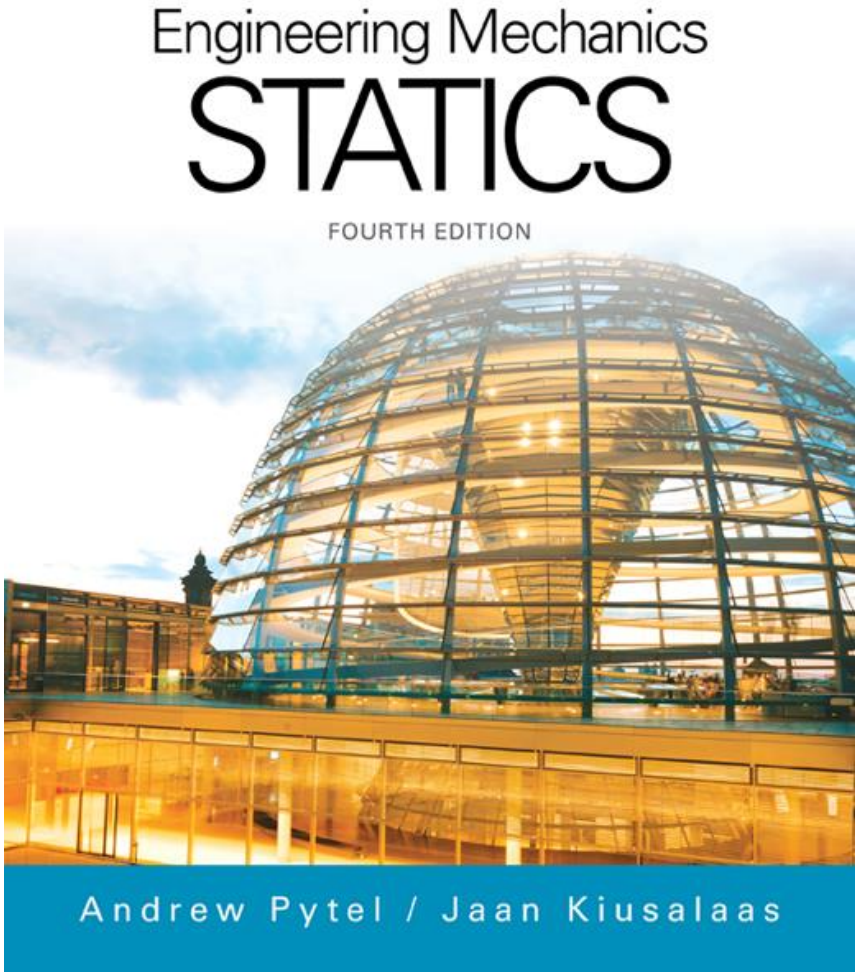
# An Instructor's Solutions Manual to Accompany

# ENGINEERING MECHANICS: STATICS, 4TH EDITION

**ANDREW PYTEL**
**JAAN KIUSALAAS**

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Instructor's Solutions Manual

to Accompany

# Engineering Mechanics: Dynamics

4th EDITION

ANDREW PYTEL

JAAN KIUSALAAS

# Contents

# Chapter 1

## 1.1

**(a)** $m = \dfrac{30 \text{ lb}}{5.32 \text{ ft/s}^2} = 5.639 \text{ slugs}$ ♦

**(b)** $W = mg = (5.639)(32.2) = 181.6 \text{ lb}$ ♦

## 1.2

$$\begin{aligned} W &= \rho g V = (7850)(9.81)\left[\pi(0.04^2)(0.110)\right] = 42.58 \text{ N} \\ W &= 42.58 \text{ N} \times \frac{0.2248 \text{ lb}}{1.0 \text{ N}} = 9.57 \text{ lb} \quad ◄ \end{aligned}$$

## 1.3

**(a)** $400 \text{ lb·ft} = 400 \text{ lb·ft} \times \dfrac{4.448 \text{ N}}{1.0 \text{ lb}} \times \dfrac{0.3048 \text{ m}}{1.0 \text{ ft}} = 542 \text{ N·m}$ ◄

**(b)** $6 \text{ m/s} = \dfrac{6 \text{ m}}{\text{s}} \times \dfrac{0.3048 \text{ ft}}{1.0 \text{ m}} \times \dfrac{1.0 \text{ mi}}{5280 \text{ ft}} \times \dfrac{3600 \text{ s}}{1.0 \text{ h}} = 1.247 \text{ mi/h}$ ◄

**(c)** $20 \text{ lb/in.}^2 = \dfrac{20 \text{ lb}}{\text{in.}^2} \times \dfrac{4.448 \text{ N}}{1.0 \text{ lb}} \times \dfrac{1.0 \text{ in.}^2}{645.2 \times 10^{-6} \text{ m}^2} = 1.379 \times 10^5 \text{ N/m}^2$
$= 137.9 \text{ kPa}$ ◄

**(d)** $500 \text{ slug/in.} = \dfrac{500 \text{ slug}}{\text{in.}} \times \dfrac{14.593 \text{ kg}}{1.0 \text{ slug}} \times \dfrac{39.37 \text{ in.}}{1.0 \text{ m}} = 2.87 \times 10^5 \text{ kg/m}$ ◄

## 1.4

$$\begin{aligned} 30 \text{ mi/gal} &= \frac{30 \text{ mi}}{\text{gal}} \times \frac{5280 \text{ ft}}{1.0 \text{ mi}} \times \frac{0.3048 \text{ m}}{1.0 \text{ ft}} \times \frac{1.0 \text{ gal}}{3.785 \text{ L}} \\ &= 12\ 760 \text{ m/L} = 12.76 \text{ km/L} \quad ◄ \end{aligned}$$

## 1.5

**(a)** $E = \dfrac{1}{2}(1000 \text{ kg})\left(6\dfrac{\text{m}}{\text{s}}\right)^2 = 18\ 000\ \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 18\ 000\left(\dfrac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)(\text{m})$
$= 18\ 000 \text{ N·m} = 18 \text{ kN·m}$ ◄

**(b)** $E = 18\ 000 \text{ N·m} = 18\ 000 \text{ N·m} \times \dfrac{0.2248 \text{ lb}}{1.0 \text{ N}} \times \dfrac{3.281 \text{ ft}}{1.0 \text{ m}}$
$= 13\ 280 \text{ lb·ft}$ ◄

## 1.6

The dimensions of $\frac{gkx}{W}$ are: $[g][k][x]\left[\frac{1}{W}\right] = \left[\frac{L}{T^2}\right]\left[\frac{F}{L}\right][L]\left[\frac{1}{F}\right] = \left[\frac{L}{T^2}\right] = [a]$ Q.E.D.

## 1.7

The dimensions of $k = \frac{F}{x}$ are: $[k] = \left[\frac{F}{x}\right] = \left[\frac{ML}{T^2}\right]\left[\frac{1}{L}\right] = \left[\frac{M}{T^2}\right]$ ♦

## 1.8

(a) $8\ \text{mm}/\mu\text{s} = \frac{8\ \text{mm}}{\mu\text{s}} \times \frac{1.0\ \text{m}}{1000\ \text{mm}} \times \frac{1.0\ \mu\text{s}}{10^{-6}\ \text{s}} = 8000\ \text{m/s}$ ◄

(b) $8000\ \text{m/s} = \frac{8000\ \text{m}}{\text{s}} \times \frac{3.281\ \text{ft}}{1.0\ \text{m}} \times \frac{1.0\ \text{mi}}{5280\ \text{ft}} \times \frac{3600\ \text{s}}{1.0\ \text{h}} = 17\ 900\ \text{mi/h}$ ◄

## 1.9

$y = kx^2$ (where $k = 1.0$)

The dimensions of $k = \frac{y}{x^2}$ are: $\therefore\ [k] = \left[\frac{y}{x^2}\right] = \left[\frac{L}{L^2}\right] = \left[\frac{1}{L}\right]$

$y = x^2$ can be dimensionally correct if the units of the constant 1.0 (not shown explicity) are understood to be $\text{in}^{-1}$.

## 1.10

$$\left[\frac{L}{T^2}\right] = [A]\left[L^2\right] + [B][L][T]$$

$$\therefore\ [A] = \left[\frac{1}{LT^2}\right] \blacktriangleleft \qquad [B] = \left[\frac{1}{T^3}\right] \blacktriangleleft$$

## 1.11

(a) The dimensions of $x = At^2 - Bvt$ are

$$[L] = [A][T^2] - [B][LT^{-1}][T]$$

$$\therefore [A] = [LT^{-2}] \blacktriangleleft \qquad [B] = [1]\ \text{(dimensionless)} \blacktriangleleft$$

**(b)** The dimensions of $x = Avte^{-Bt}$ are

$$\begin{aligned} [L] &= [A][LT^{-1}][T]e^{[B][T]} \\ [B][T] &= [1] \qquad \therefore [B] = [T^{-1}] \ \blacktriangleleft \\ [L] &= [A][LT^{-1}][T] \qquad \therefore [A] = [1] \ \blacktriangleleft \end{aligned}$$

## 1.12

$$\begin{aligned} \left[\frac{d^4y}{dx^4}\right] &= \left[\frac{L}{L^4}\right] = [L^{-3}] \\ \left[\frac{\omega^2\gamma}{D}y\right] &= \frac{[T^{-2}][ML^{-1}]}{[FL^2]}[L] = \left[\frac{M}{T^2FL^2}\right] \end{aligned}$$

Substituting $[F] = \left[MLT^{-2}\right]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2\gamma}{D}y\right] = \left[\frac{M}{T^2L^2}\right]\left[\frac{T^2}{ML}\right] = [L^{-3}] \ \text{Q.E.D.}$$

Substituting $[F] = \left[MLT^{-2}\right]$ —see Eq. (1.2b)— we get

$$\left[\frac{\omega^2\gamma}{D}y\right] = \left[\frac{M}{T^2L^2}\right]\left[\frac{T^2}{ML}\right] = [L^{-3}] \ \text{Q.E.D.}$$

## 1.13

The argument of the sine function must be dimensionless:

$$\begin{aligned} \left[\frac{Bx}{k}\right] &= [1] \qquad [B][L]\left[\frac{L}{F}\right] = [1] \qquad [B] = [FL^{-2}] \ \blacktriangleleft \\ [F] &= [Akx^2] = [A][FL^{-1}][L^2] \qquad [A] = [L^{-1}] \ \blacktriangleleft \end{aligned}$$

## 1.14

**(a)** $110 \text{ hp} = 110 \text{ hp} \times \dfrac{550 \text{ lb} \cdot \text{ft/s}}{1.0 \text{ hp}} = 60\ 500 \text{ lb}\cdot\text{ft/s} \ \blacktriangleleft$

**(b)** $110 \text{ hp} = 110 \text{ hp} \times \dfrac{0.7457 \text{ kW}}{1.0 \text{ hp}} = 82.0 \text{ kW} \ \blacktriangleleft$

## 1.15

$$\begin{aligned} F &= G\frac{m_Am_B}{R^2} = (6.67 \times 10^{-11})\frac{(12)(12)}{0.4^2} = 6.003 \times 10^{-8} \text{ N} \\ W &= mg = (12)(9.81) = 117.7 \text{ N} \\ \% \text{ of weight} &= \frac{F}{W} \times 100\% = \frac{6.003 \times 10^{-8}}{117.7} \times 100\% = 5.10 \times 10^{-8}\ \% \ \blacktriangleleft \end{aligned}$$

## 1.16

$$F = G\frac{m_A m_B}{R^2} = \left(3.44 \times 10^{-8}\right)\frac{\left(\frac{2}{32.2}\right)\left(\frac{2}{32.2}\right)}{(16/12)^2} = 7.46 \times 10^{-11} \text{ lb} \quad \blacklozenge$$

## 1.17

$$h = (28\ 000 \text{ ft})\left(\frac{0.3048 \text{ m}}{1.0 \text{ ft}}\right) = 8534 \text{ m} = 8.534 \text{ km}$$

$$\text{On earth:} \quad W_e = \frac{GM_e m}{R_e^2} \qquad \text{At elevation } h\text{:} \quad W = \frac{GM_e m}{(R_e + h)^2}$$

$$W = W_e \frac{R_e^2}{(R_e + h)^2} = 170\frac{6378^2}{(6378 + 8.534)^2} = 169.5 \text{ lb} \quad \blacktriangleright$$

## 1.18

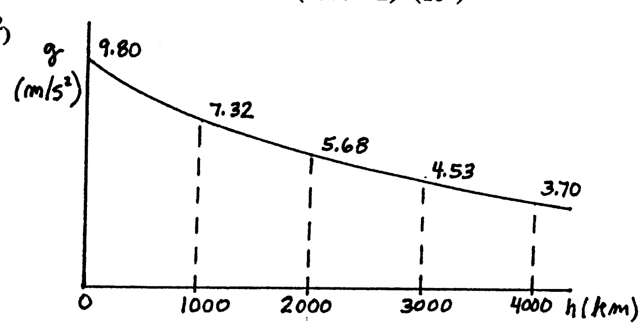
$$g_m = \frac{GM_m}{R_m^2} \qquad g_e = \frac{GM_e}{R_e^2}$$

$$\frac{g_m}{g_e} = \frac{M_m R_e^2}{M_e R_m^2} = \frac{0.07348(6378)^2}{5.974(1737)^2} = 0.1658 \approx \frac{1}{6} \text{ Q.E.D.}$$

## 1.19

Shown below is the plot of $g = \frac{GM_e}{R^2} = \frac{\left(6.67 \times 10^{-11}\right)\left(5.9742 \times 10^{24}\right)}{(6378 + h)^2\,(10^6)}$

| h (km) | g (m/s$^2$) |
|---|---|
| 0 | 9.80 |
| 1000 | 7.32 |
| 2000 | 5.68 |
| 3000 | 4.53 |
| 4000 | 3.70 |




## 1.20

$$\text{On earth:} \quad W_e = \frac{GM_e m}{R_e^2} \qquad \text{At elevation } h\text{:} \quad W = \frac{GM_e m}{(R_e + h)^2}$$

$$W = \frac{W_e}{10} \qquad \frac{GM_e m}{(R_e+h)^2} = \frac{GM_e m}{10R_e^2} \qquad (R_e+h)^2 = 10R_e^2$$

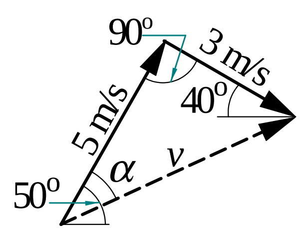
$$(6378+h)^2 = 10(6378)^2 \qquad h = 13\ 790 \text{ km} \blacktriangleleft$$

**1.21**

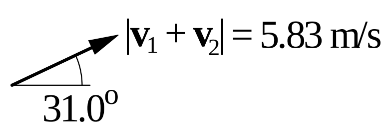
$$\begin{aligned} R &= R_e + R_m + d = 6378 + 1737 + 384 \times 10^3 \\ &= 392.1 \times 10^3 \text{ km} = 392.1 \times 10^6 \text{ m} \end{aligned}$$

$$\begin{aligned} F &= G\frac{M_e M_m}{R^2} = \left(6.67 \times 10^{-11}\right) \frac{\left(5.974 \times 10^{24}\right)\left(0.07348 \times 10^{24}\right)}{\left(392.1 \times 10^6\right)^2} \\ &= 1.904 \times 10^{20} \text{ N} \blacktriangleleft \end{aligned}$$

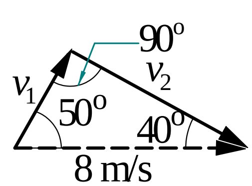
**1.22**




$$v = \sqrt{5^2 + 3^2} = 5.83 \text{ m/s} \qquad \alpha = \tan^{-1}\frac{3}{5} = 31.0°$$

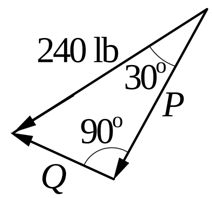



**1.23**



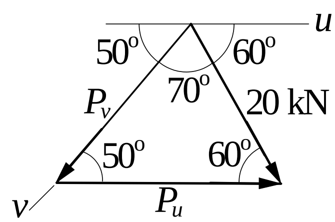

$$v_1 = 8\sin 40° = 5.14 \text{ m/s} \blacktriangleleft \qquad v_2 = 8\sin 50° = 6.13 \text{ m/s} \blacktriangleleft$$

**1.24**



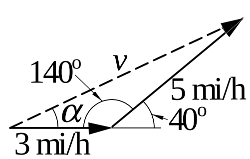

$$\text{Component parallel to } AB \quad : \quad P = 240\cos 30^\circ = 208 \text{ lb} \blacktriangleleft$$
$$\text{Component pependicular to } AB \quad : \quad Q = 240\sin 30^\circ = 120 \text{ lb} \blacktriangleleft$$

**1.25**




$$\frac{P_v}{\sin 60^\circ} = \frac{P_u}{\sin 70^\circ} = \frac{20}{\sin 50^\circ}$$
$$P_v = 20\frac{\sin 60^\circ}{\sin 50^\circ} = 22.6 \text{ kN} \blacktriangleleft$$
$$P_u = 20\frac{\sin 70^\circ}{\sin 50^\circ} = 24.5 \text{ kN} \blacktriangleleft$$

**1.26**

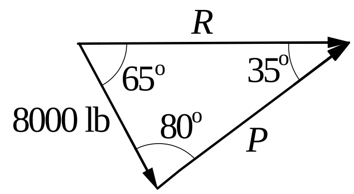



Law of cosines: $v = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 140^\circ} = 7.549$ mi/h

Law of sines: $\frac{5}{\sin\alpha} = \frac{7.549}{\sin 140^\circ} \qquad \sin\alpha = 0.4257 \qquad \alpha = 25.2^\circ$

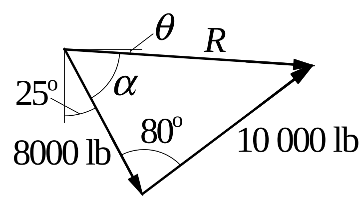

◄

**1.27**




$$\frac{P}{\sin 65^\circ} = \frac{8000}{\sin 35^\circ} \qquad P = 8000\frac{\sin 65^\circ}{\sin 35^\circ} = 12\ 640 \text{ lb} \blacktriangleleft$$

**1.28**

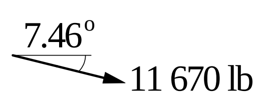



Law of cosines:

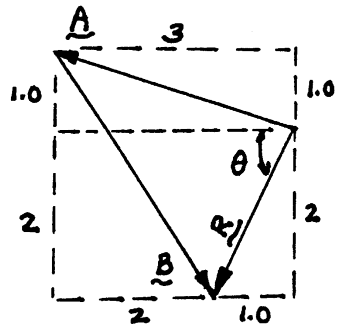
$$\begin{aligned} R &= \sqrt{8000^2 + 10\ 000^2 - 2(8000)(10\ 000)\cos 80^\circ} \\ &= 11\ 671 \text{ lb} \blacktriangleleft \end{aligned}$$

Law of sines:

$$\begin{aligned} \frac{10\ 000}{\sin\alpha} &= \frac{11\ 671}{\sin 80^\circ} \qquad \sin\alpha = \frac{10\ 000}{11\ 671}\sin 80^\circ = 0.8438 \\ \alpha &= \sin^{-1}(0.8438) = 57.54^\circ \\ \theta &= 90^\circ - 25^\circ - 57.54^\circ = 7.46^\circ \blacktriangleleft \end{aligned}$$



## 1.29

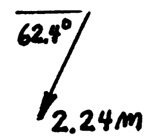



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

**By inspection of the triangle (dimensions in meters**

$R = \sqrt{2^2 + 1.0^2} = 2.24$ m and $\theta = \tan^{-1} 2 = 62.4$

**Therefore, the resultant of A and B is:**

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## 1.30

$\mathbf{R} = \mathbf{P} + \mathbf{Q}$

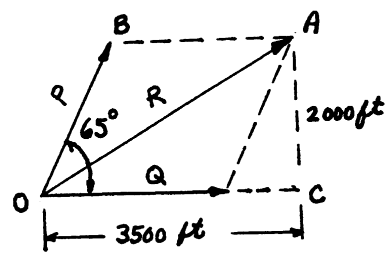
$$P = \frac{2000}{\sin 65^\circ} = 2207 \text{ ft}$$

$$Q = 3500 - P\cos 65^\circ$$

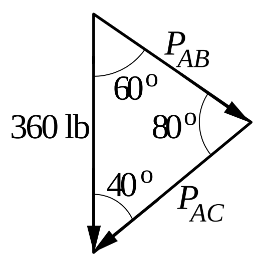
$$= 3500 - 2207\cos 65^\circ = 2567 \text{ ft}$$

**Therefore, the components are:**

**2210 ft along OB and 2570 ft along OC** ◆




## 1.31



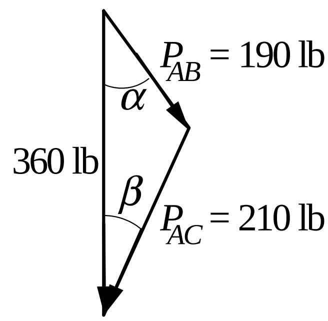

Law of sines:

$$\frac{360}{\sin 80^\circ} = \frac{P_{AB}}{\sin 40^\circ} = \frac{P_{AC}}{\sin 60^\circ}$$

$$P_{AB} = \frac{360 \sin 40^\circ}{\sin 80^\circ} = 235 \text{ lb} \blacktriangleleft$$

$$P_{AC} = \frac{360 \sin 60^\circ}{\sin 80^\circ} = 317 \text{ lb} \blacktriangleleft$$

## 1.32




Law of cosines:

$$
\begin{aligned}
210^2 &= 360^2 + 190^2 - 2(360)(190)\cos\alpha \\
\alpha &= 27.3^\circ \quad \blacktriangleleft \\
190^2 &= 360^2 + 210^2 - 2(360)(210)\cos\beta \\
\beta &= 24.5^\circ \quad \blacktriangleleft
\end{aligned}
$$

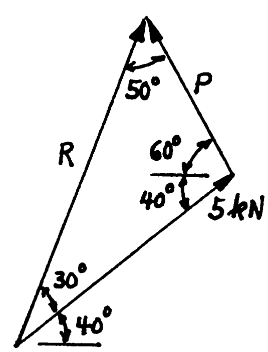
## 1.33

**Law of sines:**

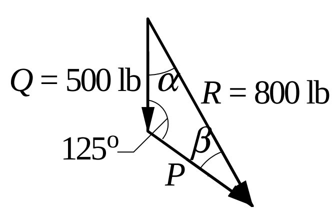
$$\frac{P}{\sin 30^\circ} = \frac{5}{\sin 50^\circ}$$

**which gives**

$$P = \frac{5 \sin 30^\circ}{\sin 50^\circ} = 3.26 \text{ kN} \quad \blacklozenge$$



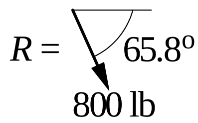

## 1.34



Law of sines:

$$\frac{500}{\sin\beta} = \frac{800}{\sin 125^\circ} \qquad \beta = 30.8^\circ$$

$$\alpha = 180^\circ - (125^\circ + 30.8^\circ) = 24.2^\circ$$

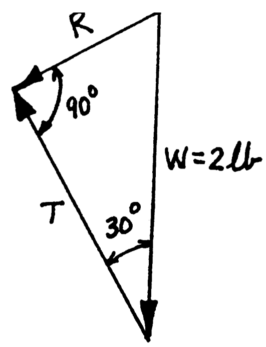



$$\frac{800}{\sin 125^\circ} = \frac{P}{\sin 24.2^\circ} \qquad P = 400 \text{ lb} \blacktriangleleft$$

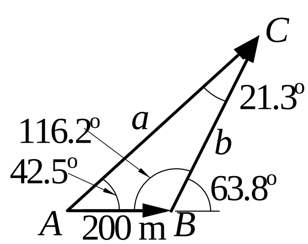
**1.35**

**R = W + T**

**T = 2 cos30° = 1.732 lb** ◆




**1.36**

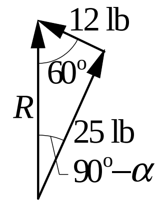



Law of sines: $$\frac{200}{\sin 21.3^\circ} = \frac{a}{\sin 116.2^\circ} = \frac{b}{\sin 42.5^\circ}$$

$$\therefore a = \frac{200 \sin 116.2^\circ}{\sin 21.3^\circ} = 494 \text{ m} \blacktriangleleft$$

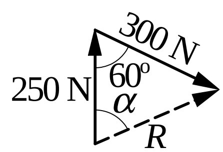
$$b = \frac{200 \sin 42.5^\circ}{\sin 21.3^\circ} = 372 \text{ m} \blacktriangleleft$$

**1.37**




$$\begin{aligned}\frac{12}{\sin(90^\circ-\alpha)} &= \frac{25}{\sin 60^\circ}\\ \sin(90^\circ-\alpha) &= \frac{12\sin 60^\circ}{25} = 0.4157\\ 90^\circ-\alpha &= 24.56^\circ \qquad \alpha = 65.4^\circ \quad ◀\end{aligned}$$

***1.38**

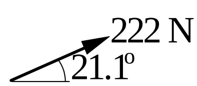



First compute the resultant **R** of the two known forces. The smallest required **F** has the same direction as **R** and its magnitude is 500 N $-R$.

$$\begin{aligned}\text{Law of cosines:}\quad R &= \sqrt{250^2+300^2-2(250)(300)\cos 60^\circ}\\ &= 278.4\ \text{N}\\ &\therefore\ F = 500 - 278.4 = 222\ \text{N}\end{aligned}$$

$$\text{Law of sines:}\qquad \frac{300}{\sin\alpha} = \frac{278.4}{\sin 60^\circ}$$

$$\alpha = \sin^{-1}\frac{300\sin 60^\circ}{278.4} = 68.9^\circ$$

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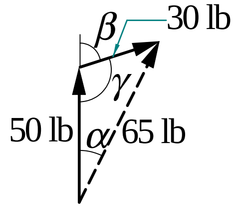
**1.39**



Law of cosines: $65^2 = 50^2 + 30^2 - 2(50)(30)\cos\gamma$

$$\gamma = \cos^{-1}\frac{-65^2 + 50^2 + 30^2}{2(50)(30)} = 105.96^\circ$$

$$\beta = 180^\circ - \gamma = 180^\circ - 105.96^\circ = 74.0^\circ \quad \blacktriangleleft$$

Law of sines: $\dfrac{30}{\sin\alpha} = \dfrac{65}{\sin 105.96^\circ}$

$$\alpha = \sin^{-1}\frac{30\sin 105.96^\circ}{65} = 26.3^\circ \quad \blacktriangleleft$$

## 1.40

$$\mathbf{P} = -30\cos 50^\circ \sin 30^\circ\,\mathbf{i} + 30\cos 50^\circ \cos 30^\circ\,\mathbf{j} + 30\sin 50^\circ\,\mathbf{k}\ \text{lb}$$

$$\therefore\ \mathbf{P} = -9.64\mathbf{i} + 16.70\mathbf{j} + 22.98\mathbf{k}\ \text{lb} \ \blacklozenge$$

## 1.41

**(a)** $\mathbf{r} = 240\sin 40^\circ \cos 50^\circ\,\mathbf{i} + 240\sin 40^\circ \sin 50^\circ\,\mathbf{j} + 240\cos 40^\circ\,\mathbf{k}$ mm

$$\therefore\ \mathbf{r} = 99.16\mathbf{i} + 118.2\mathbf{j} + 183.8\mathbf{k}\ \text{mm} \ \blacklozenge$$

**(b)** $\lambda_x = \dfrac{r_x}{r} = \dfrac{99.16}{240} = 0.413 \qquad \lambda_y = \dfrac{r_y}{r} = \dfrac{118.2}{240} = 0.493 \qquad \lambda_z = \dfrac{r_z}{r} = \dfrac{183.8}{240} = 0.766$

$$\therefore\ \vec{\lambda} = 0.413\mathbf{i} + 0.493\mathbf{j} + 0.766\mathbf{k} \ \blacklozenge$$

## 1.42

$$\overrightarrow{AB} = -5\mathbf{i} + 3\mathbf{j}\ \text{ft} \qquad \left|\overrightarrow{AB}\right| = \sqrt{5^2 + 3^2} = 5.831\ \text{ft}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{-5\mathbf{i} + 3\mathbf{j}}{5.831} = -0.8575\mathbf{i} + 0.5145\mathbf{j}$$

$$\mathbf{F} = F\boldsymbol{\lambda} = 560(-0.8575\mathbf{i} + 0.5145\mathbf{j}) = -480\mathbf{i} + 288\mathbf{j}\ \text{lb} \quad \blacktriangleleft$$

## 1.43

**(a)** $\overrightarrow{\mathbf{AB}} = 7\mathbf{i} + \mathbf{j} + 5\mathbf{k}$ ft $\quad \therefore \left|\overrightarrow{\mathbf{AB}}\right| = \sqrt{7^2 + 1^2 + 5^2} = 8.66$ ft ♦

**(b)** $\vec{\lambda}_{AB} = \dfrac{\overrightarrow{\mathbf{AB}}}{\left|\overrightarrow{\mathbf{AB}}\right|} = \dfrac{7\mathbf{i} + \mathbf{j} + 5\mathbf{k}}{8.66} = 0.808\,\mathbf{i} + 0.115\,\mathbf{j} + 0.577\,\mathbf{k}$ ♦

## 1.44

$$\text{(a)}\quad \boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{-2.2\mathbf{i} + 7.5\mathbf{j} + 3\mathbf{k}}{8.372} = -0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k} \blacktriangleleft$$

$$\begin{aligned}\text{(b)}\quad \mathbf{v} &= 8\boldsymbol{\lambda}_{AB} = 8(-0.2628\mathbf{i} + 0.8958\mathbf{j} + 0.3583\mathbf{k}) \\ &= -2.10\mathbf{i} + 7.17\mathbf{j} + 2.87\mathbf{k}\ \text{m/s} \blacktriangleleft\end{aligned}$$

## 1.45

$$\begin{aligned}\boldsymbol{\lambda}_{OA} &= \frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|} = \frac{-3\mathbf{i} + 4\mathbf{j} + 2.5\mathbf{k}}{5.590} = -0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k} \\ \mathbf{F} &= F\boldsymbol{\lambda}_{OA} = 320(-0.5367\mathbf{i} + 0.7156\mathbf{j} + 0.4472\mathbf{k}) \\ &= -172\mathbf{i} + 229\mathbf{j} + 143\mathbf{k}\ \text{N} \blacktriangleleft\end{aligned}$$

## 1.46

$$\begin{aligned}\boldsymbol{\lambda}_{BA} &= \frac{\overrightarrow{BA}}{\left|\overrightarrow{BA}\right|} = \frac{14\mathbf{i} - 10\mathbf{j} - 18\mathbf{k}}{24.90} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k} \\ \mathbf{F} &= F\boldsymbol{\lambda}_{AB} = 160(0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}) \\ &= 90.0\mathbf{i} - 64.3\mathbf{j} - 115.7\mathbf{k}\ \text{lb} \blacktriangleleft\end{aligned}$$

## 1.47

$$\begin{aligned}\overrightarrow{AB} &= 160\mathbf{i} + 220\mathbf{j} - 70\mathbf{k}\ \text{ft} \\ \boldsymbol{\lambda} &= \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{160\mathbf{i} + 220\mathbf{j} - 70\mathbf{k}}{\sqrt{160^2 + 220^2 + 70^2}} = 0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k} \\ \mathbf{v} &= v\boldsymbol{\lambda} = 1400(0.5696\mathbf{i} + 0.7832\mathbf{j} - 0.2492\mathbf{k}) \\ &= 797\mathbf{i} + 1096\mathbf{j} - 349\mathbf{k}\ \text{ft/s} \blacktriangleleft\end{aligned}$$

**1.48**

**(a)**

$$\overrightarrow{BA} = -20\mathbf{i} + 60\mathbf{j} - 90\mathbf{k}\text{ ft} \qquad \left|\overrightarrow{BA}\right| = \sqrt{20^2 + 60^2 + 90^2} = 110.0\text{ ft}$$

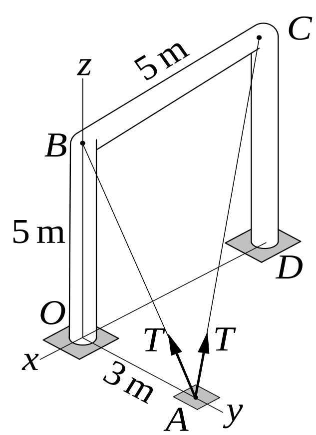
$$\boldsymbol{\lambda} = \frac{\overrightarrow{BA}}{\left|\overrightarrow{BA}\right|} = \frac{-20\mathbf{i} + 60\mathbf{j} - 90\mathbf{k}}{110.0} = -0.1818\mathbf{i} + 0.5455\mathbf{j} - 0.8182\mathbf{k}$$

$$F_x = F\lambda_x = 600\,(-0.1818) = -109\text{ lb} \blacktriangleleft$$
$$F_y = F\lambda_y = 600(0.5455) = 327\text{ lb} \blacktriangleleft$$
$$F_z = F\lambda_z = 600(-0.8182) = -491\text{ lb} \blacktriangleleft$$

**(b)**

$$\theta_x = \cos^{-1}\lambda_x = \cos^{-1}(-0.1818) = 100.5^\circ \blacktriangleleft$$
$$\theta_y = \cos^{-1}\lambda_y = \cos^{-1}(0.5455) = 56.9^\circ \blacktriangleleft$$
$$\theta_z = \cos^{-1}\lambda_z = \cos^{-1}(-0.8182) = 144.9^\circ \blacktriangleleft$$

**1.49**




$$\overrightarrow{AB} = -3\mathbf{j} + 5\mathbf{k}\text{ m} \qquad \left|\overrightarrow{AB}\right| = \sqrt{3^2 + 5^2} = 5.831\text{ m}$$

$$\mathbf{T}_{AB} = T\frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = 35\frac{-3\mathbf{j} + 5\mathbf{k}}{5.831} = -18.01\mathbf{j} + 30.01\mathbf{k}\text{ kN}$$

$$\begin{aligned}
\overrightarrow{AC} &= -5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} \text{ m} \qquad \left|\overrightarrow{AC}\right| = \sqrt{5^2 + 3^2 + 5^2} = 7.681 \text{ m} \\
\mathbf{T}_{AC} &= T\frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = 35\frac{-5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}}{7.681} = -22.78\mathbf{i} - 13.67\mathbf{j} + 22.78\mathbf{k} \text{ kN}
\end{aligned}$$

$$\begin{aligned}
\mathbf{R} &= \mathbf{T}_{AB} + \mathbf{T}_{AC} \\
&= -22.78\mathbf{i} + (-18.01 - 13.67)\mathbf{j} + (30.01 + 22.78)\mathbf{k} \\
&= -22.8\mathbf{i} - 31.7\mathbf{j} + 52.8\mathbf{k} \text{ kN} \blacktriangleleft
\end{aligned}$$

## 1.50

$$\begin{aligned}
\overrightarrow{AB} &= 8\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \text{ ft} \qquad \left|\overrightarrow{AB}\right| = \sqrt{8^2 + 8^2 + 4^2} = 12.0 \text{ ft} \\
\mathbf{F}_{AB} &= F\frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = F\frac{8\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}}{12.0}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{AC} &= -4\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \text{ ft} \qquad \left|\overrightarrow{AC}\right| = \sqrt{4^2 + 8^2 + 4^2} = 9.798 \text{ ft} \\
\mathbf{F}_{AC} &= 200\frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = 200\frac{-4\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}}{9.798}
\end{aligned}$$

The resultant $\mathbf{R}$ lies in the $yz$-plane if

$$\begin{aligned}
R_x &= (F_{AB})_x + (F_{AC})_x = 0 \qquad F\frac{8}{12.0} - 200\frac{4}{9.798} = 0 \\
F &= 122.5 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 1.51

**(a)**

$$\begin{aligned}
\mathbf{R} &= (F_1 + F_2\sin 35^\circ)\mathbf{i} + (F_2\cos 35^\circ + F_3\cos 65^\circ)\mathbf{j} + (F_3\sin 65^\circ)\mathbf{k} \\
&= (1.6 + 1.2\sin 35^\circ)\mathbf{i} + (1.2\cos 35^\circ + 1.0\cos 65^\circ)\mathbf{j} + (1.0\sin 65^\circ)\mathbf{k} \\
&= 2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k} \text{ kN} \blacktriangleleft
\end{aligned}$$

**(b)**

$$\begin{aligned}
R &= \sqrt{2.288^2 + 1.4056^2 + 0.9063^2} = 2.834 \text{ kN} \\
\boldsymbol{\lambda} &= \frac{\mathbf{R}}{R} = \frac{2.288\mathbf{i} + 1.4056\mathbf{j} + 0.9063\mathbf{k}}{2.834} \\
&= 0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k}
\end{aligned}$$

$$\therefore R = 2.83(0.807\mathbf{i} + 0.496\mathbf{j} + 0.320\mathbf{k}) \text{ kN} \blacktriangleleft$$

### 1.52

$$\mathbf{P} = 120\left(\frac{4\mathbf{i}+3\mathbf{j}}{5}\right) = 96\mathbf{i}+72\mathbf{j}\ \text{lb} \qquad \mathbf{Q} = 130\left(\frac{5\mathbf{i}-12\mathbf{j}}{13}\right) = 50\mathbf{i}-120\mathbf{j}\ \text{lb}$$

$$\therefore\ \mathbf{P}+\mathbf{Q} = (96\mathbf{i}+72\mathbf{j})+(50\mathbf{i}-120\mathbf{j}) = 146\mathbf{i}-48\mathbf{j}\ \text{lb} \quad \blacklozenge$$

### 1.53

$$\mathbf{P} = 90\frac{4\mathbf{i}+3\mathbf{j}}{5} = 72\mathbf{i}+54\mathbf{j}\ \text{lb} \qquad \mathbf{Q} = Q\frac{5\mathbf{i}-12\mathbf{j}}{13}$$

Because $\mathbf{R} = \mathbf{P}+\mathbf{Q}$ lies in $x$-direction, we have

$$R_y = 0 \qquad 54 - \frac{12}{13}Q = 0 \qquad Q = 58.5\ \text{lb} \ \blacktriangleleft$$

$$R = R_x = 72 + 58.5\frac{5}{13} = 94.5\ \text{lb} \ \blacktriangleleft$$

### 1.54

$$P_x + Q_x = R_x \qquad P\cos 30^\circ - Q\sin 30^\circ = 360\cos 25^\circ$$

$$P_y + Q_y = R_y \qquad P\sin 30^\circ - Q\cos 30^\circ = -360\sin 25^\circ$$

Solution is: $P = 717$ lb ◄ $\qquad Q = 590$ lb ◄

### 1.55

$$P_x + Q_x = R_x \qquad 3\cos\theta = 2\sin 55^\circ$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\sin 55^\circ\right) = 56.90^\circ \ \blacktriangleleft$$

$$P_y + Q_y = R_y \qquad 3\sin\theta - Q = -2\sin 55^\circ$$

$$Q = 2\sin 55^\circ + 3\sin 56.90^\circ = 4.15\ \text{kN} \ \blacktriangleleft$$

### 1.56

$$\boldsymbol{\lambda}_P = \frac{6\mathbf{i}+8\mathbf{j}-12\mathbf{k}}{\sqrt{6^2+8^2+(-12)^2}} = 0.3841\mathbf{i}+0.5121\mathbf{j}-0.7682\mathbf{k}$$

$$\boldsymbol{\lambda}_Q = \frac{-6\mathbf{i}+6\mathbf{j}-12\mathbf{k}}{\sqrt{(-6)^2+6^2+(-12)^2}} = -0.4082\mathbf{i}+0.4082\mathbf{j}-0.8165\mathbf{k}$$

$$\boldsymbol{\lambda}_F = \frac{-8\mathbf{j}-12\mathbf{k}}{\sqrt{(-8)^2+(-12)^2}} = -0.5547\mathbf{j}-0.8321\mathbf{k}$$

$$\begin{aligned} P_x + Q_x + F_x &= 0 \qquad 0.3841P - 0.4082Q + 0 = 0 \\ P_y + Q_y + F_y &= 0 \qquad 0.5121P + 0.4082Q - 0.5547(120) = 0 \end{aligned}$$

Solution is: $P = 74.3$ lb ◄ $Q = 69.9$ lb ◄

## 1.57

$$\begin{aligned} \text{(a) } \mathbf{A}\cdot\mathbf{B} &= 12(-2) + 8(3) = 0 \;◄ \\ \text{(b) } \mathbf{A}\cdot\mathbf{B} &= 5(7) = 35 \text{ N}\cdot\text{m} \;◄ \\ \text{(c) } \mathbf{A}\cdot\mathbf{B} &= 3(-6) + 2(2) + (-1)(-8) = -6 \text{ m}^2 \;◄ \end{aligned}$$

## 1.58

$$\text{(a) } \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 12 & 8 \\ 4 & -2 & 3 \end{vmatrix} = 52\mathbf{i} + 32\mathbf{j} - 48\mathbf{k} \text{ ft}^2 \;◄$$

$$\text{(b) } \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 7 & 0 & -12 \end{vmatrix} = -36\mathbf{i} + 60\mathbf{j} - 21\mathbf{k} \text{ N}\cdot\text{m} \;◄$$

$$\text{(c) } \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ -6 & 2 & -8 \end{vmatrix} = -14\mathbf{i} + 30\mathbf{j} + 18\mathbf{k} \text{ m}^2 \;◄$$

## 1.59

$$\mathbf{r}\times\mathbf{F}\cdot\boldsymbol{\lambda} = \begin{vmatrix} 4 & -6 & 2 \\ 20 & 40 & -30 \\ 0 & 0.8 & 0.6 \end{vmatrix} = 296 \text{ N}\cdot\text{m} \;◄$$

$$\boldsymbol{\lambda}\times\mathbf{r}\cdot\mathbf{F} = \begin{vmatrix} 0 & 0.8 & 0.6 \\ 4 & -6 & 2 \\ 20 & 40 & -30 \end{vmatrix} = 296 \text{ N}\cdot\text{m} \;◄$$

## 1.60

$\mathbf{A} = 2\mathbf{i} + 1.2\mathbf{j}$ m $\qquad \mathbf{B} = 2\mathbf{i} + 1.2\mathbf{j} + 1.5\mathbf{k}$ m $\qquad \mathbf{C} = -1.5\mathbf{k}$ m

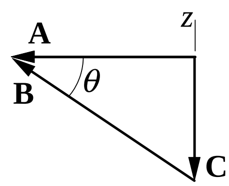
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.2 & 0 \\ 2 & 1.2 & 1.5 \end{vmatrix} = 1.8\mathbf{i} - 3\mathbf{j} \text{ m}^2 \;◄$$

$$\mathbf{C}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1.5 \\ 2 & 1.2 & 1.5 \end{vmatrix} = 1.8\mathbf{i} - 3\mathbf{j} \text{ m}^2 \;◄$$

## 1.61

$$\begin{aligned} \mathbf{A} &= 2\mathbf{i} + 1.2\mathbf{j} \text{ m} \qquad \mathbf{B} = 2\mathbf{i} + 1.2\mathbf{j} + 1.5\mathbf{k} \text{ m} \\ A &= \sqrt{2^2 + 1.2^2} = 2.332 \text{ m} \qquad B = \sqrt{2^2 + 1.2^2 + 1.5^2} = 2.773 \text{ m} \end{aligned}$$

$$\begin{aligned} \cos\theta &= \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{2(2) + 1.2(1.2)}{(2.332)\,(2.773)} = 0.8412 \\ &\therefore \quad \theta = 32.7^\circ \ \blacktriangleleft \end{aligned}$$




Because the three vectors form a right triangle, we have in this case

$$\theta = \cos^{-1}\frac{A}{B} = \cos^{-1}\frac{2.332}{2.773} = 32.8^\circ$$

The difference in the results is due to round-off error.

## 1.62

$$\begin{aligned} B_z &= A_z = \sqrt{14^2 + 9^2}\csc 50^\circ = 21.73 \text{ ft} \\ \mathbf{A} &= 9\mathbf{i} + 14\mathbf{j} + 21.73\mathbf{k} \text{ ft} \qquad \mathbf{B} = 6\mathbf{i} + 21.73\mathbf{k} \text{ ft} \end{aligned}$$

$$\begin{aligned} A &= \sqrt{9^2 + 14^2 + 21.73^2} = 27.37 \text{ ft} \\ B &= \sqrt{6^2 + 21.73^2} = 22.54 \text{ ft} \\ \cos\theta &= \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{9(6) + 21.73(21.73)}{27.37(22.54)} = 0.8529 \\ \theta &= 31.5^\circ \ \blacktriangleleft \end{aligned}$$

## 1.63

**Statement (ii) is true.** ◆

**<u>Proof</u>** **A × B is perpendicular to A and B, that is, normal to plane S. Therefore, C = A × (A × B) is "normal to the normal" of S. Then C lies in plane S. (Note: C is also normal to A.)**

## 1.64

$$\begin{aligned} \mathbf{P} &= 3\mathbf{i} + 4\mathbf{k} \text{ in.} \qquad \mathbf{Q} = -2\mathbf{j} + 4\mathbf{k} \text{ in.} \\ P &= \sqrt{3^2 + 4^2} = 5 \text{ in.} \qquad Q = \sqrt{2^2 + 4^2} = 4.472 \text{ in.} \end{aligned}$$

**(a)**

$$\begin{aligned} \cos\theta &= \frac{\mathbf{P}\cdot\mathbf{Q}}{PQ} = \frac{3(-2) + 4(4)}{5(4.472)} = 0.4472 \\ \theta &= 63.4^\circ \ \blacktriangleleft \end{aligned}$$

**(b)**

$$\begin{aligned} \mathbf{P}\times\mathbf{Q} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 4 \\ 0 & -2 & 4 \end{vmatrix} = 8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k} \text{ in.} \\ \boldsymbol{\lambda} &= \frac{\mathbf{P}\times\mathbf{Q}}{|\mathbf{P}\times\mathbf{Q}|} = \frac{8\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}}{\sqrt{8^2 + 12^2 + 6^2}} \\ &= 0.512\mathbf{i} - 0.768\mathbf{j} - 0.384\mathbf{k} \ \blacktriangleleft \end{aligned}$$

## 1.65

$$\begin{aligned} \mathbf{A}\times\mathbf{B} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & -2 \\ -2 & -4 & 3 \end{vmatrix} = -17\mathbf{i} - 8\mathbf{j} - 22\mathbf{k} \text{ m}^2 \\ \boldsymbol{\lambda} &= \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|} = \pm\frac{-17\mathbf{i} - 8\mathbf{j} - 22\mathbf{k}}{\sqrt{17^2 + 8^2 + 22^2}} \\ &= \pm(-0.588\mathbf{i} - 0.277\mathbf{j} - 0.760\mathbf{k}) \ \blacktriangleleft \end{aligned}$$

## 1.66

$$\begin{aligned} \overrightarrow{CA} &= (0-3)\mathbf{i} + (-2-0)\mathbf{j} + (2-0)\mathbf{k} = -3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ in.} \\ \overrightarrow{CB} &= (-1-3)\mathbf{i} + (4-0)\mathbf{j} + (1-0)\mathbf{k} = -4\mathbf{i} + 4\mathbf{j} + \mathbf{k} \text{ in.} \end{aligned}$$

$$\begin{aligned} \left|\overrightarrow{CA}\right| &= \sqrt{(-3)^2 + (-2)^2 + 2^2} = 4.123 \text{ in.} \\ \left|\overrightarrow{CB}\right| &= \sqrt{(-4)^2 + 4^2 + 1^2} = 5.745 \text{ in.} \end{aligned}$$

$$\overrightarrow{CA}\times\overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 2 \\ -4 & 4 & 1 \end{vmatrix} = -10\mathbf{i} - 5\mathbf{j} - 20\mathbf{k} \text{ in.}^2$$

$$\boldsymbol{\lambda} = \pm\frac{\overrightarrow{CA}\times\overrightarrow{CB}}{\left|\overrightarrow{CA}\times\overrightarrow{CB}\right|} = \pm\frac{-10\mathbf{i} - 5\mathbf{j} - 20\mathbf{k}}{4.123(5.745)} = \pm(-0.422\mathbf{i} - 0.211\mathbf{j} - 0.844\mathbf{k}) \ \blacktriangleleft$$

## 1.67

$$\mathbf{P} = 3\mathbf{i} + 4\mathbf{k}\ \text{m} \qquad \mathbf{Q} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}\ \text{m}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|} = \frac{3\mathbf{i} + 4\mathbf{j}}{\sqrt{3^2 + 4^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$

The component of $\mathbf{P} \times \mathbf{Q}$ in direction of $\boldsymbol{\lambda}$ is

$$\mathbf{P} \times \mathbf{Q} \cdot \boldsymbol{\lambda} = \begin{vmatrix} 3 & 0 & 4 \\ 3 & 4 & 5 \\ 0.6 & 0.8 & 0 \end{vmatrix} = -12.0\ \text{m} \blacktriangleleft$$

## 1.68

$$\vec{\lambda}_A = \frac{\mathbf{A}}{A} = \frac{2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}}{\sqrt{38}} = 0.3244\mathbf{i} - 0.4867\mathbf{j} + 0.8111\mathbf{k}$$

$$F_A = \mathbf{F} \bullet \vec{\lambda}_A = 6(0.3244) + 20(-0.4867) + (-12)(0.8111) = -17.52\ \text{lb} \ \blacklozenge$$

## 1.69

$$\mathbf{A} \cdot \mathbf{B} = 0$$

$$3(4) - a(1) - 2(1) = 0 \qquad a = 10.0 \blacktriangleleft$$

## *1.70

$$\vec{\lambda}_B = \frac{\mathbf{B}}{B} = \frac{6\mathbf{i} + 2\mathbf{k}}{\sqrt{6^2 + 2^2}} = 0.9487\mathbf{i} + 0.3162\mathbf{k}$$

The component of **A** parallel to **B** is: $A_B = \mathbf{A} \bullet \vec{\lambda}_B = 3(0.9487) - 4(0.3162) = 1.5813$ in.

$$\therefore \mathbf{A}_B = 1.5813(0.9487\mathbf{i} + 0.3162\mathbf{k})\ \text{in.} \ \blacklozenge$$

Letting $\mathbf{A}_C$ be the component of **A** that is perpendicular to **B**, we have

$$\mathbf{A}_C = \mathbf{A} - \mathbf{A}_B$$

$$\mathbf{A}_C = (3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}) - 1.5813(0.9487\mathbf{i} + 0.3162\mathbf{k}) = 1.500\mathbf{i} + 5.000\mathbf{j} - 4.500\mathbf{k}\ \text{in.}$$

$$\therefore A_C = \sqrt{1.500^2 + 5.000^2 + 4.500^2} = 6.892\ \text{in.}$$

The unit vector in the direction of $\mathbf{A}_C$ is

$$\vec{\lambda}_C = \frac{\mathbf{A}_C}{A_C} = \frac{1.500\mathbf{i} + 5.000\mathbf{j} - 4.500\mathbf{k}}{6.892} = 0.2176\mathbf{i} + 0.7255\mathbf{j} - 0.6529\mathbf{k}$$

$$\therefore \mathbf{A}_C = 6.892(0.2176\mathbf{i} + 0.7255\mathbf{j} - 0.6529\mathbf{k})\ \text{in.} \ \blacklozenge$$

## 1.71

By inspection, a unit vector perpendicular to the door is

$$\boldsymbol{\lambda} = \sin 20^\circ\ \mathbf{i} + \cos 20^\circ\ \mathbf{j} = 0.3420\mathbf{i} + 0.9397\mathbf{j}$$

The component of $\mathbf{F}$ perpendicular to the plane of the door is

$$F_\perp = \mathbf{F} \cdot \boldsymbol{\lambda} = -5(0.3420) + 12(0.9397) = 9.57 \text{ lb} \blacktriangleleft$$

## 1.72

**For the three vectors to lie in the same plane, $\mathbf{A} \times \mathbf{B} \bullet \mathbf{C} = 0$.**

$$\mathbf{A} \times \mathbf{B} \bullet \mathbf{C} = \begin{vmatrix} 2 & -1 & 2 \\ 6 & 3 & a \\ 16 & 46 & 7 \end{vmatrix} = 2(21 - 46a) + 1(42 - 16a) + 2(276 - 48) = 0$$

**which gives:** $540 - 108a = 0 \quad \therefore\ a = 5 \text{ m}$ ♦

## *1.73

**We first compute a unit vector $\vec{\lambda}$ that is perpendicular to plane ABC:**

$$\overrightarrow{AB} = -2\mathbf{i} + 5\mathbf{k} \text{ in.} \qquad \overrightarrow{AC} = -2\mathbf{i} + 6\mathbf{j} \text{ in.}$$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & 0 \\ -2 & 0 & 5 \end{vmatrix} = 30\mathbf{i} + 10\mathbf{j} + 12\mathbf{k} \text{ in}^2$$

$$\therefore\ \vec{\lambda} = \frac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|} = \frac{30\mathbf{i} + 10\mathbf{j} + 12\mathbf{k}}{\sqrt{30^2 + 10^2 + 12^2}} = 0.8870\mathbf{i} + 0.2957\mathbf{j} + 0.3548\mathbf{k}$$

**The normal component of $\mathbf{F} = 20\mathbf{i} + 30\mathbf{j} + 50\mathbf{k}$ lb is:**

$$F_n = \mathbf{F} \bullet \vec{\lambda} = 20(0.8870) + 30(0.2957) + 50(0.3548)) = 44.15 \text{ lb}$$

$$\mathbf{F}_n = F_n \vec{\lambda} = 44.15(0.8870\mathbf{i} + 0.2957\mathbf{j} + 0.3548\mathbf{k}) = 39.16\mathbf{i} + 13.06\mathbf{j} + 15.66\mathbf{k} \text{ lb}$$

**Therefore, the component of $\mathbf{F}$ that lies in plane ABC is:**

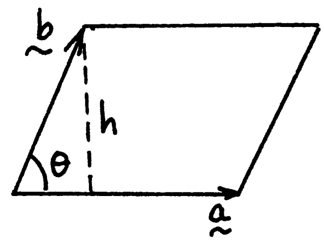
$$\mathbf{F}_t = \mathbf{F} - \mathbf{F}_n = -19.16\mathbf{i} + 16.94\mathbf{j} + 34.34\mathbf{k} \text{ lb}$$ ♦

1.74

Since **a** × **b** is perpendicular to the area, it has the correct direction.

Check of magnitude:

$|\mathbf{a} \times \mathbf{b}| = ab\sin\theta = ah = A$ It checks!



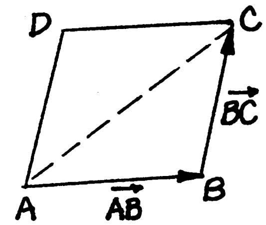

1.75

$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$ = area of parallelogram ABCD

= 2(area of triangle ABC)

$\overrightarrow{AB} = -5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ in. $\overrightarrow{BC} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ in.

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 3 \\ 2 & -2 & 1 \end{vmatrix} = 9\mathbf{i} + 11\mathbf{j} + 4\mathbf{k} \text{ in.}^2$$




$$\text{Therefore, area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \frac{1}{2}\sqrt{9^2 + 11^2 + 4^2} = 14.76 \text{ in.}^2 \ \blacklozenge$$

1.76

$|\mathbf{a} \times \mathbf{b} \bullet \mathbf{c}| = |\mathbf{a} \times \mathbf{b}|\,|\mathbf{c}|\,|\cos\theta|$

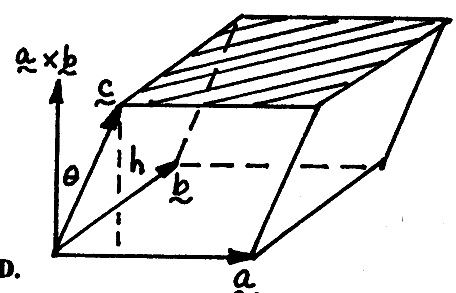
From Prob. 1.74:

$|\mathbf{a} \times \mathbf{b}|$ = area of base (shown shaded in figure)

Note that $|\mathbf{c}|\,|\cos\theta| = h$

$\therefore |\mathbf{a} \times \mathbf{b} \bullet \mathbf{c}|$ = (area of base) x h

= vol. of parallelpiped **Q.E.D.**



# Chapter 2

## 2.1

The resultant of each force system is 500N ↑.

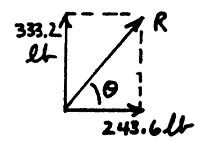
Each resultant force has the same line of action as the the force in (a), except (f) and (h)

Therefore (b), (c), (d), (e) and (g) are equivalent to (a) ◀

## 2.2

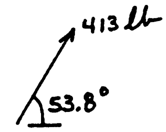
$$R_x = \Sigma F_x : \overset{+}{\rightarrow} \quad R_x = 300 \cos 70^\circ + 150 \cos 20^\circ = 243.6 \text{ lb}$$

$$R_y = \Sigma F_y : +\uparrow \quad R_y = 300 \sin 70^\circ + 150 \sin 20^\circ = 333.2 \text{ lb}$$




$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{243.6^2 + 333.2^2} = 413 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{333.2}{243.6}\right) = 53.8^\circ$$

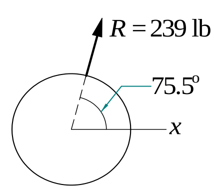
**R** = 

◆

## 2.3

$$\begin{aligned} R_x &= \Sigma F_x = -T_1 \cos 60^\circ + T_3 \cos 40^\circ \\ &= -110 \cos 60^\circ + 150 \cos 40^\circ = 59.91 \text{ lb} \\ R_y &= \Sigma F_y = T_1 \sin 60^\circ + T_2 + T_3 \sin 40^\circ \\ &= 110 \sin 60^\circ + 40 + 150 \sin 40^\circ = 231.7 \text{ lb} \end{aligned}$$

$$\begin{aligned} R &= \sqrt{59.91^2 + 231.7^2} = 239 \text{ lb} \quad ◀ \\ \theta &= \tan^{-1} \frac{231.7}{59.91} = 75.5^\circ \quad ◀ \end{aligned}$$




## 2.4

$$\begin{aligned} R_x &= \Sigma F_x \qquad + \longrightarrow \qquad R_x = 25 \cos 45^\circ + 40 \cos 60^\circ - 30 \\ R_x &= 7.68 \text{ kN} \end{aligned}$$

$$R_y = \Sigma F_y \quad +\uparrow \quad R_y = 25\sin 45^\circ - 40\sin 60^\circ$$
$$R_y = -16.96 \text{ kN}$$

$$\mathbf{R} = 7.68\mathbf{i} - 16.96\mathbf{k} \text{ kN} \blacktriangleleft$$

## 2.5

$$\mathbf{F}_1 = F_1\boldsymbol{\lambda}_{AB} = 80\frac{-120\mathbf{j}+80\mathbf{k}}{\sqrt{(-120)^2+80^2}} = -66.56\mathbf{j}+44.38\mathbf{k} \text{ N}$$
$$\mathbf{F}_2 = F_2\boldsymbol{\lambda}_{AC} = 60\frac{-100\mathbf{i}-120\mathbf{j}+80\mathbf{k}}{\sqrt{(-100)^2+(-120)^2+80^2}}$$
$$= -34.19\mathbf{i}-41.03\mathbf{j}+27.35\mathbf{k} \text{ N}$$
$$\mathbf{F}_3 = F_3\boldsymbol{\lambda}_{AD} = 50\frac{-100\mathbf{i}+80\mathbf{k}}{\sqrt{(-100)^2+80^2}} = -39.04\mathbf{i}+31.24\mathbf{k} \text{ N}$$

$$\mathbf{R} = \Sigma\mathbf{F} = (-34.19-39.04)\mathbf{i}+(-66.56-41.03)\mathbf{j}$$
$$+(44.38+27.35+31.24)\mathbf{k}$$
$$= -73.2\mathbf{i}-107.6\mathbf{j}+103.0\mathbf{k} \text{ N} \blacktriangleleft$$

## 2.6

**(a)** $\mathbf{P}_1 = 110\mathbf{j}$ lb $\quad \mathbf{P}_2 = -200\cos 25^\circ\mathbf{i} + 200\sin 25^\circ\mathbf{j} = -181.26\mathbf{i} + 84.52\mathbf{j}$ lb

$\mathbf{P}_3 = -150\cos 40^\circ\mathbf{i} + 150\sin 40^\circ\mathbf{k} = -114.91\mathbf{i} + 96.42\mathbf{k}$ lb

$\mathbf{R} = \Sigma\mathbf{P} = (-181.26 - 114.91)\mathbf{i} + (110 + 84.52)\mathbf{j} + 96.42\mathbf{k}$

$= -296.17\mathbf{i} + 194.52\mathbf{j} + 96.42\mathbf{k}$ lb

$\therefore \; R = \sqrt{(-296.17)^2 + 194.52^2 + 96.42^2} = 367.2$ lb ♦

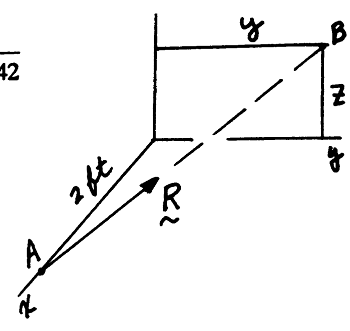
$$\frac{\overline{AB}_x}{|R_x|} = \frac{\overline{AB}_y}{|R_y|} = \frac{\overline{AB}_z}{|R_z|} : \quad \frac{2}{296.17} = \frac{y}{194.52} = \frac{z}{96.42}$$

$$y = \frac{2(194.52)}{296.17} = 1.314 \text{ ft}$$

$$z = \frac{2(96.42)}{296.17} = 0.651 \text{ ft}$$

$\therefore$ **R** passes through the point

(b) (0, 1.314 ft, 0.651 ft) ♦



## 2.7

$$\begin{aligned}\mathbf{R} &= (-P_2\cos 25^\circ - P_3\cos 40^\circ)\mathbf{i} + (P_1 + P_2\sin 25^\circ)\mathbf{j} + P_3\sin 40^\circ\mathbf{k} \\ &= -800\mathbf{i} + 700\mathbf{j} + 500\mathbf{k}\ \text{lb}\end{aligned}$$

Equating like coefficients:

$$\begin{aligned}-P_2\cos 25^\circ - P_3\cos 40^\circ &= -800 \\ P_1 + P_2\sin 25^\circ &= 700 \\ P_3\sin 40^\circ &= 500\end{aligned}$$

Solution is

$$P_1 = 605\ \text{lb} \blacktriangleleft \qquad P_2 = 225\ \text{lb} \blacktriangleleft \qquad P_3 = 778\ \text{lb} \blacktriangleleft$$

## 2.8

$$\begin{aligned}\mathbf{T}_1 &= 90\frac{-\mathbf{i}+2\mathbf{j}+6\mathbf{k}}{\sqrt{(-1)^2+2^2+6^2}} = -14.06\mathbf{i} + 28.11\mathbf{j} + 84.33\mathbf{k}\ \text{kN} \\ \mathbf{T}_2 &= 60\frac{-2\mathbf{i}-3\mathbf{j}+6\mathbf{k}}{\sqrt{(-2)^2+(-3)^2+6^2}} = -17.14\mathbf{i} - 25.71\mathbf{j} + 51.43\mathbf{k}\ \text{kN} \\ \mathbf{T}_3 &= 40\frac{2\mathbf{i}-3\mathbf{j}+6\mathbf{k}}{\sqrt{2^2+(-3)^2+6^2}} = 11.43\mathbf{i} - 17.14\mathbf{j} + 34.29\mathbf{k}\ \text{kN}\end{aligned}$$

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = (-14.06 - 17.14 + 11.43)\mathbf{i} \\ &\quad +(28.11 - 25.71 - 17.14)\mathbf{j} + (84.33 + 51.43 + 34.29)\mathbf{k} \\ &= -19.77\mathbf{i} - 14.74\mathbf{j} + 170.05\mathbf{k}\ \text{kN} \blacktriangleleft\end{aligned}$$

## 2.9

$$\begin{aligned}\mathbf{T}_1 &= T_1\frac{-\mathbf{i}+2\mathbf{j}+6\mathbf{k}}{\sqrt{(-1)^2+2^2+6^2}} = T_1(-0.15617\mathbf{i} + 0.3123\mathbf{j} + 0.9370\mathbf{k}) \\ \mathbf{T}_2 &= T_2\frac{-2\mathbf{i}-3\mathbf{j}+6\mathbf{k}}{\sqrt{(-2)^2+(-3)^2+6^2}} = T_2(-0.2857\mathbf{i} - 0.4286\mathbf{j} + 0.8571\mathbf{k}) \\ \mathbf{T}_3 &= T_3\frac{2\mathbf{i}-3\mathbf{j}+6\mathbf{k}}{\sqrt{2^2+(-3)^2+6^2}} = T_3(0.2857\mathbf{i} - 0.4286\mathbf{j} + 0.8571\mathbf{k})\end{aligned}$$

$$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = \mathbf{R}$$

Equating like components, we get

$$\begin{gathered}-0.15617T_1 - 0.2857T_2 + 0.2857T_3 = 0 \\ 0.3123T_1 - 0.4286T_2 - 0.4286T_3 = 0 \\ 0.9370T_1 + 0.8571T_2 + 0.8571T_3 = 210\end{gathered}$$

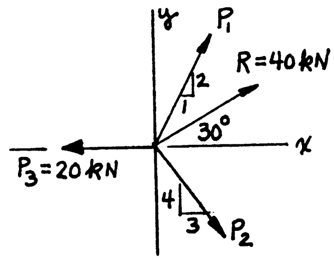
Solution is

$$T_1 = 134.5\ \text{kN} \blacktriangleleft \qquad T_2 = 12.24\ \text{kN} \blacktriangleleft \qquad T_3 = 85.8\ \text{kN} \blacktriangleleft$$

## 2.10

$R_x = \Sigma F_x$: $\overset{+}{\rightarrow}$ $\frac{1}{\sqrt{5}}P_1 + \frac{3}{5}P_2 - 20 = 40\cos 30^\circ$ (1)

$R_y = \Sigma F_y$: $+\uparrow$ $\frac{2}{\sqrt{5}}P_1 - \frac{4}{5}P_2 = 40\sin 30^\circ$ (2)




Solving (1) and (2) gives:

$P_1 = 62.3$ kN ◆

$P_2 = 44.6$ kN ◆

## 2.11

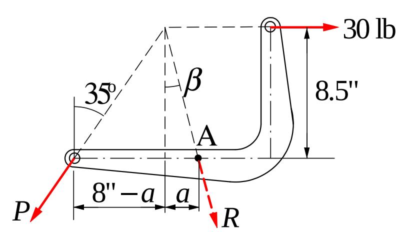
$$\mathbf{F}_1 = -10\cos 20^\circ\mathbf{i} - 10\sin 20^\circ\mathbf{j} = -9.397\mathbf{i} - 3.420\mathbf{j} \text{ lb}$$
$$\mathbf{F}_2 = F_2(\sin 60^\circ\mathbf{i} + \cos 60^\circ\mathbf{j}) = F_2(0.8660\mathbf{i} + 0.5\mathbf{j})$$
$$\mathbf{R} = \Sigma\mathbf{F} = (-9.397 + 0.8660F_2)\mathbf{i} + (-3.420 + 0.5F_2)\mathbf{j}$$

$$\overrightarrow{AB} = -4\mathbf{i} + 6\mathbf{j} \text{ in.}$$

Because $\mathbf{R}$ and $\overrightarrow{AB}$ are parallel, their components are proportional:

$$\frac{-9.397 + 0.8660F_2}{-4} = \frac{-3.420 + 0.5F_2}{6}$$
$$F_2 = 9.74 \text{ lb} \blacktriangleleft$$

## 2.12




First find the direction of $\mathbf{R}$ from geometry (the 3 forces must intersect at a common point).

$$8 - a = 8.5\tan 35^\circ \qquad \therefore a = 2.048 \text{ in.}$$
$$\beta = \tan^{-1}\frac{a}{8.5} = \tan^{-1}\frac{2.048}{8.5} = 13.547^\circ$$

$$R_x = \Sigma F_x \quad +\longrightarrow \quad R\sin 13.547^\circ = -P\sin 35^\circ + 30$$
$$R_y = \Sigma F_y \quad +\downarrow \quad R\cos 13.547^\circ = P\cos 35^\circ$$

Solution is

$$P = 38.9 \text{ lb} \blacktriangleleft \qquad R = 32.8 \text{ lb} \blacktriangleleft$$

## 2.13

$$\mathbf{F}_{AB} = 15\frac{12\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}}{\sqrt{12^2 + (-6)^2 + 9^2}} = 11.142\mathbf{i} - 5.571\mathbf{j} + 8.356\mathbf{k} \text{ lb}$$
$$\mathbf{F}_{AC} = -11.142\mathbf{i} - 5.571\mathbf{j} + 8.356\mathbf{k} \text{ lb (by symmetry)}$$

$$\Sigma F_y = 0: \quad 2(-5.571) + T = 0$$
$$T = 11.14 \text{ lb} \blacktriangleleft$$

## 2.14

$$\mathbf{P}_1 = 100\frac{3\mathbf{i} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}} = 60\mathbf{i} + 80\mathbf{k} \text{ lb}$$
$$\mathbf{P}_2 = 120\frac{3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 3^2 + 4^2}} = 61.74\mathbf{i} + 61.74\mathbf{j} + 82.32\mathbf{k} \text{ lb}$$
$$\mathbf{P}_3 = 60\mathbf{j} \text{ lb}$$

$$\mathbf{Q}_1 = Q_1\mathbf{i}$$
$$\mathbf{Q}_2 = Q_2\frac{-3\mathbf{i} - 3\mathbf{j}}{\sqrt{3^2 + 3^2}} = Q_2(-0.7071\mathbf{i} - 0.7071\mathbf{j})$$
$$\mathbf{Q}_3 = Q_3\frac{3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}} = Q_3(0.6\mathbf{j} + 0.8\mathbf{k})$$

Equating similar components of $\Sigma\mathbf{Q} = \Sigma\mathbf{P}$:

$$Q_1 - 0.7071Q_2 = 60 + 61.74$$
$$-0.7071Q_2 + 0.6Q_3 = 61.74 + 60$$
$$0.8Q_3 = 80 + 82.32$$

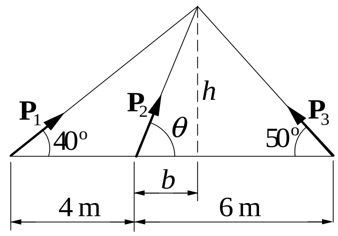
Solution is

$$Q_1 = 121.7 \text{ lb} \blacktriangleleft \qquad Q_2 = 0 \qquad Q_3 = 203 \text{ lb} \blacktriangleleft$$

## 2.15

$$R_x = \Sigma F_x \quad +\longrightarrow \quad 8 = 40\sin 45^\circ - Q\sin 30^\circ \qquad Q = 40.57 \text{ lb}$$
$$R_y = \Sigma F_y \quad +\uparrow \quad 0 = 40\cos 45^\circ - W + 40.57\cos 30^\circ$$
$$\therefore \quad W = 63.4 \text{ lb} \blacktriangleleft$$

## 2.16




The forces must be concurrent. From geometry:

$$h = (4+b)\tan 40^\circ = (6-b)\tan 50^\circ \qquad \therefore b = 1.8682 \text{ m} \blacktriangleleft$$

$$\therefore \quad h = (4+1.8682)\tan 40^\circ = 4.924 \text{ m}$$

$$\theta = \tan^{-1}\frac{h}{b} = \tan^{-1}\frac{4.924}{1.8682} = 69.22^\circ \blacktriangleleft$$

$$\begin{aligned}\mathbf{R} = \Sigma\mathbf{F} &= (25\cos 40^\circ + 60\cos 69.22^\circ - 80\cos 50^\circ)\mathbf{i} \\ &\quad + (25\sin 40^\circ + 60\sin 69.22^\circ + 80\sin 50^\circ)\mathbf{j} \\ &= -10.99\mathbf{i} + 133.45\mathbf{j} \text{ kN} \blacktriangleleft\end{aligned}$$

## 2.17

**The three forces intersect at C.**

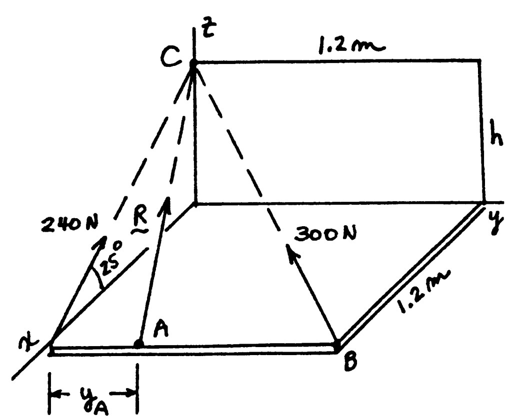
**$h = 1.2 \tan 25^\circ = 0.5596$ m**

**For the 240-N force :**

$$-240(\cos 25^\circ\,\mathbf{i} - \sin 25^\circ\,\mathbf{k}) = -217.5\,\mathbf{i} + 101.4\,\mathbf{k} \text{ N}$$

**For the 300-N force $\left(300\,\overrightarrow{\lambda_{BC}}\right)$:**

$$300\left(\frac{-1.2\,\mathbf{i} - 1.2\,\mathbf{j} + 0.5596\,\mathbf{k}}{1.787}\right) = -201.5\,\mathbf{i} - 201.5\,\mathbf{j} + 93.95\,\mathbf{k} \text{ N}$$




**$\mathbf{R} = \Sigma\mathbf{F}$**

$$= (-217.5 - 201.5)\mathbf{i} - 201.5\,\mathbf{j} + (101.4 + 93.95)\mathbf{k} = -419.0\,\mathbf{i} - 201.5\,\mathbf{j} + 195.4\,\mathbf{k} \text{ N} \blacklozenge$$

**Since $\mathbf{R}$ acts along $\overline{AC}$:** $\dfrac{|R_y|}{y_A} = \dfrac{|R_x|}{1.2}$ $\qquad \therefore y_A = \dfrac{|R_y|}{|R_x|}(1.2) = \dfrac{201.5}{419.0}(1.2) = 0.577$ m $\blacklozenge$

## 2.18

$$\begin{aligned}
\mathbf{T}_1 &= 180\frac{3\mathbf{i}-2\mathbf{j}-6\mathbf{k}}{\sqrt{3^2+(-2)^2+(-6)^2}} = 77.14\mathbf{i}-51.43\mathbf{j}-154.29\mathbf{k}\text{ lb}\\
\mathbf{T}_2 &= 250\frac{3\mathbf{j}-6k}{\sqrt{3^2+(-6)^2}} = 111.80\mathbf{j}-223.61\mathbf{k}\text{ lb}\\
\mathbf{T}_3 &= 400\frac{-4\mathbf{i}-6\mathbf{k}}{\sqrt{(-4)^2+(-6)^2}} = -221.88\mathbf{i}-332.82\mathbf{k}\text{ lb}\\
\mathbf{R} &= \Sigma\mathbf{T} = (77.14-221.88)\mathbf{i}+(-51.43+111.80)\mathbf{j}\\
&\quad +(-154.29-223.61-332.82)\mathbf{k}\\
&= -144.7\mathbf{i}+60.4\mathbf{j}-710.7\mathbf{k}\text{ lb} \blacktriangleleft \text{ acting through point } A.
\end{aligned}$$

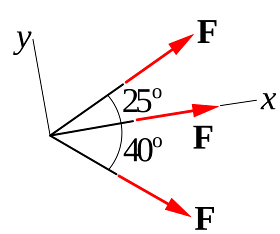
## 2.19

$$\begin{aligned}
\mathbf{T}_{AB} &= T_{AB}\boldsymbol{\lambda}_{AB} = 120\frac{3\mathbf{i}-12\mathbf{j}+10\mathbf{k}}{\sqrt{3^2+(-12)^2+10^2}}\\
&= 22.63\mathbf{i}-90.53\mathbf{j}+75.44\mathbf{k}\text{ lb}\\
\mathbf{T}_{AC} &= T_{AC}\boldsymbol{\lambda}_{AC} = 160\frac{-8\mathbf{i}-12\mathbf{j}+3\mathbf{k}}{\sqrt{(-8)^2+(-12)^2+3^2}}\\
&= -86.89\mathbf{i}-130.34\mathbf{j}+32.59\mathbf{k}\text{ lb}
\end{aligned}$$

$$\begin{aligned}
\mathbf{R} &= \mathbf{T}_{AB}+\mathbf{T}_{AC}-W\mathbf{k}\\
&= (22.63-86.89)\mathbf{i}+(-90.53-130.34)\mathbf{j}+(75.44+32.59-108)\mathbf{k}\\
&= -64.3\mathbf{i}-220.9\mathbf{j}+0.0\mathbf{k}\text{ lb} \blacktriangleleft
\end{aligned}$$

## 2.20

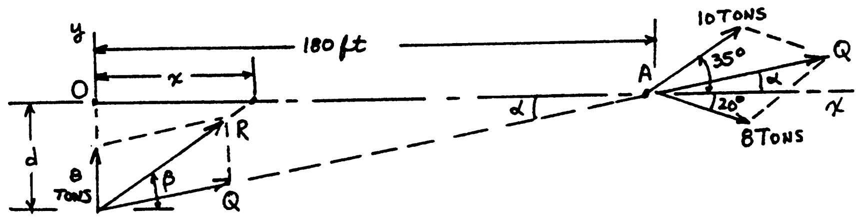
Choose the line of action of the middle force as the $x$-axis.




$$\begin{aligned}
R_x &= \Sigma F_x = F(\cos 25^\circ + 1 + \cos 40^\circ) = 2.672F\\
R_y &= \Sigma F_y = F(\sin 25^\circ - \sin 40^\circ) = -0.2202F
\end{aligned}$$

$$\begin{aligned}
R &= F\sqrt{2.672^2+(-0.2202)^2} = 2.681F\\
400 &= 2.681F \qquad \therefore F = 149.2\text{ lb} \blacktriangleleft
\end{aligned}$$

***2.21**




Let **Q** be the resultant of the two forces at A.

$\xrightarrow{+}$ $Q_x = \Sigma F_x = 10 \cos 35^\circ + 8 \cos 20^\circ = 15.71$ tons

$+\uparrow$ $Q_y = \Sigma F_y = 10 \sin 35^\circ - 8 \sin 20^\circ = 3.00$ tons

$\therefore$ $\tan \alpha = Q_y/Q_x = 3.00/15.71 = 0.1910$

Let **R** be the resultant of **Q** and the 8-ton vertical force.

$\xrightarrow{+}$ $R_x = \Sigma F_x = Q_x = 15.71$ tons

$+\uparrow$ $R_y = \Sigma F_y = 8 + Q_y = 8 + 3 = 11$ tons

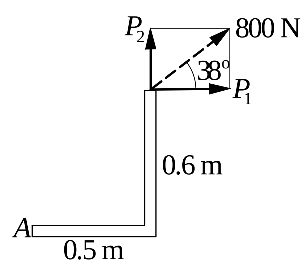
$\therefore$ $\mathbf{R} = 15.71\,\mathbf{i} + 11.00\,\mathbf{j}$ tons ♦

(Note that $\tan \beta = R_y/R_x = 11.00/15.71 = 0.7002$)

To find **x**: $d = 180 \tan \alpha = 180(0.1910) = 34.38$ ft

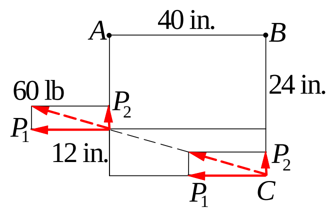
$x = d/\tan \beta = 34.38/0.7002 = 49.1$ ft ♦

**2.22**




$$
\begin{aligned}
+\circlearrowleft \quad M_A &= -0.6P_1 + 0.5P_2 \\
&= -0.6(800 \cos 38^\circ) + 0.5(800 \sin 38^\circ) = -132.0\ \text{N}\cdot\text{m} \\
&\therefore\ M_A = 132.0\ \text{N}\cdot\text{m} \circlearrowright \quad \blacktriangleleft
\end{aligned}
$$

## 2.23




$$P_1 = 60\frac{40}{\sqrt{40^2 + 12^2}} = 57.47 \text{ lb}$$

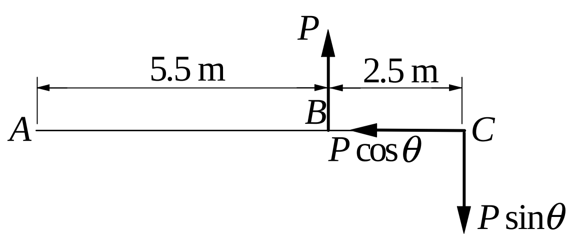
With the force in the original position:

$$M_A = 24P_1 = 24(57.47) = 1379 \text{ lb} \cdot \text{in.} \circlearrowright \quad ◀$$

With the force moved to point $C$:

$$M_B = 36P_1 = 36(57.47) = 2070 \text{ lb} \cdot \text{in.} \circlearrowright ◀$$

## 2.24

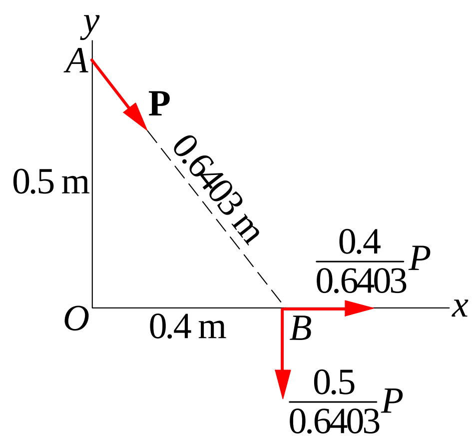



Resolve the force at $C$ into components as shown. Adding the moments of the forces about $A$ yields

$$+ \circlearrowleft \quad M_A = 5.5P - 8P\sin\theta = 0$$

$$\sin\theta = \frac{5.5}{8} = 0.6875 \qquad \theta = 43.4° \quad ◀$$

**2.25**




Since $M_A = M_B = 0$, the force **P** passes through $A$ and $B$, as shown.

$$+\ \circlearrowleft \quad M_O = \frac{0.5}{0.6403}P(0.4) = 350\ \text{kN}\cdot\text{m} \qquad P = 1120.5\ \text{N}$$

$$P = \frac{0.4}{0.6403}1120.5\mathbf{i} - \frac{0.5}{0.6403}1120.5\mathbf{j} = 700\mathbf{i} - 875\mathbf{j}\ \text{N} \ ◄$$

**2.26**

Since $M_B = 0$, **P** passes though B.

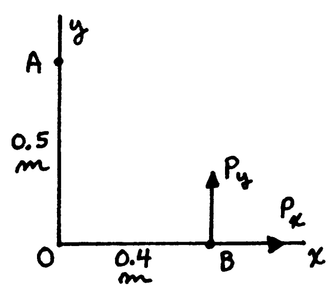
$\circlearrowleft +$ $M_O = 0.4\,P_y = 80$ N•m

$P_y = 200$ N

$\circlearrowleft +$ $M_A = 0.4(200) + 0.5\,P_x = -200$ N•m

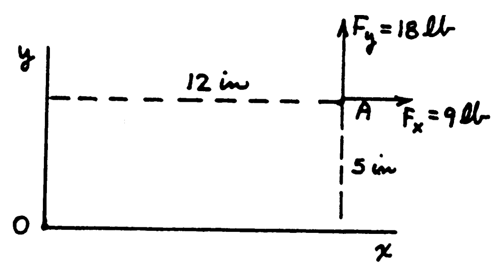
$P_x = -280/0.5 = -560$ N

$\therefore$ $\mathbf{P} = -560\mathbf{i} + 200\mathbf{j}$ N ◆




**2.27**

$\mathbf{F} = 9\,\mathbf{i} + 18\,\mathbf{j}$ lb



(a) $\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 5 & 0 \\ 9 & 18 & 0 \end{vmatrix}$

$= \mathbf{k}\left[18(12) - 5(9)\right] = 171\,\mathbf{k}$ lb•in. ♦

(b) $\circlearrowleft +$ $M_O = 18(12) - 9(5) = 171$ lb•in.  $\therefore M_O = 171$ lb•in CCW ♦

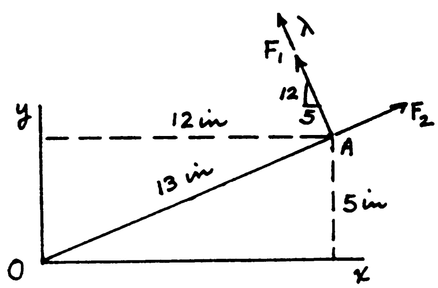
(c) Unit vector perpendicular to OA is

$\vec{\lambda} = -\frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j}$

$F_1 = \mathbf{F} \bullet \vec{\lambda}$

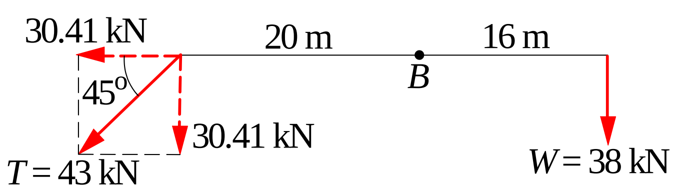
$= (9\,\mathbf{i} + 18\,\mathbf{j}) \bullet (-\frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j})$

$= \frac{-45 + 216}{13} = 13.15$ lb•in.




$\circlearrowleft +$ $M_O = 13\,F_1 = 13(13.15) = 171$ lb•in.  $\therefore M_O = 171$ lb•in CCW ♦

**2.28**




(a) Moment of **T**:

$$+\circlearrowleft \; M_B = 30.41(20) = 608 \text{ kN}\cdot\text{m CCW} \;◀$$

(b) Moment of $W$:

$$+\circlearrowleft \; M_B = 38(16) = 608 \text{ kN}\cdot\text{m CW} \;◀$$

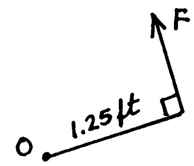
(c) Combined moment:

$$+\circlearrowleft \; M_B = 608 - 608 = 0 \;◀$$

## 2.29

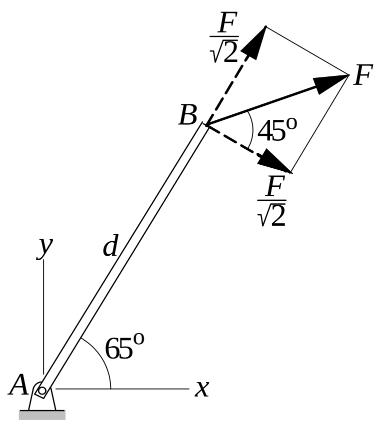
**The moment of F about O is maximum when $\theta = 90^{\circ}$** ◆

$M_O = F(1.25) = 50$ lb•ft $\quad \therefore\ F = \dfrac{50}{1.25} = 40$ lb ◆




## 2.30

(a)



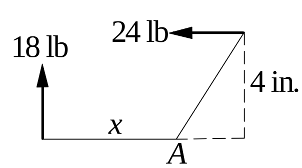

$$M_A = \frac{Fd}{\sqrt{2}} \circlearrowright \quad \blacktriangleleft$$

(b)

$$\begin{aligned}\mathbf{F} &= F\cos 20^{\circ}\mathbf{i} + F\sin 20^{\circ}\mathbf{j} \\ \mathbf{r} &= \overrightarrow{AB} = d\cos 65^{\circ}\mathbf{i} + d\sin 65^{\circ}\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_A &= \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 65^{\circ} & \sin 65^{\circ} & 0 \\ \cos 20^{\circ} & \sin 20^{\circ} & 0 \end{vmatrix} Fd \\ &= (\sin 20^{\circ}\cos 65^{\circ} - \cos 20^{\circ}\sin 65^{\circ})\, Fd\ \mathbf{k} = -\,0.707Fd\mathbf{k} \quad \blacktriangleleft\end{aligned}$$

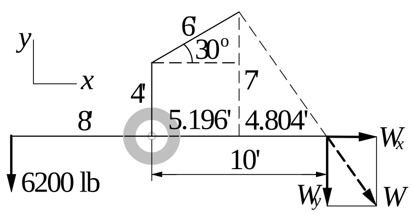
## 2.31



Because the resultant passes through point $A$, we have

$$\Sigma M_A = 0 \qquad + \circlearrowleft \quad 24(4) - 18x = 0 \qquad x = 5.33 \text{ in.} \blacktriangleleft$$

**2.32**



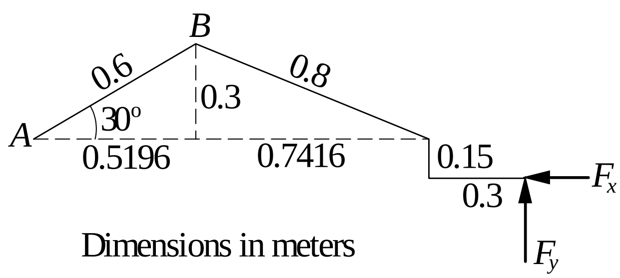

$$W_y = W \frac{7}{\sqrt{7^2 + 4.804^2}} = 0.8245W$$

Largest $W$ occurs when the moment about the rear axle is zero.

$$+ \circlearrowleft \qquad M_{\text{axle}} = 6200(8) - (0.8245W)(10) = 0$$

$$\therefore \quad W = 6020 \text{ lb} \blacktriangleleft$$

**2.33**




$$\begin{aligned} + \circlearrowleft \quad M_A &= -F_x(0.15) + F_y(0.5196 + 0.7416 + 0.3) \\ 310 &= -0.15F_x + 1.5612F_y \qquad \text{(a)} \\ + \circlearrowleft \quad M_B &= -F_x(0.3 + 0.15) + F_y(0.7416 + 0.3) \\ 120 &= -0.45F_x + 1.0416F_y \qquad \text{(b)} \end{aligned}$$

$$\begin{aligned} 310 &= -0.15F_x + 1.5612F_y \\ 120 &= -0.45F_x + 1.0416F_y \end{aligned}$$

Solution of Eqs. (a) and (b) is $F_x = 248.1$ N and $F_y = 222.4$ N

$$\therefore \quad F = \sqrt{248.1^2 + 222.4^2} = 333 \text{ N} \blacktriangleleft$$

$$\theta = \tan^{-1}\frac{F_x}{F_y} = \tan^{-1}\frac{248.1}{222.4} = 48.1° \blacktriangleleft$$

## 2.34

$$\begin{aligned}\mathbf{P} &= P\frac{-70\mathbf{i}-100\mathbf{k}}{\sqrt{(-70)^2+(-100)^2}} = (-0.5735\mathbf{i}-0.8192\mathbf{k})P \\ \mathbf{r} &= \overrightarrow{AB} = -0.07\mathbf{i}+0.09\mathbf{j}\ \text{m}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_A &= \mathbf{r}\times\mathbf{P} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.07 & 0.09 & 0 \\ -0.5735 & 0 & -0.8192\end{vmatrix} P \\ &= (-73.73\mathbf{i}-57.34\mathbf{j}+51.62\mathbf{k})\times 10^{-3}P\end{aligned}$$

$$\begin{aligned}M_A &= \sqrt{(-73.73)^2+(-57.34)^2+51.62^2}(10^{-3}P) \\ &= 106.72\times 10^{-3}P\end{aligned}$$

Using $M_A = 15$ N·m, we get

$$15 = 106.72\times 10^{-3}P \qquad P = 140.6\ \text{N}\ \blacktriangleleft$$

## 2.35

$$\begin{aligned}\mathbf{P} &= 160\boldsymbol{\lambda}_{AB} = 160\frac{-0.5\mathbf{i}-0.6\mathbf{j}+0.36\mathbf{k}}{\sqrt{(-0.5)^2+(-0.6)^2+0.36^2}} \\ &= -93.02\mathbf{i}-111.63\mathbf{j}+66.98\mathbf{k}\ \text{N}\end{aligned}$$

(a)

$$\mathbf{M}_O = \mathbf{r}_{OB}\times\mathbf{P} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.36 \\ -93.02 & -111.63 & 66.98\end{vmatrix} = 40.2\mathbf{i}-33.5\mathbf{j}\ \text{N}\cdot\text{m}\ \blacktriangleleft$$

(b)

$$\mathbf{M}_C = \mathbf{r}_{CB}\times\mathbf{P} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.6 & 0 \\ -93.02 & -111.63 & 66.98\end{vmatrix} = -40.2\mathbf{i}-55.8\mathbf{k}\ \text{N}\cdot\text{m}\ \blacktriangleleft$$

## 2.36

$$\mathbf{Q} = 250\,\vec{\boldsymbol{\lambda}}_{BD} = 250\left(\frac{-0.500\mathbf{i}+0.360\mathbf{k}}{0.6161}\right) = -202.9\,\mathbf{i}+146.1\,\mathbf{k}\ \ \mathbf{N}$$

**(a)** $\mathbf{M}_O = \mathbf{r}_{OB}\times\mathbf{Q}$ $\quad \mathbf{r}_{OB} = 0.360\,\mathbf{k}$ **m** $\quad$ **($\mathbf{r}_{OD}$ is also convenient)**

$$\therefore\ \mathbf{M}_O = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.360 \\ -202.9 & 0 & 146.1\end{vmatrix} = -73.0\mathbf{j}\ \text{N}\bullet\text{m}\ \blacklozenge$$

**(b)** $\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{Q}$ $\quad \mathbf{r}_{CB} = -0.600\mathbf{j}$ m $\quad$ ($\mathbf{r}_{CD}$ is also convenient)

$$\therefore \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.600 & 0 \\ -202.9 & 0 & 146.1 \end{vmatrix} = -87.7\mathbf{i} - 121.7\mathbf{k} \text{ N}\cdot\text{m} \ \blacklozenge$$

## 2.37

$$\mathbf{r}_{OC} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \text{ m} \qquad \mathbf{P} = P(-\cos 25°\mathbf{i} + \sin 25°\mathbf{k})$$

$$\mathbf{M}_O = P\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -3 \\ -\cos 25° & 0 & \sin 25° \end{vmatrix} = P(1.6905\mathbf{i} + 1.8737\mathbf{j} + 3.6252\mathbf{k})$$

$$M_0 = P\sqrt{1.6905^2 + 1.8737^2 + 3.6252^2} = 4.417P = 350 \text{ kN} \cdot \text{m}$$

$$P = 79.2 \text{ kN} \ \blacktriangleleft$$

## 2.38

$$\mathbf{P} = 50(-\cos 25^{\circ}\mathbf{i} + \sin 25^{\circ}\mathbf{k}) = -45.32\mathbf{i} + 21.13\mathbf{k} \text{ kN}$$

**(a)** $\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{P}$ $\qquad \mathbf{r}_{AC} = 4\mathbf{j} - 3\mathbf{k}$ m

$$\therefore \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -45.32 & 0 & 21.13 \end{vmatrix} = 84.52\mathbf{i} + 135.96\mathbf{j} + 181.28\mathbf{k} \text{ kN}\cdot\text{m}$$

**(b)** $\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{P}$ $\qquad \mathbf{r}_{BC} = 4\mathbf{j}$ m

$$\therefore \mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ -45.32 & 0 & 21.13 \end{vmatrix} = 84.52\mathbf{i} + 181.28\mathbf{k} \text{ kN}\cdot\text{m}$$

## 2.39

$$\mathbf{P} = P\boldsymbol{\lambda}_{BA} = 20\frac{-2\mathbf{j} + 4\mathbf{k}}{\sqrt{(-2)^2 + 4^2}} = -8.944\mathbf{j} + 17.889\mathbf{k} \text{ kN}$$

$$\mathbf{Q} = Q\boldsymbol{\lambda}_{AC} = 20\frac{-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{\sqrt{(-2)^2 + 2^2 + (-1)^2}} = -13.333\mathbf{i} + 13.333\mathbf{j} - 6.667\mathbf{k} \text{ kN}$$

$$\mathbf{r} = \overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{k} \text{ m}$$

$$\begin{aligned}\mathbf{P}+\mathbf{Q} &= -13.333\mathbf{i}+(-8.944+13.333)\mathbf{j}+(17.889-6.667)\mathbf{k} \\ &= -13.333\mathbf{i}+4.389\mathbf{j}+11.222\mathbf{k}\text{ kN}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_O &= \mathbf{r}\times(\mathbf{P}+\mathbf{Q}) = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 4 \\ -13.333 & 4.389 & 11.222\end{vmatrix} \\ &= -17.56\mathbf{i}-75.78\mathbf{j}+8.78\mathbf{k}\text{ kN}\cdot\text{m} \blacktriangleleft\end{aligned}$$

## 2.40

Noting that both **P** and **Q** pass through $A$, we have

$$\mathbf{M}_O = \mathbf{r}_{OA}\times(\mathbf{P}+\mathbf{Q}) \qquad \mathbf{r}_{OA} = 2\mathbf{k}\text{ ft}$$

$$\begin{aligned}\mathbf{P} &= 60\frac{-4.2\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{\sqrt{(-4.2)^2+(-2)^2+2^2}} = -49.77\mathbf{i}-23.70\mathbf{j}+23.70\mathbf{k}\text{ lb} \\ \mathbf{Q} &= 80\frac{-2\mathbf{i}-3\mathbf{j}+2\mathbf{k}}{\sqrt{(-2)^2+(-3)^2+2^2}} = -38.81\mathbf{i}-58.21\mathbf{j}+38.81\mathbf{k}\text{ lb}\end{aligned}$$

$$\begin{aligned}\mathbf{P}+\mathbf{Q} &= -88.58\mathbf{i}-81.91\mathbf{j}+62.51\mathbf{k}\text{ lb} \\ \therefore\quad \mathbf{M}_O &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ -88.58 & -81.91 & 62.51\end{vmatrix} = 163.8\mathbf{i}-177.2\mathbf{j}\text{ lb}\cdot\text{ft} \blacktriangleleft\end{aligned}$$

## 2.41

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} \qquad \mathbf{r} = -8\mathbf{i}+12\mathbf{j}\text{ in.} \qquad \mathbf{F} = -120\mathbf{k}\text{ lb}$$

$$\therefore\ \mathbf{M}_O = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 12 & 0 \\ 0 & 0 & -120\end{vmatrix} = -1440\mathbf{i}-960\mathbf{j}\text{ lb}\bullet\text{in.} = -120\mathbf{i}-80\mathbf{j}\text{ lb}\bullet\text{ft} \blacklozenge$$

## 2.42

$\mathbf{P} = -16\cos 40^\circ \mathbf{i} + 16\sin 40^\circ \mathbf{k} = -12.257\mathbf{i} + 10.285\mathbf{k}$ lb $\qquad \mathbf{Q} = -22.00\mathbf{j}$ lb

$\therefore\ \mathbf{P} + \mathbf{Q} = -12.257\mathbf{i} - 22.00\mathbf{j} + 10.285\mathbf{k}$ lb

$\mathbf{M}_O = \mathbf{r}_{OA} \times (\mathbf{P} + \mathbf{Q}) \qquad \mathbf{r}_{OA} = -(3 + 8\cos 40^\circ)\mathbf{i} + (8\sin 40^\circ)\mathbf{k} = -9.128\mathbf{i} + 5.142\mathbf{k}$ in.

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -9.128 & 0 & 5.142 \\ -12.257 & -22.00 & 10.285 \end{vmatrix} = 113.12\mathbf{i} + 30.86\mathbf{j} + 200.82\mathbf{k} \text{ lb}\bullet\text{in.}$$

$$M_O = \sqrt{113.12^2 + 30.86^2 + 200.82^2} = 232.5 \text{ lb}\bullet\text{in.} \ \blacklozenge$$

$$\cos\theta_x = \frac{113.12}{232.5} = 0.4865; \quad \cos\theta_y = \frac{30.86}{232.5} = 0.1327; \quad \cos\theta_z = \frac{200.82}{232.5} = 0.8637 \ \blacklozenge$$

## 2.43

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 50 & -100 & -70 \end{vmatrix} = 100z\mathbf{i} + (70x + 50z)\mathbf{j} - 100x\mathbf{k}$$

Equating the $x$- and $z$-components of $\mathbf{M}_O$ to the given values yields

$$100z = 400 \qquad \therefore z = 4 \text{ ft} \ \blacktriangleleft$$
$$-100x = -300 \qquad \therefore x = 3 \text{ ft} \ \blacktriangleleft$$

Check $y$-component:

$$70x + 50z = 70(3) + 50(4) = 410 \text{ lb}\cdot\text{ft} \quad \text{O.K.}$$

## 2.44

$$\mathbf{F} = 150\cos 60^\circ\mathbf{j} + 150\sin 60^\circ\mathbf{k} = 75\mathbf{j} + 129.90\mathbf{k} \text{ N}$$
$$\mathbf{r} = \overrightarrow{OB} = -50\mathbf{i} - 60\mathbf{j} \text{ mm}$$
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & -60 & 0 \\ 0 & 75 & 129.90 \end{vmatrix} = -7794\mathbf{i} + 6495\mathbf{j} - 3750\mathbf{k} \text{ N}\cdot\text{mm}$$
$$M_O = \sqrt{(-7794)^2 + 6495^2 + (-3750)^2} = 10\ 816 \text{ N}\cdot\text{mm} = 10.82 \text{ N}\cdot\text{m} \ \blacktriangleleft$$
$$d = \frac{M_O}{F} = \frac{10\ 816}{150} = 72.1 \text{ mm} \ \blacktriangleleft$$

## 2.45

$$\mathbf{P}_1 = \frac{P}{\sqrt{2}}(\mathbf{j}-\mathbf{k}) \qquad \mathbf{r}_1 = -d\mathbf{i} \qquad\qquad \mathbf{P}_2 = \frac{P}{\sqrt{3}}(\mathbf{i}+\mathbf{j}-\mathbf{k}) \qquad \mathbf{r}_2 = (a-d)\mathbf{i}$$

$$\mathbf{M}_A = \mathbf{r}_1 \times \mathbf{P}_1 + \mathbf{r}_2 \times \mathbf{P}_2 = \frac{P}{\sqrt{2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -d & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} + \frac{P}{\sqrt{3}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (a-d) & 0 & 0 \\ 1 & 1 & -1 \end{vmatrix} = \mathbf{0}$$

**Canceling P and expanding the determinants gives:** $\frac{d}{\sqrt{2}}(-\mathbf{j}-\mathbf{k}) + \frac{a-d}{\sqrt{3}}(\mathbf{j}+\mathbf{k}) = \mathbf{0}$

**Equating either the j-components or the k-components yields:** $\frac{d}{\sqrt{2}} = \frac{a-d}{\sqrt{3}}$

**from which we find:** $d = \frac{a\sqrt{2}}{\sqrt{2}+\sqrt{3}} = 0.449a$ ◆

## 2.46

$$\begin{aligned}
\mathbf{F} &= 2\mathbf{i} - 12\mathbf{j} + 5\mathbf{k} \text{ lb} \\
\mathbf{r} &= \overrightarrow{BA} = (-x+2)\mathbf{i} + 3\mathbf{j} - z\mathbf{k} \\
\mathbf{M}_B &= \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x+2 & 3 & -z \\ 2 & -12 & 5 \end{vmatrix} \\
&= (-12z+15)\mathbf{i} + (5x-2z-10)\mathbf{j} + (12x-30)\mathbf{k}
\end{aligned}$$

Setting **i** and **k** components to zero:

$$\begin{aligned}
-12z + 15 &= 0 \qquad z = 1.25 \text{ ft} \; ◄ \\
12x - 30 &= 0 \qquad x = 2.5 \text{ ft} \; ◄
\end{aligned}$$

Check **j** component:

$$5x - 2z - 10 = 5(2.5) - 2(1.25) - 10 = 0 \text{ Checks!}$$

## 2.47

**(a)**

$$\begin{aligned}
M_x &= -75(0.85) = -63.75 \text{ kN} \cdot \text{m} \; ◄ \\
M_y &= 75(0.5) = 37.5 \text{ kN} \cdot \text{m} \; ◄ \\
M_z &= 160(0.5) - 90(0.85) = 3.5 \text{ kN} \cdot \text{m} \; ◄
\end{aligned}$$

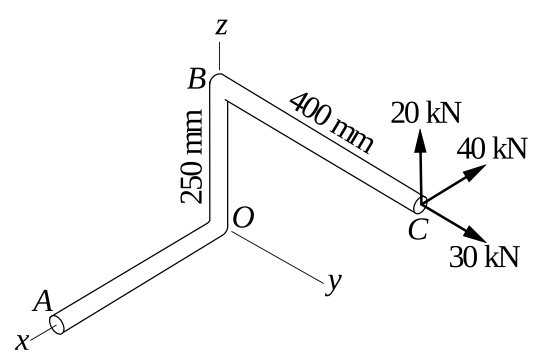
**(b)**

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.85 & 0 \\ 90 & 160 & -75 \end{vmatrix} = -63.75\mathbf{i} + 37.5\mathbf{j} + 3.5\mathbf{k} \text{ kN} \cdot \text{m}$$

The components of $\mathbf{M}_O$ agree with those computed in part (a).

## 2.48

**(a)**




$$M_{OA} = 20(400) - 30(250) = 500 \text{ kN} \cdot \text{mm} = 500 \text{ N} \cdot \text{m} \blacktriangleleft$$

**(b)**

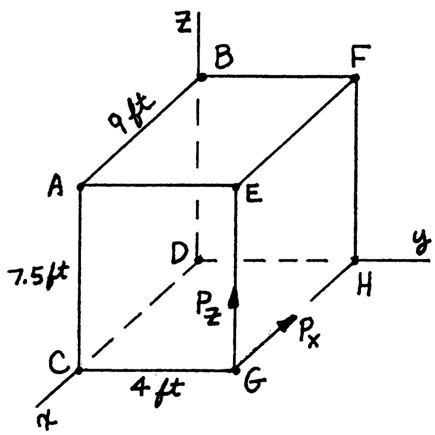
$$\begin{aligned} \mathbf{F} &= -40\mathbf{i} + 30\mathbf{j} + 20\mathbf{k} \text{ kN} \\ \mathbf{r} &= \overrightarrow{OC} = 400\mathbf{j} + 250\mathbf{k} \text{ mm} \\ M_{OA} &= \mathbf{r} \times \mathbf{F} \cdot \mathbf{i} = \begin{vmatrix} 0 & 400 & 250 \\ -40 & 30 & 20 \\ 1 & 0 & 0 \end{vmatrix} = 500 \text{ kN} \cdot \text{mm} \\ &= 500 \text{ N} \cdot \text{m} \blacktriangleleft \end{aligned}$$

## 2.49

$\overline{FG} = \sqrt{9^2 + 7.5^2} = 11.715$ ft

$P_x = 400\left(\frac{9}{11.715}\right) = 307.3$ lb

$P_z = 400\left(\frac{7.5}{11.715}\right) = 256.1$ lb




**(a)** $\mathbf{M}_{AB} = P_z(\overline{AE})\mathbf{i} = 256.1(4)\mathbf{i}$

$= 1024\mathbf{i}$ lb•ft ◆

**(b)** $\mathbf{M}_{CD} = P_z(\overline{CG})\mathbf{i} = 256.1(4)\mathbf{i}$

$= 1024\mathbf{i}$ lb•ft ◆

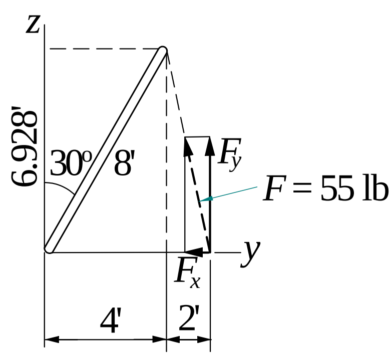
**(c)** $\mathbf{M}_{BF} = \mathbf{0}$ (because the force passes through F) ◆

**(d)** $\mathbf{M}_{DH} = -P_z(\overline{GH})\mathbf{j} = -256.1(9)\mathbf{j} = -2305\mathbf{j}$ lb•ft ◆

**(e)** $\mathbf{M}_{BD} = P_x(\overline{DH})\mathbf{k} = 307.3(4)\mathbf{k} = 1229\mathbf{k}$ lb•ft ◆

## 2.50

(a)




Only $F_y$ has a moment about $x$-axis (since $F_x$ intersects $x$-axis, it has no moment about that axis).

$$F_y = 55\frac{6.928}{\sqrt{6.928^2 + 2^2}} = 52.84 \text{ lb}$$

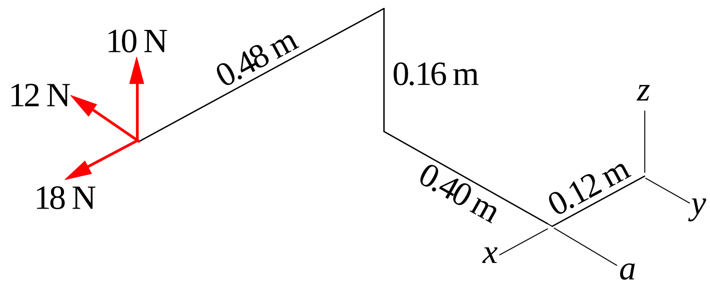
$$+ \circlearrowleft \quad M_x = 6F_y = 6(52.84) = 317 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

**(b)**

$$\mathbf{F} = 55\frac{-2\mathbf{i} + 6.928\mathbf{k}}{\sqrt{6.928^2 + 2^2}} = -15.26\mathbf{j} + 52.84\mathbf{k} \text{ lb} \qquad \mathbf{r} = 6\mathbf{j} \text{ ft}$$

$$M_x = \mathbf{r} \times \mathbf{F} \cdot \boldsymbol{\lambda} = \begin{vmatrix} 0 & 6 & 0 \\ 0 & -15.26 & 52.84 \\ 1 & 0 & 0 \end{vmatrix} = 317 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

**2.51**




**(a)**

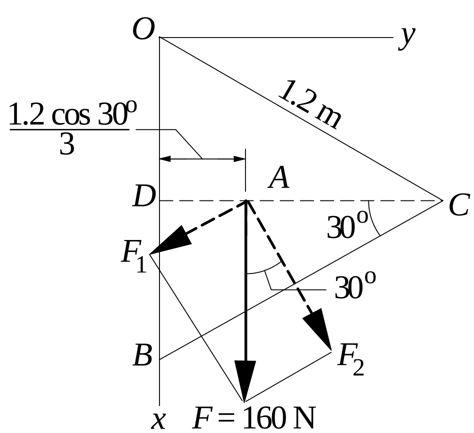
$$\mathbf{M}_a = [-10(0.48) + 18(0.16)]\,\mathbf{j} = -1.920\mathbf{j} \text{ N} \cdot \text{m} \blacktriangleleft$$

**(b)**

$$\mathbf{M}_z = [-12(0.48 + 0.12) + 18(0.4)]\,\mathbf{k} = \mathbf{0} \blacktriangleleft$$

**2.52**

**(a)**




We resolve **F** into components $F_1$ and $F_2$, which are parallel and perpendicular to $BC$, respectively. Only $F_2$ contributes to $M_{BC}$:

$$M_{BC} = 1.8F_2 = 1.8(160\cos 30°) = 249 \text{ N} \cdot \text{m} \blacktriangleleft$$

**(b)**

$$
\begin{aligned}
\mathbf{F} &= 160\mathbf{i} \text{ N} \\
\mathbf{r} &= \overrightarrow{BA} = -0.6\mathbf{i} + \frac{1.2\cos 30^\circ}{3}\mathbf{j} + 1.8\mathbf{k} = -0.6\mathbf{i} + 0.3464\mathbf{j} + 1.8\mathbf{k} \text{ m} \\
\boldsymbol{\lambda}_{BC} &= -\sin 30^\circ\mathbf{i} + \cos 30^\circ\mathbf{j} = -0.5\mathbf{i} + 0.8660\mathbf{j} \\
M_{BC} &= \mathbf{r}\times\mathbf{F}\cdot\boldsymbol{\lambda}_{BC} = \begin{vmatrix} -0.6 & 0.3464 & 1.8 \\ 160 & 0 & 0 \\ -0.5 & 0.8660 & 0 \end{vmatrix} = 249 \text{ N}\cdot\text{m} \blacktriangleleft
\end{aligned}
$$

## 2.53

$$
\begin{aligned}
\mathbf{F} &= -40\mathbf{i} - 8\mathbf{j} + 5\mathbf{k} \text{ N} \\
\mathbf{r} &= 350\sin 20^\circ\mathbf{i} - 350\cos 20^\circ\mathbf{k} = 119.7\mathbf{i} - 328.9\mathbf{k} \text{ mm} \\
M_y &= \mathbf{r}\times\mathbf{F}\cdot\mathbf{j} = \begin{vmatrix} 119.7 & 0 & -328.9 \\ -40 & -8 & 5 \\ 0 & 1 & 0 \end{vmatrix} = 12\ 560 \text{ N}\cdot\text{mm} \\
&= 12.56 \text{ N}\cdot\text{m} \blacktriangleleft
\end{aligned}
$$

## 2.54

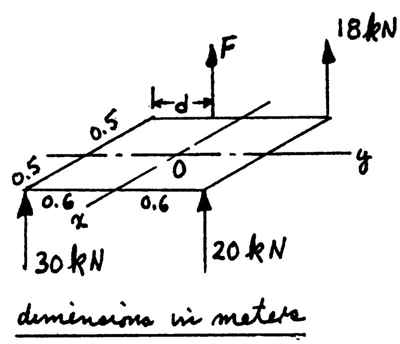
$$M_y = (F + 18)(0.5) - (30 + 20)(0.5) = 0$$

$$\therefore F = \frac{25 - 9}{0.5} = 32.0 \text{ N} \ \blacklozenge$$

$$M_x = (20 + 18)(0.6) - 30(0.6) - F(0.6 - d) = 0$$

Substituting F = 32.0 N, and solving for d gives:

$$\therefore d = \frac{-22.8 + 18 + 32.0(0.6)}{32.0} = 0.450 \text{ m} \ \blacklozenge$$




## 2.55

$M_{aa} = 30(4 - y_0) + 20(6 - y_0) - 40y_0 = 0$ Solving gives: $y_0 = 2.67$ ft ◆

$M_{bb} = (20 + 40)x_0 - 30(6 - x_0) = 0$ Solving gives: $x_0 = 2.00$ ft ◆

## 2.56

With $T$ acting at $A$, only the component $T_z$ has a moment about the $y$-axis: $M_y = -4T_z$.

$$T_z = T\frac{\overline{AB}_z}{\overline{AB}} = 60\frac{3}{\sqrt{4^2+4^2+3^2}} = 28.11 \text{ lb}$$

$$\therefore \quad M_y = -4(28.11) = -112.40 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

## 2.57

Only the $x$-component of each force has a moment about the $z$-axis.

$$\therefore M_z = (P\cos 30^\circ + Q\cos 25^\circ)\,15$$
$$= (32\cos 30^\circ + 36\cos 25^\circ)\,15 = 905 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

## 2.58

$$\mathbf{P} = 360\frac{-0.42\mathbf{i} - 0.81\mathbf{j} + 0.54\mathbf{k}}{\sqrt{(-0.42)^2 + (-0.81)^2 + 0.54^2}} = -142.6\mathbf{i} - 275.0\mathbf{j} + 183.4\mathbf{k} \text{ N}$$

$$\mathbf{r}_{CA} = 0.42\mathbf{i} \text{ m} \qquad \boldsymbol{\lambda}_{CD} = \frac{0.42\mathbf{i} + 0.54\mathbf{k}}{\sqrt{0.42^2 + 0.54^2}} = 0.6139\mathbf{i} + 0.7894\mathbf{k}$$

$$M_{CD} = \mathbf{r}_{CA}\times\mathbf{P}\cdot\boldsymbol{\lambda}_{\mathbf{CD}} = \begin{vmatrix} 0.42 & 0 & 0 \\ -142.6 & -275.0 & 183.4 \\ 0.6139 & 0 & 0.7894 \end{vmatrix} = -91.18 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_{CD} = M_{CD}\boldsymbol{\lambda}_{CD} = -91.18(0.6139\mathbf{i} + 0.7894\mathbf{k})$$
$$= -56.0\mathbf{i} - 72.0\mathbf{k} \text{ N}\cdot\text{m} \blacktriangleleft$$

## 2.59

**Let the 20-lb force be Q:**

$$\mathbf{Q} = 20\,\vec{\lambda}_{ED} = 20\left(\frac{-12\mathbf{j} - 4\mathbf{k}}{12.649}\right) = -18.974\mathbf{j} - 6.324\mathbf{k} \text{ lb}$$

$$\mathbf{P} = P\,\vec{\lambda}_{AF} = P\left(\frac{-4\mathbf{i} + 4\mathbf{k}}{4\sqrt{2}}\right) = P(-0.7071\mathbf{i} + 0.7071\mathbf{k}) \text{ lb}$$

$$\mathbf{M}_{GB} = \mathbf{r}_{BE}\times\mathbf{Q}\cdot\vec{\lambda}_{GB} + \mathbf{r}_{BA}\times\mathbf{P}\cdot\vec{\lambda}_{GB} = 0$$

$$\mathbf{r}_{BE} = 4\mathbf{i} + 4\mathbf{k} \text{ in.} \qquad \mathbf{r}_{BA} = 4\mathbf{i} \text{ in.} \qquad \vec{\lambda}_{GB} = \frac{12\mathbf{j} - 4\mathbf{k}}{12.649}$$

$$M_{GB} = \frac{1}{12.649}\begin{vmatrix} 4 & 0 & 4 \\ 0 & -18.974 & -6.324 \\ 0 & 12 & -4 \end{vmatrix} + \frac{P}{12.649}\begin{vmatrix} 4 & 0 & 0 \\ -0.7071 & 0 & 0.7071 \\ 0 & 12 & -4 \end{vmatrix} = 0$$

**Expanding the determinants gives:** $\frac{607.1}{12.649} + \frac{P}{12.649}(-33.94) = 0 \quad \therefore\ P = 17.89\ \text{lb}$ ◆

## 2.60

$$M_{BC} = \mathbf{r}_{BA} \times \mathbf{F} \cdot \boldsymbol{\lambda}_{BC}$$

$$\mathbf{r}_{BA} = 5\mathbf{i} \qquad \mathbf{F} = F\frac{-3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + 3^2 + (-3)^2}} = 0.5774F(-\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$\boldsymbol{\lambda}_{BC} = \frac{4\mathbf{j} - 2\mathbf{k}}{\sqrt{4^2 + (-2)^2}} = 0.8944\mathbf{j} - 0.4472\mathbf{k}$$

$$M_{BC} = \mathbf{r}_{BA} \times \mathbf{F} \cdot \boldsymbol{\lambda}_{BC} = 0.5774F\begin{vmatrix} 5 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0.8944 & -0.4472 \end{vmatrix} = 1.2911F$$

$$M_{BC} = 150\ \text{lb}\cdot\text{ft} \qquad 1.2911F = 150\ \text{lb}\cdot\text{ft} \qquad F = 116.2\ \text{lb}$$ ◄

## 2.61

The unit vector perpendicular to plane $ABC$ is

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|}$$

$$\overrightarrow{AB} = (0.3\mathbf{i} - 0.5\mathbf{k}) \qquad \overrightarrow{AC} = (0.4\mathbf{j} - 0.5\mathbf{k})\ \text{m}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0 & -0.5 \\ 0 & 0.4 & -0.5 \end{vmatrix} = 0.2\mathbf{i} + 0.15\mathbf{j} + 0.12\mathbf{k}$$

$$\mathbf{F} = F\boldsymbol{\lambda} = 200\frac{0.2\mathbf{i} + 0.15\mathbf{j} + 0.12\mathbf{k}}{\sqrt{0.2^2 + 0.15^2 + 0.12^2}}$$
$$= 144.24\mathbf{i} + 108.18\mathbf{j} + 86.55\mathbf{k}\ \text{N}\cdot\text{m}$$

$$M_x = \overrightarrow{OA} \times \mathbf{F} \cdot \mathbf{i} = \begin{vmatrix} 0 & 0 & 0.5 \\ 144.24 & 108.18 & 86.55 \\ 1 & 0 & 0 \end{vmatrix} = -54.1\ \text{N}\cdot\text{m}$$

$$|M_x| = 54.1\ \text{N}\cdot\text{m}$$ ◄

## 2.62

$$\mathbf{P} = 240\,\vec{\lambda}_{CE} = 240\left(\frac{-3\mathbf{i}+2\mathbf{j}-7\mathbf{k}}{\sqrt{62}}\right)\text{ lb} \qquad \vec{\lambda}_{AD} = \frac{-3\mathbf{i}+6\mathbf{j}+7\mathbf{k}}{\sqrt{94}}$$

**(a)** $\mathbf{r} = \mathbf{r}_{AC} = 6\mathbf{j} + 7\mathbf{k}$ ft

$$M_{AD} = \mathbf{r}_{AC} \times \mathbf{P} \bullet \vec{\lambda}_{AD} = \frac{240}{\sqrt{62}\sqrt{94}}\begin{vmatrix} 0 & 6 & 7 \\ -3 & 2 & -7 \\ -3 & 6 & 7 \end{vmatrix} = \frac{240}{\sqrt{62}\sqrt{94}}(168) = 528\text{ lb}\bullet\text{ft} \;\blacklozenge$$

**(b)** $\mathbf{r} = \mathbf{r}_{DC} = 3\mathbf{i}$ ft

$$M_{AD} = \mathbf{r}_{DC} \times \mathbf{P} \bullet \vec{\lambda}_{AD} = \frac{240}{\sqrt{62}\sqrt{94}}\begin{vmatrix} 3 & 0 & 0 \\ -3 & 2 & -7 \\ -3 & 6 & 7 \end{vmatrix} = \frac{240}{\sqrt{62}\sqrt{94}}(168) = 528\text{ lb}\bullet\text{ft} \;\blacklozenge$$

## 2.63

Equating moments about the $x$- and $y$- axis:

$$\begin{aligned} 600(1.5) + 400(2) + 200(4) &= 1200y \qquad y = 2.08\text{ ft} \;\blacktriangleleft \\ -600(3) - 200(3) &= -1200x \qquad x = 2.00\text{ ft} \;\blacktriangleleft \end{aligned}$$

## 2.64

$$M_{BC} = \mathbf{M}_B \bullet \vec{\lambda}_{BC} = \mathbf{r}_{BD} \times \mathbf{F} \bullet \vec{\lambda}_{BC} = 0 \qquad \mathbf{r}_{BD} = -1.6\mathbf{j} - (1.2 - z_D)\mathbf{k}\text{ m}$$

$$\mathbf{F} = F(0.6\mathbf{i} + 0.8\mathbf{j}) \qquad \vec{\lambda}_{BC} = \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} = \frac{1.2\mathbf{i} - 0.6\mathbf{j} - 1.2\mathbf{k}}{1.8}$$

$$\therefore\; M_{BC} = \frac{F}{1.8}\begin{vmatrix} 0 & -1.6 & -(1.2 - z_D) \\ 0.6 & 0.8 & 0 \\ 1.2 & -0.6 & -1.2 \end{vmatrix} = 0$$

**Expanding the determinant:** $1.6(0.6)(-1.2) - (1.2 - z_D)(-0.36 - 0.96) = 0$

**which gives:** $z_D = 0.327$ m $\blacklozenge$

2.65

$$\vec{\lambda}_{AB} = \frac{-3\mathbf{i} + 4\mathbf{j}}{5} = -0.600\mathbf{i} + 0.800\mathbf{j}$$

**For the pulley at A:**

$$M_A = M_x = 20(0.5) - 60(0.5) = -20 \text{ kN}\bullet\text{m} \qquad \therefore\ \mathbf{M}_A = -20\mathbf{i} \text{ kN}\bullet\text{m}$$

**For the pulley at B:**

$$M_B = M_y = 40(0.8) - 20(0.8) = 16 \text{ kN}\bullet\text{m} \qquad \therefore\ \mathbf{M}_B = 16\mathbf{j} \text{ kN}\bullet\text{m}$$

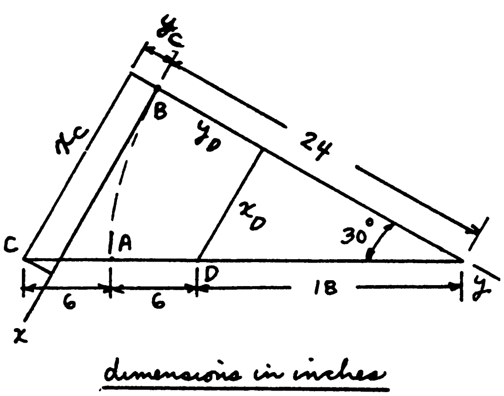
**For both pulleys combined:**

$$M_{AB} = \left(\mathbf{M}_A + \mathbf{M}_B\right) \bullet \vec{\lambda}_{AB} = (-20\mathbf{i} + 16\mathbf{j}) \bullet (-0.600\mathbf{i} + 0.800\mathbf{j})$$

$$= 12 + 12.8 = 24.8 \text{ kN}\bullet\text{m} \ \blacklozenge$$

2.66

**From the figure at the right:**




$$x_C = 30 \sin 30^\circ = 15.000 \text{ in.}$$

$$y_C = 30 \cos 30^\circ - 24 = 1.981 \text{ in.}$$

$$x_D = 18 \sin 30^\circ = 9.000 \text{ in.}$$

$$y_D = 24 - 18 \cos 30^\circ = 8.412 \text{ in.}$$

$$(M_B)_x = \mathbf{r}_{BC} \times \mathbf{P}_C \bullet \mathbf{i} + \mathbf{r}_{BD} \times \mathbf{P}_D \bullet \mathbf{i}$$

$$\mathbf{P}_C = 20\mathbf{k} \text{ lb} \qquad \mathbf{P}_D = -20\mathbf{k} \text{ lb}$$

$$\mathbf{r}_{BC} = x_C\mathbf{i} - y_C\mathbf{j} = 15.000\mathbf{i} - 1.981\mathbf{j} \text{ in.}$$

$$\mathbf{r}_{BD} = x_D\mathbf{i} + y_D\mathbf{j} = 9.000\mathbf{i} + 8.412\mathbf{j} \text{ in.}$$

$$\therefore\ (M_B)_x = \begin{vmatrix} 15.000 & -1.981 & 0 \\ 0 & 0 & 20 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 9.000 & 8.412 & 0 \\ 0 & 0 & -20 \\ 1 & 0 & 0 \end{vmatrix} = -39.62 - 168.2 = -208 \text{ lb}\bullet\text{in}$$

Written in vector form: $(\mathbf{M}_B)_x = (M_B)_x\,\mathbf{i} = -208\,\mathbf{i}$ lb•in $\blacklozenge$

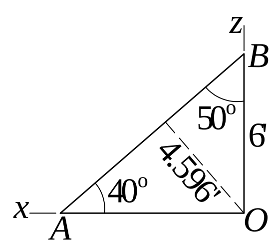
## 2.67

**(a)**

$$\begin{aligned}\mathbf{F} &= 180\frac{4\mathbf{i}+8\mathbf{j}+10\mathbf{k}}{\sqrt{4^2+8^2+10^2}} = 53.67\mathbf{i}+107.33\mathbf{j}+134.16\mathbf{k}\text{ lb}\\ \mathbf{r}_{BO} &= -6\mathbf{k}\text{ ft} \qquad \boldsymbol{\lambda}_{AB} = \frac{(-6\cot 40°)\,\mathbf{i}+6\mathbf{k}}{\sqrt{(-6\cot 40°)^2+6^2}} = -0.7660\mathbf{i}+0.6428\mathbf{k}\end{aligned}$$

$$M_{AB} = \mathbf{r}_{BO}\times\mathbf{F}\cdot\boldsymbol{\lambda}_{AB} = \begin{vmatrix} 0 & 0 & -6 \\ 53.67 & 107.33 & 134.16 \\ -0.7660 & 0 & 0.6428 \end{vmatrix} = -493\text{ lb}\cdot\text{ft} \quad ◄$$

**(b)**




Note that only $F_y = 107.33$ lb has a moment about $AB$. From trigonometry, the moment arm is $d = 6\sin 50° = 4.596$ ft.

$$\therefore M_{AB} = -F_y d = -107.33(4.596) = -493\text{ lb}\cdot\text{ft} \quad ◄$$

## 2.68

**Assume counterclockwise couples are positive.**

**(a) C = − 10(0.6)= − 6 N•m**

**(b) C = − 6 N•m**

**(c) C = − 15(0.4) = − 6 N•m**

**(d) C = − 6 N•m**

**(e) C = 9 − 3 = 6 N•m**

**(f) C = − 5(O.6) − 7.5(0.4) = − 6 N•m**

**(g) C = − 22.5(0.4) + 5(0.6) = − 6 N•m**

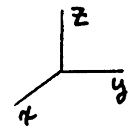
**(h) C = − 5 + 5(0.3) = − 3.5 N•m**

**(i) C = 3 − 4 − 6 + 3 = − 4 N•m**

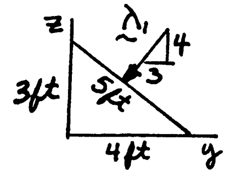
## 2.69

(a) $\mathbf{C} = -60(5)\mathbf{k} = -300\mathbf{k}$ lb•ft

(b) $\mathbf{C} = -75(4)\mathbf{k} = -300\mathbf{k}$ lb•ft




(c) $\mathbf{C}_1 = 75(5)\vec{\lambda}_1 = 375\left(-\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = -225\mathbf{j} - 300\mathbf{k}$ lb•ft

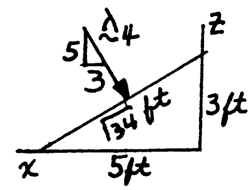



(d) $\mathbf{C} = 100(3)\mathbf{i} = 300\mathbf{i}$ lb•ft

(e) 75-lb forces: $\mathbf{C}_1 = -225\mathbf{j} - 300\mathbf{k}$ lb•ft [as in (c)]

45-lb forces: $\mathbf{C}_2 = 45(5)\mathbf{j} = 225\mathbf{j}$ lb•ft

$\mathbf{C}_1 + \mathbf{C}_2 = -300\mathbf{k}$ lb•ft




(f) 45-lb forces: $\mathbf{C}_3 = 45(4)\mathbf{i} = 180\mathbf{i}$ lb•ft

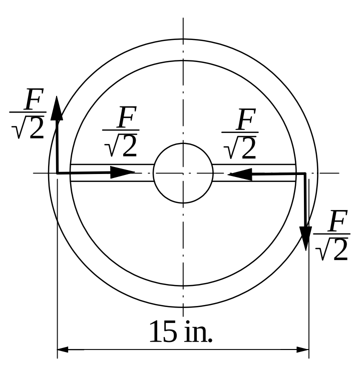
50-lb forces: $\mathbf{C}_4 = 50\left(\sqrt{34}\right)\vec{\lambda}_4$

$$= 50\left(\sqrt{34}\right)\left(\frac{-3\mathbf{i} - 5\mathbf{k}}{\sqrt{34}}\right) = -150\mathbf{i} - 250\mathbf{k} \text{ lb•ft}$$

$\mathbf{C}_3 + \mathbf{C}_4 = 30\mathbf{i} - 250\mathbf{k}$ lb•ft

Comparing the above results: (b) and (e) are equivalent to (a). ♦

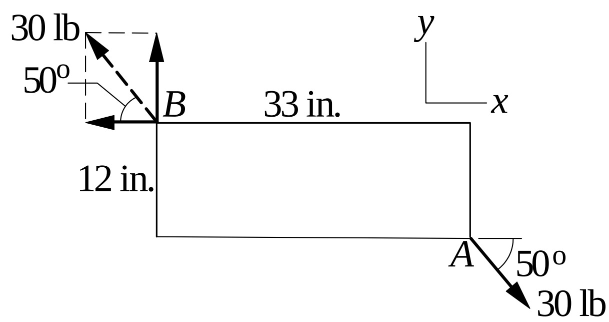
## 2.70




$$C = 15\frac{F}{\sqrt{2}}$$

$$F = \frac{\sqrt{2}}{15}C = \frac{\sqrt{2}}{15}(120) = 11.31 \text{ lb} \blacktriangleleft$$

**2.71**




Choosing $A$ as the moment center, we get

$$+\ \circlearrowleft \quad C = M_A = (30\sin 50^\circ)(33) - (30\cos 50^\circ)(12)$$
$$= 527 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

**2.72**

Choosing $A$ as the moment center, we get

$$\mathbf{C} = \mathbf{M}_A = 60(3)\mathbf{i} + 60(2)\mathbf{j} - 30(2)\mathbf{j} - 30(3)\mathbf{k}$$
$$= 180\mathbf{i} + 60\mathbf{j} - 90\mathbf{k} \text{ lb} \cdot \text{ft} \blacktriangleleft$$

**2.73**

$$\mathbf{C} = 60\boldsymbol{\lambda}_{DB} = 60\frac{0.4\mathbf{i} - 0.3\mathbf{j} + 0.4\mathbf{k}}{\sqrt{0.4^2 + (-0.3)^2 + 0.4^2}} = 37.48\mathbf{i} - 28.11\mathbf{j} + 37.48\mathbf{k} \text{ N} \cdot \text{m}$$

$$\mathbf{P} = -300\mathbf{k} \text{ N} \qquad \mathbf{r}_{AD} = -0.4\mathbf{i} \text{ m} \qquad \boldsymbol{\lambda}_{AB} = \frac{-0.3\mathbf{i} + 0.4\mathbf{k}}{0.5} = -0.6\mathbf{j} + 0.8\mathbf{k}$$

Moment of the couple:

$$(M_{AB})_C = \mathbf{C} \cdot \boldsymbol{\lambda}_{AB} = -28.11(-0.6) + 37.48(0.8) = 46.85 \text{ N} \cdot \text{m}$$

Moment of the force:

$$(M_{AB})_P = \mathbf{r}_{AD} \times \mathbf{P} \cdot \boldsymbol{\lambda}_{AB} = \begin{vmatrix} -0.4 & 0 & 0 \\ 0 & 0 & -300 \\ 0 & -0.6 & 0.8 \end{vmatrix} = 72.0 \text{ N} \cdot \text{m}$$

Combined moment:

$$M_{AB} = (M_{AB})_C + (M_{AB})_P = 46.85 + 72.0 = 118.9 \text{ N} \cdot \text{m} \blacktriangleleft$$

## *2.74

$\mathbf{C}_1 = -200\mathbf{i}$ lb•in. $\mathbf{C}_2 = 140\mathbf{k}$ lb•in.

