B: ‘*Brownie is an every-donkey-seeing-thing*’ is logically equivalent to ‘*Brownie is of every donkey a seeing-thing*’, where the former sentence in addition to ‘*Brownie*’ contains one (complex) denoting phrase and the latter contains two.

The first form of this rule is of special interest because it meets the goal of section 5.4. This is because it makes a singular term followed by any denoting phrase equivalent to a single complex term. Since the new complex term is a common term, all of our old rules apply to it. Recall the issue of how to validate this inference:

*An animal is of-Socrates a seeing thing*  
 $\therefore$  *of-Socrates a seeing thing is an animal*

1.	$(\cdot \text{animal } \alpha)(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)$ $\alpha$ is $\gamma$	
2.	$(n_\alpha)(\cdot \text{animal } \beta)$ $\alpha$ is $\beta$	1 EX
3.	$(n_\alpha)(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)$ $\alpha$ is $\gamma$	1 EX
4.	$(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)(\cdot \text{animal } \alpha)$ $\alpha$ is $\gamma$	2 3 (?????)

This can be achieved as follows:

1.	$(\cdot \text{animal } \alpha)(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)$ $\alpha$ is $\gamma$	
2.	$(n_\alpha)(\cdot \text{animal } \beta)$ $\alpha$ is $\beta$	1 EX
3.	$(n_\alpha)(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)$ $\alpha$ is $\gamma$	1 EX
4.	$(n_\alpha)(\cdot \{(\text{Socrates } \beta)\text{seeing-}\beta\text{-thing}\}_\gamma)$ $\alpha$ is $\gamma$	3 Complex Term 2
5.	$(\cdot \{(\text{Socrates } \beta)\text{seeing-}\beta\text{-thing}\}_\gamma)(\cdot \text{animal } \alpha)$ $\alpha$ is $\gamma$	2 4 ES
6.	$(\text{Socrates } \beta)(\cdot \text{seeing-}\beta\text{-thing } \gamma)(\cdot \text{animal } \alpha)$ $\alpha$ is $\gamma$	5 Complex Term 2

It is apparent from this example that it is an acceptable shortcut to treat a combination of a singular denoting phrase binding the marker in a parasitic term immediately following it as a unit so far as rule ES is concerned. And the analogous technique should also apply to rules EX and UA. So we have the following shortcut rules:



EX+       $(t_\beta)(\text{some Ting-}\beta\text{-thing } \alpha) \phi$   
              $\langle (t_\beta)\text{T-}\beta\text{-thing is non-empty} \rangle$   
              $\therefore (n_\alpha) \phi$   
              $\therefore (n_\alpha)(t_\beta)(\text{some Ting-}\beta\text{-thing } \gamma) \alpha \text{ is } \gamma$   
             where  $n$  is a name that does not already occur in the derivation

ES+       $(n_\alpha) \phi$   
              $(n_\alpha)(t_\beta)(\text{some Ting-}\beta\text{-thing } \gamma) \alpha \text{ is } \gamma$   
              $\therefore (t_\beta)(\text{some Ting-}\beta\text{-thing } \alpha) \phi$       where 'n' is any singular term

UA+       $(t_\beta)(\text{every Ting-}\beta\text{-thing } \alpha) \phi$        $(t_\beta)(\text{no Ting-}\beta\text{-thing } \alpha) \phi$   
              $(n_\alpha)(t_\beta)(\cdot \text{Ting-}\beta\text{-thing } \gamma) \alpha \text{ is } \gamma$        $(n_\alpha)(t_\beta)(\cdot \text{Ting-}\beta\text{-thing } \gamma) \alpha \text{ is } \gamma$   
              $\therefore (n_\alpha) \phi$        $\therefore \text{not } (n_\alpha) \phi$

#### APPLICATIONS

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

- Plato is an of-Socrates<sub>acc</sub> seeing thing*  
 $\therefore$  *Of-Socrates<sub>acc</sub> a seeing thing is Plato*
- Some of-Socrates<sub>acc</sub> seeing thing is a running thing*  
 $\therefore$  *Some running thing is of-Socrates<sub>acc</sub> a seeing thing*
- Some donkey sees an of-Plato<sub>acc</sub> seeing thing<sub>acc</sub>*  
 $\therefore$  *Of-Plato<sub>acc</sub> a seeing thing<sub>acc</sub> some donkey sees*

#### 5.5.3 Terms modified by complex terms

Once the complex terms in section 5.5.2 are formed, they in turn can modify nouns. They go in the same positions as adjectives and participles discussed previously. Examples are:

- Every {donkey {seeing some horse}} is running*  
*Every {{seeing some horse} donkey} is running*

In addition, it is possible for a negation to appear, as in:

- Every {donkey not {seeing some horse}} is running*  
*Every {not {seeing some horse} donkey} is running*

### Attributive adjectives and participle constructions (updated)

If 'P' is a common term and 'X' is a combination of a transitive verb with its direct object, then '{PX}' and '{P not X}' are common terms.

'{PX}' suppositis (with respect to time t) for whatever 'P' and 'X' both supposit for (with respect to t). '{P not X}' suppositis (with respect to time t) for whatever 'P' suppositis for and 'X' does not supposit for (with respect to t).

Some logical forms are:

**(every {donkey {(some horse<sub>a</sub>) seeing-<sub>a</sub>-thing}}<sub>β</sub>)<sub>β</sub> runs**

*Every donkey of-some horse<sub>acc</sub> seeing thing runs*

**(every {donkey {seeing-<sub>a</sub>-thing (some horse<sub>a</sub>)}}<sub>β</sub>)<sub>β</sub> runs**

*Every donkey seeing thing of-some horse<sub>acc</sub> runs*

**(every {{(some horse<sub>a</sub>) seeing-<sub>a</sub>-thing} donkey}<sub>β</sub>)<sub>β</sub> runs**

*Every of-some horse<sub>acc</sub> seeing thing donkey runs*

**(every {{seeing-<sub>a</sub>-thing (some horse<sub>a</sub>)} donkey}<sub>β</sub>)<sub>β</sub> runs**

*Every seeing thing of-some horse<sub>acc</sub> donkey runs*

To accommodate these in Linguish we add a clause to the inference rule given earlier for adjectives and participles modifying common terms:

### Rules of inference for attributive adjectives and participle constructions (updated)

Rule for Complex Term 3:

If 'P' and 'X' are as previously, then these are valid:

$(n_a)(\cdot \{PX\} \beta) \alpha \text{ is } \beta$

$\therefore (n_a)(\cdot P \beta) \alpha \text{ is } \beta$

$\therefore (n_a)(\cdot X \beta) \alpha \text{ is } \beta$

$(n_a)(\cdot P \beta) \alpha \text{ is } \beta$

$(n_a)(\cdot X \beta) \alpha \text{ is } \beta$

$\therefore (n_a)(\cdot \{PX\} \beta) \alpha \text{ is } \beta$

$(n_a)(\cdot \{P \text{ not } X\} \beta) \alpha \text{ is } \beta$

$\therefore (n_a)(\cdot P \beta) \alpha \text{ is } \beta$

$\therefore (n_a) \text{ not } (\cdot X \beta) \alpha \text{ is } \beta$

$(n_a)(\cdot P \beta) \alpha \text{ is } \beta$

$(n_a) \text{ not } (\cdot X \beta) \alpha \text{ is } \beta$

$\therefore (n_a)(\cdot \{P \text{ not } X\} \beta) \alpha \text{ is } \beta$

Again, these rules were not stated by medieval logicians, but I think that they would endorse instances of them if they were pointed out.

#### APPLICATIONS

For each of the following arguments, represent the argument in Linguish notation and show that it is valid.

*Plato is a farmer of-Socrates<sub>acc</sub> seeing thing*

<Plato is a farmer seeing Socrates>

∴ *A farmer sees Socrates<sub>acc</sub>*

*Every seeing thing of-some horse<sub>acc</sub> donkey runs*

∴ *Some donkey sees some horse*

*No donkey not a seeing thing of a horse<sub>acc</sub> sees Plato<sub>acc</sub>*

*Brownie is a donkey*

*Brownie doesn't see a horse<sub>acc</sub>*

∴ *Brownie doesn't see Plato<sub>acc</sub>*

## 5.6 Relative clauses

Relative clauses come in two forms, namely restrictive and non-restrictive.<sup>14</sup>

*The woman who left early was miffed.*      RESTRICTIVE

*The woman, who left early, was miffed.*      NON-RESTRICTIVE

We will only discuss restrictive relative clauses. Restrictive relative clauses make complex terms. In distinguishing restrictive from non-restrictive relative clauses Buridan says:

[In 'A man who is white is colored'] there is one predicate here, namely, 'colored,' which by the mediation of the copula is predicated of the whole of the rest as of its subject, namely, of the whole phrase: 'man who is white'; for the whole phrase: 'who is white' functions as a determination of the subject 'man.' And the case is not similar to 'A man is colored, who is white,' for there are two separate predicates here, which are predicated separately of their two subjects, and there is not a predicate here which would be predicated by the mediation of one copula of the whole of the rest. And although these [propositions] are equivalent, they are not equivalent if we add a universal sign. For positing the case that every white man runs and there are many others who do not run, the proposition 'Every man who is white runs' is true, and is equivalent to: 'Every white man runs'; but the proposition 'Every man, who is white, runs' is false, for it is equivalent to: 'Every man runs and he is white.' (Buridan *SD* 1.3.2)

<sup>14</sup> Sherwood *S*1.10 (28): "When 'every' or 'all' is added to a term involving a clause or phrase, the [resultant] locution is ambiguous in that the distribution can be united either for the whole term together with the clause or phrase or [for the term] without it."

With few exceptions, a relative clause takes the form of a relative pronoun followed by a sentence with a gap in it. That is, it is followed by what would be a sentence if a denoting phrase were inserted into it in a certain place. Examples are:

*dog which [Sam lost \_\_\_]*  
*book which [I gave \_\_\_ to Martha]*  
*dog which [\_\_\_ chased my cat]*

The *case* of the relative pronoun (e.g. nominative ‘*who*’ versus objective ‘*whom*’) is whatever case a denoting phrase would have if it were placed in the gap. Thus we have:

*woman whom [he loved \_\_\_]*  
 \**woman whom [\_\_\_ loved him]* <\*ungrammatical>

The common explanation for both the gap and the case agreement is that the relative pronoun originates inside the sentence (where it has case) and moves to the front, leaving a gap in the place where it was. For our purposes we needn’t adopt the movement account; what is important for us is that a relative pronoun, like an ordinary denoting phrase, needs to play a grammatical role in the sentence making up the relative clause. Then, once formed, the relative clause combines with a common term, immediately preceding it to make a complex term. So to accommodate restrictive relative clauses we can simply add a new rule for complex terms:

#### Complex terms:

If  $T$  is a common term, and  $\phi$  is a formula with exactly one free marker  $\alpha$ , then ‘ $\{T \text{ which}_\alpha \phi\}$ ’ is a common term.

Since a relative pronoun is grammatically like a term, I could symbolize the ‘*which*’ with parentheses and a non-subscripted grammatical role marker, to write:  $\{T (\text{which } \alpha) \phi\}$ . I use the parenthesis-free notation partly for readability, and because putting a relative pronoun on the front of a relative clause produces something special: a modifier instead of a sentence. Those who prefer the representation that is more like other denoting phrases can understand ‘*which* <sub>$\alpha$</sub> ’ to abbreviate ‘ $(\text{which } \alpha)$ ’.

On one modern view, the ‘ $\alpha$ ’ in ‘ $\{T \text{ which}_\alpha \phi\}$ ’ is a bound variable, bound by ‘*which*’, and ‘*which*’ plays a role similar to ‘ $\lambda$ ’ in  $\lambda$ -abstracts.<sup>15</sup> The whole phrase ‘ $\{T \text{ which}_\alpha \phi\}$ ’ then consists of two one-place predicates next to each other, ‘ $T$ ’ and ‘ $\text{which}_\alpha \phi$ ’, enclosed in curly brackets; the interpretation of the whole is the conjunction of those predicates. An example of a complex common term made by modifying a common term with a restrictive relative clause is:

$\{\text{donkey which}_\alpha \text{ runs}\}$       a complex common term  
*donkey which runs*

<sup>15</sup> See Andrews 2009 for an exposition of the  $\lambda$ -calculus.

### 5.6.1 Semantics of relative clauses

Since a term modified by a relative clause is itself a (complex) term, what is needed for the semantics of these terms is to specify what they supposit for relative to an assignment to any temporary name:

**Supposita of complex terms modified by relative clauses:**

{T **which**<sub>α</sub> φ}' supposit<sub>σ</sub> (with respect to time t) for a thing o iff  
T supposit<sub>σ</sub> (with respect to t) for o and (∑n α) φ is true<sub>σ[∑n/o]</sub>

Although this gives the right truth conditions for complex terms made using relative clauses, it is not expressed in a way that is faithful to some medieval authors. This is because the rule as stated ignores the suppositional status of the relative pronoun, treating it as a piece of logical apparatus. This is *prima facie* inappropriate because relative pronouns have grammatically prominent characters; for example, they have grammatical case just as other terms do. Some medieval authors seem to have thought of relative pronouns as terms that have logically significant roles. It is not certain what those roles should be. One natural idea is that relative pronouns by themselves modify the terms that they follow, and the role of the relative clause itself is to constrain what the relative pronoun supposits for.<sup>16</sup> That view can be encapsulated in this rule:

**Supposita of complex terms modified by relative clauses:**

{T **which**<sub>α</sub> φ}' supposit<sub>σ</sub> (with respect to time t) for a thing o iff T supposit<sub>σ</sub> (with respect to t) for o and **which**<sub>α</sub>, in that context, also supposit<sub>σ</sub> (with respect to t) for o.

This puts modification of a term by a relative pronoun on a par with its modification by adjectives. We just need to explain what it is that the relative pronoun supposits for. That is straightforward:

**Supposita of relative pronouns**

In the context '{T **which**<sub>α</sub> φ}', '**which**<sub>α</sub>' supposit<sub>σ</sub> (with respect to time t) for a thing o iff '(∑n α) φ' is true<sub>σ[∑n/o]</sub>

<sup>16</sup> Though the quote from Buridan which begins this section seems to endorse the analysis we have already given.

These two conditions together are equivalent to the simpler one given first. They do, however, supply more meaningful parts of the construction; these will be discussed in Chapter 10.

On either account, a single two-way rule of derivation suffices for the logic of restrictive relative clauses:

**RelClause:**

$$\begin{aligned} & (\mathbf{n}_\alpha)(\cdot \{\mathbf{T} \text{ which}_\gamma \phi\} \beta) \alpha \text{ is } \beta \\ \therefore & (\mathbf{n}_\alpha)(\cdot \mathbf{T} \beta) \alpha \text{ is } \beta \\ \therefore & (\mathbf{n}_\gamma) \phi \\ & (\mathbf{n}_\alpha)(\cdot \mathbf{T} \beta) \alpha \text{ is } \beta \\ & (\mathbf{n}_\gamma) \phi \\ \therefore & (\mathbf{n}_\alpha)(\cdot \{\mathbf{T} \text{ which}_\gamma \phi\} \beta) \alpha \text{ is } \beta \end{aligned}$$

To expand the completeness theorem to include sentences containing relative clauses, it should be enough to add the rules given in this section, and to use the semantic provision given earlier as a guide to how to interpret the complex term when constructing the interpretation I. I think the changes are straightforward, but they are not discussed here.

### 5.6.2 Representing ordinary language Latin (and English)

Only one addition is required for turning sentences with relative clauses into their natural language forms. We have been indicating the case of a denoting phrase by putting a case subscript on its main term; the case is determined by the grammatical marker that the denoting phrase binds.<sup>17</sup> We extend this practice so as to put a case subscript on the word ‘which’; the subscript is again the one determined by the marker that ‘which<sub>x</sub>’ binds. In Latin, relative pronouns take various forms for the different cases; except for ‘who/whom/whose’ this is invisible in English.

From the logical form:

$$(\text{Some } \{\text{donkey which}_\alpha (\text{Plato } \beta) \beta \text{ sees } \alpha\} \gamma) (\text{Socrates } \delta) \gamma \text{ strikes } \delta$$

we generate:

$$\text{Some donkey which}_{acc} \text{ Plato sees Socrates}_{acc} \text{ strikes}$$

and then the more natural form:

$$\text{Some donkey which}_{acc} \text{ Plato sees strikes Socrates}_{acc}$$

<sup>17</sup> Our practice is that a marker preceding a verb, or following the verb ‘is’ determines nominative case, and a marker following a non-copular verb or a participle determines accusative case. These are the only two cases discussed so far. Genitive case will be discussed in the next section.

Some additional sentences containing relative clauses follow. (In these examples the verbs have been moved forward just far enough to yield the natural word order for English. Any of the word orders are grammatical in Latin.)

(Some {donkey which<sub>α</sub> (every animal β) α sees β} γ)(Socrates δ) γ strikes δ

*Some donkey which every animal<sub>acc</sub> sees Socrates<sub>acc</sub> strikes*

*Some donkey which sees every animal<sub>acc</sub> strikes Socrates<sub>acc</sub>*

(Some {donkey which<sub>α</sub> (every animal β) β sees α} γ)(Socrates<sub>acc</sub> δ) γ strikes δ

*Some donkey which<sub>acc</sub> every animal sees Socrates<sub>acc</sub> strikes*

*Some donkey which<sub>acc</sub> every animal sees strikes Socrates<sub>acc</sub>*

(Some {donkey which<sub>α</sub> (every animal β) α sees β} γ)(Socrates δ) δ strikes γ

*Some donkey<sub>acc</sub> which every animal<sub>acc</sub> sees Socrates strikes*

*Some donkey<sub>acc</sub> which sees every animal<sub>acc</sub> Socrates strikes*

(Socrates δ)(Some {donkey which<sub>α</sub> (every animal β) α sees β} γ) δ strikes γ

*Socrates some donkey<sub>acc</sub> which every animal<sub>acc</sub> sees strikes*

*Socrates strikes some donkey<sub>acc</sub> which sees every animal<sub>acc</sub>*

As a test for adequacy we can derive either of Buridan's sentences from the other:

*Every man who is white runs*

*Every white man runs*

Another ordinary language complication: in Latin, relative clauses can be located apart from the noun that they are modifying. Our rules do not generate such sentences, but one might ask whether our formation rules should permit them to be generated from constructions in which the relative clause immediately follows the term that is modified. It is not clear to me at the moment whether this offers special possibilities for scope relations, as in 'Some man sees every donkey who kicks it,' where the 'who' refers back to 'man' and the 'it' to 'donkey.' I will assume that when a relative clause is separated from the term that it modifies, it is to be interpreted semantically as if it were not separated. I don't think that this was a unanimous view, since at least one author who insists on it says that others have denied it.<sup>18</sup> But I don't know what else to say here. At the moment such constructions are not generated, and they will be ignored.

<sup>18</sup> Anonymous, *About Univocation*, p. 570: "However, there can be a doubt, when the order of a locution on the front is contrary to the principle of the construction. By which in locutions of this sort an appellative name which supposes ought to contract appellation. As when it is said: 'Something will be Antichrist which is not a human,' the principle of the construction requires that the order of the locution is this: 'Something which isn't a human will be Antichrist'; and thus this name 'human' contracts the appellation from this verb 'is.' However the order of the locution on the front requires that it contract the appellation from this verb: 'will be,' since it is spoken first in that locution.

Some say to this that the appellative name always contracts the appellation from the near one, so that if the one nearer to it be a present tense verb it contracts the appellation from that, if it is a past tense verb, from that, if of future time, from that. However we say that it always contracts the appellation from that with which it is construed intransitively, whether that is spoken first, or not. And if therefore in the foregoing locution the appellative name is construed with this verb 'will be' intransitively, it contracts the appellation from that, no matter how the locution is ordered."

## APPLICATIONS

Give the logical forms underlying Buridan's sentences, and derive each from the other:

*Every man who is white runs*

*Every white man runs*

Give the logical forms for these arguments and show that they are valid.

*Brownie is an animal which<sub>acc</sub> Plato sees*

∴ *Plato sees Brownie<sub>acc</sub>*

*Brownie is an animal which<sub>acc</sub> Plato sees*

∴ *Plato sees an animal<sub>acc</sub>*

*Some animal which runs sees a donkey<sub>acc</sub>*

∴ *Some donkey<sub>acc</sub> an animal sees*

*No animal which runs sees a donkey<sub>acc</sub>*

*Brownie is an animal which sees a donkey<sub>acc</sub>*

∴ *Brownie doesn't run*

## 5.7 Genitives

The genitive shows up in English in two different forms, as a genitive ending and as a prepositional phrase construction with 'of':

*This is the woman's donkey*

*This is the donkey of the woman.*

In the Latin of the logic texts there is only one construction commonly discussed:<sup>19</sup> the genitive noun (the one that in English gets the possessive ending or is object of the preposition 'of') is inflected for genitive case. The difference from the English use of "- 's" is that in Latin the possessive noun needn't immediately precede what it modifies. The following word orders are all OK in Latin:

*Every woman's donkey is lame.*

*A donkey every woman's is lame.*

*A donkey is lame every woman's.*

These forms are confusing in English; to increase comprehension I will usually use the 'of' form in the English transliterations. This is a convention sometimes used in the literature, because it is useful to English readers.<sup>20</sup> I will assume that if an 'of-' occurs on

<sup>19</sup> I am not aware of discussion by logicians of the use of the dative case to indicate possession.

<sup>20</sup> See Karger 1997 for an illustration of this convenient usage.



the front of a denoting phrase whose term is not marked as accusative, then the term is in the genitive case. (I refrain from introducing genitive subscripts merely to avoid clutter.) Thus, although the following are not standard English, you should easily be able to figure out what they mean:

*Of-every woman a donkey is lame.*

*A donkey of-every woman is lame.*

The first means that for each woman *w*, some donkey of *w*'s is lame. The second means that there is a donkey that is every woman's, and it is lame. It is apparent that there is more freedom to order the denoting phrases than in English. (I will continue occasionally to use the English apostrophe 's' construction when this produces a more natural rendition.)

Genitives are important to medieval logicians because they afford a natural means to make propositions with multiple quantifiers with interacting scopes.<sup>21</sup> Some examples are:

*Of-every man some donkey sees every horse.*

*Of-some man every donkey sees some horse.*

*Of-some man every donkey sees of-every woman no horse.*

with the last meaning that there is some man such that every donkey of his is such that for every woman, the donkey sees none of her horses. Each of those sentences is naturally read as containing three or more simple terms. It is equally easy to make sentences which are naturally read as containing complex terms such as:

*Some donkey of every man sees every horse of a woman.*

Reading scopes from left to right this is a sentence with two complex terms: '*donkey of every man*' and '*horse of a woman*.' Clearly there are plenty of potential complexities for logicians to untangle.

### 5.7.1 *What the genitive means*

Some common nouns, such as '*mother*,' '*owner*,' . . . are inherently relational; others such as '*woman*,' '*chair*' are not. Typically the genitive construction used with a *non*-relational noun indicates some kind of possession ('*a horse of Fred's*') or something analogous to possession ('*Mary's job*'), or almost any other kind of relation that can be doped out from context ('*Mary's hill*' = the hill Mary has been assigned to climb).<sup>22</sup> Usually the genitive construction used with a relational noun indicates the relation

<sup>21</sup> The same potential exists with parasitic terms that are participles of transitive verbs, but these were not so widely discussed. An example would be '*Every man some donkey seeing sees no horse*' meaning that for every man there is a donkey seeing him who doesn't see any horse (if '*man*' is in the accusative case) or meaning that for every man who sees some donkey he sees no horse (if '*donkey*' is in the accusative case).

<sup>22</sup> Buridan *SD* 4.4.7 (287) distinguishes similar kinds of use of the genitive; he adds to these the use of the genitive to indicate efficient cause, and he states that the genitive is "taken in many ways." I lump together all of the ways that are not uses of relational nouns, leaving it to context to determine how they are to be interpreted.

conventionally associated with the noun; ‘*Mary’s mother*’ usually means the female person who gave birth to Mary. Relational nouns also have non-relational uses, as in: ‘*Four mothers showed up*.’ I assume that these non-relational uses are best construed as paronyms; they are non-relational nouns whose meaning is derived from a relational use of the same noun, as in ‘*mother*’, meaning ‘*mother of someone*.’<sup>23</sup> These non-relational nouns then also enter into the first kind of genitive construction, so that ‘*Mary’s mother*’ can mean the mother (of someone) to whom Mary has been assigned as a case worker. The difference between these two different genitive relations is illustrated by the popular medieval sophism:

*That dog is yours*  
*That dog is a father*  
*So that dog is your father*

On the most natural reading, the genitive relation indicated by ‘*yours*’ in the first premise is ownership and that in the conclusion is fatherhood; and ‘*father*’ is used non-relationally in the second premise and relationally in the conclusion. So there is a fallacy of ambiguity, though it is not equivocation or amphibole or composition/division; at best it falls under Aristotle’s fallacy of form of expression. (There is also a less natural reading on which the argument is valid; this is the reading on which ‘*father*’ in the conclusion has its non-relational use, and the genitive in the conclusion is interpreted as the generic possession relation.)

On either the relational noun or the possession interpretation, one ingredient of interpreting the genitive in a sentence is to mark the grammatical relation between the denoting phrase which is marked as genitive and the denoting phrase that is the “possessed.” We will do this as usual by including a grammatical marker with the “possessed” term; that marker will be bound by the “possessor” denoting phrase. For example, in ‘*some owner of Brownie*’ or ‘*of Brownie some owner*’ the relational noun ‘*owner*’ is the one that must be logically special; it will include a grammatical marker which will be bound by a denoting phrase containing ‘*Brownie*’, which fills that grammatical role and is thus in the genitive case, and which is logically ordinary. We will consider first the relational noun uses of the genitive, and then the possession uses; finally we look briefly at complex terms made using the genitive.

### 5.7.2 *Relational common nouns*

We begin with relational common nouns, such as ‘*mother*.’ It seems natural to suppose that each of these signifies a relation, such as the relation of being a mother of. The presence of such a noun provides a grammatical role for the denoting phrase that picks

<sup>23</sup> Ockham *SL* 1.52 (171–2) says: “But there are . . . names which cannot be truly predicated of anything unless it is possible to add to them names which are not their abstract forms; the added names are in one of the oblique cases, . . . Examples are names like ‘*master*’ and ‘*slave*’ and ‘*father*’ and ‘*son*’; for no one is a father unless he is someone’s father, nor is anything similar unless it is similar to something.” I take him here to be saying that any non-relational use of ‘*father*’ is equivalent to ‘*someone’s father*.’

out which thing the mother is a mother of. This is similar to participles of transitive verbs providing grammatical roles for their direct objects. We will add a new category of terms to the vocabulary of Linguish:

**Relational nouns:**

If ‘N’ is a relational noun, then ‘N-of- $\alpha$ ’ is a common term. The term ‘N-of- $\alpha$ ’ supposits for a thing  $o$  relative to  $o'$  iff  $o$  is related to  $o'$  by the relation that ‘N’ signifies.

E.g. relative to Socrates, the common term ‘**mother-of- $\alpha$** ’ supposits for those things that are Socrates’ mother. So on this assignment, ‘(Socrates  $\alpha$ )(**every mother-of- $\alpha$   $\beta$** )’ will mean something like ‘every mother of Socrates. . . .’ So we have sentences like:

(**every donkey  $\alpha$** )(**· mother-of- $\alpha$   $\beta$** )  **$\beta$  speaks**

*Of-every donkey a mother speaks*      <Every donkey’s mother speaks>

In Latin the relational noun needn’t immediately follow the term it is construed with:

(**every donkey  $\alpha$** )(Socrates  $\beta$ )(**· mother-of- $\alpha$   $\gamma$** )  **$\beta$  sees  $\gamma$**

*Of-every donkey Socrates sees a mother*

Unlike English,<sup>24</sup> the terms related to relational nouns in Latin easily take quantifier expressions other than indefinites, as in:

(**some woman  $\alpha$** )(**every child-of- $\alpha$   $\beta$** )  **$\beta$  speaks**

*Of-some woman every child speaks*      <Some woman’s every child speaks>

### 5.7.3 Non-relational uses

We can provide for the non-relational use of relational nouns by defining as follows:

**Non-relational occurrences of relational nouns:**

If ‘N’ is a relational noun, then ‘N- $\exists$ ’ is a common term. ‘N- $\exists$ ’ supposits (with respect to time  $t$ ) for a thing  $o$  iff  $o$  is related to something by the relation that ‘N’ signifies (with respect to  $t$ ).

The ‘ $\exists$ ’ symbol is heuristic only. We suppose that in the transition to natural language, the ‘- $\exists$ ’ vanishes. An example is:

<sup>24</sup> It is a common view that genitives in English are part of the quantificational structure of the “possessed” noun. So ‘*Every donkey’s mother speaks*’ contains a single complex denoting phrase, where the “quantifier” of ‘*mother*’ is ‘*every donkey’s*’. As a result, something like ‘*Some woman’s every child speaks*’ is ungrammatical in English. I take it, however, that it can be understood, and that it is clear what it means. It is grammatical in Latin. Because of the way in which genitives in English are formed, it may not be right to see them as cases of role-filling, as we are doing here. I think there is no consensus on whether this is the right analysis for Latin.

(every mother- $\exists \beta$ ) $_{\beta}$  speaks  
*Every mother speaks*

We have a rule of inference relating non-relational uses of relational nouns to those nouns:

**Non-relational uses of relational nouns**

$(\mathbf{m}_{\alpha})(\cdot \mathbf{N}\text{-of-}\alpha \beta) \phi$   
 $\therefore (\cdot \mathbf{N}\text{-}\exists \beta) \phi$                       when  $\phi$  is affirmative

If we had a “universal” term ‘*thing*’ we could add a two-way rule, giving:

**Non-relational uses of relational nouns**

$(\cdot \mathbf{thing}_{\alpha})(\cdot \mathbf{N}\text{-of-}\alpha \beta) \phi$   
 $\therefore (\cdot \mathbf{N}\text{-}\exists \beta) \phi$                       *and vice versa*

To flesh this out would require some additional constraints on ‘*thing*’, requiring something like “everything that is is a thing”:

**Things**

<P is non-empty>  
 $\therefore (\mathbf{every P}_{\alpha})(\cdot \mathbf{thing}_{\beta})_{\alpha} \text{ is } \beta$                       *and vice versa*

However this is speculative, and we will not pursue it further here.

**APPLICATIONS**

Give the logical forms for these arguments and show that they are valid.

*Xanthippe is of-some woman an aunt*    <Xanthippe is some woman’s aunt>  
*Every woman is an animal*  
 $\therefore$  *Xanthippe is of-some animal an aunt*

*Every woman is of-some woman a daughter*  
*Every woman is a human*  
 $\therefore$  *Every woman is of-some human a daughter*

*Some woman is of-Socrates a wife*  
 $\therefore$  *Some woman is a wife*

#### 5.7.4 Non-relational possessives

Suppose that a sentence contains a word in the genitive case, but there is no relational noun, as in ‘*Some woman’s donkey runs.*’ Then there is a possessor and a possessed. In English there is a use of ‘*have*’ that stands for this abstract relation, whatever it is in a given context. Mary’s book is a book that she *has*; and if she has been assigned to climb hill number 413 she can say either ‘*Hill 413 is my hill*’ or ‘*I have hill 413.*’ This means that non-relational possessives can be explained in terms of a highly context-sensitive form of the verb ‘*have*.’

##### Non-relational nouns and having:

If  $N$  is a non-relational noun, then ‘ $N$ -poss- $\alpha$ ’ is a common term that supposit $_{\sigma}$  (with respect to time  $t$ ) for a thing  $o$  that ‘ $N$ ’ signifies (relative to  $t$ ) relative to  $o'$  iff  $o'$  is had (with respect to  $t$ ) by  $o'$ .

The verb ‘**has**’ holds of a pair of things iff its first member has its second member.

The sequence of denoting phrases ‘(Socrates  $\alpha$ )(every horse-poss- $\alpha$   $\beta$ )’ will mean ‘Socrates’ every horse,’ or ‘Of Socrates, every horse.’ The ‘poss’ that is hyphenated between the noun and the grammatical marker is there to indicate generalized possession. It is part of the logical apparatus, and is thus to be erased in the transition from Linguish to natural language.

In Latin the possessor phrase may precede the possessed with quantifier words on the possessed. A typical Latin construction would be:

*Of-some farmer every donkey is brown*

meaning that there is a farmer such that every one of his/her donkeys is brown. In English this would be worded somewhat awkwardly as:

*Some farmer’s every donkey is brown.*

The logical form underlying this sentence is:

(Some farmer  $\alpha$ ) (every donkey-poss- $\alpha$   $\beta$ )  $\beta$  is brown.

Erasing the markers and parentheses yields the desired:

*Of-some farmer every donkey is brown.*

Unlike the genitive associated with a relational noun, the genitive of possession is logically complex: Brownie is Socrates’ donkey if and only if Brownie is a donkey and Socrates has Brownie. We have this inference rule for ‘ $N$ -poss- $\alpha$ ’:

**Possession:**

$(\mathbf{m} \delta)(\mathbf{n} \alpha)(\cdot\mathbf{N}\text{-poss-}\alpha \epsilon) \delta \text{ is } \epsilon$	$(\mathbf{m} \delta)(\cdot\mathbf{N} \epsilon) \delta \text{ is } \epsilon$
$\therefore (\mathbf{m} \delta)(\cdot\mathbf{N} \epsilon) \delta \text{ is } \epsilon$	$(\mathbf{n} \alpha)(\mathbf{m} \beta) \alpha \text{ has } \beta$
$\therefore (\mathbf{n} \alpha)(\mathbf{m} \beta) \alpha \text{ has } \beta$	$\therefore (\mathbf{m} \delta)(\mathbf{n} \alpha)(\cdot\mathbf{N}\text{-poss-}\alpha \epsilon) \delta \text{ is } \epsilon$

This will let us make inferences that are not possible with relational nouns. For example, given that every donkey is an animal, we can infer:

*Some farmer's donkey is running*  
 $\therefore$  *Some farmer's animal is running*

But given that every mother is a daughter,<sup>25</sup> we may not infer:

*Some farmer's mother is running*  
 $\therefore$  *Some farmer's daughter is running*

(That is, we may not infer this if the genitives in the premise and conclusion are understood as meaning the motherhood and daughterhood relations, as opposed to some kind of possession.)

But since every mother of someone is an ancestor *of theirs*, this inference *is* good:

*Some farmer's mother is running*  
 $\therefore$  *Some farmer's ancestor is running*

I am not aware of any discussion of these inferences by medieval authors. For that reason I will not comment on them further.

**APPLICATIONS**

Give the logical forms for these arguments and show that they are valid.

*Brownie is of-some woman a donkey* <Brownie is some woman's donkey>  
*Every donkey is an animal*  
 $\therefore$  *Brownie is an animal*

*Every woman has a donkey*  
*Xanthippe is a woman*  
 $\therefore$  *Some donkey is of-Xanthippe a donkey*

*Some animal is of-Socrates a donkey*  
 $\therefore$  *Socrates has a donkey*

<sup>25</sup> Of course, the claim that every mother is a daughter makes sense only if the terms 'mother' and 'daughter' are used in their non-relational senses. In any event, the inference is not valid.

### 5.7.5 Complex terms with genitives

As with participles, genitives form complex terms. Their constructions mirror those with participles. Examples are:

*every donkey of Socrates is running*  
*every father of some donkey is running*  
*every donkey's father is running*

#### Complex genitive terms

If 'P' is a relational common term, 'T' a non-relational common term, 't' a singular term, and 'Q' a quantifier sign, then ' $\{(t \gamma)P\text{-of-}\gamma\}$ ', ' $\{(t \gamma)P\text{-poss-}\gamma\}$ ', ' $\{(Q T \gamma)P\text{-of-}\gamma\}$ ', and ' $\{(Q T \gamma)P\text{-poss-}\gamma\}$ ' are common terms.

Supposition conditions: These are similar to those for complex terms formed using the participles of transitive verbs.

These terms are subject to the same rules as those made up with participles:

#### Rule Complex Term 2:

$(Q\{(t \gamma)P\text{-of-}\gamma\} \beta)$  is equipollent to  $(t \gamma)(Q P\text{-of-}\gamma \beta)$

$(Q\{(t \gamma)P\text{-poss-}\gamma\} \beta)$  is equipollent to  $(t \gamma)(Q P\text{-poss-}\gamma \beta)$

$(\cdot\{(\cdot T \gamma)P\text{-of-}\gamma\} \beta)$  is equipollent to  $(\cdot T \gamma)(\cdot P\text{-of-}\gamma \beta)$

$(\cdot\{(\cdot T \gamma)P\text{-poss-}\gamma\} \beta)$  is equipollent to  $(\cdot T \gamma)(\cdot P\text{-poss-}\gamma \beta)$

Similarly, permutations are possible.

#### The derived rule **Modifier-Permute**:

Any proposition containing ' $\{(Q T \gamma)P\text{-of-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-of-}\gamma\text{-thing} (Q T \gamma)\}$ '

Any proposition containing ' $\{(t \gamma)P\text{-of-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-of-}\gamma\text{-thing} (t \gamma)\}$ '

Any proposition containing ' $\{(Q T \gamma)P\text{-poss-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-poss-}\gamma\text{-thing} (Q T \gamma)\}$ '

Any proposition containing ' $\{(t \gamma)P\text{-poss-}\gamma\text{-thing}\}$ ' is logically equivalent to the result of replacing it by ' $\{P\text{-poss-}\gamma\text{-thing} (t \gamma)\}$ '

Given our rules for forming sentences with genitives, when the possessed term in a genitive relation precedes the possessor term they must both be part of a single complex term. For if the terms are in independent denoting phrases, there is no way for a marker attached to the possessed term to be bound by a denoting phrase to its right. But when the possessed term follows the possessor term then those terms may either be parts of independent denoting phrases, with the possessor denoting phrase binding the marker attached to the possessed term, or they may be parts of the same complex term by the rule just given for complex genitive terms. If they are part of a single complex term then there will be two quantifier signs in a row, as in ‘some every farmer’s donkey,’ meaning some donkey possessed by all the farmers. In Latin, where the indefinite construction does not involve a visible indefinite sign, the surface wording may be ambiguous, so that ‘every farmer’s donkey’ may mean either “for every farmer there is a donkey such that,” or “for every donkey-belonging-to-some-farmer.” Given the various options for including genitive constructions in sentences, it may not always be obvious which construction is present, given only the surface order of the words and their grammatical cases. Late medieval logicians put some energy into clarifying this; see section 6.3 for discussion.

#### APPLICATIONS

Give the logical forms for these arguments and show that they are valid.

*Every every farmer’s donkey is grey*

∴ *Some donkey of every farmer is grey*

*Every some farmer’s donkey is grey*

*Brownie is of some farmer a donkey*

∴ *Brownie is grey*

*Some every farmer’s donkey is grey*

∴ *Of every farmer some donkey is grey*

## 5.8 Demonstratives

Medieval authors make frequent use of two kinds of demonstrative terms. There are simple demonstratives, such as ‘*this*’ and ‘*that*,’ which function on their own as singular terms. And there are complex demonstrative terms consisting of a demonstrative plus a common term, such as ‘*this donkey*’ and ‘*that stone*.’ Each of these supposits for at most one thing. In the tradition, a demonstrative supposits for whatever is demonstrated by a use of a word like ‘*this*.’ (Note that the demonstration could be virtual, since people spoke of demonstrating non-perceivable things, such as God.) Simple demonstratives



supposit for whatever it is that they are used to demonstrate;<sup>26</sup> if they do not in fact demonstrate anything, they supposit for nothing. A complex demonstrative supposits for whatever its demonstrative part demonstrates provided that its common term supposits for the thing demonstrated; so ‘*this donkey*’ supposits for the donkey that is demonstrated if a donkey is demonstrated; otherwise it supposits for nothing.

We can incorporate simple demonstrative singular terms in the language by using: ‘**this<sub>1</sub>**’, ‘**this<sub>2</sub>**’, ‘**this<sub>3</sub>**’, . . . Complex demonstratives will be of the form ‘**this<sub>n</sub> T**’ where ‘**T**’ is any common term:

### Simple demonstratives

Each positive numeral indicates a possible demonstration of a thing. For each  $n$ , ‘**this<sub>n</sub>**’ is a singular term which supposits (with respect to time  $t$ ) for the  $n$ th thing demonstrated (or for nothing if the  $n$ th demonstration does not succeed in picking out a thing).

### Complex demonstratives

If ‘**T**’ is a common term then ‘**this<sub>n</sub> T**’ is a singular term which supposits (with respect to time  $t$ ) for whatever ‘**this<sub>n</sub>**’ supposits for (with respect to  $t$ ) if ‘**T**’ also supposits for that thing (with respect to  $t$ ); otherwise ‘**this<sub>n</sub> T**’ supposits for nothing (with respect to  $t$ ).<sup>27</sup>

#### Rule Demonstrative:

$$\begin{array}{ll} (\mathbf{this}_k \mathbf{T} \alpha)(n \beta) \alpha \text{ is } \beta & (\mathbf{this}_k \alpha)(n \beta) \alpha \text{ is } \beta \\ \therefore (\mathbf{this}_k \alpha)(n \beta) \alpha \text{ is } \beta & (\cdot \mathbf{T} \alpha)(n \beta) \alpha \text{ is } \beta \\ \therefore (\cdot \mathbf{T} \alpha)(n \beta) \alpha \text{ is } \beta & \therefore (\mathbf{this}_k \mathbf{T} \alpha)(n \beta) \alpha \text{ is } \beta \end{array}$$

The completeness theorem given earlier can be extended to sentences using mixed demonstratives by including the rules just given, and adding the provision that in constructing the interpretation  $\Gamma$  a simple demonstrative is assigned a supposition just as a name is assigned one, and the term ‘**this<sub>k</sub> F**’ supposits for whatever ‘**this<sub>k</sub>**’ supposits for (if anything) if ‘**F**’ supposits for that thing; otherwise it supposits for nothing.

<sup>26</sup> This is the right condition if singular terms are immune to restriction by tenses, etc. Otherwise one must say that simple demonstratives supposit for whatever they are demonstrating providing that that thing meets the restricting conditions.

<sup>27</sup> An anonymous comment on restrictions (in Marsilius of Inghen *TPT* appendix 2) gives an equivalent condition, phrased in terms of how the demonstrative restricts the supposition of the common term that it precedes: “a demonstrative pronoun or a relative [i.e. anaphoric pronoun] restricts a term to standing for that which is assigned to it by the pronoun or relative, as *this man runs*.”

## APPLICATIONS

Give the logical forms for these arguments and show that they are valid.

*Socrates sees that<sub>3</sub> donkey*

∴ *Socrates sees a donkey*

*Some horse sees that<sub>4</sub> donkey*

*Every donkey is an animal*

∴ *Some horse sees an animal*

*No woman sees that<sub>5</sub>*

∴ *No woman sees that<sub>5</sub> donkey*

## 5.9 Molecular propositions

Aristotle did not formulate logical principles governing propositions containing connectives. Whatever his view on this matter (assuming that he had a view),<sup>28</sup> he managed to validate conversions and syllogisms without ever himself using molecular propositions containing connectives, as we have seen. Principles such as modus ponens or disjunctive syllogism were just not relevant.

Propositional logic in fact existed, even in ancient times. Its development seems to be primarily due to the Stoics. Martianus Capella, writing apparently before 429, describes that part of the liberal arts curriculum called dialectic, or logic. It includes Aristotle's conversions and syllogisms, though not his proofs of conversion or reductions of syllogisms to the first figure. It also contains a set of seven principles of propositional logic, including various "modes" (in which disjunction is treated as exclusive). The first five of these, which eventually became known as the "five indemonstrables," are:

From antecedents:	P; if P then Q ∴ Q
From consequents:	P; if not Q then not P ∴ Q
From incompatibles:	not (P and not Q); P ∴ Q
By disjunction:	P or Q; P ∴ not Q
By disjunction:	P or Q; not P ∴ Q

(For more on Stoic logic see Bobzien 2006 or Mates 1961.)

Boethius wrote about molecular propositions, and they formed a standard part of the curriculum that was inherited by medieval logicians. Eventually, six types of molecular proposition were typically identified:

<sup>28</sup> One of Aristotle's commentators, Apuleius (p. 95), seems to hold that molecular inferences are superfluous "because they do not infer beyond what has been accepted." He gives as an example of such a superfluous inference '*If it is day, it is light; but it is day, therefore it is light.*' (Ascription of this text to Apuleius is disputed.)

Conjunctions  
 Disjunctions  
 Conditionals  
 Causals  
 Temporal Propositions  
 Locational Propositions

All of these are called *hypotheticals*, not just the conditionals.

Conjunctions and disjunctions are just what one expects; they are combinations of propositions made with ‘*and*’ (*et*) and ‘*or*’ (*vel*), or similar words. A conjunction is true if both of its conjuncts are true, and false if either or both are false. In Stoic logic disjunction was taken to be exclusive, but in medieval logic a disjunction is usually taken to be true if either or both of its disjuncts are true, and false if both disjuncts are false. So they were usually given modern truth conditions.<sup>29</sup>

Conditionals are made with ‘*if*’ (*si*), or equivalent words. Conditionals were given various analyses; a common one is that a conditional is taken to be true if it is necessary that the antecedent not be true without the consequent also being true.<sup>30</sup> This makes the truth of a conditional equivalent to the goodness of the related argument, as some writers observed.<sup>31</sup> Indeed, when an author is discussing “consequences” it is sometimes unclear whether it is arguments or conditionals that are meant—unclear because the discussion makes sense read either way. Some authors also discuss “*ut nunc*” (*as of now*) conditionals; as usually explained, these turn out to be equivalent to our material implication.<sup>32</sup> But the commonest unqualified meaning of a conditional is that it is true if and only if it is impossible for the antecedent to be true and the consequent false. One can get a feel for medieval theorizing about conditionals from Walter Burley’s “Consequences” in Kretzmann and Stump 1988.

Causal, temporal, and locational propositions are not generally integrated with the other logical principles, and they will not be discussed here.

<sup>29</sup> Sherwood SXXI.1 (141) holds that “‘*or*’ is taken sometimes as a disjunctive and at other times as a sub-disjunctive. In the first case it indicates that one is true and the other is false; in the second case it indicates solely that one is true while touching on nothing regarding the other part.” Later, Paul of Venice *LPI*.14 (132) gives inclusive truth conditions: “For the truth of an affirmative disjunctive it is required and it suffices that one of the parts is true; e.g. ‘you are a man or you are a donkey.’ For the falsity of an affirmative disjunctive it is required that both parts be false; e.g., ‘you are running or no stick is standing in the corner.’”

<sup>30</sup> Ockham *SL* II.31; Peter of Spain *LS* I.17. Sherwood *IL* I.18 gives truth conditions that are not clearly modal: “whenever the antecedent is [true], the consequent is [true].”

<sup>31</sup> Ockham *SL* II.31.

<sup>32</sup> Buridan *TC* I.4.7 (185) gives as the conditions of an *ut nunc* consequence: “it is completely impossible with things related as they are now related for the antecedent to be true without the consequent <being true as well>.” This is understood so that if the antecedent is false, the antecedent cannot be true as things are now related (since they are so related as to make the antecedent false), and the *ut nunc* reading is true; likewise, if the consequent is true then, again, with things as they are now, it can’t but be true. So the result is our material conditional. Buridan I.4.11 (186) points out that “if the antecedent is false, though not impossible, the consequence is acceptable *ut nunc*.” At I.4.12 he gives a case in which the antecedent is true, and whether or not the consequence is acceptable *ut nunc* depends on whether or not the consequent is actually true.

The Stoics pursued propositional logic through giving axioms and deriving theorems from them. In medieval logic tautological relations were often stated without justification. For example, versions of De Morgan's laws were stated as if pointing them out is justification enough.

**Conjunctions, disjunctions, and ut nunc conditionals:** It is easy to include molecular sentences which have no free markers in them in Linguish. Just add the following:

If ' $\phi$ ' and ' $\psi$ ' are formulas with no free variables, so are ' $[\phi \text{ and } \psi]$ ' and ' $[\phi \text{ or } \psi]$ ' and ' $[\text{if } \phi \text{ then } \psi]$ .'

' $[\phi \text{ and } \psi]$ ' is true<sub>o</sub> iff ' $\phi$ ' is true<sub>o</sub> and ' $\psi$ ' is true<sub>o</sub>

' $[\phi \text{ or } \psi]$ ' is true<sub>o</sub> iff ' $\phi$ ' is true<sub>o</sub> or ' $\psi$ ' is true<sub>o</sub> (or both)

' $[\text{if } \phi \text{ then } \psi]$ ' is true<sub>o</sub> iff ' $\phi$ ' is false<sub>o</sub> or ' $\psi$ ' is true<sub>o</sub> (or both) <the as-of-now reading>

**Rules:** All tautological implications, including e.g.:

From ' $[\phi \text{ and } \psi]$ ' infer ' $\phi$ ,' and infer ' $\psi$ '

From ' $\phi$ ' and ' $\psi$ ' infer ' $[\phi \text{ and } \psi]$ .'

From ' $[\phi \text{ or } \psi]$ ' and '**not**  $\phi$ ,' infer ' $\psi$ ,' and

from ' $[\phi \text{ or } \psi]$ ' and '**not**  $\psi$ ,' infer ' $\phi$ .'

From ' $\phi$ ' or from ' $\psi$ ' infer ' $[\phi \text{ or } \psi]$ .'

From ' $[\text{if } \phi \text{ then } \psi]$ ' and ' $\phi$ ,' infer ' $\psi$ ,' and

from ' $[\text{if } \phi \text{ then } \psi]$ ' and '**not**  $\psi$ ,' infer '**not**  $\phi$ .'

From '**not**  $\phi$ ' or from ' $\psi$ ' infer ' $[\text{if } \phi \text{ then } \psi]$ .'

With the addition of molecular propositions we should expand our account of when a term occurs in an affirmative or negative context. We can say the following:

If a term ' $\alpha$ ' occurs affirmatively in ' $\phi$ ,' then it also occurs affirmatively in ' $[\phi \text{ and } \psi]$ ' and in ' $[\psi \text{ and } \phi]$ .'

If a term ' $\alpha$ ' occurs negatively in ' $\phi$ ,' then it also occurs negatively in ' $[\phi \text{ or } \psi]$ ' and in ' $[\psi \text{ or } \phi]$ .'

If a term ' $\alpha$ ' occurs negatively in '**not**  $\phi$ ,' then it also occurs negatively in ' $[\text{if } \phi \text{ then } \psi]$ ' and if ' $\alpha$ ' occurs negatively in ' $\psi$ ,' then it also occurs negatively in ' $[\text{if } \phi \text{ then } \psi]$ .'

This still holds:

If a term ' $\alpha$ ' occurs affirmatively (negatively) in ' $\phi$ ,' then it occurs negatively (affirmatively) in '**not**  $\phi$ .'

It is clear from this that some occurrences of terms are no longer either affirmative or negative. For example, this is true of '*donkey*' in '*Some donkey runs or some horse runs*';

it does not occur affirmatively because its emptiness does not make the whole sentence false, and it does not occur negatively because its emptiness does not make the whole sentence true.

### 5.9.1 *Constructions with free markers*

There is a problem with propositions containing conjunctions and disjunctions of sentences with free markers: it is not clear that there are such things. Recall that markers mark grammatical roles. If a whole conjunct, say, contains a free marker, then it must be bound by some denoting phrase not within that conjunct. But then the main term of that denoting phrase is occupying a grammatical role in that conjunct while not itself occurring in the conjunct. If we were not concerned to stick to the resources recognizable to medieval logicians, this would be easy; just allow conjunctions and disjunctions with free markers to be bound by additional denoting phrases, and don't worry about it. The truth conditions are no problem. But we would have a genuinely artificial language that corresponds to nothing in natural language. For example, suppose we conjoin the formulas:

(some donkey  $\alpha$ )  $\alpha$  runs  
 $\beta$  sits

into the conjunction:

[(some donkey  $\alpha$ )  $\alpha$  runs and  $\beta$  sits]

Then we add a denoting phrase to the front:

(every horse  $\beta$ ) [(some donkey  $\alpha$ )  $\alpha$  runs and  $\beta$  sits]

The truth conditions are straightforward; the sentence is true if and only if every horse is such that some donkey runs and it (the horse) sits. But that logical form of Linguish yields a natural language sentence that doesn't actually seem to be a sentence at all:

(every horse  $\beta$ ) [(some donkey  $\alpha$ )  $\alpha$  runs and  $\beta$  sits]  $\Rightarrow$   
*every horse some donkey runs and sits*

This is not grammatical English, and its analogue in Latin is not grammatical either.

It thus appears that we must—at least for the time being—forbid the formation of conjunctions and disjunctions containing free grammatical markers. This topic will be explored further, after we have considered how medieval logicians treated anaphoric expressions in Chapter 8.

## APPLICATIONS

Give the logical forms for these arguments and show that they are valid.

*Socrates sees a horse or Brownie is grey*

*Brownie isn't grey*

*Every horse is an animal*

∴ *Socrates sees an animal*

*If some horse sees a donkey then no donkey sees a horse*

*Some donkey sees a horse*

∴ *No horse sees a donkey*

*No horse which sees a donkey is grey*

*Brownie is a donkey and some horse sees Brownie*

∴ *Some horse isn't grey*

In the next two chapters molecular propositions will be ignored, since nothing else to be discussed depends on them. They will reappear in Chapter 8.

# 6

## Some Illustrative Topics

In the last chapter a number of linguistic constructions were introduced with a minimum of discussion. This chapter discusses a few particular cases in which those constructions are employed. Section 6.1 is about relational verbs, parasitic terms, and complex terms. Section 6.2 focuses on some particular views about subjects and predicates; this section contains some test cases for the new constructions. Section 6.3 focuses on how one tells in particular cases whether one is dealing with independent terms or complex terms with terms as parts.

### 6.1 Relational expressions and De Morgan's challenge

In De Morgan 1847, Augustus De Morgan states that there are certain good inferences that are not syllogistically good. He says:

There is another process which is often necessary, in the formation of the premises of a syllogism, involving a transformation which is neither done by syllogism, nor immediately reducible to it. It is the substitution, in a compound phrase, of the name of the genus for that of the species, when the use of the name is particular. For example, 'man is animal, therefore the head of a man is the head of an animal' is inference, but not syllogism. (114)

He says that this is an application of the *dictum de omni et nullo*,<sup>1</sup> which he explains as:

*dictum de omni et nullo* . . . is as follows. For every term used universally *less* may be substituted, and for every term used particularly, *more*. (114–15)<sup>2</sup>

This principle will be discussed in section 7.5, where it will be seen to be a good one. However, the principle can only be used in reasoning when one can *show* that the term in question is indeed used universally, or used particularly. In the example cited by De Morgan, that is not at all trivial. In this section, we will see how to derive some

<sup>1</sup> The *dictum de omni et nullo* has been interpreted differently by different writers over the centuries. De Morgan's version is typical.

<sup>2</sup> In De Morgan 1966 he repeats the point, saying "I gave a challenge in my work on formal logic to deduce syllogistically from 'Every man is an animal' that 'every head of a man is the head of an animal'" (29; written in 1850). The same point is also made on page 216 (written in 1860). On page 252–3 (written in 1860) he sets a related but different problem: "let A be an existing animal; it follows that the tail of X is the tail of an animal," and he says that the consequence is formal, not material.

inferences of the kind that De Morgan is interested in. Later writers interpret the significance of De Morgan's challenge as one involving relational expressions (the relation "head of" in the quotation from De Morgan). In fact, his challenge raises a number of different issues. Since the example he himself gives is fairly complicated, I'll start with a simpler example involving relations, and then turn to De Morgan's specific example.

6.1.1 *Dealing with explicit relational expressions*

A typical illustration of how relational expressions interrelate with quantifiers is the following valid argument:

Some A sees every B  
 ∴ Every B is seen by some A

The argument is not reversible; the premise does not follow from the conclusion. This kind of argument uses an unanalyzed relational term, 'sees,' instead of the copula. Arguments of this sort are straightforward to deal with, not syllogistically, but using the logical techniques that Aristotle used to validate syllogisms together with some 13th-century doctrines about quantifiers and singular terms. Here is a derivation for the argument in Linguish notation. One posits the premise, and then the negation of the conclusion to set up a reductio. The derivation appeals to Aristotle's technique of exposition and universal application and then quantifier equipollences and, finally, principles for permuting singular terms and negations (which are laid out in section 4.3).