**Identify the three points at the corners of the triangle:**

**A(9 in., 3 in., 6 in.); B(3 in., 7 in., 6 in.); C(9 in., 7 in., 2 in.)**

$\mathbf{C}_3 = 220\,\vec{\lambda}$ lb•in. where $\vec{\lambda}$ is the unit vector that is perpendicular to triangle ABC, with its sense consistent with the sense of $\mathbf{C}_3$.

$$\vec{\lambda} = \frac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|} \quad \text{where } \overrightarrow{AC} = 4\mathbf{j} - 4\mathbf{k} \text{ in. and } \overrightarrow{AB} = -6\mathbf{i} + 4\mathbf{j} \text{ in.}$$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -4 \\ -6 & 4 & 0 \end{vmatrix} = 16\mathbf{i} + 24\mathbf{j} + 24\mathbf{k} \text{ in.}^2$$

$$\therefore\ \vec{\lambda} = \frac{16\mathbf{i} + 24\mathbf{j} + 24\mathbf{k}}{37.52} = 0.4264\mathbf{i} + 0.6397\mathbf{j} + 0.6397\mathbf{k}$$

$$\mathbf{C}_3 = 220(0.4264\mathbf{i} + 0.6397\mathbf{j} + 0.6397\mathbf{k}) = 93.81\mathbf{i} + 140.73\mathbf{j} + 140.73\mathbf{k} \text{ lb•in.}$$

$$\therefore\ \mathbf{C}^R = \mathbf{C}_1 + \mathbf{C}_2 + \mathbf{C}_3 = -200\mathbf{i} + 140\mathbf{k} + (93.81\mathbf{i} + 140.73\mathbf{j} + 140.73\mathbf{k})$$

$$= -106.2\mathbf{i} + 140.7\mathbf{j} + 280.7\mathbf{k} \text{ lb•in.} \ \blacklozenge$$

## 2.75

Moment of a couple is the same about any point. Choosing $B$ as the moment center, we have

$$\mathbf{F} = -30\mathbf{i} \text{ kN} \qquad \mathbf{r}_{BA} = -1.8\mathbf{j} - 1.2\mathbf{k} \text{ m}$$

$$\mathbf{C} = \mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1.8 & -1.2 \\ -30 & 0 & 0 \end{vmatrix} = 36.0\mathbf{j} - 54.0\mathbf{k} \text{ kN} \cdot \text{m} \ \blacktriangleleft$$

## 2.76

Moment of a couple is the same about any point. Choosing $B$ as the moment center, we have

$$\mathbf{r}_{BA} = 180\mathbf{i} - b\mathbf{j} \text{ mm}$$

$$C_z = (M_B)_z = \mathbf{r}_{BA} \times \mathbf{F} \cdot \mathbf{k} = \begin{vmatrix} 180 & -b & 0 \\ 150 & -90 & 60 \\ 0 & 0 & 1 \end{vmatrix} = 150b - 16\ 200 \text{ kN} \cdot \text{mm}$$

$$\therefore\ 150b - 16\ 200 = 0 \qquad b = 108.0 \text{ mm} \ \blacktriangleleft$$

**2.77**

$$\begin{aligned}\mathbf{C} &= \mathbf{M}_A = 20(24)\mathbf{i} - 80(16)\mathbf{j} + 50(24)\mathbf{k} \\ &= 480\mathbf{i} - 1280\mathbf{j} + 1200\mathbf{k} \text{ lb} \cdot \text{in.} \quad \blacktriangleleft\end{aligned}$$

**2.78**

$$\mathbf{C} = -360\cos 30^\circ\,\mathbf{i} - 360\sin 30^\circ\,\mathbf{j} = -311.8\,\mathbf{i} - 180.0\,\mathbf{j} \text{ lb}\bullet\text{ft}$$

$$\vec{\lambda}_{CD} = -\cos 30^\circ\,\mathbf{i} - \sin 30^\circ \cos 40^\circ\,\mathbf{j} + \sin 30^\circ \sin 40^\circ\,\mathbf{k} = -0.8660\,\mathbf{i} - 0.3830\,\mathbf{j} + 0.3214\,\mathbf{k}$$

$$\therefore\ M_{CD} = \mathbf{C} \bullet \vec{\lambda}_{CD} = (-311.8)(-0.8660) + (-180.0)(-0.3830) = 339 \text{ lb}\bullet\text{ft} \quad \blacklozenge$$

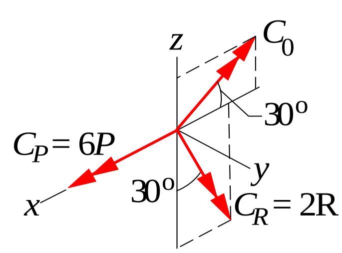
**2.79**

$$\vec{\lambda}_{DC} = \sin 30^\circ \sin 40^\circ\,\mathbf{i} - \sin 30^\circ \cos 40^\circ\,\mathbf{j} + \cos 30^\circ\,\mathbf{k} = 0.3214\,\mathbf{i} - 0.3830\,\mathbf{j} + 0.8660\,\mathbf{k}$$

**(a)** $\mathbf{C} = 52\,\vec{\lambda}_{DC} = 16.71\,\mathbf{i} - 19.92\,\mathbf{j} + 45.03\,\mathbf{k}$ lb•ft ◆

**(b)** $\mathbf{M}_z = \mathbf{C}_z = 45.03\,\mathbf{k}$ lb•ft ◆

**2.80**




$$\begin{aligned}\mathbf{C}_P &= 6P\mathbf{i} = 6(750)\mathbf{i} = 4500\mathbf{i} \text{ lb} \cdot \text{in.} \\ \mathbf{C}_0 &= C_0(-\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{k}) = C_0(-0.8660\mathbf{i} + 0.50\mathbf{k}) \\ \mathbf{C}_R &= 2R(-\sin 30^\circ \mathbf{i} - \cos 30^\circ \mathbf{k}) = -R(\mathbf{i} + 1.7321\mathbf{k})\end{aligned}$$

$$\Sigma\mathbf{C} = (4500 - 0.8660C_0 - R)\mathbf{i} + (0.5C_0 - 1.7321R)\mathbf{k} = \mathbf{0}$$

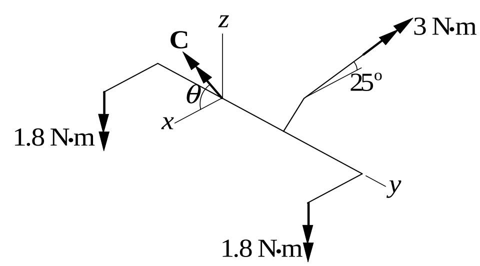
Equating like components:

$$\begin{aligned}4500 - 0.8660C_0 - R &= 0 \\ 0.5C_0 - 1.7321R &= 0\end{aligned}$$

The solution is:

$$R = 1125 \text{ lb} \ \blacktriangleleft \qquad C_0 = 3900 \text{ lb} \cdot \text{in.} \ \blacktriangleleft$$

## 2.81




The system consists of the four couples shown, where

$$\mathbf{C} = 0.36F(\mathbf{i}\cos\theta + \mathbf{k}\sin\theta)\ \text{N}\cdot\text{m}$$

$$\Sigma\mathbf{C} = -2(1.8)\mathbf{k} + 3(-\mathbf{i}\cos 25^\circ + \mathbf{k}\sin 25^\circ) + 0.36F(\mathbf{i}\cos\theta + \mathbf{k}\sin\theta) = \mathbf{0}$$

Equating like components:

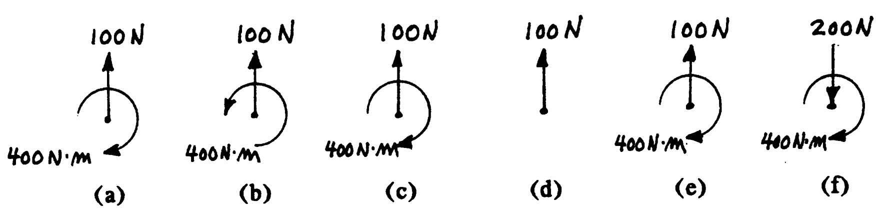
$$\begin{aligned} -3\cos 25^\circ + 0.36F\cos\theta &= 0 \\ -3.6 + 3\sin 25^\circ + 0.36F\sin\theta &= 0 \end{aligned}$$

$$\begin{aligned} F\cos\theta &= \frac{3\cos 25^\circ}{0.36} = 7.553 \\ F\sin\theta &= \frac{3.6 - 3\sin 25^\circ}{0.36} = 6.478 \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{6.478}{7.553} = 0.8577 \qquad \theta = 40.6^\circ \ \blacktriangleleft \\ F &= \sqrt{7.553^2 + 6.478^2} = 9.95\ \text{N} \ \blacktriangleleft \end{aligned}$$

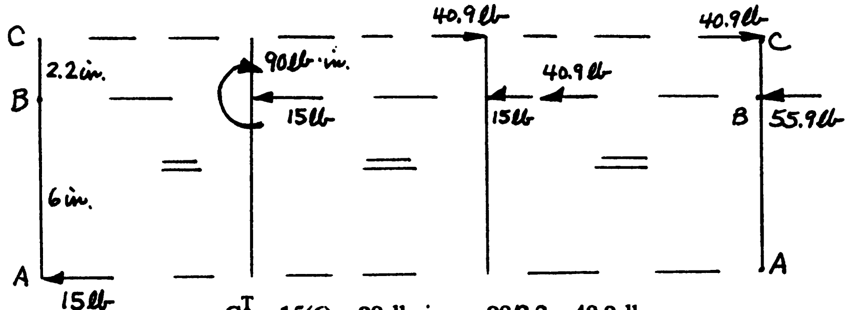
## 2.82

**Represent each of the systems by an eqivalent force-couple system with the force acting at the upper left corner of the figure.**




**By inspection, the systems in (c) and (e) are equivalent to the system in (a).** ♦

## 2.83




$C^T = 15(6) = 90$ lb•in. $\quad 90/2.2 = 40.9$ lb

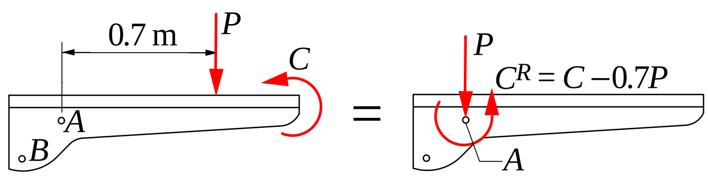
**Original system** $\quad$ **(i) Equivalent system with force at B.** $\quad$ **(ii) Equivalent system: one force at B and one force at C.**

**(a) Fig. (i): A 15-lb force acting to the left at B, and a 90 lb-in. clockwise couple.** ◆

**(b) Fig. (ii): A 55.9-lb force acting to the left at B, and a 40.9-lb force acting to the right at C.** ◆

## 2.84

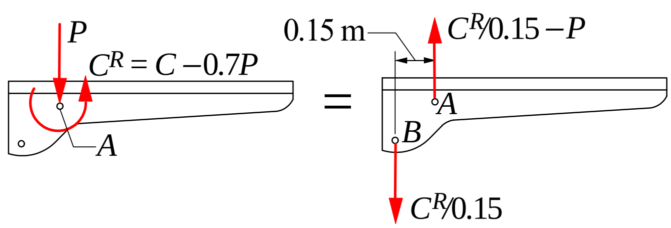
**(a)**




$+\downarrow \quad R = P = 140$ N down ◀

$+\circlearrowleft \quad C^R = \Sigma M_A = C - 0.7P = 180 - 0.7(140) = 82.0$ N · m CCW ◀

**(b)**




$$F_A = \frac{C^R}{0.15} - P = \frac{82}{0.15} - 140 = 407 \text{ N up} \blacktriangleleft$$

$$F_B = \frac{C^R}{0.15} = \frac{82}{0.15} = 547 \text{ N down} \blacktriangleleft$$

## 2.85

$$\downarrow \quad R = \Sigma F = 15 - 20 + 20 = 15 \text{ kN} \blacktriangleleft$$
$$+ \circlearrowleft \quad C^R = \Sigma M_A = 15(3) - 20(6) + 20(8) = 85 \text{ kN} \cdot \text{m} \blacktriangleleft$$

## 2.86

$$\mathbf{R} = -90\mathbf{j} + 50(\mathbf{i}\sin 30^\circ - \mathbf{j}\cos 30^\circ) = 25.0\mathbf{i} - 133.3\mathbf{j} \text{ lb} \blacktriangleleft$$
$$+ \circlearrowleft \quad C^R = 90(9) - 50(12) = 210 \text{ lb} \cdot \text{in.} \qquad \mathbf{C}^R = 210\mathbf{k} \text{ lb} \cdot \text{in.} \blacktriangleleft$$

## 2.87

The resultant force $R$ equals $V$.

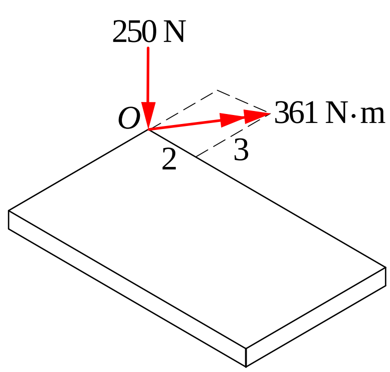
$$\therefore V = R = 1400 \text{ lb} \blacktriangleleft$$

$$C^R = \Sigma M_D = 0: \qquad 20V - 10H - C = 0$$
$$20\,(1400) - 10H - 750\,(12) = 0 \qquad H = 1900 \text{ lb} \blacktriangleleft$$

## 2.88

$$\mathbf{R} = -250\mathbf{k} \text{ N} \blacktriangleleft$$
$$\mathbf{C}^R = \mathbf{M}_O = -250(1.2)\mathbf{i} + 250(0.8)\mathbf{j}$$
$$= -300\mathbf{i} + 200\mathbf{j} \text{ N} \cdot \text{m} \blacktriangleleft$$

$$C^R = \sqrt{(-300)^2 + 200^2} = 361 \text{ N} \cdot \text{m}$$



## 2.89

$$\begin{aligned}\mathbf{F} &= 270\boldsymbol{\lambda}_{AB} = 270\frac{-2.2\mathbf{i}+2.0\mathbf{j}-2.0\mathbf{k}}{\sqrt{(-2.2)^2+2.0^2+(-2.0)^2}} \\ &= -165.8\mathbf{i}+150.7\mathbf{j}-150.7\mathbf{k}\text{ kN} \blacktriangleleft \\ \mathbf{C}^R &= \mathbf{r}_{OB}\times\mathbf{F} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 2 & 0\\ -165.8 & 150.7 & -150.7\end{vmatrix} = -301\mathbf{i}+332\mathbf{k}\text{ kN}\cdot\text{m} \blacktriangleleft\end{aligned}$$

## 2.90

$$\begin{aligned}\text{40-lb force:}\quad \mathbf{P} &= 40\frac{-3\mathbf{i}-2\mathbf{k}}{\sqrt{(-3)^2+(-2)^2}} = -33.28\mathbf{i}-22.19\mathbf{k}\text{ lb} \\ \text{90-lb}\cdot\text{ft couple:}\quad \mathbf{C} &= 90\frac{-3\mathbf{i}-5\mathbf{j}}{\sqrt{(-3)^2+(-5)^2}} = -46.30\mathbf{i}-77.17\mathbf{j}\text{ lb}\cdot\text{ft} \\ \mathbf{r}_{OA} &= 3\mathbf{i}+5\mathbf{j}\text{ ft}\end{aligned}$$

$$\begin{aligned}\mathbf{R} &= \mathbf{P} = -33.28\mathbf{i}-22.19\mathbf{k}\text{ lb} \blacktriangleleft \\ \mathbf{C}^R &= \mathbf{C}+\mathbf{r}_{OA}\times\mathbf{P} = -46.30\mathbf{i}-77.17\mathbf{j}+\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 5 & 0\\ -33.28 & 0 & -22.19\end{vmatrix} \\ &= -157.3\mathbf{i}-10.6\mathbf{j}+166.4\mathbf{k}\text{ lb}\cdot\text{ft} \blacktriangleleft\end{aligned}$$

## *2.91

**(a)**

$$\begin{aligned}\mathbf{R} &= \mathbf{F} = -2800\mathbf{i}+1600\mathbf{j}+3000\mathbf{k}\text{ lb} \blacktriangleleft \\ \mathbf{r}_{OA} &= 10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\text{ in.} \\ \mathbf{C}^R &= \mathbf{r}_{OA}\times\mathbf{F} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 10 & 5 & -4\\ -2800 & 1600 & 3000\end{vmatrix} \\ &= 21\ 400\mathbf{i}-18\ 800\mathbf{j}+30\ 000\mathbf{k}\text{ lb}\cdot\text{in.} \blacktriangleleft\end{aligned}$$

**(b)**

$$\begin{aligned}\text{Normal component of }\mathbf{R}\ &:\quad P = |R_y| = 1600\text{ lb} \blacktriangleleft \\ \text{Shear component of }\mathbf{R}\ &:\quad V = \sqrt{R_x^2+R_z^2} = \sqrt{(-2800)^2+3000^2} = 4100\text{ lb} \blacktriangleleft\end{aligned}$$

**(c)**

$$\begin{aligned}\text{Torque:}\quad T &= \left|C_y^R\right| = 18\ 800\text{ lb}\cdot\text{in.} \blacktriangleleft \\ \text{Bending moment:}\quad M &= \sqrt{(C_x^R)^2+(C_z^R)^2} = \sqrt{21\ 400^2+30\ 000^2} \\ &= 36\ 900\text{ lb}\cdot\text{in.} \blacktriangleleft\end{aligned}$$

## 2.92

$$\vec{\lambda}_{DC} = \sin 30^\circ \sin 40^\circ\, \mathbf{i} - \sin 30^\circ \cos 40^\circ\, \mathbf{j} + \cos 30^\circ \mathbf{k}$$

$$= 0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}$$

**The force at O equals the original force:**

$$\mathbf{F} = 9.8\,\vec{\lambda}_{DC} = 9.8(0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}) = 3.150\mathbf{i} - 3.753\mathbf{j} + 8.487\mathbf{k} \text{ lb}$$

**The given couple is:**

$$\mathbf{C} = 52\,\vec{\lambda}_{DC} = 52(0.3214\mathbf{i} - 0.3830\mathbf{j} + 0.8660\mathbf{k}) = 16.71\mathbf{i} - 19.92\mathbf{j} + 45.03\mathbf{k} \text{ lb}\bullet\text{ft}$$

**Moving the force to O, and letting $\mathbf{C}^R$ be the resultant couple, we have:** $\mathbf{C}^R = \mathbf{C} + \mathbf{M}_O$

$$\mathbf{M}_O = \mathbf{r}_{OD} \times \mathbf{F} \qquad \mathbf{r}_{OD} = -4.2\sin 40^\circ\, \mathbf{i} + 4.2\cos 40^\circ\, \mathbf{j} + 2.800\mathbf{k}$$

$$= -2.700\mathbf{i} + 3.217\mathbf{j} + 2.800\mathbf{k} \text{ ft}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.700 & 3.217 & 2.800 \\ 3.150 & -3.753 & 8.487 \end{vmatrix} = 37.81\mathbf{i} + 31.73\mathbf{j} \text{ lb}\bullet\text{ft}$$

$$\therefore\ \mathbf{C}^R = \mathbf{C} + \mathbf{M}_O = (16.71\mathbf{i} - 19.92\mathbf{j} + 45.03\mathbf{k}) + (37.81\mathbf{i} + 31.73\mathbf{j})$$

$$= 54.52\mathbf{i} + 11.81\mathbf{j} + 45.03\mathbf{k} \text{ lb}\bullet\text{ft}$$

**The equivalent force-couple system with the force acting at O is:**

**Force: $3.150\mathbf{i} - 3.753\mathbf{j} + 8.487\mathbf{k}$ lb; Couple: $54.52\mathbf{i} + 11.81\mathbf{j} + 45.03\mathbf{k}$ lb•ft** ♦

## 2.93

$$\begin{aligned}
\mathbf{F} &= 600\frac{-1.2\mathbf{i} + 0.8\mathbf{k}}{\sqrt{(-1.2)^2 + 0.8^2}} = -499.2\mathbf{i} + 332.8\mathbf{k} \text{ N} \\
\mathbf{C} &= 1200\frac{-1.2\mathbf{i} + 1.8\mathbf{j}}{\sqrt{(1.2)^2 + 1.8^2}} = -665.6\mathbf{i} + 998.5\mathbf{k} \text{ N}\cdot\text{m} \\
\mathbf{r}_{BA} &= 1.2\mathbf{i} - 1.8\mathbf{j} \text{ m}
\end{aligned}$$

$$\begin{aligned}
\mathbf{R} &= \mathbf{F} = -499.2\mathbf{i} + 332.8\mathbf{k} \text{ N} \blacktriangleleft \\
\mathbf{C}^R &= \mathbf{r}_{BA} \times \mathbf{F} + \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & -1.8 & 0 \\ -499.2 & 0 & 332.8 \end{vmatrix} + \mathbf{C} \\
&= (-599.0\mathbf{i} - 399.4\mathbf{j} - 898.6\mathbf{k}) + (-665.6\mathbf{i} + 998.5\mathbf{k}) \\
&= -1265\mathbf{i} - 399\mathbf{j} + 100\mathbf{k} \text{ N}\cdot\text{m} \blacktriangleleft
\end{aligned}$$

## 2.94

$$\begin{aligned}
M_{AB} &= \mathbf{r}_{AO} \times \mathbf{P} \cdot \boldsymbol{\lambda}_{AB} = 850 \text{ lb} \cdot \text{ft} \qquad \mathbf{r}_{AO} = -8\mathbf{j} \text{ ft} \\
\mathbf{P} &= P\left(\cos 20^\circ \mathbf{i} + \sin 20^\circ \mathbf{k}\right) \qquad \boldsymbol{\lambda}_{AB} = -\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{k} \\
M_{AB} &= P \begin{vmatrix} 0 & -8 & 0 \\ \cos 20^\circ & 0 & \sin 20^\circ \\ -\cos 30^\circ & 0 & \sin 30^\circ \end{vmatrix} = 6.128P \\
6.128P &= 850 \text{ lb} \cdot \text{ft} \qquad P = 138.7 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 2.95

Given force and couple:

$$\begin{aligned}
\mathbf{F} &= 32\frac{-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + 6^2}} = -12.292\mathbf{i} - 16.389\mathbf{j} + 24.58\mathbf{k} \text{ kN} \\
\mathbf{C} &= 180\frac{3\mathbf{i} - 4\mathbf{j}}{\sqrt{3^2 + (-4)^2}} = 108.0\mathbf{i} - 144.0\mathbf{j} \text{ kN} \cdot \text{m}
\end{aligned}$$

Equivalent force-couple ststem at $A$:

$$\begin{aligned}
\mathbf{R} &= \mathbf{F} = -12.29\mathbf{i} - 16.39\mathbf{j} + 24.6\mathbf{k} \text{ kN} \blacktriangleleft \\
\mathbf{C}^R &= \mathbf{C} + \mathbf{r}_{AB} \times \mathbf{F} = 108.0\mathbf{i} - 144.0\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ -12.292 & -16.389 & 24.58 \end{vmatrix} \\
&= 206\mathbf{i} - 70.3\mathbf{j} + 98.3\mathbf{k} \text{ kN} \cdot \text{m} \blacktriangleleft
\end{aligned}$$

## 2.96

$$\begin{aligned}
\mathbf{T}_1 &= 60\frac{-3\mathbf{i} - 7\mathbf{j}}{\sqrt{(-3)^2 + (-7)^2}} = -23.64\mathbf{i} - 55.15\mathbf{j} \text{ kN} \\
\mathbf{T}_2 &= 60\frac{6\mathbf{i} - 7\mathbf{j}}{\sqrt{6^2 + (-7)^2}} = 39.05\mathbf{i} - 45.56\mathbf{j} \text{ kN} \\
\mathbf{T}_3 &= 60\frac{-3\mathbf{i} - 2\mathbf{j}}{\sqrt{(-3)^2 + (-2)^2}} = -49.92\mathbf{i} - 33.28\mathbf{j} \text{ kN}
\end{aligned}$$

$$\begin{aligned}
\mathbf{R} &= \Sigma\mathbf{T} = (-23.64 + 39.05 - 49.92)\mathbf{i} + (-55.15 - 45.56 - 33.28)\mathbf{j} \\
&= -34.51\mathbf{i} - 133.99\mathbf{j} \text{ kN} \blacktriangleleft
\end{aligned}$$

Noting that only the $x$-components of the tensions contribute to the moment about $O$:

$$\mathbf{C}^R = \Sigma\mathbf{M}_O = [7(23.64) - 7(39.05) + 2(49.92)]\,\mathbf{k} = -8.03\mathbf{k} \text{ kN} \cdot \text{m} \blacktriangleleft$$

## 2.97

$$\begin{aligned}\mathbf{M}_O &= \mathbf{r}_{OA}\times\mathbf{F} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ b & 0.25 & 0.3\\ 10 & 20 & -5\end{vmatrix}\\ &= -7.25\mathbf{i} + (3+5b)\mathbf{j} + (-2.5+20b)\mathbf{k}\ \text{kN}\cdot\text{m}\\ M_y &= 3+5b = 8 \qquad \therefore b = 1.0\ \text{m}\ \blacktriangleleft\end{aligned}$$

$$\mathbf{M}_O = -7.25\mathbf{i} + 8\mathbf{j} + 17.5\mathbf{k}\ \text{kN}\cdot\text{m}\ \blacktriangleleft$$

## 2.98

$\mathbf{M_{CD}} = \mathbf{r_{CA}} \times \mathbf{P} \bullet \vec{\lambda}_{CD} = 50$ **lb•in.**

$\mathbf{r_{CA}} = 6\mathbf{i} - 2\mathbf{j}$ **in.** $\quad \mathbf{P} = P\vec{\lambda}_{AB} = P\left(\dfrac{-3\mathbf{i}-2\mathbf{j}+5\mathbf{k}}{\sqrt{38}}\right)$ **lb** $\quad \vec{\lambda}_{CD} = \dfrac{-4\mathbf{j}+5\mathbf{k}}{\sqrt{41}}$

**Using the determinant form of the scalar triple product:**

$$M_{CD} = \frac{P}{\sqrt{38}\sqrt{41}}\begin{vmatrix}6 & -2 & 0\\ -3 & -2 & 5\\ 0 & -4 & 5\end{vmatrix} = \frac{P}{\sqrt{38}\sqrt{41}}\left[6(-10+20)+2(-15)\right] = 50\ \text{lb•in.}$$

**Solving for P gives:** $P = \dfrac{50\sqrt{38}\sqrt{41}}{30} = 65.8$ **lb** ♦

## 2.99

$$\begin{aligned}\mathbf{F} &= -160\mathbf{i} - 120\mathbf{j} + 90\mathbf{k}\ \text{N}\\ \mathbf{r} &= \overrightarrow{BA} = -0.36\mathbf{i} + 0.52\mathbf{j} - 0.48\mathbf{k}\ \text{m}\\ \mathbf{C} &= \mathbf{M}_B = \mathbf{r}\times\mathbf{F} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -0.36 & 0.52 & -0.48\\ -160 & -120 & 90\end{vmatrix}\\ &= -10.80\mathbf{i} + 109.2\mathbf{j} + 126.4\mathbf{k}\ \text{N}\cdot\text{m}\ \blacktriangleleft\end{aligned}$$

## 2.100

**(a)**

$$\begin{aligned}\mathbf{M}_O &= \mathbf{r}_{OA}\times\mathbf{P} + \mathbf{C} \qquad \mathbf{r}_{OA} = 4\mathbf{k}\ \text{ft}\\ \mathbf{P} &= 800\frac{3\mathbf{i}-4\mathbf{k}}{5} = 480\mathbf{i} - 640\mathbf{k}\ \text{lb} \qquad \mathbf{C} = 1400\mathbf{k}\ \text{lb}\cdot\text{ft}\\ \mathbf{M}_O &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 4\\ 480 & 0 & -640\end{vmatrix} + 1400\mathbf{k} = 1920\mathbf{j} + 1400\mathbf{k}\ \text{lb}\cdot\text{ft}\ \blacktriangleleft\end{aligned}$$

**(b)**

$$M_{OF} = \mathbf{M}_O \cdot \boldsymbol{\lambda}_{OF} = (1920\mathbf{j} + 1400\mathbf{k}) \cdot \frac{3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}}{13}$$
$$= \frac{1920(12) + 1400(4)}{13} = 2200 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

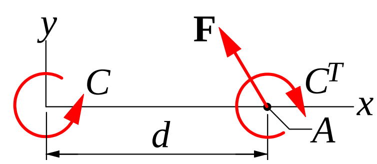
## 2.101

$$R_x = \Sigma F_x = T_1 \sin 45^\circ - T_3 \sin 30^\circ = 0$$
$$R_y = \Sigma F_y = T_1 \cos 45^\circ + T_3 \cos 30^\circ + 250 = 750$$

The solution is

$$T_1 = 259 \text{ lb} \blacktriangleleft \qquad T_3 = 366 \text{ lb} \blacktriangleleft$$

## 2.102




Transferring **F** to point $A$ introduces the couple of transfer $C^T$ which is equal to the moment of the original **F** about point $A$:

$$C^T = F_y d = 300d$$

The couples $C$ and $C^T$ cancel out if

$$C = C^T \qquad 600 = 300d \qquad d = 2 \text{ ft} \blacktriangleleft$$

## 2.103

$$\mathbf{R} = \Sigma \mathbf{F} = 40\mathbf{i} + 30\mathbf{k} \text{ kN} \blacktriangleleft$$
$$\mathbf{r}_{OA} = 0.8\mathbf{i} + 1.2\mathbf{j} \text{ m}$$
$$\mathbf{C}^R = \Sigma \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{R} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.8 & 1.2 & 0 \\ 40 & 0 & 30 \end{vmatrix} = 36\mathbf{i} - 24\mathbf{j} - 48\mathbf{k} \text{ kN} \cdot \text{m} \blacktriangleleft$$

2.104

$\xrightarrow{+}$ $R_x = \Sigma F_x = P - P = 0$

$+\uparrow$ $R_y = \Sigma F_y = P$

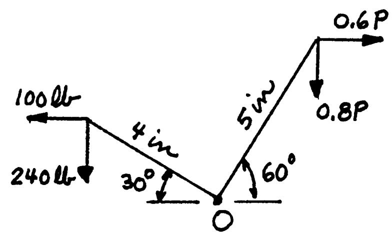
Therfore, the force acting at A is R = P (acting upward) ♦

Because **R** passes through point A, the moment of the three forces about A is zero.

$\circlearrowleft +$ $\Sigma M_A = P(L - x) - P(L/2) = 0$ which gives $x = L/2$ ♦

2.105

Because the resultant force passes through O and there is no resultant couple, the combined moment of the two forces about O is zero.




$\circlearrowleft +$ $\Sigma M_O = 240(4\cos 30^\circ) + 100(4\sin 30^\circ) - 0.8P(5\cos 60^\circ) - 0.6P(5\sin 60^\circ) = 0$

Solving for P gives: P = 224 lb ♦

2.106

$\overrightarrow{BA} = -3\mathbf{i} - 3\cos 20^\circ\mathbf{j} + (4 - 3\sin 20^\circ)\mathbf{k} = -3\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}$ lb

$\overrightarrow{CA} = 2\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}$ lb

$$\mathbf{T}_1 = 30\,\vec{\lambda}_{BA} = 30\left(\frac{-3\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}}{5.0785}\right) = -17.722\mathbf{i} - 16.653\mathbf{j} + 17.568\mathbf{k} \text{ lb}$$

$$\mathbf{T}_2 = 90\,\vec{\lambda}_{CA} = 90\left(\frac{2\mathbf{i} - 2.819\mathbf{j} + 2.974\mathbf{k}}{4.5600}\right) = 39.474\mathbf{i} - 55.638\mathbf{j} + 58.697\mathbf{k} \text{ lb}$$

$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2 = 21.752\mathbf{i} - 72.291\mathbf{j} + 76.265\mathbf{k}$ lb

$$\therefore\ R = \sqrt{21.752^2 + (-72.291)^2 + 76.265^2} = 107.3 \text{ lb}$$ ♦

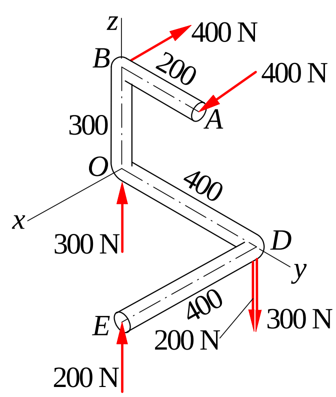
## 2.107

$$\begin{aligned}\mathbf{F} &= -400\mathbf{i} + 300\mathbf{j} + 250\mathbf{k} \text{ lb} \\ \mathbf{C} &= C\frac{-3\mathbf{j}+4\mathbf{k}}{5} = C(-0.6\mathbf{j}+0.8\mathbf{k}) \\ \mathbf{r}_{DA} &= 3\mathbf{j} \text{ ft} \qquad \boldsymbol{\lambda}_{DE} = -0.6\mathbf{i} + 0.8\mathbf{k}\end{aligned}$$

$$\begin{aligned}(M_{DE})_P &= \mathbf{r}_{DA} \times \mathbf{P} \cdot \boldsymbol{\lambda}_{DE} = \begin{vmatrix} 0 & 3 & 0 \\ -400 & 300 & 250 \\ -0.6 & 0 & 0.8 \end{vmatrix} = 510 \text{ lb} \cdot \text{ft} \\ (M_{DE})_C &= \mathbf{C} \cdot \boldsymbol{\lambda}_{DE} = C(-0.6\mathbf{j}+0.8\mathbf{k}) \cdot (-0.6\mathbf{i}+0.8\mathbf{k}) = 0.64C\end{aligned}$$

$$\begin{aligned}M_{DE} &= (M_{DE})_P + (M_{DE})_C = 1200 \text{ lb} \cdot \text{ft} \\ 510 + 0.64C &= 1200 \qquad C = 1078 \text{ lb} \cdot \text{ft} \ \blacktriangleleft\end{aligned}$$

## 2.108




Split the 500-N force at $D$ into the 200-N and 300-N forces as shown. We now see that the force system consists of three couples.

$$\begin{aligned}\mathbf{C}^R &= \Sigma\mathbf{C} = -300(0.4)\mathbf{i} - 200(0.4)\mathbf{j} - 400(0.2)\mathbf{k} \\ &= -120\mathbf{i} - 80\mathbf{j} - 80\mathbf{k} \text{ N} \cdot \text{m} \ \blacktriangleleft\end{aligned}$$

# Chapter 3

## 3.1

**Let point A be the point where the 10 lb force is applied in Fig. (a).**

**(a):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 0.8(10) = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 0.6(10) = 6$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -50$ lb•ft

**(b):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 10 - 4 = 6$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -10(5) = -50$ lb•ft

**(c):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 0.8(10) = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 0.6(10) = 6$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -50 - 0.6(10)(1) = -56$ lb•ft

**(d):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 10 - 4 = 6$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -24 - 10(5) + 8(3) = -50$ lb•ft

**(e):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 6$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -80 - 6(5) + 36 + 8(3) = -50$ lb•ft

**(f):** $\xrightarrow{+}$ $R_x = \Sigma F_x = 8$ lb ; $+\uparrow$ $R_y = \Sigma F_y = 5$ lb ;

$\circlearrowleft +$ $\Sigma M_A = -5(5) = -25$ lb•ft

**Comparing the above results, we see that (b), (d), and (e) are equivalent to (a).** ◆

## 3.2

**(a)**

$$+\uparrow \quad R = \Sigma F = 0 \blacktriangleleft$$

$$+\circlearrowleft \quad C^R = \Sigma M_A = 94 + 20(7) - 36(2) = 162.0 \text{ kN}\cdot\text{m} \blacktriangleleft$$

**(b)**

From part (a): $R = 0$ ◄

$$+\circlearrowleft \; C^R = \Sigma M_B = 94 + 36(5) - 16(7) = 162.0 \text{ kN}\cdot\text{m} \blacktriangleleft$$

The answers make sense: since the resultant is a couple, its moment is the same about any point.

## 3.3

$$\mathbf{R} = \Sigma\mathbf{F} = -75\mathbf{i} - (20 + 60)\mathbf{j} = -75\mathbf{i} - 80\mathbf{j} \text{ lb} \blacktriangleleft$$

$$+\circlearrowleft \quad C^R = \Sigma M_O = 75(6\cos 30^\circ) + 20(9\cos 30^\circ) - 60(9\cos 30^\circ)$$

$$= 77.9 \text{ lb}\cdot\text{in.} \quad \text{CCW} \blacktriangleleft$$

## 3.4

$\mathbf{R} = \Sigma \mathbf{F}$: $+\uparrow$ $R = 18 + 30 + 46 + 60 = 154$ lb

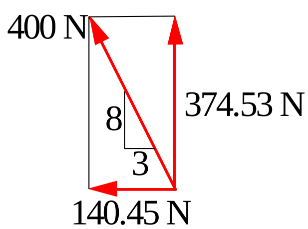
$C^R = \Sigma M_O$: $\circlearrowleft +$ $C^R = 60(3.2) - 46(3.5 - 3.2) - 30(8.0 - 3.2) - 18(11.5 - 3.2)$

$= -115.2$ lb•in.

Therefore, the equivalent force-couple system with the force acting at O is

$R = 154$ lb $\uparrow$; $C^R = 115.2$ lb•in. CW ◆

## 3.5




$$
\begin{aligned}
+ \longrightarrow \quad & R_x = \Sigma F_x = 200 - 140.45 = 59.6 \text{ N} \\
+ \uparrow \quad & R_y = \Sigma F_y = 374.53 + 300 = 674.5 \text{ N} \\
+ \circlearrowleft \quad & C^R = \Sigma M_O = (374.53 + 300)(0.6) - 200(0.4) \\
= \quad & 324.7 \text{ N} \cdot \text{m}
\end{aligned}
$$

The equivalent force-couple system with the force acing at $O$ is

$$\mathbf{R} = 59.6\mathbf{i} + 674.5\mathbf{j} \text{ N} \blacktriangleleft \qquad C^R = 324.7 \text{ N} \cdot \text{m} \ \ \text{CCW} \blacktriangleleft$$

## 3.6

$$
\begin{aligned}
\Sigma F_x &= R_x \quad + \longrightarrow \quad -200 + 120\cos 30^\circ + P_x = 80 \quad \therefore P_x = 176.1 \text{ lb} \\
\Sigma F_y &= R_y \quad + \uparrow \quad 120\sin 30^\circ + 80 + P_y = 20 \quad \therefore P_y = -120 \text{ lb} \\
\therefore \ \ & \mathbf{P} = 176.1\mathbf{i} - 120\mathbf{j} \text{ lb} \blacktriangleleft
\end{aligned}
$$

$$
\begin{aligned}
\Sigma M_O &= 0 \quad + \circlearrowleft \quad -176.1(6 + b) + 200(6) + 80(3) = 0 \\
\therefore \ \ & b = 2.18 \text{ in.} \blacktriangleleft
\end{aligned}
$$

## 3.7

The equivalent force-couple system with the force acing at $O$ is

$$
\begin{aligned}
\mathbf{R} &= \Sigma \mathbf{F} = (400 + 300 - 500)\mathbf{k} = 200\mathbf{k} \text{ lb} \blacktriangleleft \\
\mathbf{C}^R &= \Sigma \mathbf{M}_O = [8\mathbf{j} \times (-500\mathbf{k})] + (6\mathbf{i} \times 400\mathbf{k}) \\
&= -400\mathbf{i} - 2400\mathbf{j} \text{ lb} \cdot \text{ft} \blacktriangleleft
\end{aligned}
$$

**3.8**

$$\mathbf{R} = \Sigma\mathbf{F} = (240 + 200 - 400)\,\mathbf{k} = 40\mathbf{k} \text{ lb} \blacktriangleleft$$

$$\begin{aligned}
\mathbf{r}_{DA} &= 6(1-\cos 40^\circ)\mathbf{i} - 6\sin 40^\circ\mathbf{j} = 1.4037\mathbf{i} - 3.857\mathbf{j} \text{ ft} \\
\mathbf{r}_{DB} &= -6\cos 40^\circ\mathbf{i} + 6(1-\sin 40^\circ)\mathbf{j} = -4.596\mathbf{i} + 2.143\mathbf{j} \text{ ft} \\
\mathbf{r}_{DO} &= -6\cos 40^\circ\mathbf{i} - 6\sin 40^\circ\mathbf{j} = -4.596\mathbf{i} - 3.857\mathbf{j} \text{ ft}
\end{aligned}$$

$$\begin{aligned}
\mathbf{C}^R &= \Sigma\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.4037 & -3.857 & 0 \\ 0 & 0 & 240 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4.596 & 2.143 & 0 \\ 0 & 0 & 200 \end{vmatrix} \\
&\quad + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4.596 & -3.857 & 0 \\ 0 & 0 & -400 \end{vmatrix} \\
&= (-925.7\mathbf{i} - 336.9\mathbf{j}) + (428.6\mathbf{i} + 919.2\mathbf{j}) + (1542.8\mathbf{i} - 1838.4\mathbf{j}) \\
&= 1046\mathbf{i} - 1256\mathbf{j} \text{ lb}\cdot\text{ft} \blacktriangleleft
\end{aligned}$$

## 3.9

$$\mathbf{C}^R = \Sigma\mathbf{M}_D = \mathbf{0} = [\mathbf{r}_{DO} \times (-P\mathbf{k})] + (\mathbf{r}_{DB} \times 200\,\mathbf{k}) + (\mathbf{r}_{DA} \times 240\,\mathbf{k})$$

$\mathbf{r}_{DO} = -6\cos\theta\mathbf{i} - 6\sin\theta\mathbf{j}$ ft $\qquad \mathbf{r}_{DB} = -6\cos\theta\,\mathbf{i} + (6 - 6\sin\theta)\mathbf{j}$ ft

$\mathbf{r}_{DA} = (6 - 6\cos\theta)\mathbf{i} - 6\sin\theta\mathbf{j}$ ft

$$\therefore\ \mathbf{C}^R = (6)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\cos\theta & -\sin\theta & 0 \\ 0 & 0 & -P \end{vmatrix} + (6)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\cos\theta & 1-\sin\theta & 0 \\ 0 & 0 & 200 \end{vmatrix}$$

$$+ (6)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1-\cos\theta & -\sin\theta & 0 \\ 0 & 0 & 240 \end{vmatrix} = \mathbf{0}$$

**Expanding the determinants gives:**

$$(P\sin\theta\mathbf{i} - P\cos\theta\mathbf{j}) + [200(1-\sin\theta)\mathbf{i} + 200\cos\theta\mathbf{j}] + [-240\sin\theta\mathbf{i} - 240(1-\cos\theta)\mathbf{j}] = \mathbf{0}$$

**Equating like components and simplifying, we obtain:**

**(i-components):** $P\sin\theta + 200 - 200\sin\theta - 240\sin\theta = 0$

which gives: $\sin\theta(P - 440) = -200$ lb (1)

**(j-components):** $-P\cos\theta + 200\cos\theta - 240 + 240\cos\theta = 0$

which gives: $\cos\theta(P - 440) = -240$ lb (2)

Dividing (1) by (2) yields: $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{200}{240}$ $\quad \therefore\ \theta = 39.8^\circ$ ◆

Using (1), we get: $P = -\dfrac{200}{\sin 39.8^\circ} + 440 = 127.6$ lb ◆

## 3.10

$\mathbf{R} = \Sigma\mathbf{F}$ and $\mathbf{C}^R = \Sigma\mathbf{M}_A$

(a) $\mathbf{R} = 2\mathbf{i} + 6\mathbf{j}$ lb; $\mathbf{C}^R = -6\mathbf{j}$ lb•in. (b) $\mathbf{R} = 4\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ lb; $\mathbf{C}^R = \mathbf{0}$

(c) $\mathbf{R} = 2\mathbf{i} + 6\mathbf{j}$ lb; $\mathbf{C}^R = -6\mathbf{j}$ lb•in.

(d) $\mathbf{R} = \mathbf{0}$; $\mathbf{C}^R = -15\mathbf{i} + (9 + 12 - 13)\mathbf{j} = -15\mathbf{i} + 8\mathbf{j}$ lb•in.

(e) $\mathbf{R} = \mathbf{0}$; $\mathbf{C}^R = -15\mathbf{i} + 8\mathbf{j}$ lb•in. (f) $\mathbf{R} = \mathbf{0}$; $\mathbf{C}^R = (25 - 25)\mathbf{i} + (16 - 16)\mathbf{j} = \mathbf{0}$

**Comparing the above results: (a) and (c) are equivalent; (d) and (e) are equivalent.** ◆

## 3.11

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} = (-8\mathbf{i} + 10\mathbf{j}) + 8\mathbf{i} = 10\mathbf{j} \text{ lb} \quad \text{O.K.}$$

$$\Sigma\mathbf{M}_O = \mathbf{C}^R \qquad a\mathbf{i} \times \mathbf{P} + (8\mathbf{j} - b\mathbf{k}) \times \mathbf{Q} = -110\mathbf{j}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & 0 & 0 \\ -8 & 10 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -b \\ 8 & 0 & 0 \end{vmatrix} = -110\mathbf{j}$$

$$\begin{aligned} 10a\mathbf{k} + (-8b\mathbf{j} - 64\mathbf{k}) &= -110\mathbf{j} \\ 10a - 64 &= 0 \qquad a = 6.4 \text{ in.} \blacktriangleleft \\ -8b &= -110 \qquad b = 13.75 \text{ in.} \blacktriangleleft \end{aligned}$$

## 3.12

$$\begin{aligned} \mathbf{F}_{AB} &= F\frac{3\mathbf{i} - 4\mathbf{k}}{5} = (0.6\mathbf{i} - 0.8\mathbf{k})F \\ \mathbf{F}_{AC} &= F\frac{3\mathbf{j} - 4\mathbf{k}}{5} = (0.6\mathbf{j} - 0.8\mathbf{k})F \\ \mathbf{C}^R &= \Sigma\mathbf{M}_O = \mathbf{r}_{OA} \times (\mathbf{F}_{AB} + \mathbf{F}_{AC}) - 30(6)\mathbf{j} \\ &= F\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0.6 & 0.6 & -1.6 \end{vmatrix} - 180\mathbf{j} = -2.4F\mathbf{i} + (2.4F - 180)\mathbf{j} \\ 2.4F - 180 &= 0 \qquad F = 75 \text{ kN} \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \mathbf{C}^R &= -2.4(75)\mathbf{i} = 180\mathbf{i} \text{ kN}\cdot\text{m} \blacktriangleleft \\ \mathbf{R} &= \Sigma\mathbf{F} = 75(0.6\mathbf{i} + 0.6\mathbf{j} - 1.6\mathbf{k}) - 30\mathbf{i} = 15\mathbf{i} + 45\mathbf{j} - 120\mathbf{k} \text{ kN} \blacktriangleleft \end{aligned}$$

## 3.13

$$\begin{aligned} \mathbf{P}_A &= 120\frac{-1.5\mathbf{i} + 2\mathbf{j}}{\sqrt{(-1.5)^2 + 2^2}} = -72.0\mathbf{i} + 96.0\mathbf{j} \text{ N} \\ \mathbf{P}_B &= 100\frac{2\mathbf{j} - 2\mathbf{k}}{\sqrt{2^2 + (-2)^2}} = 70.71\mathbf{j} - 70.71\mathbf{k} \text{ N} \\ \mathbf{C} &= 180\frac{-1.5\mathbf{i} + 2\mathbf{k}}{\sqrt{(-1.5)^2 + 2^2}} = -108.0\mathbf{i} + 144.0\mathbf{k} \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \mathbf{R} &= \mathbf{P}_A + \mathbf{P}_B = (-72.0\mathbf{i} + 96.0\mathbf{j}) + (70.71\mathbf{j} - 70.71\mathbf{k}) \\ &= -72.0\mathbf{i} + 166.71\mathbf{j} - 70.71\mathbf{k} \text{ N} \blacktriangleleft \end{aligned}$$

$$
\begin{aligned}
\mathbf{C}^R &= \Sigma\mathbf{M}_C = \mathbf{r}_{CD}\times\mathbf{P}_A + \mathbf{r}_{CD}\times\mathbf{P}_B + \mathbf{C} = \mathbf{r}_{CD}\times\mathbf{R} + \mathbf{C} \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ -72.0 & 166.71 & -70.71 \end{vmatrix} + (-108.0\mathbf{i} + 144.0\mathbf{k}) \\
&= 225\mathbf{i} + 144\mathbf{j} + 144.0\mathbf{k}\ \text{N}\cdot\text{m} \ \blacktriangleleft
\end{aligned}
$$

## 3.14

$$\mathbf{R} = \Sigma\mathbf{F} = \left[-12 - \frac{12}{13}(26)\right]\mathbf{i} + 0\mathbf{j} + \left[\frac{5}{13}(26) - 32 - 8\right] = -36\mathbf{i} - 30\mathbf{k}\ \text{lb} \ \blacklozenge$$

$\mathbf{C}^R = \Sigma\mathbf{M}_D$

$$C_x^R = \Sigma M_{Dx} = 32(1.5) + 8(1.5) - \frac{5}{13}(26)(4.5) = 15\ \text{lb}\cdot\text{ft}$$

$$C_y^R = \Sigma M_{Dy} = 32(0.75) - 8(0.75) - 12(1.0) + 26(1.0) - 32 = 0$$

$$C_z^R = \Sigma M_{Dz} = -\frac{12}{13}(26)(4.5) - 12(4.5) = -162\ \text{lb}\cdot\text{ft}$$

$$\therefore \mathbf{C}^R = 15\mathbf{i} - 162\mathbf{k}\ \text{lb}\cdot\text{ft} \ \blacklozenge$$

## 3.15

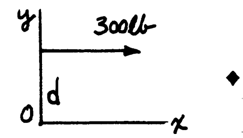
$$
\begin{aligned}
\mathbf{F}_{AC} &= 20\frac{-180\mathbf{i} + 150\mathbf{j} + 210\mathbf{k}}{\sqrt{(-180)^2 + 150^2 + 210^2}} = -11.442\mathbf{i} + 9.535\mathbf{j} + 13.348\mathbf{k}\ \text{N} \\
\mathbf{C}_{BC} &= 6\frac{300\mathbf{i} - 200\mathbf{j} + 210\mathbf{k}}{\sqrt{300^2 + (-200)^2 + 210^2}} = 4.314\mathbf{i} - 2.876\mathbf{j} + 3.020\mathbf{k}\ \text{N}\cdot\text{m} \\
\mathbf{r}_{BA} &= 0.48\mathbf{i} - 0.35\mathbf{j}\ \text{m}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{C}^R &= \mathbf{r}_{BA}\times\mathbf{F}_{AC} + \mathbf{C}_{BC} \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.48 & -0.35 & 0 \\ -11.442 & 9.535 & 13.348 \end{vmatrix} + 4.314\mathbf{i} - 2.876\mathbf{j} + 3.020\mathbf{k} \\
&= -0.358\mathbf{i} - 9.28\mathbf{j} + 3.59\mathbf{k}\ \text{N}\cdot\text{m} \ \blacktriangleleft \\
\mathbf{R} &= \mathbf{F}_{AC} = -11.44\mathbf{i} + 9.54\mathbf{j} + 13.35\mathbf{k}\ \text{N} \ \blacktriangleleft
\end{aligned}
$$

## 3.16

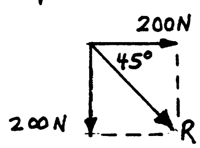
**(1)** $\circlearrowleft+$ $\Sigma M_O = -300d = -900$ lb•in.

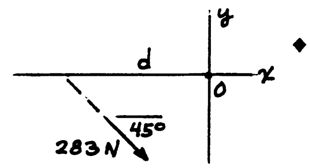
$\therefore$ d = 3 in.




**R** = 300**i** lb intersecting the y axis at y = 3 in.

**(2)** $R = \sqrt{200^2 + 200^2} = 283$ N

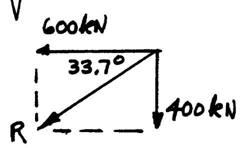


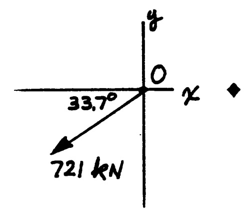




$\circlearrowleft+$ $\Sigma M_O = 200d = 800$ N•m $\quad\therefore$ d = 4.00 m

**R** = 200**i** – 200**j** N intersecting the x axis at x = – 4 m

**(3)** $R = \sqrt{600^2 + 400^2} = 721$ kN

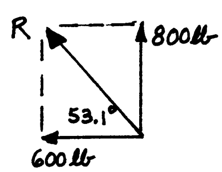


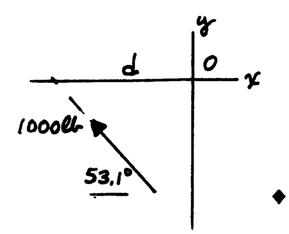




Because $\Sigma M_O = 0$, **R** passes through O.

**R** = – 600**i** – 400**j** kN passing through the origin O

**(4)** $R = \sqrt{600^2 + 800^2} = 1000$ lb




$\circlearrowleft+$ $\Sigma M_O = -800d = -24\,000$ lb•ft $\quad\therefore$ d = 30 ft

**R** = – 600**i** + 800**j** lb intersecting the x axis at x = – 30 ft

## 3.17

$$+\uparrow \quad \Sigma F = 3200 \qquad T_1 \sin 45^\circ + T_2 \sin 25^\circ = 3200$$
$$+\rightarrow \quad \Sigma F = 0 \qquad -T_1 \cos 45^\circ + T_2 \cos 25^\circ = 0$$

The solution is

$T_1 = 3090$ lb ◄ $\qquad T_2 = 2410$ lb ◄

## 3.18

$$\begin{aligned} R_y &= \Sigma F_y \qquad +\uparrow \ 600 = 800\sin\theta + \frac{3}{5}(400) \qquad \theta = 26.74^\circ \ \blacktriangleleft \\ R_x &= \Sigma F_x \qquad + \longrightarrow \ 0 = -800\cos\theta + \frac{4}{5}(400) + P \\ P &= 800\cos 26.74^\circ - 320 = 394 \text{ lb} \ \blacktriangleleft \end{aligned}$$

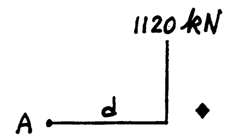
## 3.19

**(a)** $\mathbf{R = \Sigma F}$ $\quad +\downarrow$ $\mathbf{R = 200 + 120 + 300 + 500 = 1120}$ **kN**

$\mathbf{\Sigma M_A = R d}$ **(using x = 5 m):**

(+) **300(2) + 120(10) + 200(14) + 500(5) = 1120d kN•m**

$\therefore$ **d = 6.34 m**




◆

**(b)** $\mathbf{\Sigma M_A = R d}$ **(using R = 1120 kN and d = 17/2 = 8.50 m):**

(+) **300(2) + 120(10) + 200(14) + 500x = 1120(8.50) kN•m**

$\therefore$ **x = 9.84 m** ◆

## 3.20

$$\begin{aligned} R_x &= \Sigma F_x = 50\cos 60^\circ = 25 \text{ lb} \\ R_y &= \Sigma F_y = -150 - 50\sin 60^\circ = -193.3 \text{ lb} \end{aligned}$$

Let $A$ be the point where the line of action of $\mathbf{R}$ crosses the $x$-axis.

$$\begin{aligned} + \ \circlearrowleft \quad \Sigma M_O = R_y x_A \qquad & 875 - 150(4) - (50\sin 60^\circ)(7) = -193.3 x_A \\ x_A &= 0.1454 \text{ ft} \end{aligned}$$

Resultant force $\mathbf{R} = 25\mathbf{i} - 193.3\mathbf{j}$ lb intersects the $x$-axis at $x = 0.1454$ ft. ◄

## 3.21

$$\begin{aligned} R_x &= \Sigma F_x = 8 - 6 = 2 \text{ kN} \\ R_y &= \Sigma F_y = 6 + 15 = 21 \text{ kN} \end{aligned}$$

Let the resultant intersect the $x$-axis at $x = x_A$.

$$\begin{aligned} + \ \circlearrowleft \quad \Sigma M_O = R_y x_A \qquad & 12 - 8(1.5) + 15(2) = 21 x_A \\ x_A &= 1.429 \text{ m} \end{aligned}$$

Resultant force $\mathbf{R} = 2\mathbf{i} + 21\mathbf{j}$ kN intersects the $x$-axis at $x = 1.429$ m. ◄

**3.22**

**(a)** $\theta = 30°$

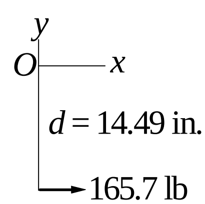
$$\begin{aligned} R_x &= \Sigma F_x = -400 + 2(400)\sin 30° = 0 \\ R_y &= \Sigma F_y = 0 \text{ by inspection} \\ C^R &= \Sigma M_O = 3(400)(2) = 2400 \text{ lb} \cdot \text{in. CCW} \end{aligned}$$

$\therefore$ Resultant is the couple $C^R = 2400$ lb·in. CCW ◄

**(b)**

$\theta = 45°$

$$\begin{aligned} R_x &= \Sigma F_x = -400 + 2(400)\sin 45° = 165.69 \text{ lb} \\ R_y &= \Sigma F_y = 0 \text{ by inspection} \\ \Sigma M_O &= R_x d \qquad 3(400)(2) = 165.69d \qquad d = 14.49 \text{ in.} \end{aligned}$$




Resultant force $\mathbf{R} = 165.7\mathbf{i}$ lb intersects $y$-axis at $y = -14.49$ in. ◄

**3.23**

$$\begin{aligned} R_x &= \Sigma F_x = 0 \qquad R_y = \Sigma F_y = -1200 + 600 = -600 \text{ lb} \\ +\circlearrowleft \quad \Sigma M_A &= 1000(4.5) + 1200(8.2) - 600(13.7) = 6120 \text{ lb} \cdot \text{ft} \end{aligned}$$

$$x = \frac{\Sigma M_A}{|R_y|} = \frac{6120}{600} = 10.20 \text{ ft}$$

$\therefore$ Resultant is $\mathbf{R} = -600\mathbf{j}$ lb intersecting the $x$-axis at $x = 10.2$ ft ◄

**3.24**

$$\begin{aligned} R_x &= \Sigma F_x = 300\left(\frac{4}{5}\right) - 240 = 0 \\ R_y &= \Sigma F_y = 300\left(\frac{3}{5}\right) - 120 - 60 = 0 \end{aligned}$$

The resultant is not a force.

$$+\circlearrowleft C^R = \Sigma M_O = -60(0.12) = -7.2 \text{ N} \cdot \text{m}$$

The resultant is the couple $C^R = 7.2$ N·m CW ◄

## 3.25

**Since R passes through B, the sum of the moments about B is zero.**

$$\circlearrowleft^{+}\ \Sigma M_B = 30.5(3) - 36(1.5) - \frac{5}{13}F(2.5) = 0 \quad \text{which gives} \quad F = 39.0 \text{ lb} \ \blacklozenge$$

$$R_x = \Sigma F_x: \quad \overset{+}{\rightarrow}\ R_x = -30.5 - \frac{12}{13}(39.0) = -66.5 \text{ lb}$$

$$R_y = \Sigma F_y: \quad +\uparrow\ R_y = 36 - \frac{5}{13}(39.0) = 21.0 \text{ lb}$$

$$\therefore\ \mathbf{R} = -66.5\mathbf{i} + 21.0\mathbf{j} \text{ lb} \ \blacklozenge$$

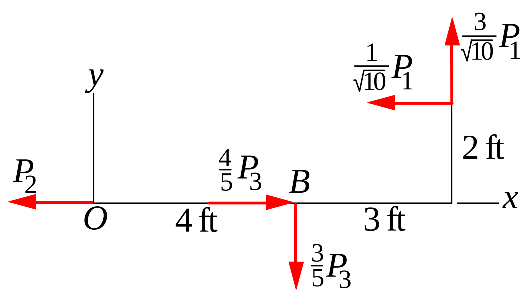
## 3.26

$$\Sigma M_O = 0 \quad +\circlearrowleft \quad C + (160 - 40)(12) + (80 - 200)(15) = 0$$

$$C = 360 \text{ lb} \cdot \text{in.} = 30 \text{ lb} \cdot \text{ft} \circlearrowleft \ \blacktriangleleft$$

$$R = \Sigma F = 200 + 80 - 160 - 40 = 80 \text{ lb} \longrightarrow \ \blacktriangleleft$$

## 3.27




$$+ \longrightarrow \quad R_x = \Sigma F_x = 0 \quad -\frac{1}{\sqrt{10}}P_1 - P_2 + \frac{4}{5}P_3 = 0$$

$$+ \uparrow \quad R_y = \Sigma F_y = 0 \quad \frac{3}{\sqrt{10}}P_1 - \frac{3}{5}P_3 = 0$$

$$+ \circlearrowleft \quad \Sigma M_B = 120 \quad 3\left(\frac{3}{\sqrt{10}}P_1\right) + 2\left(\frac{1}{\sqrt{10}}P_1\right) = 120$$

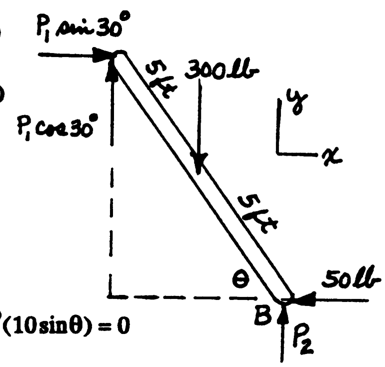
Solution is $P_1 = 34.5$ lb ◀ $\quad P_2 = 32.7$ lb ◀ $\quad P_3 = 54.5$ lb ◀

### 3.28

$$\Sigma F_x = 0 \qquad + \longrightarrow \qquad -P_2 + \frac{3}{5}P_3 = 0$$
$$\Sigma F_y = -220 \qquad + \downarrow \quad P_1 + \frac{4}{5}P_3 = 220$$
$$\Sigma M_B = 0 \qquad + \circlearrowleft \quad \frac{4}{5}P_1(2) - 2P_3 = 0$$

Solution is $P_1 = 134.1$ kN ◄ $\qquad P_2 = 64.4$ kN ◄ $\qquad P_3 = 107.3$ kN ◄

### *3.29




**$R_x = \Sigma F_x = 0$: $\underrightarrow{+}$ $P_1 \sin 30° - 50 = 0$ (1)**

**$R_y = \Sigma F_y = 0$: $+\uparrow$ $P_1 \cos 30° + P_2 - 300 = 0$ (2)**

**Solving (1) and (2) gives:**

**$P_1 = 100$ lb, $P_2 = 213.40$ lb ◆**

**$\Sigma M_B = 0$:**

**$\circlearrowleft +$ $300(5\cos\theta) - P_1\cos 30°(10\cos\theta) - P_1 \sin 30°(10\sin\theta) = 0$**

$$300(5\cos\theta) - 100\cos 30°(10\cos\theta) - 100\sin 30°(10\sin\theta) = 0$$
$$1500\cos\theta - 866.03\cos\theta - 500\sin\theta = 0$$
$$633.97\cos\theta = 500\sin\theta$$

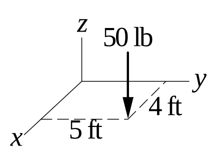
**$\tan\theta = \sin\theta/\cos\theta = 633.97/500 = 1.2679 \qquad \therefore\ \theta = 51.7°$ ◆**

### 3.30

All three cases represent parallel force systems.

(a)

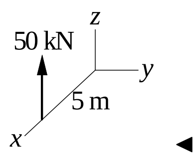
$$\Sigma M_x = Ry \qquad -250 = -50y \qquad y = 5.0 \text{ ft}$$
$$\Sigma M_y = -Rx \qquad 200 = -(-50)x \qquad x = 4.0 \text{ ft}$$




◄

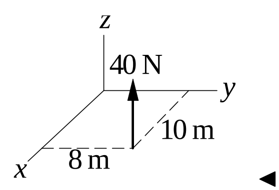
(b)

$$\Sigma M_x = Ry \qquad 0 = 50y \qquad y = 0$$
$$\Sigma M_y = -Rx \qquad -250 = -50x \qquad x = 5.0 \text{ m}$$



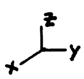

(c)

$$\Sigma M_x = Ry \qquad 320 = 40y \qquad y = 8.0 \text{ m}$$
$$\Sigma M_y = -Rx \qquad -400 = -40x \qquad x = 10.0 \text{ m}$$




## 3.31

**(a): The given forces are concurrent. Therefore, the resultant is a force. ♦**

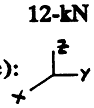
**(b):** 


$$\mathbf{R} = \left[\frac{3}{5}(15) - \frac{3}{5}(15)\right]\mathbf{i} + \left[\frac{4}{5}(15) + \frac{4}{5}(15) - 24\right]\mathbf{j} = \mathbf{0}$$

**Because R = 0, the resultant is not a force. The 24-kN force and one of the 15-kN forces intersect. The sum of the moments about this point of intersection is not zero. Therefore, the resultant is a couple. ♦**

**(c): The 25-kN force is perpendicular to the couple-vector that is equivalent to the two 20-kN forces. This force and couple-vector can be reduced to a single force. Therfore, the resultant is a force. ♦**

**(d): The 16 kN-force is not perpendicular to the couple-vector that is equivalent to the two 12-kN forces. Therefore, the resultant is a wrench. ♦**

**(e):** 


**The given force is** $\mathbf{F} = 15\left(-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}\right) = -9\mathbf{i} + 12\mathbf{j}$ **kN.**

**The sum of the two couples is** $\mathbf{C} = 400\mathbf{i} + 300\mathbf{j}$ **kN. Since F and C are perpendicular (they have negative reciprocal slopes), they can be reduced to a single force. Therefore, the resultant is a force. ♦**

### 3.32

$$
\begin{aligned}
\mathbf{T}_1 &= T_1\boldsymbol{\lambda}_1 = 800\frac{3\mathbf{i}-8\mathbf{j}+4\mathbf{k}}{\sqrt{89}} = 254.4\mathbf{i}-678.4\mathbf{j}+339.2\mathbf{k}\text{ lb}\\
\mathbf{T}_2 &= T_2\boldsymbol{\lambda}_2 = 600\frac{-4\mathbf{i}-8\mathbf{j}+4\mathbf{k}}{\sqrt{96}} = -244.9\mathbf{i}-489.9\mathbf{j}+244.9\mathbf{k}\text{ lb}\\
\mathbf{T}_3 &= T_3\boldsymbol{\lambda}_3 = 400\frac{-8\mathbf{j}-6\mathbf{k}}{10} = -320.0\mathbf{j}-240.0\mathbf{k}\text{ lb}
\end{aligned}
$$

$$
(254.4\mathbf{i}-678.4\mathbf{j}+339.2\mathbf{k})+(-244.9\mathbf{i}-489.9\mathbf{j}+244.9\mathbf{k})
$$
$$
+(-320.0\mathbf{j}-240.0\mathbf{k})
$$

$$
\mathbf{R} = \Sigma\mathbf{T} = 9.5\mathbf{i}-1488\mathbf{j}+344\mathbf{k}\text{ lb} \quad ◄
$$

### 3.33

$$
\begin{aligned}
\mathbf{T}_1 &= T_1\boldsymbol{\lambda}_1 = T_1\frac{3\mathbf{i}-8\mathbf{j}+4\mathbf{k}}{\sqrt{89}} = T_1(0.3180\mathbf{i}-0.8480\mathbf{j}+0.4240\mathbf{k})\\
\mathbf{T}_2 &= T_2\boldsymbol{\lambda}_2 = 620\frac{-4\mathbf{i}-8\mathbf{j}+4\mathbf{k}}{\sqrt{96}} = -253.1\mathbf{i}-506.2\mathbf{j}+253.1\mathbf{k}\text{ lb}\\
\mathbf{T}_3 &= T_3\boldsymbol{\lambda}_3 = T_3\frac{-8\mathbf{j}-6\mathbf{k}}{10} = T_3(-0.8\mathbf{j}-0.6\mathbf{k})
\end{aligned}
$$

$$
\mathbf{R} = R\mathbf{j} = \Sigma\mathbf{T}
$$

Equating like components:

$$
\begin{aligned}
0 &= 0.3180T_1 - 253.1\\
R &= -0.8480T_1 - 506.2 - 0.8T_3\\
0 &= 0.4240T_1 + 253.1 - 0.6T_3
\end{aligned}
$$

Solution is $R = 1969$ lb $\quad T_1 = 796$ lb ◄ $\quad T_3 = 984$ lb ◄

### 3.34

Due to symmetry $P_2 = P_3$.

$$
\begin{aligned}
\mathbf{P}_1 &= P_1\frac{-20\mathbf{i}+32\mathbf{k}}{\sqrt{(-20)^2+32^2}} = (-0.5300\mathbf{i}+0.8480\mathbf{k})P_1\\
\mathbf{P}_2 &= P_2\frac{20\mathbf{i}+15\mathbf{j}}{\sqrt{20^2+15^2}} = (0.8000\mathbf{i}+0.6000\mathbf{j})P_2
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= 0 \qquad -0.5300P_1 + 2(0.8000)P_2 = 0\\
\Sigma F_z &= 200\text{ lb} \qquad 0.8480P_1 = 200
\end{aligned}
$$

Solution is: $P_1 = 236$ lb ◄ $\quad P_2 = P_3 = 78.1$ lb ◄

## 3.35

$\mathbf{\Sigma F = 0}$: $\mathbf{P} - 80\mathbf{i} - 120\mathbf{j} + 90\mathbf{k} = \mathbf{0}$ $\quad \therefore\ \mathbf{P} = 80\mathbf{i} + 120\mathbf{j} - 90\mathbf{k}$ **N** ◆

**Choosing point O as the moment center (any point could be used):**

$$\mathbf{C}^R = \Sigma \mathbf{M}_O = 90(0.3)\,\mathbf{i} + 120(0.2)\,\mathbf{i} = 51.0\mathbf{i}\ \text{N·m} \quad ◆$$

## 3.36

Let the resultant intersect the plate at $(x, y)$.

$$\begin{aligned} R &= \Sigma F_z = -6P \\ x &= -\frac{\Sigma M_y}{R} = -\frac{2Pa}{-6P} = \frac{a}{3} \\ y &= \frac{\Sigma M_x}{R} = \frac{-3Pb}{-6P} = \frac{b}{2} \end{aligned}$$

The resultant is $\mathbf{R} = -6P\mathbf{k}$ intersecting the plate at $(a/3, b/2)$.

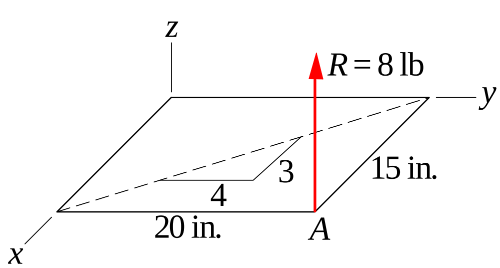
## 3.37

Let the resultant $\mathbf{R}$ intersect the plate at $(x, y)$.

$$\begin{aligned} R &= \Sigma F_z = 60 + 70 - 10 = 120 \text{ lb} \\ x &= -\frac{\Sigma M_y}{R} = -\frac{(-60)(6)}{120} = 3.0 \text{ ft} \\ y &= \frac{\Sigma M_x}{R} = \frac{60(6) + 70(0.75) - 10(5.25)}{120} = 3.0 \text{ ft} \end{aligned}$$

$\therefore$ Resultant is $\mathbf{R} = 120\mathbf{k}$ lb acting at the center of the plate ◄

## 3.38

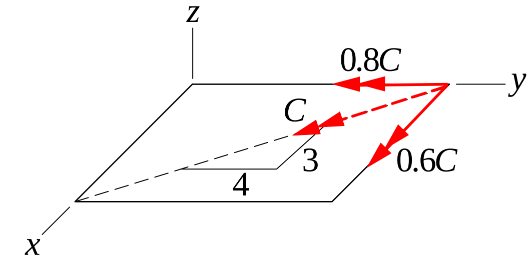



$$\begin{aligned} R &= \Sigma F_x \qquad 8 = P_1 + P_2 \\ 20R &= \Sigma M_x \qquad 20(8) = P_2(5) + \frac{3}{5}C \\ 15R &= \Sigma M_y \qquad -15(8) = -P_2(15) - \frac{4}{5}C \end{aligned}$$

Solution is $\quad C = 360.0$ lb·in. ◄ $\qquad P_1 = 19.2$ lb ◄ $\qquad P_2 = -11.2$ lb ◄

## 3.39

Let $x$ and $y$ be the coordinates of the point where the resultant intersects the $xy$-plane




$$\begin{aligned} R &= \Sigma F_z = 35 + 20 = 55 \text{ lb} \\ \Sigma M_x &= Ry \qquad P_2(5) + 0.6C = Ry \\ y &= \frac{5P_2 + 0.6C}{R} = \frac{5(20) + 0.6(80)}{55} = 2.69 \text{ in.} \\ \Sigma M_y &= -Rx \qquad -P_2(15) - 0.8C = -Rx \\ x &= \frac{15P_2 + 0.8C}{R} = \frac{15(20) + 0.8(80)}{55} = 6.62 \text{ in.} \end{aligned}$$

The resultant force $\mathbf{R} = 55\mathbf{k}$ lb passes through the point $(6.62, 2.69, 0)$.◄

## 3.40

Let the resultant $\mathbf{R}$ intersect the plate at $(x, y)$.

$$\begin{aligned} R &= \Sigma F_z = -300 + 120 - 200 = -380 \text{ lb} \\ x &= -\frac{\Sigma M_y}{R} = -\frac{-500 - 120(3) + 200(3\sin 30^\circ)}{-380} = -1.474 \text{ ft} \\ y &= \frac{\Sigma M_x}{R} = \frac{300(3) - 200(3\cos 30^\circ)}{-380} = -1.001 \text{ ft} \end{aligned}$$

$\therefore$ Resultant is $\mathbf{R} = -380\mathbf{k}$ lb acting at $(-1.474 \text{ ft}, -1.001 \text{ ft}, 0)$ ◄

## 3.41

$$\begin{aligned} \mathbf{R} &= \mathbf{F} = -6\mathbf{i} + 8\mathbf{j} + 5\mathbf{k} \text{ kN} \;◄ \\ \mathbf{C} &= -18\mathbf{i} + 24\mathbf{j} + 15\mathbf{k} \text{ kN}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \mathbf{C}^R &= \mathbf{C} + \mathbf{r}_{OA} \times \mathbf{F} = -18\mathbf{i} + 24\mathbf{j} + 15\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 2 \\ -6 & 8 & 5 \end{vmatrix} \\ &= -14\mathbf{i} + 27\mathbf{j} + 15\mathbf{k} \text{ kN}\cdot\text{m} \;◄ \end{aligned}$$

## 3.42

Note symmetry about the $y$-axis. $\therefore M_y = 0$.

$$\begin{aligned} \mathbf{R} &= \Sigma \mathbf{F} = 60\mathbf{j} + 80\mathbf{k} \text{ lb} \\ \mathbf{M}_O &= 20(4)\mathbf{i} = 80\mathbf{i} \text{ lb} \cdot \text{ft} \end{aligned}$$

Because $\mathbf{R}$ and $\mathbf{M}_O$ are perpendicular, the resultant is a force. Due to symmetry, the resultant must intersect the $y$-axis. Let the coordinate of the crossing point be $y$. Equating moments about the $x$-axis of the original force system and the resultant, we get

$$M_x = R_z y \qquad 80 = 80y \qquad y = 1.0 \text{ ft}$$

Resultant force $\mathbf{R} = 60\mathbf{j} + 80\mathbf{k}$ lb crosses the $y$-axis at $y = 1.0$ ft. ◄

## 3.43

The couple of the wrench is

$$\begin{aligned} \mathbf{C} &= 1200\frac{\mathbf{R}}{|\mathbf{R}|} = 1200\frac{600\mathbf{i} + 1400\mathbf{j} + 700\mathbf{k}}{\sqrt{600^2 + 1400^2 + 700^2}} \\ &= 429.5\mathbf{i} + 1002.2\mathbf{j} + 501.1\mathbf{k} \text{ lb} \cdot \text{ft} \end{aligned}$$

$$\begin{aligned} \mathbf{C}^R &= \mathbf{C} + \mathbf{r}_{OA} \times \mathbf{R} \\ &= 429.5\mathbf{i} + 1002.2\mathbf{j} + 501.1\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 600 & 1400 & 700 \end{vmatrix} \\ &= 1830\mathbf{i} - 1098\mathbf{j} + 3501\mathbf{k} \text{ lb} \cdot \text{ft} \quad ◄ \end{aligned}$$

### 3.44

Unit vector in the direction of $\mathbf{R}$ is

$$\boldsymbol{\lambda} = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{250\mathbf{i} + 360\mathbf{j} - 400\mathbf{k}}{\sqrt{250^2 + 360^2 + (-400)^2}} = 0.4213\mathbf{i} + 0.6067\mathbf{j} - 0.6741\mathbf{k}$$

The component of $\mathbf{C}^R$ in the direction of $\boldsymbol{\lambda}$ is

$$\begin{aligned} C_t^R &= \mathbf{C}^R \cdot \boldsymbol{\lambda} = 1200(0.4213) + 750(0.6067) + 560(-0.6741) = 583.1 \text{ N} \cdot \text{m} \\ \mathbf{C}_t^R &= C_t^R \boldsymbol{\lambda} = 583.1(0.4213\mathbf{i} + 0.6067\mathbf{j} - 0.6741\mathbf{k}) \\ &= 245.7\mathbf{i} + 353.8\mathbf{j} - 393.1\mathbf{k} \text{ N} \cdot \text{m} \end{aligned}$$

The component of $\mathbf{C}^R$ that is normal to $\boldsymbol{\lambda}$ is

$$\begin{aligned} \mathbf{C}_n^R &= \mathbf{C}^R - \mathbf{C}_t^R \\ &= (1200\mathbf{i} + 750\mathbf{j} + 560\mathbf{k}) - (245.7\mathbf{i} + 353.8\mathbf{j} - 393.1\mathbf{k}) \\ &= 954.3\mathbf{i} + 396.2\mathbf{j} + 953.1\mathbf{k} \text{ N} \cdot \text{m} \end{aligned}$$

Let $A(x, y, 0)$ be the point where the wrench intersects the $xy$-plane.

$$\mathbf{r}_{OA} \times \mathbf{R} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 250 & 360 & -400 \end{vmatrix} = -400y\mathbf{i} + 400x\mathbf{j} + (360x - 250y)\mathbf{k}$$

Equating $x$- and $y$-components of $\mathbf{r}_{OA} \times \mathbf{R} = \mathbf{C}_n^R$ yields

$$\begin{aligned} -400y &= 954.3 \qquad y = -2.39 \text{ m} \\ 400x &= 396.2 \qquad x = 0.991 \text{ m} \end{aligned}$$

$\therefore$ The wrench consists of:

$$\begin{aligned} \text{Force} \quad \mathbf{R} &= 250\mathbf{i} + 360\mathbf{j} - 400\mathbf{k} \text{ N} \;\blacktriangleleft \\ \text{Couple} \quad \mathbf{C}_t^R &= 245.7\mathbf{i} + 353.8\mathbf{j} - 393.1\mathbf{k} \text{ N} \cdot \text{m} \;\blacktriangleleft \end{aligned}$$

The axis of the wrench passes through the point (0.991 m, −2.39 m, 0) ◀

## 3.45

**(a)**

$$\begin{aligned}\mathbf{R} &= \Sigma\mathbf{F} = 8\mathbf{k}\text{ kN} \blacktriangleleft \\ \mathbf{C}^R &= \Sigma\mathbf{M}_O = -6(1.5)\mathbf{i} + 6(1.2)\mathbf{k} = -9.0\mathbf{i} + 7.2\mathbf{k}\text{ kN}\cdot\text{m} \blacktriangleleft\end{aligned}$$

**(b)** Note that $\boldsymbol{\lambda} = \mathbf{k}$ is the unit vector in the direction of $\mathbf{R}$. The component of $\mathbf{C}^R$ in the direction of $\mathbf{k}$ is $\mathbf{C}_t^R = 7.2\mathbf{k}$ kN·m. ∴ The equivalent wrench consists of

$$\text{Force} = \mathbf{R} = 8\mathbf{k}\text{ kN} \qquad \text{Couple} = \mathbf{C}_t^R = 7.2\mathbf{k}\text{ kN}\cdot\text{m} \blacktriangleleft$$

The component of $\mathbf{C}^R$ that is normal to $\boldsymbol{\lambda}$ is

$$\mathbf{C}_n^R = \mathbf{C}^R - \mathbf{C}_t^R = (-9.0\mathbf{i} + 7.2\mathbf{k}) - 7.2\mathbf{k} = 9.0\mathbf{i}\text{ kN}\cdot\text{m}$$

Let $A(x, y, 0)$ be the point where the wrench intersects the $xy$-plane.

$$\mathbf{r}_{OA} \times \mathbf{R} = \mathbf{C}_n^R \qquad \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ x & y & 0 \\ 0 & 0 & 8\end{vmatrix} = 9.0\mathbf{i} \qquad 8.0y\mathbf{i} - 8.0x\mathbf{j} = 9.0\mathbf{i}$$

$$\therefore \quad x = 0 \blacktriangleleft \qquad y = 1.125\text{ m} \blacktriangleleft$$

## 3.46

Area of sign: $A = 800^2 - 4\left(\dfrac{250^2}{2}\right) = 515 \times 10^3\text{ mm}^2 = 0.515\text{ m}^2$

Resultant: $R = pA = 110(0.515) = 56.7\text{ N}$

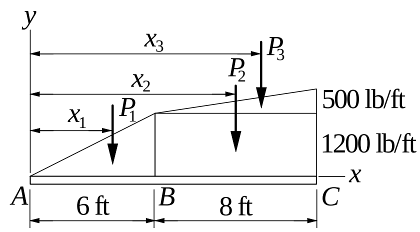
Resultant is a 56.7 N normal force acting at the center of the sign. ◄

## 3.47

**R = volume under the load surface = 0.5(58.9)(32)(6) = 5650 kN**

**The resultant is a 5650 kN normal force, acting at $\bar{x}$ = 16 m, $\bar{z}$ = 2 m (the centroid of the load volume).** ♦

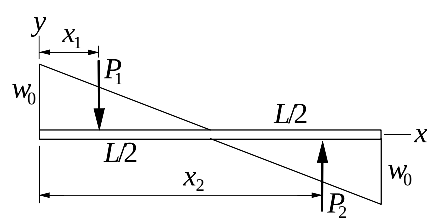
**3.48**




$$P_1 = \frac{1}{2}(1200)(6) = 3600 \text{ lb} \qquad x_1 = \frac{2}{3}(6) = 4 \text{ ft}$$
$$P_2 = 1200(8) = 9600 \text{ lb} \qquad x_2 = 6 + \frac{1}{2}(8) = 10 \text{ ft}$$
$$P_3 = \frac{1}{2}(500)(8) = 2000 \text{ lb} \qquad x_3 = 6 + \frac{2}{3}(8) = 11.333 \text{ ft}$$

$$R = \Sigma P = 3600 + 9600 + 2000 = 15\ 200 \text{ lb} \downarrow \quad \blacktriangleleft$$
$$\Sigma M_A = R\bar{x}: \quad 3600(4) + 9600(10) + 2000(11.333) = 15\ 200\bar{x}$$
$$\therefore \quad \bar{x} = 8.75 \text{ ft} \quad \blacktriangleleft$$

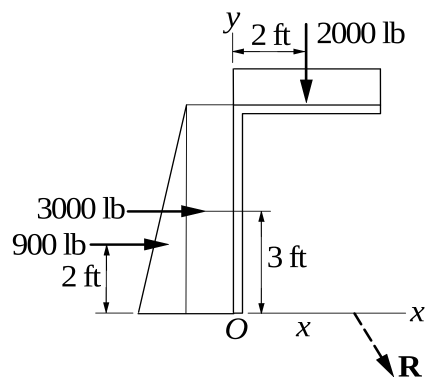
**3.49**




$$P_1 = P_2 = \frac{1}{2}w_0\left(\frac{L}{2}\right) = \frac{1}{4}w_0 L$$
$$x_1 = \frac{1}{3}\left(\frac{L}{2}\right) = \frac{L}{6} \qquad x_2 = \frac{L}{2} + \frac{2}{3}\left(\frac{L}{2}\right) = \frac{5}{6}L$$

$$R = \Sigma P = 0 \quad \therefore \text{ Resultant is a couple}$$
$$+\circlearrowleft \quad C^R = \Sigma M_A = -\frac{1}{4}w_0 L\left(\frac{L}{6}\right) + \frac{1}{4}w_0 L\left(\frac{5}{6}L\right) = \frac{1}{6}w_0 L^2 \ \circlearrowleft \quad \blacktriangleleft$$

## 3.50




$$\mathbf{R} = 3900\mathbf{i} - 2000\mathbf{j} \text{ lb} \blacktriangleleft$$

$$+ \circlearrowleft \quad \Sigma M_O = R_y x \qquad - 900(2) - 3000(3) - 2000(2) = -2000x$$

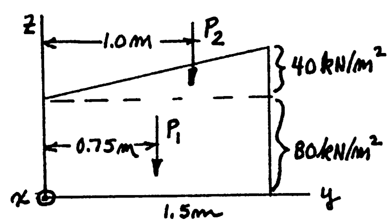
$$x = 7.40 \text{ ft} \blacktriangleleft$$

## 3.51

**By symmetry, $\bar{x} = 0.4$ m.**
**Using the fact that the length of the loading along the x axis is 0.8 m:**

$P_1 = 80(1.5)(0.8) = 96$ kN

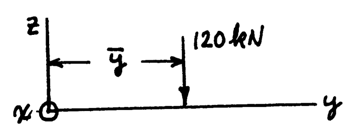
$P_2 = 0.5(40)(1.5)(0.8) = 24$ kN




$R_z = \Sigma F_z$ :

$+\uparrow \; R = -P_1 - P_2 = -96 - 24 = -120$ kN

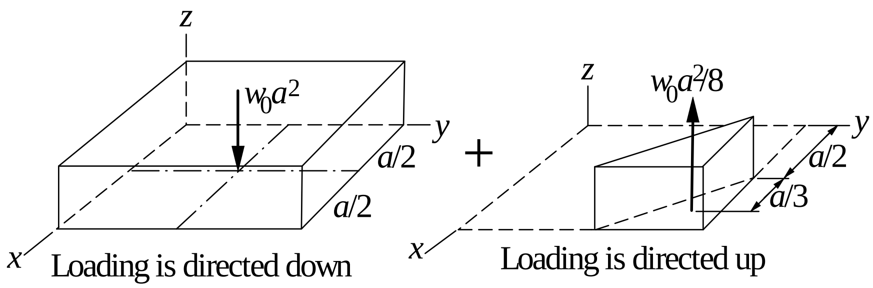
$\Sigma M_x = R\bar{y}$ : $\quad -96(0.75) - 24(1.0) = -120\bar{y}$




**which gives $\bar{y} = 0.800$ m**

**The resultant is the force $\mathbf{R} = -120\mathbf{k}$ kN passing through the point (0.4 m, 0.8 m, 0)**

**3.52**




Use the superposition shown in the figure.

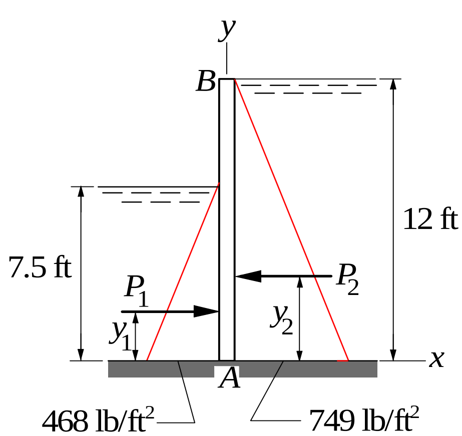
$$\mathbf{R} = \left(-w_0a^2 + \frac{1}{8}w_0a^2\right)\mathbf{k} = -\frac{7}{8}w_0a^2\mathbf{k} \blacktriangleleft$$

Let $(x, y)$ be the point where $\mathbf{R}$ crosses the $xy$-plane. Due to symmetry $x = y$.

$$\Sigma M_y = -Rx \qquad w_0a^2\left(\frac{a}{2}\right) - \frac{1}{8}w_0a^2\left(\frac{a}{2} + \frac{a}{3}\right) = -\left(-\frac{7}{8}w_0a^2x\right)$$

$$x = y = \frac{19}{42}a \blacktriangleleft$$

**3.53**

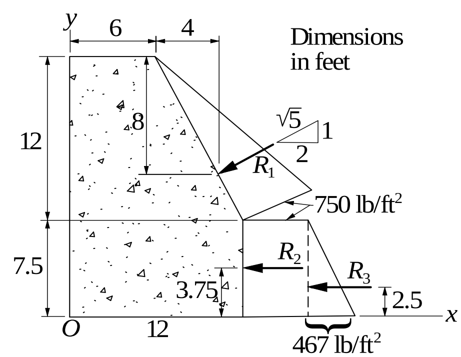



$$P_1 = \frac{1}{2}(468)(7.5)(22) = 38\ 610 \text{ lb} \qquad y_1 = \frac{1}{3}(7.5) = 2.50 \text{ ft}$$

$$P_2 = \frac{1}{2}(749)(12)(22) = 98\ 870 \text{ lb} \qquad y_2 = \frac{1}{3}(12) = 4.0 \text{ ft}$$

$$
\begin{aligned}
\overset{+}{\leftarrow}\ R = P_2 - P_1 = 98\ 870 - 38\ 610 &= 60\ 260 \text{ lb} \ \blacktriangleleft \\
+\circlearrowleft\ \Sigma M_A = R\overline{y} \qquad P_2 y_2 - P_1 y_1 &= R\overline{y} \\
98\ 870(4) - 38\ 610(2.5) &= 60\ 260\overline{y} \\
\overline{y} &= 4.96 \text{ ft} \ \blacktriangleleft
\end{aligned}
$$

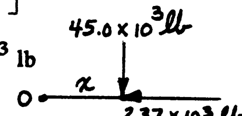
**3.54**




$$
\begin{aligned}
R_1 &= \frac{1}{2}(750)(20)\sqrt{6^2 + 12^2} = 100.6 \times 10^3 \text{ lb} \\
R_2 &= 750(20)(7.5) = 112.5 \times 10^3 \text{ lb} \\
R_3 &= \frac{1}{2}(7.5)(467)(20) = 35.0 \times 10^3 \text{ lb}
\end{aligned}
$$

$\mathbf{R_x = \Sigma F_x}$ :

$$\overset{+}{\rightarrow}\ R_x = -\frac{2}{\sqrt{5}} R_1 - R_2 - R_3 = \left[ -\frac{2}{\sqrt{5}}(100.6) - 112.5 - 35.0 \right] \times 10^3 = -237 \times 10^3 \text{ lb}$$



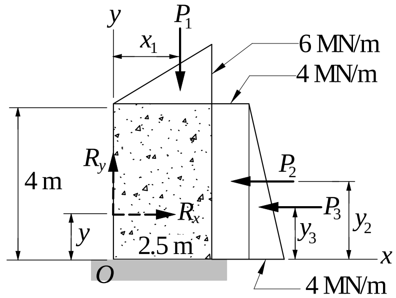

$\mathbf{R_y = \Sigma F_y}$ : $+\uparrow\ R_y = -\frac{1}{\sqrt{5}} R_1 = -\frac{1}{\sqrt{5}}(100.6 \times 10^3) = -45.0 \times 10^3 \text{ lb}$

$$\Sigma M_O = Rd: \quad \circlearrowleft^{+} \quad \frac{1}{\sqrt{5}} R_1(10) - \frac{2}{\sqrt{5}} R_1(11.5) - R_2(3.75) - R_3(2.5) = (45.0 \times 10^3)x$$

$$x = \frac{1}{45}\left[ \frac{1}{\sqrt{5}}(100.6)(10) - \frac{2}{\sqrt{5}}(100.6)(11.5) - 112.5(3.75) - 35.0(2.5) \right] = -24.3 \text{ ft}$$

**The resultant force is $\mathbf{R} = (-237 \times 10^3)\mathbf{i} - (45.0 \times 10^3)\mathbf{j}$ lb acting through the point: (– 24.3 ft, 0).** ◆

## 3.55




$$P_1 = \frac{1}{2}(6)(2.5) = 7.5 \text{ MN} \qquad x_1 = \frac{2}{3}(2.5) = 1.6667 \text{ m}$$
$$P_2 = 4(4) = 16 \text{ MN} \qquad y_2 = \frac{1}{2}(4) = 2 \text{ m}$$
$$P_3 = \frac{1}{2}(4)(4) = 8 \text{ MN} \qquad y_3 = \frac{1}{3}(4) = 1.3333 \text{ m}$$

$$\mathbf{R} = -(P_2 + P_3)\mathbf{i} - P_1\mathbf{j} = -24\mathbf{i} - 7.5\mathbf{j} \text{ MN} \blacktriangleleft$$
$$\Sigma M_O = -R_x y \qquad +\circlearrowleft \quad P_2 y_2 + P_3 y_3 - P_1 x_1 = -R_x y$$
$$16(2) + 8(1.3333) - 7.5(1.6667) = -(-24y)$$
$$y = 1.257 \text{ m} \blacktriangleleft$$

## 3.56

$$R_x = \Sigma F_x = 520\cos 50^\circ + 340\cos 20^\circ - 140 = 513.7 \text{ lb}$$
$$R_y = \Sigma F_y = 520\sin 50^\circ - 340\sin 20^\circ = 282.1 \text{ lb}$$
$$\mathbf{R} = 514\mathbf{i} + 282\mathbf{j} \text{ lb} \blacktriangleleft$$

## 3.57

**$R_x = \Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $0.6P - 0.8Q = 0$ (1)**

**$R_y = \Sigma F_y = 0$: $+\uparrow$ $0.8P - 0.6Q - 20 = 0$ (2)**

**Solving (1) and (2) gives: $P = 57.1$ lb and $Q = 42.9$ lb ◆**

**$C^R = \Sigma M_{origin}$ $\circlearrowleft^{+}$ $50 = C - 0.6P(3) - 0.6Q(4) - 20(2)$**

**$\therefore$ $C = 296$ lb•ft ◆**

## 3.58

$+\uparrow \quad R = \Sigma F_z = -200 - 50 - 150 = -400 \text{ kN} \qquad \therefore \mathbf{R} = -400\mathbf{k} \text{ kN}$ ◄

The coordinates of the point where $R$ crosses the $xy$-plane are

$$x = -\frac{\Sigma M_y}{R} = -\frac{200(2)}{(-400)} = 1.0 \text{ m} \blacktriangleleft$$

$$y = \frac{\Sigma M_x}{R} = \frac{-150(3)}{(-400)} = 1.125 \text{ m} \blacktriangleleft$$

## 3.59

$\mathbf{T}_1 = T_1 \vec{\lambda}_{AB} = T_1(-0.6\mathbf{i} - 0.8\mathbf{j}) \qquad \mathbf{T}_2 = T_2 \vec{\lambda}_{AC} = T_2(-\cos 30^\circ \mathbf{j} + \sin 30^\circ \mathbf{k})$

$\mathbf{T}_3 = T_3 \vec{\lambda}_{AD} = T_3\left(\dfrac{4\mathbf{i} - 8\mathbf{j}}{\sqrt{80}}\right)$

$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + 60\mathbf{j} - 20\mathbf{k} = \mathbf{0}$

**Equating each of the components to zero, we obtain:**

$$\text{(i-component)} \quad -0.6T_1 + \frac{4}{\sqrt{80}}T_3 = 0 \quad (1)$$

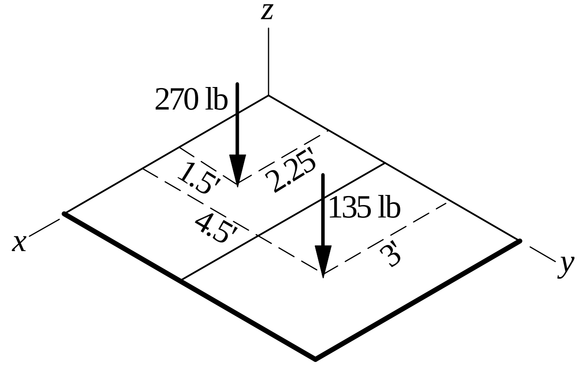
$$\text{(j-component)} \quad -0.8T_1 - \cos 30^\circ T_2 - \frac{8}{\sqrt{80}}T_3 + 60 = 0 \quad (2)$$

$$\text{(k-component)} \quad \sin 30^\circ T_2 - 20 = 0 \quad (3)$$

**Solving (1), (2), and (3) gives:**

$T_1 = 12.68 \text{ kN};\ T_2 = 40.00 \text{ kN};\ T_3 = 17.01 \text{ kN}$ ◆

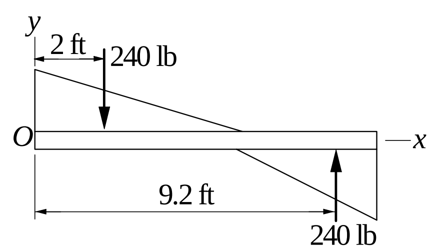
## 3.60



$$\mathbf{R} = (-270 - 135)\,\mathbf{k} = -405\mathbf{k}\text{ lb} \blacktriangleleft$$

$$\begin{aligned} \Sigma M_y &= -Rx \qquad 270(2.25) + 135(3) = -(-405)x \\ x &= 2.50\text{ ft} \blacktriangleleft \\ \Sigma M_x &= Ry \qquad -270(1.5) - 135(4.5) = -405y \\ y &= 2.50\text{ ft} \blacktriangleleft \end{aligned}$$

## 3.61




The resultant is the couple

$$C^R = 240(7.2) = 1728\text{ lb}\cdot\text{ft} \quad \text{CCW} \blacktriangleleft$$

## 3.62

$$\begin{aligned} \mathbf{R} &= -15\mathbf{i} + 18\mathbf{j} + 15\mathbf{k}\text{ kN} \blacktriangleleft \\ \mathbf{C}^R &= \Sigma\mathbf{M}_O = 25(1.2)\mathbf{i} + 30(0.9)\mathbf{j} + 15(1.2)\mathbf{k}\text{ kN}\cdot\text{m} \\ &= 30\mathbf{i} + 27\mathbf{j} + 18\mathbf{k}\text{ kN}\cdot\text{m} \blacktriangleleft \end{aligned}$$

## 3.63

$R_z = \Sigma F_z$: $+\uparrow$ $30 = T_1 + T_2 + T_3 = 6 + 14 + T_3$ $\quad\therefore$ $T_3 = 30 - 20 = 10$ lb ◆

$\Sigma M_x = 0$: $T_1(2) - T_2(3) + T_3 y = 0$

$6(2) - 14(3) + 10y = 0$ $\quad\therefore$ $y = 30/10 = 3.00$ in. ◆

$\Sigma M_y = 0$: $T_1(6) - T_2(2) - T_3 x = 0$

$6(6) - 14(2) - 10x = 0$ $\quad\therefore$ $x = 8/10 = 0.80$ in. ◆

## 3.64

$$R_x = 140\text{ kN} \qquad R_y = 10\text{ kN}$$

Find the $y$-coordinate of the point where the resultant crosses the $y$-axis:

$$\begin{aligned} + \circlearrowleft \quad \Sigma M_O = -R_x y & \qquad -140 - 40(0.4) + 60(0.2) = -140y \\ y &= 1.029 \text{ m} \end{aligned}$$

Resultant force $\mathbf{R} = 140\mathbf{i} + 10\mathbf{j}$ kN intersects the $y$-axis at $y = 1.029$ m. ◄

## 3.65

$$\begin{aligned} R &= \Sigma F_z = -20 - 60 - 40 = -120 \text{ kN} \\ \Sigma M_x &= R_y \qquad 60(4) + 40(4\sin 40^\circ) - 20(4\sin 30^\circ) = -120y \\ y &= -2.52 \text{ m} \blacktriangleleft \\ \Sigma M_y &= -Rx \qquad 40(4\cos 40^\circ) - 20(4\cos 30^\circ) = -(-120x) \\ x &= 0.444 \text{ m} \blacktriangleleft \end{aligned}$$

## 3.66

$$\boldsymbol{\lambda}_{BE} = \frac{50\mathbf{i} + 80\mathbf{j} - 30\mathbf{k}}{\sqrt{50^2 + 80^2 + (-30)^2}} = 0.5051\mathbf{i} + 0.8081\mathbf{j} - 0.3030\mathbf{k}$$

$$\begin{aligned} \mathbf{F}_E &= F_E \boldsymbol{\lambda}_{BE} = 180(0.5051\mathbf{i} + 0.8081\mathbf{j} - 0.3030\mathbf{k}) \\ &= 90.92\mathbf{i} + 145.46\mathbf{j} - 54.54\mathbf{k} \text{ lb} \\ \mathbf{F}_A &= 250\mathbf{k} \text{ lb} \\ \mathbf{C} &= C(-\boldsymbol{\lambda}_{BE}) = -620(0.5051\mathbf{i} + 0.8081\mathbf{j} - 0.3030\mathbf{k}) \\ &= -313.2\mathbf{i} - 501.0\mathbf{j} + 187.9\mathbf{k} \text{ lb} \cdot \text{in.} \end{aligned}$$

$$\begin{aligned} \mathbf{R} &= \mathbf{F}_A + \mathbf{F}_E = 250\mathbf{k} + (90.92\mathbf{i} + 145.46\mathbf{j} - 54.54\mathbf{k}) \\ &= 90.9\mathbf{i} + 145.5\mathbf{j} + 195.5\mathbf{k} \text{ lb} \blacktriangleleft \\ \mathbf{C}^R &= \Sigma \mathbf{M}_D = \mathbf{C} + \mathbf{r}_{DE} \times \mathbf{F}_E \\ &= -313.2\mathbf{i} - 501\mathbf{j} + 187.9\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 0 \\ 90.92 & 145.46 & -54.54 \end{vmatrix} \\ &= -4676\mathbf{i} - 501\mathbf{j} - 7086\mathbf{k} \text{ lb} \cdot \text{in.} \blacktriangleleft \end{aligned}$$

**3.67**

**(a)**

$$+ \quad \rightarrow \quad R_x = \Sigma F_x = 260 - \frac{3}{5}(200) - \frac{4}{5}(120) = 44 \text{ lb}$$
$$+ \quad \uparrow \quad R_y = \Sigma F_y = \frac{4}{5}(200) - \frac{3}{5}(120) = 88 \text{ lb}$$
$$+ \quad \circlearrowleft \quad C^R = \Sigma M_O = 600 + 200(2.5) = 1100 \text{ lb} \cdot \text{ft}$$

The equivalent force-couple system is

$$\mathbf{R} = 44\mathbf{i} + 88\mathbf{j} \text{ lb acting at } O \qquad C^R = 1100 \text{ lb} \cdot \text{ft CCW} \blacktriangleleft$$

**(b)**

$$\mathbf{R} = 44\mathbf{i} + 88\mathbf{j} \text{ lb as in Part (a)}$$
$$+ \quad \circlearrowleft \quad C^R = \Sigma M_A = 600 + 260(3) - 200(2.5) = 880 \text{ lb} \cdot \text{ft}$$

The equivalent force-couple system is

$$\mathbf{R} = 44\mathbf{i} + 88\mathbf{j} \text{ lb acting at } A \qquad C^R = 880 \text{ lb} \cdot \text{ft CCW} \blacktriangleleft$$

$$-96.0z_D\mathbf{i} - 72.0z_D\mathbf{j} + (96.0x_D - 144.0)\mathbf{k} = -69.12\mathbf{i} - 51.84\mathbf{j} + 144.0\mathbf{k}$$

Equating **i** and **k** components:

$$-96.0z_D = -69.12 \qquad z_D = 0.720 \text{ m}$$
$$(96.0x_D - 144.0) = 144.0 \qquad x_D = 3.00 \text{ m}$$

Wrench consists of the force $P = -72.0\mathbf{i} + 96.0\mathbf{j}$ N and the couple $C_t^R = -38.88\mathbf{i} + 51.84\mathbf{j}$ N·m; the axis of the wrench passes through the point (3.00, 0, 0.720) m. ◀

## 3.68

$$+ \rightarrow \quad R_x = \Sigma F_x = 300\cos 60^\circ + 100 + 200\cos 30^\circ = 423.2 \text{ lb}$$
$$+ \uparrow \quad R_y = \Sigma F_y = 300\sin 60^\circ - 200\sin 30^\circ = 159.8 \text{ lb}$$

$$R = \sqrt{423.2^2 + 159.8^2} = 452 \text{ lb} \blacktriangleleft$$

## 3.69

$$\begin{aligned}\mathbf{P} &= 120\boldsymbol{\lambda}_{AC} = 120\frac{-1.5\mathbf{i} + 2\mathbf{j}}{2.5} = -72.0\mathbf{i} + 96.0\mathbf{j} \text{ N} \\ \mathbf{C}^R &= 180\boldsymbol{\lambda}_{AB} = 180\frac{-1.5\mathbf{i} + 2\mathbf{k}}{2.5} = -108.0\mathbf{i} + 144.0\mathbf{k} \text{ N}\cdot\text{m}\end{aligned}$$

Component of $\mathbf{C}^R$ in direction of $\boldsymbol{\lambda}_{AC}$ is

$$\begin{aligned}C_t^R &= \mathbf{C}^R \cdot \boldsymbol{\lambda}_{AC} = (-108.0\mathbf{i} + 144.0\mathbf{k}) \cdot (-0.6\mathbf{i} + 0.8\mathbf{j}) = 64.80 \text{ N}\cdot\text{m} \\ \mathbf{C}_t^R &= C_t^R\boldsymbol{\lambda}_{AC} = 64.80(-0.6\mathbf{i} + 0.8\mathbf{j}) = -38.88\mathbf{i} + 51.84\mathbf{j} \text{ N}\cdot\text{m}\end{aligned}$$

Component of $\mathbf{C}^R$ normal to $\boldsymbol{\lambda}_{AC}$ is

$$\begin{aligned}\mathbf{C}_n^R &= \mathbf{C}^R - \mathbf{C}_t^R = (-108.0\mathbf{i} + 144.0\mathbf{k}\ ) - (-38.88\mathbf{i} + 51.84\mathbf{j}) \\ &= -69.12\mathbf{i} - 51.84\mathbf{j} + 144.0\mathbf{k} \text{ N}\cdot\text{m}\end{aligned}$$

Let $D$ be the point where the axis of the wrench crosses the $xz$-plane.

$$\begin{aligned}\mathbf{r}_{AD} &= (x_D - 1.5)\mathbf{i} + z_D\mathbf{k} \text{ m} \\ \mathbf{r}_{AC} \times \mathbf{P} &= \mathbf{C}_n^R \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_D - 1.5 & 0 & z_D \\ -72.0 & 96.0 & 0 \end{vmatrix} = -69.12\mathbf{i} - 51.84\mathbf{j} + 144.0\mathbf{k}\end{aligned}$$

$$-96.0z_D\mathbf{i} - 72.0z_D\mathbf{j} + (96.0x_D - 144.0)\mathbf{k} = -69.12\mathbf{i} - 51.84\mathbf{j} + 144.0\mathbf{k}$$

Equating **i** and **k** components:

$$\begin{aligned}-96.0z_D &= -69.12 \qquad z_D = 0.720 \text{ m} \\ (96.0x_D - 144.0) &= 144.0 \qquad x_D = 3.00 \text{ m}\end{aligned}$$

Wrench consists of the force $P = -72.0\mathbf{i} + 96.0\mathbf{j}$ N and the couple $C_t^R = -38.88\mathbf{i} + 51.84\mathbf{j}$ N·m; the axis of the wrench passes through the point (3.00, 0, 0.720) m. ◀

**3.70**

$$\begin{aligned}
\mathbf{T}_1 &= T_1\boldsymbol{\lambda}_1 = T_1\frac{3\mathbf{i}-2\mathbf{j}-8\mathbf{k}}{\sqrt{77}} = T_1(0.3419\mathbf{i}-0.2279\mathbf{j}-0.9117\mathbf{k}) \\
\mathbf{T}_2 &= T_2\boldsymbol{\lambda}_2 = T_2\frac{3\mathbf{j}-8\mathbf{k}}{\sqrt{73}} = T_2(0.3511\mathbf{j}-0.9363\mathbf{k}) \\
\mathbf{T}_3 &= T_3\boldsymbol{\lambda}_3 = 500\frac{-4\mathbf{i}-8\mathbf{k}}{\sqrt{80}} = -223.6\mathbf{i}-447.2\mathbf{k}\ \text{N}
\end{aligned}$$

$$\mathbf{R} = R\mathbf{k} = \Sigma\mathbf{T}_i$$

Equating like components:

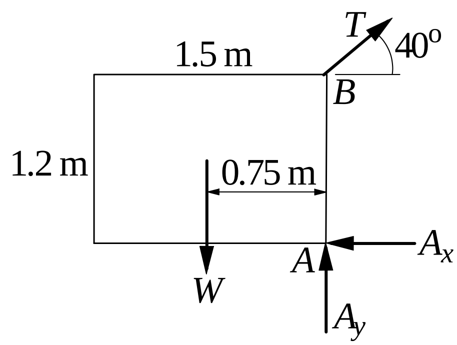
$$\begin{aligned}
0 &= 0.3419T_1 - 223.6 \\
0 &= -0.2279T_1 + 0.3511T_2 \\
R &= -0.9117T_1 - 0.9363T_2 - 447.2
\end{aligned}$$

Solution is

$T_1 = 654$ N ◀ $T_2 = 425$ N ◀ $R = -1441$ N ◀

# Chapter 4

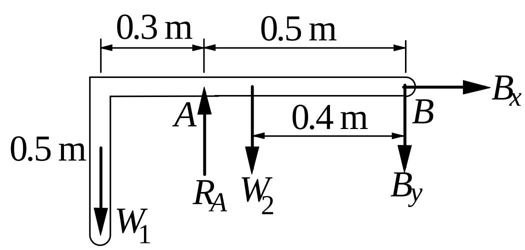
## 4.1




$$W = 30(9.81) = 294.3 \text{ N}$$

Three unknowns: $T$, $A_x$, $A_y$ ◀

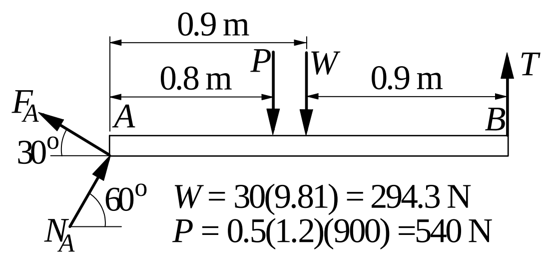
## 4.2



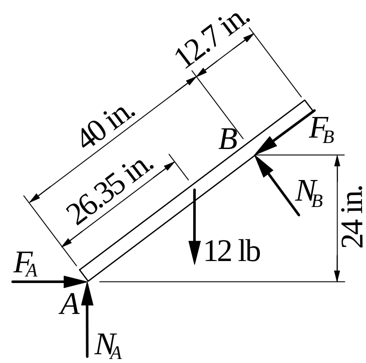

$$W_1 = \frac{0.5}{1.3}(30)(9.81) = 113.19 \text{ N}$$
$$W_2 = \frac{0.8}{1.3}(30)(9.81) = 181.11 \text{ N}$$

Three unknowns: $R_A$, $B_x$, $B_y$ ◀

**4.3**



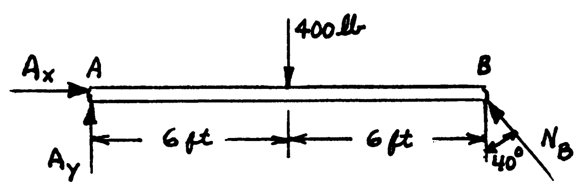

**4.4**




4 unknowns ($N_A, F_A, N_B$ and $F_B$) ◄

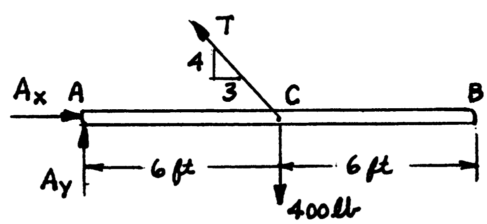
**4.5**

**(a)**



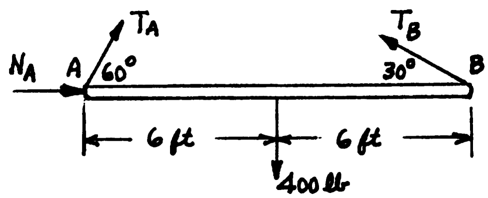

**3 unknowns** ◆
**($A_x$, $A_y$, $N_B$)**

**(b)**



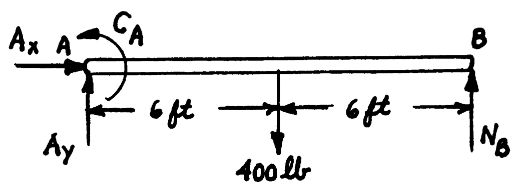

**3 unknowns** ◆
**($A_x$, $A_y$, T)**

(c)




3 unknowns ◆
($N_A$, $T_A$, $T_B$)

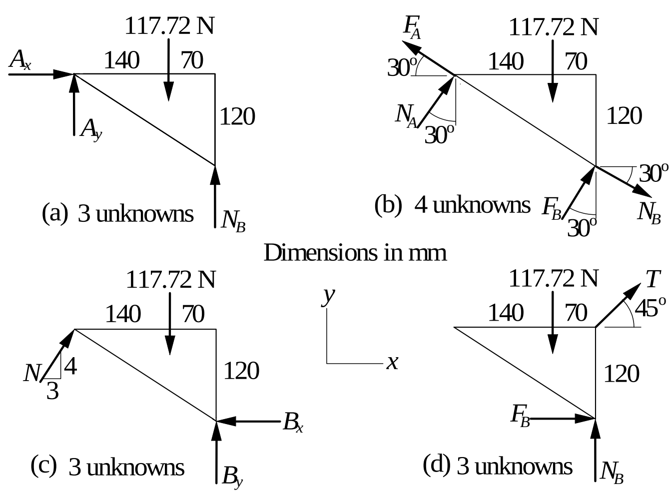
(d)



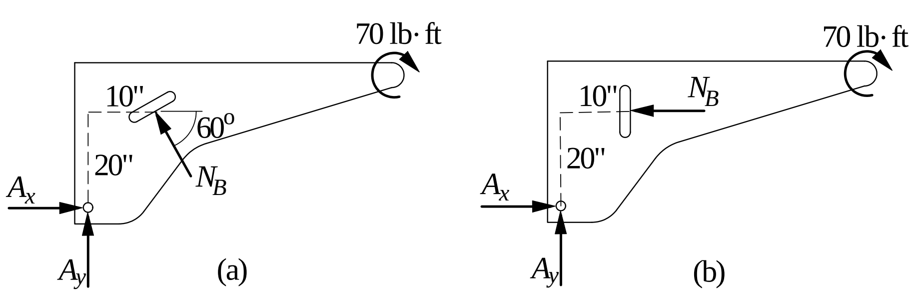

4 unknowns ◆
($A_x$, $A_y$, $N_B$, $C_A$)

## 4.6

Weight of the plate is $W = 12(9.81) = 117.72$ N

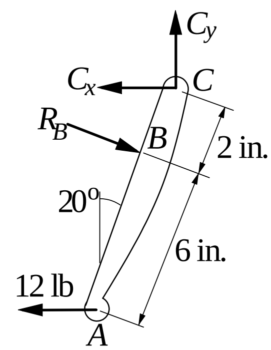



## 4.7



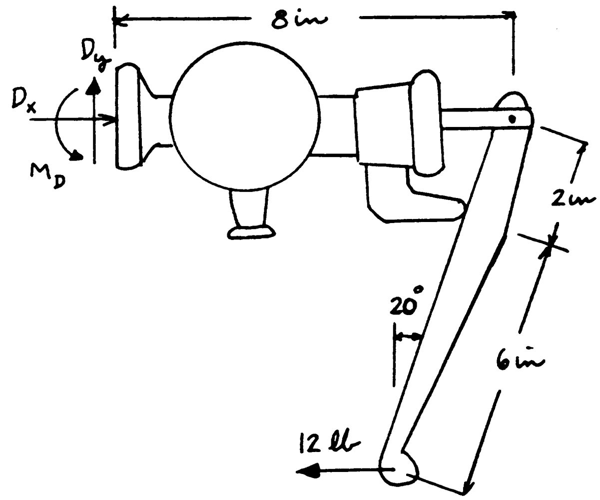
**(a)** 3 unknowns ($A_x, A_y$ and $N_B$) ◄
**(b)** 3 unknowns ($A_x, A_y$ and $N_B$) ◄

**4.8**



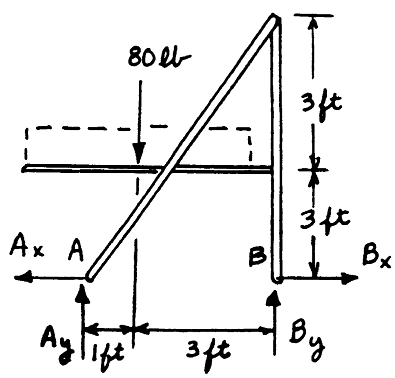

Three unknowns: $R_B$, $C_x$, $C_y$ ◄

**4.9**



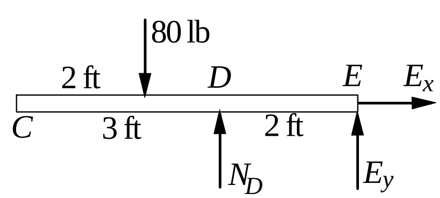

**3 unknowns** ◆
**($D_x$, $D_y$, $M_D$)**

**4.10**




**4 unknowns** ◆
**($A_x$, $A_y$, $B_x$, $B_y$)**

**4.11**




3 unknowns ( $N_D$, $E_x$, $E_y$) ◄

## 4.12

**Refer to the given FBD. Let the x-axis be parallel to the inclined plane and the y-axis be perpendicular to the inclined plane.**

$\Sigma F_x = 0$: $+\nearrow$ $N_A - W\sin\theta = 0$ $\qquad N_A = W\sin\theta \qquad (1)$

$\Sigma F_y = 0$: $\nwarrow +$ $N_B - W\cos\theta = 0$ $\qquad N_B = W\cos\theta \qquad (2)$

Dividing (1) by (2): $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{N_A}{N_B}$. Using $N_B = 1.5\,N_A$, $\theta = \tan^{-1}(1/1.5) = 33.7^\circ$ ♦

$N_A = W\sin 33.7^\circ = 0.555\,W$ ♦ $\qquad N_B = W\cos 33.7^\circ = 0.832\,W$ ♦

## 4.13

Refer to the given FBD.

$$\begin{aligned} \Sigma M_A &= 0 \qquad +\circlearrowleft \quad 4P - 120(4\sin 15^\circ) = 0 \\ P &= 120\sin 15^\circ = 31.1 \text{ lb} \ \blacktriangleleft \end{aligned}$$

## 4.14

Refer to the given FBD.

$$\begin{aligned} \Sigma M_O &= 0 \qquad +\circlearrowleft \ 18T_C - 18T_B = 0 \\ \Sigma F_y &= 0 \qquad +\uparrow \ T_B\cos 25^\circ + T_C - 60 = 0 \\ \Sigma F_x &= 0 \qquad +\longrightarrow \ F - T_B\sin 25^\circ = 0 \end{aligned}$$

Solution is

$$T_B = T_C = 31.5 \text{ lb} \qquad F = 13.30 \text{ lb} \ \blacktriangleleft$$

## 4.15

Refer to the given FBD.

$$\begin{aligned} \text{Weight of boom} &= 180(9.81) = 1766 \text{ N} = 1.766 \text{ kN} \\ \text{Weight of load} &= 320(9.81) = 3139 \text{ N} = 3.139 \text{ kN} \end{aligned}$$

$$\begin{aligned} \Sigma M_A &= 0 \qquad +\circlearrowleft \ T(2\sin 30^\circ) - 1.766(2\cos 30^\circ) - 3.139(4\cos 30^\circ) = 0 \\ T &= 13.933 \text{ kN} \ \blacktriangleleft \\ \Sigma F_x &= 0 \qquad +\rightarrow \ A_x - T = 0 \qquad A_x = T = 13.933 \text{ kN} \\ \Sigma F_y &= 0 \qquad +\uparrow \ A_y - 1.766 - 3.139 = 0 \qquad A_y = 4.905 \text{ kN} \end{aligned}$$

$$A = \sqrt{13.933^2 + 4.905^2} = 14.77 \text{ kN} \ \blacktriangleleft$$

## 4.16

Refer to the given FBD.

$$\Sigma M_A = 0 \qquad +\circlearrowleft \; C_A + 0.6(10\sin 15^\circ) + 0.6(20\sin 15^\circ - 10)$$
$$+1.8(20\sin 15^\circ - 24) = 0$$
$$C_A = 35.2 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

## 4.17

Refer to the given FBD.

$$\Sigma M_A = 0 \qquad +\circlearrowleft \; 0.6(10\sin\theta) + 0.6(20\sin\theta - 10) + 1.8(20\sin\theta - 24) = 0$$
$$54\sin\theta - 49.2 = 0 \qquad \theta = 65.7^\circ \blacktriangleleft$$

## 4.18

Refer to the given FBD.

$$\begin{aligned}
\Sigma M_B &= 0 \qquad +\circlearrowleft \; 1.2(0.25) - N_C(0.25) = 0 \\
\Sigma F_x &= 0 \qquad +\rightarrow \; -B_x + \frac{3}{5}N_C = 0 \\
\Sigma F_y &= 0 \qquad +\uparrow \; B_y - 1.2 - \frac{4}{5}N_C = 0
\end{aligned}$$

Solution is

$$\begin{aligned}
B_x &= 0.72 \text{ kN} \qquad B_y = 2.16 \text{ kN} \qquad N_C = 1.2 \text{ kN} \blacktriangleleft \\
R_B &= \sqrt{0.72^2 + 2.16^2} = 2.28 \text{ kN} \blacktriangleleft
\end{aligned}$$

## 4.19

Refer to the given FBD.

$$\Sigma M_A = 0 \qquad +\circlearrowleft \; \frac{12}{\sqrt{18^2+12^2}}T(18) - 300(9.6) + C_A = 0$$
$$C_A = 2880 - 9.985T$$

$$\begin{aligned}
\Sigma F_x &= 0 \qquad +\longrightarrow \; A_x - \frac{12}{\sqrt{18^2+12^2}}T = 0 \qquad A_x = 0.5547T \\
\Sigma F_y &= 0 \qquad +\uparrow \; A_y - 300 - \frac{18}{\sqrt{18^2+12^2}}T = 0 \\
& A_y = 300 + 0.8321T
\end{aligned}$$

**(a)** With $T = 490$ lb:

$$\begin{aligned}
C_A &= 2880 - 9.985(490) = -2013 \text{ lb}\cdot\text{ft} \blacktriangleleft \\
A_x &= 0.5547(490) = 272 \text{ lb} \blacktriangleleft \qquad A_y = 300 + 0.8321(490) = 708 \text{ lb} \blacktriangleleft
\end{aligned}$$

**(b)** With $T = 0$:

$$C_A = 2880 \text{ lb} \cdot \text{ft} \blacktriangleleft \qquad A_x = 0 \blacktriangleleft \qquad A_y = 300 \text{ lb} \blacktriangleleft$$

## 4.20

Refer to the given FBD.

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad 400(2) - 35(9.81)(3\cos\theta) = 0 \quad \theta = 39.0^\circ \blacktriangleleft$$

## 4.21

Refer to the given FBD.

$$\begin{aligned}
\Sigma F_x &= 0 \quad +\nearrow \; P - 1200\sin 10^\circ = 0 \\
\Sigma F_y &= 0 \quad +\nwarrow \; N_A + N_B - 1200\cos 10^\circ = 0 \\
\Sigma M_A &= 0 \quad +\circlearrowleft \; 5N_B - 3.6P + 1200(-1.5\cos 10^\circ + 1.8\sin 10^\circ) = 0
\end{aligned}$$

Solution is

$$P = 208 \text{ lb} \blacktriangleleft \qquad N_A = 752 \text{ lb} \blacktriangleleft \qquad N_B = 430 \text{ lb} \blacktriangleleft$$

## 4.22

Refer to the given FBD.

$$\begin{aligned}
\Sigma M_A &= 0 \quad +\circlearrowleft \quad T(3 + 2.5\sin 70^\circ) - 400(1.5) = 0 \\
T &= 112.17 \text{ N} \blacktriangleleft \\
\Sigma F_x &= 0 \quad +\longrightarrow \quad A_x + 112.17\cos 70^\circ = 0 \qquad A_x = -38.4 \text{ N} \blacktriangleleft \\
\Sigma F_y &= 0 \quad +\uparrow \quad A_y + 112.17(1 + \sin 70^\circ) - 400 = 0 \\
A_y &= 182.4 \text{ N} \blacktriangleleft
\end{aligned}$$

## 4.23

Refer to the given FBD.

$$\begin{aligned}
\Sigma M_D &= 0 \quad +\circlearrowleft \quad 850b + (388\sin 30^\circ)(0.6 - b) - (388\cos 30^\circ)(1.2) = 0 \\
b &= 0.437 \text{ m} \blacktriangleleft
\end{aligned}$$

## 4.24

Refer to the given FBD.

$$\begin{aligned}
\Sigma M_B &= 0 \quad +\circlearrowright \quad 8T_2 - 340(4) = 0 \qquad T_2 = 170 \text{ lb} \blacktriangleleft \\
\Sigma M_A &= 0 \quad +\circlearrowright \quad \frac{3}{5}T_3(8) - 340(4) = 0 \qquad T_3 = 283.3 \text{ lb} \blacktriangleleft \\
\Sigma F_x &= 0 \quad +\longleftarrow \quad T_1 - \frac{4}{5}(283.3) = 0 \qquad T_1 = 226.6 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 4.25

Refer to the given FBD. Let the y-axis be vertical.

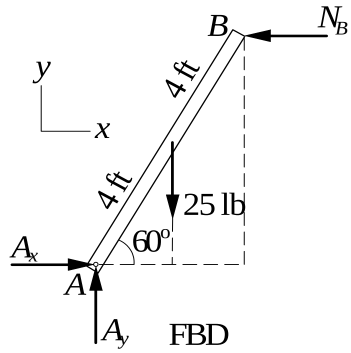
$\Sigma F_y = 0$: $+\uparrow$ $2N_B + N_B - \frac{W}{5} - \frac{4W}{5} - 6000 = 0$ $\therefore$ $3N_B = 6000 + W$ lb (1)

$\Sigma M_B = 0$: (+ counterclockwise) $\frac{W}{5}(248) + \frac{4W}{5}(152) + 6000(60) - N_A(200) = 0$

Using $N_A = 2N_B$, and simplifying: $N_B = 0.4280W + 900$ lb (2)

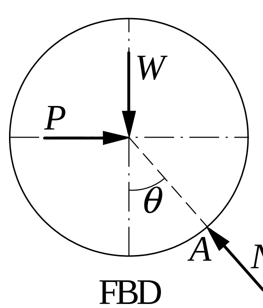
Solving (1) and (2) gives: $W = 11\,620$ lb ♦

## 4.26




$$
\begin{aligned}
\Sigma M_A &= 0 \quad +\circlearrowleft \quad N_B(8\sin 60^\circ) - 25(4\cos 60^\circ) = 0 \\
&\phantom{=} N_B = 7.217 \text{ lb} \;\blacktriangleleft \\
\Sigma F_x &= 0 \quad +\longrightarrow \quad A_x - 7.217 = 0 \qquad A_x = 7.217 \text{ lb} \\
\Sigma F_y &= 0 \quad +\uparrow \quad A_y - 25 = 0 \qquad A_y = 25 \text{ lb} \\
A &= \sqrt{7.217^2 + 25^2} = 26.0 \text{ lb} \;\blacktriangleleft
\end{aligned}
$$

## 4.27

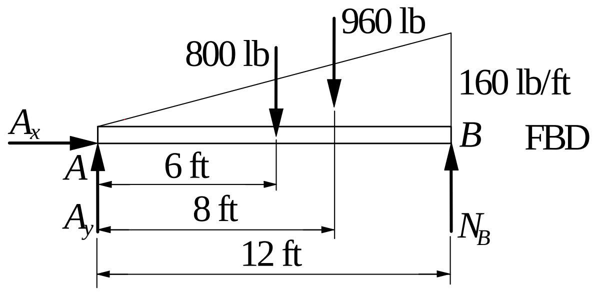


$$\begin{aligned} \Sigma F_x &= 0 \qquad +\rightarrow\ P - N\sin\theta = 0 \\ \Sigma F_y &= 0 \qquad +\uparrow\ N\cos\theta - W = 0 \end{aligned}$$

Solution is

$$N = \frac{W}{\cos\theta} \qquad P = W\tan\theta \ \blacktriangleleft$$

**4.28**




Resultant of distributed load is $\frac{1}{2}(160)(12) = 960$ lb

$$\begin{aligned} \Sigma M_A &= 0 \qquad +\circlearrowleft \quad 12N_B - 800(6) - 960(8) = 0 \qquad N_B = 1040 \text{ lb} \ \blacktriangleleft \\ \Sigma F_x &= 0 \qquad A_x = 0 \ \blacktriangleleft \\ \Sigma F_y &= 0 \qquad +\uparrow \quad A_y + 1040 - 800 - 960 = 0 \qquad A_y = 720 \text{ lb} \ \blacktriangleleft \end{aligned}$$

**4.29**

Weight of the bar is $40(9.81) = 392.4$ N

$$\Sigma F_x = 0 \qquad +\rightarrow \quad T - N_B\sin 30^\circ = 0 \qquad N_B = 2T$$

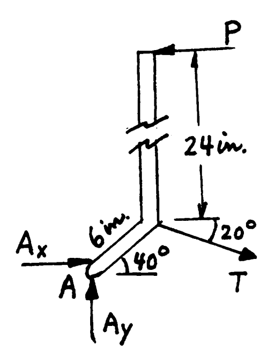
$$\begin{aligned} \Sigma M_A = 0 \qquad +\circlearrowleft \quad T(3.25\sin 30^\circ) &- N_B(2.0) + 392.4(1.625\cos 30^\circ) = 0 \\ 1.625T - 2.0N_B + 552.2 &= 0 \\ 1.625T - 4.0T + 552.2 &= 0 \qquad T = 233 \text{ N} \ \blacktriangleleft \end{aligned}$$

**4.30**

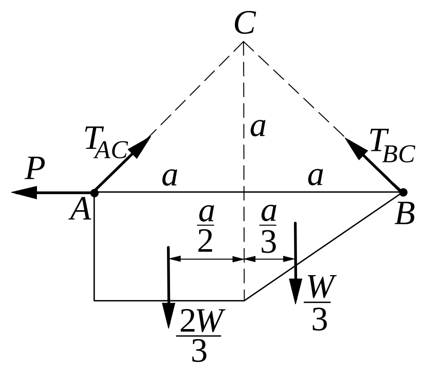
$\Sigma M_A = 0$:

$$\overset{+}{\circlearrowleft}\ P(6\sin 40^\circ + 24) - T\cos 20^\circ(6\sin 40^\circ) - T\sin 20^\circ(6\cos 40^\circ) = 0$$

$$\therefore\ T = \frac{(6\sin 40^\circ + 24)P}{6(\sin 40^\circ\cos 20^\circ + \cos 40^\circ\sin 20^\circ)} = 5.36P \ \blacklozenge$$



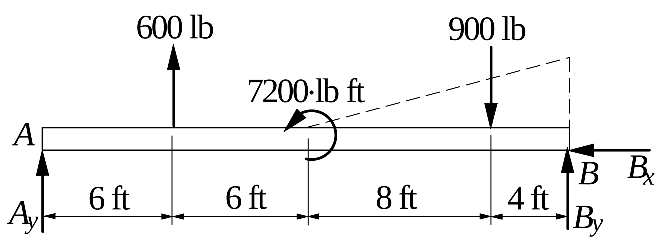
**4.31**




$$\Sigma M_C = 0 \qquad +\circlearrowleft \ \frac{2W}{3}\frac{a}{2} - \frac{W}{3}\frac{a}{3} - Pa = 0$$

$$P = \frac{2}{9}W \ ◂$$

**4.32**




$$\Sigma F_x = 0 \qquad +\leftarrow \ B_x = 0$$

$$\Sigma M_B = 0 \qquad +\circlearrowright \ 24A_y + 600(18) - 900(4) - 7200 = 0 \qquad A_y = 0$$

$$\Sigma F_y = 0 \qquad B_y + 600 - 900 = 0 \qquad B_y = 300 \text{ lb}$$

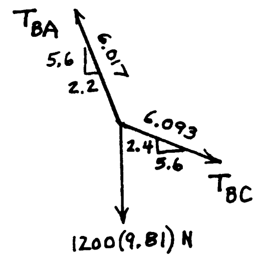
The reactions are: $R_A = 0$ ◂ $R_B = 300 \text{ lb} \uparrow$ ◂

## 4.33

$\Sigma F_x = 0$:

$$\overset{+}{\rightarrow} \quad \frac{5.6}{6.093} T_{BC} - \frac{2.2}{6.017} T_{BA} = 0$$

$$\therefore \quad T_{BC} = 0.3978\, T_{BA} \qquad (1)$$




$\Sigma F_y = 0$:

$$+\uparrow \quad \frac{5.6}{6.017} T_{BA} - \frac{2.4}{6.093} T_{BC} - 1200(9.81) = 0 \qquad (2)$$

Solving (1) and (2) gives: $T_{BA} = 15\,210$ N ♦ ; $T_{BC} = 6050$ N ♦

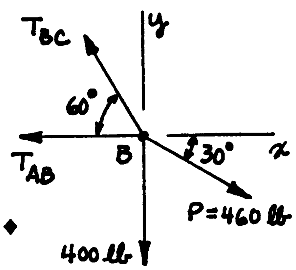
## 4.34

$\Sigma F_y = 0$: $+\uparrow$ $T_{BC}\sin 60^\circ - 400 - 460\sin 30^\circ = 0$

$$\therefore \quad T_{BC} = 727.46 \text{ lb} \quad ♦$$

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $460\cos 30^\circ - T_{AB} - T_{BC}\cos 60^\circ = 0$

$$\therefore \quad T_{AB} = 460\cos 30^\circ - 727.46\cos 60^\circ = 34.6 \text{ lb} \quad ♦$$




## 4.35

$\Sigma F_y = 0$: $+\uparrow$ $T_{BC}\sin 60^\circ - 400 - P\sin 30^\circ = 0$

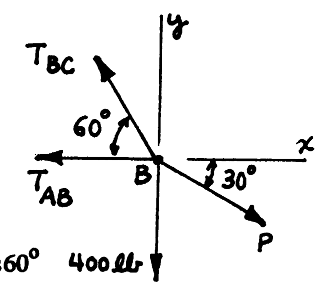
$$\therefore \quad T_{BC} = 461.9 + 0.5774P \text{ lb}$$

Note that $T_{BC}$ is always positive.

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P\cos 30^\circ - T_{AB} - T_{BC}\cos 60^\circ = 0$

$$\therefore \quad T_{AB} = P\cos 30^\circ - (461.9 + 0.5774P)\cos 60^\circ$$

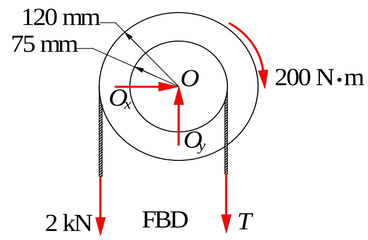
$$T_{AB} = -231.0 + 0.5773P \text{ lb}$$




Note that $T_{AB}$ is positive only if $P > \dfrac{231.0}{0.5773} = 400$ lb

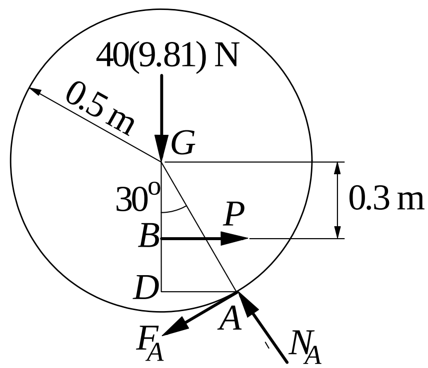
Therefore, $P_{min} = 400$ lb ♦

**4.36**




$$\Sigma M_O = 0 \qquad + \circlearrowleft \quad 0.075T + 200 - 2000(0.12) = 0 \qquad T = 533 \text{ N} \blacktriangleleft$$

**4.37**




**(a)**

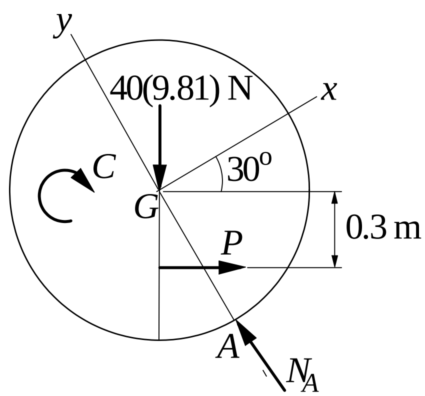
$$\Sigma M_A = 0 \qquad + \circlearrowleft \quad 40(9.81)\left(\overline{DA}\right) - P\left(\overline{BD}\right) = 0$$

$$\overline{DA} = 0.5 \sin 30^\circ = 0.25 \text{ m}$$
$$\overline{BD} = 0.5 \cos 30^\circ - 0.3 = 0.133\ 01 \text{ m}$$

$$40(9.81)(0.25) - P\,(0.133\ 01) = 0 \qquad P = 738 \text{ N} \blacktriangleleft$$

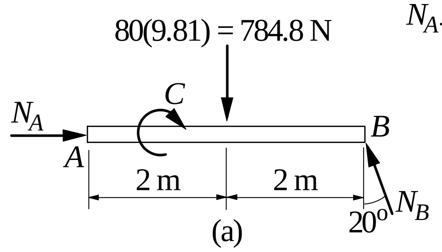
(b) If the inclined surface is smooth ($F_A = 0$), the equation $\Sigma M_G = 0$ cannot be satisfied. Hence equilbrium is not possible. ◀

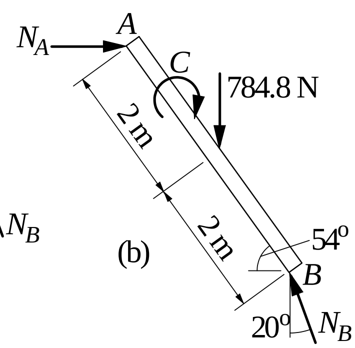
**4.38**




$$\Sigma F_x = 0 \qquad +\nearrow \; P\cos 30^\circ - 40(9.81)\sin 30^\circ = 0 \qquad P = 226.6 \text{ N} \blacktriangleleft$$
$$\Sigma M_G = 0 \qquad +\circlearrowleft \; P(0.3) - C = 0 \qquad 226.6(0.3) - C = 0$$
$$C = 68.0 \text{ N}\cdot\text{m} \blacktriangleleft$$

**4.39**



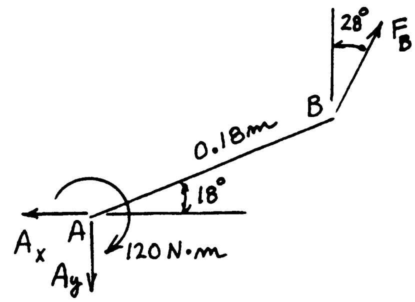

**(a)**

$$\Sigma M_B = 0 \qquad +\circlearrowleft \; 784.8(2) - C = 0 \qquad C = 1570 \text{ N}\cdot\text{m}$$

**(b)**

$$\Sigma F_y = 0 \qquad +\uparrow \; N_B\cos 20^\circ - 784.8 = 0 \qquad N_B = 835.2 \text{ N}$$
$$\Sigma F_x = 0 \qquad +\rightarrow \; N_A - N_B\sin 20^\circ = 0 \qquad N_A - 835.2\sin 20^\circ = 0$$
$$N_A = 285.7 \text{ N}$$
$$\Sigma M_B = 0 \qquad +\circlearrowright \; C + N_A(4\sin 54^\circ) - 784.8(2\cos 54^\circ) = 0$$
$$C + 285.7(4\sin 54^\circ) - 784.8(2\cos 54^\circ) = 0$$
$$C = -1.96 \text{ N}\cdot\text{m} \blacktriangleleft$$

4.40




$\Sigma M_A = 0$:

(+↺) $F_B \cos 28^o(0.18 \cos 18^o)$

$- F_B \sin 28^o(0.18 \sin 18^o) - 120 = 0$

$\therefore\ F_B = 959.7$ N ◆

$\Sigma F_x = 0$:

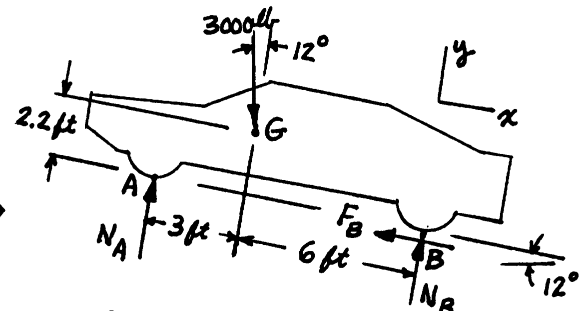
$\overset{+}{\rightarrow}$ $- A_x + 959.7 \sin 28^o = 0$ gives: $A_x = 450.6$ N

$\Sigma F_y = 0$:

+↑ $- A_y + 959.7 \cos 28^o = 0$ gives: $A_y = 847.4$ N

$\therefore\ R_A = \sqrt{450.6^2 + 847.4^2} = 960$ N ◆

4.41




(a) $\Sigma M_B = 0$:

(+↺) $3000 \cos 12^o(6)$

$- 3000 \sin 12^o(2.2)$

$- N_A(9) = 0$

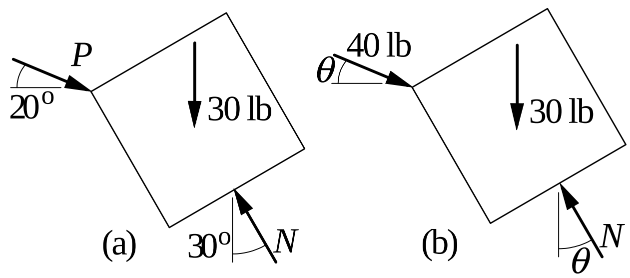
$\therefore\ N_A = 1803.8$ lb ◆

$\Sigma F_y = 0$:

$N_B = 3000 \cos 12^o - N_A = 3000 \cos 12^o - 1803.8 = 1130.6$ lb ◆

(b) $\Sigma F_x = 0$: $F_B = 3000 \sin 12^o = 624$ lb ◆

**4.42**




**(a)**

$$\Sigma F_x = 0 \qquad +\rightarrow \; P\cos 20^\circ - N\sin 30^\circ = 0 \qquad N = \frac{P\cos 20^\circ}{\sin 30^\circ} = 1.8794P$$

$$\Sigma F_y = 0 \qquad +\uparrow \; N\cos 30^\circ - P\sin 20^\circ - 30 = 0$$

$$(1.8794P)\cos 30^\circ - P\sin 20^\circ - 30 = 0 \qquad P = 23.3 \text{ lb} \;◄$$

**(b)**

$$\Sigma F_x = 0 \qquad +\rightarrow \; 40\cos\theta - N\sin 30^\circ = 0 \qquad N = \frac{40\cos\theta}{\sin 30^\circ} = 80\cos\theta$$

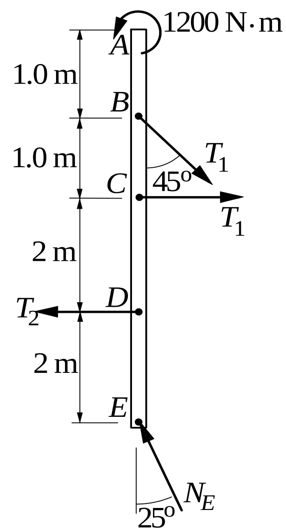
$$\Sigma F_y = 0 \qquad +\uparrow \; N\cos 30^\circ - 40\sin\theta - 30 = 0$$

$$80\cos\theta\cos 30^\circ - 40\sin\theta - 30 = 0$$

$$69.282\cos\theta - 40\sin\theta - 30 = 0$$

Solving numerically: $\theta = 38.0^\circ$ ◄

**4.43**




$$\Sigma F_y = 0 \qquad +\uparrow \; N_E \cos 25^\circ - T_1 \cos 45^\circ = 0$$

$$N_E = \frac{T_1 \cos 45^\circ}{\cos 25^\circ} = 0.7802 T_1$$

$$\Sigma M_D = 0 \qquad +\circlearrowleft \; 1200 - (T_1 \sin 45^\circ)(3) - T_1(2) - (N_E \sin 25^\circ)(2) = 0$$

$$1200 - 4.121 T_1 - 0.8452 N_E = 0$$

$$1200 - 4.121 T_1 - 0.8452(0.7802 T_1) = 0 \qquad T_1 = 251.0 \text{ N} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad +\rightarrow \; T_1(1 + \sin 45^\circ) - T_2 - N_E \sin 25^\circ = 0$$

$$1.7071 T_1 - T_2 - 0.4226 N_E = 0$$

$$1.7071(251.0) - T_2 - 0.4226(0.7802 \times 251.0) = 0$$

$$T_2 = 346 \text{ N} \blacktriangleleft$$

## 4.44

$\Sigma M_A = 0$:

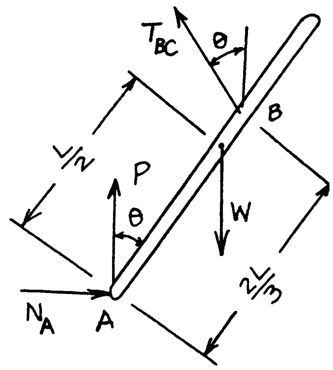
$$\circlearrowleft+\; T_{BC}\sin\theta\left(\frac{2L}{3}\cos\theta\right) + T_{BC}\cos\theta\left(\frac{2L}{3}\sin\theta\right) - W\left(\frac{L}{2}\sin\theta\right) = 0$$

which simplifies as $T_{BC} = \dfrac{3W}{8\cos\theta}$

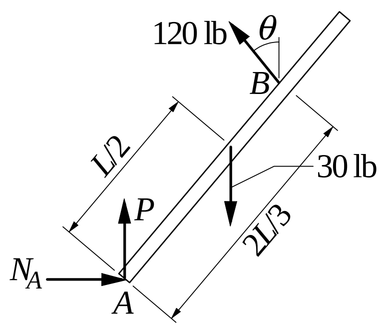
$\Sigma F_y = 0$: $+\uparrow$ $P + T_{BC}\cos\theta - W = 0$

$$P = W - \frac{3W}{8} = \frac{5W}{8}$$

$\therefore$ **P is independent of θ. Q.E.D.**




## 4.45




$$\Sigma M_A = 0 \quad +\circlearrowleft$$

$$(120\sin\theta)\left(\frac{2L}{3}\cos\theta\right) + (120\cos\theta)\left(\frac{2L}{3}\sin\theta\right) - 30\left(\frac{L}{2}\sin\theta\right) = 0$$

$$160\cos\theta - 15 = 0 \qquad \theta = \cos^{-1}\frac{15}{160} = 84.6^\circ \;◄$$

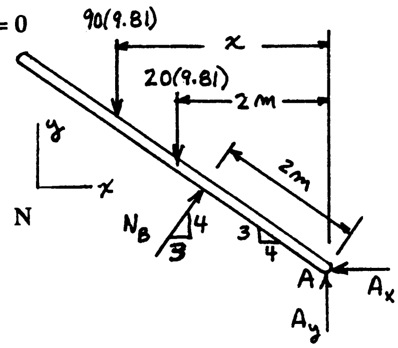
**4.46**

**(a)** $x = 1.5$ m

$\Sigma M_A = 0$:

$\circlearrowleft +\; -N_B(2) + 90(9.81)(1.5) + 20(9.81)(2) = 0$

$\therefore\; N_B = 858.4$ N ◆




$\Sigma F_y = 0$: $\quad +\uparrow\; A_y + \frac{4}{5}N_B - 110(9.81) = 0$

$\therefore\; A_y = 110(9.81) - \frac{4}{5}(858.4) = 392.4$ N

$\Sigma F_x = 0$: $\quad A_x = \frac{3}{5}N_B = \frac{3}{5}(858.4) = 515.0$ N

$R_A = \sqrt{515.0^2 + 392.4^2} = 647$ N ◆

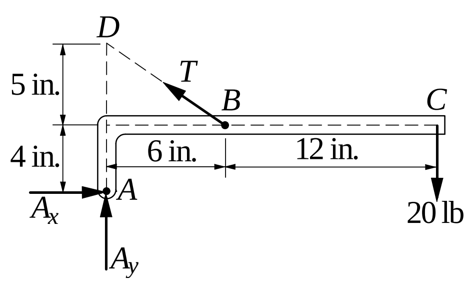
**(b) The ladder is ready to fall when $A_y = 0$.**

$\Sigma F_y = 0$: $\quad +\uparrow\; \frac{4}{5}N_B - 110(9.81) = 0 \quad \therefore\; N_B = 1349$ N

$\Sigma M_A = 0$: $\quad \circlearrowleft +\; 90(9.81)x + 20(9.81)(2) - N_B(2) = 0$

$$\therefore\; x = \frac{-20(9.81)(2) + 1349(2)}{90(9.81)} = 2.61 \text{ m} \;\blacklozenge$$

**4.47**




$$\Sigma M_A = 0 \qquad +\circlearrowleft\; \left(\frac{6}{\sqrt{61}}T\right)(9) - 20(18) = 0 \qquad T = 52.1 \text{ lb} \;\blacktriangleleft$$

$$\Sigma M_D = 0 \qquad +\circlearrowleft\; 9A_x - 20(18) = 0 \qquad A_x = 40.0 \text{ lb}$$

$$\Sigma M_B = 0 \qquad +\circlearrowleft\; 6A_y - 4A_x + 20(12) = 0$$

$$6A_y - 4(40.0) + 20(12) = 0 \qquad A_y = -13.33 \text{ lb}$$

$$A = \sqrt{40.0^2 + 13.33^2} = 42.2 \text{ lb} \;\blacktriangleleft$$

**4.48**

$\Sigma M_A = 0$:

$\circlearrowleft +$ $T(20) - T(120) + 14(120) - 0.40(9.81)(80) = 0$

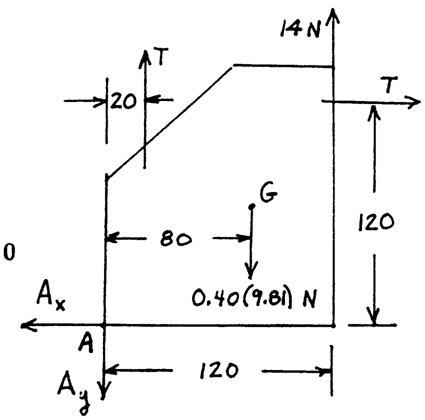
$\therefore\ T = 13.66\ \text{N}$ ◆

$\Sigma F_x = 0$: $A_x = T = 13.66\ \text{N}$

$\Sigma F_y = 0$: $+\uparrow\ T - A_y + 14 - 0.40(9.81) = 0$

$A_y = 13.66 + 14 - 0.40(9.81) = 23.74\ \text{N}$

$\therefore\ R_A = \sqrt{13.66^2 + 23.74^2} = 27.4\ \text{N}$ ◆

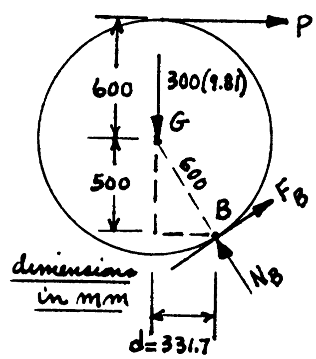



**4.49**

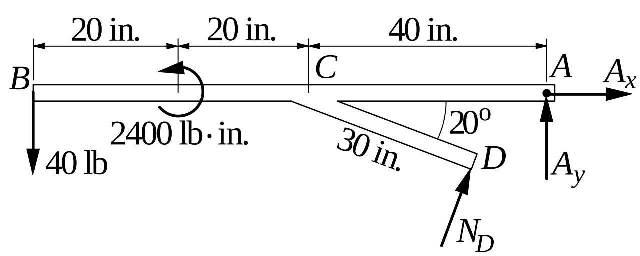
$\Sigma M_B = 0$:

$\circlearrowleft +$ $300(9.81)(331.7) - P(600 + 500) = 0$

$\therefore\ P = 887\ \text{N}$ ◆



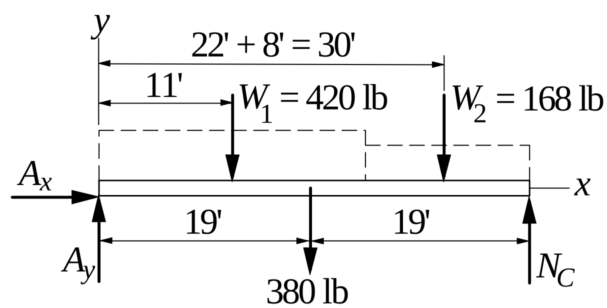

**4.50**




$$\Sigma M_A = 0 \qquad +\circlearrowleft\ 40(80) + 2400 + (N_D \sin 20^\circ)(30 \sin 20^\circ)$$
$$-(N_D \cos 20^\circ)(40 - 30\cos 20^\circ) = 0$$
$$5600 - 7.588 N_D = 0 \qquad N_D = 738.0\ \text{lb}\ ◄$$

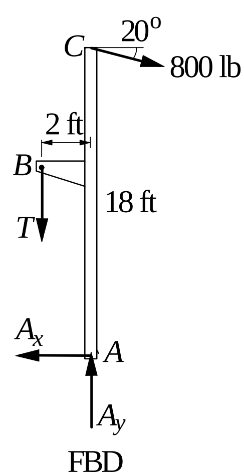
$$\Sigma F_y = 0 \quad +\uparrow \; N_D \cos 20^\circ + A_y - 40 = 0$$
$$738.0 \cos 20^\circ + A_y - 40 = 0 \qquad A_y = -653.5 \text{ lb}$$
$$\Sigma F_x = 0 \quad +\rightarrow \; N_D \sin 20^\circ + A_x = 0$$
$$738.0 \sin 20^\circ + A_x = 0 \qquad A_x = -252.4 \text{ lb}$$
$$A = \sqrt{252.4^2 + 653.5^2} = 701 \text{ lb} \;\blacktriangleleft$$

**4.51**



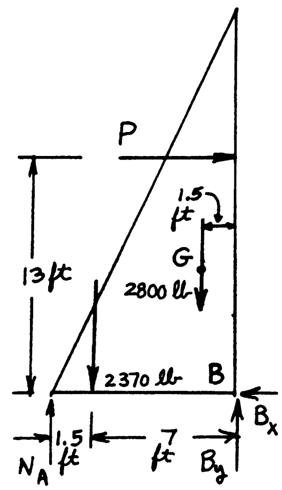

$$\Sigma F_x = 0 \quad +\rightarrow \; A_x = 0$$
$$\Sigma M_A = 0 \quad +\circlearrowleft \; 38N_C - 420(11) - 168(30) - 380(19) = 0$$
$$N_C = 444.2 \text{ lb}$$
$$\Sigma F_y = 0 \quad +\uparrow \; A_y - 420 - 168 - 380 + N_C = 0$$
$$A_y - 420 - 168 - 380 + 444.2 = 0 \qquad A_y = 523.8 \text{ lb}$$
$$\mathbf{A} = 524\mathbf{j} \text{ lb} \;\blacktriangleleft \qquad \mathbf{N}_C = 444\mathbf{j} \text{ lb} \;\blacktriangleleft$$

**4.52**



$$\begin{aligned}
\Sigma M_A &= 0 \qquad +\circlearrowleft\ 2T - 18(800\cos 20^\circ) = 0 \qquad T = 6766 \text{ lb} \blacktriangleleft \\
\Sigma F_x &= 0 \qquad +\rightarrow\ 800\cos 20^\circ - A_x = 0 \qquad A_x = 752 \text{ lb} \\
\Sigma F_y &= 0 \qquad +\uparrow\ A_y - T - 800\sin 20^\circ = 0 \qquad A_y = 7040 \text{ lb} \\
R_A &= \sqrt{752^2 + 7040^2} = 7080 \text{ lb} \blacktriangleleft
\end{aligned}$$

**4.53**




**(a)** $P = 1200$ lb

$\Sigma M_B = 0$:

$\circlearrowleft + \quad -N_A(8.5) + 2370(7) + 2800(1.5) - 1200(13) = 0$

$\therefore\ N_A = 611$ lb ♦

$\Sigma F_x = 0$: $B_x = P = 1200$ lb

$\Sigma F_y = 0$: $+\uparrow\ N_A + B_y - 2370 - 2800 = 0$

$\therefore\ B_y = 5170 - N_A = 5170 - 611 = 4559$ lb

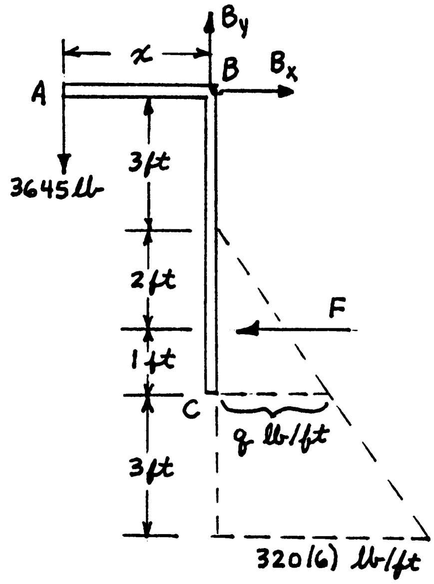
$\therefore\ R_B = \sqrt{1200^2 + 4559^2} = 4710$ lb ♦

**(b)** $P = 10\,q_{min}$ and $N_A = 0$ (for impending tipping)

$\Sigma M_B = 0$: $\circlearrowleft + \quad 2370(7) + 2800(1.5) - 10\,q_{min}(13) = 0 \qquad \therefore\ q_{min} = 160$ lb/ft ♦

4.54

When h = 6 ft, the contact force at C is zero.




Using similar triangles:

$$\frac{q}{3} = \frac{320(6)}{6}$$

$\therefore$ q = 960 lb/ft

F = 0.5(3)(960) = 1440 lb

$\Sigma M_B = 0$:

(+) 3645 x − 1440(5) = 0

$\therefore$ x = 1.975 ft ♦

4.55

$\Sigma F_x = 0$:

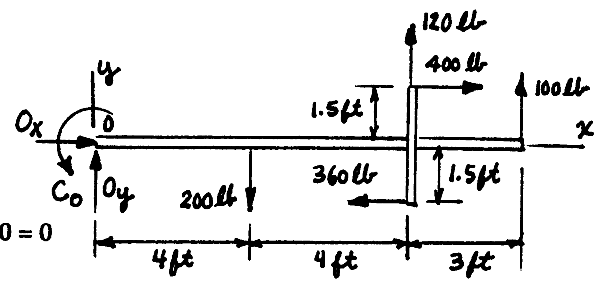
$\xrightarrow{+}$ $O_x + 400 - 360 = 0$

$\therefore$ $O_x = -40$ lb

$\Sigma F_y = 0$:

+↑ $O_y - 200 + 120 + 100 = 0$

$\therefore$ $O_y = -20$ lb

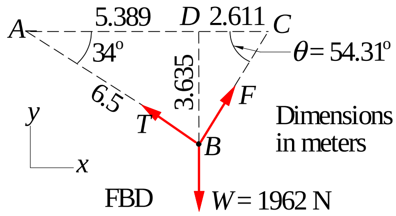



$\Sigma M_O = 0$: (+) $C_O - 200(4) + 120(8) - 400(1.5) - 360(1.5) + 100(11) = 0$

$\therefore$ $C_O = -120$ lb•ft

Therefore, the reactions at O are: $\mathbf{C}_O = -120\mathbf{k}$ lb•ft and $\mathbf{O} = -40\mathbf{i} - 20\mathbf{j}$ lb ♦

## 4.56




$$\overline{AD} = 6.5\cos 34^\circ = 5.389 \text{ m} \qquad \overline{BD} = 6.5\sin 34^\circ = 3.635 \text{ m}$$

$$\overline{DC} = \overline{AC} - \overline{AD} = 8 - 5.389 = 2.611 \text{ m}$$

$$\theta = \tan^{-1}\frac{\overline{BD}}{\overline{DC}} = \tan^{-1}\frac{3.635}{2.611} = 54.31^\circ$$

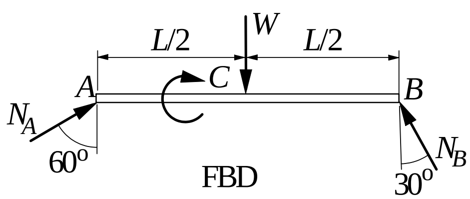
$$W = 200(9.81) = 1962 \text{ N}$$

$$\Sigma F_x = 0 \quad +\rightarrow \quad F\cos 54.31^\circ - T\cos 34^\circ = 0 \qquad T = 0.7037F$$

$$\Sigma F_y = 0 \quad +\uparrow \quad T\sin 34^\circ + F\sin 54.31^\circ - 1962 = 0$$

$$0.7037F\sin 34^\circ + F\sin 54.31^\circ - 1962 = 0 \qquad F = 1627 \text{ N} \blacktriangleleft$$

## 4.57




$$\Sigma F_x = 0 \quad +\rightarrow \quad N_A\sin 60^\circ - N_B\sin 30^\circ = 0$$

$$\Sigma F_y = 0 \quad +\uparrow \quad N_A\cos 60^\circ + N_B\cos 30^\circ - W = 0$$

Solution is

$$N_A = 0.5W \qquad N_B = 0.8660W$$

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad (N_B\cos 30^\circ)\,L - W\frac{L}{2} - C = 0$$

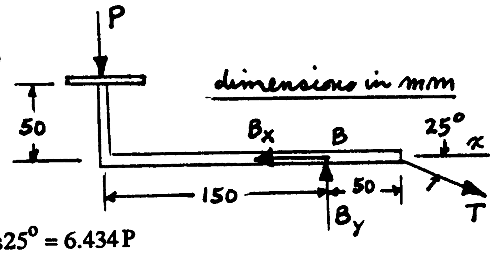
$$C = 0.8660WL\cos 30^\circ - W\frac{L}{2} = 0.250\ WL \blacktriangleleft$$

## 4.58

$\Sigma M_B = 0$:

$\circlearrowleft + \; P(0.150) - T\sin 25^\circ(0.05) = 0$

$T = 7.0986P$




$\Sigma F_x = 0$: $\xrightarrow{+}$ $T\cos 25^\circ - B_x = 0$

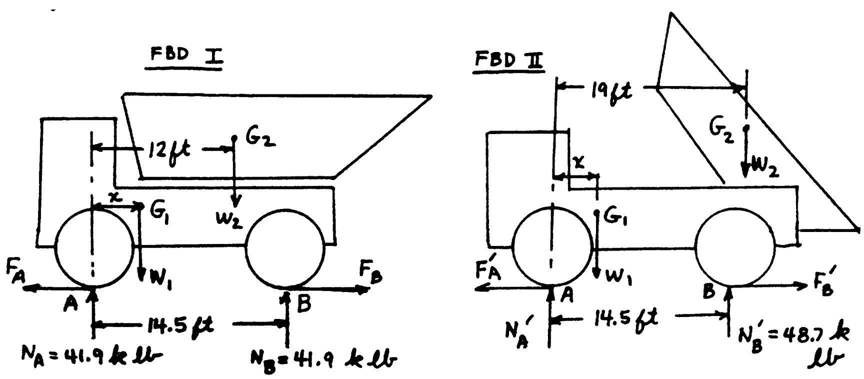
$B_x = T\cos 25^\circ = (7.0986P)\cos 25^\circ = 6.434P$

$\Sigma F_y = 0$: $+\uparrow$ $B_y - P - T\sin 25^\circ = 0$

$B_y = P + T\sin 25^\circ = P + (7.0986P)\sin 25^\circ = 4.000P$

$\therefore\; B = \sqrt{B_x^2 + B_y^2} = P\sqrt{6.434^2 + 4.000^2} = 7.576P = 1800\text{ N}$ gives $P = 238\text{ N}$ ◆

## 4.59



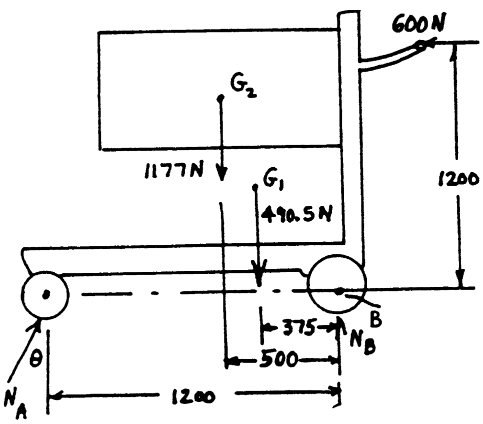

**FBD I** $\quad \Sigma M_A = 0$: $\circlearrowleft +$ $W_1 x + W_2(12) - 41.9(14.5) = 0 \quad (1)$

**FBD II** $\quad \Sigma M_A = 0$: $\circlearrowleft +$ $W_1 x + W_2(19) - 48.7(14.5) = 0 \quad (2)$

**FBD I** $\quad \Sigma F_y = 0$: $+\uparrow$ $2(41.9) - W_1 - W_2 = 0 \quad (3)$

Solving (1), (2) and (3) gives: $x = 6.29$ ft ◆ ; $W_1 = 69.7$ kips ◆ ; $W_2 = 14.1$ kips ◆

4.60




$W_1 = 50(9.81) = 490.5$ N

$W_2 = 120(9.81) = 1177$ N

Find the largest acceptable value of θ

$\Sigma F_x = 0$: $N_A \sin\theta = 600$ (1)

$\Sigma M_B = 0$:

$\circlearrowleft +$ $N_A \cos\theta(1200) - 1177(500) - 490.5(375) - 600(1200) = 0$

$\therefore N_A \cos\theta = 1244$ (2)

Dividing (1) by (2) gives: $\theta = \tan^{-1}(600/1244) = 25.75^\circ$

Check tipping

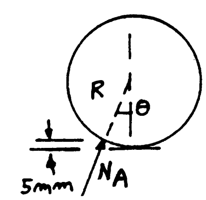
$\Sigma F_y = 0$: $+\uparrow$ $N_A \cos\theta + N_B - 1177 - 490.5 = 0$ $\therefore N_B = 1177 + 490.5 - 1244 = 423.5$ N

Because $N_B$ is not negative, the truck will not tip.

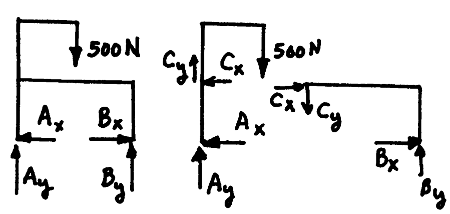
Find R

$R - R\cos\theta = 5$ mm

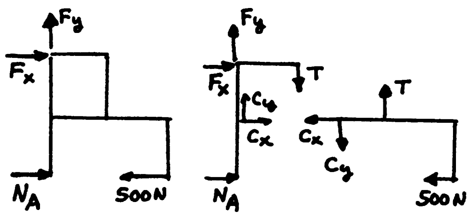
which gives: $R = \dfrac{5}{1 - \cos 25.75^\circ} = 50.4$ mm ♦



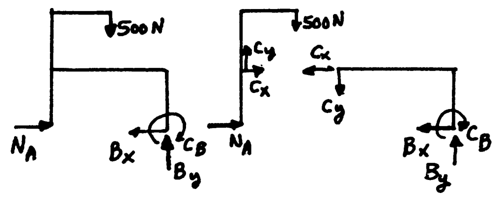

4.61




(a) 6 unknowns, 6 independent equations ♦

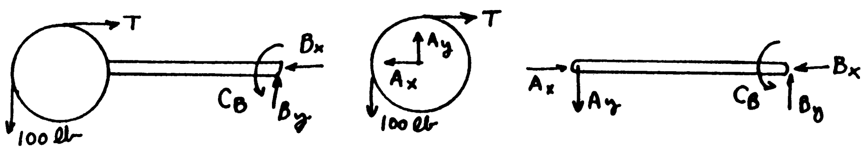



**(b)** 6 unknowns, 6 independent equations ♦

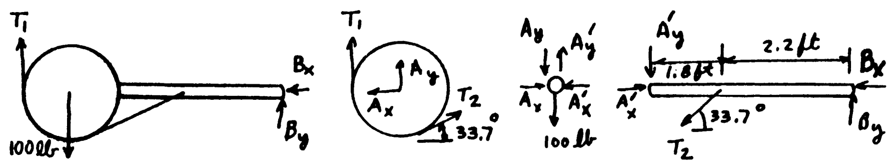



**(c)** 6 unknowns, 6 independent equations ♦

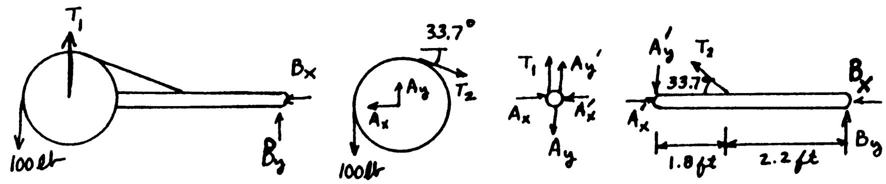
4.62




**(a)** 6 unknowns, 6 independent equations ♦

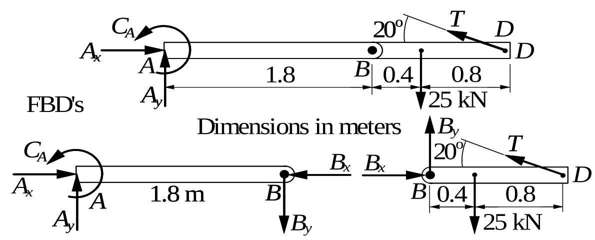



**(b)** 8 unknowns, 8 independent equations ♦



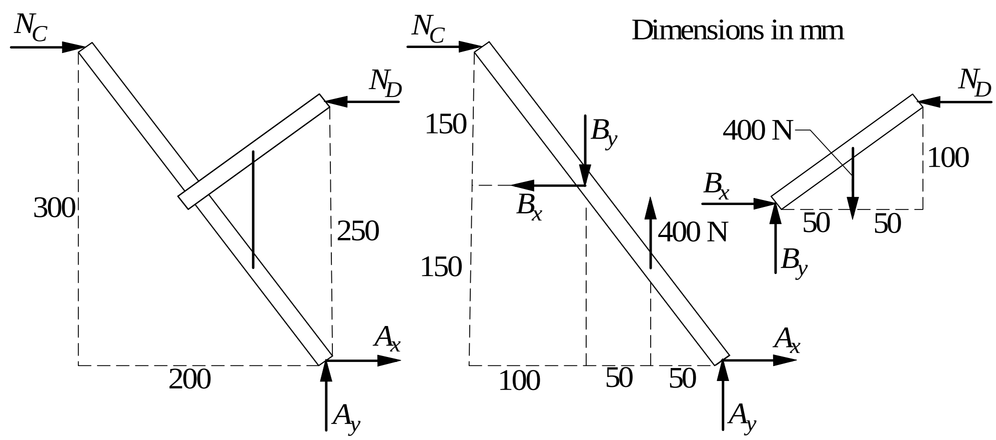

**(c)** 8 unknowns, 8 independent equations ♦

**4.63**



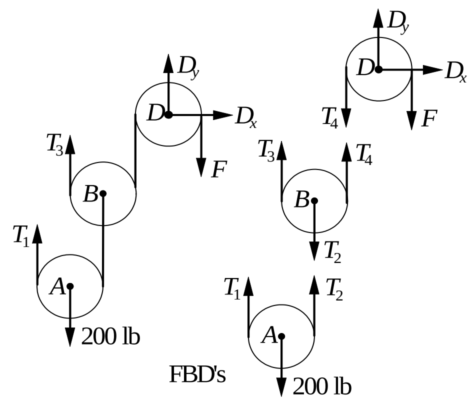

6 unknowns, 6 independent equations ◄

**4.64**



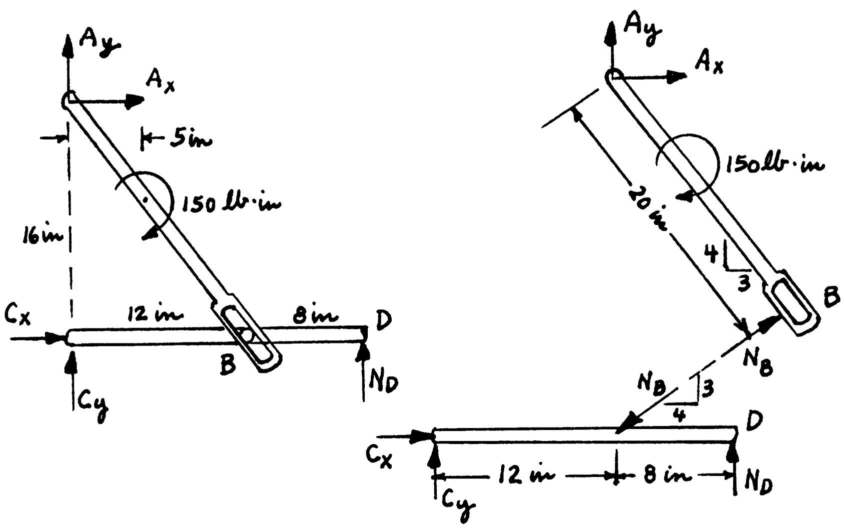

6 unknowns, 6 independent equations ◄

**4.65**



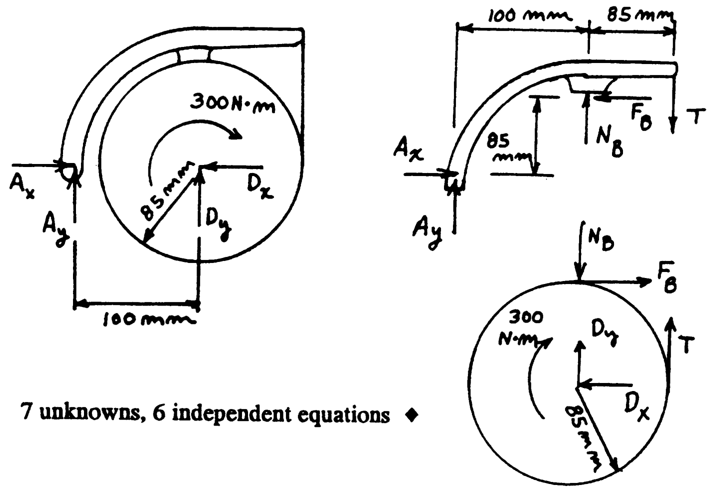

7 unknowns, 7 independent equations ◀

**4.66**



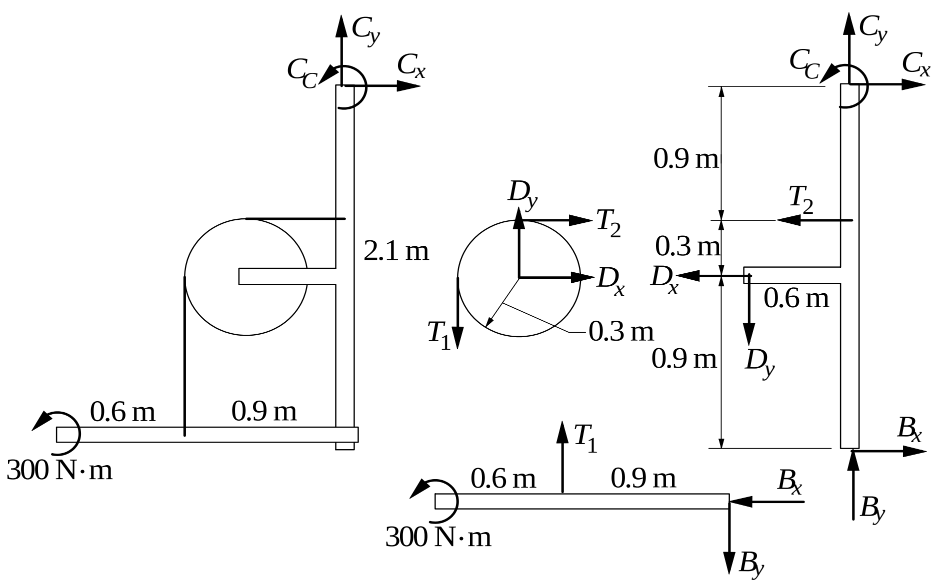

**6 unknowns, 6 independent equations** ◆

**4.67**



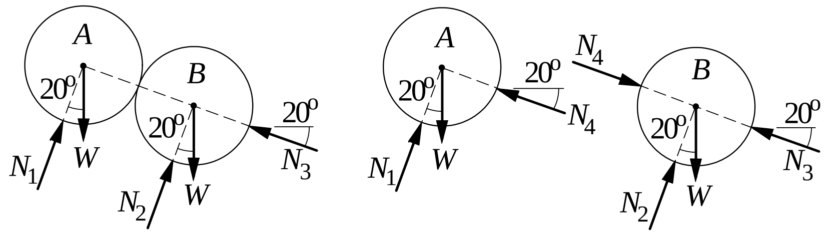

**7 unknowns, 6 independent equations** ◆

**4.68**



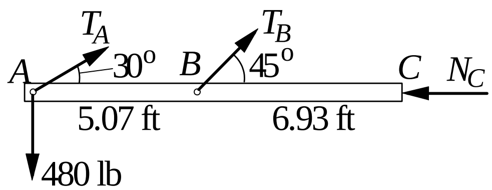

9 unknowns, 9 independent equations. ◄

**4.69**

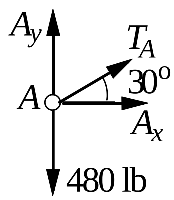



4 unknowns, 4 independent equations.

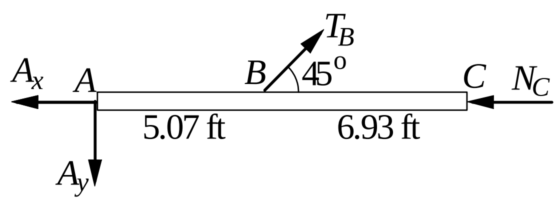
**4.70**




(a) Bar *ABC* with pin *A* in bar *ABC*

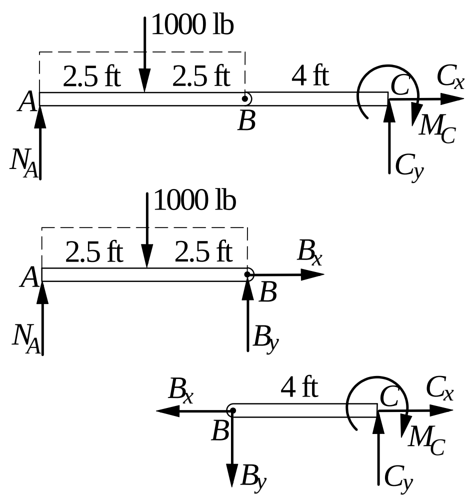



(c) Pin *A*



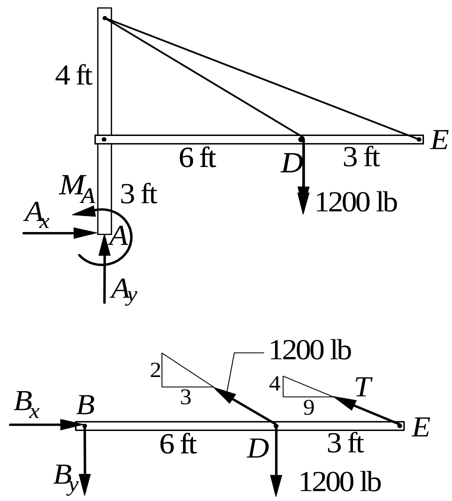

(b) Bar *ABC* with pin *A* removed

**4.71**



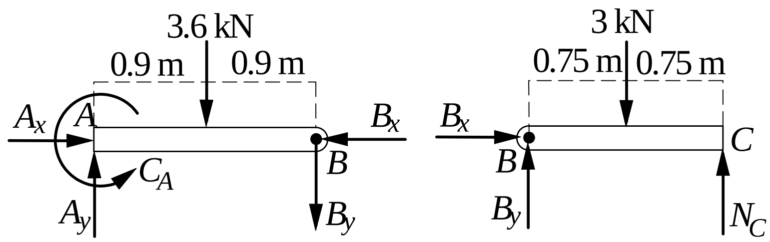

6 unknowns, 6 independent equations.

**4.72**




6 unknowns, 6 independent equilibrium equations.

**4.73**



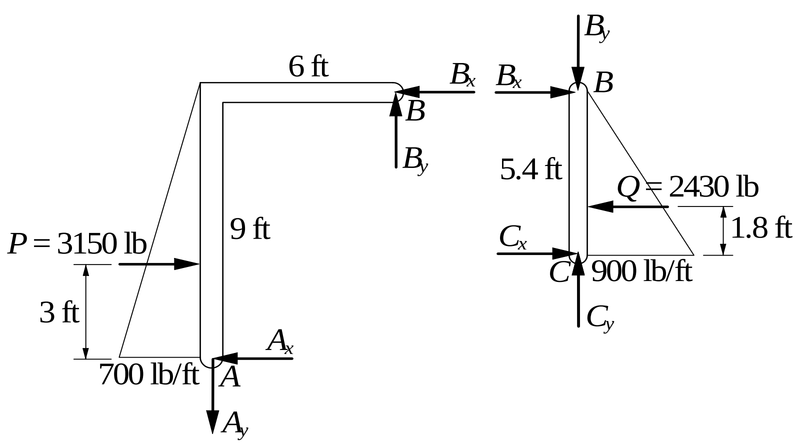

From FBD of $BC$:

$$\begin{aligned} \Sigma M_B &= 0 \quad +\circlearrowleft\ 1.5N_C - 3(0.75) = 0 \qquad N_C = 1.5 \text{ kN} \\ \Sigma F_y &= 0 \quad +\uparrow\ B_y + N_C - 3 = 0 \qquad B_y = 1.5 \text{ kN} \\ \Sigma F_x &= 0 \quad +\rightarrow\ B_x = 0 \end{aligned}$$

From FBD of $AB$:

$$\begin{aligned} \Sigma F_x &= 0 \quad +\rightarrow\ A_x - B_x = 0 \qquad A_x = 0 \ \blacktriangleleft \\ \Sigma M_A &= 0 \quad +\circlearrowleft\ C_A - 3.6(0.9) - 1.8B_y = 0 \qquad C_A = 5.94 \text{ kN}\cdot\text{m} \ \blacktriangleleft \\ \Sigma F_y &= 0 \quad +\uparrow\ A_y - 3.6 - B_y = 0 \qquad A_y = 5.1 \text{ kN} \ \blacktriangleleft \end{aligned}$$

**4.74**




$$P = \frac{1}{2}(700)(9) = 3150 \text{ lb} \qquad Q = \frac{1}{2}(900)(5.4) = 2430 \text{ lb}$$

FBD of $BC$:

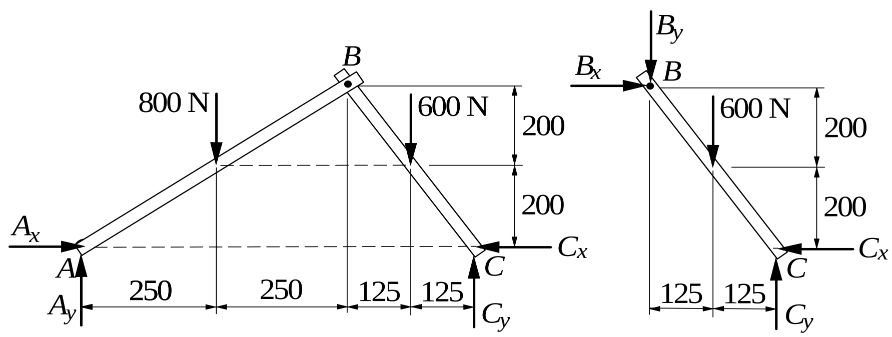
$$\Sigma M_C = 0 \qquad +\circlearrowleft\ 5.4B_x - 2430(1.8) = 0 \qquad B_x = 810 \text{ lb}$$

FBD of $AB$:

$$\Sigma M_A = 0 \qquad +\circlearrowleft\ 6B_y + 9B_x - 3150(3) = 0$$
$$6B_y + 9(810) - 3150(3) = 0 \qquad B_y = 360 \text{ lb}$$

$$B = \sqrt{810^2 + 360^2} = 886 \text{ lb} \blacktriangleleft$$

**4.75**




From FBD of $ABC$:

$$\Sigma M_A = 0 \qquad +\circlearrowleft\ 750C_y - 800(250) - 600(625) = 0$$
$$\Sigma M_C = 0 \qquad +\circlearrowright\ 750A_y - 800(500) - 600(125) = 0$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ A_x - C_x = 0$$

From FBD of $BC$:

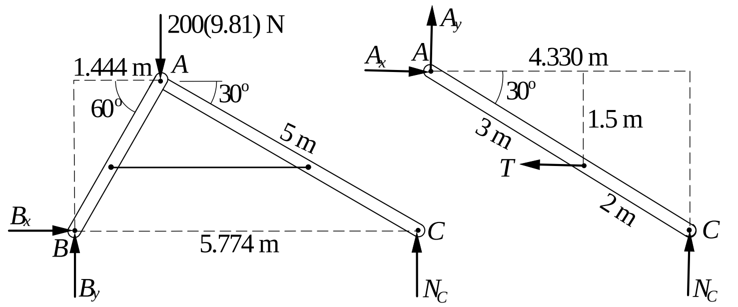
$$\Sigma M_B = 0 \qquad +\circlearrowleft\ 250C_y - 400C_x - 600(125) = 0$$

Solution is

$$A_x = 291.7 \text{ N} \qquad A_y = 633.3 \text{ N} \qquad C_x = 291.7 \text{ N} \qquad C_y = 766.7 \text{ N}$$

$$R_A = \sqrt{291.7^2 + 633.3^2} = 697 \text{ N} \blacktriangleleft$$
$$R_C = \sqrt{291.7^2 + 766.7^2} = 820 \text{ N} \blacktriangleleft$$

## 4.76



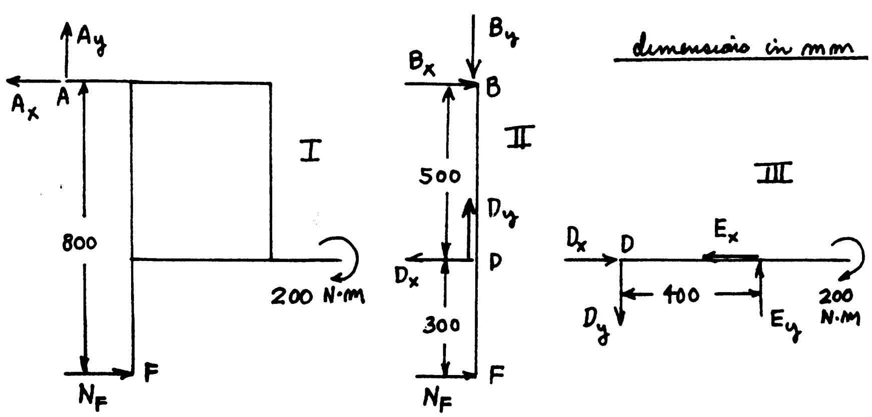

FBD of entire structure:

$$\Sigma M_B = 0 \qquad + \circlearrowleft \quad 5.774 N_C - 200(9.81)(1.444) = 0 \qquad N_C = 490.7 \text{ N}$$

FBD of bar $AC$ (with the pin at $A$ removed):

$$\Sigma M_A = 0 \qquad + \circlearrowleft \quad 490.7(4.330) - 1.5T = 0 \qquad T = 1416 \text{ N} \blacktriangleleft$$

## 4.77



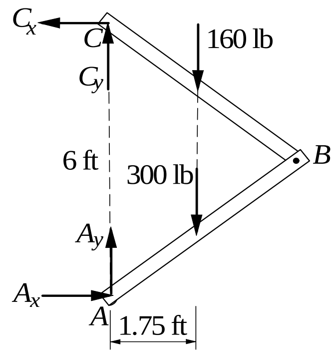

**FBD I** $\quad \Sigma M_A = 0$: $\circlearrowleft +$ $0.8\, N_F - 200 = 0 \qquad \therefore\ N_F = 250$ N

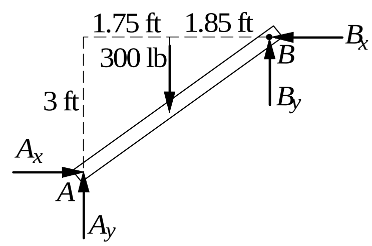
**FBD II** $\quad \Sigma M_B = 0$: $\circlearrowleft +$ $0.8\, N_F - 0.5\, D_x = 0 \qquad \therefore\ D_x = 400$ N

**FBD III** $\quad \Sigma M_E = 0$: $\circlearrowleft +$ $0.4\, D_y - 200 = 0 \qquad \therefore\ D_y = 500$ N

$$D = \sqrt{400^2 + 500^2} = 640 \text{ N} \ \blacklozenge$$

**4.78**




From FBD of $ABC$:

$$\begin{aligned} \Sigma M_A &= 0 \qquad +\circlearrowleft\ 6C_x - 460(1.75) = 0 \\ \Sigma F_x &= 0 \qquad +\rightarrow\ A_x - C_x = 0 \\ \Sigma F_y &= 0 \qquad +\uparrow\ A_y + C_y - 460 = 0 \end{aligned}$$

From FBD of $AB$:

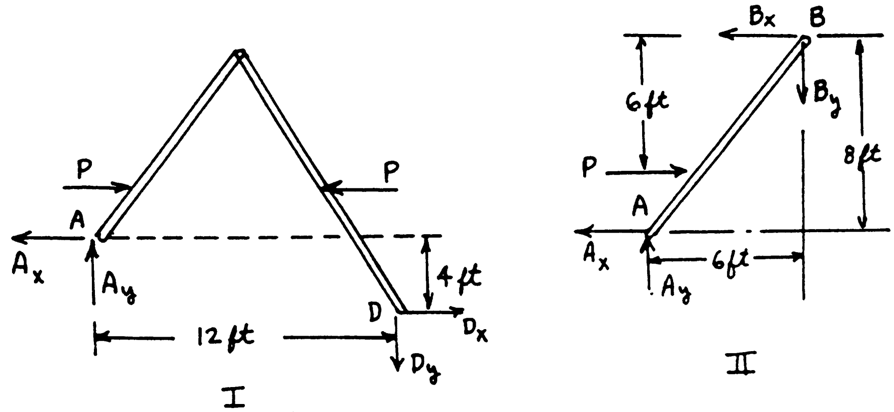
$$\Sigma M_B = 0 \qquad +\circlearrowleft\ 3.60A_y - 3A_x - 300(1.85) = 0$$

Solution of these equations is

$A_x = 134.17$ lb $\quad A_y = 266.0$ lb $\quad C_x = 134.17$ lb $\quad C_y = 194.0$ lb

$$\begin{aligned} R_A &= \sqrt{134.17^2 + 266.0^2} = 298 \text{ lb} \blacktriangleleft \\ R_C &= \sqrt{134.17^2 + 194.0^2} = 236 \text{ lb} \blacktriangleleft \end{aligned}$$

**4.79**



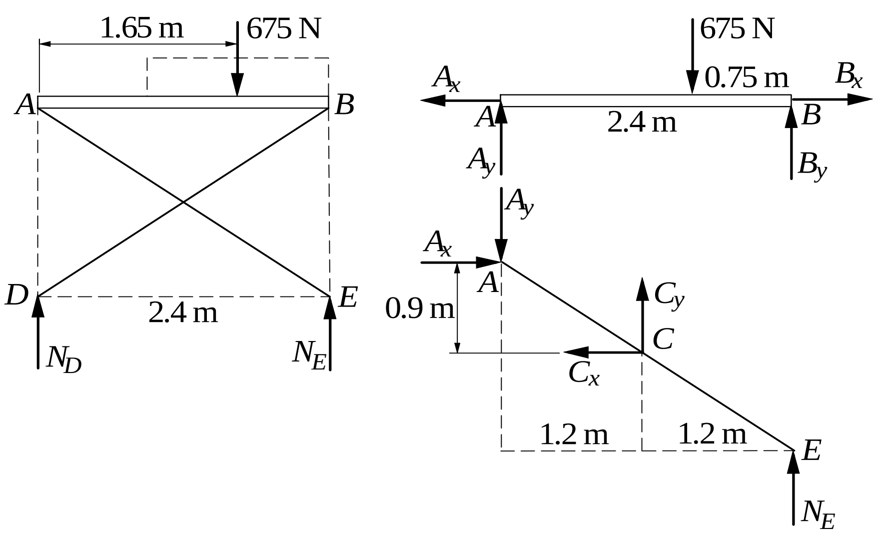

**FBD I** $\Sigma M_D = 0$: $\circlearrowleft +$ $A_x(4) - A_y(12) = 0$ $\therefore$ $A_x = 3A_y$

**FBD II** $\Sigma M_B = 0$: $\circlearrowleft +$ $P(6) - A_y(6) - A_x(8) = 0$

$P(6) - A_y(6) - 3A_y(8) = 0$ $\therefore$ $A_y = 0.2P$ $(A_x = 0.6P)$

$$R_A = P\sqrt{0.2^2 + 0.6^2} = 0.632P \quad \blacklozenge$$

**4.80**




From FBD of entire table:

$$\Sigma M_D = 0 \qquad + \circlearrowleft \; 2.4N_E - 675(1.65) = 0 \qquad N_E = 464.1 \text{ N}$$

From FBD of member $AB$:

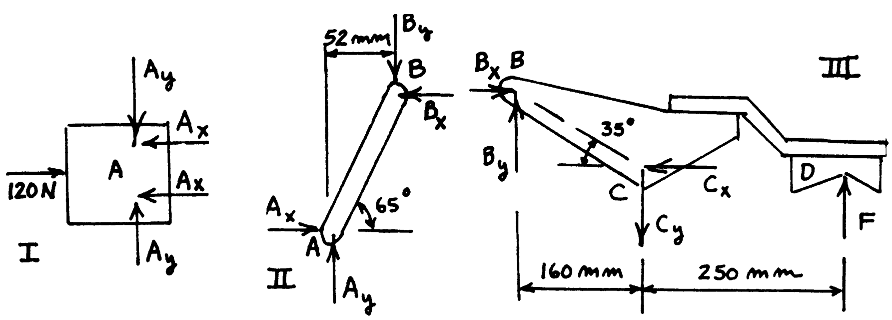
$$\Sigma M_B = 0 \qquad + \circlearrowleft \; 675(0.75) - 2.4A_y = 0 \qquad A_y = 210.9 \text{ N}$$

From FBD of member $ACE$:

$$\Sigma M_C = 0 \qquad + \circlearrowleft \; 1.2N_E - 0.9A_x + 1.2A_y = 0$$
$$1.2(464.1) - 0.9A_x + 1.2(210.9) = 0 \qquad A_x = 900 \text{ N}$$
$$\Sigma F_y = 0 \qquad + \uparrow \; C_y + N_E - A_y = 0 \qquad C_y + 464.1 - 210.9 = 0$$
$$C_y = -253.2 \text{ N}$$
$$\Sigma F_x = 0 \qquad + \rightarrow \; A_x - C_x = 0 \qquad C_x = A_x = 900 \text{ N}$$

$$A = \sqrt{900^2 + 210.9^2} = 924 \text{ N} \blacktriangleleft \qquad C = \sqrt{900^2 + 253.2^2} = 935 \text{ N} \blacktriangleleft$$

**4.81**




**FBD I** $\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $120 - 2A_x = 0$ $\therefore$ $A_x = 60$ N

**FBD II** $\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $A_x - B_x = 0$ $\therefore$ $B_x = A_x = 60$ N

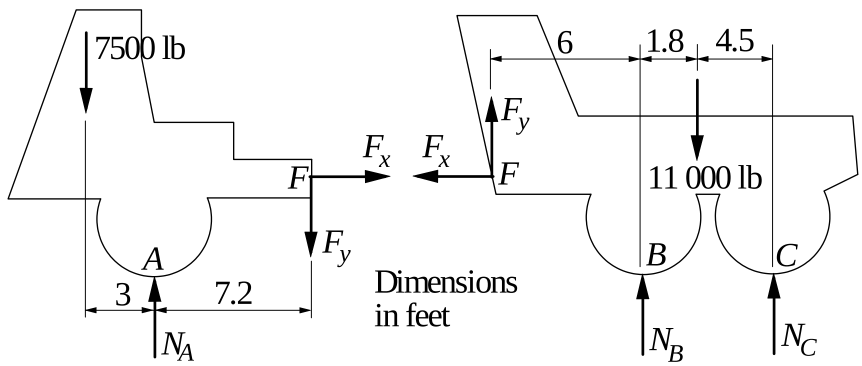
$\Sigma M_A = 0$: $\circlearrowleft +$ $B_x(52 \tan 65^\circ) - B_y(52) = 0$

$\therefore$ $B_y = B_x \tan 65^\circ = 60 \tan 65^\circ = 128.7$ N

**FBD III** $\Sigma M_C = 0$: $\circlearrowleft +$ $F(250) - B_y(160) - B_x(160 \tan 35^\circ) = 0$

$\therefore$ $F = (1/250)[128.7(160) + 60(160 \tan 35^\circ)] = 109.3$ N ◆

**4.82**



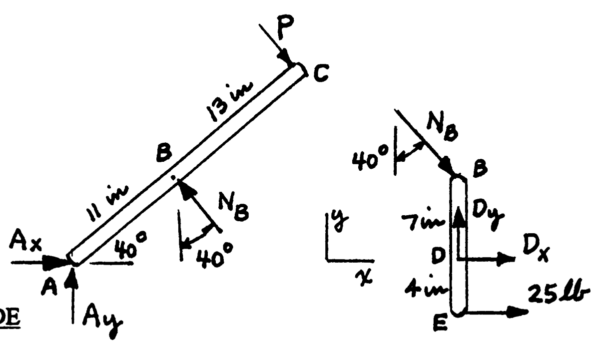

From FBD of the cab:

$$\Sigma M_F = 0 \qquad +\circlearrowleft\ 7500(10.2) - 7.2N_A = 0 \qquad N_A = 10\ 625 \text{ lb} \blacktriangleleft$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ F_x = 0$$
$$\Sigma F_y = 0 \qquad +\uparrow\ N_A - F_y - 7500 = 0 \qquad 10\ 625 - F_y - 7500 = 0$$
$$F_y = 3125 \text{ lb}$$

From FBD of the trailer:

$$\Sigma M_B = 0 \qquad +\circlearrowleft\ 6.3N_C - 6F_y - 11\ 000(1.8) = 0$$
$$6.3N_C - 6(3125) - 11\ 000(1.8) = 0 \qquad N_C = 6119 \text{ lb} \blacktriangleleft$$
$$\Sigma M_F = 0 \qquad +\circlearrowleft\ 6N_B + 12.3N_C - 11\ 000(7.8) = 0$$
$$6N_B + 12.3(6119) - 11\ 000(7.8) = 0 \qquad N_B = 1756 \text{ lb} \blacktriangleleft$$

**4.83**



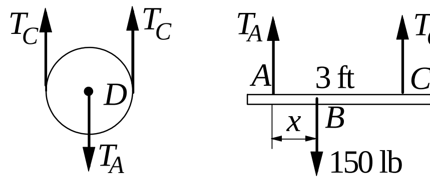

**FBD of bar BDE**

$\Sigma M_D = 0$: $\circlearrowleft\!+$ $25(4) - N_B \sin 40^\circ(7) = 0 \quad \therefore\ N_B = 22.22$ lb

**FBD of bar ABC**

$\Sigma M_A = 0$: $\circlearrowleft\!+$ $N_B(11) - P(24) = 0 \quad \therefore\ P = (11/24)N_B = (11/24)(22.22) = 10.19$ lb ◆

## 4.84



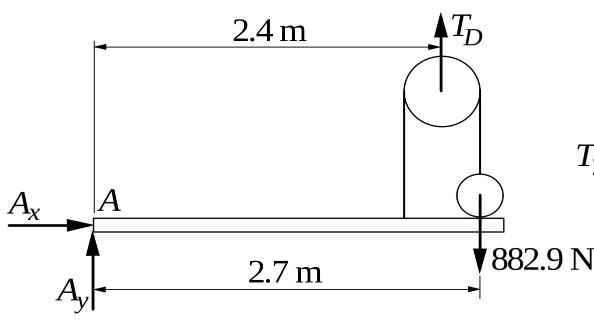

From FBD of pulley $D$:

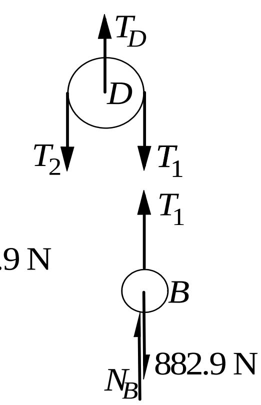
$$\Sigma F = 0 \qquad +\uparrow\ 2T_C - T_A = 0 \qquad T_A = 2T_C$$

From FBD of bar $ABC$:

$$\Sigma F = 0 \quad +\uparrow\ T_A + T_C - 150 = 0 \qquad T_C = 50 \text{ lb} \blacktriangleleft \qquad T_A = 100 \text{ lb} \blacktriangleleft$$
$$\Sigma M_A = 0 \quad +\circlearrowleft\ 3T_C - 150x = 0 \qquad x = 1.0 \text{ ft} \blacktriangleleft$$

## 4.85




$$W = 90(9.81) = 882.9 \text{ N}$$

From FBD of the assembly:

$$\Sigma M_A = 0 \qquad +\circlearrowleft\ 2.4T_D - 882.9(2.7) = 0 \qquad T_D = 993.3 \text{ N}$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ A_x = 0$$
$$\Sigma F_y = 0 \qquad +\uparrow\ A_y + T_D - 882.9 = 0 \qquad A_y + 993.3 - 882.9 = 0$$
$$A_y = -110.4 \text{ N} \qquad \therefore A = 110.4 \text{ N} \blacktriangleleft$$

From FBD of the pulley:

$$\Sigma M_D = 0 \qquad T_1 = T_2$$
$$\Sigma F_y = 0 \qquad +\uparrow\ T_D - T_1 - T_2 = 0$$
$$T_1 = T_2 = 0.5T_D = 0.5(993.3) = 496.7 \text{ N}$$

From FBD of the ball:

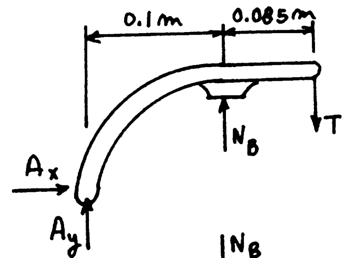
$$\Sigma F_y = 0 \qquad +\uparrow\ T_1 + N_B - 882.9 = 0 \qquad 496.7 + N_B - 882.9 = 0$$
$$N_B = 386 \text{ N} \ \blacktriangleleft$$

## 4.86

**FBD of arm**

$\Sigma M_A = 0$: (+ counterclockwise) $N_B(0.1) - T(0.185) = 0$

$\therefore\ N_B = 1.85\,T$

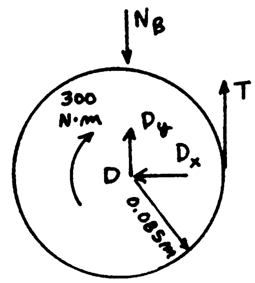



**FBD of cylinder**

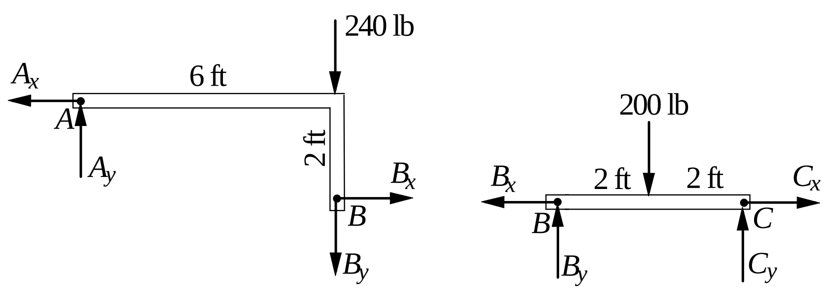
$\Sigma M_D = 0$: (+ counterclockwise) $T(0.085) - 300 = 0$

$\therefore\ T = 3530$ N ◆

**Therefore,** $N_B = 1.85(3530) = 6531$ N ◆




## 4.87



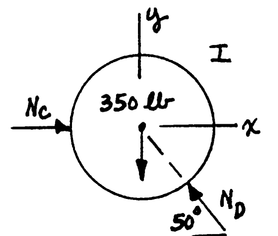

From FBD of $BC$ :

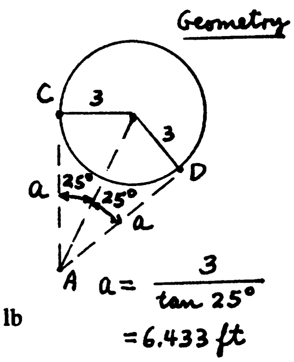
$$\Sigma M_B = 0 \qquad + \circlearrowleft\ 4C_y - 200(2) = 0 \qquad C_y = 100 \text{ lb}$$
$$\Sigma F_y = 0 \qquad +\uparrow\ B_y + C_y - 200 = 0 \qquad B_y = 100 \text{ lb}$$

From FBD of $AB$:

$$\Sigma M_A = 0 \qquad + \circlearrowleft\ (240 + B_y)(6) - 2B_x = 0 \qquad B_x = 1020 \text{ lb}$$
$$R_B = \sqrt{1020^2 + 100^2} = 1025 \text{ lb} \ \blacktriangleleft$$

## 4.88




**FBD I**

$\Sigma F_y = 0$: $N_D \sin 50^\circ = 350 \quad \therefore\ N_D = 456.9 \text{ lb}$

$\Sigma F_x = 0$: $N_C = N_D \cos 50^\circ = 456.9 \cos 50^\circ = 293.7 \text{ lb}$

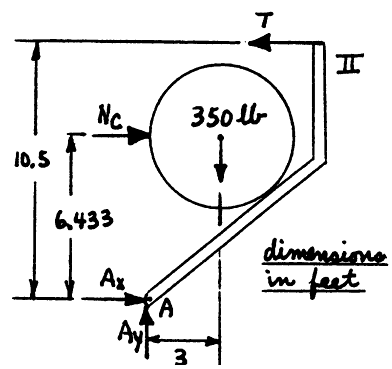
**FBD II**

**(Using the above geometry, we have found that the vertical distance between point A and the line of action of $N_C$ is 6.433 ft.)**

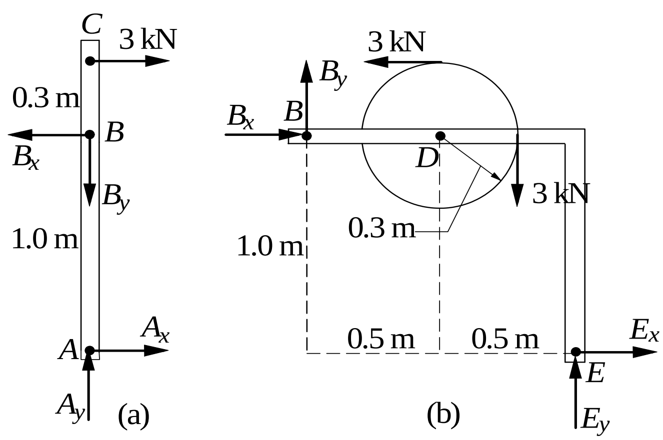
$\Sigma M_A = 0$:

$$\circlearrowleft + \quad T(10.5) - 350(3) - N_C(6.433) = 0$$

$$\therefore\ T = \frac{350(3) + 293.7(6.433)}{10.5} = 280 \text{ lb} \ \blacklozenge$$




## 4.89




From FBD (a):

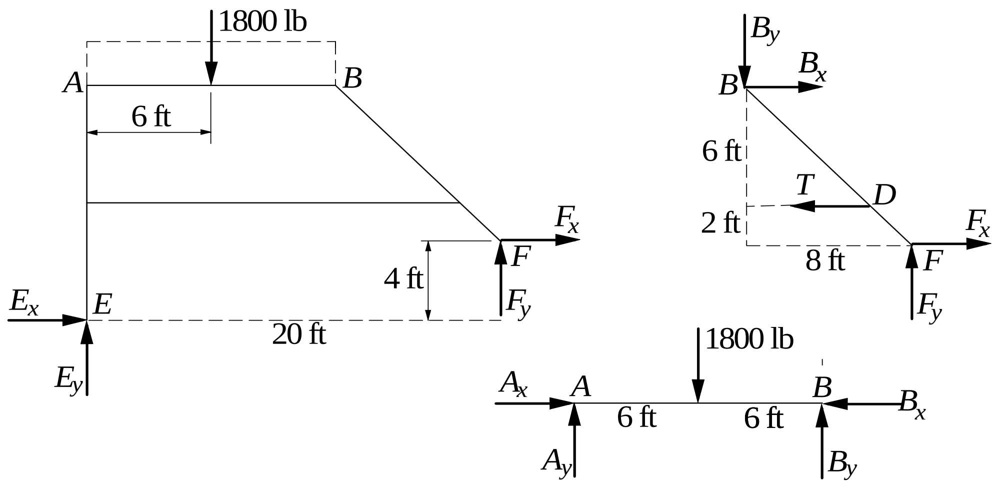
$$\Sigma M_A = 0 \qquad + \circlearrowleft \ 1.0B_x - 3(1.3) = 0 \qquad B_x = 3.9 \text{ kN}$$

From FBD (b):

$$\Sigma M_E = 0 \qquad + \circlearrowleft \; 1.0B_x + 1.0B_y - 3(1.3) - 3(0.2) = 0 \qquad B_y = 0.6 \text{ kN}$$

$$R_B = \sqrt{3.9^2 + 0.6^2} = 3.95 \text{ kN} \blacktriangleleft$$

**4.90**




From FBD of member $AB$:

$$\Sigma M_A = 0 \qquad + \circlearrowleft \; 12B_y - 1800(6) = 0 \qquad B_y = 900 \text{ lb}$$

From FBD of member $BDF$:

$$\Sigma F_y = 0 \qquad + \uparrow \; F_y - B_y = 0 \qquad F_y = 900 \text{ lb}$$

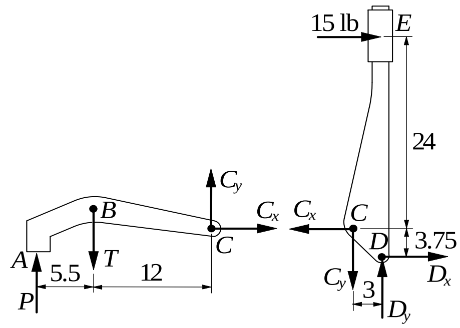
From FBD of entire frame:

$$\Sigma M_E = 0 \qquad + \circlearrowleft \; 20F_y - 4F_x - 1800(6) = 0$$
$$20(900) - 4F_x - 1800(6) = 0 \qquad F_x = 1800 \text{ lb}$$

From FBD of member $BDF$:

$$\Sigma M_B = 0 \qquad + \circlearrowleft \; 8F_x + 8F_y - 6T = 0 \qquad 8(1800 + 900) - 6T = 0$$
$$T = 3600 \text{ lb} \blacktriangleleft$$

## 4.91




FBD of $ABC$:

$$\Sigma F_x = 0 \qquad + \longrightarrow \qquad C_x = 0$$

FBD of $DCE$:

$$\Sigma M_D = 0 \qquad + \circlearrowleft \quad 3C_y - 15(27.75) = 0 \qquad C_y = 138.75 \text{ lb}$$

FBD of $ABC$:

$$\Sigma M_B = 0 \qquad + \circlearrowleft \quad 5.5P - 138.75\,(12) = 0 \qquad P = 303 \text{ lb} \;◄$$

## 4.92

**FBD I**

**Using similar triangles:**

$$\frac{d}{5} = \frac{2}{7.5} \quad \text{or} \quad d = 4/3 \text{ ft}$$

$\Sigma M_A = 0$:

$$(+)\; N_E(4) - 165(4/3) = 0$$

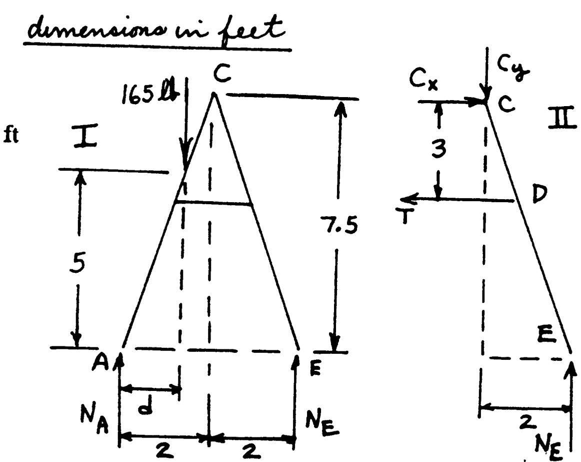
$$\therefore\; N_E = 55.00 \text{ lb}$$

**FBD II**

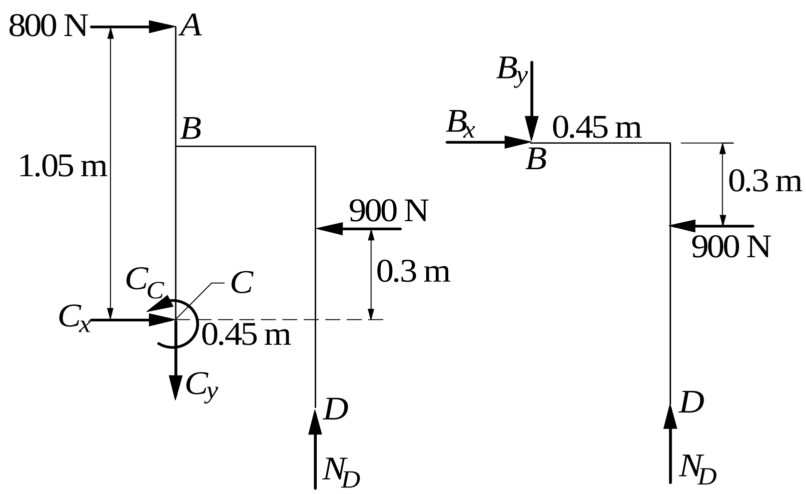
$\Sigma M_C = 0$:

$$(+)\; N_E(2) - T(3) = 0$$

$$\therefore\; T = (2/3)55.00 = 36.7 \text{ lb} \;◆$$



**4.93**



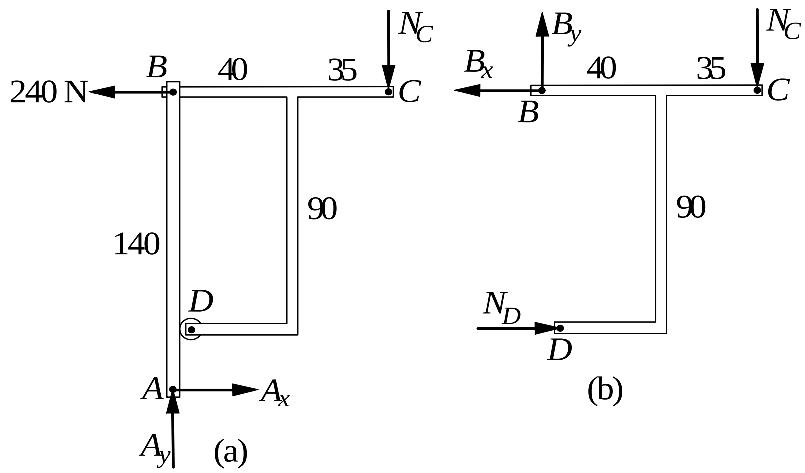

From FBD of member $BD$:

$$\Sigma M_B = 0 \qquad +\circlearrowleft\ 0.45N_D - 900(0.3) = 0 \qquad N_D = 600 \text{ N}$$

From FBD of entitre frame:

$$\begin{aligned} \Sigma M_C &= 0 \qquad +\circlearrowleft\ C_C + 0.45N_D + 900(0.3) - 800(1.05) = 0 \\ & \quad C_C + 0.45(600) + 900(0.3) - 800(1.05) = 0 \qquad C_C = 300 \text{ N} \blacktriangleleft \\ \Sigma F_x &= 0 \qquad +\rightarrow\ C_x + 800 - 900 = 0 \qquad C_x = 100 \text{ N} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad +\uparrow\ N_D - C_y = 0 \qquad C_y = N_D = 600 \text{ N} \blacktriangleleft \end{aligned}$$

**4.94**




From FBD (a):

$$\Sigma M_A = 0 \qquad + \circlearrowleft \; 75N_C - 240(140) = 0 \qquad N_C = 448 \text{ N} \blacktriangleleft$$

From FBD (b):

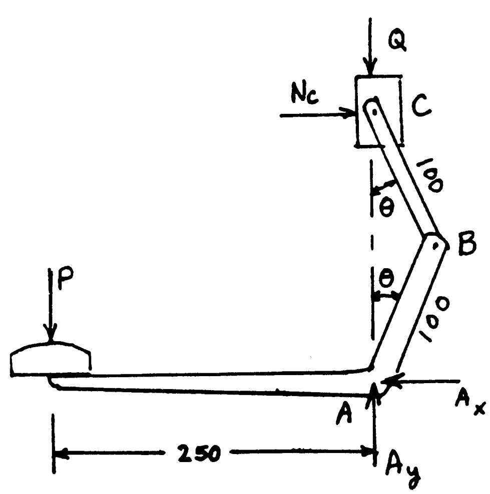
$$\Sigma M_B = 0 \qquad + \circlearrowleft \; 90N_D - 75N_C = 0 \qquad N_D = 373 \text{ N} \blacktriangleleft$$

**4.95**

**$\Sigma M_A = 0$:**

**$\circlearrowleft +$ $P(250) - N_C(200 \cos \theta) = 0$**

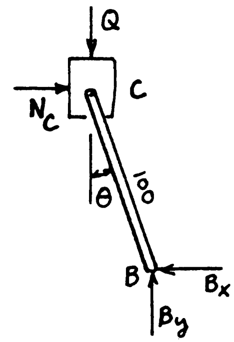
**$\therefore \; N_C = \dfrac{1.25\,P}{\cos \theta}$**



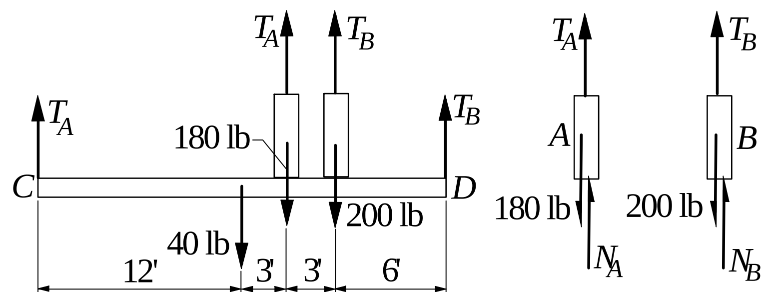
$\Sigma M_B = 0$: $\circlearrowleft +$ $Q(100 \sin\theta) - N_C(100\cos\theta) = 0$

$\therefore\ Q\sin\theta = N_C\cos\theta = \dfrac{1.25\,P}{\cos\theta}\cos\theta$ or $\dfrac{Q}{P} = \dfrac{1.25}{\sin\theta}$

Using $Q/P = 4$, $\theta = \sin^{-1}\left(\dfrac{1.25}{4}\right) = 18.2^\circ$ ◆




**4.96**




**(a)** FBD of entire system:

$$\begin{aligned}
\Sigma F_y &= 0 \qquad +\uparrow\ 2T_A + 2T_B - (40+180+200) = 0 \\
&\quad\ \ T_A + T_B = 210 \\
\Sigma M_D &= 0 \qquad +\circlearrowleft\ 40(12) + 180(9) + 200(6) - T_A(24+9) - T_B(6) = 0 \\
&\quad\ \ 33T_A + 6T_B = 3300
\end{aligned}$$

Solving : $T_A = 75.6$ lb ◄ $\qquad T_B = 134.4$ lb ◄

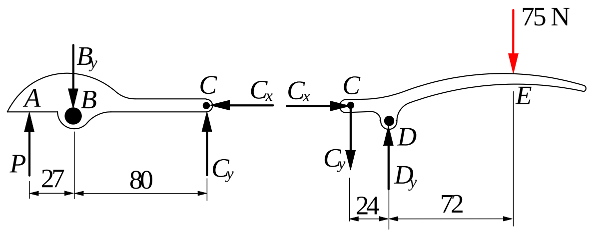
**(b)** FBD of man $A$:

$$\begin{aligned}
\Sigma F_y &= 0 \qquad +\uparrow\ N_A + T_A - 180 = 0 \qquad N_A + 75.6 - 180 = 0 \\
N_A &= 104.4 \text{ lb} ◄
\end{aligned}$$

FBD of man $B$:

$$\begin{aligned}
\Sigma F_y &= 0 \qquad +\uparrow\ N_B + T_B - 200 = 0 \qquad N_B + 134.4 - 200 = 0 \\
N_B &= 65.6 \text{ lb} ◄
\end{aligned}$$

## 4.97



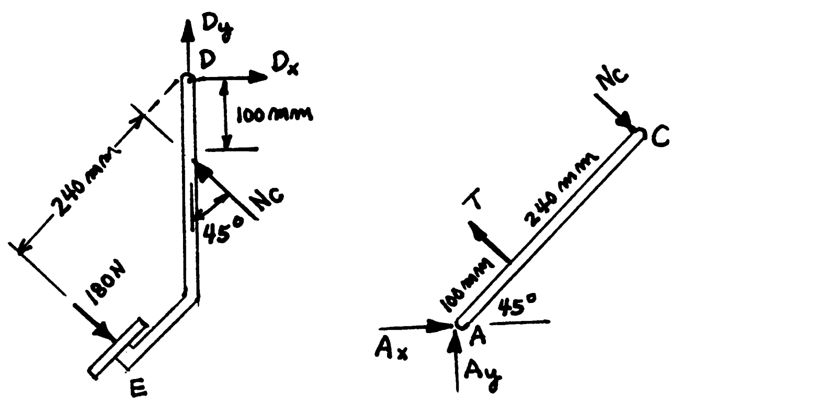

FBD of $CDE$:

$$\Sigma F_x = 0 \qquad + \longrightarrow \qquad C_x = 0$$
$$\Sigma M_D = 0 \qquad + \circlearrowleft \qquad 24C_y - 75(72) = 0 \qquad C_y = 225.0 \text{ N}$$

FBD of $ABC$:

$$\Sigma M_B = 0 \qquad + \circlearrowright \qquad 27P - 225.0(80) = 0 \qquad P = 667 \text{ N} \blacktriangleleft$$

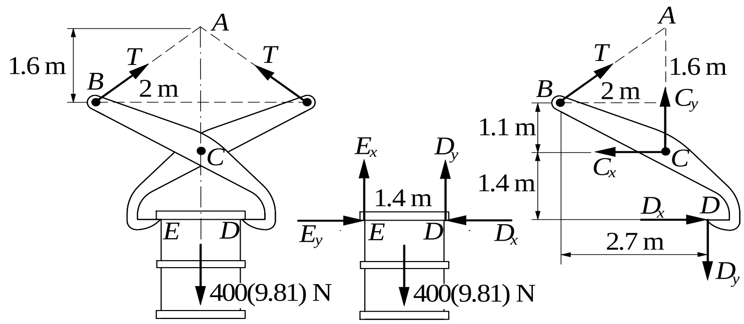
## 4.98




**FBD I** $\Sigma M_D = 0$: $\circlearrowleft +$ $180(240 + 100\sin 45^{\circ}) - N_C \sin 45^{\circ}(100)$ $\therefore$ $N_C = 790.9$ N

**FBD II** $\Sigma M_A = 0$: $\circlearrowleft +$ $T(100) - N_C(340) = 0$ $\therefore$ $T = (340/100)(790.9) = 2690$ N ♦

## 4.99




FBD of the drum:

$$\Sigma M_E = 0 \qquad + \circlearrowleft \quad 1.4D_y - 400(9.81)(0.7) = 0 \qquad D_y = 1962 \text{ N}$$

FBD of tongs with drum:

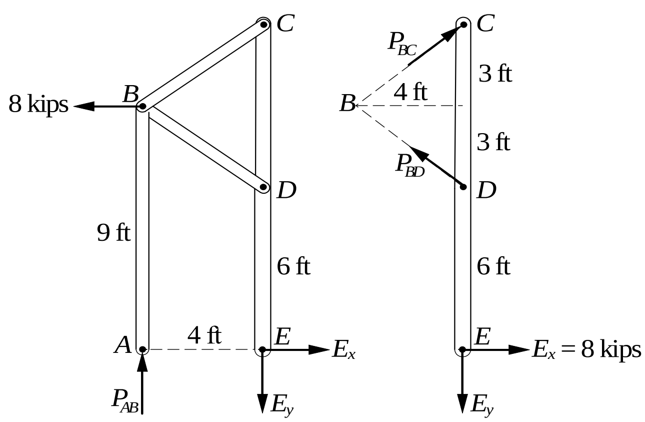
$$\Sigma F_y = 0 \qquad + \uparrow \quad 2T\frac{1.6}{\sqrt{2^2 + 1.6^2}} - 400(9.81) = 0 \qquad T = 3141 \text{ N}$$

FBD of $BCD$:

$$\begin{aligned} \Sigma M_C = 0 \quad + \circlearrowleft \quad 1.4D_x - T\frac{1.6}{\sqrt{2^2 + 1.6^2}}(2) - T\frac{2}{\sqrt{2^2 + 1.6^2}}(1.1) - 0.7D_y &= 0 \\ 1.4D_x - (3141)\frac{1.6}{\sqrt{2^2 + 1.6^2}}(2) - (3141)\frac{2}{\sqrt{2^2 + 1.6^2}}(1.1) - 0.7(1962) &= 0 \\ D_x = 5711 \quad &\text{N} \end{aligned}$$

$$D = \sqrt{5711^2 + 1962^2} = 6040 \text{ N} \blacktriangleleft$$

## 4.100



FBD of entire frame ($AB$ is a 2-force member):

$$\Sigma F_x = 0 \qquad + \longrightarrow \qquad E_x - 8 = 0 \qquad E_x = 8.0 \text{ kips}$$

FBD of member $CDE$ ($BC$ and $BD$ are 2-force members):

$$\Sigma M_D = 0 \qquad + \circlearrowright \quad \frac{4}{5}P_{BC}(6) - 8.0(6) = 0 \qquad P_{BC} = 10.0 \text{ kips} \blacktriangleleft$$

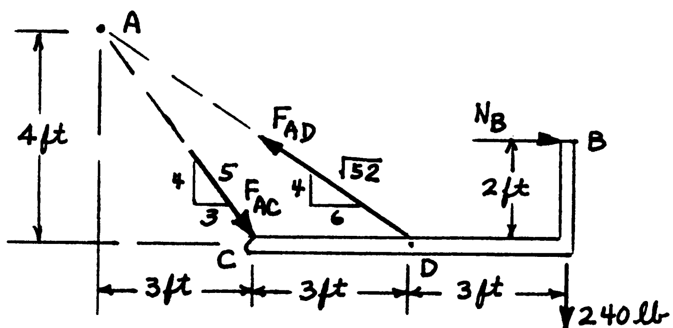
$$\Sigma M_C = 0 \qquad + \circlearrowright \quad \frac{4}{5}P_{BD}(6) - 8.0(12) = 0 \qquad P_{BD} = 20.0 \text{ kips} \blacktriangleleft$$

$$\Sigma F_y = 0 \qquad + \downarrow \quad E_y - \frac{3}{5}(20) - \frac{3}{5}(10) = 0 \qquad E_y = 18.0 \text{ kips}$$

$$E = \sqrt{8.0^2 + 18.0^2} = 19.70 \text{ kips} \blacktriangleleft$$

## 4.101

**AC and AD are two-force members.**




$$\Sigma M_A = 0: \quad \circlearrowleft + \quad N_B(2) - 240(9) = 0 \qquad \therefore\ N_B = 1080 \text{ lb} \ \blacklozenge$$

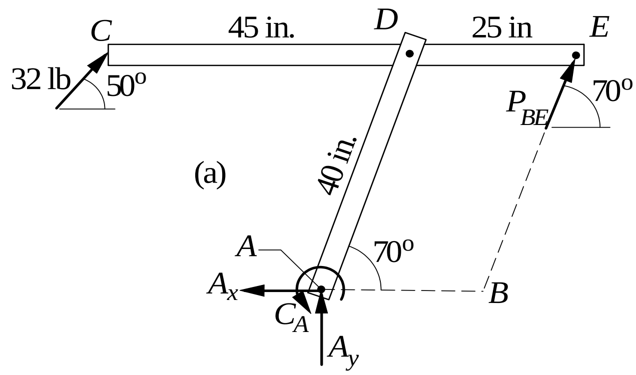
$$\Sigma M_C = 0: \quad \circlearrowleft + \quad \frac{4}{\sqrt{52}} F_{AD}(3) - 240(6) - N_B(2) = 0$$

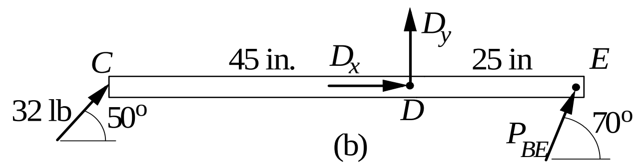
$$\therefore\ F_{AD} = \left(\sqrt{52}/12\right)[240(6) + 1080(2)] = 2160 \text{ lb} \ \blacklozenge$$

$$\Sigma M_D = 0: \quad \circlearrowleft + \quad \frac{4}{5} F_{AC}(3) - N_B(2) - 240(3) = 0$$

$$\therefore\ F_{AC} = (5/12)[1080(2) + 240(3)] = 1200 \text{ lb} \ \blacklozenge$$

## 4.102




From FBD (b) ($BE$ is a two-force member):

$$\Sigma M_D = 0 \quad + \circlearrowleft \quad (P_{BE} \sin 70^\circ)(25) - (32 \sin 50^\circ)(45) = 0$$

$$P_{BE} = 46.96 \text{ lb}$$

From FBD (a):

$$\Sigma M_A = 0 \quad + \circlearrowleft \quad C_A + (P_{BE} \sin 70^\circ)(25) - (32 \cos 50^\circ)(40 \sin 70^\circ)$$

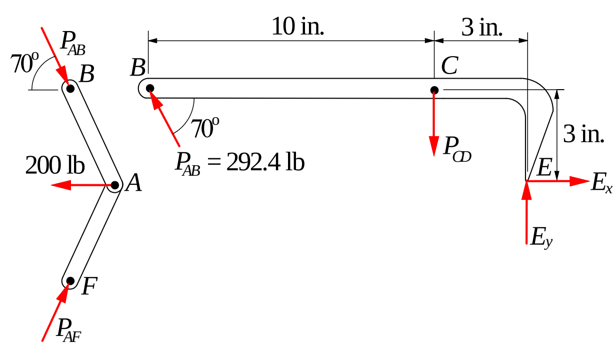
$$-(32 \sin 50^\circ)(45 - 40 \cos 70^\circ) = 0$$

$$C_A + 23.492 P_{BE} - 1540.9 = 0.0$$

$$C_A + 23.49(46.96) - 1540.9 = 0$$

$$C_A = 438 \text{ lb} \cdot \text{in.} \quad ◀$$

## 4.103



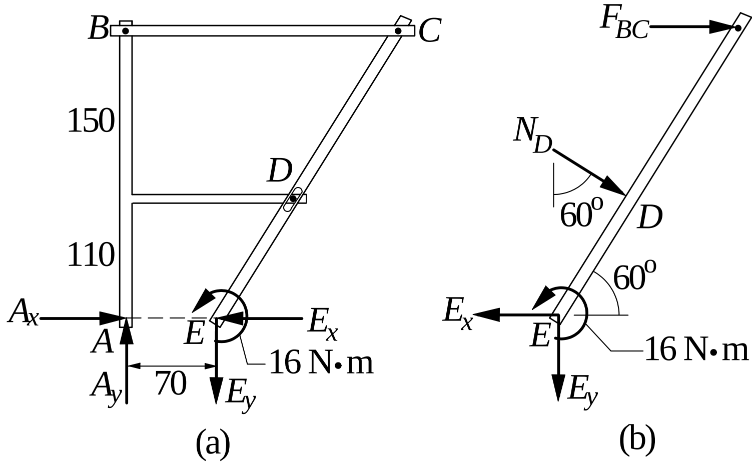
FBD of links $AB$ and $AF$ (both are 2-force members):

$$\begin{aligned}\Sigma F_y &= 0 \text{ yields } P_{AF} = P_{AB} \\ \Sigma F_x &= 0 \quad + \longrightarrow \quad 2P_{AB}\cos 70^\circ - 200 = 0 \qquad P_{AB} = 292.4 \text{ lb}\end{aligned}$$

FBD of $BCE$ ($CD$ is a 2-force member):

$$\begin{aligned}\Sigma F_x &= 0 \quad + \longrightarrow \quad E_x - 292.4\cos 70^\circ = 0 \qquad E_x = 100 \text{ lb} \\ \Sigma M_C &= 0 \quad + \circlearrowleft \quad 3E_y + 100(3) - (292.4\sin 70^\circ)(10) = 0 \qquad E_y = 816 \text{ lb} \blacktriangleleft\end{aligned}$$

**4.104**




From FBD (a):

$$\Sigma M_A = 0 \quad + \circlearrowleft \;\; 0.07E_y - 16 = 0 \qquad E_y = 228.6 \text{ N}$$

From FBD (b) ($BC$ is a two-force member):

$$\begin{aligned}\Sigma F_y &= 0 \quad + \downarrow \;\; E_y + N_D\cos 60^\circ = 0 \qquad N_D = -457.2 \text{ N} \\ \Sigma M_E &= 0 \quad + \circlearrowleft \;\; 0.26F_{BC} + N_D\frac{0.11}{\sin 60^\circ} = 0 \qquad F_{BC} = 223 \text{ N} \blacktriangleleft\end{aligned}$$

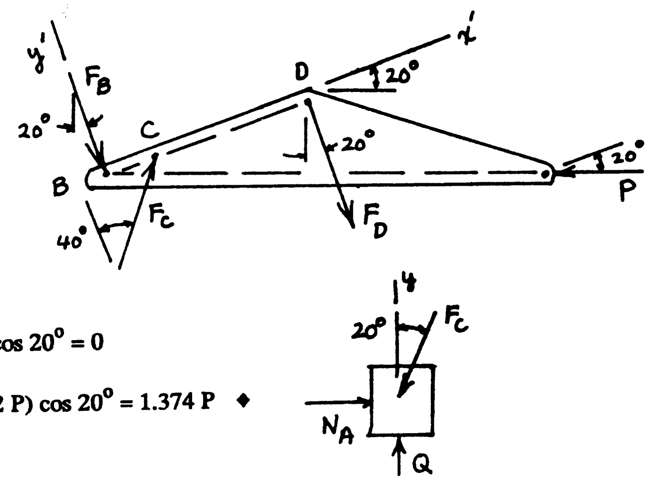
## 4.105

**All bars are two-force members.**

$\Sigma F_{x'} = 0$:

$+\nearrow$ $F_C \sin 40^\circ - P \cos 20^\circ = 0$

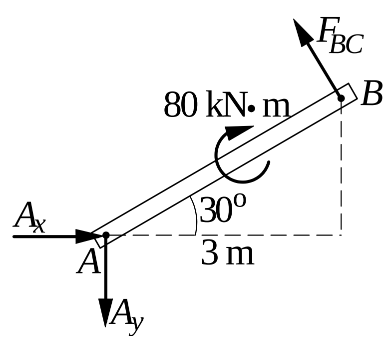
$\therefore$ $F_C = 1.462\ P$




$\Sigma F_y = 0$: $+\uparrow$ $Q - F_C \cos 20^\circ = 0$

$\therefore$ $Q = (1.462\ P) \cos 20^\circ = 1.374\ P$ ◆

## 4.106

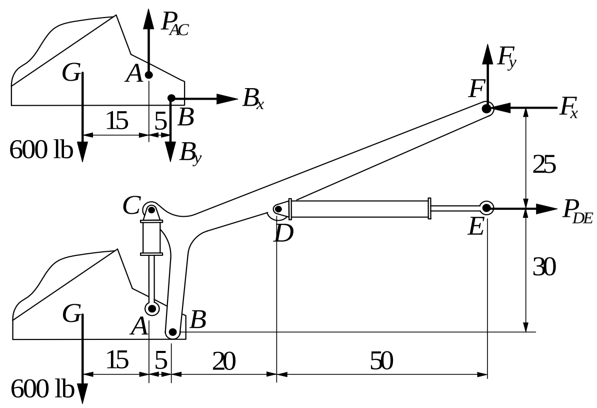



Bar $BC$ is a two-force member.

$$
\begin{aligned}
\Sigma M_A &= 0 \quad + \circlearrowleft\ F_{BC}\frac{3}{\cos 30^\circ} - 80 = 0 \qquad F_{BC} = 23.09\ \text{kN} \\
\Sigma F_x &= 0 \quad + \rightarrow\ A_x - F_{BC}\sin 30^\circ = 0 \qquad A_x = 11.55\ \text{kN} \\
\Sigma F_y &= 0 \quad + \downarrow\ A_y - F_{BC}\cos 30^\circ = 0 \qquad A_y = 20.00\ \text{kN}
\end{aligned}
$$

$$
\begin{aligned}
R_A &= \sqrt{11.55^2 + 20.00^2} = 23.1\ \text{kN} \blacktriangleleft \\
R_C &= F_{BC} = 23.1\ \text{kN} \blacktriangleleft
\end{aligned}
$$

## 4.107




Both cylinders are 2-force members.

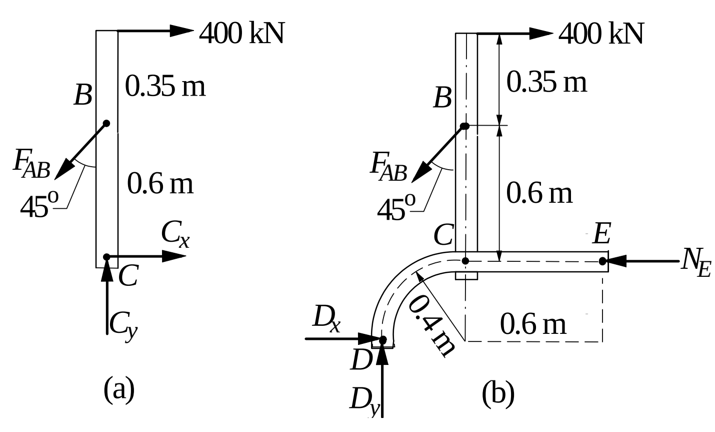
FBD of bucket:

$$\Sigma F_x = 0 \quad + \longrightarrow \quad B_x = 0$$
$$\Sigma M_B = 0 \quad + \circlearrowright \quad 5P_{AC} - 600(20) = 0 \qquad P_{AC} = 2400 \text{ lb} \blacktriangleleft$$

From FBD of mechanism:

$$\Sigma M_F = 0 \quad + \circlearrowleft \quad 25P_{DE} - 600(90) = 0 \quad P_{DE} = 2160 \text{ lb} \blacktriangleleft$$

## 4.108




From FBD (a) (bar $AB$ is a two-force member):

$$\Sigma M_C = 0 \quad + \circlearrowleft \quad (F_{AB} \sin 45°)\,(0.6) - 400(0.95) = 0 \qquad F_{AB} = 895.7 \text{ kN}$$

From FBD (b):

$$\Sigma M_D = 0$$
$$+ \circlearrowleft \quad 0.4N_E - 400(1.35) + 1.0\,(F_{AB}\sin 45^\circ) - 0.4\,(F_{AB}\cos 45^\circ) = 0$$
$$N_E = 400 \text{ kN} \blacktriangleleft$$

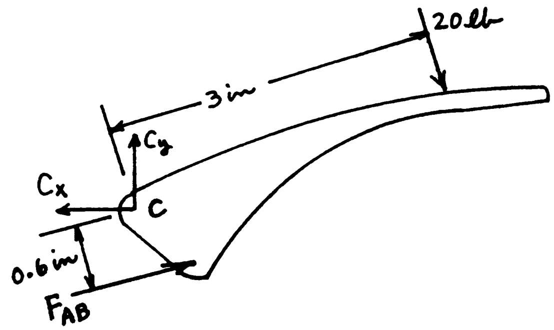
## 4.109

**AB is a two-force member.**

**$\Sigma M_C = 0$:**

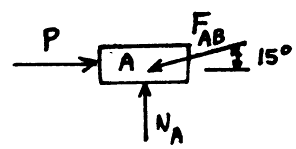
$\circlearrowleft +$ $F_{AB}(0.6) - 20(3) = 0$

$\therefore\ F_{AB} = 100$ lb

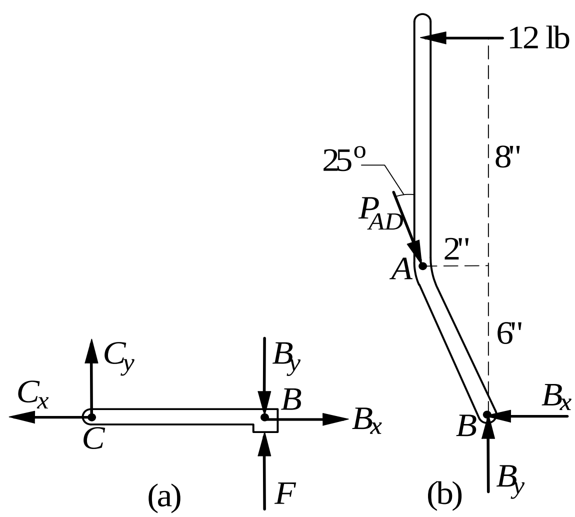



$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P - F_{AB}\cos 15^\circ = 0$

$\therefore\ P = 100 \cos 15^\circ = 96.6$ lb ♦




## 4.110




*AD* is a two-force member.

From FBD (b):

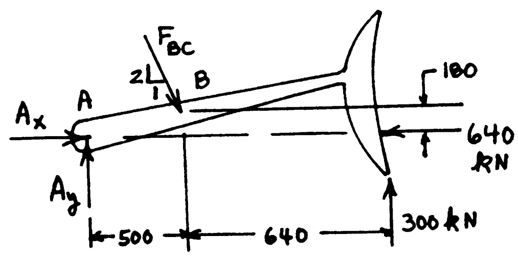
$$\Sigma M_B = 0 \quad +\circlearrowleft \; (P_{AD}\cos 25°)(2) - (P_{AD}\sin 25°)(6) + 12(14) = 0$$
$$P_{AD} = 232.3 \text{ lb}$$
$$\Sigma F_y = 0 \quad +\uparrow \; B_y - P_{AD}\cos 25° = 0 \qquad B_y - 232.3\cos 25° = 0$$
$$B_y = 210.5 \text{ lb}$$

From FBD (a):

$$\Sigma M_C = 0 \qquad +\circlearrowleft \; F - B_y = 0 \qquad F = B_y = 210.5 \text{ lb} \;◄$$

## 4.111

**BC is a two-force member.**




$\Sigma M_A = 0$:

$$+\circlearrowleft \; 300(1140) - \frac{2}{\sqrt{5}}F_{BC}(500) - \frac{1}{\sqrt{5}}F_{BC}(180) = 0$$

$$\therefore \; F_{BC} = 648 \text{ kN}$$

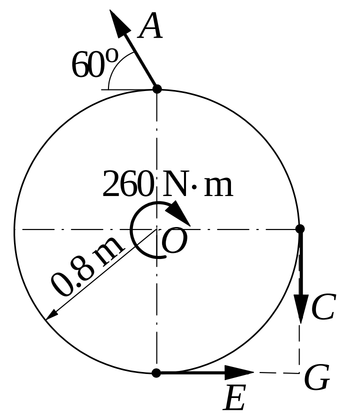
**The pin reactions at B and C each equal 648 kN ◆**

$$\Sigma F_x = 0: \; \overset{+}{\rightarrow} \; A_x + \frac{1}{\sqrt{5}}F_{BC} - 640 = 0 \qquad \therefore \; A_x = -\frac{1}{\sqrt{5}}(648) + 640 = 350.2 \text{ kN}$$

$$\Sigma F_y = 0: \; +\uparrow \; A_y - \frac{2}{\sqrt{5}}F_{BC} + 300 = 0 \qquad \therefore \; A_y = \frac{2}{\sqrt{5}}(648) - 300 = 279.6 \text{ kN}$$

**The pin reaction at A is** $\sqrt{350.2^2 + 279.6^2} = 448$ kN ◆

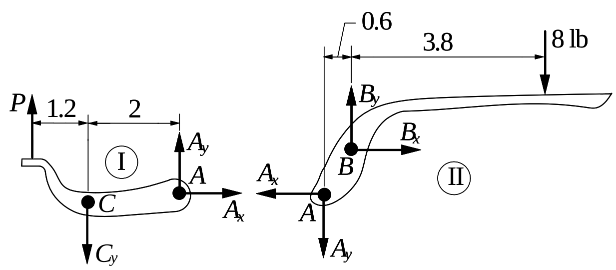
**4.112**




The three bars are two-force members.

$$\begin{aligned}\Sigma M_G &= 0 \quad +\circlearrowleft \;(A\cos 60^\circ)(1.6) - (A\sin 60^\circ)(0.8) - 260 = 0\\ &\quad A = 2426 \text{ N} \;◄\\ \Sigma F_x &= 0 \quad E = A\cos 60^\circ = 2426\cos 60^\circ = 1213 \text{ N} \;◄\\ \Sigma F_y &= 0 \quad C = A\sin 60^\circ = 2426\sin 60^\circ = 2101 \text{ N} \;◄\end{aligned}$$

**4.113**




$CD$ is 2 two-force member.

From FBD I:

$$\begin{aligned}\Sigma F_x &= 0 \quad +\longrightarrow \quad A_x = 0\\ \Sigma M_A &= 0 \quad +\circlearrowleft \quad 2C_y - 3.2P = 0 \qquad C_y = 1.6P\end{aligned}$$

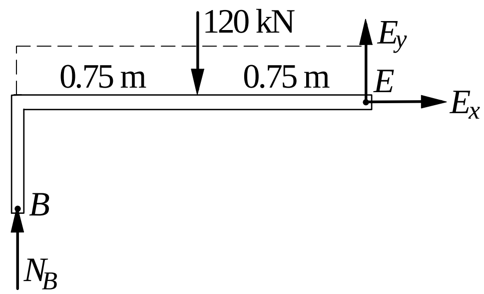
From FBD II:

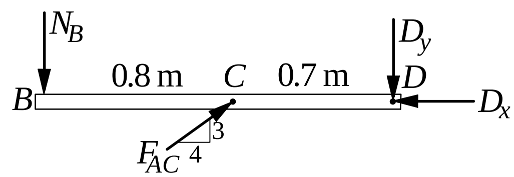
$$\Sigma M_B = 0 \qquad +\circlearrowleft \quad 0.6A_y - 8(3.8) = 0 \qquad A_y = 50.67 \text{ lb}$$

From FBD I:

$$\Sigma F_y = 0 \qquad +\uparrow \quad P - C_y + A_y = 0 \qquad P - 1.6P + 50.67 = 0 \qquad P = 84.5 \text{ lb} \;◄$$

## 4.114




From FBD of bar $BE$:

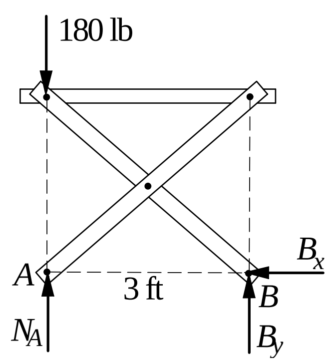
$$\Sigma M_E = 0 \qquad +\circlearrowright\ 1.5N_B - 120(0.75) = 0 \qquad N_B = 60 \text{ kN}$$
$$\Sigma F_y = 0 \qquad +\uparrow\ E_y + N_B - 120 = 0 \qquad E_y = 60 \text{ kN}$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ E_x = 0$$

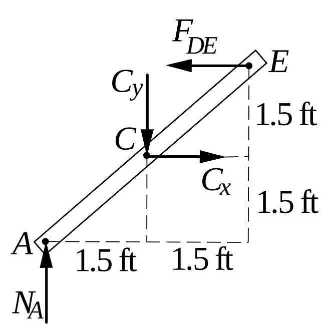
From FBD of bar $BCD$ (bar $AC$ is a two-force member):

$$\Sigma M_D = 0 \qquad +\circlearrowright\ 1.5N_B - \frac{3}{5}F_{AC}(0.7) = 0 \qquad F_{AC} = 214.3 \text{ kN}$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ \frac{4}{5}F_{AC} - D_x = 0 \qquad D_x = 171.4 \text{ kN}$$
$$\Sigma F_y = 0 \qquad +\downarrow\ D_y + N_B - \frac{3}{5}F_{AC} = 0 \qquad D_y = 68.6 \text{ kN}$$

$$R_A = F_{AC} = 214 \text{ kN} \blacktriangleleft$$
$$R_D = \sqrt{171.4^2 + 68.6^2} = 184.6 \text{ kN} \blacktriangleleft$$
$$R_E = 60 \text{ kN} \blacktriangleleft$$

## 4.115




From FBD of entire frame:

$$\Sigma M_B = 0 \qquad +\circlearrowright\ 3N_A - 180(3) = 0 \qquad N_A = 180 \text{ lb}$$

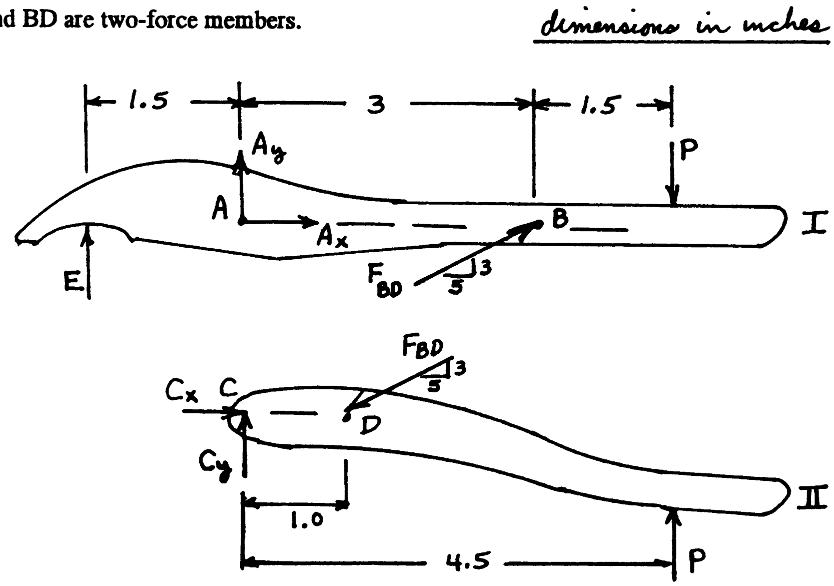
From FBD of $ACE$ (bar $DE$ is a two-force member):

$$\Sigma M_C = 0 \qquad +\circlearrowleft\ 1.5F_{DE} - 1.5N_A = 0 \qquad F_{DE} = 180 \text{ lb}$$
$$\Sigma F_x = 0 \qquad +\rightarrow\ C_x - F_{DE} = 0 \qquad C_x = 180 \text{ lb}$$
$$\Sigma F_y = 0 \qquad +\downarrow\ C_y - N_A = 0 \qquad C_y = 180 \text{ lb}$$

$$R_C = 180\sqrt{2} = 255 \text{ lb} \ \blacktriangleleft$$

## 4.116

**E and BD are two-force members.**

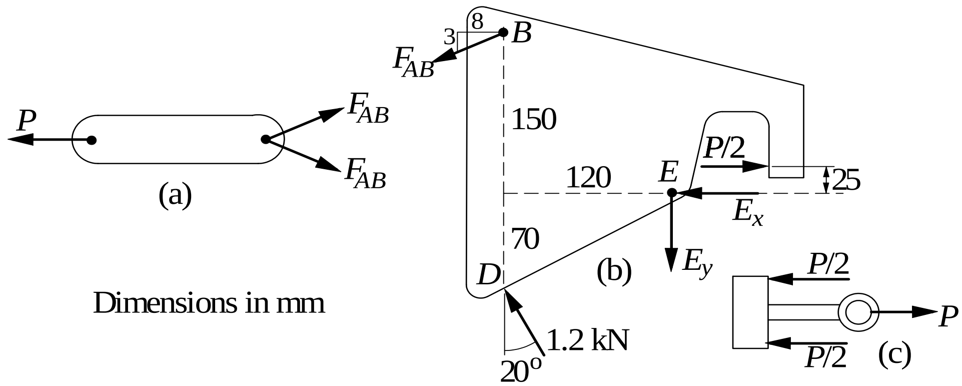



**FBD II** $\Sigma M_C = 0$: $\circlearrowleft + \ P(4.5) - \dfrac{3}{\sqrt{34}} F_{BD}(1.0) = 0 \qquad \therefore\ F_{BD} = \dfrac{3}{2}\sqrt{34}\,P$

**FBD I** $\Sigma M_A = 0$: $\circlearrowleft + \ \dfrac{3}{\sqrt{34}} F_{BD}(3) - E(1.5) - P(4.5) = 0$

$\therefore\ \dfrac{27}{2} P - E(1.5) - P(4.5) = 0 \qquad$ **Solving gives** $E = 6P$ ◆

## 4.117




The two bars connected to pin $A$ are two-force members.

From FBD (a):

$$\Sigma F_x = 0 \qquad 2\left(\frac{8}{\sqrt{73}}F_{AB}\right) - P = 0 \qquad F_{AB} = 0.5340P$$

From FBD (b):

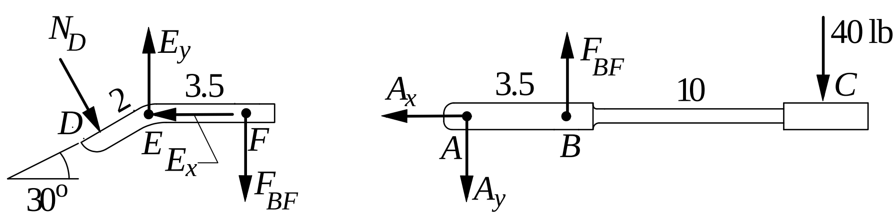
$$\Sigma M_E = 0 \qquad + \circlearrowleft \quad \left(\frac{8}{\sqrt{73}}F_{AB}\right)(150) + \left(\frac{3}{\sqrt{73}}F_{AB}\right)(120)$$

$$-\frac{P}{2}(25) - (1.2\cos 20^\circ)(120) - (1.2\sin 20^\circ)(70) = 0$$

$$182.58F_{AB} - 12.5P - 164.05 = 0$$

$$182.58(0.5340P) - 12.5P - 164.05 = 0 \qquad P = 1.930 \text{ kN} \blacktriangleleft$$

## 4.118




From FBD of $ABC$ (link $BF$ is a two-force member):

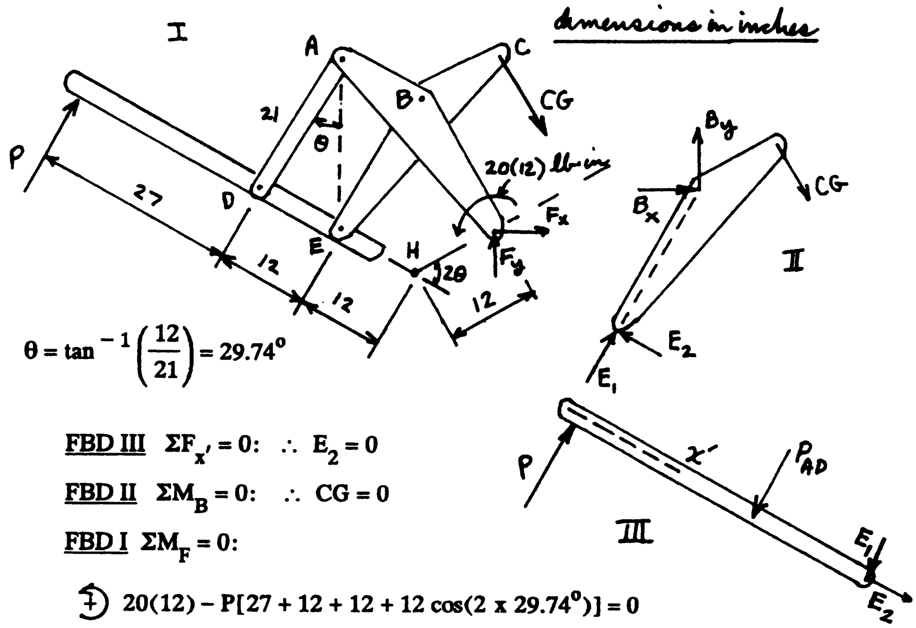
$$\Sigma M_A = 0 \qquad + \circlearrowleft \; 3.5F_{BF} - 40(13.5) = 0 \qquad F_{BF} = 154.29 \text{ lb}$$

From FBD of $DEF$:

$$\Sigma M_E = 0 \qquad + \circlearrowleft \; 2N_D - 3.5F_{BF} = 0 \qquad N_D = 270 \text{ lb} \blacktriangleleft$$

## *4.119

CG and AD are two-force members.




$\theta = \tan^{-1}\left(\frac{12}{21}\right) = 29.74^o$

**FBD III** $\Sigma F_{x'} = 0$: $\therefore\ E_2 = 0$

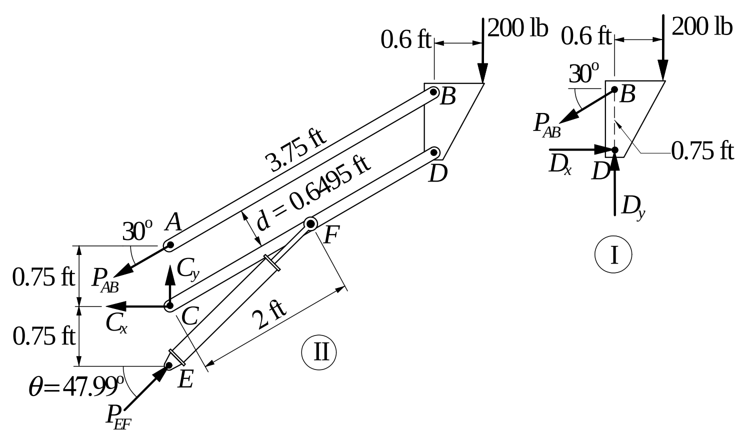
**FBD II** $\Sigma M_B = 0$: $\therefore\ CG = 0$

**FBD I** $\Sigma M_F = 0$:

$$20(12) - P[27 + 12 + 12 + 12\cos(2 \times 29.74^o)] = 0$$

$\therefore\ P = 4.20$ lb ◆

## 4.120




FBD I ($AB$ is a 2-force member):

$\Sigma M_D = 0 \qquad + \circlearrowleft \quad (P_{AB}\cos 30°)(0.75) - 200(0.6) = 0 \qquad P_{AB} = 184.75$ lb

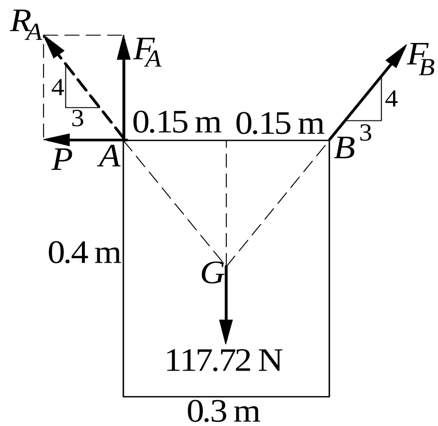
Geometry (see FBD II):

$$d = 0.75\cos 30^\circ = 0.6495 \text{ ft}$$
$$\theta = 30^\circ + \tan^{-1}\frac{d}{2} = 30^\circ + \tan^{-1}\frac{0.6495}{2} = 47.99^\circ$$

FBD II ($EF$ is a 2-force member):

$$\Sigma M_C = 0 \quad + \circlearrowleft \quad (P_{EF}\cos 47.99^\circ)(0.75) + (184.75\cos 30^\circ)(0.75) - 200(3.75\cos 30^\circ + 0.6) = 0$$
$$P_{EF} = 1294 \text{ lb} \blacktriangleleft$$

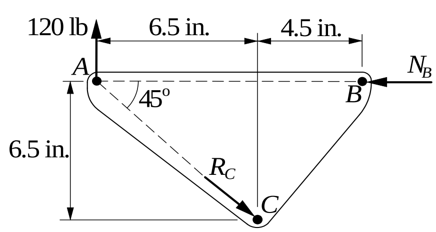
**4.121**




The two links are two-force bodies. Let $R_A$ be the resultant of $P$ and $F_A$. Because the plate is a three-force body, $R_A$ and $F_B$ intersect at $G$ (the mass center of the plate).

$$\Sigma F_x = 0 \qquad R_A = F_B$$
$$\Sigma F_y = 0 \qquad \frac{4}{5}(R_A + F_B) = 117.72 \qquad \frac{8}{5}R_A = 117.72 \qquad R_A = 73.58 \text{ N}$$
$$P = \frac{3}{5}R_A = \frac{3}{5}(73.58) = 44.1 \text{ N} \blacktriangleleft$$

**4.122**

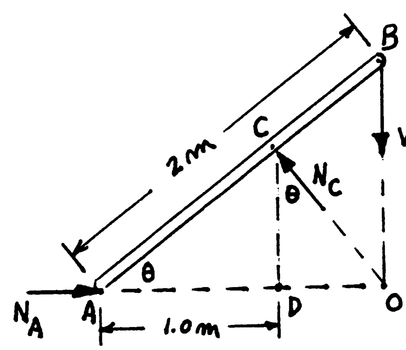


The bracket is a 3-force member. Hence the forces intersect at $A$.

$$\Sigma F_y = 0 \quad +\downarrow \quad R_C \sin 45^\circ - 120 = 0 \qquad R_C = 169.71 \text{ lb} \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\longleftarrow \quad N_B - R_C \cos 45^\circ = 0 \qquad N_B - 169.71 \cos 45^\circ = 0$$
$$N_B = 120.0 \text{ lb} \blacktriangleleft$$

## 4.123

**Bar ACB is a three-force body. Let O refer to the point of intersection of the three forces.**




$\overline{AD} = 1.0$ m $\qquad \overline{AB} = 2.0$ m

$\overline{CD} = \overline{AD}\tan\theta = 1.0\tan\theta$ m

$\overline{DO} = \overline{CD}\tan\theta = \tan^2\theta$ m

By geometry: $\overline{AD} + \overline{DO} = \overline{AB}\cos\theta$

which gives:

$$1 + \tan^2\theta = 2\cos\theta$$

$$1 + \frac{\sin^2\theta}{\cos^2\theta} = 2\cos\theta$$

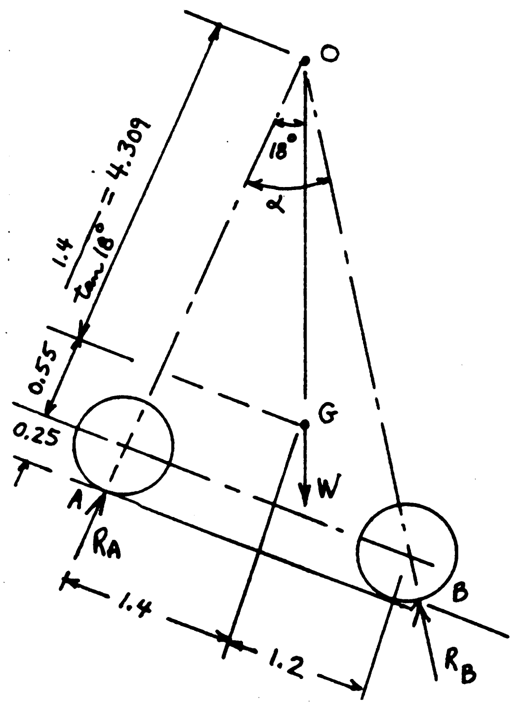
$$\cos^2\theta + \sin^2\theta = 2\cos^3\theta$$

$$1 = 2\cos^3\theta$$

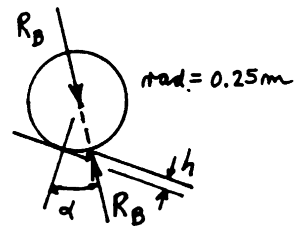
$\therefore \cos^3\theta = 1/2$ which gives $\theta = 37.5^\circ$ ♦

## 4.124

**When the car is about to mount the curb, all wheels are two force members. Therefore, the reactions pass through the axles. The car is a three force body.**




$$\alpha = \tan^{-1}\left(\frac{2.6}{4.309 + 0.550}\right) = 28.15^\circ$$

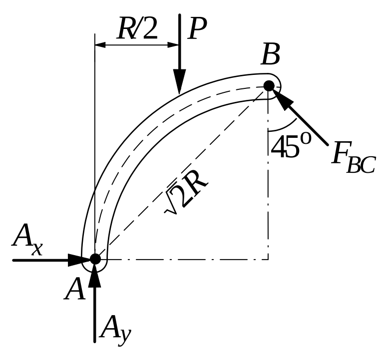



$$\cos\alpha = \frac{0.25 - h}{0.25}$$

$$\therefore \quad h = 0.25(1 - \cos 28.15^\circ)$$

$$= 0.0296 \text{ m} = 29.6 \text{ mm} \quad ◆$$

**4.125**



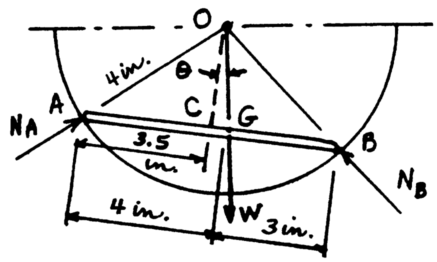

From FBD of $AB$ (segment $BC$ is a two-force member):

$$\Sigma M_A = 0 \qquad + \circlearrowleft \; F_{BC}\left(\sqrt{2}R\right) - P\frac{R}{2} = 0 \qquad F_{BC} = \frac{P}{2\sqrt{2}}$$

$$\Sigma F_x = 0 \qquad + \rightarrow \; A_x - \frac{F_{BC}}{\sqrt{2}} = 0 \qquad A_x = \frac{P}{4} \; ◄$$

## 4.126

The bar is a three-force member. Therefore, $N_A$, $N_B$ and W intersect at O.




From triangle COG,

$$\tan\theta = \frac{\overline{CG}}{\overline{CO}}$$

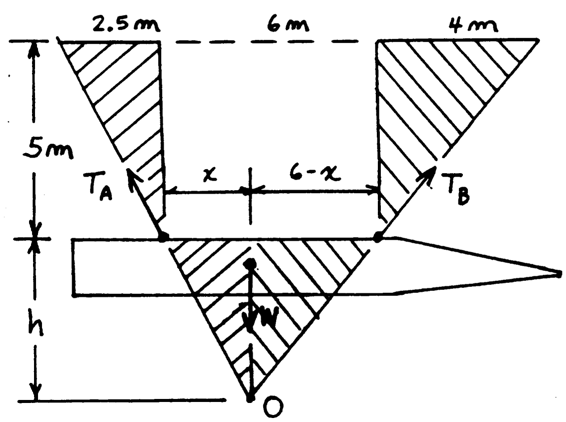
Note that $\overline{CG} = 4.0 - 3.5 = 0.5$ in.

and from triangle AOC, we have

$$\overline{CO} = \sqrt{4^2 - 3.5^2} = 1.936 \text{ in.} \qquad \therefore\ \theta = \tan^{-1}\left(\frac{0.5}{1.936}\right) = 14.48^\circ \ ♦$$

## 4.127

The rocket is a three-force member. Therefore, $T_A$, $T_B$ and W intersect at O.




Using similar triangles,

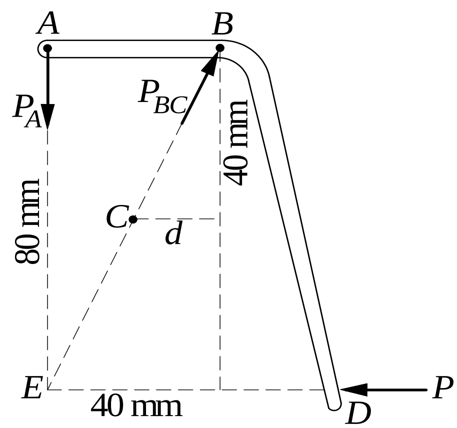
$$\frac{2.5}{5} = \frac{x}{h} \qquad \therefore\ h = 2x$$

Also,

$$\frac{4}{5} = \frac{6-x}{h} = \frac{6-x}{2x}$$

Solving gives: $x = 2.31$ m ♦

**4.128**

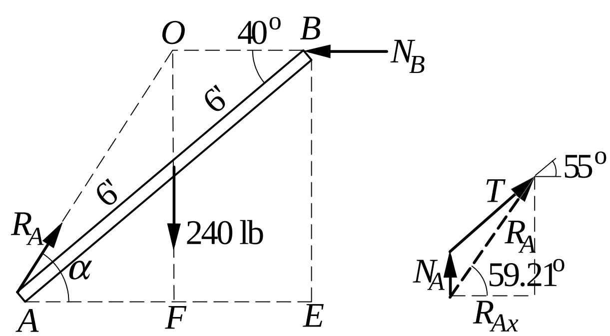



$BC$ is a two-force member. Since $ABD$ is a three-force member, all three forces intersect at point $E$.

From similar triangles:

$$\frac{40}{80} = \frac{d}{40} \qquad d = 20 \text{ mm} \blacktriangleleft$$

**4.129**




Bar $AB$ is a three-force body. All three forces acting on it intersect at point $O$.