1.	<b>(Some A <math>\alpha</math>)(every B <math>\beta</math>) <math>\alpha</math> sees <math>\beta</math></b>	Premise
2.	<b>not (every B <math>\beta</math>)(some A <math>\alpha</math>) <math>\alpha</math> sees <math>\beta</math></b>	Hypothesis (for reductio)
3.	<b>(some B <math>\beta</math>)(every A <math>\alpha</math>) not <math>\alpha</math> sees <math>\beta</math></b>	2 quantifier equipollences
4.	<b>(b <math>\gamma</math>)(<math>\cdot</math> B <math>\beta</math>) <math>\gamma</math> is <math>\beta</math></b>	1 3 EX <line 1 guarantees non-emptiness of 'B'>
5.	<b>(b <math>\beta</math>)(every A <math>\alpha</math>) not <math>\alpha</math> sees <math>\beta</math></b>	1 3 EX <line 1 guarantees non-emptiness of 'B'>
6.	<b>(every A <math>\alpha</math>)(b <math>\beta</math>) not <math>\alpha</math> sees <math>\beta</math></b>	5 permute singular term
7.	<b>(a <math>\alpha</math>)(<math>\cdot</math> A <math>\beta</math>) <math>\alpha</math> is <math>\beta</math></b>	1 1 EX <line 1 guarantees non-emptiness of 'A'>
8.	<b>(a <math>\alpha</math>)(every B <math>\beta</math>) <math>\alpha</math> sees <math>\beta</math></b>	1 1 EX < line 1 guarantees non-emptiness of 'A'>
9.	<b>(every B <math>\beta</math>)(a <math>\alpha</math>) <math>\alpha</math> sees <math>\beta</math></b>	8 permute singular term
10.	<b>(b <math>\beta</math>)(a <math>\alpha</math>) <math>\alpha</math> sees <math>\beta</math></b>	4 9 UA
11.	<b>(a <math>\alpha</math>)(b <math>\beta</math>) not <math>\alpha</math> sees <math>\beta</math></b>	6 7 UA
12.	<b>not (b <math>\beta</math>)(a <math>\alpha</math>) <math>\alpha</math> sees <math>\beta</math></b>	11 permute negation and singular terms
13.	<b>(every B <math>\beta</math>)(some A <math>\alpha</math>) <math>\alpha</math> sees <math>\beta</math></b>	reductio; line 12 contradicts line 10

Attempting to derive the premise from the conclusion just results in a dead end. For a medieval type counterexample to inferring the premise from the conclusion, suppose that Plato and Socrates are all of the As and also all of the Bs. Suppose that Plato sees only Socrates, Socrates sees only Plato. Then every B is seen by some A, but it's not true that some A sees every B.



## APPLICATIONS

Produce the Linguish logical forms underlying the following arguments and (1) provide derivations to validate the good ones and (2) for the invalid ones, provide an argument with the same form having true premises and a false conclusion. (In the first two arguments ignore the readings in which dogs are said to own or not to own women.)

- Some woman owns no dog*  
 $\therefore$  *Every dog some woman doesn't own*
- Every dog some woman doesn't own*  
 $\therefore$  *Some woman owns no dog*
- Some animal is every phoenix*  
 $\therefore$  *Every phoenix is an animal*
- Every phoenix is an animal*  
 $\therefore$  *Some animal is every phoenix*

## 6.1.2 Dealing with parasitic terms

De Morgan's own example is different from the one just given, because, as he says, it involves complex terms ("compound phrases"): '*head of a man*,' '*head of an animal*.' In contemporary logic this would not be considered a problem for logic, since such phrases do not occur in standard logical notation. If it is a problem, it is a problem for ordinary language, and the problem is to be solved by finding a way to mimic the logic of the complex term of ordinary language by using some combination of quantifiers, connectives, etc. For example, one might decide to use a relational predicate, as in 'x is-of y.' Then the ordinary language sentence:

every head of a man is a head of an animal

could be symbolized as:

$$\forall x[\text{head}(x) \ \& \ \exists y[\text{man}(y) \ \& \ x \text{ is-of } y] \rightarrow \text{head}(x) \ \& \ \exists y[\text{animal}(y) \ \& \ x \text{ is-of } y]]$$

or one could use the relational predicate 'x is-a-head-of y,' and write:

$$\forall x[\exists y[\text{man}(y) \ \& \ x \text{ is-a-head-of } y] \rightarrow \exists y[\text{animal}(y) \ \& \ x \text{ is-a-head-of } y]]$$

Neither of these formulas contain any complex terms. Either of these can easily be deduced from the symbolization of '*every man is an animal*':

$$\forall x[\text{man}(x) \rightarrow \text{animal}(x)].$$

In medieval logic there is no such clear distinction between what is ordinary language and what is logical notation, and that is why I included complex terms in the

notation in section 5.8, so that 'head of a man' would be represented as the complex Linguish term '{head-of- $\alpha$  ( $\cdot$  man  $\alpha$ )}' which is pronounced, as usual, by erasing logical notation: 'head of a man.' Using this notation, and taking the existence of a head of a man as an additional premise (since otherwise the argument is invalid), the derivation can be done using the principle for complex terms from section 5.7.3, as follows.

To show: (every {head-of- $\alpha$  ( $\cdot$  man  $\alpha$ )}) $\beta$ )( $\cdot$  {head-of- $\delta$  ( $\cdot$  animal  $\delta$ )}) $\epsilon$ )  $\beta$  is  $\epsilon$

1.	(every man $\alpha$ )( $\cdot$ animal $\beta$ ) $\alpha$ is $\beta$	
2.	<{head-of- $\alpha$ ( $\cdot$ man $\alpha$ )} is non-empty>	
3.	<u>not (every {head-of-<math>\alpha</math> (<math>\cdot</math> man <math>\alpha</math>)})<math>\beta</math>)(<math>\cdot</math> {head-of-<math>\delta</math> (<math>\cdot</math> animal <math>\delta</math>)})<math>\epsilon</math>) <math>\beta</math> is <math>\epsilon</math></u>	
4.	(some {head-of- $\alpha$ ( $\cdot$ man $\alpha$ )}) $\beta$ )(every {head-of- $\delta$ ( $\cdot$ animal $\delta$ )}) $\epsilon$ ) not $\beta$ is $\epsilon$	3 Equipoll
5.	(h $\beta$ )( $\cdot$ {head-of- $\alpha$ ( $\cdot$ man $\alpha$ )}) $\epsilon$ ) $\beta$ is $\epsilon$	2 4 EX
6.	(h $\beta$ )(every {head-of- $\delta$ ( $\cdot$ animal $\delta$ )}) $\epsilon$ ) not $\beta$ is $\epsilon$	2 4 EX
7.	(h $\beta$ )( $\cdot$ man $\alpha$ )( $\cdot$ head-of- $\alpha$ $\epsilon$ ) $\beta$ is $\epsilon$	5 complex term 2
8.	( $\cdot$ man $\alpha$ )( $\cdot$ head-of- $\alpha$ $\epsilon$ )(h $\beta$ ) $\beta$ is $\epsilon$	7 permute
9.	(a $\alpha$ )( $\cdot$ head-of- $\alpha$ $\epsilon$ )(h $\beta$ ) $\beta$ is $\epsilon$	8 8 EX
10.	(a $\alpha$ )( $\cdot$ man $\beta$ ) $\alpha$ is $\beta$	8 8 EX
11.	(a $\alpha$ )( $\cdot$ animal $\beta$ ) $\alpha$ is $\beta$	1 10 UA
12.	( $\cdot$ animal $\alpha$ )( $\cdot$ head-of- $\alpha$ $\epsilon$ )(h $\beta$ ) $\beta$ is $\epsilon$	9 11 ES
13.	(h $\beta$ )( $\cdot$ animal $\alpha$ )( $\cdot$ head-of- $\alpha$ $\epsilon$ ) $\beta$ is $\epsilon$	12 permute
14.	(h $\beta$ )( $\cdot$ {head-of- $\delta$ ( $\cdot$ animal $\delta$ )}) $\epsilon$ ) $\beta$ is $\epsilon$	13 complex term 2
15.	(every {head-of- $\delta$ ( $\cdot$ animal $\delta$ )}) $\epsilon$ )(h $\beta$ ) not $\beta$ is $\epsilon$	6 permute
16.	(h $\epsilon$ )(h $\beta$ ) not $\beta$ is $\epsilon$	14 15 UA
17.	not (h $\epsilon$ )(h $\beta$ ) $\beta$ is $\epsilon$	16 permute
18.	(h $\epsilon$ )(h $\beta$ ) $\beta$ is $\epsilon$	13 self-iden
19.	(every {head-of- $\alpha$ ( $\cdot$ man $\alpha$ )}) $\beta$ )( $\cdot$ {head-of- $\delta$ ( $\cdot$ animal $\delta$ )}) $\epsilon$ ) $\beta$ is $\epsilon$	17 18 reductio

Although somewhat lengthy, there are no surprises. Relational notions, per se, are not problematic in medieval logic.

#### APPLICATIONS

Some of these arguments are valid. Produce the Linguish logical forms underlying them and (1) provide derivations to validate the good ones and (2) for the invalid ones, provide an argument with the same form having true premises and a false conclusion.

*Every woman is an animal*  
*Some head of a woman is a head of a woman*  
 $\therefore$  *Some head of an animal is a head of a woman*

*Every woman is an animal*  
*Some head of an animal is a head of an animal*  
 $\therefore$  *Some head of an animal is a head of a woman*

## 6.2 Buridan on subjects and predicates

In the original Aristotelian treatment, each proposition contains exactly one term used as subject and exactly one term used as predicate. In the expanded notation of medieval logic things become more complicated. The most sophisticated treatment of these complications is due to John Buridan. This section discusses some details of his views.

### 6.2.1 Identifying subjects and predicates

The logic that Buridan inherited contained a number of principles involving subjects and predicates of categorical propositions:

#### Quantity

A categorical proposition is universal, particular, indefinite, or singular iff its subject is universal, particular, indefinite, or singular. So you have to be able to identify the subject in order to determine the quantity of the proposition.

#### Conversions

Conversions are the result of interchanging subject and predicate. So you have to know what the subject and predicate are to determine what a conversion is.

#### Syllogisms

Syllogistic inferences are given in terms of figures, and the figures are defined in terms of whether the middle term is a subject or a predicate. So you have to know what the subject and predicate are in order to apply the theory of the syllogism to particular examples.

#### Ampliations

Tenses and modal words and words about conceiving, etc, can alter what the terms in a proposition supposit for. (Chapter 10.) Medieval logicians agree that these affect subject and predicate terms differently. So you need to know what the subject and predicate are to know how ampliation works.

These issues become interesting when categorical propositions get complicated. For example, what are the subject and predicate of ‘*Every farmer<sub>acc</sub> some donkey sees?*’ (Buridan would say that this proposition contains a copula, ‘*is*’, a subject, ‘*donkey*’, and a predicate, ‘*farmer seeing*.’) We can get no guidance about subjects and predicates from contemporary logicians, because they have abandoned the notions of subject and predicate as understood by ancient and medieval logicians.<sup>3</sup> But Buridan retained the notions of subject and predicate, and thought it vital to determine what these are in complicated cases. He also retained the idea that every categorical proposition consists of one subject and one predicate, together with the copula and some syncategorematic

<sup>3</sup> Frege, *Begriffsschrift*, Section 3: “A distinction of *subject* and *predicate* finds *no place* in my way of representing a judgment . . . In my first draft of a formalized language I was misled by the example of ordinary language into compounding judgments out of subject and predicate. But I soon convinced myself that this was obstructive of my special purpose and merely led to useless prolixity.”

signs—and nothing else. Thus, when a proposition has more than two terms, either the subject or the predicate has to contain more than two terms. This is unproblematic when the extra terms get into the proposition as parts of complex terms. But there are propositions with more than two main terms. Examples are:

*Of-every farmer some donkey is running*  
*Some donkey every farmer seeing is*  
*Of-every farmer some donkey sees every leg of-some horse*

It is also necessary in identifying subjects and predicates that one be able to tell when a string of terms constitutes a single complex term (such as ‘*leg of-some horse*’ in our last example) and when not, and this can be difficult as well.

Buridan defines a subject and predicate as follows: “A subject is that of which something is said; a predicate is that which is said of something else, namely, of the subject” (Buridan *SD* 1.3.3 (25)). This is too vague to be of much help in difficult cases. Mostly, what Buridan does is to say what he thinks the right analysis into subject and predicate is in particular cases, sometimes with no argument at all, and sometimes by arguing that this yields the right answer as to which pairs of propositions are contradictories and which are contraries—generally assuming that such pairs contain the same units as subjects and as predicates (though, in the case of simple conversions, the units may switch from one to the other status). I will not try to review the reasoning that he gives (I find much of it inconclusive). Instead, I will try to give a precise account of a method that agrees with his classifications, and then compare the results of applying this method with his own judgments in the cases that he discusses.

There is one important constraint on any choice. A spoken or written proposition signifies a mental proposition, and a mental proposition must consist of some simple concepts, which correspond to the subject and predicate of the spoken proposition, together with a *complexive concept* “by means of which the intellect affirms or denies the one of the other” (*SD* 1.3.2 (24)). The complexive concept corresponds to the copula<sup>4</sup> of the spoken proposition. This is usually not helpful in detail, since if it is unclear what are the subject and predicate of a spoken proposition, we won’t know more about the mental one. But it does absolutely require that a categorical proposition have the copula ‘*is*’ as the verb, and have a unique subject and a unique predicate. Buridan says that in order to determine the subject and predicate of a proposition with a non-copular verb, one needs to re-express the proposition by replacing the verb with a copula and the participle of the verb. He also sometimes does this replacement without comment. I will follow him here, rephrasing when necessary so that all propositions have the copula as their main verb.

Here is how to determine the subject and predicate of any categorical proposition. We begin by analyzing any verbs other than the copula into a copula and a participle,

<sup>4</sup> The copula of the spoken proposition, in simple cases anyway, is either the affirmative copula ‘*is*’ or the negative copula ‘*is not*’; these may be wholly or partly incorporated into another verb.

as discussed in sections 5.2–5.3. Then we “link” the main terms of the proposition with one another.

- If a main term is in the genitive case then there is another term in the proposition that is related to the first as possessed to possessor; when this is so, link those terms. (It may be a matter of interpretation which pairs are so related. If so, the proposition is ambiguous, and we need to consider its different readings separately.)
- If a main term is in the accusative case, this is because it is serving as the direct object of a transitive verb, which will appear in the proposition in participle form; link that term with the participle term as well. (Again, there may be more than one way to analyze the proposition.)

When no more linkings are possible, make sequences out of the mutually linked terms. (A sequence will have only one member if there is a term that is not linked to any other.) In any categorial proposition you will find that you have exactly two sequences. Each sequence will contain exactly one term in the nominative case (it’s the rightmost term in the sequence). The subject will be the string of expressions formed from one of the sequences, and the predicate will be the other.

Which string is the subject, and which the predicate? One natural response might be that so long as the verb is the copula, there is no answer to this question, since, speaking logically, it treats its terms the same. A similar response is that such a proposition is ambiguous; it may be read with either term as grammatical subject and either term as predicate. At least, this is true when both terms are in the third person singular; in that case there is no grammatical difference between subject and predicate. But if the persons are mixed, there may be a grammatical difference. For example, these are grammatically different propositions:

*You are Marcus.*<sup>5</sup>  
*Marcus is you.*

In the first proposition, ‘*you*’ is the subject, and in the second proposition ‘*Marcus*’ is the subject; you can tell this by the fact that the verb agrees with its subject in (grammatical) person. The same thing happens in Latin if you put the verb at the end:

*Marcus you are.*  
*Marcus you is.*

(In English the latter is ill formed, but it’s OK in Latin.) One might think then that the same possibilities of grammatical analysis are possible if both terms are third-person:

*Marcus Tully is*            <‘*Tully*’ is subject.>  
*Marcus Tully is*            <‘*Marcus*’ is subject.>

<sup>5</sup> Buridan *SD* 4.2.6 gives the example ‘White am I’, where the subject follows the predicate.

This approach has the unintuitive consequence that all categorical propositions in the third person whose verb is ‘is’ are ambiguous. For example, ‘*Every donkey is an animal*’ can be read with ‘*donkey*’ as the subject and ‘*animal*’ as the predicate, or the other way round. This is contrary to the tradition, which tends to take the first option for granted.

There is a Buridanian tradition, explained in Karger 1997, which holds that when all terms are in the third person, the grammar works differently than when they are mixed. It holds in particular that such a proposition is unambiguous, and you can tell which (string) is subject and which is not by the rule that the subject string is that string whose nominative term precedes the nominative term of the other string. This has the advantage of yielding natural unique readings in simple cases, where authors seem to see no ambiguity.

Some examples to illustrate Buridan’s view about subjects and predicates:

*Every donkey is an animal.*

Subject = *donkey*                      Predicate = *animal*

(No terms are linked. The subject is the term that comes first.)

*Of every farmer a donkey is running*

Subject = *of-farmer donkey*                      Predicate = *running*

(The first two terms are linked; they precede the predicate term.)

*Some donkey<sub>acc</sub> every farmer is seeing*

Subject = *farmer*                      Predicate = *donkey<sub>acc</sub> seeing*

*Some donkey every farmer<sub>acc</sub> is seeing*

Subject = *donkey*                      Predicate = *farmer<sub>acc</sub> seeing*

(This proposition is ambiguous in English because either term can be the seer. In Latin the nouns would have observable case inflections which would often eliminate any ambiguity.)

### 6.2.2 *Subjects and predicates in Linguish*

In this section we formulate precise criteria for identifying subjects and predicates in Linguish propositions, and in the surface sentences that are generated from them.

Some observations: When we build up any proposition in Linguish whose main verb is ‘is’ we begin with a copular formula with exactly two free markers, say  $\alpha$  and  $\beta$ . There are two ways of making a propositional formula  $\phi$  into a more complex categorical formula. One way is to add a **not**, which leaves the free markers unchanged; that is, the free markers in **not**  $\phi$  are the same as those in  $\phi$ . The other way is to add to  $\phi$  a denoting phrase, D, which binds one of the free markers in  $\phi$ . If D itself contains no free marker, the number of free markers in  $D\phi$  will be one less than the number in  $\phi$ . If D contains a free marker (as it will if its main term originates from a (participle of a) transitive verb, or is the “object” of a genitive denoting phrase) the number of free markers in  $D\phi$

will be the same as the number in  $\phi$ . So the total number of free markers is never more than two, and it eventually reaches zero when we end up with a proposition (which by definition has no free markers).

To generate Linguish subjects and predicates: Given a Linguish categorical propositional logical form  $\phi$ , make a sequence of denoting phrases starting with the first main denoting phrase, D, in  $\phi$ . The next member of the sequence is whatever denoting phrase, if any, contains a free marker that is bound by D. Continue this process. The last member of this first sequence is whatever member binds a marker in the copular formula itself. Then form another sequence starting with the first denoting phrase in  $\phi$  that is not in the first sequence. It is easy to show that for any categorical proposition  $\phi$  exactly two sequences may be formed in this way, and every main denoting phrase in  $\phi$  is a member of exactly one of them. (This yields essentially the same sequences as discussed earlier for natural language examples.)

I will stipulate that the sequence whose last term binds the marker preceding the copula is the subject sequence, and the other is the predicate. This seems like a fundamental assumption of how the markers used as indices of grammatical position work. If we also wish to go along with Buridan and avoid the ambiguity of surface sentences with regard to subject and predicate discussed in the last section, we can additionally stipulate that the term binding the subject marker must precede the term binding the predicate marker. That is, the Linguish logical form is ill formed when the term binding the subject marker follows the term binding the predicate marker. I'll suppose that this stipulation is in effect for the purpose of discussing examples in this chapter. This issue will re-arise in Chapter 10.

Some examples follow in which the subjects and predicates are identified. We express what the subject and the predicate are using ordinary language terminology, as Buridan does. This can be done by first taking the Linguish subjects and predicates and applying our previously given rules for generating natural language. Then we eliminate the commas that punctuate the sequence, and also erase all overt quantifier signs:

**(Every donkey <sub>$\alpha$</sub> )(· animal <sub>$\beta$</sub> ) <sub>$\alpha$</sub>  is  <sub>$\beta$</sub>**

*Every donkey is an animal*

Subject:       **(Every donkey <sub>$\alpha$</sub> )**

*Every donkey  
donkey*

Predicate:     **(· animal <sub>$\beta$</sub> )**

*an animal  
animal*

**(Every farmer <sub>$\alpha$</sub> )(some donkey-poss- <sub>$\alpha$</sub>  <sub>$\beta$</sub> )(· running-thing <sub>$\gamma$</sub> ) <sub>$\beta$</sub>  is  <sub>$\gamma$</sub>**

*Of-every farmer some donkey a running-thing is  
Of-every farmer some donkey is running*

Subject: (Every farmer  $\alpha$ ), (some donkey-poss- $\alpha\beta$ )  
*Of-every farmer, some donkey*  
*of-farmer donkey*  
 (or: *farmer's donkey*)

Predicate: ( $\cdot$  running-thing  $\gamma$ )  
*running-thing* in English; *running* in Latin.

(Some donkey  $\alpha$ )(every farmer  $\beta$ )( $\cdot$  seeing- $\alpha$ -thing  $\gamma$ )  $\alpha$  is  $\gamma$   
*Some donkey<sub>acc</sub> every farmer a seeing-thing is*

Subject: (Every farmer  $\beta$ )  
*every farmer*  
*farmer*

Predicate: (some donkey  $\alpha$ ), ( $\cdot$  seeing- $\alpha$ -thing  $\gamma$ )  
*some donkey<sub>acc</sub> seeing-thing*  
*donkey<sub>acc</sub> seeing-thing*

#### APPLICATIONS

Determine the subject and predicate of the Linguish logical forms underlying each of the following propositions.

*Buridan is no fool.*

*Some horse sees a donkey.*

*Every farmer's donkey sees a horse's leg.*

*Every farmer's donkey sees a leg of a horse.*

*Every farmer which owns a donkey owns an animal.*

### 6.2.3 Agreeing with Buridan (mostly)

Here I review some judgments that Buridan makes about the subjects and predicates of categorical propositions, and some of his reasons. The examples are mostly from Buridan *SD* Treatise 1, Chapter 3. (Unless otherwise noted, our theory agrees with Buridan's opinions.)

Buridan considers the view that the subject and predicate in

(1) *A man runs*

are 'man' and 'runs,' and rejects that answer. He says "the verb is not the predicate, strictly speaking." Instead, an intransitive verb needs to be analyzed into the copula plus a participle, as in 'A man is running.' In 'a man runs,' 'man' is the subject and 'running' is predicated (*SD* 1.3.2 (22)).



The analysis of an intransitive verb into a copula plus participle applies also to any use of ‘is’ as “second adjacent,” as in

(2) *A man is*

where the verb needs to be analyzed into a copula and participle, as in ‘*A man is [a] being*’ (SD 1.3.2 (22)).

Buridan considers (SD 1.3.2 (22))

(3) *The one lecturing and disputing is a master or a bachelor*

which he classifies as categorical, having complex subjects and subjects which are made using connectives. We have not implemented these in Linguish, but if we did so the natural way of doing so would probably agree with Buridan.

Buridan considers

(4) *A man who is white is colored*

and judges that its subject is ‘*man who is white*’ and its predicate is ‘*colored*’ (SD 1.3.2 (23)). This agrees with our account which would generate the sentence and analyze it as:

$(\cdot \{\mathbf{man\ who}_\alpha (\cdot \mathbf{white}_\beta)_\alpha \mathbf{is}_\beta\}_\gamma) (\cdot \mathbf{colored}_\delta)_\gamma \mathbf{is}_\delta$

*A man who white is colored is*

*A man who is white is colored*

Subject:         $(\cdot \{\mathbf{man\ who}_\alpha (\cdot \mathbf{white}_\beta)_\alpha \mathbf{is}_\beta\}_\gamma)$   
                   *man who white is*  
                   *man who is white*

Predicate:      $(\cdot \mathbf{colored}_\delta)_\gamma$   
                   *colored*

Buridan claims that

(5) *A man is colored, who is white*

is not a categorical proposition at all, because it contains two main verbs. (This proposition contains a non-restrictive relative clause. Our analysis does not apply to it.)

In the inherited tradition, categorical propositions are classified in terms of quantity: universal, particular, indefinite, or singular. When subjects are sequences of terms, things are more complicated. Buridan decides that a proposition can have more than one quantity; e.g. it may be universal with respect to one term and particular with respect to another. In discussing one example he says:

in the divided sense it is partly universal and partly indefinite, for not the whole subject is distributed, but one part is distributed while the other part remains indefinite. And this is not unacceptable concerning propositions whose subjects are composed of several substantives, whether in the nominative or in any of the oblique cases, as in ‘Every-man’s donkey runs’ [*cujuslibet hominis asinus currit*—‘Of every man a donkey runs’] or even in ‘A man’s every

donkey runs' [*hominis quilibet asinus currit*]; for the first is universal with respect to its oblique term and indefinite with respect to its nominative term, while the second is the other way round. The proposition 'Every man's-donkey runs' [*quilibet hominis asinus currit*] is absolutely universal, however, for its total subject is distributed together; and the same goes for the proposition 'Every donkey of a man runs' [*quilibet asinus hominis currit*], for they are equivalent, both with respect to grammaticality, as far as grammar is concerned, and with respect to truth and falsity, as far as logic is concerned. (SD 1.3.3 (27))

Our theory analyzes these sentences as follows:

(6) *Of every man a donkey runs*

(every man  $\alpha$ )(· donkey-poss- $\alpha$   $\beta$ )(· running-thing  $\gamma$ )  $\beta$  is  $\gamma$   
*of-every man a donkey a running-thing is*  
*of-every man a donkey runs*

Subject: (every man  $\alpha$ ), (· donkey-poss- $\alpha$   $\beta$ )  
*of-every man, a donkey*

Predicate: (· running-thing  $\gamma$ )  
*running-thing* English  
*running* Latin

The subject has two parts. The first is preceded by 'every' and so Buridan says that the proposition is universal with respect to it (with respect to the oblique term) and the second is preceded by '·', so the proposition is indefinite with respect to it (to the nominative term). He compares this to the similar sentence:

(7) *Of a man every donkey runs*

which is like the first but with its quantifier signs reversed, and he naturally classifies this as "the other way round."

He compares those two with the structure:

(8) *Every man's-donkey runs*

He says that in this case the proposition is universal because its total subject is. And he says the same of

(9) *Every donkey of a man runs*

In fact, he says that these propositions are both grammatically and logically the same. Our analysis generates these propositions from equivalent Linguish sources, given in section 5.8.3, validating his claim that the propositions are logically equivalent. One is:

(Every {(· man  $\alpha$ ) donkey-poss- $\alpha$ })  $\delta$   $\delta$  runs  
*Every man's donkey runs*

or:

*Every donkey of-a man runs*

The other:

(Every {donkey-poss- $\alpha$  ( $\cdot$  man  $\alpha$ )})  $\delta$   $\delta$  runs  
*Every donkey-poss-a-man runs*

Next example: Buridan says that in

(10) [A] *man's donkey runs*

the whole phrase '*man's donkey*' is the subject. This could be generated exactly the same as the previous example, except for having an indefinite quantifier instead of the '*every*.' That would yield:

( $\cdot$  {( $\cdot$  man  $\alpha$ ) donkey-poss- $\alpha$ }  $\delta$ )  $\delta$  runs  
*A man's donkey runs*

The subject is obviously '*man's donkey*.' However, it could also be generated from the form:

( $\cdot$  man  $\alpha$ )( $\cdot$  donkey-poss- $\alpha$   $\delta$ )  $\delta$  runs  
*Of-a man a donkey runs*

Our theory says that the subject of this sentence is:

*of-a man donkey*

These are different constructions. The two are logically equivalent, but they are different. Because there are no articles in Latin, the surface sentences are the same in Latin. This may be a case in which the lack of articles in Latin covers up a logical phenomenon. I am not sure of the significance of this.

Buridan's reason for classifying '*man's donkey*' as the subject is "it is obviously of this whole phrase that the predicate 'running' is predicated by the mediation of the verb 'is' implied in the verb 'runs'" (SD 1.3.3). I guess this is obvious. In any event, our theory predicts it.

Contrasted with this is the proposition

(11) *Every horse a man is seeing*

whose subject Buridan says is '*man*' and whose predicate is '*horse seeing*.'<sup>6</sup> Our analysis would be:

<sup>6</sup> Buridan SD 1.3.3: Buridan also discusses the very same example at 4.2.6 (245) in which he identifies the predicate as '[someone] seeing [a] horse'; he argues that these ingredients go together because "the determination should be united with its determinable." The theory I have given does not produce that order from the sentence '*Every horse a man is seeing*.' Apparently Buridan is casual about the ordering of the parts of a subject or a predicate if the orderings are logically equivalent.

(every horse<sub>α</sub>)(·man<sub>β</sub>)(·seeing-<sub>α</sub>-thing<sub>γ</sub>)<sub>β</sub> is<sub>γ</sub>

*every horse<sub>acc</sub> a man a seeing-thing is*

*every horse<sub>acc</sub> a man is a seeing-thing*

*every horse<sub>acc</sub> a man is seeing*

The subject is: (·man<sub>β</sub>)

*man*

The predicate is: (every horse<sub>α</sub>), (· seeing-<sub>α</sub>-thing<sub>γ</sub>)

*horse, seeing*

*horse seeing*

Buridan notes that part of the predicate, namely ‘horse’ is placed before the verb, and indicates that this is OK. In fact, he notes, sometimes the whole predicate occurs before the verb, as in ‘every man an animal is’ or ‘every man an animal is not’ (SD 1.3.3).

Buridan also opines that infinitizing negations are always integral parts of the subjects or predicates of propositions (SD 1.3.3). This also falls out of our analysis, simply because the only elements that are not included in the subject and predicate are the copula, negation, and the quantifier signs, and ‘non-’ cannot be a part of any of these.

#### 6.2.4 *The asymmetry of subjects and predicates*

Buridan has another view in addition to those discussed already. He believes that parts of predicates that occur to the right of the verb are to be treated differently from the parts of those predicates occurring to the left of the verb. (This has not been relevant so far.) This happens in two ways.

##### 6.2.4.1 WAY 1: QUANTIFIER SIGNS

First, he says that the parts of the predicate that occur to the right of the verb are to include their quantifier signs. His reason for this is his view that contradictories should have the same predicate. And the following are contradictory:

*Every man is every man*

*Some man isn’t every man*

likewise for:

*No man is no man*

*Some man is no man*

This is odd reasoning because if you leave off the quantifier signs in the predicate the contradictories already have the same predicates, so he has not given examples that illustrate what he is trying to show. Also, his general principle that contradictories must have the same predicates is violated in the simplest of cases, for, since these are contradictories:

*No S is P*  
*Some S is P*

so are these by simple conversion:

*No S is P*  
*Some P is S*

Probably Buridan has more in mind than I have interpreted him as saying. A clue may come from his discussion of conversion. He discusses the following principle of simple conversion:

the sign of quantity in the subject of the proposition to be converted has to be added to the predicate and the resulting whole has to be placed on the side of the subject, for example: ‘Some man is an animal; therefore, some animal is a man.’ (SD 1.6.5 (56))

This leads to an objection:

it was said earlier, and it is commonly held, that the whole which is placed on the side of the predicate is an integral part of that predicate (for example, if I say ‘Some man is some animal.’ the whole phrase ‘some animal’ is the predicate, and similarly if I say ‘No man is no animal.’ the whole phrase ‘no animal’ is the predicate); therefore, when adding the sign of the subject to the predicate, in performing the conversion we are committed to saying ‘Some some animal is a man’ and ‘No no animal is a man.’ (SD 1.6.5 (56))

Clearly this incongruity could be avoided by retracting the view that a predicate to the right of the copula contains its own quantifier signs as part of it. Buridan does not do this. Instead, he argues that the form of conversion sometimes needs to be altered:

Solution: I say that if this appears to be an unacceptable locution, then in such a case conversion may take place by applying the name ‘which’ [*quod*]. For example, ‘Some man is some animal; therefore, something which is some animal is a man.’ and similarly ‘No man is no animal; therefore, nothing which is no animal is a man.’<sup>7</sup> (SD 1.6.5 (56))

This shows that he is consistent in his thinking about the ingredients of predicates. But the argument is not very informative, since conversion only holds for some forms of propositions, and it is not clear whether the propositions he considers are ones for which conversion *should* hold.

In general, I have ignored this view of Buridan’s. In fact, it’s hard not to ignore it, since apart from conversions, whether a quantifier sign is part of a predicate doesn’t seem to interact with any other logical issues. This is because none of our rules of inference or semantic principles employ the notions ‘subject’ or ‘predicate.’ (This will change in Chapter 10.)

<sup>7</sup> Introducing a relative clause in this way was quite common in discussions of conversions of complex propositions.

## 6.2.4.2 WAY 2: NEGATIVE SIGNS

Buridan holds that if a negative sign comes after the verb then it cannot actually be negating; it must be “infinetizing” (see section 3.5). For example, if the Latin phrase ‘*non homo*’ comes after the copula, then it cannot be interpreted as ‘*not a man*’; it must be interpreted as ‘*a non-man*.’ This view is problematic in part because it is incompatible with applications of the patterns of quantifier equipollences. He explains his view when commenting on the equipollence rules that lead to these consequences:

Therefore ‘Nothing is nothing’ is equipollent to ‘Anything is something,’ for, by the second rule, ‘Nothing . . . is not . . .’ is equivalent to ‘Anything,’ just as ‘None . . . is not . . .’ is equivalent to ‘Everything,’ and, by the first rule, ‘Not nothing’ and ‘Something’ are equipollent. (*SD* 1.5.5 (47))

Buridan thinks those conclusions are, strictly, wrong when one of the negation signs follows the copula:

But if the second negation does not precede the copula, then the rule is not formally valid. For the one is then negative with an infinite predicate, as with ‘Nothing is nothing’ or ‘No chimera is no chimera,’ and the second then is affirmative with a finite predicate, as for example, ‘Anything is something’ or ‘Every chimera is some chimera,’ and such are not convertible between themselves, except supposing the constancy of terms. For the proposition ‘No chimera is no man’ is true, since its contradictory ‘Some chimera is no man’ is false, for it is an affirmative whose subject supposit for nothing; and the proposition ‘Every chimera is some man’ is false; therefore this consequence is not valid: ‘No chimera is no man; therefore every chimera is some man’ for a falsity does not follow from a truth. But the consequence would be valid assuming the constancy of its terms, namely, that each of the terms did in fact supposit for something. (*SD* 1.5.5 (48))

The negative signs ‘*no*’ and ‘*not*’ in the equipollence rules are understood to be negating. But on Buridan’s view, a proposition that appears to have a negative sign ‘*no*’ after the verb, as in ‘*Some A is no B*,’ does not really contain a negative sign; as a result, it is not of a form where the equipollences apply. If you do apply the equipollences to get ‘*Some A is not some B*’ then you either get an ungrammatical proposition, since ‘*not*’ may not immediately follow the copula in Latin, or the proposition is equivalent to ‘*Some A not is some B*’ which is not equivalent to the original sentence, because the original sentence is affirmative (the ‘*no*’ occurs after the verb, and so it does not make the proposition negative) and the resulting proposition is negative (the ‘*not*’ occurs before the verb).

The common view is that the Latin word ‘*non*’ can combine with a term to form a complex term, and when it does this it is called infinitizing negation. But Buridan is here discussing the quantifier word ‘*no*’ (Latin, ‘*nullus*’). So his view needs to be fleshed out with an explanation of what it means for a ‘*no*’ to be infinitizing. We get such an explanation—or at least an illustrative example—at *SD* 4.2.2 (230):

as regards ‘nobody’ and ‘nothing,’ we should hold that a negation following the copula of a proposition cannot constitute a negating [i.e. propositional] negation, but only an infinitizing

[i.e. term-]negation, which infinitizes the term to which it is prefixed. Hence the proposition 'A stone is no man' is affirmative with an infinite predicate term and is equivalent to 'A stone is a non-man'; for an infinitizing negation is quite correctly [taken to be] a part of the subject or of the predicate term.

The pattern seems to be that after the copula we analyze 'no F' as 'a non-F'. (And we analyze 'not an F' as 'a non-F'.) In the example Buridan gives, this seems unproblematic; 'A stone is no man' does seem to say about the same as 'A stone is a non-man,' the only difference being that the former appears to be negative, and so true if there are no stones, and the latter is affirmative, and so false if there are no stones. I don't see how one could argue this one way or the other. However, things change if the verb is something other than the copula. Consider

*Socrates sees no donkey*

i.e. replacing the verb with the copula plus participle:

*Socrates is seeing no donkey* or *Socrates is no donkey seeing*

The pattern just mentioned would analyze this as

*Socrates is seeing a non-donkey* or *Socrates is a non-donkey seeing*

But if Socrates sees both a donkey and a horse, 'Socrates sees no donkey' is clearly false, and 'Socrates is seeing a non-donkey' is clearly true, quite apart from issues of affirmative and negative. So the pattern breaks down here.

Unfortunately, I do not know why Buridan holds the view he does. More work needs to be done on this matter. I will ignore it in further discussion here.

## 6.3 Simple and complex terms

Previous discussion has all been based on the assumption that one can tell what the main terms of a categorical proposition are. In Linguish notation this is easy: A main term is any term that does not occur within curly brackets. But in Latin the problem is not so clear. Some substantial discussion went into this question. I begin with some problematic sentences and how they were viewed. Then I turn to a general theory.

### 6.3.1 Some interesting constructions

(This subsection uses some of the terminology of the theory of modes of supposition, which is presented in Chapter 7. Most of the discussion can be followed without reading that chapter, but some details will need filling in.<sup>8</sup>)

<sup>8</sup> Roughly, a mode of supposition is a kind of quantificational status. Determinate supposition ≈ Existentially quantified with wide scope; Merely confused supposition ≈ Existentially quantified with scope inside a universal; Distributive supposition ≈ Universally quantified.

Paul of Venice, in a discussion of what we would call truth conditions of propositions with multiple quantifiers, discusses the proposition (*LM, Tractatus on Terms*, chapter 4 in Kretzmann 1979, 257–9):<sup>9</sup>

*Of-every man a donkey runs*

or

*Every man's donkey runs*

He suggests that the proposition is ambiguous. There are two interpretations:

(1) 'donkey' belongs to the predicate

On this interpretation, the proposition is universal; 'man' has distributive supposition; 'donkey' has merely confused supposition. The proposition means that for every man there is a donkey of his that runs.

(2) 'donkey' belongs to the subject

On this interpretation, the proposition is indefinite; 'man' has distributive supposition; 'donkey' has determinate supposition. He says that in this case, the proposition is equivalent to: *a donkey of every man runs*.

(The notion of subject and predicate used here are not Buridan's. For Buridan, 'donkey' will belong to the subject on any reading.)

Understood in the first way, the sentence has two main terms: 'man' and 'donkey,' and they are not part of a single complex term; understood in the second way, the sentence has one main term, the complex term 'every man's donkey.'

Using our formulation of Linguish, the sentence may be generated in two different ways. One way corresponds to (1):

(1') (every man<sub>α</sub>)(· donkey-poss<sub>-α β</sub>) β runs  
*of-every man a donkey runs*

or:

*Every man's donkey runs*

On this construal, 'man' has distributive supposition, and 'donkey' is merely confused, as Paul says. (Or 'donkey' would have merely confused supposition if it were not for the fact that it is here a parasitic term, and parasitic terms do not have modes of supposition. See section 7.4.)

<sup>9</sup> The Latin version is '*Cuiuslibet hominis asinus currit*.' The translator translates the genitive as 'belonging to'; I have changed this to 'of'; also, he translates '*currit*' as '*is running*', instead of 'runs' as I have it. This is arguably better than 'runs' in terms of capturing its meaning, so long as the '*is running*' is read in English as the present progressive form of the verb 'run.' One must avoid interpreting it as the English translation of '*est currens*,' which is held by logicians to be the copula followed by a substantive common term (a participle). Note that 'every' is in the genitive case, so it goes with 'man' and not with 'donkey.' So the sentence does not permit a reading that says roughly 'Every belonging-to-a-man donkey runs.'



The other way of generating the sentence corresponds to (2):

- (2')  $(\cdot \{(\text{every man } \alpha) \text{ donkey-poss-}\alpha\} \beta) \beta$  runs  
*An of-every man donkey runs*

or

*An every man's donkey runs*

(Note that in Latin, without an indefinite article, the two surface sentences are identical.) For English the first wording is not grammatical. The second probably isn't either, though it seems to make sense to me, and to give one of the meanings of the corresponding Latin (which transliterates into '*Every man's donkey runs*'). With this structure, the proposition is indefinite, as Paul says. The complex term '*every man's donkey*' ( $\{(\text{every man } \alpha) \text{ donkey-poss-}\alpha\}$ ) has determinate supposition. On the popular view that the supposition of a complex term is also attributed to its head noun (see section 7.6.1 on modified supposition), '*donkey*' would be said to have determinate supposition, as Paul says. And '*man*' has distributive supposition here. This is because, given that a donkey owned by every man runs, then a donkey owned by this man runs, for any man:

- $(\cdot \{(\text{this man } \alpha) \text{ donkey-poss-}\alpha\} \beta) \beta$  runs, and  
 $(\cdot \{(\text{that man } \alpha) \text{ donkey-poss-}\alpha\} \beta) \beta$  runs, and  
 $(\cdot \{(\text{that man } \alpha) \text{ donkey-poss-}\alpha\} \beta) \beta$  runs, and  
 . . . and so on for every man

Paul notes further that understood in way (2) the following propositions are not contraries, but subcontraries:

- Of-every man a donkey runs*  
*Of-every man a donkey does not run*

They aren't contraries because both propositions would be true if there were two donkeys, each owned by every man, one of which runs and one of which doesn't. They are subcontraries because if the former is not true, either there is a donkey owned by every man, but no such donkey runs, and this will make the latter true, or there isn't a donkey owned by every man, in which case the latter (which is negative) is vacuously true. He also states that these are logically independent of one another (still reading the constructions in way 2):

- Of-every man a donkey runs*  
*Of-some man a donkey does not run*

This seems right to me. It holds for their Linguish logical forms. William Sherwood (S I.15) discusses the same example ('*Every man's donkey runs*'), though I am not certain what he has to say about it. My impression is that he denies that the second way of interpreting the proposition is a possible interpretation of it. (This is what Kretzmann

says in a footnote.)<sup>10</sup> This interpretation is uncertain because soon after this point (at S I.20) he gives another example which seems to have exactly the same grammatical structure so far as the initial portion of the proposition is concerned, and he takes it to be ambiguous. His example differs only in that he uses the relational noun ‘*possessor*’ instead of the non-relational term ‘*donkey*’:

*Every head’s possessor is only one head’s possessor*

He holds that this proposition is ambiguous, and that it is true on one reading and false on another:

‘Every head’s possessor is only one head’s possessor’ (*omne caput habens est unum solum caput habens*) (where the word ‘every’ is taken accusatively<sup>11</sup> and as attached to ‘head’ rather than to ‘possessor’) is proved as follows: ‘this head’s possessor [is only one head’s possessor]’ and so on with respect to the single things. But on the contrary, an opposite is here predicated of an opposite, and so the locution is false.

It must be said that the word ‘possessor’ in the subject can either be compounded with the predicate or divided [from it], but it is always named under one and the same construction. If it is compounded the locution signifies that the force of the distribution extends to the predicate as well as to the subject. In that case the phrase ‘only one [head’s possessor]’ in the predicate is [merely] confused, and it is not true, as was claimed above, that an opposite is predicated of an opposite. If it is divided the division signifies that the force of the distribution does not go over to the predicate, in which case the phrase ‘only one [head’s possessor]’ is not confused [but determinate] and the locution is false. (S I.20, p. 38)

This appears to be the same as Paul’s analysis of the similar example given earlier, with the compounded reading being way (1) and the divided reading being way (2). (When he says “the force of the distribution does not go over to the predicate” this means that the scope of the universal quantifier does not include the predicate.)

Notice that the examples we are discussing are ones in which a certain string of words, ‘*every man’s donkey*’ or ‘*every head’s possessor*’ can be taken either as consisting of two simple main terms, each having scope over the remainder of the sentence, or as a single complex main term containing a universal quantifier whose scope is confined

<sup>10</sup> Sherwood says: “Suppose that each man owns one ass and it is running, and that Brownie is the ass they own in common and it is not running. Then each man’s ass is running. But whatever is each man’s ass is Brownie; therefore Brownie is running—because the supposition of the word ‘ass’ is changed in a similar way.

Nevertheless, some maintain that these expressions are ambiguous in that the locution can be judged either in connection with the subject of the locution or in connection with the subject of the attribution. (They call the nominative itself the subject of the attribution and the oblique case itself the subject of the locution, but others name them the other way around.)

But this is nothing, because when the phrase ‘each man’s’ precedes the word ‘ass’ it has power over it (i.e. the nominative), and so the locution is to be judged in relation to it (i.e. the [distributive] sign). It is not a function of the discourse that it is judged this way or that way, but a function of ourselves only” (Sherwood S I.15).

<sup>11</sup> A literal translation of the Latin example would be ‘*Every head possessing is one only head possessing*’ in which ‘*every head*’ is taken accusatively because it functions as the direct object of the participle ‘*possessing*’. Kretzmann changes ‘*possessing*’ to ‘*possessor*’ to produce a more natural English sentence, and this is why ‘*every head*’ is in the genitive case in the translation.

to the complex subject term. As we will see in the next subsection, some theorists tried to develop a theory that would systematize the question of how to tell when we have two simple terms versus when we have a single complex term.

### 6.3.2 *Simple and complex terms*

Buridan is one of several logicians to concentrate on the logical structures of categorical propositions containing more than two terms. His work is part of a tradition that continued for centuries. That tradition includes a general theory of what those structures are. This section discusses that general theory.

All of the historical information in this section and much of the analysis is based on the very useful paper, Karger 1997.

As I understand the theory, it contains instructions for how to tell when two contiguous terms, or a term and a contiguous grammatical expression, are part of a single complex term, and when not.

The theory begins with a grammatical task, which is to identify when two terms bear a certain relation to one another; this relation is summed up by saying that one of them is a determinable, and the other is a determinant of that determinable. Examples are a common noun and an adjective modifying (and “determining”) it, a term in the genitive case determining (“possessing”) another term, a term in the accusative case being the direct object of (and thereby determining) a present participle, and so on. These relations are all grammatical, and the theory supposes that they are all identifiable by grammatical means. Sometimes the surface form of a proposition can be analyzed in different ways vis-à-vis what determines what; in such a case the proposition is grammatically ambiguous, and we are to apply the theory to be presented shortly to each of its readings.

Determinable/determinant pairs are of two sorts. One sort contains determinants that must grammatically follow the term that they determine. The three examples of this that Karger culls from the theory are: the relation of an adjective (including a participle) to the preceding noun that it modifies (which is the normal position for such an adjective in Latin), the relation of a relative clause to the noun that it modifies, and the relation of a complex phrase which “may be regarded as derived from a relative clause by substituting for the [relative] pronoun and the verb a participle of that verb (as in ‘man seeing every horse’).”<sup>12</sup> (That is, we regard ‘man seeing every horse’ as generated from ‘man who sees every horse’ by replacing ‘who sees’ by ‘seeing.’)

The other sort of determinable/determinant pair may occur in either order in a proposition. Such a pair either consists of a term in the genitive case determining the other term, which is what I have elsewhere called the (grammatical) possessed term, or it consists of a term in the accusative case determining the other term, which is a participle, by being its direct object.

<sup>12</sup> My terminology differs from Karger’s. She uses ‘complex term’ to stand for a string of terms; these may or may not themselves be part of a larger term. I instead use ‘complex term’ only for a string that itself forms a term. When a complex term in her sense is “taken as one” then it does indeed form part of a single complex term in my sense. When it is “taken as several” it does not.

## 6.3.2.1 THE FIRST TYPE OF DETERMINABLE/DETERMINANT PAIRS

Regarding the first type of combination, Karger says:

In terms of the first type, the determinant necessarily follows the determinable. Whenever such a term forms the subject of a basic categorial, it is taken as one. In the sentence “a man seeing every horse is an animal,” for example, the subject “man seeing every horse,” which is a term of that type, is taken as one. [Underlined terms are in a grammatical case other than the nominative.] (1997, 68)

Karger here uses ‘term’ to include phrases that are potentially either a single term or more than one term in a row. To say that the “term” is taken as one amounts to saying that it forms a single complex term in my sense. One can show that the Linguish formation rules given in Chapters 4 and 5 produce this result. Except for a change in the position of the verb, surface sentences generated by logical forms of Linguish have their words in exactly the same order as their occurrences in the logical form. So I can concentrate on those forms. They are:

Adjective + noun: The only way provided in Linguish for an adjective to modify a noun is for it to be generated by the rule for making complex terms (section 5.6.1), which has the adjective following the noun.

Common term + relative clause: These combinations can only be formed into a single term (section 5.7).

Phrases like ‘man seeing every horse’: These phrases are defined in section 5.6.2. They are all complex terms.

## 6.3.2.2 THE SECOND TYPE OF DETERMINABLE/DETERMINANT PAIRS

Regarding constructions of the second type, Karger says:

In complex terms of the other type, the determinant may follow or precede the determinable. These terms form the sensitive and interesting cases. When used as *subject* of a basic categorial, a complex term of that type is, in some cases, taken as one, in others, severally. According to the Hagenau commentary, the rule is as follows.

*If the determinable precedes the determinant or if the determinant precedes the determinable both terms being preceded by a sign of quantity which agrees grammatically with the determinable, the complex term is taken as one. In all other cases, i.e. if the determinant precedes the determinable and is not preceded by a sign of quantity agreeing with the determinable, the complex term is taken severally.*

The examples given of cases where the complex term, subject of the sentence, is to be taken as one are: “a donkey of-a-man runs” and “every of-a-king animal runs.” The example given of a case where the component parts of the term are to be taken separately is: “of-a-man a donkey runs.” (1997, 68)

Recall that these are cases in which the determinant is a genitive whose “possessed” is the other term, or the determinant is in the accusative case and is the direct object of the other term, which is a present participle.

It is easy to see that when either of these kinds of pair occurs in the Linguish logical form and the determinant follows the determinable the two terms must be part of a single complex term. This is because for these pairs the determinable is a parasitic term with a free marker that needs to be bound by the (denoting phrase containing the) determinant. If the terms are taken as independent, with the determinant occurring second, there is no way for it to bind the marker in the determinable. So the only option is for them to combine by means of a rule for producing complex terms.

It is also clear that when the determinant precedes the determinable and both terms are preceded by a quantifier sign agreeing with (and construed with) the determinable, then the terms must be taken as one. For this is the only way that anything at all (the determinant, in this case) can get in between a quantifier sign and its term (the determinable, in this case).

It is also clear that when the determinant precedes the determinable, then it is *possible* for those terms (actually, their denoting phrases) to be independent. For then the determinable with its free marker falls within the scope of the determinant, which binds that marker. The rule cited earlier says that this is the *only* way for things to be. However, our rules for constructing complex terms allow the formation of complex terms consisting of a term in the genitive or accusative case binding a marker in the determinable. So our rules are more liberal than the opinion in the text. In fact, this is a matter on which medieval authors disagreed. If we recall the previous section, we find there two examples of the sort we are discussing here. And both Paul of Venice and William Sherwood hold that it is possible to understand the construction as a single complex term. My linguistic instincts go along with Paul and William here, so I am inclined to be satisfied with the fact that the Linguish construction rules permit these structures. I am not aware of any argument against the existence of such constructions, and so I will leave the issue there.

I have been discussing cases in which the verb occurs at the end of the proposition. The rules that Karger cites also specify that:

If the verb comes between two terms, they must not be taken as making up one complex term.

The Linguish construction rules agree with this. The verb always starts out at the end, and it moves toward the beginning only by permuting with a contiguous term. I understand that if a verb is contiguous with a complex term, it is not thereby contiguous with any of its parts. As a result, a verb can never move into a complex term.

One more part of the rules cited by Karger is not accommodated here. Following Buridan, these authors hold that when the terms in question are part of the predicate, and when they occur to the right of the verb, they are always taken to be parts of a single complex term. This is an extension of Buridan's idea that any negation occurring to the right of a verb must be infinitizing, and not negating. As with Buridan's views on negation, the Linguish rules do not treat words or phrases differently when they follow the verb than when they precede it. It would be easy enough to invoke Buridan's ideas

here, but the resulting theory is quite odd. For example, it interferes with natural principles of logic. Consider the following argument:

*Brownie sees some man's every donkey*  
*Every donkey is an animal*  
 ∴ *Brownie sees some man's animal.*

On the theory under discussion, 'some man's every donkey' is a single complex term. We cannot break it into parts by moving it in front of the verb, because that is tantamount to saying that its position after the verb is irrelevant to its logical behavior. But so long as it remains a unit, there is no way by known logical rules to exploit the quantificational behavior of the 'every.' Of course, there might be additional rules, but I am not aware of any. It is also unclear what the difference in logical import is as a result of shifting the position of the main verb in a proposition. Clearly, there is more research to do on this matter.

#### APPLICATIONS

Give the Linguish logical form for this argument and show that it is valid.

*An every woman's donkey runs*  
 ∴ *Of-every woman a donkey runs*

Give the Linguish logical forms for two readings of this sentence and decide whether either reading entails the other.

*Every head's possessor is one head's possessor*  
 <supposing 'one' is a quantifier word>

In the following sentences identify the complex terms and the separate terms. Say if the sentence is ambiguous.

*Every grey donkey sees a lion which runs*  
*A dog chasing every cat is brown*  
*Some woman's every donkey is running*  
*Every some woman's donkey is running*  
*An animal seeing every woman is a dog*

# 7

## Modes of Personal Supposition

This chapter has two parts. The first part, sections 7.1–7.7, explains an historical version of the theory of modes of personal supposition. The second part, sections 7.8–7.12, deals with a refined version of that theory.

### 7.1 Introduction to the medieval theory

A common term which is used personally<sup>1</sup> has what is called a *mode* of (common) personal supposition.<sup>2</sup> A mode of supposition is something like a kind of quantificational status. It is a status that a term has in a proposition based on where it occurs in the proposition and what quantifier word occurs with it and with other terms in the proposition. Three modes of common personal supposition are widely discussed:

- Determinate Supposition
- (Confused and) Distributive<sup>3</sup> Supposition
- Merely Confused Supposition

Determinate supposition has something to do with a term's being existentially quantified; a paradigm example is '*donkey*' in '*Some donkey is spotted*.' Distributive

<sup>1</sup> Medieval logicians held that expressions can be used in three ways. An expression can be used *materially* (with "material supposition") as in '*Man is a noun*'; used in this way the expression supposits for itself or for a similar expression. An expression can be used *simply* (with "simple supposition") as in '*Man is a species*'; used in this way the expression supposits for a related form in the external world or for a mental concept. (Realists held the former; nominalists something like the latter.) Finally, an expression can be used *personally* (with "personal supposition") as in '*A man is an animal*'; to supposit for things that "fall under" the term. (Realists would say that in personal supposition a term supposits for things that fall under the form that the word supposits for when used with simple supposition; some nominalists would say that in personal supposition a term supposits for the things that its associated mental concept naturally signifies.) The topic of this chapter concerns words used personally.

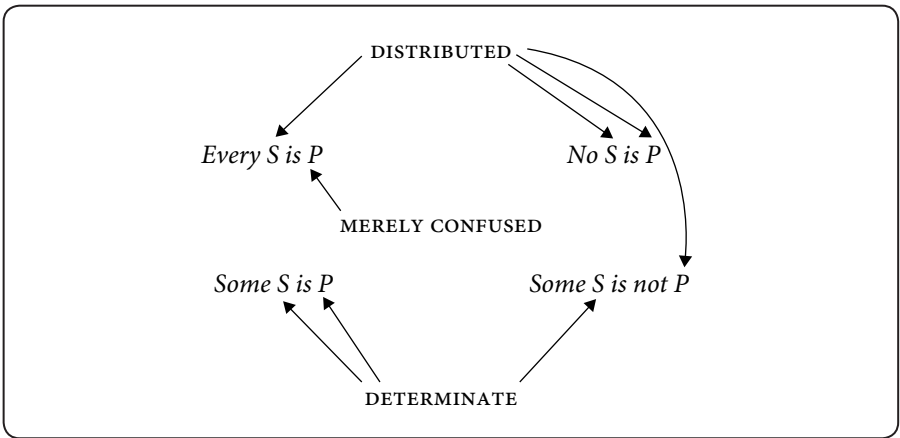
<sup>2</sup> Some authors (see Marsilius of Inghen 1 (57–9)) believe that terms used *materially* can also be assigned a mode of supposition, since they are common terms. (Dutilh Novaes 2007, 62 suggests that he was the first to hold this.) For example, in '*Every donkey in that sentence is bisyllabic*' the word '*donkey*' is a common term that stands for any word spelled d-o-n-k-e-y. On this analysis, '*donkey*' has distributive material supposition in that sentence. According to some, even discrete terms used *materially* are common terms, and can have modes of supposition, as in '*Every this uttered this morning was the subject of the sentence in which it occurred*.' Most authors either disagree with this view, or ignore it.

<sup>3</sup> Some authors use 'distributed' and some use 'distributive'; I use these interchangeably. (The term 'distributed' may presuppose that a term cannot have that mode of supposition unless it has been distributed by some distributing sign; see section 7.4.)

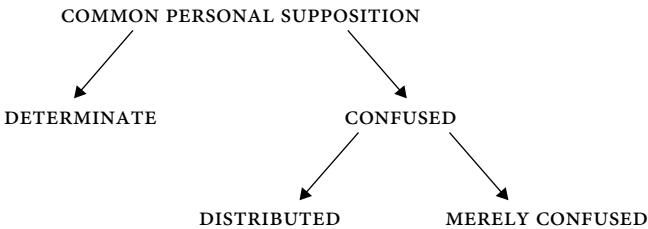
supposition has something to do with a term's being universally quantified; a paradigm of distributive supposition is 'donkey' in 'Every donkey is spotted.' Merely confused supposition is neither of these; it needs to be discussed. (An example of a term with merely confused supposition is 'donkey' in 'Every animal is a donkey'.)