From geometry:

$$\begin{aligned}\overline{OF} &= \overline{BE} = 12\sin 40^\circ = 7.714 \text{ ft}\\ \overline{AF} &= 6\cos 40^\circ = 4.596 \text{ ft}\\ \alpha &= \tan^{-1}\frac{\overline{OF}}{\overline{AF}} = \tan^{-1}\frac{7.714}{4.596} = 59.21^\circ\end{aligned}$$

From FBD of bar $AB$:

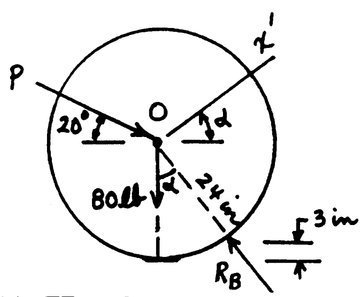
$$\Sigma F_y = 0 \qquad +\uparrow \; R_A \sin\alpha - 240 = 0 \qquad R_A \sin 59.21^\circ - 240 = 0$$
$$R_A = 279.4 \text{ lb}$$

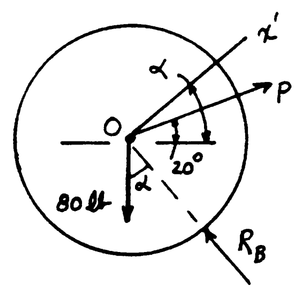
$R_A$ is the resultant of $N_A$ (the normal reaction at $A$) and cable tension $T$. Referring to the figure on the right:

$$R_{Ax} = R_A \cos 59.21^\circ = 279.4 \cos 59.21^\circ = 143.02 \text{ lb}$$
$$T = \frac{R_{Ax}}{\cos 55^\circ} = \frac{143.02}{\cos 55^\circ} = 249 \text{ lb} \blacktriangleleft$$

## 4.130

**In both cases, the roller is a three-force member. Therefore, P, $R_B$ and the 80-lb force intersect at O. $\therefore$ $\alpha = \cos^{-1}(21/24) = 28.96^\circ$**




**(a)** $\Sigma F_{x'} = 0$:

$$+\nearrow \; P\cos(28.96^\circ + 20^\circ) - 80\sin 28.96^\circ = 0$$

$\therefore$ $P = 59.0$ lb ♦

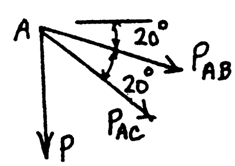
**(b)** $\Sigma F_{x'} = 0$:

$$+\nearrow \; P\cos(28.96^\circ - 20^\circ) - 80\sin 28.96^\circ = 0$$

$\therefore$ $P = 39.2$ lb ♦

## 4.131

**<u>FBD of joint A</u>**




$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P_{AB}\cos 20^\circ + P_{AC}\cos 40^\circ = 0$

$\therefore$ $P_{AC} = -1.2267\,P_{AB}$ (1)

$\Sigma F_y = 0$: $+\downarrow$ $P + P_{AB}\sin 20^\circ + P_{AC}\sin 40^\circ = 0$ (2)

**Substituting (1) into (2) gives:**

$$P + P_{AB}\sin 20^\circ + (-1.2267\,P_{AB})\sin 40^\circ = 0$$

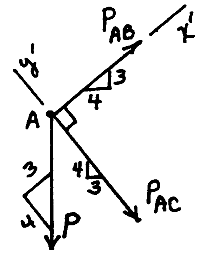
$\therefore$ $P_{AB} = 2.24\,P = 2.24\,P$ (T) ♦ and $P_{AC} = -1.2267(2.24) = -2.75\,P = 2.75\,P$ (C) ♦

## 4.132

**FBD of joint A**

$\Sigma F_{x'} = 0$: $P_{AB} = 0.6\,P = 0.6\,P$ (T) ◆

$\Sigma F_{y'} = 0$: $P_{AC} = -0.8\,P = 0.8\,P$ (C) ◆



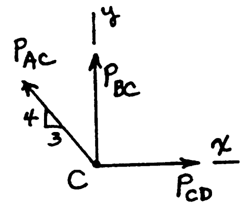

**FBD of joint C**

$\Sigma F_y = 0$: $P_{BC} = -0.8\,P_{AC}$

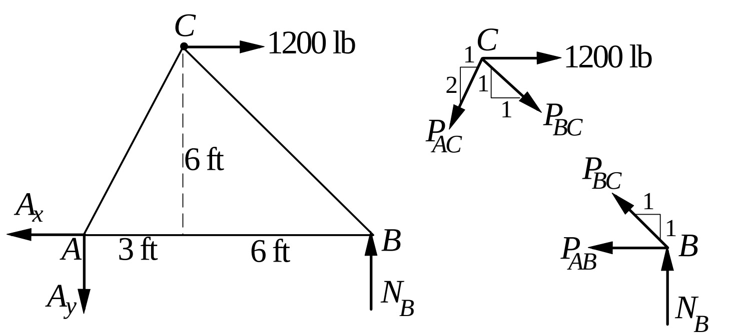
$= -0.8(-0.8\,P) = 0.64\,P = 0.64\,P$ (T) ◆

$\Sigma F_x = 0$: $P_{CD} = 0.6\,P_{AC} = 0.6(-0.8P)$

$= -0.48\,P = 0.48\,P$ (C) ◆




## 4.133




FBD of entire truss:

$$\Sigma M_A = 0 \qquad + \circlearrowleft\; 9N_B - 1200(6) = 0 \qquad N_B = 800 \text{ lb}$$

FBD of joint $B$:

$$\Sigma F_y = 0 \qquad +\uparrow\; \frac{1}{\sqrt{2}}P_{BC} + N_B = 0 \qquad \frac{1}{\sqrt{2}}P_{BC} + 800 = 0$$

$$P_{BC} = -1131.4 \text{ lb} = 1131.4 \text{ lb (C)} \;◄$$

$$\Sigma F_x = 0 \qquad +\longleftarrow\; P_{AB} + \frac{1}{\sqrt{2}}P_{BC} = 0 \qquad P_{AB} + \frac{1}{\sqrt{2}}(-1131.4) = 0$$

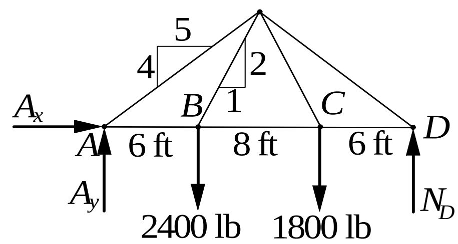
$$P_{AB} = 800 \text{ lb (T)} \;◄$$

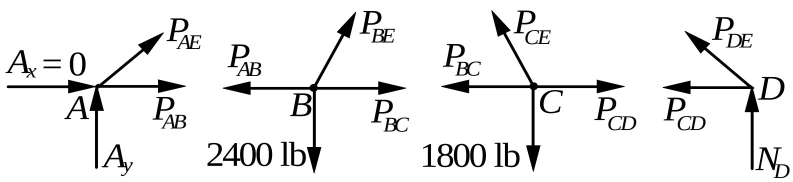
FBD of joint $C$:

$$\Sigma F_y = 0 \qquad +\downarrow\; \frac{2}{\sqrt{5}}P_{AC} + \frac{1}{\sqrt{2}}P_{BC} = 0 \qquad \frac{2}{\sqrt{5}}P_{AC} + \frac{1}{\sqrt{2}}(-1131.4) = 0$$

$$P_{AC} = 894 \text{ lb (T)} \;◄$$

**4.134**




FBD of entire truss:

$$\Sigma M_D = 0 \quad +\circlearrowright \quad 20A_y - 2400(14) - 1800(6) = 0 \qquad A_y = 2220 \text{ lb}$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad A_x = 0$$

FBD of joint $A$:

$$\Sigma F_y = 0 \quad +\uparrow \quad \frac{4}{\sqrt{41}}P_{AE} + A_y = 0 \qquad \frac{4}{\sqrt{41}}P_{AE} + 2220 = 0$$
$$P_{AE} = -3554 \text{ lb} = 3554 \text{ lb (C)} \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{AB} + \frac{5}{\sqrt{41}}P_{AE} = 0 \qquad P_{AB} + \frac{5}{\sqrt{41}}(-3554) = 0$$
$$P_{AB} = 2775 \text{ lb (T)} \blacktriangleleft$$

FBD of joint $B$:

$$\Sigma F_y = 0 \quad +\uparrow \quad \frac{2}{\sqrt{5}}P_{BE} - 2400 = 0 \qquad P_{BE} = 2683 \text{ lb (T)} \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{BC} - P_{AB} + \frac{1}{\sqrt{5}}P_{BE} = 0$$
$$P_{BC} - 2775 + \frac{1}{\sqrt{5}}2683 = 0 \qquad P_{BC} = 1575 \text{ lb (T)} \blacktriangleleft$$

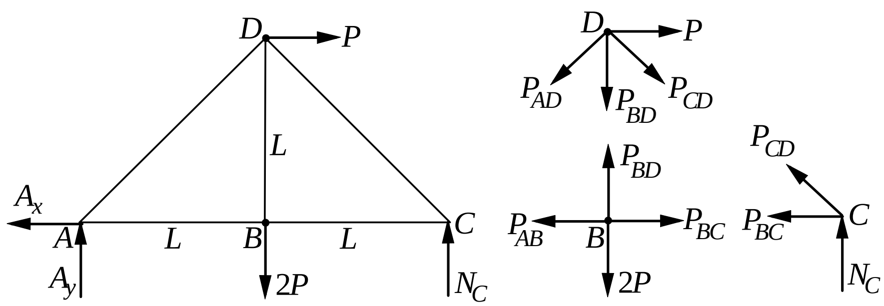
FBD of joint $C$:

$$\Sigma F_y = 0 \quad +\uparrow \quad \frac{2}{\sqrt{5}}P_{CE} - 1800 = 0 \qquad P_{CE} = 2012 \text{ lb (T)} \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{CD} - P_{BC} - \frac{1}{\sqrt{5}}P_{CE} = 0$$
$$P_{CD} - 1575 - \frac{1}{\sqrt{5}}2012 = 0 \qquad P_{CD} = 2475 \text{ lb (T)} \blacktriangleleft$$

FBD of joint $D$:

$$\Sigma F_x = 0 \quad +\leftarrow \quad \frac{5}{\sqrt{41}}P_{DE} + P_{CD} = 0 \qquad \frac{5}{\sqrt{41}}P_{DE} + 2475 = 0$$
$$P_{DE} = -3170 \text{ lb} = 3170\,\text{lb (C)} \blacktriangleleft$$

**4.135**




FBD of entire truss:

$$\Sigma M_A = 0 \qquad +\circlearrowleft \; N_C(2L) - 2PL - PL = 0 \qquad N_C = 1.5P$$

FBD of joint $C$:

$$\Sigma F_y = 0 \quad +\uparrow \; N_C + \frac{1}{\sqrt{2}}P_{CD} = 0 \qquad 1.5P + \frac{1}{\sqrt{2}}P_{CD} = 0$$
$$P_{CD} = -2.121P = 2.121P \text{ (C)} \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\leftarrow \; P_{BC} + \frac{1}{\sqrt{2}}P_{CD} = 0 \qquad P_{BC} + \frac{1}{\sqrt{2}}(-2.121P) = 0$$
$$P_{BC} = 1.500P \text{ (T)} \blacktriangleleft$$

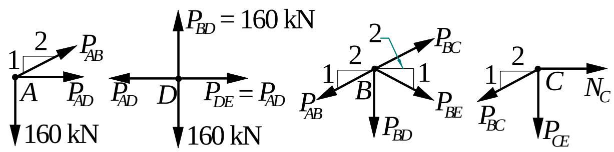
FBD of joint $B$:

$$\Sigma F_x = 0 \quad +\leftarrow \; P_{AB} - P_{BC} = 0 \qquad P_{AB} = P_{BC} = 1.500P \text{ (T)} \blacktriangleleft$$
$$\Sigma F_y = 0 \quad +\uparrow \; P_{BD} - 2P = 0 \qquad P_{BD} = 2P \text{ (T)} \blacktriangleleft$$

FBD of joint $D$:

$$\Sigma F_x = 0 \quad +\leftarrow \; \frac{1}{\sqrt{2}}P_{AD} - \frac{1}{\sqrt{2}}P_{CD} - P = 0$$
$$\frac{1}{\sqrt{2}}P_{AD} - \frac{1}{\sqrt{2}}(-2.121P) - P = 0$$
$$P_{AD} = -0.7068P = 0.7068P \text{ (C)} \blacktriangleleft$$

## 4.136




FBD of joint $A$:

$$\Sigma F_y = 0 \qquad +\uparrow \quad \frac{1}{\sqrt{5}}P_{AB} - 160 = 0 \qquad P_{AB} = 357.8 \text{ kN (T)} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad +\rightarrow \quad P_{AD} + \frac{2}{\sqrt{5}}P_{AB} = 0 \qquad P_{AD} + \frac{2}{\sqrt{5}}357.8 = 0$$

$$P_{AD} = -320 \text{ kN} = 320 \text{ kN (C)} \blacktriangleleft$$

FBD of joint $D$:

$$P_{DE} = P_{AD} = 320 \text{ kN (C)} \blacktriangleleft \qquad P_{BD} = 160 \text{ kN (T)} \blacktriangleleft$$

FBD of joint $B$:

$$\Sigma F_y = 0 \qquad +\uparrow \quad \frac{1}{\sqrt{5}}(P_{BC} - P_{BE} - P_{AB}) - P_{BD} = 0$$

$$\frac{1}{\sqrt{5}}(P_{BC} - P_{BE} - 357.8) - 160 = 0 \qquad (1)$$

$$\Sigma F_x = 0 \qquad +\rightarrow \quad \frac{2}{\sqrt{5}}(P_{BC} + P_{BE} - P_{AB}) = 0$$

$$P_{BC} + P_{BE} - 357.8 = 0 \qquad (2)$$

Solving Eqs. (1) and (2):

$$P_{BC} = 536.7 \text{ kN (T)} \blacktriangleleft \qquad P_{BE} = -178.9 \text{ kN} = 178.9 \text{ kN (C)} \blacktriangleleft$$

FBD of joint $C$:

$$\Sigma F_y = 0 \qquad +\downarrow \quad P_{CE} + \frac{1}{\sqrt{5}}P_{BC} = 0 \qquad P_{CE} + \frac{1}{\sqrt{5}}536.7 = 0$$

$$P_{CE} = -240 \text{ kN} = 240 \text{ kN (C)} \blacktriangleleft$$

## 4.137

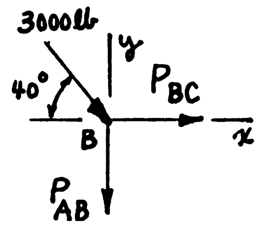
**<u>FBD of entire truss</u> (not shown here)**

**$\Sigma M_A = 0$: $3000\cos 40^\circ(6) = N_D(8)$ $\therefore$ $N_D = 1724$ lb ($\uparrow$)**

**$\Sigma F_y = 0$: $A_y = 3000\sin 40^\circ - N_D = 3000\sin 40^\circ - 1724 = 204.4$ lb ($\uparrow$)**

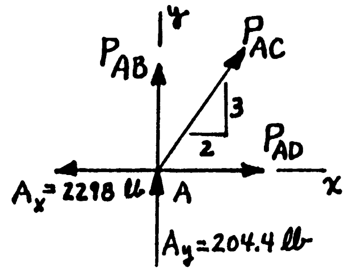
**$\Sigma F_x = 0$: $A_x = 3000\cos 40^\circ = 2298$ lb ($\leftarrow$)**

**FBD of joint B**



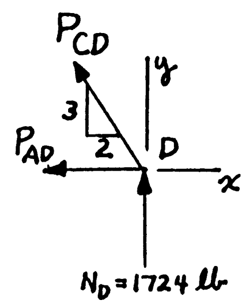

$$\Sigma F_x = 0: \quad P_{BC} = -3000\cos 40^\circ$$
$$= -2298 \text{ lb} = 2298 \text{ lb (C)} \ \blacklozenge$$
$$\Sigma F_y = 0: \quad P_{AB} = -3000\sin 40^\circ$$
$$= -1928 \text{ lb} = 1928 \text{ lb (C)} \ \blacklozenge$$

**FBD of joint A**



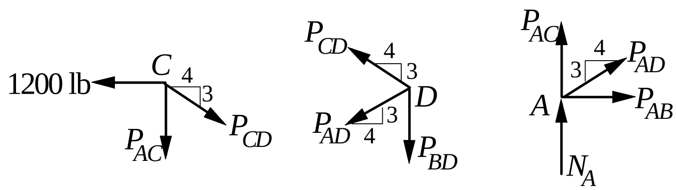

$$\Sigma F_y = 0: \quad +\uparrow \ \frac{3}{\sqrt{13}}P_{AC} + P_{AB} + 204.4 = 0$$
$$\therefore \ P_{AC} = \frac{\sqrt{13}}{3}(-P_{AB} - 204.4)$$
$$= \frac{\sqrt{13}}{3}(1928 - 204.4) = 2072 \text{ lb (T)} \ \blacklozenge$$
$$\Sigma F_x = 0: \quad \overset{+}{\rightarrow} \ P_{AD} + \frac{2}{\sqrt{13}}P_{AC} - 2298 = 0$$
$$\therefore \ P_{AD} = -\frac{2}{\sqrt{13}}(2072) + 2298 = 1149 \text{ lb (T)} \ \blacklozenge$$

**FBD of joint D**




$$\Sigma F_y = 0: \quad +\uparrow \ \frac{3}{\sqrt{13}}P_{CD} + 1724 = 0$$
$$\therefore \ P_{CD} = -\frac{\sqrt{13}}{3}(1724) = -2072 \text{ lb} = 2072 \text{ lb (C)} \ \blacklozenge$$

**4.138**



FBD of joint $C$:

$$\Sigma F_x = 0 \quad +\rightarrow \quad \frac{4}{5}P_{CD} - 1200 = 0 \qquad P_{CD} = 1500 \text{ lb (T)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\downarrow \quad P_{AC} + \frac{3}{5}P_{CD} = 0 \qquad P_{AC} + \frac{3}{5}1500 = 0$$

$$P_{AC} = -900 \text{ lb} = 900 \text{ lb (C)} \blacktriangleleft$$

FBD of joint $D$:

$$\Sigma F_x = 0 \quad +\leftarrow \quad \frac{4}{5}P_{AD} + \frac{4}{5}P_{CD} = 0 \qquad P_{AD} + 1500 = 0$$

$$P_{AD} = -1500 \text{ lb} = 1500 \text{ lb (C)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\downarrow \quad P_{BD} + \frac{3}{5}(P_{AD} - P_{CD}) = 0$$

$$P_{BD} + \frac{3}{5}(-1500 - 1500) = 0 \qquad P_{BD} = 1800 \text{ lb (T)} \blacktriangleleft$$

FBD of joint $A$:

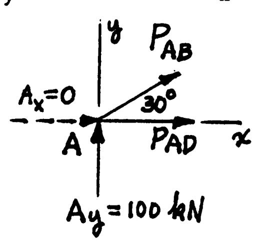
$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{AB} + \frac{4}{5}P_{AD} = 0 \qquad P_{AB} + \frac{4}{5}(-1500) = 0$$

$$P_{AB} = 1200 \text{ lb (T)} \blacktriangleleft$$

## 4.139

**FBD of entire truss** (not shown here) $A_y = C_y = 100$ kN (↑) and $A_x = 0$

**FBD of joint A**



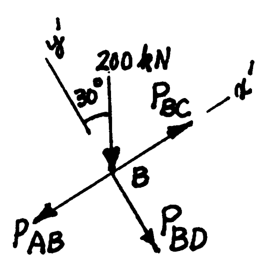

$\Sigma F_y = 0$: $+\uparrow$ $100 + P_{AB}\sin 30^\circ = 0$

$\therefore$ $P_{AB} = -200$ kN $= 200$ kN (C) ♦

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P_{AB}\cos 30^\circ + P_{AD} = 0$

$\therefore$ $P_{AD} = -P_{AB}\cos 30^\circ = -(-200)\cos 30^\circ = 173.2$ kN (T) ♦

**FBD of joint B**




$\Sigma F_{y'} = 0$: $+\nwarrow$ $-200\cos 30^\circ - P_{BD} = 0$

$\therefore$ $P_{BD} = -173.2$ kN $= 173.2$ kN (C) ♦

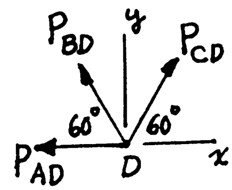
$\Sigma F_{x'} = 0$: $+\nearrow$ $P_{BC} - P_{AB} - 200\sin 30^\circ = 0$

$\therefore$ $P_{BC} = -200 + 100 = -100$ kN $= 100$ kN (C) ♦

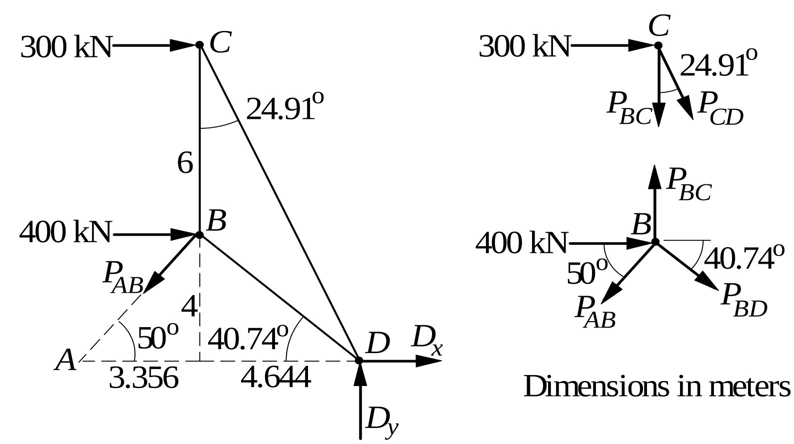
**FBD of joint D**

$\Sigma F_y = 0$: $+\uparrow$ $P_{CD}\sin 60^\circ + P_{BD}\sin 60^\circ = 0$

$\therefore$ $P_{CD} = -P_{BD} = -(-173.2) = 173.2$ kN (T) ◆




## 4.140




FBD of entire truss:

$$\Sigma M_D = 0 \quad +\circlearrowleft \quad (P_{AB}\sin 50^\circ)(8) - 400(4) - 300(10) = 0$$

$$P_{AB} = 750.6 \text{ kN (T)} \blacktriangleleft$$

FBD of joint $B$:

$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{BD}\cos 40.74^\circ - P_{AB}\cos 50^\circ + 400 = 0$$

$$P_{BD}\cos 40.74^\circ - 750.6\cos 50^\circ + 400 = 0$$

$$P_{BD} = 108.9 \text{ kN (T)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\uparrow \quad P_{BC} - P_{AB}\sin 50^\circ - P_{BD}\sin 40.74^\circ = 0$$

$$P_{BC} - 750.6\sin 50^\circ - 108.9\sin 40.74^\circ = 0$$

$$P_{BC} = 646.1 \text{ kN (T)} \blacktriangleleft$$

FFD of joint $C$:

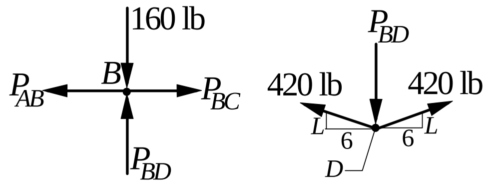
$$\Sigma F_x = 0 \quad +\rightarrow \quad 300 + P_{CD}\sin 24.91^\circ = 0$$

$$P_{CD} = -712 \text{ kN} = 712 \text{ kN (C)} \blacktriangleleft$$

## 4.141

(a) $BD$ and $CD$; (b) $AE$, $BE$ and $CE$; (c) $BD$; (d) $DF$

## 4.142




FBD of joint $B$:

$$\Sigma F_y = 0 \qquad +\uparrow \;\; P_{BD} - 160 = 0 \qquad P_{BD} = 160 \text{ lb}$$

FBD of joint $D$:

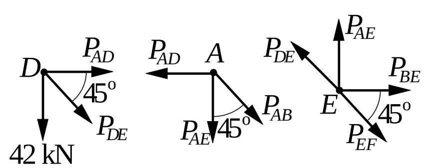
$$\Sigma F_y = 0 \qquad +\uparrow \;\; 2(420)\frac{L}{\sqrt{L^2+6^2}} - P_{BD} = 0$$

$$840\frac{L}{\sqrt{L^2+36}} - 160 = 0$$

$$5.25L = \sqrt{L^2+36} \qquad 27.56L^2 = L^2 + 36$$

$$L = \sqrt{\frac{36}{26.56}} = 1.164 \text{ ft} \;\; \blacktriangleleft$$

## 4.143




FBD of joint $D$:

$$\Sigma F_y = 0 \qquad +\downarrow \;\; P_{DE}\sin 45^\circ + 42 = 0 \qquad P_{DE} = -59.40 \text{ kN}$$

$$\Sigma F_x = 0 \qquad +\rightarrow \;\; P_{AD} + P_{DE}\cos 45^\circ = 0$$

$$P_{AD} + (-59.40)\cos 45^\circ = 0 \qquad P_{AD} = 42.0 \text{ kN}$$

FBD of joint $A$:

$$\Sigma F_x = 0 \qquad +\rightarrow \;\; P_{AB}\sin 45^\circ - P_{AD} = 0 \qquad P_{AB}\sin 45^\circ - 42.0 = 0$$

$$P_{AB} = 59.40 \text{ kN}$$

$$\Sigma F_y = 0 \qquad +\downarrow \;\; P_{AE} + P_{AB}\cos 45^\circ = 0 \qquad P_{AE} + 59.40\cos 45^\circ = 0$$

$$P_{AE} = -42.0 \text{ kN}$$

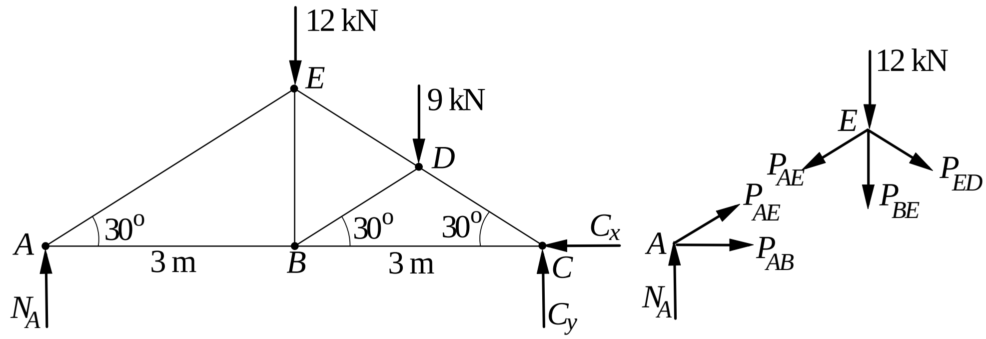
FBD of joint $E$:

$$\Sigma F_y = 0 \qquad +\uparrow \;\; (P_{DE} - P_{EF})\sin 45^\circ + P_{AE} = 0$$

$$(-59.40 - P_{EF})\sin 45^\circ + (-42.0) = 0$$

$$P_{EF} = -118.8 \text{ kN} = 118.8 \text{ kN (C)} \;\; \blacktriangleleft$$

**4.144**




FBD of entire truss:

$$\Sigma M_C = 0 \qquad + \circlearrowleft \; 6N_A - 12(3) - 9(1.5) = 0 \qquad N_A = 8.25 \text{ kN}$$

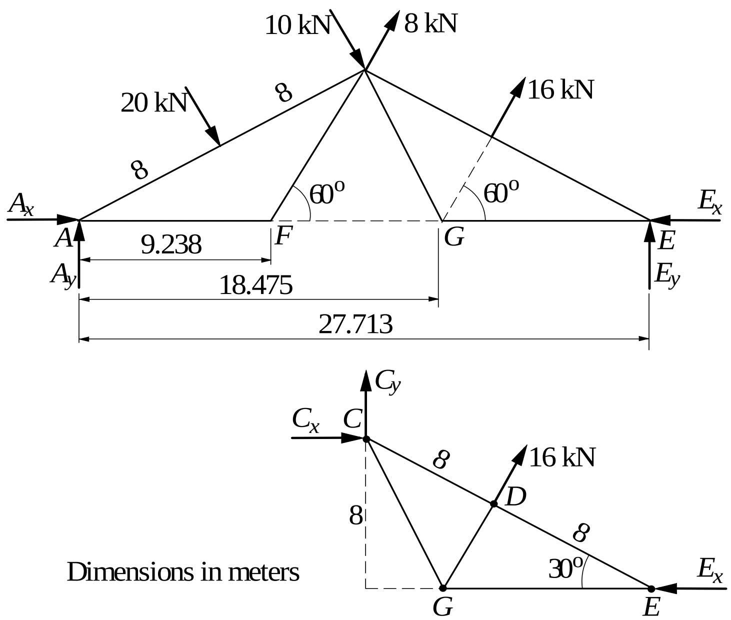
FBD of joint $A$:

$$\Sigma F_y = 0 \qquad + \uparrow \; P_{AE} \sin 30^\circ + N_A = 0 \qquad P_{AE} \sin 30^\circ + 8.25 = 0$$
$$P_{AE} = -16.5 \text{ kN} = 16.5 \text{ kN (C)} \; \blacktriangleleft$$

FBD of joint $E$:

$$\Sigma F_x = 0 \qquad + \rightarrow \; (P_{ED} - P_{AE}) \cos 30^\circ = 0 \qquad P_{ED} - (-16.5) = 0$$
$$P_{ED} = -16.5 \text{ kN} = 16.5 \text{ kN (C)} \; \blacktriangleleft$$
$$\Sigma F_y = 0 \qquad + \downarrow \; P_{BE} + (P_{ED} + P_{AE}) \sin 30^\circ + 12 = 0$$
$$P_{BE} + 2(-16.5) \sin 30^\circ + 12 = 0 \qquad P_{BE} = 4.5 \text{ kN (T)} \; \blacktriangleleft$$

**4.145**

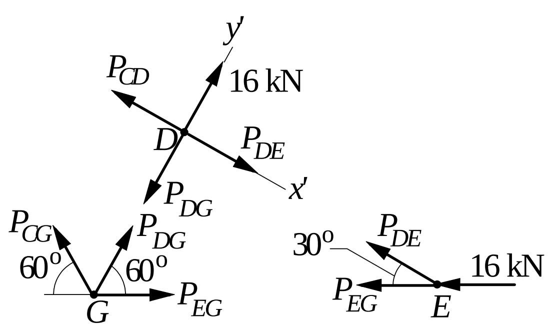



FBD of entire truss:

$$\Sigma M_A = 0 \qquad + \circlearrowleft \ 27.713E_y - 20(8) - 10(16) + (8\sin 60^\circ)(9.238)$$
$$+(16\sin 60^\circ)(18.475) = 0 \qquad E_y = 0$$

FBD of right half of trusss:

$$\Sigma M_C = 0 \qquad + \circlearrowleft \ 16(8) - 8E_x = 0 \qquad E_x = 16 \text{ kN}$$




FBD of joint $E$:

$$\Sigma F_y = 0 \qquad + \uparrow \ P_{DE} = 0 \ \blacktriangleleft$$
$$\Sigma F_x = 0 \qquad + \longleftarrow \ P_{EG} + 16 = 0 \qquad P_{EG} = -16 \text{ kN} = 16 \text{ kN (C)} \ \blacktriangleleft$$

FBD of joint $D$:

$$\Sigma F_{y'} = 0 \quad +\swarrow \; P_{DG} - 16 = 0 \qquad P_{DG} = 16 \text{ kN (T)} \blacktriangleleft$$
$$\Sigma F_{x'} = 0 \quad +\searrow \; P_{DE} - P_{CD} = 0 \qquad 0 - P_{CD} = 0 \qquad P_{CD} = 0 \blacktriangleleft$$

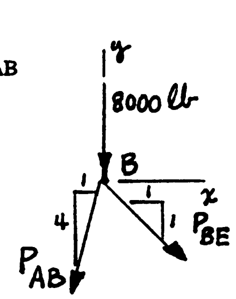
FBD of joint $G$:

$$\Sigma F_y = 0 \quad +\uparrow \; P_{CG} + P_{DG} = 0 \qquad P_{CG} + 16 = 0$$
$$P_{CG} = -16 \text{ kN} = 16 \text{ kN (C)} \blacktriangleleft$$

## 4.146

**From the FBD of the entire truss (not shown here): $A_x = 0$; $A_y = N_D = 8000$ lb ↑**

**FBD of joint B**



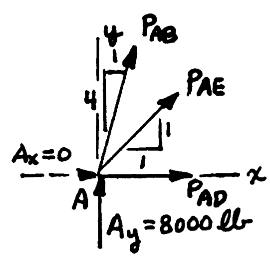

$$\Sigma F_x = 0: \; \xrightarrow{+} \; \frac{1}{\sqrt{2}} P_{BE} - \frac{1}{\sqrt{17}} P_{AB} = 0 \quad \therefore \; P_{BE} = \frac{\sqrt{2}}{\sqrt{17}} P_{AB}$$

$$\Sigma F_y = 0: \; +\uparrow \; -8000 - \frac{4}{\sqrt{17}} P_{AB} - \frac{1}{\sqrt{2}} P_{BE} = 0$$

$$-8000 - \frac{4}{\sqrt{17}} P_{AB} - \frac{1}{\sqrt{17}} P_{AB} = 0$$

$$\therefore \; P_{AB} = -6597 \text{ lb}$$

**FBD of jointA**




$$\Sigma F_y = 0: \; +\uparrow \; 8000 + \frac{4}{\sqrt{17}} P_{AB} + \frac{1}{\sqrt{2}} P_{AE} = 0$$

$$8000 + \frac{4}{\sqrt{17}}(-6597) + \frac{1}{\sqrt{2}} P_{AE} = 0$$

which gives: $P_{AE} = -2263$ lb

$$\Sigma F_x = 0: \; \xrightarrow{+} \; P_{AD} + \frac{1}{\sqrt{2}} P_{AE} + \frac{1}{\sqrt{17}} P_{AB} = 0$$

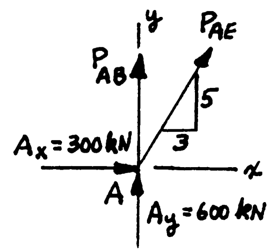
$$\therefore \; P_{AD} = -\frac{1}{\sqrt{2}}(-2263) - \frac{1}{\sqrt{17}}(-6597) = 3200 \text{ lb (T)} \; \blacklozenge$$

## 4.147

From the FBD of the entire truss (not shown here):

$A_x = 300$ kN $\rightarrow$ ; $A_y = 600$ kN $\uparrow$ ; $D_y = 200$ kN $\downarrow$

<u>FBD of joint A</u>




$$\Sigma F_x = 0: \quad \overset{+}{\rightarrow} \quad \frac{3}{\sqrt{34}} P_{AE} + 300 = 0$$

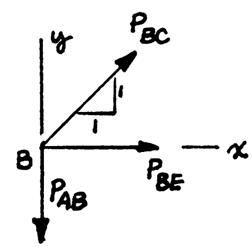
$$\therefore \; P_{AE} = -\frac{300\sqrt{34}}{3}$$

$$= -583.1 \text{ kN} = 583.1 \text{ kN (C)} \; \blacklozenge$$

$$\Sigma F_y = 0: \quad +\uparrow \quad P_{AB} + \frac{5}{\sqrt{34}} P_{AE} + 600 = 0$$

$$\therefore \; P_{AB} = -\frac{5}{\sqrt{34}}(-583.1) - 600 = -100 \text{ kN} = 100 \text{ kN (C)} \; \blacklozenge$$

<u>FBD of joint B</u>




$$\Sigma F_y = 0: \quad +\uparrow \quad \frac{1}{\sqrt{2}} P_{BC} - P_{AB} = 0$$

$$\therefore \; P_{BC} = \sqrt{2}\, P_{AB} = \sqrt{2}\,(-100)$$

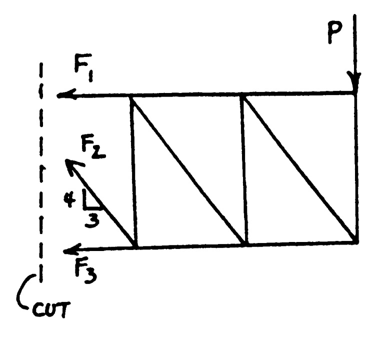
$$= -141.4 \text{ kN} = 141.4 \text{ kN (C)} \; \blacklozenge$$

$$\Sigma F_x = 0: \quad \overset{+}{\rightarrow} \quad P_{BE} + \frac{1}{\sqrt{2}} P_{BC} = 0$$

$$\therefore \; P_{BE} = -\frac{1}{\sqrt{2}}(-141.4) = 100 \text{ kN (T)} \; \blacklozenge$$

## 4.148

<u>FBD of a section to the right of a vertical cut</u>

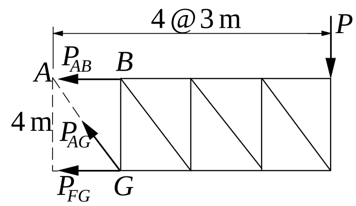



$$\Sigma F_y = 0: \quad \frac{4}{5} F_2 = P$$

$$\therefore \; F_2 = 1.25\,P = 1.25\,P \text{ (T)} \; \blacklozenge$$

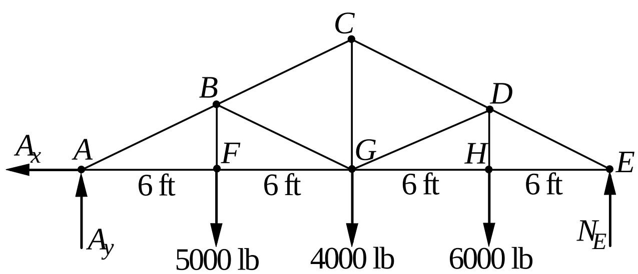
Since the FBD is the same regardless of the panel in which the cut is made, all diagonal members carry the same force: 1.25 P (T). ◆

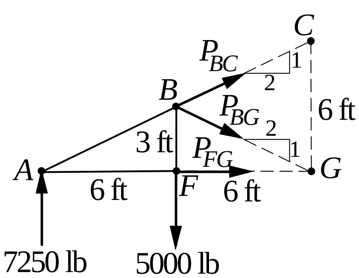
## 4.149




$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad 4P_{FG} + 12P = 0 \qquad P_{FG} = -3P = 3P \text{ (C)} \blacktriangleleft$$
$$\Sigma M_G = 0 \qquad +\circlearrowleft \quad 4P_{AB} - 9P = 0 \qquad P_{AB} = 2.25P \text{ (T)} \blacktriangleleft$$

## 4.150



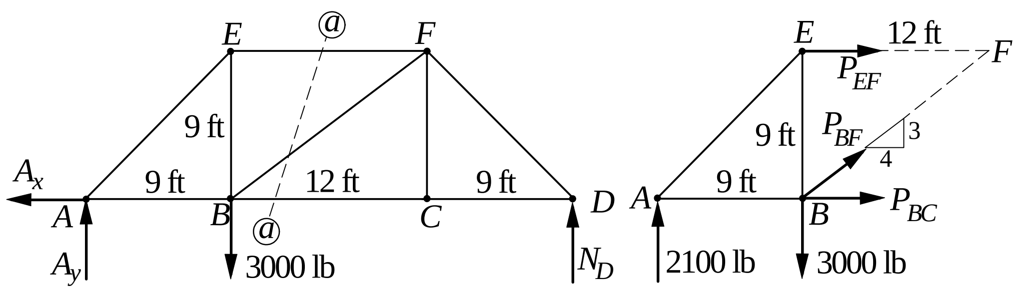

FBD of entire truss:

$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad 24N_E - 5000(6) - 4000(12) - 6000(18) = 0$$
$$N_E = 7750 \text{ lb}$$
$$\Sigma F_x = 0 \qquad +\longleftarrow \quad A_x = 0$$
$$\Sigma F_y = 0 \qquad +\uparrow \quad A_y + N_E - 5000 - 4000 - 6000 = 0$$
$$A_y + 7750 - 5000 - 4000 - 6000 = 0 \qquad A_y = 7250 \text{ lb}$$

FBD of part $ABF$:

$$\Sigma M_B = 0 \qquad +\circlearrowleft \quad 3P_{FG} - 7250(6) = 0 \qquad P_{FG} = 14\,500 \text{ lb (T)} \blacktriangleleft$$
$$\Sigma M_G = 0 \qquad +\circlearrowleft \quad \frac{2}{\sqrt{5}}P_{BC}(6) + 7250(12) - 5000(6) = 0$$
$$P_{BC} = -10\,620 \text{ lb} = 10\,620 \text{ lb (C)} \blacktriangleleft$$
$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad \frac{1}{\sqrt{5}}P_{BG}(12) + 5000(6) = 0$$
$$P_{BG} = -5590 \text{ lb} = 5590 \text{ lb (C)} \blacktriangleleft$$

**4.151**



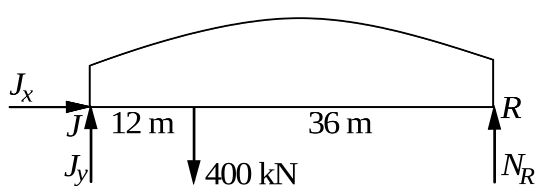

FBD of entire truss:

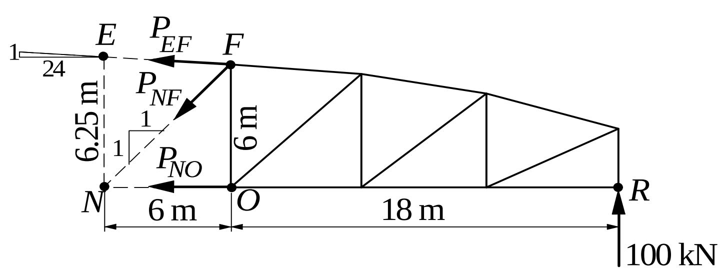
$$\Sigma M_D = 0 \quad +\circlearrowright \; 30A_y - 3000(21) = 0 \qquad A_y = 2100 \text{ lb}$$
$$\Sigma F_x = 0 \quad +\leftarrow \; A_x = 0$$

FBD of truss left of section $a-a$:

$$\Sigma M_B = 0 \quad +\circlearrowright \; 9P_{EF} + 2100(9) = 0$$
$$P_{EF} = -2100 \text{ lb} = 2100 \text{ lb (C)} \blacktriangleleft$$
$$\Sigma F_y = 0 \quad +\uparrow \; \frac{3}{5}P_{BF} + 2100 - 3000 = 0 \qquad P_{BF} = 1500 \text{ lb (T)} \blacktriangleleft$$
$$\Sigma M_F = 0 \quad +\circlearrowleft \; 9P_{BC} - 2100(21) + 3000(12) = 0$$
$$P_{BC} = 900 \text{ lb (T)} \blacktriangleleft$$

**4.152**




FBD of entire truss:

$$\Sigma M_J = 0 \quad +\circlearrowleft \; 48N_R - 400(12) = 0 \qquad N_R = 100 \text{ kN}$$

FBD of right portion of truss:

$$\Sigma M_F = 0 \quad +\circlearrowright \; 6P_{NO} - 100(18) = 0 \qquad P_{NO} = 300 \text{ kN (T)} \blacktriangleleft$$

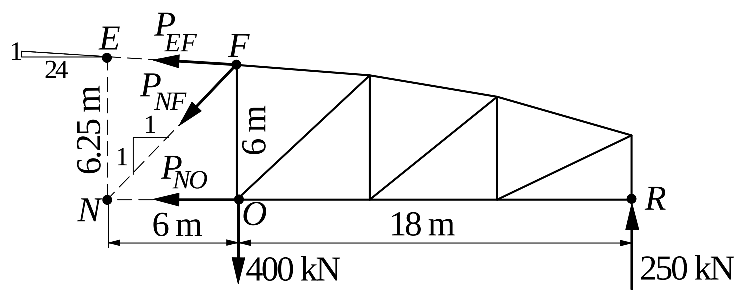
$$\Sigma M_N = 0 \quad +\circlearrowleft \; \left(\frac{24}{\sqrt{577}} P_{EF}\right)(6.25) + 100(24) = 0$$

$$P_{EF} = -384.3 \text{ kN} = 384.3 \text{ kN (C)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\downarrow \; \frac{1}{\sqrt{2}} P_{NF} - \frac{1}{\sqrt{577}} P_{EF} - 100 = 0$$

$$\frac{1}{\sqrt{2}} P_{NF} - \frac{1}{\sqrt{577}} 384.3 - 100 = 0 \qquad P_{NF} = 164.1 \text{ kN (T)} \blacktriangleleft$$

**4.153**




From FBD of entire truss (not shown):

$$\Sigma M_J = 0 \qquad +\circlearrowleft \; 48N_R - 400(30) = 0 \qquad N_R = 250 \text{ kN}$$

From FBD of right portion of truss:

$$\Sigma M_F = 0 \quad +\circlearrowright \; 6P_{NO} - 250(18) = 0 \qquad P_{NO} = 750 \text{ kN (T)} \blacktriangleleft$$

$$\Sigma M_N = 0 \quad +\circlearrowleft \; \left(\frac{24}{\sqrt{577}} P_{EF}\right)(6.25) - 400(6) + 250(24) = 0$$

$$P_{EF} = -576.5 \text{ kN} = 576.5 \text{ kN (C)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\uparrow \; \frac{1}{\sqrt{577}} P_{EF} - \frac{1}{\sqrt{2}} P_{NF} + 250 - 400 = 0$$

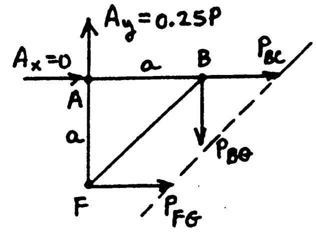
$$\frac{1}{\sqrt{577}}(-576.5) - \frac{1}{\sqrt{2}} P_{NF} - 150 = 0$$

$$P_{NF} = 246.1 \text{ kN} = 246.1 \text{ kN (C)} \blacktriangleleft$$

4.154

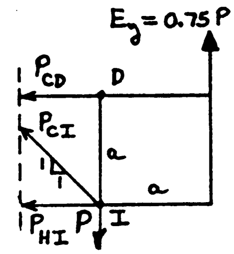
From the analysis of the FBD of the entire truss (not shown here), we obtain

$$A_x = 0,\ A_y = 0.250\ P\ \uparrow,\ \text{and}\ E_y = 0.750\ P\ \uparrow$$




FBD of section shown

$\Sigma F_y = 0$: $P_{BG} = 0.250\ P = 0.250\ P$ (T) ◆




FBD of section shown

$\Sigma F_y = 0$: $+\uparrow\ 0.75\ P - P + \left(1/\sqrt{2}\right) P_{CI} = 0$

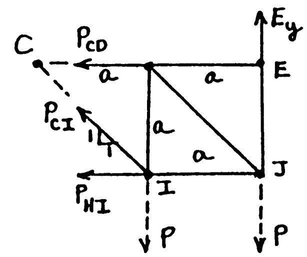
$\therefore\ P_{CI} = \sqrt{2}\ (P - 0.750\ P) = 0.354\ P = 0.354\ P$ (T) ◆

$\Sigma M_I = 0$: $\circlearrowleft +$ $0.750\ P(a) + P_{CD}(a) = 0$

$\therefore\ P_{CD} = -0.750\ P = 0.750\ P$ (C) ◆

4.155

From the analysis of the entire truss (not shown here), the vertical reaction at E depends on the location of P as follows : P at J: $E_y = P$; P at I: $E_y = 0.750\ P$; P at H: $E_y = 0.500\ P$; P at G: $E_y = 0.250\ P$; P at F: $E_y = 0$. (In each case, $E_y$ is directed upward.)




FBD of the section shown

If P acts at J ($E_y = P$)

$\Sigma F_y = 0$ gives $P_{CI} = 0$

$\Sigma M_C = 0$ gives $P_{HI} = 0$

If P acts at I ($E_y = 0.750\ P$)

$\Sigma F_y = 0$: $+\uparrow\ E_y - P + \left(1/\sqrt{2}\right) P_{CI} = 0$

$\therefore\ P_{CI} = \sqrt{2}\left(P - E_y\right) = \sqrt{2}\left(P - 0.750\ P\right) = 0.3536\ P = 0.3536\ P$ (T)

$\Sigma M_C = 0$: $\circlearrowleft +$ $E_y(2\,a) - P(a) - P_{HI}(a) = 0$

$\therefore\ P_{HI} = 2(0.750)P - P = 0.500\ P = 0.500\ P$ (T)

**<u>If P acts at H, G or F</u>**

$\Sigma F_y = 0$: $+\uparrow$ $E_y + (1/\sqrt{2})P_{CI} = 0$ $\therefore$ $P_{CI} = -\sqrt{2}\ E_y = \sqrt{2}\ E_y$ (C)

$\Sigma M_C = 0$: $\circlearrowleft +$ $E_y(2\ a) - P_{HI}(a) = 0$ $\therefore$ $P_{HI} = 2\ E_y = 2\ E_y$ (T)

**(a)** The maximum tension occurs in HI when P acts at H ($E_y = 0.500$ P).

$\therefore$ Max tension in HI = $2\ E_y = 2(0.500\ P) = P = 48$ kips ♦

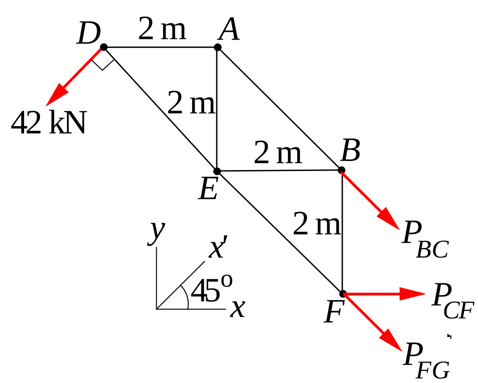
**(b)** The maximum compression in CI occurs when P acts at H ($E_y = 0.500$ P).

$\therefore$ Max compression in CI = $\sqrt{2}\ E_y = \sqrt{2}\ (0.500\ P) = \sqrt{2}\ (0.500)(48) = 33.9$ kips ♦

**(c)** The maximum tension in CI occurs when P acts at I.

$\therefore$ Max tension in CI = $0.3536\ P = 0.3536(48) = 16.97$ kips ♦

**4.156**



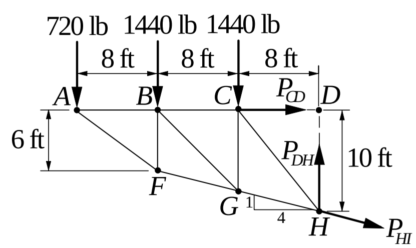

$$\Sigma M_F = 0 \qquad + \circlearrowright \ P_{BC}(\sqrt{2}) - 42(4\sqrt{2}) = 0 \qquad P_{BC} = 168.0 \text{ kN (T)} \ \blacktriangleleft$$

$$\Sigma F_y = 0 \qquad + \downarrow \ \frac{1}{\sqrt{2}}(P_{FG} + P_{BC} + 42) = 0 \qquad P_{FG} + 168.0 + 42 = 0$$

$$P_{FG} = -210.0 \text{ kN} = 210.0 \text{ kN (C)} \ \blacktriangleleft$$

$$\Sigma F_{x'} = 0 \qquad + \nearrow \ \frac{1}{\sqrt{2}}P_{CF} - 42 = 0 \qquad P_{CF} = 59.4 \text{ kN (T)} \ \blacktriangleleft$$

**4.157**



$$\Sigma M_H = 0 \quad +\circlearrowleft \quad 10P_{CD} - 720(24) - 1440(16+8) = 0$$

$$P_{CD} = 5184 \text{ lb (T)} \blacktriangleleft$$

$$\Sigma M_D = 0 \quad +\circlearrowleft \quad \left(\frac{4}{\sqrt{17}}P_{HI}\right)(10) + 720(24) + 1440(16+8) = 0$$

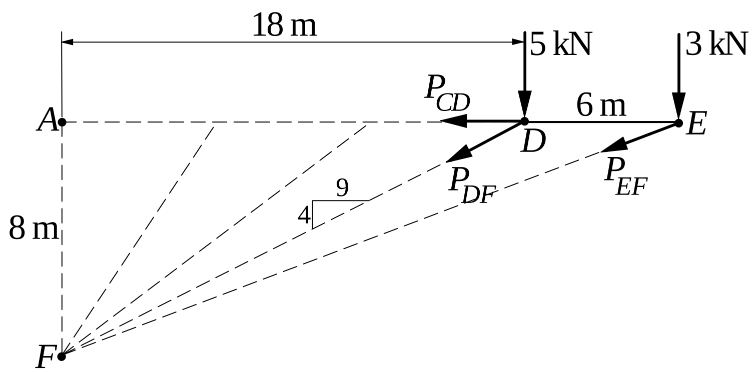
$$P_{HI} = -5344 \text{ lb} = 5344 \text{ lb (C)} \blacktriangleleft$$

$$\Sigma F_y = 0 \quad +\uparrow \quad P_{DH} - \frac{1}{\sqrt{17}}P_{HI} - 720 - 2(1440) = 0$$

$$P_{DH} - \frac{1}{\sqrt{17}}(-5344) - 720 - 2(1440) = 0$$

$$P_{DH} = 2300 \text{ lb (T)} \blacktriangleleft$$

**4.158**

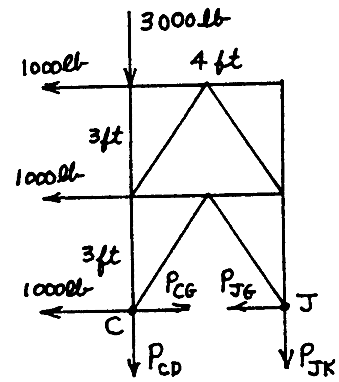



$$\Sigma M_F = 0 \quad +\circlearrowleft \quad 8P_{CD} - 5(18) - 3(24) = 0 \qquad P_{CD} = 20.25 \text{ kN (T)} \blacktriangleleft$$

$$\Sigma M_E = 0 \quad +\circlearrowleft \quad \left(\frac{4}{\sqrt{97}}P_{DF}\right)(6) + 5(6) = 0$$

$$P_{DF} = -12.31 \text{ kN} = 12.31 \text{ kN (C)} \blacktriangleleft$$

4.159




FBD of section shown

$\Sigma M_J = 0$:

$\circlearrowleft + \; P_{CD}(4) + 3000(4) + 1000(6) + 1000(3) = 0$

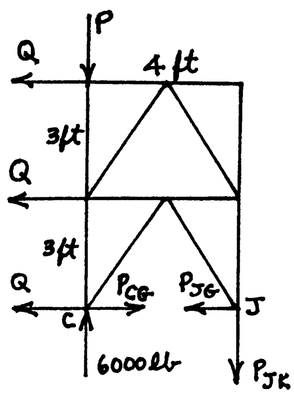
$\therefore \; P_{CD} = -5250 \text{ lb} = 5250 \text{ lb (C)}$ ◆

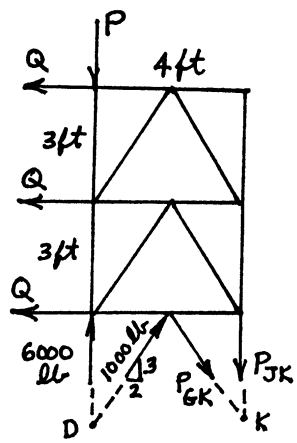
$\Sigma M_C = 0$:

$\circlearrowleft + \; -P_{JK}(4) + 1000(6) + 1000(3) = 0$

$\therefore \; P_{JK} = 2250 \text{ lb} = 2250 \text{ lb (T)}$ ◆

4.160




FBD of section shown above

$\Sigma M_J = 0$:

$\circlearrowleft + \; P(4) + Q(3+6) - 6000(4) = 0$

$\therefore \; 4P + 9Q = 24\,000 \text{ lb}\cdot\text{ft} \quad (1)$

FBD of section shown above

$\Sigma M_K = 0$:

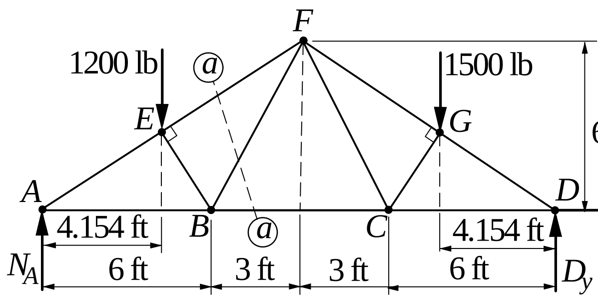
$\circlearrowleft + \; P(4) + Q(3+6+9) - 6000(4) - \frac{3}{\sqrt{13}}(1000)(4) = 0$

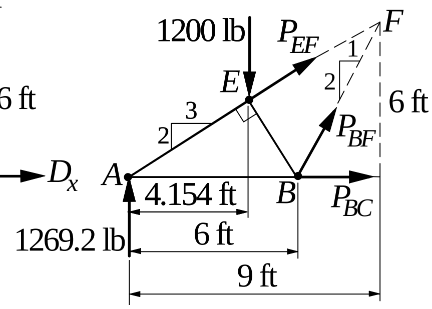
$\therefore \; 4P + 18Q = 27\,328 \text{ lb}\cdot\text{ft} \quad (2)$

Solving (1) and (2) gives:

$P = 5170 \text{ lb}$ ◆ and $Q = 370 \text{ lb}$ ◆

**4.161**



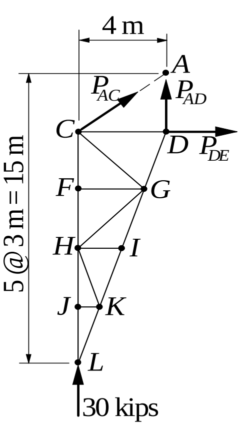

FBD of entire truss:

$$\Sigma M_D = 0 \quad + \circlearrowleft \; 18N_A - 1200(18 - 4.154) - 1500(4.154) = 0$$
$$N_A = 1269.2 \text{ lb}$$

FBD of truss to left of section $a - a$:

$$\Sigma M_A = 0 \quad + \circlearrowleft \; \frac{2}{\sqrt{5}} P_{BF}(6) - 1200(4.154) = 0$$
$$P_{BF} = 929 \text{ lb (T)} \blacktriangleleft$$
$$\Sigma M_B = 0 \quad + \circlearrowleft \; \frac{2}{\sqrt{13}} P_{EF}(6) - 1200(6 - 4.154) + 1269.2(6) = 0$$
$$P_{EF} = -1622 \text{ lb} = 1622 \text{ lb (C)} \blacktriangleleft$$
$$\Sigma M_F = 0 \quad + \circlearrowleft \; 6P_{BC} + 1200(9 - 4.154) - 1269.2(9) = 0$$
$$P_{BC} = 935 \text{ lb (T)} \blacktriangleleft$$

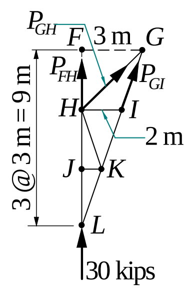
**4.162**



Due to symmetry, the reaction at $L$ is 30 kips.

$$\Sigma M_A = 0 \quad +\circlearrowright \quad 3P_{DE} - 30(4) = 0 \qquad P_{DE} = 40 \text{ kips (T)} \blacktriangleleft$$
$$\Sigma M_C = 0 \quad +\circlearrowright \quad P_{AD} = 0 \blacktriangleleft$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad P_{DE} + \frac{4}{5}P_{AC} = 0 \qquad 40 + \frac{4}{5}P_{AC} = 0$$
$$P_{AC} = -50 \text{ kips } = 50 \text{ kips (C)} \blacktriangleleft$$

## 4.163




Due to symmetry, the reaction at $L$ is 30 kips.

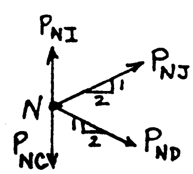
$$\Sigma M_H = 0 \quad P_{GI} = 0 \blacktriangleleft$$
$$\Sigma M_G = 0 \quad +\circlearrowright \quad 3(P_{FH} + 30) = 0 \qquad P_{FH} = -30 \text{ kips} = 30 \text{ kips (C)} \blacktriangleleft$$
$$\Sigma M_L = 0 \quad P_{GH} = 0 \blacktriangleleft$$

## 4.164

**From the FBD of the entire truss (not shown here): $A_x = 0$ and $A_y = 2.5P \uparrow$**

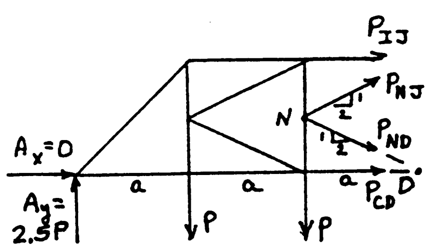
**<u>FBD of joint N</u>**

**$\Sigma F_x = 0$: $P_{NJ} = -P_{ND}$**




**<u>FBD of section shown</u>**

**$\Sigma F_y = 0$:**




$$+\uparrow \quad 2.5P - P - P + \frac{1}{\sqrt{5}}(P_{NJ} - P_{ND}) = 0$$

$$0.5P + \frac{1}{\sqrt{5}}(2P_{NJ}) = 0$$

$$\therefore P_{NJ} = -0.559P = 0.559P \text{ (C)} \blacklozenge \quad \text{and} \quad P_{ND} = 0.559P = 0.559P \text{ (T)}$$

$$\Sigma M_D = 0: \quad \circlearrowleft\!+ \quad P(2a) + P(a) - 2.5\,P(3a) - P_{IJ}(a) - \frac{1}{\sqrt{5}}\,P_{NJ}(a) - \frac{2}{\sqrt{5}}\,P_{NJ}\left(\frac{a}{2}\right) = 0$$

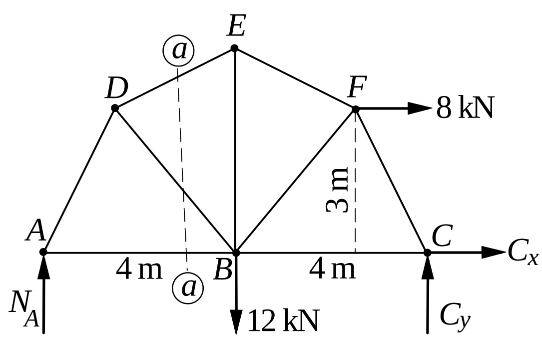
$$\therefore\ -4.5\,P - P_{IJ} - \frac{2}{\sqrt{5}}\,P_{NJ} = 0$$

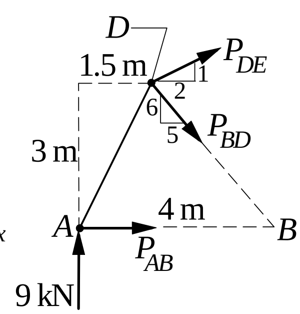
$$\text{which gives:} \quad P_{IJ} = -4.5\,P - \frac{2}{\sqrt{5}}\,(-0.559\,P) = -4.00\,P = 4.00\,P\ \text{(C)} \quad \blacklozenge$$

$$\Sigma M_N = 0: \quad \circlearrowleft\!+ \quad P(a) - 2.5\,P(2\,a) - P_{IJ}\left(\frac{a}{2}\right) + P_{CD}\left(\frac{a}{2}\right) = 0$$

$$\therefore\ P_{CD} = 2\,[-P + 5\,P + 0.5(-4.00\,P)] = 4.00\,P = 4.00\,P\ \text{(T)} \quad \blacklozenge$$

**4.165**




FBD of entire truss:

$$\Sigma M_C = 0 \qquad +\circlearrowleft\ 8N_A - 12(4) - 8(3) = 0 \qquad N_A = 9\ \text{kN}$$

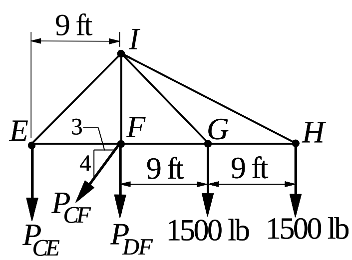
FBD of truss left of section $a-a$:

$$\Sigma M_D = 0 \qquad +\circlearrowleft\ 3P_{AB} - 9(4) = 0 \qquad P_{AB} = 12\ \text{kN (T)} \ \blacktriangleleft$$

$$\Sigma M_B = 0 \qquad \circlearrowleft\ \frac{2}{\sqrt{5}}P_{DE}(3) + \frac{1}{\sqrt{5}}P_{DE}(2.5) + 9(4) = 0$$

$$P_{DE} = -9.470\ \text{kN} = 9.47\ \text{kN (C)} \ \blacktriangleleft$$

**4.166**



**(a)**

$$\Sigma F_x = 0 \quad +\leftarrow \quad P_{CF} = 0 \ ◂$$

$$\Sigma M_E = 0 \quad +\circlearrowleft \quad 9P_{DF} + 1500(18) + 1500(27) = 0$$

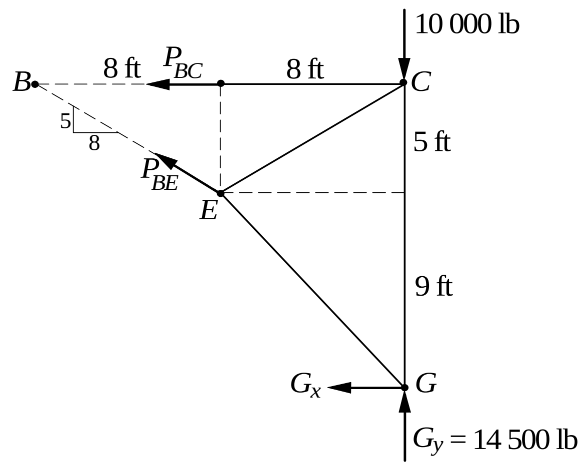
$$P_{DF} = -7500 \text{ lb} = 7500 \text{ lb (C)} \ ◂$$

$$\Sigma M_F = 0 \quad +\circlearrowleft \quad 9P_{CE} - 1500(9+18) = 0 \qquad P_{CE} = 4500 \text{ lb (T)} \ ◂$$

**(b)**

The zero-force members are $CF$, $CD$, $BC$ and $AB$ ◂

## 4.167




From FBD of entire truss (not shown): $G_y = 14\ 500$ lb ↑

$$\Sigma M_B = 0 \quad +\circlearrowleft \quad 14G_x - (14\ 500 - 10\ 000)(16) = 0 \qquad G_x = 5143 \text{ lb}$$

$$\Sigma F_y = 0 \quad +\uparrow \quad \frac{5}{\sqrt{89}}P_{BE} + 14\ 500 - 10\ 000 = 0$$

$$P_{BE} = -8491 \text{ lb} = 8491 \text{ lb (C)} \ ◂$$

$$\Sigma F_x = 0 \quad +\longleftarrow \quad P_{BC} + \frac{8}{\sqrt{89}}P_{BE} + G_x = 0$$

$$P_{BC} + \frac{8}{\sqrt{89}}(-8491) + 5143 = 0 \qquad P_{BC} = 2057 \text{ lb (T)} \ ◂$$

**4.168**

$\Sigma M_L = 0$:

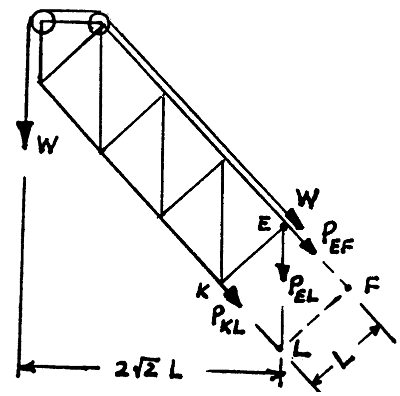
$\circlearrowleft + \; W(2\sqrt{2}\,L) - W(L) - P_{EF}(L) = 0$

$\therefore\; P_{EF} = (2\sqrt{2} - 1)W = 1.828\,W\ (T)$ ◆

$\Sigma M_E = 0$:

$\circlearrowleft + \; W(2\sqrt{2}\,L) + P_{KL}(L) = 0$

$\therefore\; P_{KL} = -2\sqrt{2}\,W = 2.83\,W\ (C)$ ◆




**4.169**

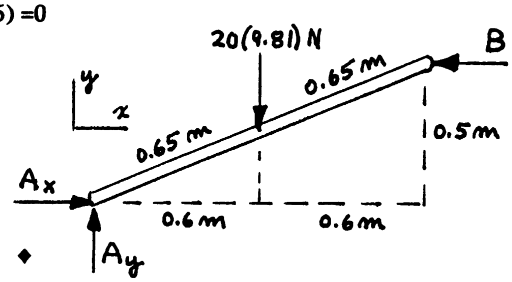
$\Sigma M_A = 0$: $\circlearrowleft + \; B(0.5) - 20(9.81)(0.6) = 0$

$\therefore\; B = 235.4\ \text{N}$ ◆

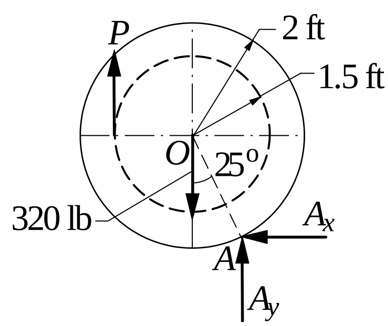
$\Sigma F_x = 0$: $A_x = B = 235.4\ \text{N}$

$\Sigma F_y = 0$: $A_y = 20(9.81) = 196.2\ \text{N}$

$\therefore\; A = \sqrt{235.4^2 + 196.2^2} = 306\ \text{N}$ ◆

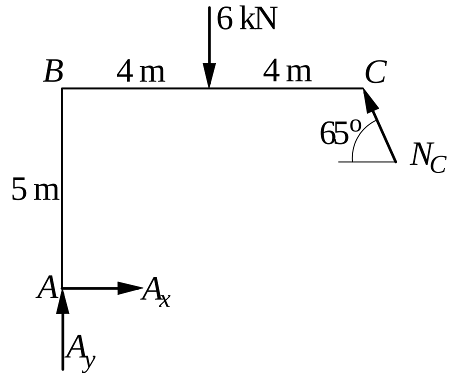



**4.170**




$\Sigma M_A = 0 \qquad + \circlearrowright \; P(1.5 + 2\sin 25°) - 320(2\sin 25°) = 0 \qquad P = 115.3\ \text{lb}$ ◀

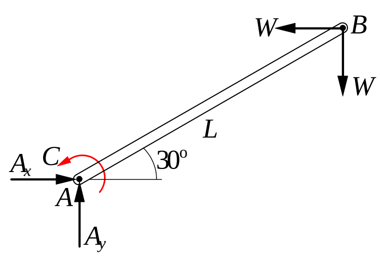
**4.171**




$$\begin{aligned}
\Sigma M_A &= 0 \qquad +\circlearrowleft \;\; (N_C \cos 65^\circ)(5) + (N_C \sin 65^\circ)(8) - 6(4) = 0 \\
& \qquad N_C = 2.563 \text{ kN} \\
\Sigma F_y &= 0 \qquad +\uparrow \;\; A_y + N_C \sin 65^\circ - 6 = 0 \\
& \qquad A_y + 2.563 \sin 65^\circ - 6 = 0 \qquad A_y = 3.677 \text{ kN} \\
\Sigma F_x &= 0 \qquad +\rightarrow \;\; A_x - N_C \cos 65^\circ = 0 \\
& \qquad A_x - 2.563 \cos 65^\circ = 0 \qquad A_x = 1.083 \text{ kN}
\end{aligned}$$

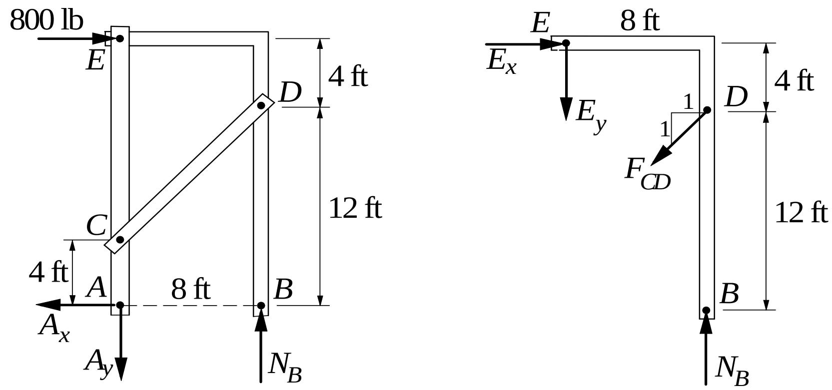
$$A = \sqrt{1.083^2 + 3.677^2} = 3.83 \text{ kN} \blacktriangleleft$$

**4.172**




$$\begin{aligned}
\Sigma M_A &= 0 \qquad +\circlearrowleft \;\; C + W(L \sin 30^\circ) - W(L \cos 30^\circ) = 0 \\
& \qquad C = 0.366WL \blacktriangleleft
\end{aligned}$$

**4.173**




FBD of entire structure:

$$\Sigma M_A = 0 \qquad + \circlearrowleft \; 8N_B - 800(16) = 0 \qquad N_B = 1600 \text{ lb}$$

FBD of member $BDE$:

$$\Sigma M_E = 0 \qquad + \circlearrowleft \; 8N_B - \frac{1}{\sqrt{2}}F_{CD}(4) - \frac{1}{\sqrt{2}}F_{CD}(8) = 0$$

$$8(1600) - \frac{12}{\sqrt{2}}F_{CD} = 0 \qquad F_{CD} = 1508.5 \text{ lb}$$

$$\Sigma F_x = 0 \qquad + \rightarrow \; E_x - \frac{1}{\sqrt{2}}F_{CD} = 0$$

$$E_x - \frac{1}{\sqrt{2}}(1508.5) = 0 \qquad E_x = 1066.7 \text{ lb}$$

$$\Sigma F_y = 0 \qquad + \downarrow \; E_y + \frac{1}{\sqrt{2}}F_{CD} - N_B = 0$$

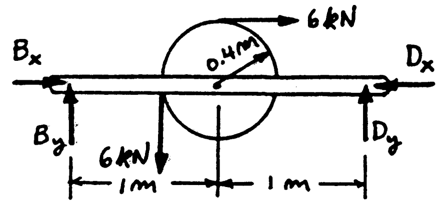
$$E_y + \frac{1}{\sqrt{2}}(1508.5) - 1600 = 0 \qquad E_y = 533.3 \text{ lb}$$

$$E = \sqrt{1066.7^2 + 533.3^2} = 1193 \text{ lb} \blacktriangleleft$$

4.174

**From the FBD of the entire structure assuming $N_E$ acts upward (not shown here):**

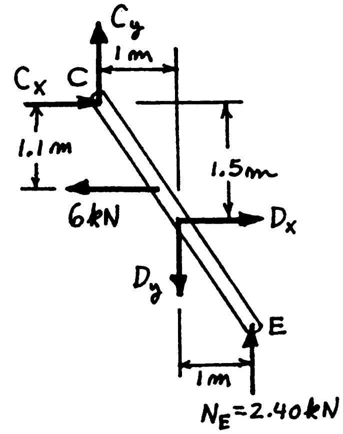
$\Sigma M_A = 0$: $N_E(4) = 6(1.6)$ $N_E = 2.40$ kN




$\Sigma M_B = 0$:

$\circlearrowleft +$ $D_y(2) - 6(0.4) - 6(0.6) = 0$

$\therefore$ $D_y = 3.00$ kN




$\Sigma M_C = 0$:

$\circlearrowleft +$ $2.40(2) + D_x(1.5) - D_y(1) - 6(1.1) = 0$

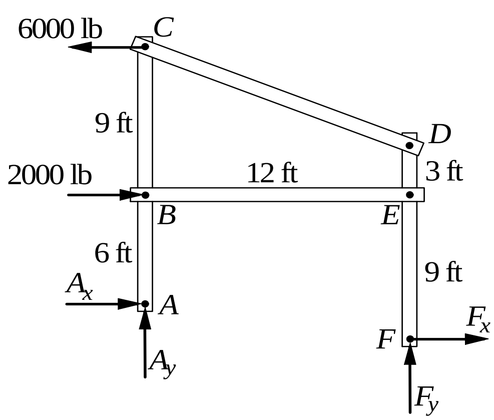
Substituting $D_y = 3.00$ kN, and solving

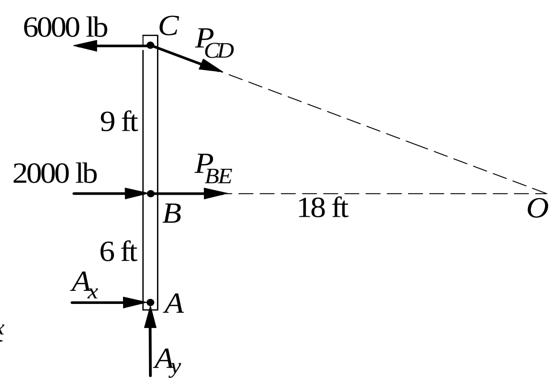
gives: $D_x = 3.20$ kN

The magnitude of the pin reaction at D is

$$D = \sqrt{3.20^2 + 3.00^2} = 4.39 \text{ kN} \;\blacklozenge$$

4.175




$BE$ and $CD$ are two-force members.

FBD of entire frame:

$$\Sigma M_F = 0 \qquad +\circlearrowleft\ 3A_x + 12A_y + 2000(9) - 6000(18) = 0$$
$$A_x + 4A_y = 30\ 000 \qquad \text{(a)}$$

FBD of member $ABC$:

$$\Sigma M_O = 0 \qquad +\circlearrowleft\ 6A_x - 18A_y + 6000(9) = 0$$
$$A_x - 3A_y = -9000 \qquad \text{(b)}$$

Solving Eqs. (a) and (b):

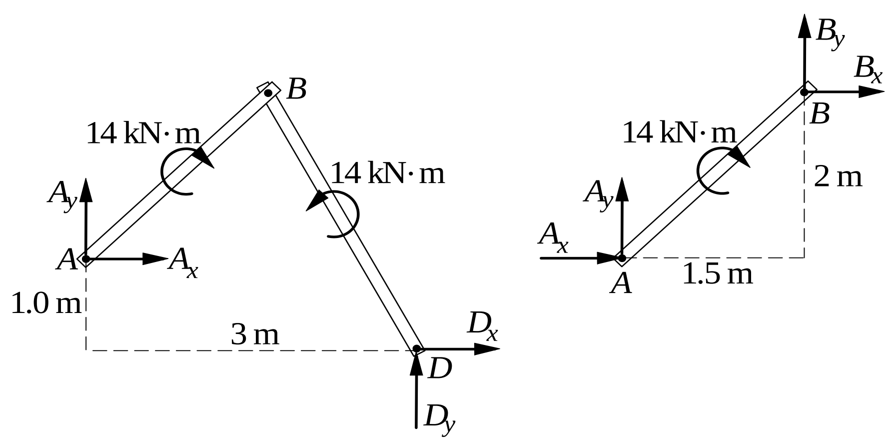
$$A_x = 7714 \text{ lb} \qquad A_y = 5571 \text{ lb}$$

FBD of entire frame:

$$\Sigma F_x = 0 \qquad +\rightarrow\ F_x + A_x + 2000 - 6000 = 0$$
$$F_x + 7714 + 2000 - 6000 = 0 \qquad F_x = -3714 \text{ lb}$$
$$\Sigma F_y = 0 \qquad +\uparrow\ A_y + F_y = 0 \qquad F_y = -A_y = -5571 \text{ lb}$$

$$A = \sqrt{7714^2 + 5571^2} = 9520 \text{ lb} \ \blacktriangleleft$$
$$F = \sqrt{3714^2 + 5571^2} = 6700 \text{ lb} \ \blacktriangleleft$$

## 4.176




FBD of entire frame:

$$\Sigma M_D = 0 \qquad +\circlearrowleft\ 1.0A_x + 3A_y = 0 \qquad A_x = -3A_y \qquad \text{(a)}$$

FBD of member $AB$:

$$\Sigma M_B = 0 \qquad +\circlearrowleft\ 2A_x - 1.5A_y - 14 = 0 \qquad \text{(b)}$$

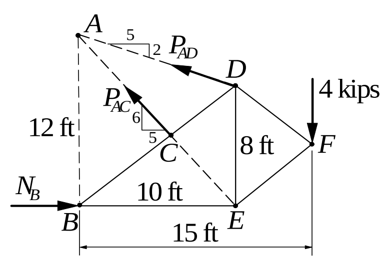
Solving Eqs. (a) and (b):

$$A_x = 5.60 \text{ kN} \qquad A_y = -1.8667 \text{ kN}$$
$$\therefore A = \sqrt{5.60^2 + 1.8667^2} = 5.90 \text{ kN} \blacktriangleleft$$

From FBD of entire frame:

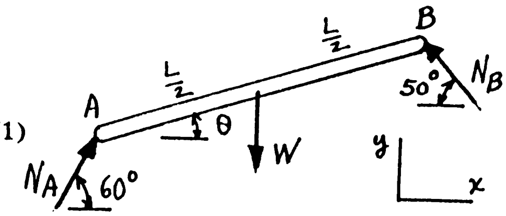
$$\Sigma F_x = 0 \quad +\longrightarrow \; D_x + A_x = 0 \qquad D_x = -A_x$$
$$\Sigma F_y = 0 \quad +\uparrow \; D_y + A_y = 0 \qquad D_y = -A_y$$
$$\therefore \quad D = A = 5.90 \text{ kN} \blacktriangleleft$$

## 4.177




$$\Sigma M_E = 0 \quad +\circlearrowleft \quad \frac{5}{\sqrt{29}}P_{AD}(8) - 4(5) = 0 \qquad P_{AD} = 2.693 \text{ kips (T)} \blacktriangleleft$$
$$\Sigma F_y = 0 \quad +\uparrow \quad \frac{2}{\sqrt{29}}P_{AD} + \frac{6}{\sqrt{61}}P_{AC} - 4 = 0$$
$$\frac{2}{\sqrt{29}}(2.693) + \frac{6}{\sqrt{61}}P_{AC} - 4 = 0 \qquad P_{AC} = 3.90 \text{ kips (T)} \blacktriangleleft$$

## 4.178




$\Sigma F_x = 0$:

$\overset{+}{\rightarrow}$ $N_A \cos 60^\circ - N_B \cos 50^\circ = 0$ (1)

$\Sigma F_y = 0$:

$+\uparrow$ $N_A \sin 60^\circ + N_B \sin 50^\circ - W = 0$ (2)

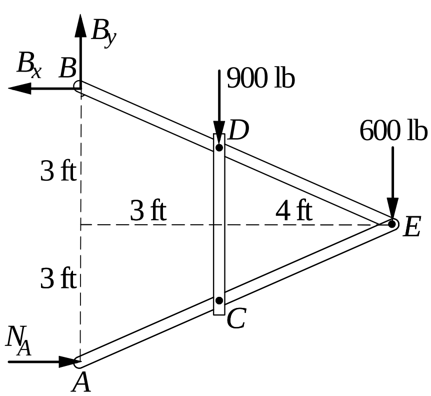
Solving (1) and (2) gives: $N_A = 0.6840\ W$ and $N_B = 0.5321\ W$

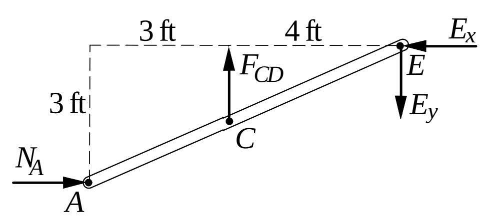
$\Sigma M_A = 0$: $+$ $N_B \sin 50^\circ (L \cos\theta) + N_B \cos 50^\circ (L \sin\theta) - W(L/2)\cos\theta = 0$

Substituting $N_B = 0.5321\ W$, and simplifying gives

$0.3420 \sin\theta = 0.0924 \cos\theta$ from which $\theta = \tan^{-1}(0.0924/0.3420) = 15.12^\circ$ ◆

## 4.179




FBD of entire frame:

$$\Sigma M_B = 0 \qquad + \circlearrowleft\ 6N_A - 900(3) - 600(7) = 0 \qquad N_A = 1150 \text{ lb}$$

FBD of member $ACE$:

$$\Sigma M_E = 0 \qquad + \circlearrowleft\ 4F_{CD} - 3N_A = 0 \qquad 4F_{CD} - 3(1150) = 0$$

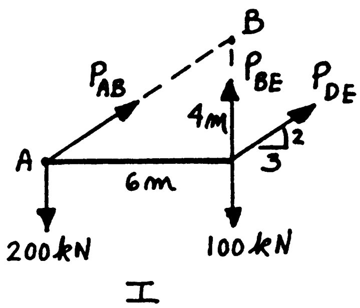
$$F_{CD} = 862.5 \text{ lb}$$

$$\Sigma F_x = 0 \qquad + \leftarrow\ E_x - N_A = 0 \qquad E_x - 1150 = 0 \qquad E_x = 1150 \text{ lb}$$

$$\Sigma F_y = 0 \qquad + \downarrow\ E_y - F_{CD} = 0 \qquad E_y - 862.5 = 0 \qquad E_y = 862.5 \text{ lb}$$

$$E = \sqrt{1150^2 + 862.5^2} = 1438 \text{ lb} \ \blacktriangleleft$$

4.180




(a) <u>FBD I</u> $\Sigma M_B = 0$:

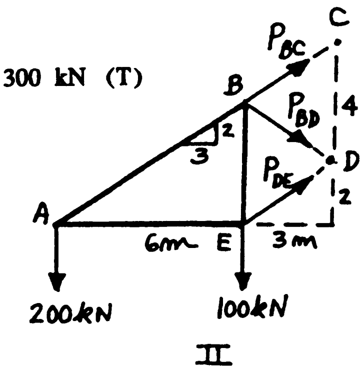
$$\circlearrowleft + \quad \frac{3}{\sqrt{13}} P_{DE}(4) + 200(6) = 0$$

$\therefore\ P_{DE} = -\,360.5\ \text{kN} = 360.5\ \text{kN (C)}$ ♦

(b) <u>FBD I</u> $\Sigma M_A = 0$:

$$\circlearrowleft + \quad \frac{2}{\sqrt{13}} P_{DE}(6) + P_{BE}(6) - 100(6) = 0$$

$\therefore\ P_{BE} = 100 - \frac{2}{\sqrt{13}} P_{DE} = 100 - \frac{2}{\sqrt{13}}(-\,360.5) = 300\ \text{kN (T)}$

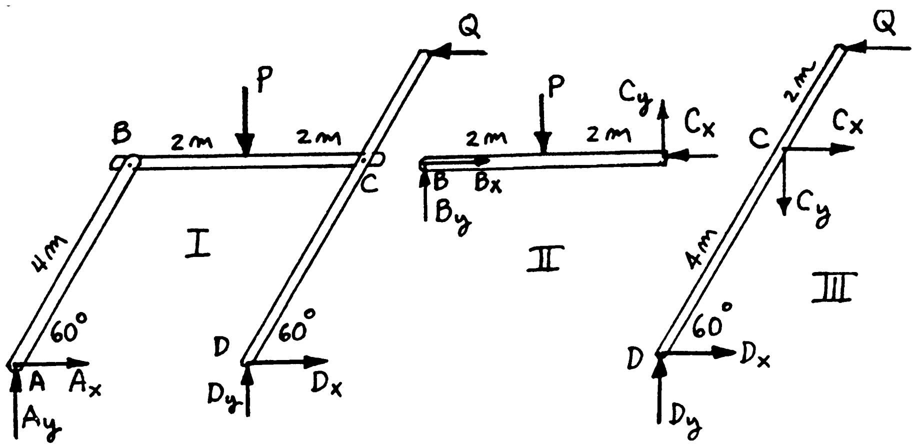



(c) <u>FBD II</u> $\Sigma M_D = 0$:

$$\circlearrowleft + \quad 200(9) + 100(3) - \frac{3}{\sqrt{13}} P_{BC}(4) = 0$$

$\therefore\ P_{BC} = 631\ \text{kN (T)}$ ♦

4.181




<u>FBD I</u> $\Sigma M_A = 0$: $\circlearrowleft + \quad D_y(4) - P(4\cos 60^\circ + 2) + Q(6\sin 60^\circ) = 0$

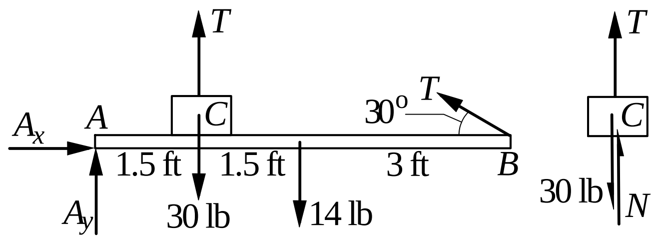
$$D_y = P - 1.299\,Q \qquad (1)$$

**FBD II** $\Sigma M_B = 0$: $\circlearrowleft +$ $C_y(4) - P(2) = 0$ $\quad \therefore\ C_y = \frac{P}{2}$ (2)

**FBD III** $\Sigma F_y = 0$: $D_y = C_y$ (3)

Combining (1), (2), and (3): $\frac{P}{2} = P - 1.299\,Q$ which gives $P = 2.60\,Q$ ◆

## 4.182



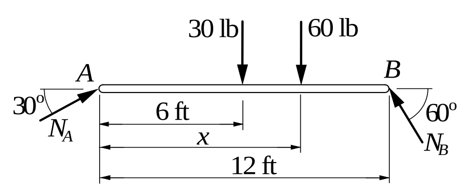

FBD of entire assembly:

$$\Sigma M_A = 0 \qquad + \circlearrowleft\ (T\sin 30^\circ)(6) + 1.5T - 30(1.5) - 14(3) = 0$$
$$T = 19.333 \text{ lb}$$

FBD of block $C$:

$$\Sigma F_y = 0 \qquad + \uparrow\ N + T - 30 = 0 \qquad N + 19.333 - 30 = 0$$
$$N = 10.67 \text{ lb}$$ ◄

## 4.183

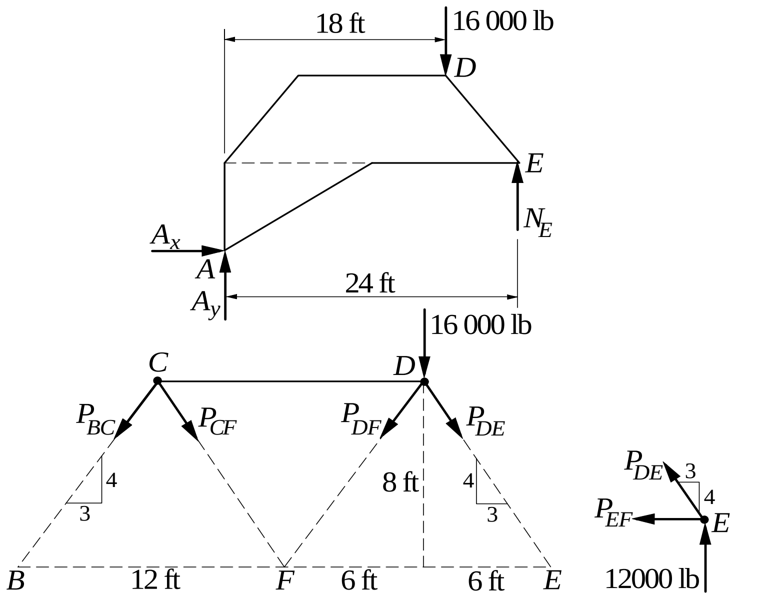



$$\Sigma F_x = 0 \qquad + \longrightarrow \quad N_A\cos 30^\circ - N_B\cos 60^\circ = 0$$
$$\Sigma F_y = 0 \qquad + \uparrow \quad N_A\sin 30^\circ + N_B\sin 60^\circ - 90 = 0$$

The solution is $N_A = 45.0$ lb $\qquad N_B = 77.94$ lb

$$\Sigma M_A = 0 \qquad + \circlearrowleft \quad (N_B\sin 60^\circ)(12) - 30(6) - 60x = 0$$
$$(77.94\sin 60^\circ)(12) - 30(6) - 60x = 0 \qquad x = 10.50 \text{ ft}$$ ◄

**4.184**




FBD of entire truss:

$$\Sigma M_A = 0 \qquad + \circlearrowleft \; 24N_E - 16\ 000(18) = 0 \qquad N_E = 12\ 000 \text{ lb}$$

**(a)** FBD of joint $E$:

$$\Sigma F_y = 0 \qquad + \uparrow \; \frac{4}{5}P_{DE} + 12\ 000 = 0 \qquad P_{DE} = -15\ 000 \text{ lb}$$

$$\Sigma F_x = 0 \qquad + \leftarrow \; P_{EF} + \frac{3}{5}P_{DE} = 0 \qquad P_{EF} + \frac{3}{5}(-15\ 000) = 0$$

$$P_{EF} = 9000 \text{ lb (T)} \quad \blacktriangleleft$$

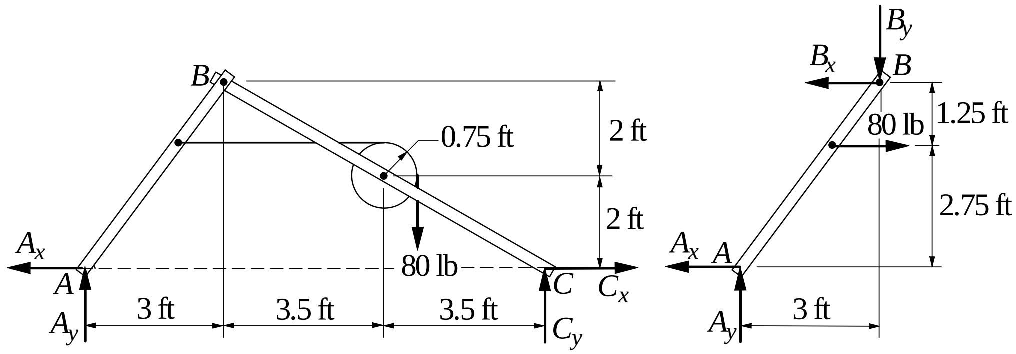
**(b)** FBD of member $CD$:

$$\Sigma M_F = 0 \qquad + \circlearrowleft \; \frac{4}{5}P_{BC}(12) - \frac{4}{5}P_{DE}(12) - 16\ 000(6) = 0$$

$$\frac{4}{5}P_{BC}(12) - \frac{4}{5}(-15\ 000)(12) - 16\ 000(6) = 0$$

$$P_{BC} = -5000 \text{ lb} = 5000 \text{ lb (C)} \quad \blacktriangleleft$$

**4.185**




FBD of entire frame:

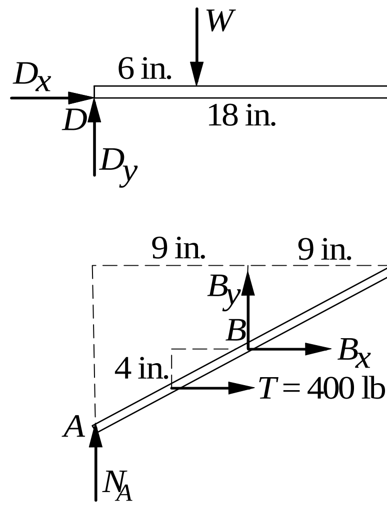
$$\Sigma M_C = 0 \qquad +\circlearrowleft \; 10A_y - 80(2.75) = 0 \qquad A_y = 22 \text{ lb}$$

FBD of member $AB$:

$$\begin{aligned} \Sigma M_B &= 0 \qquad +\circlearrowleft \; 4A_x + 3A_y - 80(1.25) = 0 \\ & \quad\; 4A_x + 3(22) - 80(1.25) = 0 \qquad A_x = 8.5 \text{ lb} \\ \Sigma F_x &= 0 \qquad + \leftarrow \; B_x + A_x - 80 = 0 \qquad B_x + 8.5 - 80 = 0 \\ & \quad\; B_x = 71.5 \text{ lb} \\ \Sigma F_y &= 0 \qquad + \downarrow \; B_y - A_y = 0 \qquad B_y - 22 = 0 \qquad B_y = 22 \text{ lb} \end{aligned}$$

$$B = \sqrt{71.5^2 + 22^2} = 74.8 \text{ lb} \;\blacktriangleleft$$

**4.186**




FBD of entire stool (not shown):

$$\Sigma M_E = 0 \qquad +\circlearrowleft\ 12W - 18N_A = 0 \qquad N_A = \frac{2}{3}W$$

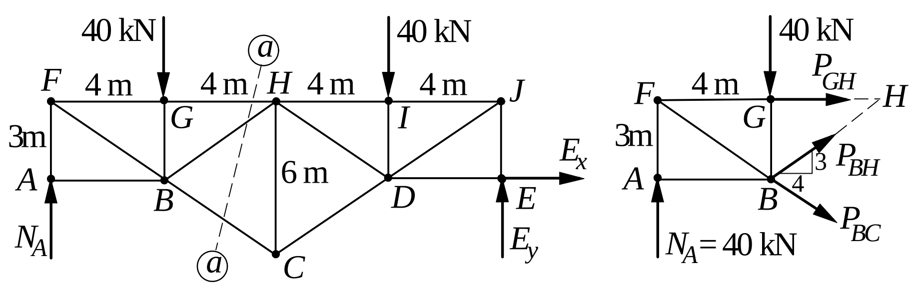
FBD of seat $CD$:

$$\Sigma M_D = 0 \qquad +\circlearrowleft\ 18N_C - 6W = 0 \qquad N_C = \frac{1}{3}W$$

FBD of member $ABC$:

$$\begin{aligned} \Sigma M_B &= 0 \qquad +\circlearrowleft\ 400(4) - 9N_A - 9N_C = 0 \\ 1600 - 6W - 3W &= 0 \qquad W = 177.8 \text{ lb} \blacktriangleleft \end{aligned}$$

**4.187**




FBD of entire truss:

$$N_A = E_y = 40 \text{ kN} \quad \text{by symmetry}$$

FBD of truss left of section $a - a$:

$$\Sigma M_B = 0 \qquad + \circlearrowleft \; 3P_{GH} + 40(4)$$

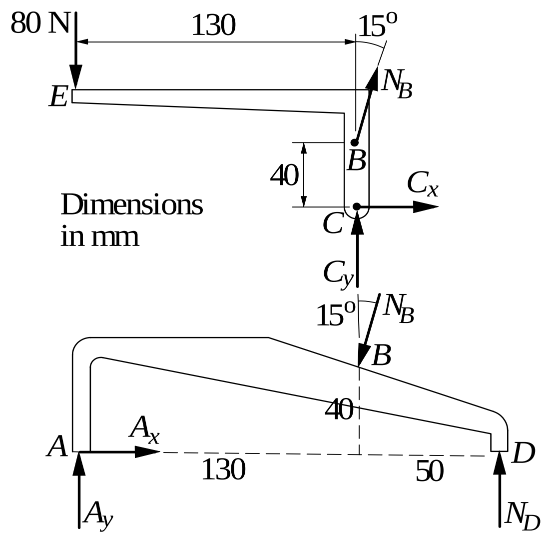
$$P_{GH} = -53.3 \text{ kN} = 53.3 \text{ kN (C)} \; ◄$$

$$\Sigma M_H = 0 \qquad + \circlearrowleft \; \frac{3}{5}P_{BC}(8) + 40(4) - 40(8) = 0$$

$$P_{BC} = 33.3 \text{ kN (T)} \; ◄$$

$$\Sigma M_F = 0 \qquad + \circlearrowleft \; \frac{3}{5}P_{BH}(8) - 40(4) = 0 \qquad P_{BH} = 33.3 \text{ kN (T)} \; ◄$$

**4.188**



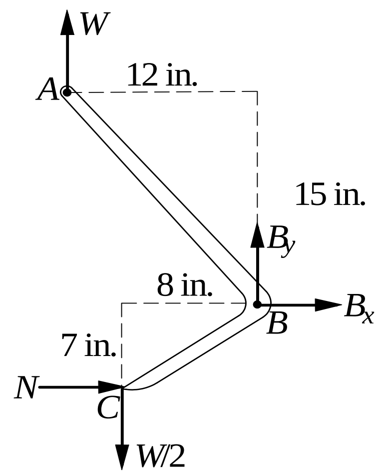

FBD of member $CBE$:

$$\Sigma M_C = 0 \qquad +\circlearrowleft\ 80(130) - (N_B \sin 15^\circ)(40) = 0 \qquad N_B = 1004.6 \text{ N}$$

FBD of member $AD$:

$$\begin{aligned} \Sigma M_A &= 0 \qquad +\circlearrowleft\ 180N_D - (N_B \cos 15^\circ)(130) + (N_B \sin 15^\circ)(40) = 0 \\ & \quad 180N_D - (1004.6 \cos 15^\circ)(130) + (1004.6 \sin 15^\circ)(40) = 0 \\ N_D &= 643 \text{ N} \ \blacktriangleleft \end{aligned}$$

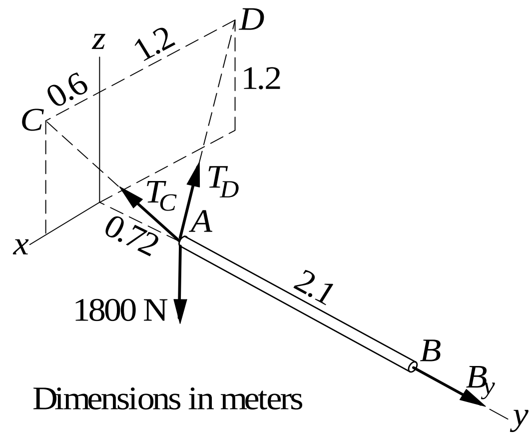
**4.189**




$$\Sigma M_B = 0 \qquad + \circlearrowleft \; \frac{W}{2}(8) - W(12) + N(7) = 0 \qquad N = \frac{8}{7}W \blacktriangleleft$$

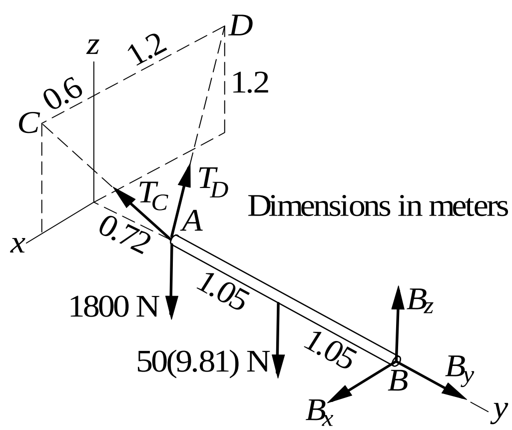
# Chapter 5

**5.1**



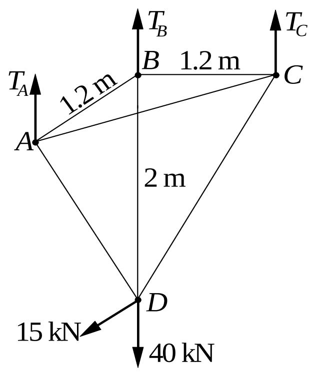

3 unknowns ◄

**5.2**



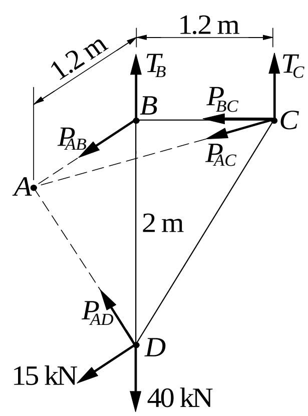

5 unknowns ◄

**5.3**



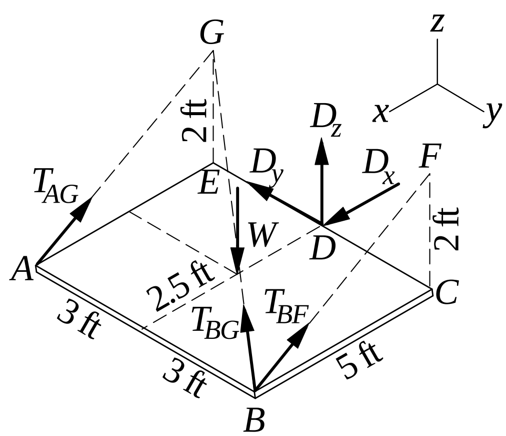

3 unknowns ◄

**5.4**



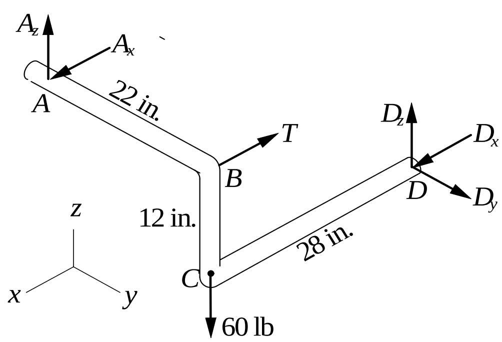

6 unknowns ◄

**5.5**



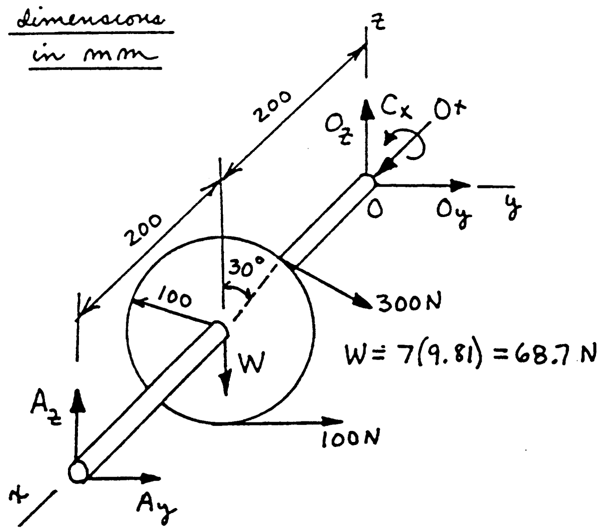

6 unknowns ◄

**5.6**




6 unknowns ◄

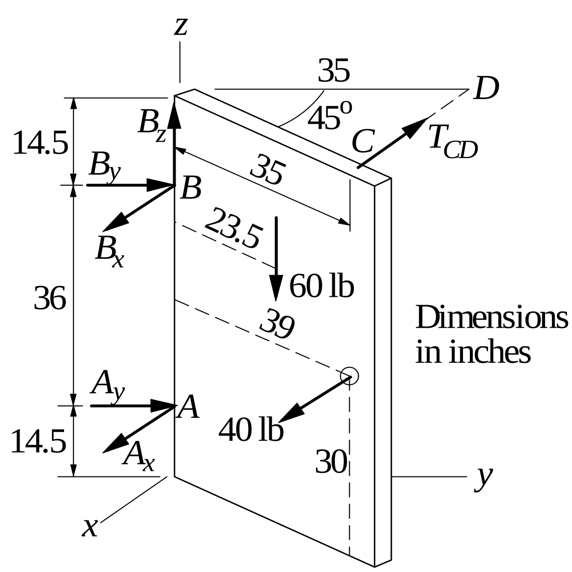
**5.7**




**6 unknowns** ◆

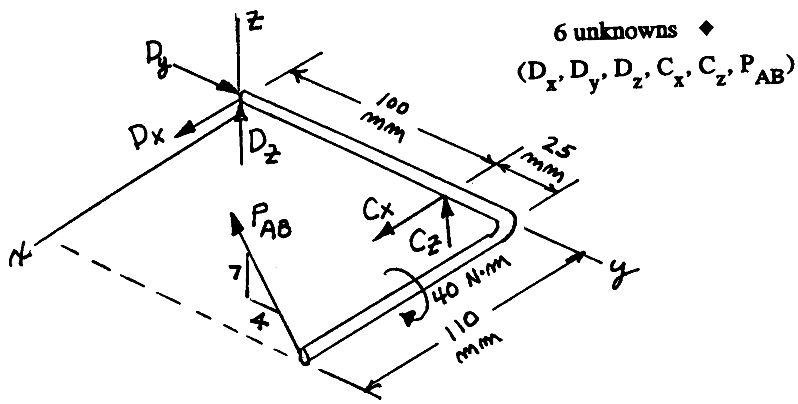
**($O_x$, $O_y$, $O_z$, $C_x$, $A_y$, $A_z$)**

**5.8**

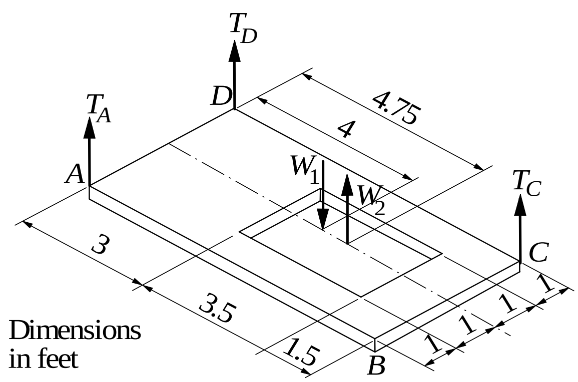



6 unknowns ◄

**5.9**




**5.10**



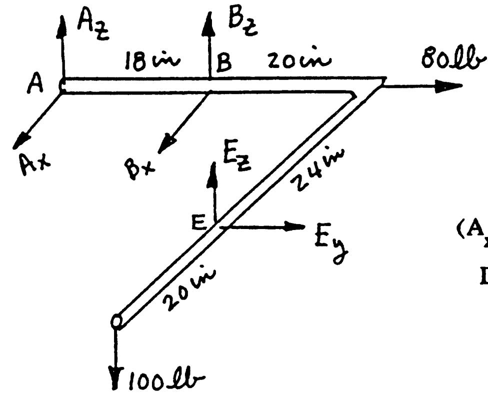

$$\text{Weight of plate per unit area is } \gamma = \frac{W}{A} = \frac{360}{(4)(8) - (2)(3.5)} = 14.40 \text{ lb/ft}^2$$

$$\text{Weight of plate without cutout is } W_1 = \gamma A_1 = 14.40(4)(8) = 460.8 \text{ lb}$$

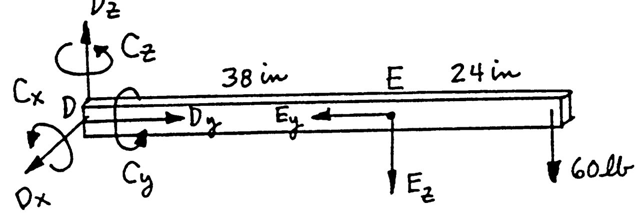
$$\text{Weight of cutout is } W_2 = \gamma A_2 = 14.40(2)(3.5) = 100.8 \text{ lb}$$

3 unknowns ◄

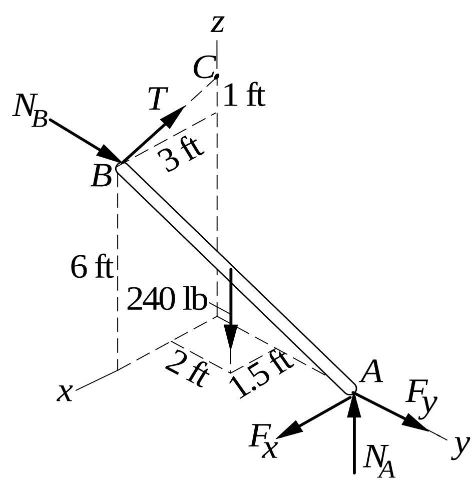
**5.11**




**12 unknowns** ◆
**($A_x$, $A_z$, $B_x$, $B_z$, $E_y$, $E_z$,**
**$D_x$, $D_y$, $D_z$, $C_x$, $C_y$, $C_z$)**




**5.12**




5 unknowns ◄

5.13

$$\Sigma M_{DB} = \left(\mathbf{r}_{BA} \times \mathbf{T}_{AC} \bullet \vec{\lambda}_{DB}\right) + \left(\mathbf{r}_{BA} \times \mathbf{W} \bullet \vec{\lambda}_{DB}\right) = 0 \qquad \text{(gives } T_{AC}\text{)}$$

$$\mathbf{r}_{BA} = -2\mathbf{i} - 2.5\mathbf{k} \text{ m} \qquad \mathbf{W} = -1500\mathbf{k} \text{ kN} \qquad \vec{\lambda}_{DB} = \frac{2\mathbf{i} - 1.5\mathbf{j}}{2.5}$$

$$\mathbf{T}_{AC} = T_{AC}\left(\frac{-3\mathbf{i} - 1.2\mathbf{j} + 2.5\mathbf{k}}{4.085}\right) = T_{AC}(-0.7344\mathbf{i} - 0.2938\mathbf{j} + 0.6120\mathbf{k})$$

$$\therefore \ \Sigma M_{DB} = \frac{T_{AC}}{2.5}\begin{vmatrix} -2 & 0 & -2.5 \\ -0.7344 & -0.2938 & 0.6120 \\ 2 & -1.5 & 0 \end{vmatrix} + \frac{1}{2.5}\begin{vmatrix} -2 & 0 & -2.5 \\ 0 & 0 & -1500 \\ 2 & -1.5 & 0 \end{vmatrix} = 0$$

Cancelling 2.5, and expanding the determinants: $-6.059T_{AC} + 4500 = 0$

which gives $T_{AC} = 743$ kN ♦

5.14

$$\Sigma M_{OE} = \left(\mathbf{r}_{OA} \times \mathbf{T}_{AD} \bullet \vec{\lambda}_{OE}\right) + \left(\mathbf{r}_{OB} \times \mathbf{P} \bullet \vec{\lambda}_{OE}\right) = 0 \qquad \text{(gives } T_{AD}\text{)}$$

$$\mathbf{r}_{OA} = 3\mathbf{j} \text{ ft} \qquad \mathbf{r}_{OB} = 5\mathbf{j} \text{ ft} \qquad \mathbf{P} = -8000\mathbf{k} \text{ lb} \qquad \vec{\lambda}_{OE} = \frac{-3.5\mathbf{i} + 3\mathbf{k}}{4.610}$$

$$\mathbf{T}_{AD} = T_{AD}\left(\frac{2.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}}{4.924}\right) = T_{AD}(0.5077\mathbf{i} - 0.6093\mathbf{j} + 0.6093\mathbf{k})$$

$$\therefore \ \Sigma M_{OE} = \frac{T_{AD}}{4.610}\begin{vmatrix} 0 & 3 & 0 \\ 0.5077 & -0.6093 & 0.6093 \\ -3.5 & 0 & 3 \end{vmatrix} + \frac{1}{4.610}\begin{vmatrix} 0 & 5 & 0 \\ 0 & 0 & -8000 \\ -3.5 & 0 & 3 \end{vmatrix} = 0$$

Cancelling 4.610, and expanding the determinants: $-10.97T_{AD} + 140\,000 = 0$

which gives $T_{AD} = 12\,760$ lb ♦

5.15

$\Sigma M_{DE} = 0$ (gives $O_y$)

$-8000(5) + O_y(3) = 0 \qquad \therefore \ O_y = 13\,330$ lb ♦

## 5.16

$$\Sigma M_{AC} = \left(\mathbf{r}_{AB} \times \mathbf{T}_B \cdot \vec{\lambda}_{AC}\right) + \left(\mathbf{r}_{AG} \times \mathbf{W} \cdot \vec{\lambda}_{AC}\right) = 0 \qquad \text{(gives } T_B\text{)}$$

$$\mathbf{r}_{AB} = -3.6\mathbf{i} + 0.8\mathbf{j} \text{ m} \qquad \mathbf{r}_{AG} = -2.4\mathbf{i} + 1.0\mathbf{j} \text{ m} \qquad \mathbf{W} = -60\mathbf{k} \text{ kN}$$

$$\vec{\lambda}_{AC} = \frac{-3.6\mathbf{i} + 3.0\mathbf{j}}{4.686} \qquad \mathbf{T}_B = T_B\mathbf{k} \text{ kN}$$

$$\therefore\ \Sigma M_{AC} = \frac{T_B}{4.686}\begin{vmatrix} -3.6 & 0.8 & 0 \\ 0 & 0 & 1 \\ -3.6 & 3 & 0 \end{vmatrix} + \frac{1}{4.686}\begin{vmatrix} -2.4 & 1.0 & 0 \\ 0 & 0 & -60 \\ -3.6 & 3 & 0 \end{vmatrix} = 0$$

Cancelling 4.686, and expanding the determinants: $7.920T_B - 216 = 0$

which gives $T_B = 27.3$ kN ♦

## 5.17

$$\Sigma M_{OD} = \left(\mathbf{r}_{OA} \times \mathbf{T}_{AE} \cdot \vec{\lambda}_{OD}\right) + \left(\mathbf{r}_{OB} \times \mathbf{P} \cdot \vec{\lambda}_{OD}\right) + (\mathbf{C} \cdot \vec{\lambda}_{OD}) = 0 \qquad \text{(gives } T_{AE}\text{)}$$

$$\mathbf{r}_{OA} = 4\mathbf{i} \text{ ft} \qquad \mathbf{r}_{OB} = 4\mathbf{i} + 2\mathbf{j} \text{ ft} \qquad \vec{\lambda}_{OD} = \frac{4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}}{8.307}$$

$$\mathbf{P} = -2000\mathbf{k} \text{ lb} \qquad \mathbf{C} = -6000\mathbf{k} \text{ lb}\cdot\text{ft}$$

$$\mathbf{T}_{AE} = T_{AE}\left(\frac{-4\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}}{9}\right) = T_{AE}(-0.4444\mathbf{i} + 0.7778\mathbf{j} + 0.4444\mathbf{k})$$

$$\therefore\ \Sigma M_{OD} = \frac{T_{AE}}{8.307}\begin{vmatrix} 4 & 0 & 0 \\ -0.4444 & 0.7778 & 0.4444 \\ 4 & 7 & 2 \end{vmatrix}$$

$$+\ \frac{1}{8.307}\begin{vmatrix} 4 & 2 & 0 \\ 0 & 0 & -2000 \\ 4 & 7 & 2 \end{vmatrix} + (-6000\mathbf{k}) \cdot \left(\frac{4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}}{8.307}\right) = 0$$

Cancelling 8.307, and expanding: $-6.221T_{AE} + 40\,000 - 12\,000 = 0$

which gives $T_{AE} = 4500$ lb ♦

## 5.18

$$\begin{aligned}
\Sigma F_y &= 0 \qquad T_A = T_B \\
\Sigma M_{CD} &= 0 \qquad \frac{20}{\sqrt{20^2+22^2}}\left(T_A+T_B\right)(30) - 80(15) = 0
\end{aligned}$$

$$\frac{20}{\sqrt{20^2+22^2}}\left(2T_A\right)(30) - 1200 = 0$$

$$T_A = T_B = 29.73 \text{ lb} \blacktriangleleft$$

$$\Sigma M_x = 0 \qquad T_C(32) + \frac{20}{\sqrt{20^2+22^2}}T_B(44) - 80(22) = 0$$

$$T_C(32) + \frac{20}{\sqrt{20^2+22^2}}(29.73)(44) - 1760 = 0$$

$$T_C = 27.50 \text{ lb} \blacktriangleleft$$

$$\Sigma F_z = 0 \qquad T_D + T_C + \frac{20}{\sqrt{20^2+22^2}}(T_A+T_B) - 80 = 0$$

$$T_D + 27.50 + \frac{20}{\sqrt{20^2+22^2}}(29.73+29.73) - 80 = 0$$

$$T_D = 12.50 \text{ lb} \blacktriangleleft$$

## 5.19

$$\begin{aligned}
\Sigma F_x &= 0 \qquad B_x - C_x = 0 && \text{(a)}\\
\Sigma F_y &= 0 \qquad B_y - A_y = 0 && \text{(b)}\\
\Sigma F_z &= 0 \qquad A_z + C_z = 0 && \text{(c)}\\
\Sigma M_x &= 0 \qquad 12C_z - 12B_y = 0 && \text{(d)}\\
\Sigma M_y &= 0 \qquad 12B_x - 10A_z - 120 = 0 && \text{(e)}\\
\Sigma M_z &= 0 \qquad 12C_x - 10A_y = 0 && \text{(f)}
\end{aligned}$$

Equations (a)-(d) yield

$$B_x = C_x \qquad A_y = B_y = C_z = -A_z$$

Equations (e) and (f) can now be written as

$$\begin{aligned}
12B_x - 10A_z &= 120\\
12B_x + 10A_z &= 0
\end{aligned}$$

The solution is

$$A_z = -6 \text{ lb} \qquad B_x = 5 \text{ lb}$$

Therefore,

$$A_y = B_y = C_z = -A_z = 6 \text{ lb} \qquad B_x = C_x = 5 \text{ lb}$$

$$A = \sqrt{6^2+6^2} = 8.49 \text{ lb} \blacktriangleleft \qquad B = C = \sqrt{6^2+5^2} = 7.81 \text{ lb} \blacktriangleleft$$

## 5.20

$$
\begin{aligned}
\Sigma M_x &= 0 \qquad \left(\frac{2.8}{\sqrt{2^2+3.5^2+2.8^2}}T\right)(3.5) - 270(3.5) = 0 \\
& \quad T = 473.3 \text{ lb} \blacktriangleleft \\
\Sigma F_x &= 0 \qquad A_x + \frac{2}{\sqrt{2^2+3.5^2+2.8^2}}T = 0 \\
& \quad A_x + \frac{2}{\sqrt{2^2+3.5^2+2.8^2}}(473.3) = 0 \qquad A_x = -192.86 \text{ lb} \\
\Sigma M_{BC} &= 0 \qquad 5.5A_z - 270(2) = 0 \qquad A_z = 98.18 \text{ lb} \\
\Sigma (M_C)_z &= 0 \qquad 5.5A_y + 3.5A_x = 0 \qquad 5.5A_y + 3.5(-192.86) = 0 \\
& \quad A_y = 122.73 \text{ lb}
\end{aligned}
$$

$$
A = \sqrt{(-192.86)^2 + 122.73^2 + 98.18^2} = 249 \text{ lb} \blacktriangleleft
$$

## 5.21

$$
\begin{aligned}
\Sigma M_x &= 0 \qquad 1200P_{BC} - 196.2(600) = 0 \qquad P_{BC} = 98.10 \text{ N} \blacktriangleleft \\
\Sigma M_z &= 0 \qquad A_y = 0 \blacktriangleleft \\
\Sigma F_y &= 0 \qquad A_y + O_y = 0 \qquad O_y = 0 \blacktriangleleft \\
\Sigma M_y &= 0 \qquad -1200A_z + 196.2(600) - 450P_{BC} = 0 \\
& \quad 196.2(600) - 1200A_z - 450(98.10) = 0 \qquad A_z = 61.31 \text{ N} \blacktriangleleft \\
\Sigma F_z &= 0 \qquad O_z + A_z - 196.2 + P_{BC} = 0 \\
& \quad O_z + 61.31 - 196.2 + 98.10 = 0 \qquad O_z = 36.79 \text{ N} \blacktriangleleft \\
\Sigma F_x &= 0 \qquad O_x = 0 \blacktriangleleft
\end{aligned}
$$

## 5.22

The force system is concurrent. Therefore,

$$
\Sigma \mathbf{F} = \mathbf{T}_{BC} + \mathbf{T}_{BD} + \mathbf{R}_A + \mathbf{P} = \mathbf{0}
$$

$$
\begin{aligned}
\mathbf{T}_{BC} &= T_{BC}\frac{4\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{\sqrt{4^2+12^2+12^2}} = T_{BC}(0.2294\mathbf{i} - 0.6883\mathbf{j} + 0.6883\mathbf{k} \\
\mathbf{T}_{BD} &= T_{BD}\frac{-8\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}}{\sqrt{8^2+12^2+4^2}} = T_{BD}(-0.5345\mathbf{i} - 0.8018\mathbf{j} + 0.2673\mathbf{k}) \\
\mathbf{R}_A &= R_A\frac{-4\mathbf{i} + 12\mathbf{j} + 6\mathbf{k}}{\sqrt{4^2+12^2+6^2}} = R_A(-0.2857\mathbf{i} + 0.8571\mathbf{j} + 0.4286\mathbf{k}) \\
\mathbf{P} &= 2200\mathbf{i} - 2800\mathbf{k} \text{ lb}
\end{aligned}
$$

$\Sigma\mathbf{F} = \mathbf{0}$ results in the following equations:

$$\begin{aligned} 0.2294T_{BC} - 0.5345T_{BD} - 0.2857R_A + 2200 &= 0 \\ -0.6883T_{BC} - 0.8018T_{BD} + 0.8571R_A &= 0 \\ 0.6883T_{BC} + 0.2673T_{BD} + 0.4286R_A - 2800 &= 0 \end{aligned}$$

The solution is

$$R_A = 3320 \text{ lb} \blacktriangleleft \qquad T_{BC} = 936 \text{ lb} \blacktriangleleft \qquad T_{BD} = 2740 \text{ lb} \blacktriangleleft$$

## 5.23

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD}\frac{-1.8\mathbf{i} - 1.2\mathbf{j} + 1.2\mathbf{k}}{\sqrt{1.8^2 + 1.2^2 + 1.2^2}} = T_{AD}\left(-0.7276\mathbf{i} - 0.4851\mathbf{j} + 0.4851\mathbf{k}\right) \\ \mathbf{T}_{BC} &= T_{BC}\frac{1.8\mathbf{i} - 2.4\mathbf{j}}{\sqrt{1.8^2 + 2.4^2}} = T_{BC}(0.6\mathbf{i} - 0.8\mathbf{j}) \\ \mathbf{r}_{OA} &= 1.2\mathbf{j} \text{ m} \qquad \mathbf{r}_{OB} = 2.4\mathbf{j} \text{ m} \qquad \mathbf{W} = -784.8\mathbf{k} \text{ N} \end{aligned}$$

$$\Sigma\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AD} + \mathbf{r}_{OB} \times \mathbf{T}_{BC} + \mathbf{r}_{OA} \times \mathbf{W}$$

$$= T_{AD}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.2 & 0 \\ -0.7276 & -0.4851 & 0.4851 \end{vmatrix} + T_{BC}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 0 \\ 0.6 & -0.8 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.2 & 0 \\ 0 & 0 & -784.8 \end{vmatrix} = \mathbf{0}$$

$$(0.5821\mathbf{i} + 0.8731\mathbf{k})\, T_{AD} - (1.440\mathbf{k})\, T_{BC} - 941.8\mathbf{i} = \mathbf{0}$$

Equating like components, we get

$$\begin{aligned} 0.5821T_{AD} - 941.8 &= 0 \qquad T_{AD} = 1618\text{N} \blacktriangleleft \\ 0.8731T_{AD} - 1.440T_{BC} &= 0 \qquad T_{BC} = \frac{0.8731(1618)}{1.440} = 981 \text{ N} \blacktriangleleft \end{aligned}$$

## 5.24

The 800-lb applied force in vector form is

$$\mathbf{F} = 800\frac{-3\mathbf{i} - 6\mathbf{j} + 1.5\mathbf{k}}{\sqrt{(-3)^2 + (-6)^2 + 1.5^2}} = -349.2\mathbf{i} - 698.3\mathbf{j} + 124.6\mathbf{k} \text{ lb}$$

$$\begin{aligned} \Sigma\mathbf{M}_A &= \mathbf{r}_{AB} \times \mathbf{F} - 10C_x\mathbf{k} - 10C_z\mathbf{i} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -349.2 & -698.3 & 124.6 \end{vmatrix} - 10C_x\mathbf{k} - 10C_z\mathbf{i} \\ &= (747.6 - 10C_z)\,\mathbf{i} + (2095 - 10C_x)\,\mathbf{k} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} C_x &= 209.5 \text{ lb} \qquad C_z = 74.76 \text{ lb} \\ C &= \sqrt{209.5^2 + 74.76^2} = 222 \text{ lb} \blacktriangleleft \end{aligned}$$

**5.25**

$$\begin{aligned} \Sigma M_x &= 0 \qquad 600(5) - 6P_{BG} = 0 \qquad P_{BG} = 500 \text{ lb (T)} \blacktriangleleft \\ \Sigma M_y &= 0 \qquad P_{AF} = 0 \blacktriangleleft \\ \Sigma M_z &= 0 \qquad -600(2) - 6P_{AE} = 0 \qquad P_{AE} = -200 \text{ lb} = 200 \text{ lb (C)} \blacktriangleleft \end{aligned}$$

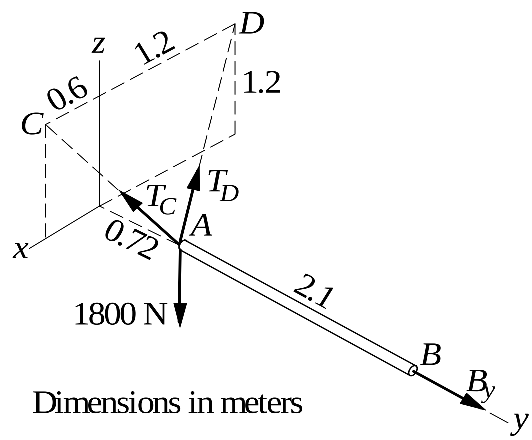
$$\begin{aligned} \Sigma F_x &= 0 \qquad C_x = 0 \\ \Sigma F_y &= 0 \qquad C_y - 600 - P_{AE} = 0 \qquad C_y - 600 - (-200) = 0 \\ & C_y = 400 \text{ lb} \\ \Sigma F_z &= 0 \qquad C_z - P_{BG} - P_{AF} = 0 \qquad C_z - 500 - 0 = 0 \\ C_z &= 500 \text{ lb} \end{aligned}$$

$$C = \sqrt{400^2 + 500^2} = 640 \text{ lb} \blacktriangleleft$$

**5.26**

$$\begin{aligned} \mathbf{P}_{BD} &= P_{BD}\frac{-2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}}{\sqrt{2^2 + 4^2 + 5^2}} = (-0.2981\mathbf{i} + 0.5963\mathbf{j} - 0.7454\mathbf{k})\, P_{BD} \\ \Sigma M_x &= \mathbf{r}_{CB} \times \mathbf{P}_{BD} \cdot \mathbf{i} + 600(5) \\ &= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ -0.2981 & 0.5963 & -0.7454 \end{vmatrix} P_{BD} + 3000 \\ &= -4.472P_{BD} + 3000 = 0 \\ P_{BD} &= 671 \text{ lb (T)} \blacktriangleleft \end{aligned}$$

**5.27**




$$\mathbf{T}_C = T_C \frac{0.6\mathbf{i} - 0.72\mathbf{j} + 1.2\mathbf{k}}{\sqrt{0.6^2 + 0.72^2 + 1.2^2}} = T_C(0.3941\mathbf{i} - 0.4729\mathbf{j} + 0.7881\mathbf{k}$$

$$\mathbf{T}_D = T_D \frac{-1.2\mathbf{i} - 0.72\mathbf{j} + 1.2\mathbf{k}}{\sqrt{1.2^2 + 0.72^2 + 1.2^2}} = T_D(-0.6509\mathbf{i} - 0.3906\mathbf{j} + 0.6509\mathbf{k}$$

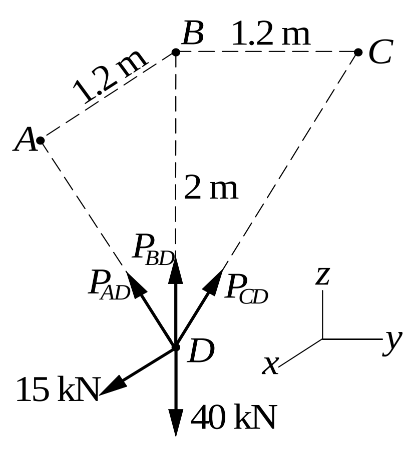
$$\Sigma F_x = 0 \qquad 0.3941 T_C - 0.6509 T_D = 0$$

$$\Sigma F_z = 0 \qquad 0.7881 T_C + 0.6509 T_D - 1800 = 0$$

Solution is: $T_C = 1522.6$ N ◄ $\qquad T_D = 921.9$ N ◄

$\Sigma M_{CD} = 0 \qquad 1.2 B_y - 1800(0.72) = 0 \qquad B_y = 1080$ N ◄

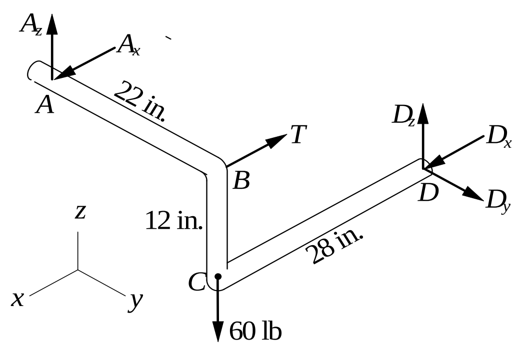
**5.28**



$$\mathbf{P}_{AD} = \frac{1.2\mathbf{i}+2\mathbf{k}}{\sqrt{1.2^2+2^2}}P_{AD} = (0.5145\mathbf{i}+0.8575\mathbf{k})\,P_{AD}$$
$$\mathbf{P}_{CD} = (0.5145\mathbf{j}+0.8575\mathbf{k})\,P_{CD}$$

$$\Sigma F_x = 0 \qquad 0.5145P_{AD}+15=0 \qquad P_{AD}=-29.15\text{ kN}$$
$$P_{AD} = 29.2\text{ kN (C)} \;\blacktriangleleft$$
$$\Sigma F_y = 0 \qquad P_{CD}=0 \;\blacktriangleleft$$
$$\Sigma F_z = 0 \qquad 0.8575P_{AD}+0.8575P_{CD}+P_{BD}-40=0$$
$$0.8575(-29.15)+0.8575(0)+P_{BD}-40=0$$
$$P_{BD} = 65.0\text{ kN (T)} \;\blacktriangleleft$$

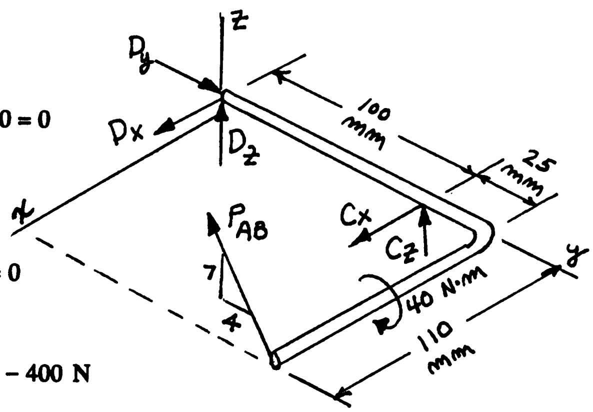
**5.29**




$$\boldsymbol{\lambda}_{AD} = \frac{-28\mathbf{i}+22\mathbf{j}-12\mathbf{k}}{\sqrt{(-28)^2+22^2+(-12)^2}} = -0.7451\mathbf{i}+0.5855\mathbf{j}-0.3193\mathbf{k}$$
$$\mathbf{r}_{DC} = 28\mathbf{i}\text{ in.} \qquad \mathbf{r}_{AB}=22\mathbf{j}\text{ in.}$$

$$\Sigma M_{AD}=0 \qquad \mathbf{r}_{AB}\times(-T\mathbf{i})\cdot\boldsymbol{\lambda}_{AD}+\mathbf{r}_{DC}\times(-60\mathbf{k})\cdot\boldsymbol{\lambda}_{AD} = 0$$
$$\begin{vmatrix} 0 & 22 & 0 \\ -T & 0 & 0 \\ -0.7451 & 0.5855 & -0.3193 \end{vmatrix} + \begin{vmatrix} 28 & 0 & 0 \\ 0 & 0 & -60 \\ -0.7451 & 0.5855 & -0.3193 \end{vmatrix} = 0$$
$$-7.025T+983.6=0 \qquad T=140.0\text{ lb} \;\blacktriangleleft$$

## 5.30




$\Sigma M_y = 0$: $P_{AB} = 0$

$\Sigma M_x = 0$: $C_z(0.1) - 40 = 0$

$\therefore\ C_z = 400$ N

$\Sigma M_z = 0$: $C_x = 0$

$\Sigma F_x = 0$: $D_x = -C_x = 0$

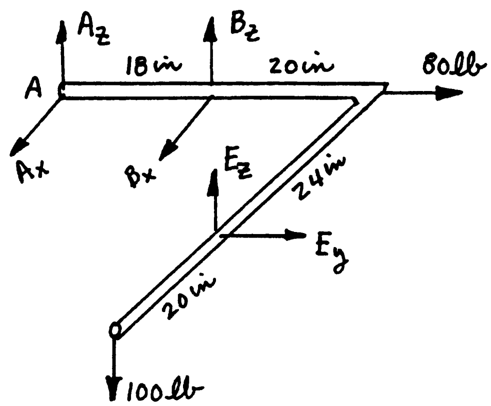
$\Sigma F_y = 0$: $D_y = 0$

$\Sigma F_z = 0$: $D_z = -C_z = -400$ N

Therefore, the reactions at C and D are:

**C** = 400**k** N; **D** = − 400**k** N ♦

## 5.31




<u>On bar ABE</u>

$\Sigma F_y = 0$: $E_y = -80$ lb

$\Sigma M_{AB} = 0$: $E_z(24) = 100(44)$

$\therefore\ E_z = 183.3$ lb

<u>On bar DE</u>

$\Sigma F_y = 0$: $D_y = E_y = -80$ lb

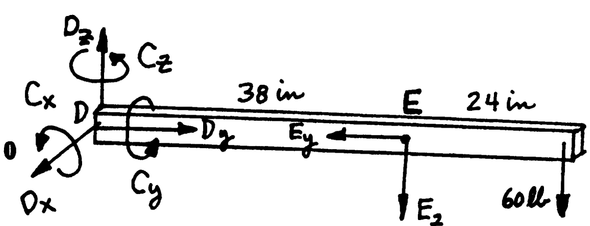
$\Sigma F_z = 0$: $D_z - E_z - 60 = 0$

$\therefore\ D_z = 60 + 183.3 = 243$ lb

$\Sigma F_x = 0$: $D_x = 0$

Therefore, the force at D is

**D** = − 80**j** + 243**k** lb ♦



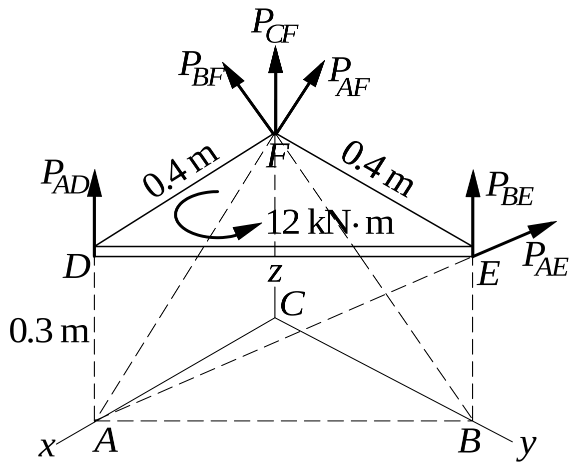

$\Sigma M_C = 0$:

**C** + [− $E_z$(38) − 60(62)]**i** = **0**

Therefore the couple at D is

**C** = 10685**i** lb•in = 890**i** lb•ft ♦

**5.32**




$$\mathbf{P}_{AE} = P_{AE}\frac{-0.4\mathbf{i}+0.4\mathbf{j}+0.3\mathbf{k}}{\sqrt{0.4^2+0.4^2+0.3^2}} = P_{AE}\left(-0.6247\mathbf{i}+0.6247\mathbf{j}+0.4685\mathbf{k}\right)$$

$$\begin{aligned}
\Sigma\mathbf{M}_F &= \mathbf{r}_{FE}\times\mathbf{P}_{AE}+0.4P_{BE}\mathbf{i}-0.4P_{AD}\mathbf{j}+12\mathbf{k}\\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0.4 & 0\\ -0.6247 & 0.6247 & 0.4685 \end{vmatrix} P_{AE}+0.4P_{BE}\mathbf{i}-0.4P_{AD}\mathbf{j}+12\mathbf{k}\\
&= \left(0.4P_{BE}+0.1874P_{AE}\right)\mathbf{i}-0.4P_{AD}\mathbf{j}+\left(12+0.2499P_{AE}\right)\mathbf{k}=\mathbf{0}
\end{aligned}$$

$$\begin{aligned}
P_{AE} &= -\frac{12}{0.2499} = -48.02 \text{ kN} = 48.0 \text{ kN (C)} \quad \blacktriangleleft\\
P_{AD} &= 0 \quad \blacktriangleleft\\
P_{BE} &= -\frac{0.1874(-48.02)}{0.4} = 22.5 \text{ kN (T)} \quad \blacktriangleleft
\end{aligned}$$

**5.33**

$$\begin{aligned}
\mathbf{T}_{AB} &= T_{AB}\frac{6\mathbf{i}-6\mathbf{j}+10.5\mathbf{k}}{\sqrt{6^2+6^2+10.5^2}} = T_{AB}(0.4444\mathbf{i}-0.4444\mathbf{j}+0.7778\mathbf{k})\\
\mathbf{T}_{AC} &= T_{AC}\frac{-6\mathbf{i}-3\mathbf{j}+10.5\mathbf{k}}{\sqrt{6^2+3^2+10.5^2}} = T_{AC}(-0.4815\mathbf{i}-0.2408\mathbf{j}+0.8427\mathbf{k}\\
\mathbf{T}_{AD} &= T_{AD}\frac{2.4\mathbf{i}+6\mathbf{j}+10.5\mathbf{k}}{\sqrt{2.4^2+6^2+10.5^2}} = T_{AD}(0.1947\mathbf{i}+0.4867\mathbf{j}+0.8516\mathbf{k}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_x &= 0 \qquad 0.4444T_{AB}-0.4815T_{AC}+0.1947T_{AD}=0\\
\Sigma F_y &= 0 \qquad -0.4444T_{AB}-0.2408T_{AC}+0.4867T_{AD}=0\\
\Sigma F_z &= 0 \qquad 0.7778T_{AB}+0.8427T_{AC}+0.8516T_{AD}-800=0
\end{aligned}$$

Solution is

$$T_{AB} = 222 \text{ lb} \blacktriangleleft \qquad T_{AC} = 359 \text{ lb} \blacktriangleleft \qquad T_{AD} = 380 \text{ lb} \blacktriangleleft$$

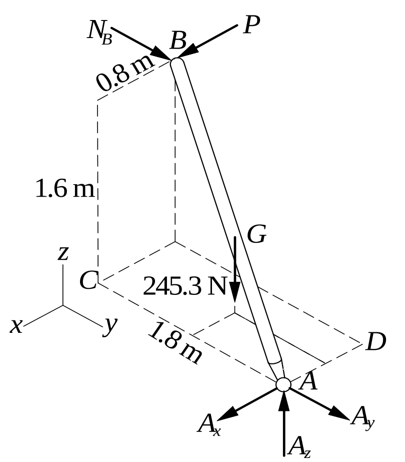
## 5.34

$$\Sigma M_{BC} = \mathbf{r}_{BA} \times \mathbf{T}_{AD} \cdot \boldsymbol{\lambda}_{BC} + \mathbf{r}_{BA} \times \mathbf{W} \cdot \boldsymbol{\lambda}_{BC} = 0$$

$$\begin{aligned}
\mathbf{T}_{AD} &= T_{AD}\frac{2.4\mathbf{i} + 6\mathbf{j} + 10.5\mathbf{k}}{\sqrt{2.4^2 + 6^2 + 10.5^2}} = T_{AD}(0.1947\mathbf{i} + 0.4867\mathbf{j} + 0.8516\mathbf{k} \\
\mathbf{W} &= -800\mathbf{k} \text{ lb} \qquad \mathbf{r}_{BA} = -6\mathbf{i} + 6\mathbf{j} - 10.5\mathbf{k} \text{ ft} \\
\boldsymbol{\lambda}_{BC} &= \frac{-12\mathbf{i} + 3\mathbf{j}}{\sqrt{12^2 + 3^2}} = -0.9701\mathbf{i} + 0.2425\mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\Sigma M_{BC} &= \begin{vmatrix} -6 & 6 & -10.5 \\ 0.1947 & 0.4867 & 0.8516 \\ -0.9701 & 0.2425 & 0 \end{vmatrix} T_{AD} + \begin{vmatrix} -6 & 6 & -10.5 \\ 0 & 0 & -800 \\ -0.9701 & 0.2425 & 0 \end{vmatrix} \\
&= -9.171T_{AD} + 3492 = 0 \qquad T_{AD} = 381 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 5.35




$$\begin{aligned}
\Sigma M_{AC} &= 0 \qquad 1.6P - 245.3(0.4) = 0 \qquad P = 61.3 \text{ N} \blacktriangleleft \\
\Sigma M_{AD} &= 0 \qquad 1.6N_B - 245.3(0.9) = 0 \qquad N_B = 138.0 \text{ N} \blacktriangleleft
\end{aligned}$$

5.36

$$\mathbf{T} = T\left(\frac{-60\mathbf{i} - 30\mathbf{j}}{\sqrt{4500}}\right)$$

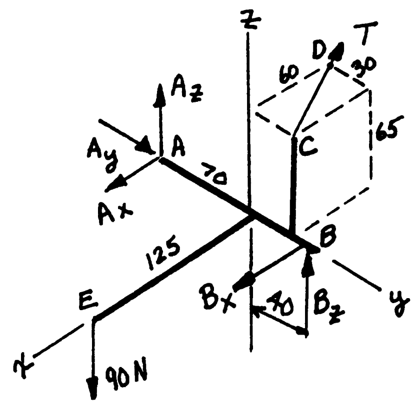
$= T(-0.8944\mathbf{i} - 0.4472\mathbf{j})$

$\mathbf{r}_{AC} = 100\mathbf{j} + 65\mathbf{k}$ mm

$\mathbf{r}_{AE} = 125\mathbf{i} + 70\mathbf{j}$ mm

$\mathbf{r}_{AB} = 110\mathbf{j}$ mm

$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$ $\quad$ $\mathbf{P} = -90\mathbf{k}$ N




$$\Sigma\mathbf{M}_A = (\mathbf{r}_{AC} \times \mathbf{T}) + (\mathbf{r}_{AE} \times \mathbf{P}) + (\mathbf{r}_{AB} \times \mathbf{B}) = \mathbf{0}$$

$$\Sigma\mathbf{M}_A = T\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 100 & 65 \\ -0.8944 & -0.4472 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 125 & 70 & 0 \\ 0 & 0 & -90 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 110 & 0 \\ B_x & 0 & B_z \end{vmatrix} = \mathbf{0}$$

Expanding the determinants, and equating like components:

(**i** component) $\quad 29.07T - 6300 + 110B_z = 0 \quad (1)$

(**j** component) $\quad -58.14T + 11250 = 0 \quad (2)$

(**k** component) $\quad 89.44T - 110B_x = 0 \quad (3)$

Solving (1), (2) and (3) gives: $T = 193.50$ N; $B_x = 157.33$ N; $B_z = 6.14$ N

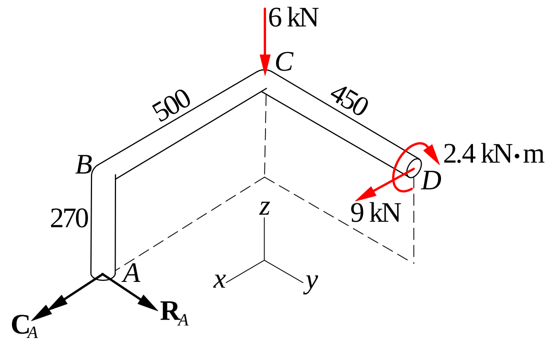
$\Sigma F_x = A_x + B_x + T_x = 0 \quad \therefore\ A_x = -B_x - T_x = -157.33 - (-0.8944)(193.50) = 15.74$ N

$\Sigma F_y = A_y + T_y = 0 \quad \therefore\ A_y = -T_y = -(-0.4472)(193.5) = 86.53$ N

$\Sigma F_z = A_z + B_z - 90 = 0 \quad \therefore\ A_z = 90 - B_z = 90 - 6.14 = 83.86$ N

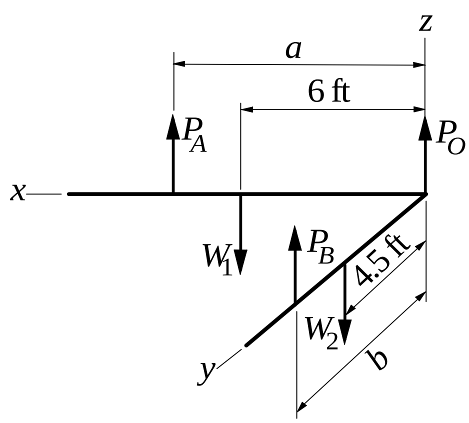
Therefore, $T = 193.5$ N; $\mathbf{A} = 15.7\mathbf{i} + 86.5\mathbf{j} + 83.9\mathbf{k}$ N; $\mathbf{B} = 157.3\mathbf{i} + 6.14\mathbf{k}$ N ◆

**5.37**




$$\begin{aligned}
\Sigma \mathbf{F} &= \mathbf{0} \qquad \mathbf{R}_A + 9\mathbf{i} - 6\mathbf{k} = \mathbf{0} \qquad \mathbf{R}_A = -9\mathbf{i} + 6\mathbf{k} \text{ kN} \blacktriangleleft \\
\Sigma \mathbf{M}_A &= \mathbf{0} \qquad \mathbf{C}_A - 2.4\mathbf{j} - 6(0.5)\mathbf{j} + 9(0.27)\mathbf{j} - 9(0.45)\mathbf{k} = \mathbf{0} \\
\mathbf{C}_A &= 2.97\mathbf{j} + 4.05\mathbf{k} \text{ N} \cdot \text{m} \blacktriangleleft
\end{aligned}$$

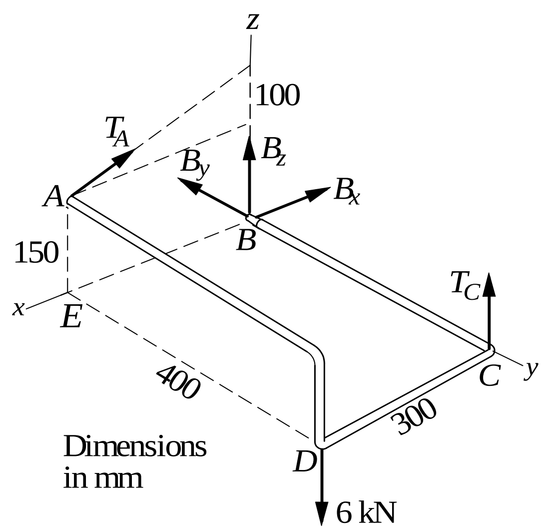
**5.38**




$$\begin{aligned}
P_A &= P_B = P_O = \frac{2700}{3} = 900 \text{ lb} \\
W_1 &= \frac{12}{21}(2700) = 1542.9 \text{ lb} \qquad W_2 = \frac{9}{21}(2700) = 1157.1 \text{ lb}
\end{aligned}$$

$$\begin{aligned}
\Sigma M_y &= 0 \qquad 6W_1 - aP_A = 0 \\
& \qquad a = \frac{6W_1}{P_A} = \frac{6(1542.9)}{900} = 10.29 \text{ ft} \blacktriangleleft \\
\Sigma M_x &= 0 \qquad bP_B - 4.5W_2 = 0 \\
b &= \frac{4.5W_2}{P_B} = \frac{4.5(1157.1)}{900} = 5.79 \text{ ft} \blacktriangleleft
\end{aligned}$$

**5.39**




$$\Sigma M_x = 0 \qquad 400T_C - 6(400) = 0 \qquad T_C = 6 \text{ kN} \blacktriangleleft$$

$$\Sigma M_y = 0 \qquad 6(300) - \frac{3}{\sqrt{10}}T_A(150) - \frac{1}{\sqrt{10}}T_A(300) = 0$$

$$T_A = 7.590 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad -B_x - \frac{3}{\sqrt{10}}T_A = 0 \qquad -B_x - \frac{3}{\sqrt{10}}(7.590) = 0$$

$$B_x = -7.20 \text{ kN}$$

$$\Sigma F_y = 0 \qquad B_y = 0$$

$$\Sigma F_z = 0 \qquad B_z + T_C + \frac{1}{\sqrt{10}}T_A - 6 = 0$$

$$B_z + 6 + \frac{1}{\sqrt{10}}7.590 - 6 = 0 \qquad B_z = -2.40 \text{ kN}$$

$$B = \sqrt{7.20^2 + 2.40^2} = 7.590 \text{ kN} \blacktriangleleft$$

5.40

$\Sigma M_y = 0$: $120(12) - 28P = 0$ $\therefore$ $P = 51.43$ lb

$\Sigma M_x = 0$:

$120(24) + 50(28)$
$+ P(71) - B_z(59) = 0$

$\therefore$ $B_z = 134.43$ lb

$\Sigma M_z = 0$ $\therefore$ $B_x = 0$

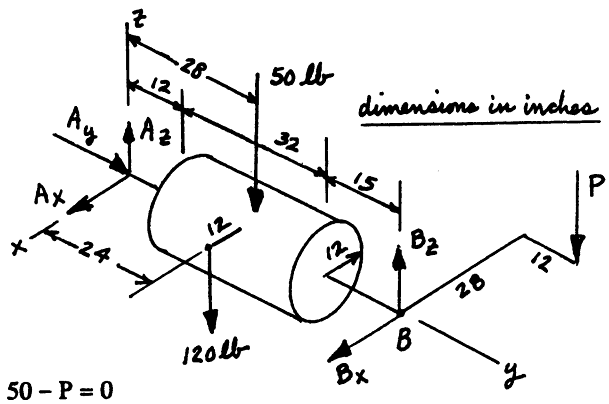
$\Sigma F_x = 0$: $A_x + B_x = 0$

$\therefore$ $A_x = 0$

$\Sigma F_y = 0$: $A_y = 0$

$\Sigma F_z = 0$: $A_z + B_z - 120 - 50 - P = 0$

$\therefore$ $A_z = 120 + 50 + 51.43 - 134.43 = 87.00$ lb




Therefore, the answers are: $P = 51.4$ lb; $\mathbf{A} = 87.0\mathbf{k}$ lb; $\mathbf{B} = 134.4\mathbf{k}$ lb ♦

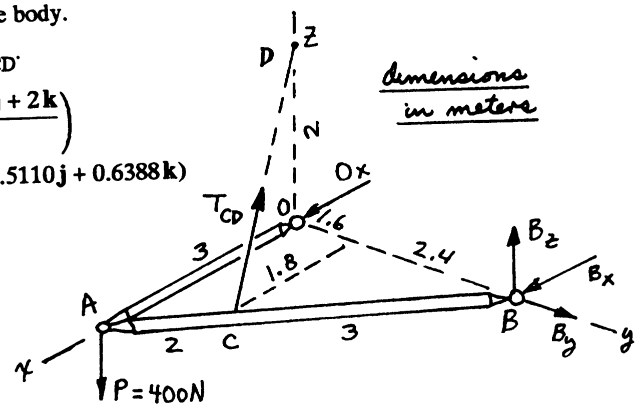
5.41

Note that OA is a two-force body.