Almost all authors agreed on the classification of terms of the Aristotelian standard forms of categorical propositions:<sup>4</sup>

- The subjects of universal propositions and the predicates of negative propositions have distributive supposition.
- The subjects and predicates of particular affirmatives, and the subjects of particular negatives, have determinate supposition.
- The predicate of a universal affirmative has merely confused supposition.



Usually this three-part classification of terms results from two bifurcations: personal supposition is divided into *determinate* versus *confused*, and *confused supposition* is divided into *distributed-and-confused* versus *merely confused*:<sup>5</sup>



In Parsons 2008a I argue that the theory of modes of supposition took different forms in the 13th and 14th centuries. Only the 14th century form will be considered here.

<sup>4</sup> Exception: Peter of Spain *LS* VI.6 (70–1) holds that all predicates have simple supposition, and do not have any mode of personal supposition. Also, some authors did not discuss the predicates of particular negatives.

<sup>5</sup> So the 'merely' in 'merely confused' contrasts with the 'distributed' in 'distributed and confused.' The term 'confused' does not here mean 'bewildered'; it means something more like 'taken all together.'



## 7.2 The 14th-century definitions of the modes

In the 14th century, Walter Burley, William Ockham, and John Buridan (among others) developed a systematic approach to the analysis of the modes of common personal supposition.<sup>6</sup> The new accounts define the mode of supposition of a term in a proposition<sup>7</sup> in terms of conditions on ascent and descent under that term in the proposition.<sup>8</sup> A descent is similar to a quantifier instantiation step in modern logic; an ascent is similar to a quantifier generalization step. Each of the three modes has a distinctive pattern of allowable ascents and descents.

What follows is an account that takes what is most common to the views of Walter Burley, William Ockham, and John Buridan—the best of BOB. The theory is based on three definitions; they give necessary and sufficient conditions for a term's having determinate, distributive, or merely confused supposition. We begin (as medieval authors usually do) with determinate supposition:

### Determinate supposition<sup>9</sup>

An occurrence of a term *F* has determinate supposition in a proposition *P* if and only if

[Descent]: you may descend under that occurrence of *F* to a disjunction of propositional instances about all *F*s, and

[Ascent]: from any single propositional instance you may ascend back to the original proposition *P*.

A propositional instance of a proposition with respect to *F* is the proposition you get by replacing the quantifier word of the denoting phrase containing *F* by '*this*' or '*that*,'

<sup>6</sup> For an historical comparison with earlier views, see Parsons 2008a.

<sup>7</sup> Some authors held that only main terms have personal supposition in a proposition (where a main term is one that is not part of another term). Burley *DS* para 5–15 argues this. But the definitions to be given of modes of supposition apply meaningfully to any term in a proposition. In practice, authors sometimes apply the definitions to parts of terms, even when denying that this is possible.

<sup>8</sup> The notions of ascent and descent were present in many earlier writings, though not in a systematic way.

<sup>9</sup> This account is common to both Ockham and Buridan. Burley omits the ascent clause.

Buridan *SD* 4.3.5 (262–3) says “there are two conditions for the determinate supposition of some common term. The first is that from any suppositum of that term it is possible to infer the common term, the other parts of the proposition remaining unchanged. For example, since, in ‘A man runs,’ the term ‘man’ supposits determinately, it follows that ‘Socrates runs; therefore, a man runs,’ ‘Plato runs; therefore, a man runs,’ and so on for any singular contained under the term ‘man.’ The second condition is that from a common term suppositing in this manner all singulars can be inferred disjunctively, by a disjunctive proposition. For example, ‘A man runs; therefore, Socrates runs, or Plato runs or John runs . . .’ and so on for the rest.”

Ockham *SL* I.70 (200) says: “whenever it is possible to descend to the particulars under a general term by way of a disjunctive proposition and whenever it is possible to infer such a proposition from a particular, the term in question has personal determinate supposition.”

Burley *PAL* 1.1.3 para. 82 (102) says: “Supposition is determinate when a common term supposits disjunctively for its supposita in such a way that one can descend to all of its supposita under a disjunction, as is plain with ‘Some man runs.’ For it follows: ‘Some man runs; therefore, Socrates runs, or Plato runs, and so on.’”

and adding a ‘*not*’ if the quantifier word is negative. Descent and ascent are inferences, picturesquely expressed in terms of the directions in which the inferences go. As an example, we validate the claim that ‘*donkey*’ has determinate supposition in ‘*Some donkey is spotted*’ by establishing these two claims:

DESCENT: You may descend under ‘*donkey*’ in ‘*Some donkey is spotted*’ to a disjunction of instances about all donkeys. That is, from:

*Some donkey is spotted*

you may infer:

*This donkey is spotted or that donkey is spotted or . . . and so on for all donkeys.*

ASCENT: You may ascend back to the original proposition. From any instance of the form:

*This donkey is spotted*

you may infer the original proposition:

*Some donkey is spotted.*

Distributive supposition has a parallel explanation:

### Distributive supposition<sup>10</sup>

An occurrence of a term F has distributive supposition in a proposition P if and only if

[Descent]: you may descend under that occurrence of F to a conjunction of propositional instances about all Fs, and

[Ascent]: from any single propositional instance you may not ascend back to the original proposition P.

So ‘*donkey*’ has distributive supposition in ‘*Every donkey is spotted*’ because

DESCENT: You may descend under ‘*donkey*’ in ‘*Every donkey is spotted*’ to a conjunction of instances about all donkeys. That is, from:

*Every donkey is spotted*

you may infer:

*This donkey is spotted and that donkey is spotted and . . . and so on for all donkeys.*

<sup>10</sup> Buridan’s account omits the non-ascent condition. Ockham includes additional options for immobile distribution. Burley does not define distributive supposition; instead he defines two kinds of distributive supposition. This will be discussed later.

Buridan *SD* 4.3.6 (264) says: “Distributive supposition is that in accordance with which from a common term any of its supposita can be inferred separately, or even all of them at once conjunctively, in terms of a conjunctive proposition. For example, from ‘Every man runs’ it follows that therefore ‘Socrates runs,’ [therefore ‘Plato runs,’ or even that] therefore ‘Socrates runs and Plato runs’ and so on for the rest.”

Ockham *SL* I.70 (201) says: “Confused and distributive supposition occurs when, assuming that the relevant term has many items contained under it, it is possible in some way to descend by a conjunctive proposition and impossible to infer the original proposition from any of the elements in the conjunction.”

ASCENT: You may not ascend back to the original proposition. From an instance of the form:

*This donkey is spotted*

you may not infer the original proposition:

*Every donkey is spotted.*

Finally, merely confused supposition:

### Merely confused supposition<sup>11</sup>

An occurrence of a term F has merely confused supposition in a proposition P if and only if

[Descent]: you may not descend under that occurrence of F to either a conjunction or a disjunction of propositional instances about all Fs, but

[Ascent]: from any propositional instance you may ascend back to the original proposition P.

The term ‘*mammal*’ has merely confused supposition in ‘*Every donkey is a mammal*’ because:

DESCENT: You may not descend under ‘*mammal*’ in ‘*Every donkey is a mammal*’ to either:

*Every donkey is this mammal and every donkey is that mammal and . . . , and so on for all donkeys*

or to:

*Every donkey is this mammal or every donkey is that mammal or . . . , and so on for all donkeys.*

ASCENT: You may ascend back to the original proposition from any instance. From:

*Every donkey is this mammal*

you may infer the original proposition:

*Every donkey is a mammal.*

<sup>11</sup> The account given is that of Burley *PAL*. Burley 1.1.4 para 85 (103): “Supposition is merely confused when a common term suppositis (a) for several things in such a way that (b) the proposition is inferred from any one of them and (c) one cannot descend to any of them either copulatively or disjunctively. The predicate suppositis in this way in ‘Every man is an animal,’ because: (a) the term ‘animal’ suppositis for several things. For if it suppositis for some determinate one, the proposition would be false. (b) The proposition is inferred from any of its singulars. For it follows: ‘Every man is this animal; therefore, every man is an animal.’ And (c) one cannot descend under ‘animal’ either disjunctively or copulatively. For it does not follow: ‘Every man is an animal’ therefore, every man is this animal or every man is that animal.’ Neither does it follow: ‘Every man is an animal; therefore, every man is this animal and every man is that animal,’ and so on.” He gives an equivalent account in *TKS* para 2.41 (see Spade 1997, para 34).

Buridan agrees, but omits the ascent condition. Buridan *SD* 4.3.6 (264): “But merely confused supposition is that in accordance with which none of the singulars follows separately while retaining the other parts of the proposition, and neither do the singulars follow disjunctively, in terms of a disjunctive proposition, although perhaps they do follow by a proposition with a disjunct term.”

Ockham adds a condition on descent which is discussed in Parsons 2008a, section 8.9.3, and here in section 7.6.

In illustrating the definitions we have shown that the subjects of affirmative propositions and the predicate of the universal affirmative have the modes that they were assigned throughout the tradition. All of the other terms in the standard categorical propositions are like this. For example, we can show that by these definitions the predicate term of a particular negative proposition has distributive supposition:

The term '*donkey*' has distributive supposition in '*Some animal is not a donkey*'

because

DESCENT: You may descend under '*donkey*' to a conjunction of instances of all donkeys. That is, from:

*Some animal is not a donkey*

you may infer:

*Some animal is not this donkey and some animal is not that donkey and . . . and so on for all donkeys.*

ASCENT: You may not ascend back to the original proposition; from an instance of the form:

*Some animal is not this donkey*

you may not infer the original proposition:

*Some animal is not a donkey.*

#### APPLICATIONS

Determine which modes of supposition are possessed by the subject and predicate terms in the following propositions. (Recall that when producing a propositional instance under a term with a negative quantifier such as '*no*' you need to place a '*not*' in front of the demonstrative term that replaces the quantifier.)

*No donkey is a stone*

*Some animal is every phoenix*

*No animal is every donkey*

*Socrates sees every donkey*

<singular terms do not have modes of supposition>

*Every donkey sees every horse*

*Some donkey sees no horse*

### 7.3 Clarification of the definitions

Some clarifications of the definitions are necessary.

### 7.3.1 *The nature of ascent and descent*

The first point has to do with the nature of the inferences involved in ascent and descent. Consider the following explanation of why ‘*donkey*’ is determinate in ‘*Some donkey is an animal*’:

From:

*Some donkey is an animal*

one may infer

*This donkey is an animal* or *that donkey is an animal* or . . . for all donkeys.

This does *not* mean that if the displayed conjunction contains a term for every donkey then ‘*Some donkey is an animal*’ entails ‘*This donkey is an animal* or *that donkey is an animal* or . . .’ For the former sentence does *not* entail the latter. What is meant instead is that from the truth of ‘*Some donkey is an animal*’, together with the information that ‘*This donkey is an animal or that donkey is an animal or . . .*’ contains a term for each donkey, and only terms for donkeys, one may infer that the disjunction is true. The inference is from the generalization *plus* the information about the exhaustiveness of the disjunction to the truth of the disjunction. This is how the test should be understood.

To keep matters straight in what follows it will be helpful to have a term for one of these conjunctions or disjunctions which have been supplemented by a ‘*for all the Fs*’ clause. I’ll refer to them as augmented conjunctions or disjunctions, with the understanding that an augmented conjunction is not a special sort of conjunction, but a construction having a conjunction as a central part.

### 7.3.2 *Occurrences of terms have modes of supposition*

Second, the classification into modes of suppositions is a classification of term-occurrences, not of term-types. In the proposition

*Every donkey is a donkey*

there are two terms (two term-occurrences) each of which has its own mode of supposition: the subject term has distributed supposition and the predicate term has merely confused supposition. You can’t just ask for the mode of supposition of ‘*donkey*’ in the proposition without specifying which occurrence is meant.

### 7.3.3 *Repeated occurrences must be ignored*

Third, the classification is meant to be a classification of an occurrence of a term independent of the particular nature of any other terms in the sentence. In particular, if a term is repeated, as in the example just discussed, you don’t just apply the test for modes of supposition to an occurrence of the term in that very sentence. For if you do, the results may depend on the very same term occurring elsewhere, and that

may give unintended results. For example, the following is an incorrect way to try to show (falsely) that the subject term of ‘*Every donkey is a donkey*’ has determinate supposition:

The subject term of ‘*Every donkey is a donkey*’ seems to have determinate supposition, because that proposition does indeed entail ‘*This donkey is a donkey or that donkey is a donkey or . . .*’, and any one of these disjuncts alone entails the original proposition.<sup>12</sup>

The subject term is supposed to have distributive, not determinate, supposition. To avoid the bad consequence just illustrated, one must ignore the fact that a repeated term is the same term. To test the subject term of

*Every donkey is a donkey*

you should apply the test to

*Every donkey is a blah*

And although the descent condition is satisfied—that proposition does indeed entail

*This donkey is a blah or that donkey is a blah or . . .*

—the ascent condition fails; the original proposition is not entailed by any of the disjuncts alone. So the subject term does not have determinate supposition. It is straightforward to show that it has distributive supposition.

A similar provision is needed regarding complex terms that contain repeated terms within themselves. Consider what kind of supposition is possessed by the subject term of:

*Every donkey which is not a donkey is running*

The sentence is logically false, and each instance (‘*this donkey which is not a donkey is running*’) is also logically false. Logically false propositions entail anything.<sup>13</sup> So if the term is left unchanged, it is possible to descend to a disjunction of instances, and, because each instance is logically false, and anything follows from a proposition which is logically false, it is also possible to ascend back from any instance. This means that the subject term has determinate supposition. On the other hand, the proposition is a universal affirmative proposition, and we said that the subject term of a universal affirmative has distributive supposition. Which is right?

It is not clear from the texts what to say about this. The issue is similar to the modern question of whether one may instantiate ‘ $\exists x[F(x) \ \& \ \neg F(x)]$ ’ to get ‘ $F(\text{Socrates}) \ \&$

<sup>12</sup> Each disjunct is affirmative and so it entails that ‘*donkey*’ isn’t empty, and non-emptiness is all that is required for the truth of ‘*Every donkey is a donkey*.’

<sup>13</sup> This was a common view. For example, Buridan *TC* I.8.3 (196): “From any impossible sentence any other follows.” Paul of Venice *LP* III.1 (65) “From the impossible follows anything, i.e. from any impossible proposition any other proposition follows.”

—F(Socrates).’ Certainly one may *infer* the latter from the former, since the former is logically false. But can one infer the latter *by instantiating*? Either answer is possible. (My intuition is that we should say that the inference, though valid, is not an application of existential instantiation.) The analogue for the medieval example given a moment ago is to say that the subject term has distributive supposition, and that the ascent condition is not met because although the original proposition may be inferred from the instance, this is not a proper case of ascent. However, it is not clear how to define ascent so as to rule out this inference. Probably the best approach is to extend the prohibition on repeated terms to include terms that occur within a complex term. For the medieval example, one would then test:

*Every donkey which is not a blah is running*

This does indeed entail the conjunction of all instances of the form:

*This donkey which is not a blah is running and . . . and so on for all donkeys*

and the original proposition may not be inferred from any instance. So the conditions for distributive supposition are met.

#### 7.3.4 *Empty terms*

There is a problem about how to apply the tests for ascent and descent when the term under which one is ascending or descending is empty. If the tests are taken literally, this seems to require an augmented conjunction or disjunction with no conjuncts or disjuncts. But there is no such thing. For example, one cannot make a conjunction or disjunction of the true instantiations of the true proposition ‘*No chimera is living*,’ because there are no true instantiations of the form ‘*This chimera is not living*’ where the subject term refers to one of the chimeras.

Some writers took this to establish that empty terms cannot have modes of supposition. Ockham is more cautious in the wording of his account of distributive supposition:

Confused and distributive supposition occurs when, *assuming that the relevant term has many items contained under it* [my italics], it is possible in some way to descend by a conjunctive proposition and impossible to infer the original proposition from any of the elements in the conjunction. (SLI.70 (201))

This is subject to different interpretations; on one of them he appears to say that the test is to be applied on the assumption that the term is not empty. Just suppose that there are some chimeras, and make an augmented conjunction with instances of these:

*This chimera is not living and that chimera is not living, and . . . and so on for all chimeras.*

Then if the original were true, the augmented conjunction would be true (and no ascent would be possible). So this device seems to work fine (though it is not completely clear how to spell it out).

Another option suggests itself; it is somewhat artificial, but it seems to fall within the stated conditions. First, we are to understand ‘a disjunction of propositional instances’ to be a disjunction, that is, a proposition containing at least one ‘or,’ and thus containing at least two disjuncts. (Similarly for conjunctions.) And we are to understand ‘about all Fs’ to mean that for each F there is a disjunct (or conjunct) with a term standing for it. These two conditions together allow that there may be several disjuncts with distinct occurrences of the term ‘*that F*’ standing for the same entity. (In fact, where there is exactly one F this will be required, for there will have to be at least two disjuncts and there will be only one F for the terms to stand for.) When there are no Fs at all, we can suppose that each term of the form ‘*this F*’ stands “vacuously” for one of the Fs—that is, it stands for nothing at all. So the terms will all be empty. (Though this is understood to be allowed only when ‘F’ is empty.) Then the definitions of the modes are adequate as they stand, even when the term in question is empty. For example, the term ‘*chimera*’ has determinate supposition in ‘*Some chimera is not running*’ because from ‘*Some chimera is not running*’ we can indeed descend to:

*This chimera is not running or that chimera is not running . . . etc. for all chimeras*

and we can ascend back to the original from the augmented disjunction. In this case both the original proposition and the descended form are true; if the example were ‘*Some chimera is running*’ both would be false. But the actual truth values are irrelevant.

#### APPLICATIONS

Determine which modes of supposition are possessed by the subject and predicate terms in the following propositions.

*No donkey isn't a donkey*

*Some animal isn't an animal*

*No animal sees every animal*

*Every donkey which sees a horse sees a horse*

*Some donkey sees no grey donkey*

## 7.4 Causes of the modes

The theory under consideration here continues an earlier tradition that modes of supposition other than determinate must be *caused*. The theory of causation takes the form of rules that can be applied recursively to the ingredients of a proposition, working from right to left. For example, the ‘*every*’ in ‘*Every donkey is running*’ causes the subject term ‘*donkey*’ to have distributive status; embedding this within a ‘*not*’ allows for the ‘*not*’ to reclassify the mode of supposition of ‘*donkey*’ as determinate in ‘*Not every donkey is running*.’ This process as a whole is an algorithm for determining modes of personal supposition almost by inspection.



Here are some rules that are common to several authors:<sup>14</sup>

DEFAULT: A main term of a proposition has determinate supposition unless something causes it not to.<sup>15</sup> A particular affirmative sign, such as ‘*some*,’ adjoined to a term makes it have determinate supposition (or, equivalently, has no effect).

UA:<sup>16</sup> A universal affirmative sign, such as ‘*every*,’ distributes the term it is adjoined to and confuses any other main term in its scope if that term is not already confused.

UN:<sup>17</sup> A universal negative sign, such as ‘*no*,’ distributes the term it is adjoined to, and regarding any other main term in its scope:

If that term is determinate or merely confused, the negative sign confuses and distributes it.

If that term is already distributed, the negation sign makes it merely confused

NEG:<sup>18</sup> A negating negation, ‘*not*,’ has the following effect on any main term in its scope:

<sup>14</sup> Paul of Venice *LP* II.5 (45–52) has similar rules plus a host of others. For example, the superlative and comparative constructions distribute their terms; an exceptive expression (as in ‘*every man except Socrates*’) makes its term be merely confused, as does a reduplicative expression (as in ‘*Every man as a man*’), and likewise for expressions concerning an act of the mind, such as ‘*know*,’ ‘*believe*.’ Paul speaks here of signs “confounding” terms, and thereby affecting their mode of supposition. Marsilius *TPT* 1 (65–71) gives 19 rules.

<sup>15</sup> Buridan *SD* 4.3.5 (263): “If you ask ‘How do I know when the supposition is determinate?. I say . . . you will know if you see that there is no cause for confusion.”

Ockham *SL* I.71 (202): “when in a categorical proposition no universal sign distributing the whole extreme is added to a term, either mediately, or immediately, . . . , and when no negation or any expression equivalent to a negative or a universal sign is added to a common term, that common term supposit determinately.”

<sup>16</sup> Buridan *SD* 4.3.7.1 (265) says: “a universal affirmative sign distributes the term immediately following and construed with it,” and 4.3.8.1 (273): “there are many causes of nondistributive confusion. The first obtains when the universal affirmative sign confuses a common term following upon it, but not immediately, as when in the proposition ‘Every man is an animal’ the term ‘animal’ supposit by nondistributive confused supposition.”

Ockham *SL* I.74 (213) says: “in every universal affirmative and universal negative proposition that is neither exclusive nor exceptive, the subject has confused and distributive mobile supposition.” I.73 (211): “where a common term mediately follows an affirmative sign of universality, it has merely confused supposition.”

<sup>17</sup> Buridan *SD* 4.3.7.2 (269–70) says: “a negative universal sign is nothing else but a word that implies in itself a negation with a particular affirmative sign. For ‘no-one’ is equivalent to ‘not one,’ ‘nothing’ to ‘not something.’” Since Buridan takes the universal negative sign to be equivalent to negations plus the particular affirmative sign, his views must follow in part from his view about these two items. These provisions seem to yield the right answers.

Ockham *SL* I.74 (213) says: “The first rule is that in every universal affirmative and universal negative proposition that is neither exclusive nor exceptive, the subject has confused and distributive mobile supposition.” For the predicate term, Ockham only mentions the first subcase: “The second rule is that in every such universal negative proposition the predicate stands confusedly and distributively.” However he probably intends his treatment of negation to apply to the universal negative sign.

<sup>18</sup> Buridan 4.3.7.2 (269): “a negating negation distributes every common term following it that without it would not be distributed and does not distribute anything that precedes it.” This applies to the first two subcases. For the third case, see note 19.

If the term is determinate or merely confused, the negation confuses and distributes it.

If the term is distributed, the negation makes it merely confused (according to Buridan)<sup>19</sup> or determinate (according to Ockham)<sup>20</sup>

The main terms of standard categorical propositions are correctly classified by these rules:

<i>Every A is B</i>	<i>A</i> is distributed and <i>B</i> is merely confused by UA
<i>No A is B</i>	<i>A</i> and <i>B</i> are both distributed by UN
<i>Some A is B</i>	<i>A</i> and <i>B</i> are both determinate by DEFAULT
<i>Some A is not B</i>	<i>A</i> is determinate by DEFAULT; <i>B</i> is distributed by NEG

Applied to non-standard categorical propositions, almost all classifications agree with the definitions of the modes in terms of ascent and descent:

<i>Every A is every B</i>	<i>A</i> and <i>B</i> are both distributed by UA. (The first ‘ <i>every</i> ’ has no effect on <i>B</i> because <i>B</i> is already confused by the second ‘ <i>every</i> .’)
<i>No A is every B</i>	<i>A</i> is distributed by UN; <i>B</i> is distributed by UA but then made merely confused by UN.
<i>Some A is every B</i>	<i>A</i> is determinate by default and <i>B</i> is distributed by UA.
<i>Some A is not every B</i>	<i>A</i> is determinate by DEFAULT; <i>B</i> is distributed by UA but this is then made by NEG to be merely confused according to Buridan, or determinate according to Ockham.

The question remains how to resolve the difference between Buridan and Ockham concerning the mode of supposition of a term that is apparently distributed by two signs. Buridan says that it ends up merely confused, and Ockham says that it ends up determinate.<sup>21</sup> In the example just given, ‘*Some A is not every B*’, the right answer in terms of ascent and descent is that ‘*B*’ should be determinate. But in other cases the term must be merely confused, as Buridan says; an example is ‘*donkey*’ in

<sup>19</sup> Buridan 4.3.8.2 (275): “a common term is confused nondistributively by two distributive [parts of speech] preceding it, either of which would distribute it without the other.” Examples include the predicate terms of ‘No man is no man’ and ‘No man is every man.’ He also (277) cites the predicate of ‘No man sees every donkey’ as an example, and notes that the predicate term of ‘Every man is every donkey’ is not an application of the rule, since the first ‘every’ by itself would not distribute the predicate term. He adds that the rule must have an exception for “two negations taken together, relating in the same way to what follows upon them. For these only cancel each other out, and so the supposition remains the same, as it would be if the negations were removed. Thus, in ‘Not: no man runs’ or in ‘Socrates does not see no man,’ ‘man’ suppositis determinately.”

<sup>20</sup> Ockham *SL* I.74 (214): “if the term were to stand confusedly and distributively when one of these expressions were taken away, then with the addition of such an expression it would stand determinately.” His example is ‘Socrates is not every man,’ wherein ‘man’ stands determinately.

<sup>21</sup> Ashworth, *Double Distribution and Conjoint Predicates* (unpublished manuscript) states: “Ockham and Buridan do not disagree, for in the passages cited, Buridan is talking about universal propositions, and Ockham is talking about singular propositions.” If Ashworth is correct, instead of disagreement we have a lack of stated opinions, and we have even less to go on.

*Not: some farmer sees every donkey*

Both the ‘every’ and the ‘not’ would distribute ‘donkey’ on their own; together they make it merely confused. (Descent is not possible, but ascent is possible from ‘*Not: some farmer sees this donkey.*’) So neither Buridan nor Ockham is right, and we have no other option.

There is a way around this problem, by using a technique which was practiced by several logicians.<sup>22</sup> It is based on the assumption that modes of supposition are not affected by the quantifier equipollences discussed in section 3.2. For example, the subject term of ‘*No A is B*’ has distributive supposition, since one can descend to:

*This A is not B, and that A is not B, and so on for all As.*

(and one cannot ascend back from any single conjunct). We can apply one of the quantifier equipollences to the ‘no A’ and turn it into ‘not some A,’ thus yielding ‘*Not some A is B.*’ And ‘A’ also has distributive supposition here, because one may descend to:

*Not this A is B, and not that A is B, and so on for all As.*

(and one cannot ascend back from any single conjunct). Now consider the first case mentioned in which it is not clear what the rules from causes should say, namely the case of the mode of supposition of ‘B’ in ‘*Some A is not every B.*’ If we apply a quantifier equipollence to ‘not every B’ it becomes ‘some B not,’ and so the original proposition turns into ‘*Some A some B isn’t.*’ And in this proposition the term ‘B’ has determinate supposition according to the rules from causes. Likewise the second case mentioned, namely ‘*not some farmer sees every donkey*’; here applying the equipollences to ‘not some farmer’ turns the proposition into ‘*every farmer not every donkey sees*,’ and then the ‘not every donkey’ can be turned into ‘*some donkey not*,’ so that the original proposition becomes ‘*every farmer some donkey doesn’t see*,’ in which ‘donkey’ has merely confused supposition according to the rules from causes of mode of supposition.

So the rules are not completely formulated as is, though we can get around this problem.<sup>23</sup> I will discuss this further a few sections later, where we consider a revision of the theory in which distribution comes in two sorts. Negating one sort produces determinate supposition, while negating the other sort produces merely confused supposition. This permits rules that are complete without a work-around.

<sup>22</sup> See Read 1991, 75 ff. He summarizes thus: “It is likely that the medievals did not believe that such a procedure always worked—that corresponding terms in equivalent propositions always have the same mode of supposition. Nonetheless, it was a standard procedure in certain cases.” In fact, when the equivalents are produced by merely applying the quantifier equipollences it always works.

<sup>23</sup> I should also mention an idea that had currency as late as the 17th century (I do not know when it arose). This is, when two signs act on a term and either sign alone would distribute it, you remove the outer sign and then ask whether the distributed term is distributed “with reference to a term having determinate supposition, or with reference to a term having merely confused supposition.” In the former case the result of the two signs together is that the term ends up merely confused; in the latter case it ends up determinate. Cf. John Poinsot OFL 2.12 rule 5. This could perhaps be developed into a completely accurate procedure. See also Dutilh Novaes 2008, section 2.1 (465).

## APPLICATIONS

Determine which modes of supposition are possessed by the subject and predicate terms in the following propositions according to the theory of causes of modes.

- No donkey isn't a donkey*  
*Some animal isn't an animal*  
*No animal sees every animal*  
*Every donkey which sees a horse sees a horse*  
*Some donkey sees no grey horse*

## 7.5 Restricted descent and parasitic terms

Recall that in order to handle relational common nouns, transitive verbs, and genitive constructions we need to use parasitic terms; intuitively, terms that supply a grammatical role to be occupied by another term. An example is '*Some farmer's every donkey is running*'.

(Of-some farmer  $\alpha$ )(every donkey-poss- $\alpha$   $\beta$ )  $\beta$  is running

*Of-some farmer every donkey is running*

So far in this chapter we have not looked at propositions with such terms. When we do, we find they are problematic. For, some of them have distributive supposition according to the rules for causes of the modes of supposition, but they do not satisfy the descent/ascent conditions for distributed supposition. The example just given is of this sort. The term '*donkey*' is distributed by its '*every*' (according to rule UA). However, its mode of supposition is not well defined. The problem is apparent even with the informal statement of the conditions for descent. The test requires that one can descend from the original proposition to:

*Of some farmer this donkey is running and of some farmer that donkey is running and . . . , and so on for all donkeys.*

So stated, the descent fails, for there may be donkeys that are not owned by any farmer, and that are not running, even though all of the donkeys of some specific farmer are running.<sup>24</sup>

In this case it appears as if we should not descend to all of the donkeys that there are, but to only those that are owned by . . . ? By what? By *some* farmer? This also would make the descent fail, since there may be donkeys that are owned by a farmer and are

<sup>24</sup> Tests for determinate and merely confused supposition also fail, because the ascent condition does not hold.

not running, even though all of *some* farmer's donkeys are running. The term would still not have distributive supposition.

Buridan discusses an example just like this, and he says:

But concerning the third proposition, 'A man's any donkey runs,' I say that the word 'man's' is not distributed at all, and remains in its determinate acceptation. But the word 'donkey' is distributed, not absolutely, however, but restricted by the word 'man's' taken determinately. And so if we need to make a subsumption under this distribution in a syllogism, then we have to syllogize as follows: 'A man's any donkey runs, Brownie is that man's donkey; therefore Brownie runs.' (SD 4.3.7.1 (266))

There are two issues here. One is "how to syllogize." Buridan does indeed give a good syllogism. We will discuss this syllogism in Chapter 8 when we discuss anaphoric terms, terms that have grammatical antecedents ('that man's donkey'). The other issue is a different one; it is that however plausible his remarks about syllogizing are, they do not justify the claim that the descent condition for distributive supposition is satisfied in a modified way. As we saw earlier, one cannot descend to:

*Of some man this donkey is running and of some man that donkey is running and . . . , and so on for all men*

for this will fail for unowned donkeys. Buridan suggests that we need to descend to a restricted set of donkeys—those belonging to "that man." That wording would be:

*Of some man this donkey is running and of some man that donkey is running and . . . , and so on for all donkeys of that man.*

but here there is clearly no possible antecedent for the anaphoric '*that man*'.

When Buridan says that '*donkey*' is not distributed "absolutely" we must agree that it is not distributed at all, according to the definition of distribution given earlier. In fact, it has no mode of supposition at all, as these have been defined. We can say that it has *restricted* supposition, but this stands for an idea that we have not spelled out, and that perhaps cannot be spelled out.

There is an exception to this, which is when a singular term binds the free marker in the parasitic term. Suppose the example were:

*Of Socrates any donkey runs*

Then we could descend to a conjunction containing a conjunct for each one of *Socrates*' donkeys:

*Of Socrates this donkey runs and of Socrates that donkey runs and . . . and so on for all donkeys of Socrates*

But although this is coherent, it is a technique that is applicable only when it is a singular term that the parasitic term is parasitic on. And even in this case, the term '*donkey*' still lacks supposition as we have defined it previously. Since the singular term case is indeed coherent, I will assume that it makes sense to discuss restricted descent in this case. This will turn out to be useful in section 7.11.

In general, a parasitic term will have no mode of supposition at all as defined in section 7.2. This will not prevent other terms from having modes of supposition; for example, in Buridan's sample sentence '*man*' is indeed determinate, as Buridan says, and '*running*' is merely confused; both of these are predicted by the rules for causes of supposition. But parasitic terms do not have these modes according to their definitions in terms of ascent and descent.

## 7.6 A variant account of merely confused supposition

We have been using this definition of merely confused supposition:

A term F has merely confused supposition in a proposition P if and only if

[Descent]: you may not descend under F to either a conjunction or a disjunction of propositional instances of all Fs, and

[Ascent]: from any instance you may ascend back to the original proposition P.

There is a famous alternative definition of this mode, apparently devised by Ockham. Ockham noted that in some paradigm cases of merely confused supposition, although one may not descend to a conjunction or disjunction of propositional instances under the term, you may instead descend to a disjunctive term. This results in:

A term F has merely confused supposition in a proposition P if and only if

[Descent]: you may not descend under F to either a conjunction or a disjunction of propositional instances of all Fs, but you may descend to a proposition with a term in place of F that enumerates all of the Fs disjunctively, and

[Ascent]: from any instance you may ascend back to the original proposition P.

For example, in '*Every horse is an animal*' one may not descend under '*animal*' either to a disjunction or conjunction of instances of the form '*Every horse is this animal*'. But one may descend to:

*Every horse is (this animal or that animal or . . . for all animals).*

(Buridan also mentions this option, but does not endorse it as a requirement.)

Ockham's proposal was very popular, and several other writers adopted it.<sup>25</sup> This was a somewhat peculiar development. The reason is that the possibility of descent to a disjunctive term is a grammatical issue, not a logical one. This is because:

<sup>25</sup> For example, Albert of Saxony, *L II.3* says: "Merely confused personal supposition is the interpretation of a term for each thing it signifies by its imposition, or which it signifies naturally (if it is a mental term), in such manner that a descent to its singulars can be made by a proposition of disjunct predicate, but not by a disjunctive or conjunctive proposition." And Paul of Venice *LP 2.4* (150) gives: "Common mobile personal supposition which is merely confused is the acceptance of a common term standing personally beneath which one descends to all of its referents in disjuncts, as in '*every man is [an] animal and these are all animals; therefore, every man is this animal or that animal and thus of singulars.*'"

*some P*

can *always* be paraphrased as

*this P or that P or . . .*

but it cannot be paraphrased as '*this P and that P and . . .*'

(Likewise for '*a P*' and so on.)

Conversely:

*every P*

can *always* be paraphrased as

*this P and that P and . . .*

though not as '*this P or that P or . . .*'<sup>26</sup>

So the paraphraseability of a denoting phrase using a disjunctive term is a matter of which quantifier word it contains. But it was generally presumed that the quantifier can be changed (if additional changes are made) so as to preserve the mode of supposition. Several of Sherwood's equipollences do this. For example, '*donkey*' has determinate supposition in

*Some donkey is not spotted*

and also in the logically equivalent

*Not every donkey is spotted*

The former can be paraphrased as

*This donkey or that donkey or . . . is not spotted*

but the latter certainly cannot be paraphrased as

*Not (this donkey or that donkey or . . .) is spotted.*

This might not be important when the term already has determinate or distributive supposition, for paraphrasing a common term into a disjunctive term is not relevant to these classifications. But consider the following example. This inference is a case of a descent under '*animal*' to a disjunctive term:

*Not some donkey every animal isn't*

∴ *Not some donkey (this animal or that animal . . .) isn't*

<sup>26</sup> Likewise, '*no P*' can always be paraphrased as

*this P and that P and . . . not*

but not as *this P or that P or . . . not*.

The inference fails,<sup>27</sup> thus establishing that ‘*animal*’ does not have merely confused supposition in the premise according to Ockham’s account. But it should have merely confused supposition. The premise is the result of applying equipollences to a universal affirmative proposition:

*every donkey an animal is*  $\Rightarrow$  *not some donkey not an animal is*  $\Rightarrow$  *not some donkey every animal isn’t*

The other modes of supposition are preserved through these equipollences, and merely confused supposition is also preserved on all accounts other than Ockham’s. A related example concerns the predicate term in ‘*No animal is every man*.’ Marilyn Adams<sup>28</sup> points out that ‘*man*’ does not have either determinate or distributive supposition, and on Ockham’s account it does not have merely confused supposition either, because one cannot descend to a proposition with a disjunctive term. From:

*No animal is every man*

we may not infer

*No animal is this man or that man or . . .*<sup>29</sup>

It thus appears that Ockham’s account does not yield the classifications that people were after.

John Dorp<sup>30</sup> has a solution to this. He proposes that in order to tell what mode of supposition a term has we should first move the verb to the end, and then move any negation to the right of all of the terms (first changing ‘*no A*’ to ‘*every A not*’). (Applications of Sherwood’s equipollences given in section 3.2 will do this.) This needs to be done before applying Ockham’s test. Then, every term in the categorical will end up with ‘*every*’ or ‘*some*’ as its quantifier sign. If its sign is ‘*every*’ it has distributive supposition. If the sign is ‘*some*’, then either it will satisfy the conditions for determinate supposition, or not. If it does not, it may be paraphrased by a disjunctive term. This renders Ockham’s test for merely confused supposition accurate. However, it may also render it redundant, because based on cases, it seems that this convoluted way of applying Ockham’s provision yields that same classification you would get by not having it at all, as in the account we discussed originally.

Some authors<sup>31</sup> defined merely confused supposition in terms of descent to either a disjunctive or a conjunctive term. Since one or the other of these descents is always possible, this also appears to be a redundant addition to the conditions given in section 7.2.

<sup>27</sup> In assessing the inference it is essential to keep the scopes straight in the conclusion. The ‘*not*’ in ‘*isn’t*’ does not have scope over anything else with scope in the proposition.

<sup>28</sup> Adams 1987, 366–7.

<sup>29</sup> Similarly, Buridan *SD* 4.3.8.2 (277) says that ‘*donkey*’ is merely confused in ‘*No man sees every donkey*.’ But ‘*every donkey*’ cannot be paraphrased here as ‘*this donkey or that donkey or . . .*’

<sup>30</sup> Cited in Karger 1993, 418–20; also described in Dutilh Novaes 2008, who also cites another solution proposed in Poinset *OFL*.

<sup>31</sup> Maulfelt; see Read 1991, 77–82.



It should also be mentioned that quite a few authors held that certain verbs, mainly ones that we think of as creating non-extensional contexts, merely confuse terms following them. The commonest example is ‘promise’ in constructions like ‘I promise you a horse,’ understood in such a way that there is no particular horse that I promise you. Some, including Ockham, held that this sort of confusion satisfies Ockham’s account, so that we can say ‘I promise you (this horse or that horse or . . .).’ This phenomenon will be discussed in Chapter 10. The remainder of the present chapter is restricted to discussing the syntactic structures introduced in Chapters 4 and 5.

## 7.7 Useful inferences

The discovery and development of useful ways to judge inferences is distinctive of the medieval era. When the notions of modes of supposition were introduced, they brought along with them new and useful ways to assess inferences. This already occurred within the 13th-century theory (see Parsons 2008a for details). Some of these applications are clear and compelling.

### 7.7.1 *Superiors and inferiors*

One is the case of inference “from a superior to an inferior” with a distributed term.

**From a superior to an inferior:** If a term *A* is distributed in a proposition *P*, then from *P* together with a proposition indicating that term *B* is inferior to *A*, the proposition that results from *P* by replacing *A* by *B* follows.<sup>32</sup>

A common term *B* is *inferior* to a common term *A* iff ‘every *B* is *A*’ is true; this is also the condition for *A* being *superior* to *B*. And a singular term *a* is inferior to a common term *A* iff ‘*a* is *A*’ is true; again, in this case *A* is superior to *a*. (When replacing a common term by a singular term one must delete the quantifier sign accompanying the common term.)

<sup>32</sup> Buridan *TC* 3.7.8 (283): “For any given proposition with a distributed term, whether nominative or oblique, an acceptable syllogism can be constructed by taking another term under the given term as the minor proposition.” Paul of Venice *LP* III.3 (71) “From a higher-level term to its corresponding lower-level term distributed affirmatively the argument is not solid unless with the due mean, because it does not follow: ‘every animal runs; therefore, every man runs’. . . . But with the due mean, the argument is solid.” (The “due mean” would be ‘Every man is an animal.’)

Parasitic terms are not subject to the rule because they do not have modes of supposition. If such terms were assigned modes of supposition in accord with the rules for the causes of modes of supposition the rule discussed here would be fallacious. Here are two counterexamples:

- Of some farmer every animal is running  
∴ Of some farmer every donkey is running
- Of every farmer every animal is running  
∴ Of every farmer every donkey is running

The simplest illustration of this rule is Aristotle's syllogism BARBARA:

Every M is P  
 Every S is M  
 ∴ Every S is P

According to the rules for causes of modes, the term M is distributed in the first premise. The second premise states that S is inferior to M. The conclusion follows by replacing M with S in the first premise. The same principle also validates CELARENT:

No M is P  
 Every S is M  
 ∴ No S is P

Again, according to the rules, M is distributed in the first premise, and the second states that S is inferior to M. The conclusion again follows by replacing M with S in the first premise.<sup>33</sup>

This principle goes beyond Aristotle's syllogistic. Buridan points out that this inference is a good one (*SD* 4.2.6 (244)):

*Socrates is seeing every horse*  
*Brownie is a horse*  
 ∴ *Socrates is seeing Brownie.*<sup>34</sup>

Our rule Universal Application is an instance of applying this principle when replacing a common term with its quantifier with an inferior singular term. But applications of the principle outrun that particular rule. For example, the following argument:

*Every donkey runs*  
*Brownie is a donkey*  
 ∴ *Brownie runs*

is an instance of Universal Application, and also of the principle "From a superior to an inferior." But the following inference is an equally good application of the latter principle, but not of the former:

<sup>33</sup> Recall that Aristotle reduced all of the other moods of syllogisms to BARBARA and CELARENT. There is a long tradition of interpreting Aristotle which has him reasoning in the way just described. In *PA* 1.1 (2) he says "For one thing to be in another as a whole is the same as for one thing to be predicated of every one of another. We use the expression 'predicated of every' when none of the subject can be taken of which the other term cannot be said, and we use 'predicated of none' likewise." Keynes *S&E* 1884, 126 (p. 157), for example, calls this the *dictum de omni et nullo* ("principle of all and none") and interprets it as saying "Whatever is predicated, whether affirmatively or negatively, of a term distributed, may be predicated in like manner of everything contained under it." This principle is supposed to directly yield Barbara and Celarent; so interpreted it is a special case of the principle "From a superior to an inferior." In this tradition, when Aristotle calls the first figure deductions perfect, he means that they follow from the *dictum de omni et nullo*. This does not strike me as what Aristotle means, but the principles cited are certainly good ones.

<sup>34</sup> Actually, in this example '*seeing every horse*' is a complex term, and one needs the principles of section 5.6.2 to eliminate the complex term. If the proposition is understood as '*Socrates is every horse seeing*' then the principle "From a superior to an inferior" directly applies.

*Plato doesn't see a donkey*  
*Brownie is a donkey*  
 $\therefore$  *Plato doesn't see Brownie*

This is like having a rule of universal instantiation that applies within formulas, and can be applied also to existential quantifiers, as in:

$\neg \exists x Fx$   
 $\therefore \neg Fa$  by "universal instantiation" applied to ' $\exists x Fx$ ' in this context

This is not the way we usually think of things, but we could, and it would be useful in shortening derivations.

The reverse principle holds for determinate and merely confused supposition:<sup>35</sup>

**From an inferior to a superior:** If a term B has determinate or merely confused supposition in a proposition P, then from P together with a proposition stating that A is superior to B, the proposition that results by replacing B by A follows.

Another of Aristotle's first figure syllogisms follows by this principle:

#### Darii

Every M is P  
 Some S is M  
 $\therefore$  Some S is P

The first premise states that 'P' is superior to 'M,' which is determinate in the second premise; the conclusion follows by replacing 'M' by 'P' in the second premise.

This principle also applies when replacing a singular term b by a superior term B with a quantifier sign provided that B ends up having determinate or merely confused supposition. For example:

<sup>35</sup> Paul of Venice *LP* III.3 (70): from a lower-level term to its corresponding higher-level term affirmatively and without a sign of distribution and without any confounding signs impeding there is a solid inference. E.g. 'man runs; therefore, animal runs.'" (I don't know why Paul does not require a "due mean" here, as he does for the inference from a higher-level term to its corresponding lower-level term.) Ockham *SL* III.3–6 (600) cites this rule: "*ab inferiori ad superius sine distributione et affirmative est bona consequentia et simplex.*" He then gives a number of counterexamples to it, such as examples in which terms do not have personal supposition. He then qualifies the rule (page 601): "from an inferior to a superior without distribution and affirmatively is a good consequence if the terms are suppositing personally and significantly" (*ab inferiori ad superius sine distributione et affirmative est bona consequentia si termini supponant personaliter et significative*)" (cited in Moody 1955, 288 note 1).

Counterexamples to the principle for parasitic terms that (apparently) have merely confused and determinate supposition are:

Of no farmer every donkey is running  
 $\therefore$  Of no farmer every animal is running

Of some farmer not every donkey is running      <premise is true if there are no donkeys>  
 $\therefore$  Of some farmer not every animal is running

*Plato owns Brownie*  
*Brownie is a donkey*  
 ∴ *Plato owns a donkey*

Of course, this inference is easily validated by combining permutation for singular terms with expository syllogism. Other examples are less obvious:

*Plato doesn't own Brownie*  
*Brownie is a donkey*  
 ∴ *Plato doesn't own every donkey*

The inference is good because ‘*donkey*’ has determinate supposition in the conclusion.

We have stated these useful principles, but we have not justified them in any way. Such a justification will be given at the end of section 7.11.

#### APPLICATIONS

Say which of the following inferences can be justified by one of the rules just discussed; also indicate how the principles for the causes of modes of supposition establish that the rule in question applies.

*Some farmer sees every donkey*  
*Every farmer is a woman*  
 ∴ *Some woman sees every donkey*

*Some farmer sees every animal*  
*Every farmer is a woman*  
*Every donkey is an animal*  
 ∴ *Some woman sees every donkey*

*Some donkey is not a pet*  
*Every pet is a grey-thing*  
 ∴ *Some donkey is not a grey-thing*

*Some donkey is not a pet*  
*Every grey-thing is a pet*  
 ∴ *Some donkey is not a grey-thing*

### 7.7.2 Monotonicity

These two principles allowing one to move from inferior to superior or vice versa are generalizations of the notion of monotonicity discussed in section 2.7, with monotonicity down being an inference from a superior to an inferior, and monotonicity up being an inference from an inferior to a superior. (Since the definitions of superior

and inferior given in section 7.7.1 require that the terms in question be non-empty, the differences between monotonicity and qualified monotonicity are not relevant here.) What we see here is that the monotonicity behavior of terms that are the immediate arguments of determiners can be altered by other signs in the proposition. An example of this effect is:

*Some A is not a B*  
*Every C is a B*  
 ∴ *Some A is not a C*

The term *B* in the first premise is immediately governed by an indefinite determiner, which is monotonic up regarding that term. But the presence of the ‘*not*’ reverses the direction of the inference, so that ‘*B*’ is in an overall monotonic-down context. In medieval terms, the ‘*not*’ distributes ‘*B*’ (and nothing else interferes with this) so that one may make an inference from a superior to an inferior.

Similarly, in this inference:

*No A is no B*  
*Every B is a C*  
 ∴ *No A is no C*

since ‘*no*’ is monotonic down on the right, this puts ‘*B*’ in a monotonic down position, but the ‘*no*’ in front of ‘*A*’ reverses this so that ‘*B*’ is in an overall monotonic up context. In medieval terms, ‘*B*’ ends up having merely confused supposition, and so one may make an inference from an inferior to a superior.

### 7.7.3 Parasitic terms

Note that these principles do not apply to parasitic terms, for such terms do not have modes of supposition. But we saw earlier that a parasitic term with a singular term as antecedent has a kind of restricted mode of supposition. One might try to apply the principles in those cases. This can occasionally be done. For example, consider ‘*Plato’s donkey is running*’. Since ‘*donkey*’ has restricted determinate supposition here—restricted by ‘*Plato’s*’—and since ‘*donkey*’ is inferior to ‘*animal*’ it seems that we can go from an inferior to a superior and infer that Plato’s animal is running. This is in fact a good inference. But the technique does not generalize. We need only consider ‘*Plato’s donkey isn’t running*’. Since this is a negative proposition it will be true if Plato doesn’t have a donkey. And at the same time ‘*Plato’s animal isn’t running*’ will be false if Plato owns only one animal, say a horse, that is running. So an inference from an inferior to a superior is not in general trustworthy for parasitic terms, even when they have restricted supposition.

## APPLICATIONS

Produce a derivation (without using Superior to Inferior, or vice versa) to show the following argument valid. (You will need to use the **Possession** rule from section 5.7.2.)

*Plato's donkey is running*  
*Every donkey is an animal*  
 ∴ *Plato's animal is running*

Try producing similar derivations for these invalid arguments to see where they fail.

*Plato's donkey is not running*  
*Every donkey is an animal*  
 ∴ *Plato's animal is not running*

*Plato's father is running*  
*Every father is a son*  
 ∴ *Plato's son is running*

#### 7.7.4 Additional useful inferences and non-inferences

In addition to the Superior/Inferior inferences, several other patterns were discussed involving modes of supposition. The following are given by William Sherwood (*IL* V.13.2 (118–19)). They are presented here with minimal discussion; we will return to them in section 7.10.

Rule II *An argument from merely confused supposition to distributive confused supposition does not follow.*

Thus when every man sees only himself this does not follow: ‘every man a man does not see; therefore every man does not see a man.’

The point here seems straightforward: the second ‘*man*’ in the premise is merely confused (by the ‘*every*’) but it is distributed in the conclusion. Sherwood sees this as an instance of a general pattern involving a term’s changing its mode of supposition from merely confused to distributive.

Another rule:

Rule III *An argument from many cases of determinate supposition to one of determinate supposition does not follow, but [only] to one of confused supposition.*

Thus when every man sees only himself this does not follow: ‘a man is seen by Socrates, and [a man is seen] by Plato (and so on with respect to [all] individual [men]); therefore a man is seen by every man.’ But this does follow: ‘therefore by every man a man is seen,’ for a distribution has force in a succeeding phrase but not in a preceding phrase.

The inference resembles what was elsewhere called “induction”: concluding that a universal generalization is true because each of its instances are. But this principle holds only when the generalized ‘every man’ ends up with wide scope, and this is not the case in the bad example that Sherwood discusses. The good example, the one that *does* follow, differs exactly in that ‘every man’ ends up with wide scope. Instead of scope, Sherwood discusses symptoms of scope: the resulting modes of supposition; in the good example ‘a man’ has merely confused supposition (because the ‘*every man*’ has scope over it), and in the bad one it has determinate supposition. So the rule is formulated in terms of whether you move from many cases of determinate supposition to determinate supposition, or to merely confused supposition.

Rule IV *An argument from determinate supposition to distributive confused supposition does not follow, but only to merely confused supposition.*

Thus this does not follow: ‘a man is not seen by Socrates; therefore Socrates does not see a man’—e.g. if Socrates sees one man only. But this follows correctly: ‘a man is seen by every man; therefore every man sees a man.’<sup>36</sup>

The badness of the first inference and goodness of the second are obvious; the rule identifies the difference in terms of a principle about modes of supposition.

Rule V *An argument from distributive confused supposition to determinate supposition does follow, but not from merely confused supposition.*

Thus this follows: ‘Socrates does not see a man; therefore a man is not seen by Socrates.’ But this does not: ‘every man sees a man (e.g. every man sees only himself); therefore a man is seen by every man.’

In the first inference ‘*man*’ goes from distributive supposition to determinate supposition, and in the second it goes from merely confused to determinate. The rule blames the goodness/badness of these inferences on the pattern of the modes of supposition.

These rules all seem to have insight behind them, and they look promising as part of an overall theory of inference in terms of modes of supposition. But for them to be useful we need to be able to apply the rules in other cases, and in every case the description that Sherwood gives is too terse for us to do this. In general the rule cites a change of mode of supposition without describing in detail the setting in which it occurs. For example, in this example of subalternation:

*Every P is Q ∴ Some P is Q*

the term ‘*P*’ goes from distributive supposition to determinate supposition, so the inference should be good according to the first half of rule V, but the term ‘*Q*’ goes from merely confused to determinate supposition, so the inference should be bad according to the second half of rule V. I think that there are in fact some useful generalizations here, but we will be better able to formulate them (in section 7.10) after a refinement in the theory of modes of supposition.

<sup>36</sup> The translation quoted here has been altered in conformity with discussion in the Preface to Kretzmann 1968.

## 7.8 Refining the theory: Distinguishing two kinds of distributive supposition

Various writers have noticed that there seems to be an asymmetry in the account of the modes of supposition. On Ockham's account, which is the best known, there is descent to a disjunction of propositions, descent to a conjunction of propositions, and descent to a disjunctive term. Various proposals have been made about how to make the account more balanced by adding a fourth mode of supposition.<sup>37</sup> I will suggest that four modes do make a better theory, but this should be accomplished not by adding a fourth mode independent of the ones that are traditionally discussed, but rather by subdividing the mode of distributive supposition. For reasons to be given later, I will call one sort of distribution "wide distribution" and the other "narrow distribution."

Wide distributive supposition is traditional distributive supposition restricted to cases in which one can ascend back to the original proposition from the *whole conjunction* of propositional instances under the term. (Not from a single instance, but from the whole conjunction.) For example, from this proposition:

*Every donkey is running*

one may descend to:

*This donkey is running, and that donkey is running, and so on for all donkeys*

And one may, from that entire conjunction, ascend back to the original proposition. So 'donkey' has wide distribution in that proposition.

Narrow distributive supposition is distribution where one cannot make this ascent. An example is the term 'vegetarian' in

*Some philosopher is not a vegetarian.*

From this proposition one may descend to the conjunction:

*Some philosopher is not this vegetarian, and some philosopher is not that vegetarian, and so on for all vegetarians.*

But from that whole conjunction one may not ascend back to the original proposition.

With this bifurcation of distributed supposition the four modes are:

Determinate

Descent to a disjunction of propositions, and ascent back from the whole disjunction.

Merely confused

No descent to a disjunction of propositions but ascent back from such a disjunction.

<sup>37</sup> Read 1991, section 6, argues that a proposal that there be a fourth mode of supposition may have occurred in 1370 for the first time. Spade 1988a argues that there cannot be a fourth mode in addition to the three already recognized. My proposal for four modes is in agreement with Spade's conclusion.



## Wide distributive

Descent to a conjunction of propositions and ascent back from the whole conjunction.

## Narrow distributive

Descent to a conjunction of propositions and no ascent back from the conjunction.

There is precedent for the category of wide distribution; it is what Paul of Venice (*LM: TS 3.11a (95)*)<sup>38</sup> proposes for what he calls mobile distribution. So far as I know, narrow distribution has not been proposed as a distinct category.<sup>39</sup> (Immobile distribution generally refers to cases in which no descent is possible.)

If we think of denoting phrases as restricted quantifiers, then this is equivalent to saying:

## Determinate

Existential Instantiation holds, and so does Existential Generalization.

## Merely confused

Existential Instantiation does not hold, but Existential Generalization does.

## Wide distributive

Universal Instantiation holds, and so does Universal Generalization.

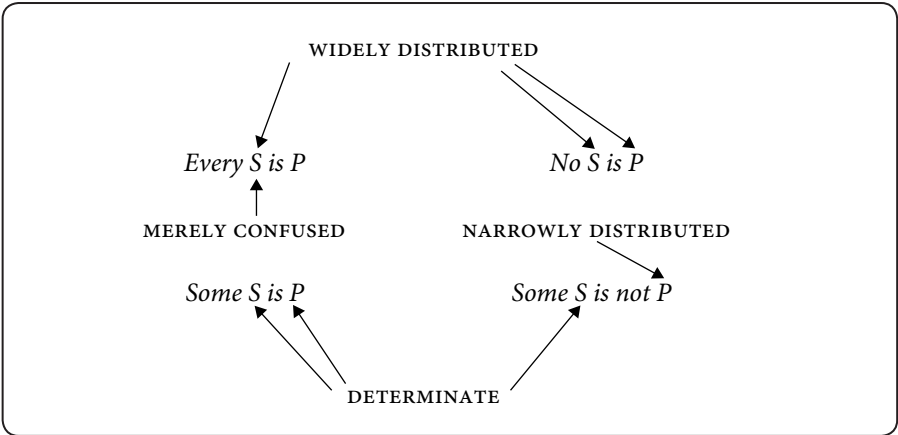
## Narrow distributive

Universal Instantiation holds, but Universal Generalization does not hold.

This more refined classification of terms yields a more symmetrical pattern. Let us call wide distribution and determinate supposition *opposites*, and likewise narrow distribution and merely confused supposition. Then when a proposition is negated, each term has its mode of supposition reversed. Since diagonally opposite propositions in the square of opposition are (logically equivalent to) negations of one another, the modes of supposition of the terms in any proposition mirror (in reverse) those in its opposite. The modes of supposition in the traditional square are now:

<sup>38</sup> “Distributive general reference is twofold because some is mobile, some immobile. Distributive mobile general reference is the meaning of a common term beneath which one can infer to all of its singulars conjunctively on the condition of a proper middle and, conversely, with the same middle. Thus this follows: ‘This animal runs and this animal runs and thus of each individual and these are all animals; therefore, every animal runs.’” Paul correctly includes subjects of universal affirmatives, and both subjects and predicates of universal negatives as having this mode of supposition.

<sup>39</sup> Spade 1976 argues that distributive supposition should be restricted to what I am here calling wide distribution, and the category of merely confused supposition should be expanded to include the remaining instances of distributive supposition (i.e. narrow distribution). Priest and Read 1980 argue that distributive supposition should be restricted to wide distribution, and the category of merely confused supposition should be abolished. I think that each of these proposals would yield a less useful theory. Priest and Read also argue that Ockham’s definition of distributive supposition is a definition of wide distribution. This has been disputed e.g. by Matthews 1984, and it is not now a widely held view.



## APPLICATIONS

Say which terms have which kinds of refined modes of supposition.

*Every donkey sees every horse*

*Some donkey sees every horse*

*Some donkey sees of every farmer a horse*

*Every donkey sees of some farmer every horse.*

## 7.9 Causes of the refined modes

With these new distinctions we can refine the theory of the causes of the modes of personal supposition developed previously. The principles used then yielded mostly intended results, except for the problem about what mode a term has if it is acted upon by two distributing signs. Ockham and Buridan disagreed about this, and neither of their views is completely correct. Their theory lacked the resources for giving a simple correct answer, since it does not subdivide the category of distributive supposition.

The revised theory makes a new set of rules possible. The refined rules in what follows are to be applied recursively to the signs of a sentence that have scope over the verb; as before, they apply first to the sign that is rightmost and has scope over the verb.

**DEFAULT:** A main term of a proposition has determinate supposition unless something causes it not to. A particular affirmative sign adjoined to a term gives it determinate supposition (or, equivalently, has no effect). In either case, any terms to the right and within the scope of the denoting phrase containing the term retain the mode of supposition they already have, except that a wide distributed term becomes narrow distributed.

UA: A universal affirmative sign widely distributes the term it is adjoined to and makes any other term to its right merely confused if it is determinate, leaving terms with the other modes unchanged.

UN: A universal negative sign widely distributes the term it is adjoined to; if a term mediately following the universal negative sign has determinate supposition, it becomes wide distributed; if the term has wide distribution it becomes merely confused; if the term has merely confused supposition it becomes narrowly distributed, and if it has narrow distribution it becomes merely confused.

NEG: A negating negation has the following effect on any main term following it and in its scope:

If the term is determinate it becomes wide distributed, and vice versa.

If the term is merely confused it becomes narrowly distributed, and vice versa.

All provisions except for the first and last are consistent with the earlier rules, though they provide more detailed information. The previous version of the last rule said that if a term has distributed supposition then the negation makes it merely confused (according to Buridan) or determinate (according to Ockham). On the new account if the term has narrow distribution it becomes merely confused, as Buridan says, and if it has wide distribution it becomes determinate, as Ockham says. Two relevant examples are these:

*Not no man runs*

*Not some farmer sees every donkey*

The new rules correctly classify ‘*man*’ as having determinate supposition in the first proposition, and they correctly classify ‘*donkey*’ as having merely confused supposition in the second proposition.

#### APPLICATIONS

Say which terms have which kinds of refined modes of supposition according to the causes of modes of supposition.

*Every donkey sees every horse*

*Some donkey sees every horse*

*Some donkey sees of every farmer a horse*

*Every donkey sees of some farmer every horse.*

### 7.9.1 Modes of supposition in Linguish

Following out this theory of causes of modes of supposition we can define the modes of supposition for all non-parasitic main terms in propositions of Linguish, as follows

(where it is understood that the term ‘T’ is not a parasitic term, and ‘R’ may or may not be a parasitic term):

‘T’ has wide distributive supposition in ‘(every  $T_\alpha$ )  $\phi$ ’ and in ‘(no  $T_\alpha$ )  $\phi$ .’

‘T’ has determinate supposition in ‘(some  $T_\alpha$ )  $\phi$ ’ and in ‘( $\cdot$   $T_\alpha$ )  $\phi$ .’

Whatever mode of supposition ‘T’ has in ‘ $\phi$ ’, it has that same mode of supposition in:

( $t_\alpha$ )  $\phi$

[ $\psi$  and  $\phi$ ] and in [ $\phi$  and  $\psi$ ]

[ $\psi$  or  $\phi$ ] and in [ $\phi$  or  $\psi$ ]

If ‘T’ has wide distributive supposition in ‘ $\phi$ ’ then:

‘T’ has wide distributive supposition in

(every  $R_\alpha$ )  $\phi$

‘T’ has narrow distributive supposition in:

(some  $R_\alpha$ )  $\phi$

‘T’ has determinate supposition in:

not  $\phi$

‘T’ has merely confused supposition in:

(no  $R_\alpha$ )  $\phi$

If ‘T’ has determinate supposition in ‘ $\phi$ ’ then:

‘T’ has determinate supposition in

(some  $R_\alpha$ )  $\phi$

‘T’ has merely confused supposition in:

(every  $R_\alpha$ )  $\phi$

‘T’ has wide distributive supposition in:

not  $\phi$

(no  $R_\alpha$ )  $\phi$

If ‘T’ has narrow distributive supposition in ‘ $\phi$ ’ then:

‘T’ has narrow distributive supposition in

(every  $R_\alpha$ )  $\phi$

(some  $R_\alpha$ )  $\phi$

‘T’ has merely confused supposition in:

not  $\phi$

(no  $R_\alpha$ )  $\phi$

If ‘T’ has merely confused supposition in ‘ $\phi$ ’ then:

‘T’ has merely confused supposition in

(some  $R_\alpha$ )  $\phi$

(every  $R_\alpha$ )  $\phi$

‘T’ has narrow distributive supposition in:

not  $\phi$

(no  $R_\alpha$ )  $\phi$

## 7.10 Useful inferences again

With these refined modes in mind, we can look again at some useful inferences expressible with the terminology of modes of supposition. It is clear that all of the instances of inferences from Superior to Inferior and from Inferior to Superior still hold good, since subdividing the mode of distributive supposition has no effect on these. In addition Sherwood's special rules lead to some interesting applications.

### 7.10.1 Complete induction

Let us look first at Sherwood's Rule III:

Rule III *An argument from many cases of determinate supposition to one of determinate supposition does not follow, but [only] to one of confused supposition.*

Thus when every man sees only himself this does not follow: 'a man is seen by Socrates, and [a man is seen] by Plato (and so on with respect to [all] individual [men]); therefore a man is seen by every man.' But this does follow: 'therefore by every man a man is seen,' for a distribution has force in a succeeding phrase but not in a preceding phrase.

This rule deals with "complete" induction, which has a form something like this:

*... man #1 ...*  
*... man #2 ...*  
*... man #3 ...*            *where these are all men*  
 $\therefore$  *... every man ...*

At first glance it seems that we can say that when we have premises of the given form, we can just replace 'man #n' by 'every man' to get the conclusion. And we can do this in simple cases, such as:

*Socrates sees man #1*  
*Socrates sees man #2*  
*Socrates sees man #3*            *etc. for all men*  
 $\therefore$  *Socrates sees every man*

But Sherwood shows us that this will not always work. This inference is clearly fallacious:

*A horse is seen by man #1*  
*A horse is seen by man #2*  
*A horse is seen by man #3*            *etc. for all men*  
 $\therefore$  *A horse is seen by every man*

From a modern point of view we would like to say that complete induction works when the generalized term in the conclusion has wide scope. Sherwood's rule is phrased in terms of a *symptom* of this: if there is a term in the proposition with determinate supposition and the generalized term is not given wide scope then that term still has

determinate supposition in the conclusion, whereas if the generalized term appeared on the front it would confuse that term, so that it would have merely confused supposition.

So why don't we just say that the conclusion of a complete induction must have 'every P' on the very front, instead of being located where the singular terms were? In the case in point this would work fine; we would have:

*Every man a horse is seen by*

However, there are circumstances in which an induction should be OK even though it is not grammatical to put the generalized term on the front. A simple example of this is

*The sun rose and man #1 got up*  
*The sun rose and man #2 got up*  
*The sun rose and man #3 got up*      *etc. for all men*  
 $\therefore$  *\*Every man the sun rose and got up*      *<\*ungrammatical>*

We can generally avoid the problem of ungrammaticality by saying that any argument of the following form is a good complete induction:

$\dots$  *man #1*  $\dots$   
 $\dots$  *man #2*  $\dots$   
 $\dots$  *man #3*  $\dots$       *where these are all men*  
 $\therefore$   $\phi$  where  $\phi$  is got by inserting 'Quant man' any place in ' $\dots$ ' where it occupies the same grammatical role as '*man #n*,' and where 'Quant' is an affirmative quantifier sign,<sup>40</sup> namely 'every' or 'some' or 'a,' and where '*man*' ends up with wide distribution.

So these would be good inductions:

*The sun rose and man #1 got up*  
*The sun rose and man #2 got up*  
*The sun rose and man #3 got up*      *etc. for all men*  
 $\therefore$  *The sun rose and every man got up*

*Not man #1 ran*  
*Not man #2 ran*  
*Not man #3 ran*      *etc. for all men*  
 $\therefore$  *Not some man ran*

We can see Sherwood's rule as a consequence of this one for the special case of categorical propositions. Suppose that there are one or more determinate terms in the inductive premises, which are categorical propositions. Then replacing the varying term with the term '*man*' with a quantifier that yields wide distributive supposition

<sup>40</sup> If we applied the suggested rule to a case with 'no' we could get a faulty inference: *Man#1 ran*, and *Man#2 ran*, and so on, therefore *No man ran*.

would convert those determinate terms to merely confused terms. So one cannot have a complete induction with determinate terms in the premises and a determinate term in the conclusion (except for molecular cases, which are not the cases Sherwood had in mind). So Sherwood's rule is a correct special case of a correct general principle about concluding with a term with wide distributive supposition in complete inductions.

I do not know at this point if there is always a way to provide a grammatical conclusion for any proposed induction. For example, it is not clear to me how to conclude:

*Every man who owns donkey #1 is running*  
*Every man who owns donkey #2 is running*  
*Every man who owns donkey #3 is running*                      *etc. for all donkeys*  
 ?????

It seems that we can conclude:

*Every man who owns a donkey is running*

But this is weaker than the conclusion we want to draw, which requires every donkey to be owned. (Since each premise is affirmative, it requires that 'man who owns donkey #n' to be non-empty, and that entails that donkey #n is owned.) What we want to conclude is something like:

*For each donkey, every man who owns it is running*

However, this makes essential use of a pronoun with a grammatical antecedent. These will be discussed in the next chapter.

### 7.10.2 *Switching scopes (thereby switching modes of supposition)*

Sherwood's rule II and the first half of rule IV are easy to sum up: It is fallacious to move a negation from the right of a determinate (rule IV) or merely confused (rule II) term to the left of that term. If this is done then you have changed the term from determinate or merely confused supposition to distributive supposition, which is how Sherwood words the prohibition. Since our rules of inference do not contain such a provision for moving negations, Sherwood's rules prohibit something which our rules of inference—fortunately—cannot produce. The prohibition of these negation moves is complemented by the first half of rule V, which illustrates the principle that a move in the opposite direction *is* always OK—you can move a negation from the left of a term to the right of that term if that term ends up with determinate or merely confused supposition. If this is done then you will have changed a term from distributive supposition to determinate or merely confused supposition, which is how Sherwood words the permitted cases. (Actually he only states the first half.) I think this is a good principle, though I haven't proved it here.

The second half of rule IV is more interesting. We can formulate it in general as this: If you move a denoting phrase from one place to another in the formula in such a way that it does not change its grammatical role, then if it had determinate supposition

before the move and has merely confused supposition (and if its quantifier sign remains with it and unchanged) after the move then the resulting proposition follows from the first. Is this principle a good one? Well, if you change a term from determinate to merely confused, then you have moved it to the right, past a term with distributive supposition. And in general this is a good technique; it is like the valid quantifier switch from  $\exists x \forall y$  to  $\forall y \exists x$ . In fact, sometimes this move is a good one even if you change the term's quantifier's sign, as in:

<i>Some A not some B<sub>acc</sub> sees</i>	'A' has determinate supposition
<i>∴ not some B<sub>acc</sub> every A sees</i>	'A' has merely confused supposition

There seems to be a good generalization here, but I am not sure how to state it.

### 7.10.3 Algorithms

Elizabeth Karger (1993) points out (424) that Buridan gives some rules similar to those just discussed, and (423–4) that those rules can be turned into an algorithm for entailment between categorical sentences with the same quality and exactly the same two (non-parasitic) terms in each.<sup>41</sup> A paraphrase of the algorithm that she gives is this:

Let P and Q be categorical sentences of the same quality and containing the same two (non-parasitic) terms. Then:

(i) P and Q mutually entail one another iff their terms have the same mode of supposition in both P and Q

(ii) P entails Q but not vice versa iff these three conditions hold:

- some term has a different mode of supposition in P than in Q
- no term has non-distributive supposition in P and distributive supposition in Q
- no term has merely confused supposition in P and determinate supposition in Q, unless it is preceded by a term in P which has distributive supposition in P and determinate supposition in Q

Karger suggests (427) that it was a major goal of the theory of modes of supposition to provide such algorithms, since no author had provided a general theory of truth conditions even for categorical sentences, and inferences between categoricals outrun the familiar principles of conversion and subalternation. She may be right about this motivation. She also points out (429) that this type of approach cannot be extended to categoricals with three terms, and that an attempt to do so led John Dorp to mistakenly judge that the following two sentences, whose terms have the same modes of supposition, mutually entail each other:

*At every time an animal every man is*  
*At every time every man an animal is*

<sup>41</sup> See Dutilh Novaes 2007, section 2.4 for further discussion.



With our refined theory of modes of supposition we could point out that ‘*man*’ in fact has narrow distributive supposition in the former sentence and wide distributive supposition in the latter, and a parallel to Sherwood’s rules would include something like:

An inference from narrow to wide distribution holds, but not from wide to narrow.

However, even though this might save the algorithm for propositions with three terms, it is only a stopgap method, since it will fail for propositions with four terms, such as:

*At every time a woman sells an animal to every farmer*

*At every time a woman sells to every farmer an animal*

## 7.11 Modes of supposition as analyses of quantification

There has been a great deal of discussion in the secondary literature about the hypothesis that modes of supposition were introduced by medieval logicians in an effort to explain quantification.<sup>42</sup> The idea is that the semantic effect of a quantifier in a given position in a proposition is explained by doing a descent under that quantifier. For example, the semantics of the word ‘some’ in this proposition:

*Some donkey is grey*

is explained by the fact that that proposition is a priori equivalent in truth value to this descended form:

*This donkey is grey or that donkey is grey or . . . etc. for all donkeys*

In fact, this disjunction looks a lot like some proposals 20th-century philosophers have made to analyze quantifiers in terms of combinations of connectives. The primary stumbling block for this proposal is that in some cases—indeed, always in a case of merely confused supposition—descent is not to an equivalent proposition, but only to one that entails or is entailed by the original proposition. But this problem may be avoided by a more refined idea which is based on a set of principles that were developed by medieval authors, principles that are meant to govern the order in which one should analyze the terms in a proposition. These principles came to be called in the secondary literature principles of “priority of analysis.”<sup>43</sup> Three principles were usually included:

- One should descend first under a determinate term if there is one.
- One should descend under a distributed term before descending under a merely confused term.

<sup>42</sup> Cf. Matthews 1973 and references therein; Priest and Read 1977, 1980; Matthews 1984. (Note that Matthews denies that modes of supposition provide an analysis of quantification.)

<sup>43</sup> Cf. Swiniarski 1970, Matthews 1973, Spade 1976, Ashworth 1978.

- When terms are grammatically related as determinable to determinant, one should descend under the determining term before descending under the determinable term.

(Some examples of determinable terms are participles of transitive verbs that are determined by their direct object terms in the accusative case, or grammatically “possessed” terms that are determined by their “possessors” in the genitive case.) A key question to ask about these principles is why they should be obeyed; why *should* one do things in this order? Suppose that I have the proposition:

*Some donkey is not a pet*

which contains both a term, ‘*donkey*,’ with determinate supposition and a term, *pet*, with distributive supposition, and suppose that I violate the first principle by descending under the distributed term to get:

*Some donkey is not this pet, and some donkey is not that pet, . . . and so on for all pets*<sup>44</sup>

This seems like a valid inference, so what is wrong with it? I think the intended answer is that if you do this you will indeed infer a consequence of the proposition you are dealing with, but you will not thereby produce an *analysis* of it. You won’t produce an analysis because you haven’t descended to an equivalent proposition. If instead you obey the rule, and descend under the term with determinate supposition, you get an *analysis* of the original proposition:

*This donkey is not a pet or that donkey is not a pet or . . . etc. for all donkeys.*

Of course these conjuncts still contain a common term, ‘*pet*’ which has distributive supposition. And now the priority rules allow us to analyze it, getting:

*This donkey is not this pet and this donkey is not that pet . . . etc. for all pets*  
or

*That donkey is not this pet and that donkey is not that pet . . . etc. for all pets*  
or

. . . . .  
*and so on for all donkeys.*

So the original proposition turns out to be equivalent to an augmented disjunction of augmented conjunctions of propositions containing only singular terms. Ideally, if the priority principles are obeyed, it will always be possible to analyze a proposition into one containing only singular terms. There are many interesting questions about what this sort of analysis achieves, but I will not try to settle them here; instead I will concentrate on exploring just how the theory works. I will also avoid discussing the significance of the widely discussed fact that if these rules are obeyed one never

<sup>44</sup> In order to claim that this follows from ‘*Some donkey is not a pet*’ we may need to appeal to the “other option” in section 7.3.4 for handling the possibly empty term ‘*donkey*.’

descends under a merely confused term *when it is merely confused*. If you have a merely confused term it will be confused by the presence of a distributed term with wider scope. By the second principle, that distributed term is analyzed first, and when all such distributed terms are analyzed they are replaced by singular terms, which confuse nothing—and so the original merely confused term becomes determinate, and is then a candidate for analysis. This fact has led some in the secondary literature to suggest that it was a theoretical mistake for medieval authors to include merely confused as a status for terms; I will ignore this issue here.<sup>45</sup> (Though one should keep in mind that inference rules such as “From an inferior to a superior” apply to terms with merely confused supposition.)

How well do the three priority principles work? Let us consider applications of them, restricting ourselves to the notations developed so far. Clearly molecular propositions can be “analyzed” into conjunctions and disjunctions, since they are already in such forms. (And “as-of-now” conditionals, explained in section 5.9 are equivalent to disjunctions of the negation of the antecedent with the consequent.) So that leaves categorical propositions. Let us begin with the simplest cases: categorical propositions all of whose main terms are non-parasitic and simple.

### 7.11.1 *Categorical propositions whose terms are non-parasitic and simple*

For categorical propositions the rules given earlier are close to having the principle that you only analyze a term which is on the front and has widest scope. For if the rule about descending under a term which is determinate in the current proposition is applicable, then that term can't be within the scope of a distributed term, for that would confuse it, and it cannot have a determinant term grammatically determining it, for this is ruled out by the third principle. So it is either on the front, or it is within the scope of other determinate terms and can be interchanged with them. Suppose instead that the second rule is applicable, and the term identified to descend under is distributed. Must this be on the front? No, and this is a case for which the rules need adjusting. Consider a proposition like:<sup>46</sup>

*Of every farmer some donkey sees every horse.*

Now the rules as currently formulated permit us to descend under ‘horse,’ which is distributed. But if we do so, we do not reach an equivalent proposition. (Because, intuitively, ‘every horse’ does not have a wide enough scope.) The adjustment that is called for is clear; the rules should be expanded to say that one descends under a term with *wide* distribution before descending under a term that is merely confused

<sup>45</sup> One might note that if medieval authors had distinguished wide and narrow distribution, narrow distribution would be subject to the same charge. This would not worry some writers, such as Priest and Read 1980, who have also argued, in effect, that distribution ought to have been limited to wide distribution all along.

<sup>46</sup> This example contains a parasitic term, but that is not essential. If we choose a verb that takes both direct and indirect objects we get ‘Every woman shows some farmer every horse.’

or narrowly distributed. Since 'horse' has narrow distribution in the example, the problem vanishes.

### 7.11.2 *Categorical propositions whose terms are simple with one or more parasitic terms*

Now let us consider parasitic terms. When there are no complex terms, parasitic terms are simple and they follow and are within the scopes of the terms on which they are parasitic. If those other terms are not singular, then by the discussion earlier about determinables we will descend under those other terms first, until all terms which have the given parasitic term within their scopes are singular. Here is where restricted supposition becomes important. If a parasitic term has wider scope than any other common term, and if its determinant is a singular term, then we have seen earlier that it makes sense to descend under it by the principle of restricted descent discussed in section 7.5. As a result, parasitic terms behave under analysis just like other terms. This may be part of the reason why medieval logicians have so little to say about them. From a modern point of view parasitic terms are inherently relational, and so they are very special, but for many medieval purposes, such as analysis, this specialness doesn't matter.

### 7.11.3 *Categorical propositions with complex terms*

Finally let us consider complex terms. The most common examples are common terms modified by adjectives (or participles), and common terms modified by relative clauses. In each of these cases principles of analysis can be applied to them as well. (Although I am not aware of cases in which this was done.)

Consider first the term 'grey donkey' and suppose that we have reached the point where all quantifier signs have been eliminated, and so this occurs in a proposition in the context '... this<sub>i</sub> grey donkey ...'. Since there are no quantifier signs, this proposition will be equivalent to a conjunction of this form: '... this<sub>i</sub> grey-thing ... and ... this<sub>j</sub> donkey ...' (See the rule of inference in section 5.5.1.) Thus the complex term is eliminated.<sup>47</sup>

In the case of a relative clause, it will help to consider the Linguish form, since this allows us to keep the relevant grammatical roles straight. We will have a proposition of the form

'... (this<sub>i</sub>, {F which<sub>γ</sub> ψ} α) φ ...?'

This will be equivalent to a conjunction of the form:

'... [(this<sub>i</sub>, F<sub>γ</sub>) ψ and (this<sub>i</sub>, F α) φ] ...?'

<sup>47</sup> In the actual case given, it would not be important to eliminate the complex term, since there are no quantifiers within it. This would be different for 'this grey donkey which ψ' if it had the logical form '(this<sub>i</sub>, {grey {donkey which<sub>γ</sub> ψ}} α)' when there are quantifier expressions within 'ψ'. In this case, separating the adjective from the complex term that it modifies yields an expression which is treated by other rules.

An example would be that ‘*this donkey which some woman feeds is running*’ becomes ‘*this donkey some woman feeds and this donkey is running*’, from the logical forms ‘... (this<sub>j</sub> {donkey which<sub>γ</sub> (some woman<sub>β</sub>) β feeds<sub>γ</sub>} α) α is running ...’ which is equivalent to ‘... [(this<sub>j</sub> donkey<sub>γ</sub>)(some woman<sub>β</sub>) β feeds<sub>γ</sub> and (this<sub>j</sub> donkey<sub>α</sub>) α is running] ...’ (See the rule of inference in section 5.6.)

Some similar eliminations are possible with other sorts of complex terms. It is not clear how far this idea can be pushed. But since we have already gone beyond work done by medieval logicians, I will not pursue this further here.

#### 7.11.4 Rules from inferior to superior and from superior to inferior

It is now possible to validate the inferior/superior principles discussed in section 7.7. Consider first a common term T with determinate supposition in this context:

... (Q T α) ...

Since T has determinate supposition, that proposition is equivalent to this disjunction:

... (this T α) ... or ... (that T α) or ... etc. for all T’s

Suppose that S is a superior of T. That means that the augmented disjunction is equivalent to

... (this S α) ... or ... (that S α) or ... etc. for all S’s that are T

There may, of course, be more propositions involving S’s of the same form. By the logic of ‘or,’ the proposition just displayed entails the result of extending it with more disjuncts:

... (this S α) ... or ... (that S α) or ... etc. for all S’s (including S’s that aren’t T)

Notice that the original proposition with S instead of T will be a proposition in which S has determinate supposition, and it will be equivalent to the disjunction just displayed. So:

... (Q S α) ...

will follow from the original proposition. The superior is thus inferable from the inferior when the term has determinate supposition.

Suppose now that T has wide distributive supposition. Then the argument here applies except that we are dealing with conjunctions instead of disjunctions, and we will be going from a conjunction about all the Ts to a sub-conjunction about all the Ss.

Suppose now that the term T has merely confused or narrow distributive supposition. If we follow the pattern described earlier in this section for analyzing the proposition containing T, we will end up with a disjunction of conjunctions of ... of propositions involving T, in which it is not determinate or wide distributive. At this point the arguments just given apply.

## 7.12 Global quantificational import

### 7.12.1 What are modes of common personal supposition?<sup>48</sup>

This is a long-standing problem in the secondary literature: We have definitions of the modes, but what are we defining?

What we have is a theory of what I call *global quantificational import*. This is the import a quantified denoting phrase actually has, described in terms of the import it would have if it had scope over the whole global context (or almost the whole context).

This idea can be made precise within the theory of “prenex forms.” In contemporary symbolic logic, if no biconditional sign appears in a formula of quantification theory then you can take any quantifier in that formula and move it in stages toward the front of the formula, each stage being equivalent to the original formula, provided that you switch the quantifier from universal to existential (or vice versa) whenever you move it past a negation sign or out of the antecedent of a conditional, and provided that you do not move it past a quantifier of opposite quantity (i.e. you don’t move a universal past an existential, or vice versa). For example, you can take the universal quantifier in:

$$\neg(Gy \rightarrow \forall x Px)$$

and move it onto the front of the conditional to get:

$$\neg \forall x (Gy \rightarrow Px),$$

and then the resulting universal sign can be moved further front, turning into an existential:

$$\exists x \neg (Gy \rightarrow Px).$$

This chain of equivalences can be interpreted as the movement of a quantifier to the front, retaining its identity while sometimes changing its quantity. If you do this systematically to all the quantifiers in a formula, the result is a formula in “prenex normal form,” in which the quantifiers are all on the front in a row, each of them having scope over the rest of the formula to its right. In terms of these prenex forms you can define the global quantificational import of any quantifier in a main term in any categorical formula. Let us take this idea and use it to analyze the terminology of supposition theory. The subject matter here is terms, not quantifiers, but each main term comes with its own quantifier, so we can treat the theory as if it is a theory of restricted quantification (with denoting phrases being the restricted quantifiers). We then give this account for terms in categorical sentences:

A prenex string for a formula  $\phi$  is a string of affirmative denoting phrases on the very front of  $\phi$ , with no other signs between them.

(That is, each is of the form ‘Every  $T$ ’ or ‘Some  $T$ ’ or ‘ $d$ ’, and there are no negations, and each denoting phrase has scope over the rest of  $\phi$  to its right.)

<sup>48</sup> This section covers material also discussed in Parsons 2008a.

An example with the prenex string underlined:

Every dog some donkey isn't

There is a systematic way to convert any categorical proposition into another one in which all of the main terms in the original one are in prenex position in the new one, and the converted proposition is logically equivalent to the original. Here is the process:

Change every main denoting phrase of the form 'No T' into 'every T not,' and every main denoting phrase of the form 'a T' into 'some T'. This leaves 'every' and 'some' as the only quantifier signs on main terms.

Remove any double *not*'s anywhere in the formula whenever they appear.

Starting at the left, replace each '*not every T*' by '*some T not*,' and each '*not some T*' by '*every T not*,' and each '*not d*' by '*d not*.' Remove double *not*'s whenever they appear.

Every categorical proposition has a unique prenex-convert produced by these rules.

Call the quantifier signs 'every' and 'some' opposites. We can then define:

a main term has (wide)/(narrow) quantificational import in a proposition iff when the proposition is converted into prenex form the term (is not)/(is) preceded by a main term with the opposite quantifier sign

a main term has (universal)/(existential) global quantificational import in a proposition iff when the proposition is converted into prenex form the term ends up with ('every')/('some') as its quantifier sign

This defines global quantificational import for all main terms in any categorical proposition.

Note that the discussion here applies only to categorical propositions. If a term occurs in a conjunct of a conjunct, say, it may not be possible to move it to the front because the result is not grammatical. For example, if we move '*every horse*' to the front of '*some donkey is running and Socrates owns every horse*' we get '*every horse some donkey is running and Socrates owns*.' We could make this work by altering the language, but I'm not sure what the significance of that would be.

#### APPLICATIONS

Say which terms have which kinds of quantificational import.

*Every donkey sees every horse*

*Some donkey sees every horse*

*Some donkey sees of every farmer a horse*

*Every donkey sees of some farmer every horse.*

### 7.12.2 *Causes of the modes and global import*

One can now establish the following equivalence between the previous classifications in terms of global quantificational import and the refined modes of supposition that are yielded by the rules governing causes of the modes in the last section, at least so far as categorical propositions are concerned. If these rules are applied to the categorical forms we have been discussing:

A term has determinate supposition according to the rules for causing modes of supposition iff it has wide existential quantificational import

A term has merely confused supposition according to the rules iff it has narrow existential quantificational import

A term has wide distributive supposition according to the rules iff it has wide universal quantificational import

A term has narrow distributive supposition according to the rules iff it has narrow universal quantificational import

Illustration: Let us test ‘*donkey*’ for its mode of supposition in ‘*Some donkey is a predator*.’ ‘*Donkey*’ has determinate supposition here, because it is already in prenex form, existentially quantified:

*Some donkey is a predator*

It has wide distributive supposition here, for the same reason:

*Every donkey is a predator*

The term ‘*predator*’ in the sentence just displayed has merely confused supposition because it is existentially quantified with scope inside that of ‘*every donkey*.’

Now consider:

*Some predator is not a donkey*

in its equivalent form

*Some predator not a donkey is*

Here the ‘*not a donkey*’ is equivalent to ‘*every donkey not*,’ yielding:

*Some predator every donkey isn’t*

The original ‘*a donkey*’ has now become universal, thus classifying it as having distributive supposition. This illustrates the importance of looking at things globally; although ‘*donkey*’ is not preceded by any universal quantifying sign here, it has universal import. You could universally instantiate it! This is why it is classified in this theory as distributive.



### 7.12.3 *Parasitic terms*

We have just established that a main term in a categorical proposition has a certain mode of supposition according to the rules for the causes of supposition iff it has a corresponding quantificational import. But, as we have noted earlier, both of these disagree with the modes of supposition as defined in terms of ascent and descent in the case of parasitic terms. Such terms do not admit of either ascent or descent, and so they have no mode of supposition at all. The situation is summed up by:

A main term that is not parasitic has the mode of supposition that is attributed to it by the refined rules for the causes of modes of supposition—equivalently, if it has the corresponding global quantificational import.

Parasitic main terms have no modes of supposition, though they are classified as having modes by the rules from causes of modes, and they do have global quantificational import.

As noted earlier, the useful rules “From a superior to an inferior” and “From an inferior to a superior” do not in general apply to parasitic terms.

# 8

## Relatives (Anaphoric Words)

Medieval logicians use the grammatical term ‘relative’ for what we would call an anaphor—for an expression that has a grammatical antecedent.<sup>1</sup> Relatives are divided into relatives of substance and relatives of attribute. An example of a relative of attribute is the ‘*such*’ in ‘*Socrates is white and Plato is such*.’ I will not discuss relatives of attribute here. So what follows is about relatives of substance. As Ockham and many others point out, relatives of substance do not always concern substances. For example, in ‘*Every attribute inheres in whatever has it*’ the ‘*it*’ is a paradigm of what is called a relative of substance, even though it is attributes, and not substances, that are under discussion. Buridan *SD* 4.4.2 (282) explains:

a relative term is not called [a relative term] ‘of substance’ for the reason that its antecedent has to be a term of the category of substance; on the contrary, its antecedent can be equally of any other category, as in ‘A white [thing] is a wall and it is seen,’ ‘A whiteness is in a wall and it is seen,’ ‘A six-foot-tall [thing] sees itself,’ ‘My master rides his horse.’ Rather, it is called a relative term of substance, because it does not signify what it refers to, or that which it is taken for, to be of quality, to be of some quantity, or to be somewhere, and so on for the rest of accidental predications, but only to be something; relative terms of accidents, on the other hand, signify something’s being of some quality or being of some quantity or being somewhere, and so forth.

Relatives are divided into relatives of identity and relatives of diversity. An example of a relative of identity is the ‘*it*’ in ‘*Socrates owns a donkey and Plato sees it*.’ An example of a relative of diversity is the ‘*other*’ in ‘*Socrates owns a donkey and Plato owns some other donkey*.’ I will discuss relatives of identity and then (very briefly) relatives of diversity.

### 8.1 Relatives of identity

The commonest accounts of relatives are rooted in a conception according to which a relative is a term which has supposition of its own; what that supposition is is determined by the term’s antecedent. The antecedent of a term determines both the kind of supposition the term has and what it supposits for. The most basic view is that a relative has the same kind of supposition its antecedent has—material, simple, or personal;

<sup>1</sup> The same term, ‘relative,’ was used for relational terms, such as ‘*father*.’ The two uses were kept distinct.

ampliated or restricted; determinate, distributive, or merely confused—and (if it is a relative of identity) it supposit for exactly the same things as its antecedent. This view gets substantially qualified and modified, but the core idea persists.

Anonymous, *Treatise on Univocation*, 347: “an anaphoric pronoun must supposit for the same and in the same way as its antecedent.”

Peter of Spain *SL* 8.3: “A relative of identity is what refers to and supposes for the same item.”

Anonymous *Cum sit nostra*, 448: “the same supposition is in the relative and in its antecedent.”

There is a good bit of discussion of relatives whose antecedents have material or simple supposition, as in ‘Man is a noun and it has three letters.’ For simplicity I will skip those discussions. So for the remainder of this chapter I will discuss only relatives that clearly have personal supposition.

A fundamental principle seems to be generally agreed on; I call it the replacement principle.

The Replacement Principle: A relative of identity may always be replaced by its antecedent if the antecedent is singular; otherwise it may not always be so replaced.

[I]t is not always permissible to put the antecedent in place of the relative. For saying ‘A man runs and he argues’ is not the same as saying ‘A man runs and a man argues,’ because for the truth of ‘a man runs and a man argues’ it suffices that one man run and another one argues. Instead, the rule ‘It is permissible to put the antecedent in place of the relative’ is to be understood as holding when the antecedent is singular and not common to [several] supposita. For saying ‘Socrates runs and he argues’ is the same as saying ‘Socrates runs and Socrates argues.’ (Burley *PAL* longer treatise para 117 (112))

Several other authors make this same point. I will take for granted that the positive part of this principle is included in all of the theories under discussion.

### Rule SA (Singular Antecedents)

A proposition containing a relative of identity with a singular antecedent is equivalent to the proposition that results from replacing that relative with its antecedent.

### APPLICATIONS

Suppose that a reflexive pronoun is symbolized just as a proper name is, with something in the notation to indicate which denoting phrase is its antecedent. Just using the rule for Singular Antecedents provide derivations for the following arguments.

- Socrates sees Socrates*  
 ∴ *Socrates sees himself*
- Socrates sees his donkey*      (*Socrates sees of himself a donkey*)  
 ∴ *Socrates sees Socrates' donkey*
- Socrates is a man who sees every man*  
 ∴ *Socrates sees himself*

Some relatives are reflexive, like *'herself'*, and some are not. I discuss reflexive pronouns in the next section.

## 8.2 Reflexive pronouns

Medieval authors use the Latin term *'reciprocus'* for what we today call reflexive pronouns. If a pronoun and its antecedent occur as the main terms of the same categorical proposition, then the pronoun must be reflexive. In English a reflexive third-person pronoun takes the form *'itself/himself/herself'* (unless it is genitive, when it takes the form *'his/her/its'*). In Latin a reflexive third-person singular pronoun is of one of the forms *'se/sui/sibi'*. Reflexive pronouns occur as direct objects, such as *'himself'* in *'Socrates sees himself'*, and they also occur as genitives related to direct or indirect objects, as in *'Each donkey sees its owner'* and in *'Socrates sees his donkey'*.

The Latin *'reciprocus'* is often translated as 'reciprocal'; however, 'reciprocal' has a technical meaning in modern linguistics, where it includes expressions like *'each other'*, but it does not include e.g. *'itself'*, which is a paradigm reflexive pronoun. I have thus changed 'reciprocal' to 'reflexive' in the translations, when necessary.

Writers often say that a relative has the same mode of supposition as its antecedent: determinate if the antecedent is determinate, distributive if the antecedent is distributive, and similarly for merely confused supposition. For reflexive pronouns this needs a major qualification. Walter Burley (*PAL*, longer treatise, paras 125–6) explains a special mode of supposition that these pronouns have:

You need to know that a reflexive relative referring to a term in the same categorical has the same kind of supposition as its antecedent has. But the relative adds 'singulation' onto the supposition its antecedent has, so that if its antecedent supposits confusedly and distributively, the relative has confused and distributive 'singled' supposition. And if its antecedent supposits particularly, the relative supposits particularly 'singly.' For example, when someone says 'Every man sees himself,' 'himself' supposits confusedly and distributively singly. . . . [Confused and distributive singled supposition] differs from absolute confused and distributive supposition, because under a term that supposits absolutely and confusedly distributively one can descend to anything for which the distribution is made. But under

a term that suppositis confusedly and distributively singly one cannot descend absolutely to any suppositum. Rather, to any suppositum one can descend *with respect to itself*. Therefore, it is called ‘singled’ supposition because it assigns singulars to singulars. For it does not follow: ‘Every man sees himself; therefore, every man sees Socrates.’ But it quite well follows: ‘Every man sees himself; therefore, Socrates sees Socrates.’<sup>2</sup>

It is not clear exactly what the principle of descent is here. Burley speaks as if he is discussing a descent “with respect to itself” under the relative ‘*himself*.’ But the illustration that he gives also involves a descent under its antecedent, ‘*man*,’ which is replaced by ‘*Socrates*.’ It appears that a descent cannot be made under the relative without also descending under its antecedent. Ockham (*SL* 1.76) makes this explicit:

It should also be noted that a relative of this sort has the same kind of supposition and suppositis for the same things as its antecedent. However, when its antecedent suppositis either confusedly and distributively or determinately, it has a similar form of supposition but exhibits this singularly—by referring particulars to particulars. Therefore, it is not possible to descend either conjunctively or disjunctively or in any way other than with respect to something contained under the antecedent. For example, in ‘Every man sees himself,’ the word ‘himself’ suppositis for every man by means of confused and distributed mobile supposition; but it does this singularly since it is not possible to descend without altering the other extreme. It does not follow that if every man sees himself every man sees Socrates. Nonetheless, it is possible here to descend to Socrates with respect to Socrates. Thus, ‘Every man sees himself; therefore, Socrates sees Socrates.’ . . . Likewise, in ‘A man sees himself’ the word ‘himself’ suppositis determinately yet singularly, for it is possible to make the descent only in the following way: a man sees himself; therefore, Socrates sees Socrates or Plato sees Plato or . . .’ (and so on for all the relevant particulars). It is also possible to ascend, but not in the following way: a man sees Plato; therefore a man sees himself. The ascent operates as follows: Socrates sees Socrates; therefore, a man sees himself.

Here, so far as inference is concerned, the point seems to be that one cannot descend under a reflexive without also descending under its antecedent, descending simultaneously to the same thing, and one cannot ascend to a reflexive without also ascending from its antecedent, and from the same thing.

<sup>2</sup> This view is not original with Burley. Lambert (*pt* 8q(ii)) says: “sometimes a [reflexive] relative refers to a common term . . . But if the common term to which it refers is taken universally, then the [reflexive] relative refers to its distributive antecedent singularly or one by one for each and every one of the single things.” The same idea may be present in Sherwood (*S* 1.16) “although ‘himself’ is the same as ‘every man’ [accusative] with respect to supposita, nevertheless ‘himself’ and ‘every man’ [accusative] relate in different ways to ‘every man’ [nominative]. Thus they differ in respect of it, since ‘himself’ relates one of its supposita to one belonging to ‘every man’ [nominative], and another to another.” The view became quite widespread. In addition to citations in the body of this chapter, it also occurs e.g. in Paul of Pergula *L* 2.4 (37–8): “when it is said: ‘Every man sees himself,’ the relative suppositis distributively, just as the antecedent, not simply, but referring one each to one each, and on that account it does not signify that every man sees every man, but that this sees himself, and this himself, and thus of singulars.” It lasts at least until the 17th century; Poinset *OFL* 2.13 (72) says it is called “imaged” supposition.

Buridan makes the same point in *SD* 4.4.6.<sup>3</sup> See also Albert of Saxony *QCL*, Question 18, especially paragraph 332.

Since in the 14th-century tradition modes of supposition are defined in terms of allowable descents and ascents, these discussions seem straightforwardly to characterize modes of supposition. As a by-product, these accounts are very informative about the effects of singled supposition regarding inferences. Marsilius of Inghen goes further in describing how one can accomplish this by descending under the antecedent of a relative alone *while letting the relative remain unchanged*. Although his case deals with a non-reflexive relative, there is a similar technique of descent:

The third rule is: a relative of identity has the same kind of supposition in the proposition in which it supposits as its antecedent: viz. materially, if its antecedent supposits materially, and personally, if its antecedent supposits personally, determinately, discretely, confusedly, or confusedly distributively in completely the same way as its antecedent supposits. I prove this rule solely on the basis of the intention of thought itself. The common way to understand the proposition *every man is an animal and he runs*, is that, just as the term *man* has confused distributive supposition, so the term *he* has confused distributive supposition. Therefore, in all such propositions the valid inference is: *every man is an animal and he runs, therefore this man is an animal and he runs*, and so on. (*TPT* 1 (75))

The theory that is stated is worded so that it treats the ‘*he*’ in the example as a distributed term whose supposita are the same as the supposita of its antecedent ‘*man*.’ A descent under such a term would take the form:

*Every man is an animal and he runs, therefore: every man is an animal and this man runs* and *every man is an animal and that man runs* and . . . , and so on.

But Marsilius does not give a descent under the pronoun; he gives a descent under the antecedent term ‘*man*,’ with the relative *left totally unchanged*. (Paul of Pergula *L* 2.4 (38) also illustrates a descent in which the relative is preserved unchanged after descent.) They seem to be employing the principle that rules of inference that normally apply to terms (or their denoting phrases) remain valid when those denoting phrases are antecedents of pronouns. This is indeed a very natural thing to do; we just apply all of our familiar rules—exposition, expository syllogisms, the quantifier equipollences—even when the denoting phrase in question is the antecedent of a pronoun. You just “leave the relative alone,” with the understanding that when the antecedent is modified in some way, its new form is the antecedent of the same pronoun.<sup>4</sup>

<sup>3</sup> Buridan: “Of the relative term ‘him/her/itself’ [*se*] we have to assert that it has the property of always being posited in the same categorical proposition as its antecedent, as in ‘Socrates likes himself.’ Its other property is that, if it is taken distributively, it is impossible to descend to one determinate suppositum thereof, the others remaining the same, rather it is necessary to descend one by one (*sigillatim*); for the inference ‘Every man likes himself; therefore, every man likes Socrates’ is invalid, but you can infer, therefore, ‘Socrates likes Socrates and Plato likes Plato;’ and so on for the rest” (*SD* 4.4.6).

<sup>4</sup> This “leave the relative alone” can be justified on the grounds that, in a sense, the relative doesn’t really have supposition of its own. Poinsot *OFL* 2.13 (72) says “The [reflexive] relative supposes through the supposition of its antecedent, so that by descending or ascending from the antecedent the descent from the [reflexive] relative takes place.”

In fact, we can do some reverse engineering and conclude that Burley, Ockham, and Buridan are committed to this principle in the examples they give. For suppose that this is a good inference:

Every man sees himself  
 Socrates is a man  
 ∴ Socrates sees Socrates

By the rule for singular antecedents, these two are equivalent:

Socrates sees himself  
 Socrates sees Socrates

Therefore any argument with the latter as a conclusion is just as valid as one having the former as a conclusion. So we have:

Every man sees himself  
 Socrates is a man  
 ∴ Socrates sees himself

It is most natural to see the Burley–Ockham–Buridan inference then as involving a two-step inference: first, descend in ‘*Every man sees himself*’ under the antecedent ‘*man*’ in the normal way to get ‘*Socrates sees himself*’ and then substitute ‘*Socrates*’ for ‘*himself*’ by the replacement principle cited earlier, that any relative of identity may be replaced by its antecedent if its antecedent is singular. If that is how things work, singled supposition does not require any new principles of inference at all; it is a consequence of already existing methods, none of which ever involve descending under a relative. (This conclusion will be sustained in discussion later.)

Before proceeding it may be of interest to expand somewhat the class of pronouns under discussion. The medieval authors focus specifically on reflexives, but there is a broader class of anaphoric pronouns of identity that also seem to work by the principles just discussed: this is the class of such pronouns that already fall within the scope of their antecedents. All reflexives do this, but there are other cases as well. An example is the pronoun in:

Every man owns a donkey which sees him.

The pronoun is not reflexive, because it is not the “whole predicate,” but on the most natural construal it certainly falls within the scope of its antecedent. And the two principles we have been discussing seem to hold for it. Given that sentence together with ‘*Socrates is a man*’ we can infer:

Socrates owns a donkey which sees him.

And then by the replacement principle for pronouns with singular antecedents we infer:

Socrates owns a donkey which sees Socrates.

I do not recall any medieval discussion of such examples, but they are so similar to the examples of reflexives that they do discuss, I will assume that the same techniques may be applied to them. It is also relevant that the main alternative account discussed later which is designed for non-reflexives cannot be applied coherently to these examples, since they do not have an applicable form (the antecedent is not in a categorical independent of the pronoun).

#### APPLICATIONS

Provide informal derivations for the following arguments.

At *SL I.76* Ockham says that the first of these is a good argument. Show that each argument is good using the techniques just discussed.

*Socrates sees Socrates*

*Socrates is a man*

∴ *A man sees himself*

*A donkey sees itself*

*Brownie is a donkey*

∴ *Brownie sees Brownie*

*Some donkey sees itself*

∴ *Some donkey sees a donkey*

### 8.3 Relatives in Linguish

Let me explore how to formalize this idea—that the notion of singled supposition can be captured in Linguish by already existing methods, together with the idea that a relative with a singular antecedent is equivalent to that antecedent. First, we need some way to encode the anaphoric relation between an expression and its antecedent. This is handled in contemporary linguistics by marking such pairs of expressions with indices:  $i, j, k, \dots$ , as superscripts or subscripts. I'll do this. Any denoting phrase may have one index added to it so long as that index is not used on any other denoting phrase in the sentence. A potential anaphor, such as a pronoun, that is within the scope of such a denoting phrase, can have that index added to it as a superscript; So the sentence:

*Socrates sees himself*

will be generated by taking the form:

(Socrates <sub>$\alpha$</sub> ) (it <sub>$\beta$</sub> ) <sub>$\alpha$</sub>  sees  $\beta$



and adding an index as follows:

$(\text{Socrates}_\alpha)^i (\text{it}^i_\beta)_\alpha \text{ sees}_\beta$

We also stipulate that in its transition to surface form, any pronoun agrees with its antecedent in gender, and if it appears as a main term in the same categorical proposition as its antecedent, it must take the grammatically reflexive form. So the logical form just given yields ‘*Socrates himself sees*’, or, in Latin, ‘*Sortes se videt*’.

This particular use of superscripts involves two theoretical choices: we consider pronouns themselves (such as ‘it’) as anaphors, instead of considering their whole denoting phrases (such as ‘(it<sub>β</sub>)’) as anaphors, and we consider whole denoting phrases (such as ‘(Socrates<sub>α</sub>)’) as antecedents instead of considering the terms within them (such as ‘Socrates’) as antecedents. These choices minimize confusion when a pronoun occurs both as an anaphor and an antecedent; an example is:

*Brownie is a donkey and he sees himself*

where ‘he’ is an anaphor with ‘Brownie’ as its antecedent and it is also the antecedent of the anaphor ‘himself.’ Our Linguish representation will be:<sup>5</sup>

$(\text{Brownie}_\alpha)^i [(\cdot \text{donkey}_\beta)_\alpha \text{ is}_\beta \text{ and } (\text{it}^i_\delta)^j (\text{it}^j_\varepsilon)_\delta \text{ sees}_\varepsilon]$

Of course, medieval authors always took the terms themselves as being antecedents and anaphors, but this is consistent with our notation; we need only understand that when we place a superscript on a denoting phrase this is the equivalent of taking the head noun of that denoting phrase as the antecedent discussed by the medieval authors.

We will use the letters ‘i,’ ‘j,’ ‘k,’ ‘l,’ ‘m,’ and ‘n’ as indexes. Our specific rules for adding indices to fully formed propositions are:

### Indexing propositions

Any denoting phrase may have an index added to it as a right superscript, so long as no other denoting phrase has the same index.

Any pronoun may have an index added to it as a right superscript if it is within the scope of a denoting phrase with that superscript.

### 8.3.1 The semantics of relatives that fall within the scope of their antecedents

The medieval descriptions of singled supposition did not say what their semantics is to be. We will do so by describing the effect of the indexing just described. To accomplish this we stipulate that when an indexed antecedent is changed to a temporary name in its semantic analysis, any pronoun anaphorically co-indexed with that antecedent is

<sup>5</sup> The use of square brackets in this example will be discussed shortly.

to be replaced by a copy of the same name, and the indices are dropped. An example with 'every':

Semantic rules for indexed denoting phrases:

If T is a common term that supposit for something, then ' $\tau_i(\text{every } T \alpha)^j \phi$ ' is  $\text{true}_\sigma$  iff for every thing a that T supposit for,  $\tau_i(\$k \alpha) \phi^*$  is  $\text{true}_{\sigma[\$k/a]}$ , where  $\phi^*$  is got from  $\phi$  by replacing every occurrence of 'it'<sup>j</sup> by '\$k'.

If T does not supposit for anything, ' $\tau_i(\text{every } T \alpha)^j \phi$ ' is  $\text{true}_\sigma$  iff ' $\tau_i(\$k \alpha) \phi^*$ ' is  $\text{true}_{\sigma[\$k/-]}$ .

and similarly for denoting phrases with other quantifier signs.

As a result, the sentence '*Socrates sees himself*' has the following truth conditions with respect to an arbitrary assignment  $\sigma$ :

'(Socrates  $\alpha$ )<sup>i</sup> (it<sup>i</sup>  $\beta$ )  $\alpha$  sees  $\beta$ ' is  $\text{true}_\sigma$   
 iff  
 '(\$1  $\alpha$ ) (\$1  $\beta$ )  $\alpha$  sees  $\beta$ ' is  $\text{true}_{\sigma[\$1/s]}$  <where s is Socrates>  
 iff  
 'sees' holds of  $\langle \sigma[\$1/s](\$1), \sigma[\$1/s](\$1) \rangle$   
 iff  
 'sees' holds of  $\langle s, s \rangle$   
 iff  
 s sees s

The same device works when the reflexive pronoun is possessive, as in '*Socrates sees his donkey*'.<sup>6</sup>

'(Socrates  $\alpha$ )<sup>i</sup> (it<sup>i</sup>  $\beta$ ) ( $\cdot$  donkey-poss- $\beta$   $\gamma$ )  $\alpha$  sees  $\gamma$ ' is  $\text{true}_\sigma$   
 iff  
 '(\$1  $\alpha$ ) (\$1  $\beta$ ) ( $\cdot$  donkey-poss- $\beta$   $\gamma$ )  $\alpha$  sees  $\gamma$ ' is  $\text{true}_{\sigma[\$1/s]}$  <where s is Socrates>  
 iff  
 for some donkey d had by  $\sigma[\$1/s](\$1)$ , '(\$1  $\alpha$ ) (\$2  $\gamma$ )  $\alpha$  sees  $\gamma$ ' is  $\text{true}_{\sigma[\$1/s][\$2/d]}$ <sup>7</sup>  
 iff  
 for some donkey d had by s, 'sees' holds of  $\langle \sigma[\$1/s][\$2/d](\$1), \sigma[\$1/s][\$2/d](\$2) \rangle$   
 iff  
 for some donkey d had by s, s sees d

<sup>6</sup> There is a complication here to be taken care of somehow. In Latin, the possessive would be an adjective, which would typically follow the possessed in surface order. But the Linguish logical form requires that the term for the possessor precede the term for the possessed, since the possessor term binds the extra variable in the possessed term. This requires a special provision in the generation of the surface Latin from the logical form that has not been worked out here.

<sup>7</sup> This derivation cheats at this point by assuming that 'donkey-poss- $\alpha$ ' is not empty when Socrates is assigned to '\$1'. Cheating can be avoided by a slightly more complicated derivation which arrives at the same conclusion.

## 8.3.2 Rules of inference for indexed expressions

The main innovation required in rules of inference to handle singled supposition is one that is so obvious it almost escapes notice. This is the principle that when a term is instantiated, if it is the antecedent of a pronoun, the term that it is instantiated to receives the index of the instantiated term, and thus it becomes the new antecedent of the pronoun. For example, in the inference:

*Every donkey sees a horse that sees it.*  
*Brownie is a donkey*  
 $\therefore$  *Brownie sees a horse that sees it.*

if ‘donkey’ is the antecedent of the pronoun ‘it’ in the first premise, then ‘Brownie’ is the antecedent of that pronoun in the conclusion. The inference takes the form:

$(\text{Every donkey } \alpha)^i (\text{a } \{\text{horse which } \gamma (\text{it}^i \delta) \gamma \text{ sees } \delta\} \beta) \alpha \text{ sees } \beta$   
 $(\text{Brownie } \alpha) (\cdot \text{donkey } \beta) \alpha \text{ is } \beta$   
 $\therefore (\text{Brownie } \alpha)^i (\text{a } \{\text{horse which } \gamma (\text{it}^i \delta) \gamma \text{ sees } \delta\} \beta) \alpha \text{ sees } \beta$  by rule UA

Although this seems almost trivial, it needs to be made explicit when we specify our rules of inference. A similar pattern occurs with the quantifier equipollences. This is an example of a quantifier equipollence:

*Not every donkey sees a horse that sees it.*  
*Some donkey doesn't see a horse that sees it.*

When ‘not every donkey’ turns into ‘some donkey not,’ if the original antecedent of ‘it’ is ‘every donkey,’ the resulting antecedent is ‘some donkey.’

$\text{not } (\text{every donkey } \alpha)^i (\cdot \{\text{horse which } \gamma (\text{it}^i \delta) \gamma \text{ sees } \delta\} \beta) \alpha \text{ sees } \beta$   
 $(\text{some donkey } \alpha)^i \text{not } (\cdot \{\text{horse which } \gamma (\text{it}^i \delta) \gamma \text{ sees } \delta\} \beta) \alpha \text{ sees } \beta$

Here are several rules from Chapter 4 with indices added:

**Quantifier equipollences:**

$(\text{every } T \alpha)^i \approx (\text{no } T \alpha)^i \text{not} \approx \text{not } (\text{some } T \alpha)^i \text{not}$   
 etc.

**Singular terms permute:**

$(t \beta)^i (\text{quant } T \alpha)^j \approx (\text{quant } T \alpha)^j (t \beta)^i$ , with **quant: every, some, no**  
 etc.

EX  $(\text{some } T \alpha)^i \phi$   
 $\langle T \text{ is non-empty} \rangle$

$\therefore (\mathbf{n} \alpha)^i \phi$

$\therefore (\mathbf{n} \alpha) (\text{some } T \beta) \alpha \text{ is } \beta$

where **n** is a name that does not already occur in the derivation

ES  $(n_\alpha)^i \phi$   
 $(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$   
 $\therefore (\text{some } T_\alpha)^i \phi$  where 'n' is any singular term

**Non-emptiness:**

$(\text{some } T_\alpha)^i \phi$   
 $\therefore \langle T \text{ is non-empty} \rangle$  if ' $(\text{some } T_\alpha)^i \phi$ ' is affirmative  
 etc.

**Subalternation**

$(\text{every } T_\alpha)^i \phi$   $(\text{no } T_\alpha)^i \phi$   
 $\therefore (\text{some } T_\alpha)^i \phi$   $\therefore (\text{some } T_\alpha)^i \text{ not } \phi$

**Substitutivity of identity:**

$(n_\alpha)(m_\beta)_\alpha \text{ is } \beta$   
 $(n_\alpha)^i \phi$   
 $\therefore (m_\alpha)^i \phi$

**Universal application [derived rule]**

$(\text{every } T_\alpha)^i \phi$   $(\text{no } T_\alpha)^i \phi$   
 $(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$   $(n_\alpha)(\text{some } T_\beta)_\alpha \text{ is } \beta$   
 $\therefore (n_\alpha)^i \phi$   $\therefore \text{not } (n_\alpha)^i \phi$

**Interchange of indices** (this may already be a derived rule)

Any proposition is equivalent to the result of replacing any index in it (in all its occurrences) by an index that does not occur in it.