$\Sigma \mathbf{M}_B = \mathbf{0}$ gives $O_x$ and $T_{CD}$.

$$\mathbf{T}_{CD} = T_{CD}\left(\frac{-1.8\mathbf{i} - 1.6\mathbf{j} + 2\mathbf{k}}{3.131}\right)$$

$$= T_{CD}(-0.5749\mathbf{i} - 0.5110\mathbf{j} + 0.6388\mathbf{k})$$




$\Sigma M_{B_y} = 0$: $400(3) - (T_{CD})_z(1.8) = 0$

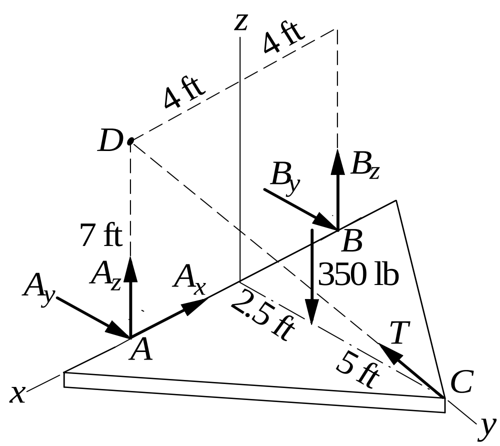
$1200 = 0.6388\,T_{CD}(1.8)$ $\therefore$ $T_{CD} = 1044$ N ♦

$\Sigma M_{B_z} = 0$: $O_x(4) - (T_{CD})_x(2.4) - (T_{CD})_y(1.8) = 0$

$O_x(4) - (0.5749)(1044)(2.4) - (0.5110)(1044)(1.8) = 0$

$\therefore$ $O_x = 600$ N which gives $\mathbf{O}_x = 600\mathbf{i}$ N ♦

**5.42**




$$\mathbf{T} = \frac{4\mathbf{i} - 7.5\mathbf{j} + 7\mathbf{k}}{\sqrt{4^2 + 7.5^2 + 7^2}} T = (0.3633\mathbf{i} - 0.6811\mathbf{j} + 0.6357\mathbf{k})\,T$$

$$\Sigma M_x = 0 \qquad (0.6357T)\,(7.5) - 350(2.5) = 0 \qquad T = 183.52 \text{ lb} \blacktriangleleft$$

$$\therefore \quad \mathbf{T} = (0.3633\mathbf{i} - 0.6811\mathbf{j} + 0.6357\mathbf{k})\,(183.52)$$

$$= 66.67\mathbf{i} - 125.0\mathbf{j} + 116.66\mathbf{k} \text{ lb}$$

$$\Sigma F_x = 0 \qquad -A_x + T_x = 0 \qquad A_x = T_x = 66.67 \text{ lb}$$

$$\Sigma M_{AD} = 0 \qquad B_y = 0$$

$$\Sigma F_y = 0 \qquad A_y + B_y + T_y = 0 \qquad A_y + 0 - 125.0 = 0$$

$$A_y = 125.0 \text{ lb}$$

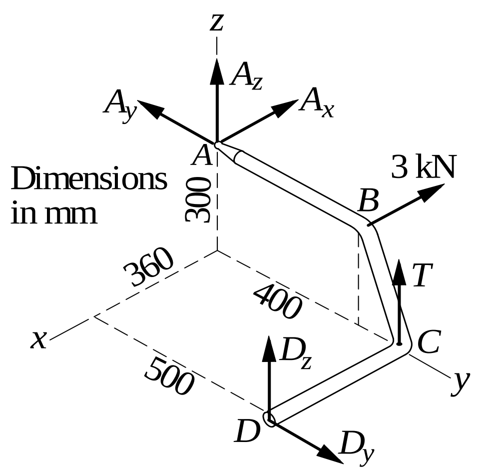
$$\Sigma M_y = 0 \qquad A_z = B_z$$

$$\Sigma F_z = 0 \qquad A_z + B_z + T_z - 350 = 0 \qquad 2A_z + 116.66 - 350 = 0$$

$$A_z = B_z = 116.67 \text{ lb}$$

$$A = \sqrt{66.67^2 + 125.0^2 + 116.67^2} = 183.5 \text{ lb} \blacktriangleleft \qquad B = 116.7 \text{ lb} \blacktriangleleft$$

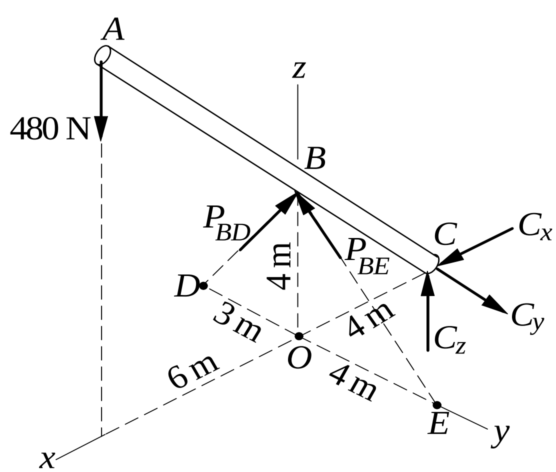
**5.43**




$$\Sigma \mathbf{M}_A = (500T + 500D_z + 300D_y)\,\mathbf{i} - 360D_z\mathbf{j} + [3(400) + 360D_y]\,\mathbf{k} = \mathbf{0}$$

$$D_z = 0 \;\blacktriangleleft \qquad D_y = -3.33 \text{ kN} \;\blacktriangleleft \qquad T = 2.00 \text{ kN} \;\blacktriangleleft$$

**5.44**



$$\Sigma M_x = 0 \qquad \frac{4}{5}P_{BD}(3) - \frac{1}{\sqrt{2}}P_{BE}(4) = 0 \qquad P_{BD} = 1.179P_{BE}$$

$$\Sigma M_{C_y} = 0 \qquad 480(10) - 4\left(\frac{4}{5}P_{BD} + \frac{1}{\sqrt{2}}P_{BE}\right) = 0$$

$$4800 - 3.20P_{BD} - 2.828P_{BE} = 0$$

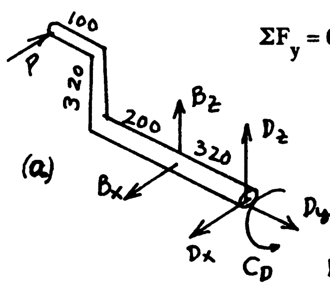
$$4800 - 3.20(1.179P_{BE}) - 2.828P_{BE} = 0 \qquad P_{BE} = 727 \text{ N} \blacktriangleleft$$

$$P_{BD} = 1.179(727) = 857 \text{ N} \blacktriangleleft$$

## 5.45

**<u>FBD of entire winch</u> (not shown here)** $\Sigma M_y = 0$: $360(200) = P(320)$ $\therefore$ $P = 225$ N ◆

**<u>FBD (a)</u>**



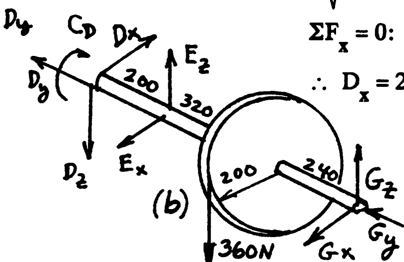

$\Sigma F_y = 0$: $D_y = 0$ $\qquad \Sigma M_{D_x} = 0$: $B_z = 0$ $\qquad \Sigma F_z = 0$: $D_z = 0$

$\Sigma M_{D_z} = 0$: $225(620) = B_x(320)$ gives $B_x = 435.9$ N

$\therefore$ $B = \sqrt{B_x^2 + B_z^2} = 436$ N ◆

$\Sigma F_x = 0$: $D_x = 225 - B_x$

$\therefore$ $D_x = 225 - 436 = -211$ N




**<u>FBD (b)</u>**

$\Sigma M_{G_x} = 0$:

$360(240) = E_z(560)$ $\therefore$ $E_z = 154.3$ N

$\Sigma M_{G_z} = 0$:

$E_x(560) = D_x(760)$

$\therefore$ $E_x = (76/56)(-210.9) = -286.2$ N

$\Sigma F_y = 0$: $G_y = 0$

$\Sigma F_z = 0$: $G_z = 360 - E_z$

$\therefore$ $G_z = 360 - 154.3 = 205.7$ N

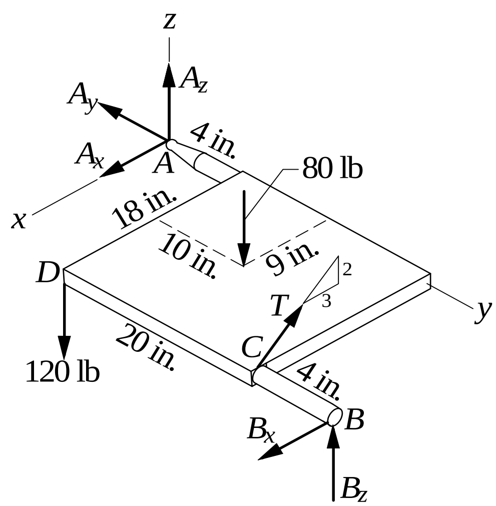
$\Sigma F_x = 0$: $G_x = D_x - E_x$

$= -210.9 + 286.2$

$= 75.3$ N

$E = \sqrt{154.3^2 + (-286.2)^2} = 325$ N ◆ $\qquad G = \sqrt{75.3^2 + 205.7^2} = 219$ N ◆

**5.46**




$$\Sigma M_x = 0 \qquad \frac{2}{\sqrt{13}}T(24) + 28B_z - 80(14) - 120(4) = 0$$

$$13.313T + 28B_z - 1600 = 0$$

$$\Sigma M_y = 0 \qquad -\frac{2}{\sqrt{13}}T(18) - 18B_z + 80(9) + 120(18) = 0$$

$$-9.985T - 18B_z + 2880 = 0$$

Solution is: $T = 1297.8$ lb ◀ $\qquad B_z = -559.9$ lb

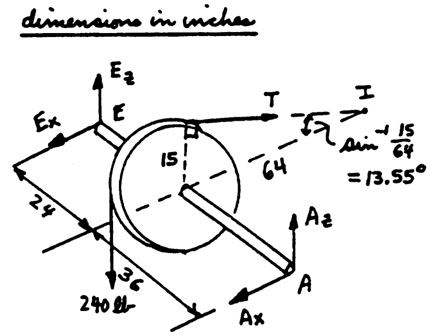
$$\Sigma M_z = 0 \qquad \frac{3}{\sqrt{13}}T(24) - 28B_x = 0$$

$$\frac{3}{\sqrt{13}}(1297.8)(24) - 28B_x = 0 \qquad B_x = 925.6 \text{ lb}$$

$$B = \sqrt{559.9^2 + 925.6^2} = 1082 \text{ lb} \blacktriangleleft$$

5.47

$\Sigma M_{AE} = 0$ gives T = 240 lb




$\Sigma M_{E_z} = 0$:

$$T\cos 13.55^o(24) - A_x(60) = 0$$

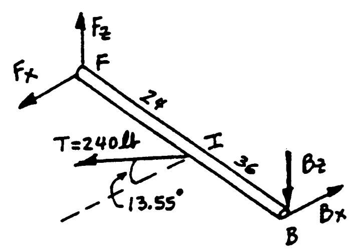
$$\therefore\ A_x = 93.33\ \text{lb}$$

$\Sigma M_{E_x} = 0$:

$$-(240 + T\sin 13.55^o)(24) + A_z(60) = 0$$

$$\therefore\ A_z = 118.5\ \text{lb}$$

$\Sigma M_{F_z} = 0$:



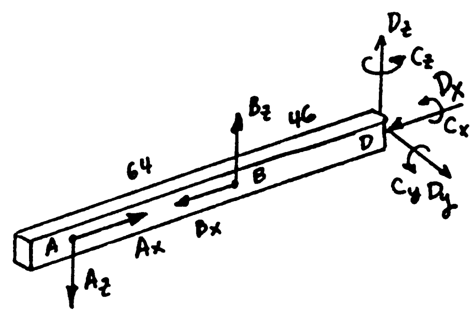

$$-240\cos 13.55^o(24) + B_x(60) = 0$$

$$\therefore\ B_x = 93.33\ \text{lb}$$

$\Sigma M_{F_x} = 0$:

$$240\sin 13.55^o(24) - B_z(60) = 0$$

$$\therefore\ B_z = 22.49\ \text{lb}$$




$\Sigma M_x = 0$: $C_x = 0$

$\Sigma M_{D_z} = 0$: $C_z = 0$

$\Sigma F_x = 0$: $D_x = A_x - B_x = 0$

$\Sigma F_y = 0$: $D_y = 0$

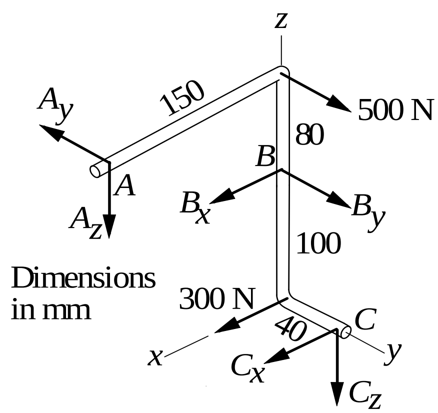
$\Sigma F_z = 0$: $D_z = A_z - B_z$

$$= 118.5 - 22.49 = 96.0\ \text{lb}$$

$\Sigma M_{D_y} = 0$ $\quad C_y = B_z(46) - A_z(110) = 22.49(46) - 118.5(110) = -12\,000$ lb•in

The reactions at D acting on ABD are: $\mathbf{D} = 96.0\,\mathbf{k}$ lb and $\mathbf{C} = -1000\,\mathbf{j}$ lb•ft ♦

**5.48**



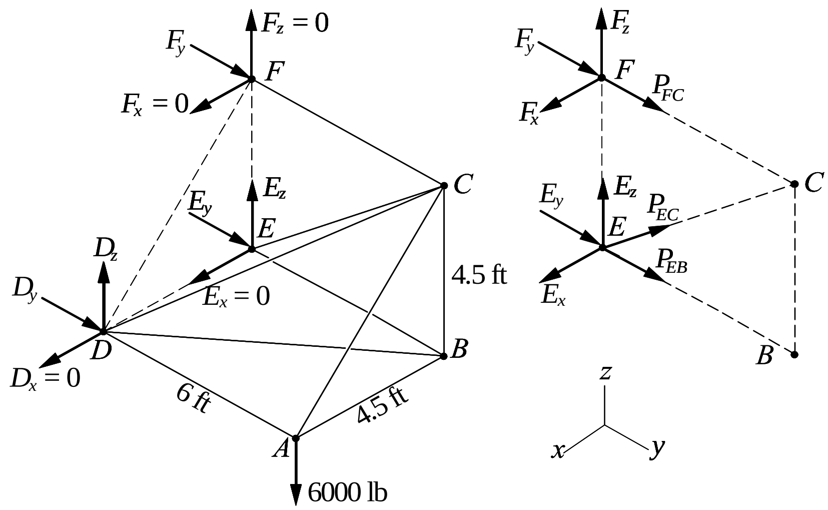

$$\begin{aligned}
\Sigma F_x &= 0 \qquad B_x + C_x + 300 = 0 \\
\Sigma F_y &= 0 \qquad -A_y + B_y + 500 = 0 \\
\Sigma F_z &= 0 \qquad -A_z - C_z = 0 \\
\Sigma M_x &= 0 \qquad 180A_y - 100B_y - 40C_z - 500(180) = 0 \\
\Sigma M_y &= 0 \qquad 100B_x + 150A_z = 0 \\
\Sigma M_z &= 0 \qquad -150A_y - 40C_x = 0
\end{aligned}$$

The solution is

$$\begin{aligned}
A_y &= -1600 \text{ N} \qquad A_z = 4200 \text{ N} \\
B_x &= -6300\text{N} \qquad B_y = -2100 \text{ N} \\
C_x &= 6000 \text{ N} \qquad C_z = -4200\text{N}
\end{aligned}$$

$$\begin{aligned}
A &= \sqrt{1600^2 + 4200^2} = 4490 \text{ N} \blacktriangleleft \\
B &= \sqrt{6300^2 + 2100^2} = 6640 \text{ N} \blacktriangleleft \\
C &= \sqrt{600^2 + 4200^2} = 4240 \text{ N} \blacktriangleleft
\end{aligned}$$

**5.49**




FBD of joint $F$:

$$\Sigma F_x = 0 \qquad F_x = 0$$
$$\Sigma F_z = 0 \qquad F_z = 0$$

FBD of joint $E$:

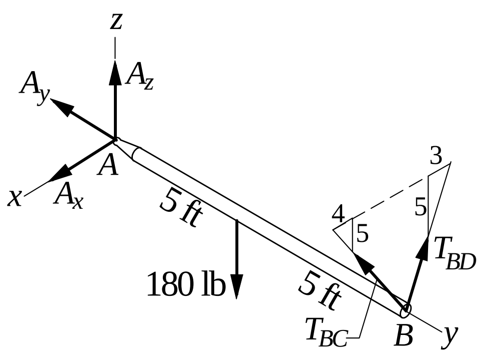
$$\Sigma F_x = 0 \qquad E_x = 0$$

FBD of entire truss:

$$\Sigma F_x = 0 \qquad D_x + E_x + F_x = 0 \qquad D_x + 0 + 0 = 0 \qquad D_x = 0$$
$$\Sigma M_{DE} = 0 \qquad 4.5F_y + 6000(6) = 0 \qquad F_y = -8000 \text{ lb}$$
$$\Sigma M_{AD} = 0 \qquad 4.5(E_z + F_z) = 0 \qquad 4.5(E_z + 0) = 0 \qquad E_z = 0$$
$$\Sigma (M_D)_z = 0 \qquad 4.5(E_y + F_y) = 0 \qquad 4.5(E_y - 8000) = 0$$
$$E_y = 8000 \text{ lb}$$
$$\Sigma M_{EF} = 0 \qquad D_y = 0$$
$$\Sigma F_z = 0 \qquad D_z + E_z + F_z - 6000 = 0 \qquad D_z + 0 + 0 - 6000 = 0$$
$$D_z = 6000 \text{ lb}$$

$$\therefore \mathbf{D} = 6000\mathbf{k} \text{ lb} \blacktriangleleft \qquad \mathbf{E} = 8000\mathbf{j} \text{ lb} \blacktriangleleft \qquad \mathbf{F} = -8000\mathbf{j} \text{ lb} \blacktriangleleft$$

**5.50**




$$\Sigma M_x = 0 \qquad \left(\frac{5}{\sqrt{41}}T_{BC} + \frac{5}{\sqrt{34}}T_{BD}\right)(10) - 180(5) = 0$$

$$7.809T_{BC} + 8.575T_{BD} - 900 = 0$$

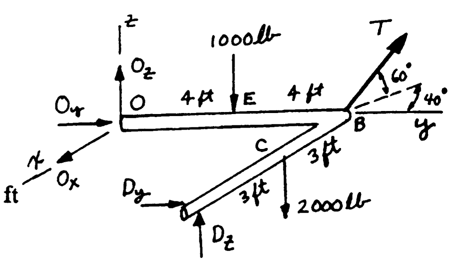
$$\Sigma M_z = 0 \qquad \left(-\frac{4}{\sqrt{41}}T_{BC} + \frac{3}{\sqrt{34}}T_{BD}\right)(10) = 0$$

$$-6.247T_{BC} + 5.145T_{BD} = 0$$

Solution is: $T_{BC} = 49.4$ lb ◄ $\qquad T_{BD} = 60.0$ lb ◄

**5.51**

$$\Sigma \mathbf{M}_O = \left[\mathbf{r}_{OE} \times (-1000\,\mathbf{k})\right] + \left[\mathbf{r}_{OC} \times (-2000\,\mathbf{k})\right] + (\mathbf{r}_{OD} \times \mathbf{D}) + (\mathbf{r}_{OB} \times \mathbf{T}) = \mathbf{0}$$




$\mathbf{r}_{OE} = 4\,\mathbf{j}$ ft $\quad \mathbf{r}_{OC} = 3\,\mathbf{i} + 8\,\mathbf{j}$ ft $\quad \mathbf{r}_{OB} = 8\,\mathbf{j}$ ft

$\mathbf{r}_{OD} = 6\,\mathbf{i} + 8\,\mathbf{j}$ ft $\quad \mathbf{D} = D_y\,\mathbf{j} + D_z\,\mathbf{k}$

$$\mathbf{T} = T(-\cos 60^\circ \sin 40^\circ\,\mathbf{i} + \cos 60^\circ \cos 40^\circ\,\mathbf{j} + \sin 60^\circ\,\mathbf{k})$$
$$= T(-0.3214\,\mathbf{i} + 0.3830\,\mathbf{j} + 0.8660\,\mathbf{k})$$

$$\Sigma \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ 0 & 0 & -1000 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 8 & 0 \\ 0 & 0 & -2000 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 8 & 0 \\ 0 & D_y & D_z \end{vmatrix}$$

$$+ T \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ -0.3214 & 0.3830 & 0.8660 \end{vmatrix} = 0$$

**Expanding the determinants, the components of the moment equation are:**

**(i component)** $\quad -4000 - 16\,000 + 8 D_z + 6.9280\, T = 0 \quad (1)$

**(j component)** $\quad 6 D_z - 6000 = 0 \quad (2)$

**(k component)** $\quad 6 D_y + 2.571\, T = 0 \quad (3)$

**Solving (1)-(3) gives:** $T = 1732$ lb ◆ $\quad D_y = -742.2$ lb $\quad D_z = 1000$ lb

$$D = \sqrt{(-742.2)^2 + 1000^2} = 1245 \text{ lb} \quad ◆$$

## 5.52

$$\begin{aligned}
\mathbf{P}_{BA} &= P_{BA}\frac{-2.4\mathbf{j}+3.2\mathbf{k}}{\sqrt{2.4^2+3.2^2}} = P_{BA}(-0.6\mathbf{j}+0.8\mathbf{k}) \\
\mathbf{P}_{CA} &= P_{CA}\frac{2.4\mathbf{i}+1.2\mathbf{j}+3.2\mathbf{k}}{\sqrt{2.4^2+1.2^2+3.2^2}} = P_{CA}(0.5747\mathbf{i}+0.2874\mathbf{j}+0.7663\mathbf{k}) \\
\mathbf{P}_{DA} &= P_{DA}\frac{-2.4\mathbf{i}+1.2\mathbf{j}+3.2\mathbf{k}}{\sqrt{2.4^2+1.2^2+3.2^2}} = P_{DA}(-0.5747\mathbf{i}+0.2874\mathbf{j}+0.7663\mathbf{k}) \\
\mathbf{P} &= -P\mathbf{k}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_x &= 0 \qquad 0.5747P_{CA} - 0.5747P_{DA} = 0 \\
\Sigma F_y &= 0 \qquad -0.6P_{BA} + 0.2874P_{CA} + 0.2874P_{DA} = 0 \\
\Sigma F_z &= 0 \qquad 0.8P_{BA} + 0.7663P_{CA} + 0.7663P_{DA} - P = 0
\end{aligned}$$

Solution is

$$P_{BA} = 0.4167P \qquad P_{CA} = P_{DA} = 0.4350P$$

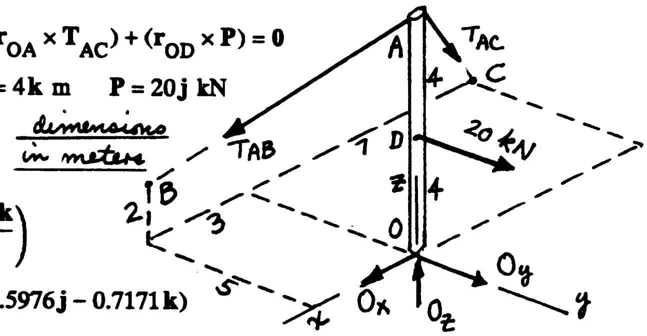
The limiting condition is

$$0.4350P = 8 \text{ kN} \qquad P = 18.39 \text{ kN} \blacktriangleleft$$

5.53

$$\Sigma \mathbf{M}_O = (\mathbf{r}_{OA} \times \mathbf{T}_{AB}) + (\mathbf{r}_{OA} \times \mathbf{T}_{AC}) + (\mathbf{r}_{OD} \times \mathbf{P}) = \mathbf{0}$$

$\mathbf{r}_{OA} = 8\mathbf{k}$ m $\quad \mathbf{r}_{OD} = 4\mathbf{k}$ m $\quad \mathbf{P} = 20\mathbf{j}$ kN




$$\mathbf{T}_{AB} = T_{AB}\left(\frac{3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k}}{\sqrt{70}}\right)$$

$$= T_{AB}(0.3586\mathbf{i} - 0.5976\mathbf{j} - 0.7171\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\left(\frac{-7\mathbf{i} - 5\mathbf{j} - 8\mathbf{k}}{\sqrt{138}}\right) = T_{AC}(-0.5959\mathbf{i} - 0.4256\mathbf{j} - 0.6810\mathbf{k})$$

$$\Sigma \mathbf{M}_O = T_{AB}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 8 \\ 0.3586 & -0.5976 & -0.7171 \end{vmatrix} + T_{AC}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 8 \\ -0.5959 & -0.4256 & -0.6810 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 20 & 0 \end{vmatrix} = \mathbf{0}$$

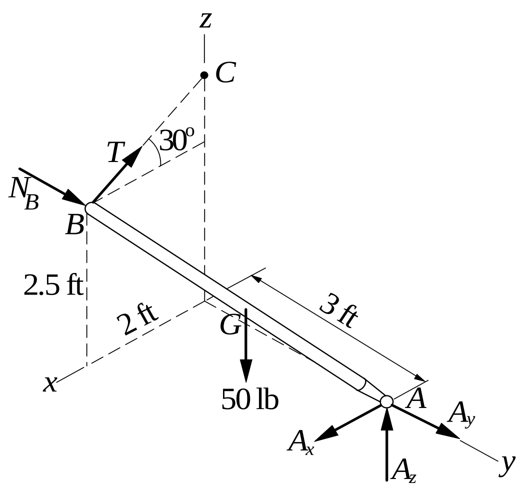
**Expanding the determinants, and equating like components gives:**

**(i-component)** $\quad 4.781\,T_{AB} + 3.405\,T_{AC} - 80 = 0 \qquad (1)$

**(j-component)** $\quad 2.869\,T_{AB} - 4.767\,T_{AC} = 0 \qquad (2)$

**Solving (1) and (2) yields:** $T_{AB} = 11.71$ kN **and** $T_{AC} = 7.05$ kN ♦

**5.54**




$$\Sigma M_y = 0 \qquad -(T\sin 30°)(2) - (T\cos 30°)(2.5) + 50(1.0) = 0$$

$$T = 15.80 \text{ lb} \blacktriangleleft$$

$$\Sigma M_{Ax} = 0 \qquad 50(1.5) - N_B(2.5) - (T\sin 30°)(3) = 0$$

$$50(1.5) - N_B(2.5) - (15.80\sin 30°)(3) = 0$$

$$N_B = 20.5 \text{ lb} \blacktriangleleft$$

**5.55**

$$\Sigma \mathbf{M}_{BC} = \left(\mathbf{r}_{BA} \times \mathbf{T}_{AD} \cdot \vec{\lambda}_{BC}\right) + \left(\mathbf{r}_{BO} \times \mathbf{W} \cdot \vec{\lambda}_{BC}\right) = 0 \qquad \text{(gives } \mathbf{T}_{AD}\text{)}$$

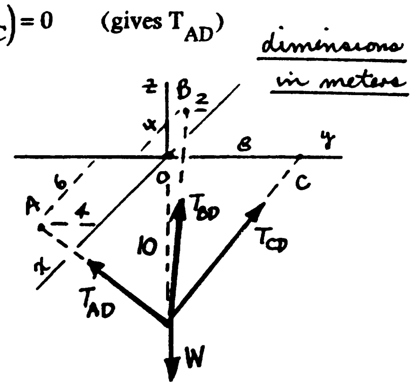
$\mathbf{r}_{BA} = 10\,\mathbf{i} - 2\,\mathbf{j}$ m; $\mathbf{r}_{BO} = 4\,\mathbf{i} + 2\,\mathbf{j}$ m

$\mathbf{W} = -500(9.81)\,\mathbf{k} = -4905\,\mathbf{k}$ N

$$\vec{\lambda}_{BC} = \frac{4\,\mathbf{i} + 10\,\mathbf{j}}{10.77}$$

$$\mathbf{T}_{AD} = T_{AD}\left(\frac{6\,\mathbf{i} - 4\,\mathbf{j} + 10\,\mathbf{k}}{12.33}\right)$$

$$= T_{AD}(0.4866\,\mathbf{i} - 0.3244\,\mathbf{j} + 0.8110\,\mathbf{k})$$



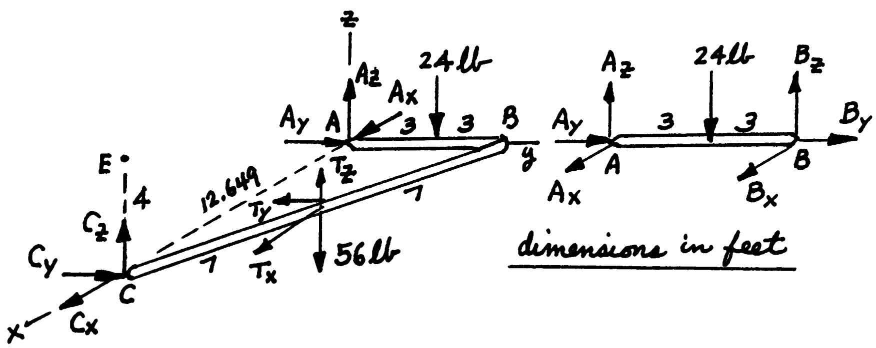
$$\therefore\ \Sigma M_{BC} = \frac{T_{AD}}{10.77}\begin{vmatrix} 10 & -2 & 0 \\ 0.4866 & -0.3244 & 0.8110 \\ 4 & 10 & 0 \end{vmatrix} + \frac{1}{10.77}\begin{vmatrix} 4 & 2 & 0 \\ 0 & 0 & -4905 \\ 4 & 10 & 0 \end{vmatrix} = 0$$

**Cancelling 10.77, and expanding the determinants:** $-87.59\,T_{AD} + 4905(32) = 0$

**which gives** $T_{AD} = 1792\text{ N}$ ◆

5.56

$$\mathbf{T} = T\vec{\lambda}_{DE} = T\left(\frac{6.325\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}}{\sqrt{65}}\right) = T(0.7845\mathbf{i} - 0.3721\mathbf{j} + 0.4961\mathbf{k})$$




**Using FBD of entire assembly**

$\Sigma M_x = 0$: $(56 + 24)(3) - T_z(3) = 80(3) - (0.4961\,T)(3) = 0 \quad \therefore\ T = 161.26\text{ lb}$ ◆

For the directions shown on the FBD:

$T_x = 0.7845(161.26) = 126.51\text{ lb}$; $T_y = 0.3721(161.26) = 60.00\text{ lb}$; $T_z = 80.00\text{ lb}$

$\Sigma M_y = 0$: $(56 - T_z)(6.325) - C_z(12.649) = 0 \quad \therefore\ C_z = \dfrac{(56 - 80)(6.325)}{12.649} = -12\text{ lb}$

$\Sigma M_z = 0$: $C_y(12.649) - T_y(6.325) - T_x(3) = 0 \quad \therefore\ C_y = \dfrac{60(6.325) + 126.51(3)}{12.649} = 60\text{ lb}$ ◆

$\Sigma M_{CE} = 0$: $\quad \therefore\ A_y = 0$

$\Sigma F_z = 0$: $A_z + C_z + T_z - 24 - 56 = 0 \quad \therefore\ A_z = 80 - C_z - T_z = 80 - (-12) - 80 = 12\text{ lb}$

**Using FBD of bar AB**

$\Sigma M_{A_x} = 0: \quad B_z(6) = 24(3) \quad \therefore\ B_z = 12.00 \text{ lb}$

$\Sigma F_y = 0: \quad B_y = -A_y = 0 \qquad \Sigma M_{A_z} = 0: \quad B_x = 0 \qquad \Sigma F_x = 0: \quad A_x = -B_x = 0$

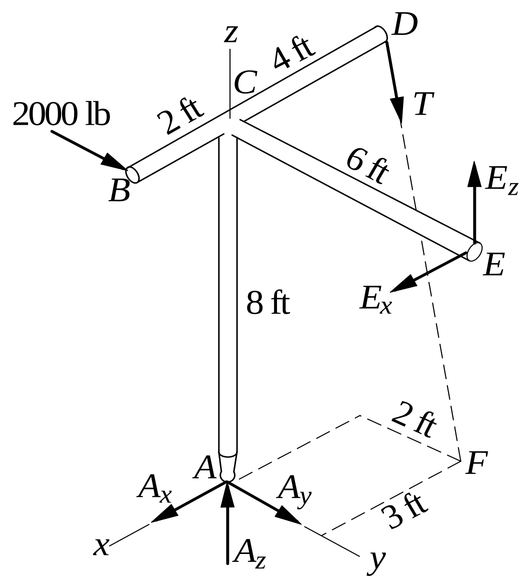
**Using FBD of entire assembly**

$\Sigma F_x = 0: \quad A_x + C_x + T_x = 0 \quad \therefore\ C_x = -A_x - T_x = 0 - 126.51 = -126.51 \text{ lb}$

**Using the above results, the magnitudes of the reactions are:**

$A = B = 12.00 \text{ lb}$ ◆ $\qquad C = \sqrt{(-126.51)^2 + 60^2 + (-12)^2} = 140.5 \text{ lb}$ ◆

**5.57**

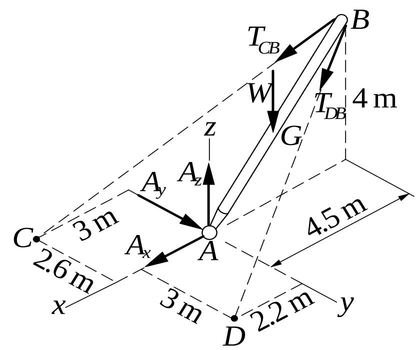



$$\mathbf{T} = T\frac{1.0\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}}{\sqrt{1.0^2 + 2^2 + 8^2}} = (0.1204\mathbf{i} + 0.2408\mathbf{j} - 0.9631\mathbf{k})T$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{j} + 8\mathbf{k}}{\sqrt{6^2 + 8^2}} = 0.6\mathbf{j} + 0.8\mathbf{k}$$

$$\begin{aligned}\Sigma M_{AE} &= \mathbf{r}_{AF} \times \mathbf{T} \cdot \boldsymbol{\lambda}_{AE} + \mathbf{r}_{AB} \times 2000\mathbf{j} \cdot \boldsymbol{\lambda}_{AE} \\ &= \begin{vmatrix} -3 & 2 & 0 \\ 0.1204 & 0.2408 & -0.9631 \\ 0 & 0.6 & 0.8 \end{vmatrix} T + \begin{vmatrix} 2 & 0 & 8 \\ 0 & 2000 & 0 \\ 0 & 0.6 & 0.8 \end{vmatrix} \\ &= -2.504T + 3200 = 0 \\ & \quad T = 1278 \text{ lb} \blacktriangleleft \end{aligned}$$

**5.58**




$$\Sigma M_{AD} = \mathbf{r}_{AC} \times \mathbf{T}_{CB} \cdot \boldsymbol{\lambda}_{AD} + \mathbf{r}_{AG} \times \mathbf{W} \cdot \boldsymbol{\lambda}_{AD} = 0$$

$$\mathbf{T}_{CB} = T_{CB}\frac{7.5\mathbf{i} - 2.6\mathbf{j} - 4\mathbf{k}}{\sqrt{7.5^2 + (-2.6)^2 + (-4)^2}} = (0.8438\mathbf{i} - 0.2925\mathbf{j} - 0.4500\mathbf{k})T_{CB}$$

$$\mathbf{W} = -860(9.81)\mathbf{k} = -8437\mathbf{k} \text{ N}$$

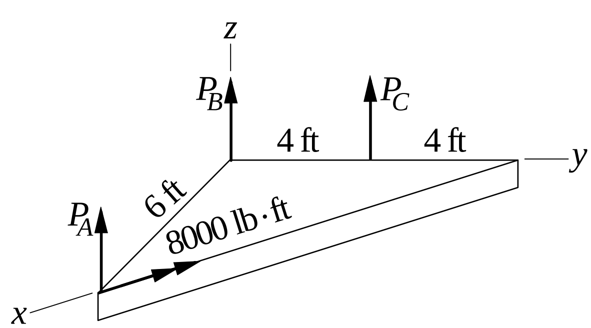
$$\mathbf{r}_{AC} = 3\mathbf{i} - 2.6\mathbf{j} \text{ m} \qquad \mathbf{r}_{AG} = -2.25\mathbf{i} + 2\mathbf{k} \text{ m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{2.2\mathbf{i} + 3\mathbf{j}}{\sqrt{2.2^2 + 3^2}} = 0.5914\mathbf{i} + 0.8064\mathbf{j}$$

$$\Sigma M_{AD} = \begin{vmatrix} 3 & -2.6 & 0 \\ 0.8438 & -0.2925 & -0.4500 \\ 0.5914 & 0.8064 & 0 \end{vmatrix} T_{CB} + \begin{vmatrix} -2.25 & 0 & 2 \\ 0 & 0 & -8437 \\ 0.5914 & 0.8064 & 0 \end{vmatrix} = 0$$

$$1.7806T_{CB} - 15308 = 0 \qquad T_{CB} = 8600 \text{ N} \blacktriangleleft$$

**5.59**



$$\Sigma M_x = 0 \qquad 4P_C - \frac{3}{5}(8000) = 0 \qquad P_C = 1200 \text{ lb (T)} \blacktriangleleft$$

$$\Sigma M_y = 0 \qquad -6P_A + \frac{4}{5}(8000) = 0 \qquad P_A = 1067 \text{ lb (T)} \blacktriangleleft$$

$$\Sigma F_z = 0 \qquad P_A + P_B + P_C = 0$$

$$P_B = -P_A - P_C = -1067 - 1200 = -2267 \text{ lb} = 2270 \text{ lb (C)} \blacktriangleleft$$

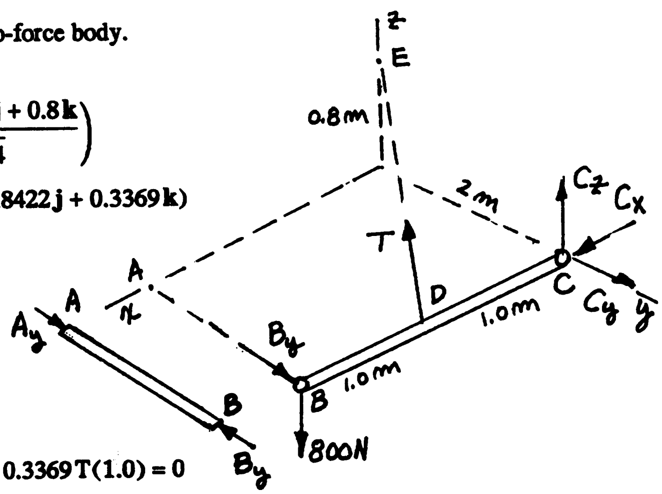
## 5.60

Note that AB is a two-force body.

<u>Using FBD for BC</u>:

$$\mathbf{T} = T\left(\frac{-1.0\mathbf{i} - 2.0\mathbf{j} + 0.8\mathbf{k}}{\sqrt{5.64}}\right)$$

$$= T(-0.4211\mathbf{i} - 0.8422\mathbf{j} + 0.3369\mathbf{k})$$




$$\Sigma(M_C)_y = 800(2) - 0.3369T(1.0) = 0$$

$$\therefore \; T = 4749 \text{ N}$$

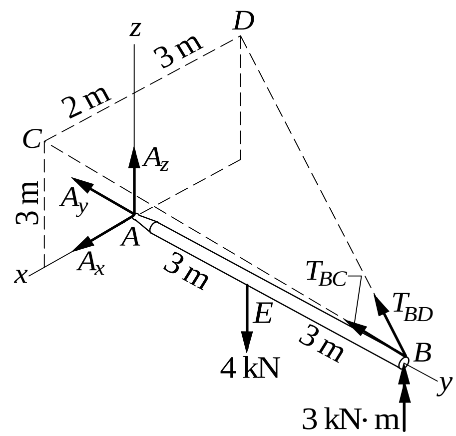
$$\Sigma(M_C)_z = 2.0B_y - 0.8422T(1.0) = 0$$

$$\therefore \; B_y = 0.8422(4749)/2.0 = 2000 \text{ N}$$

<u>Using FBD for AB</u>: $\Sigma F_y = 0$: $A_y = B_y = 2000$ N

Therefore the answers are: $T = 4750$ N; $\mathbf{A} = 2000\mathbf{j}$ N ◆

**5.61**

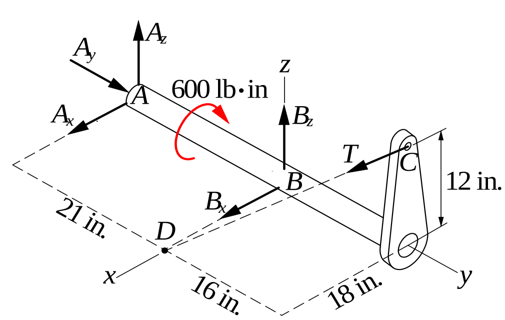



$$\mathbf{T}_{BD} = T_{BD}\frac{-3\mathbf{i}-6\mathbf{j}+3\mathbf{k}}{\sqrt{3^2+6^2+3^2}} = (-0.4083\mathbf{i}-0.8165\mathbf{j}+0.4083\mathbf{k})\,T_{BD}$$

$$\boldsymbol{\lambda}_{AC} = \frac{2\mathbf{i}+3\mathbf{k}}{\sqrt{2^2+3^2}} = 0.5547\mathbf{i}+0.8321\mathbf{k}$$

$$\begin{aligned}
\Sigma M_{AC} &= \mathbf{r}_{CD}\times\mathbf{T}_{BD}\cdot\boldsymbol{\lambda}_{AC}+\mathbf{r}_{AE}\times(-4\mathbf{k})\cdot\boldsymbol{\lambda}_{AC}+3\mathbf{k}\cdot\boldsymbol{\lambda}_{AC}\\
&= \begin{vmatrix} -5 & 0 & 0\\ -0.4083 & -0.8165 & 0.4083\\ 0.5547 & 0 & 0.8321\end{vmatrix} T_{BD} + \begin{vmatrix} 0 & 3 & 0\\ 0 & 0 & -4\\ 0.5547 & 0 & 0.8321\end{vmatrix}\\
&\quad +3(0.8321)\\
&= 3.397T_{BD}-6.656+2.496=0 \qquad T_{BD}=1.225\text{ kN} \blacktriangleleft
\end{aligned}$$

**5.62**



$$\Sigma M_y = 0 \qquad 12T_x - 600 = 0 \qquad T_x = 50 \text{ lb}$$

$$\mathbf{T} = T\frac{18\mathbf{i} - 16\mathbf{j} - 12\mathbf{k}}{\sqrt{18^2 + (-16)^2 + (-12)^2}} = (0.6690\mathbf{i} - 0.5946\mathbf{j} - 0.4460\mathbf{k})T$$

$$\therefore \quad 0.6690T = 50 \qquad T = 74.74 \text{ lb} \blacktriangleleft$$

$$\Sigma \mathbf{M}_A = \mathbf{0} \qquad \mathbf{r}_{AB} \times \mathbf{B} + \mathbf{r}_{AD} \times \mathbf{T} - 600\mathbf{j} = \mathbf{0}$$

$$\mathbf{r}_{AB} = 21\mathbf{j} \text{ in.} \qquad \mathbf{r}_{AD} = 18\mathbf{i} + 21\mathbf{j} \text{ in.}$$

$$\Sigma \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 21 & 0 \\ B_x & 0 & B_z \end{vmatrix} + 74.74 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 18 & 21 & 0 \\ 0.6690 & -0.5946 & -0.4460 \end{vmatrix} - 600\mathbf{j} = \mathbf{0}$$

$$21(B_z\mathbf{i} - B_x\mathbf{k}) + (-700\mathbf{i} + 600\mathbf{j} - 1850\mathbf{k}) - 600\mathbf{j} = \mathbf{0}$$

Equating like components:

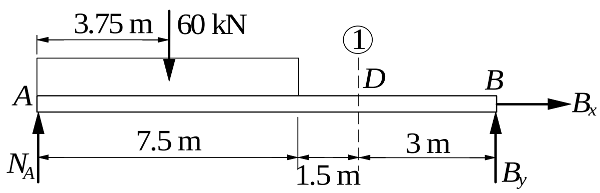
$$(\mathbf{i}\text{-component}) \quad 21B_z - 700 = 0 \qquad B_z = 33.33 \text{ lb}$$

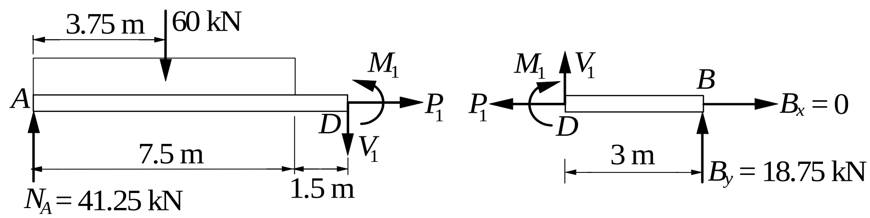
$$(\mathbf{k}\text{-component}) \quad -21B_x - 1850 = 0 \qquad B_x = -88.10 \text{ lb}$$

$$\therefore B = \sqrt{33.33^2 + (-88.10)^2} = 94.2 \text{ lb} \blacktriangleleft$$

# Chapter 6

**6.1**




FBD of entire beam:

$$\begin{aligned} \Sigma M_A &= 0 \qquad B_y(12) - 60(3.75) = 0 \qquad B_y = 18.75 \text{ kN} \\ \Sigma F_x &= 0 \qquad B_x = 0 \\ \Sigma F_y &= 0 \qquad N_A + 18.75 - 60 = 0 \qquad N_A = 41.25 \text{ kN} \end{aligned}$$

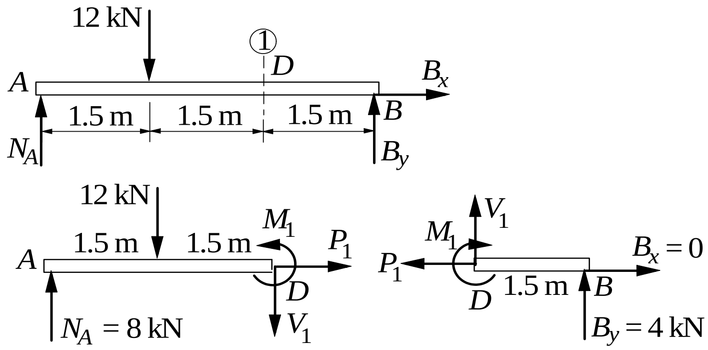
FBD of segment $AD$:

$$\begin{aligned} \Sigma F_x &= 0 \qquad P_1 = 0 \ \blacktriangleleft \\ \Sigma F_y &= 0 \qquad 41.25 - V_1 - 60 = 0 \qquad V_1 = -18.75 \text{ kN} \ \blacktriangleleft \\ \Sigma M_D &= 0 \qquad M_1 - 41.25(9) + 60(5.25) = 0 \\ & M_1 = 56.25 \text{ kN} \cdot \text{m} \ \blacktriangleleft \end{aligned}$$

FBD of segment $DB$

$$\begin{aligned} \Sigma F_x &= 0 \qquad P_1 = 0 \ \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_1 + 18.75 = 0 \qquad V_1 = -18.75 \text{ kN} \ \blacktriangleleft \\ \Sigma M_D &= 0 \qquad M_1 - 18.75(3) = 0 \qquad M_1 = 56.25 \text{ kN} \cdot \text{m} \ \blacktriangleleft \end{aligned}$$

**6.2**




FBD of entire beam:

$$\begin{aligned} \Sigma M_A &= 0 \quad 4.5B_y - 12(1.5) = 0 \quad B_y = 4 \text{ kN} \\ \Sigma F_x &= 0 \quad B_x = 0 \\ \Sigma F_y &= 0 \quad N_A + 4 - 12 = 0 \quad N_A = 8 \text{ kN} \end{aligned}$$

FBD of segment $AD$:

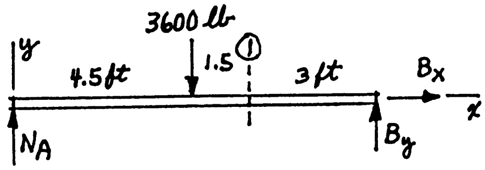
$$\begin{aligned} \Sigma F_x &= 0 \quad P_1 = 0 \blacktriangleleft \\ \Sigma F_y &= 0 \quad 8 - 12 - V_1 = 0 \quad V_1 = -4 \text{ kN} \blacktriangleleft \\ \Sigma M_D &= 0 \quad M_1 - 8(3) + 12(1.5) = 0 \quad M_1 = 6 \text{ kN} \cdot \text{m} \blacktriangleleft \end{aligned}$$

FBD of segment $DB$:

$$\begin{aligned} \Sigma F_x &= 0 \quad P_1 = 0 \blacktriangleleft \\ \Sigma F_y &= 0 \quad V_1 + 4 = 0 \quad V_1 = -4 \text{ kN} \blacktriangleleft \\ \Sigma M_D &= 0 \quad M_1 - 4(1.5) = 0 \quad M_1 = 6 \text{ kN} \cdot \text{m} \blacktriangleleft \end{aligned}$$

## 6.3

**FBD of entire beam**

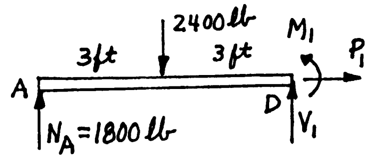



By symmetry:

$N_A = B_y = 1800$ lb

$\Sigma F_x = 0$: $B_x = 0$

**FBD of segment AD**




$\Sigma F_x = 0$: $P_1 = 0$ ◆

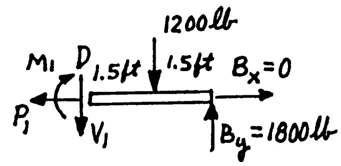
$\Sigma F_y = 0$: $V_1 = 2400 - N_A$

$\therefore$ $V_1 = 2400 - 1800 = 600$ lb ◆

$\Sigma M_D = 0$: (+↺) $-N_A(6) + 2400(3) + M_1 = 0$

$\therefore$ $M_1 = 1800(6) - 7200 = 3600$ lb•ft ◆

**FBD of segment DB**




$\Sigma F_x = 0$: $P_1 = B_x = 0$ ◆

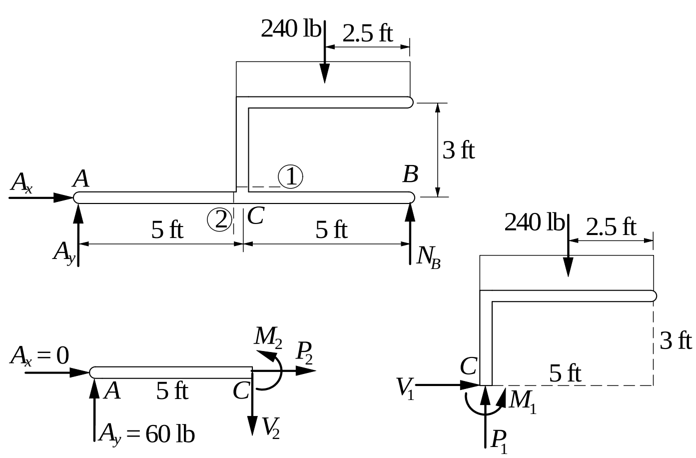
$\Sigma F_y = 0$: $V_1 = B_y - 1200$

$\therefore$ $V_1 = 1800 - 1200 = 600$ lb ◆

$\Sigma M_D = 0$: (+↺) $B_y(3) - 1200(1.5) - M_1 = 0$

$\therefore$ $M_1 = 1800(3) - 1800 = 3600$ lb•ft ◆

## 6.4



FBD of entire structure:

$$\Sigma M_B = 0 \qquad A_y(10) - 240(2.5) = 0 \qquad A_y = 60 \text{ lb}$$
$$\Sigma F_x = 0 \qquad A_x = 0$$

FBD of segment above section 1:

$$\Sigma F_x = 0 \qquad V_1 = 0 \blacktriangleleft$$
$$\Sigma F_y = 0 \qquad P_1 - 240 = 0 \qquad P_1 = 240 \text{ lb} \blacktriangleleft$$
$$\Sigma M_C = 0 \qquad M_1 - 240(2.5) = 0 \qquad M_1 = 600 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

FBD of segment to the left of section 2:

$$\Sigma F_x = 0 \qquad P_2 = 0 \blacktriangleleft$$
$$\Sigma F_y = 0 \qquad V_2 = 60 \text{ lb} \blacktriangleleft$$
$$\Sigma M_C = 0 \qquad M_2 - 60(5) = 0 \qquad M_2 = 300 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

## 6.5

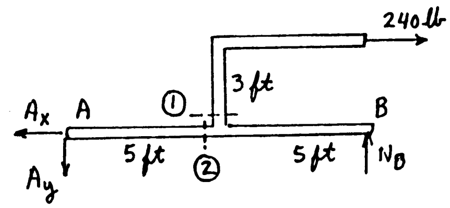
**FBD of entire beam**

**$\Sigma M_A = 0$: $N_B(10) = 240(3)$**

**$\therefore\ N_B = 72$ lb**

**$\Sigma F_y = 0$: $A_y = N_B = 72$ lb**

**$\Sigma F_x = 0$: $A_x = 240$ lb**

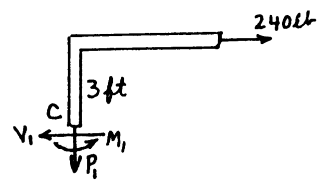



**FBD of segment above section 1**

**$\Sigma F_x = 0$: $V_1 = 240$ lb ♦**

**$\Sigma F_y = 0$: $P_1 = 0$ ♦**

**$\Sigma M_C = 0$: $M_1 = 240(3) = 720$ lb•ft ♦**

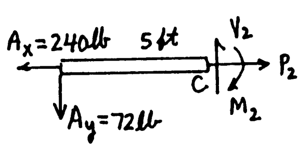



**FBD of segment to the left of section 2**

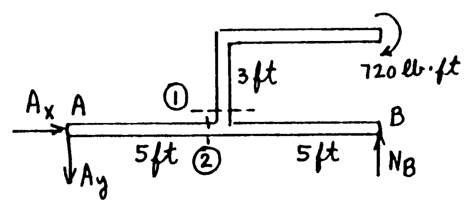
**$\Sigma F_x = 0$: $P_2 = 240$ lb (tension) ♦**

**$\Sigma F_y = 0$: $V_2 = A_y = 72$ lb ♦**

**$\Sigma M_C = 0$: $M_2 = 72(5) = 360$ lb•ft ♦**



## 6.6




FBD of entire beam

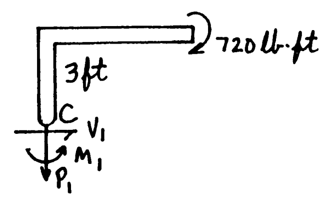
$\Sigma M_A = 0$: $N_B(10) = 720$ lb•ft

$\therefore$ $N_B = 72$ lb

$\Sigma F_y = 0$: $A_y = N_B = 72$ lb

$\Sigma F_x = 0$: $A_x = 0$

FBD of segment above section 1

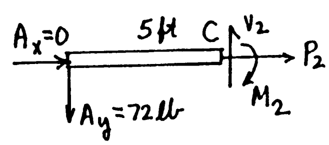



$\Sigma F_x = 0$: $V_1 = 0$ ♦

$\Sigma F_y = 0$: $P_1 = 0$ ♦

$\Sigma M_C = 0$: $M_1 = 720$ lb•ft ♦

FBD of segment to the left of section 2



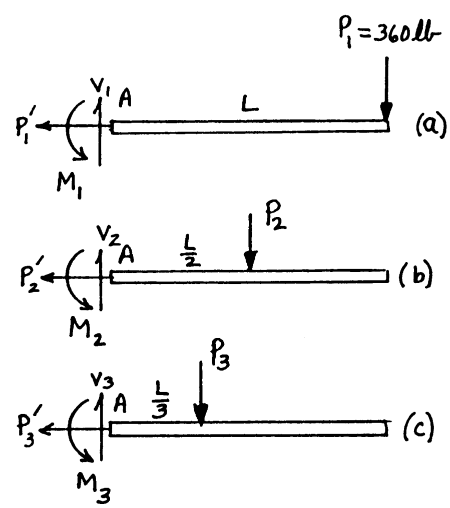

$\Sigma F_x = 0$: $P_2 = 0$ ♦

$\Sigma F_y = 0$: $V_2 = A_y = 72$ lb ♦

$\Sigma M_C = 0$: $M_2 = 72(5) = 360$ lb•ft ♦

## 6.7

In each case, the maximum bending moment occurs at the support.




Case (a)

$\Sigma M_A = 0$:

$M_1 = P_1 L = 360L = M_{max}$

Case (b)

$\Sigma M_A = 0$:

$M_2 = P_2(L/2) = M_{max} = 360L$

$\therefore$ $P_2 = 720$ lb ♦

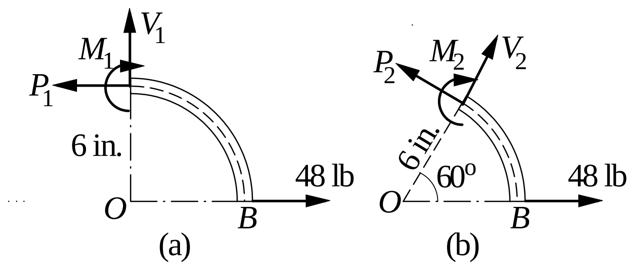
Case (c)

$\Sigma M_A = 0$:

$M_3 = P_3(L/3) = M_{max} = 360L$

$\therefore$ $P_3 = 1080$ lb ♦

**6.8**



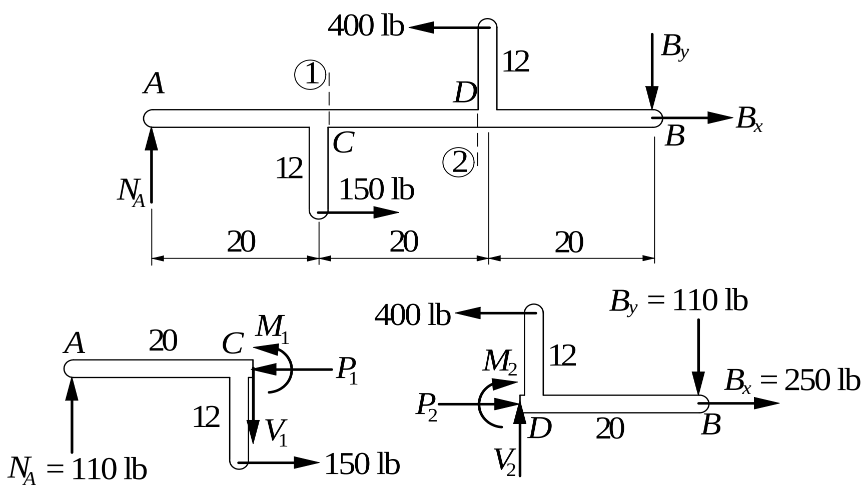

FBD (a):

$$\begin{aligned}\Sigma F_x &= 0 \qquad P_1 = 48 \text{ lb} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_1 = 0 \blacktriangleleft \\ \Sigma M_O &= 0 \qquad 6P_1 - M_1 = 0 \qquad M_1 = 6(48) = 288 \text{ lb}\cdot\text{in.} \blacktriangleleft\end{aligned}$$

FBD (b):

$$\begin{aligned}\Sigma F_{P_2} &= 0 \qquad P_2 - 48\sin 60^\circ = 0 \qquad P_2 = 41.6 \text{ lb} \blacktriangleleft \\ \Sigma F_{V_1} &= 0 \qquad V_2 + 48\cos 60^\circ = 0 \qquad V_2 = 24.0 \text{ lb} \blacktriangleleft \\ \Sigma M_O &= 0 \qquad 6P_2 - M_2 = 0 \qquad M_2 = 6(41.6) = 250 \text{ lb}\cdot\text{in.} \blacktriangleleft\end{aligned}$$

**6.9**




FBD of entire stucture:

$$\begin{aligned}\Sigma M_A &= 0 \qquad B_y(60) - (150 + 400)(12) = 0 \qquad B_y = 110 \text{ lb} \\ \Sigma F_x &= 0 \qquad B_x - 400 + 150 = 0 \qquad B_x = 250 \text{ lb} \\ \Sigma F_y &= 0 \qquad N_A - B_y = 0 \qquad N_A = B_y = 110 \text{ lb}\end{aligned}$$

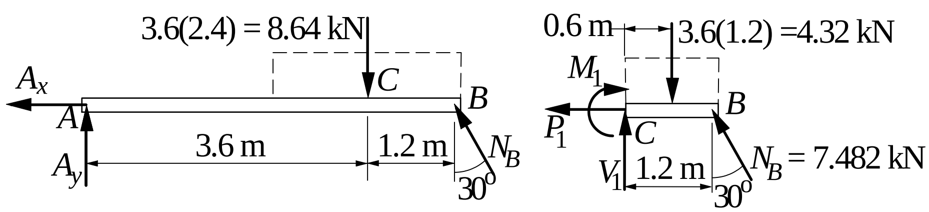
FBD of portion $AC$:

$$\begin{aligned} \Sigma M_C &= 0 \qquad M_1 + 150(12) - 110(20) = 0 \\ &M_1 = 400 \text{ lb} \cdot \text{in.} \blacktriangleleft \\ \Sigma F_x &= 0 \qquad P_1 = 150 \text{ lb} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_1 = 110 \text{ lb} \blacktriangleleft \end{aligned}$$

FBD of portion $DB$:

$$\begin{aligned} \Sigma M_D &= 0 \qquad M_2 - 400(12) + 110(20) = 0 \\ &M_2 = 2600 \text{ lb} \cdot \text{in.} \blacktriangleleft \\ \Sigma F_x &= 0 \qquad P_2 - 400 + 250 = 0 \qquad P_2 = 150 \text{ lb} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_2 = 110 \text{ lb} \blacktriangleleft \end{aligned}$$

**6.10**



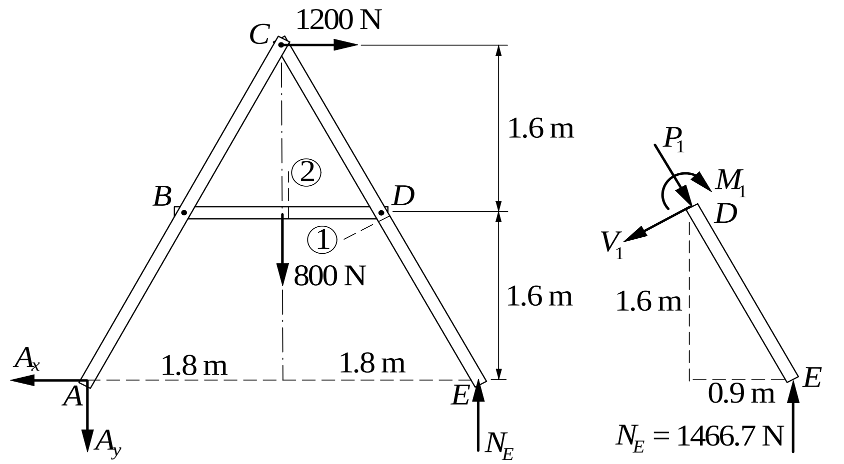

FBD of entire beam:

$$\Sigma M_A = 0 \qquad (N_B \cos 30^\circ)(4.8) - 8.64(3.6) = 0 \qquad N_B = 7.482 \text{ kN}$$

FBD of segment $CB$:

$$\begin{aligned} \Sigma M_C &= 0 \qquad M_1 + 4.32(0.6) - (7.842 \cos 30^\circ)(1.2) = 0 \\ &M_1 = 5.56 \text{ kN} \cdot \text{m} \blacktriangleleft \\ \Sigma F_x &= 0 \qquad P_1 + 7.482 \sin 30^\circ = 0 \qquad P_1 = -3.74 \text{ kN} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_1 + 7.482 \cos 30^\circ - 4.32 = 0 \qquad V_1 = -2.16 \text{ kN} \blacktriangleleft \end{aligned}$$

**6.11**




FBD of entire frame:

$$\Sigma M_A = 0 \qquad N_E(3.6) - 800(1.8) - 1200(3.2) = 0 \qquad N_E = 1466.7 \text{ N}$$

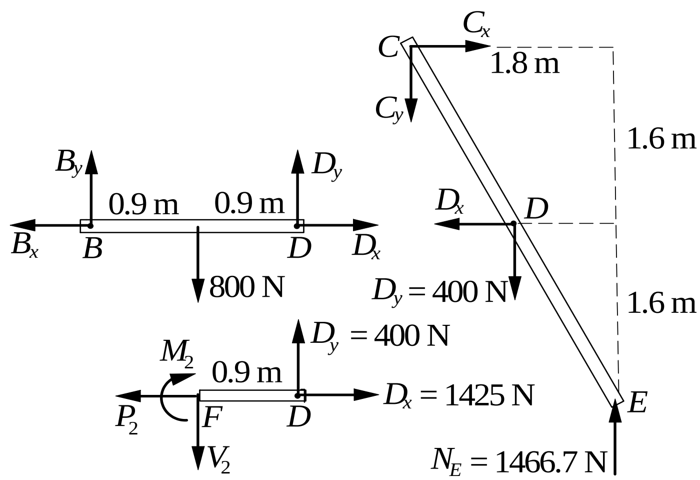
FBD of segment $DE$:

$$\Sigma M_D = 0 \qquad M_1 - 1466.7(0.9) = 0 \qquad M_1 = 1320 \text{ N}\cdot\text{m} \blacktriangleleft$$

$$\Sigma F_{V_1} = 0 \qquad V_1 - 1466.7\left(\frac{0.9}{\sqrt{0.9^2 + 1.6^2}}\right) \qquad V_1 = 719 \text{ N} \blacktriangleleft$$

$$\Sigma F_{P_1} = 0 \qquad P_1 - 1466.7\left(\frac{1.6}{\sqrt{0.9^2 + 1.6^2}}\right) \qquad P_1 = 1278 \text{ N} \blacktriangleleft$$

## 6.12




FBD of member $BD$:

$$\Sigma M_B = 0 \qquad D_y(1.8) - 800(0.9) = 0 \qquad D_y = 400 \text{ N}$$

FBD of member $CE$ ($N_E$ was computed in the solution of Prob. 6.11):

$$\Sigma M_C = 0 \qquad D_x(1.6) + 400(0.9) - 1466.7(1.8) = 0 \qquad D_x = 1425 \text{ N}$$

FBD of segment $FD$:

$$\begin{aligned} \Sigma M_F &= 0 \qquad M_2 - 400(0.9) = 0 \qquad M_2 = 360 \text{ N}\cdot\text{m} \blacktriangleleft \\ \Sigma F_x &= 0 \qquad P_2 = 1425 \text{ N} \blacktriangleleft \\ \Sigma F_y &= 0 \qquad V_2 = 400 \text{ N} \blacktriangleleft \end{aligned}$$

## 6.13

<u>FBD of entire frame</u> (recognizing that BC is a two-force body)

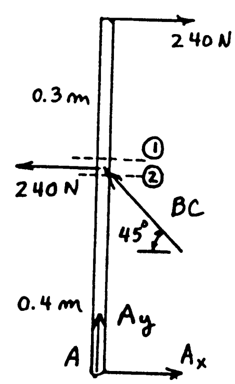
$\Sigma M_A = 0$:

(+) $BC \cos 45^\circ(0.4) - 240(0.7) + 240(0.4) = 0$

$\therefore$ BC = 254.6 N

which gives: $BC_x = 254.6 \cos 45^\circ = 180$ N

$BC_y = 254.6 \sin 45^\circ = 180$ N

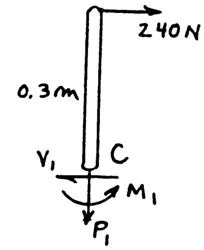


**FBD of segment above section 1**

$\Sigma F_x = 0$: $V_1 = 240$ N ♦

$\Sigma F_y = 0$: $P_1 = 0$ ♦

$\Sigma M_C = 0$: $M_1 = 240(0.3) = 72$ N•m ♦

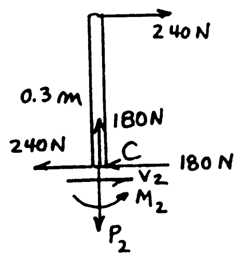



**FBD of segment above section 2**

$\Sigma F_x = 0$: $V_2 = 240 - 240 + 180 = 180$ N ♦

$\Sigma F_y = 0$ : $P_2 = 180$ N (tension) ♦

$\Sigma M_C = 0$: $M_2 = 240(0.3) = 72$ N•m ♦




## 6.14

**FBD of entire frame** (recognizing that BC is a two-force body)

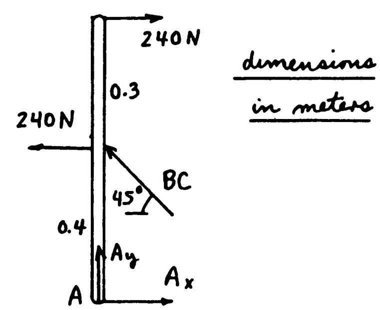
$\Sigma M_A = 0$:

(+↺) $BC \cos 45^\circ(0.4) - 240(0.7) + 240(0.4) = 0$

$\therefore$ BC = 254.6 N

which gives: $BC_x = 254.6 \cos 45^\circ = 180$ N

$BC_y = 254.6 \sin 45^\circ = 180$ N




**FBD of segment of BC that lies to the left of section 3**

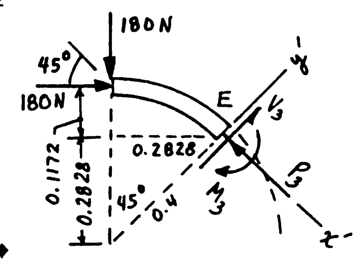
$\Sigma F_{x'} = 0$:

$P_3 = 180 \cos 45^\circ + 180 \cos 45^\circ = 255$ N (C) ♦

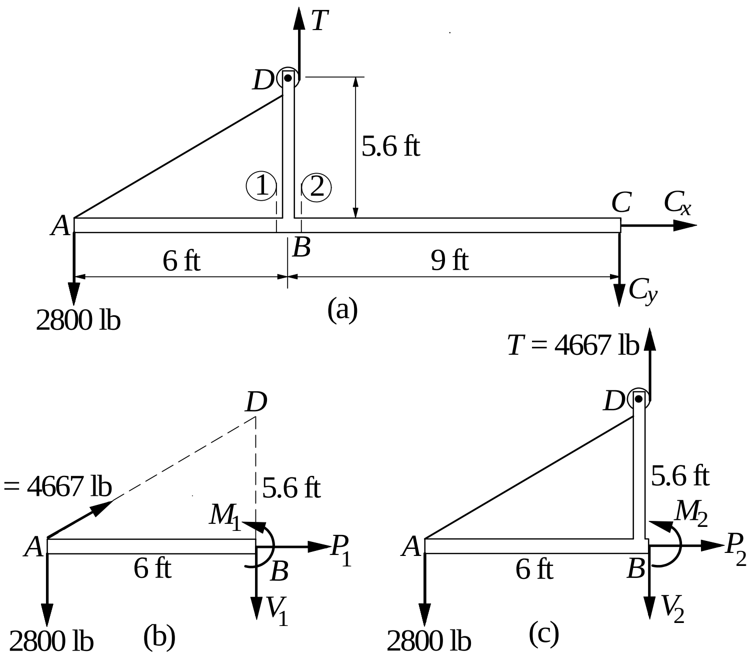
$\Sigma F_{y'} = 0$: $V_3 = 180 \sin 45^\circ - 180 \sin 45^\circ = 0$ ♦

$\Sigma M_E = 0$:

$M_3 = 180(0.2828) - 180(0.1172) = 29.8$ N•m ♦



**6.15**




FBD (a):

$$\Sigma M_C = 0 \qquad 2800(15) - 9T = 0 \qquad T = 4667 \text{ lb}$$

FBD (b):

$$\Sigma M_B = 0 \qquad 2800(6) - \frac{5.6}{\sqrt{5.6^2 + 6^2}}(4667) + M_1 = 0$$

$$M_1 = -13\ 629 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad \frac{6}{\sqrt{5.6^2 + 6^2}}(4667) + P_1 = 0 \qquad P_1 = -3410 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0 \qquad \frac{5.6}{\sqrt{5.6^2 + 6^2}}(4667) - V_1 = 0 \qquad V_1 = 3180 \text{ lb} \blacktriangleleft$$

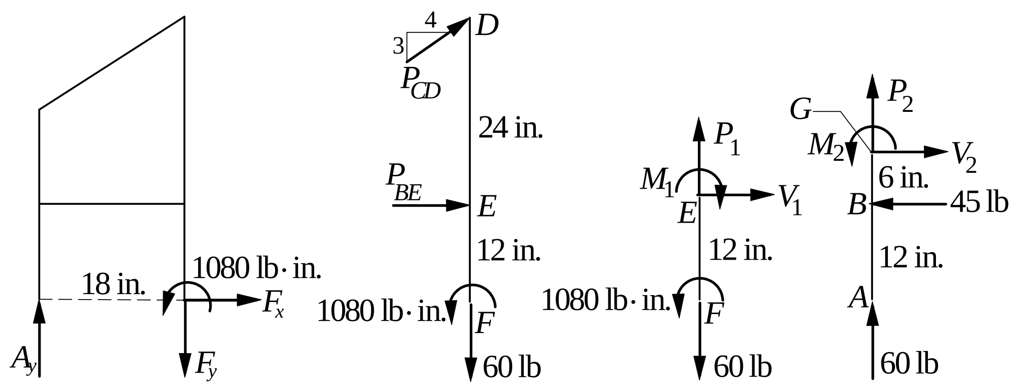
FBD (c):

$$\Sigma M_B = 0 \qquad 2800(6) + M_2 = 0 \qquad M_2 = -16\ 800 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad P_2 = 0 \blacktriangleleft$$

$$\Sigma F_y = 0 \qquad 4667 - 2800 - V_2 = 0 \qquad V_2 = 1867 \text{ lb} \blacktriangleleft$$

**6.16**




FBD of entire frame:

$$\Sigma M_A = 0 \qquad 1080 - 18A_y = 0 \qquad A_y = 60 \text{ lb}$$
$$\Sigma F_x = 0 \qquad F_x = 0$$
$$\Sigma F_y = 0 \qquad F_y - A_y = 0 \qquad F_y = A_y = 60 \text{ lb}$$

FBD of member $FED$:

$$\Sigma M_D = 0 \qquad 24P_{BE} - 1080 = 0 \qquad P_{BE} = 45 \text{ lb}$$

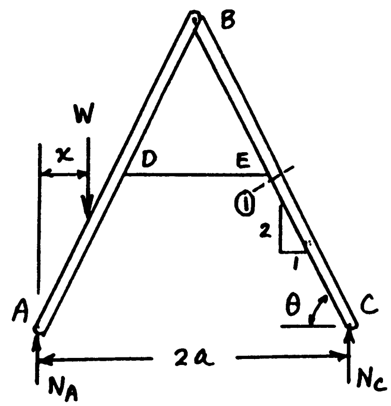
FBD of segment $DEF$ below section 1:

$$\Sigma F_x = 0 \qquad V_1 = 0 \blacktriangleleft$$
$$\Sigma F_y = 0 \qquad P_1 - 60 = 0 \qquad P_1 = 60 \text{ lb} \blacktriangleleft$$
$$\Sigma M_E = 0 \qquad M_1 - 1080 = 0 \qquad M_1 = 1080 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

FBD of segment $ABC$ below section 2:

$$\Sigma F_x = 0 \qquad V_2 - 45 = 0 \qquad V_2 = 45 \text{ lb} \blacktriangleleft$$
$$\Sigma F_y = 0 \qquad P_2 + 60 = 0 \qquad P_2 = -60 \text{ lb} \blacktriangleleft$$
$$\Sigma M_G = 0 \qquad M_2 - 45(6) = 0 \qquad M_2 = 270 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

## 6.17

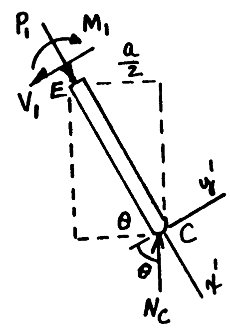



**FBD of entire frame**

$\Sigma M_A = 0$: $Wx = N_C(2a)$

$$\therefore\ N_C = \frac{Wx}{2a}$$

Note: $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$



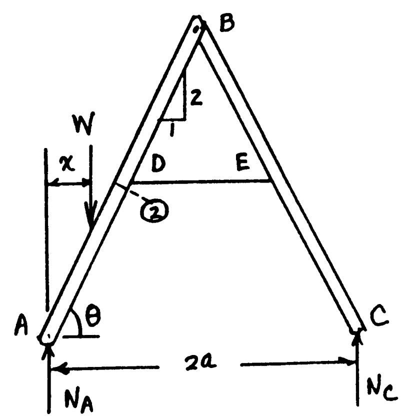

**FBD of segment of BC that is below section 1**

$\Sigma F_{x'} = 0$: $P_1 = N_C \sin\theta = \frac{Wx}{2a}\frac{2}{\sqrt{5}} = \frac{Wx}{a\sqrt{5}}$ ◆

$\Sigma F_{y'} = 0$: $V_1 = N_C \cos\theta = \frac{Wx}{2a}\frac{1}{\sqrt{5}} = \frac{Wx}{2a\sqrt{5}}$ ◆

$\Sigma M_E = 0$: $M_1 = N_C\left(\frac{a}{2}\right) = \frac{Wx}{2a}\left(\frac{a}{2}\right) = \frac{Wx}{4}$ ◆

## 6.18



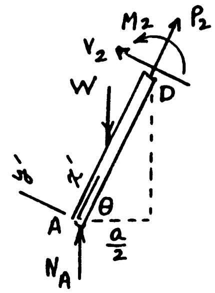

**FBD of entire frame**

$\Sigma M_C = 0$: $W(2a - x) = N_A(2a)$

$$\therefore\ N_A = \frac{W(2a-x)}{2a} = W - \frac{Wx}{2a}$$

Note: $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$

**FBD of segment of AB that is below section 2**




$\Sigma F_{x'} = 0$:

$$P_2 = W\sin\theta - N_A\sin\theta = \frac{2}{\sqrt{5}}\left[W - \left(W - \frac{Wx}{2a}\right)\right]$$

$$\therefore\ P_2 = \frac{Wx}{a\sqrt{5}} \quad \text{(tension)} \ \blacklozenge$$

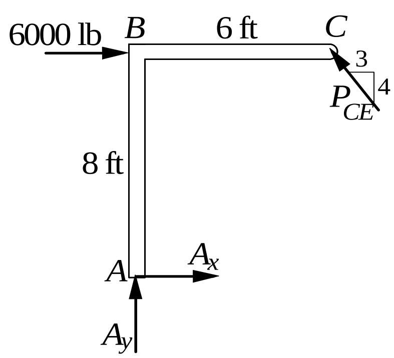
$\Sigma F_{y'} = 0$:

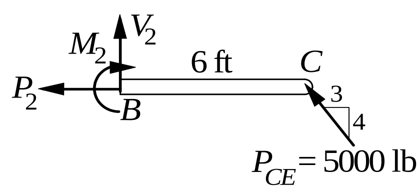
$$V_2 = W\cos\theta - N_A\cos\theta = \frac{1}{\sqrt{5}}\left[W - \left(W - \frac{Wx}{2a}\right)\right] = \frac{Wx}{2a\sqrt{5}} \ \blacklozenge$$

$\Sigma M_D = 0$:

$$M_2 = N_A\left(\frac{a}{2}\right) - W\left(\frac{a}{2} - x\right) = \frac{a}{2}\left[W - \frac{Wx}{2a}\right] - W\left(\frac{a}{2} - x\right) = \frac{3Wx}{4} \ \blacklozenge$$

**6.19**




Note that $CDE$ is a two-force body.

FBD of $ABC$:

$$\Sigma M_A = 0 \qquad \frac{3}{5}P_{CE}(8) + \frac{4}{5}P_{CE}(6) - 6000(8) = 0 \qquad P_{CE} = 5000 \text{ lb}$$

FBD of $BD$:

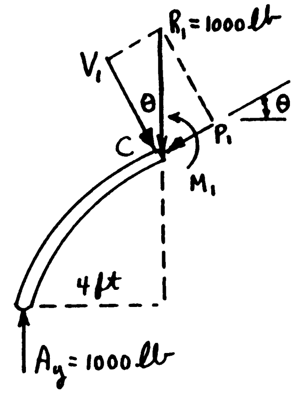
$$\Sigma M_B = 0 \qquad 4000(6) - M_2 = 0 \qquad M_2 = 24\,000 \text{ lb}\cdot\text{ft} \ \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad -P_2 - 3000 = 0 \qquad P_2 = -3000 \text{ lb} \ \blacktriangleleft$$

$$\Sigma F_y = 0 \qquad V_2 + 4000 = 0 \qquad V_2 = -4000 \text{ lb} \ \blacktriangleleft$$

## *6.20

From FBD of entire parabolic arch (not shown here), $A_y = 1000$ lb ↑ .




FBD of arch segment below section 1

$\Sigma M_C = 0$: $M_1 = 1000(4) = 4000$ lb•ft ◆

$\Sigma F_y = 0$: $R_1 = A_y = 1000$ lb

Note that $P_1$ and $V_1$ are components of $R_1$.

We must find θ, the slope of the arch at section 1 (i.e., at x = – 2 ft).

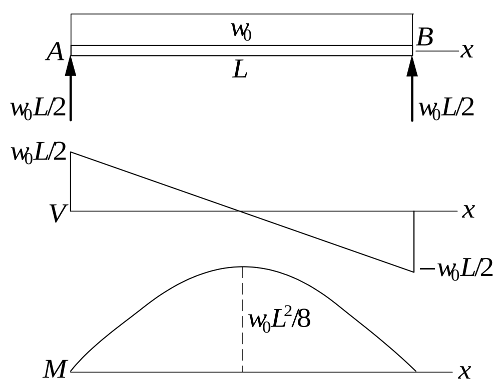
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{36 - x^2}{6}\right) = -\frac{x}{3}$$

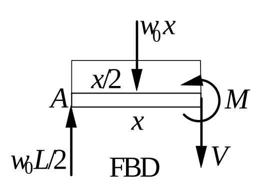
At x = – 2 ft, $\frac{dy}{dx} = \frac{2}{3}$, which gives θ = 33.69°.

Therefore, $P_1 = 1000 \sin\theta = 1000 \sin 33.69^\circ = 555$ lb ◆

$V_1 = 1000 \cos\theta = 1000 \cos 33.69^\circ = 832$ lb ◆

## 6.21

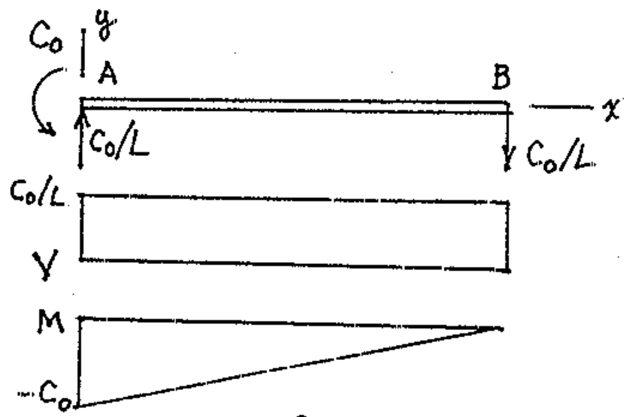


$$V = \frac{w_0 L}{2} - w_0 x \blacktriangleleft \qquad M = \frac{w_0 L x}{2} - \frac{w_0 x^2}{2} \blacktriangleleft$$

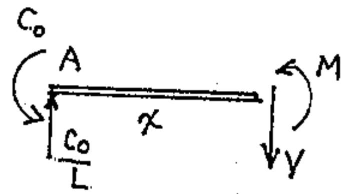
**6.22**

Derive expressions for $V$ and $M$; draw $V$- and $M$-diagrams

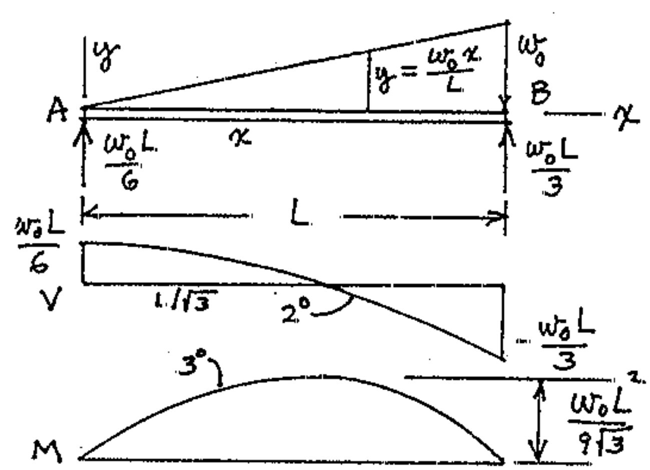



$$V = \frac{C_0}{L} \blacktriangleleft$$

$$M = -C_0 + \frac{C_0 x}{L} \blacktriangleleft$$



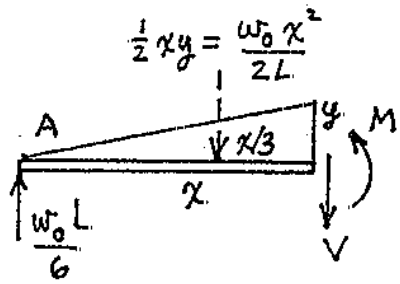

**6.23**



$$V = \frac{w_0 L}{6} - \frac{w_0 x^2}{2L} \blacktriangleleft$$

$$M = \frac{w_0 Lx}{6} - \frac{w_0 x^2}{2L}\left(\frac{x}{3}\right)$$

$$M = \frac{w_0 Lx}{6} - \frac{w_0 x^3}{6L} \blacktriangleleft$$

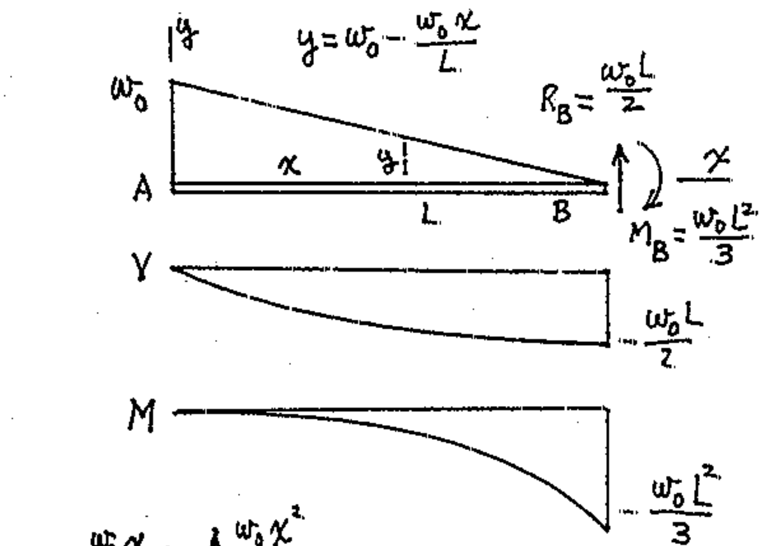


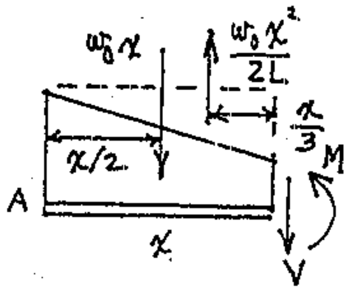

*Find where shear is zero, which is also where M is maximum*

$$V = \frac{w_0 L}{6} - \frac{w_0 x^2}{2L} = 0 \quad \text{gives } x = \frac{L}{\sqrt{3}}$$

$$M_{\max} = \frac{w_0 L}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{w_0}{6L}\left(\frac{L}{\sqrt{3}}\right)^3 = \frac{w_0 L^2}{9\sqrt{3}}$$

**6.24**



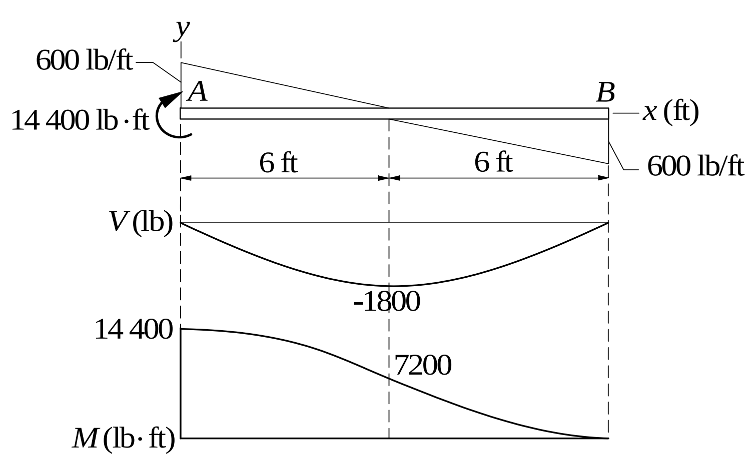



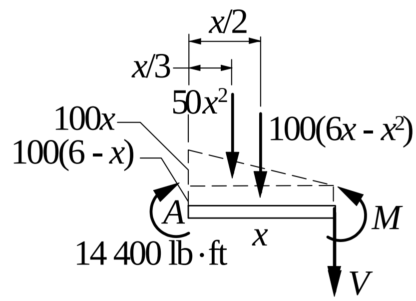

$$V = -w_0 x + \frac{w_0 x^2}{2L}$$ ◄

$$M = -w_0 x\left(\frac{x}{2}\right) + \frac{w_0 x^2}{2L}\left(\frac{x}{3}\right)$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L}$$ ◄

**6.25**



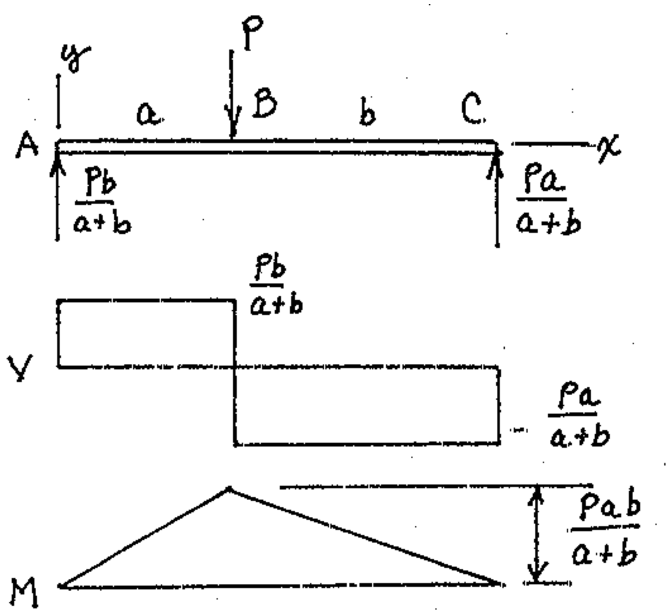



$$V = -50x^2 - 100(6x - x^2) = -600x + 50x^2 \text{ lb} \blacktriangleleft$$

$$M = 14\ 400 - 50x^2\left(\frac{2}{3}x\right) - 100(6x - x^2)\left(\frac{1}{2}x\right)$$

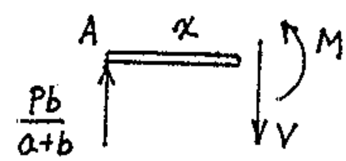
$$= -300x^2 + \frac{50}{3}x^3 + 14400 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

**6.26**

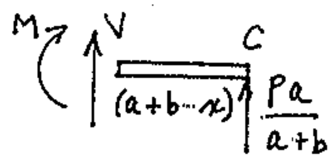



$AB$ $\quad V = \dfrac{Pb}{a+b} \blacktriangleleft$

$M = \dfrac{Pbx}{a+b} \blacktriangleleft$



$$BC \quad V = -\frac{Pa}{a+b} \blacktriangleleft$$

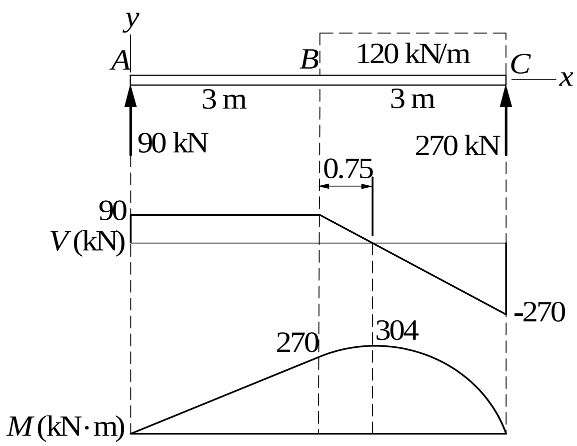


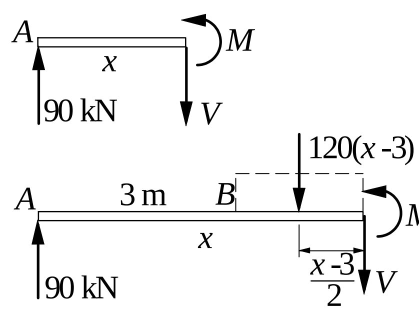

$$M = \frac{Pa(a+b-x)}{a+b}$$

$$M = \frac{Pa(a+b)}{a+b} - \frac{Pax}{a+b}$$

$$M = Pa\left(1 - \frac{x}{a+b}\right) \blacktriangleleft$$

**6.27**




Segment $AB$:

$$V = 90 \text{ kN} \blacktriangleleft$$

$$M = 90x \text{ kN} \cdot \text{m} \blacktriangleleft$$

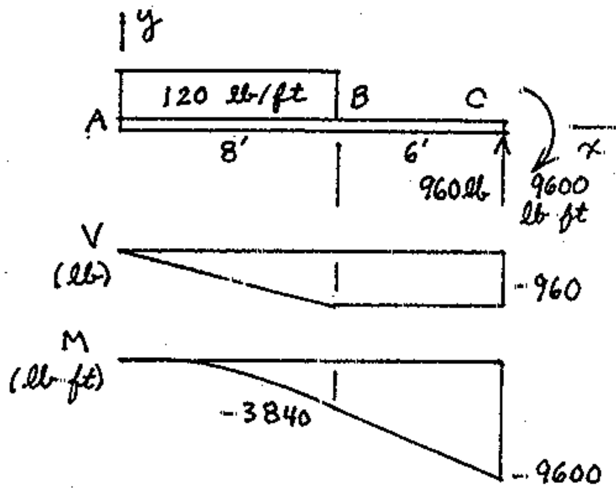
Segment $BC$:

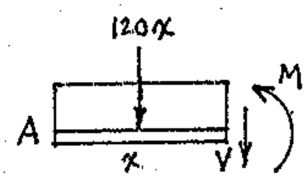
$$V = 90 - 120(x-3) = 450 - 120x \text{ kN} \blacktriangleleft$$

$$M = 90x - 120(x-3)\frac{x-3}{2} = -60x^2 + 450x - 540 \text{ kN} \cdot \text{m} \blacktriangleleft$$

$$M_{\max} = M|_{3.75 \text{ m}} = -60(3.75^2) + 450(3.75) - 540 = 304 \text{ kN} \cdot \text{m}$$

**6.28**



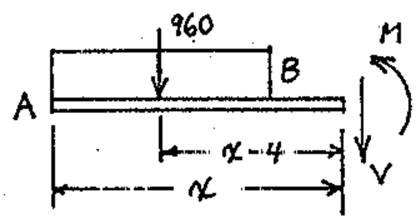

$$AB \quad V = -120x \text{ lb} \blacktriangleleft$$

$$M = -120x\left(\frac{x}{2}\right)$$

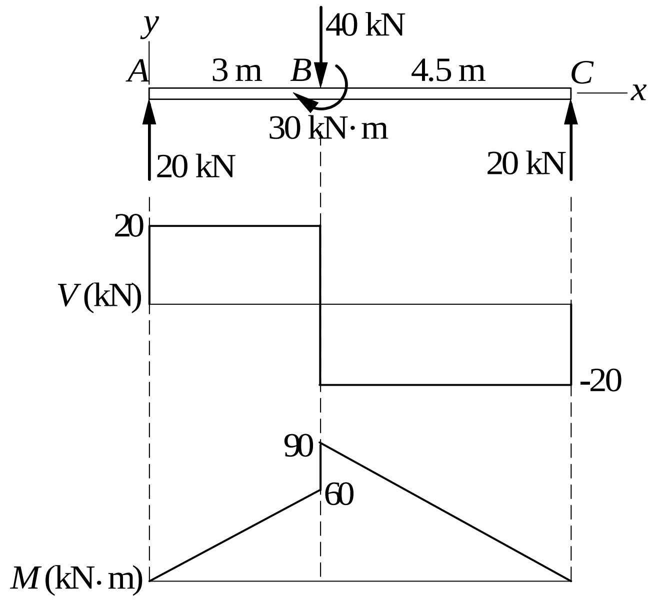
$$M = -60x^2 \text{ lb ft} \blacktriangleleft$$

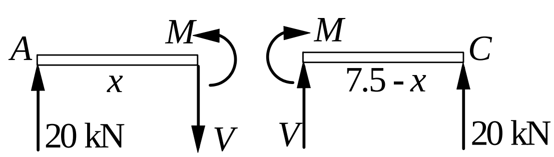
$$BC \quad V = -960 \text{ lb} \blacktriangleleft$$

$$M = -960(x-4) = -960x + 3840 \text{ lb ft} \blacktriangleleft$$



**6.29**

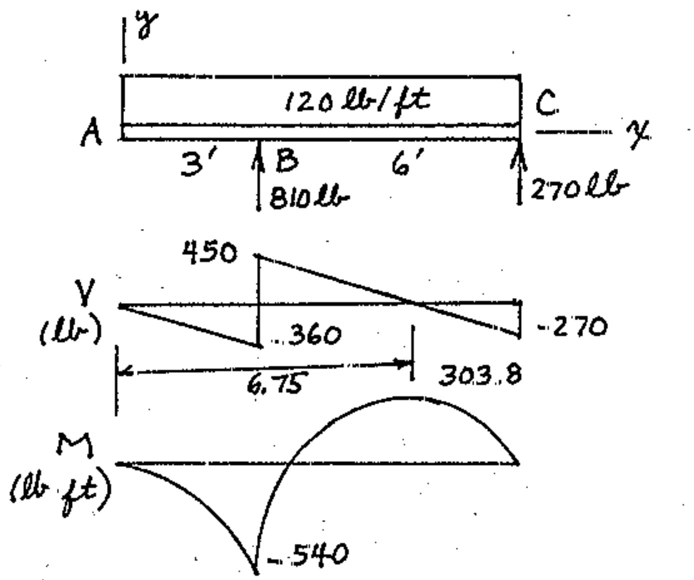


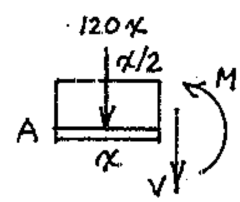




$$V = \begin{cases} 20 \text{ kN} & \text{if} \quad x \leq 3 \text{ m} \\ -20 \text{ kN} & \text{if} \quad x \geq 3 \text{ m} \end{cases} \quad \blacktriangleleft$$

$$M = \begin{cases} 20x \text{ kN} \cdot \text{m} & \text{if} \quad x \leq 3 \text{ m} \\ 20(7.5 - x) \text{ kN} \cdot \text{m} & \text{if} \quad x \geq 3 \text{ m} \end{cases} \quad \blacktriangleleft$$

**6.30**

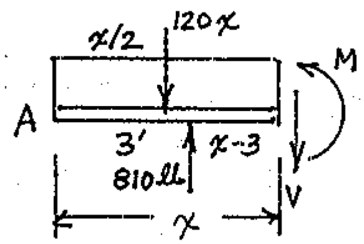



$$AB \quad V = -120x \text{ lb} \blacktriangleleft$$

$$M = -120x\left(\frac{x}{2}\right)$$

$$M = -60x^2 \text{ lb} \cdot \text{ft} \blacktriangleleft$$




$$BC \quad V = 810 - 120x \text{ lb} \blacktriangleleft$$

$$M = 810(x-3) - 120x\left(\frac{x}{2}\right)$$

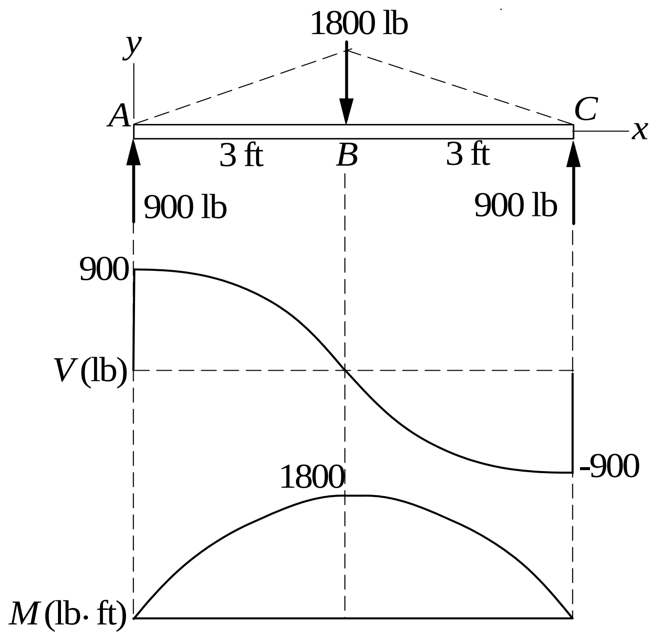
$$M = -60x^2 + 810x - 2430 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

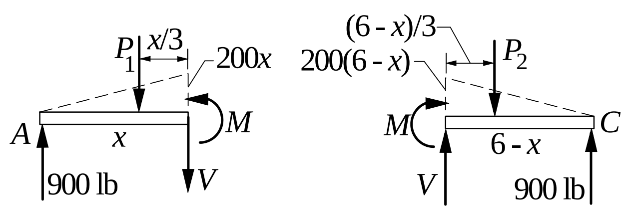
*Find where shear is zero, which is also where M is maximum*

$$V = 810 - 120x = 0 \quad \text{gives } x = 6.75 \text{ ft}$$

$$M_{max} = -60(6.75)^2 + 810(6.75) - 2430 = 303.8 \text{ lb} \cdot \text{ft}$$

**6.31**



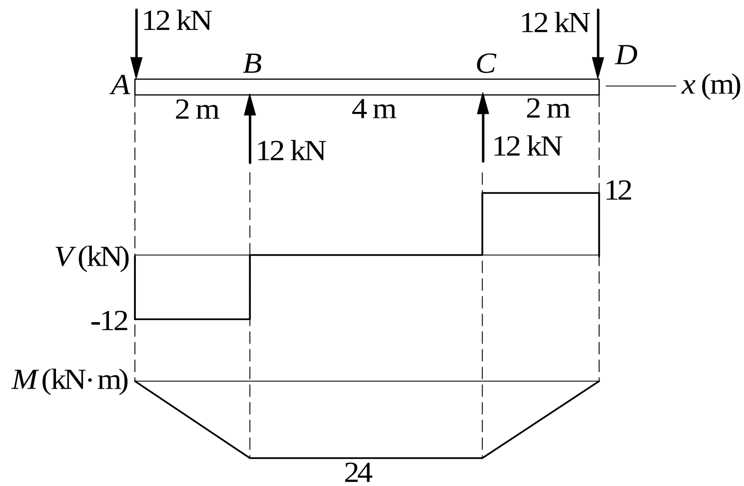

Segment $AB$:

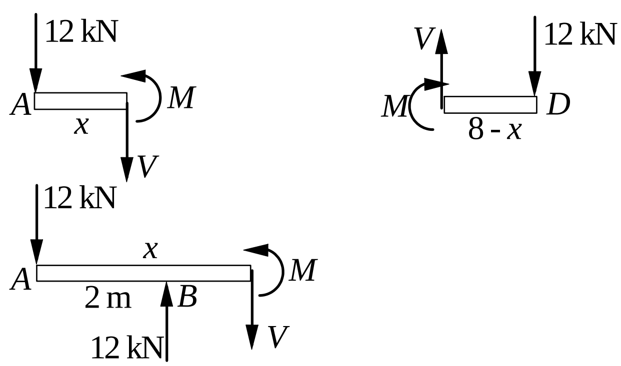
$$\begin{aligned} P_1 &= 100x^2 \text{ lb} \\ V &= 900 - P_1 = 900 - 100x^2 \text{ lb} \blacktriangleleft \\ M &= 900x - P_1\frac{x}{3} = 900x - \frac{100}{3}x^3 \text{ lb}\cdot\text{ft} \blacktriangleleft \end{aligned}$$

Segment $BC$:

$$\begin{aligned} P_2 &= 100(6-x)^2 \text{ lb} \\ V &= -900 + P_2 = -900 + 100(6-x)^2 \text{ lb} \blacktriangleleft \\ M &= 900(6-x) - P_2\frac{6-x}{3} = 900(6-x) - \frac{100}{3}(6-x)^3 \text{ lb}\cdot\text{ft} \blacktriangleleft \end{aligned}$$

**6.32**

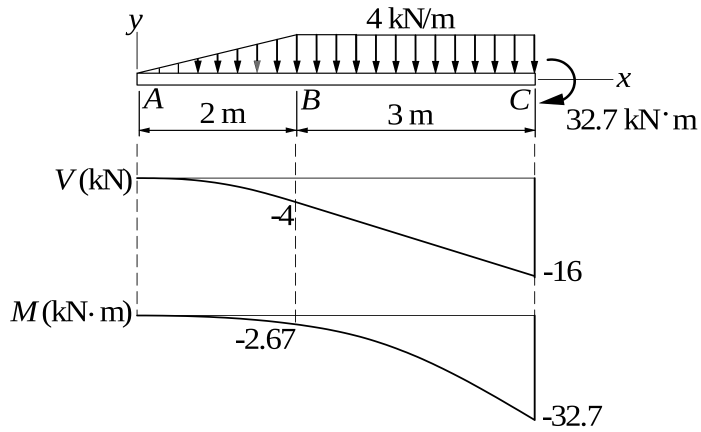


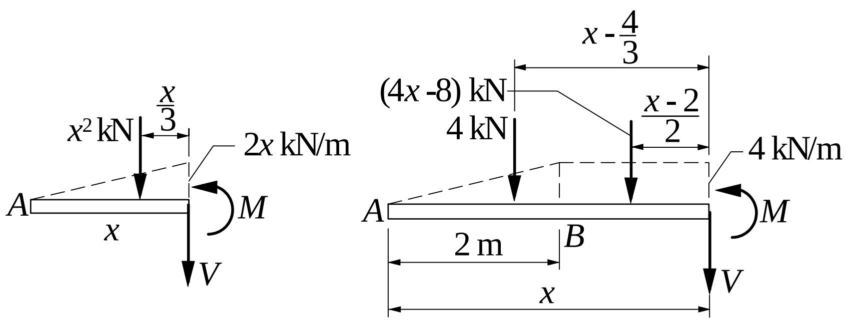




$$V = \begin{cases} -12 \text{ kN} & \text{if} \quad x \le 2 \text{ m} \\ 0 & \text{if} \quad 2 \text{ m} \le x \le 6 \text{ m} \\ 12 \text{ kN} & \text{if} \quad x \ge 6 \text{ m} \end{cases} \quad ◀$$

$$M = \begin{cases} -12x \text{ kN} \cdot \text{m} & \text{if} \quad x \le 2 \text{ m} \\ -24 \text{ kN} \cdot \text{m} & \text{if} \quad 2 \text{ m} \le x \le 6 \text{ m} \\ -12(8-x) \text{ kN} \cdot \text{m} & \text{if} \quad x \ge 6 \text{ m} \end{cases} \quad ◀$$

**6.33**




Segment $AB$:

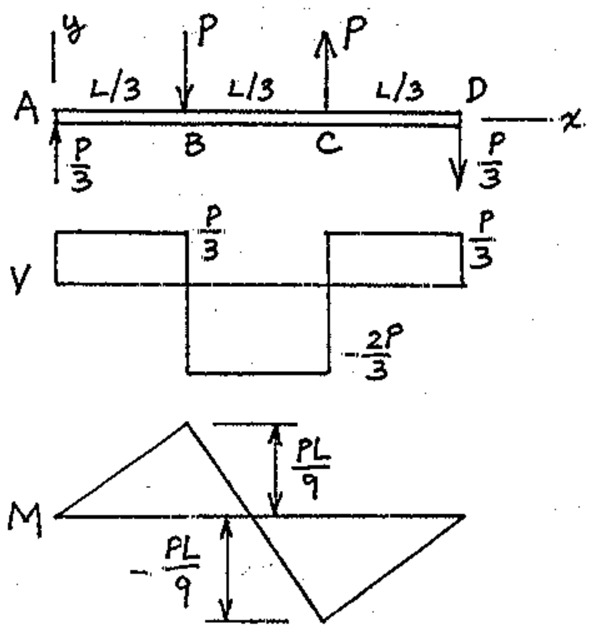
$$V = -x^2 \text{ kN} \quad \blacktriangleleft$$
$$M = -\frac{1}{3}x^3 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

Segment $BC$:

$$V = -4 - (4x - 8) = 4(1 - x) \text{ kN} \quad \blacktriangleleft$$
$$M = -4(x - \frac{4}{3}) - (4x - 8)\left(\frac{x - 2}{2}\right) = -2x^2 + 4x - \frac{8}{3} \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

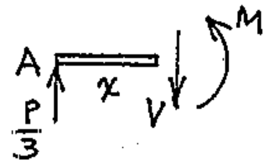
$$M_{\max} = M|_{x=5 \text{ m}} = -2(5^2) + 4(5) - \frac{8}{3} = -32.7 \text{ kN} \cdot \text{m}$$

**6.34**




$$AB \quad V = \frac{P}{3} \blacktriangleleft$$

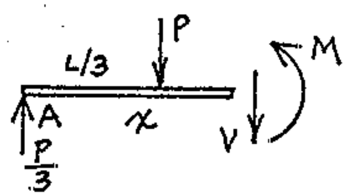
$$M = \frac{Px}{3} \blacktriangleleft$$




$$BC \quad V = \frac{P}{3} - P = -\frac{2P}{3} \blacktriangleleft$$

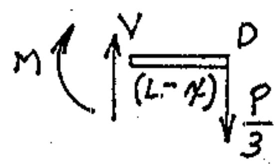
$$M = \frac{Px}{3} - P\left(x - \frac{L}{3}\right)$$

$$M = \frac{P}{3}(L - 2x) \blacktriangleleft$$

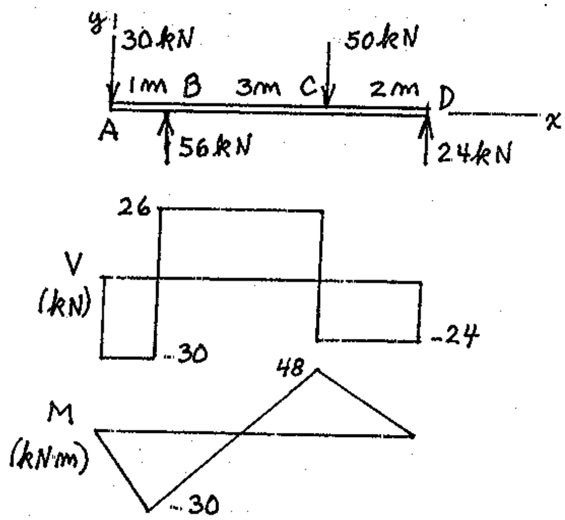



$$CD \quad V = \frac{P}{3} \blacktriangleleft$$

$$M = -\frac{P}{3}(L - x) \blacktriangleleft$$

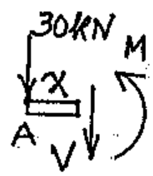


**6.35**




AB $V = -30 \text{ kN}$ ◀

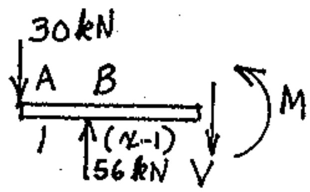
$M = -30x \text{ kN m}$ ◀




BC $V = -30 + 56 = 26 \text{ kN}$ ◀

$M = -30x + 56(x-1)$

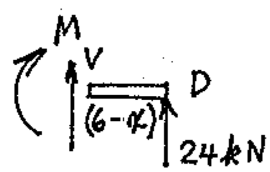
$M = 26x - 56 \text{ kN m}$ ◀



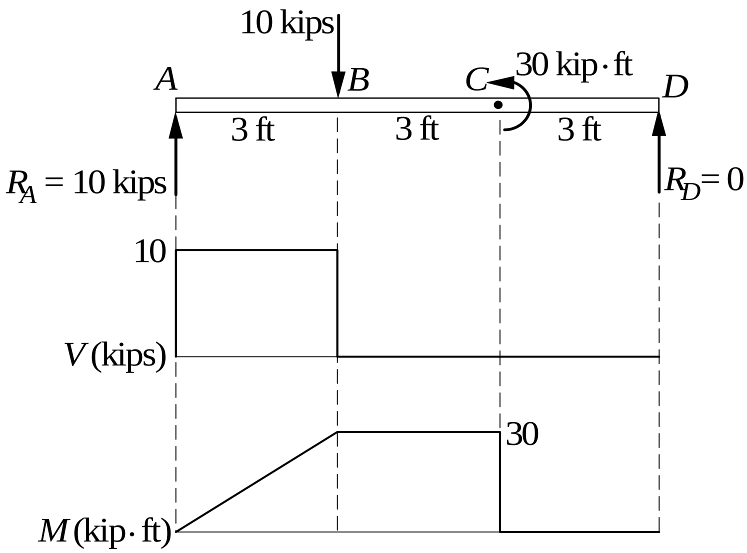

CD $V = -24 \text{ kN}$ ◀

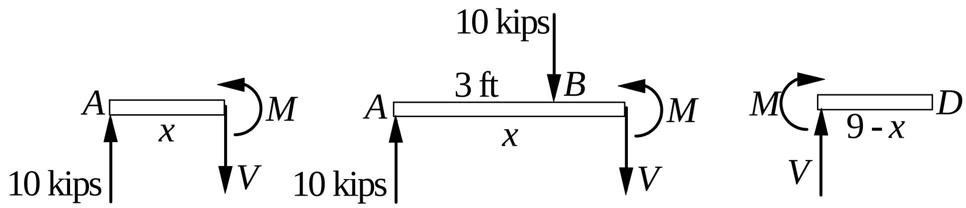
$M = 24(6-x)$

$M = -24x + 144 \text{ kN m}$ ◀



**6.36**

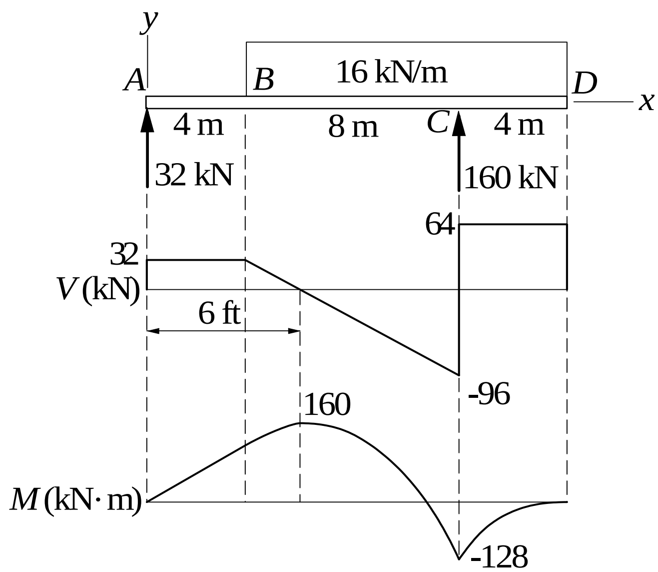


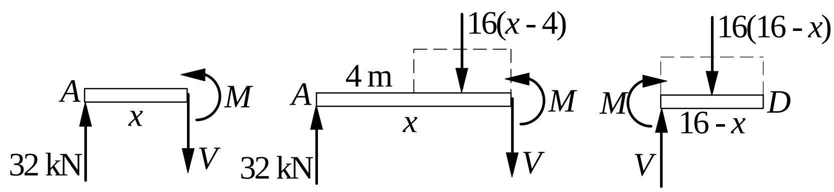




$$V = \begin{cases} 10 \text{ kips} & \text{if} \quad x \leq 3 \text{ ft} \\ 0 & \text{if} \quad 3 \text{ ft} \leq x \leq 6 \text{ ft} \\ 0 & \text{if} \quad x \geq 6 \text{ ft} \end{cases} \quad \blacktriangleleft$$

$$M = \begin{cases} 10x \text{ kip}\cdot\text{ft} & \text{if} \quad x \leq 3 \text{ ft} \\ 30 \text{ kip}\cdot\text{ft} & \text{if} \quad 3 \text{ ft} \leq x \leq 6 \text{ ft} \\ 0 & \text{if} \quad x \geq 6 \text{ ft} \end{cases} \quad \text{kip}\cdot\text{ft} \quad \blacktriangleleft$$

**6.37**



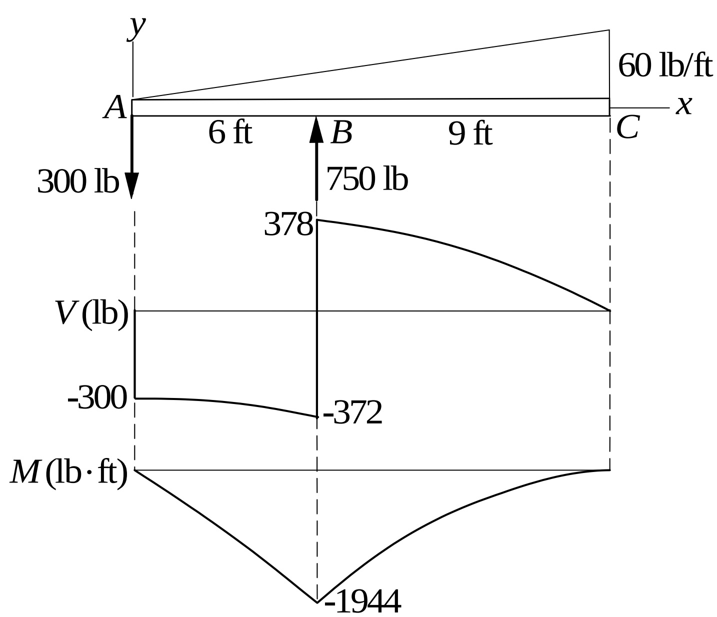



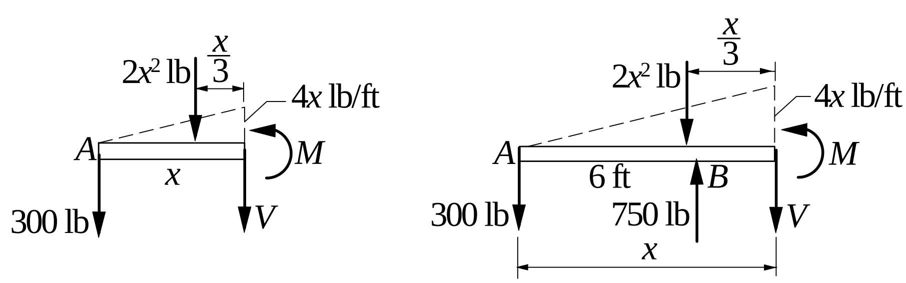

$$V = \begin{cases} 32 \text{ kN} & \text{if} \quad x \le 4 \text{ m} \\ 32 - 16(x-4) \text{ kN} & \text{if} \quad 4 \text{ m} \le x \le 12 \text{ m} \\ 16(16-x) \text{ kN} & \text{if} \quad x \ge 12 \text{ m} \end{cases} \blacktriangleleft$$

$$M = \begin{cases} 32x \text{ kN}\cdot\text{m} & \text{if} \quad x \le 4 \text{ m} \\ 32x - 8(x-4)^2 \text{ kN}\cdot\text{m} & \text{if} \quad 4 \text{ m} \le x \le 12 \text{ m} \\ -8(16-x)^2 \text{ kN}\cdot\text{m} & \text{if} \quad x \ge 12 \text{ m} \end{cases} \blacktriangleleft$$

$$M_{\max} = M|_{x=6 \text{ m}} = 32(6) - 8(6-4)^2 = 160 \text{ kN}\cdot\text{m}$$

**6.38**




Segment $AB$:

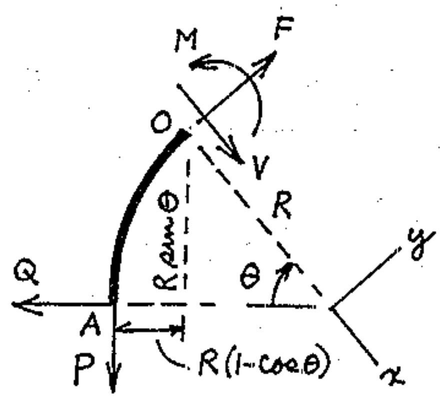
$$V = -300 - 2x^2 \text{ lb} \blacktriangleleft$$
$$M = -300x - \frac{2}{3}x^3 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

Segment $BC$:

$$V = 450 - 2x^2 \text{ lb} \blacktriangleleft$$
$$M = -300x + 750(x-6) - \frac{2}{3}x^3 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

**6.39**

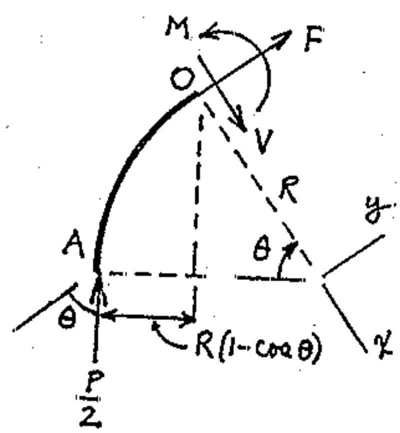
$$\Sigma F_x = 0$$
$$V = Q\cos\theta - P\sin\theta \blacktriangleleft$$
$$\Sigma M_O = 0$$
$$M = QR\sin\theta - PR(1-\cos\theta) \blacktriangleleft$$




**6.40**

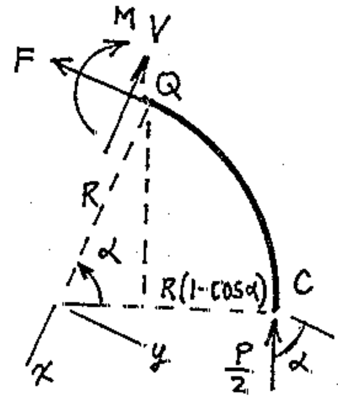
*Section AB*

$$\Sigma F_x = 0$$
$$V = \frac{P}{2}\sin\theta \blacktriangleleft$$
$$\Sigma M_O = 0$$
$$M = \frac{PR}{2}(1-\cos\theta) \blacktriangleleft$$

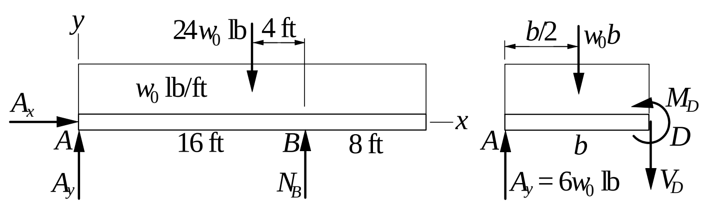


*Section BC* (Note: $\alpha = \pi - \theta$)

$$\Sigma F_x = 0$$
$$V = -\frac{P}{2}\sin\alpha$$
$$V = -\frac{P}{2}\sin(\pi - \theta)$$
$$V = -\frac{P}{2}\sin\theta \quad \blacktriangleleft$$
$$\Sigma M_Q = 0$$
$$M = \frac{PR}{2}(1 - \cos\alpha)$$
$$M = \frac{PR}{2}[1 - \cos(\pi - \theta)] = \frac{PR}{2}(1 + \cos\theta) \quad \blacktriangleleft$$




**6.41**




FBD of entire beam:

$$\Sigma F_x = 0 \qquad A_x = 0$$
$$\Sigma M_B = 0 \qquad 16A_y - 24w_0(4) = 0 \qquad A_y = 6w_0$$

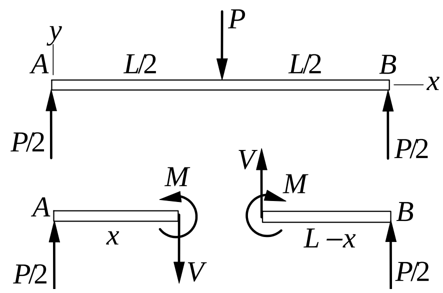
FBD of segment $AD$:

$$\Sigma M_D = 0 \qquad M_D - 6w_0(b) + w_0 b\left(\frac{b}{2}\right) = 0 \qquad M_D = \frac{w_0 b}{2}(12 - b)$$

Note that $M_D = 0$ when $b = 12$ ft. Therefore, the most advantageous position for the joint is at $b = 12$ ft ◄

## 6.42

**Case 1**



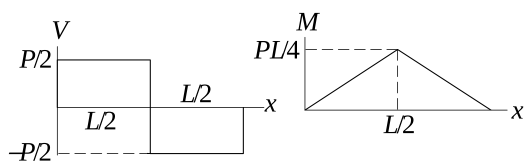

The reactions were determined by symmetry (each support carries half the load).