And here is an example from Chapter 5 of a case in which a relative pronoun is itself an antecedent:

$(\text{every } \{ \text{donkey which}_\gamma^i (\text{it}^i \delta)_\gamma \text{ sees } \delta \}_\beta)_\beta \text{ is grey}$   
*Every donkey which sees itself is grey*

**RelClause:**

$(n_\alpha)(\cdot \{ T \text{ which}_\gamma^i \phi \}_\beta)_\alpha \text{ is } \beta$   
 $\therefore (n_\alpha)(\cdot T_\beta)_\alpha \text{ is } \beta$   
 $\therefore (n_\gamma)^i \phi$   
 $(n_\alpha)(\cdot T_\beta)_\alpha \text{ is } \beta$   
 $(n_\gamma)^i \phi$   
 $\therefore (n_\alpha)(\cdot \{ T \text{ which}_\gamma^i \phi \}_\beta)_\alpha \text{ is } \beta$

Finally, we need to officially adopt the special rule of inference that lets relatives with singular antecedents be replaced by those antecedents, and vice versa:

**Replacement rule for Singular Antecedents:**

Suppose that  $\phi$  contains a relative of the form '(it<sup>i</sup> <sub>$\gamma$</sub> )' and no denoting phrase in  $\phi$  is indexed with 'i'. Then '(t <sub>$\alpha$</sub> )<sup>i</sup>  $\phi$ ' is equivalent to '(t <sub>$\alpha$</sub> )  $\phi^*$ ', where ' $\phi^*$ ' is the result of replacing any '(it<sup>i</sup> <sub>$\gamma$</sub> )' in ' $\phi$ ' by '(t <sub>$\gamma$</sub> )'.

It is straightforward to see that the examples of good inferences involving reflexive relatives of identity given in the passages quoted earlier are validated by these rules. An example is 'Every man sees himself; therefore, Socrates sees Socrates.' This inference (with the added premise that Socrates is a man) would go:

- |    |   |                        |
|----|---|------------------------|
| 1. | (Every man $\alpha$ ) <sup>i</sup> (it <sup>i</sup> <sub><math>\beta</math></sub> ) $\alpha$ sees $\beta$ |                        |
| 2. | (Socrates $\alpha$ )(· man $\beta$ ) $\alpha$ is $\beta$  |                        |
| 3. | (Socrates $\alpha$ ) <sup>i</sup> (it <sup>i</sup> <sub><math>\beta</math></sub> ) $\alpha$ sees $\beta$  | 1 2 UA                 |
| 4. | (Socrates $\alpha$ )(Socrates $\beta$ ) $\alpha$ sees $\beta$   | 3 Singular Antecedents |

This account also seems to work well for cases in which non-reflexives fall within the scope of their antecedents. Some examples are:

(Every man  $\alpha$ )<sup>i</sup>(· {horse which <sub>$\gamma$</sub>  (it<sup>i</sup> <sub>$\beta$</sub> )  $\gamma$  saw  $\beta$ }  $\delta$ )  $\alpha$  saw  $\delta$   
*Every man saw a horse which saw him*

(Every {man which <sub>$\epsilon$</sub>  (· {horse which <sub>$\gamma$</sub>  (it<sup>i</sup> <sub>$\delta$</sub> )  $\gamma$  saw  $\delta$ }  $\beta$ )  $\epsilon$  saw  $\beta$ }  $\alpha$ )  $\alpha$  is running  
*Every man who saw a horse which saw him is running*

(note that the antecedent of 'it' in the second example is 'which <sub>$\epsilon$</sub> ', which is considered to be a denoting phrase for present purposes.)

Here is another example; a statement of transitivity of the exceeds relation:

(Every number  $\alpha$ )<sup>i</sup>(every {number which <sub>$\gamma$</sub>  (· {number which <sub>$\eta$</sub>  (it<sup>i</sup> <sub>$\epsilon$</sub> )  $\epsilon$  exceeds  $\eta$ }  $\delta$ )  $\delta$  exceeds  $\gamma$ }  $\beta$ )  $\alpha$  exceeds  $\beta$   
*Every number exceeds every number which a number which it exceeds exceeds.*

**APPLICATIONS**

Produce the Linguish logical forms that generate these sentences and provide formal derivations for the following arguments.

(At SL I.76 Ockham says that the first of these is a good argument.)

- |                                    |                               |
|------------------------------------|-------------------------------|
| <i>Socrates sees Socrates</i>      | <i>A donkey sees itself</i>   |
| <i>Socrates is a man</i>           | <i>Brownie is a donkey</i>    |
| ∴ <i>A man sees himself</i>        | ∴ <i>Brownie sees Brownie</i> |
| <i>Some donkey sees itself</i>     |                               |
| ∴ <i>Some donkey sees a donkey</i> |                               |

We now turn to the cases in which the pronoun is (at least apparently) not within the scope of its antecedent.

## 8.4 Non-reflexive relatives of identity

Most medieval theorizing deals with non-reflexive relatives of identity. The standard theory of non-reflexive relatives gives content to relative terms of identity. It identifies the mode of supposition of such a term as being the same as that of its antecedent. And the theory also identifies what the relative supposits for. Sometimes it seems that a non-reflexive relative term is thought to supposit for exactly what its antecedent supposits for. Peter of Spain says:

On the relative of identity the following rule is given:

every non-reflexive relative of identity should have the same supposition that its antecedent has.

For example, when someone says ‘every man runs, and he is Sortes,’ the relative ‘he’ supposes for every man, and the meaning is ‘he is Sortes’ or ‘every man is Sortes.’ (LS 8.15)

This seems pretty clear. And on this account non-reflexive relatives can be paraphrased in terms of their antecedent coupled with whatever quantifier sign is needed to give it the same mode of supposition as the antecedent. The sentence ‘*every man is running and he is Socrates*’ does indeed seem to be logically equivalent to saying that every man is running and every man is Socrates. So the stated account at least has the intended truth conditions in this case. However, this simple account will not work in other cases. This is pointed out in several places, such as this comment by Walter Burley (already quoted):

You need to know that it is not always permissible to put the antecedent in place of the relative. For saying ‘A man runs and he argues’ is not the same as saying ‘A man runs and a man argues,’ because for the truth of ‘A man runs and a man argues’ it suffices that one man runs and another one argues. (PAL, longer treatise, para. 117 (112))<sup>8</sup>

A modification of the theory is needed. Buridan SD 4.4.3 states a theory that he suggests is already widely held:

The universal rule is that a relative term of identity need not supposit or stand in a proposition for all those things which its antecedent supposits or stands for. On the contrary, the relative term refers back to its antecedent only with respect to those of the antecedent’s supposita for

<sup>8</sup> Peter himself seems to know about this example and to agree with Burley’s point. In LS 8.6 (165) he says: “From what was said it is clear that all relatives of identity recall a substance identical with its antecedent and that they recall and stand for the numerically same thing. It is also clear from this, that more certainty is effected by a relative of identity than by its antecedent substituted for a relative. In ‘a man is running, a man is debating,’ it is dubious whether the same man is spoken of or not. But in saying: ‘a man is running and that one is debating,’ it is certain that the same man is spoken of.”

Ockham SL I.76 uses the same example, while making a sweeping claim: “when the antecedent of a relative is a common term with personal supposition, one can never, by substituting the antecedent for the relative, generate a proposition which is convertible with and equivalent to the original proposition. Thus, ‘A man runs and he disputes’ and ‘A man runs and a man disputes’ are not equivalent.”

which the categorical proposition in which the antecedent occurred was verified; hence, the proposition 'An animal is a man and he is a donkey' is false.<sup>9</sup>

From this rule it is immediately evident that a relative term cannot be more amplified than its antecedent is amplified. Thus, if I say 'A man runs and he disputed,' the relative term 'he' supposits only for present[ly existing men], just as 'man' does. But [the relative term] can be more restricted, for if I say 'A man runs and he is white,' then the antecedent 'man' supposits for all men, although indefinitely, but the relative term 'he' does not so supposit for all men, but only for those for which the proposition 'A man runs' is true, and if it were true for none, then the relative term would supposit for nothing, whence both members of the previous conjunctive proposition would be false if no man were running, even if all men were white.

Marsilius of Inghen *TPT* 1 (73, 75) makes pretty much the same point.<sup>10</sup> He then goes on to clarify:

A relative does not necessarily supposit for as many instances as its antecedent, . . . For it does not have supposition in a proposition other than for those instances for which its antecedent is verified in the proposition in which it is used, whereas an antecedent often supposits for many more instances. As in the case *an ass is an animal*, the term *animal* stands distributively<sup>11</sup> for all animals, but is verified only of brayers.

So the relative is supposed to supposit for only those animals for which the proposition containing the antecedent is verified. Hülsen 1994 and 2000 (43) points out that this is almost exactly the same as a proposal put forward in the 20th century by Gareth Evans 1977 (499) in his discussion of what became known as E-type pronouns. Evans says: "Roughly, the pronoun denotes those objects which *verify* (or that object which verifies) the sentence containing the quantifier antecedent." This has been a very influential idea in the last few decades, though there is no agreement on its details.

In order to implement this idea, we need to be clear about which things a term is verified for in a proposition. In the simple cases discussed here, it appears that this may be characterized as follows: a main term 'P' in a proposition '. . . P . . .' is verified for exactly those things that the complex term 'P which . . . . .' supposits for (dropping any quantifier sign in front of 'P'). For example, in:

<sup>9</sup> Both Burley (*PAL*, longer treatise, Chapter 4, para. 117) and Ockham (*SL* I.76, 218) also speak of relatives being limited to what their antecedents are verified for, but it is unclear to me exactly what they are saying.

<sup>10</sup> "A relative of identity does not have supposition unless its antecedent is verified concerning something. . . . otherwise it would be true to say *an ass is an animal and that is a man*, because if *that* were to stand for animals other than for those concerning which *animal* is verified in the proposition *an ass is an animal*, then it would stand for man, because it would stand for all animals; therefore the subject and the predicate of the proposition *and that is a man* would stand for the same thing. Therefore the proposition would be true. . . . [but] on hearing the [original] proposition, it is understood that the same animal that is an ass is a man. . . . From this the corollary is inferred: it does not follow that, if an antecedent, used instead of a relative, were to cause a proposition to be true, a relative used in the same place, would also cause it to be true. For whilst it is true to say *an ass is an animal and an animal is a man*, it is false to say: *an ass is an animal and the same is a man*. The reason is that the antecedent of the relative supposits for more cases than the relative itself."

<sup>11</sup> I don't know why he says "distributively." It is clear that the term has determinate supposition. (If the indefinite article is read universally, then '*animal*' still does not stand distributively; it has merely confused supposition.)

*An ass is an animal*

the term 'animal' is verified for whatever this term supposits for:

*animal which an ass is*

That is, it is verified for all those animals that are asses. So in the example '*an ass is an animal and the same is a man*' the term '*the same*' supposits for animals which are asses. In the example, that means that

*an ass is an animal and the same is a man*

is equivalent to:

*an ass is an animal and an animal which an ass is is a man*

(where the 'an' preceding 'animal' in the underlined passage is there to cause the relative to have determinate supposition, as does the antecedent of '*the same*'). This seems to have the right truth conditions. (If this isn't obvious, note that the second conjunct of the lower sentence is logically equivalent to the whole upper sentence, and the first conjunct of the lower sentence is redundant.)

Can the theory be extended to other sorts of examples? Burley discusses the proposition

*A man argues and his ass runs*

The theory under discussion would replace the relative '*his*' with a term suppositing for those things that the term 'man' is verified for in the first conjunct. This yields (changing 'which' to 'who'):

*A man argues and an ass of a man who argues runs*

This too seems to be logically equivalent to the proposition under discussion.

It turns out that this example is typical, in the sense that the vast majority of examples of non-reflexive relatives of identity discussed by writers have this syntactic form: a conjunction of two simple categorical propositions in which a relative in the second conjunct has a quantified antecedent in the first conjunct.

Some forms of this sort are a bit odd, though they were taken to be coherent by medieval writers. Here are a few natural combinations each followed by a paraphrase equivalent to what is yielded by the analysis of relatives under discussion:

*A man sees a donkey and he speaks*

*A man sees a donkey and a man who sees a donkey speaks*

*A man sees a donkey and it is grey*

*A man sees a donkey and a donkey which a man sees is grey*

*A man sees every donkey and he speaks*

*A man sees every donkey and a man who sees every donkey speaks*



*A man sees every donkey and it is grey*

*A man sees every donkey and every donkey that a man sees is grey*

*Every man sees every donkey and he speaks*

*Every man sees every donkey and every man who sees every donkey speaks*

*Every man sees every donkey and it is grey*

*Every man sees every donkey and every donkey which every man sees is grey*

*Every man sees a donkey and he speaks*

*Every man sees a donkey and every man who sees a donkey speaks*

*Every man sees a donkey and it is grey*

*Every man sees a donkey and a donkey which every man sees is grey*

It is easy to tell by inspection that in each of these cases the paraphrase is what is intended as the analysis of a relative as having a mode of supposition the same as its antecedent and suppositing for the things that its antecedent is verified for. And in every case but the last, the result of the analysis is a proposition having truth conditions equivalent to the proposition containing the relative. (At least, that is my judgment.) So one can see why this approach was taken to be plausible. The last example, however, seems to me to be clearly incorrect. And it is just the tip of an iceberg.

In the second half of the 14th century a number of writers discussed relatives, and a major theme of some of their writings was the many faults in the view that relatives supposit for the same things as their antecedents. These writers also go on to find fault with the revised view that we have been discussing, that a non-reflexive relative supposits for whatever its antecedent is verified for. A common type of counterexample uses disjunctions or conditionals instead of conjunctions. For example:

*A man is a donkey or he is a man*<sup>12</sup>

In this example the antecedent is verified for men who are donkeys, namely, for nothing. As a result, the second disjunct contains a relative without supposition, and it is thus false. Since the first disjunct is clearly false, the whole proposition must be false on the theory in question. But it is taken to be true. Another example is:

*God is not, if he is*<sup>13</sup>

The first clause is verified for gods which are not, namely for nothing. So the 'he' in the second clause lacks supposition, and thus the second clause is false. Since that clause is the antecedent of a conditional, the whole conditional, which is the proposition in question, is true. (The proposition is true if the conditional is read "as-of-now." I think

<sup>12</sup> Bernhard Berwart, *L*, 277 in Hülsen 1994. Similar examples also appear in other works edited in Hülsen: Hugo Kym, *STR*, 301; Anonymous, *Q*, 426.

<sup>13</sup> Bernhard Berwart, *L*, 277.

it would also be true on the reading under discussion even if it were read as a strict conditional.) But it is taken to be false.

Some authors, such as Bernhard Berwart, conclude that the principle that a non-reflexive relative supposits for those things that the antecedent is verified for:

should be understood to be about relatives put in conjunctions, not in disjunctions or conditionals; rather, a relative in a disjunction or in a conditional refers to its antecedent without restriction. (*L*, 278)<sup>14</sup>

With this provision, ‘he’ in the first example supposits for men in general, and ‘he’ in the second example supposits for God, and both examples turn out to have the intended truth values. And this seems to have evolved into a general view, for it is endorsed by one author over a century later.<sup>15</sup> But this success does not extend to other examples. For example, the ‘it’ in:

*Every donkey is grey or it is not grey*

would supposit for every donkey, and would have distributed supposition, so the proposition would be equivalent to:

*Every donkey is grey or every donkey is not grey*

This is false, though I think that the original proposition would be taken to be true. And:

*Every animal is grey if it is a donkey*

would be equivalent to:

*Every animal is grey if every animal is a donkey*

But this is true—because it has a false antecedent—whereas the original proposition is false.

I think that some of these counterexamples might be got around, at least by tinkering with the details of the theory. But I don’t think it can be saved in the end. For consider this example:

*Every thing is a being, and either it is God or it is created by God*

where the antecedent of each ‘it’ is ‘thing.’ The basic theory would yield truth conditions something like:

<sup>14</sup> A similar diagnosis appears in Hugo Kym, *STR*, 301.

<sup>15</sup> Broadie 1985 (74) quotes a discussion by David Cranston (early 16th century) which states this generalization, concluding: “in the affirmative conjunction it is not permissible, in place of the relative, to put its antecedent without restriction, but . . . it is permissible in the case of the affirmative disjunction, the negative conjunction, and the conditional.”

*Every thing is a being and either every thing which is a being is God or every thing which is a being is created by God*

This is false, while the original sentence is true. The problem is one of principle: by parity both 'it's' must be treated alike, and no matter what they supposit for the analysis will come out false. (Examples of this sort also cause trouble for modern attempts to treat these pronouns as E-type pronouns.)

It seems to me that the theory under discussion is not likely to have any kind of workable version. (Though I hope I am wrong.)

## 8.5 Applying the singulation theory

At this point a suggestion in Hülsen 2000 (40) is pertinent. He notes

Surprisingly, the concept of *supposition singillatim* [what I am calling singled supposition] was diagnosed only in the case of reflexive relatives. It should have appeared natural to analyze sentences such as ['A man is debating and he is running'] with its help.

I agree completely with Hülsen. I think that the theory developed earlier for reflexive relatives can handle most of the problem cases, including all of those mentioned in section 8.4. For example, a paraphrase for

*A man is a donkey or he is a man*

would be:

A man  $x^k$  is a donkey or  $x^k$  is a man

And

*Every animal is grey if it is a donkey*

would be:

Every animal  $x^k$  is grey if  $x^k$  is a donkey

which is false as desired.

In fact, there is some evidence that some medieval logicians did exactly this. We have already seen (in section 8.2) that Marsilius of Inghen illustrates what appears to be the singulation theory applied to a non-reflexive example, 'every man is an animal and he runs.' This sort of example is also illustrated by Richard Campsall *L*, Chapter 60, p. 229:

just as the antecedent in a universal affirmative proposition which is part of a conjunction supposits confusedly and distributively, so does the relative which refers to the antecedent also supposit distributively, for example, in saying thus: 'every man runs and he disputes,' for just as 'man' supposits confusedly and distributively, so does this relative 'he.' Nevertheless this ought not to be understood so that a descent is made under the antecedent for one individual and

under the relative for another, but under both for the same; for instance in saying ‘every man runs and he disputes’; therefore Socrates runs and disputes, and Plato runs and Plato disputes, and thus of the others. And this is to be understood properly and in virtue of the words, although thanks to the matter it could be done otherwise.

Indeed, another medieval logician seems, in essence, to propose as a perfectly general principle that regardless of how the mode of supposition of a relative is described, one can never descend under the relative itself. Richard Lavenham says (Spade 1974, page 98, paragraph 20, my translation):

it should be known that no relative has descent to its supposita, because it does not have supposition from its own self, but from its antecedent. And for that reason if it is asked in what way ‘he’ supposits in this proposition ‘Any man runs and he is white,’ it should be said [to supposit] confusedly and distributively. But if it is asked further in what way it has [descent], it should be said that it doesn’t have descent, because a relative lacks descent to supposita.

So although relatives in general are *described* as having the same mode of supposition as their antecedents, this is not an exact statement, for they have their modes of supposition in a kind of parasitic way. And as far as inference is concerned, you can’t descend under them at all. And this applies to non-reflexives as well as reflexives, as Lavenham’s example illustrates. I think that this approach is workable.

The theory seems relatively clear, but we have not yet seen how to implement its logical forms. For our particular idea for making non-reflexive relatives depend on their antecedents just as reflexives do requires those relatives to be within the scopes of their antecedents. So we need to face a question that was postponed from section 5.10: how can we get a term in an earlier categorical proposition to have scope over a relative that occurs in a later categorical proposition? These are examples like ones we have already seen in earlier sections:

*Every man is running and he is Socrates*  
*An ass is an animal and the same [animal] is a man.*  
*Every man is an animal and every risible is it [i.e. is the animal]*

The simplest way to give the antecedents in these examples scope over their anaphors would be to expand our conditions for generating conjunctions and disjunctions from Chapter 5 so as to form molecular formulas which contain unfilled roles. For example, in addition to generating the conjunction:

$[(\text{Every man } \alpha) \alpha \text{ runs and } (\cdot\text{farmer } \beta)(\text{Socrates } \gamma) \beta \text{ is } \gamma]$   
*Every man runs and a farmer is Socrates*

we could instead generate a structure like:

$(\text{Every man } \alpha)[\alpha \text{ runs and } (\cdot\text{farmer } \beta)(\text{Socrates } \gamma) \beta \text{ is } \gamma]$   
*Every man [runs and a farmer is Socrates]*

where the initial denoting phrase has scope over both categoricals within the brackets.

A similar example with anaphora would be:

(·man<sub>α</sub>)<sup>1</sup>[<sub>α</sub> runs and (it<sup>1</sup><sub>β</sub>)(Socrates<sub>γ</sub>)<sub>β</sub> is<sub>γ</sub>]  
*A man [runs and he is Socrates]*

This raises a problem of grammatical principle. Aren't the natural language sentences under discussion conjunctions (or disjunctions, or conditionals)? It would seem that putting a denoting phrase on the front with scope over the whole means that we do not have a conjunction; we have a denoting phrase in front of something like a conjunction. Isn't that wrong? This very problem already occurs in English with a sentence like:

*Every woman showed up and she brought a book.*

Grammatically this seems like a paradigm conjunction. But there is something odd about the second conjunct:

*she brought a book*

since it seems to have no self-contained meaning on its own. It gets its meaning from the denoting phrase on the front of the sentence. It appears perhaps that the surface syntax of the sentence is at odds with its semantics. This is an option worth considering. Linguists already take seriously the idea that there is more to logical form than surface syntactical form. For example, surface syntactical form often leaves quantifier scopes unsettled; it is common to hold that a sentence in addition to its surface form has a logical form that includes displaced quantifiers which have scope. In most of the theory we have been dealing with, medieval theorists have taken advantage of the flexible word order of Latin to make a usable syntax where logical form and grammatical form are the same. Perhaps we need to extend that idea a bit to have an adequate theory of relatives. It is easy to develop such a theory using Linguish notation. We just expand our formation rules from section 5.10 by inserting the word 'partial' as follows:

If  $\phi$  is a partial categorical proposition and  $\psi$  is a categorical proposition with no free markers, then these are partial categorical propositions:

[ $\phi$  and  $\psi$ ]

[ $\phi$  or  $\psi$ ]

[ $\phi$  if  $\psi$ ]

Our old formation rule lets us generate these "partial" categorical propositions:

<sub>α</sub> runs

(·farmer<sub>β</sub>)(Socrates<sub>γ</sub>)<sub>β</sub> is<sub>γ</sub>

Our new one lets us put them together to get this partial categorical proposition:

[<sub>α</sub> runs and (·farmer<sub>β</sub>)(Socrates<sub>γ</sub>)<sub>β</sub> is<sub>γ</sub>]

By a previous generation rule we can put a denoting phrase on the front to get our first sample sentence:

(Every man  $\alpha$ )[ $\alpha$  runs and ( $\cdot$ farmer  $\beta$ )](Socrates  $\gamma$ )  $\beta$  is  $\gamma$ ]  
*Every man runs and a farmer is Socrates*

Starting instead with the partial propositions:

$\alpha$  runs  
 (it  $\beta$ )](Socrates  $\gamma$ )  $\beta$  is  $\gamma$

we can put them together by our new provision to make:

[ $\alpha$  runs and (it  $\beta$ )](Socrates  $\gamma$ )  $\beta$  is  $\gamma$ ]

and then we add a denoting phrase to the front using our old rules:

(Every man  $\alpha$ )[ $\alpha$  runs and (it  $\beta$ )](Socrates  $\gamma$ )  $\beta$  is  $\gamma$ ]

Our rules for adding indices then lets us make:

(Every man  $\alpha$ )<sup>i</sup>[ $\alpha$  runs and (it<sup>i</sup>  $\beta$ )](Socrates  $\gamma$ )  $\beta$  is  $\gamma$ ]  
*Every man runs and he is Socrates*

in which the initial denoting phrase is the antecedent of the pronoun.

Notice that the surface sentences we are generating are indistinguishable in wording from ordinary conjunctions, which explains perhaps why their special forms would not be noticed. We may need to restrict the formation of such sentences so as to disallow the occurrences of negatives, so as not to produce:

*No donkey [is grey and it is running]*  
*A donkey not [is grey and it is running]*

This is because many people tend to see such wordings as ill formed. However, within the medieval tradition this is not completely clear. So I will simply ignore examples of this sort.<sup>16</sup>

<sup>16</sup> As stated, our generation principles may be too permissive. This is pointed out in Anonymous, *Treatise on Univocation* (340): “we also say that a [pronominal] reference cannot be made to a predicate preceded by a negative particle, a particle which is confused and which makes the predicate be held confused, as in: ‘Antichrist is not an animal, and that [animal] exists’ or ‘. . . does not exist.’” The example does seem odd, so maybe we should prohibit it. However, writers disagreed about this. For example, Lambert, *PT*, section 80, says, “when one says ‘A man is not a donkey, and he is rational,’ ‘he’ can refer to ‘man’ (in which case it is true) or to ‘donkey’ (in which case it is false). Another example concerns antecedents with ‘no.’” Later in the *Treatise on Univocation* (349), it is stated: “To the subject of a negative proposition a pronominal anaphoric reference can be made, if (the negation) is placed after the subject, as in ‘A man does not run, and he moves.’ Because if the negation is placed before, it negates the whole, and therefore a reference cannot be made to it (the subject), just as we have said of the predicate.” The quotes taken from that work seem to imply that a denoting phrase whose quantifier sign is ‘no’ cannot be an antecedent, as in ‘No man runs, and he moves.’ I am not sure how to decide which are the good cases, and which not, nor to state a proper restriction that would permit the good cases and prevent the others.

An advantage of this new formation rule is that we need no new semantic provisions at all. The semantic principles we already have apply to the new forms. However, there will have to be additional rules of inference to exploit the new forms. Two sorts of provisions come to mind. The first is that our semantics validates versions of contemporary confinement rules for affirmative denoting phrases, and we have to add rules for these:

### Confinement

The following equivalences hold whenever they are well formed:

$(t_\alpha)[\phi \text{ and } \psi]$  is equivalent to  $[(tx) \phi \text{ and } \psi]$

$(\text{some } T_\alpha)[\phi \text{ and } \psi]$  is equivalent to  $[(\text{some } T_\alpha) \phi \text{ and } \psi]$

$(\text{every } T_\alpha)[\phi \text{ and } \psi]$  is equivalent to  $[(\text{every } T_\alpha) \phi \text{ and } \psi]$

and also when the denoting phrase has a parasitic term, as in:

$(t_\alpha)(\cdot T\text{-of-}\alpha_\beta)[\phi \text{ and } \psi] \approx (t_\alpha)[(\cdot T\text{-of-}\alpha_\beta) \phi \text{ and } \psi]$

In addition these principles hold when ‘and’ is replaced by ‘or’ or ‘if’

In addition, these principles all hold when the “confined” denoting phrase bears an anaphoric index and there is no pronoun using that index in ‘ $\psi$ ’.

(The well-formedness constraint prohibits using these equivalences when the denoting phrase bears an anaphoric index that appears free in  $\psi$ .)

The other sort of rule we need concerns the conditions under which our rule for exposition applies. In order to apply that rule, we need to show that the term being instantiated is non-empty. Previously that could be shown only by its occurrence as a main term in an affirmative proposition. But now there are additional positions that guarantee non-emptiness. An example is when the term has scope over a conjunction in which it (and whatever else is outside the conjunction along with it) along with the left conjunct would make up an affirmative proposition if so assembled. An example is the term ‘donkey’ in:

$(\text{some donkey}_\alpha)^i[\alpha \text{ runs and not } (it^i_\beta)(\cdot \text{horse}_\gamma)_\beta \text{ sees } \gamma]$

*some donkey runs and it doesn't see a horse*

which occurs here as a main term, but not in an affirmative proposition. (The proposition is not defined as either affirmative or negative.) If there were no anaphora, we could apply a confinement rule and then derive the affirmative proposition:

$(\text{some donkey}_\alpha)_\alpha \text{ runs}$

in which ‘donkey’ appears as a main term. But the anaphora prevents this. So we need a provision such as

**Affirmative contexts**

If a term ‘ $\alpha$ ’ occurs as the main term of an affirmative proposition,  $\phi$ , it is in an affirmative context in  $\phi$ .

If a term ‘ $\alpha$ ’ occurs in an affirmative context in  $\phi$ , it also occurs in an affirmative context in ‘ $[\phi \text{ and } \psi]$ ’ and in ‘ $\phi^* \text{ and } \psi$ ’, where ‘ $\phi^*$ ’ consists of ‘ $\phi$ ’ with a left bracket in it that matches the bracket to the right of ‘ $\psi$ ’.

If a denoting phrase ‘ $(n_\alpha)^i$ ’ occurs in an affirmative context in  $\psi$ , then it also occurs affirmatively in ‘ $(n_\beta)^i[\phi \text{ and } \psi^*]$ ’, where ‘ $\beta$ ’ occurs in ‘ $\phi$ ’ and where ‘ $\psi^*$ ’ is the result of replacing ‘ $(n_\alpha)^i$ ’ by ‘ $(it^i_\alpha)$ ’ in ‘ $\psi$ ’.

I think that this last condition should be expanded. The point is to let an antecedent of a pronoun occur affirmatively under certain conditions if the pronoun does. Further, recall our proof in section 4.3 of the principle called substitutivity of empties; that proof relied on the assumption that every sentence is either affirmative or negative. That assumption no longer holds. As a result, that substitutivity principle may need to be posited. More work is needed here.

As a test of adequacy we can check some of the conclusions drawn earlier.

One had to do with the claim ‘*a man runs and he argues*’, about which Burley says:

In order that ‘a man runs and he argues’ be true, ‘a man runs’ has to be made true for some suppositum of ‘man’ and the second part made true for the same suppositum. (Burley, *PAL*, longer treatise, para. 118 (112))

The truth conditions for:

$(\cdot \text{man } \alpha)^i[\alpha \text{ runs and } (it^i_\beta) \beta \text{ argues}]$   
*A man [runs and he argues]*

are indeed that there is something for which ‘man’ supposits and that thing runs and that thing argues. Another conclusion was that the sentence ‘*every man is running and he is Socrates*,’ seems to be equivalent to saying that every man is running and every man is Socrates. In our Linguish notation, the logical forms are:

$(\text{Every man } \alpha)^i[(\cdot \text{running } \beta) \alpha \text{ is } \beta \text{ and } (it^i_\gamma)(\text{Socrates } \delta) \gamma \text{ is } \delta]$   
 $[(\text{Every man } \alpha)(\cdot \text{running } \beta) \alpha \text{ is } \beta \text{ and } (\text{Every man } \gamma)(\text{Socrates } \delta) \gamma \text{ is } \delta]$

It is straightforward to prove that these are equivalent. Here are derivations which show this:



1.	<u>(Every man <math>\alpha</math>)<sup>i</sup>[(· running <math>\beta</math>) <math>\alpha</math> is <math>\beta</math> and (it<sup>i</sup> <math>\gamma</math>)(Socrates <math>\delta</math>) <math>\gamma</math> is <math>\delta</math>]</u>	
2.	<u>not [(Every man <math>\alpha</math>)(· running <math>\beta</math>) <math>\alpha</math> is <math>\beta</math> and (Every man <math>\gamma</math>)(Socrates <math>\delta</math>) <math>\gamma</math> is <math>\delta</math>]</u>	
3.	not (Every man $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ or not (Every man $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ] 2 Taut	
4.	<u>not (Every man <math>\alpha</math>)(· running <math>\beta</math>) <math>\alpha</math> is <math>\beta</math></u>	
5.	(some man $\alpha$ ) not (· running $\beta$ ) $\alpha$ is $\beta$	4 Equipoll
6.	(m $\alpha$ )(· man $\beta$ ) $\alpha$ is $\beta$	1 5 EX
7.	(m $\alpha$ ) not (· running $\beta$ ) $\alpha$ is $\beta$	1 5 EX
8.	(m $\alpha$ ) <sup>i</sup> [(· running $\beta$ ) $\alpha$ is $\beta$ and (it <sup>i</sup> $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	6 1 UA
9.	(m $\alpha$ )[(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	8 Sing Ante
10.	(m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	9 Conf
11.	(m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$	10 Simp
12.	not (m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$	7 Permute
13.	(Every man $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$	11 12 Reductio
14.	not (Every man $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$	13 3 Taut
15.	(some man $\gamma$ ) not (Socrates $\delta$ ) $\gamma$ is $\delta$	14 Equipoll
16.	(n $\gamma$ )(· man $\beta$ ) $\gamma$ is $\beta$	1 15 EX
17.	(n $\gamma$ ) not (Socrates $\delta$ ) $\gamma$ is $\delta$	1 15 EX
18.	(n $\alpha$ ) <sup>i</sup> [(· running $\beta$ ) $\alpha$ is $\beta$ and (it <sup>i</sup> $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	1 16 UA
19.	(n $\alpha$ )[(· running $\beta$ ) $\alpha$ is $\beta$ and (n $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	18 Sing Ante
20.	[(n $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ and (n $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	19 Conf
21.	(n $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$	20 Simp
22.	not (n $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$	17 Permute
23.	[(Every man $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ and (Every man $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	21 22 Reductio
1.	<u>[(Every man <math>\alpha</math>)(· running <math>\beta</math>) <math>\alpha</math> is <math>\beta</math> and (Every man <math>\gamma</math>)(Socrates <math>\delta</math>) <math>\gamma</math> is <math>\delta</math>]</u>	
2.	(Every man $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$	1 Simp
3.	(Every man $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$	1 Simp
4.	<u>not (Every man <math>\alpha</math>)<sup>i</sup>[(· running <math>\beta</math>) <math>\alpha</math> is <math>\beta</math> and (it<sup>i</sup> <math>\gamma</math>)(Socrates <math>\delta</math>) <math>\gamma</math> is <math>\delta</math>]</u>	
5.	(some man $\alpha$ ) <sup>i</sup> not [(· running $\beta$ ) $\alpha$ is $\beta$ and (it <sup>i</sup> $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	4 Equipoll
6.	(m $\alpha$ )(· man $\beta$ ) $\alpha$ is $\beta$	2 5 EX
7.	(m $\alpha$ ) <sup>i</sup> not [(· running $\beta$ ) $\alpha$ is $\beta$ and (it <sup>i</sup> $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	2 5 EX
8.	(m $\alpha$ ) not [(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	7 SA
9.	not (m $\alpha$ )[(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	8 Permute
10.	not [(m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	9 Conf
11.	(m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$	2 6 UA
12.	(m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$	3 6 UA
13.	[(m $\alpha$ )(· running $\beta$ ) $\alpha$ is $\beta$ and (m $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	12 Adjoin
14.	(Every man $\alpha$ ) <sup>i</sup> [(· running $\beta$ ) $\alpha$ is $\beta$ and (it <sup>i</sup> $\gamma$ )(Socrates $\delta$ ) $\gamma$ is $\delta$ ]	10 13 Reductio

Another example—one that illustrates the importance of what the antecedent of an anaphor is verified for:

*an ass is an animal and the same is a man*

is equivalent to:

*an ass is an animal and an animal which an ass is is a man*

The (equivalent) Linguish logical forms will be:

$(\cdot \text{ass } \alpha)(\cdot \text{animal } \beta)^i [\alpha \text{ is } \beta \text{ and } (\text{it}^i \gamma)(\cdot \text{man } \delta) \gamma \text{ is } \delta]$   
 $[(\cdot \text{ass } \alpha)(\cdot \text{animal } \beta) \alpha \text{ is } \beta \text{ and } (\cdot \{\text{animal which}_\varepsilon (\cdot \text{ass } \eta) \eta \text{ is } \varepsilon\} \gamma)(\cdot \text{man } \delta) \gamma \text{ is } \delta]$

These too can be proved equivalent by our rules. Another example is the equivalence of:

*A man argues and his ass runs*

and

*A man argues and an ass of a man who argues runs*

The logical forms would be:

$(\cdot \text{man } \alpha)^i [\alpha \text{ argues and } (\text{it}^i \beta)(\cdot \text{ass-poss-}\beta \gamma) \gamma \text{ runs}]$   
 $[(\cdot \text{man } \alpha) \alpha \text{ argues and } (\cdot \{\text{man which}_\varepsilon \varepsilon \text{ argues}\} \beta)(\cdot \text{ass-poss-}\beta \gamma) \gamma \text{ runs}]$

Finally recall the problem example

*Socrates is a donkey or he is a man.* (antecedent of ‘he’ is ‘Socrates’)

It can be generated as follows:

$(\text{Socrates } \alpha)^i [(\cdot \text{donkey } \beta) \alpha \text{ is } \beta \text{ or } (\text{it}^i \gamma)(\cdot \text{man } \eta) \gamma \text{ is } \eta]$

If we apply the rule for singular antecedents we get the equivalent form:

$(\text{Socrates } \alpha) [(\cdot \text{donkey } \beta) \alpha \text{ is } \beta \text{ or } (\text{Socrates } \gamma)(\cdot \text{man } \eta) \gamma \text{ is } \eta]$

which by confinement becomes:

$[(\text{Socrates } \alpha)(\cdot \text{donkey } \beta) \alpha \text{ is } \beta \text{ or } (\text{Socrates } \gamma)(\cdot \text{man } \eta) \gamma \text{ is } \eta]$

This is clearly true, as desired.

## APPLICATIONS

Produce the Linguish logical forms that generate these sentences and provide formal derivations for the following arguments.

*Socrates owns Brownie*

*Socrates feeds Brownie*

*Socrates is a man*

*Brownie is a donkey*

∴ *A man owns a donkey and he feeds it*

*Every man who is not running is sitting*

∴ *Every man is running or he is sitting*

*Every man who is not running is sitting*

∴ *Every man is running if he is not sitting*

*Every man is Socrates or he is Plato*

*Socrates is running and Plato is running*

∴ *Every man is running*

## 8.6 An application of relatives to syllogistic

Buridan thought that relatives of identity are important because they permit one to get around certain traditional prohibitions on what kind of syllogisms may be valid. For example, a traditional rule says that a syllogism cannot be valid if neither of its premises is universal. Buridan (*SD* 5.1.8) points out that this rule does not hold when relatives of identity are present:

But we should also realize that by the same principle an affirmative syllogism would be valid even with a common middle term in every figure, if in the minor proposition we were to add a relative [pronoun] of identity, and even if neither of the premises were universal, as for example: ‘A man is running, and a white thing is that man; therefore, a white thing is running’; similarly, in the second figure: ‘A running thing is a man, and a white thing is that man; therefore, a white thing is running’; similarly, in the third figure: ‘A man is white, and that man is running; therefore, a running thing is white.’ It is clear that all these syllogisms are valid; and they should be called ‘expository’, as though their middle term were singular, for the relative [pronoun] of identity ensures that, if the premises are true, the minor is true for the same suppositum of the middle term for which the major was true, and thus the extremities are said to be the same as the middle term in respect of numerically the same thing; whence it has to be concluded that they are themselves the same.

Here is a proof of Buridan’s second example, where I understand the premise to be a “conjunction” with anaphora:

*A running thing is a man, and a white thing is that man; therefore, a white thing is running*

$(\cdot \text{running-thing } \alpha)(\cdot \text{man } \beta)^i[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(it^i \epsilon) \delta \text{ is } \epsilon]$   
 $\therefore (\cdot \text{white-thing } \alpha)(\cdot \text{running-thing } \beta) \alpha \text{ is } \beta$

1.	$(\cdot \text{running-thing } \alpha)(\cdot \text{man } \beta)^i[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(it^i \epsilon) \delta \text{ is } \epsilon]$	
2.	$(r \alpha)(\cdot \text{running-thing } \beta) \alpha \text{ is } \beta$	1 1 EX
3.	$(r \alpha)(\cdot \text{man } \beta)^i[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(it^i \epsilon) \delta \text{ is } \epsilon]$	1 1 EX
4.	$(\cdot \text{man } \beta)^i(r \alpha)[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(it^i \epsilon) \delta \text{ is } \epsilon]$	3 Permute
5.	$(m \alpha)(\cdot \text{man } \beta) \alpha \text{ is } \beta$	4 4 EX
6.	$(m \beta)^i(r \alpha)[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(it^i \epsilon) \delta \text{ is } \epsilon]$	4 4 EX
7.	$(m \beta)(r \alpha)[\alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(m \epsilon) \delta \text{ is } \epsilon]$	6 Singular antecedent
8.	$(m \beta)(r \alpha) \alpha \text{ is } \beta \text{ and } (\cdot \text{white-thing } \delta)(m \epsilon) \delta \text{ is } \epsilon$	Confinement (twice)
9.	$(m \beta)(r \alpha) \alpha \text{ is } \beta$	8 simp
10.	$(\cdot \text{white-thing } \delta)(m \epsilon) \delta \text{ is } \epsilon$	8 simp
11.	$(r \alpha)(m \beta) \alpha \text{ is } \beta$	9 Permute
12.	$(m \alpha)(\cdot \text{running-thing } \beta) \alpha \text{ is } \beta$	2 11 LL
13.	$(m \epsilon)(\cdot \text{white-thing } \delta) \delta \text{ is } \epsilon$	10 Permute
14.	$(\cdot \text{white-thing } \alpha)(\cdot \text{running-thing } \beta) \alpha \text{ is } \beta$	12 13 ES

This indeed yields a syllogism with a singular middle term, as Buridan says. Then a little bit of rearranging together with an expository syllogism yields:

*A white thing is running*

Another example from Buridan is this one:

For example, we truly say: ‘The first cause exists and it is God,’ or even ‘The almighty is not evil and he is God.’ But this is not so with ‘chimera,’ for even if we said ‘A chimera is not, and she is a chimera,’ the proposition ‘She is a chimera’ is false. (SD 4.1.2)

Of course, ‘*she is a chimera*’ is not a free-standing part of ‘*A chimera is not, and she is a chimera*’ with a truth value of its own. Buridan’s point is that the truth of ‘*A chimera is not*’ does not license one to add ‘*and she is a chimera*.’ If we do, ‘*A chimera is not, and she is a chimera*’ must be false. We can show this.

To disprove: *A chimera is not and she is a chimera*

i.e. to disprove:  $(\cdot C \alpha)^i[\text{not } \alpha \text{ is and } (it^i \delta)(\cdot C \epsilon) \delta \text{ is } \epsilon]$

<assuming that Buridan’s ‘x is’ means the same as ‘x is x’>.

1.	$(\cdot C \alpha)^i[\text{not } \alpha \text{ is and } (\text{it}^i \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon]$	
2.	$(c \alpha)^i[\text{not } \alpha \text{ is and } (\text{it}^i \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon]$	1 EX <'C' occurs affirmatively>
3.	$(c \alpha)[\text{not } \alpha \text{ is and } (c \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon]$	2 Singular antecedent
4.	$[(c \alpha) \text{ not } \alpha \text{ is and } (c \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon]$	3 Confinement
5.	$(c \alpha) \text{ not } \alpha \text{ is}$	4 Simp
6.	$(c \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon$	4 Simp
7.	$(c \alpha)(c \beta) \alpha \text{ is } \beta$	6 is affirmative
8.	$\text{not } (c \alpha) \alpha \text{ is}$	5 Permutation
9.	$\text{not } (c \alpha)(c \beta) \alpha \text{ is } \beta$	8 <assured equivalence>
10.	$\text{not } (\cdot C \alpha)^i[\text{not } \alpha \text{ is and } (\text{it}^i \delta)(\cdot C \varepsilon) \delta \text{ is } \varepsilon]$	7 9 reductio

## APPLICATIONS

Produce the Linguish logical forms that generate these sentences and provide formal derivations for the following arguments.

- A man owns a donkey and it is running*  
*Every donkey is an animal*  
 $\therefore$  *An animal is running*
- Every man owns a donkey and it is not running*  
 $\therefore$  *Not every donkey is running*
- Every man sees a donkey*  
*Every donkey sees a horse*  
*Every donkey is an animal*  
 $\therefore$  *Every man sees an animal and it sees a horse*

## 8.7 Donkey anaphora

The theory that we have been discussing deals with relatives of identity that occur within the scopes of their antecedents. But there are grammatically well-formed sentences in which a relative does not lie within the scope of its antecedent. (These were the kinds of cases that Evans' discussion was aimed at.) Here is a case that was considered by Walter Burley in the early 14th century (*PAL*, longer treatise, para. 130–2):

*Every man having an ass sees it*

In the 20th century this example was changed by Peter Geach (1962, 117) into:

*Any man who owns a donkey beats it*

He discussed this example in Geach 1962, and in this donkey-beating form it entered the field of modern linguistics as a problematic kind of construction. For years the problem was referred to as the problem of “donkey sentences.” It is now called “unbound anaphora,” because it deals with pronouns which apparently cannot be bound by their antecedents (because they do not fall within the scopes of their antecedents). I think it is fair to say that there is no consensus on how to handle the semantics of such pronouns. So it will be no surprise if medieval accounts fall short. I will confine myself to discussing the views of Walter Burley on this matter.

Upon inspection it is clear that the methods given so far do not generate this sentence with a reading in which ‘*it*’ has ‘*ass*’ as its antecedent. This is because ‘*man having an ass*’ is a complex common term which occurs totally within the subject of the sentence. So ‘*it*’ cannot be within the scope of ‘*ass*.’ What is to be done?

There is a problem in understanding this sentence. What does the sentence have to say about men who own more than one donkey? Most people (not all; and not Burley) take it that for the sentence to be true, any man having many donkeys must see every one of them. There is no argument for this (or against it); it is just a matter of how the sentence is understood in natural language. However, this view is undercut when we consider other sentences with the same form, where the intuitions go differently. A standard example is:

*Everyone who had a credit card used it.*

Hardly anyone thinks that this sentence requires for its truth that everyone who had more than one credit card used them all. The sentence only requires that everyone with at least one credit card used at least one of his/her credit cards.

Burley understood his sentence in the credit-card sense, to require only that men with multiple donkeys must see at least one of them. He took this to be problematic because he observed (*PAL*, longer treatise, para. 128–32) that on that understanding the following two sentences apparently do *not* contradict one another:

*Every man having an ass sees it*

*Some man having an ass does not see it*

If he is right, this means that the fundamental equipollences discussed in Chapter 3 do not hold for such sentences. So the significance for formulating a general theory of logic is substantial.

The problem in detail: Burley thinks that the sentences can both be true if some man owns more than one ass and sees one without seeing another. And that is the case if the second sentence requires only that some man not see one of his donkeys. But consider a credit-card variant:

*Someone who had a credit card didn't use it*

This might naturally be understood as meaning that someone who had one or more credit cards didn't use any credit card. Burley's example might be understood in this

way too. It appears to me, then, that there are two ways to understand each of these sentences, and so Burley's problematic pair of sentences can be read four ways altogether. Two ways make them contradictory and two don't, so Burley's problem about contradictoriness is not forced on us.

I do not want to try to decide who is right here. Instead I will discuss one possible way to handle these sentences that validates Burley's point of view regarding the first sentence. This *may* be one of those examples in which the theory of relatives gives the right answer exactly as it was articulated by medieval authors. On this view, the '*it*' in

*Every man who has an ass sees it*

is supposed to have the same mode of supposition as its antecedent, and it supposit for those things that its antecedent supposit for, for which the previous clause is verified. This means that it supposit for asses seen by the man in question. Now the previous clause is a relative clause, which has independent meaning only if the relation of the relative pronoun to the man is handled in some way. If the '*who*' indicates anaphora to the man, then it appears that the sentence will have the truth conditions of:

*Every man who has an ass sees an ass that he has*

If the '*he*' is anaphorically linked to the term '*man who has an ass*' then this gives exactly the truth conditions that Burley thinks it has. (When this account is applied to the example '*Some man having an ass does not see it*' there are different ways to take the scope of the negation, and so the sentence is structurally ambiguous.)

My analysis of this example is somewhat ad hoc (for example, in the choice of the anaphora for '*he*') and it is not at all easy to spell this out in a systematic way. Nor do I know how to generalize this case.

Many questions are now left open. There is no really satisfactory theory of all relatives, either in the medieval period or today, and I can't improve on that. Perhaps a question to keep in mind is whether there is any theoretically unavoidable need for purposes of logic for pronouns of natural language which do not occur within the scopes of their antecedents. The next chapter will suggest that there is not.

## 8.8 Common term relatives of identity and diversity

Two very useful common relative terms are '*same*' and '*other*.' When used as relatives,<sup>17</sup> they have antecedents. The first supposit for whatever its antecedent supposit for, and the second for whatever its antecedent does not supposit for—though these are rough statements, because of the unclarity as to what an antecedent supposit for. A general account that seems to work well is:

<sup>17</sup> Words like '*same*' and '*other*' can occur not as relatives, as in '*Socrates is other than Plato*.' These uses are not under discussion here. (In the case of a related word, '*differ*,' there is a well-established treatment of it as an *exponible* term. In particular, a proposition of the form '*A differs from B*' is to be expounded in terms of a conjunction of three propositions: '*A is, and B is, and A is not B*,' with it being controversial whether the second conjunct should be present. Cf. Marsilius of Inghen, *TPT*, *Suppositions* (85).)

‘same’<sup>b</sup> supposit<sub>σ</sub> for  $\sigma(\$k)$ , where ‘ $(\$k)_i$ ’<sup>b</sup> is the antecedent  
 ‘other’<sup>b</sup> supposit<sub>σ</sub> for everything that is not  $\sigma(\$k)$ , where ‘ $(\$k)_i$ ’<sup>b</sup> is the antecedent

An example of the use of ‘other’ is:

*No number’s successor is some other number’s successor*

*Of-no number<sup>i</sup> a successor some successor of-some other<sup>i</sup> number is*

(no number<sub>α</sub>)<sup>i</sup>(·successor-of-<sub>α</sub> δ)(some {successor-of-<sub>β</sub> (some {other<sup>i</sup> number} β)}  
 γ) δ is γ

Such words appear as independent terms, or modifying other common terms. When used independently they can be taken to be equivalent to headless relative clauses:

‘same’<sup>k</sup> is equivalent to ‘{which<sub>γ</sub> (it<sup>k</sup> δ) γ is δ}’

‘other’<sup>k</sup> is equivalent to ‘{which<sub>γ</sub> (it<sup>k</sup> δ) not γ is δ}’

When used to modify common terms, they also seem to be equivalent to constructions with relative clauses:

‘same’<sup>k</sup> T’ is equivalent to ‘{T which<sub>γ</sub> (it<sup>k</sup> δ) γ is δ}’

‘other’<sup>k</sup> T’ is equivalent to ‘{T which<sub>γ</sub> (it<sup>k</sup> δ) not γ is δ}’

With these equivalences, no additional rules of inference are necessary. An illustration of their use may be based on another example of Buridan’s. In his *TC* there is a passage parallel to the one cited earlier, concerning how syllogisms with relative terms are exempt from certain general principles governing syllogisms.

Sixth Conclusion: no syllogism is valid in which the middle is distributed in neither premise, unless the middle is used in the minor with a relative of identity.

For the rules by which syllogisms hold require that if the middle is a general [term] the extremes are linked by reason of the same thing for which this general term supposit, as explained earlier. Since the middle is not distributed in either [premise] it is possible that its conjunction with the major extreme is true for one thing and its conjunction with the minor is true for another; and from this no conjunction of the extremes with one another can be inferred unless the middle is brought together by a relative of identity to hold for the same thing in the minor premise as that for which it was verified in the major. But then the syllogism is valid, and clearly holds by the rules given above, and it is effectively an expository syllogism; for example, ‘Some B is an A and a C is the same B, so some C is A.’ So such syllogisms hold in all moods in which expository syllogisms hold. (*TC* 89–90)

The relevant syllogism is:

Some B is an A and a C is the same B, so some C is an A

In Linguish notation:

(some B<sub>α</sub>)<sup>i</sup>[(·A β)<sub>α</sub> is β and (·C δ)(·same<sup>i</sup> B ε) δ is ε]

∴ (some C<sub>α</sub>)(·A β)<sub>α</sub> is β



1.	$(\text{some } B_\alpha)^i[(\cdot A_\beta)_\alpha \text{ is } \beta \text{ and } (\cdot C_\delta)(\cdot \text{same}^i B_\varepsilon)_\delta \text{ is } \varepsilon]$	
2.	$(b_\alpha)(\cdot B_\beta)_\alpha \text{ is } \beta$	1 EX
3.	$(b_\alpha)^i[(\cdot A_\beta)_\alpha \text{ is } \beta \text{ and } (\cdot C_\delta)(\cdot \text{same}^i B_\varepsilon)_\delta \text{ is } \varepsilon]$	1 EX
4.	$(b_\alpha)^i[(\cdot A_\beta)_\alpha \text{ is } \beta \text{ and } (\cdot C_\delta)(\cdot \{B \text{ which}_\gamma (it^i)_\gamma \text{ is } \eta\}_\varepsilon)_\delta \text{ is } \varepsilon]$	3 rule 'Same'
5.	$(b_\alpha)[(\cdot A_\beta)_\alpha \text{ is } \beta \text{ and } (\cdot C_\delta)(\cdot \{B \text{ which}_\gamma (b_\eta)_\gamma \text{ is } \eta\}_\varepsilon)_\delta \text{ is } \varepsilon]$	4 Sing Ante
6.	$[(b_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta \text{ and } (\cdot C_\delta)(\cdot \{B \text{ which}_\gamma (b_\eta)_\gamma \text{ is } \eta\}_\varepsilon)_\delta \text{ is } \varepsilon]$	5 Conf
7.	$(b_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta$	6 Simp
8.	$(\cdot A_\beta)(b_\alpha)_\alpha \text{ is } \beta$	7 permute
9.	$(a_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta$	8 EX
10.	$(a_\beta)(b_\alpha)_\alpha \text{ is } \beta$	8 EX
11.	$(\cdot C_\delta)(\cdot \{B \text{ which}_\gamma (b_\eta)_\gamma \text{ is } \eta\}_\varepsilon)_\delta \text{ is } \varepsilon$	6 Simp
12.	$(c_\alpha)(\cdot C_\delta)_\delta \text{ is } \alpha$	11 EX
13.	$(c_\delta)(\cdot \{B \text{ which}_\gamma (b_\eta)_\gamma \text{ is } \eta\}_\varepsilon)_\delta \text{ is } \varepsilon$	11 EX
14.	$(c_\delta)(\cdot B_\varepsilon)_\delta \text{ is } \varepsilon$	13 RelClause
15.	$(c_\gamma)(b_\eta)_\gamma \text{ is } \eta$	13 RelClause
16.	$(c_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta$	7 15 LL
17.	$(\text{some } \cdot C_\alpha)(\cdot A_\beta)_\alpha \text{ is } \beta$	12 16 ES

The proof is, as Buridan says, “effectively an expository syllogism,” in the sense that that is the form of the major (last) inference in it.

Additional illustrations would be helpful. Some are given in section 9.5 on first-order arithmetic.

APPLICATIONS

Produce the Linguish logical forms that generate these sentences and provide formal derivations for the following arguments.

- A man and another man are running*
- ∴ *No man is every man*
- A donkey is sitting*
- A donkey is running*
- No donkey which is sitting is running*
- ∴ *A donkey is sitting and another donkey is running*
- No number’s successor is another number’s successor*
- Zero is a number and one is another number*
- ∴ *Zero’s successor is not one’s successor*

# 9

## Comparison of Medieval Logic with Contemporary Logic

The goal of this chapter is to explore the logical potential of medieval logic. Since medieval logic is couched in a regimented form of natural language, it is also a good test case for exploring the logical potential of natural language. A crucial step in Frege's 1879 effort to provide a logical foundation for arithmetic was to set aside natural language and to use an invented language instead, a *conceptual notation*. Clearly his notation is more useful than natural language for this task. But it is not clear—to me, anyway—whether it is essential, or only useful, to dispense with natural language in this way. The purpose of this chapter is to evaluate the expressive power of the natural language based systems of logic that were developed in medieval times.

In section 9.1, I discuss the expressive power of medieval logic as developed in previous chapters. Without anaphoric pronouns, the system is logically weak (it is probably decidable); with anaphoric pronouns it seems to have a power similar to that of modern logic. Section 9.2 discusses how to represent medieval logic within modern logic; this is easy, though somewhat clumsy. The next two sections discuss how to represent modern logic within medieval logic. In 9.3 problems of existential import are discussed and dealt with, and in 9.4 problems due to the restrictive grammar of medieval logic are discussed, and an algorithm is given for representing modern logic within medieval logic. The method is artificial, but it works. Finally, in section 9.5 we consider what it would be like to use medieval logic in a natural way to formulate a straightforward modern theory—first-order arithmetic.

### 9.1 The expressive power of medieval logic

The simplest system of logic that we looked at (Chapter 1) was Aristotle's, confined to the four standard forms of categorical propositions. This is pretty much the system that is taught today in sections of texts on "Aristotelian" logic (though often with altered truth conditions for the propositions, to make them conform to texts on symbolic logic). This is equivalent to a very simple fragment of monadic predicate logic. It is decidable by simple methods such as the use of Venn diagrams or the use of rules (such as "nothing may be distributed in the conclusion unless it is distributed in a premise"). Aristotle himself proved all of the principles of this logic using *reductio*, *exposition*,

expository syllogism, and two first figure syllogisms (and these are superfluous; see Chapter 2 for details). Including singular propositions in syllogistic (that is, propositions whose subject terms are singular terms) changes very little, though it complicates the use of Venn diagrams.<sup>1</sup>

The early medieval expansion of Aristotle's logic (Chapter 3) is equivalent to a fragment of the monadic predicate calculus with identity. Quantifier signs occur ineliminably within the scopes of other quantifier signs. The system of logic is still decidable, but it is harder to test for validity; e.g. Venn diagrams are inadequate. Medieval authors also invoked principles of supposition theory, but these do not go beyond what can be proved by Aristotle's basic means of *reductio*, exposition, and expository syllogism, now supplemented with substitutivity of identity and a number of other rules such as quantifier equipollences. (See Chapters 4 and 5 for details.)