From FBD of left segment ($0 < x < L/2$):

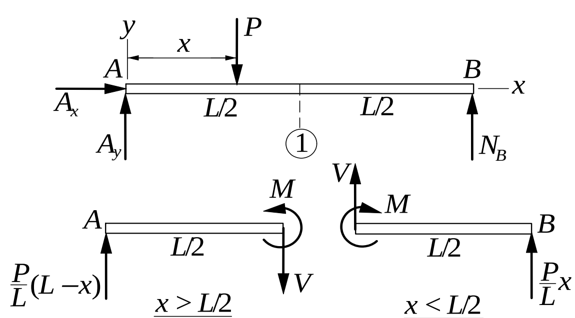
$$M = \frac{P}{2}x \quad ◀ \qquad V = \frac{P}{2} \quad ◀$$

From FBD of right segment ($L/2 < x < L$):

$$M = \frac{P}{2}(L - x) \quad ◀ \qquad V = -\frac{P}{2} \quad ◀$$




**Case 2**



FBD of entire beam:

$$\Sigma M_A = 0 \qquad N_B L - Px = 0 \qquad N_B = \frac{P}{L}x$$

$$\Sigma F_x = 0 \qquad A_x = 0$$

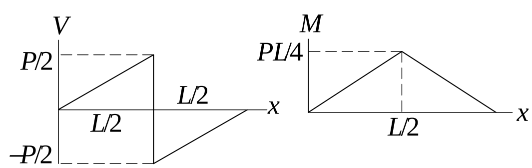
$$\Sigma F_y = 0 \qquad A_y + N_B - P = 0 \qquad A_y + \frac{P}{L}x - P = 0 \qquad A_y = \frac{P}{L}(L - x)$$

When $P$ is in the range $0 < x < L/2$ (use FBD of right half):

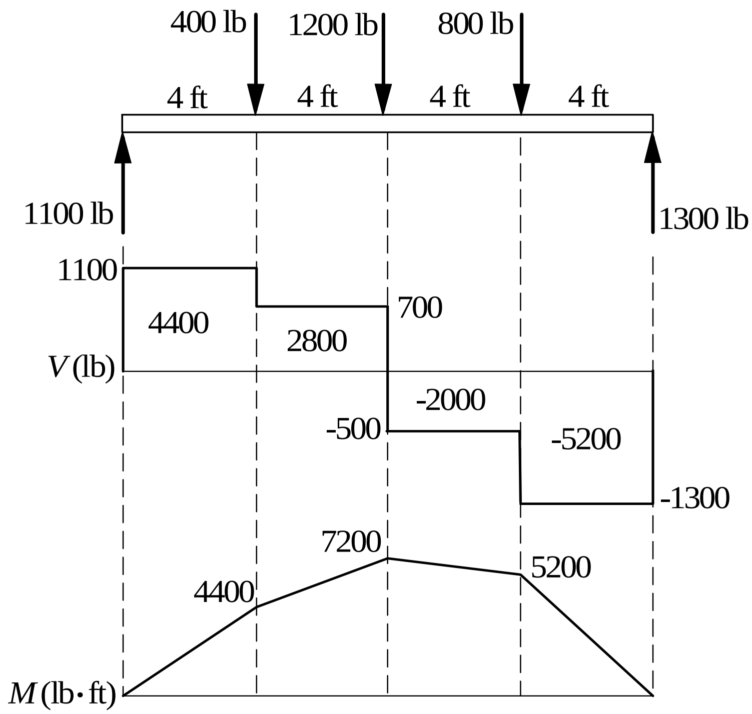
$$M = \frac{P}{L}x\left(\frac{L}{2}\right) = \frac{P}{2}x \blacktriangleleft \qquad V = -\frac{P}{L}x$$

When $P$ is in the range $L/2 < x < L$ (use FBD of left half):

$$M = \frac{P}{L}(L - x)\left(\frac{L}{2}\right) = \frac{P}{2}(L - x) \blacktriangleleft \qquad V = \frac{P}{L}(L - x) \blacktriangleleft$$

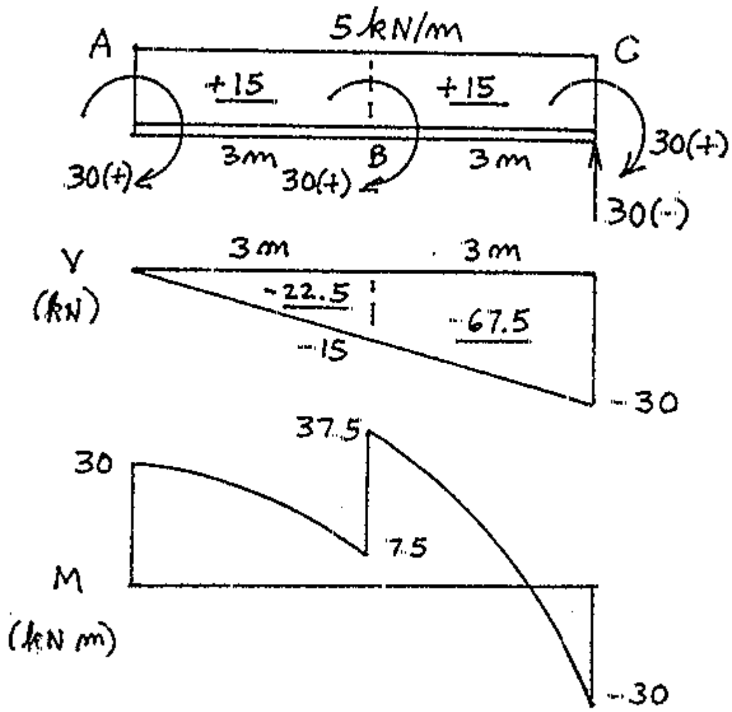


**6.43**



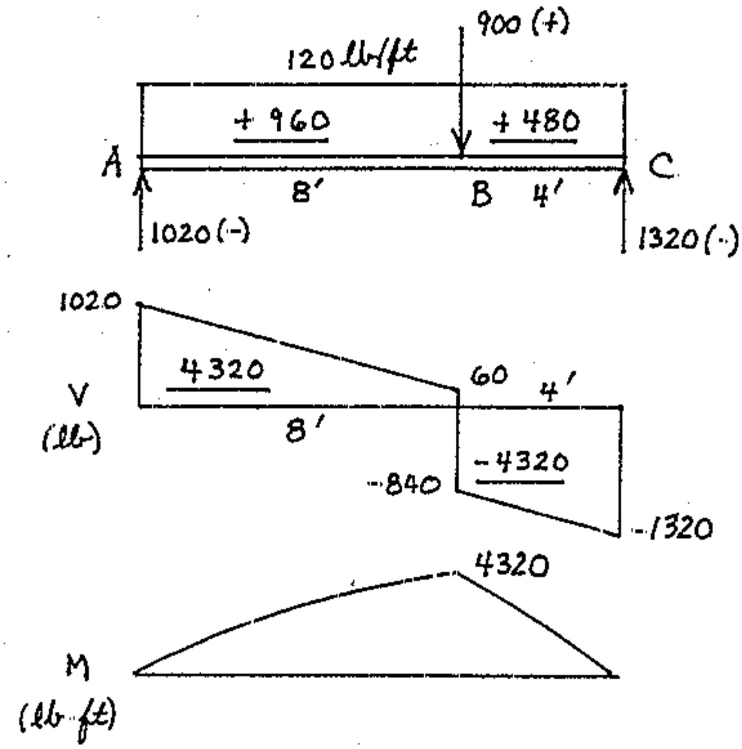

$V_{\text{max}} = -1300$ lb ◄ $M_{\text{max}} = 7200$ lb · ft ◄

**6.44**



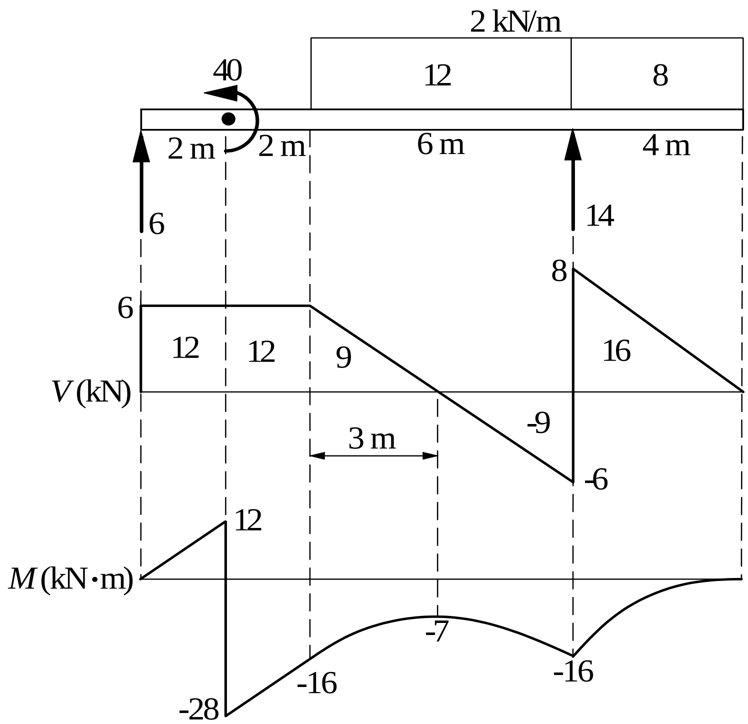

$V_{max} = -30$ kN; $M_{max} = 37.5$ kN m ◀

**6.45**



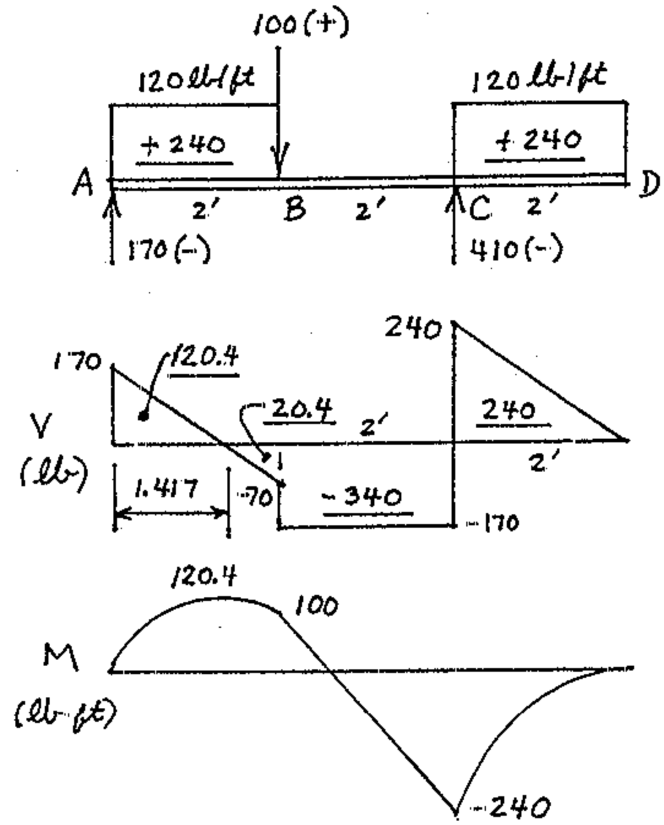

$V_{max} = -1320$ lb; $M_{max} = 4320$ lb · ft ◄

**6.46**



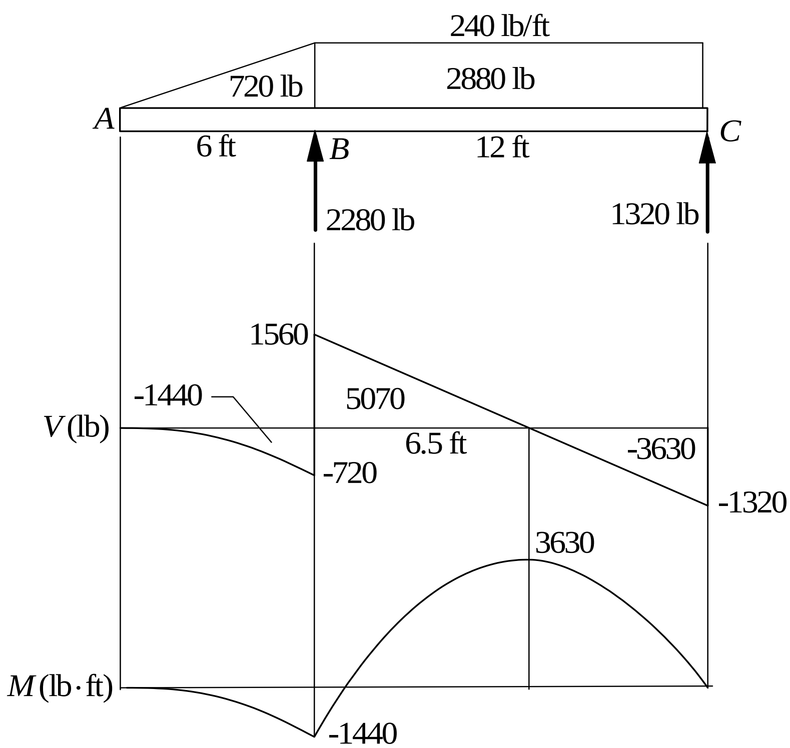

$V_{\max} = 8$ kN ◀ $M_{\max} = -28$ kN · m ◀

**6.47**



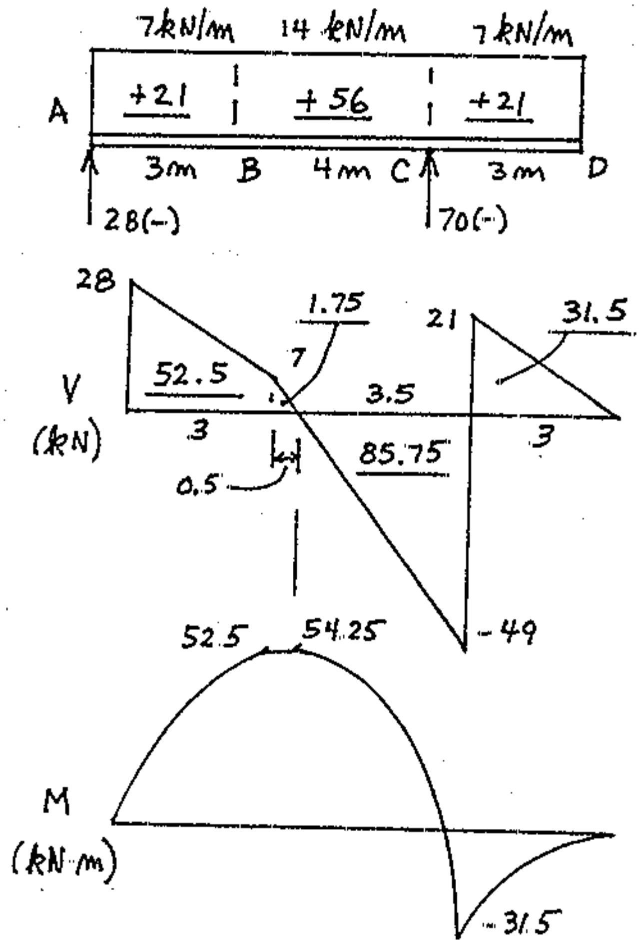

$V_{max} = 240$ lb; $M_{max} = -240$ lb ft ◄

**6.48**



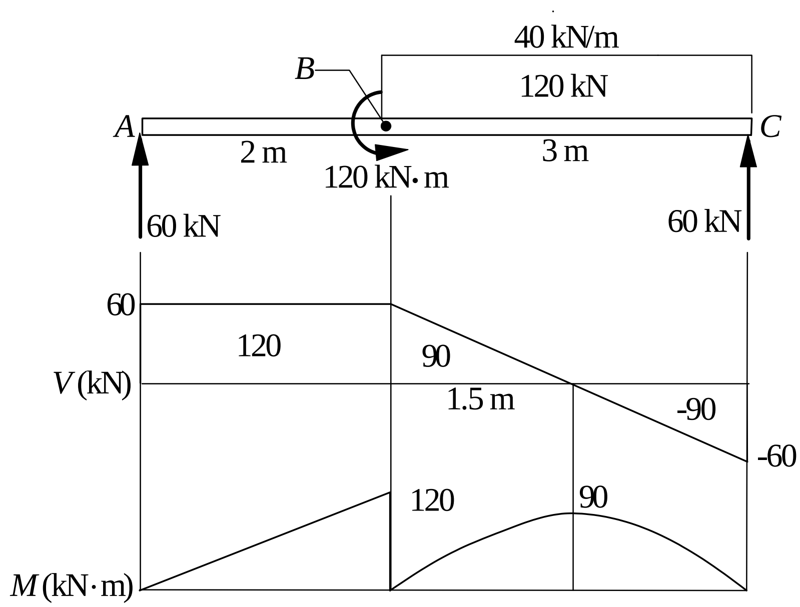

$V_{\max} = 1560$ lb ◄ $\qquad M_{\max} = 3630$ lb · ft ◄

**6.49**



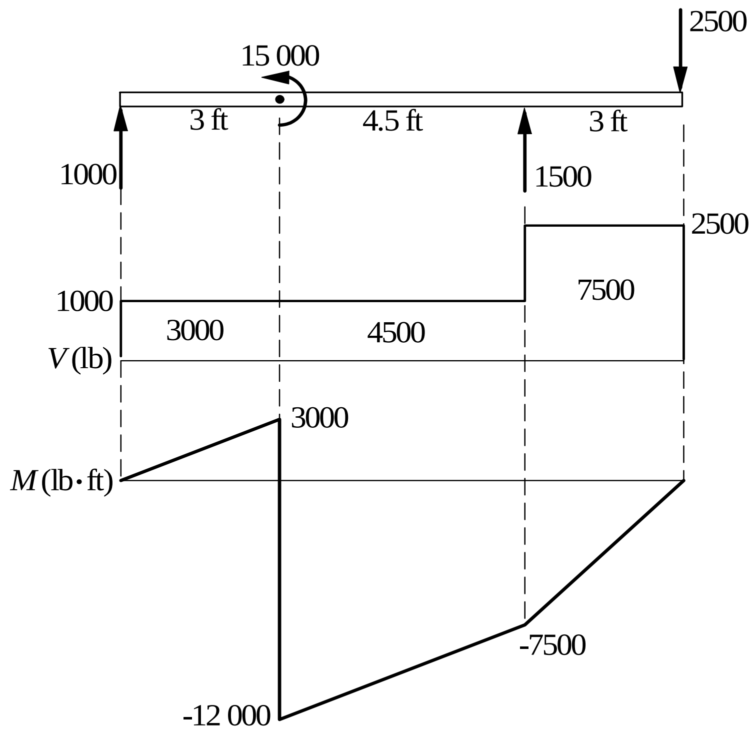

$V_{max} = -49$ kN; $M_{max} = 54.25$ kN · m ◀

**6.50**



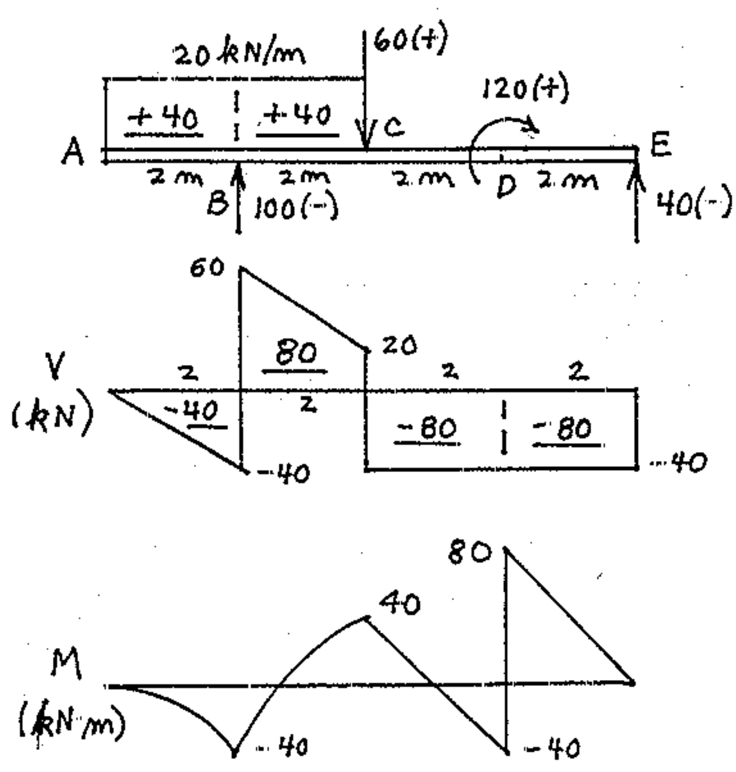

$V_{\max} = 60 \text{ kN}$ ◄ $M_{\max} = 120 \text{ kN} \cdot \text{m}$ ◄

**6.51**



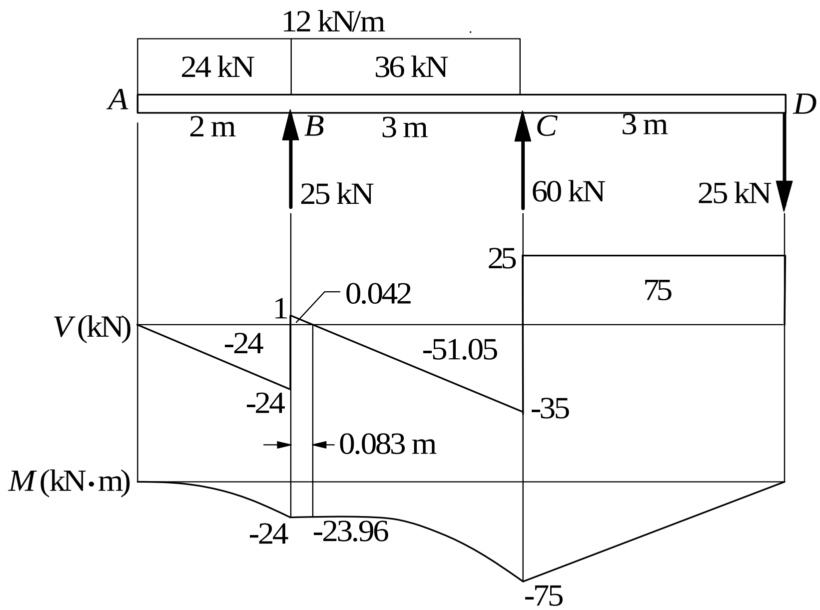

$V_{\max} = 2500$ lb ◄ $M_{\max} = -12\ 000$ lb · ft ◄

**6.52**



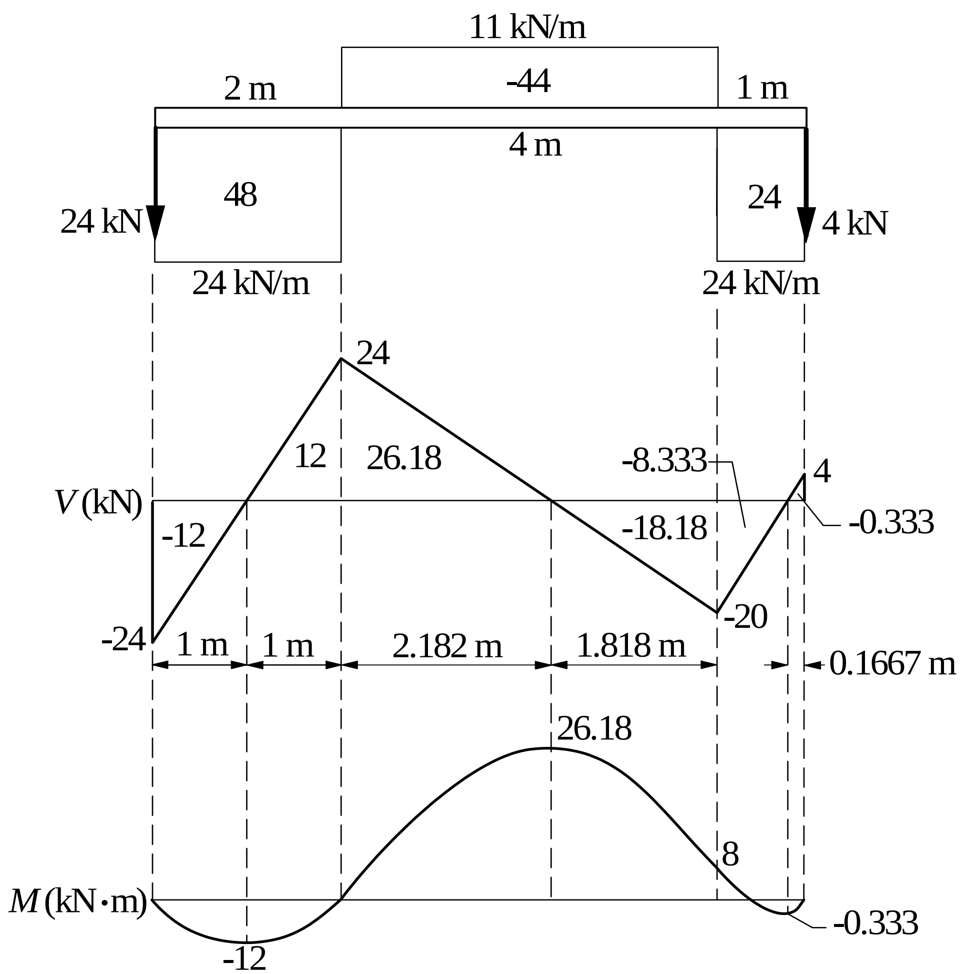

$V_{max} = 60$ kN; $M_{max} = 80$ kN · m ◄

**6.53**



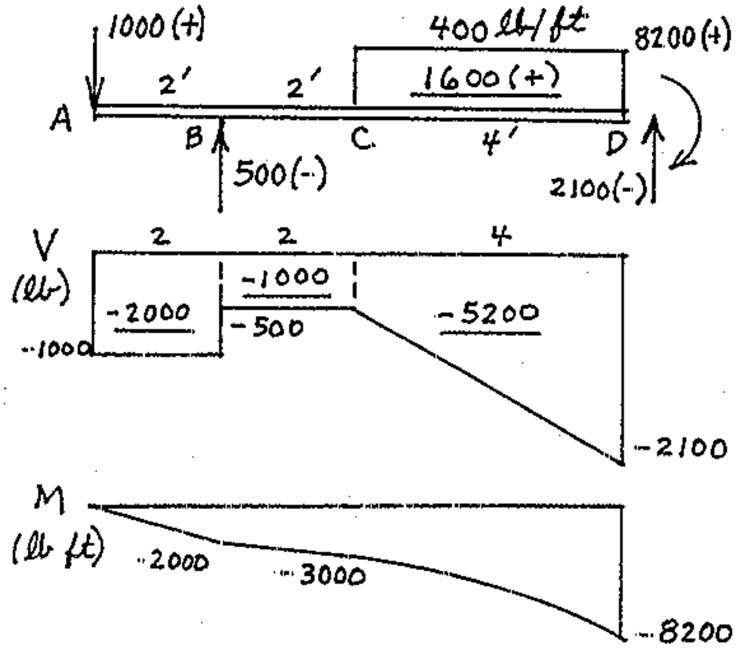

$V_{\max} = -35$ kN ◄ $M_{\max} = -75$ kN · m ◄

**6.54**



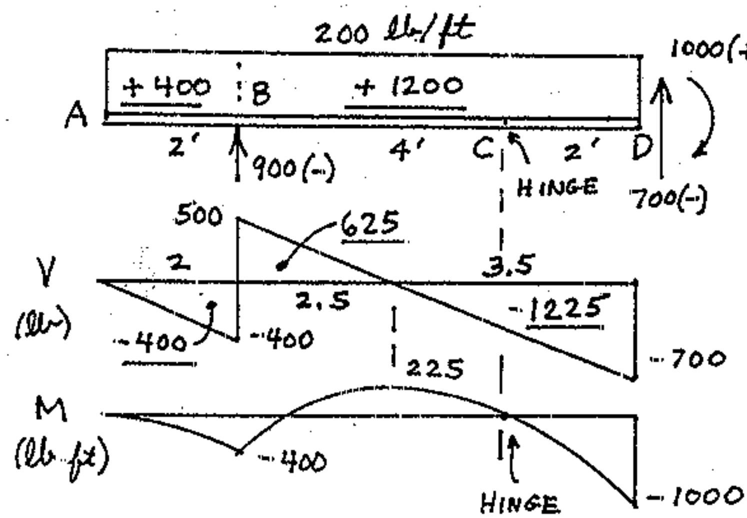

$V_{\max} = \pm 24$ kN ◄ $M_{\max} = 26.2$ kN · m ◄

**6.55**



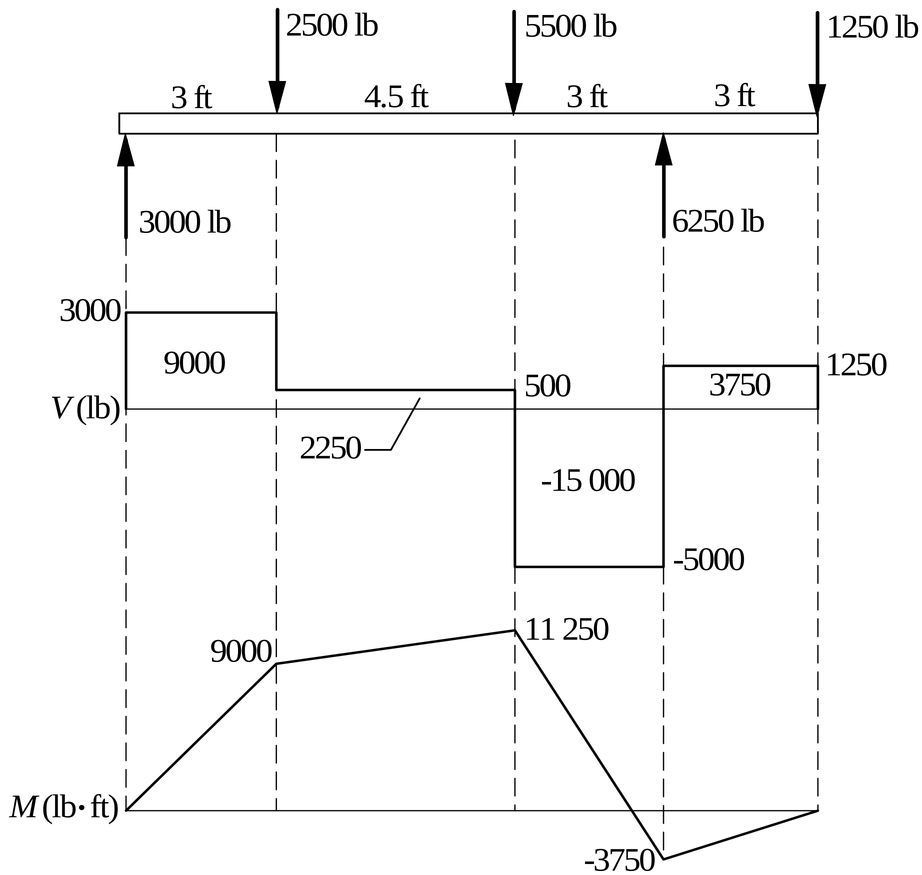

$V_{max} = -2100$ lb; $M_{max} = -8200$ lb ft ◄

**6.56**



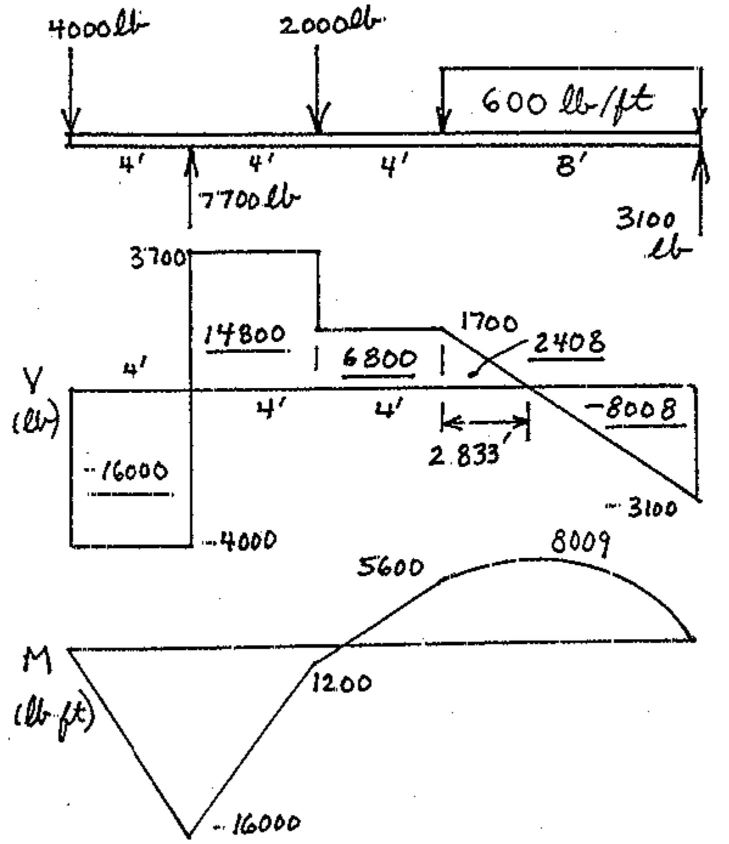

$V_{max} = -700$ lb; $M_{max} = -1000$ lb · ft ◄

**6.57**



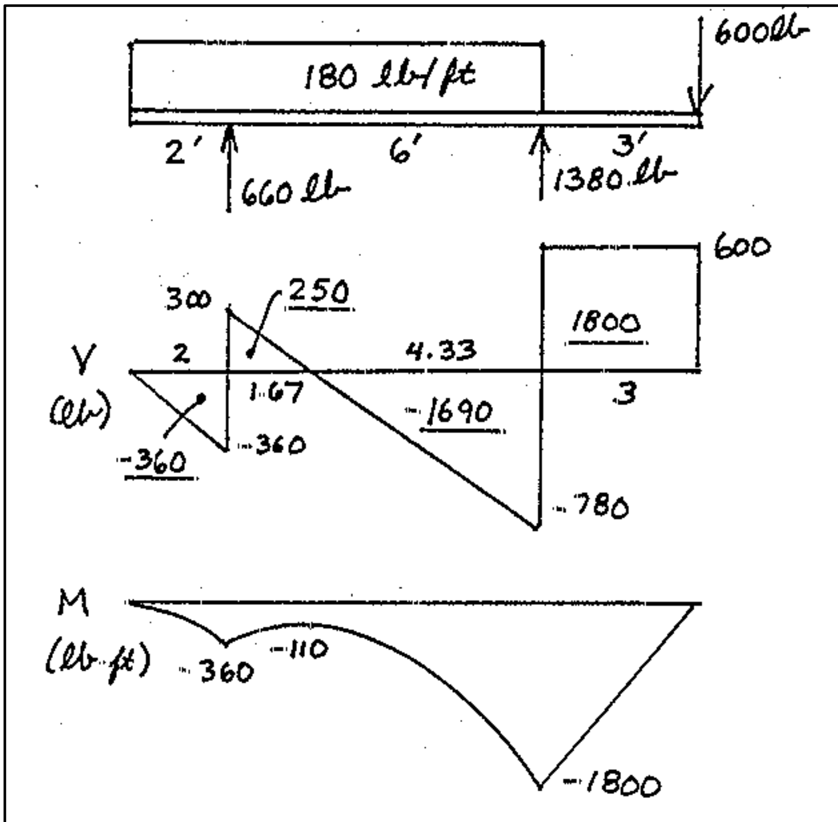

$V_{\max} = -5000$ lb ◄ $M_{\max} = 11\ 250$ lb · ft ◄

**6.58**



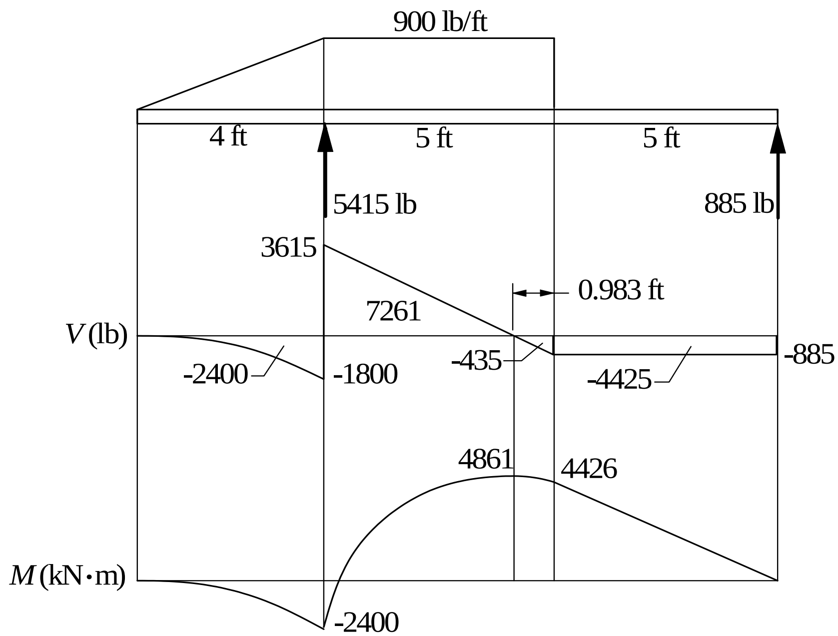

$V_{max} = -4000$ lb; $M_{max} = -16\,000$ lb · ft ◄

**6.59**



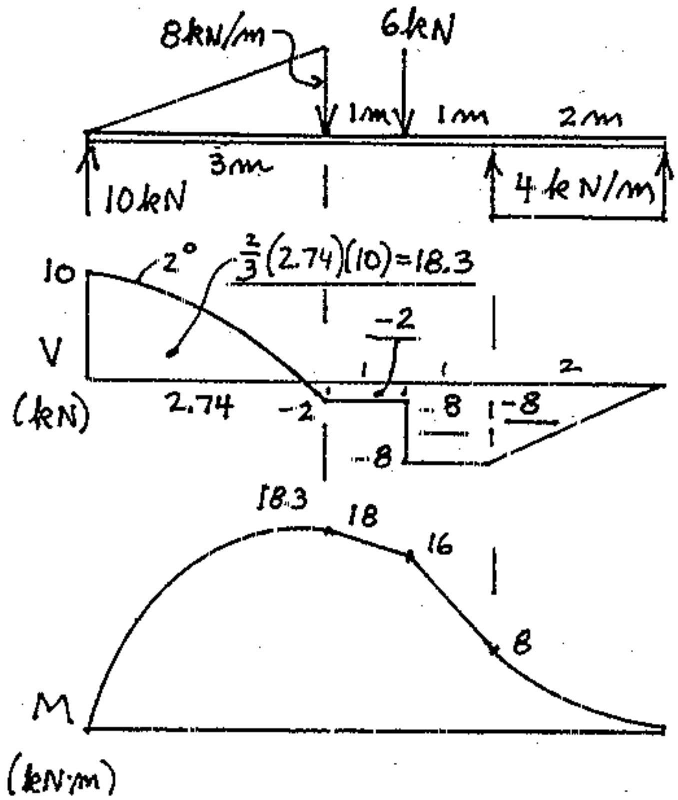

$V_{\max} = -780$ lb; $M_{\max} = -1800$ lb · ft ◄

**6.60**

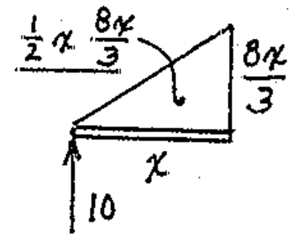


900 lb/ft
4 ft
5 ft
5 ft
5415 lb
885 lb
3615
0.983 ft
7261
V (lb)
-2400
-1800
-435
-4425
-885
4861
4426
M (kN·m)
-2400

**6.61**




*Find where shear is zero, which is also where M is maximum.*




$$10 = \frac{1}{2}x\left(\frac{8x}{3}\right) \quad \text{gives} \quad x = 2.74 \text{ m}$$

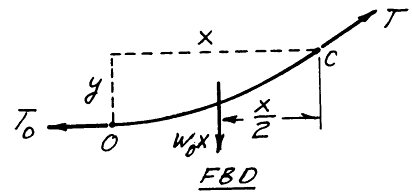
$$M_{\max} = 10(2.74) - \frac{8(2.74)^2}{6}\left(\frac{2.74}{3}\right) = 18.3 \text{ kN}\cdot\text{m}$$

$$V_{\max} = 10 \text{ kN};\ M_{\max} = 18.3 \text{ kN}\cdot\text{m} ◀$$

## 6.62

**From the FBD**

$$\Sigma M_C = 0: \;(+)\; (w_0 x)\frac{x}{2} - T_0 y = 0$$




$$\therefore\ T_0 = \frac{w_0 x^2}{2y}$$

**Equation (6.9) is**

$$T = \sqrt{T_0^2 + (w_0 x)^2} = \sqrt{\left(\frac{w_0 x^2}{2y}\right)^2 + (w_0 x)^2} = w_0 x\sqrt{\left(\frac{x}{2y}\right)^2 + 1} \qquad \textbf{Q.E.D.}$$

## 6.63

The shape of the cable is parabolic. From Eq. (6.10):

$$H = y|_{x=L/2} = \frac{w_0 (L/2)^2}{2T_0} \qquad T_0 = \frac{L^2}{8H} w_0 = \frac{140^2}{8(20)} w_0 = 122.50 w_0 \text{ N}$$

From Eq. (6.9):

$$\begin{aligned} T_{\max} &= T|_{x=L/2} = \sqrt{T_0^2 + \left(\frac{w_0 L}{2}\right)^2} = w_0\sqrt{122.50^2 + \left(\frac{140.0}{2}\right)^2} \\ &= 141.09 w_0 \text{ N} \\ \therefore\ & w_0 = \frac{T_{\max}}{141.09} = \frac{4\times 10^6}{141.09} = 28\ 400 \text{ N/m} \ \blacktriangleleft \end{aligned}$$

## 6.64

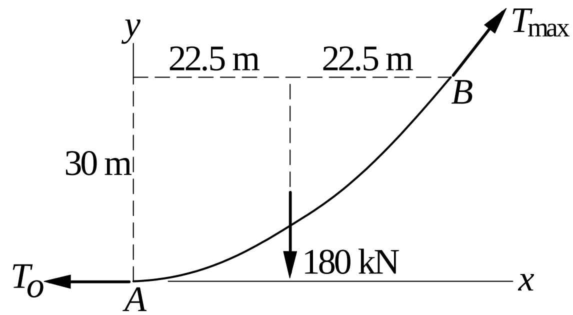
The shape of the cable is parabolic. From Eq. (6.10):

$$\begin{aligned} H &= y|_{x=L/2} = \frac{w_0 (L/2)^2}{2T_0} \\ T_0 &= \frac{L^2}{8H} w_0 = \frac{1990^2}{8(233)}(444.7\times 10^3) = 944.8 \times 10^6 \text{ N} \end{aligned}$$

From Eq. (6.9):

$$\begin{aligned} T_{\max} &= T|_{x=L/2} = \sqrt{T_0^2 + \left(\frac{w_0 L}{2}\right)^2} \\ &= \sqrt{(944.8\times 10^6)^2 + \left(\frac{(444.7\times 10^3)1990}{2}\right)^2} \\ &= 1.043 \times 10^9 \text{ N} = 1.043 \text{ GN} \ \blacktriangleleft \end{aligned}$$

## 6.65




The shape of the cable is parabolic.

**(a)**

$$W = w_0 L = 4(45) = 180 \text{ kN}$$

$$\Sigma M_B = 0 \qquad 180(22.5) - 30T_0 = 0 \qquad T_0 = 135 \text{ kN}$$

From Eq. (6.12):

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{135^2 + 180^2} = 225 \text{ kN} \blacktriangleleft$$

**(b)** From Eq. (6.12):

$$s(L) = \frac{L}{2}\sqrt{1 + \left(\frac{w_0 L}{T_0}\right)^2} + \frac{1}{2}\frac{T_0}{w_0}\ln\left[\frac{w_0 L}{T_0} + \sqrt{1 + \left(\frac{w_0 L}{T_0}\right)^2}\right]$$

$$\frac{w_0 L}{T_0} = \frac{4(45)}{135} = \frac{4}{3}$$

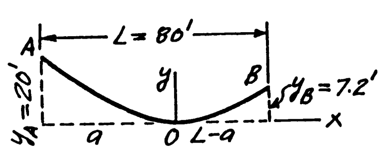
$$\therefore \quad s(L) = \frac{45}{2}\sqrt{1 + \left(\frac{4}{3}\right)^2} + \frac{1}{2}\frac{135}{4}\ln\left(\frac{4}{3} + \sqrt{1 + \left(\frac{4}{3}\right)^2}\right)$$

$$= 56.0 \text{ m} \blacktriangleleft$$

## 6.66

**The cable is parabolic, the distributed loading being**

$$w_0 = \frac{W}{L} = \frac{960}{80} = 12 \text{ lb/ft}$$




**Location of point O can be found from Eq. (6.10):**

$$y = \frac{w_0 x^2}{2T_0} \qquad \therefore \frac{y_A}{y_B} = \frac{y|_{x=-a}}{y|_{x=L-a}} = \frac{a^2}{(L-a)^2} \qquad \therefore \frac{20}{7.2} = \frac{a^2}{(80-a)^2} \qquad \therefore a = 50 \text{ ft}$$

**At point A, Eq. (6.10) becomes** $y_A = \dfrac{w_0 a^2}{2T_0}$ $\quad \therefore T_0 = \dfrac{w_0 a^2}{2y_A} = \dfrac{(12)(50^2)}{2(20)} = 750 \text{ lb}$

Equation (6.12): $s(x) = \frac{x}{2}\sqrt{1 + (w_0x/T_0)^2} + \frac{1}{2}(T_0/w_0)\ln\left[(w_0x/T_0) + \sqrt{1 + (w_0x/T_0)^2}\right]$

With x = 50 ft: $\frac{w_0x}{T_0} = \frac{(12)(50)}{750} = 0.8$

$\therefore s_{OA} = s(50 \text{ ft}) = \frac{50}{2}\sqrt{1 + 0.8^2} + \frac{1}{2}\frac{750}{12}\ln\left[0.8 + \sqrt{1 + 0.8^2}\right] = 54.9 \text{ ft}$

With x = 30 ft: $\frac{w_0x}{T_0} = \frac{(12)(30)}{750} = 0.48$

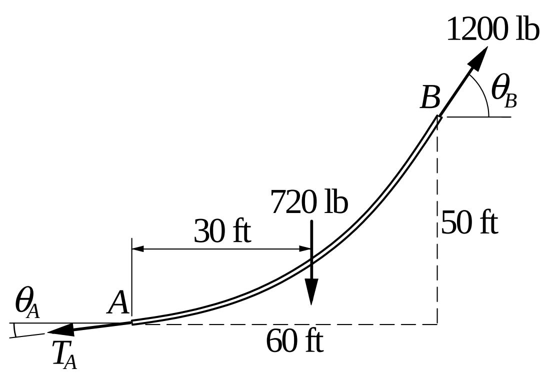
$\therefore s_{OB} = s(30 \text{ ft}) = \frac{30}{2}\sqrt{1 + 0.48^2} + \frac{1}{2}\frac{750}{12}\ln\left[0.48 + \sqrt{1 + 0.48^2}\right] = 31.1 \text{ ft}$

Total length of the cable is: $s = s_{OA} + s_{OB} = 54.9 + 31.1 = 86.0$ ft ◆

The maximum value of T occurs at A. Substituting x = 50 ft in Eq. (6.9), we get

$T_{max} = \sqrt{T_0^2 + (w_0a)^2} = \sqrt{750^2 + (12 \times 50)^2} = 960$ lb ◆

**6.67**



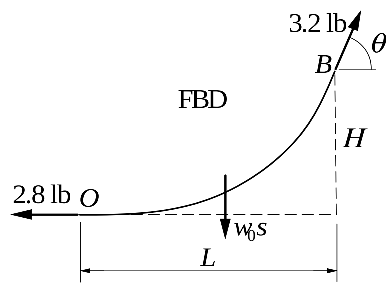

$$\Sigma M_A = 0 \qquad -(1200\cos\theta_B)(50) + (1200\sin\theta_B)(60) - 720(30) = 0$$
$$72\sin\theta_B - 60\cos\theta_B = 21.6 \qquad \theta_B = 53.13^\circ \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad 1200\cos\theta_B - T_A\cos\theta_A = 0$$
$$T_A\cos\theta_A = 1200\cos 53.13^\circ = 720.0$$
$$\Sigma F_y = 0 \qquad 1200\sin\theta_B - T_A\sin\theta_A - 720 = 0$$
$$T_A\sin\theta_A = 1200\sin 53.13^\circ - 720 = 240.0$$

$$\tan\theta_A = \frac{240}{720} = 0.3333 \qquad \theta_A = 18.43^\circ \blacktriangleleft$$
$$T_A = 1200\frac{\cos\theta_B}{\cos\theta_A} = 1200\frac{\cos 53.13^\circ}{\cos 18.43^\circ} = 759 \text{ lb} \blacktriangleleft$$

## 6.68

The shape of the string is a catenary.




$$w_0 = \frac{0.4}{16} = 0.025 \text{ lb/ft}$$

From FBD:

$$\begin{aligned} \Sigma F_x &= 0 \quad +\rightarrow \quad 3.2\cos\theta - 2.8 = 0 \quad \theta = 28.96^\circ \\ \Sigma F_y &= 0 \quad +\downarrow \quad w_0 s - 3.2\sin\theta = 0 \quad 0.025s - 3.2\sin 28.96^\circ = 0 \\ & s = 61.98 \text{ ft} \quad \blacktriangleleft \end{aligned}$$

From Eq. (6.15):

$$\begin{aligned} s &= \frac{T_0}{w_0}\sinh\frac{w_0 L}{T_0} \\ \therefore L &= \frac{T_0}{w_0}\sinh^{-1}\frac{s w_0}{T_0} = \frac{2.8}{0.025}\sinh^{-1}\frac{61.98(0.025)}{2.8} = 59.19 \text{ ft} \end{aligned}$$

From Eq. (6.16):

$$\begin{aligned} H &= y|_{x=L} = \frac{T_0}{w_0}\left(\cosh\frac{w_0 L}{T_0} - 1\right) = \frac{2.8}{0.025}\left(\cosh\frac{0.025(59.19)}{2.8} - 1\right) \\ &= 16.01 \text{ ft} \quad \blacktriangleleft \end{aligned}$$

## 6.69

**Equation (6.17):** $\mathbf{T = T_0 \cosh\frac{w_0 x}{T_0} \qquad \therefore \cosh\frac{w_0 x}{T_0} = \frac{T}{T_0}}$

**Equation (6.16):** $\mathbf{y = \frac{T_0}{w_0}\left(\cosh\frac{w_0 x}{T_0} - 1\right) = \frac{T_0}{w_0}\left(\frac{T}{T_0} - 1\right) \qquad \therefore T = T_0 + w_0 y}$ **Q.E.D.**

## 6.70

This is a catenary cable.

**(a)** From Prob. 6.69:

$$\begin{aligned} T &= T_0 + w_0 y \\ \therefore\ & T_B = T_0 + w_0 y_B \qquad 400 = T_0 + 16(6) \qquad T_0 = 304 \text{ N} \\ \therefore\ & T_A = T_0 + w_0 y_A = 304 + 16(3) = 352 \text{ N} \blacktriangleleft \end{aligned}$$

**(b)** From Eq. (6.17):

$$\begin{aligned} T &= T_0 \cosh \frac{w_0 x}{T_0} \qquad \therefore x = \frac{T_0}{w_0} \cosh^{-1} \frac{T}{T_0} \\ L &= x_A + x_B = \frac{T_0}{w_0} \left( \cosh^{-1} \frac{T_A}{T_0} + \cosh^{-1} \frac{T_B}{T_0} \right) \\ &= \frac{304}{16} \left( \cosh^{-1} \frac{352}{304} + \cosh^{-1} \frac{400}{304} \right) = 25.3 \text{ m} \blacktriangleleft \end{aligned}$$

## 6.71

From Eq. (6.17):

$$\begin{aligned} T_B &= T_O \cosh \frac{w_0 x_B}{T_0} \qquad 1800 = 1200 \cosh \frac{w_0 (22.5)}{1200} \\ 1.5 &= \cosh \frac{w_0}{53.33} \qquad w_0 = 53.33 \cosh^{-1} 1.5 = 51.33 \text{ N/m} \end{aligned}$$

From Eq. (6.15):

$$\begin{aligned} s_B &= \frac{T_O}{w_0} \sinh \frac{w_0 x_B}{T_0} = \frac{1200}{51.33} \sinh \frac{51.33(22.5)}{1200} = 26.14 \text{ m} \\ W &= w_0 (2 s_B) = 51.33(2)(26.14) = 2680 \text{ N} \blacktriangleleft \end{aligned}$$

## 6.72

Eq. (6.16): $$y = \frac{T_O}{w_O} \left( \cosh \frac{w_O x}{T_O} - 1 \right) \qquad \therefore \frac{dy}{dx} = \sinh \frac{w_O x}{T_O}$$

Given: $$\left. \frac{dy}{dx} \right|_{x=22.5} = \tan 20^\circ \qquad \therefore \tan 20^\circ = \sinh \frac{24(22.5)}{T_O}$$

$$\sinh^{-1}(\tan 20^\circ) = \frac{24(22.5)}{T_O}$$

$$T_O = \frac{24(22.5)}{\sinh^{-1}(\tan 20^\circ)} = 1515.2 \text{ N}$$

$$\begin{aligned} H &= y|_{x=22.5} = \frac{1515.2}{24} \left( \cosh \frac{24(22.5)}{1515.2} - 1 \right) = 4.05 \text{ m} \blacktriangleleft \\ T_{\max} &= T_B = \frac{T_O}{\cos 20^\circ} = \frac{1515.2}{\cos 20^\circ} = 1612 \text{ N} \blacktriangleleft \end{aligned}$$

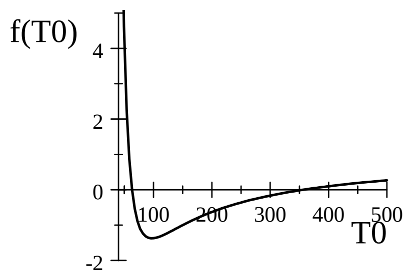
**6.73**

$$w_0 = 1.8(9.81) = 17.658 \text{ N/m}$$

From Eq. (16.17):

$$T_{\max} = T_0 \cosh \frac{w_0(L/2)}{T_0} \qquad 40(9.81) = T_0 \cosh \frac{17.658(9)}{T_0}$$

$$f(T_0) = \cosh \frac{158.92}{T_0} - \frac{392.4}{T_0} = 0$$




The plot of $f(T_0)$ shows two roots which can be computed numerically. They are $T_0 = 63.38$ N and 356.4 N.

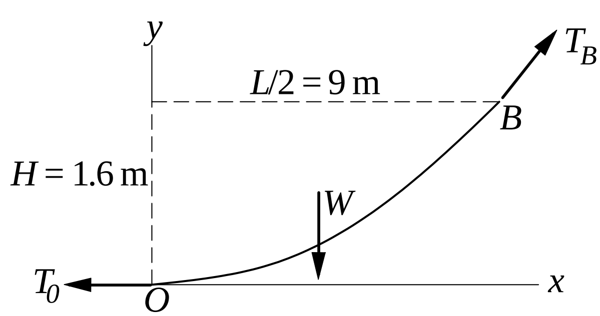
From Eq. (6.16):

$$H = y|_{x=L/2} = \frac{T_0}{w_0}\left(\cosh \frac{w_0 L}{2T_0} - 1\right)$$

$$\therefore H_1 = \frac{63.38}{17.658}\left[\cosh \frac{17.658(18)}{2(63.38)} - 1\right] = 18.58 \text{ m} \blacktriangleleft$$

$$H_2 = \frac{356.4}{17.658}\left[\cosh \frac{17.658(18)}{2(356.4)} - 1\right] = 2.04 \text{ m} \blacktriangleleft$$

**6.74**




The shape of the cable is caternary.

$$w_0 = 1.2(9.81) = 11.772 \text{ N/m}$$

Equation (6.16):

$$y = \frac{T_0}{w_0}\left(\cosh\frac{w_0 x}{T_0} - 1\right) \qquad \therefore H = \frac{T_0}{w_0}\left[\cosh\frac{w_0(L/2)}{T_0} - 1\right]$$

$$1.6 = \frac{T_0}{w_0}\left[\cosh\left(9\frac{w_0}{T_0}\right) - 1\right] \qquad 1.6\frac{w_0}{T_0} = \cosh\left(9\frac{w_0}{T_0}\right) - 1$$

Solving numerically for $\omega_0/T_0$ yields

$$\frac{w_0}{T_0} = 0.03910 \qquad \therefore T_0 = \frac{11.772}{0.03910} = 301.1 \text{ N}$$

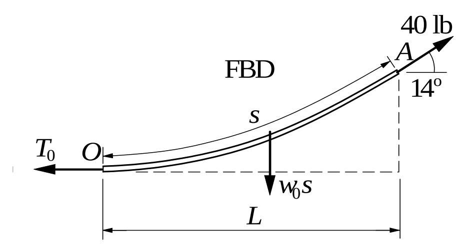
Equation (6.17):

$$T = T_0 \cosh\frac{w_0 x}{T_0}$$

$$\therefore \quad T_B = T_0 \cosh\left(\frac{w_0}{T_0}\frac{L}{2}\right) = 301.1 \cosh[0.03910(9)] = 319.9 \text{ N}$$

$$M = \frac{T_B}{g} = \frac{319.9}{9.81} = 32.6 \text{ kg} \blacktriangleleft$$

## 6.75




The shape of the hose is a catenary. From FBD:

$$\Sigma F_y = 0 \qquad +\downarrow \quad w_0 s = 40\sin 14^\circ$$

$$s = \frac{40\sin 14^\circ}{w_0} = \frac{40\sin 14^\circ}{0.5} = 19.35 \text{ ft} \blacktriangleleft$$

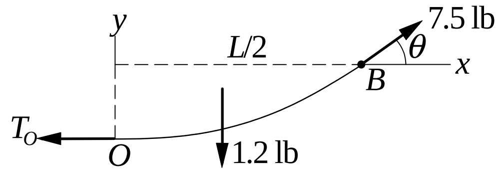
$$\Sigma F_x = 0 \qquad +\leftarrow \quad T_0 - 40\cos 14^\circ = 0 \qquad T_0 = 38.81 \text{ lb}$$

From Eq. (6.15):

$$s = \frac{T_0}{w_0}\sinh\frac{w_0 L}{T_0}$$

$$\therefore L = \frac{T_0}{w_0}\sinh^{-1}\frac{s w_0}{T_0} = \frac{38.81}{0.5}\sinh^{-1}\frac{19.35(0.5)}{38.81} = 19.15 \text{ ft} \blacktriangleleft$$

## 6.76




$$w_0 = \frac{W}{\text{Length}} = \frac{2.4}{50} = 0.048 \text{ lb/ft}$$

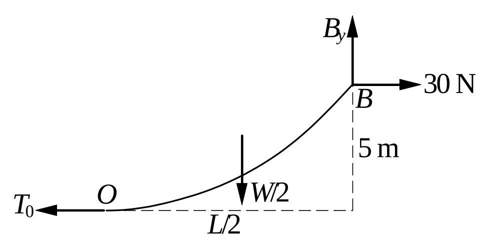
From FBD:

$$\begin{aligned}
\Sigma F_y &= 0 \qquad 7.5\sin\theta - 1.2 = 0 \qquad \theta = \sin^{-1}\frac{1.2}{7.5} = 9.207^\circ \\
\Sigma F_x &= 0 \qquad 7.5\cos\theta - T_O = 0 \qquad T_O = 7.5\cos 9.207^\circ = 7.403 \text{ lb} \\
\frac{T_O}{w_0} &= \frac{7.403}{0.048} = 154.23 \text{ ft}
\end{aligned}$$

Eq. (6.15):

$$\begin{aligned}
s &= \frac{T_O}{w_0}\sinh\frac{w_0 x}{T_O} \qquad \therefore x = \frac{T_O}{w_0}\sinh^{-1}\frac{w_0 s}{T_O} \\
\frac{L}{2} &= x|_{s=25} = 154.23\sinh^{-1}\frac{25}{154.23} = 24.892 \text{ ft} \\
L &= 2\,(24.892) = 49.78 \text{ ft} \;\blacktriangleleft
\end{aligned}$$

## 6.77



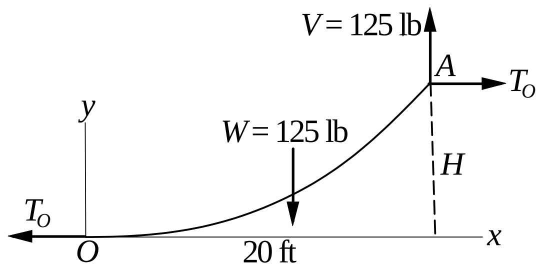

From FBD:

$$\Sigma F_x = 0 \qquad +\leftarrow \quad T_0 - 30 = 0 \qquad T_0 = 30 \text{ N}$$

From Eq. (6.16):

$$\begin{aligned}
y|_{x=L/2} &= \frac{T_0}{w_0}\left(\cosh\frac{w_0 L}{2T_0} - 1\right) \qquad 5 = \frac{30}{5.2}\left[\cosh\frac{5.2L}{2(60)} - 1\right] \\
& L = 28.5 \text{ m} \;\blacktriangleleft
\end{aligned}$$

**6.78**




$$W = w_0 s_A = 5(25) = 125 \text{ lb}$$

From Eq. (6.15):

$$s_A = \frac{T_O}{w_0}\sinh\frac{w_0 x_A}{T_O} \qquad 25 = \frac{T_O}{w_0}\sinh\frac{20w_0}{T_O} \qquad \therefore \frac{T_O}{w_0} = 16.91 \text{ ft}$$

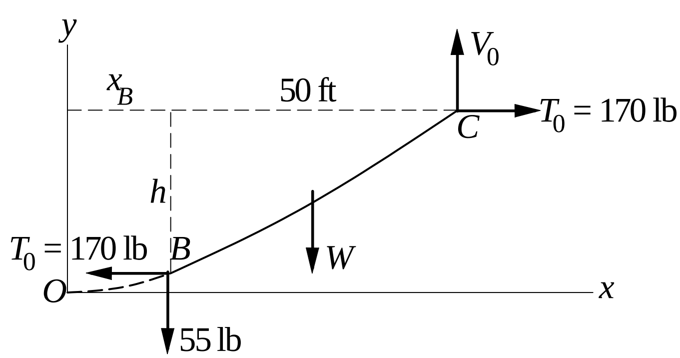
$$T_O = 16.91w_0 = 16.91(5) = 84.55 \text{ lb}$$

$$T_{\max} = T_A = \sqrt{T_O^2 + V^2} = \sqrt{125^2 + 84.55^2} = 150.9 \text{ lb} \blacktriangleleft$$

From Eq. (6.16):

$$H = \frac{T_O}{w_0}\left(\cosh\frac{w_0 x_A}{T_O} - 1\right) = 16.91\left(\cosh\frac{20}{16.91} - 1\right) = 13.27 \text{ ft} \blacktriangleright$$

**6.79**




Each cable is a catenary.

$$T_B = \sqrt{170^2 + 55^2} = 178.68 \text{ lb}$$

Equation (6.17):

$$T_B = T_0\cosh\frac{w_0 x_B}{T_0}$$

$$x_B = \frac{T_0}{w_0}\cosh^{-1}\frac{T_B}{T_0} = \frac{170}{0.8}\cosh^{-1}\frac{178.68}{170} = 67.62 \text{ ft}$$

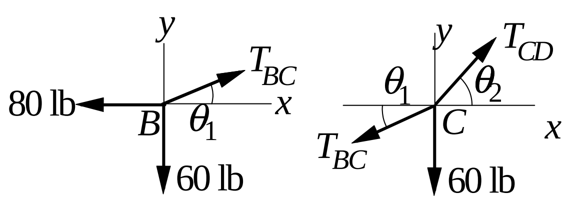
Equation (6.15):

$$\begin{aligned}
s &= \frac{T_0}{w_0}\sinh\frac{w_0 x}{T_0} \\
s_{BC} &= s_C - s_B = \frac{T_0}{w_0}\left(\sinh\frac{w_0 x_C}{T_0} - \sinh\frac{w_0 x_B}{T_0}\right) \\
&= \frac{170}{0.8}\left(\sinh\frac{0.8(117.62)}{170} - \sinh\frac{0.8(67.62)}{170}\right) = 55.0 \text{ ft} \blacktriangleleft
\end{aligned}$$

Equation (6.16):

$$\begin{aligned}
y &= \frac{T_0}{w_0}\left(\cosh\frac{w_0 x}{T_0} - 1\right) \\
h &= y_C - y_B = \frac{T_0}{w_0}\left(\cosh\frac{w_0 x_C}{T_0} - \cosh\frac{w_0 x_B}{T_0}\right) \\
&= \frac{170}{0.8}\left(\cosh\frac{0.8(117.62)}{170} - \cosh\frac{0.8(67.62)}{170}\right) = 22.5 \text{ ft} \blacktriangleleft
\end{aligned}$$

**6.80**




FBD of joint $B$:

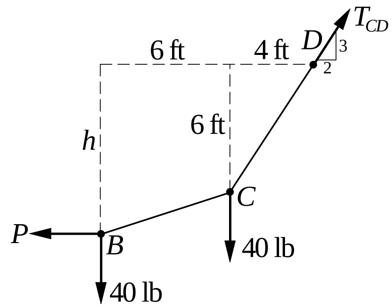
$$\begin{aligned}
\Sigma F_x &= 0 \qquad T_{BC}\cos\theta_1 - 80 = 0 \\
\Sigma F_y &= 0 \qquad T_{BC}\sin\theta_1 - 60 = 0 \\
\theta_1 &= \tan^{-1}\frac{60}{80} = 36.87^\circ \qquad T_{BC} = \sqrt{80^2 + 60^2} = 100 \text{ lb} \blacktriangleleft
\end{aligned}$$

FBD of joint $C$:

$$\begin{aligned}
\Sigma F_x &= 0 \qquad T_{CD}\cos\theta_2 - T_{BC}\cos\theta_1 = 0 \qquad T_{CD}\cos\theta_2 - 80 = 0 \\
\Sigma F_y &= 0 \qquad T_{CD}\sin\theta_2 - T_{BC}\sin\theta_1 - 60 = 0 \qquad T_{CD}\sin\theta_2 - 120 = 0 \\
\theta_2 &= \tan^{-1}\frac{120}{80} = 56.31^\circ \qquad T_{CD} = \sqrt{80^2 + 120^2} = 144.2 \text{ lb} \blacktriangleleft
\end{aligned}$$

$$h = s_{BC}\sin\theta_1 + s_{CD}\sin\theta_2 = 8(\sin 36.87^\circ + \sin 56.31^\circ) = 11.46 \text{ ft} \blacktriangleleft$$

## 6.81




From FBD:

$$\Sigma F_y = 0 \qquad +\uparrow \qquad \frac{3}{\sqrt{13}}T_{CD} - 80 = 0 \qquad T_{CD} = 96.15 \text{ lb} \blacktriangleleft$$

$$\Sigma F_x = 0 \qquad +\leftarrow \qquad P - \frac{2}{\sqrt{13}}T_{CD} = 0 \qquad P - \frac{2}{\sqrt{13}}(96.15) = 0$$

$$P = 53.33 \text{ lb} \blacktriangleleft$$

$$\Sigma M_D = 0 \qquad +\circlearrowright \qquad Ph - 40(10) - 40(4) = 0 \qquad 55.33h - 560 = 0$$

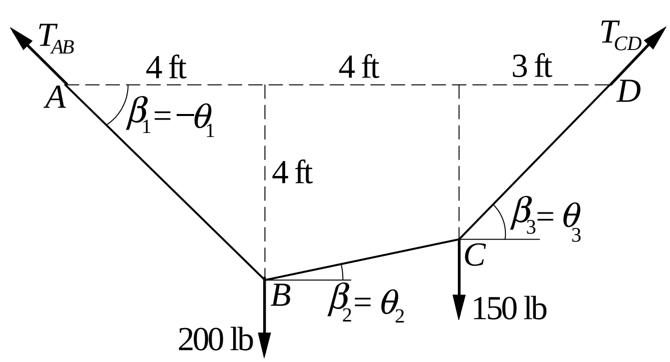
$$h = 10.121 \text{ ft} \blacktriangleleft$$

Horizontal component of force in the cable is $P$.

$$\therefore \quad \frac{6}{\sqrt{6^2 + (h-6)^2}}T_{BC} = P \qquad \frac{6}{\sqrt{6^2 + (10.121-6)^2}}T_{BC} = 53.33$$

$$T_{BC} = 64.7 \text{ lb} \blacktriangleleft$$

## 6.82




Geometry: $\beta_1 = 45°$ ◄

$$\Sigma M_D = 0 \qquad +\circlearrowright \quad (T_{AB}\sin 45°)(11) - 200(7) - 150(3) = 0$$

$$T_{AB} = 237.9 \text{ lb} \blacktriangleleft$$

$$\therefore \quad T_0 = 237.9\cos 45° = 168.22 \text{ lb}$$

Using Eqs. (6.19):

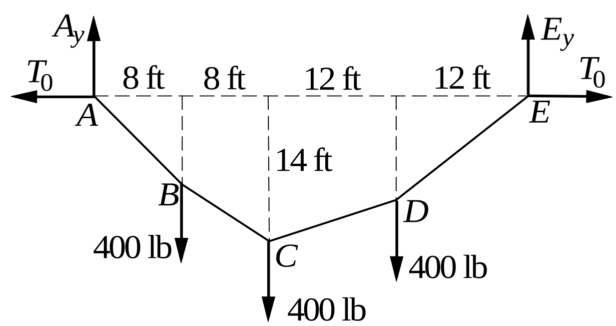
$$T_0(\tan\theta_2 - \tan\theta_1) = W_1 \qquad 168.22(\tan\beta_2 + 1) = 200$$
$$\tan\beta_2 = 0.18892 \qquad \beta_2 = 10.698^\circ \blacktriangleleft$$

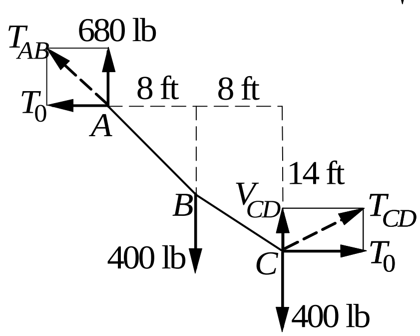
$$T_0(\tan\theta_3 - \tan\theta_2) = W_2 \qquad 168.22(\tan\beta_3 - 0.18892) = 150$$
$$\tan\beta_3 = 1.0806 \qquad \beta_3 = 47.22^\circ \blacktriangleleft$$

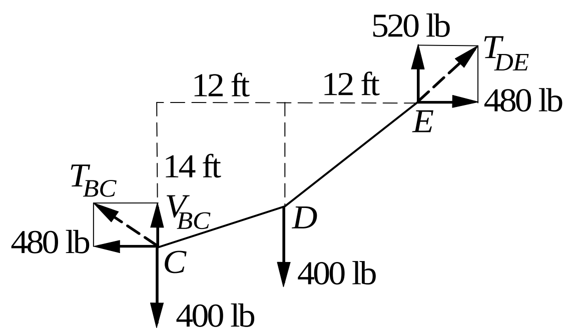
Using Eqs.(6.18):

$$T_{BC} = \frac{T_0}{\cos\beta_2} = \frac{168.22}{\cos 10.698^\circ} = 171.2 \text{ lb} \blacktriangleleft$$
$$T_{CD} = \frac{T_0}{\cos\beta_3} = \frac{168.22}{\cos 47.22^\circ} = 247.7 \text{ lb} \blacktriangleleft$$

**6.83**




FBD of entire cable:

$$\Sigma M_E = 0 \qquad 400(12 + 24 + 32) - 40A_y = 0 \qquad A_y = 680 \text{ lb}$$
$$\Sigma F_y = 0 \qquad A_y + E_y - 3(400) = 0 \qquad 680 + E_y - 1200 = 0$$
$$E_y = 520 \text{ lb}$$

FBD of portion $ABC$:

$$\Sigma F_y = 0 \qquad V_{CD} + 680 - 2(400) = 0 \qquad V_{CD} = 120 \text{ lb}$$
$$\Sigma M_A = 0 \qquad 16V_{CD} + 14T_0 - 400(8+16) = 0$$
$$16(120) + 14T_0 - 400(8+16) = 0 \qquad T_0 = 548.6 \text{ lb}$$

FBD of portion $CDE$:

$$\Sigma F_y = 0 \qquad V_{BC} + 520 - 2(400) = 0 \qquad V_{BC} = 280 \text{ lb}$$

$$T_{AB} = \sqrt{T_0^2 + A_y^2} = \sqrt{548.6^2 + 680^2} = 874 \text{ lb} \blacktriangleleft$$
$$T_{BC} = \sqrt{T_0^2 + V_{BC}^2} = \sqrt{548.6^2 + 280^2} = 616 \text{ lb} \blacktriangleleft$$
$$T_{CD} = \sqrt{T_0^2 + V_{CD}^2} = \sqrt{548.6^2 + 120^2} = 562 \text{ lb} \blacktriangleleft$$
$$T_{DE} = \sqrt{T_0^2 + E_y^2} = \sqrt{548.6^2 + 520^2} = 756 \text{ lb} \blacktriangleleft$$

## 6.84

**Assume maximum tension to occur in segment AB, i.e. assume $T_{AB} = 900$ lb**

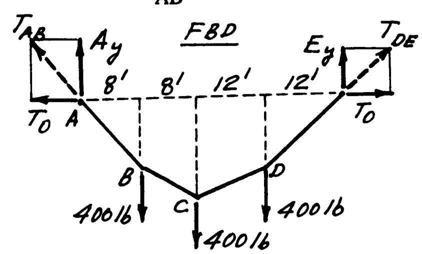
**From the FBD of the cable**

$\Sigma M_E = 0$: (+) $400(12 + 24 + 32) - 40A_y = 0$

$\Sigma F_y = 0$: $+\uparrow$ $A_y + E_y - 3(400) = 0$

**The solution is:** $A_y = 680$ lb $\quad E_y = 520$ lb

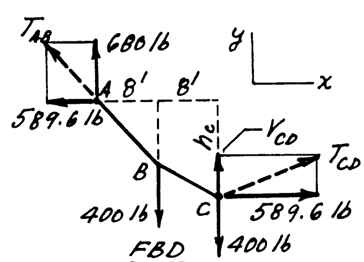
$\therefore T_0 = \sqrt{T_{AB}^2 - A_y^2} = \sqrt{900^2 - 680^2} = 589.6$ lb




**From FBD of portion ABC**

$\Sigma M_C = 0$: (+) $589.6h_C - (680)(16) + 400(8) = 0$

$\therefore h_C = 13.03$ ft ◆




**Check tension in DE**

$T_{DE} = \sqrt{T_0^2 + E_y^2} = \sqrt{589.6^2 + 520^2} = 786 \text{ lb} < T_{AB}$ $\quad\therefore$ **Assumption was O.K.**

## 6.85

**(a)** From geometry

$$2\cos\beta_1 + 3\cos\beta_2 = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(a)}$$

$$2\sin\beta_1 - 3\sin\beta_2 = 0 \qquad \therefore \sin\beta_2 = \frac{2}{3}\sin\beta_1 \ldots \text{(b)}$$

Substituting Eq. (b) in the trigonometric identity

$\cos\beta_2 = \sqrt{1 - \sin^2\beta_2}$ we get

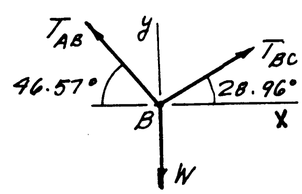
$$\cos\beta_2 = \sqrt{1 - \frac{4}{9}\sin^2\beta_1} = \sqrt{1 - \frac{4}{9}(1 - \cos^2\beta_1)} = \frac{1}{3}\sqrt{5 + 4\cos^2\beta_1}$$

Substitution in Eq. (a) yields

$$2\cos\beta_1 + \sqrt{5 + 4\cos^2\beta_1} = 4 \qquad \therefore 5 + 4\cos^2\beta_1 = (4 - 2\cos\beta_1)^2$$

$$\therefore 16\cos\beta_1 = 11 \qquad \therefore \beta_1 = 46.57° \ \blacklozenge$$

From Eq. (b): $\sin\beta_2 = \frac{2}{3}\sin 46.57° \quad \therefore \beta_2 = 28.96°$ ♦



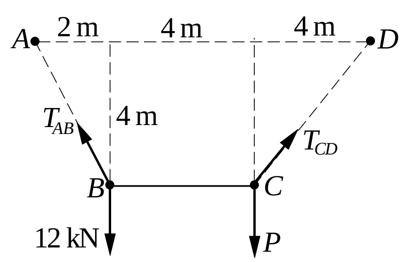

**(b)** From FBD of joint B

$$\Sigma F_x = 0: \quad \overset{+}{\rightarrow} \quad T_{BC}\cos 28.96° - T_{AB}\cos 46.57° = 0$$

$$\Sigma F_y = 0: \quad +\uparrow \quad T_{BC}\sin 28.96° + T_{AB}\sin 46.57° - W = 0$$

The solution is: $T_{AB} = 0.904\ W$ ♦ $\qquad T_{BC} = 0.621\ W$ ♦

## 6.86



$$\Sigma M_C = 0 \quad +\circlearrowleft \quad \frac{2}{\sqrt{20}}T_{AB}(4) - 12(4) = 0 \qquad T_{AB} = 26.83 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0 \quad +\rightarrow \quad \frac{4}{\sqrt{32}}T_{CD} - \frac{2}{\sqrt{20}}T_{AB} = 0$$

$$\frac{4}{\sqrt{32}}T_{CD} - \frac{2}{\sqrt{20}}(26.83) = 0 \qquad T_{CD} = 16.969 \text{ kN} \blacktriangleleft$$

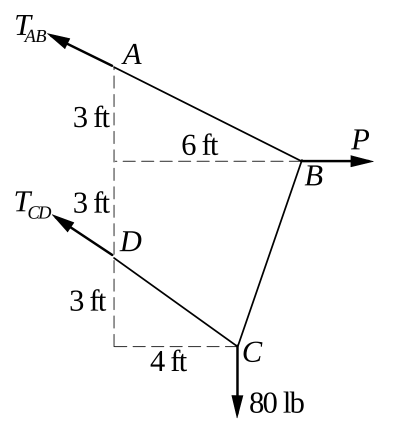
$$\Sigma F_y = 0 \quad +\downarrow \quad P - \frac{4}{\sqrt{20}}T_{AB} - \frac{4}{\sqrt{32}}T_{CD} - 12 = 0$$

$$P - \frac{4}{\sqrt{20}}(26.83) - \frac{4}{\sqrt{32}}(16.969) - 12 = 0 \qquad P = 48.0 \text{ kN} \blacktriangleleft$$

Horizontal component of force in the cable is constant.

$$\therefore T_{BC} = \frac{2}{\sqrt{20}}T_{AB} = \frac{2}{\sqrt{20}}(26.83) = 12.00 \text{ kN} \blacktriangleleft$$

**6.87**




$$\Sigma M_B = 0 \quad +\circlearrowleft \quad 80(2) - \frac{4}{5}T_{CD}(3) - \frac{3}{5}T_{CD}(6) = 0$$

$$T_{CD} = 26.67 \text{ lb}$$

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad 3P - 80(4) - \frac{4}{5}T_{CD}(6) = 0$$

$$3P - 80(4) - \frac{4}{5}(26.67)(6) = 0 \qquad P = 149.3 \text{ lb} \blacktriangleleft$$

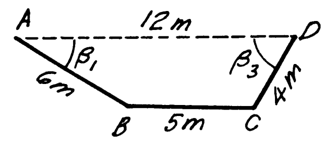
## 6.88

From geometry

$6 \sin\beta_1 = 4 \sin\beta_3 \quad \therefore \sin\beta_3 = 1.5 \sin\beta_1 \ldots$ (a)

$6 \cos\beta_1 + 5 + 4 \cos\beta_3 = 12$

$\therefore 6 \cos\beta_1 + 4 \cos\beta_3 - 7 = 0 \ldots\ldots\ldots\ldots$ (b)




Eliminate $\beta_3$ from the equations and solve for $\beta_1$

$$\cos\beta_3 = \sqrt{1 - \sin^2\beta_3} = \sqrt{1 - (1.5 \sin\beta_1)^2} = \sqrt{1 - 1.5^2(1 - \cos^2\beta_1)}$$

$$= \sqrt{2.25 \cos^2\beta_1 - 1.25}$$

Substitute in Eq. (b): $6 \cos\beta_1 + 4\sqrt{2.25 \cos^2\beta_1 - 1.25} - 7 = 0$

Numerical solution is (the equation could be turned into a quadratic equation in $\cos\beta_1$):

$\cos\beta_1 = 0.8214 \quad \therefore \beta_1 = 34.77°$

From Eq. (a): $\sin\beta_3 = 1.5 \sin 34.77 \quad \therefore \beta_3 = 58.81°$

Apply Eq. (6.19) to joints B and C

$T_0(\tan\theta_2 - \tan\theta_1) = W_1 \quad \therefore T_0\left[\tan 0 - \tan(-\beta_1)\right] = W_1$

$T_0(\tan\theta_3 - \tan\theta_2) = W_2 \quad \therefore T_0(\tan\beta_3 - \tan 0) = W_2$

$$\therefore \frac{W_1}{W_2} = \frac{\tan\beta_1}{\tan\beta_3} = \frac{\tan 34.77°}{\tan 58.81°} = 0.420 \ \blacklozenge$$

## 6.89

**(a)** From geometry: $6 \sin\beta_1 - 5 \sin\beta_2 - 4 \sin\beta_3 = 0 \ldots\ldots\ldots\ldots$ (a) ◆

$6 \cos\beta_1 + 5 \cos\beta_2 + 4 \cos\beta_3 = 12 \ldots\ldots\ldots\ldots$ (b) ◆

Apply Eq. (6.19) to joints B and C:

$T_0(\tan\theta_2 - \tan\theta_1) = W \quad \therefore T_0\left[\tan\beta_2 - \tan(-\beta_1)\right] = W \ldots$ (c)

$T_0(\tan\theta_3 - \tan\theta_2) = W \quad \therefore T_0(\tan\beta_3 - \tan\beta_2) = W$

Equating the LHS's yields: $\tan\beta_1 + 2 \tan\beta_2 - \tan\beta_3 = 0 \ldots\ldots$ (d) ◆

Equations (a), (b) and (d) are the simultaneous equations for the $\beta$'s

**(b)** Equation (a): $6 \sin 41.0° - 5 \sin 9.8° - 4 \sin 50.5° = -0.001 \approx 0 \quad \therefore$ O.K.

Equation (b): $6 \cos 41.0° + 5 \cos 9.8° + 4 \cos 50.5° = 12.00 \quad \therefore$ O.K.

Equation (d): $\tan 41.0° + 2 \tan 9.8° - \tan 50.5° = 0.002 \approx 0 \quad \therefore$ O.K.

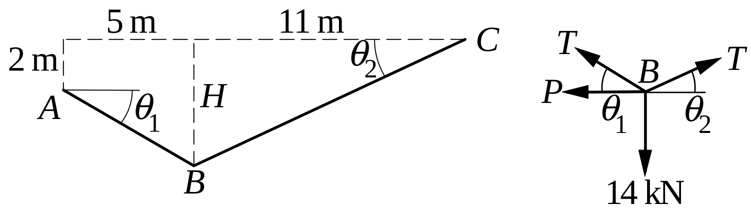
**(c)** From Eq. (c): $T_0 = \dfrac{W}{\tan\beta_1 + \tan\beta_2} = \dfrac{W}{\tan 41.0° + \tan 9.8°} = 0.9597W$

Applying Eq. (6.18) to joints B, C and D (note that $\theta_1 = -\beta_1$)

$$T_{AB} = \frac{T_0}{\cos(-\beta_1)} = \frac{0.9597W}{\cos(-41.0°)} = 1.272W \ \blacklozenge$$

$$T_{BC} = \frac{T_0}{\cos\beta_2} = \frac{0.9597W}{\cos 9.8°} = 0.974W \ \blacklozenge \qquad T_{CD} = \frac{T_0}{\cos\beta_3} = \frac{0.9597W}{\cos 50.5°} = 1.509W \ \blacklozenge$$

## 6.90




Geometry:

$$\begin{aligned}
s_{AB} + s_{BC} &= 20 \text{ m} \qquad \sqrt{5^2 + (H-2)^2} + \sqrt{11^2 + H^2} = 2 \\
&\phantom{=} \text{Solution is } H = 6.912 \text{ m} \\
\theta_1 &= \tan^{-1}\frac{H-2}{5} = \tan^{-1}\frac{6.912-2}{5} = 44.49° \\
\theta_2 &= \tan^{-1}\frac{H}{11} = \tan^{-1}\frac{6.912}{11} = 32.14°
\end{aligned}$$

FBD of joint $B$:

$$\begin{aligned}
\Sigma F_y &= 0 \qquad T(\sin\theta_1 + \sin\theta_2) - 14 = 0 \\
&\phantom{=} T(\sin 44.49° + \sin 32.14°) - 14 = 0 \qquad T = 11.356 \text{ kN} \\
\Sigma F_x &= 0 \qquad T(\cos\theta_2 - \cos\theta_1) - P = 0 \\
&\phantom{=} 11.356(\cos 32.14° - \cos 44.49°) - P = 0 \\
&\phantom{=} P = 1.515 \text{ kN} \ \blacktriangleleft
\end{aligned}$$

## 6.91

$$T_0 = 20\cos 32° = 16.961 \text{ kN}$$

**(a)** From Eqs. (6.19):

$$\begin{aligned} T_0(\tan\theta_2 - \tan\theta_1) &= 12 \qquad 16.961\,[\tan\theta_2 - \tan(-32^\circ)] = 12 \\ \theta_2 &= 4.72^\circ \;\blacktriangleleft \\ T_0(\tan\theta_3 - \tan\theta_2) &= 18 \qquad 16.961(\tan\theta_3 - \tan 4.72^\circ) = 18 \\ \theta_3 &= 48.84^\circ \;\blacktriangleleft \end{aligned}$$

**(b)**

$$\begin{aligned} T_{AB} &= 20 \text{ kN} \;\blacktriangleleft \\ T_{BC} &= \frac{T_0}{\cos\theta_2} = \frac{16.961}{\cos 4.72^\circ} = 17.02 \text{ kN} \;\blacktriangleleft \\ T_{CD} &= \frac{T_0}{\cos\theta_3} = \frac{16.961}{\cos 48.84^\circ} = 25.77 \text{ kN} \;\blacktriangleleft \end{aligned}$$

**(c)**

$$\begin{aligned} L &= 6(\cos\theta_1 + \cos\theta_2 + \cos\theta_3) \\ &= 6\,[\cos(-32^\circ) + \cos 4.72^\circ + \cos 48.84^\circ] = 15.02 \text{ m} \;\blacktriangleleft \\ H &= 6(-\sin\theta_1 + \sin\theta_2 + \sin\theta_3) \\ &= 6\,[-\sin 32^\circ + \sin 4.72^\circ + \sin 48.84^\circ] = 1.83 \text{ m} \;\blacktriangleleft \end{aligned}$$

# Chapter 7

## 7.1

Assume equilibrium.

From FBD of system

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $-N_1 + F_2 = 0$ . . . . . . . . . . . (a)

$\Sigma F_y = 0$: $+\uparrow$ $N_2 - 120 - 80 = 0$ . . . . . . .(b)

From FBD of block A

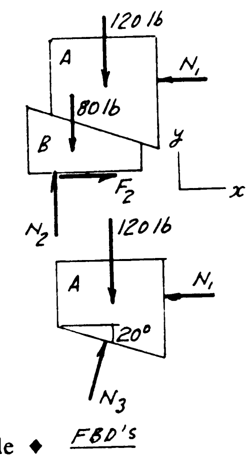
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $N_3 \sin 20° - N_1 = 0$ . . . . . . (c)

$\Sigma F_y = 0$: $+\uparrow$ $N_3 \cos 20° - 120 = 0$ . . . . .(d)

Solution of Eqs. (a)-(d) is

$N_1 = F_2 = 43.68$ lb $N_2 = 200$ lb $N_3 = 127.70$ lb

$\therefore \dfrac{F_2}{N_2} = \dfrac{43.68}{200} = 0.218 > \mu_s$ $\therefore$ Equilibrium is not possible ♦




## 7.2

Assume impending motion of block B <u>to the left</u>.

From FBD of the system

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P + 0.2N_2 - N_1 = 0$ . . . . . . (a)

$\Sigma F_y = 0$: $+\uparrow$ $N_2 - 120 - 80 = 0$ . . . . . . (b)

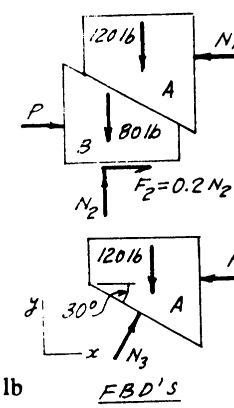
From FBD of block A

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $N_3 \sin 30° - N_1 = 0$ . . . . . (c)

$\Sigma F_y = 0$: $+\uparrow$ $N_3 \cos 30° - 120 = 0$ . . . . (d)

Solution of Eqs. (a)-(d) is

$P = 29.28$ lb $N_1 = 69.28$ lb $N_2 = 200$ lb $N_3 = 138.56$ lb




Assume impending motion of block B <u>to the right</u>.

The sense of $F_2$ on the FBD of the system must be reversed. The above equations remain valid, with the exception of Eq. (a), which becomes

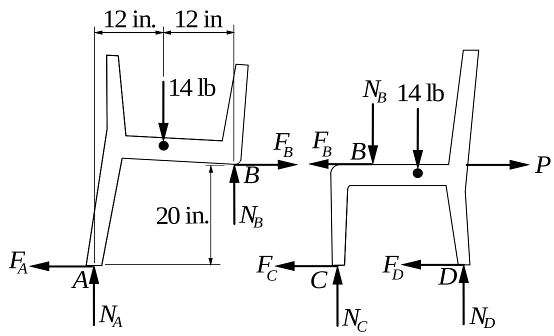
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P - 0.2N_2 - N_1 = 0$ . . . . . . . (e)

The solution of Eqs. (b)-(e) is

$P = 109.28$ lb $N_1 = 69.28$ lb $N_2 = 200$ lb $N_3 = 138.56$ lb

$\therefore$ The equilibrium range of P is $29.3 \text{ lb} \le P \le 109.3 \text{ lb}$ ♦

**7.3**




Assume impending sliding at $B$, $C$ and $D$.

$$\therefore F_B = 0.2N_B \qquad F_C = 0.35N_C \qquad F_D = 0.35N_D$$

From FBD of left chair:

$$\Sigma M_A = 0 \qquad + \circlearrowright \quad 0.2N_B(20) - N_B(24) + 14(12) = 0 \qquad N_B = 8.400 \text{ lb}$$

From FBD of right chair:

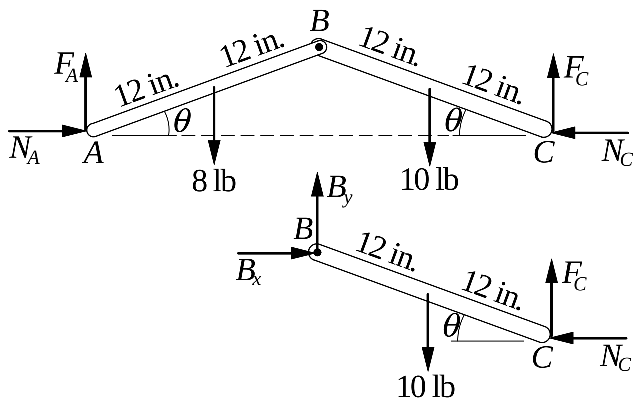
$$\begin{aligned} \Sigma F_y &= 0 \qquad +\uparrow \quad N_C + N_D - N_B - 14 = 0 \\ & N_C + N_D - 8.400 - 14 = 0 \qquad N_C + N_D = 22.40 \text{ lb} \\ \Sigma F_x &= 0 \qquad +\rightarrow \quad P - 0.35(N_C + N_D) - 0.2N_B = 0 \\ & P - 0.35(22.40) - 0.2(8.400) = 0 \qquad P = 9.52 \text{ lb} \blacktriangleleft \end{aligned}$$

Check for sliding at $A$.
From FBD of left chair:

$$\begin{aligned} \Sigma F_y &= 0 \qquad +\uparrow \quad N_A + N_B - 14 = 0 \qquad N_A + 8.400 - 14 = 0 \\ & N_A = 5.600 \text{ lb} \\ \Sigma F_x &= 0 \qquad +\leftarrow \quad F_A - 0.2N_B = 0 \qquad F_A - 0.2(8.400) = 0 \\ & F_A = 1.6800 \text{ lb} \end{aligned}$$

$$\frac{F_A}{N_A} = \frac{1.6800}{5.600} = 0.3 < \mu_A \quad \therefore \text{The assumption was correct}$$

**7.4**




From FBD of assembly:

$$\begin{aligned}
\Sigma M_A &= 0 \quad +\circlearrowleft \quad F_C(48\cos\theta) - 8(12\cos\theta) - 10(36\cos\theta) = 0 \\
& F_C(48) - 96 - 360 = 0 \qquad F_C = 9.50 \text{ lb} \\
\Sigma F_y &= 0 \quad +\uparrow \quad F_A + F_C - 8 - 10 = 0 \\
& F_A + 9.50 - 8 - 10 = 0 \qquad F_A = 8.50 \text{ lb} \\
\Sigma F_x &= 0 \qquad N_A = N_C
\end{aligned}$$

Impending sliding will first occur at $C$ since $F_C/N_C > F_A/N_A$ ($\mu_s$ being the same at $C$ and $A$).

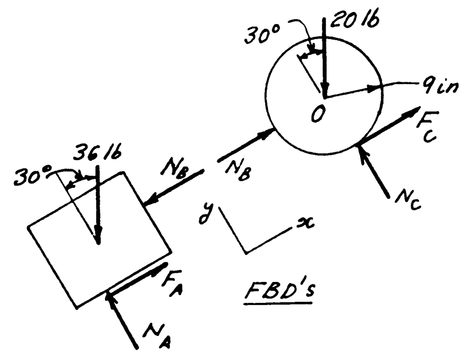
$$\therefore F_C = 0.25N_C \qquad N_C = \frac{F_C}{0.25} = \frac{9.50}{2.5} = 38.0 \text{ lb}$$

From FBD of bar $BC$:

$$\begin{aligned}
\Sigma M_B &= 0 \quad +\circlearrowleft \quad F_C(24\cos\theta) - N_C(24\sin\theta) - 10(12\cos\theta) = 0 \\
& 9.50(24\cos\theta) - 38.0(24\sin\theta) - 10(12\cos\theta) = 0 \\
& 108\cos\theta - 912\sin\theta = 0 \qquad \theta = \tan^{-1}\frac{108}{912} = 6.75^\circ \quad \blacktriangleleft
\end{aligned}$$

**7.5**

Assume equilibrium.




From FBD of cylinder

$\Sigma F_x = 0$: (+ ↗) $F_C + N_B - 20 \sin 30° = 0 \ldots$ (a)

$\Sigma M_O = 0$: (↺+) $9F_C = 0 \ldots\ldots\ldots\ldots$ (b)

From FBD of block

$\Sigma F_x = 0$: (+ ↗) $F_A - N_B - 36 \sin 30° = 0 \ldots$ (c)

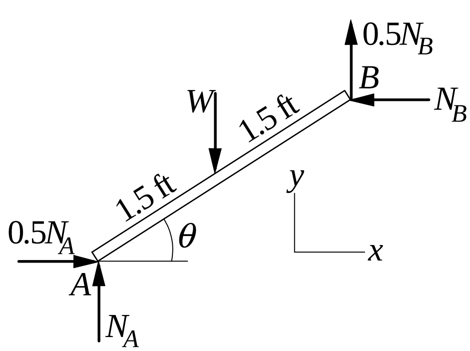
$\Sigma F_y = 0$: (↖+) $N_A - 36 \cos 30° = 0 \ldots\ldots$ (d)

Solution of Eqs. (a)-(d) is

$N_A = 31.18$ lb  $F_A = 28.0$ lb  $N_B = 10.0$ lb  $F_C = 0$

$\therefore \dfrac{F_A}{N_A} = \dfrac{28.0}{31.18} = 0.898 > \mu_s$  ∴ Block is <u>not in equilibrium</u> ◆

**7.6**



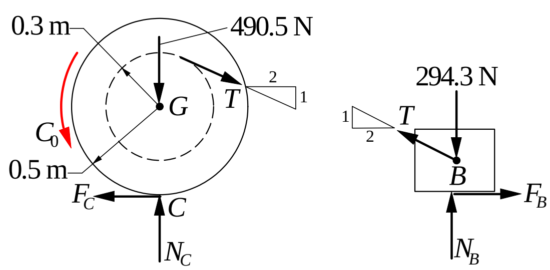
$$\Sigma F_x = 0 \qquad 0.5N_A - N_B = 0$$
$$\Sigma F_y = 0 \qquad N_A + 0.5N_B - W = 0$$

Solution is

$$N_A = 0.8W \qquad N_B = 0.4W$$

$$\Sigma M_A = 0 \qquad N_B(3\sin\theta) + 0.5N_B(3\cos\theta) - W(1.5\cos\theta) = 0$$
$$0.4W(3\sin\theta) + 0.5(0.4W)(3\cos\theta) - W(1.5\cos\theta) = 0$$
$$1.2\sin\theta - 0.9\cos\theta = 0$$
$$\theta = \tan^{-1}\frac{0.9}{1.2} = 36.9^\circ \blacktriangleleft$$

**7.7**




Assume impending sliding of the block. $\therefore F_B = 0.2N_B$

FBD of block:

$$\Sigma F_x = 0 \qquad +\rightarrow \qquad 0.2N_B - \frac{2}{\sqrt{5}}T = 0$$
$$\Sigma F_y = 0 \qquad +\uparrow \qquad N_B + \frac{1}{\sqrt{5}}T - 294.3 = 0$$

The solution is

$$T = 59.83 \text{ N} \qquad N_B = 267.5 \text{ N}$$

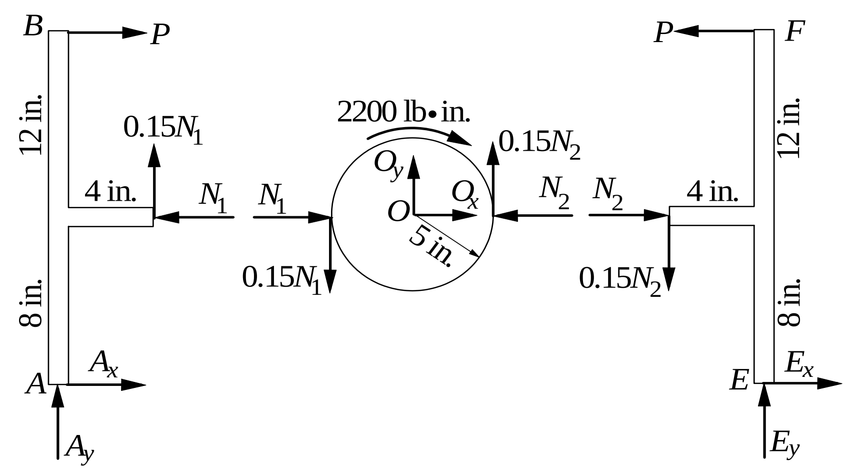
FBD of spool:

$$\Sigma F_x = 0 \qquad +\leftarrow \qquad F_C = \frac{2}{\sqrt{5}}T = 0 \qquad F_C - \frac{2}{\sqrt{5}}(59.83) = 0$$
$$F_C = 53.51 \text{ N}$$
$$\Sigma M_G = 0 \qquad +\circlearrowleft \qquad C_0 - F_C(0.5) - T(0.3) = 0$$
$$C_0 - (53.51)(0.5) - (59.83)(0.3) = 0 \qquad C_0 = 44.7 \text{ N}\cdot\text{m} \blacktriangleleft$$

Check for sliding at $C$:

$$\Sigma F_y = 0 \qquad +\uparrow \quad N_C - \frac{1}{\sqrt{5}}T - 490.5 = 0$$
$$N_C - \frac{1}{\sqrt{5}}(59.83) - 490.5 = 0 \qquad N_C = 517.3 \text{ N}$$
$$\frac{F_C}{N_C} = \frac{53.51}{517.3} = 0.1034 < 0.15 \qquad \text{Spool does not slide}$$

**7.8**




FBD of $EF$:

$$\Sigma M_E = 0 \qquad 20P - 8N_2 + 4(0.15N_2) = 0 \qquad N_2 = 2.703P$$

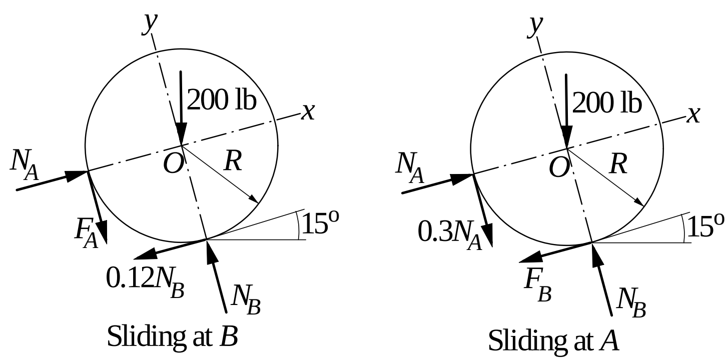
FBD of $AB$:

$$\Sigma M_A = 0 \qquad 20P - 8N_1 - 4(0.15N_1) = 0 \qquad N_1 = 2.326P$$

FBD of cylinder:

$$\Sigma M_O = 0 \qquad 5(0.15N_1) + 5(0.15N_2) - 2200 = 0 \qquad N_1 + N_2 = 2933 \text{ lb}$$
$$2.326P + 2.703P = 2933 \qquad P = 583 \text{ lb} \blacktriangleleft$$

**7.9**




Assume sliding at $B$.

$$\begin{aligned} \Sigma F_x &= 0 \qquad N_A - 0.12N_B - 200\sin 15^\circ = 0 \\ \Sigma F_y &= 0 \qquad N_B - F_A - 200\cos 15^\circ = 0 \\ \Sigma M_O &= 0 \qquad F_A - 0.12N_B = 0 \end{aligned}$$

Solution is:

$$F_A = 26.34 \text{ lb} \qquad N_A = P = 78.11 \text{ lb} \qquad N_B = 219.5 \text{ lb}$$

Assume sliding at $A$.

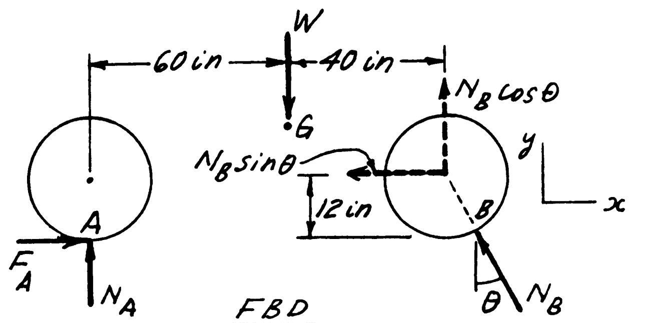
$$\begin{aligned} \Sigma F_x &= 0 \qquad N_A - F_B - 200\sin 15^\circ = 0 \\ \Sigma F_y &= 0 \qquad N_B - 0.3N_A - 200\cos 15^\circ = 0 \\ \Sigma M_O &= 0 \qquad 0.3N_A - F_B = 0 \end{aligned}$$

Solution is:

$$F_B = 22.18 \text{ lb} \qquad N_A = P = 73.95 \text{ lb} \qquad N_B = 215.4 \text{ lb}$$

The smallest force that initiates motion is $P = 74.0$ lb ◀

7.10




From the FBD of the truck (note that $N_B$ was moved from B to the axle)

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_A - N_B \sin\theta = 0$ . . . . . . . . . . . . . . . . . . . . . . . (a)

$\Sigma F_y = 0$: $+\uparrow$ $N_A + N_B \cos\theta - W = 0$ . . . . . . . . . . . . . . . . . . (b)

$\Sigma M_A = 0$: (↻+) $(N_B \cos\theta)(100) + (N_B \sin\theta)(12) - 60W = 0$ . . . (c)

Solution of Eqs. (a)-(c) is

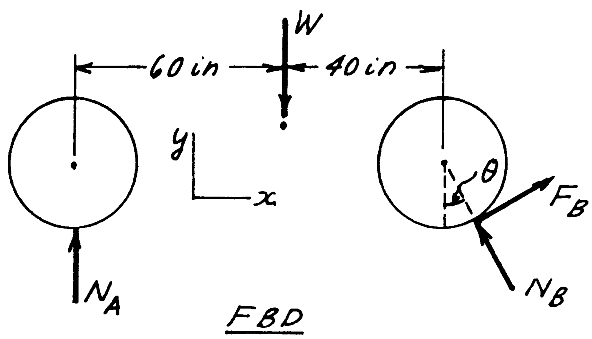
$$N_B = \frac{15W}{25\cos\theta + 3\sin\theta} \qquad F_A = \frac{15W\sin\theta}{25\cos\theta + 3\sin\theta} \qquad N_A = \frac{(10\cos\theta + 3\sin\theta)W}{25\cos\theta + 3\sin\theta}$$

Set $F_A = \mu N_A$: $15W\sin\theta = \mu(10\cos\theta + 3\sin\theta)W$ $\quad\therefore\ \theta = \tan^{-1}\dfrac{10\mu}{15 - 3\mu}$

**(a)** With $\mu = \mu_k = 0.15$, we get $\theta = \tan^{-1}\dfrac{10(0.15)}{15 - 3(0.15)} = 5.89°$ ◆

**(b)** With $\mu = \mu_s = 0.18$, we get $\theta = \tan^{-1}\dfrac{10(0.18)}{15 - 3(0.18)} = 7.10°$ ◆

**7.11**




From the FBD of the truck

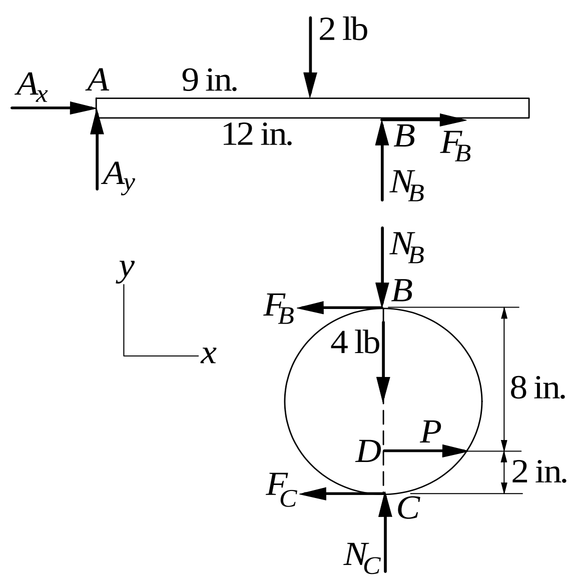
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_B \cos\theta - N_B \sin\theta = 0$

Set $F_B = \mu N_B$: $N_B(\mu \cos\theta - \sin\theta) = 0$ $\therefore\ \theta = \tan^{-1}\mu$

**(a)** With $\mu = \mu_k = 0.15$ we get $\theta = \tan^{-1}(0.15) = 8.53°$ ◆

**(b)** With $\mu = \mu_s = 0.18$ we get $\theta = \tan^{-1}(0.18) = 10.20°$ ◆

**7.12**



FBD of bar:

$$\Sigma M_A = 0 \qquad 12N_B - 2(9) = 0 \qquad N_B = 1.50 \text{ lb}$$

FBD of spool:

$$\begin{aligned} \Sigma F_y &= 0 \qquad N_C - 4 - N_B = 0 \qquad N_C - 4 - 1.50 = 0 \qquad N_C = 5.50 \text{ lb} \\ \Sigma M_B &= 0 \qquad 8P - 10F_C = 0 \qquad P = 1.25F_C \\ \Sigma M_C &= 0 \qquad 2P - 10F_B = 0 \qquad P = 5F_B \end{aligned}$$

Assume impending sliding at $B$:

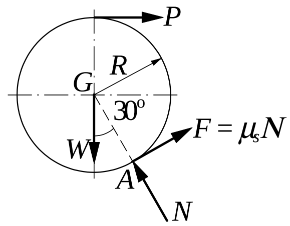
$$F_B = \mu_s N_B = 0.3(1.50) = 0.45 \text{ lb} \qquad P = 5(0.45) = 2.25 \text{ lb}$$

Assume impending sliding at $C$:

$$F_C = \mu_s N_C = 0.3(5.50) = 1.65 \text{ lb} \qquad P = 1.25(1.65) = 2.06 \text{ lb}$$

Largest $P$ that does not cause sliding is $P = 2.06$ lb ◀

## 7.13



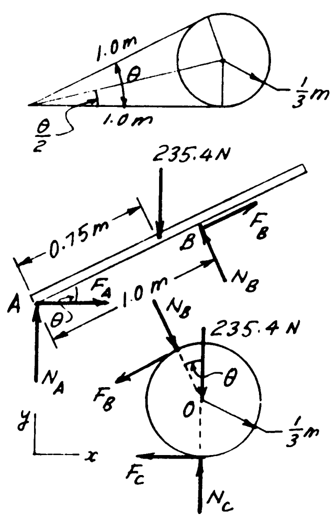

Consider impending slipping at $A$.

$$\Sigma M_G = 0 \quad +\circlearrowleft \quad (\mu_s N) R - PR = 0 \qquad P = \mu_s N$$

$$\Sigma F_x = 0 \quad +\rightarrow \quad \mu_s N \cos 30^\circ - N \sin 30^\circ + P = 0$$

$$\mu_s N \cos 30^\circ - N \sin 30^\circ + \mu_s N = 0 \qquad \mu_s = 0.268 \;◄$$

## 7.14




W = 24(9.81) = 235.4 N

Geometry: $\frac{\theta}{2} = \tan^{-1}\frac{1}{3}$ $\quad\therefore\ \theta = 36.87°$

**(a)** From the FBD of the bar

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_A + F_B\cos\theta - N_B\sin\theta = 0$

$\Sigma F_y = 0$: $+\uparrow$ $N_A + N_B\cos\theta + F_B\sin\theta - 235.4 = 0$

$\Sigma M_A = 0$: $(+)$ $N_B(1.0) - (235.4\cos\theta)(0.75) = 0$

From the FBD of the cylinder

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $N_B\sin\theta - F_C - F_B\cos\theta = 0$

$\Sigma F_y = 0$: $+\uparrow$ $N_C - N_B\cos\theta - F_B\sin\theta - 235.4 = 0$

$\Sigma M_O = 0$: $(+)$ $F_B - F_C = 0$

Solution of the above equations is, after substituting for θ

$F_A = F_B = F_C = 47.1$ N ♦

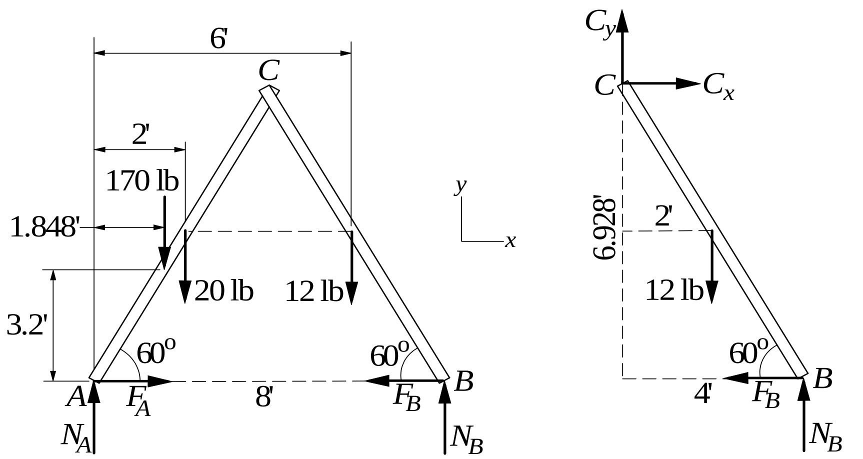
$N_A = 94.2$ N ♦ $\quad N_B = 141.2$ N ♦ $\quad N_C = 376.6$ N ♦

**(b)** The largest ratio $\frac{F}{N}$ occurs at A: $\frac{F_A}{N_A} = \frac{47.1}{94.2} = 0.500$.

$\therefore$ Smallest coefficient of friction required for equilibrium is $\mu_s = 0.500$ ♦

## 7.15

Assume equilibrium.




FBD of entire ladder:

$$
\begin{aligned}
\Sigma M_A &= 0 \qquad 8N_B - 12(6) - 20(2) - 170(1.848) = 0 \qquad N_B = 53.27 \text{ lb} \\
\Sigma F_y &= 0 \qquad N_A + N_B - 12 - 20 - 170 = 0 \qquad N_A + 53.27 - 202 = 0 \\
& \quad N_A = 148.73 \text{ lb} \\
\Sigma F_x &= 0 \qquad F_A - F_B = 0 \qquad F_A = F_B
\end{aligned}
$$

FBD of $BC$:

$$
\begin{aligned}
\Sigma M_C &= 0 \qquad 4N_B - 12(2) - 6.928F_B = 0 \\
& \quad 4(53.27) - 12(2) - 6.928F_B = 0 \qquad F_B = F_A = 27.29 \text{ lb}
\end{aligned}
$$

$$
\begin{aligned}
\frac{F_A}{N_A} &= \frac{27.29}{148.73} = 0.1835 < 0.4 \qquad \text{Ladder will not slide at } A \\
\frac{F_B}{N_B} &= \frac{27.29}{53.27} = 0.152 > 0.4 \qquad \text{Ladder will slide at } B \blacktriangleleft
\end{aligned}
$$

**7.16**

**Assume equilibrium.**

$\Sigma F_y = 0$:

$\nwarrow +$ $N - W\cos 20° = 0 \quad \therefore N = 0.9397W$

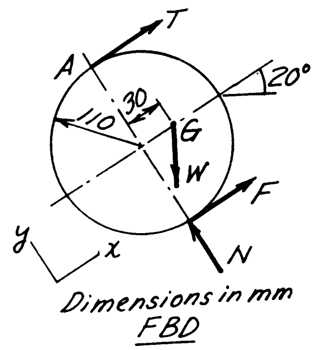
$\Sigma M_A = 0$:

$\circlearrowleft +$ $F(220) - (W\sin 20°)(110) - (W\cos 20°)(30) = 0$

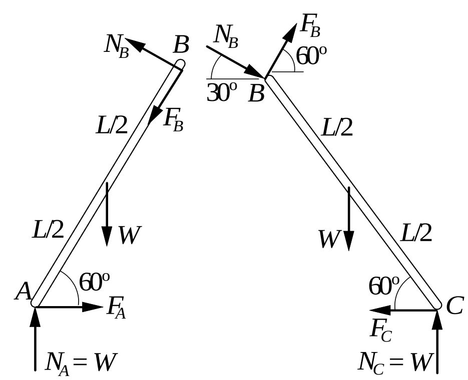
$\therefore F = 0.2992W$

$$\frac{F}{N} = \frac{0.2992W}{0.9397W} = 0.318 > \mu_s$$

$\therefore$ Cylinder cannot be at rest ◆




**7.17**




Due to symmetry of the assembly $N_A = N_C = W$.

FBD of sheet $AB$:

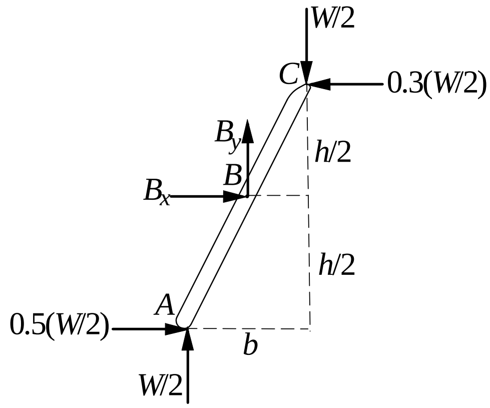
$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad N_B(L) - W\frac{L}{2}\cos 60° = 0 \qquad N_B = 0.25W$$

$$\Sigma F_y = 0 \qquad +\downarrow \quad F_B \sin 60° - N_B \cos 60° - N_A + W = 0$$

$$F_B \sin 60° - (0.25W)\cos 60° - W + W = 0 \qquad F_B = 0.14434W$$

$$\frac{F_B}{N_B} = \frac{0.14434W}{0.25W} = 0.5774 > 0.5 \qquad \therefore \text{Equilibrium is impossible}$$

**7.18**

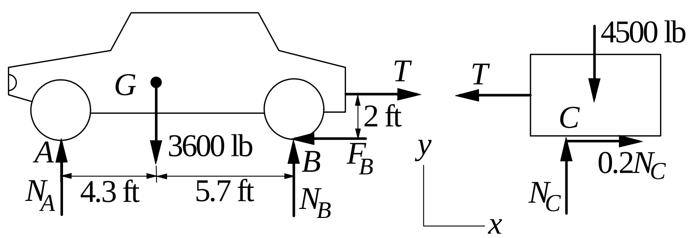



Equilibrium is lost when sliding impends at $A$ and $C$.

$$\Sigma M_B = 0 \qquad \left(0.5\frac{W}{2}\right)\frac{h}{2} + \left(0.3\frac{W}{2}\right)\frac{h}{2} - \frac{W}{2}b = 0$$

$$0.2h - 0.5b = 0 \qquad \frac{b}{h} = 0.4 \blacktriangleleft$$

**7.19**




Assume impending sliding of the crate.

FBD of the crate:

$$\Sigma F_y = 0 \qquad N_C - 4500 = 0 \qquad N_C = 4500 \text{ lb}$$
$$\Sigma F_x = 0 \qquad 0.2N_C - T = 0 \qquad T = 0.2N_C = 0.2(4500) = 900 \text{ lb}$$

FBD of car:

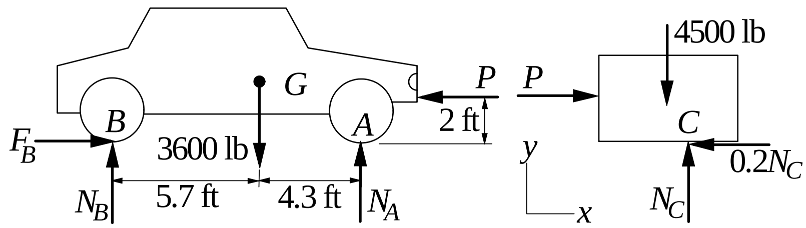
$$\Sigma M_A = 0 \qquad 10N_B - 3600(4.3) - 2T = 0$$
$$10N_B - 3600(4.3) - 2(900) = 0 \qquad N_B = 1728.0 \text{ lb}$$
$$\Sigma F_x = 0 \qquad T - F_B = 0 \qquad F_B = T = 900 \text{ lb}$$

Check for slipping at $B$:

$$\frac{F_B}{N_B} = \frac{900}{1728.0} = 0.521 < 0.6 \text{ Tires will not slip.}$$

Assumption was O.K. Crate will slide. ◀

## 7.20




Assume impending sliding of the crate.
FBD of the crate:

$$\Sigma F_y = 0 \qquad N_C - 4500 = 0 \qquad N_C = 4500 \text{ lb}$$
$$\Sigma F_x = 0 \qquad P - 0.2N_C = 0 \qquad P = 0.2N_C = 0.2(4500) = 900 \text{ lb}$$

FBD of car:

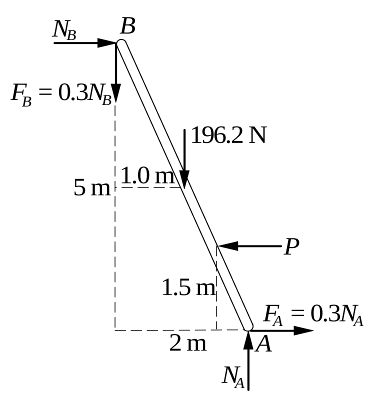
$$\Sigma M_A = 0 \qquad 3600(4.3) + 2P - 10N_B = 0$$
$$3600(4.3) - 2(900) - 10N_B = 0 \qquad N_B = 1368.0 \text{ lb}$$
$$\Sigma F_x = 0 \qquad F_B - P = 0 \qquad F_B = P = 900 \text{ lb}$$

Check for slipping at $B$:

$$\frac{F_B}{N_B} = \frac{900}{1368.0} = 0.658 > 0.6 \qquad \text{Tires will slip.}$$

Assumption was incorrect. Crate will not slide. ◀

## 7.21



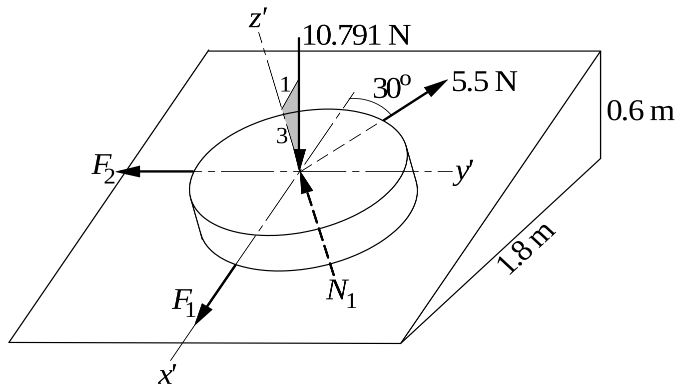
Consider impending slipping at $A$ and $B$.

$$\begin{aligned}
\Sigma M_A &= 0 \qquad +\circlearrowright \quad N_B(5) - 0.3N_B(2) - 196.2(1.0) - P(1.5) = 0 \\
& \qquad 4.4N_B - 1.5P - 196.2 = 0 \\
\Sigma F_x &= 0 \qquad +\rightarrow \quad 0.3N_A + N_B - P = 0 \\
\Sigma F_y &= 0 \qquad +\uparrow \quad N_A - 0.3N_B - 196.2 = 0
\end{aligned}$$

Solution is

$$N_A = 227 \text{ N} \qquad N_B = 102.9 \text{ N} \qquad P = 171.0 \text{ N} \blacktriangleleft$$

## 7.22




Let the $x'$ and $y'$ axes lie on the inclined plane, and let $z'$ be perpendicular to the plane.

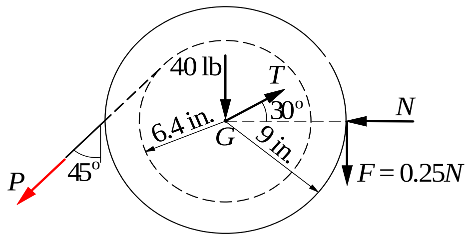
$$W = 1.1(9.81) = 10.791 \text{ N}$$

Assume equilibrium.

$$\begin{aligned}
\Sigma F_{x'} &= 0 \qquad F_1 + \frac{1}{\sqrt{10}}(10.791) - 5.5\cos 30^\circ = 0 \qquad F_1 = 1.3507 \text{ N} \\
\Sigma F_{y'} &= 0 \qquad -F_2 + 5.5\sin 30^\circ = 0 \qquad F_2 = 2.75 \text{ N} \\
\Sigma F_{z'} &= 0 \qquad N_1 - \frac{3}{\sqrt{10}}(10.791) = 0 \qquad N_1 = 10.237 \text{ N}
\end{aligned}$$

$$\frac{F}{N_1} = \frac{\sqrt{1.3507^2 + 2.75^2}}{10.237} = 0.299 < 0.35 \qquad \text{Disk is in equilibrium} \blacktriangleleft$$

## 7.23




The spool will slip against the wall. $\therefore F = \mu_s N = 0.25N$.

$$\begin{aligned}
\Sigma M_G &= 0 \quad +\circlearrowright \quad 0.25N(9) - P(6.4) = 0 \\
\Sigma F_x &= 0 \quad +\rightarrow \quad T\cos 30^\circ - N - P\sin 45^\circ = 0 \\
\Sigma F_y &= 0 \quad +\uparrow \quad T\sin 30^\circ - 0.25N - P\cos 45^\circ - 40 = 0
\end{aligned}$$

Solution is

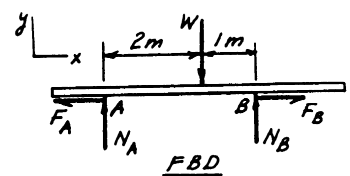
$$N = 180.0 \text{ lb} \qquad T = 259 \text{ lb} \qquad P = 63.3 \text{ lb} \blacktriangleleft$$

## 7.24

$\Sigma M_A = 0$: (+) $3N_B - 2W = 0$ . . . . . (a)

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_B - F_A = 0$ . . . . . . .(b)

$\Sigma F_y = 0$: $+\uparrow$ $N_A + N_B - W = 0$ . . (c)




Solution of Eqs. (a)-(c) is

$$N_A = \frac{W}{3} \quad N_B = \frac{2W}{3} \quad F_A = F_B$$

**(a)** If the plank is to remain stationary, it must resist the maximum <u>static</u> friction force applied to it by the drum at B (this force occurs just when the drum starts rotating).