The expanded form of Linguish is a stronger system. Here I will discuss the version of Linguish from Chapter 4 (basic Linguish) supplemented with the expansions given in sections

- 5.1 Adjectives
- 5.2 Intransitive verbs
- 5.3 Transitive verbs
- 5.6 Relative clauses
- 5.7 Genitives
- 5.9 Molecular propositions

Recall that participles of transitive verbs and genitives introduce parasitic ("relational") terms. I omit

- 5.5 Some Complex Terms
- 5.8 Demonstratives

just for simplicity. (I also assume that there are no past or future tenses, or modal expressions, or any other sources of ampliation. These are discussed in Chapter 10.)

### 9.1.1 *Medieval logic without anaphoric pronouns*

Notice that the system of logic demarcated here does not yet include the anaphoric pronouns from Chapter 8. Some examples of sentences that can be expressed with its resources are:

*Every horse sees some donkey*  
*Some farmer's every donkey is running*  
*Some farmer's every grey donkey sees a horse which no woman who owns a donkey sees.*

<sup>1</sup> For example, one needs to decide how to diagram a sentence such as 'n isn't F' keeping in mind that 'n' might be empty.

Although one can express some rather complex claims, I believe that this system of medieval logic, like its simpler predecessors, is decidable. That is, for any finite set of sentences in the symbolism, there is a mechanical way to decide whether or not there is an interpretation which makes them all true. This is a bit surprising, because the notation includes quantifiers and relational verbs. The reason for the decidability is the constraint on the use of markers, which in this notation represent grammatical roles. If we were to express in modern logic the claim that exceeding is transitive, the notation would look something like this:

$$\forall x \forall y \forall z [xEy \ \& \ yEz \rightarrow xEz]$$

The variables inside the parentheses look something like our grammatical role markers. But they can't serve that purpose, since they each occur in multiple places, which grammatical markers cannot do. But anaphoric pronouns can accomplish this purpose.

### 9.1.2 Medieval logic with anaphoric expressions

Because the system of logic without anaphoric expressions is limited, we should see if adding pronouns and other anaphoric expressions helps. Some examples of sentences that can be expressed with the addition of anaphoric pronouns (Chapter 8) are:

- Some donkey sees itself*
- Some donkey sees some other donkey*
- Every horse sees some donkey which it likes*

Things are quite different when we consider this system of logic with anaphoric expressions added to it. With this notation it is easy to produce a small set of sentences that has an infinite model, but no finite model. An example is this set, whose third member expresses that the relation of exceeding is transitive:

- No number exceeds itself.*
- Every number is exceeded by some number.*
- Every number exceeds every number which is exceeded by some number which it (i.e. the first number) exceeds.*

The first and third sentences contain anaphoric pronouns, which are underlined. In Linguish notation:

- (no number  $\alpha$ )<sup>i</sup>(it<sup>i</sup>  $\beta$ )  $\alpha$  exceeds  $\beta$
- (every number  $\alpha$ )(some number  $\beta$ )  $\beta$  exceeds  $\alpha$
- (every number  $\alpha$ )<sup>i</sup>(every {number which $\beta$  (some {number which $\gamma$  (it<sup>i</sup>  $\delta$ )  $\delta$  exceeds  $\gamma$ )  $\epsilon$ )  $\epsilon$  exceeds  $\beta$ }  $\eta$ )  $\alpha$  exceeds  $\eta$

Notice that this set of sentences does not contain any parasitic terms, though it uses a verb other than the copula.

This system of logic is not decidable. This will follow from the fact, discussed in section 9.4, that the first-order predicate calculus is representable in it.

## 9.2 Representing medieval logic within modern predicate logic with identity

It is no surprise that medieval logic can be emulated within modern logic; the only question is how complicated the procedure is to produce a formula in predicate logic that is equivalent to a given one in medieval logic. The procedure is a bit clumsy. For example, medieval logic and contemporary logic differ with respect to how they represent generalities. Consider the symbolizations of ‘Every A is  $\phi$ ’. The natural representations are:

$$(\text{Every } A_a) \phi \quad \text{vs} \quad \forall x(Ax \rightarrow \phi^*)$$

(where ‘ $\phi$ ’ and ‘ $\phi^*$ ’ are the respective symbolizations of ‘ $\phi$ ’). These forms, in isolation, are incommensurable. This is because when ‘A’ is empty, the Linguish version is automatically false if  $\phi$  is affirmative, but automatically true if  $\phi$  is negative, whereas the symbolic logic sentence is automatically true when there are no A’s no matter what the character of  $\phi^*$  is. There is, however, a natural way to go about representing sentences one at a time: one can mirror the medieval denoting phrases differently depending on whether the context is affirmative or negative. In particular, these are equivalent:

$$(\text{Every } A_a) \phi \quad \approx \quad \begin{array}{ll} \exists xAx \ \& \ \forall x(Ax \rightarrow \phi^*) & \text{if } \phi \text{ is affirmative} \\ \forall x(Ax \rightarrow \phi^*) & \text{if } \phi \text{ is negative} \end{array}$$

However, there are cases in which the formula  $\phi$  is neither affirmative nor negative. Consider:

$$(\text{Every } A_a)^1 [a \text{ is running or } [(it^i \gamma)(\text{Socrates } \beta) \gamma \text{ sees } \beta \text{ or } (\text{Brownie } \delta) \delta \text{ runs}]]$$

Here ‘(Every A<sub>a</sub>)’ is in neither an affirmative nor a negative position, and confinement does not apply. Probably there is an algorithm that will yield an equivalent formula in all cases, but I have not produced one.

## 9.3 Representing modern logic within medieval logic: The problem of existential import

Suppose that we wish to represent modern predicate logic within medieval logic. The first step seems clear: whereas medieval logic has an unlimited number of denoting phrases, modern logic has only two: ‘everything’ and ‘something’—‘ $\forall_a$ ’ and ‘ $\exists_a$ ’. According to medieval logicians, ‘everything’ means ‘every thing’. So we can apparently mimic modern quantifiers by using ‘(every thing<sub>a</sub>)’ and ‘(some thing<sub>a</sub>)’ for the quantifiers, provided that we add as a logical axiom that there is at least one thing: ‘(some thing<sub>a</sub>)(some thing <sub>$\beta$</sub> )<sub>a</sub> is  $\beta$ .’<sup>2</sup> This is the analogue of the assumption made in classical logic that the domain of quantification is non-empty. Names are also easy to handle:

<sup>2</sup> Medieval logicians would be happy to accept this, since God is a thing, and God necessarily exists.

for the name 's' just insert '(s<sub>α</sub>)' with smallest scope over the atomic formula of modern logic within which 's' occurs, and replace 's' itself by 'α.' So for the sentence '*Socrates sees everything*,' symbolized:

$$\forall y sSy$$

we would have:

$$\forall y (s_{\alpha})_{\alpha} \text{ sees } y$$

and then:

$$(\text{every thing } \beta)(s_{\alpha})_{\alpha} \text{ sees } \beta$$

For classical logic we must also add as a logical axiom that the name is not empty, and that it falls under the medieval quantifiers: '(s<sub>α</sub>)(some thing β)<sub>α</sub> is β.' The result is a fragment of medieval logic with no empty simple terms.

However, one cannot just take any old sentence of modern logic, change its quantifiers in this way, and end up with an equivalent sentence of medieval logic. This is because in medieval logic the markers are used only to mark grammatical roles, and there is no guarantee that such a transformation will produce a string of symbols that corresponds to a grammatical sentence. Take, for example, a modern symbolization of the transitivity of the relation of exceeding. If 'xEy' means that x exceeds y, then one naturally says that this relation is transitive by writing:

$$\forall x \forall y \forall z [xEy \ \& \ yEz \rightarrow xEz]$$

Using the device outlined a moment ago, one gets a proposed "sentence" of medieval logic by changing the quantifiers as indicated already and writing 'E' as 'exceeds'; this produces:

$$(\text{every thing } \alpha)(\text{every thing } \beta)(\text{every thing } \gamma)[\text{if } \alpha \text{ exceeds } \beta \text{ and } \beta \text{ exceeds } \gamma \text{ then } \alpha \text{ exceeds } \gamma]$$

But this is not well formed in medieval logic since it makes each denoting phrase be the subject or direct object of two different sentences (because the marker accompanying the common term occurs in two grammatical role-indicating positions).

Instead, if one wants to represent the transitivity of exceeding in medieval logic one needs to use a different form, such as the one used in section 9.1.2 (changing 'number' to 'thing'):

$$(\text{every thing } \alpha)^i(\text{every } \{\text{thing which } \beta \text{ (some } \{\text{thing which } \gamma \text{ (it}^i \delta \text{ ) } \delta \text{ exceeds } \gamma \} \epsilon \} \epsilon \text{ exceeds } \beta \} \eta)_{\alpha} \text{ exceeds } \eta$$

However, although this formula does require that exceeding be transitive, it also requires something more. This is because the sentence is affirmative, and so it has existential import for its terms. Consider now adding the truth that zero is something:

$$(\text{zero } \alpha)(\cdot \text{ thing } \beta)_{\alpha} \text{ is } \beta$$

If we then apply UA we infer the following:

(zero  $\alpha$ )(every {thing which  $\beta$  (some {thing which  $\gamma$  (zero  $\delta$ )  $\delta$  exceeds  $\gamma$ )  $\epsilon$  exceeds  $\beta$ }  $\eta$ )  $\alpha$  exceeds  $\eta$ )

The second main term of this is ‘*thing which exceeds some thing which zero exceeds*’ and since it is in a main term of an affirmative proposition it must be non-empty, which means that ‘*thing which zero exceeds*’ must also be non-empty. So from the statement that exceeding is transitive we infer that there is something which zero exceeds, which is not true. So this idea for symbolizing transitivity does not work in this case.<sup>3</sup>

There seem to me to be two ways to proceed here. One is to show how to alter the truth conditions of the medieval logic forms when that seems desirable, and the other is to find some way to mimic predicate calculus notation within medieval logic that bypasses the issue of existential import. The present section deals with the first of these ideas: finding a way to eliminate the existential import of universal affirmative denoting phrases that occur in affirmative propositions.

There is a way to convert sentences of affirmative form into a negative form which is completely equivalent except that it is true when the main terms are empty. Recall the idea that the only true verb is the copula, and that other verbs are to be analyzed in terms of a copula plus a participle. We can replace ‘ $\alpha$  exceeds  $\beta$ ’ by ‘(- exceeding- $\beta$ -thing  $\gamma$ )  $\alpha$  is  $\gamma$ ’ (“ *$\alpha$  is an exceeding- $\beta$ -thing*”) as we did in section 5.3. It is then possible to write propositions in which this introduced parasitic term combines with infinitizing negation. E.g. we can say that every number non-exceeds a number: ‘(every number  $\alpha$ ) (- number  $\beta$ )(- non-exceeding- $\beta$ -thing  $\gamma$ )  $\alpha$  is  $\gamma$ .’ And we can also say that for every number there’s a number that it “exceeds” by saying there’s a number that it *doesn’t non-exceed*: ‘(every number  $\alpha$ )(- number  $\beta$ ) not (- non-exceeding- $\beta$ -thing  $\gamma$ )  $\alpha$  is  $\gamma$ .’ Suppose now that we take our original sentence attempting to define transitivity and change it to:

(every thing  $\alpha$ )<sup>i</sup>(every {thing which  $\beta$  (some {thing which  $\gamma$  (it<sup>i</sup>  $\delta$ )  $\delta$  exceeds  $\gamma$ )  $\epsilon$ )  $\epsilon$  exceeds  $\beta$ }  $\eta$ ) not (- non-exceeding- $\eta$ -thing  $\delta$ )  $\alpha$  is  $\delta$

This is just like the original sentence except that it has ‘*isn’t a thing non-exceeding*’ instead of ‘*exceeds*,’ and it is now a negative sentence, not an affirmative one. So it is true for instances of ‘**every thing**’ that make the second denoting phrase empty. This then avoids the problem of existential import. So the problem can be got around in this case, though it is quite clumsy and I am not sure how systematic it can be made.

<sup>3</sup> This also raises a problem for a suggestion that was made in section 1.4.1. There I suggested that different systems of logic might naturally differ on the interpretation of universal affirmative sentences so long as each system can find some way to duplicate the truth conditions yielded by the other theory. And in simple cases this is easy. For example, if you want ‘Every A is B’ to be true when ‘A’ is empty, and you are confined to using the medieval forms with their truth conditions, just write:

*Every A is B or no A is an A*

which is true if ‘A’ is empty. But the example of transitivity shows that this simple idea may not automatically extend to more complex cases.

## 9.4 Representing modern logic within medieval logic: Grammatical issues

There is an additional limitation on our ability to represent modern logic in medieval logic. In medieval logic the marker within a denoting phrase links to the unique grammatical position in the sentence which that denoting phrase fills. Since each denoting phrase has a unique grammatical role, a medieval denoting phrase must bind exactly one occurrence of a marker. Now consider how to represent a sentence such as ‘*everything that sees Socrates is running*’ in modern logic:

$$\forall x(xSs \rightarrow Rx).$$

It is essential to this representation that the initial universal quantifier bind two occurrences of the variable ‘x,’ one in the antecedent and one in the consequent of the conditional. No denoting phrase in medieval logic can do this, because binding markers in two different propositions would make that denoting phrase fill grammatical roles in different propositions, and natural language just doesn’t work like that. Of course, if your goal is to symbolize the English sentence ‘*everything that sees Socrates is running*’ you can do that:

**(every {thing which <sub>$\gamma$</sub>  (Socrates <sub>$\beta$</sub> )  $\gamma$  sees  $\beta$ }  $\alpha$ )  $\alpha$  is running**

This achieves the same purpose as the modern symbolization of that particular English sentence. But it is not obvious how to generalize this case to cover sentences of modern logic that do not obviously come from sentences of natural language, sentences such as:

$$\forall x(\exists yRy \vee \sim Qx)$$

There is a natural way to try to accomplish this goal. This is to give each denoting phrase the grammatical role of some one selected marker that it binds, and use anaphoric pronouns for the rest. For example, we might decide to assign to each denoting phrase the *first* occurrence of its marker as its grammatical role identifier, and use pronouns for the rest. The symbolic form:

$$\forall x xRx$$

could be represented as:

**(every thing  $\alpha$ )<sup>i</sup>(it<sup>i</sup>  $\beta$ )  $\alpha$ R $\beta$**

(Recall section 8.5 on anaphora.) The more complicated sentence would be represented as:

**(every thing  $\alpha$ )<sup>i</sup>[(some thing  $\beta$ )  $\alpha$ R $\beta$  or not (it<sup>i</sup>  $\gamma$ )Q $\gamma$ ]**

This approach seems artificial in that the grammatical role of the initial quantifier seems capricious. But some instances of it are natural. For example, the analogue of:

$$\forall x[\text{donkey } x \rightarrow \text{grey } x]$$



could be the logical form:

(every thing  $\alpha$ )<sup>i</sup>[ $\alpha$  is grey if ( $\cdot$  donkey  $\beta$ )(it<sup>i</sup>  $\gamma$ )  $\gamma$  is  $\beta$ ]

This then goes over into natural language as:

*every thing is grey if it is a donkey*

But other examples don't work this well. Consider:

$\forall x[\text{donkey } x \rightarrow \sim(\text{grey } x \ \& \ \text{young } x)]$

(every thing  $\alpha$ )<sup>i</sup>[if  $\alpha$  is a donkey not [(it<sup>i</sup>  $\gamma$ )  $\gamma$  is grey and (it<sup>i</sup>  $\delta$ )  $\delta$  is young]]

This would yield

*everything if is a donkey not it is grey and it is young*<sup>4</sup>

A slightly different idea works better. This is to give the quantificational term *no* grammatical role in the sentence it combines with. Instead, it occurs in a prepositional phrase modifying the whole sentence. The preposition itself contributes no content; it just allows the term to combine with a sentence that the term is not a part of. To accomplish this, we read the ' $\forall \alpha$ ' of symbolic logic as 'for every thing.' Then we read

$\forall x[\text{donkey } x \rightarrow \text{grey } x]$

as

**for-(every thing)**<sup>i</sup>[if (it<sup>i</sup>  $\alpha$ )( $\cdot$  donkey  $\beta$ )  $\alpha$  is  $\beta$  then (it<sup>i</sup>  $\gamma$ )  $\gamma$  is grey]

Applying our rules for turning logical forms into sentences of natural language, we get:

*for every thing if it is a donkey then it is grey*

This appears to be grammatical, and it seems to say just what is needed. We just need to add something like '**for-(every T)**' to our language with the semantic rule:

'**for-(every T)**<sup>i</sup>  $\phi$ ' is true <sub>$\sigma$</sub>  if and only if either 'T' is non-empty and for every thing  $\alpha$  for which 'T' supposes  $\phi$  is true <sub>$\sigma(i/\alpha)$</sub>  or 'T' is empty and ' $\phi$ ' is true <sub>$\sigma(i/-)$</sub>

Notice that there is no marker in '**for-(every T)**.' This is because '*every T*' does not have a grammatical role *within* the sentence *following* it; rather, it is the object of a prepositional phrase that modifies the whole sentence. The term (strictly, its denoting phrase) does have a grammatical role; it is the object of the preposition '**for**.' So it has a grammatical role in the *whole* sentence, as every denoting phrase must.<sup>5</sup> This device apparently

<sup>4</sup> Writing the consequent first is also problematic:

(every thing  $\alpha$ )<sup>i</sup>[not  $\alpha$  is grey and (it<sup>i</sup>  $\delta$ )  $\delta$  is young] if (it<sup>i</sup>  $\eta$ )  $\eta$  is a donkey]

It produces '*everything isn't grey and it is young if it is a donkey*' which has the wrong meaning.

<sup>5</sup> We could let the preposition supply a grammatical role in its prepositional object position, by writing '**for**  $\alpha$ ' and then writing the denoting phrase before it, as '(every donkey  $\alpha$ )<sup>i</sup> for  $\alpha$  (some horse  $\beta$ )(it<sup>i</sup>  $\gamma$ )  $\beta$  sees  $\gamma$ .' We would then need to specify that when pronounced, the denoting phrase is to follow its preposition, to yield e.g. '*for every donkey, some horse sees it*.'

works well, though it looks ad hoc. There may well be problems with it that I do not currently understand.

Is this an idea that is part of medieval logic? I am not aware of any medieval discussion of this construction; it appears instead to be a 21st-century construction. So it is not ideal.

Fortunately, there is a similar way to emulate modern logic within medieval logic using grammatical structures already existing in medieval logic. This is to use the fact that a term in one categorical can be the antecedent of another term occurring in a categorical conjoined to it. We saw in Chapter 8 how to do this. As previously, we use **thing** as the common term in all denoting phrases that represent quantifiers of modern logic. Then instead of representing quantified sentences by

*for every thing  $\phi$*

we represent them by

*every thing is a thing and  $\phi$*

In this form, *every thing* has a grammatical role; it is the subject of the verb *'is'*, and it can be the antecedent for any number of terms in *' $\phi$ '*. For example, as in:

**(every thing  $\alpha$ )<sup>i</sup>[(**thing**  $\beta$ )  $\alpha$  is  $\beta$  and (**it**<sup>i</sup>  $\delta$ )(**it**<sup>i</sup>  $\varepsilon$ )  $\delta$  sees  $\varepsilon$ ]**  
*every thing is a thing and it sees itself*

Here then is a recipe for converting any formula of modern logic (without names, for simplicity) into one in the medieval logic notation which is logically equivalent to it. Given any formula of predicate logic, make the following changes (one at a time, starting with the most embedded formulas):

- Eliminate all biconditional signs using combinations of the other connectives.
- Delete all vacuous quantifiers.
- One at a time replace each atomic formula of the form ' $xRy$ ' with one of the form '**(it<sup>x</sup>  $\delta$ ) (it<sup>y</sup>  $\varepsilon$ )  $\delta R \varepsilon$ ' where ' $\delta$ ' and ' $\varepsilon$ ' are so far distinct unused grammatical markers. If the same variable occurs more than once flanking ' $R$ ', use it as a superscript on each pronoun, but introduce a new marker for each pronominal denoting phrase itself. E.g. ' $xRx$ ' becomes '**(it<sup>x</sup>  $\delta$ ) (it<sup>x</sup>  $\varepsilon$ )  $\delta R \varepsilon$ '.****
- Replace ' $\neg\phi$ ' by **not  $\phi$** .
- Replace  $[\phi \& \psi]$  by  **$[\phi$  and  $\psi]$** .
- Replace  $[\phi \vee \psi]$  by  **$[\phi$  or  $\psi]$** .
- Replace  $[\phi \rightarrow \psi]$  by  **$[\psi$  if  $\phi]$** .
- Replace ' $\forall x\phi$ ' by '**(every thing  $\delta$ )<sup>x</sup>[(**thing**  $\varepsilon$ )  $\delta$  is  $\varepsilon$  and  $\phi$ ]**', where ' $\varepsilon$ ' and ' $\delta$ ' are new markers.
- Replace ' $\exists x\phi$ ' by '**(some thing  $\delta$ )<sup>x</sup>[(**thing**  $\varepsilon$ )  $\delta$  is  $\varepsilon$  and  $\phi$ ]**', where ' $\varepsilon$ ' and ' $\delta$ ' are new markers.

(One may wish then to change the variables ' $x$ ,' ' $y$ ,' etc. to ' $i$ ,' ' $j$ ,' etc.)

For an illustration, consider the formula:

$$\exists x[Px \& \sim Qx]$$

The atomic formulas yield:

$$\exists x[(it^x_\delta) P_\delta \& \sim (it^x_\epsilon) Q_\epsilon]$$

the connectives are standard:

$$\exists x[(it^x_\delta) P_\delta \text{ and not } (it^x_\epsilon) Q_\epsilon]$$

and the quantifier is then analyzed, yielding:

$$(\text{some thing } \alpha)^x[(\cdot \text{ thing } \eta)_\alpha \text{ is } \eta \text{ and } [(it^x_\delta) P_\delta \text{ and not } (it^x_\epsilon) Q_\epsilon]]$$

Its pronunciation in natural language will be:

$$\begin{aligned} &(\text{some thing } \alpha)^x[(\cdot \text{ thing } \eta)_\alpha \text{ is } \eta \text{ and } [(it^x_\delta) P_\delta \text{ and not } (it^x_\epsilon) Q_\epsilon]] \\ &\text{some thing a thing is and it P and not it Q} \\ &\text{something is a thing and it P and not it Q} \end{aligned}$$

If we suppose that ‘P’ means, say, ‘is grey’ and ‘Q’ means ‘is running’ we would get:

$$\text{something is a thing and it is grey and not it is running}$$

(For normal logicians’ usage the ‘not’ should be read ‘it is not the case that.’)<sup>6</sup>

The results are well-formed propositions of medieval logic. They are artificial and not ideal to read, but they are precise and logically fully expressive. And the required resources are from the core of medieval logic—what is used is notation that was discussed by medieval authors and the inference rules were almost all discussed by medieval authors. In particular, what is required by this technique is:

- Notation and rules from basic Linguish as introduced in Chapter 4
- Molecular constructions from section 5.10
- Anaphoric pronouns as introduced in sections 8.3–8.5

It is not necessary to use any parasitic terms, or complex terms, or infinitizing negation. It can be done only with logical apparatus that was widely used and understood. And existential import is not a problem because the only common term is ‘thing.’

In the next section, we look at whether certain mathematical theories can be formalized within medieval logic without the kind of artificiality appealed to here.

<sup>6</sup> One can view the construction given here as implementing the previous idea of using “quantifiers” on the fronts of formulas, but in place of the form ‘for every thing,’ we use ‘every thing is a thing and.’ This strikes some people as unnecessarily prolix, and in need of shortening. I don’t know how to shorten it in a way that will work in general. The ‘is a thing and’ portion is there to provide a location for the grammatical role that the initial ‘every thing’ is playing, and such a place is necessary. It is blatantly artificial, but that is no objection since the task is to find natural language wordings for idioms of a patently artificial notation—that of symbolic logic.

## 9.5 First-order arithmetic in medieval logic

We have been comparing medieval logic with modern symbolic logic. But why should modern symbolic logic be the standard of comparison? Historically, symbolic logic did not come with a seal of approval on it. Instead, it achieved its present status by providing a system in which it was possible to formulate central claims of mathematics and (less evidently) science, in which the valid derivations correspond to what mathematicians already recognize as valid reasoning. It is possible then that some other system of logic might accomplish the same goal, while being quite different from modern predicate logic. It need only provide for the formulation of central parts of mathematics and science. This section provides a sample test: the formulation of first-order arithmetic in medieval logic.

The goal of this section is to see what it would be like to formulate first-order arithmetic within medieval logic in a natural way. The challenge is to use a logical language (an extension of Linguish) to formulate the theory—to state axioms and rules, and derive theorems. There will continue to be a way to “pronounce” the logical notation in natural language, a way that is mechanically generated from the logical forms of Linguish, and that, in a transparent fashion, permits one to represent the logical structure of symbolic notation (although not without ambiguity). One constraint that must be maintained is that every denoting phrase in the logical notation gives rise to a surface denoting phrase which occupies a unique grammatical role. As long as this constraint is satisfied, we can continue to view grammatical markers as merely encoding the type of grammatical information that is taken into account by medieval theorists.

For readability of the natural language representations, I find that it is natural to use ‘**any**’ rather than ‘**every**’ here, because the ‘**any**’ naturally takes wide scope. So I’ll use ‘**any**’ with the understanding that it has the logical properties of ‘**every**’ in previous chapters. I will continue the policy of generating natural language translations of the logical forms by erasing all logical notation, and optionally moving verbs from the end of a proposition to an earlier position, as discussed and practiced previously. We also avail ourselves of the use of “relatives,” that is, of pronouns with antecedents, as developed in section 8.5.

We begin with the core of first-order arithmetic, the axioms commonly called “Peano’s Postulates.”

### 9.5.1 Peano’s postulates

In order to symbolize the Dedekind/Peano axioms we need a notion of *successor*. I will introduce ‘**successor**’ as a one-place parasitic singular term. (Call it a one-place function symbol if you like.) This is just like a parasitic common noun, except that it is a singular term, not a common one. We could use a common term instead; it is quite a bit simpler to use a singular one because we thereby avoid the need to introduce uniqueness clauses. It is to be pronounced ‘*the successor*,’ unless it is immediately

preceded by a genitive construction with apostrophe 's,' in which case it is simply pronounced 'successor.' The axioms will guarantee that every number has a successor. It is neatest to suppose that non-numbers don't have successors (so 'the successor of Caesar' is an empty term). However, that will not be relevant to any of the sentences we are interested in.

Here are the traditional axioms in Linguish notation:

**AX1** (zero<sub>α</sub>)(· number<sub>β</sub>)<sub>α</sub> is β  
*Zero is a number*

**AX2** (Zero<sub>α</sub>)(no number<sub>β</sub>)(successor-of-β<sub>γ</sub>)<sub>α</sub> is γ  
*Zero is no number's successor*

**AX3** (any number<sub>α</sub>)(successor-of-α<sub>β</sub>)(· number<sub>γ</sub>)<sub>β</sub> is γ  
*Any number's successor is a number*

**AX4** (any number<sub>α</sub>)<sup>i</sup>(successor-of-α<sub>β</sub>)(no {other<sup>i</sup> number<sub>γ</sub>})<sub>β</sub> is δ  
*Any number's successor is no other number's successor*

The induction rule is normally stated as an axiom schema, using quantification, conjunction, and implication. We could state it that way here, but it is more convenient to introduce it as a rule schema, which holds for any formula  $\phi$ :

**Induction** (zero<sub>α</sub>) φ  
(any {number which<sub>α</sub> φ}<sub>β</sub>)(successor-of-β<sub>α</sub>) φ  
∴ (any number<sub>α</sub>) φ  
  
*zero is φ*  
*of any number which is φ, the successor is φ*  
∴ *any number is φ*

(of course 'is φ' is a stand-in for the pronunciation of 'φ,' whatever it might be).

We also allow anaphora in the sentences used, as in section 8.3. So this form is OK:

(zero<sub>α</sub>)<sup>i</sup> φ  
(any {number which<sub>α</sub><sup>i</sup> φ}<sub>β</sub>)(successor-of-β<sub>α</sub>)<sup>i</sup> φ  
∴ (any number<sub>α</sub>)<sup>i</sup> φ

(Note that the scope of the first superscript 'i' in the second premise is confined to the complex term that it occurs in.) Allowing for anaphora in this particular way is just for convenience; we can get along without it, if necessary, by complicating φ. For example, instead of proving '(any number<sub>α</sub>)<sup>i</sup> φ,' we could prove: '(any number<sub>γ</sub>)(some {number which<sub>α</sub><sup>i</sup> φ}<sub>δ</sub>)<sub>γ</sub> is δ.'

The only logical innovation here is the introduction of parasitic singular terms. When an immediately preceding singular term binds the argument marker, the combination of the two terms is treatable as a unit for purposes of applying rules of

inference; this is in analogy to the enhanced rules for parasitic terms described in section 5.5.2. So one may apply the rules ES+ and UA+ to their combination. E.g. this inference is allowable:

$$\begin{array}{ll}
 (n_\alpha)(\text{successor-of-}\alpha\beta)(\cdot \text{number } \gamma) \beta \text{ is } \gamma & \\
 n\text{'s successor is a number} & \\
 (n_\alpha)(\text{successor-of-}\alpha\beta)(\cdot \text{prime } \gamma) \beta \text{ is } \gamma & \\
 n\text{'s successor is a prime} & \\
 \therefore (\text{some number } \beta)(\cdot \text{prime } \gamma) \beta \text{ is } \gamma & \text{by rule ES+} \\
 \text{some number is a prime} &
 \end{array}$$

Peano's postulates alone form a very weak system. To get first-order arithmetic, one normally adds axioms for addition and multiplication.

### 9.5.2 Definition of addition

For addition we will introduce 'added-to,' which is a two-place parasitic singular term. (Call it a two-place function symbol if you like.) It is tricky to pronounce this so as to clearly indicate the order of the arguments of 'added-to,' because the denoting phrases which bind the markers that accompany 'added-to' must occur in an order that indicates their quantificational scope. In the predicate calculus notation, this problem does not occur, since '+' can be flanked by variables, with the first argument to the left of the sign and the second to its right, where the quantifiers that bind the variables occur elsewhere; the positions of the quantifiers indicate their scope, independent of which binds the first, and which the second, variable flanking '+'. That is not the case in the natural language pronunciations, which contain no variables. So I have hit upon this artifice: The denoting phrases which bind the markers accompanying 'added-to' will be pronounced in the order in which they occur in the logical notation; the 'to' will immediately precede the denoting phrase that binds the second marker. For English, when the order of the markers following 'added-to' is the same as the order of the denoting phrases binding those markers, the participle 'added' will be pronounced between the two denoting phrases, giving a natural ordering (such as 'some number added to any number'); otherwise the 'added' remains where it is, giving an unnatural ordering (such as 'to some number any number added'). As an example, when the antecedents are names, the pronunciations will be:

$$\begin{array}{ll}
 (n_\alpha)(m_\beta)(\text{added-to-}\alpha\beta\gamma) & n \text{ added to } m \\
 (m_\alpha)(n_\beta)(\text{added-to-}\beta\alpha\gamma) & \text{to } m, n \text{ added}
 \end{array}$$

These logical forms are provably equivalent by permuting the terms, and then changing the bound markers. The surface forms, 'n added-to m' and 'to m, n added,' are thus logically equivalent as well.

Like 'successor,' 'added-to' immediately preceded by two singular terms is equivalent to having a complex singular term so far as the logical rules are concerned.

The usual axioms for addition (often called “recursive definitions”) in modern notation, using the prime mark for successor, look like this:

$$\begin{aligned} n+0 &= n \\ n+m' &= (n+m)' \end{aligned}$$

In our notation they are:

Added-to<sub>1</sub> (any number  $\alpha$ )<sup>i</sup>(zero  $\beta$ )( $\alpha$ -added-to- $\beta$   $\gamma$ )(it<sup>i</sup>  $\delta$ )  $\gamma$  is  $\delta$   
*any number added to zero is itself*<sup>7</sup>

Added-to<sub>2</sub> (any number  $\alpha$ )<sup>i</sup>(any number  $\beta$ )<sup>j</sup>( $\alpha$ -added-to- $\beta$   $\gamma$ )(successor-of- $\gamma$   $\delta$ )  
 (it<sup>i</sup>  $\epsilon$ )(it<sup>j</sup>  $\eta$ )(successor-of- $\eta$   $\varsigma$ )( $\epsilon$ -added-to- $\varsigma$   $\delta$ ) is  $t$   
*of (any number added to any number) the successor is it added to its successor*  
*i.e. the successor of (any number added to any number) is it [the former number] added to its successor [the latter number's successor]*

(In transition to natural language, the genitive produced by the markers accompanying ‘successor’ is attributed to the whole phrase of which ‘added-to’ is the center.)

Axiom Added-to<sub>1</sub> says that zero leaves a number unchanged if it is added on the right. It is customary to begin development of the theory by proving a theorem saying that zero also leaves a number unchanged if it is added on the left. This is done here to illustrate how the theoretical development would go when the sentences in it become technically complex.

**Theorem:**

(zero  $\beta$ )(any number  $\alpha$ )<sup>i</sup>( $\beta$ -added-to- $\alpha$   $\gamma$ )(it<sup>i</sup>  $\delta$ )  $\gamma$  is  $\delta$   
*zero added to any number is it [is that number]*

To use the induction schema, we need a proposition that begins with a universal quantifier over numbers. So we will prove our theorem by proving this Lemma, which is equivalent to the theorem by permutation of the terms:

**Lemma:**

(any number  $\alpha$ )<sup>i</sup>(zero  $\beta$ )( $\beta$ -added-to- $\alpha$   $\gamma$ )(it<sup>i</sup>  $\delta$ )  $\gamma$  is  $\delta$   
*to any number, zero added is it*

We prove this by induction, where ‘ $\phi$ ’ is ‘(zero  $\beta$ )( $\beta$ -added-to- $\alpha$   $\gamma$ )(it<sup>i</sup>  $\delta$ )  $\gamma$  is  $\delta$ .’

<sup>7</sup> This ‘*itself*’ should probably just be ‘*it*,’ because its antecedent is the first part of the subject, and not all of the subject. I think that in the vernacular ‘*itself*’ is natural, which is why I use it here.

**Base Case:**

To show:  $(\text{zero } \alpha)^i(\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$   
*to zero zero added is itself*

- |   |  |
|---|--|
| 1. $(\text{zero } \alpha)^i(\text{zero } \beta)(\alpha\text{-added-to-}\beta \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$ | AX1 Added-to <sub>1</sub> UA                   |
| 2. $(\text{zero } \beta)(\text{zero } \alpha)^i(\alpha\text{-added-to-}\beta \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$ | 1 permutation                                  |
| 3. $(\text{zero } \alpha)(\text{zero } \beta)^i(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$ | 2 change bound mkrs                            |
| 4. $(\text{zero } \alpha)(\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{zero } \delta) \gamma \text{ is } \delta$  | 3 relative term with singular term antecedent. |
| 5. $(\text{zero } \alpha)^i(\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$ | 4 relative term with singular term antecedent  |

Pronounced in natural language:

- |  |                              |
|--|------------------------------|
| 1. <i>zero added to zero is itself</i> | AX1 Added-to <sub>1</sub> UA |
| 2. <i>to zero zero added is it</i>     | 1 permutation                |
| 3. <no change>                         |                              |
| 4. <i>to zero zero added is zero</i>   | 3 singular antecedent        |
| 5. <i>to zero zero added is itself</i> | 4 singular antecedent        |

**The Induction Step:** With modern semiformal notation, the strategy behind the inductive step is:

- |   |                                    |
|---|------------------------------------|
| [A] Assume the inductive hypothesis:<br>$0+n = n$ |                                    |
| [B] $(0+n)' = (0+n)'$                             | Self-identity                      |
| [C] $(0+n)' = n'$                                 | [A] [B] Substitutivity of Identity |
| [D] $(0+n)' = 0+n'$                               | Added-to <sub>2</sub>              |
| [E] $0+n' = n'$                                   | [C] [D] Transitivity of Identity   |

This reasoning takes for granted that all terms are non-empty, a well-known assumption of mainstream classical logic. Reproducing the reasoning in Linguish requires quite a few additional steps to establish non-emptiness. We will also need to show, e.g. that  $0+n$  is a number, since we are not making the common assumption that everything is a number.

In the proof to be given, I will freely interchange bound markers without comment. I introduce the labels of the parts of the previous argument in the derivation to follow as an aid in tracking the reasoning. (Since the proof will be by reductio, the assumption of the inductive hypothesis is actually made in the assumption for reductio, line 1; it surfaces in an identifiable form only at line 7.)



## Induction step:

To show:  $(\text{any } \{\text{number which}_\alpha^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta\} \beta)$   
 $(\text{successor-of-}\beta \alpha)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$

1.	$\text{not } (\text{any } \{\text{number which}_\alpha^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta\} \beta)$ $(\text{successor-of-}\beta \alpha)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$	
2.	$(\text{some } \{\text{number which}_\alpha^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta\} \beta)$ $\text{not } (\text{successor-of-}\beta \alpha)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$	1 Equipoll
3.	$(\text{n } \gamma)(\cdot\{\text{number which}_\alpha^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta\} \beta) \gamma \text{ is } \beta$	2 EX
4.	$(\text{n } \beta) \text{not } (\text{successor-of-}\beta \alpha)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$	2 EX
5.	$(\text{n } \alpha)(\cdot\text{number } \beta) \alpha \text{ is } \beta$	3 RelClause
6.	$(\text{n } \alpha)^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$	3 RelClause
7. [A]	$(\text{n } \alpha)(\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{n } \delta) \gamma \text{ is } \delta$	6 Sing Ante
8.	$(\text{any number } \alpha)(\text{successor-of-}\alpha \beta)(\cdot\text{number } \gamma) \beta \text{ is } \gamma$	AX3
9.	$(\text{n } \alpha)(\text{successor-of-}\alpha \beta)(\cdot\text{number } \gamma) \beta \text{ is } \gamma$	5 8 UA
10.	$(\text{n } \alpha)(\text{zero } \delta)(\delta\text{-added-to-}\alpha \gamma)(\text{successor-of-}\gamma \beta)(\cdot\text{number } \varepsilon) \beta \text{ is } \varepsilon$	7 9 Subst-Iden
11. [B]	$(\text{n } \alpha)(\text{zero } \delta)(\delta\text{-added-to-}\alpha \gamma)(\text{successor-of-}\gamma \beta)$ $(\text{n } \alpha)(\text{zero } \delta)(\delta\text{-added-to-}\alpha \eta)(\text{successor-of-}\eta \varepsilon) \beta \text{ is } \varepsilon$	10 Self-Iden
12. [C]	$(\text{n } \alpha)(\text{zero } \delta)(\delta\text{-added-to-}\alpha \gamma)(\text{successor-of-}\gamma \beta)$ $(\text{n } \eta)(\text{successor-of-}\eta \varepsilon) \beta \text{ is } \varepsilon$	7 11 Subst-Iden
13.	$(\text{zero } \alpha)(\cdot\text{number } \beta) \alpha \text{ is } \beta$	AX1
14.	$(\text{zero } \alpha)^i (\text{any number } \beta)^i (\alpha\text{-added-to-}\beta \gamma)(\text{successor-of-}\gamma \delta)$ $(\text{it}^i \varepsilon)(\text{it}^i \eta)(\text{successor-of-}\eta \sigma)(\varepsilon\text{-added-to-}\sigma \tau) \delta \text{ is } \tau$	13 Added-to <sub>2</sub>
15. [D]	$(\text{zero } \alpha)^i (\text{n } \beta)^i (\alpha\text{-added-to-}\beta \gamma)(\text{successor-of-}\gamma \delta)$ $(\text{it}^i \varepsilon)(\text{it}^i \eta)(\text{successor-of-}\eta \sigma)(\varepsilon\text{-added-to-}\sigma \tau) \delta \text{ is } \tau$	5 14 UA+
16. [D]	$(\text{zero } \alpha)(\text{n } \beta)(\alpha\text{-added-to-}\beta \gamma)(\text{successor-of-}\gamma \delta)$ $(\text{zero } \alpha)(\text{n } \eta)(\text{successor-of-}\eta \sigma)(\varepsilon\text{-added-to-}\sigma \tau) \delta \text{ is } \tau$	15 Sing Ante
17. [D]	$(\text{n } \beta)(\text{zero } \alpha)(\alpha\text{-added-to-}\beta \gamma)(\text{successor-of-}\gamma \delta)$ $(\text{zero } \varepsilon)(\text{n } \eta)(\text{successor-of-}\eta \sigma)(\varepsilon\text{-added-to-}\sigma \tau) \delta \text{ is } \tau$	16 permute
18. [E]	$(\text{zero } \varepsilon)(\text{n } \eta)(\text{successor-of-}\eta \sigma)(\varepsilon\text{-added-to-}\sigma \tau)$ $(\text{n } \eta)(\text{successor-of-}\eta \delta) \tau \text{ is } \delta$	12 17 trans of identity
19.	$\text{not } (\text{n } \delta)(\text{successor-of-}\delta \beta)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\beta \gamma)(\text{it}^i \eta) \delta \text{ is } \eta$	4 Permute
20.	$\text{not } (\text{n } \delta)(\text{successor-of-}\delta \beta)(\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\beta \gamma)$ $(\text{n } \delta)(\text{successor-of-}\delta \eta) \delta \text{ is } \eta$	19 Sing Ante
21.	$\text{not } (\text{zero } \varepsilon)(\text{n } \delta)(\text{successor-of-}\delta \beta)(\varepsilon\text{-added-to-}\beta \gamma)$ $(\text{n } \delta)(\text{successor-of-}\delta \eta) \delta \text{ is } \eta$	20 Permute
22.	$(\text{any } \{\text{number which}_\alpha^i (\text{zero } \beta)(\beta\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta\} \beta)$ $(\text{successor-of-}\beta \alpha)^i (\text{zero } \varepsilon)(\varepsilon\text{-added-to-}\alpha \gamma)(\text{it}^i \delta) \gamma \text{ is } \delta$	18 21 Reductio

It is apparent that, although completely rigorous, if this derivation were pronounced in natural language it would be extremely difficult to follow.

### 9.5.3 Multiplication

The usual axioms for multiplication (often called “recursive definitions”) in modern notation look like this:

$$n \times 0 = 0 \quad n \times m' = (n \times m) + m$$

In our notation they are:

Times<sub>1</sub> (any number  $\alpha$ )<sup>i</sup>(zero  $\beta$ )( $\alpha$ -multiplied-by- $\beta$   $\gamma$ )(zero  $\delta$ )  $\gamma$  is  $\delta$   
*any number multiplied by zero is zero*

Times<sub>2</sub> (any number  $\alpha$ )<sup>i</sup>(any number  $\beta$ )<sup>j</sup>(successor-of- $\beta$   $\epsilon$ )( $\alpha$ -multiplied-by- $\epsilon$   $\gamma$ )  
 (it<sup>i</sup> s)(it<sup>j</sup>  $\eta$ )(s-multiplied-by- $\eta$ )(it<sup>i</sup>  $\tau$ )( $\eta$ -added-to- $\tau$   $\rho$ )  $\gamma$  is  $\rho$   
*any number multiplied by any number's successor is itself multiplied by it added to itself*

i.e. *any number multiplied by any number's successor is it [the first number] multiplied by it [the second number] added to it [the first number]*

Without exploring theorems at this point, it may be useful to see a few things that can be expressed in our notation. Here are some definitions:

(One  $\alpha$ ) (zero  $\gamma$ )(successor-of- $\gamma$   $\alpha$ )  
*one zero's successor*

(Two  $\alpha$ ) (one  $\gamma$ )(successor-of- $\gamma$   $\alpha$ )  
*two one's successor*

PosNumber {number which <sub>$\gamma$</sub>  not (zero  $\beta$ )  $\gamma$  is  $\beta$ }  
*positive (nmbr) number which is not zero*

$n < m$  ( $n$   $\alpha$ )( $\cdot$ Posnumber  $\gamma$ )( $\alpha$ -added-to- $\gamma$   $\delta$ )( $m$   $\beta$ )  $\delta$  is  $\beta$   
 *$n$  added to a positive number is  $m$*

$n$  is prime (no {PosNumber which <sub>$\gamma$</sub>  not (one  $\beta$ )  $\gamma$  is  $\beta$ }  $\epsilon$ )  
 ( $\cdot$  {PosNumber which <sub>$\gamma$</sub>  not (one  $\beta$ )  $\gamma$  is  $\beta$ }  $\eta$ )( $\epsilon$ -multiplied-by- $\eta$   $\delta$ )( $n$   $\alpha$ )  $\delta$  is  $\alpha$   
 *$n$  is prime no positive number which is not one multiplied by a positive number which is not one is  $n$*

$n$  is even ( $\cdot$  number  $\beta$ )(two  $\gamma$ )( $\beta$ -multiplied-by- $\gamma$   $\delta$ )( $n$   $\alpha$ )  $\delta$  is  $\alpha$   
 *$n$  is even a number multiplied by two is  $n$*

Every even number is the sum of two primes:

(every {number which <sub>$\gamma$</sub>  is even}  $\alpha$ )( $\cdot$ {number which <sub>$\gamma$</sub>  is prime}  $\delta$ )  
 ( $\cdot$ {number which <sub>$\gamma$</sub>  is prime}  $\epsilon$ )( $\delta$ -added-to- $\epsilon$   $\eta$ )  $\alpha$  is  $\eta$   
*every number which is even is a number which is prime added to a number which is prime*

Of course, much more than this is needed to demarcate the logical power of medieval logic. What has been given here is only a first step.

# 10

## Ampliation and Restriction

### 10.1 Univocation as the source of ampliation and restriction

Aristotle uses ‘equivocation’ for a case in which things are called by the same name but the name does not have the same definition in each case. An example is ‘*dog*’, which can apply to one thing because it is a canine, and to another because it is a certain star. So if you have inferred that something is a dog, and is not a dog, you haven’t inferred contradictory propositions<sup>1</sup> unless ‘*dog*’ has the same definition in each proposition.

According to Boethius,<sup>2</sup> there is a way in which apparently contradictory propositions are not actually contradictory, even when you use the same word with the same definition in each application. His illustration is:

[A] man walks	<i>Homo ambulat</i>
Man does not walk	<i>Homo non ambulat</i>

These propositions are not contradictory if ‘*homo*’ in the first proposition stands for an individual man, and in the second proposition refers to the species man. This is not equivocation, because the word ‘*homo*’ does not have different definitions in the two propositions. Boethius calls this phenomenon “univocation.” Some medieval treatises were written explicitly about univocation. Their understanding is that univocation occurs when a word is used twice with the same signification, but what it stands for (its “supposition” or its “appellation”) changes. This kind of explanation is given e.g. in the (Anonymous) *Treatise on Univocation*, second paragraph:

Univocation therefore is when the appellation of a name varies and the signification remains the same.

And in (Anonymous) *About Univocation* (562) we read:

Univocation is [a case of] the supposition of a name having varied, the signification having remained the same.

<sup>1</sup> Equivocation is discussed along with other fallacies in Aristotle’s *Sophistical Refutations*. A “refutation” is achieved when the respondent admits a proposition that contradicts the proposition being defended. So the relation of contradictoriness is central to that essay.

<sup>2</sup> Boethius’s commentary on Aristotle’s *On Interpretation*, cited in de Rijk *LM II.I.XV.1* (492).

When a word is used as in our example to stand for the species, this is called *simple supposition*. When a word is used to stand for itself or for a related expression, as in ‘*Donkey is a noun*,’ this is called *material supposition*. Normal kinds of supposition, including all of the cases discussed so far in this book, are called *personal supposition*.<sup>3</sup> Even when words are used personally it is still possible for univocation to occur: their supposition may vary—be “restricted” or “ampliated” differently—when their signification does not vary. That is the topic of the present chapter.

In a non-modal present tense categorical proposition, common terms typically do not supposit for everything that they signify. Instead they supposit only for the presently existing things that they signify. As a result, in

*Every donkey is running*

the subject is restricted to suppositing for presently existing donkeys. If all of them are now running, then the proposition is true, even though there were or will be donkeys that are not now running. Similarly, in the proposition:

*Socrates is a philosopher*

the predicate is restricted to presently existing things. Since Socrates does not presently exist, the term ‘*Socrates*’ does not supposit for something for which the predicate supposits. (The predicate only supposits for presently existing things.) So the affirmative proposition is false.<sup>4</sup>

This restriction of supposition in a present tense non-modal categorical proposition to presently existing things is an example of the ‘Restriction’ in ‘Ampliation and Restriction.’

Terms may also supposit for things in addition to their presently existing significates. In

*Some donkey was running*

the subject term is taken to stand for present or past donkeys;<sup>5</sup> the proposition is true if one of them was running sometime in the past. This expansion of what the terms may supposit for is the ‘Ampliation’ in ‘Ampliation and Restriction.’ It is somewhat arbitrary what is to be called restriction and what is to be called ampliation. So long as terms supposit only for things that they signify, one may call any use of a term a restriction if the term does not supposit for everything that it signifies. And any use of a term to supposit for things outside of some reference class of things, such as the presently existing things, can be called an ampliation. The custom is generally to say that terms are restricted when they are required to supposit exactly for the presently existing

<sup>3</sup> For discussion of these types of supposition see Parsons 2008a, section 4.

<sup>4</sup> Some writers held that singular terms are also restricted by a tense (we return to this later). If that is so the proposition in question is false also because the subject term is empty.

<sup>5</sup> My wording here is meant to be ambiguous. Some writers took the term to supposit disjunctively for all present and all past donkeys; others took the sentence to be ambiguous with the term suppositing for present donkeys on one reading and for past ones on the other reading. See section 10.2.2.

things that they presently signify, or for some subset of these, and to call any extension of this range of things ampliation.

Some kinds of restriction and ampliation are:<sup>6</sup>

1. Supposition restricted to presently existing things. (All previous chapters have taken this to be the default case.) Examples are:

*Every donkey is grey*

*No donkey is a stone*

In these cases the underlined words are “restricted” to the presently existing things that they signify. This restriction is caused by the present tense on the verb, when the verb itself is not special (we return to this later).

2. Supposition for past things (for formerly existing things) or for future things. Such supposition is caused by past and future tenses, as in:

*A donkey was grey*

*A donkey will be grey*

In these examples, the underlined words supposit for present or past or future things (details to be given shortly). Such ampliation is also caused by past or future participles used as predicates with present tense copulas, as in ‘*A donkey is dead*’ {*Asinus mortuus est*}.<sup>7</sup>

3. Supposition for never-existing but possible things, caused by an alethic modal word:

*A mountain can be golden*

*Every pink donkey is necessarily an animal.*<sup>8</sup>

Since no past, present, or future mountain is golden, ‘*mountain*’ must supposit for possible mountains in addition to actual ones in the first proposition. Likewise, one must be discussing all possible pink donkeys in the second. This ampliation is caused by the modal words ‘*can*’ and ‘*necessarily*.’

4. Supposition for impossible but conceivable things, caused by words that “pertain to the soul”:

*A chimera is believed to be an animal.*

*A bishop conceives [of] a donkey which is a stone.*<sup>9</sup>

<sup>6</sup> Cf. Anonymous, *About Univocation*, 563–4. The theory is developed in detail in Anonymous, *Treatise on Univocation*, sections 1 and 2, and Anonymous, *On the Properties of Discourse*, in the second major section, “Concerning Appellation.” It is discussed in Buridan, *SD* 4.6 and in chapter 5 of his treatise on *Sophismata*. It is mentioned by every writer on the properties of terms.

<sup>7</sup> In English ‘*dead*’ is not a participle; here it translates ‘*mortuus*,’ which is a participle in Latin.

<sup>8</sup> The examples given here are to be read *de re*, not *de dicto*. In Aristotelian terminology, the examples are given the “divided” reading, not the “composed” reading.

<sup>9</sup> The subject term is not amplified in this last example, since it occurs with a present tense non-modal verb. The verb pertains to an act of the soul, and this affects the supposition of terms that come after it, but the subject of such a verb is not thereby amplified.

Authors disagree about the truth values of these cases, but they seem to agree with the semantic mechanism. Certain verbs such as ‘believe’ or ‘want’ are naturally used with terms that appear to supposit for things that are not possible. Since it is impossible for a chimera to exist, one must take the first proposition to contain supposition for not just actual or possible chimeras, but for impossible ones (if there are any) too. Some authors reject this ampliation because they hold that there are no impossible things, and so terms cannot supposit for them. For simplicity, I will interpret these latter authors as agreeing with the semantic principle that in such contexts the correct semantics is to let the terms supposit for absolutely all things that they signify,<sup>10</sup> and this will include impossible things if, and only if, there are any. The sentences will then be taken to have different truth values for those who disagree about whether there are any impossible things. For example, Buridan does not accept impossible beings, so he would consider the proposition about chimeras to be false; since chimeras are impossible, there aren’t any chimeras to have beliefs about. The subject term is amplified to supposit for impossible things, but since there aren’t any impossible things, the subject is empty and the sentence is false.

The following subsections will discuss temporal ampliation by tenses, ampliation to the merely possible by (alethic) modal words, and ampliation to all things whatsoever by semantic verbs or by verbs which indicate an act of the soul.

The medieval accounts discussed here are remarkable in that they assume that tenses, modals, etc. (when given divided readings<sup>11</sup>), although they affect what the terms in the proposition supposit for, do not change the logical forms of the propositions in which they occur. A sentence of the form:

*Every A is/was/will be/can be/is thought to be understood*

is a universal affirmative proposition, and as such it is true iff the subject has supposition, and everything for which the subject supposits is something for which the predicate supposits. Likewise:

*Some A is not/was not/will not be/can not be understood*

is a particular negative proposition, and it is true iff either the subject lacks supposition, or it supposits for something that the predicate does not supposit for. Although the past tense shows up on the copula ‘was’ in ‘*Some donkey was grey*’, it has no effect on the verb; the verb just represents identity in every case. The verb’s tense only affects (ampliates or restricts) what the subject and predicate terms supposit for. Likewise (apparently) for modal propositions: ‘*every donkey possibly runs*’ is true iff every possible donkey *is* (tenselessly) a possible runner. And ‘*every chimera is conceivable*’ is true

<sup>10</sup> Not just verbs that pertain to the soul do this; according to many writers, semantic words such as ‘signify’ and ‘supposit’ ampliater terms following them so that they supposit for everything that they signify. See section 10.6.

<sup>11</sup> The divided reading is similar to what we call today the *de re* reading. E.g. Some A is such that it can be that it is understood.

iff every chimera—possible or not—is (tenselessly) something able-to-be-conceived. (The term ‘conceivable’ does the ampliating here because it pertains to the soul.)

## 10.2 Ampliation and restriction by tenses

### 10.2.1 Tenses

For tenses, the account usually goes:

In a present tense proposition both the subject and predicate terms supposit only for the presently existing things which they presently signify.<sup>12</sup>

In a past tense proposition the subject supposits for presently and formerly existing things presently or formerly signified by the term, whereas the predicate supposits only for formerly existing things formerly signified by the term.<sup>13</sup>

In a future tense proposition the subject supposits for presently existing things presently signified by the term and things that will exist and will be signified by the term, whereas the predicate supposits only for things that will be signified by the term that will exist.

This account is found as early as the 12th century and it is preserved throughout the rest of the medieval tradition. The present wording is similar to Ockham’s (*SL* 1.72, response to the first difficulty).

### 10.2.2 A Complexity: Ambiguity or disjunction?

A sentence like ‘*Some bishop was running*’ can be true in two different ways. In one way it is true if something that is a bishop now was running in the past, even if it was not a bishop then. The other way is that something which may not be a bishop now (indeed, may not even exist now) was a bishop in the past and ran in the past. Theorists agree that there are these two options. They disagree, however, about whether this is because the sentence is ambiguous between these two readings (as Ockham says<sup>14</sup>), or whether the sentence is unambiguous and has disjunctive truth conditions (as much of the

<sup>12</sup> I am ignoring a rather bizarre view, discussed by very many authors, according to which a term that does not signify any presently existing things “reverts to non-existents”; that is, it supposits for past and future things that it signified or will signify. (According to some authors, this even happens when the term fails to signify at least three presently existing things.) Ockham famously argues against this view in *SL* 2.4 (97–9).

<sup>13</sup> This is the commonest option. In Anonymous, *Properties of Discourse* (726) it is held that when the predicate term is a substantive (though not when it is an accidental term, such as an adjective) it supposits both for presently existing and formerly existing things. In Anonymous, *Treatise on Univocation* (345) a view saying that accidental terms are never amplified is mentioned, and rejected.

<sup>14</sup> Ockham, *SL* II.22 (158): “every past-tense or future tense proposition in which the subject is a common term must be distinguished as equivocal . . . For if the proposition is past-tense, then the subject can supposit for that which is such-and-such or for that which was such-and-such.” Sherwood, *IL* 5.16.3 seems also to hold that the construction is ambiguous, due to an ambiguity of composition and division.

tradition says<sup>15</sup>) so that it is univocally true in either of these cases. I don't know how to choose between these options.<sup>16</sup> I will discuss the disjunctive truth condition option; it should be relatively straightforward to turn this account into one that appeals to ambiguity instead of disjunction.

Theorists generally assume that this dual interpretation is not available for the predicate. The proposition '*A bishop was running*' must be made true by something that ran in the past, not by something that only runs at present. So the subject and predicate of a past tensed sentence are affected differently by the tense. The predicate is amplified/restricted so as to supposit only for things which it formerly signified. The subject however is amplified so as to supposit for things which it signifies with respect to the present or with respect to some past time.

### 10.2.3 *Coordination of times in tensed sentences*

How tenses work in natural language is a complex matter, which is made difficult by unclarity in the data—that is, unclarity due to speakers of the language not associating clear truth conditions with tensed sentences. As a result, it is not easy to assess the ultimate success of the medieval theory. It conflicts with some standard paradigms of late 20th-century tense logic, but it is often not completely clear which view better matches ordinary language usage. Since the intent is to develop the semantics of a somewhat artificial regimented use of Latin, I will be reluctant to draw many firm conclusions about its adequacy in terms of capturing Latin usage.

#### 10.2.3.1 COORDINATION OF TIMES BETWEEN SUBJECT AND PREDICATE

Consider the sentence:

*A bishop was grey*

This is true iff something that is now or was sometime a bishop was sometime grey. One part of these truth conditions is clearly right: if some present bishop was grey previously, whether it was then a bishop or not, the sentence has a true reading. But suppose that something was once a bishop, and was once grey, although that thing was never a bishop *while* being grey. In this situation the medieval reading comes out true. It is unclear to me whether this accords with how speakers of natural language treat such sentences.<sup>17</sup> In any event, these are the official truth conditions given to the sentence by the theory.

<sup>15</sup> Buridan, *SD* 4.5.2 (293): “a term put before the verb appellates its form in a disjunctive manner, for the present and for the tense of the verb.” Paul of Venice, *PL* II.8 (58): “every term standing in initial position and with respect to a verb of past time or its participle is amplified in order to stand for that which is or which was, e.g. in “white was black,” “white” does not stand for that which was white precisely or alone for that which is white; but in disjuncts [it stands for] that which is or was white.”

<sup>16</sup> I thus endorse the agnostic view that Burley states (*Longer Treatise*, para. 209): “a proposition about the past in which a common term supposits has two causes of truth, or two senses of an ambiguity.” (Later, at para. 225, Burley takes the second option to be the right one.)

<sup>17</sup> I think it is closer to actual current English usage than many present logicians think. Of course, this may be because speakers tend to speak loosely, and perhaps the meanings of their sentences should not reflect that looseness.



A typical 20th-century tense logician might treat the sentence as if it is ambiguous, having the following three readings:<sup>18</sup>

$\exists x(\text{bishop } x \ \& \ \text{Past}\{\text{grey } x\})$   
 $\exists x(\text{Past}\{\text{bishop } x\} \ \& \ \text{Past}\{\text{grey } x\})$   
 $\text{Past}\{\exists x(\text{bishop } x \ \& \ \text{grey } x)\}$

The last option is not a reading that the medieval theory attributes to the sentence. On the medieval reading, the proposition cannot be read so as to entail that there is a time in the past such that the subject and predicate both stand for the same thing then; this is because the ampliation of the subject and predicate terms are independent of one another. This is a limit on the theory as articulated in the texts. To many modern philosophical logicians this will seem wrong. Others will hold that the simultaneity of past times of being a bishop and of being grey is at most a possible implicature of the sentence, and not a constraint on any of its readings.<sup>19</sup>

This lack of coordination of times of subject and predicate was sometimes featured by “truths” like:

*A virgin was pregnant*  
*A boy was an old man*  
*An old man will be a boy*

The first is true because someone who was a virgin sometime in the past was pregnant sometime in the past. And so on. These examples and ones like them are featured by many different medieval writers,<sup>20</sup> who seem to take them to be obviously correct.

### 10.2.3.2 TENSES WITH RELATIVE CLAUSES

We have not yet said how to handle complex terms with relative clauses occurring in tensed sentences. If we follow our instructions so far, we end up with an amplified complex term, but with no account of how the amplified complex term is to be understood in terms of how its parts work. In the case of modification by a relative clause there are two natural options. Option 1: We suppose that ampliating a complex term made with a relative clause ampliates only the simple term that is modified by the relative clause. The terms in the relative clause itself, including the relative pronoun itself, are already taken care of, since that clause has a verb of its own, and the tense on that

<sup>18</sup> These would be read: Some present bishop is such that there is a past time at which it was grey. For some presently existing thing, it was a bishop at some earlier time, and it was grey at some earlier time. At some earlier time it was the case that something was then a bishop and was then grey.

<sup>19</sup> In favor of the latter, one might note that it is coherent to say ‘*A bishop was grey until just before he became bishop*’. However, when ambiguity is at issue, reasoning of this sort is usually inconclusive. This is because of the difficulty in telling whether the additional clause with ‘*until*’ cancels an implicature of a non-ambiguous proposition, or merely eliminates one of the readings of an ambiguous proposition.

<sup>20</sup> E.g. by Paul of Venice, *LP* II.8 (59), who also gives ‘*A prostitute will be a virgin*’, ‘*A decapitated person will sing*’. These examples were staples of the theory, and their apparent incorrectness was used in the early 16th century by Juan Luis Vives (Guerlac 1979) to ridicule scholastic theorizing.

verb takes care of ampliating or restricting the terms in it, including the relative pronoun. This may be how the medieval theory was intended to work. Option 2: Like option 1 but we suppose that the relative pronoun itself is amplified by the same verb that ampliates the common noun that is modified by the relative clause. The contents of the relative clause, of course, will restrict what the relative pronoun supposits for, and this, in affirmative cases, will have much the same effect as restricting the relative pronoun based on the verb within the clause. But in some cases there will be a difference. Notice that option 2 is equivalent to the principle that a relative pronoun is amplified as is its antecedent, holding (stipulating?) that the antecedent of a relative pronoun is the common term modified by the relative clause. Marsilius of Inghen suggests this, and also suggests that the rules proposed for ampliation and restriction needn't apply to anaphoric terms.<sup>21</sup> This suggests option 3: relative pronouns are never restricted (or amplified) at all. An example is the following sentence. It has a main verb in the past tense, and its subject term contains a relative clause in the future tense.

*A donkey which will be grey was brown*

The options are that (1) 'which' is amplified by 'will be' to present and future things, but it is restricted by the clause following it to future grey things only; the sentence is true if a donkey that was brown is a future grey thing; (2) 'which' is amplified by 'was' to stand for present and past things, and restricted by the clause to stand for things which will be grey; the sentence is true if a donkey that was brown is a thing which will be grey; (3) 'which' is not amplified at all; it is restricted by its clause to stand for things which will be grey; the sentence is true if a donkey that was brown is a thing which will be grey.

The treatment just given accords with the view that a proposition with a past or future tense verb can be expounded in terms of one with a relative clause that spells out the effects of ampliation. Marsilius of Inghen, *TPT*, 115:

in the proposition *a man will be*, the term *man* is amplified to have supposition for men who are or will be. Therefore the sense is as follows: 'a man who is or will be, will be.' Similarly in the proposition *a man will be generated*, the term *man* supposits for those that are or will be. For the sense is: 'what is or will be, will be generated.'<sup>22</sup>

If the contents of the relative clauses were amplified by the tense on the main verb of the relative clause, the relative clauses would be obviously redundant. (Of course, if the rephrasals are truly equivalent to the originals, the relative clauses must be actually redundant. The point is supposed to be that the paraphrases are informative.)

<sup>21</sup> Marsilius of Inghen, *TPT*, 91: "From this it is evident that the masters' rules on ampliatiions and restrictions are to be understood to apply to non-relative [i.e. non-anaphoric] terms."

<sup>22</sup> Buridan, *SD* 4.6.2 also gives examples: "From these rules it follows that such propositions are expounded by propositions with disjunctive subjects. For example, 'A will run' is equivalent to 'What is or will be A will run.' Similarly, 'A is dead' equals 'What is or was A is dead.' Similarly, 'A can run' equals 'What is or can be A can run.' Similarly, 'The one creating is of necessity God' equals 'What is or can be the one creating is of necessity God.' Similarly, 'A is thought of' equals 'What is or was or can be A is thought of.'"

There remains a question about the coordination (or lack thereof) of times within the complex phrase. Consider:

*A bishop which was grey was running*

This sentence is true according to the account iff there is some present or past time at which someone was a bishop, and some past time at which that person was grey, and some past time at which that person was running. It is not required that the times of being a bishop and being grey are the same. This may, arguably, be one correct reading of the sentence.

Buridan discusses a simple example which looks on the surface to be a clear counterexample to the theory:

*Everything which will be is*<sup>23</sup>

The proposed truth conditions are that every presently existing thing which exists now or will exist in the future exists now. Because the ‘*thing*’ is restricted to presently existing things by the present tense of ‘*is*’ the sentence is true. This seems to be a straightforward application of the theory.<sup>24</sup> Many readers will think the sentence should be false, since they will take it to say that every future thing exists now. Buridan anticipates this problem and suggests that people who think this are mistakenly reading the sentence as saying:

*Whatever will be is*

This seems to be a universally quantified “headless” relative clause, followed by ‘*is*.’ Since only the head noun of a relative clause is restricted by the tense of the main verb, and since it lacks a head, the ‘*what*’ is unaffected by the tense of the main verb, and so it supposits for every present or future thing, and the whole proposition is false so long as something exists in the future which doesn’t exist now—which is the reading we are inclined, perhaps wrongly, to attribute to the first example.

Buridan seems to have a different idea about the grammatical structure of the latter proposition. He says:

the word ‘whatever’ is here construed with the verb ‘will be,’ and it is necessary to add in thought a relative [i.e. an anaphoric pronoun] which should be construed with the ‘is,’ so that one has ‘Whatever will be, that is.’ And so the term ‘whatever’ is amplified to future things, and, consequently,

<sup>23</sup> Buridan, *SD, Sophismata*, chapter 5, eighth sophism, 925. Buridan has a somewhat intricate discussion. The point at issue is what things the complex term ‘*thing which will be*’ supposits for. I think that his point is that the ‘*thing*’ is initially restricted to the present by the verb ‘*is*’ and that the ‘*which*’ is restricted to the present-or-future by the ‘*will be*,’ and since each restricts the other, they both supposit (modifiedly) for presently existing things. As a consequence, the complex term ‘*thing which will be*’ supposits for presently existing things. The Linguish logical form displays the preliminary supposition for the individual terms (not yet restricted by each other), and the truth conditions for the whole relative clause are then equivalent to Buridan’s.

<sup>24</sup> Paul of Venice, *PL IV.2* (85) gives several similar examples, including e.g. ‘*everything which was, is*.’ He presents these as correct results of logical theory.

so is its relative, namely, ‘that.’ And thus we have counterinstances to the proposition ‘Whatever will be is’ in all future things which not yet are. (*SD, Sophismata*, chapter 5, eighth sophism, 925)

Buridan here seems to be suggesting that the ‘*whatever will be*’ is not construed with the main verb at all, and this would seem to give it no grammatical role in the proposition at all. Since it is not the subject of the main verb, you need to insert an anaphoric pronoun into the sentence to be subject. This would give the sentence a grammatical structure like that which quantified sentences in modern logic have: there is an initial (restricted) quantifier which has no grammatical role in the sentence it has scope over; it only affects the interpretation of the anaphoric expressions in that sentence that it is linked to. We could easily add such “quantifiers” to the Linguish notation. They would lack a marker, since they do not occupy a grammatical role in the sentence they combine with, and we are using markers only to indicate grammatical roles. The Linguish sentence would be:

(every {which<sub>γ</sub> will be})<sup>i</sup>(that<sup>i</sup> α) α is  
*whatever will be, that is*

where we generate ‘*whatever*’ (by stipulation) from ‘*every which*’, and where ‘*that*’ is a pronoun like ‘*it*.’

Having explained what I take Buridan’s view to be, I admit that I don’t know why he holds it. All that seems to be necessary to get an intuitively plausible reading is to deny that the tense of the main verb affects ‘*whatever*.’

### 10.2.3.3 COORDINATION OF TIMES AMONG PARTS OF THE SUBJECT OR PREDICATE

Buridan says that in certain cases parts of the subject or predicate must all be treated as simultaneous (*SD, Soph.* 3.1 (887)). An example is *I was eating an apple in Paris*, where, he says, there must be some time in the past such that I was in Paris then, and I was then eating, and it was an apple then that I was eating. This is a natural intuition, but it is not clear how this idea can be combined with the fundamental theory. The problem is that ‘*eating-thing*’ and ‘*in Paris*’ and ‘*apple*’ are all independent terms. The first stands for all past eating things, the second for all things that were in Paris, and the third for all things that were apples. And they all can be independently quantified: *I ate every apple in Paris*, meaning *I some-past-time-ate every some-past-time-apple that was some-past-time-in Paris*. The issue is how to make this turn into *At some past time I then-ate every then-apple that was then-in Paris*. This would require some development of the semantics; whether it would be a revision or a refinement is unclear. But it is clear that this is not how the current theory goes.

### 10.2.3.4 SUBCLAUSES AND COORDINATION OF TENSE

There is another way in which tenses are coordinated in natural language. Consider the sentence:

*Socrates complained that Plato hit him.*

This seems to have a reading in which the past tense in the subordinate clause (the tense of ‘*hit*’) is parasitic on the tense of the main clause (the tense of ‘*complained*’). On this reading, the sentence alleges that there was a past time at which Socrates made a complaint, and the content of the complaint concerns something that took place prior to the complaining. Not prior to *now*, but prior to the time of the complaining.

Note that the sentence contains an embedded that-clause. The medieval semantic account we have of this seems to be that it is true iff Socrates is a present or past thing who is a past complainer regarding a certain proposition: one that existed in the past and was of the form:

*Plato hit Socrates*

A proposition of that past tense form evaluated at a certain time would say that Plato hit Socrates at some time before then (before the time at which it was uttered). So the whole sentence is true iff at some past time Socrates complained, and his complaint was the content of a mental proposition existing then saying in the past tense that Plato hit him. Thus the coordination of tenses seems to work out exactly right.

The reading where one takes the past tense in the subordinate clause to be past relative to the past tense in the main clause is one option for the “past-under-past” reading. Some sentences with the past tense in both clauses have another reading. For example, this proposition:

*Socrates thought that Euthyphro was a fool*

has a natural reading according to which at some time in the past Socrates had a thought whose content was that Euthyphro was *then* a fool. This may be taken care of by stipulating that a past tense in a subordinate clause sometimes behaves semantically as a present tense. That is, the sentence is true iff Socrates at some time in the past thought a mental proposition whose content was that of:

*Euthyphro is a fool*

(The general rule of thumb seems to be that past-of-past reading occurs with event sentences and the present-of-past occurs with state sentences.) The interaction of tenses in main and subordinate clauses is not well understood. There is some hope of the medieval account doing as well in capturing the semantics of sentences with embedded subclauses as most modern theories. However, this topic was not discussed in detail by medieval writers.

#### 10.2.4 Buridan’s special use of appellation

The theory we have been discussing makes what a term supposits for in a sentence dependent on its signification and the time of utterance. It is patterned after Ockham’s view in that it makes use of a temporally relevant relation of signification. For example, in a past tense sentence the word or concept ‘grey’ supposits for things that it signified at some earlier time. Buridan has a different view. For him, signification is a relation

between a term and some things, and this relation does not vary with time.<sup>25</sup> For example, the concept donkey signifies all donkeys, past, present, and future, and never-existing but possible donkeys (if there are any), and even impossible but conceivable donkeys (if there are any—Buridan thinks that there aren't any). This is how an *absolute* concept works; it does not vary its signification with time. To fill in the picture we need to distinguish between simple concepts and complex concepts. Examples of simple concepts are genus and species concepts such as “donkey,” as well as differentia, such as risibility for humans. It is important to Buridan's account that it is not possible for an entity to fall under one of these concepts at one time and under another at another time; e.g. it is not possible for an animal to become a stone, or for a donkey to become a horse. If you're a donkey at any time, you're a donkey throughout your existence; you cease being a donkey only by ceasing to be. A simple mental concept, or a word that is a sign of a simple concept signifies things absolutely, that is without respect to time. For words that indicate simple concepts we can then simplify these conditions from section 10.2.1 by eliminating the underlined words:

In a present tense proposition both the subject and predicate terms supposit only for presently existing things which they presently signify.

In a past tense proposition the subject supposits for presently and formerly existing things presently or formerly signified by the term, whereas the predicate supposits only for formerly existing things formerly signified by the term.

In a future tense proposition the subject supposits for presently existing things presently signified by the term and things that will exist and will be signified by the term, whereas the predicate supposits only for things that will be signified by the term that will exist.

The underlined terminology was intended to deal with terms such as ‘*white-thing*’ or ‘*monarch*.’ For Buridan, these terms are not absolute; they are signs of *appellative* concepts. This means that their supposition is relative not just to what they signify, but also to what they appellate, and how. The simplest case of this is when a term has a nominal definition, that is, a complex phrase which is synonymous with the term in question where the relations among the parts of the phrase show how those simple concepts combine to make the complex one.

Some examples will be helpful. The word ‘*white*’ expresses an appellative concept. It applies to a thing at a time if that thing has a whiteness in it then. The term thus signifies whitenesses, but it does not supposit for them; instead it supposits for things (substances) which have a whiteness in them, not for whitenesses. Since having or not having a whiteness occurs in time, what the term supposits for varies with time, as expected, even though the conceptual part of the term, ‘*whiteness*,’ does not signify with

<sup>25</sup> Buridan, *SD Soph.*, chapter 2, sixth conclusion: “every present, past, and future man is signified indifferently by the term ‘man,’ for it signifies without time.”

respect to times; once a whiteness always a whiteness (so long as it exists). The time factor becomes relevant in explaining the supposition of the term by means of the account of relative clauses in Chapter 5. So that in a present tense sentence ‘*white*’ means ‘*thing which a whiteness is in,*’ and in a past tense sentence it means ‘*thing which a whiteness is or was in.*’ The word ‘*vacuum*’ is also an appellative term; it has the nominal definition ‘*place not filled with body.*’ Presumably both ‘*place*’ and ‘*body*’ are simple terms that signify apart from time.

It seems that on this theory the notion of signification is only needed for simple concepts, because the supposition of a term that expresses a complex concept is completely determined by its parts. Buridan, however, has much to say about this. In rough terms, his view is that an explicitly or implicitly complex expression signifies everything that is signified by any of its parts, with the exception of those things for which it supposits. So that ‘*white*’ signifies all whitenesses, while suppositing for white things, and ‘*vacuum*’ signifies all places and all bodies, etc. And ‘*chimera*’ signifies several things without suppositing for anything at all. This even extends to whole sentences, so that ‘*A donkey is an animal*’ would be said to signify all donkeys and all animals, and the sentences ‘*God is God*’ and ‘*God is not God*’ both signify the same thing, namely God. So far as I can see this extension of the notion of signification to such complex expressions does not interact with anything else in the logic. More work needs to be done to understand the significance of these ideas.

### 10.2.5 *Tenses in Linguish*

Adding tenses to Linguish is fairly straightforward, because the account itself is clear. First, we add tenses to the verbs of the language. That is simplest if we just stick to using familiar words such as ‘*is,*’ ‘*was,*’ ‘*will be,*’ ‘*sees,*’ ‘*saw,*’ ‘*will see,*’ . . . , where the tense is obvious from the spelling. We then embellish the terms in the logical form of each proposition indicating what effect the tense of the verb has on its supposition. That is, we prefix a sign to the term indicating how its supposition is restricted or amplified. This notation is strictly redundant, since restriction and ampliation is predictable from other ingredients of the sentence, such as the tense; it only makes it easier to keep track. One can then take the embellished term to be a kind of term in its own right which has supposition independent of its context.

If the main verb is in the present tense, we embellish the subject and predicate terms of that verb with ‘ $\approx$ ’ to indicate that the embellished term is to supposit<sub>o</sub> at each time *t* for exactly those things existing at *t* that the unembellished term supposits<sub>o</sub> for at *t*.

If the main verb is in the past tense, we embellish the subject term of that verb with ‘ $\leq$ ’ to indicate that the embellished term is to supposit<sub>o</sub> at each time *t* for exactly those things that the term embellished by ‘ $\approx$ ’ supposits<sub>o</sub> for at *t* or some time prior to *t* (when it existed), and we embellish the predicate term of that verb with ‘ $<$ ’ to indicate that the embellished term is to supposit<sub>o</sub> at each time *t* for exactly those things that the term embellished by ‘ $\approx$ ’ supposits<sub>o</sub> for at some time prior to *t* (when it existed).

If the main verb is in the future tense, we embellish the subject term of that verb with '≥' to indicate that the term is to supposit<sub>σ</sub> at each time *t* for exactly those future things that the term embellished by '≈' supposit<sub>σ</sub> for at *t* or some time after *t*, and we embellish the predicate term of that verb with '>' to indicate that the term is to supposit<sub>σ</sub> at each time *t* for exactly those things that the term embellished by '≈' supposit<sub>σ</sub> for at some time after *t*.<sup>26</sup>

The verbs themselves work as follows:

Copulas of any tense relate a thing *n* and a thing *m* iff *n* tenselessly is *m*.<sup>27</sup>

For this reason I will sometimes write the copula in a logical form using the identity sign.

Other verbs are not explicitly discussed in the literature, but it is easy to see how they would go. Consider a transitive verb *V* as an example, and let *V*<sub>past</sub>, *V*<sub>present</sub>, and *V*<sub>future</sub> be its tensed forms. The proposition '(§1 α)(§2 β) α *V*<sub>present</sub> β' is the same as before:

'(§1 α)(§2 β) α *V*<sub>present</sub> β' is true<sub>σ</sub> at *t* iff σ(§1) bears the relation that 'V' signifies to σ(§2) at *t*.

For the other tensed forms:

'(§1 α)(§2 β) α *V*<sub>past</sub> β' is true<sub>σ</sub> at *t* iff σ(§1) bears the relation that 'V' signifies to σ(§2) at some time before *t*.

'(§1 α)(§2 β) α *V*<sub>future</sub> β' is true<sub>σ</sub> at *t* iff σ(§1) bears the relation that 'V' signifies to σ(§2) at some time after *t*.

(These are the conditions that result from making a verb equivalent to its participle-plus-copula form when the substantivated participle binds the marker following the copula.)

<sup>26</sup> These conditions work as intended only if the term is univocal across time. Without such a constraint, a term could supposit for certain unintended past things because the word used to have a different meaning. For example, '*Roses stank*' would be true because '*stink*' used to supposit for roses (it used to supposit for roses because it used to mean '*smell strongly*'). It is not completely clear how to avoid this problem. One attempt is this: A term either is or is subordinated to a mental term (concept) *T*; in a past tense proposition the term supposit for things presently existing and presently signified by *T*, or things previously existing and previously signified by *T*; etc. This assumes that a concept cannot change what it is a concept of as time goes by. Probably that assumption is hard to justify within the medieval framework. A concept is an accident in the soul of the person who possesses it, and so it is subject to the persistence conditions of accidents. But what things it signifies depends on what it represents; and that depends in turn on its connections with the environment. Perhaps they could change without it becoming a new accident. Or perhaps not. This goes beyond matters discussed in the logic texts with which I am familiar.

<sup>27</sup> This interpretation of the theory is based primarily on the fact that tensed identity is not appealed to in the texts in stating the semantics. In general, it would be redundant to appeal to tensed identity, since the terms in question are themselves confined to certain times. One might wonder however how to analyze '*Marcus will be Tully*' if one takes the option that singular terms are not restricted or amplified. The theory as I have stated it would make this true since both singular terms would supposit for the same thing. But one might think that it should be false because *Marcus/Tully* do not now exist. Here is where one might want to have tensed versions of the copula as well. But this is speculation.



It is convenient to take the absence of an embellishing symbol to abbreviate embellishment by ‘≈’. If we do this, then all of the Linguish logical forms discussed in previous chapters are already properly embellished, and no changes are needed in them. (I will occasionally use the ‘≈’ to call attention to the restriction to the present.)

This theory relies on identifying which term is subject and which predicate. I assume that in Linguish logical forms, the subject term is the one whose denoting phrase binds the marker preceding the verb, as discussed in section 6.2. This means that a surface sentence whose verb is a third-person singular form of the copula will usually be ambiguous (in Latin, anyway) regarding which term is subject and which predicate. That did not matter previously when discussing only non-ampliative contexts, since the readings are equivalent. With ampliative contexts the readings are not equivalent, and so for surface sentences the ambiguity matters.

Some additional things can be said concerning even simple examples. One point is that as soon as tenses other than the present are allowed into the symbolism, many arguments that are valid when confined to the present tense are no longer formally valid in other tenses. An example is simple conversion:

$$\begin{array}{ll} \text{Some white was black} & (\text{some} \leq \text{white}_\alpha)(\cdot < \text{black}_\beta)_{\alpha=\beta} \\ \therefore \text{Some black was white} & (\text{some} \leq \text{black}_\alpha)(\cdot < \text{white}_\beta)_{\alpha=\beta} \end{array}$$

Suppose that something is white now, and it was black in the past, and that nothing was white in the past. Then the premise is true and the conclusion false, for the conclusion requires a past white thing and the premise does not.

Interestingly, all of Aristotle’s direct syllogisms (the direct ones from figure 1 plus all of figures 2 and 3) remain valid when past or future tense is uniformly added (providing that there are no non-tense sources of ampliation—to be discussed later). An example is past-tense Barbara:

$$\begin{array}{ll} \text{Every B was C} & (\text{every} \leq \text{B}_\alpha)(\cdot < \text{C}_\beta)_{\alpha \text{ was } \beta} \\ \text{Every A was B} & (\text{every} \leq \text{A}_\alpha)(\cdot < \text{B}_\beta)_{\alpha \text{ was } \beta} \\ \therefore \text{Every A was C} & (\text{every} \leq \text{A}_\alpha)(\cdot < \text{C}_\beta)_{\alpha \text{ was } \beta} \end{array}$$

When tenses are added to a direct syllogism the embellishments yield an argument with four terms, with the “middle terms” related as ‘≤F’ and ‘<F’ or as ‘≥F’ and ‘>F.’ One could easily construct rules of proof for the new arguments if it were possible to express these relationships. But that would require propositions such as:

$$\begin{array}{l} (\text{every} < \text{F}_\alpha)(\cdot \leq \text{F}_\beta)_{\alpha \text{ is } \beta} \\ (\text{every} > \text{F}_\alpha)(\cdot \geq \text{F}_\beta)_{\alpha \text{ is } \beta} \end{array}$$

and these are not correctly embellished, and would not be correctly embellished even if ‘is’ were changed to ‘was’ or ‘will be.’

The indirect figure 1 moods are all formally invalid. For example, this form of Baralipton can have true premises and a false conclusion if there are present Ps but no past ones:

Every M was S      (**every**  $\leq M_\alpha$ )( $\cdot$   $< S_\beta$ )  $\alpha=\beta$   
 Every P was M      (**every**  $\leq P_\alpha$ )( $\cdot$   $< M_\beta$ )  $\alpha=\beta$   
 $\therefore$  Some S was P      (**some**  $\leq S_\alpha$ )( $\cdot$   $< P_\beta$ )  $\alpha=\beta$

(Embellishing these forms yields a syllogism with six distinct terms. It’s an uphill fight to get validity out of that many terms.)

These facts hold for the disjunction treatment of tense. If past- and future-tensed propositions are held to be ambiguous as outlined earlier then validity of the direct forms would depend on how the ambiguity is resolved. Sometimes resolving them in one way yields a good inference while resolving them in another way yields an invalid one. Ockham discusses extensively how various of these options bear on conversions in *SL* 2.22 (158–62), including options where  $<$  conversion is valid if a relative clause is introduced, as in ‘No white thing was a man; therefore nothing which was a man is white.’

We might consider how to expand the rules for Linguish given in Chapters 4–5 to include propositions with embellished terms. One change that needs to be made is that we need an expanded condition for when a term is non-empty. We could define this by cases, including e.g.

$\approx P$  is non-empty      (**some**  $P_\alpha$ )( $\cdot$   $P_\beta$ )  $\alpha$  is  $\beta$   
 $< P$  is non-empty      (**some**  $\leq P_\alpha$ )( $\cdot$   $< P_\beta$ )  $\alpha$  was  $\beta$   
 $\leq P$  is non-empty       $\approx P$  is non-empty or  $< P$  is non-empty<sup>28</sup>  
 $> P$  is non-empty      (**some**  $\geq P_\alpha$ )( $\cdot$   $> P_\beta$ )  $\alpha$  will be  $\beta$   
 $\geq P$  is non-empty       $\approx P$  is non-empty or  $> P$  is non-empty

Reductio is a basic rule that needs no alteration. Likewise, conditions for when two propositions are contradictories or contraries, the rule of double negation, and all of the quantifier equipollences are unchanged. If singular terms are susceptible to ampliation and restriction, permutation for singular terms with other denoting phrases still holds, though somewhat similar processes are no longer valid—namely, those that change which term is the subject and which the predicate. This, for example, is a good inference:

*A runner Socrates was*      ( $\cdot$   $< \text{runner}_\beta$ )( $\leq \text{Socrates}_\alpha$ )  $\alpha$  was  $\beta$   
 $\therefore$  *Socrates a runner was*      ( $\leq \text{Socrates}_\alpha$ )( $\cdot$   $< \text{runner}_\beta$ )  $\alpha$  was  $\beta$

But this is fallacious:

*A runner Socrates was*      ( $\cdot$   $\leq \text{runner}_\alpha$ )( $< \text{Socrates}_\beta$ )  $\alpha$  was  $\beta$   
 $\therefore$  *Socrates a runner was*      ( $\leq \text{Socrates}_\alpha$ )( $\cdot$   $< \text{runner}_\beta$ )  $\alpha$  was  $\beta$

<sup>28</sup> This use of the connective ‘or’ can be avoided by using inference rules, as I have done throughout. The rules could be (taking ‘ $\leq$ ’ as an example):

$\approx P$ is non-empty	$< P$ is non-empty
$\therefore \leq P$ is non-empty	$\therefore \leq P$ is non-empty
$\leq P$ is non-empty, $\approx P$ is not non-empty	$\leq P$ is non-empty, $< P$ is not non-empty
$\therefore < P$ is non-empty	$\therefore \approx P$ is not non-empty

If Socrates existed in the past without running and exists and runs now, the first is true, but the second is false.

Interestingly, the reverse inference seems to be valid:

*Socrates a runner was*                     $(\leq \text{Socrates}_\alpha)(\cdot < \text{runner}_\beta)_\alpha \text{ was } \beta$   
 $\therefore$  *A runner Socrates was*             $(\cdot \leq \text{runner}_\alpha)( < \text{Socrates}_\beta)_\alpha \text{ was } \beta$

This is because in order for the premise to be true Socrates must have run in the past, and so he must have existed in the past, so changing ' $\leq \text{Socrates}$ ' to ' $< \text{Socrates}$ ' does not provide an additional constraint.

So we have the following new rule for tense permutations:

### Tense permutation

If  $m$  and  $n$  are singular terms and  $P$  a common term, these inferences are valid:

$(\leq m_\alpha)(\cdot < P_\beta)_\alpha \text{ was } \beta$   
 $\therefore (\cdot < m_\beta)(\cdot \leq P_\alpha)_\alpha \text{ was } \beta$

$(\leq m_\alpha)(\cdot < n_\beta)_\alpha \text{ was } \beta$   
 $\therefore (\cdot < m_\beta)(\leq n_\alpha)_\alpha \text{ was } \beta$

Similarly for the future tense.

There are additional constraints on Exposition. For example, taking a past tense form for illustration, this form of Exposition is OK:

$(\text{Some } \leq A_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$   
 $\therefore (\cdot < n_\beta)(\cdot \leq A_\alpha)_\alpha \text{ was } \beta$                     EX  
 $\therefore (\leq n_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$                     EX

But this is not even well formed:

$(\text{Some } \leq A_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$   
 $\therefore (\cdot < n_\alpha)(\cdot \leq A_\beta)_\alpha \text{ was } \beta$                     EX            ← not well formed  
 $\therefore (\leq n_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$                     EX

A similar constraint holds for Expository Syllogism, which, as usual, is the reverse of Exposition:

$(\cdot < n_\beta)(\cdot \leq A_\alpha)_\alpha \text{ was } \beta$   
 $(\leq n_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$   
 $\therefore (\text{Some } \leq A_\alpha)(\cdot < B_\beta)_\alpha \text{ was } \beta$                     ES

In its general form, Exposition is:

$(\text{Some } \leq A_\alpha) \phi$   
 $\therefore (\cdot < n_\beta)(\cdot \leq A_\alpha)_\alpha \text{ was } \beta$                     EX            provided that ' $\leq A$ ' is non-empty  
 $\therefore (\leq n_\alpha) \phi$                     EX            provided that ' $\leq A$ ' is non-empty

and Expository Syllogism is:

$$\begin{array}{l} (\langle n \beta \rangle)(\cdot \leq A \alpha) \alpha \text{ was } \beta \\ (\leq n \alpha) \phi \\ \therefore (\text{Some } \leq A \alpha) \phi \end{array} \quad \text{ES}$$

The derived rule, Universal Application, takes the form:

$$\begin{array}{l} (\text{every } \leq T \alpha) \phi \\ (\langle n \beta \rangle)(\cdot \leq T \alpha) \alpha \text{ is } \beta \\ / \therefore (\leq n \alpha) \phi \end{array}$$

Proof of UA:

1.	(every $\leq T \alpha$ ) $\phi$	
2.	( $\langle n \beta \rangle$ )( $\cdot \leq T \alpha$ ) $\alpha$ is $\beta$	
3.	not ( $\leq n \alpha$ ) $\phi$	
4.	(some $\leq T \alpha$ ) not $\phi$	2 3 ES
5.	not (every $\leq T \alpha$ ) $\phi$	4 Equipollence
6.	( $\leq n \alpha$ ) $\phi$	Reductio; 5 Contradicts 1

To illustrate the updated rules we give a proof of the past tense version of Barbara. The proof is like that given in Chapter 2, with an additional appeal to the new rule of Tense Permutation:

BARBARA

1.	(every $\leq B \alpha$ )( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	
2.	(every $\leq C \alpha$ )( $\cdot \langle B \beta$ ) $\alpha$ was $\beta$	
3.	(some $\leq C \alpha$ ) not ( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	
4.	( $\langle c \beta$ )( $\cdot \leq C \alpha$ ) $\alpha$ was $\beta$	3 EX
5.	( $\leq c \alpha$ ) not ( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	3 EX
6.	( $\leq c \alpha$ ) ( $\cdot \langle B \beta$ ) $\alpha$ was $\beta$	4 2 UA
7.	( $\langle c \beta$ )( $\cdot \leq B \alpha$ ) $\alpha$ was $\beta$	6 Tense Permutation
8.	(some $\leq B \alpha$ ) not ( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	5 7 ES
9.	not (every $\leq B \alpha$ )( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	8 Equipollence
10.	(every $\leq C \alpha$ )( $\cdot \langle A \beta$ ) $\alpha$ was $\beta$	reductio: 9 contradicts 1

Naturally, propositions in the future tense are to be handled similarly to those in the past tense.

Looking ahead, it is unclear whether a complete set of rules can be formulated for the system of Linguish as expanded here. This is because embellishments constrain the formation of formulas that can be used in stating the rules. It is not at all clear that all

valid arguments can be proved by derivations using some set of rules where the derivations consist entirely of correctly embellished formulas. (An example of a good derivation was given earlier in connection with Barbara.) I am not certain of much at this point. Additional work is needed here.

#### 10.2.5.1 IF SINGULAR TERMS ARE NOT SUBJECT TO RESTRICTION AND AMPLIATION

In section 4.3 it was noted that according to many earlier writers, singular terms are not subject to ampliation and restriction. In our framework, this means that they would not be embellished, so that they would be the same in all tensed propositions. I believe that everything that was said earlier about the embellishment option holds if singular terms are not embellished. Whenever there is only one singular term in a proposition, the embellishment of the other term is sufficient to require the special provisions discussed earlier. If all the terms in a proposition are singular, then the only difference now is that certain new rules are redundant. For example, the second pattern listed under “Tense Permutation” earlier is already provable when the terms are not embellished at all.

#### 10.2.6 *Infinitizing negation*

Our treatment of infinitizing negation from section 5.5 is:

The term ‘**non-T**’ supposit<sub>o</sub> (with respect to t) for everything that ‘T’ does not supposit<sub>o</sub> for (with respect to t).

But if we are to calculate the supposition of **non-T** from that of T in a sentence, we must know what T supposit<sub>o</sub> for there. We cannot assume that it supposit<sub>o</sub> for the presently existing things that it signifies, unless we can identify some cause of restricting the supposition to those present things. It thus appears that the tense of a verb must restrict or amplify the sub-term ‘T’ when ‘**non-T**’ occurs as a main term. In terms of our present methods, this means that embellishing the term ‘**non-T**’ is accomplished by embellishing the ‘T’ itself. This leaves the ‘**non**’ to act on the embellished supposition of ‘T’, as in the rule that we have stated. So in:

*A non-bishop was running*

the term ‘*bishop*’ supposit<sub>o</sub> for present and past bishops, and ‘*non-bishop*’ supposit<sub>o</sub> for absolutely everything that isn’t a present or past bishop. Since ‘*running*’ is amplified to include all past running things, only the supposita of ‘*non-bishop*’ that are past things are relevant to the truth of the proposition; other things don’t matter. When other sources of ampliation are present, the other supposita may be relevant; we return to these cases in section 10.4.

One might wonder whether the term should be doubly amplified/restricted, that is, the negated term and the whole negative term should both be amplified/restricted. So that, for example, in the sentence ‘A non-white thing will run’ the term ‘non-white’ is

required to supposit for presently existing and future things that are not presently nor will be white. This is certainly an option. We will see in section 10.4 that Buridan, at least, did not do this. His account appears to be as we have taken it to be in the stated rule (i.e. the rule that the predicate governed by ‘*non*’ is amplified or restricted, and not the whole complex predicate).

## 10.3 Ampliation by modal terms

### 10.3.1 *What are modal propositions?*

Today, a “modal proposition” is a proposition containing a modal word, like ‘*necessary*’ or ‘*possible*’ together with a non-extensional context, such as ‘*It is necessary that no donkey is a stone.*’ Such propositions are called modal by some in the medieval tradition; see Peter of Spain, *LS* 1.20–1.23; Ockham, *SL* 2.9 (109). Others, such as Buridan (*SD* 1.8.2 (67–70)) reserve the term ‘modal’ for a proposition in which the copula is *modified* by a word such as ‘*necessarily*’ or ‘*possibly*.’ There is no such modification in ‘*It is necessary that no donkey is a stone.*’ since the ‘*necessary*’ is a predicate of the whole that-clause, not a modifier of the copula. Such propositions containing embedded that-clauses<sup>29</sup> were taken to attribute some property, such as necessity or possibility, to the thing that the that-clause stands for. For many writers (e.g. Ockham) a that-clause typically stands for the proposition that constitutes the contents of the clause. This might be a written or spoken proposition, or, more frequently, a mental proposition (a sentence of mental language). A minority of writers also think that that-clauses stand for abstract non-linguistic things. Propositions with embedded that-clauses are not discussed (much) in this text. In any event, they are not considered to be modal for present purposes.

For Buridan, a real modal proposition is one in which a modal word applies to the copula. These come in at least three varieties.

- (i) Modal adverbs modifying the copula alone

*Some animal possibly-is grey*  
*Every donkey necessarily-is an animal*  
*Every donkey of-necessity-is an animal*  
*She possibly-is grey*

- (ii) Combinations with the helping verb ‘*can*’

*Some animal can be grey*  
*No donkey can be a stone*  
*She can be grey*

<sup>29</sup> When I speak in general of that-clauses I include the constructions of Latin that are typically translated into English as that-clauses. These are usually accusative-infinitive clauses, which also occur in restricted contexts in English. An example is the underlined clause in ‘She considers him to be a fool,’ in which the subject of the embedded clause (‘*him*’) appears in the accusative case and in which the verb (‘*to be*’) takes the infinitive form.

## (iii) Combining a modal adjective with the copula

The third sort is more complex, with a syntax different from that of English. Here a modal adjective together with a new copula combines with the existing copula, which assumes its infinitive form. The subject and predicate terms also take the objective case. Transliterations of some Latin examples are:

*Some animal possible is [to] be grey*

*No donkey possible is [to] be a stone*

*Her possible is [to] be a stone*

My own opinion is that examples of type (iii) are really accusative-infinitive constructions with raised subjects, as evidenced by the accusative case of their subjects.<sup>30</sup> But this is not the way that Buridan (and probably others) viewed them. So I will take them to be special cases of genuinely modal propositions.

### 10.3.2 Modal propositions: Semantics

Discussion in medieval texts leaves the truth conditions for modal propositions somewhat unclear. The unclarity that we face here has to do with the logical forms of such propositions. One option works fairly clearly, and matches most of the discussion in the texts. This is to make modal propositions behave semantically as much like temporal ones as possible. In particular, in modal propositions we assume that the copula itself is unaffected by the modal sign, and the terms themselves are amplified by the modal expression to possible things, or to necessary things. In particular:

The subject is amplified to possible things (including things that exist at any time).

The predicate is also amplified; what it is amplified to depends on the mode.

There is no other effect.

Examples:

*Every animal possibly-is [a] donkey* ≈ Every possible-animal is a possible-donkey

*Every animal necessarily-is [a] donkey* ≈ Every possible-animal is a necessary-donkey

Which things does a term like '*possible donkey*' supposit for? Presumably a possible donkey is an entity that is possibly a donkey. This will include all actual donkeys, past, present, and future—since modal words are explicitly said to amplify their supposition to the past, present, and future.<sup>31</sup> It will probably not include any other actual things, since e.g. a tree has a kind of essence that excludes being a donkey. (This depends on what view you have about necessary properties. I am assuming what I think is the most usual view, attributed to Aristotle.) Will it include never-existing

<sup>30</sup> E.g. an underlying form is '*It is possible for some animal to be grey*'; the '*some animal*' raises to replace the dummy word '*it*' to yield '*Some animal is possible to be grey*'. The underlying form is one that Buridan would not regard as being modal.

<sup>31</sup> Some theorists held that the ampliation is only to the present and future, since the past is necessary.

things that are possible donkeys? So far as I can see, the semantic theory is neutral on this. If there are such things, then ‘donkey’ signifies them, and it supposits for them in modal propositions. Buridan, *SD* 4.6.2 (299) says:

a term put before the verb ‘can’ or before the copula of a proposition about possibility [de possibili] in the divided sense is amplified to stand for possible things, even if they do not and did not exist. Therefore the proposition ‘A golden mountain can be as large as the Mont Ventoux’ is true.

So he seems to think that in some alethic modal contexts terms do stand for never-existing things.

Since existing Fs are included among the possible Fs, in a proposition of possibility the subject and predicate are amplified in the same way. As a result, conversions are possible (‘some A possibly is a B; therefore some B possibly is an A’), as are all of Aristotle’s syllogisms (e.g. ‘Every B possibly is a C, every A possibly is a B; therefore every A possibly is a C’). But conversion fails for propositions of necessity (‘some A necessarily is a B; therefore some B necessarily is an A’), although the direct syllogisms, but not the indirect ones, are valid (e.g. ‘Every B necessarily is a C, every A necessarily is a B; therefore every A necessarily is a C’). (This assumes that every necessary P is a possible P.)

What then about necessary donkeys, which are needed for the second sentence. This will include all actual donkeys if each donkey is necessarily a donkey. (Though there may be none, since perhaps something that is necessarily a donkey necessarily exists, as Buridan argues early in *TC* 4 (Latin page 112).) Again, it will probably not include any other actual things. Again, there is a question as to whether there are also non-actual necessary donkeys. Perhaps these coincide with the non-actual possible donkeys (on Aristotle’s view—though probably not on contemporary views). Again, that is a metaphysical view that the semantic theory is neutral on.

(Perhaps some of the uncertainty stems from the fact that one tradition has it that ‘necessarily’ means ‘essentially,’ and some later writers interpret it more like 20th-century philosophers do.)

### 10.3.3 Differences between medieval and modern readings

Medieval and modern logicians have different and conflicting paradigms. A clear example of this is given in Marsilius of Inghen, *TPT*, 119, discussing the ampliation of subject terms by modal words:

From this it is evident that a consequence from a proposition with *is* to a proposition with *can* by way of a distributive sign is never valid, as in the case: *every B is A, therefore every B can be A*. And the reason is that it is an argument from a less ample to a more ample suppositing term. In propositions of the present the term stands only for those things that are, because there is no ampliation. And in a proposition with *can*, it stands for every thing that is or can be.

An example would be:

*Every animal is a donkey*  
 ∴ *Every animal can be a donkey*



From a modern point of view, this argument is obviously valid, since when ‘*can*’ is read *de re* it has no effect on ‘*animal*’; ‘*animal*’ stands for the same in both propositions. But on the medieval reading the premise would be true if all actual animals were donkeys, but the conclusion would be false in that case, since it means essentially “every possible animal can be a donkey,” and so far as logic is concerned there may be possible animals, such as possible humans, which cannot be donkeys. Clearly the disagreement in theory depends highly on different intuitions about what the data are.

### 10.3.4 Modal propositions in Linguish

At a superficial level it is straightforward to add modal propositions to Linguish. For simplicity let us confine ourselves to necessity and possibility. We allow copulas to be modified by them, so the modified copulas now include ‘possibly-is’ and ‘necessarily-is.’ Either of these ampliates the subject of the sentence to possibles, indicated by diamond-shaped embellishments. They affect the predicate term differently. A couple of examples are:

(No  $\diamond$  donkey  $\alpha$ )( $\cdot \diamond$  stone  $\beta$ )  $\alpha$  possibly-is  $\beta$   
*No donkey possibly is a stone.*

(every  $\diamond$  donkey  $\alpha$ )( $\cdot \square$  animal  $\beta$ )  $\alpha$  necessarily-is  $\beta$   
*Every donkey necessarily is an animal*

The semantics of the embellished terms requires explanation, and this is a problem, because of uncertainty about how signification is to be understood. There are fundamentally two themes found in the texts (as already discussed).

- One theme is the view that Ockham appears to have. His explanation is that in a modal context a term supposits for what it “can signify” (*SL* 1.72).<sup>32</sup> Presumably things that a term *can* signify are in addition to those that it *does* signify.
- The other theme is governed by the idea, discussed earlier, that non-appellative terms are amplified to supposit for everything they signify which can exist, whereas the story is more complicated for appellative terms.

New rules of inference are needed to handle terms in modal contexts. I have not pursued these matters here because there is so little medieval discussion of the details that development would be mostly speculation. (One exception is Book 4 of Buridan’s *TC*.)

A special case: Medieval writers took some non-alethic words to create modal readings as well; these are modal because they *modify* the copula, and they amplify the terms in the proposition. A special case is ‘*always*,’ mentioned in Lambert (*PT* 6c)

<sup>32</sup> This requires a caution similar to that mentioned in connection with tenses: since written and spoken terms have their signification by convention, such a word *can* signify anything at all. Perhaps Ockham’s intent is better captured by saying something like “a word either is or is subordinated to a mental term (a concept) T, and in a modal context the term supposits for whatever T can signify.”

which ampliates the terms in the proposition to all times, though not to non-actual things.

## 10.4 Ampliation due to semantic words

Semantic terminology ampliates terms, just as do words that pertain to the soul. For example, ‘*signify*’ ampliates terms as widely as they can be amplified. This is clear from examples like ‘*In a position following “conceive,” “Dodo” signifies all dodos*,’ which is intended to apply to all dodos, not just to presently existing ones. This is consistent with no dodo presently existing. Some authors would accept “*chimera*” *signifies chimeras and no chimera is possible*.’ (Those who oppose this view do so because they think that there are no chimeras at all to be signified.) The same goes for ‘*supposit*,’ as in “*“dodo” supposits for dodos in past tense sentences*.’

One might want to ask about the ontological commitments of such a theory. The logic and semantics alone don’t commit you to much. Saying that ‘*donkey*’ signifies all donkeys is, by itself, a truism. Since it is an affirmative sentence it commits you to there being at least one actual, possible, or imaginable donkey. But no more. It is possible to state various commitments within the theory. For example, one might say ‘*Some donkey which is not will be grey*.’ This commits you to future donkeys that do not exist now. But only if you endorse it. If you thought that God had just destroyed all donkeys, and would not permit there to be any more, you wouldn’t assert that sentence. Likewise ‘*Some donkey which was not and which is not and which will not be is possible*’ commits you to possible donkeys that are not actual at any time; if you don’t believe in them, you needn’t endorse the proposition.

Consider however ‘*A dodo lived*.’ Presumably this is true, and so the semantics will say that in ‘*A dodo lived*’ the term ‘*dodo*’ supposits for dodos. However, one cannot argue from this to ‘*Some dodos are*,’ since in the first sentence the term ‘*dodo*’ is amplified to supposit for past dodos and in the second sentence it is not amplified. This is a classic case of the fallacy of univocation.

### 10.4.1 Looking ahead

In previous sections we have been studying propositions involving tenses and modalities. There has been much contemporary work in these areas, and the subject matter is relatively well behaved and relatively well understood. Much of this work falls under “intensional logic,” wherein logical structures can be given semantics involving alternative times or alternative (possible) worlds. The same cannot be said for propositions containing locutions for believing, wishing, saying, imagining, owing, and so on. These are all notions that medieval logicians classified as “pertaining to the soul.” In the remaining sections I will touch briefly on some of the many things they had to say in this area. We begin with ampliation, but things fairly quickly branch out. I will consider the following sections successful if I succeed in communicating a glimpse at some of the rich ideas that were discussed.

## 10.5 Ampliation due to words which pertain to the soul

This doctrine is simple: the use of words which pertain to the soul amplify certain terms so that they may supposit for everything that those terms signify (or did or will or can or can be imagined to signify).<sup>33</sup> For example, in *'The antichrist is opinable'* the adjective permits the subject to supposit for future things, so the subject is not empty, and presumably the term *'opinable'* is able to supposit for anything—past, present, future, possible, conceivable—that one may have an opinion about. Since one may have opinions about the antichrist, the proposition is true. *'Some person thinks of a gold mountain'* can be true because the verb ampliates the term *'gold mountain.'* However, *'Socrates thinks of the antichrist'* is not true when Socrates himself does not exist, since the subject is not included in the pertaining to the soul.<sup>34</sup>

Buridan, *SD* 5.6.8 notes that when the term *'believed'* is used, all syllogisms fail.

Buridan, *SD Sophismata*, chapter 5, 923 gives an example that applies this sort of ampliation to a negative term that uses infinitizing negation. Regarding the sophism *A NON-BEING IS UNDERSTOOD*, he says:

I respond that the sophism is false, for the term [*'NON-BEING'*] supposits for nothing. And this is clear in the following manner: for the verb *'to understand'* or *'to be understood'* ampliates supposition to past, and future, and even all possible things. Therefore, if I say, *'A being is understood,'* the term *'being'* supposits indifferently for every present or past or future or possible thing. But the rule is that an infinitizing negation added to a term removes its supposition for everything for which it supposited, and makes it supposit for everything for which it did not supposit, if there are any such things. Therefore, in the proposition *'A non-being is understood,'* the term *'non-being'* does not supposit for some present, nor for some past, nor for some future, nor for some possible being; therefore, it supposits for nothing; and so the proposition is false.

<sup>33</sup> Marsilius of Inghen, *TPT*, 125: "a term governed by terms that signify an inner act of the soul is amplified to supposit for what is or was or will be or can be or can be imagined to be. Verbs of this kind are: *to understand, to love, to think, to strive for, to long for, to foresee, to imagine, to know* and so on."

p. 111: "when it is asked if *void* is amplified in this proposition *the void is understood*, I agree. The reason is that it supposits for its significates with respect to different times, for this verb *is understood* ampliates it to have supposition for all the significates which it has. But it should be noted that it has significates corresponding to it with respect to different copulas, because it signifies what can be and what can be imagined to be."

Anonymous, *Properties of Discourse*, 729–30: "As when it is said *'Something is true,' 'Something is false,' 'Something is imaginable,'* that term *'something'* is restricted as much for an existent [thing] as for a nonexistent. And this happens because of this, that the form of the predicate is not joined to the content of the subject, but to something with respect to it. Hence when it is said *'A chimera is imaginable,'* the imaginability is not joined to the chimera, but to something with respect to the chimera. Whence it should be observed generally that whenever the form of a predicate is not joined with the content of a subject, although any restriction may have to be made through the form, no form is joined to it.

Again it should be noted that verbs pertaining to the approving power of the soul have the force of ampliating the supposition of a common term. As when it is said: *'Socrates praises a man,' 'A man disdains a man,' 'A man thinks about a man,'* that term *'man'* in each supposits as much for an existent as for a nonexistent. Similarly, nominals deriving from those verbs have the force of ampliating. As when it is said *'A man is laudable,'* that term *'man'* supposits for a nonexistent. And this happens because of this: that nothing is joined to the praised thing, but to be praised with respect to it"

<sup>34</sup> Marsilius of Inghen, *TPT*, 127: "in the proposition *I understand*, the pronoun *I* is not amplified because the act of understanding does not transmit to me."

(This was the example alluded to in section 10.2.10 regarding how ampliation works with negative terms.) He goes on to give a very similar proposition that seems almost equivalent to the one under discussion, but is true. It illustrates the lack of equivalence between ‘non-*P*’ and ‘*what is not P*’ in the context of ampliation. He argues:

And I say that ‘A non-being is understood’ and ‘What is not a being is understood’ are not equivalent, for by the verb ‘is’ you restrict the infinity [*infinitatem*] to present things. Therefore, the supposition for past and future [and possible] things remains, and thus this has to be conceded: ‘What is not [a being] is understood.’ If, therefore, we are to give an equivalent analysis of ‘A non-being is understood,’ then it will be the following: ‘What neither is, nor was, nor will be, nor can be is understood,’ and this is false, just as the sophism was. We should say the same about the proposition ‘A non-being will be.’ For it is false, although this is true: ‘What is not a being will be.’

But there is a strong objection to this way of settling the matter [*determinatio*]: for in the proposition ‘What is not a being is understood’ or even in ‘what is not a being will be’ the verb ‘is’ restricts the term ‘what’ so that it supposits only for things that exist [at present]. And then, since of everything that exists it is false to say that it is not a being, it follows that the whole subject ‘what is not a being’ implies contradiction, therefore, it supposits for nothing; e.g., the whole [phrase] ‘man who is not a man’ supposits for nothing.

Also, ‘something that is not a being’ supposits for nothing, but to say ‘what is not a being’ and to say ‘something that is not a being’ amount to the same; therefore, this is false: ‘What is not a being is understood’ or ‘What is not a being will be,’ for the subject supposits for nothing.

To this I reply as before, that these propositions are true. For this reason we have to realize that such a proposition, namely, ‘What is not will be,’ or ‘What is not is understood,’ is not complete, except on account of what is added [to it] in thought, and this is so on two accounts. First, because here the relative [pronoun] ‘what’ is posited without an antecedent. Secondly, there are two verbs here, for which a single [name in the] nominative [case] cannot provide a suppositum; therefore, we have to supplement it [as follows]: ‘What is not will be,’ that is, ‘Something that is not will be.’ But in this proposition, the term ‘something’ is not construed with the verb ‘is,’ but with the verb ‘will be’; therefore, it is not restricted to present things, but is amplified to future ones. Furthermore, although the relative [pronoun] ‘what’ [or ‘that,’ *quod*] is construed with the verb ‘is,’ nevertheless, it is not restricted by [this verb] to present things, for a characteristic feature of a relative [pronoun] is that it should supposit as does its antecedent. Therefore, the whole subject ‘something that is not a being’ does indeed supposit for something not present but future; therefore, the proposition is true. (*SD Sophismata*, chapter 5, 923–4)

Again, Buridan takes the proposition in question to have a grammatical form different from its apparent grammatical form. But he treats this case differently from the one about what will be. Patterned after that other case, we would expect Buridan to say that the actual grammatical form of ‘*What is not a being is understood*’ is: ‘*What is not a being, that is understood*.’ But this would not accomplish his purpose, for the ‘*what is not a being*’ in this sentence supposits for nothing (according to the theory), and the verb cannot alter this. Instead, he proposes ‘*something that is not a being is understood*.’ This would seem to have the same problem, but he argues not. He seems to say that ‘*thing*’ is

ampliated by the nature of the phrase ‘*is understood*’, which seems to be what the theory forces us to say. But then he argues that the relative pronoun ‘*that*’ has ‘*thing*’ as its antecedent, and pronouns with antecedents “should supposit as their antecedents do.” This then seems to require that we choose the option 2 semantics for relative pronouns as discussed in section 10.2.6, namely, the relative pronoun is not to be construed with the verb within the relative clause, at least so far as ampliation and restriction are concerned. Instead, it is amplified as is the noun that is modified by the relative clause. The relative clause still restricts the relative pronoun, but only by what it says. In the case in question, the relative pronoun is amplified to potentially supposit for absolutely everything, and the relative clause restricts these things to those that are not (present, actual) beings. The example then works as Buridan intends. In Linguish notation it is (using ‘ $\wedge$ ’ to indicate ampliation of a term for all of its significates):

(some { $\wedge$ thing  $\wedge$ which <sub>$\gamma$</sub>  not ( $\cdot \approx$ being <sub>$\beta$</sub> )  $\gamma$  is  $\beta$ } <sub>$\alpha$</sub> )  $\alpha$  is understood

Here the relative clause ‘ $\wedge$ which <sub>$\gamma$</sub>  not ( $\cdot \approx$ being <sub>$\beta$</sub> )  $\gamma$  is  $\beta$ ’ supposits for everything that is not a being (that does not presently actually exist); ‘ $\wedge$ thing’ supposits for everything; and so the subject term ‘{ $\wedge$ thing  $\wedge$ which <sub>$\gamma$</sub>  not ( $\cdot \approx$ being <sub>$\beta$</sub> )  $\gamma$  is  $\beta$ }’ supposits for everything that does not presently exist. Presumably, one of those is understood, and so the proposition is true.