The maximum friction force at B is $F_B = (\mu_s)_B N_B = 0.18N_B = 0.18(2W/3) = 0.12W$

$\therefore F_A = 0.12W \quad \therefore \dfrac{F_A}{N_A} = \dfrac{0.12W}{W/3} = 0.36 > (\mu_s)_A$ $\quad \therefore$ Plank will slide at A ♦

**(b)** If the plank remains stationary, it must slip on the drum at B.

$\therefore F_B = (\mu_k)_B N_B = 0.15(2W/3) = 0.10W \quad \therefore F_A = 0.10W$

$\therefore \dfrac{F_A}{N_A} = \dfrac{0.10W}{W/3} = 0.30 < (\mu_s)_A$ $\quad \therefore$ The plank will not move ♦

## 7.25

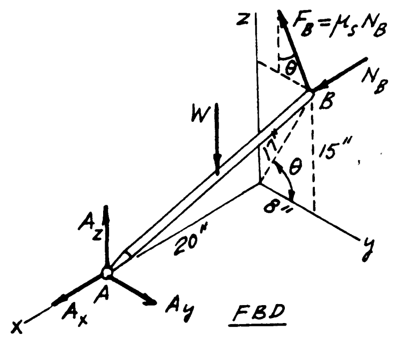
Note that $F_B$ on the FBD is drawn opposite to the direction of impending sliding.

$\Sigma(M_A)_z = 0$:

$20(\mu_s N_B \sin\theta) - 8N_B = 0$

$\therefore 20\left(\mu_s N_B \frac{15}{17}\right) - 8N_B = 0$

$\therefore \mu_s = 0.453$ ♦




## 7.26

Assume that the plank comes to rest in the position shown below.

During the period of sliding $F_A = (\mu_k)_A N_A = 0.28N_A$ and $F_B = (\mu_s)_B N_B = 0.36N_B$

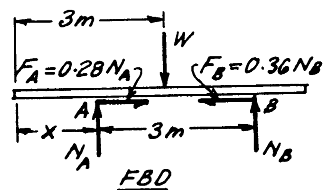
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $0.28N_A - 0.36N_B = 0$

$\Sigma F_y = 0$: $+\uparrow$ $N_A + N_B - W = 0$

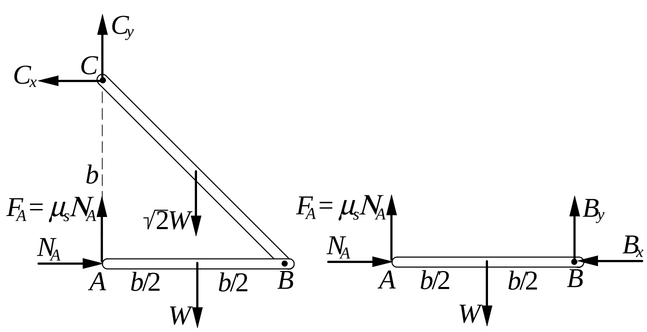
$\Sigma M_A = 0$: (+) $3N_B - (3 - x)W = 0$

Solution of the equations is: $N_A = 0.5625W$

$N_B = 0.4375W$ $x = 1.6875$ m ♦




## 7.27




Consider impending slipping at $A$.

FBD of entire assembly:

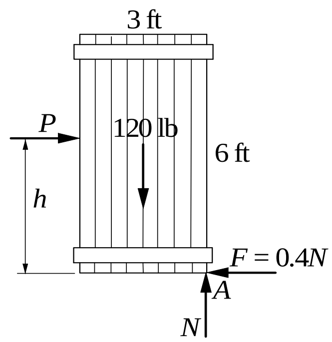
$$\Sigma M_C = 0 \qquad + \circlearrowleft \quad N_A b - (\sqrt{2}+1)W\left(\frac{b}{2}\right) = 0 \qquad N_A = 1.2071W$$

FBD of bar $AB$:

$$\Sigma M_B = 0 \qquad +\circlearrowright \quad \mu_s N_A b - W\frac{b}{2} = 0 \qquad 1.2071\mu_s - \frac{1}{2} = 0$$

$$\mu_s = 0.414 \blacktriangleleft$$

**7.28**



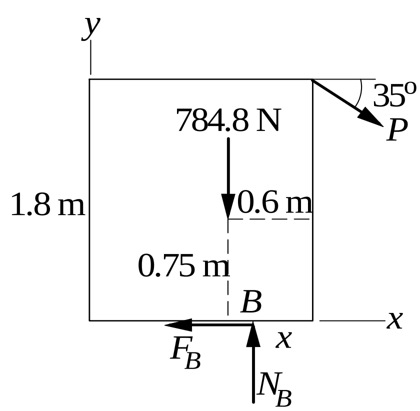

Consider simultaneous impending sliding and tipping about $A$.

$$\Sigma F_y = 0 \qquad +\uparrow \; N - 120 = 0 \qquad N = 120 \text{ lb}$$

$$\Sigma F_y = 0 \qquad +\rightarrow \quad P - 0.4N = 0 \qquad P = 0.4(120) = 48 \text{ lb}$$

$$\Sigma M_A = 0 \qquad +\circlearrowright \quad Ph - 120(1.5) = 0 \qquad h = \frac{120(1.5)}{48} = 3.75 \text{ ft} \blacktriangleleft$$

**7.29**




$$W = 80(0.81) = 784.8 \text{ N}$$

Assume impending sliding ($F_B = 0.3N_B$):

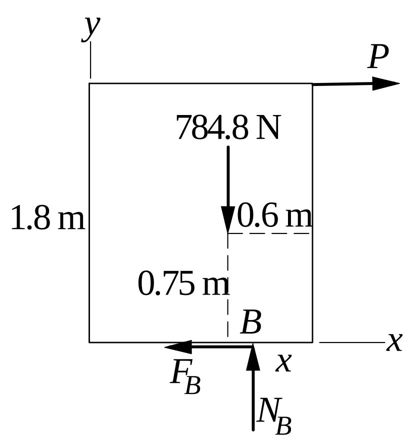
$$\Sigma F_x = 0 \qquad P\cos 35^\circ - 0.3N_B = 0$$
$$\Sigma F_y = 0 \qquad -P\sin 35^\circ + N_B - 784.8 = 0$$
Solution is $N_B = 993.5$ N $\qquad P = 363.8$ N

Assume impending tipping ($x = 0$):

$$\Sigma M_B = 0 \qquad (P\cos 35^\circ)(1.8) - 784.8(0.6) = 0$$
$$P = 319 \text{ N}$$

$P = 319$ N determined by tipping ◄

**7.30**




Assume impending sliding ($F_B = 0.3N_B$):

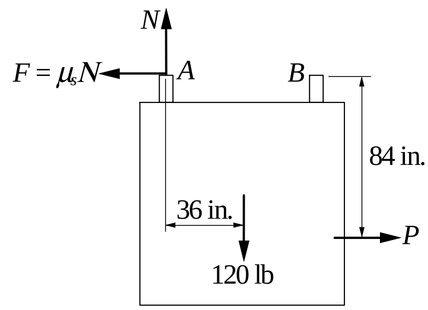
$$\Sigma F_y = 0 \qquad N_B - 784.8 = 0 \qquad N_B = 784.8 \text{ N}$$
$$\Sigma F_x = 0 \qquad P - 0.3N_B = 0 \qquad P - 0.3(784.8) = 0 \qquad P = 235 \text{ N}$$

Assume impending tipping ($x = 0$):

$$\Sigma M_B = 0 \qquad 1.8P - 784.8(0.6) = 0 \qquad P = 262 \text{ N}$$

$P = 235$ N determined by sliding ◄

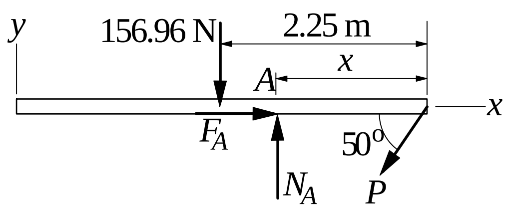
**7.31**




Consider simultaneous impending sliding and tipping (door lifting off the track at $B$).

$$\begin{aligned}
\Sigma F_y &= 0 \quad +\uparrow \quad N - 120 = 0 \qquad N = 120 \text{ lb} \\
\Sigma M_A &= 0 \quad +\circlearrowleft \quad P(84) - 120(36) = 0 \qquad P = 51.43 \text{ lb} \blacktriangleleft \\
\Sigma F_x &= 0 \quad +\rightarrow \quad P - \mu_s N = 0 \qquad 51.43 - \mu_s(120) = 0 \\
& \mu_s = 0.429 \blacktriangleleft
\end{aligned}$$

**7.32**




$$W = 16(9.81) = 156.96 \text{ N}$$

Assume impending sliding ($F_A = 0.3N_A$):

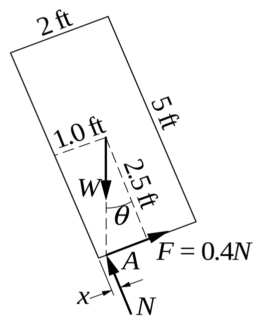
$$\begin{aligned}
\Sigma F_x &= 0 \quad 0.3N_A - P\cos 50^\circ = 0 \\
\Sigma F_y &= 0 \quad N_A - P\sin 50^\circ - 156.96 = 0 \\
& \text{Solution is} \quad N_A = 244.3 \text{ N} \qquad P = 114.02 \text{ N}
\end{aligned}$$

Assume impending tipping about corner $A$ ($x = 1.5$ m):

$$\begin{aligned}
\Sigma M_A &= 0 \quad 156.96(2.25 - 1.5) - (P\sin 50^\circ)(1.5) = 0 \\
& P = 102.4 \text{ N}
\end{aligned}$$

$$P = 102.4 \text{ N determined by tipping} \blacktriangleleft$$

## 7.33




Assume impending sliding.

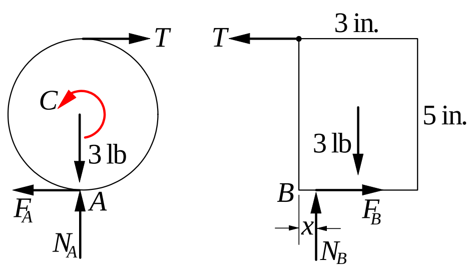
$$\Sigma F_x = 0 \qquad + \rightarrow \qquad 0.4N\cos\theta - N\sin\theta = 0 \qquad \tan\theta = 0.4$$
$$\theta = 21.80^\circ \quad \blacktriangleleft$$

Check for tipping:

$$x = 1.0 - 2.5\tan\theta = 1.0 - 2.5\tan 21.80^\circ = 7.134 \times 10^{-5} \text{ ft}$$

Since $x \approx 0$, tipping and sliding impend simultaneously.

## 7.34




Assume impending tipping of the block. From FBD of block using $x = 0$:

$$\Sigma F_y = 0 \qquad + \uparrow \qquad N_B - 3 = 0 \qquad N_B = 3 \text{ lb}$$
$$\Sigma M_A = 0 \qquad + \circlearrowleft \qquad T(5) - 3(1.5) = 0 \qquad T = 0.9 \text{ lb}$$
$$\Sigma F_x = 0 \qquad + \rightarrow \qquad F_B - T = 0 \qquad F_B = T = 0.9 \text{ lb}$$

Assume impending sliding of the block. From FBD of block using $F_B = 0.35N_B$:

$$\Sigma F_y = 0 \qquad + \uparrow \qquad N_B - 3 = 0 \qquad N_B = 3 \text{ lb}$$
$$\Sigma F_x = 0 \qquad + \leftarrow \qquad T - 0.35N_B = 0 \qquad T = 1.05 \text{ lb}$$

Assume impending sliding of the cylinder. From FBD of cylinder with $F_A = 0.35N_A$:

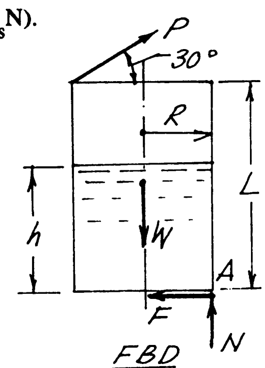
$$\Sigma F_y = 0 \quad +\uparrow \quad N_A - 3 = 0 \quad N_A = 3 \text{ lb}$$
$$\Sigma F_x = 0 \quad +\rightarrow \quad T - 0.35N_A = 0 \quad T = 1.05 \text{ lb}$$

Equilibrium is lost when $T = 0.9$ lb due to tipping of the block. From FBD of cylinder:

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad C - 5T = 0 \quad C = 5T = 5(0.9) = 4.5 \text{ lb} \cdot \text{in} \blacktriangleleft$$

## 7.35

**(a) Assume simultaneous impending tipping and sliding ($F = \mu_s N$).**




Full tank: $W = \gamma(\pi R^2 L) = 62.4\pi(1.4)^2(4) = 1536.9$ lb

$\Sigma M_A = 0$:

$(+)\ WR - (P \cos 30°)L - (P \sin 30°)(2R) = 0$

$$\therefore P = \frac{WR}{L \cos 30° + 2R \sin 30°} = \frac{(1536.9)(1.4)}{4 \cos 30° + 2.8 \sin 30°}$$
$= 442.4$ lb ♦

$\Sigma F_y = 0$: $+\uparrow$ $P \sin 30° + N - W = 0$

$\therefore N = W - P \sin 30° = 1536.9 - 442.4 \sin 30°$
$= 1315.7$ lb

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P \cos 30° - F = 0$ $\quad \therefore F = P \cos 30° = 442.4 \cos 30° = 383.1$ lb

$$\therefore \mu_s = \frac{F}{N} = \frac{383.1}{1315.7} = 0.291 \ ♦$$

**(b) Assume impending tipping.**

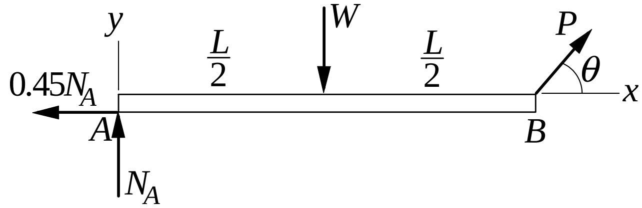
Partially filled tank: $W = \gamma(\pi R^2 h) = 62.4\pi(1.4)^2 h = 384.2h$

$\Sigma M_A = 0$: $(+)$ $WR - (P \cos 30°)L - (P \sin 30°)(2R) = 0$

$\therefore (384.2h)(1.4) - 200 \cos 30° (4) - 200 \sin 30° (2.8) = 0 \quad \therefore h = 1.809$ ft ♦

## 7.36

Assume simultaneous impending tipping and sliding.

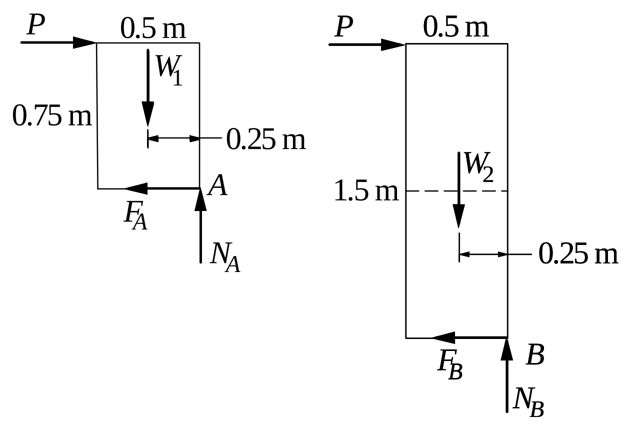


$$\Sigma M_B = 0 \qquad W\frac{L}{2} - N_A L = 0 \qquad N_A = \frac{W}{2}$$
$$\Sigma F_x = 0 \qquad P\cos\theta - 0.45N_A = 0 \qquad P\cos\theta = 0.45\frac{W}{2}$$
$$\Sigma M_A = 0 \qquad (P\sin\theta)\,L - W\frac{L}{2} = 0 \qquad P\sin\theta = \frac{W}{2}$$

$$\tan\theta = \frac{1}{0.45} = 2.222 \qquad \theta = \tan^{-1}(2.222) = 65.8^\circ \blacktriangleleft$$
$$P = \frac{W}{2\sin\theta} = \frac{W}{2\sin 65.8^\circ} = 0.548W \blacktriangleleft$$

**7.37**

Assume tipping




$$W_1 = 20(9.81) = 196.2 \text{ N} \qquad W_2 = (20+45)(9.81) = 637.7 \text{ N}$$

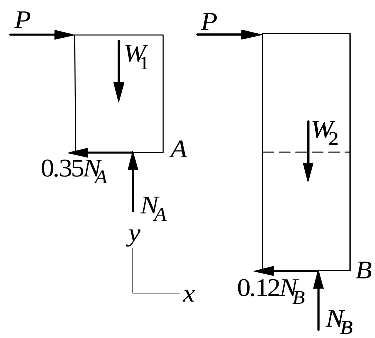
Top box:

$$\Sigma M_A = 0 \qquad 0.25W_1 - 0.75P = 0 \qquad 0.25(196.2) - 0.75P = 0$$
$$P = 65.4 \text{ N}$$

Both boxes together:

$$\Sigma M_B = 0 \qquad 0.25W_2 - 1.5P = 0 \qquad 0.25(637.7) - 1.5P = 0$$
$$P = 106.3 \text{ N}$$

Assume slipping




Top box:

$$\begin{aligned}\Sigma F_y &= 0 \qquad N_A - W_1 = 0 \qquad N_A = W_1 = 196.2 \text{ N}\\ \Sigma F_x &= 0 \qquad P - 0.35N_A = 0 \qquad P = 0.35N_A = 0.35(196.2) = 68.7 \text{ N}\end{aligned}$$

Both boxes:

$$\begin{aligned}\Sigma F_y &= 0 \qquad N_B - W_2 = 0 \qquad N_B = W_2 = 637.7 \text{ N}\\ \Sigma F_x &= 0 \qquad P - 0.12N_B = 0 \qquad P = 0.12N_B = 0.12(637.7) = 76.5 \text{ N}\end{aligned}$$

Smallest $P$ that results in impending motion (tipping of top box) is

$$P = 65.4 \text{ N} \quad ◄$$

## 7.38

**Assume equilibrium.**

**From FBD of upper block**

$R_1 = \frac{1}{2}(16)(18) = 144$ lb

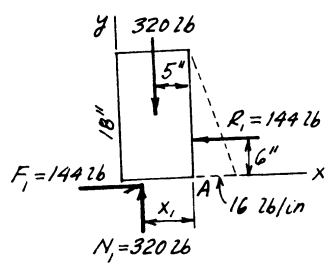
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_1 - 144 = 0$ $\therefore F_1 = 144$ lb

$\Sigma F_y = 0$: $+\uparrow$ $N_1 - 320 = 0$ $\therefore N_1 = 320$ lb

$\frac{F_1}{N_1} = \frac{144}{320} = 0.450 < 0.5$ $\therefore$ **Block does not slide**

$\Sigma M_A = 0$: $(\circlearrowleft +)$ $(320)(5) + (144)(6) - (320)x_1 = 0$

$\therefore x_1 = 7.70$ in $< 10$ in $\therefore$ **Block does not tip**



From FBD of both blocks

$R_2 = \frac{1}{2}(32)(36) = 576 \text{ lb}$

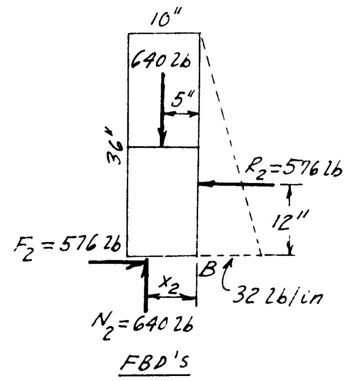
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $F_2 - 576 = 0$ $\quad \therefore F_2 = 576 \text{ lb}$

$\Sigma F_y = 0$: $+\uparrow$ $N_2 - 640 = 0$ $\quad \therefore N_2 = 640 \text{ lb}$

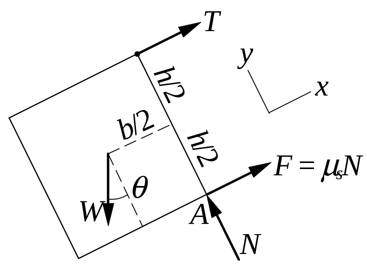
$\frac{F_2}{N_2} = \frac{576}{640} = 0.900 < 1.0$ $\quad \therefore$ Blocks do not slide

$\Sigma M_B = 0$: (+) $(640)(5) + (576)(12) - (640)x_2 = 0$

$\therefore x_2 = 15.8 \text{ in} > 10 \text{ in}$ $\quad \therefore$ Blocks tip about corner B ♦




## 7.39




Consider simultaneous impending sliding and impending tipping about $A$.

$$\Sigma F_y = 0 \quad N - W\cos\theta = 0 \qquad N = W\cos\theta$$

$$\Sigma F_x = 0 \quad \mu_s N + T - W\sin\theta = 0 \qquad T = W\sin\theta - \mu_s W\cos\theta \qquad \text{(a)}$$

$$\Sigma M_A = 0 \quad (W\sin\theta)\frac{h}{2} + (W\cos\theta)\frac{b}{2} - Th = 0$$

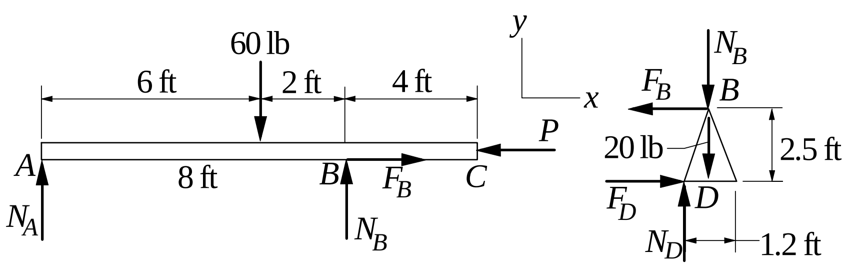
$$T = \frac{1}{2}W\sin\theta + \frac{b}{2h}W\cos\theta \qquad \text{(b)}$$

Equating the right-hand sides of Eqs. (a) and (b):

$$W\sin\theta - \mu_s W\cos\theta = \frac{1}{2}W\sin\theta + \frac{b}{2h}W\cos\theta$$

$$\frac{1}{2}\sin\theta = \left(\mu_s + \frac{b}{2h}\right)\cos\theta \qquad \theta = \tan^{-1}\left(2\mu_s + \frac{b}{h}\right) \quad \blacktriangleleft$$

**7.40**




FBD of plank $ABC$:

$$\begin{aligned} \Sigma M_A &= 0 \qquad 8N_B - 60(6) = 0 \qquad N_B = 45 \text{ lb} \\ \Sigma F_x &= 0 \qquad F_B - P = 0 \qquad F_B = P \end{aligned}$$

FBD of support $BD$:

$$\begin{aligned} \Sigma F_x &= 0 \qquad F_D - F_B = 0 \qquad F_D = F_B = P \\ \Sigma F_y &= 0 \qquad N_D - N_B - 20 = 0 \qquad N_D = N_B + 20 = 65 \text{ lb} \end{aligned}$$

Assume impending tipping of support $BD$:

$$\begin{aligned} \Sigma M_D &= 0 \qquad 2.5F_B - (N_B + 20)(0.6) = 0 \\ & \qquad 2.5P - (45 + 20)(0.6) = 0 \qquad P = 15.60 \text{ lb} \end{aligned}$$

Assume impending sliding at $B$:

$$P = \mu_B N = 0.4(45) = 18.0 \text{ lb}$$

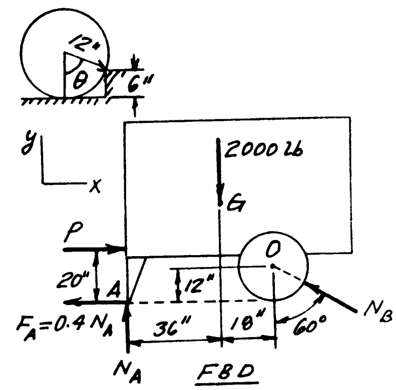
Assume impending sliding at $D$ :

$$P = F_D = \mu_D N_D = 0.3(65) = 19.5 \text{ lb}$$

The largest $P$ that can be applied without causing impending motion (tipping of the support) is

$$P = 15.60 \text{ lb} \quad ◄$$

## 7.41




**Geometry:** $12(1 - \cos\theta) = 6 \qquad \therefore \theta = 60°$

**Assume impending sliding at A.**

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $P - 0.4N_A - N_B \sin 60° = 0$

$\Sigma F_y = 0$: $+\uparrow$ $N_A + N_B \cos 60° - 2000 = 0$

$\Sigma M_O = 0$: $\circlearrowright +$ $54N_A + 12(0.4N_A) + 8P - (2000)(18) = 0$

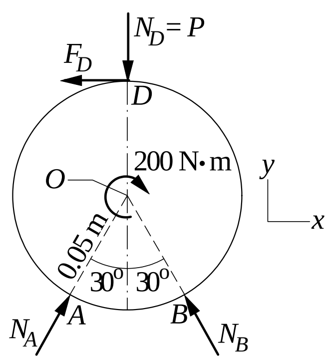
**The solution is:**

$N_A = 172$ lb $\quad N_B = 3656$ lb $\quad P = 3235$ lb

**Since $N_A > 0$, the trailer does not tip.**

**$\therefore$ Trailer can be pushed over the curb with P = 3235 lb ♦**

## 7.42




$$\Sigma F_x = 0 \qquad (N_A - N_B)\sin 30° - F_D = 0 \qquad \text{(a)}$$
$$\Sigma F_y = 0 \qquad (N_A + N_B)\cos 30° - P = 0 \qquad \text{(b)}$$
$$\Sigma M_O = 0 \qquad 0.05F_D - 200 = 0 \qquad F_D = 4000 \text{ N} \qquad \text{(c)}$$

Assume impending slipping at $D$ ($F_D = 1.5P$):

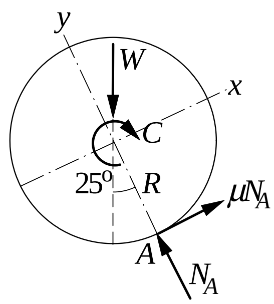
$$\text{From Eq. (c):} \quad 1.5P = 4000 \qquad P = 2667 \text{ N}$$

Assume impending rolling about $D$ ($N_B = 0$):

$$\text{From Eq. (a):} \qquad N_A \sin 30° - 4000 = 0 \qquad N_A = 8000 \text{ N}$$
$$\text{From Eq. (b):} \qquad 8000\cos 30° - P = 0 \qquad P = 6930 \text{ N}$$

Smallest force that prevents motion is $P = 6930$ N ◄

**7.43**



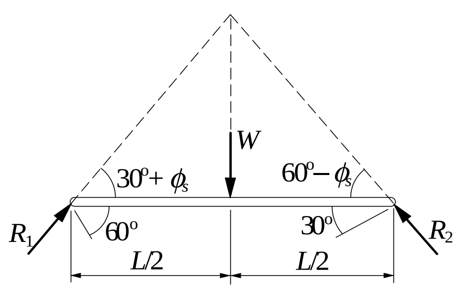

Assume simultaneous slipping and tipping about point $A$.

$$\Sigma F_y = 0 \qquad N_A - W\cos 25^\circ = 0$$
$$\Sigma F_x = 0 \qquad \mu N_A - W\sin 25^\circ = 0$$

$$\mu = \tan 25^\circ = 0.466 \quad \blacktriangleleft$$

$$\Sigma M_A = 0 \qquad WR\sin 25^\circ - C = 0$$
$$C = WR\sin 25^\circ = 0.423WR \quad \blacktriangleleft$$

**7.44**



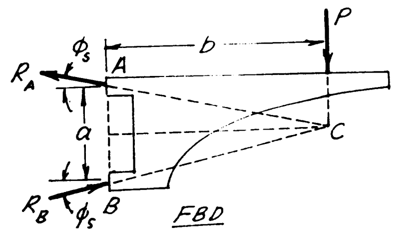

Assume impeding slipping (left end down, right end up).

Because the bar is a 3-force body, the forces intersect at a common point.

$$30^\circ + \phi_s = 60^\circ - \phi_s \qquad \phi_s = 15^\circ \qquad \mu_s = \tan 15^\circ = 0.268 \quad \blacktriangleleft$$

## 7.45

Assume impending slipping. Note that the bracket is a 3-force body with the forces intersecting at C.




From geometry of triangle BAC:

$$\frac{a}{2} = b \tan\phi_s$$

$$\therefore \frac{b}{a} = \frac{1}{2\tan\phi_s} = \frac{1}{2(0.2)} = 2.5 \ \blacklozenge$$

## 7.46

Assume impending slipping. Note that the plank is a 3-force body with the forces intersecting at C.

$(\phi_s)_A = \tan^{-1} 0.3 = 16.70°$

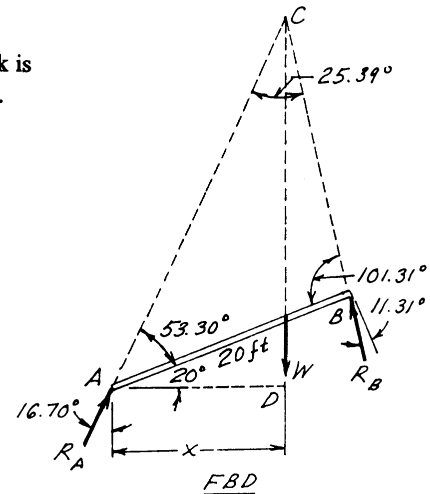
$(\phi_s)_B = \tan^{-1} 0.2 = 11.31°$

From triangle ACB: $\dfrac{20}{\sin 25.39°} = \dfrac{\overline{AC}}{\sin 101.31°}$

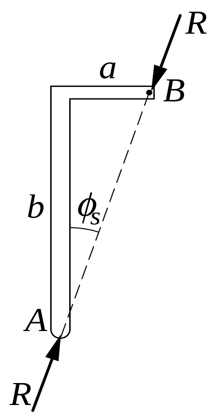
$\therefore\ \overline{AC} = 45.74$ ft

From triangle ACD: $x = \overline{AC} \cos 73.30°$

$\therefore\ x = 45.74 \cos 73.30° = 13.14$ ft ◆



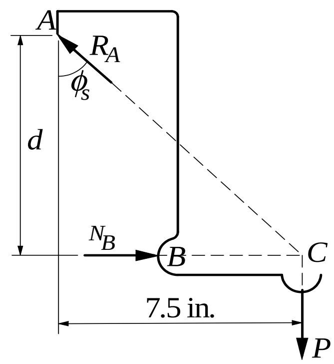

## 7.47




Assume impending sliding at $A$ (note that $AB$ is a two-force body).

$$\mu_s = \tan \phi_s = \frac{a}{b} \quad ◀$$

## 7.48




The hook is a three-force body with the forces intersecting at $C$.
Assuming impending sliding at $A$:

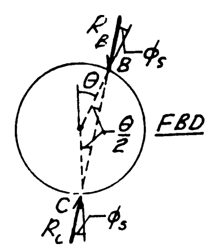
$$\tan\phi_s = \frac{7.5}{d} \qquad d = \frac{7.5}{\tan\phi_s} = \frac{7.5}{\mu_s} = \frac{7.5}{0.5} = 15 \text{ in.} \blacktriangleleft$$

## 7.49

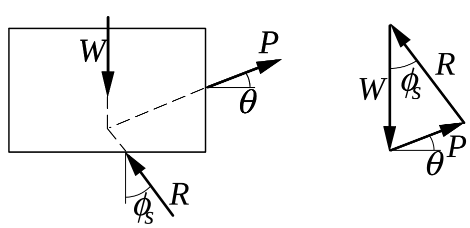
**The roller is a two-force body.**
**Assume impending sliding at B and C.**
**From geometry of the FBD:**

$$\phi_s = \frac{\theta}{2}$$

$$\therefore \theta = 2\tan^{-1}\mu_s = 2\tan^{-1} 0.24 = 27.0° \blacklozenge$$



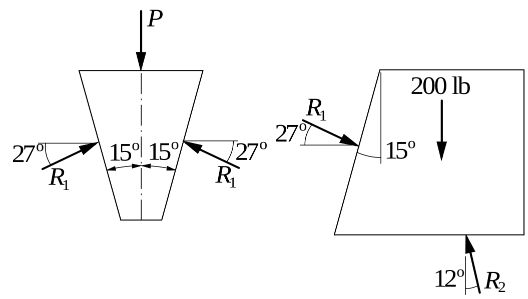

## 7.50




The block is a three-force body (the forces must be concurrent). From the force triangle we see that $P$ is minimized when it is perpendicular to $R$. Hence

$$\theta = \phi_s \blacktriangleleft$$
$$P = W\sin\phi_s \blacktriangleleft$$

## 7.51




Consider impending slipping.
FBD of wedge:

$$\Sigma F_y = 0 \qquad +\uparrow \quad 2R_1 \sin 27^\circ - P = 0$$

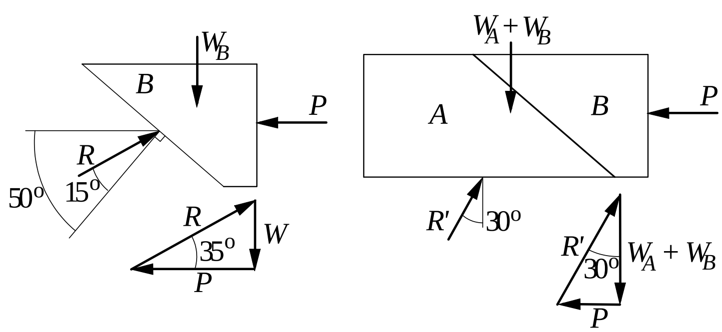
FBD of block:

$$\begin{aligned} \Sigma F_y &= 0 \qquad +\uparrow \quad R_2 \cos 12^\circ - R_1 \sin 27^\circ - 200 = 0 \\ \Sigma F_x &= 0 \qquad +\leftarrow \quad R_2 \sin 12^\circ - R_1 \cos 27^\circ = 0 \end{aligned}$$

Solution is

$$R_1 = 53.5 \text{ lb} \qquad R_2 = 229 \text{ lb} \qquad P = 48.6 \text{ lb} \blacktriangleleft$$

## 7.52




Assume impending sliding of block $B$ on block $A$. From the force triangle

$$P = \frac{W}{\tan 35^\circ} = \frac{300(9.81)}{\tan 35^\circ} = 4203 \text{ N}$$

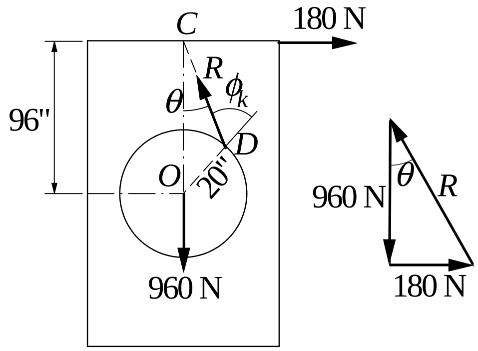
Assume impeding sliding of both blocks on ground. From the force triangle

$$P = (W_A + W_B)\tan 30^\circ = 800(9.81)\tan 30^\circ = 4531 \text{ N}$$

Smallest $P$ causing impending motion is

$$P = 4200 \text{ N} \blacktriangleleft$$

## 7.53




The collar is a three-force body with the force intersecting at $C$.

From the force triangle:

$$\theta = \tan^{-1}\frac{180}{960} = 10.620^\circ$$

From triangle $ODC$ in the FBD:

$$\frac{\sin(\pi-\phi_k)}{96} = \frac{\sin\theta}{20} \qquad \frac{\sin\phi_k}{96} = \frac{\sin 10.620^\circ}{20} \qquad \phi_k = 62.20^\circ$$

$$\mu_k = \tan\phi_k = \tan 62.20^\circ = 1.897 \blacktriangleleft$$

## 7.54

$$p = 2\pi r\tan\theta \qquad \theta = \tan^{-1}\frac{p}{2\pi r} = \tan^{-1}\frac{0.5}{2\pi(1.75)} = 2.604^\circ$$

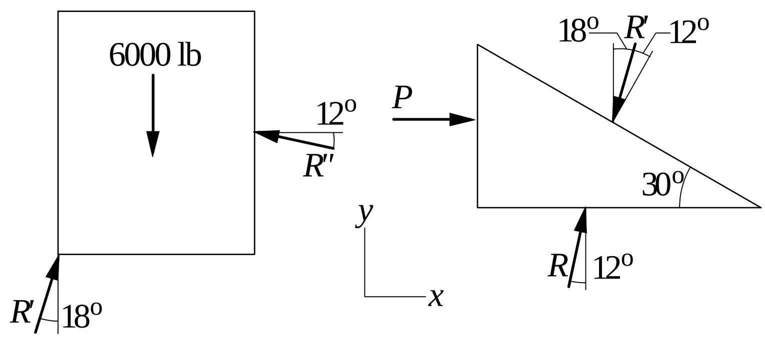
**(a)**

$$\begin{aligned} C_{\text{up}} &= Wr\tan(\phi_s+\theta) = 4000(1.75)\tan(8.5^\circ + 2.604^\circ) \\ &= 1373.9 \text{ lb}\cdot\text{in.} = 114.5 \text{ lb}\cdot\text{ft} \blacktriangleleft \end{aligned}$$

**(b)**

$$\begin{aligned} C_{\text{down}} &= Wr\tan(\phi_s-\theta) = 4000(1.75)\tan(8.5^\circ - 2.604^\circ) \\ &= 722.9 \text{ lb}\cdot\text{in.} = 60.2 \text{ lb}\cdot\text{ft} \blacktriangleleft \end{aligned}$$

## 7.55




Assume impending sliding (wedge slides to the left).
From FBD of the block:

$$\Sigma F_x = 0 \qquad R' \sin 18^\circ - R'' \cos 12^\circ = 0$$
$$\Sigma F_y = 0 \qquad R' \cos 18^\circ + R'' \sin 12^\circ - 6000 = 0$$
$$R' = 5901 \text{ lb} \qquad R'' = 1864 \text{ lb}$$

From FBD of wedge:

$$\Sigma F_y = 0 \qquad R \cos 12^\circ - R' \cos 18^\circ = 0$$
$$R \cos 12^\circ - 5901 \cos 18^\circ = 0 \qquad R = 5738 \text{ lb}$$
$$\Sigma F_x = 0 \qquad P - R' \sin 18^\circ + R \sin 12^\circ = 0$$
$$P - 5901 \sin 18^\circ + 5738 \sin 12^\circ = 0 \qquad P = 631 \text{ lb} \blacktriangleleft$$

## 7.56

Given: $r = 0.004$ m $\qquad p = 0.0016$ m $\qquad \mu_s = 0.2$

Lead angle: $$\theta = \tan^{-1} \frac{p}{2\pi r} = \tan^{-1} \frac{0.0016}{2\pi(0.004)} = 3.643^\circ$$
$$\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.2) = 11.310^\circ$$

**(a)** Clamping force from Eq. (7.7a):

$$F = \frac{C}{r \tan(\phi_s + \theta)} = \frac{1.50}{0.004 \tan(11.310^\circ + 3.643^\circ)} = 1404 \text{ N} \blacktriangleleft$$

**(b)** Unclamping torque from Eq. (7.7b):

$$C' = Fr \tan(\phi_s - \theta) = 1404(0.004) \tan(11.310^\circ - 3.643^\circ) = 0.756 \text{ N} \cdot \text{m} \blacktriangleleft$$

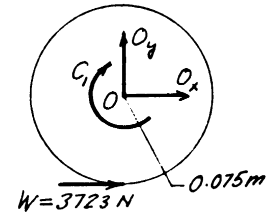
## 7.57

The lead angle is $\theta = \tan^{-1}\dfrac{p}{2\pi r} = \tan^{-1}\dfrac{10}{2\pi(18)} = 5.053°$

$\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.06 = 3.434°$

The axial thrust W on the screw is found from $C_0 = Wr\tan(\phi_k + \theta)$. Note that the coefficient of kinetic friction is used, because sliding takes place.

$$\therefore W = \frac{C_0}{r\tan(\phi_k + \theta)} = \frac{10}{0.018\tan(3.434° + 5.053°)} = 3723\text{ N}$$




From FBD of the gear

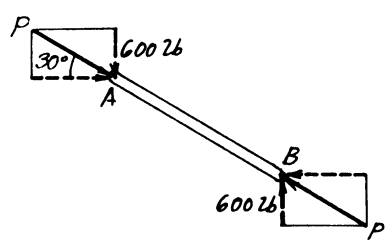
$\Sigma M_O = 0$: (+) $(3723)(0.075) - C_1 = 0$

$\therefore C_1 = 279\text{ N·m}$ ♦

## 7.58

The lead angle is $\theta = \tan^{-1}\dfrac{p}{2\pi r} = \tan^{-1}\dfrac{0.1}{2\pi(0.175)} = 5.197°$

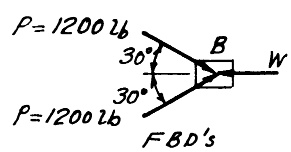
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.08 = 4.574°$




**(a)** From the FBD of the strut AB (note that the strut is a two-force body, and that it supports half of the 1200 lb vertical load)

$$P = \frac{600}{\sin 30°} = 1200\text{ lb}$$

Due to symmetry, each strut carries the same axial force.




From the FBD of the collar

$\Sigma F_x = 0$ $\overset{+}{\rightarrow}$ $2(1200\cos 30°) - W = 0$

$\therefore W = 2078.5\text{ lb}$

Note that there are two collars that must be moved by $C_0$, so that

$C_0 = 2Wr\tan(\phi_s + \theta) = 2(2078.5)(0.175)\tan(4.574° + 5.197°) = 125.3\text{ lb•in}$ ♦

**(b)** As $\theta > \phi_s$, the screw is not self-locking. Hence no couple is needed to lower the load ♦

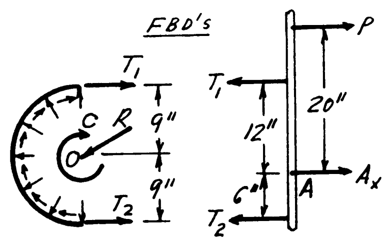
## 7.59

$$\frac{T_2}{T_1} = e^{\mu_s \theta} \qquad \theta = \frac{1}{\mu_s} \ln \frac{T_2}{T_1} = \frac{1}{0.25} \ln \frac{8000}{40} = 21.19 \text{ rad}$$

$$n = \frac{\theta}{2\pi} = \frac{21.19}{2\pi} = 3.37 \text{ turns}$$

4 turns are required to hold the ship ◄

## 7.60




Note that $T_2 > T_1$. $\therefore$ Use $T_2 = 3800$ lb

$T_1 = T_2 e^{-\mu_s \theta} = 3800\, e^{-0.2\pi} = 2027$ lb

From FBD of the belt (R and C represent the resultant force-couple at O due to the distributed normal and friction forces):

$\Sigma M_O = 0$: (+) $9(T_2 - T_1) - C = 0$

$\therefore 9(3800 - 2027) - C = 0 \qquad \therefore C = 15\,960$ lb·in ♦

From FBD of the handle

$\Sigma M_A = 0$: (+) $12T_1 - 6T_2 - 20P = 0 \quad 12(2027) - 6(3800) - 20P = 0 \quad \therefore P = 76.2$ lb ♦

## 7.61

Now $T_1 > T_2$. Therefore, use $T_1 = 3800$ lb.

Switching the roles of $T_1$ & $T_2$ results in $T_2 = T_1 e^{-\mu_s \theta} = 3800\, e^{-0.2\pi} = 2027$ lb

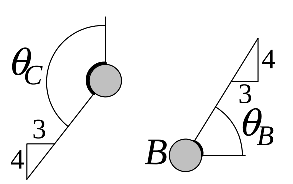
From FBD of the belt (see solution of Prob. 7.60):

$\Sigma M_O = 0$: (+) $9(T_2 - T_1) - C = 0 \quad \therefore 9(2027 - 3800) - C = 0 \quad \therefore C = -15\,960$ lb•in ♦

From FBD of the handle (see solution of Prob. 7.60):

$\Sigma M_A = 0$: (+) $12T_1 - 6T_2 - 20P = 0 \quad \therefore 12(3800) - 6(2027) - 20P = 0 \quad \therefore P = 1672$ lb ♦

## 7.62




The angles of contact are

$$\theta_B = \tan^1 \left(\frac{4}{3}\right) = 0.927 \text{ rad} \qquad \theta_C = \frac{\pi}{2} + \tan^{-1}\left(\frac{4}{3}\right) = 2.498 \text{ rad}$$

Total angle of contact is

$$\theta = \theta_B + \theta_C = 0.927 + 2.498 = 3.425 \text{ rad}$$

Assume impending motion with <u>$A$ moving up</u>:

$$P = 120(9.81)e^{0.25(3.425)} = 2770 \text{ N}$$

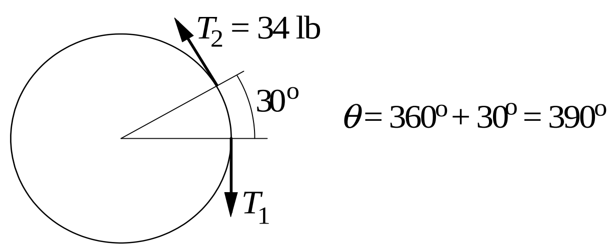
Assume impending motion with <u>$A$ moving down</u>:

$$120(9.81) = Pe^{0.25(3.425)} \qquad P = 500 \text{ N}$$

Equilibrium range is

$$500 \text{ N } \leq P \leq 2770 \text{ N} \blacktriangleleft$$

**7.63**




$$T_1 = T_2 e^{-\mu_s \theta} = 34e^{-0.6\frac{390\pi}{180}} = 0.5725 \text{ lb}$$

But $T_1$ is the weight of the free end of the rein:

$$T_1 = \frac{3.5}{16}L \qquad 0.5725 = \frac{3.5}{16}L \qquad L = 2.617 \text{ ft} = 31.4 \text{ in.} \blacktriangleleft$$

**7.64**

Assume impending sliding with the weight about to move up:

$$P = 30e^{0.3\pi} = 77.0 \text{ lb}$$

Assume impending sliding with the weight about to move down:

$$30 = Pe^{0.3\pi} \qquad P = 11.69 \text{ lb}$$

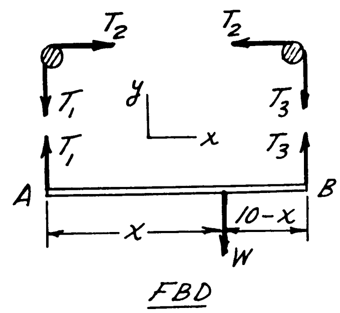
System is at rest if

$$11.69 \text{ lb} < P < 77.0 \text{ lb} \blacktriangleleft$$

## 7.65

$T_2 = T_1 e^{\mu_s \theta} = T_1 e^{0.5(\pi/2)} = 2.193 T_1$

$T_3 = T_2 e^{\mu_s \theta} = (2.193)^2 T_1 \quad \therefore T_1 = 0.2079 T_3$




From FBD of rail

$\Sigma M_A = 0$: (+) $10T_3 - Wx = 0$

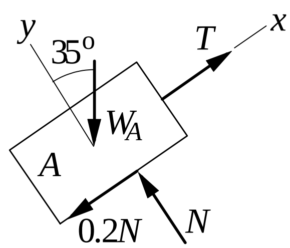
$\therefore T_3 = 0.1Wx \quad \therefore T_1 = 0.02079Wx$

$\Sigma F_y = 0$: $+\uparrow$ $T_1 + T_3 - W = 0$

$\therefore 0.02079Wx + 0.1Wx - W = 0$

$\therefore x = 8.28$ ft ◆

## 7.66




Assume impending sliding with block $A$ moving up.

$$\Sigma F_y = 0 \qquad N - W_A \cos 35^\circ = 0$$
$$\Sigma F_x = 0 \qquad T - 0.2N - W_A \sin 35^\circ = 0$$

$$\text{Solution is } W_A = 1.3561T \qquad N = 1.1109T$$

The contact angle between the rope and the peg is

$$\theta = 90^\circ + 35^\circ = 125^\circ = 2.182 \text{ rad}$$

$$W_B = Te^{\mu'\theta} = Te^{0.25(2.182)} = 1.7255T$$
$$\therefore \frac{W_B}{W_A} = \frac{1.7255}{1.3561} = 1.272$$

Assume impending sliding with block $A$ moving down. The friction force on the FBD must be reversed, yielding

$$\Sigma F_y = 0 \qquad N - W_A \cos 35^\circ = 0$$
$$\Sigma F_x = 0 \qquad T + 0.2N - W_A \sin 35^\circ = 0$$

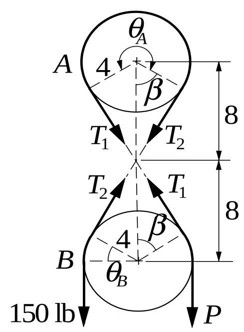
$$\text{Solution is } W_A = 2.441T \qquad N = 1.999T$$

$$\begin{aligned} T &= W_B e^{\mu'\theta} \qquad W_B = Te^{-\mu'\theta} = Te^{-0.25(2.182)} = 0.5796T \\ \therefore \quad & \frac{W_B}{W_A} = \frac{0.5796}{2.441} = 0.237 \end{aligned}$$

System is in equilibrium if

$$0.237 \le \frac{W_B}{W_A} \le 1.272 \quad \blacktriangleleft$$

**7.67**



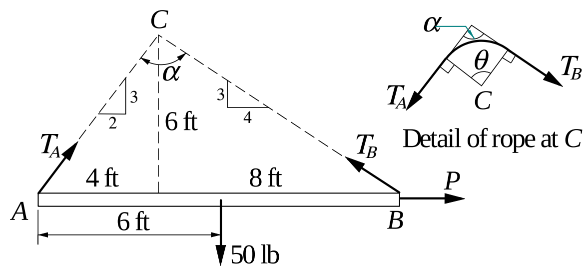

From geometry:

$$\begin{aligned} \beta &= \cos^{-1}\frac{4}{8} = \frac{\pi}{3} \text{ rad} \\ \theta_A &= 2\pi - 2\beta = 2\pi - 2\frac{\pi}{3} = \frac{4}{3}\pi \text{ rad} \\ \theta_B &= \frac{\pi}{2} - \beta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \text{ rad} \end{aligned}$$

Consider impending slipping:

$$\begin{aligned} T_2 &= 150e^{-\mu_s\theta_B} = 150e^{-0.3(\pi/6)} = 128.20 \text{ lb} \\ T_1 &= T_2 e^{-\mu_s\theta_A} = 128.20e^{-0.3(4\pi/3)} = 36.49 \text{ lb} \\ P &= T_1 e^{-\mu_s\theta_B} = 36.49e^{-0.3(\pi/6)} = 31.2 \text{ lb} \quad \blacktriangleleft \end{aligned}$$

## 7.68




From FBD of bar:

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad \left(\frac{3}{5}T_B\right)(12) - 50(6) = 0 \qquad T_B = 41.67 \text{ lb}$$

$$\Sigma M_B = 0 \quad +\circlearrowright \quad \left(\frac{3}{\sqrt{13}}T_A\right)(12) - 50(6) = 0 \qquad T_A = 30.05 \text{ lb}$$

Geometry:

$$\alpha = \tan^{-1}\frac{2}{3} + \tan^{-1}\frac{4}{3} = 1.5153 \text{ rad}$$
$$\theta = \pi - \alpha = \pi - 1.5153 = 1.6263 \text{ rad}$$

Condition for impending sliding of rope on peg:

$$T_B = T_A e^{\mu_s \theta} \qquad \mu_s = \frac{1}{\theta}\ln\frac{T_B}{T_A} = \frac{1}{1.6263}\ln\left(\frac{41.67}{30.05}\right) = 0.201 \blacktriangleleft$$

## 7.69

**For uniform pressure (new surfaces), we have**

$$C = \frac{2\mu_s P}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) = \frac{2(0.15)(400)}{3}\left(\frac{0.04^3 - 0.02^3}{0.04^2 - 0.02^2}\right) = 1.867 \text{ N·m} \blacklozenge$$

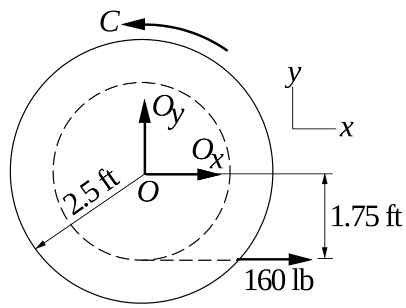
## 7.70

$$P = \int_{\mathcal{A}} p\, dA = \int_0^R p(2\pi r\, dr) = \int_0^R p_0\left(1 - \frac{r^2}{R^2}\right)(2\pi r)dr$$
$$= 2\pi p_0 \int_0^R \left(r - \frac{r^3}{R^2}\right)dr = 2\pi p_0\left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{2}\pi p_0 R^2$$
$$p_0 = \frac{2}{\pi}\frac{P}{R^2} \qquad p = \frac{2}{\pi}\frac{P}{R^2}\left(1 - \frac{r^2}{R^2}\right)$$

Consider impending slipping:

$$\begin{aligned} C &= \int_{\mathcal{A}} \mu_s pr\ dA = \int_0^R \mu_s \frac{2}{\pi}\frac{P}{R^2}\left(1-\frac{r^2}{R^2}\right) r(2\pi r\ dr) \\ &= 4\mu_s \frac{P}{R^2}\int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr = 4\mu_s \frac{P}{R^2}\left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]_0^R = \frac{8}{15}\mu_s PR \ \blacktriangleleft \end{aligned}$$

**7.71**

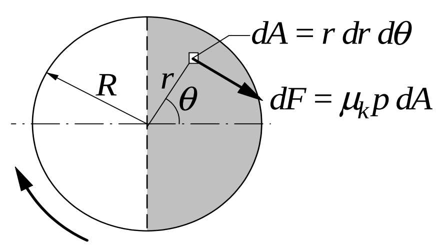



$$\Sigma M_O = 0 \qquad C - 160(1.75) = 0 \qquad C = 280 \text{ lb}\cdot\text{ft}$$

Assume unworn surfaces. From Eq. (7.12):

$$C = \frac{2}{3}\mu_s PR \qquad \mu_s = \frac{3C}{2PR} = \frac{3(280)}{2(600)(2.5)} = 0.280 \ \blacktriangleleft$$

**7.72**




$$\begin{aligned} F &= \int_0^R \int_{-\pi/2}^{\pi/2} dF \cos\theta = \mu_k p \int_0^R \int_{-\pi/2}^{\pi/2} r\cos\theta\ dr\ d\theta \\ &= 2\mu_k p \int_0^R r dr = \mu_k p R^2 \ \blacktriangleleft \end{aligned}$$

## 7.73

$$C = \mu_k \int_A pr\, dA = 0.86 \int_2^8 \int_0^{2\pi} \left( \frac{4}{3} + \frac{1}{6} r^2 \right) r\, d\theta\, dr = 0.86(2\pi) \int_2^8 \left( \frac{4}{3} r + \frac{1}{6} r^3 \right) dr$$

$$= 0.86(2\pi) \left[ \frac{2}{3} r^2 + \frac{1}{24} r^4 \right]_2^8 = 0.86(2\pi)(213.3 - 3.3) = 1135 \text{ lb} \bullet \text{in} \ \blacklozenge$$

## 7.74

From Eqs. (7.11) and (7.12):

$$\begin{aligned} C &= \frac{2\mu_A P R_0}{3} + \frac{2\mu_B (P+W)}{3} \left( \frac{R_0^3 - R_i^3}{R_0^2 - R_i^2} \right) \\ &= \frac{2(0.2)(290)(0.055)}{3} + \frac{2(0.08)(290 + 24 \times 9.81)}{3} \left( \frac{0.165^3 - 0.055^3}{0.165^2 - 0.055^2} \right) \\ &= 2.127 + 5.009 = 7.14 \text{ N} \cdot \text{m} \ \blacktriangleleft \end{aligned}$$

## 7.75

$$C = \frac{2}{3} \mu_s P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \qquad \therefore P = \frac{3C(R_o^2 - R_i^2)}{2\mu_s (R_o^3 - R_i^3)} = \frac{3(56 \times 12)(9^2 - 4^2)}{2(1.6)(9^3 - 4^3)} = 61.6 \text{ lb} \ \blacklozenge$$

## 7.76

$$C = \frac{2}{3} \mu_s P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \qquad \therefore 120 \times 12 = \frac{2}{3}(1.6)(75) \frac{R_o^3 - 4^3}{R_o^2 - 4^2}$$

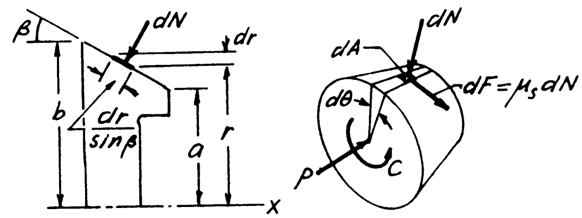
$$\therefore R_o^3 - 4^3 = \frac{3(120 \times 12)}{2(1.6)(75)} (R_o^2 - 4^2) \qquad \therefore R_o^3 - 4^3 = 18(R_o^2 - 4^2)$$

$$\therefore (R_o - 4)(R_o^2 + 4R_o + 4^2) = 18(R_o - 4)(R_o + 4)$$

After cancelling $(R_o - 4)$, we get $R_o^2 - 14R_o - 56 = 0$

The positive root is: $R_o = 17.25$ in ◆

## 7.77




**Assume impending slipping.**

$$dN = p\, dA = p \frac{dr}{\sin\beta}(r\, d\theta)$$

$$\Sigma F_x = 0{:}\ P - \int_A \sin\beta\, dN = 0$$

$$\therefore P = \int_A \sin\beta\, dN$$

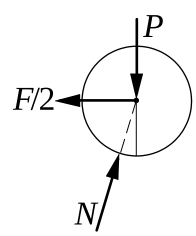
$$= p\int_a^b\int_0^{2\pi} r\, d\theta\, dr = 2\pi p\int_a^b r\, dr = \pi p(b^2 - a^2) \qquad \therefore p = \frac{P}{\pi(b^2 - a^2)} \ldots\ldots \text{(a)}$$

$$\Sigma M_x = 0{:}\ C - \int_A r(\mu_s dN) = 0$$

$$\therefore C = \mu_s\int_A r\, dN = \frac{\mu_s p}{\sin\beta}\int_a^b\int_0^{2\pi} r^2\, d\theta\, dr = \frac{2\pi\mu_s p}{\sin\beta}\int_a^b r^2\, dr = \frac{2\pi\mu_s p}{\sin\beta}\,\frac{b^3 - a^3}{3}$$

**Substitute for p from Eq. (a):** $C = \dfrac{2\mu_s P}{3\sin\beta}\,\dfrac{b^3 - a^3}{b^2 - a^2}$ ◆

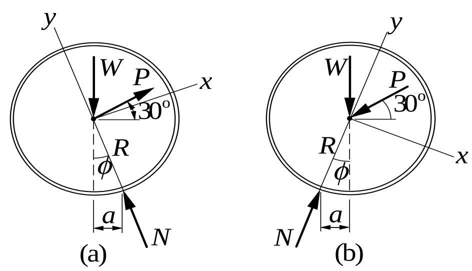
## 7.78




Applying Eq. (7.16) to the top roller:

$$\frac{F}{2} = \mu_r P \qquad F = 2\mu_r P = 2(0.016)(80) = 2.56 \text{ kN} \blacktriangleleft$$

## 7.79




$$\phi = \sin^{-1}\frac{a}{R} = \sin^{-1}\mu_d = \sin^{-1} 0.1 = 5.739°$$

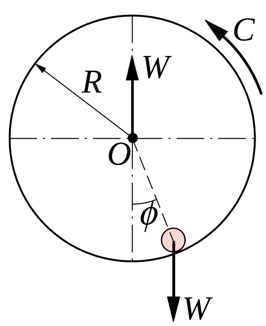
**(a)**

$$\begin{aligned} \Sigma F_x &= 0 \quad P\cos(30° - \phi) - W\sin\phi = 0 \\ & \quad\; P\cos(30° - 5.739°) - 30(9.81)(0.1) = 0 \qquad P = 32.3 \text{ N} \blacktriangleleft \end{aligned}$$

**(b)**

$$\begin{aligned} \Sigma F_x &= 0 \quad W\sin\phi - P\cos(30° + \phi) = 0 \\ & \quad\; 30(9.81)(0.1) - P\cos(30° + 5.739°) = 0 \qquad P = 36.3 \text{ N} \blacktriangleleft \end{aligned}$$

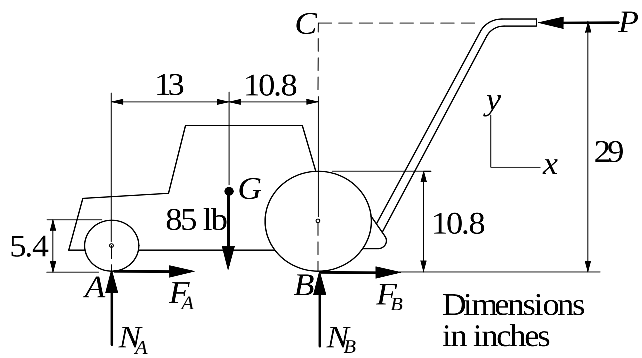
## 7.80




From the FBD of cylinder (the weight of the cylinder is not shown since it is irrelevant):

$$C = WR\sin\phi = WR\mu_r \blacktriangleleft$$

## 7.81




$$F_A = (\mu_r)_A N_A = 0.12N_A \qquad F_B = (\mu_r)_B N_B = 0.18N_B$$

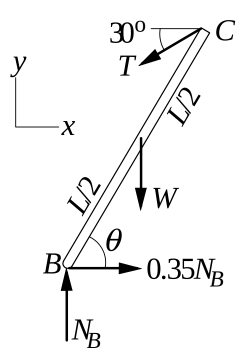
$$\begin{aligned}
\Sigma M_C &= 0 \qquad 23.8N_A - 29F_A - 29F_B - 85(10.8) = 0 \\
& \qquad 23.8N_A - 29(0.12N_A) - 29(0.18N_B) - 85(10.8) = 0 \\
& \qquad 20.32N_A - 5.22N_B - 918 = 0 \qquad \text{(a)} \\
\Sigma F_y &= 0 \qquad N_A + N_B - 85 = 0 \qquad \text{(b)}
\end{aligned}$$

Solution of Eqs. (a) and (b) is

$$N_A = 53.32 \text{ lb} \qquad N_B = 31.68 \text{ lb}$$

$$\begin{aligned}
\Sigma F_x &= 0 \qquad F_A + F_B - P = 0 \\
& \qquad 0.12(53.32) + 0.18(31.68) - P = 0 \qquad P = 12.10 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 7.82




Assume impending slipping.

$$\begin{aligned}
\Sigma F_x &= 0 \qquad 0.35N_B - T\cos 30^\circ = 0 \\
\Sigma F_y &= 0 \qquad N_B - W - T\sin 30^\circ = 0 \\
\text{Solution is } T &= 0.5065W \qquad N_B = 1.2532W
\end{aligned}$$

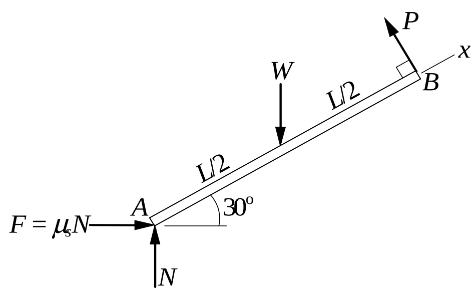
$$\Sigma M_C = 0 \qquad W(0.5L\cos\theta) + 0.35N_B(L\sin\theta) - N_B(L\cos\theta) = 0$$

$$0.5W\cos\theta + (1.2532W)(0.35\sin\theta - \cos\theta) = 0$$

$$(0.5 - 1.2532)\cos\theta + 1.2532(0.35)\sin\theta = 0$$

$$\tan\theta = \frac{1.2532 - 0.5}{1.2532(0.35)} = 1.7172 \qquad \theta = 59.8^\circ \blacktriangleleft$$

**7.83**




Consider impending sliding.

$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad PL = W\left(\frac{L}{2}\cos 30^\circ\right) \qquad P = 0.4330W$$

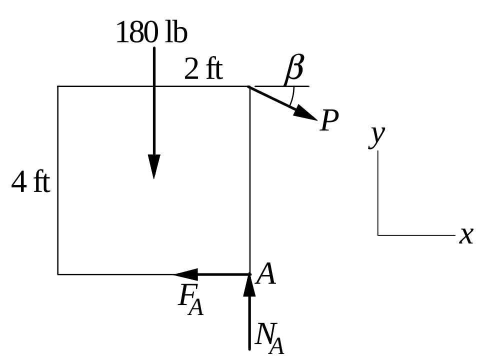
$$\Sigma F_y = 0 \qquad +\uparrow \quad P\cos 30^\circ - W + N = 0$$

$$0.4330W\cos 30^\circ - W + N = 0 \qquad N = 0.6250W$$

$$\Sigma F_x = 0 \qquad +\rightarrow \quad \mu_s N - P\sin 30^\circ = 0$$

$$\mu_s(0.6250W) - 0.4330W\sin 30^\circ = 0 \qquad \mu_s = 0.346 \blacktriangleleft$$

**7.84**



Assume simultaneous impending sliding and tipping.

$$\Sigma M_A = 0 \qquad 180(2) - 4P\cos\beta = 0 \qquad P\cos\beta = 90 \text{ lb}$$
$$\Sigma F_x = 0 \qquad P\cos\beta - 0.3N_A = 0 \qquad 90 - 0.3N_A = 0 \qquad N_A = 300 \text{ lb}$$
$$\Sigma F_y = 0 \qquad N_A - P\sin\beta - 180 = 0 \qquad 300 - \frac{90}{\cos\beta}\sin\beta - 180 = 0$$
$$\tan\beta = \frac{120}{90} = \frac{4}{3} \qquad \beta = \tan^{-1}\frac{4}{3} = 53.13° \blacktriangleleft$$
$$P = \frac{90}{\cos\beta} = \frac{90}{\cos 53.13°} = 150.0 \text{ lb} \blacktriangleleft$$

## 7.85

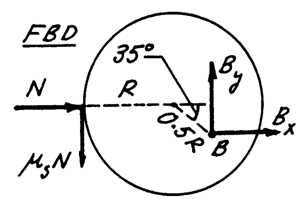
**Assume impending slipping of the belt.**

**$\Sigma M_B = 0$:**

$(\circlearrowleft +)\ \mu_s N(R + 0.5R\cos 35°) - N(0.5R\sin 35°) = 0$

$\therefore \mu_s(1 + 0.5\cos 35°) - 0.5\sin 35° = 0$

$\therefore \mu_s = 0.203$ ♦



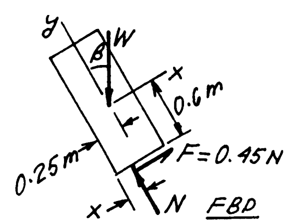

## 7.86

**Assume impending sliding.**

$\Sigma F_y = 0$: $+\nwarrow$ $N - W\cos\beta = 0 \quad \therefore N = W\cos\beta$

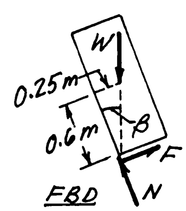
$\Sigma F_x = 0$: $+\nearrow$ $0.45N - W\sin\beta = 0$

$\therefore 0.45W\cos\beta - W\sin\beta = 0 \quad \therefore \beta = \tan^{-1} 0.45 = 24.22°$



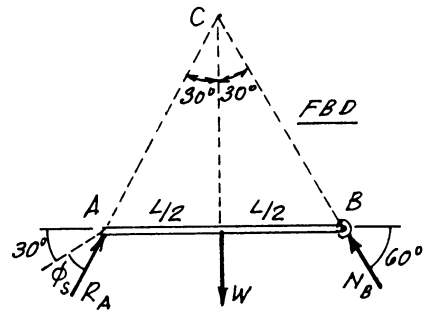

**Assume impending tipping.**

$$\beta = \tan^{-1}\frac{0.25}{0.6} = 22.62°$$




$\therefore$ **Largest possible angle for equilibrium is $\beta = 22.6°$** ♦

## 7.87



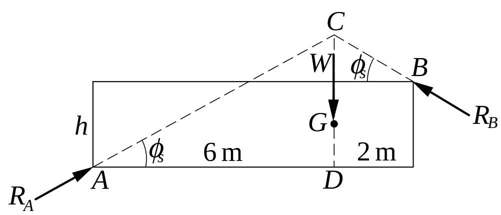

$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.6 = 30.96°$

**The bar is a three-force body, with the forces intersecting at C.**

**From geometry of the FBD: $\phi = 30°$**

**Since $\phi < \phi_s$ the bar is in equilibrium ♦**

## 7.88

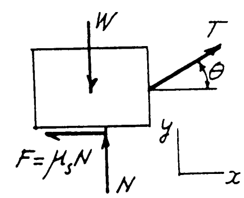



The panel is a 3-force body with the forces intersecting at $C$.

Assuming impending sliding at $A$ and $B$, we get from geometry:

$$\begin{aligned}\overline{CD} &= 6\tan\phi_s = 2\tan\phi_s + h \\ h &= 4\tan\phi_s = 4\mu_s = 4(0.5) = 2.00 \text{ m} \quad ◄\end{aligned}$$

## 7.89




**Assume impending sliding.**

$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $T\cos\theta - \mu_s N = 0$

$\Sigma F_y = 0$: $+\uparrow$ $T\sin\theta + N - W = 0$

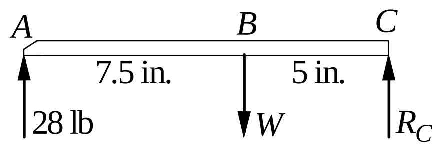
**The solution is:** $N = \dfrac{W}{1 + \mu_s \tan\theta}$ $\quad T = \dfrac{\mu_s W}{\cos\theta + \mu_s \sin\theta}$

**T is minimized if $\cos\theta + \mu_s\sin\theta$ is maximized:** $\dfrac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0$

$\therefore -\sin\theta + \mu_s\cos\theta = 0 \quad \therefore \theta = \tan^{-1}\mu_s$ ♦

$\therefore \sin\theta = \dfrac{\mu_s}{\sqrt{1+\mu_s^2}} \quad \therefore \cos\theta = \dfrac{1}{\sqrt{1+\mu_s^2}} \quad \therefore T = \dfrac{\mu_s W\sqrt{1+\mu_s^2}}{1+\mu_s^2} = \dfrac{\mu_s W}{\sqrt{1+\mu_s^2}}$ ♦

**7.90**




$$
\begin{aligned}
p &= 2\pi r \tan\theta \text{ (pitch of screw)} \\
\theta &= \tan^{-1}\frac{p}{2\pi r} = \tan^{-1}\frac{0.16}{2\pi(0.3)} = 4.852^\circ \text{ (lead angle)} \\
\phi_s &= \tanh^{-1}\mu_s = \tan^{-1} 0.4 = 21.80^\circ
\end{aligned}
$$

From FBD of member $ABC$:

$\Sigma M_C = 0 \qquad 5W - 28(12.5) = 0 \qquad W = 70$ lb (axial thrust on screw)

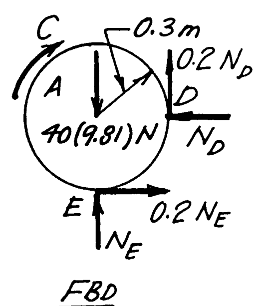
(a) $C_0 = Wr\tan(\phi_s + \theta) = 70(0.3)\tan(21.80^\circ + 4.85^\circ) = 10.54 \text{ lb}\cdot\text{in.}$ ◀

(b) $C_0 = Wr\tan(\phi_s - \theta) = 70(0.3)\tan(21.80^\circ - 4.85^\circ) = 6.40 \text{ lb}\cdot\text{in.}$ ◀

**7.91**

**Assume impending slipping at D and E (cylinder rotates; block does not move)**

**From FBD of cylinder**




$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $0.2N_E - N_D = 0$

$\Sigma F_y = 0$: $+\uparrow$ $N_E + 0.2N_D - (40)(9.81) = 0$

The solution is: $N_D = 75.46$ N, $N_E = 377.3$ N

$\Sigma M_D = 0$:

$\circlearrowleft$+ $\left[0.2N_E - N_E + (40)(9.81)\right](0.3) - C = 0$

$\therefore C = 0.3\left[(40)(9.81) - 0.8N_E\right] = 0.3\left[392.4 - 0.8(377.3)\right] = 27.17$ N·m

**Check for slippage under the block**

**From FBD of block**

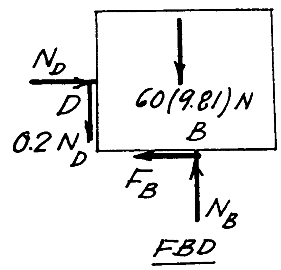
$\Sigma F_x = 0$: $\overset{+}{\rightarrow}$ $N_D - F_B = 0$

$\therefore F_B = N_D = 75.46$ N

$\Sigma F_y = 0$ $+\uparrow$ $N_B - 0.2N_D - (60)(9.81) = 0$

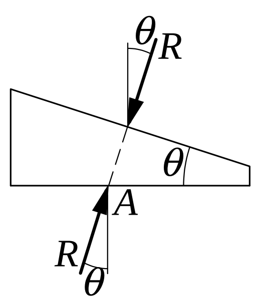
$\therefore N_B = 0.2N_D + (60)(9.81)$

$= 0.2(75.46) + (60)(9.81) = 603.7$ N




$\frac{F_B}{N_B} = \frac{75.46}{603.7} = 0.1250 < 0.2$ $\therefore$ The block does not slip $\therefore$ C = 27.2 N·m ◆

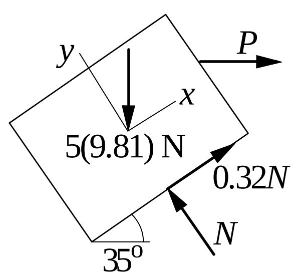
## 7.92




The wedge is a two-force body. There is no slipping at $A$ if $\phi_s > \theta$.
$\therefore$ Smallest coefficient of static friction that prevents slipping is

$$\mu_s = \tan\phi_s = \tan\theta = \tan 18^\circ = 0.325 \blacktriangleleft$$

## 7.93




Assume impending sliding down the incline:

$$\Sigma F_x = 0 \qquad P\cos 35^\circ + 0.32N - 5(9.81)\sin 35^\circ = 0$$
$$\Sigma F_y = 0 \qquad -P\sin 35^\circ + N - 5(9.81)\cos 35^\circ = 0$$

Solution is $N = 48.9\text{N} \qquad P = 15.24\text{ N}$

Assume impending sliding up the incline (the direction of the friction force is reversed):

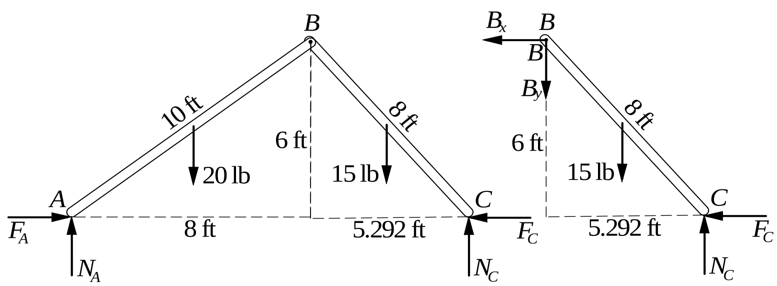
$$\Sigma F_x = 0 \qquad P\cos 35^\circ - 0.32N - 5(9.81)\sin 35^\circ = 0$$
$$\Sigma F_y = 0 \qquad -P\sin 35^\circ + N - 5(9.81)\cos 35^\circ = 0$$

Solution is: $N = 77.2\text{ N} \qquad P = 64.5\text{ N}$

Block will remain at rest if

$$15.24\text{ N} \le P \le 64.5\text{ N} \blacktriangleleft$$

**7.94**




Assume equilibrium. FBD of assembly:

$$\Sigma M_A = 0 \qquad +\circlearrowleft \quad N_C(8+5.292) - 20(4) - 15\left(8 + \frac{5.292}{2}\right) = 0$$
$$N_C = 18.033\text{ lb}$$
$$\Sigma F_y = 0 \qquad +\uparrow \quad N_A + N_C - 20 - 15 = 0$$
$$N_A + 18.033 - 20 - 15 = 0 \qquad N_A = 16.967\text{ lb}$$
$$\Sigma F_x = 0 \qquad +\rightarrow \quad F_A - F_C = 0 \qquad F_C = F_A$$

FBD of bar $BC$:

$$\Sigma M_B = 0 \qquad +\circlearrowleft \quad F_C(6) - N_C(5.292) + 15\left(\frac{5.292}{2}\right) = 0$$
$$F_C(6) - 18.033(5.292) + 15\left(\frac{5.292}{2}\right) = 0 \qquad F_C = F_A = 9.290\text{ lb}$$

Check for sliding:

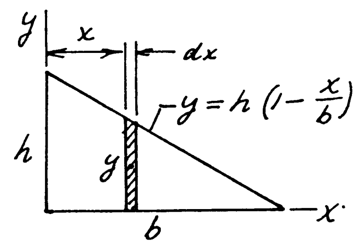
$$\frac{F_A}{N_A} = \frac{9.290}{16.967} = 0.5475 < \mu_s \quad \therefore \text{No sliding at } A$$
$$\frac{F_C}{N_C} = \frac{9.290}{18.033} = 0.5152 < \mu_s \quad \therefore \text{No sliding at } C$$

$\therefore$ Bars are in equilibrium $\blacktriangleleft$

# Chapter 8

## 8.1

We choose single integration




$$dA = y\,dx = h\left(1-\frac{x}{b}\right)dx \qquad A = \frac{1}{2}bh$$

$$\bar{x}_{el} = x \qquad \bar{y}_{el} = \frac{y}{2} = \frac{h}{2}\left(1-\frac{x}{b}\right)$$

$$Q_x = \int_A \bar{y}_{el}\,dA = \frac{h^2}{2}\int_0^b \left(1-\frac{x}{b}\right)^2 dx$$

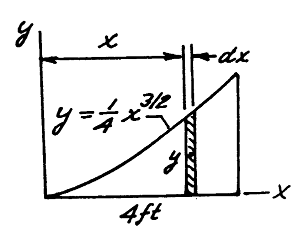
$$= \frac{h^2}{2}\left[x - \frac{x^2}{b} + \frac{x^3}{3b^2}\right]_0^b = \frac{bh^2}{6}$$

$$Q_y = \int_A \bar{x}_{el}\,dA = h\int_0^b x\left(1-\frac{x}{b}\right)dx = h\left[\frac{x^2}{2} - \frac{x^3}{3b}\right]_0^b = \frac{b^2h}{6}$$

$$\therefore\ \bar{x} = \frac{Q_y}{A} = \frac{b^2h/6}{bh/2} = \frac{b}{3}\ \blacklozenge \qquad \therefore\ \bar{y} = \frac{Q_x}{A} = \frac{bh^2/6}{bh/2} = \frac{h}{3}\ \blacklozenge$$

## 8.2

We choose single integration




$$dA = y\,dx = \frac{1}{4}x^{3/2}dx \qquad \bar{x}_{el} = x \qquad \bar{y}_{el} = \frac{y}{2} = \frac{1}{8}x^{3/2}$$

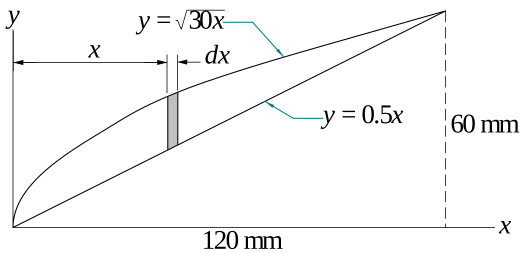
$$A = \int_A dA = \frac{1}{4}\int_0^4 x^{3/2}dx = \frac{1}{4}\left[\frac{2}{5}x^{5/2}\right]_0^4 = \frac{16}{5}\ \text{ft}^2$$

$$Q_x = \int_A \bar{y}_{el}\,dA = \frac{1}{32}\int_0^4 x^3 dx = \frac{1}{32}\left[\frac{x^4}{4}\right]_0^4 = 2\ \text{ft}^3$$

$$Q_y = \int_A \bar{x}_{el}\,dA = \frac{1}{4}\int_0^4 x^{5/2}dx = \frac{1}{4}\left[\frac{2}{7}x^{7/2}\right]_0^4 = \frac{64}{7}\ \text{ft}^3$$

$$\therefore\ \bar{x} = \frac{Q_y}{A} = \frac{64/7}{16/5} = \frac{20}{7} = 2.857\ \text{ft}\ \blacklozenge \qquad \therefore\ \bar{y} = \frac{Q_x}{A} = \frac{2}{16/5} = \frac{5}{8} = 0.625\ \text{ft}\ \blacklozenge$$

## 8.3

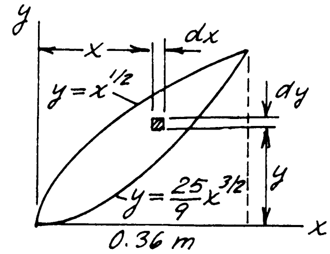



$$
\begin{aligned}
dA &= \left(\sqrt{30x} - 0.5x\right) dx \qquad \bar{y}_{\text{el}} = \frac{1}{2}\left(\sqrt{30x} + 0.5x\right) \\
A &= \int_{\mathcal{A}} dA = \int_0^{120} \left(\sqrt{30x} - 0.5x\right) dx = \left[\frac{2}{3}\sqrt{30}x^{3/2} - 0.25x^2\right]_0^{120} \\
&= 1200 \text{ mm}^2 \\
Q_x &= \int_{\mathcal{A}} \bar{y}_{\text{el}} dA = \int_0^{120} \frac{1}{2}\left(30x - 0.25x^2\right) dx = \left[\frac{30}{4}x^2 - \frac{0.25}{6}x^3\right]_0^{120} \\
&= 36\ 000 \text{ mm}^3 \\
Q_y &= \int_{\mathcal{A}} x\ dA = \int_0^{120} \left(\sqrt{30}x^{3/2} - 0.5x^2\right) dx = \left[\frac{2}{5}\sqrt{30}x^{5/2} - \frac{0.5}{3}x^3\right]_0^{120} \\
&= 57\ 600 \text{ mm}^3 \\
\bar{x} &= \frac{Q_y}{A} = \frac{57\ 600}{1200} = 48.0 \text{ mm} \blacktriangleleft \qquad \bar{y} = \frac{Q_x}{A} = \frac{36\ 000}{1200} = 30.0 \text{ mm} \blacktriangleleft
\end{aligned}
$$

## 8.4

**We choose double integration**

**dA = dx dy**




$$A = \int_A dA = \int_{x=0}^{0.36}\left(\int_{y=(25/9)x^{3/2}}^{x^{1/2}} dy\right) dx$$

$$= \int_0^{0.36} \left[y\right]_{(25/9)x^{3/2}}^{x^{1/2}} dx = \int_0^{0.36}\left(x^{1/2} - \frac{25}{9}x^{3/2}\right) dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{25}{9}\frac{2}{5}x^{5/2}\right]_0^{0.36} = 0.05760 \text{ m}^2$$

$$Q_x = \int_A y\,dA = \int_{x=0}^{0.36}\left(\int_{y=(25/9)x^{3/2}}^{x^{1/2}} y\,dy\right)dx = \int_0^{0.36}\left[\frac{y^2}{2}\right]_{(25/9)x^{3/2}}^{x^{1/2}} dx$$

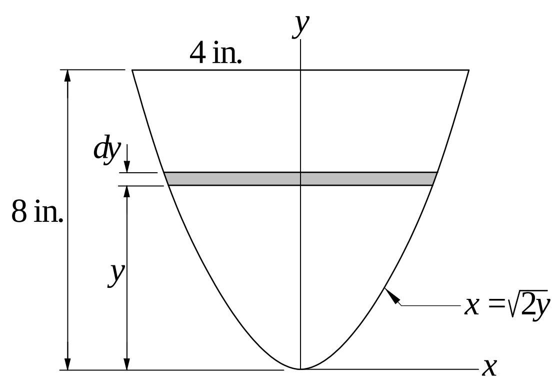
$$= \int_0^{0.36}\frac{1}{2}\left[x - \left(\frac{25}{9}\right)^2 x^3\right]dx = \frac{1}{2}\left[\frac{x^2}{2} - \left(\frac{25}{9}\right)^2\frac{x^4}{4}\right]_0^{0.36} = 0.01620\ \text{m}^3 \ \blacklozenge$$

$$Q_y = \int_A x\,dA = \int_{x=0}^{0.36}\left(\int_{y=(25/9)x^{3/2}}^{x^{1/2}} x\,dy\right)dx = \int_0^{0.36} x\left[y\right]_{(25/9)x^{3/2}}^{x^{1/2}} dx$$

$$= \int_0^{0.36}\left(x^{3/2} - \frac{25}{9}x^{5/2}\right)dx = \left[\frac{2}{5}x^{5/2} - \frac{25}{9}\frac{2}{7}x^{7/2}\right]_0^{0.36} = 0.008887\ \text{m}^3$$

$$\therefore\ \bar{x} = \frac{Q_y}{A} = \frac{0.008887}{0.05760} = 0.1543\ \text{m} \ \blacklozenge \qquad \therefore\ \bar{y} = \frac{Q_x}{A} = \frac{0.01620}{0.05760} = 0.2813\ \text{m} \ \blacklozenge$$

**8.5**



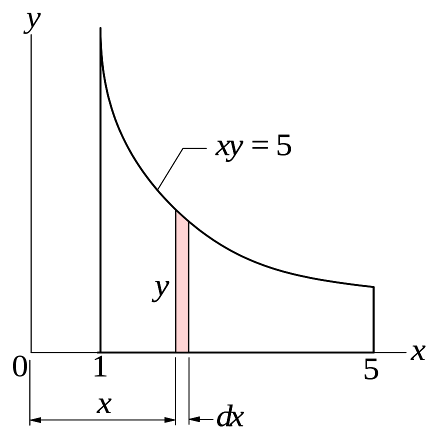

$$dA = 2\sqrt{2y}dy = 2.828\sqrt{y}\ dy$$

$$A = \int_A dA = 2.828\int_0^8 \sqrt{y}\ dy = 42.67\ \text{in}^2$$

$$Q_x = \int_A y\ dA = 2.828\int_0^8 y^{3/2}\ dy = 204.8\ \text{in}^3$$

$$\bar{x} = 0\ \text{ due to symmetry} \qquad \bar{y} = \frac{Q_x}{A} = \frac{204.8}{42.67} = 4.80\ \text{in} \ \blacktriangleleft$$

**8.6**




$$dA = y\ dx = \frac{5}{x}dx$$

$$A = \int_A dA = \int_1^5 \frac{5}{x}dx = 5\ln 5 = 8.047\ \text{in}^2$$

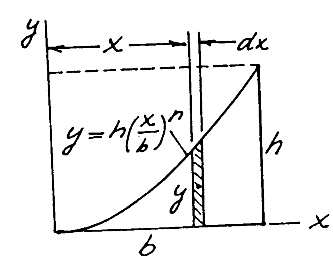
$$Q_x = \int_A \frac{y}{2}dA = \int_1^5 \frac{5}{2x}\left(\frac{5}{x}dx\right) = \int_1^5 \frac{25}{2x^2}dx = 10.0\ \text{in}^3$$

$$Q_y = \int_A x\ dA = \int_1^5 5dx = 20.0\ \text{in}^3$$

$$\bar{x} = \frac{Q_y}{A} = \frac{20.0}{8.047} = 2.49\ \text{in.} \blacktriangleleft \qquad \bar{y} = \frac{Q_x}{A} = \frac{10.0}{8.047} = 1.243\ \text{in.} \blacktriangleleft$$

**8.7**

**(a) We choose single integration**




$$dA = y\,dx = h\left(\frac{x}{b}\right)^n dx$$

$$\bar{x}_{el} = x \qquad \bar{y}_{el} = \frac{y}{2} = \frac{h}{2}\left(\frac{x}{b}\right)^n$$

$$A = \int_A dA = \frac{h}{b^n}\int_0^b x^n\,dx = \frac{h}{b^n}\left[\frac{x^{n+1}}{n+1}\right]_0^b = \frac{bh}{n+1}$$

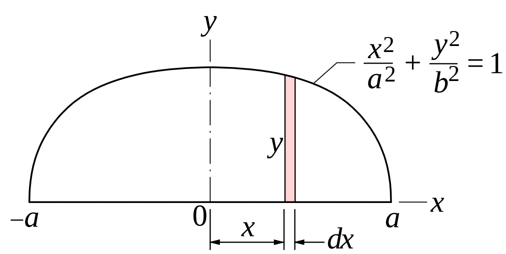
$$Q_x = \int_A \bar{y}_{el}\, dA = \frac{h^2}{2b^{2n}} \int_0^b x^{2n}\, dx = \frac{h^2}{2b^{2n}} \left[ \frac{x^{2n+1}}{2n+1} \right]_0^b = \frac{bh^2}{2(2n+1)}$$

$$Q_y = \int_A \bar{x}_{el}\, dA = \frac{h}{b^n} \int_0^b x^{n+1}\, dx = \frac{h}{b^n} \left[ \frac{x^{n+2}}{n+2} \right]_0^b = \frac{b^2 h}{n+2}$$

$$\therefore \bar{x} = \frac{Q_y}{A} = \frac{b^2h/(n+2)}{bh/(n+1)} = \frac{n+1}{n+2} b \; \blacklozenge \qquad \therefore \bar{y} = \frac{Q_x}{A} = \frac{bh^2/[2(2n+1)]}{bh/(n+1)} = \frac{n+1}{2(2n+1)} h \; \blacklozenge$$

**(b)** When n = 2, the above formulas yield $\bar{x} = \frac{3}{4} b$, $\bar{y} = \frac{3}{10} h$, which agree with Table 8.1.

## 8.8




$$y = b\sqrt{1 - \frac{x^2}{a^2}} = \frac{b}{a}\sqrt{a^2 - x^2}$$

Use symmetry in integration: $\int_{-a}^{a} ...dx = 2\int_0^a ...dx$.

$$\begin{aligned}
dA &= y\ dx = \frac{b}{a}\sqrt{a^2 - x^2}dx \\
A &= \int_A dA = \frac{b}{a}\int_{-a}^{a} \sqrt{a^2 - x^2}dx = 2\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}dx \\
&= 2\frac{b}{a}\left[\frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right)\right]_0^a = \frac{\pi ab}{2} \\
Q_x &= \int_A \frac{y}{2}dA = \left(\int_{-a}^{a} \frac{b}{2a}\sqrt{a^2 - x^2}\right)\left(\frac{b}{a}\sqrt{a^2 - x^2}dx\right) \\
&= \frac{b^2}{a^2}\int_0^a (a^2 - x^2)dx = \frac{2ab^2}{3}
\end{aligned}$$

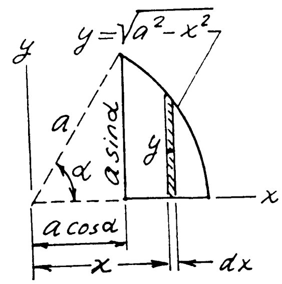
$$\bar{y} = \frac{Q_x}{A} = \frac{\frac{2ab^2}{3}}{\frac{\pi ab}{2}} = \frac{4b}{3\pi} \;\blacktriangleleft$$

## 8.9

We choose single integration

$dA = y\,dx = \sqrt{a^2 - x^2}\,dx$

$\bar{x}_{el} = x \qquad \bar{y}_{el} = \frac{y}{2} = \frac{1}{2}\sqrt{a^2 - x^2}$




$$A = A_{sector} - A_{triangle} = \frac{\alpha a^2}{2} - \frac{(a\cos\alpha)(a\sin\alpha)}{2}$$

$$= \frac{a^2}{2}(\alpha - \sin\alpha\cos\alpha) = \frac{18^2}{2}\left(\frac{\pi}{4} - \sin 45° \cos 45°\right) = 46.23 \text{ in}^2$$

$$Q_x = \int_A \bar{y}_{el}\,dA = \frac{1}{2}\int_{a\cos\alpha}^{a} (a^2 - x^2)\,dx = \frac{1}{2}\left[a^2 x - \frac{x^3}{3}\right]_{a\cos\alpha}^{a} = \frac{a^3}{2}\left[\left(1 - \frac{1}{3}\right) - \left(\cos\alpha - \frac{\cos^3\alpha}{3}\right)\right]$$

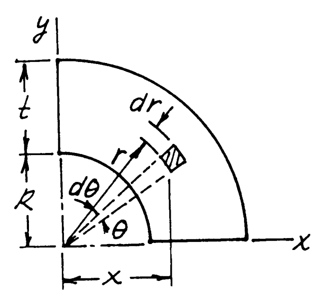
$$= \frac{a^3}{2}\left(\frac{2}{3} - \cos\alpha + \frac{\cos^3\alpha}{3}\right) = \frac{18^3}{2}\left(\frac{2}{3} - \cos 45° + \frac{\cos^3 45°}{3}\right) = 225.7 \text{ in}^3$$

$$\therefore\ \bar{y} = \frac{Q_x}{A} = \frac{225.7}{46.23} = 4.88 \text{ in} \ ♦$$

## 8.10

**(a) We choose double integration**

$dA = (dr)(r\,d\theta) \qquad x = r\cos\theta$




$$A = \frac{\pi}{4}\left[(R + t)^2 - R^2\right] = \frac{\pi}{4} t\,(2R + t)$$

$$Q_y = \int_A x\,dA = \int_{r=R}^{R+t}\int_{\theta=0}^{\pi/2} r^2 \cos\theta\,d\theta\,dr$$

$$= \int_R^{R+t} r^2 \left[\sin\theta\right]_0^{\pi/2} dr = \left[\frac{r^3}{3}\right]_R^{R+t}$$

$$= \frac{1}{3}\left[(R + t)^3 - R^3\right] = \frac{t}{3}(3R^2 + 3Rt + t^2)$$

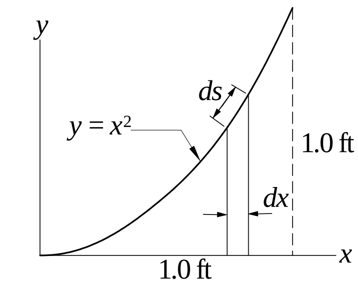
$$\therefore\ \bar{x} = \frac{Q_y}{A} = \frac{t(3R^2 + 3Rt + t^2)/3}{\pi t(2R + t)/4} = \frac{4}{3\pi}\,\frac{3R^2 + 3Rt + t^2}{2R + t} \ ♦$$

**Due to symmetry $\bar{y} = \bar{x}$**

**(b) When $t \to 0$, then $\bar{x} \to \frac{2}{\pi}R$, which agrees with Table 8.2.**

## 8.11

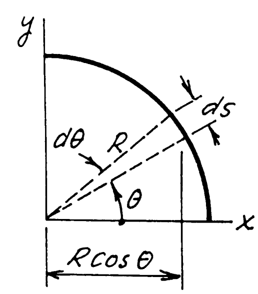
Consider half of the parabola




$$
\begin{aligned}
y &= x^2 \qquad dy = 2x\ dx \qquad ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + 4x^2}dx \\
L &= \int_{\mathcal{L}} ds = \int_0^1 \sqrt{1+4x^2}dx = \left[\frac{1}{4}\ln\left(2x + \sqrt{4x^2+1}\right) + \frac{1}{2}x\sqrt{4x^2+1}\right]_0^1 \\
&= 1.4789 \text{ ft} \\
Q_x &= \int_{\mathcal{L}} y\ ds = \int_0^1 x^2\sqrt{1+4x^2}dx \\
&= \left[\frac{1}{16}x\left(1+4x^2\right)^{3/2} - \frac{1}{32}x\sqrt{1+4x^2} - \frac{1}{64}\ln\left(2x + \sqrt{1+4x^2}\right)\right]_0^1 \\
&= 0.6063 \text{ ft}^2 \\
\bar{x} &= 0 \text{ (by symmetry)} \blacktriangleleft \qquad \bar{y} = \frac{Q_x}{L} = \frac{0.6063}{1.4789} = 0.410 \text{ ft} \ \blacktriangleleft
\end{aligned}
$$

## 8.12

$ds = R\, d\theta \qquad L = \frac{\pi}{2}R \qquad x = R\cos\theta$




$$Q_y = \int_L x\, ds = R^2\int_0^{\pi/2}\cos\theta\, d\theta = R^2\left[\sin\theta\right]_0^{\pi/2} = R^2$$

$$\therefore\ \bar{x} = \frac{Q_y}{L} = \frac{R^2}{\pi R/2} = \frac{2}{\pi}R \ \blacklozenge$$

Due to symmetry $\bar{y} = \bar{x}$

## *8.13

$x = a(\theta - \sin\theta) \qquad \therefore\ dx = a(1 - \cos\theta)d\theta$

$y = a(1 - \cos\theta) \qquad \therefore\ dy = a \sin\theta\ d\theta$

$$ds = \sqrt{dx^2 + dy^2} = a\sqrt{(1-\cos\theta)^2 + \sin^2\theta}\ d\theta = a\sqrt{2(1-\cos\theta)}\ d\theta = 2a \sin\frac{\theta}{2}\ d\theta$$

$$L = \int_L ds = 2a\int_0^{2\pi} \sin\frac{\theta}{2}\ d\theta = 2a\left[-2\cos\frac{\theta}{2}\right]_0^{2\pi} = 8a$$

$$Q_x = \int_L y\ ds = 2a^2\int_0^{2\pi}(1-\cos\theta)\sin\frac{\theta}{2}\ d\theta = 2a^2\int_0^{2\pi}\left(2\sin^2\frac{\theta}{2}\right)\sin\frac{\theta}{2}\ d\theta = 4a^2\int_0^{2\pi}\sin^3\frac{\theta}{2}\ d\theta$$

Let $\varphi = \frac{\theta}{2} \qquad \therefore\ d\theta = 2\ d\varphi$

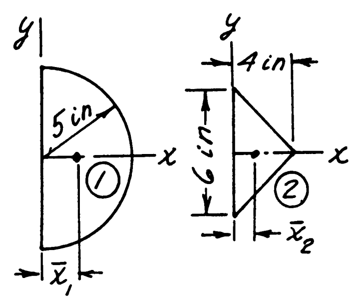
$$\therefore\ Q_x = 8a^2\int_0^{\pi}\sin^3\varphi\ d\varphi = 8a^2\left(\frac{4}{3}\right) = \frac{32}{3}a^2$$

$\therefore\ \bar{y} = \frac{Q_x}{L} = \frac{32a^2/3}{8a} = \frac{4}{3}a$ ◆ By symmetry $\bar{x} = \pi a$ ◆

## 8.14

$A_1 = \frac{\pi R^2}{2} = \frac{\pi(5)^2}{2} = 39.27\ in^2$

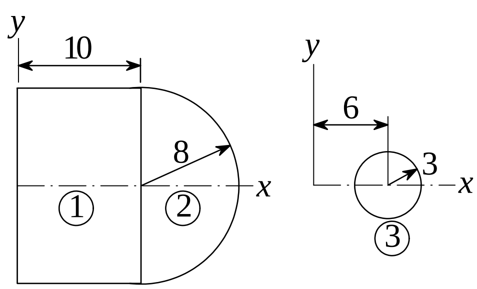
$\bar{x}_1 = \frac{4R}{3\pi} = \frac{4(5)}{3\pi} = 2.122\ in$




| Part | A ($in^2$) | $\bar{x}$ (in) | $A\bar{x}$ ($in^3$) |
|---|---|---|---|
| 1 | 39.27 | 2.122 | 83.33 |
| 2 | −12.00 | 1.333 | −16.00 |
| Sum | 27.27 | | 67.33 |

$\therefore\ \bar{x} = \frac{67.33}{27.27} = 2.47\ in$ ◆ Due to symmetry $\bar{y} = 0$ ◆

**8.15**



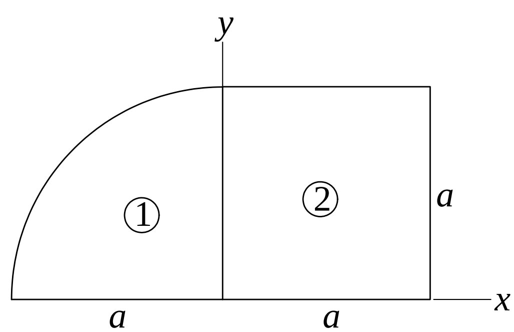

$$A_1 = 10(16) = 160 \text{ in}^2 \qquad \bar{x}_1 = 5 \text{ in.}$$

$$A_2 = \frac{\pi}{2}(8^2) = 100.53 \text{ in}^2 \qquad \bar{x}_2 = 10 + \frac{4}{3\pi}(8) = 13.395 \text{ in.}$$

$$A_3 = -\pi(3^2) = -28.27 \text{ in}^2 \qquad \bar{x}_3 = 6 \text{ in.}$$

$$x = \frac{\Sigma A_i \bar{x}_i}{A} = \frac{160(5) + 100.53(13.395) - 28.27(6)}{160 + 100.53 - 28.27} = 8.51 \text{ in.} \blacktriangleleft$$

**8.16**




$$A = A_1 + A_2 = \frac{\pi a^2}{4} + a^2 = 1.7854a^2$$

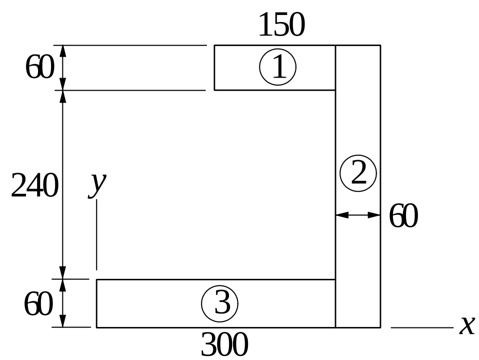
$$Q_x = A_1\bar{y}_1 + A_2\bar{y}_2 = \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right) + a^2\left(\frac{a}{2}\right) = 0.8333a^3$$

$$Q_y = A_1\bar{x}_1 + A_2\bar{x}_2 = \frac{\pi a^2}{4}\left(-\frac{4a}{3\pi}\right) + a^2\left(\frac{a}{2}\right) = 0.166\ 67a^3$$

$$\bar{x} = \frac{Q_y}{A} = \frac{0.166\ 67a^3}{1.7854a^2} = 0.0934a \blacktriangleleft$$

$$\bar{y} = \frac{Q_x}{A} = \frac{0.8333a^3}{1.7854a^2} = 0.467a \blacktriangleleft$$

**8.17**




$$A_1 = 150(60) = 9000 \text{ mm}^2 \qquad \bar{x}_1 = 225 \text{ mm} \qquad \bar{y}_1 = 330 \text{ mm}$$
$$A_2 = 360(60) = 21\ 600 \text{ mm}^2 \qquad \bar{x}_2 = 330 \text{ mm} \qquad \bar{y}_2 = 180 \text{ mm}$$
$$A_3 = 300(60) = 18\ 000 \text{ mm}^2 \qquad \bar{x}_3 = 150 \text{ mm} \qquad \bar{y}_3 = 30 \text{ mm}$$
$$A = \Sigma A_i = (9 + 21.6 + 18) \times 10^3 \text{ mm}^2 = 48.6 \times 10^3 \text{ mm}^2$$

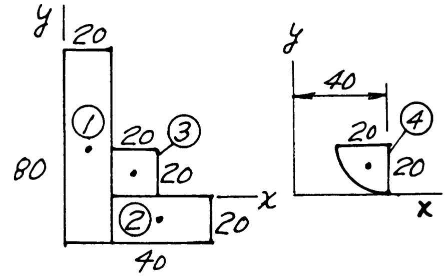
$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{A} = \frac{9(225) + 21.6(330) + 18(150)}{48.6} = 244 \text{ mm} \blacktriangleleft$$
$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{9(330) + 21.6(180) + 18(30)}{48.6} = 152.2 \text{ mm} \blacktriangleleft$$

**8.18**

$$A_4 = -\frac{\pi}{4}(20^2) = -314 \text{ mm}^2$$

$$\bar{x}_4 = 40 - \frac{4}{3\pi}(20) = 31.51 \text{ mm}$$

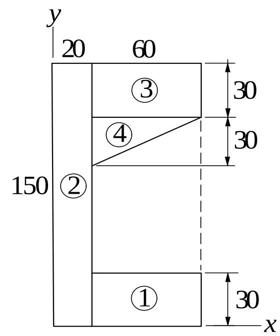
$$\bar{y}_4 = 20 - \frac{4}{3\pi}(20) = 11.51 \text{ mm}$$




| Part | $A(\text{mm}^2)$ | $\bar{y}$ (mm) | $A\bar{y}$ ($\text{mm}^3$) | $\bar{x}$ (mm) | $A\bar{x}$ ($\text{mm}^3$) |
|---|---|---|---|---|---|
| 1 | 1600 | 20.00 | 32 000 | 10.00 | 16 000 |
| 2 | 800 | −10.00 | −8 000 | 40.00 | 32 000 |
| 3 | 400 | 10.00 | 4 000 | 30.00 | 12 000 |
| 4 | −314 | 11.51 | −3 614 | 31.51 | −9 894 |
| Sum | 2486 | | 24 386 | | 50 106 |

$$\bar{x} = \frac{50\ 106}{2486} = 20.16 \text{ mm} \ \blacklozenge \qquad \bar{y} = \frac{24\ 386}{2486} = 9.81 \text{ mm} \ \blacklozenge$$

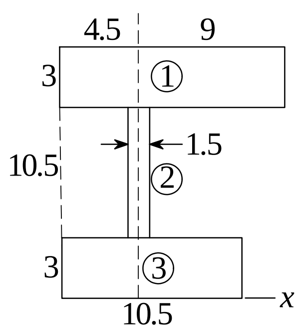
**8.19**




| Part | $A$ (mm$^2$) | $\bar{x}$ (mm) | $A\bar{x}$ (mm$^3$) | $\bar{y}$ (mm) | $A\bar{y}$ (mm$^3$) |
|---|---|---|---|---|---|
| 1 | 1800 | 50 | 90 000 | 15 | 27 000 |
| 2 | 3000 | 10 | 30 000 | 75 | 225 000 |
| 3 | 1800 | 50 | 90 000 | 135 | 243 000 |
| 4 | 900 | 40 | 36 000 | 110 | 99 000 |
| Sum | 7500 | | 246 000 | | 594 000 |

$$\bar{x} = \frac{246\ 000}{7500} = 32.8 \text{ mm} \blacktriangleleft \qquad \bar{y} = \frac{594\ 000}{7500} = 79.2 \text{ mm} \blacktriangleleft$$

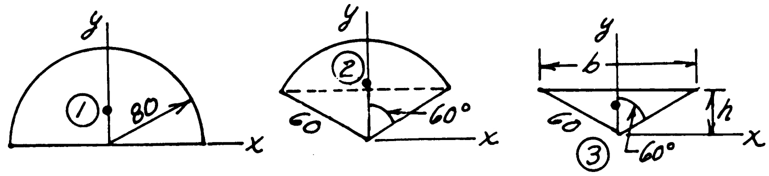
**8.20**




$$\begin{aligned}
A_1 &= 3(13.5) = 40.5 \text{ in}^2 \qquad \bar{x}_1 = 2.25 \text{ in.} \qquad y_1 = 15 \text{ in.} \\
A_2 &= 10.5(1.5) = 15.75 \text{ in}^2 \qquad \bar{x}_2 = 0 \qquad \bar{y}_2 = 8.25 \text{ in.} \\
A_3 &= 3(10.5) = 31.5 \text{ in}^2 \qquad \bar{x}_3 = 0.75 \text{ in.} \qquad \bar{y}_3 = 1.5 \text{ in.} \\
A &= \Sigma A_i = 40.5 + 15.75 + 31.5 = 87.75 \text{ in}^2
\end{aligned}$$

$$\begin{aligned}
x &= \frac{\Sigma A_i x_i}{A} = \frac{40.5(2.25) + 0 + 31.5(0.75)}{87.75} = 1.308 \text{ in.} \blacktriangleleft \\
y &= \frac{\Sigma A_i y_i}{A} = \frac{40.5(15) + 15.75(8.25) + 31.5(1.5)}{87.75} = 8.94 \text{ in.} \blacktriangleleft
\end{aligned}$$

## 8.21




$h = 60 \cos 60° = 30$ mm $\quad b = 2(60 \sin 60°) = 103.92$ mm

$A_1 = \frac{1}{2}\pi(80)^2 = 10\,053$ mm$^2$ $\quad A_2 = -\frac{1}{3}\pi(60)^2 = -3770$ mm$^2$

$A_3 = \frac{1}{2}hb = \frac{1}{2}(30)(103.92) = 1558.8$ mm$^2$

$\bar{y}_1 = \frac{4(80)}{3\pi} = 33.95$ mm $\quad \bar{y}_2 = \frac{2(60)\sin 60°}{3(\pi/3)} = 33.08$ mm $\quad \bar{y}_3 = \frac{2h}{3} = \frac{2(30)}{3} = 20$ mm

| Part | A(mm$^2$) | $\bar{y}$ (mm) | $A\bar{y}$ (mm$^3$) |
|---|---|---|---|
| 1 | 10 053 | 33.95 | 341 299 |
| 2 | −3 770 | 33.08 | −124 712 |
| 3 | 1 559 | 20.00 | 31 180 |
| Sum | 7 842 | | 247 767 |

$\bar{y} = \frac{247\,767}{7842} = 31.6$ mm ♦ $\quad$ Due to symmetry $\bar{x} = 0$ ♦

## 8.22

Semicircle:

$$A_1 = \frac{\pi R^2}{2} \qquad \bar{y}_1 = -\frac{4}{3\pi}R \qquad A_1\bar{y}_1 = -\frac{2}{3}R^3$$

Parabola:

$$A_2 = \frac{2}{3}(2R)h \qquad \bar{y}_2 = \frac{2}{5}h \qquad A_2\bar{y}_2 = \frac{8}{15}Rh^2$$

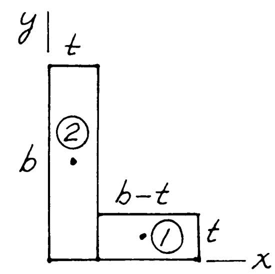
$$A_1\bar{y}_1 + A_2\bar{y}_2 = 0 \qquad -\frac{2}{3}R^3 + \frac{8}{15}Rh^2 = 0 \qquad h = \frac{\sqrt{5}}{2}R \ ◀$$

## 8.23

$$Q_x = \Sigma A_i\bar{y}_i = \left(\frac{\pi R^2}{2}\right)\left(\frac{4R}{3\pi}\right) + (Rh)\left(-\frac{h}{3}\right) = \frac{2R^3}{3} - \frac{Rh^2}{3}$$

$$Q_x = 0 \qquad h = \sqrt{2}R \ ◀$$

**8.24**




$A_1 = (b - t)t \qquad A_2 = bt$

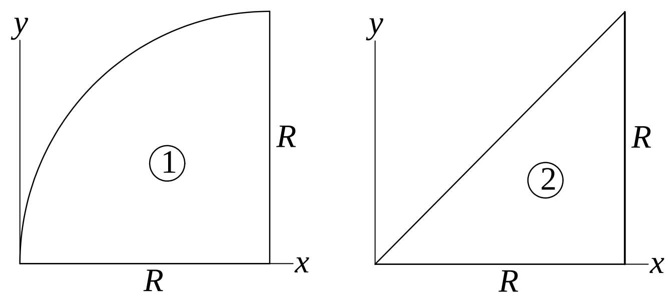
$\bar{x}_1 = \dfrac{b + t}{2} \qquad \bar{x}_2 = \dfrac{t}{2}$

$A_1\bar{x}_1 = \dfrac{t}{2}(b^2 - t^2) \qquad A_2\bar{x}_2 = \dfrac{1}{2}bt^2$

$$\therefore \bar{x} = \frac{\frac{t}{2}(b^2 - t^2) + \frac{1}{2}bt^2}{(b - t)t + bt} = \frac{b^2 + bt - t^2}{2(2b - t)} \; ◆$$

Due to symmetry $\bar{y} = \bar{x}$ ◆

**8.25**




$$
\begin{aligned}
A &= A_1 - A_2 = \frac{\pi R^2}{4} - \frac{R^2}{2} = \frac{R^2}{2}\left(\frac{\pi}{2} - 1\right) \\
Q_x &= A_1\bar{y}_1 - A_2\bar{y}_2 = \left(\frac{\pi R^2}{4}\right)\left(\frac{4R}{3\pi}\right) - \left(\frac{R^2}{2}\right)\left(\frac{R}{3}\right) = \frac{R^3}{6} \\
Q_y &= A_1\bar{x}_1 - A_2\bar{x}_2 = \left(\frac{\pi R^2}{4}\right)\left(R - \frac{4R}{3\pi}\right) - \left(\frac{R^2}{2}\right)\left(\frac{2R}{3}\right) \\
&= \frac{1}{12}(3\pi - 8)R^3 \\
\bar{x} &= \frac{Q_y}{A} = \frac{\frac{1}{12}(3\pi - 8)R^3}{\frac{R^2}{2}\left(\frac{\pi}{2} - 1\right)} = \frac{3\pi - 8}{3\pi - 6}R = 0.416R \; ◀ \\
\bar{y} &= \frac{Q_x}{A} = \frac{\frac{R^3}{6}}{\frac{R^2}{2}\left(\frac{\pi}{2} - 1\right)} = \frac{2R}{3(\pi - 2)} = 0.584R \; ◀
\end{aligned}
$$

## 8.26

$$A = 4ab - 2ah \qquad Q_x = 4ab\left(\frac{b}{2}\right) - 2ah\left(\frac{h}{2}\right) = 2ab^2 - ah^2$$

$$\bar{y} = \frac{Q_x}{A} = \frac{2ab^2 - ah^2}{4ab - 2ah} = \frac{2b^2 - h^2}{4b - 2h}$$

$\bar{y}$ is maximized when

$$\frac{d\bar{y}}{dh} = 0 \qquad \frac{(4b-2h)(-2h) - (2b^2 - h^2)(-2)}{(4b-2h)^2} = 0$$

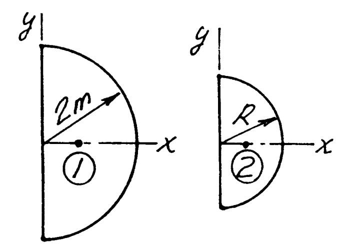
$$\frac{4b^2 - 8bh + 2h^2}{(4b-2h)^2} = 0 \qquad h = 0.586b \;\blacktriangleleft$$

## 8.27

$$A_1 = \frac{\pi}{2}(2^2) = 2\pi \text{ m}^2 \qquad A_2 = -\frac{\pi}{2}R^2$$

$$\bar{x}_1 = \frac{4}{3\pi}(2) = \frac{8}{3\pi}\text{ m} \qquad \bar{x}_2 = \frac{4}{3\pi}R$$

$$A_1\bar{x}_1 = \frac{16}{3}\text{ m}^3 \qquad A_2\bar{x}_2 = -\frac{2}{3}R^3$$



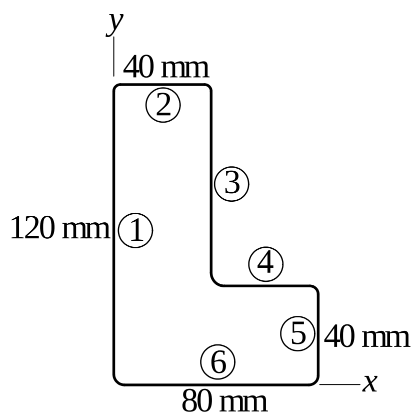

$$\therefore\ \bar{x} = \frac{\frac{16}{3} - \frac{2}{3}R^3}{\pi\left(2 - \frac{1}{2}R^2\right)} = \frac{4(8 - R^3)}{3\pi(4 - R^2)}$$

Setting $\bar{x} = R$ yields the cubic equation $3\pi R(4 - R^2) = 4(8 - R^3)$

$\therefore\ (3\pi - 4)R^3 - 12\pi R + 32 = 0 \qquad \therefore\ 5.425\,R^3 - 37.70\,R + 32 = 0$

Solving by a numerical method gives us $R = 0.987$ m ◆

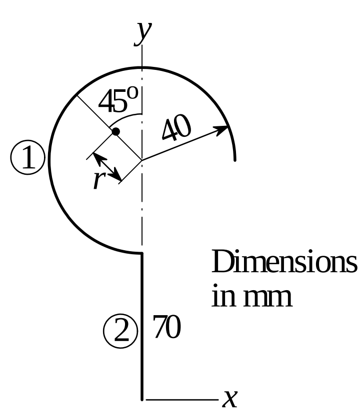
**8.28**




$$
\begin{aligned}
L &= \Sigma L_i = 2(120) + 2(80) = 400 \text{ mm} \\
Q_x &= \Sigma L_i \bar{y}_i = 120(60) + 40(120) + 80(80) + 40(40) + 40(20) + 0 \\
&= 20\ 800 \text{ mm}^2 \\
Q_y &= \Sigma L_i \bar{x}_i = 0 + 40(20) + 80(40) + 40(60) + 40(80) + 80(40) \\
&= 12\ 800 \text{ mm}^2
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{Q_y}{L} = \frac{12\ 800}{400} = 32 \text{ mm} \blacktriangleleft \\
y &= \frac{Q_x}{L} = \frac{20\ 800}{400} = 52 \text{ mm} \blacktriangleleft
\end{aligned}
$$

**8.29**

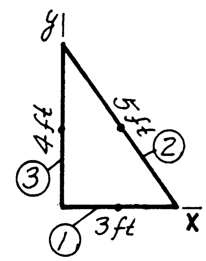


$$
\begin{aligned}
r &= \frac{R\sin\alpha}{\alpha} = \frac{40\sin 135^\circ}{0.75\pi} = 12.004 \text{ mm} \\
L_1 &= 0.75(2\pi R) = 0.75(2\pi)(40) = 188.50 \text{ mm} \\
\bar{x}_1 &= -r\sin 45^\circ = -12.004\sin 45^\circ = -8.488 \text{ mm} \\
\bar{y}_1 &= 110 + r\cos 45^\circ = 110 + 12.004\cos 45^\circ = 118.49 \text{ mm}
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{\Sigma L_i\bar{x}_i}{L} = \frac{188.50(-8.488)+0}{188.50+70} = -6.19 \text{ mm} \blacktriangleleft \\
\bar{y} &= \frac{\Sigma L_i\bar{y}_i}{L} = \frac{188.50(118.49)+70(35)}{188.50+70} = 95.9 \text{ mm} \blacktriangleleft
\end{aligned}
$$

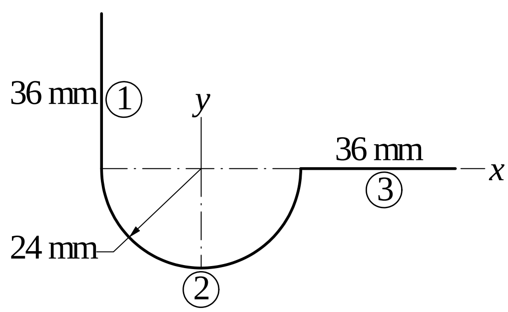
**8.30**

| Part | L (ft) | $\bar{x}$ (ft) | $L\bar{x}$ (ft$^2$) | $\bar{y}$ (ft) | $L\bar{y}$ (ft$^2$) |
|---|---|---|---|---|---|
| 1 | 3 | 1.5 | 4.5 | 0 | 0 |
| 2 | 5 | 1.5 | 7.5 | 2 | 10 |
| 3 | 4 | 0 | 0 | 2 | 8 |
| Sum | 12 | | 12 | | 18 |




$\therefore\ \bar{x} = \frac{12}{12} = 1.0$ ft ♦ $\quad \therefore\ \bar{y} = \frac{18}{12} = 1.5$ ft ♦

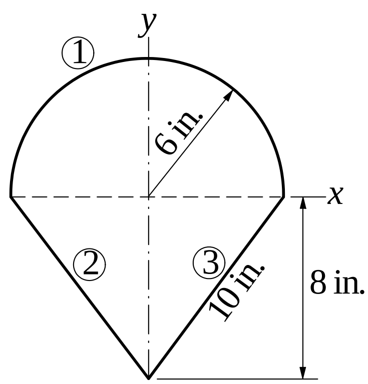
**8.31**




$$
\begin{aligned}
L &= \Sigma L_i = 36 + 24\pi + 36 = 147.40 \text{ mm} \\
Q_x &= \Sigma L_i\bar{y}_i = 36(18) + 24\pi\left[-\frac{2(24)}{\pi}\right] + 0 = -504 \text{ mm}^2 \\
Q_y &= \Sigma L_i\bar{x}_i = 36(-24) + 0 + 36(42) = 648 \text{ mm}^2
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{Q_y}{L} = \frac{648}{147.40} = 4.40 \text{ mm} \blacktriangleleft \\
y &= \frac{Q_x}{L} = \frac{-504}{147.40} = -3.42 \text{ mm} \blacktriangleleft
\end{aligned}
$$

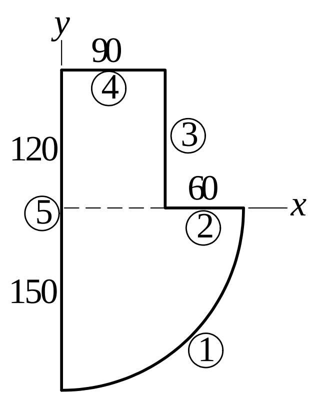
**8.32**




$$\bar{y} = \frac{\Sigma L_i \bar{y}_i}{\Sigma L_i} = \frac{6\pi(\frac{2}{\pi}6) + 10(-4) + 10(-4)}{6\pi + 10 + 10} = -0.206 \text{ in.} \blacktriangleleft$$

Due to symmetry $\bar{x} = 0$ ◀

**8.33**




$$L_1 = \frac{\pi}{2}(150) = 75\pi \text{ mm} \qquad \bar{x}_1 = -\bar{x}_2 = \frac{2}{\pi}(150) = \frac{300}{\pi} \text{ mm}$$

$$L = \Sigma L_i = \frac{\pi}{2}(150) + 60 + 120 + 90 + 270 = 775.6 \text{ mm}$$

$$\bar{x} = \frac{\Sigma L_i \bar{x}_i}{L} = \frac{75(300) + 60(120) + 120(90) + 90(45) + 0}{775.6}$$

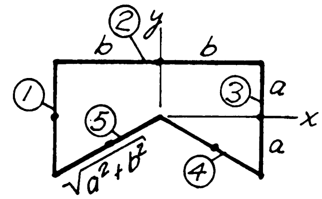
$$= 57.4 \text{ mm} \blacktriangleleft$$

$$\bar{y} = \frac{\Sigma L_i \bar{y}_i}{L} = \frac{75(-300) + 0 + 120(60) + 90(120) + 270(-15)}{775.6}$$

$$= -11.02 \text{ mm} \blacktriangleleft$$

## 8.34

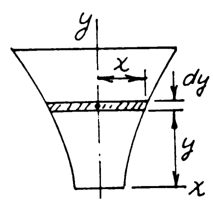
| Part | L | $\bar{y}$ | $L\bar{y}$ |
|---|---|---|---|
| 1 | 2a | 0 | 0 |
| 2 | 2b | a | 2ab |
| 3 | 2a | 0 | 0 |
| 4 | $\sqrt{a^2+b^2}$ | $-\frac{a}{2}$ | $-\frac{a}{2}\sqrt{a^2+b^2}$ |
| 5 | $\sqrt{a^2+b^2}$ | $-\frac{a}{2}$ | $-\frac{a}{2}\sqrt{a^2+b^2}$ |
| Sum | | | $2ab - a\sqrt{a^2+b^2}$ |




$\Sigma L\bar{y} = 0$ yields $2b - \sqrt{a^2+b^2} = 0 \quad \therefore (2b)^2 = a^2 + b^2 \quad \therefore 3b^2 = a^2 \quad \therefore \frac{a}{b} = \sqrt{3}$ ♦

## 8.35

Using single integration:




$dA = 2x\,dy \qquad \bar{y}_{el} = y$

$A = \int_A dA = 2\int_A x\,dy$

$Q_x = \int_A y_{el}\,dA = 2\int_A xy\,dA$

Computing the integrals with Simpson's rule:

$$A \approx 2\frac{\Delta y}{3}\sum_i W_i x_i = 2\frac{20}{3}\left[20 + 4(22) + 2(27) + 4(35) + 2(46) + 4(60) + 77\right] = 9480 \text{ mm}^2$$

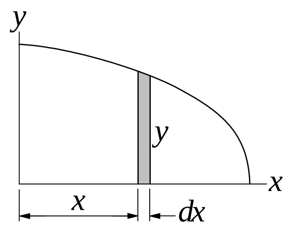
$$Q_x \approx 2\frac{\Delta y}{3}\sum_i W_i x_i y_i$$

$$= 2\frac{20}{3}\left[(20)(0) + 4(22)(20) + 2(27)(40) + 4(35)(60) + 2(46)(80) + 4(60)(100) + 77(120)\right]$$

$$= 705\,600 \text{ mm}^3$$

$\bar{y} = \frac{Q_x}{A} = \frac{705\,600}{9480} = 74.4$ mm ♦ Due to symmetry $\bar{x} = 0$ ♦

## 8.36




Use single integration:

$$dA = y\ dx \qquad \bar{x}_{\text{el}} = x \qquad \bar{y}_{\text{el}} = \frac{y}{2}$$

$$A = \int_{\mathcal{A}} dA = \int_{\mathcal{A}} y\ dx$$

$$Q_x = \int_A \bar{y}_{\text{el}} dA = \frac{1}{2}\int_{\mathcal{A}} y^2 dA \qquad Q_y = \int_{\mathcal{A}} \bar{x}_{\text{el}} dA = \int_{\mathcal{A}} xy\ dA$$

Evaluate integrals with Simpson's rule:

$$A = \frac{2}{3}[8.5 + 4(8.0) + 2(7.2) + 4(5.4) + 0] = 51.0\text{ in}^2$$

$$Q_x = \frac{1}{3}\left[8.5^2 + 4(8.0)^2 2 + 2(7.2)^2 + 4(5.4)^2 + 0\right] = 182.86\text{ in}^3$$

$$Q_y = \frac{2}{3}[(0)(8.5) + 4(2)(8.0) + 2(4)(7.2) + 4(6)(5.4) + (8)(0)]$$

$$= 167.47\text{ in}^3$$

$$\bar{x} = \frac{Q_y}{A} = \frac{167.47}{51.0} = 3.28\text{ in.} \blacktriangleleft \qquad \bar{y} = \frac{Q_x}{A} = \frac{182.86}{51.0} = 3.59\text{ in.} \blacktriangleleft$$

## 8.37

$$y = e^{-x^2} \qquad \frac{dy}{dx} = -2xe^{x^2} = -2xy$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + (2xy)^2}$$

$$L = \int_0^1 \frac{ds}{dx}dx \qquad Q_x = \int_0^1 y\frac{ds}{dx}dx$$

| $x$ (m) | $y$ (m) | $dy/dx = -2xy$ | $ds/dx$ | $y(ds/dx)$ (m) |
|---|---|---|---|---|
| 0 | 1.0 | 0.0 | 1.0 | 1.0 |
| 0.25 | 0.9394 | −0.4697 | 1.1048 | 1.0379 |
| 0.5 | 0.7788 | −0.7788 | 1.2675 | 0.9871 |
| 0.75 | 0.5698 | −0.8547 | 1.3155 | 0.7496 |
| 1.0 | 0.3679 | −0.7358 | 1.2415 | 0.4567 |

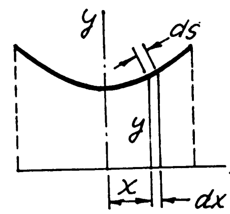
$$L = [1.0 + 4(1.1048) + 2(1.2675) + 4(1.3155) + 1.2415]\,\frac{0.25}{3} = 1.2048 \text{ m}$$

$$Q_x = [1.0 + 4(1.0379) + 2(0.9871) + 4(0.7496) + 0.4567]\,\frac{0.25}{3} = 0.8817 \text{ m}^2$$

$$\bar{y} = \frac{Q_x}{L} = \frac{0.8817}{1.2048} = 0.732 \text{ m} \blacktriangleleft$$

## 8.38

$\bar{y}_{el} = y = 100\cosh\frac{x}{100} \quad \therefore \frac{dy}{dx} = \sinh\frac{x}{100}$




$\frac{ds}{dx} = \sqrt{1 + (dy/dx)^2} = \sqrt{1 + \sinh^2\frac{x}{100}}$

**The following are integrated with Simpson's rule:**

$$L = \int_L ds = \int_L \frac{ds}{dx}dx \approx \frac{\Delta x}{3}\sum_i W_i\left(\frac{ds}{dx}\right)_i$$

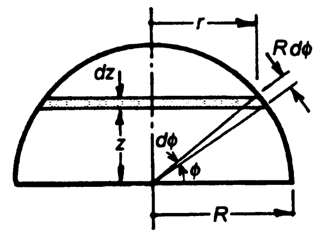
$$Q_x = \int_L \bar{y}_{el}\,ds = \int_L y\frac{ds}{dx}dx \approx \frac{\Delta x}{3}\sum_i W_i y_i\left(\frac{ds}{dx}\right)_i$$

| x (ft) | y(ft) | $\frac{dy}{dx}$ | $\frac{ds}{dx}$ | $y\frac{ds}{dx}$ (ft) | W | $W\frac{ds}{dx}$ | $Wy\frac{ds}{dx}$ (ft) |
|---|---|---|---|---|---|---|---|
| 0 | 100.0 | 0.0000 | 1.0000 | 100.00 | 1 | 1.0000 | 100.00 |
| 25 | 103.1 | 0.2526 | 1.0314 | 106.38 | 4 | 4.1256 | 425.52 |
| 50 | 112.8 | 0.5211 | 1.1276 | 127.15 | 2 | 2.2552 | 254.30 |
| 75 | 129.5 | 0.8223 | 1.2947 | 167.62 | 4 | 5.1788 | 670.48 |
| 100 | 154.3 | 1.1752 | 1.5431 | 238.11 | 1 | 1.5431 | 238.11 |
| Sum | | | | | | 14.1027 | 1688.41 |

$$\bar{y} = \frac{Q_y}{A} = \frac{1688.41}{14.1027} = 119.7 \text{ ft} \blacklozenge$$

## 8.39

The volume element is the shaded thin disk.




$r = R\cos\phi \qquad \bar{z}_{el} = z = R\sin\phi$

$\therefore\ dz = R\cos\phi\, d\phi \qquad dV = \pi r^2 dz = \pi R^3 \cos^3\phi\, d\phi$

$$V = \int_V dV = \pi R^3 \int_0^{\pi/2} \cos^3\phi\, d\phi = \frac{2\pi}{3} R^3$$

$$Q_{xy} = \int_V \bar{z}_{el}\, dV = \pi R^4 \int_0^{\pi/2} \sin\phi \cos^3\phi\, d\phi = \pi R^4 \int_0^{\pi/2} d\left(-\frac{1}{4}\cos^4\phi\right) = \frac{\pi}{4} R^4$$

$\therefore\ \bar{z} = \dfrac{Q_{xy}}{V} = \dfrac{\pi R^4/4}{2\pi R^3/3} = \dfrac{3}{8} R$ ♦ Due to symmetry $\bar{x} = \bar{y} = 0$ ♦ Agrees with Table 8.3

## 8.40

The surface element dA is the ring formed by rotating the arc R dϕ about the z-axis (see figure in solution of Prob. 8.39).

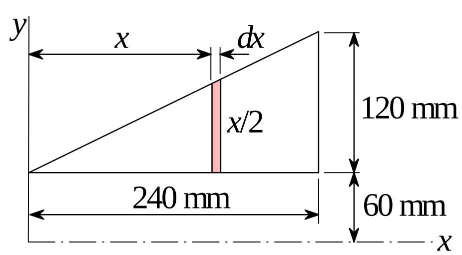
$r = R\cos\phi \qquad \bar{z}_{el} = z = R\sin\phi \qquad dA = (2\pi r)(R\, d\phi) = 2\pi R^2 \cos\phi\, d\phi$

$$A = \int_A dA = 2\pi R^2 \int_0^{\pi/2} \cos\phi\, d\phi = 2\pi R^2$$

$$Q_{xy} = \int_A \bar{z}_{el}\, dA = 2\pi R^3 \int_0^{\pi/2} \sin\phi \cos\phi\, d\phi = \pi R^3 \int_0^{\pi/2} \sin 2\phi\, d\phi = \pi R^3$$

$\therefore\ \bar{z} = \dfrac{Q_{xy}}{A} = \dfrac{\pi R^3}{2\pi R^2} = \dfrac{1}{2} R$ ♦ Due to symmetry $\bar{x} = \bar{y} = 0$ ♦ Agrees with Table 8.4

## 8.41



Volume element is a thin disk obtained by rotating the shaded are about $x$-axis.