The meaning of the caret shares a problem with that of the tense and modal embellishments discussed earlier. The two options from before would extend to the present case:

- On the first option, we would say something like: in the presence of a word pertaining to the soul, a term supposits for everything for which it signifies and for which it has signified and for which it will signify and for which it can signify and for which it can be thought to signify.
- On the second option, we say simply that in the presence of a word pertaining to the soul, a non-appellative term supposits for everything which it signifies (and the story for appellative terms needs articulation).

## 10.6 Promising and owing

Suppose that I promise to give you a particular horse, Blackie. Then most researchers took this fact to verify the sentence:

*A horse I promised you*

In this position, the term horse was generally thought to satisfy the conditions for determinate supposition, because we can infer:

*Horse<sub>1</sub> I promised you or horse<sub>2</sub> I promised you or horse<sub>3</sub> . . .*

and the original sentence follows from:

*This horse I promised you*

This position licenses substitution; if all and only horses are running, then it follows:

*A running thing I promised you*

The interesting case, however, is if I do not specify a particular horse, but I merely promise to give you some horse or other. Then most researchers took that to verify the sentence:

*I promised you a horse*

In this position it is not clear what mode of supposition the term 'horse' has. It is generally agreed that it does not have determinate supposition. And in particular the first sentence, '*A horse I promised you*' does not follow from it.

Some thought that in this position the term does not have personal supposition at all; it has simple supposition, as in '*Horse is a species*.' This is initially problematic because it seems that this would validate the sentence '*I promised you a universal*,' but others were not convinced of this inference, or were not bothered by it. Some opposed the analysis on other grounds: if what is promised is the universal, then the promise cannot be paid off by giving a particular horse. But this was subject to the rejoinder that giving a particular horse is *how* you pay off a promise when what is promised is a universal. This is explained in an early anonymous discussion:

It is customary to say that in this locution 'this one promises you a horse' the term 'horse' may have simple supposition. If that be true, that term 'horse' is not able to be taken for some inferior, but only for [a] common. Hence, whoever is thus obligated is not able to be released from the promise unless he were to give a horse in common. Which is impossible.

To this it should be said that this verb 'promise' brings in two acts, namely an act of obligating and an act of giving. That is evident in this exposition: 'I promise' is the same as 'I oblige myself to give.' But of these acts, one is mental, the other is corporeal. And on account of this, this verb 'promise' insofar as it is said with respect to those two reflections, reflects upon this term 'horse.' Because with respect to the act of obligating, which is an act of the mind, it reflects upon it simply, and with respect to this it confers on it simple supposition. But with respect to the act of giving, which is a reflection of some inferior, it reflects upon the same term personally. Hence whoever is thus obligated is able to give some particular horse and to be released from his/her promise. (Anonymous, *On Significant Words*, 610)

This analysis is suggestive, though it is not part of a systematic theory. More needs to be said.

Many people took the term instead to have personal supposition. Burley held that the term has determinate supposition here, but "under a disjunction." This is not very informative since he did not explain what it means to be determinate under a disjunction.

Burley may have had in mind merely confused supposition. Ockham took the term to have merely confused supposition, because the descent described earlier for determinate supposition is not satisfied. The term '*horse*' is confused by the verb '*promise*'

which pertains to the soul. The verb also ampliates the term at least to all future horses; otherwise there would be a possible impediment to the analysis. For Ockham thought that terms with merely confused supposition licensed descent to a disjunctive term, so that ‘*I promised you a horse*’ would entail:

*I promised you horse<sub>1</sub> or horse<sub>2</sub> or horse<sub>3</sub> . . .*

Suppose that there exist now only three horses,  $h_1$ ,  $h_2$ , and  $h_3$ . Then if the term were not amplified to the future, we could infer:

*I promised you  $h_1$  or  $h_2$  or  $h_3$  . . .*

This would mean that if I wait for other horses to be born, and give you one of them, there would still be a promise of mine that was unfulfilled, namely the promise to give you  $h_1$  or  $h_2$  or  $h_3$ . Ampliating to future horses avoids this problem.

Even this may not be enough, for the previous argument could perhaps be iterated, naming all future horses. So perhaps ampliation would have to be to possible horses as well. In fact, this was one of the standard views. In another early anonymous work, we see:

This verb ‘is promised’ has a force similar in extension to the foregoing verbs [i.e. verbs pertaining to enunciations]. For when ‘A horse is promised’ is said, this name ‘horse’ is held there confusedly, so that it not only draws its appellation around to all times, but also is related to those which are able to be. (Anonymous, *About Univocation*, 569)

Buridan gave an interestingly different solution which seems to grow out of the tradition of simple supposition. Buridan (and others) thought that when there is a verb pertaining to the soul a term that comes after the verb is made to “appellate” its own concept,<sup>35</sup> and the verb is sensitive to the concept. This provides a solution to a puzzle originated by Aristotle: Suppose that someone is approaching, but they are too far off for you to identify them. Then Aristotle thought, and medieval authors agreed, you do not know the one approaching. But if the one approaching is in fact your father, it would seem to follow that you do not know your father. Since the verb ‘*know*’ pertains to the soul, Buridan would construe the parts of the puzzle as actually having the form:

*You do not know the one approaching under the concept “approaching.”*

*Your father is the one approaching.*

*You know your father under the concept “father.”*

You may perhaps use the identity to substitute, but what you will get is:

*You do not know your father under the concept “approaching.”*

<sup>35</sup> This is something like Frege’s thesis that in such contexts words come to refer to their customary senses. But on Frege’s view such words cease to have their ordinary references, whereas on the medieval account their original reference is maintained.

Verbs have this effect only on words following them, so that the facts are:

*Your father you know.*

*The one approaching you know.*

Buridan proposes to apply this technique to the promising case, so that ‘*I promised you a horse*’ means something like ‘*I promised under the concept horse to give you a horse*.’ The case is similar with ‘*I owe you a horse*.’ In his *Sophismata* he argues:

Finally, concerning this matter, we posit the fifteenth sophism in this chapter, and it is a rather difficult one, namely, ‘I owe you a horse’ . . . And I posit the case that in return for some good service that you performed for me, I promised you one good horse, and that I obligated myself before a competent judge to give you one good horse. . . . (*SD Sophismata*, chapter 4, 907–8)

This sophism appears to be difficult. First, however, I lay it down that in the case posited I would owe you a horse, but then the question arises whether I owe you Blackie. And we should reply that this is not so, and also that by promising you a horse I did not promise you Blackie; for, as was said earlier, the verbs ‘promise,’ ‘owe,’ just like the verbs ‘know’ and ‘think,’ make the terms following them appellate their concepts. Therefore, a consequence is not valid in which the concept or predicate is changed after [the verb]; indeed, it does not appear to be a valid consequence either if we descend from the species to the individual without distribution. However, it should be added that it makes a great difference whether we place ‘horse’ before or after [the verb], for the aforementioned verbs, because of the appellation of the concept, somehow confuse the [supposition of the] terms that follow them, so that it is not possible to descend to the singulars by means of a disjunctive proposition. For example, this is not valid: ‘I owe you a horse; therefore, I owe you Tawny, or I owe you Blackie, . . .’ and so forth; for each [member of this disjunction] is false. But before [the verb] the term is not thus confused; therefore, it is possible to descend by means of a disjunctive [proposition]. Therefore, if ‘A horse is owed by me to you’ is true, then it follows that either Tawny is owed by me to you or Blackie is owed by me to you, and so forth. (*SD Sophismata*, chapter 4, 909)

At this point we have only touched on a substantial set of issues. They have been much discussed by others,<sup>36</sup> and I have no doubt that new insights await us in the future.

<sup>36</sup> See Ashworth 1974b, 1976; Klima 1993; Read 1985.



# Appendix

## Artificial Quantifiers in Early 16th-Century Logic

It seems to me likely that important new insights in logic might yet be provided by the detailed investigation of those four artificial quantifiers and of others that could be added.

(Broadie 1985, 68)

It is distinctive of medieval logic that it is formulated almost entirely in natural language. Authors did not feel the same need as Frege to abandon natural language and replace it with a specially designed artificial notation. However, towards the end of the 15th century (Ashworth 1978, 601), logicians began to introduce artificial signs to alter the meanings of otherwise purely natural language sentences.<sup>1</sup> These signs were designed to affect the quantificational status of terms. The signs were introduced apparently to increase the expressive power of the logical notation already in use. I will argue that in a certain sense this goal is accomplished; nonetheless, for any proposition containing such a sign, there is a mechanical way to convert that proposition into a logically equivalent proposition not containing such a sign. So in another sense the new signs did not increase the logical expressibility of the notation.

This discussion is *tentative* because I rely mostly on secondary sources, primarily the very useful essays Ashworth 1978 (hereafter *MQ*), Broadie 1983 (hereafter *GL*), and 1985 (hereafter *CJM*) in understanding both the theory of special signs and the goals that were supposed to be accomplished by their use. Further, the theory to which the artificial signs are added, as presented in the secondary sources, contains elements that do not fit together neatly, so one must make choices. For example, Ashworth reports that by the 16th century, tests for modes of supposition had evolved so as to consist of a set of equivalences. As an illustration, a term has distributive supposition if and only if the sentence in which it occurs is *equivalent to* a conjunction of sentences each of which has a singular term in place of the term in question. ‘*Donkey*’ has distributive

<sup>1</sup> These signs are ridiculed in Vives, *APD* 61 center.

supposition in ‘*Every donkey is running*’ because it is *equivalent* to ‘*Donkey<sub>1</sub> is running and donkey<sub>2</sub> is running and . . . and donkey<sub>n</sub> is running*’ (MQ 599). But theorists also held that ‘B’ has distributive supposition in ‘Some A is not B’ (MQ 600). And it is apparent that ‘Some A is not B’ is *not* equivalent to ‘Some A is not B<sub>1</sub> and some A is not B<sub>2</sub> and . . . some A is not B<sub>n</sub>.’<sup>2</sup> In this Appendix I ignore the equivalence constraint and go with the intended application, but additional study of the original texts could show this to be the wrong tack.

## 1. The signs

There are four artificial signs explicitly introduced to affect the mode of supposition of a term. They come with the following explanations:

Sign a Makes a term have merely confused supposition

Illustration: In ‘a A is not B’ the term ‘A’ has merely confused supposition<sup>3</sup>

Sign b Makes a term have determinate supposition

Illustration: In ‘*Every A is b B*’ the term ‘B’ has determinate supposition<sup>4</sup>

Sign c Used on a main term that occurs with two other main terms; makes the term have merely confused supposition relative to the first term and determinate supposition relative to the second

Illustration: In ‘*Of every A every B is c C*’ the term ‘C’ has merely confused supposition relative to ‘A’ and determinate supposition relative to ‘B’<sup>5</sup>

Sign d Used on a main term that occurs with two other main terms; makes the term have determinate supposition relative to the first term and merely confused supposition relative to the second

Illustration: In ‘*Of every A every B is d C*’ the term ‘C’ has determinate supposition relative to ‘A’ and merely confused supposition relative to ‘B’<sup>6</sup>

<sup>2</sup> In practice attention would be diverted from this problem because a primary focus was on *analyzing* sentences, and for this purpose rules were given regarding the order in which terms were to be replaced by conjunctions and disjunctions of singulars so as to analyze the original sentence. In ‘Some A is not B’ the rules required that ‘A’ be replaced first, yielding ‘A<sub>1</sub> is not B or A<sub>2</sub> is not B or . . . or A<sub>n</sub> is not B.’ In each resulting disjunct, the term ‘B’ has distributive supposition by the equivalence test, and so no difficulty arises. This does not mean that there is no inconsistency, but it does mean that ignoring it would not lead to trouble when doing analysis.

<sup>3</sup> Broadie, *CJM* 66. The example: ‘a man is not an animal’ is given in both *GL* 51 and Ashworth, *MQ* 601. (*a. homo non est animal.*)

<sup>4</sup> Broadie, *CJM* 67. The example: ‘Every man is b animal’ is given in both *GL* 51 and Ashworth, *MQ* 602. (*omnis homo est b. animal.*)

<sup>5</sup> Broadie, *CJM* 68. The example: ‘Of every man every ass is c animal’ is given in *GL* 52, and ‘Of every man every donkey is c. donkey’ in Ashworth, *MQ* 609. (*Cuiuslibet hominis quilibet asinus est c. asinus.*)

<sup>6</sup> Broadie, *CJM* 68. The example: ‘Of every man every ass is d animal’ is given in *GL* 53, where it is attributed to Lokert. Ashworth does not give an example.

Why were these signs introduced? One purpose was to fill out the theory of conversion. In the inherited tradition, universal affirmatives convert *per accidens*, that is, from

*Every A is B*

one can infer:

*Some B is A*

but they do not convert simply; '*Every A is B*' does not convert at all to a proposition that is equivalent to it. Apparently. But you can indeed get a proposition that is logically equivalent to what you started with if you use one of the new signs, as follows:<sup>7</sup>

*Every A is B*      $\Rightarrow$      *A* *B is every A*

Since the term '*B*' in the converted form has merely confused supposition, this in essence gives it scope to the right of the term '*A*'; that is, it is to be interpreted logically just like '*Every A is B*'. So the forms are equivalent.

Broadie (*CJM* 66) also suggests that the signs were introduced to make notation more economical. He suggests that:

*Each B is something that some A or other is not*

can be expressed more briefly as

*A* *is not B*

This is certainly more economical. In general, these signs let you express old contents with new combinations of symbols, and if a theory addresses, say, the order of signs—as does the theory of conversion—use of these signs can expand the existing theory.

## 2. What the signs mean

The explanations given in section 1 do not completely specify the meanings of the new signs. This is because giving a term a new mode of supposition typically affects the modes of supposition of other terms. For example, suppose that we change the mode of supposition of '*A*' from determinate to distributive in this proposition:

*Some A is B*

by changing its initial quantifier:

*Every A is B*

<sup>7</sup> This is described in Ashworth, *MQ* 603.

This obviously makes the other term, 'B,' change from determinate to merely confused supposition. Introducing one of the new signs appears to have a similar effect. However, the explanations of the meanings of the special signs given in section 1 do not describe the effects of those special signs on other terms in the proposition. So one must speculate about the exact meaning of propositions containing the new signs, based in part on the intended applications. I do have a speculation about the intended meanings of the special signs. I will explain the meanings of these signs by giving an algorithm for converting a proposition containing the sign into an equivalent proposition without special signs. I speculate that the effect of the sign on the proposition containing it is that the sign makes the proposition equivalent to the one yielded by the algorithm. The algorithm then needs to be tested by seeing if it gives results that accord with the logicians' intent.

The idea behind the algorithm is to first put the proposition in question into a kind of normal form, and then make the obvious alteration in that form.

### The algorithm

1. Preparation: Move the verb to the rightmost position. (This isn't always necessary, but it never hurts.)<sup>8</sup>
2. Preparation: Now eliminate negative signs by moving them to the right, using the well-known equipollences for this. E.g. in '*Some A not some B is*,' move the '*not*' to get '*Some A every B not is*.' And change '*Some A no B is*' to '*Some A every B not is*.'
3. Move terms (with any quantifier signs that accompany them) as follows:
  - a. Move a term with 'a' to the right of the first universal term to its right, if there is such a term.
  - b. Move a term with 'b' all the way to the left.
  - c. Move the term with 'c' to the left of the term next to it on its left.
  - d. Move the second term in the sentence to the front of the sentence, and move the first term in the sentence to the right of the term containing 'd'.
4. Finally, replace the moved special sign by '*some*' (or, equivalently, by nothing at all, thereby producing an indefinite construction).<sup>9</sup>

For the paradigms cited earlier, the results of applying this algorithm are as follows.

**The sign 'a':**     'a A isn't B'     $\Rightarrow$     '*every B some A isn't*'

<sup>8</sup> Based on examples cited in Broadie's texts, the logicians he discusses were comfortable with such verbal constructions.

<sup>9</sup> If more special signs were added, some of them might need to have their terms replaced by '*every*.' It just happens that the signs discussed all require something equivalent to '*some*'.

This gives ‘A’ merely confused supposition, as required.

**The sign ‘b’:** ‘Every A is b B’  $\Rightarrow$  ‘some B every A is’

This gives ‘B’ determinate supposition, as required.

**The sign ‘c’:** ‘Of every A every B is c C’  $\Rightarrow$  ‘of every A some C every B is’

This gives ‘C’ merely confused supposition, because of the preceding ‘every A.’ It does not have determinate supposition, but if the ‘every A’ were removed, or if one were to descend under it, ‘C’ would then have determinate supposition. I take it that this is what is meant by saying that ‘C’ has determinate supposition *relative to* ‘B.’

**The sign ‘d’:** ‘Of every A every B is d C’  $\Rightarrow$  ‘every B some C of every A is’

This gives ‘C’ merely confused supposition, because of the preceding ‘every B.’ It does not have determinate supposition, but if the ‘every B’ were removed, or if one were to descend under it, ‘C’ would then have determinate supposition. I take it that this is what is meant by saying that ‘C’ has determinate supposition *relative to* ‘A.’

### 3. The signs are, in a sense, logically dispensable

It is clear that if this algorithm works correctly, it is not logically necessary to introduce these special signs, since any proposition containing one of them is equivalent to a proposition not containing any of them. But this goes against the judgment of some theorists. In discussing Lokert’s proposal that ‘Every A is B’ has a subcontrary, Broadie says:

What is being said here is that ‘Every A is B’ has a subcontrary in which the ‘A’ has merely confused supposition and the ‘B’ has distributive supposition. Without the use of special quantifiers such a proposition cannot be constructed, and indeed much of the impetus in early sixteenth-century logic for developing the theory of special quantifiers derived specifically from the need to describe the transformation rules by which, given any proposition, its contrary, contradictory, or subcontrary opposite could be described. Using the special quantifier ‘a’ the subcontrary proposition described by Lokert can readily be constructed. It is

12. a A is not B.” (GL 170)

I think that sentence (12) is indeed a subcontrary of ‘Every A is B,’ and it is readily and simply constructed, but a subcontrary can also be constructed without the use of special signs. The proposition ‘Every B some A isn’t’ is equivalent to (12) by the algorithm, and in any event it is clearly a subcontrary of ‘Every A is B.’ So this particular claim of indispensability does not hold.

Later, Broadie describes Lokert’s attempt to find a contradictory of

30 a A is not B.

Lokert says that ‘ $\underline{a}$  man is not white’ has a contradictory whose predicate has determinate supposition. Broadie says:

This highlights one reason why, granted the employment of the  $\underline{a}$  operator, the  $\underline{b}$  operator had to be introduced, for without the  $\underline{b}$  operator there is no way, short of using a formula with a number of singular propositions, of saying what the contradictory of 30 is. (*GL* 174)

But again this can be done without special signs; the proposition ‘Some *B* is every *A*’ contains no special signs, and it is a contradictory of 30. Broadie does not quote Lokert himself as claiming that the new signs are indispensable, and I don’t know whether Lokert, or Major, or others make the claim. I think that the use of the signs is interesting even if they are not indispensable. I think that at this point we must regard their indispensability as an open question.

#### 4. A doubt: Certain examples do not work as advertised

There is an apparent problem with the theory as described, in that use of the sign ‘ $\underline{a}$ ’ does not always give its term merely confused supposition.

Broadie (*GL* 144) illustrates a descent from the proposition:

(xiv)  $\underline{a}$  A is not B

Assuming that there are only two Bs, he says that descent under (xiv) takes us to:

(xviii)  $\underline{a}$  A is not B<sup>1</sup> &  $\underline{a}$  A is not B<sup>2</sup>

(where ‘B<sup>1</sup>’ and ‘B<sup>2</sup>’ are singular terms standing for the two Bs). But this is problematic. The sign ‘ $\underline{a}$ ’ occurring in ‘ $\underline{a}$  A is not B<sup>1</sup>’ cannot do its job of conferring merely confused supposition on its term, because it is in a proposition that contains no other common terms, and merely confused supposition is not possible in such a proposition. Suppose, contrary-to-fact, that ‘A’ were to have merely confused supposition in

$\underline{a}$  A is not B<sup>1</sup>

Then, by definition of merely confused supposition, the term ‘A’ does not have determinate supposition; so no descent under ‘ $\underline{a}$ ’ in that proposition to a disjunction of propositions is possible. That is, this does not follow:

A<sup>1</sup> is not B<sup>1</sup>  $\vee$  A<sup>2</sup> is not B<sup>1</sup>  $\vee$  A<sup>3</sup> is not B<sup>1</sup>  $\vee$  . . . etc.

But since ‘A’ has merely confused supposition, it *is* possible to descend to a proposition with a disjunct term:

(A<sup>1</sup> or A<sup>2</sup> or A<sup>3</sup> or . . .) is not B<sup>1</sup>.

However, since 'B<sup>1</sup>' is a singular term, these two propositions are equivalent; the disjunction of propositions is equivalent to the proposition with a disjunct term.<sup>10</sup> So it can't be that descent is impossible to the first but possible to the second.<sup>11</sup>

I think that the solution to this is to ignore the fact that the words 'merely confused' appear in the standard definition of the sign 'a'. The meaning of the sign 'a' is correctly given by the algorithm in section 2; it has the effect of permuting its term with a universal one to its right— if there is one—with the term then becoming particular or indefinite. In its intended central uses, the term to its right has distributive supposition, and the term marked with 'a' ends up with merely confused supposition. But if the term on the right is singular, the term with 'a' may end up with determinate supposition. So the term ends up with merely confused supposition in the featured cases, but not in simpler cases. With this understanding, Broadie's analysis is correct, and there is no problem at all with the theory. It is just not completely accurate to say that 'a' always confers merely confused supposition on its term.

The same understanding is required by Broadie's claim (*GL* 169) that this equivalence is valid:

$$A^1 \text{ is } B \leftrightarrow A^1 \text{ is } \underline{a} B$$

Since the second 'B' cannot be permuted at all (and it cannot be given merely confused supposition) we must take this to be one of the applications of 'a' in which its term ends up having determinate supposition (which is essentially what the equivalence that Broadie claims requires).<sup>12</sup>

<sup>10</sup> This is assuming that the negation has narrow scope here. If it were interpreted with wide scope the proposition would be the denial of '(A<sup>1</sup> or A<sup>2</sup> or A<sup>3</sup> or . . .) is B<sup>1</sup>'. But then it would also be the denial of 'A<sup>1</sup> is B<sup>1</sup> or A<sup>2</sup> is B<sup>1</sup> or A<sup>3</sup> is B<sup>1</sup> or . . .', so it would be equivalent to 'A<sup>1</sup> is not B<sup>1</sup> and A<sup>2</sup> is not B<sup>1</sup> and A<sup>3</sup> is not B<sup>1</sup> and . . .' which would mean that 'A' actually has distributive supposition in the proposition from which descent is made, contradicting the assumption that it has merely confused supposition there.

<sup>11</sup> I am here presupposing the following analyses of the modes of personal supposition. These are attributed to John Major by Broadie, *CJM* 52–4 and *GL* 46–7:

Determinate supposition: descent to a disjunction and ascent from any disjunct

Distributive supposition: descent to a conjunction and no ascent from a single conjunct

Merely Confused disjunctive supposition: no descent to a disjunction, but descent to a disjunctive term and ascent from any disjunct.

(Actually, Broadie does not include the proviso under Merely Confused disjunctive supposition that the descent conditions for Determinate supposition are not satisfied. This is needed because otherwise 'A' would have merely confused supposition in 'Some A is B,' which is not a view that anybody held.)

<sup>12</sup> Actually, Broadie seems to claim that the predicate of the left-hand proposition has determinate supposition without having merely confused supposition, and the predicate on the right has merely confused supposition without having determinate supposition. (E.g. he speaks of a "transformation" of a term from one having determinate supposition to one with merely confused supposition.) But modes of supposition are defined in terms of ascent and descent, and if the equivalence is valid, the predicate terms must have the same mode(s) of supposition.

## 5. Another doubt: The paradigm use of sign ‘d’ is incoherent

Perhaps the major objection to my proposed algorithm is that if my analysis of the sign ‘d’ is applied to the paradigm given in section 1, the result is incoherent. This is because the paradigm illustration contains what in earlier chapters I have called a parasitic term; this is the “possessed” term in the genitive relation, namely, the term ‘*B*’ in the paradigm example:

*Of every A every B is d C*

If this is expressed in the Linguish notation, the example is:

$(\text{Every } A_\alpha)(\text{every } B\text{-poss-}\alpha\beta)(\underline{d} C_\gamma)_\beta \text{ is } \gamma$

If this is transformed as I suggest, you get:

$(\text{every } B\text{-poss-}\alpha\beta)(\text{some } C_\gamma)(\text{every } A_\alpha)_\beta \text{ is } \gamma$

where the ‘ $\alpha$ ’ in the initial denoting phrase is not bound. So the result of applying the transformation is not a meaningful proposition.

This may show that I haven’t given a correct way to analyze the special sign. But I think that a case can be made that the paradigm example itself is incoherent in terms of the theory itself, quite apart from the algorithm.

Before explaining this I need to say something about rules of priority of ascent and descent. It was commonly said that descent should be carried out under a determinate term first, if one is present, and then under a (wide) distributive term if no determinate one is present. Why *should* one do these things in this order? If other descents are valid, why is it wrong to do them? The answer, I think, is that when priority rules are under discussion, they are being proposed within an enterprise of *analyzing* propositions with quantified terms into conjunctions and disjunctions of propositions without quantified terms. For the purpose of analysis one needs a descent to an *equivalent* conjunction or disjunction. For example, one can analyze:

*Some A is not B*

as:

$[A_1 \text{ is not } B] \vee [A_2 \text{ is not } B] \vee \dots \text{ for all the } A_s$

And one can further descend under each conjunct to get:

$[A_1 \text{ is not } B_1 \ \& \ A_1 \text{ is not } B_2 \ \& \ A_1 \text{ is not } B_3 \ \dots \text{ for all the } B_s]$

∨

$[A_2 \text{ is not } B_1 \ \& \ A_2 \text{ is not } B_2 \ \& \ A_2 \text{ is not } B_3 \ \dots \text{ for all the } B_s]$

∨

$\dots \text{ for all the } A_s$



The result is a disjunction of conjunctions, which is true if and only if the original proposition is true. It is equivalent to the original proposition, which is required if the descents are to be seen as a kind of analysis of it. Now if we were to descend under ‘B’ first, and then ‘A,’ we would validly get a conjunction of disjunctions. But this could not be an *analysis* of the original proposition since it is not equivalent to it. So if we are looking for an equivalent proposition, we must descend first under the term with determinate supposition.

Now let me return to my argument that the paradigm example is incoherent. I base this on Broadie’s explanation (which seems right to me):

In

(18) Of every A every B is  $\underline{d}$  C

descent is made under B (which is distributed) before being made under C, and under C (which is determinate) before being made under A (which is distributed). (*CJM* 68)<sup>13</sup>

Broadie says this because of the rules for priority of descent: descent must be made first under a determinate term, if there is one; then under a distributive term. (Although ‘C’ is determinate *with respect to A*, it is not determinate simpliciter.<sup>14</sup>) However, this conflicts with another rule of the theory, which is that when terms are related as determinable and determinant, then descent should be made first under the determinant if both determinant and determinable have the same kind of supposition.<sup>15</sup> In our example, the determinant is ‘A’ and the determinable is ‘B,’ and both terms are distributed. So the analysis that Broadie proposes for this case is ruled out.<sup>16</sup> And I can see no better proposal.

I suspect that this problem does not lie with the theory in general, but with the selection of the particular paradigm illustration. The problem is not with the suggested meaning of the special sign ‘ $\underline{d}$ ,’ but it is a problem with the attempt to use this sign along with a *genitive* construction, a construction which involves a determinable/determinant (“possessed”/“possessor”) pair. If we change the example from a proposition with a genitive construction to one where the terms are not so related, the problem disappears. This requires a proposition with three main terms, none of them parasitic,

<sup>13</sup> Broadie’s wording seems to say that ‘C’ is determinate in ‘Of every A every B is  $\underline{d}$  C,’ which is not right; but if descent is made first under ‘B,’ then ‘C’ will be determinate in the results of that descent, and I think that this is what he means. In any event, I don’t see an issue here.

<sup>14</sup> Strictly, a term’s being determinate or merely confused relative to another term doesn’t obviously say anything at all about it’s being determinate or merely confused in the whole proposition. The theory may be indeterminate here. Broadie’s judgment about the statuses of the terms would be correct if my proposal is right, and I don’t see anything else more plausible.

<sup>15</sup> *CJM* 64. Also *GL* 52: “in order of descent, determinant has precedence over determinable.”

<sup>16</sup> Another example of this is John Major’s claim, reported in *GL* 240, note 4, that this equivalence is valid:

Of B every A is C  $\leftrightarrow$  Of  $\underline{d}$  B every A is C

The right-hand side of this is either ill formed, or else it means ‘Every A of B is C,’ where the first two items form a complex term; the result is well formed, but it is not equivalent to the left-hand side. I am not sure what to conclude about this.

and this can be accomplished by using a verb that takes both direct and indirect objects. An example would be:

*Every woman gives to every farmer d gift*

which would mean (on my proposed analysis):

*To every farmer some gift every woman gives*

(“For every farmer *d* there is a gift *g* such that every woman gives *g* to *d*.”) If we were to analyze this proposition by appeals to descent, one would descend first under ‘*farmer*’ (for ‘*gift*’ is merely confused and ‘*woman*’ has only narrow distribution), next under ‘*gift*’ (which would then be determinate), and finally under ‘*woman*.’ There would be no problem at all. The problem with the paradigm given by medieval authors is just that they used a parasitic term—which cannot have a mode of supposition at all<sup>17</sup>—as if it could have a mode of supposition.

## 6. Some examples from John Major

There are probably many other issues lurking in the unedited writings, which are voluminous. Here are a very few examples of uses of special signs taken from a collection of John Major’s works from the early 16th century.<sup>18</sup>

At fol lxiii “**Dubium hoc est satis difficile . . .**” Major is discussing how to produce the contradictories of various propositions. He proposes certain specific propositions to be the contradictories of certain given propositions. The propositions that he claims here to be contradictories are:<sup>19</sup>

*Of a man a donkey any rudible is*

*Of any man any donkey c rudible not is*

The algorithms from section 2 turn these sentences into:

*Of a man any rudible a donkey is*

*Of any man a rudible any donkey not is*

Considering quantifier equipollences it is apparent that these propositions are indeed contradictories. (He then goes on to discuss intricacies of ascent and descent.)

At fol lxxv “**Pro quo advertere . . .**” Major is again discussing what propositions are contradictories. He proposes these slightly complicated cases as contradictories:

*Of a man a donkey not is a donkey*

*Of any man any donkey c donkey is*

<sup>17</sup> See section 7.4.

<sup>18</sup> Throughout this section I am very indebted to Dr. Thomas Ward for assistance with the (unedited) Latin text.

<sup>19</sup> In all of the examples to follow that contain genitives one needs to guess which term is related as the “possessed” to the genitive term. When it is possible I have chosen the first term to the right of the genitive term. Otherwise the selection is made to parallel other examples.

Our algorithm turns these into:

*Of a man every donkey a donkey isn't*  
*Of any man a donkey any donkey is*

These are contradictories by the quantifier equipollences.

Right after that example he claims that these are contradictories:

*Of a man a donkey not is a donkey*  
*Of any man any donkey d donkey is*

Our algorithms turn these into:

*A donkey every donkey of a man isn't*  
*Any donkey a donkey of any man is*

These are contradictories by quantifier equipollences—or they would be if they were well formed. The problem is that each of them has a parasitic term that is not within the scope of the term it is parasitic on. So these sentences are not generable with the version of Linguish discussed earlier. If we ignore this difficulty, the algorithm seems to be working, in the sense that it produces examples that work as intended in the text.

At fol lxi “*Septimo arguitur sic . . .*” Major is considering the principle that you can always produce a contradictory of any proposition by placing a ‘*not*’ on its front (with scope over the whole proposition). This is of special interest regarding how special signs are to be understood. Consider the proposition ‘*not a man not is an animal*.’ (1) One way to understand this proposition is that the special sign ‘a’ is to act *locally*; it is to be understood in the context of the sub-proposition ‘a man not is an animal’, where it gives ‘*man*’ merely confused supposition; putting a ‘*not*’ on the front of that sub-proposition would produce a larger proposition in which the negation would operate on an occurrence of ‘*man*’ having merely confused supposition, making ‘*man*’ have distributive supposition in the whole proposition. (2) Another way to understand this proposition is that the special sign ‘a’ is to be understood here *globally*, in the context of the whole proposition, ‘*not a man not is an animal*’, where it gives ‘*man*’ merely confused supposition. The propositions that result from these two interpretations are not equivalent. It turns out that Major himself adopts the local interpretation. This allows him to maintain the view that putting a ‘*not*’ on the front of ‘a man not is an animal’ does indeed produce its contradictory. His examples are purported contradictories:

*a man not is an animal*  
*Not a man not is an animal*

The algorithm from section 2 applied locally would turn these into:

*Every animal a man isn't*  
*Not every animal a man isn't*

These contain no special signs, and they are contradictories, as he says.

Major also says that the second proposition is equivalent to:

*Every man is b animal.*

The algorithm turns this into:

*An animal every man is*

and this proposition can be seen to be equivalent to ‘*Not every animal a man isn’t*’ by quantifier equipollences, as desired.

He also discusses these purported contradictories:

*Every man is b animal*

*Not every man is b animal*

The algorithm, applied locally, turns these into:

*An animal every man is*

*Not an animal every man is*

These are overt contradictories. He also says that ‘*man*’ in the second sentence is merely confused. Again, by quantifier equipollences, that sentence is equivalent to ‘*Every animal a man isn’t*’ in which ‘*man*’ obviously has merely confused supposition. He also says that if the sign ‘b’ were removed from the second sentence, ‘*man*’ would be determinate. That proposition with ‘b’ removed is ‘*Not every man is an animal*’; by quantifier equipollences, this becomes:

*A man every animal isn’t*

in which ‘*man*’ clearly has determinate supposition.

Finally, at fol lxxxvi “**con-tingenter est homo . . .**” Major is considering a rule governing infinitizing negation; the rule is that these are equivalent:

*a is non-b*

*a is and a not is b*

The equivalence does indeed hold when ‘*a*’ and ‘*b*’ are singular terms or common terms occurring alone (so that the construction is indefinite). Major discusses lots of cases for which the equivalence fails to hold when the examples are not actually of the displayed form, though they have similar forms, and he discusses cases when there are quantifier signs or modal signs present that interfere with the equivalence. An example of this that he gives is the failure of the equivalence:

*a man is a non-animal*

*a man is and a man not is an animal*

where the lower conjunction can be true without the upper proposition. He explains that the rule should not be said to hold when the subject is merely confused. Our algorithm would make the lower proposition equivalent to:

*a man is and every animal a man isn't*

which is actually true, and it would make the upper one equivalent to:

*a man is a non-animal*

which is false. (Here the sign 'a' is redundant because there is no universal term to its right.)

Not everything works this well, however. In one case Major states that the following pair of propositions are contradictories:

*Of any man a donkey isn't a donkey*

*Of some man any donkey is b donkey*

Our algorithm would move the last term in the second sentence to the front:

*A donkey of some man any donkey is*

equivalently:

*Of some man a donkey any donkey is*

This is clearly not the contradictory of the first sentence, since they could both be false. To make it a contradictory one would need to permute '*a donkey*' with '*any donkey*'. So perhaps the algorithm is wrong in this case. But maybe the problem does not lie with the algorithm, for the contradictory of the first sentence is equivalent (by quantifier equipollences) to:

*Of some man every donkey is a donkey*

And in this sentence the '*donkey*' does not have determinate supposition, as its special sign would require. So it would seem that Major is wrong when he says that the second sentence contradicts the first. The situation is not completely clear.

I have no more examples to give at this point.<sup>20</sup>

A concluding caveat: These signs were introduced late in the 15th century; they were being used at least a century and a half later. More writings need examination before we can be sure that we have the right picture. My tentative judgment must be to agree with Vives (*APD*),<sup>21</sup> that the introduction of these special signs was not necessary. But perhaps other applications will show otherwise.

<sup>20</sup> The algorithm also agrees with all of the analyses given on pp. 50–3 of Broadie's *GL*, even including one interesting example that does not work as intended. This is the example 'Of some man every ass is c white.' Broadie gives an analysis of this sentence which (he says, correctly) is equivalent to "it is true of some man that there is some one white thing that all his asses are." However, this analysis gives the term 'white thing' determinate, not merely confused, supposition. The algorithm gives the same analysis.

I don't think that there is any way to understand 'Of some man every ass is c white' that conforms to the intent of how the sign 'c' is to work. (Broadie does not attribute this example to any medieval source.) I suspect that medieval logicians just avoided such examples, but that is a speculation.

<sup>21</sup> Though I disagree strongly with Vives's contemptuous attitude toward the use of these signs.

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# Index

- Abelard 4, 13  
Adams, Marilyn 201  
addition 272–5  
adjective 7, 73, 127, 123, 296, 300  
    attributive 134, 139  
    substantivated 123–5, 127, 133, 289  
Albert of Saxony 5, 73, 74, 96, 127, 199, 231  
Alexander of Aphrodisias 27, 42  
Ammonius 9  
ampliation 74, 97, 164, 260, 276–304  
anaphora 227–58, 260–75  
Andrews, Robert 129, 141  
Anonymous 63, 74, 76, 97, 144, 154, 228, 242,  
    247, 276, 278, 280, 300, 303, 304  
appellation 144, 276, 287–8, 298, 302–5  
Aquinas 5  
Aristotle 1, 2, 5, 6–55, 56, 61–2, 64, 65, 72, 98,  
    109, 110, 123, 125, 147, 155, 161, 203, 204,  
    259, 260, 276, 290, 296, 297, 304  
arithmetic:  
    first-order 269–75  
Arnauld 15  
artificial quantifier 306–18  
artificial sign, *see* artificial quantifier  
ascent 186, 187, 188, 190, 191, 192, 195,  
    196, 197, 199, 209, 210, 226, 230, 312,  
    313, 315  
Ashworth, E. J. 97, 195, 218, 305, 306, 307, 308  
assignment 99–108, 111, 142, 148, 235  
  
Bacon, Roger 5, 14, 56, 76, 96  
Barbara 16, 19, 30, 31, 33, 39, 40, 43, 48, 49, 50,  
    51, 203, 290, 293, 294  
Barocho 16, 17, 32, 33, 34, 51  
being 65, 128  
Berwart, Bernhard 242, 243  
Billingham, Richard 45, 99  
binding 87, 130–1, 136, 143, 167–8, 182, 198,  
    235, 265, 270–1, 289–90  
bivalence 9  
Bobzien, Susanne 155  
Bocardo 16, 34, 36, 37, 39, 51, 53  
Boehner, Philotheus 5  
Boethius, Anacius Manlius Severinus 6, 13, 92,  
    155, 276  
Broadie, Alexander 5, 243, 306–14, 318  
Buridan, John 1, 5, 14, 27, 57, 61, 67, 69, 74–6, 92,  
    95–7, 110, 123–4, 126–7, 129, 134, 140, 142,  
    144, 146, 156, 164–77, 180, 182, 186–8, 191,  
    194–6, 198–99, 201, 202–3, 211–12, 217,  
    227, 231–2, 239, 252–3, 257–8, 278–9,  
    281–5, 286–8, 295–7, 298, 300–2, 304–5  
Burley, Walter 5, 73, 96, 97, 156, 186–8, 228–32,  
    239–41, 249, 254–6, 281, 303  
  
Campsall, Richard 244  
Celarent 16, 30, 33, 39, 40, 43, 48, 50, 51, 203  
Cesare 16, 32, 39, 51, 53  
chimera 42, 50, 74, 75, 92, 175, 192, 193, 253,  
    278–9, 288, 299, 300  
common noun:  
    non-relational 148  
    relational 147, 197  
completeness 77, 87, 98, 113–22, 143, 154  
concept 3, 96, 97, 165, 184, 244, 286–9, 298,  
    304–5  
conditional, as of now 156–7  
conditional, *ut nunc* 156–7  
conjunction 2, 83, 135, 141, 156, 158, 187, 188,  
    189, 190, 192, 198, 199, 209, 210, 221, 222,  
    241, 243, 244, 245, 246, 248, 252, 256, 257,  
    270, 306, 312, 313, 314, 317  
    augmented 190, 219  
consequence 5  
contradiction 2, 9, 10, 11, 12, 13, 18, 22, 23, 26,  
    41, 42, 53, 54, 64, 65, 78, 90, 92, 121, 122, 165,  
    173, 175, 256, 276, 291, 310, 311, 315,  
    316–18  
contrary 10–12, 22–3, 27–9, 31, 53, 54, 64, 78, 90,  
    116, 121, 144, 167, 227, 239, 310, 311  
conversion 1, 2, 14, 21, 25–35, 39, 41–2, 49, 50,  
    51–3, 54, 60, 65, 67–8, 72, 74, 123–4, 133,  
    155, 164, 174, 217, 290–1, 297, 308  
    by contraposition 14, 74  
    per accidens 14, 27, 28, 29, 34, 35, 54–5, 65, 67,  
    308  
Copenhaver, Brian 5  
copula 7, 19–20, 57, 59, 65, 77, 81, 88, 93–4,  
    110, 125–9, 132, 140, 161, 164–7, 168–9,  
    170, 173–6, 177, 261, 264, 279, 289, 290,  
    295–7, 298  
    symmetry of 93–4  
  
Darapti 16, 17, 27, 34, 36, 51, 67  
Darii 16, 30, 34, 39, 40, 43, 49, 50, 51, 204  
Datisi 16, 34, 39, 51  
de Morgan, Augustus 157, 160–2  
de Rijk, L. M. 45, 60, 74, 76, 99, 276

- denoting phrase 43, 57–9, 65, 82–4, 87–9, and throughout
- descent 186–201, 209–10, 218, 221, 226, 230–1, 244–5, 303, 311–15  
restricted 197
- determiner 43–50, 206  
monotonic 45–50, 206
- dictum de omni et nullo* 30, 160, 203
- dictum of all and none, see *dictum de omni et nullo*
- 'differ' 256
- direct object 67, 81–6, 129–30, 135–9, 166, 179–81, 219, 263
- disjunction 155–6, 186–93, 199, 209, 218–19, 222, 243, 280–1, 291, 303, 305, 311–14
- donkey anaphora 254
- Dorp, John 201, 217
- dot, indicating indefinite article 57, 59
- Dutilh Novaes, Catarina 5, 19, 184, 196, 201, 217
- equipollence 60–70, 74–8, 90–3, 113–14, 133, 137, 152, 161, 175, 196, 200, 201, 231, 236, 255, 260, 291, 309, 315–18
- equivocation 147, 276
- existential import 1, 9, 12–13, 17, 37, 40–1, 49, 74, 98, 105, 109, 110, 259, 262–4, 268
- exponible 45, 256
- exposition 2, 23–4, 27, 29, 34–7, 41–2, 53, 54, 65, 66, 79, 95, 123, 161, 231, 248, 252, 259, 260, 292, 303
- expository syllogism 24, 29, 41, 50, 54, 66, 79, 292, 293
- fallacies 5
- Ferio 16, 30, 39, 40, 42, 43, 49, 51
- figure 1, 15–17, 30–34, 38–42, 49, 51–2, 61, 70, 146, 155, 203, 204, 252, 260, 290
- form:  
grammatical 3, 81–9, 246–7, 265–7, 301  
logical 8, 18–20, 57–9, 65–71, 83–9, 95, 98, 124, 125, 128, 132, 139, 145, 149, 151, 153, 155, 159, 162, 163, 169, 178, 181, 183, 222, 238, 245, 249, 251, 252, 254, 258, 266, 269, 271, 279, 290, 296
- Frege, Gottlob 164, 259, 304, 306
- Fromkin, Victoria 82
- Geach, Peter 254–5
- generalized quantifier 43, 47
- genitive 145–53, 166, 167, 177, 179–82, 197, 219, 229, 270, 272, 313–15  
of possession, 147
- global quantificational import 223–6
- grammatical marker 82–7, 89, 117, 129, 131, 143, 147, 150
- Henkin modeling theorem 114
- Henkin, Leon 114
- Horn, Lawrence 48
- Hülsen, Reinhard 4
- identity  
self- 72, 94  
substitutivity of 70–1, 93, 237, 273  
transitivity of 2, 69, 94
- indefinite 7, 8, 18, 27, 56, 57, 59, 61, 62, 67, 72, 84, 89, 92, 153, 164, 170, 171, 172, 177, 178, 206, 240, 309, 312, 317
- induction 214, 216
- inference:  
algorithm 217  
from a superior to an inferior 202–6  
from an inferior to a superior 204, 226  
useful 202, 207, 214
- insoluble 5
- interpretation 110–22, 143, 154, 199, 261, 281
- Karger, Elizabeth 145, 167, 180–2, 201, 217
- Keenan, Edward 43
- Kilwardby, Robert 63
- Klima 5
- Klima, Gyula 5, 105, 305
- Kneale and Kneale 13
- Knuuttila, Simo 5
- Kretzmann, Norman 5, 96, 156, 177–9, 208
- Kym, Hugo 242–3
- Lagerlund, Henrik 5
- Lambert of Lagny 5, 96–7, 230, 247, 298
- language, natural 1, 3, 43, 45, 47–8, 59, 83, 86, 125, 127, 134–5, 143, 148, 150, 158, 168, 246, 255–6, 259, 265–6, 268–9, 271–4, 281, 285, 306
- Lavenham, Richard 245
- Leibniz's Law 51, 70
- Lokert, George 307, 310–11
- Major, John 311–18
- Malink, Marko 12, 27
- Marenbon, John 5
- marker, grammatical 67, 81, 82, 84, 87, 129, 148, 158, 221, 261, 263, 265, 285
- Marsilius of Inghen 5, 96–7, 154, 184, 231, 240, 244, 256, 283, 297, 300
- Martianus Capella 155
- Mates, Benson 24, 155
- Matthews, Gareth 210, 218
- Maulfelt, Thomas 201
- modality 5, 8, 30, 73, 97, 123, 125, 156, 164, 260, 277–9, 295–8, 302, 317
- monotonicity 45–50, 205
- mood 15–17, 21, 30, 31, 33, 34, 38–41, 49, 51–3, 203, 257, 290  
direct 15
- multiplication 275



- name, temporary 99–100, 104–5, 108, 112–13
- negation 7, 9, 14, 18, 57, 59, 62–4, 68, 72–81, 90, 102, 112, 115–19, 138, 161, 173, 175, 182, 194, 195, 201, 212, 216, 220, 223, 247, 256, 264, 268, 291, 294, 300, 309, 312, 316–17  
 double 58, 63, 73  
 indefinite, *see* negation infinitizing  
 infinitizing 18, 72–4, 173, 175–6, 182, 264, 268, 294, 300, 317
- Nicole 15
- nominalist 5
- Normore, Calvin 5
- obligation 5
- obversion 60, 75, 76
- Ockham, William 5, 56, 61, 67, 92, 95–7, 147, 156, 186–8, 192, 194–6, 199, 201, 202, 204, 209–12, 227, 230, 232, 233, 238–40, 280, 286, 291, 295, 298, 303, 304
- ‘only’ 45
- participle 126–39, 143, 146, 148, 152, 165–7, 169, 170, 176–7, 179–81, 219, 221, 260, 264, 271, 278, 281, 289
- Paul of Venice 4, 5, 29, 60, 96, 97, 127, 156, 177, 179, 182, 191, 194, 199, 202, 204, 210, 281, 282, 284
- Peano’s postulates 269
- permutation 68, 70, 71, 79, 254, 292, 293, 294  
 pertaining to the soul 280, 299, 304
- Peter of Spain 5, 8, 9, 11, 14, 16, 51, 64, 67, 68, 73, 96, 97, 156, 185, 228, 239, 295
- Poinsot, John 5, 196, 201, 230, 231
- possession, *see* genitive
- predicate 2, 7–9, 14–15, 23, 30, 40, 56, 62–3, 65, 67, 69, 72, 74–6, 83, 99, 123, 124, 127, 140, 162, 164–79, 182, 185, 188, 189, 190, 193, 194, 195, 197, 199, 201, 232, 240, 247, 259, 260, 261, 262, 264, 267, 269, 271, 277, 279, 280, 281, 282, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 300, 305, 311, 312  
 quantified 56
- predicate calculus 4, 63, 260, 261, 264, 271
- Priest, Graham 210, 218, 220
- pronoun 3, 8, 154, 180, 216, 227–58, 267, 282, 284–5, 300–1  
 reflexive 3, 228, 229–45  
 relative 141, 142, 283, 302
- proposition 6  
 affirmative 7–17, 25, 28, 35, 37, 41, 47, 49, 50, 52, 54, 61, 63, 65–7, 69, 72–8, 91–3, 98, 109, 110, 124, 149, 156, 157, 165, 175, 176, 185, 189, 191, 194, 201, 204, 211, 212, 215, 216, 223, 237, 243, 244, 248, 249, 252, 254, 262, 263, 264, 277, 279, 283, 299  
 categorical 7–9, 15, 56, 58, 59, 72, 110, 127, 164–70, 176, 194, 224, 226, 229, 231, 234, 240, 245, 246, 277  
 conditional 156  
*ut nunc* 156, 157  
 hypothetical 156  
 indefinite 7, 57–9  
 mental 165, 286, 295  
 molecular 155–9, 220  
 negative 7–17, 23, 25–30, 36, 37, 41, 49, 50, 52, 61, 63, 67, 69, 72–7, 92, 98, 108–10, 123, 157, 165, 175, 176, 178, 185, 187, 189, 194, 206, 212, 243, 247, 248, 249, 262, 264, 279, 294, 300, 301  
 particular 7–17, 23–5, 29, 35–7, 39, 41, 42, 49, 52, 56, 61, 62, 64, 67, 68, 72, 74, 75, 95, 96, 108, 124, 160, 164, 170, 185, 186, 189, 194, 211, 279, 312  
 universal 7–17, 23–9, 35, 37, 38, 39, 41, 42, 49, 50, 52, 54, 55, 56, 61, 64, 67, 68, 72, 74, 124, 140, 149, 161, 164, 170, 171, 176, 177, 179, 185, 189, 191, 194, 195, 201, 204, 208, 210, 212, 223, 224, 225, 239, 244, 252, 264, 265, 272, 279, 303, 308, 309, 312, 318
- quality 8
- quantification, analysis of 218
- quantifier, generalized 43
- quantity 8, 164
- Read, Stephen 5, 196, 201, 209, 210, 218, 220, 305
- realist 5
- reductio 22, 26–33, 42, 53, 54, 64, 71, 78, 90, 92, 122, 250, 274, 291, 293
- reduction by impossibility 52
- relational expression 161
- relative 227  
 antecedent of 3, 14, 83, 156, 198, 206, 216, 220, 223, 227–57, 265, 267, 272, 273, 283, 301, 302  
 non-reflexive 239  
 of diversity 256
- relative clause 140–4, 170, 174, 180, 181, 221, 256, 282, 283, 284, 291, 302  
 restrictive 140
- replacement principle 228, 232
- restriction 74, 97, 154, 243, 247, 276–83, 288, 290, 291, 294, 300, 302
- scope 3, 12, 44, 63, 86, 108, 109, 144, 176, 179, 182, 194, 201, 208, 211, 212, 214, 216, 220, 221, 223, 225, 232, 233, 234, 238, 239, 245, 246, 248, 254, 255, 256, 260, 263, 269, 270, 271, 285, 308, 312, 316
- Sherwood, William 5, 56, 60–4, 68, 73, 74, 96, 97, 140, 156, 178, 179, 182, 200, 201, 207, 208, 214, 215, 216, 218, 230, 280

- signification 73, 95, 96, 97, 125, 276, 277, 286, 288, 298
- Simmons, Keith 5
- Smith, Robin 13, 23, 27
- sophism 5
- Sortes 85
- Spade, Paul Vincent 5, 188, 209, 210, 218, 245
- square of opposition 10, 12, 210
- Stoic 2, 155, 156
- Stump, Eleonore 5, 96, 156
- subalternation 11, 12, 23, 29, 54, 55, 65, 92, 93, 95, 208, 217, 237
- subcontrary 11, 178
- subject 6–9, 12, 14, 15, 17, 23, 30, 37, 40–2, 44, 50, 56, 61, 65–7, 72, 74–6, 81–6, 92, 94, 96, 97, 108, 109, 123, 124, 127, 140, 164, 165–81, 184, 189–94, 196, 197, 202, 203, 220, 240, 247, 255, 260, 263, 267, 272, 277–83, 285, 287–91, 295–302, 317
- supposition 63, 96–7, 154, 176–9, 184–245, 256, 260, 276–88, 294, 296, 300–18
- determinate 176, 184, 186, 209, 210, 225, 312
- distributive 176, 184, 187, 209, 210, 211, 225, 312
- material 3, 184, 277
- merely confused 176, 184, 185, 188, 199, 209, 210, 225, 312
- mode of 4, 177, 184, 190, 193, 196, 197, 199, 202, 206, 208–17, 225, 226, 229, 231, 245, 308, 312, 315
- cause of 193, 225
- narrow distributive 210
- personal 3, 184, 185, 186, 199, 204, 223, 228, 239, 277, 303
- simple 3, 184, 185, 228, 277, 303, 304
- singled 3, 229, 244
- wide distributive 209, 210
- Swinarski, John 218
- syllogism 1, 2, 15–21, 24, 27, 30–4, 39, 41, 42, 52–5, 66, 69, 70, 99, 155, 160, 164, 198, 202–5, 231, 252, 253, 257, 258, 260, 290, 291, 297, 300
- expository 2, 27
- figure 2
- perfect 30–1
- reduction of 30, 31, 39
- syncategoremata 5
- tense 164, 280–94
- coordination of times 281, 285
- term:
- complex 73, 134–40, 141, 143, 152–3, 162–3, 165, 175, 176–83, 192, 203, 221, 240, 260, 270, 282, 284, 314
- demonstrative 7, 8, 153, 154, 189
- determinable 172, 180–2, 219, 314
- embellished 288–91, 294, 298
- empty 12, 13, 17, 37, 41, 42, 44, 47, 50, 63, 65, 66, 72, 74–80, 91–5, 104, 108, 109, 114–21, 129, 131, 138, 149, 163, 191–3, 206, 216, 219, 235–7, 248, 260, 262–4, 266, 270, 273, 277, 279, 291, 292, 300
- parasitic 130, 133, 134, 137, 146, 160, 177, 182, 197, 198, 199, 204, 206, 212, 213, 217, 220, 221, 226, 245, 248, 260, 261, 264, 268, 269, 270, 271, 286, 313, 314, 315, 316
- singular 7, 8, 23, 24, 27, 55, 57, 58, 63, 66, 68, 71, 73, 79, 87, 91, 93, 96, 104, 112, 117, 119, 129, 133, 136, 137, 138, 152, 154, 161, 198, 202, 203, 204, 206, 221, 237, 269, 270, 271, 273, 294, 306, 312
- singular, permutation of 67
- theta-criterion 82
- theta-role 82
- Thom, Paul 2, 5, 23, 39, 69
- topics 5
- truth conditions 12, 41, 42, 61, 96, 98–109, 123, 131, 132, 142, 156, 158, 177, 217, 235, 239, 241, 242, 243, 249, 256, 259, 264, 280, 281, 284, 296
- Uckelman, Sara 63
- universal application 35–6
- univocation 276, 277, 299
- validity 13, 15, 19, 21, 23, 24, 31, 63, 87, 98, 110, 113, 122, 260, 291
- formal 18–20
- van Benthem, Johan 47
- variable 24, 99, 261, 267, 271
- Venn diagram 259, 260
- verb:
- intransitive 126–8, 134, 169, 170
- transitive 67, 81, 129–36, 139, 146, 148, 152, 166, 167, 197, 219, 260, 289
- Vives, Juan Luis 282, 306, 318
- Ward, Thomas 315
- Westerstahl, Dag 50
- Whitaker, C. W. A. 9
- Wyclif, John 5, 60, 64, 70
- Yrjönsurri, Mikko 5