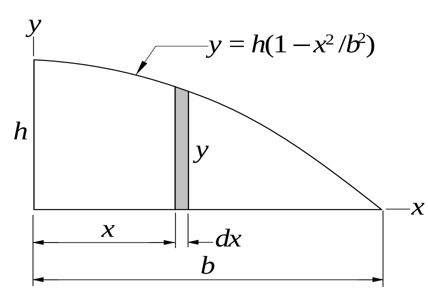
$$\begin{aligned}
\bar{x}_{\text{el}} &= x \\
dV &= \pi\left(60+\frac{x}{4}\right)^2\ dx - \pi\,(60)^2\ dx = \pi\left(60+\frac{x}{4}\right) x\ dx \\
V &= \int_{\mathcal{V}} dV = \pi\int_0^{240}\left(60+\frac{x}{4}\right) x\ dx = 9.048\times10^6\ \text{mm}^2 \\
Q_{xy} &= \int_{\mathcal{V}} \bar{x}_{\text{el}} dV = \pi\int_0^{240}\left(60+\frac{x}{4}\right) x^2\ dx = 1.520\times10^9\ \text{mm}^3 \\
\bar{x} &= \frac{Q_{xy}}{V} = \frac{1.520\times10^9}{9.048\times10^6} = 168.0\ \text{mm}\ \blacktriangleleft \qquad \bar{y}=\bar{z}=0\ \blacktriangleleft
\end{aligned}$$

## 8.42

Volume element is a thin shell obtained by rotating the shaded area about $y$-axis.

$$\begin{aligned}
\bar{x}_{\text{el}} &= x \qquad \bar{y}_{\text{el}} = 60+\frac{x}{4} \\
dV &= 2\pi\bar{x}_{\text{el}}\left(\frac{x}{2}\ dx\right) = 2\pi x\left(\frac{x}{2}\ dx\right) = \pi x^2\ dx \\
V &= \int_{\mathcal{V}} dV = \pi\int_0^{240} x^2\ dx = 14.476\times10^6\ \text{mm}^2 \\
Q_{zx} &= \int_V \bar{y}_{\text{el}}\ dV = \pi\int_0^{240}\left(60+\frac{x}{4}\right) x^2\ dx = 1.520\times10^9\ \text{mm}^3 \\
\bar{y} &= \frac{Q_{zx}}{V} = \frac{1.520\times10^9}{14.476\times10^6} = 105.0\ \text{mm}\ \blacktriangleleft \qquad \bar{x}=\bar{z}=0\ \blacktriangleleft
\end{aligned}$$

## 8.43




Volume element is a thin disk obtained by rotating the shaded area about $x$-axis.

$$\begin{aligned}
dV &= \pi y^2 dx = \pi h^2\left(1-\frac{x^2}{b^2}\right)^2 dx \qquad \bar{x}_{\text{el}} = x \\
V &= \int_{\mathcal{V}} dV = \pi h^2\int_0^b\left(1-\frac{x^2}{b^2}\right)^2 dx = \frac{8}{15}\pi b h^2
\end{aligned}$$

$$Q_{yz} = \int_{\mathcal{V}} \bar{x}_{\text{el}}\, dV = \pi h^2 \int_0^b x\left(1-\frac{x^2}{b^2}\right)^2 dx = \frac{1}{6}\pi b^2 h^2$$

$$\therefore \quad \bar{x} = \frac{Q_{yz}}{V} = \frac{5}{16}b \ \blacktriangleleft \qquad \text{By symmetry } \bar{y} = z = 0 \ \blacktriangleleft$$

## 8.44

Volume element is a thin-walled cylinder obtained by rotating the shaded area about the $y$-axis (see figure in solution of Prob. 8.43).

$$dV = 2\pi xy\, dx = 2\pi x h\left(1-\frac{x^2}{b^2}\right)dx \qquad \bar{y}_{\text{el}} = \frac{1}{2}y = \frac{h}{2}\left(1-\frac{x^2}{b^2}\right)$$

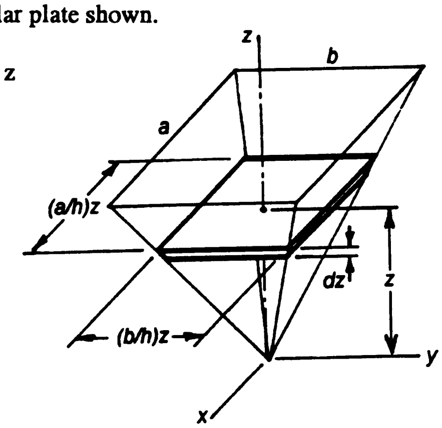
$$V = \int_{\mathcal{V}} dV = 2\pi h \int_0^b x\left(1-\frac{x^2}{b^2}\right)dx = \frac{1}{2}\pi b^2 h$$

$$Q_{zx} = \int_{\mathcal{V}} \bar{y}_{\text{el}}\, dV = \pi h^2 \int_0^b x\left(1-\frac{x^2}{b^2}\right)^2 dx = \frac{1}{6}\pi b^2 h^2$$

$$\therefore \quad \bar{y} = \frac{Q_{zx}}{V} = \frac{1}{3}h \ \blacktriangleleft \qquad \text{By symmetry } \bar{x} = \bar{z} = 0 \ \blacktriangleleft$$

## 8.45

**The volume element is the thin rectangular plate shown.**




$$dV = \left(\frac{a}{h}z\right)\left(\frac{b}{h}z\right)dz = \frac{ab}{h^2}z^2\,dz \qquad \bar{z}_{el} = z$$

$$V = \int_V dV = \frac{ab}{h^2}\int_0^h z^2\,dz = \frac{abh}{3}$$

$$Q_{xy} = \int_V \bar{z}_{el}\,dV = \frac{ab}{h^2}\int_0^h z^3\,dz = \frac{abh^2}{4}$$

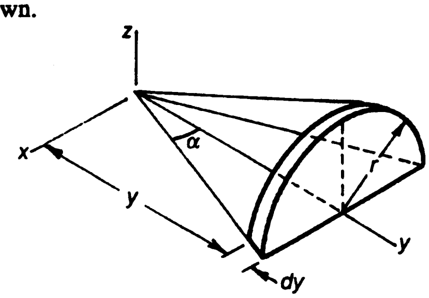
$$\therefore \bar{z} = \frac{Q_{xy}}{V} = \frac{abh^2/4}{abh/3} = \frac{3}{4}h \ \blacklozenge$$

**Due to symmetry** $\bar{x} = \bar{y} = 0$ ◆

## 8.46

The volume element is the thin "slice" shown.

$$r = \frac{R}{h}y \qquad \bar{z}_{el} = \frac{4}{3\pi}r = \frac{4R}{3\pi h}y$$




$$dV = \frac{\pi}{2}r^2\,dy = \frac{\pi R^2}{2h^2}y^2\,dy$$

$$V = \int_V dV = \frac{\pi R^2}{2h^2}\int_0^h y^2\,dy = \frac{\pi R^2 h}{6}$$

$$Q_{xy} = \int_V \bar{z}_{el}\,dV = \frac{2R^3}{3h^3}\int_0^h y^3\,dy = \frac{R^3 h}{6} \qquad \therefore\ \bar{z} = \frac{Q_{xy}}{V} = \frac{R^3h/6}{\pi R^2 h/6} = \frac{R}{\pi}\ \blacklozenge$$

## 8.47

The area element is the shaded surface of the thin slice used in Prob. 8.46.

$$r = \frac{R}{h}y \qquad \bar{z}_{el} = \frac{2}{\pi}r = \frac{2R}{\pi h}y \qquad dA = \pi r\frac{dy}{\cos\alpha} = \frac{\pi R}{h\cos\alpha}y\,dy$$

$$A = \int_A dA = \frac{\pi R}{h\cos\alpha}\int_0^h y\,dy = \frac{\pi R h}{2\cos\alpha} \qquad Q_{xy} = \int_A \bar{z}_{el}\,dA = \frac{2R^2}{h^2\cos\alpha}\int_0^h y^2\,dy = \frac{2R^2 h}{3\cos\alpha}$$

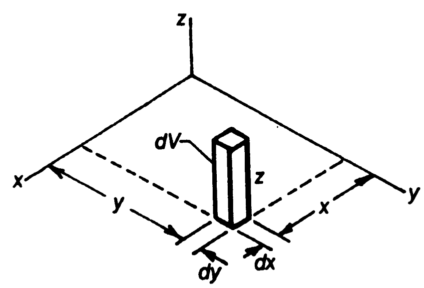
$$\therefore\ \bar{z} = \frac{Q_{xy}}{A} = \frac{2R^2h/(3\cos\alpha)}{\pi R h/(2\cos\alpha)} = \frac{4R}{3\pi}\ \blacklozenge$$

## 8.48

We use double integration.

$$dV = z\,dx\,dy = \left[h_1 - (h_1 - h_2)\frac{xy}{ab}\right]dx\,dy$$

$$\bar{x}_{el} = x \qquad \bar{y}_{el} = y$$




$$V = \int_V dV = \int_0^b\int_0^a \left[h_1 - (h_1 - h_2)\frac{xy}{ab}\right]dx\,dy$$

$$= \int_0^b \left[h_1 a - (h_1 - h_2)\frac{ay}{2b}\right]dy$$

$$= h_1 ab - (h_1 - h_2)\frac{ab}{4} = (3h_1 + h_2)\frac{ab}{4}$$

$$Q_{yz} = \int_V \bar{x}_{el}\, dA = \int_0^b \int_0^a \left[ h_1 x - (h_1 - h_2)\frac{x^2 y}{ab} \right] dx\, dy = \int_0^b \left[ h_1 \frac{a^2}{2} - (h_1 - h_2)\frac{a^2 y}{3b} \right] dy$$

$$= h_1 \frac{a^2 b}{2} - (h_1 - h_2)\frac{a^2 b}{6} = (2h_1 + h_2)\frac{a^2 b}{6}$$

$$\therefore\ \bar{x} = \frac{Q_{yz}}{V} = \frac{(2h_1 + h_2)a^2 b/6}{(3h_1 + h_2)ab/4} = \frac{2(2h_1 + h_2)}{3(3h_1 + h_2)} a \ \blacklozenge \qquad \text{By analogy } \bar{y} = \frac{2(2h_1 + h_2)}{3(3h_1 + h_2)} b \ \blacklozenge$$

## 8.49

We choose double integration using the volume element shown in the solution of Prob. 8.48.

$$\begin{aligned} dV &= z\, dx\, dy = h \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} dx\, dy \\ \bar{z}_{el} &= \frac{1}{2} z = \frac{1}{2} h \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} \\ V &= \int_{\mathcal{V}} dV = h \int_{-a}^{a} \int_{-b}^{b} \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} dx\, dy = 4h \int_0^a \int_0^b \cos\frac{\pi x}{2a} \cos\frac{\pi y}{2b} dx\, dy \\ &= 4h \left(\frac{2a}{\pi}\right)\left(\frac{2b}{\pi}\right) = \frac{16}{\pi^2} abh \end{aligned}$$

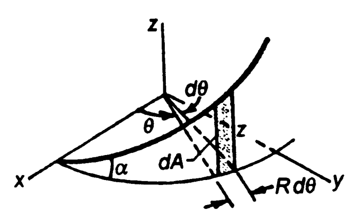
$$\begin{aligned} Q_{xy} &= \int_{\mathcal{V}} \bar{z}_{el}\, dV = \frac{1}{2} h^2 \int_{-a}^{a} \int_{-b}^{b} \cos^2\frac{\pi x}{2a} \cos^2\frac{\pi y}{2b}\, dx\, dy \\ &= 2h^2 \int_0^a \int_0^b \cos^2\frac{\pi x}{2a} \cos^2\frac{\pi y}{2b}\, dx\, dy = 2h^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) = \frac{abh^2}{2} \\ \therefore\ & \bar{z} = \frac{Q_{xy}}{V} = \frac{\pi^2}{32} h \ \blacktriangleleft \qquad \text{By symmetry } \bar{x} = \bar{y} = 0 \ \blacktriangleleft \end{aligned}$$

## 8.50

$$dA = zR\, d\theta = \frac{hR}{\pi} \theta\, d\theta \qquad A = \frac{\pi}{2} hR$$

$$\bar{x}_{el} = R\cos\theta \qquad \bar{y}_{el} = R\sin\theta \qquad \bar{z}_{el} = \frac{z}{2} = \frac{h}{2\pi}\theta$$

$$Q_{xy} = \int_A \bar{z}_{el}\, dA = \frac{h^2 R}{2\pi^2} \int_0^{\pi} \theta^2\, d\theta = \frac{\pi}{6} h^2 R$$



$$Q_{yz} = \int_A \bar{x}_{el}\, dA = \frac{hR^2}{\pi}\int_0^\pi \theta \cos\theta\, d\theta = \frac{hR^2}{\pi}\Big[\cos\theta + \theta \sin\theta\Big]_0^\pi = -\frac{2}{\pi} hR^2$$

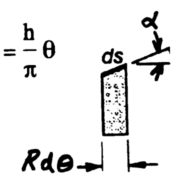
$$Q_{zx} = \int_A \bar{y}_{el}\, dA = \frac{hR^2}{\pi}\int_0^\pi \theta \sin\theta\, d\theta = \frac{hR^2}{\pi}\Big[\sin\theta - \theta \cos\theta\Big]_0^\pi = hR^2$$

$$\therefore\ \bar{x} = \frac{Q_{yz}}{A} = -\left(\frac{2}{\pi}\right)^2 R\ \blacklozenge \qquad \therefore\ \bar{y} = \frac{Q_{zx}}{A} = \frac{2}{\pi} R\ \blacklozenge \qquad \therefore\ \bar{z} = \frac{Q_{xy}}{A} = \frac{1}{3} h\ \blacklozenge$$

## 8.51

Refer to the figure in the solution of Prob. 8.50.

$$ds = \frac{R}{\cos\alpha} d\theta \qquad L = \frac{\pi R}{\cos\alpha} \qquad \bar{x}_{el} = R\cos\theta \qquad \bar{y}_{el} = R\sin\theta \qquad \bar{z}_{el} = z = \frac{h}{\pi}\theta$$




$$Q_{xy} = \int_L \bar{z}_{el}\, ds = \frac{Rh}{\pi \cos\alpha}\int_0^\pi \theta\, d\theta = \frac{\pi Rh}{2\cos\alpha}$$

$$Q_{yz} = \int_L \bar{x}_{el}\, ds = \frac{R^2}{\cos\alpha}\int_0^\pi \cos\theta\, d\theta = 0 \qquad Q_{zx} = \int_L \bar{y}_{el}\, ds = \frac{R^2}{\cos\alpha}\int_0^\pi \sin\theta\, d\theta = \frac{2R^2}{\cos\alpha}$$

$$\therefore\ \bar{x} = \frac{Q_{yz}}{L} = 0\ \blacklozenge \qquad \therefore\ \bar{y} = \frac{Q_{zx}}{L} = \frac{2}{\pi} R\ \blacklozenge \qquad \therefore\ \bar{z} = \frac{Q_{xy}}{L} = \frac{1}{2} h\ \blacklozenge$$

## 8.52

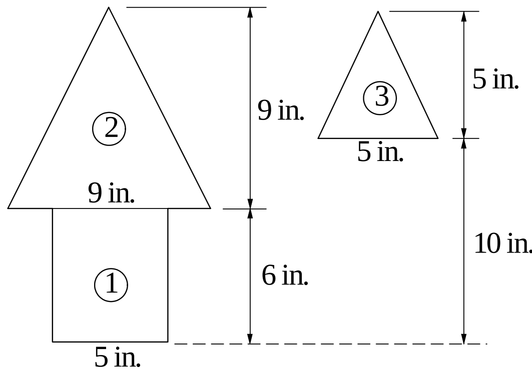
$$\text{Block:}\quad V_1 = 160(340)(70) = 3.808 \times 10^6\ \text{mm}^3 \qquad \bar{x}_1 = 170\ \text{mm}$$

$$\text{Hole:}\quad V_2 = -\frac{\pi(130^2)}{4}(70) = -0.9291 \times 10^6\ \text{mm}^3 \qquad \bar{x}_2 = 80\ \text{mm}$$

$$\bar{x} = \frac{\Sigma V_1 \bar{x}_i}{\Sigma V_i} = \frac{3.808(170) - 0.9291(80)}{3.808 - 0.9291} = 199.1\ \text{mm}\ \blacktriangleleft$$

By symmetry $\bar{y} = 80$ mm ◀ $\bar{z} = 35$ mm ◀

**8.53**



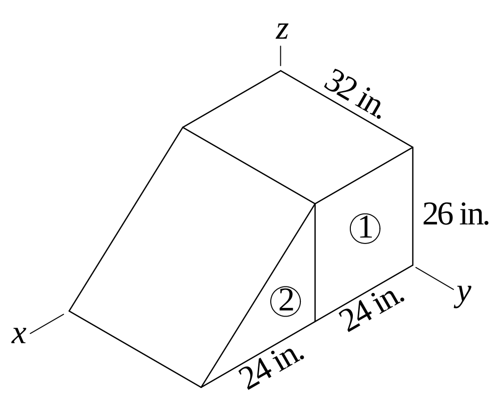

$$\begin{aligned} V_1 &= \pi\,(2.5)^2\,(6) = 117.81 \text{ in}^3 \qquad \bar{z}_1 = 3 \text{ in.} \\ V_2 &= \frac{\pi}{3}(4.5)^2(9) = 190.85 \text{ in}^3 \qquad \bar{z}_2 = 6 + \frac{9}{4} = 8.25 \text{ in.} \\ V_3 &= -\frac{\pi}{3}(2.5)^2(5) = -32.73 \text{ in}^3 \qquad \bar{z}_3 = 10 + \frac{5}{4} = 11.25 \text{ in.} \end{aligned}$$

$$\begin{aligned} V &= \Sigma V_i = 117.81 + 190.85 - 32.73 = 275.9 \text{ in}^3 \\ Q_{xy} &= \Sigma V_i \bar{z}_i = 117.81(3) + 190.85(8.25) - 32.73(11.25) \\ &= 1559.7 \text{ in}^4 \end{aligned}$$

$$\bar{z} = \frac{Q_{xy}}{V} = \frac{1559.7}{275.9} = 5.65 \text{ in.} \blacktriangleleft$$

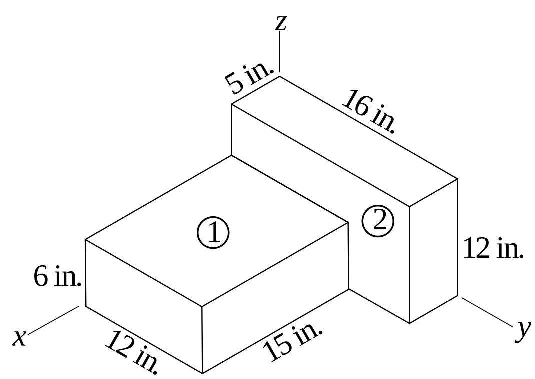
By symmetry $\overline{x} = \overline{y} = 0$ ◄

**8.54**



$$
\begin{aligned}
V_1 &= 24(26)(32) = 19.968 \times 10^3 \text{ in}^3 \qquad \bar{x}_1 = 12 \text{ in.} \qquad \bar{z}_1 = 13 \text{ in.} \\
V_2 &= \frac{1}{2}V_1 = 9.984 \times 10^3 \text{ in}^3 \qquad \bar{x}_2 = 32 \text{ in.} \qquad \bar{z}_2 = \frac{26}{3} = 8.667 \text{ in.} \\
V &= \Sigma V_i = (19.986 + 9.984) \times 10^3 = 29.97 \times 10^3 \text{ in}^3 \\
\bar{x} &= \frac{\Sigma V_i \bar{x}_i}{V} = \frac{19.968(12) + 9.984(32)}{29.97} = 18.66 \text{ in.} \blacktriangleleft \\
\bar{z} &= \frac{\Sigma V_i \bar{z}_i}{V} = \frac{19.968(13) + 9.984(8.667)}{29.97} = 11.55 \text{ in.} \blacktriangleleft \\
&\text{By symmetry } \bar{y} = 16 \text{ in.} \blacktriangleleft
\end{aligned}
$$

**8.55**

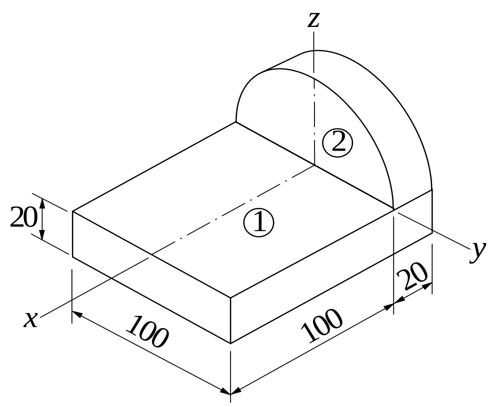



$$
\begin{aligned}
V_1 &= (15)(12)(5) = 900 \text{ in}^3 \qquad V_2 = (5)(16)(12) = 960 \text{ in}^3 \\
\bar{x}_1 &= 12.5 \text{ in.} \qquad \bar{x}_2 = 2.5 \text{ in.} \\
\bar{y}_1 &= 6 \text{ in.} \qquad \bar{y}_2 = 8 \text{ in.} \qquad \bar{z}_1 = 3 \text{ in.} \qquad \bar{z}_2 = 6 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
V &= \Sigma V_i = 900 + 960 = 1860 \text{ in}^3 \\
Q_{yz} &= \Sigma V_i \bar{x}_i = 900(12.5) + 960(2.5) = 13\ 650 \text{ in}^4 \\
Q_{zx} &= \Sigma V_i \bar{y}_i = 900(6) + 960(8) = 13\ 080 \text{ in}^4 \\
Q_{xy} &= \Sigma V_i \bar{z}_i = 900(3) + 960(8) = 10\ 380 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{Q_{yz}}{V} = \frac{13\ 650}{1860} = 7.34 \text{ in.} \blacktriangleleft \\
\bar{y} &= \frac{Q_{zx}}{V} = \frac{13\ 080}{1860} = 7.03 \text{ in.} \blacktriangleleft \\
\bar{z} &= \frac{Q_{xy}}{V} = \frac{10\ 380}{1860} = 5.58 \text{ in.} \blacktriangleleft
\end{aligned}
$$

**8.56**



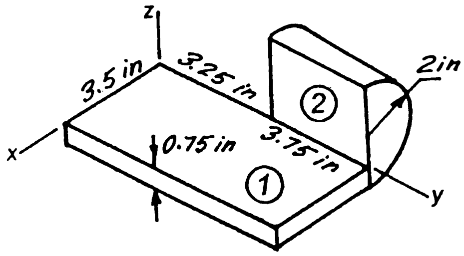

$$
\begin{aligned}
V_1 &= (120)(100)(20) = 240\ 000\ \text{mm}^3 \\
V_2 &= \frac{\pi(50^2)}{2}(20) = 78\ 540\ \text{mm}^3 \\
\bar{x}_1 &= 50\ \text{mm} \qquad \bar{x}_2 = -10\ \text{mm} \\
\bar{z}_1 &= -10\ \text{mm} \qquad \bar{z}_2 = \frac{4}{3\pi}(50) = 21.22\ \text{mm}
\end{aligned}
$$

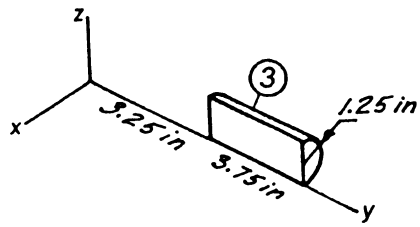
$$
\begin{aligned}
V &= \Sigma V_i = 240\ 000 + 78\ 540 = 318\ 500\ \text{mm}^3 \\
Q_{yz} &= \Sigma V_i \bar{x}_i = 240\ 000(50) + 78\ 540(-10) = 11.215 \times 10^6\ \text{mm}^4 \\
Q_{xy} &= \Sigma V_i \bar{z}_i = 240\ 000(-10) + 78\ 540(21.22) = -0.7334 \times 10^6\ \text{mm}^4
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{Q_{yz}}{V} = \frac{11.215 \times 10^6}{318\ 500} = 35.2\ \text{mm} \ ◀ \\
\bar{z} &= \frac{Q_{xy}}{V} = \frac{-0.7334 \times 10^6}{318\ 500} = -2.30\ \text{mm} \ ◀
\end{aligned}
$$

By symmetry $y = 0$ ◀

**8.57**




$V_1 = (7)(3.5)(0.75) = 18.375 \text{ in}^3$

$V_2 = \frac{\pi}{2}(2^2)(3.75) = 23.562 \text{ in}^3$

$\bar{x}_2 = -\frac{4(2)}{3\pi} = -0.8488 \text{ in}$

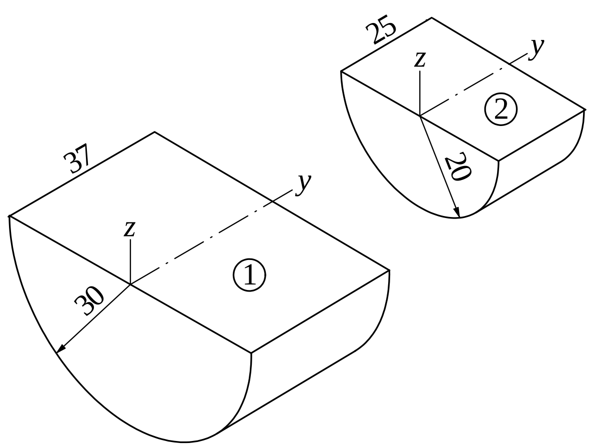
$V_3 = -\frac{\pi}{2}(1.25^2)(3.75) = -9.204 \text{ in}^3$

$\bar{x}_3 = -\frac{4(1.25)}{3\pi} = -0.5305 \text{ in}$

| Part | V (in$^3$) | $\bar{x}$ (in) | $V\bar{x}$ (in$^4$) | $\bar{y}$ (in) | $V\bar{y}$ (in$^4$) | $\bar{z}$ (in) | $V\bar{z}$ (in$^4$) |
|---|---|---|---|---|---|---|---|
| 1 | 18.375 | 1.7500 | 32.156 | 3.5000 | 64.313 | −0.3750 | −6.891 |
| 2 | 23.562 | −0.8488 | −19.999 | 5.1250 | 120.755 | 1.2500 | 29.453 |
| 3 | −9.204 | −0.5305 | 4.883 | 5.1250 | −47.171 | 1.2500 | −11.505 |
| Sum | 32.733 | | 17.040 | | 137.897 | | 11.057 |

$\therefore \bar{x} = \frac{17.040}{32.733} = 0.521 \text{ in}$ ◆ $\quad \bar{y} = \frac{137.897}{32.733} = 4.213 \text{ in}$ ◆ $\quad \bar{z} = \frac{11.057}{32.733} = 0.338 \text{ in}$ ◆

**8.58**



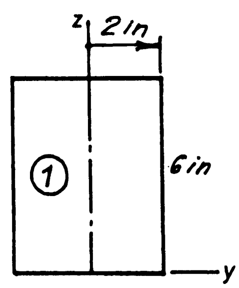
$$V_1 = \frac{\pi(30^2)}{2}(37) = 52\ 308\ \text{mm}^3 \qquad V_2 = -\frac{\pi(20^2)}{2}(25) = -15\ 708\ \text{mm}^3$$
$$\bar{y}_1 = \frac{37}{2} = 18.5\ \text{mm} \qquad \bar{y}_2 = \frac{25}{2} = 12.5\ \text{mm}$$
$$\bar{z}_1 = -\frac{4}{3\pi}(30) = -12.732\ \text{mm} \qquad \bar{z}_2 = -\frac{4}{3\pi}(20) = -8.488\ \text{mm}$$

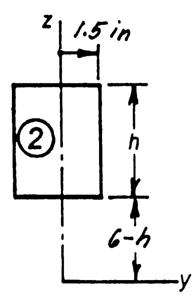
$$V = \Sigma V_i = 52\ 308 - 15\ 708 = 36\ 600\ \text{mm}^2$$
$$Q_{zx} = \Sigma V_i \bar{y}_i = 52\ 308(18.5) - 15\ 708(12.5) = 0.7714 \times 10^6\ \text{mm}^4$$
$$Q_{xy} = \Sigma V_i \bar{z}_i = 52\ 308(-12.732) - 15\ 708(-8.488) = -0.5327 \times 10^6\ \text{mm}^4$$

$$\bar{y} = \frac{Q_{zx}}{V} = \frac{0.7714 \times 10^6}{36\ 600} = 21.1\ \text{mm} \blacktriangleleft$$
$$\bar{z} = \frac{Q_{xy}}{V} = \frac{-0.5327 \times 10^6}{36\ 600} = -14.56\ \text{mm} \blacktriangleleft$$

By symmetry $\bar{x} = 0$ ◄

**8.59**

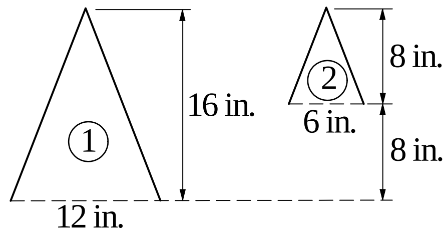



$$V_1 = \pi(2^2)(6) = 24\pi\ \text{in}^3 \qquad \bar{z}_1 = 3\ \text{in}$$
$$V_2 = -\pi(1.5^2)h = -2.25\pi h\ \text{in}^3$$
$$\bar{z}_2 = 6 - \frac{h}{2}\ \text{in}$$
$$V = V_1 + V_2 = \pi(24 - 2.25h)\ \text{in}^3$$
$$Q_{xy} = \bar{z}_1 V_1 + \bar{z}_2 V_2$$
$$= (3)(24\pi) + \left(6 - \frac{h}{2}\right)(-2.25\pi h) = \pi(72 - 13.5h + 1.125h^2)$$
$$\therefore \bar{z} = \frac{Q_{xy}}{V} = \frac{72 - 13.5h + 1.125h^2}{24 - 2.25h}$$
$$\therefore \frac{d\bar{z}}{dh} = \frac{(24 - 2.25h)(-13.5 + 2.25h) + (72 - 13.5h + 1.125h^2)(2.25)}{(24 - 2.25h)^2}$$
$$= \frac{-162 + 54h - 2.53125h^2}{(24 - 2.25h)^2}$$

$d\bar{z}/dh = 0$: $-162 + 54h - 2.53125h^2 = 0$ The positive root is $h = 3.61$ in ◆

**8.60**




$$A_1 = \pi(6)\sqrt{6^2+16^2} = 322.1 \text{ in}^2 \qquad \bar{z}_1 = \frac{16}{3} = 5.333 \text{ in.}$$

$$A_2 = -\pi(3)\sqrt{3^2+8^2} = -80.53 \text{ in}^2 \qquad \bar{z}_2 = 8 + \frac{8}{3} = 10.667 \text{ in.}$$

$$A = A_1 + A_2 = 322.1 - 80.53 = 241.6 \text{ in}^2$$

$$Q_{xy} = A_1\bar{z}_1 + A_2\bar{z}_2 = 322.1(5.333) - 80.53(10.667) = 858.8 \text{ in}^3$$

$$\bar{z} = \frac{Q_{xy}}{A} = \frac{858.8}{241.6} = 3.55 \text{ in.} \blacktriangleleft$$

By symmetry $\bar{x} = \bar{y} = 0$ ◄

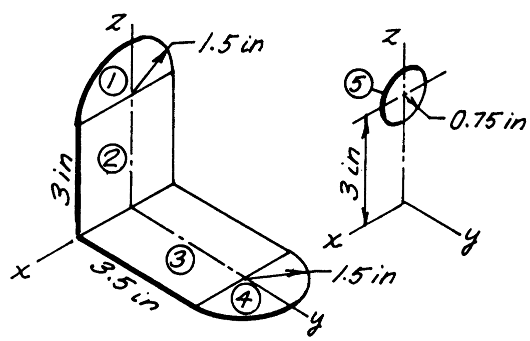
**8.61**

$A_1 = A_4 = \frac{1}{2}\pi(1.5^2) = 3.534 \text{ in}^2$

$\bar{z}_1 = 3 + \frac{4}{3\pi}(1.5) = 3.637 \text{ in}$

$\bar{y}_4 = 3.5 + \frac{4}{3\pi}(1.5) = 4.137 \text{ in}$

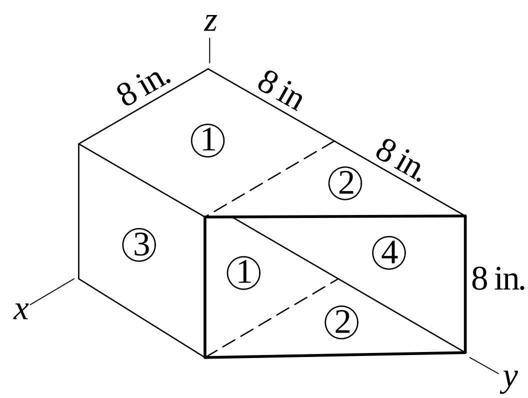
$A_5 = -\pi(0.75^2) = -1.767 \text{ in}^2$




| Part | A (in$^2$) | $\bar{y}$ (in) | A$\bar{y}$ (in$^3$) | $\bar{z}$ (in) | A$\bar{z}$ (in$^3$) |
|---|---|---|---|---|---|
| 1 | 3.534 | 0 | 0 | 3.637 | 12.853 |
| 2 | 9.000 | 0 | 0 | 1.500 | 13.500 |
| 3 | 10.500 | 1.750 | 18.375 | 0 | 0 |
| 4 | 3.534 | 4.137 | 14.620 | 0 | 0 |
| 5 | −1.767 | 0 | 0 | 3.000 | −5.301 |
| Sum | 24.801 | | 32.995 | | 21.052 |

$\therefore \bar{y} = \frac{32.995}{24.801} = 1.330$ in ◆ $\quad \therefore \bar{z} = \frac{21.052}{24.801} = 0.849$ in ◆ $\quad$ By symmetry $\bar{x} = 0$ ◆

## 8.62




$$A_1 = A_4 = 128 \text{ in}^2 \qquad A_2 = A_3 = 64 \text{ in}^2$$
$$A = \Sigma A_i = 2\,(128 + 64) = 384 \text{ in}^2$$

$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{A} = \frac{128(4) + 64(8/3) + 64(8) + 0}{384} = 3.11 \text{ in.} \blacktriangleleft$$
$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{128(4) + 64(8 + 8/3) + 64(4) + 128(8)}{384} = 6.44 \text{ in.} \blacktriangleleft$$

By symmetry $\bar{z} = 4$ in. ◄

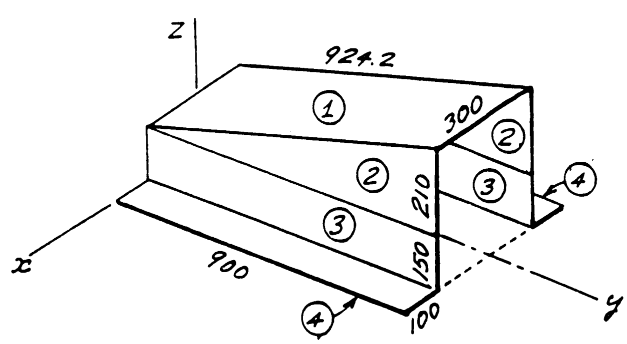
## 8.63

| Part | A ($m^2$) | $\bar{y}$ (m) | $A\bar{y}$ ($m^3$) | $\bar{z}$ (m) | $A\bar{z}$ ($m^3$) |
|---|---|---|---|---|---|
| 1 | 0.2773 | 0.450 | 0.1248 | 0.255 | 0.0707 |
| 2 | 0.1890 | 0.600 | 0.1134 | 0.220 | 0.0416 |
| 3 | 0.2700 | 0.450 | 0.1215 | 0.075 | 0.0203 |
| 4 | 0.1800 | 0.450 | 0.0810 | 0.000 | 0.0000 |
| Sum | 0.9163 | | 0.4407 | | 0.1326 |

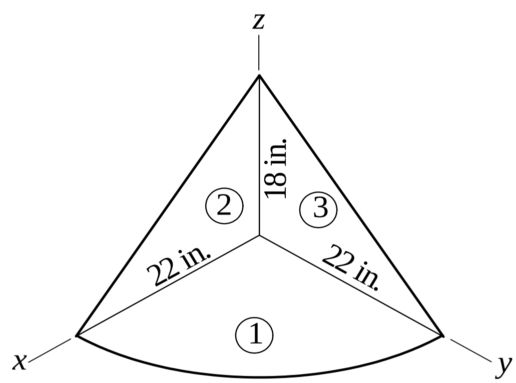
$\therefore\ \bar{y} = \dfrac{0.4407}{0.9163} = 0.481$ m ♦

$\therefore\ \bar{z} = \dfrac{0.1326}{0.9163} = 0.145$ m ♦

Due to symmetry $\bar{x} = 0$ ♦



**8.64**



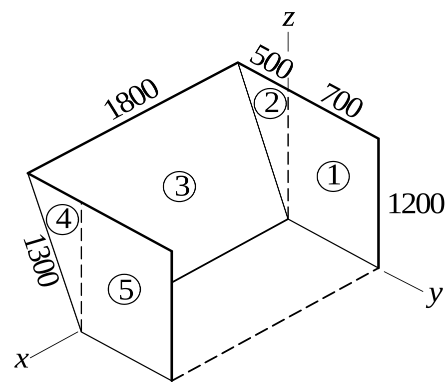

$$A_1 = \frac{\pi}{4}(22^2) = 380.1 \text{ in}^2 \qquad A_2 = A_3 = \frac{1}{2}(22)(18) = 198 \text{ in}^2$$
$$\bar{x}_1 = \frac{4}{3\pi}(22) = 9.337 \text{ in.} \qquad \bar{z}_1 = 0$$
$$\bar{x}_2 = \frac{22}{3} = 7.333 \text{ in.} \qquad \bar{z}_2 = 6 \text{ in.}$$
$$\bar{x}_3 = 0 \qquad \bar{z}_3 = 6 \text{ in.}$$

$$A = \Sigma A_i = 380.1 + 2(198) = 776.1 \text{ in}^2$$
$$Q_{yz} = \Sigma A_i \bar{x}_i = 380.1(9.337) + 198(7.333) + 0 = 5001 \text{ in}^3$$
$$Q_{xy} = \Sigma A_i \bar{z}_i = 0 + 2(198)(6) = 2376 \text{ in}^3$$

$$\bar{x} = \frac{Q_{yz}}{A} = \frac{5001}{776.1} = 6.44 \text{ in.} \blacktriangleleft$$
$$\bar{z} = \frac{Q_{xy}}{A} = \frac{2376}{776.1} = 3.06 \text{ in.} \blacktriangleleft$$

By symmetry $\bar{y} = 6.44$ in. ◀

**8.65**




$$\begin{aligned} A_1 &= A_5 = 700(1200) = 840 \times 10^3 \text{ mm}^2 \\ A_2 &= A_4 = \frac{1}{2}(500)(1200) = 300 \times 10^3 \text{ mm}^2 \\ A_3 &= 1300(1800) = 2340 \times 10^3 \text{ mm}^2 \\ A &= \Sigma A_i = 2(840 + 300) + 2340 \times 10^3 = 4620 \times 10^3 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{\Sigma A_i \bar{y}_i}{A} = \frac{2\,(840)\,(350) + 2\,(300)\,(-500/3) + 2340(-250)}{4620} \\ &= -21.0 \text{ mm} \blacktriangleleft \\ \bar{z} &= \frac{\Sigma A_i \bar{z}_i}{A} = \frac{2\,(840)\,(600) + 2(300)(800) + 2340(600)}{4620} = 626.0 \text{ mm} \blacktriangleleft \end{aligned}$$

**8.66**

$A_1 = (200)(250) = 50\,000 \text{ mm}^2 \qquad \bar{z}_1 = 125 \text{ mm}$

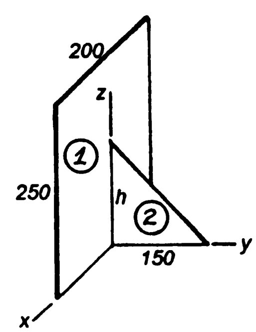
$A_2 = 75h \text{ mm}^2 \qquad \bar{z}_2 = h/3$

$A = \Sigma A_i = 50\,000 + 75h \text{ mm}^2$

$\therefore Q_{xy} = \Sigma A_i \bar{z}_i = (50\,000)(125) + (75h)(h/3)$

$= 6\,250\,000 + 25h^2 \text{ mm}^3$

$\therefore \bar{z} = \dfrac{6\,250\,000 + 25h^2}{50\,000 + 75\,h}$

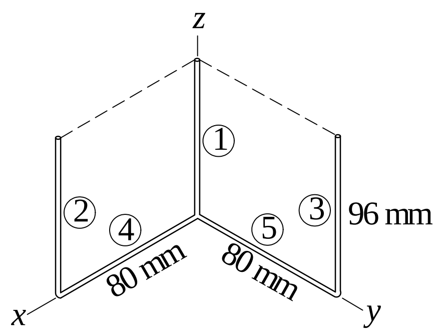


$$\therefore \frac{d\bar{z}}{dh} = \frac{(50\,000 + 75h)(50h) - (75)(6\,250\,000 + 25h^2)}{(50\,000 + 75h)^2} = \frac{1.875\,h^2 + 2500\,h - 468\,750}{(50\,000 + 75h)^2} \times 10^3$$

**$\bar{z}$ is minimized when $d\bar{z}/dh = 0$**

$\therefore$ **$1.875\,h^2 + 2500\,h - 468\,750 = 0$ The positive root is $h = 166.7$ mm ♦**

**8.67**




$$L = 3(96) + 2(80) = 448 \text{ mm}$$

$$\bar{x} = \frac{\Sigma L_i \bar{x}_i}{L} = \frac{96(80) + 80(40)}{448} = 24.3 \text{ mm} \blacktriangleleft$$

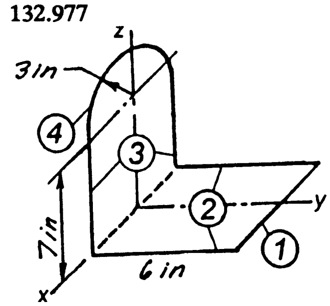
$$\bar{z} = \frac{\Sigma L_i \bar{z}_i}{L} = \frac{3(96)(48)}{448} = 30.9 \text{ mm} \blacktriangleleft$$

By symmetry $\bar{y} = \bar{x} = 24.3$ mm ◀

8.68

$L_4 = \pi(3) = 9.425$ in $\qquad \bar{z}_4 = 7 + \frac{2}{\pi}(3) = 8.910$ in

| Part | L (in) | $\bar{y}$ (in) | $L\bar{y}$ (in$^2$) | $\bar{z}$ (in) | $L\bar{z}$ (in$^2$) |
|---|---|---|---|---|---|
| 1 | 6.000 | 6.000 | 36.000 | 0.000 | 0.000 |
| 2 | 12.000 | 3.000 | 36.000 | 0.000 | 0.000 |
| 3 | 14.000 | 0.000 | 0.000 | 3.500 | 49.000 |
| 4 | 9.425 | 0.000 | 0.000 | 8.910 | 83.977 |
| Sum | 41.425 | | 72.000 | | 132.977 |

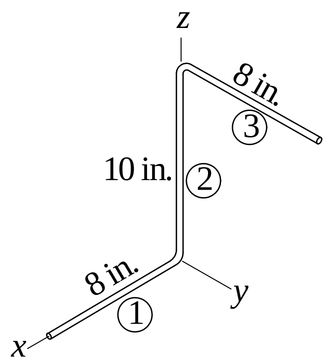



$\bar{y} = \frac{72.000}{41.425} = 1.74$ in ◆

$\bar{z} = \frac{132.977}{41.425} = 3.21$ in ◆

Due to symmetry $\bar{x} = 0$ ◆

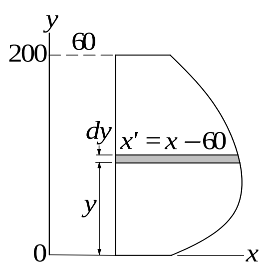
## 8.69




$$
\begin{aligned}
L &= \Sigma L_i = 26 \text{ in.} \\
Q_{yz} &= \Sigma L_i \bar{x}_i = 8(4) + 0 + 0 = 32 \text{ in}^2 \\
Q_{zx} &= \Sigma L_i \bar{y}_i = 0 + 0 + 8(4) = 32 \text{ in}^2 \\
Q_{xy} &= \Sigma L_i \bar{z}_i = 0 + 10(5) + 8(10) = 130 \text{ in}^2
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{Q_{yz}}{L} = \frac{32}{26} = 1.231 \text{ in.} \blacktriangleleft \qquad \bar{y} = \frac{Q_{zx}}{L} = \frac{32}{26} = 1.231 \text{ in.} \blacktriangleleft \\
\bar{z} &= \frac{Q_{xy}}{L} = \frac{130}{26} = 5.0 \text{ in.} \blacktriangleleft
\end{aligned}
$$

## 8.70




Volume element is the thin shell generated by rotating the shaded area about the $x$-axis.

$$
\begin{aligned}
dV &= 2\pi y x' dy \qquad \bar{x}_{\text{el}} = 60 + \frac{1}{2}x' \\
V &= \int_{\mathcal{V}} dV = 2\pi \int_0^{200} y x' \, dy \\
Q_{yz} &= \int_{\mathcal{V}} \bar{x}_{\text{el}} \, dV = 2\pi \int_0^{200} \bar{x}_{\text{el}} y x' \, dy
\end{aligned}
$$

Evaluating the integrals with Simpson's rule:

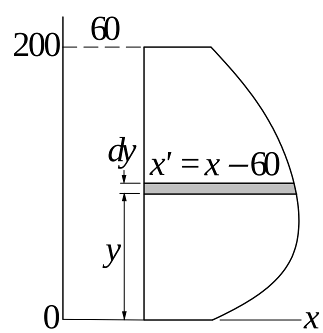
| $y$ (mm) | $x'$ (mm) | $yx'$ (mm$^2$) | $\bar{x}_{\rm el}$ (mm) | $\bar{x}_{\rm el}yx'$ ($10^6$ mm$^3$) |
|---|---|---|---|---|
| 0 | 51 | 0 | 85.5 | 0 |
| 50 | 114 | 5700 | 117.0 | 0.6669 |
| 100 | 114 | 11 400 | 117.0 | 1.3338 |
| 150 | 92 | 13 800 | 106.0 | 1.4628 |
| 200 | 51 | 10 200 | 85.5 | 0.8752 |

$$V = 2\pi\frac{50}{3}\left[0 + 4(5700) + 2(11\ 400) + 4(13\ 800) + 10\ 200\right]$$
$$= 11.624 \times 10^6 \text{ mm}^3$$
$$Q_{yz} = 2\pi\frac{50}{3}\left[0 + 4(0.6669) + 2(1.3338) + 4(1.4629) + 0.8752\right] \times 10^6$$
$$= 1.2631 \times 10^9 \text{ mm}^4$$

$$\bar{x} = \frac{Q_{yz}}{V} = \frac{1.2631 \times 10^9}{11.624 \times 10^6} = 108.7 \text{ mm} \blacktriangleleft$$

By symmetry $\bar{y} = \bar{z} = 0$ ◄

**8.71**




The volume element is a thin annular disk obtained by rotating the shaded area about the $y$-axis.

$$dV = \pi\left[(60 + x')^2 - 60^2\right] dy = \pi x'(x' + 120)dy \qquad \bar{y}_{el} = y$$
$$V = \int_{\mathcal{V}} dV = \pi\int_0^{200} x'(x' + 120)dy$$
$$Q_{zx} = \int_{\mathcal{V}} \bar{y}_{el}\ dV = \pi\int_0^{200} yx'(x' + 120)dy$$

Evaluating the integrals with Simpson's rule:

| $y$ (mm) | $x'$ (mm) | $x'(x'+120)$ (mm$^2$) | $yx'(x'+120)$ ($10^3$ mm$^3$) |
|---|---|---|---|
| 0 | 51 | 8721 | 0 |
| 50 | 114 | 26 676 | 1334 |
| 100 | 114 | 26 676 | 2668 |
| 150 | 92 | 19 504 | 2926 |
| 200 | 51 | 8721 | 1744 |

$$\begin{aligned} V &= \pi\frac{50}{3}[8721 + 4(26\ 676) + 2(26\ 676) + 4(19\ 504) + 8721] \\ &= 13.379 \times 10^6 \text{ mm}^3 \\ Q_{zx} &= \pi\frac{50}{3}[0 + 4(1334) + 2(2668) + 4(2926) + 1744]\,10^3 \\ &= 1.2629 \times 10^9 \text{ mm}^4 \end{aligned}$$

$$\bar{y} = \frac{Q_{zx}}{V} = \frac{1.2629 \times 10^9}{13.379 \times 10^6} = 94.4 \text{ mm} \blacktriangleleft$$

By symmetry $x = z = 0$ ◄

## 8.72

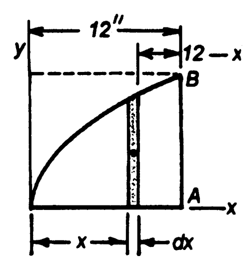
**The volume element is the thin cylindrical shell obtained by rotating the shaded area about the axis AB.**

$$dV = 2\pi(12 - x)y\,dx \qquad \bar{y}_{el} = y/2$$

$$V = \int_V dV = 2\pi\int_0^{12}(12 - x)y\,dx \qquad Q_{zx} = \int_V \bar{y}_{el}\,dV = \pi\int_0^{12}(12 - x)y^2\,dx$$

**Evaluating the integrals by Simpson's rule:**

| x (in) | y (in) | W | W(12 − x)y | W(12 − x)$y^2$ |
|---|---|---|---|---|
| 0 | 0.00 | 1 | 0.0 | 0 |
| 2 | 4.90 | 4 | 196.0 | 960 |
| 4 | 6.93 | 2 | 110.9 | 768 |
| 6 | 8.49 | 4 | 203.8 | 1730 |
| 8 | 9.80 | 2 | 78.4 | 768 |
| 10 | 10.95 | 4 | 87.6 | 959 |
| 12 | 12.00 | 1 | 0.00 | 0 |
| | | | 676.7 | 5185 |



$$V \approx 2\pi \frac{\Delta x}{3} \sum_i W_i(12 - x_i)y_i = 2\pi \frac{2}{3}(676.7) = 2835 \text{ in}^3$$

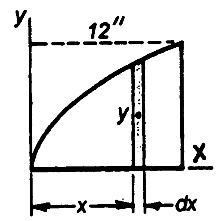
$$Q_{zx} \approx \pi \frac{\Delta x}{3} \sum_i W_i(12 - x_i)y_i^2 = \pi \frac{2}{3}(5185) = 10\,859 \text{ in}^4$$

$$\therefore \bar{y} = \frac{10\,859}{2835} = 3.83 \text{ in} \ \blacklozenge \qquad \text{Due to symmetry } \bar{x} = 12 \text{ in} \ \blacklozenge \quad \bar{z} = 0 \ \blacklozenge$$

## 8.73

The volume element is a thin disk obtained by rotating the shaded area about the x-axis.

$$dV = \pi y^2\, dx \qquad \bar{x}_{el} = x$$




$$V = \int_V dV = \pi \int_0^{12} y^2\, dx \qquad Q_{yz} = \int_V \bar{x}_{el}\, dV = \pi \int_0^{12} y^2 x\, dx$$

Evaluating the integrals by Simpson's rule:

| x (in) | y (in) | W | $Wy^2$ ($\text{in}^2$) | $Wy^2x$ ($\text{in}^3$) |
|---|---|---|---|---|
| 0 | 0.00 | 1 | 0 | 0 |
| 2 | 4.90 | 4 | 96 | 192 |
| 4 | 6.93 | 2 | 96 | 384 |
| 6 | 8.49 | 4 | 288 | 1 730 |
| 8 | 9.80 | 2 | 192 | 1 537 |
| 10 | 10.95 | 4 | 480 | 4 796 |
| 12 | 12.00 | 1 | 144 | 1 728 |
| | | | 1 296 | 10 367 |

$$V \approx \pi \frac{\Delta x}{3} \sum_i W_i y_i^2 = \pi \frac{2}{3}(1296) = 2714 \text{ in}^3$$

$$Q_{yz} = \pi \frac{\Delta x}{3} \sum_i W_i y_i^2 x_i = \pi \frac{2}{3}(10\,367) = 21\,713 \text{ in}^4$$

$$\therefore \bar{x} = \frac{21\,713}{2714} = 8.00 \text{ in} \ \blacklozenge \qquad \text{Due to symmetry } \bar{y} = \bar{z} = 0 \ \blacklozenge$$

<u>Check</u>: for the paraboloid of revolution in Table 8.3

$$V = \frac{\pi}{2} R^2 h = \frac{\pi}{2}(12^2)(12) = 2714 \text{ in}^3 \qquad \bar{x} = \frac{2}{3} h = \frac{2}{3}(12) = 8.00 \text{ in} \qquad \text{Checks O.K.}$$

## 8.74

The surface element is a ring obtained by rotating the line element of length ds about the y-axis.

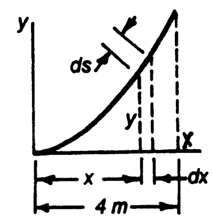
$$y = \frac{3}{16}x^2 \qquad \therefore \frac{dy}{dx} = \frac{3}{8}x \qquad \therefore \frac{ds}{dx} = \sqrt{1 + (dy/dx)^2} = \sqrt{1 + (3x/8)^2}$$

$$dA = 2\pi x\, ds = 2\pi x \frac{ds}{dx} dx = 2\pi x \sqrt{1 + (3x/8)^2}\, dx \qquad \bar{y}_{el} = y$$

$$A = \int_A dA = 2\pi \int_0^4 x \sqrt{1 + (3x/8)^2}\, dx \qquad Q_{zx} = \int_A \bar{y}_{el}\, dA = 2\pi \int_0^4 xy \sqrt{1 + (3x/8)^2}\, dx$$

Evaluating the integrals with Simpson's rule:

| x (m) | y (m) | W | $Wx\sqrt{1 + (3x/8)^2}$ | $Wxy\sqrt{1 + (3x/8)^2}$ |
|---|---|---|---|---|
| 0 | 0.0000 | 1 | 0.000 | 0.000 |
| 1 | 0.1875 | 4 | 4.272 | 0.801 |
| 2 | 0.7500 | 2 | 5.000 | 3.750 |
| 3 | 1.6875 | 4 | 18.062 | 30.480 |
| 4 | 3.0000 | 1 | 7.211 | 21.633 |
| | | | 34.545 | 56.664 |




$$A \approx 2\pi \frac{\Delta x}{3} \sum_i W_i x_i \sqrt{1 + (3x_i/8)^2} = 2\pi \frac{1}{3}(34.545) = 72.35 \text{ m}^2$$

$$Q_{zx} = 2\pi \frac{\Delta x}{3} \sum_i W_i x_i y_i \sqrt{1 + (3x_i/8)^2} = 2\pi \frac{1}{3}(56.664) = 118.68 \text{ m}^3$$

$$\therefore \bar{y} = \frac{118.68}{72.35} = 1.640 \text{ m} \; ♦ \qquad \text{Due to symmetry } \bar{x} = \bar{z} = 0 \; ♦$$

## 8.75

The surface element is a a ring obtained by rotating the line element of length ds about the x-axis.

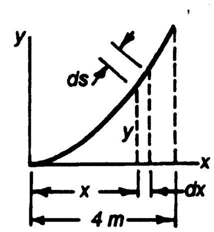
$$y = \frac{3}{16}x^2 \qquad \therefore \frac{dy}{dx} = \frac{3}{8}x \qquad \therefore \frac{ds}{dx} = \sqrt{1 + (dy/dx)^2} = \sqrt{1 + (3x/8)^2}$$

$$dA = 2\pi y\, ds = 2\pi y \frac{ds}{dx} dx = 2\pi y \sqrt{1 + (3x/8)^2}\, dx \qquad \bar{x}_{el} = x$$

$$A = \int_A dA = 2\pi \int_0^4 y \sqrt{1 + (3x/8)^2}\, dx \qquad Q_{yz} = \int_A \bar{x}_{el}\, dA = 2\pi \int_0^4 xy \sqrt{1 + (3x/8)^2}\, dx$$

Evaluating the integral for A with Simpson's rule (the expression for $Q_{yz}$ is the same as $Q_{zx}$ in Prob. 8.74):

| x (m) | y (m) | W | $Wy\sqrt{1+(3x/8)^2}$ |
|---|---|---|---|
| 0 | 0.0000 | 1 | 0.000 |
| 1 | 0.1875 | 4 | 0.801 |
| 2 | 0.7500 | 2 | 1.875 |
| 3 | 1.6875 | 4 | 10.160 |
| 4 | 3.0000 | 1 | 5.408 |
| | | | 18.244 |

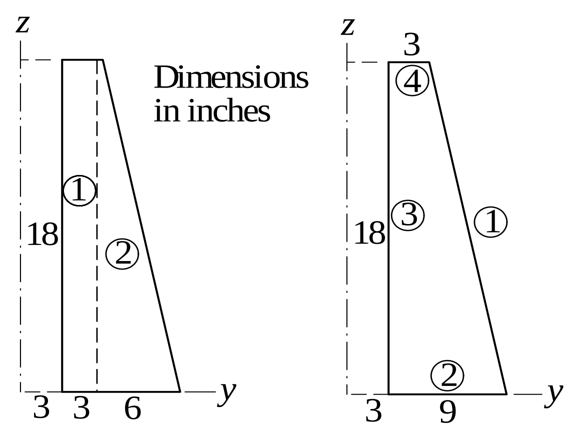



$$A \approx 2\pi \frac{\Delta x}{3} \sum_i W_i y_i \sqrt{1+(3x_i/8)^2} = 2\pi \frac{1}{3}(18.244) = 38.21 \text{ m}^2$$

$Q_{yz} \approx 118.68 \text{ m}^3$ (from solution of Prob. 8.74)

$\therefore \bar{x} = \dfrac{118.68}{38.21} = 3.11 \text{ m}$ ◆ Due to symmetry $\bar{y} = \bar{z} = 0$ ◆

**8.76**




$$V = 2\pi\Sigma A_i\bar{y}_i = 2\pi\left[(54)(4.5) + (54)(8)\right] = 4240 \text{ in}^3 \blacktriangleleft$$

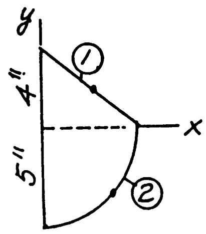
$$\begin{aligned} A &= 2\pi\Sigma L_i\bar{y}_i = 2\pi\left[\left(\sqrt{18^2+6^2}\right)(9) + 9(7.5) + 18(3) + 3(4.5)\right] \\ &= 1921 \text{ in}^2 \blacktriangleleft \end{aligned}$$

**8.77**

$$\begin{aligned} V &= 2\pi\left(Q_{\text{area}}\right)_{AB} = 2\pi(\pi b^2)a = 2\pi^2 ab^2 \blacktriangleleft \\ A &= 2\pi\left(Q_{\text{curve}}\right)_{AB} = 2\pi(2\pi b)a = 4\pi^2 ab \blacktriangleleft \end{aligned}$$

## 8.78

**Surface area**



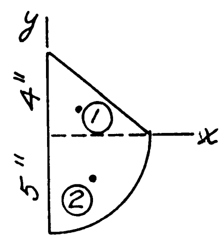

$L_1 = \sqrt{4^2 + 5^2} = 6.403 \text{ in} \qquad L_2 = \frac{\pi}{2}(5) = 7.854 \text{ in}$

$\bar{x}_2 = \frac{2}{\pi}(5) = 3.183 \text{ in}$

$Q_y = L_1\bar{x}_1 + L_2\bar{x}_2 = (6.403)(2.5) + (7.854)(3.183) = 41.01 \text{ in}^2$

$A = 2\pi Q_y = 2\pi(41.01) = 258 \text{ in}^2$ ♦

**Volume**

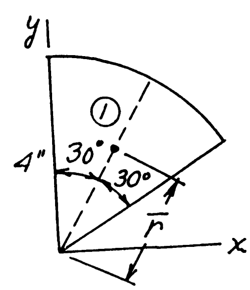


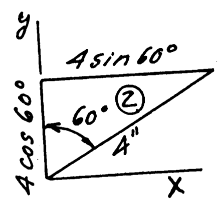

$A_2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ in}^2 \qquad \bar{x}_2 = \frac{4}{3\pi}(5) = 2.122 \text{ in}$

$Q_y = A_1\bar{x}_1 + A_2\bar{x}_2 = (10)\left(\frac{5}{3}\right) + (19.635)(2.122) = 58.33 \text{ in}^3$

$V = 2\pi Q_y = 2\pi(58.33) = 366 \text{ in}^3$ ♦

## 8.79




$A_1 = \frac{1}{2}R^2\theta = \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right) = 8.378 \text{ in}^2$

$A_2 = -\frac{1}{2}(4 \cos 60°)(4 \sin 60°) = -3.464 \text{ in}^2$

$\bar{r} = \frac{2R \sin(\theta/2)}{3(\theta/2)} = \frac{2(4)\sin 30°}{3(\pi/6)} = 2.546 \text{ in}$

$\bar{x}_1 = \bar{r} \sin 30° = 1.273 \text{ in}$

$\bar{x}_2 = \frac{1}{3}(4 \sin 60°) = 1.155 \text{ in}$

$Q_y = A_1\bar{x}_1 + A_2\bar{x}_2 = (8.378)(1.273) - (3.464)(1.155) = 6.664 \text{ in}^3$

$V = 2\pi Q_y = 2\pi(6.664) = 41.9 \text{ in}^3$ ♦

**8.80**

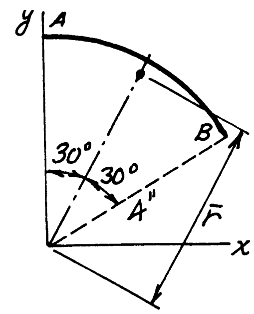
$$L = R\theta = (4)\left(\frac{\pi}{3}\right) = 4.189 \text{ in}$$

$$\bar{r} = \frac{R\sin(\theta/2)}{(\theta/2)} = \frac{(4)\sin 30°}{\pi/6} = 3.820 \text{ in}$$

$$\bar{x} = \bar{r}\sin 30° = 3.820 \sin 30° = 1.9099 \text{ in}$$

$$Q_y = L\bar{x} = (4.189)(1.9099) = 8.001 \text{ in}^2$$

$$A = 2\pi Q_y = 2\pi(8.001) = 50.3 \text{ in}^2 \blacklozenge$$




**8.81**

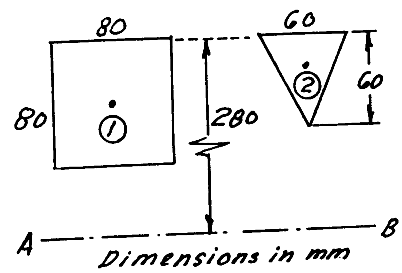
$$Q_{AB} = A_1\bar{r}_1 + A_2\bar{r}_2$$

$$= (80 \times 80)(240) - \frac{1}{2}(60 \times 60)(260)$$

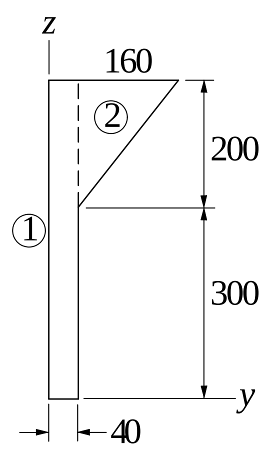
$$= 1.0680 \times 10^6 \text{ mm}^3 = 1.0680 \times 10^{-3} \text{ m}^3$$

$$m = \rho V = \rho(2\pi Q_{AB})$$

$$= (7850)(2\pi)(1.0680 \times 10^3) = 52.7 \text{ kg} \blacklozenge$$



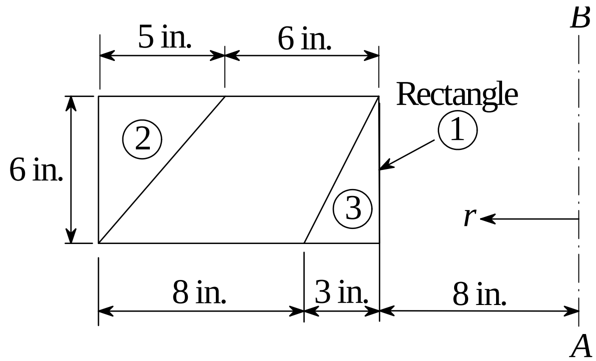

**8.82**




$$V = 2\pi\Sigma A_i\bar{y}_i = 2\pi\left[(40 \times 500)(20) + \left(\frac{1}{2} \times 160 \times 200\right)\left(40 + \frac{1}{3} \times 160\right)\right]$$
$$= 11.90 \times 10^6 \text{ mm}^3 \blacktriangleleft$$

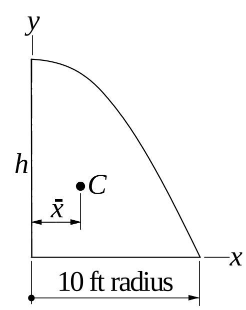
## 8.83

Properties of the generating area:




$$
\begin{aligned}
A_1 &= (11)(6) = 66 \text{ in}^2 \qquad \bar{r}_1 = 8 + \frac{11}{2} = 13.5 \text{ in.} \\
A_2 &= -\frac{1}{2}(5)(6) = -15 \text{ in}^2 \qquad \bar{r}_2 = 19 - \frac{5}{3} = 17.333 \text{ in.} \\
A_3 &= -\frac{1}{2}(3)(6) = -9 \text{ in}^2 \qquad \bar{r}_3 = 8 + \frac{3}{3} = 9.0 \text{ in.} \\
V &= 2\pi \Sigma A_i r_i = 2\pi\left[66(13.5) - 15(17.333) - 9(9.0)\right] = 3460 \text{ in}^3 \blacktriangleleft
\end{aligned}
$$

## 8.84




$$V = 2\pi A\bar{x} \qquad 2000 = 2\pi\left(\frac{2}{3}10h\right)\left(\frac{3}{8}10\right) \qquad h = 12.73 \text{ ft} \blacktriangleleft$$

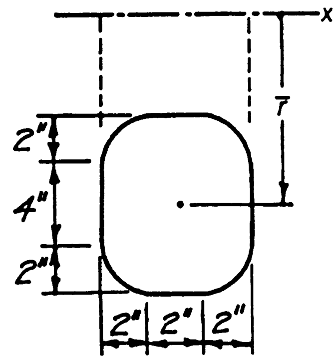
8.85

$L = 2(4) + 2(2) + 2\pi(2) = 24.57$ in

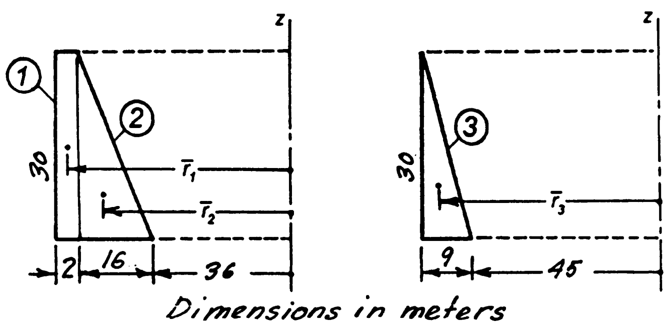
$\bar{r} = 9$ in

$Q_x = L\bar{r} = (24.57)(9) = 221.1$ in$^2$

$A = \frac{1}{4}(2\pi Q_x) = \frac{1}{4}(2\pi)(221.1) = 347$ in$^2$ ◆




8.86



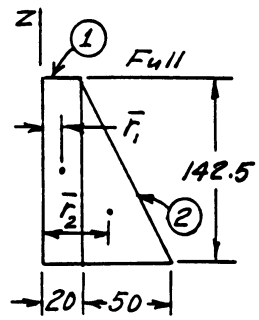

$A_1 = (30)(2) = 60$ m$^2$ $\quad A_2 = \frac{1}{2}(30)(16) = 240$ m$^2$ $\quad A_3 = -\frac{1}{2}(30)(9) = -135$ m$^2$

$\bar{r}_1 = 53$ m $\quad \bar{r}_2 = 36 + \frac{2}{3}(16) = 46.67$ m $\quad \bar{r}_3 = 45 + \frac{2}{3}(9) = 51$ m

$Q_z = \Sigma A_i\bar{r}_i = (60)(53) + (240)(46.67) - (135)(51) = 7496$ m$^3$

$V = \frac{60°}{360°}(2\pi Q_z) = \frac{1}{6}(2\pi)(7496) = 7850$ m$^3$ ◆

8.87

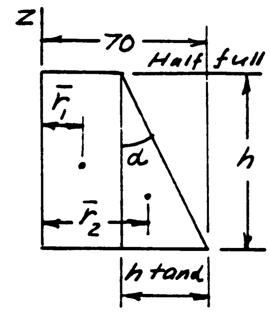



**(a)** $A_1 = (20)(142.5) = 2850 \text{ mm}^2 \qquad \bar{r}_1 = 10 \text{ mm}$

$A_2 = \frac{1}{2}(50)(142.5) = 3562 \text{ mm}^2 \qquad \bar{r}_2 = 20 + \frac{50}{3} = 36.67 \text{ mm}$

$Q_z = \Sigma A_i \bar{r}_i = (2850)(10) + (3562)(36.67) = 159.12 \times 10^3 \text{ mm}^3$

$V = 2\pi Q_z = 2\pi(159.12 \times 10^3) = 1.000 \times 10^6 \text{ mm}^3$ ♦




**(b)** $\tan\alpha = \frac{50}{142.5} = 0.3509$

$A_1 = (70 - h\tan\alpha)h \qquad A_2 = \frac{1}{2}(h\tan\alpha)h$

$\bar{r}_1 = \frac{1}{2}(70 - h\tan\alpha)$

$\bar{r}_2 = (70 - h\tan\alpha) + \frac{1}{3}h\tan\alpha = 70 - \frac{2}{3}h\tan\alpha$

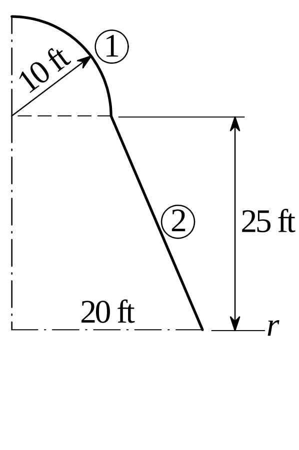
$Q_z = \Sigma A_i \bar{r}_i = \frac{h}{2}(70 - h\tan\alpha)^2 + \frac{h^2}{2}\left(70 - \frac{2}{3}h\tan\alpha\right)\tan\alpha = \frac{h}{2}\left(70^2 - 70h\tan\alpha + \frac{h^2}{3}\tan^2\alpha\right)$

Given: $V = 2\pi Q_z = 0.5 \times 10^6 \text{ mm}^3 \quad \therefore \pi h\left(70^2 - 70h\tan\alpha + \frac{1}{3}h^2\tan^2\alpha\right) = 0.5 \times 10^6$

After substituting for $\tan\alpha$, we get: $0.12894\, h^3 - 77.17\, h^2 + 15\,394\, h - 500\,000 = 0$

The solution is (a numerical method must be used) $h = 39.9$ mm ♦

8.88



$$L_1 = \frac{\pi}{2}(10) \text{ ft} \qquad r_1 = \frac{2}{\pi}(10) \text{ ft}$$
$$L_2 = \sqrt{10^2 + 25^2} = 26.93 \text{ ft} \qquad r_2 = 15 \text{ ft}$$

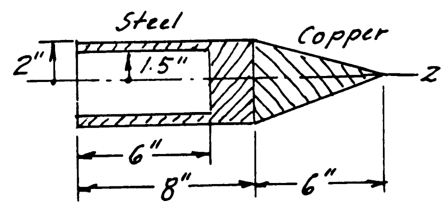
$$A = 2\pi\Sigma L_i r_i = 2\pi\left[100 + 26.93(15)\right] = 3170 \text{ ft}^2 \blacktriangleleft$$

## 8.89

**Due to symmetry:** $\bar{x} = \bar{y} = 0$ ◆

**Copper:** $\rho_c = \dfrac{556}{12^3} = 0.3218 \text{ lb/in}^3$

**Steel:** $\rho_s = \dfrac{489}{12^3} = 0.2830 \text{ lb/in}^3$




**Steel cylinder without hole:** $W_1 = \rho_s V_1 = 0.2830\left[\pi(2)^2(8)\right] = 28.45 \text{ lb} \qquad \bar{z}_1 = 4 \text{ in}$

**Steel hole:** $W_2 = \rho_s V_2 = 0.2830\left[-\pi(1.5)^2(6)\right] = -12.00 \text{ lb} \qquad \bar{z}_2 = 3 \text{ in}$

**Copper cone:** $W_3 = \rho_c V_3 = 0.3218\left[\frac{\pi}{3}(2)^2(6)\right] = 8.09 \text{ lb} \qquad \bar{z}_3 = 8 + \frac{6}{4} = 9.5 \text{ in}$

**Assembly:** $\bar{z} = \dfrac{\Sigma W_i \bar{z}_i}{\Sigma W_i} = \dfrac{28.45(4) - 12.00(3) + 8.09(9.5)}{28.45 - 12.00 + 8.09} = 6.30 \text{ in}$ ◆

## 8.90

Approximate the bowl as a thin shell.

Bowl: $W_1 = 2\pi \bar{R}^2 t \gamma_1 = 2\pi(6.15^2)(0.3)\left(\dfrac{162}{12^3}\right) = 6.684 \text{ lb}$

$\bar{y}_1 = \dfrac{1}{2}\bar{R} = \dfrac{6.15}{2} = 3.075 \text{ in.}$

Water: $W_2 = \dfrac{2\pi}{3}R^3\gamma_2 = \dfrac{2\pi}{3}(6^3)\left(\dfrac{62.4}{12^3}\right) = 16.336 \text{ lb}$

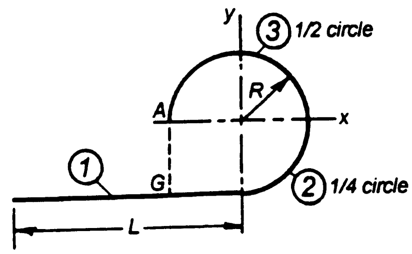
$\bar{y}_2 = \dfrac{3}{8}R = \dfrac{3}{8}(6) = 2.25 \text{ in.}$

$$\bar{y} = \frac{\Sigma W_i \bar{y}_i}{\Sigma W_i} = \frac{6.684(3.075) + 16.336(2.25)}{6.684 + 16.336} = 2.49 \text{ in.} \blacktriangleleft$$

## 8.91

The center of gravity G must lie directly below A, i.e. $\bar{x} = -R$. Since the wire is uniform, its center of gravity and centroid coincide.

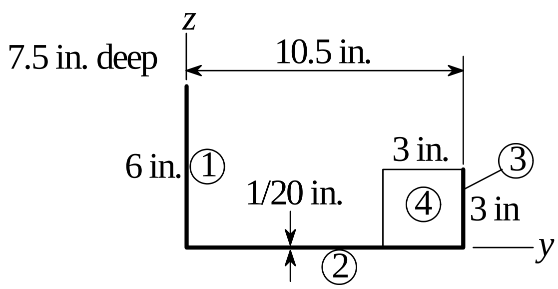
| Part | L | $\bar{x}$ | $L\bar{x}$ |
|---|---|---|---|
| 1 | L | $-L/2$ | $-L^2/2$ |
| 2 | $\pi R/2$ | $2R/\pi$ | $R^2$ |
| 3 | $\pi R$ | 0 | 0 |
| Sum | $L + 3\pi R/2$ | | $-L^2/2 + R^2$ |




$\bar{x} = -R$: $\dfrac{-L^2/2 + R^2}{L + 3\pi R/2} = -R$  $\therefore -R\left(L + \dfrac{3\pi}{2}R\right) + \dfrac{L^2}{2} - R^2 = 0$

$\therefore 0.5\left(\dfrac{L}{R}\right)^2 - \dfrac{L}{R} - 5.712 = 0$  The positive root is $\dfrac{L}{R} = 4.52$ ♦

## 8.92



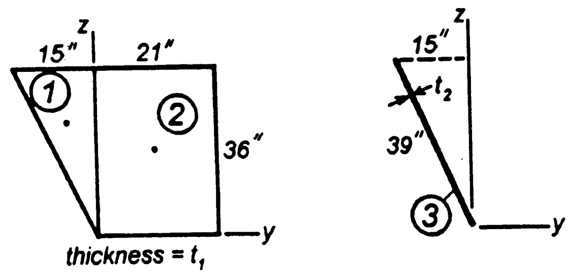

$$
\begin{aligned}
W_1 &= 0.283(6)(7.5)(1/20) = 0.6368 \text{ lb} \\
W_2 &= 0.283(10.5)(7.5)(1/20) = 1.1143 \text{ lb} \\
W_3 &= 0.283(3)(7.5)(1/20) = 0.3184\text{lb} \\
W_4 &= 0.029(3)(3)(7.5) = 1.9575 \text{ lb} \\
W &= \Sigma W_i = 0.6368 + 1.1143 + 0.3184 + 1.9575 = 4.027 \text{ lb}
\end{aligned}
$$

$$
\begin{aligned}
\bar{y} &= \frac{\Sigma W_i \bar{y}_i}{W} = \frac{0 + 1.1143(5.25) + 0.3184(10.5) + 1.9575(9)}{4.027} \\
&= 6.66 \text{ in} \blacktriangleleft \\
\bar{z} &= \frac{\Sigma W_i \bar{z}_i}{W} = \frac{0.6368(3) + 0 + 0.3184(1.5) + 1.9575(1.5)}{4.027} \\
&= 1.322 \text{ in.} \blacktriangleleft
\end{aligned}
$$

By symmetry $\bar{x} = 3.75$ in. ◄

## 8.93

**The partition does not tip if $\bar{y} > 0$, the limiting case being $\bar{y} = 0$.**




| Part | V (in$^3$) | $\bar{y}$ (in) | $V\bar{y}$ (in$^4$) |
|---|---|---|---|
| 1 (2 pcs) | $(15)(36)t_1 = 540\ t_1$ | −5.0 | −2 700 $t_1$ |
| 2 (2 pcs) | $2(21)(36)t_1 = 1512\ t_1$ | 10.5 | 15 876 $t_1$ |
| 3 | $(39)(54)t_2 = 2106\ t_2$ | −7.5 | −15 795 $t_2$ |
| Sum | | | 13 176 $t_1$ − 15 795 $t_2$ |

**If $\bar{y} = 0$, then $\Sigma V_i \bar{y}_i = 0$.** $\quad \therefore\ 13\,176\ t_1 - 15\,795\ t_2 = 0 \quad \therefore\ t_2/t_1 = 0.834$ ◆

## 8.94

$$\text{Hemisphere} \quad : \quad W_1 = \frac{2\pi}{3}R^3\gamma_{\text{st}} = \frac{2\pi}{3}(9)^3(0.283) = 432.1 \text{ lb}$$

$$\bar{y}_1 = -\frac{3}{8}R = -\frac{3}{8}(9) = -3.375 \text{ in.}$$

$$\text{Cylinder}: \ W_2 = \pi R^2 h = \pi(9)^2 h = 254.5h \text{ lb} \qquad \bar{y}_2 = \frac{1}{2}h$$

$$W_1\bar{y}_1 + W_2\bar{y}_2 = 0 \qquad 432.1(-3.375) + 254.5h\left(\frac{1}{2}h\right) = 0$$

$$h = 3.39 \text{ in.} \ ◄$$

## 8.95

$$\text{Rod:} \qquad m_1 = \frac{\pi}{4}\left(0.006^2\right)(0.4)(7850) = 0.08878 \text{ kg}$$

$$\text{Collar:} \qquad m_2 = \frac{\pi}{4}(0.04^2 - 0.006^2)(0.02)(8300) = 0.2039 \text{ kg}$$

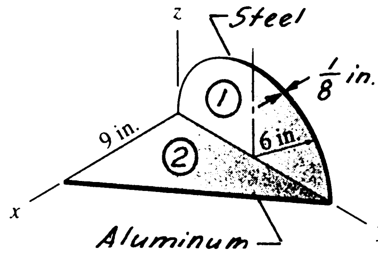
$$\Sigma m_i\bar{x}_i = 0.08878(-0.1) + 0.2039x = 0 \qquad x = 0.0435 \text{ m} = 43.5 \text{ mm} \ ◄$$

**8.96**

$$W_1 = \frac{490}{12^3}\left[\frac{\pi}{2}(6)^2\left(\frac{1}{8}\right)\right] = 2.004 \text{ lb}$$

$$W_2 = \frac{166}{12^3}\left[\frac{1}{2}(9)(12)\right] = 5.188 \text{ lb}$$

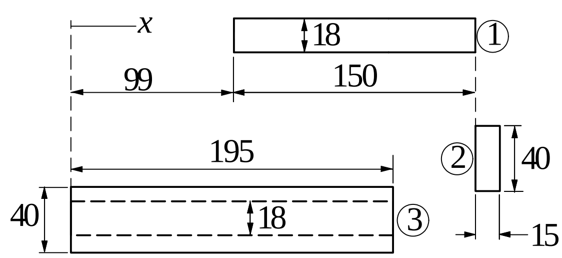
$$\bar{z}_1 = \frac{4}{3\pi}(6) = 2.546 \text{ in}$$




| Part | W (lb) | $\bar{x}$ (in) | $W\bar{x}$ (lb·in) | $\bar{y}$ (in) | $W\bar{y}$ (lb·in) | $\bar{z}$ (in) | $W\bar{z}$ (lb·in) |
|---|---|---|---|---|---|---|---|
| 1 | 2.004 | 0 | 0 | 6.000 | 12.024 | 2.546 | 5.102 |
| 2 | 5.188 | 3.000 | 15.564 | 4.000 | 20.752 | 0 | 0 |
| Sum | 7.192 | | 15.564 | | 32.776 | | 5.102 |

$\therefore \bar{x} = \frac{15.564}{7.192} = 2.16$ in ♦ $\quad \bar{y} = \frac{32.776}{7.192} = 4.56$ in ♦ $\quad \bar{z} = \frac{5.102}{7.192} = 0.71$ in ♦

**8.97**

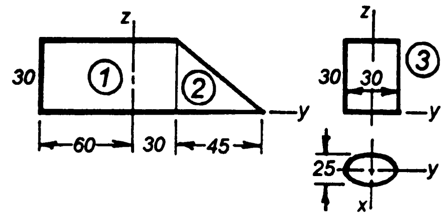



$$m_1 = \pi(9^2)(150)(7850 \times 10^{-9}) = 0.2996 \text{ kg}$$
$$m_2 = \pi(20^2)(15)(7850 \times 10^{-9}) = 0.14797 \text{ kg}$$
$$m_3 = \pi(20^2 - 9^2)(195)(2660 \times 10^{-9}) = 0.5198 \text{ kg}$$

$$\bar{x} = \frac{\Sigma m_i \bar{x}_i}{\Sigma m_i} = \frac{0.2996(174) + 0.14797(256.5) + 0.5198(97.5)}{0.2996 + 0.14797 + 0.5198}$$
$$= 145.5 \text{ mm} \blacktriangleleft$$

## 8.98

First locate the center of gravity (it coincides with centroid) of the hammerhead:




$V_1 = (90)(30)(40) = 108\ 000\ \text{mm}^3$

$V_2 = \frac{1}{2}(45)(30)(40) = 27\ 000\ \text{mm}^3$

$V_3 = -\pi(15)(12.5)(30) = -\ 17\ 671\ \text{mm}^3$

| Part | V ($mm^3$) | $\bar{y}$ (mm) | $V\bar{y}$ ($mm^4$) | $\bar{z}$ (mm) | $V\bar{z}$ ($mm^4$) |
|---|---|---|---|---|---|
| 1 | 108 000 | −15 | −1 620 000 | 15 | 1 620 000 |
| 2 | 27 000 | 45 | 1 215 000 | 10 | 270 000 |
| 3 | −17 670 | 0 | 0 | 15 | −265 100 |
| Sum | 117 330 | | −405 000 | | 1 624 900 |

$$\therefore\ \bar{y} = -\frac{405\ 000}{117\ 330} = -3.452\ \text{mm} \qquad \therefore\ \bar{z} = \frac{1\ 624\ 900}{117\ 330} = 13.849\ \text{mm}$$

For the handle: $\bar{y} = 0 \quad \bar{z} = -\ 75$ mm

For the assembly:

$$\bar{y} = \frac{(0.919)(-3.452) + (0.0990)(0)}{0.919 + 0.0990} = -3.12\ \text{mm}\ \blacklozenge$$

$$\bar{z} = \frac{(0.919)(13.849) + (0.0990)(-75)}{0.919 + 0.0990} = 5.21\ \text{mm}\ \blacklozenge$$ Due to symmetry $\bar{x} = 0$ ♦

## 8.99

$$(W\bar{x})_{assy} = (W\bar{x})_{wheel} + (W\bar{x})_{weights} = 0 \qquad \therefore\ \bar{x}_{wheel} = -\frac{(W\bar{x})_{weights}}{W_{wheel}}$$

But $(W\bar{x})_{weights} = \frac{2}{16}(7 - 7\sin 45°) = 0.2563$ lb•in $\quad W_{wheel} = 24 - \frac{2}{16}(2) = 23.75$ lb

$$\therefore\ \bar{x}_{wheel} = -\frac{0.2563}{23.75} = -0.01079\ \text{in}\ \blacklozenge$$

Similarly $\bar{y} = -\frac{(W\bar{y})_{weights}}{W_{wheel}} = -\frac{(2/16)(-7\cos 45°)}{23.75} = 0.02605$ in ♦

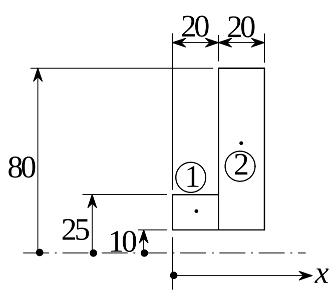
## 8.100

$$W_{\tan k} = 18\ 000 \text{ lb} \qquad W_{\text{water}} = 62.4\pi(10^2)h = 19\ 604h \text{ lb}$$
$$\bar{y}_{\tan k} = 8 \text{ ft} \qquad \bar{y}_{\text{water}} = \frac{h}{2}$$

$$\bar{y} = \frac{\Sigma W_i y_i}{\Sigma W_i} = \frac{18\ 000(8) + 19\ 604h\,(h/2)}{18\ 000 + 19\ 604h} = \frac{144\ 000 + 9802h^2}{18\ 000 + 19\ 604h}$$
$$\bar{y} = h \qquad \frac{144\ 000 + 9802h^2}{18\ 000 + 19\ 604h} = h \qquad \frac{4901h^2 + 9000h - 72\,000}{9802h + 9000} = 0$$

Positive root is $h = 3.02$ ft ◄

## 8.101




Pulley:

$$m_1 = \pi(0.025^2 - 0.01^2)(0.02)(2660) = 0.08775 \text{ kg}$$
$$m_2 = \pi(0.08^2 - 0.01^2)(0.02)(2660) = 1.0529 \text{ kg}$$
$$\bar{x}_1 = 0.01 \text{ m} \qquad \bar{x}_2 = 0.03 \text{ m}$$

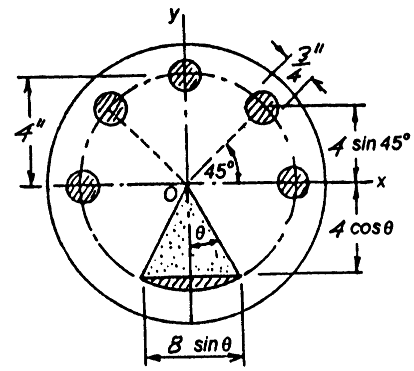
Shaft:

$$m_3 = \pi(0.01^2)(0.14)(7850) = 0.3453 \text{ kg}$$
$$\bar{x}_3 = 0.07 \text{ m}$$

$$x = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{0.08775(0.01) + 1.0529(0.03) + 0.3453(0.07)}{0.08775 + 1.0529 + 0.3453}$$
$$= 0.0381 \text{ m} = 38.1 \text{ mm} \;◄$$

## 8.102

For COG to remain at O, the first moment of all removed areas about x-axis must be zero.




For the holes: $\Sigma A_i \bar{y}_i = \pi(3/8)^2 \left[2(4 \sin 45°) + 4\right]$

$= 4.266 \text{ in}^3$

For the segment:

| Part | A (in$^2$) | $\bar{y}$ (in) | A$\bar{y}$ (in$^3$) |
|---|---|---|---|
| Sector | $\theta(4^2)$ | $-\frac{2}{3}\frac{4 \sin\theta}{\theta}$ | $-\frac{128}{3}\sin\theta$ |
| Triangle | $-\frac{1}{2}(4 \cos\theta)(8 \sin\theta)$ | $-\frac{2}{3}(4 \cos\theta)$ | $\frac{128}{3}\cos^2\theta \sin\theta$ |
| Segment | | | $-\frac{128}{3}\sin\theta(1 - \cos^2\theta)$ |

$(\Sigma A_i \bar{y}_i)_{holes} + (A\bar{y})_{segment} = 0$: $\quad 4.266 - \frac{128}{3}\sin\theta(1 - \cos^2\theta) = 0$

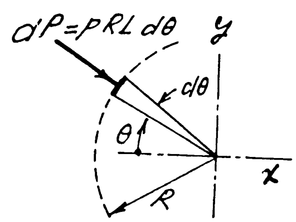
$\therefore\ 4.266 - \frac{128}{3}\sin^3\theta = 0 \qquad \therefore\ \sin^3\theta = 0.09998 \qquad \therefore\ \theta = 27.7°$ ♦

## 8.103

$dP = p\, dA = p(RL\, d\theta) = (p_0 \cos\theta)(RL\, d\theta)$

$P_x = \int_A dP \cos\theta = p_0 RL \int_0^{2\pi} \cos^2\theta\, d\theta = \pi p_0 RL$

Due to symmetry: $P_y = 0 \qquad \therefore\ P = \pi p_0 RL \rightarrow$ ♦



## 8.104

With $\Delta x = 2.5$ ft, Simpson's rule yields

$$\begin{aligned} R &= \int_{\mathcal{L}} w\ dx \\ &\approx \frac{2.5}{3}[42.5 + 4(37.5) + 2(28.3) + 4(32.4) + 2(42.3) + 4(52.1) + 58.6] \\ &= 608.6 \text{ lb} \end{aligned}$$

$$\begin{aligned} R\bar{x} &= \int_{\mathcal{L}} wx\ dx \\ &\approx \frac{2.5}{3}[0 + 4(37.5)(2.5) + 2(28.3)(5) + 4(32.4)(7.5) + 2(42.3)(10) \\ &\quad +4(52.1)(12.5) + 58.6(15)] \\ &= 4967 \text{ lb} \cdot \text{ft} \end{aligned}$$

$$R = 609 \text{ lb} \blacktriangleleft \qquad \bar{x} = \frac{4967}{608.6} = 8.16 \text{ ft} \blacktriangleleft$$

## 8.105

$$\begin{aligned} R &= \int_{\mathcal{A}} p\ dA = p_0 \int_0^b \int_0^b \left(\frac{x}{b} + \frac{xy}{b^2}\right) dx\ dy \\ &= \frac{1}{2}p_0 \int_0^b (b + y)dy = \frac{3}{4}p_0 b^2 \blacktriangleleft \end{aligned}$$

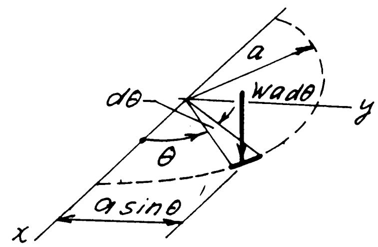
$$\begin{aligned} R\bar{x} &= \int_{\mathcal{A}} px\ dA = p_0 \int_0^b \int_0^b x\left(\frac{x}{b} + \frac{xy}{b^2}\right) dx\ dy \\ &= \frac{1}{3}p_0 b \int_0^b (b + y)dy = \frac{1}{2}p_0 b^3 \end{aligned}$$

$$\begin{aligned} R\bar{y} &= \int_{\mathcal{A}} py\ dA = p_0 \int_0^b \int_0^b y\left(\frac{x}{b} + \frac{xy}{b^2}\right) dx\ dy \\ &= \frac{1}{2}p_0 \int_0^b y(b + y)dy = \frac{5}{12}p_0 b^3 \end{aligned}$$

$$\bar{x} = \frac{\frac{1}{2}p_0 b^3}{\frac{3}{4}p_0 b^2} = \frac{2}{3}b \blacktriangleleft \qquad \bar{y} = \frac{\frac{5}{12}p_0 b^3}{\frac{3}{4}p_0 b^2} = \frac{5}{9}b \blacktriangleleft$$

## 8.106

$$R = \int_L wa\, d\theta = \int_0^{\pi} \left(w_0 \frac{y}{a}\right) a\, d\theta = \int_0^{\pi} w_0 y\, d\theta$$



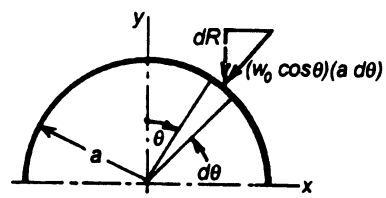

$$= \int_0^{\pi} w_0 a \sin\theta\, d\theta = 2w_0 a \quad \blacklozenge$$

$$R\bar{y} = \int_L ywa\, d\theta = \int_0^{\pi} y \left(w_0 \frac{y}{a}\right) a\, d\theta = \int_0^{\pi} w_0 y^2\, d\theta$$

$$= \int_0^{\pi} w_0 a^2 \sin^2\theta\, d\theta = \frac{\pi}{2} w_0 a^2 \qquad \therefore\ \bar{y} = \frac{(\pi/2) w_0 a^2}{2w_0 a} = \frac{\pi}{4} a \quad \blacklozenge \qquad \text{By symmetry: } \bar{x} = 0 \quad \blacklozenge$$

## 8.107

Due to symmetry: R is vertical and $\bar{x} = 0$.




$$+\downarrow\ dR = (w_0 \cos\theta)(a\, d\theta)\cos\theta = w_0 a \cos^2\theta\, d\theta$$

$$\therefore\ R = \int_L dR = w_0 a \int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta = \frac{\pi}{2} w_0 a \downarrow \quad \blacklozenge$$

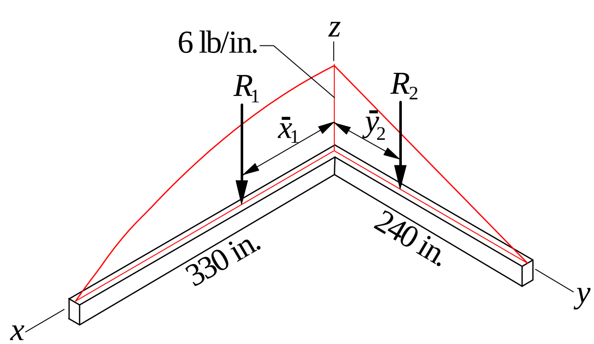
## 8.108

$$+\downarrow\ R = \int_L w\, dx = \frac{1}{40}\int_0^{60} (40x - x^2)\, dx = \frac{1}{40}\left[20x^2 - \frac{1}{3}x^3\right]_0^{60} = 0 \quad \blacklozenge$$

$$\circlearrowright\!+\ C^R = \int_L wx\, dx = \frac{1}{40}\int_0^{60} (40x^2 - x^3)dx = \frac{1}{40}\left[\frac{40}{3}x^3 - \frac{1}{4}x^4\right]_0^{60} = -9000 \text{ lb·in}$$

$$\therefore\ C^R = 9000 \text{ lb·in} \curvearrowleft \quad \blacklozenge$$

## 8.109



$$
\begin{aligned}
R_1 &= \frac{2}{3}(330)(6) = 1320 \text{ lb} \qquad \bar{x}_1 = \frac{3}{8}(330) = 123.75 \text{ in.} \\
R_2 &= \frac{1}{2}(240)(6) = 720 \text{ lb} \qquad \bar{y}_2 = \frac{1}{3}(240) = 80 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{\Sigma R_i \bar{x}_i}{\Sigma R_i} = \frac{1320(123.75) + 0}{1320 + 720} = 80.1 \text{ in.} \blacktriangleleft \\
\bar{y} &= \frac{\Sigma R_i \bar{y}_i}{\Sigma R_i} = \frac{0 + 720(80)}{1320 + 720} = 28.2 \text{ in.} \blacktriangleleft
\end{aligned}
$$

## 8.110

$$
\begin{aligned}
R &= \int_0^{\pi/2} w\ ds = \int_0^{\pi/2} \left(\frac{2w_0}{\pi}\theta\right)(a\ d\theta) = \frac{2w_0 a}{\pi}\left[\frac{\theta^2}{2}\right]_0^{\pi/2} \\
&= \frac{\pi w_0 a}{4} = 0.785 w_0 a \\
M_x &= \int_0^{\pi/2} wy\ ds = \int_0^{\pi/2} \left(\frac{2w_0}{\pi}\theta\right)(a\sin\theta)(a\ d\theta) \\
&= \frac{2w_0 a^2}{\pi}\left[\sin\theta - \theta\cos\theta\right]_0^{\pi/2} = \frac{2w_0 a^2}{\pi} = 0.6366 w_0 a^2 \\
M_y &= \int_0^{\pi/2} wx\ ds = \int_0^{\pi/2} \left(\frac{2w_0}{\pi}\theta\right)(a\cos\theta)(a\ d\theta) \\
&= \frac{2w_0 a^2}{\pi}\left[\cos\theta + \theta\sin\theta\right]_0^{\pi/2} = \frac{2w_0 a^2}{\pi}\left(\frac{\pi}{2} - 1\right) = 0.3634 w_0 a^2
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{M_y}{R} = \frac{0.3634 w_0 a^2}{0.785 w_0 a} = 0.463a \\
\bar{y} &= \frac{M_x}{R} = \frac{0.6366 w_0 a^2}{0.785 w_0 a} = 0.811a
\end{aligned}
$$

The resultant is the force $R = 0.785 w_0 a$ crossing the $xy$-plane at $x = 0.463a$, $y = 0.811a$. ◀

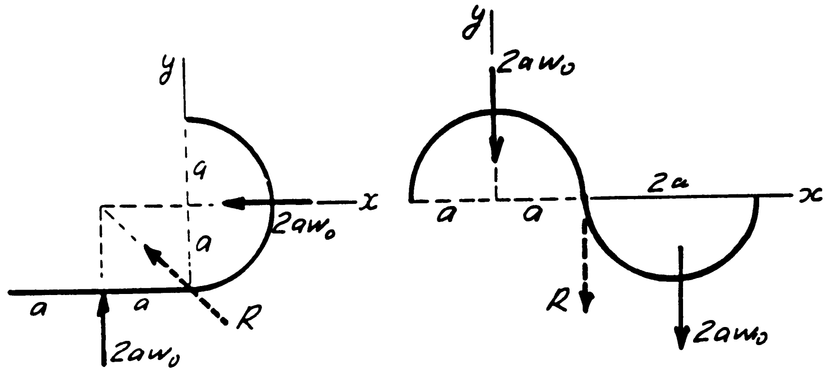
**8.111**

**(a)** $\mathbf{R} = \mathbf{0}$

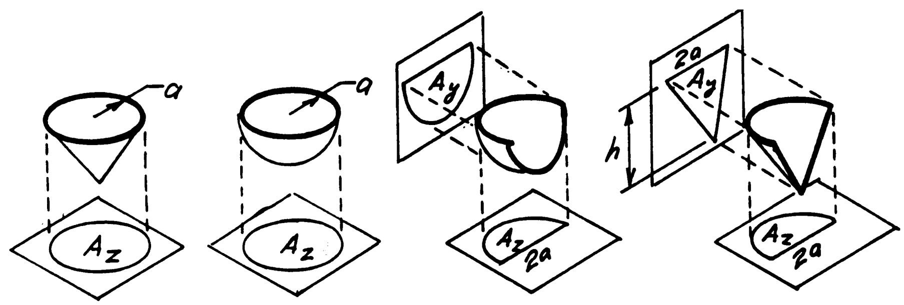
**(b)** $\mathbf{R} = -2aw_0\mathbf{j}$ passing through (0, 0)

**(c)** $\mathbf{R} = 2aw_0(-\mathbf{i} + \mathbf{j})$ passing through (−a, 0)

**(d)** $\mathbf{R} = -4aw_0\mathbf{j}$ passing through (a, 0)




**8.112**




**(a)** $A_z = \pi a^2$

$\mathbf{R} = -p_0 A_z \mathbf{k}$

$= -\pi a^2 p_0 \mathbf{k}$

$\therefore R = \pi a^2 p_0$ ♦

**(b)** $A_z = \pi a^2$

$\mathbf{R} = -p_0 A_z \mathbf{k}$

$= -\pi a^2 p_0 \mathbf{k}$

$\therefore R = \pi a^2 p_0$ ♦

**(c)** $A_y = A_z = \frac{\pi}{2} a^2$

$\mathbf{R} = p_0(-A_y \mathbf{j} - A_z \mathbf{k})$

$= -\frac{\pi}{2} a^2 p_0 (\mathbf{j} + \mathbf{k})$

$\therefore R = 2.22 a^2 p_0$ ♦

**(d)** $A_y = ah \quad A_z = \frac{\pi}{2} a^2$

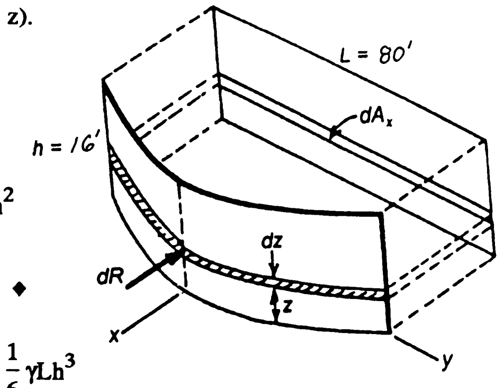
$\mathbf{R} = p_0(-A_y \mathbf{j} - A_z \mathbf{k})$

$= -\frac{ap_0}{2}(2h\mathbf{j} + \pi a \mathbf{k})$

$\therefore R = \frac{ap_0}{2}\sqrt{4h^2 + \pi^2 a^2}$ ♦

## 8.113

At depth h − z the pressure is $p = \gamma(h - z)$.
The resultant of the pressure acting on the strip of width dz is
$dR = p\, dA_x = \gamma(h - z)(L\, dz)$.




$$\therefore R = \int_A dR = \gamma L \int_0^h (h - z)\, dz = \frac{1}{2}\gamma L h^2$$

$$= \frac{1}{2}(62.4)(80)(16^2) = 639 \times 10^3 \text{ lb} \;\blacklozenge$$

$$\therefore M_y = \int_A z\, dR = \gamma L \int_0^h (hz - z^2)\, dz = \frac{1}{6}\gamma L h^3$$

$$\therefore \bar{z} = \frac{M_y}{R} = \frac{\gamma L h^3/6}{\gamma L h^2/2} = \frac{h}{3} = \frac{16}{3} = 5.33 \text{ ft} \;\blacklozenge$$

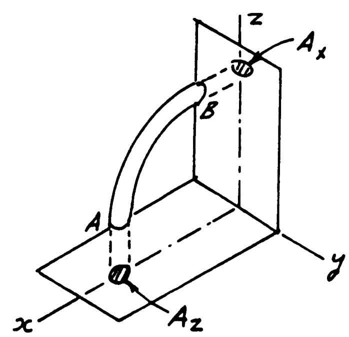
Due to symmetry $\bar{y} = 0$ ◆

## 8.114

The projected areas of the pressurized surfaces are

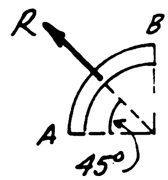
$$A_x = A_z = \frac{\pi}{4}d^2 \qquad A_y = 0$$

$$\therefore R_x = R_z = \frac{\pi}{4}p_0 d^2 \quad R_y = 0$$

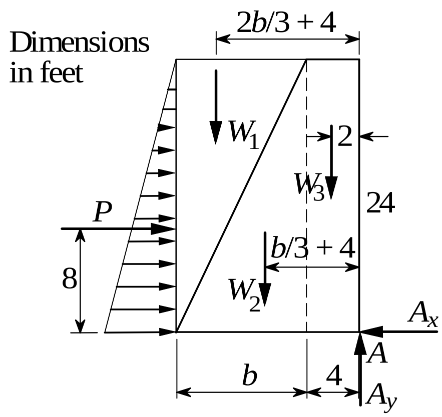



$$\therefore R = \sqrt{R_x^2 + R_z^2} = \sqrt{2}\,\frac{\pi}{4}p_0 d^2$$

$$= 1.111\, p_0 d^2 \;\blacklozenge$$



**8.115**

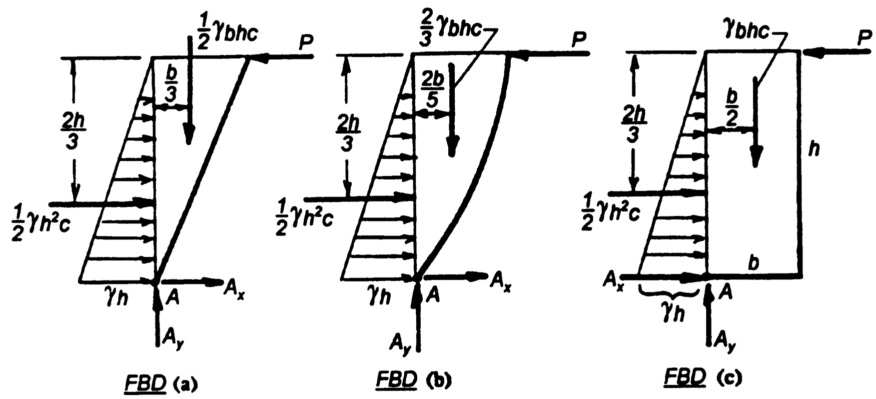



Consider 1 ft length of dam at impending tipping.

$$\begin{aligned}
P &= \frac{1}{2}\gamma_w h^2 = \frac{1}{2}(62.4)(24^2) = 17\ 971\ \text{lb/ft} \\
W_1 &= \frac{1}{2}\gamma_w hb = \frac{1}{2}(62.4)(24)b = 748.8b\ \text{lb/ft} \\
W_2 &= \frac{1}{2}\gamma_c hb = \frac{1}{2}(150)(24)b = 1800.0b\ \text{lb/ft} \\
W_3 &= \gamma_c(4h) = 150(4)(24) = 14\ 400\ \text{lb/ft}
\end{aligned}$$

$$\begin{aligned}
\Sigma M_A &= 8(17\ 971) - \left(\frac{2}{3}b + 4\right)(748.8b) - \left(\frac{1}{3}b + 4\right)(1800b) - 2(14\ 400) \\
&= -1099.2b^2 - 10195b + 0.1149\,7 \times 10^6 = 0 \\
& \quad b = 6.59\ \text{ft} \ \blacktriangleleft
\end{aligned}$$

8.116

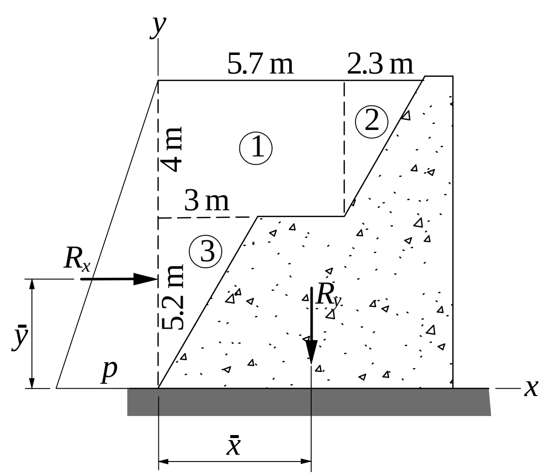



(a) $\Sigma M_A = 0$: $\left(\frac{1}{2}\gamma h^2 c\right)\frac{h}{3} + \left(\frac{1}{2}\gamma bhc\right)\frac{b}{3} - Ph = 0 \qquad \therefore P = \gamma c\left(\frac{h^2}{6} + \frac{b^2}{6}\right)$ ◆

(b) $\Sigma M_A = 0$: $\left(\frac{1}{2}\gamma h^2 c\right)\frac{h}{3} + \left(\frac{2}{3}\gamma bhc\right)\frac{2b}{5} - Ph = 0 \qquad \therefore P = \gamma c\left(\frac{h^2}{6} + \frac{4b^2}{15}\right)$ ◆

(c) $\Sigma M_A = 0$: $\left(\frac{1}{2}\gamma h^2 c\right)\frac{h}{3} + (\gamma bhc)\frac{b}{2} - Ph = 0 \qquad \therefore P = \gamma c\left(\frac{h^2}{6} + \frac{b^2}{2}\right)$ ◆

8.117



$$p = \rho g h = 1000(9.81)(9.2) = 90\ 250 \text{ Pa}$$
$$R_x = \frac{1}{2}ph = \frac{1}{2}(90\ 250)(9.2) = 415\ 000 \text{ N/m}$$
$$\bar{y} = \frac{1}{3}h = \frac{1}{3}(9.2) = 3.07 \text{ m}$$

$$W_1 = 1000(9.81)(5.7)(4) = 223\ 700 \text{ N/m}$$
$$W_2 = \frac{1}{2}(1000)(9.81)(2.3)(4) = 45\ 130 \text{ N/m}$$
$$W_3 = \frac{1}{2}(1000)(9.81)(3)(5.2) = 76\ 520 \text{ N/m}$$
$$R_y = \Sigma W_i = 223\ 700 + 45\ 130 + 76\ 520 = 345\ 400 \text{ N/m}$$
$$\bar{x} = \frac{\Sigma W_i \bar{x}_i}{R} = \frac{223\ 700(2.85) + 45\ 130(6.467) + 76\ 520(1.0)}{345\ 400} = 2.91 \text{ m}$$

The resultant force is $\mathbf{R} = 415\mathbf{i} - 345\mathbf{j}$ kN/m acting through the point (2.91 m, 3.07 m). ◄

## 8.118

$p_1 = g\rho_w h = (9.81)(1030)(4) = 40.42 \times 10^3 \text{ N/m}^2$

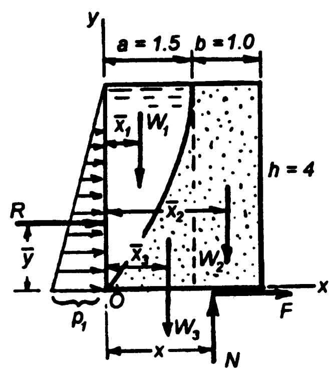
**Work with 1m length of dam from here on.**

$R = \frac{1}{2}p_1 h = \frac{1}{2}(40.42 \times 10^3)(4) = 80.83 \times 10^3 \text{ N}$

$\bar{y} = \frac{1}{3}h = \frac{4}{3} \text{ m}$

$W_1 = \frac{2}{3}g\rho_w ah = \frac{2}{3}(9.81)(1030)(1.5)(4) = 40.42 \times 10^3 \text{ N}$

$\bar{x}_1 = \frac{2}{5}a = \frac{2}{5}(1.5) = 0.6 \text{ m}$




Dimensions in meters

$W_2 = g\rho_c bh = (9.81)(2400)(1.0)(4) = 94.18 \times 10^3 \text{ N}$ $\quad \bar{x}_2 = a + \frac{b}{2} = 1.5 + 0.5 = 2.0 \text{ m}$

$W_3 = \frac{1}{3}g\rho_c ah = \frac{1}{3}(9.81)(2400)(1.5)(4) = 47.09 \times 10^3 \text{ N}$ $\quad \bar{x}_3 = \frac{7}{10}b = \frac{7}{10}(1.5) = 1.05 \text{ m}$

From FBD: $\Sigma F_y = 0$: $+\uparrow$ $N - W_1 - W_2 - W_3 = 0$

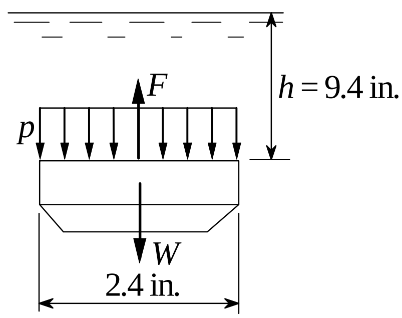
$\therefore N = (40.42 + 94.18 + 47.09)10^3 = 181.69 \times 10^3 \text{ N}$

$\Sigma M_O = 0$: $\circlearrowleft +$ $Nx - R\bar{y} - W_1\bar{x}_1 - W_2\bar{x}_2 - W_3\bar{x}_3 = 0$

$\therefore Nx = \left[(80.83)\frac{4}{3} + (40.42)(0.6) + (94.18)(2.0) + (47.09)(1.05)\right]10^3 = 369.8 \times 10^3 \text{ N}\cdot\text{m}$

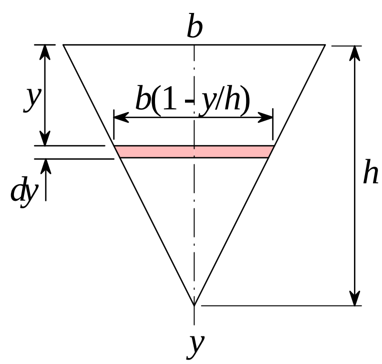
$\therefore x = \frac{Nx}{N} = \frac{369.8 \times 10^3}{181.69 \times 10^3} = 2.04 \text{ m}$ $\quad$ Safe against tipping, since $x < a + b = 2.5$ m ◆

**8.119**




$$
\begin{aligned}
p &= \gamma h = 0.036(9.4) = 0.3384 \text{ lb/in}^2 \\
\Sigma F &= 0 \qquad +\uparrow \; F - pA - W = 0 \\
F &= pA + W = 0.3384\left[\frac{\pi}{4}(2.4^2)\right] + \frac{3.5}{16} = 1.750 \text{ lb} \quad ◄
\end{aligned}
$$

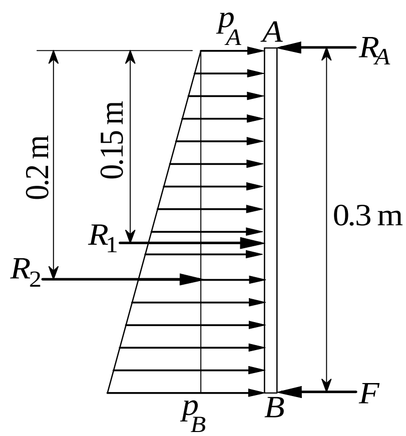
**8.120**




$$
\begin{aligned}
R &= \int_A p\, dA = \int_0^h \left(p_0 \frac{y}{h}\right) b \left(1 - \frac{y}{h}\right) dy \\
&= \frac{bp_0}{h^2}\int_0^h \left(hy - y^2\right) dy = \frac{bhp_0}{6} \quad ◄ \\
\int_A py\, dA &= \frac{bp_0}{h^2}\int_0^h \left(hy^2 - y^3\right) dy = \frac{bh^2 p_0}{12}
\end{aligned}
$$

$$
\bar{y} = \frac{\int_A py\, dA}{\int_A p\, dA} = \frac{h}{2} \quad ◄
$$

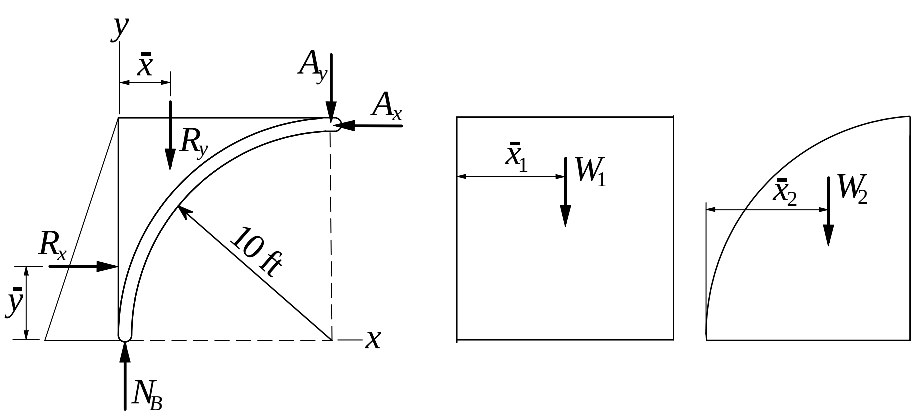
## 8.121




$$
\begin{aligned}
p_A &= \gamma h_A = 1000(9.81)(0.2) = 1962 \text{ N/m}^2 \\
p_B &= \gamma h_B = 1000(9.81)(0.5) = 4905 \text{ N/m}^2 \\
R_1 &= p_A A = 1962(0.3^2) = 176.58 \text{ N} \\
R_2 &= \frac{1}{2}(p_B - p_A)A = \frac{1}{2}(4905 - 1962)(0.3^2) = 132.44 \text{ N}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma M_A &= 0 \qquad 0.15R_1 + 0.2R_2 - 0.3F = 0 \\
& \qquad 0.15(176.58) + 0.2(132.44) - 0.3F = 0 \\
F &= 176.6 \text{ N} \blacktriangleleft
\end{aligned}
$$

## 8.122



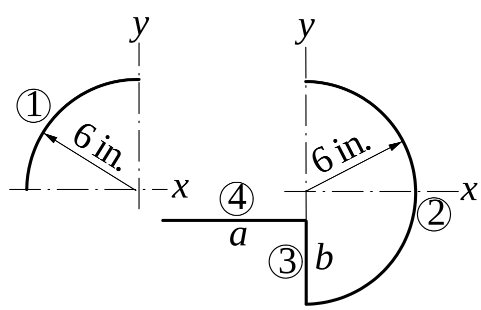
$$R_x = \frac{1}{2}(62.4)(10^2)(12) = 37\ 440 \text{ lb} \qquad \bar{y} = \frac{10}{3} = 3.333 \text{ ft}$$

$$\begin{aligned}
W_1 &= 62.4(10^2)(12) = 74\ 880 \text{ lb} \qquad \bar{x}_1 = 5 \text{ ft} \\
W_2 &= -62.4\left(\frac{\pi}{4}(10^2)\right)(12) = -58\ 810 \text{ lb} \qquad \bar{x}_2 = 10 - \frac{4(10)}{3\pi} = 5.756 \text{ ft} \\
R_y &= \Sigma W_i = 74\ 880 - 58\ 810 = 16\ 070 \text{ lb} \\
\bar{x} &= \frac{\Sigma W_i x_i}{R_y} = \frac{74\ 880(5) - 58\ 810(5.756)}{16\ 070} = 2.233 \text{ ft}
\end{aligned}$$

From FBD:

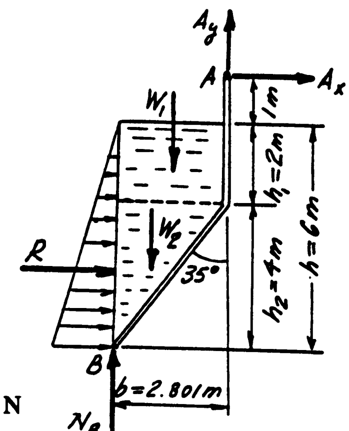
$$\begin{aligned}
\Sigma M_A &= 0 \qquad 10N_B - R_x(10 - \bar{y}) - R_y(10 - \bar{x}) = 0 \\
& \qquad 10N_B - 37\ 440(10 - 3.333) - 16\ 070(10 - 2.233) = 0 \\
N_B &= 37\ 400 \text{ lb} \ \blacktriangleleft
\end{aligned}$$

**8.123**




$$\begin{aligned}
\Sigma L_i \bar{x}_i &= 0 \qquad \frac{\pi R}{2}\left(-\frac{2R}{\pi}\right) + \pi R\left(\frac{2R}{\pi}\right) + 0 + a\left(-\frac{a}{2}\right) = 0 \\
R^2 - \frac{a^2}{2} &= 0 \qquad a = \sqrt{2}a = \sqrt{2}(6) = 8.485 \textbf{ in.} \ \blacktriangleleft \\
\Sigma L_i \bar{y}_i &= 0 \qquad \frac{\pi R}{2}\left(\frac{2R}{\pi}\right) + 0 + b\left(-6 + \frac{b}{2}\right) + a(-6 + b) = 0 \\
& b^2 + 2b(a - 6) + 2R^2 - 12a = 0 \\
& b^2 + 2b(8.485 - 6) + 2(6^2) - 12(8.485) = 0 \\
& b^2 + 4.970b - 29.82 = 0 \qquad b = 3.51 \textbf{ in.} \ \blacktriangleleft
\end{aligned}$$

8.124



FBD

$\rho = 1000$ kg/m$^3$ $\quad \therefore \gamma = 9.81 \times 10^3$ N/m$^3$

$b = 4 \tan 35° = 2.801$ m

$W_1 = \gamma b h_1 L = (9.81 \times 10^3)(2.801)(2)(10)$

$\quad = 549.6 \times 10^3$ N

$W_2 = \dfrac{1}{2} \gamma b h_2 L = \dfrac{1}{2}(9.81 \times 10^3)(2.801)(4)(10)$

$\quad = 549.6 \times 10^3$ N

$\quad = \dfrac{1}{2} \gamma h^2 L = \dfrac{1}{2}(9.81 \times 10^3)(6^2)(10) = 1765.8 \times 10^3$ N

From FBD

$$\Sigma M_A = 0: \ (+)\ \frac{b}{2} W_1 + \frac{2b}{3} W_2 + \left(1.0 + \frac{2h}{3}\right) R - b N_B = 0$$

$$\therefore \left\{\frac{1}{2}(2.801)(549.6) + \frac{2}{3}(2.801)(549.6) + \left[1 + \frac{2}{3}(6)\right](1765.8)\right\} 10^3 - 2.801\, N_B = 0$$

$$\therefore 10\,625 \times 10^3 - 2.801\, N_B = 0 \qquad \therefore N_B = 3.793 \times 10^6 \text{N} \ \blacklozenge$$

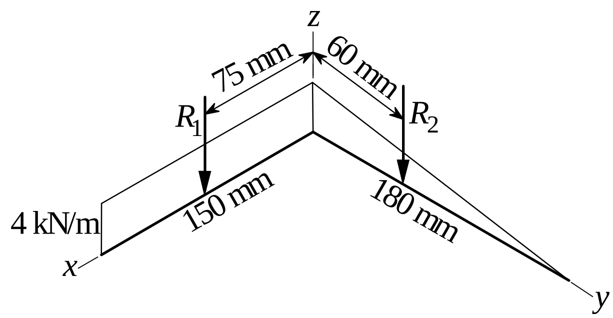
$$\Sigma F_x = 0: \ \overset{+}{\rightarrow} \ R + A_x = 0 \qquad \therefore A_x = -R = -1.7658 \times 10^6 \text{ N}$$

$$\Sigma F_y = 0: \ +\uparrow \ N_B - W_1 - W_2 + A_y = 0$$

$$\therefore A_y = -N_B + W_1 + W_2 = (-3.793 + 0.5496 + 0.5496)10^6 = -2.694 \times 10^6 \text{ N}$$

$$\therefore A = \sqrt{1.7658^2 + 2.694^2} \times 10^6 = 3.22 \times 10^6 \text{ N} \ \blacklozenge$$

**8.125**




$$R_1 = 4(0.15) = 0.60 \text{ kN} \qquad R_2 = \frac{1}{2}(4)(0.18) = 0.36 \text{ kN}$$

$$R = \Sigma R_i = 0.96 \text{ kN} \blacktriangleleft$$

$$\Sigma R_i \bar{x}_i = 0.6(0.075) + 0 = 0.045 \text{ kN} \cdot \text{m}$$

$$\Sigma R_i \bar{y}_i = 0 + 0.36(0.06) = 0.0216 \text{ kN} \cdot \text{m}$$

Line of action of $R$ crosses the $xy$-plane at

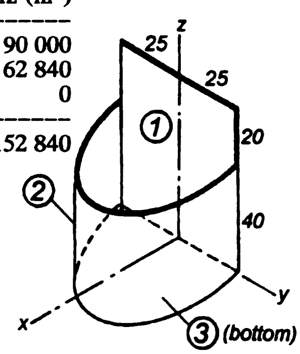
$$\bar{x} = \frac{\Sigma R_i x_i}{R} = \frac{0.045}{0.96} = 0.0469 \text{ m} = 46.9 \text{ mm} \blacktriangleleft$$

$$\bar{y} = \frac{\Sigma R_i y_i}{R} = \frac{0.0216}{0.96} = 0.0225 \text{ m} = 22.5 \text{ mm} \blacktriangleleft$$

**8.126**

**Since the thickness is uniform, the COG and centroid coincide.**

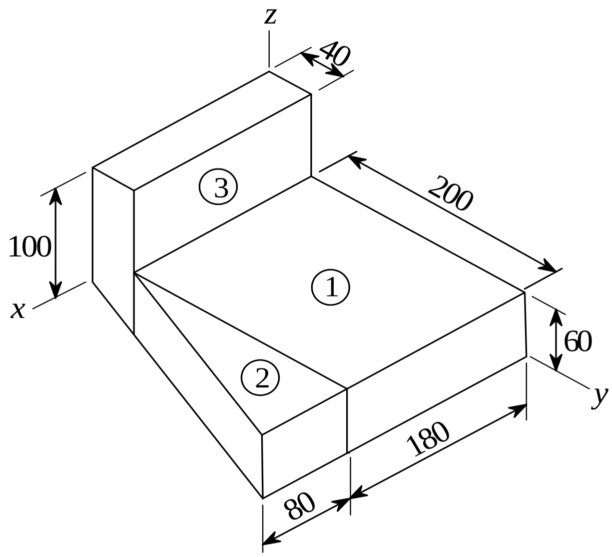
| Part | A ($in^2$) | $\bar{x}$ (in) | $A\bar{x}$ ($in^3$) | $\bar{z}$ (in) | $A\bar{z}$ ($in^3$) |
|---|---|---|---|---|---|
| 1 | 3 000 | 0.000 | 0 | 30.000 | 90 000 |
| 2 | 3 142 | 15.915 | 50 000 | 20.000 | 62 840 |
| 3 | 982 | 10.610 | 10 420 | 0.000 | 0 |
| Sum | 7 124 | | 60 420 | | 152 840 |




$$\bar{x} = \frac{60\,420}{7\,124} = 8.48 \text{ in} \;\blacklozenge \qquad \bar{z} = \frac{152\,840}{7\,124} = 21.45 \text{ in} \blacklozenge$$

**Due to symmetry** $\bar{y} = 0$ $\blacklozenge$

**8.127**




$$
\begin{aligned}
V_1 &= (0.18)(0.2)(0.06) = 2.16 \times 10^{-3}\ \text{m}^3 \\
V_2 &= \frac{1}{2}(0.08)(0.2)(0.06) = 0.48 \times 10^{-3}\ \text{m}^3 \\
V_3 &= (0.18)(0.1)(0.04) = 0.72 \times 10^{-3}\ \text{m}^3 \\
V &= \Sigma V_i = (2.16 + 0.48 + 0.72) \times 10^{-3} = 3.36 \times 10^{-3}\ \text{m}^3
\end{aligned}
$$

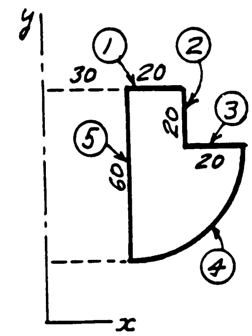
$$
\begin{aligned}
\bar{x}_1 &= 0.09\ \text{m} \qquad \bar{y}_1 = 0.14\ \text{m} \qquad \bar{z}_1 = 0.03\ \text{m} \\
\bar{x}_2 &= 0.18 + \frac{0.08}{3} = 0.2067\ \text{m} \qquad \bar{y}_2 = 0.04 + \frac{2}{3}(0.2) = 0.17333\ \text{m} \\
&\quad \bar{z}_2 = 0.03\ \text{m} \\
\bar{x}_3 &= 0.09\ \text{m} \qquad \bar{y}_3 = 0.02\ \text{m} \qquad \bar{z}_3 = 0.05\ \text{m}
\end{aligned}
$$

$$
\begin{aligned}
\bar{x} &= \frac{\Sigma V_i \bar{x}_i}{V} = \frac{2.16(0.09) + 0.48(0.20670) + 0.72(0.09)}{3.36} \\
&= 106.67 \times 10^{-3}\ \text{m} = 106.7\ \text{mm} \blacktriangleleft \\
\bar{y} &= \frac{\Sigma V_i \bar{y}_i}{V} = \frac{2.16(0.14) + 0.48(0.17333) + 0.72(0.02)}{3.36} \\
&= 119.1 \times 10^{-3}\ \text{m} = 119.1\ \text{mm} \blacktriangleleft \\
\bar{z} &= \frac{\Sigma V_i \bar{z}_i}{V} = \frac{2.16(0.03) + 0.48(0.03) + 0.72(0.05)}{3.36} \\
&= 34.3 \times 10^{-3}\ \text{m} = 34.3\ \text{mm} \blacktriangleleft
\end{aligned}
$$

## 8.128

$L_4 = \frac{\pi}{2}(40) = 62.83$ mm $\qquad \bar{x}_4 = 30 + \frac{2}{\pi}(40) = 55.46$ mm

| Part | L (mm) | $\bar{x}$ (mm) | $L\bar{x}$ (mm$^2$) |
|---|---|---|---|
| 1 | 20.00 | 40.00 | 800 |
| 2 | 20.00 | 50.00 | 1000 |
| 3 | 20.00 | 60.00 | 1200 |
| 4 | 62.83 | 55.46 | 3485 |
| 5 | 60.00 | 30.00 | 1800 |
| Sum | | | 8285 |




$\therefore A = 2\pi L\bar{x} = 2\pi(8285) = 52.1 \times 10^3 \text{ mm}^2$ ◆

## 8.129

$$R = \int_0^a \int_{-b}^{b} p\, dx\, dy = \frac{p_0}{a}\int_0^a x \left( \int_{-b}^{b} \cos\frac{\pi y}{2b}\, dy \right) dx$$

Noting that

$$\int_{-b}^{b} \cos\frac{\pi y}{2b}\, dy = 2\int_0^b \cos\frac{\pi y}{2b}\, dy = 2\left[\frac{2b}{\pi}\sin\frac{\pi y}{2b}\right]_0^b = \frac{4b}{\pi}$$

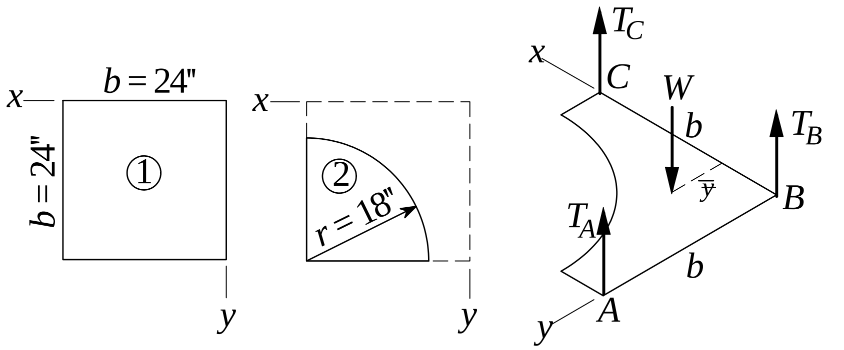
we get

$$R = p_0\frac{4b}{\pi}\int_0^a \frac{x}{a}\, dx = p_0\frac{2ab}{\pi} \quad \blacktriangleleft$$

$$\begin{aligned} R\bar{x} &= \int_0^a \int_{-b}^{b} px\, dx\, dy = p_0\int_0^a \frac{x^2}{a}\left( \int_{-b}^{b} \cos\frac{\pi y}{2b}\, dy \right) dx \\ &= p_0\frac{4b}{\pi}\int_0^a \frac{x^2}{a}\, dx = p_0\frac{4a^2 b}{3\pi} \end{aligned}$$

$$\bar{x} = \frac{p_0\dfrac{4a^2b}{3\pi}}{p_0\dfrac{2ab}{\pi}} = \frac{2}{3}a \quad \blacktriangleleft \qquad \text{By symmetry } \bar{y} = 0 \quad \blacktriangleleft$$

## 8.130




$$W_1 = \gamma t b^2 = 0.284(0.5)(24^2) = 81.79 \text{ lb}$$

$$W_2 = -\gamma t\left(\frac{\pi r^2}{4}\right) = -0.284(0.5)\frac{\pi(18^2)}{4} = -36.13 \text{ lb}$$

$$W = \Sigma W_i = 81.79 - 36.13 = 45.66 \text{ lb}$$

$$\bar{y}_1 = 12 \text{ in.} \qquad \bar{y}_2 = b - \frac{4r}{3\pi} = 24 - \frac{4(18)}{3\pi} = 16.361 \text{ in.}$$

$$W\bar{y} = \Sigma W_i \bar{y}_i = 81.79(12) - 36.13(16.361) = 390.4 \text{ lb} \cdot \text{in.}$$

From FBD:

$$\Sigma M_x = 0 \qquad T_A b - W\bar{y} = 0 \qquad T_A(24) - 390.4 = 0$$

$T_A = 16.267$ lb ◀

Due to symmetry $T_C = T_A = 16.267$ lb ◀

$$\Sigma F_z = 0 \qquad T_A + T_B + T_C - W = 0$$

$T_B = W - T_A - T_C = 45.66 - 2(16.267) = 13.13$ lb ◀

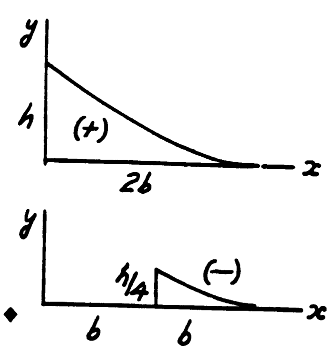
## 8.131

$$A = \frac{1}{3}(2b)h - \frac{1}{3}b\frac{h}{4} = \frac{7}{12}bh$$

$$A\bar{x} = \left[\frac{1}{3}(2b)h\right]\left[\frac{1}{4}(2b)\right] - \left(\frac{1}{3}b\frac{h}{4}\right)\left(b + \frac{b}{4}\right) = \frac{11}{48}b^2h$$

$$A\bar{y} = \left[\frac{1}{3}(2b)h\right]\frac{3h}{10} - \left(\frac{1}{3}b\frac{h}{4}\right)\left(\frac{3}{10}\frac{h}{4}\right) = \frac{31}{160}bh^2$$

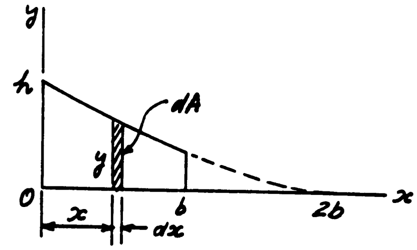
$$\therefore \bar{x} = \frac{11b^2h/48}{7bh/12} = \frac{11}{28}b \; ◆ \qquad \therefore \bar{y} = \frac{31bh^2/160}{7bh/12} = \frac{93}{280}h \; ◆$$



## 8.132

Equation of the parabola is obtained from the conditions y = h at x = 0, $y = \frac{dy}{dx} = 0$ at x = 2b:

$$y = \frac{h}{4b^2}(x - 2b)^2$$




We choose single integration.

$$dA = y\,dx = \frac{h}{4b^2}(x - 2b)^2\,dx$$

$$\bar{x}_{el} = x \qquad \bar{y}_{el} = \frac{y}{2} = \frac{h}{8b^2}(x - 2b)^2$$

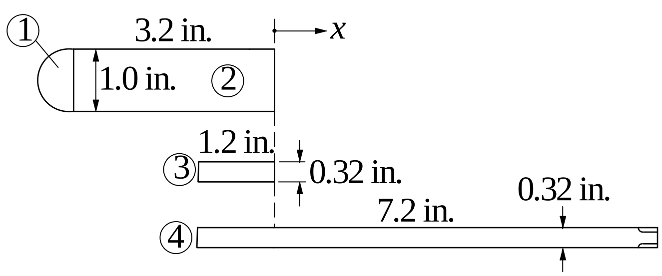
$$A = \int_A dA = \frac{h}{4b^2}\int_0^b (x - 2b)^2\,dx = \frac{7}{12}bh$$

$$Q_x = \int_A \bar{y}_{el}\,dA = \frac{h^2}{32b^4}\int_0^b (x - 2b)^4\,dx = \frac{31}{160}bh^2$$

$$Q_y = \int_A \bar{x}_{el}\,dA = \frac{h}{4b^2}\int_0^b x(x - 2b)^2\,dx = \frac{11}{48}b^2h$$

$$\bar{x} = \frac{Q_y}{A} = \frac{11b^2h/48}{7bh/12} = \frac{11}{28}b \ \blacklozenge \qquad \bar{y} = \frac{Q_x}{A} = \frac{31bh^2/160}{7bh/12} = \frac{93}{280}h \ \blacklozenge$$

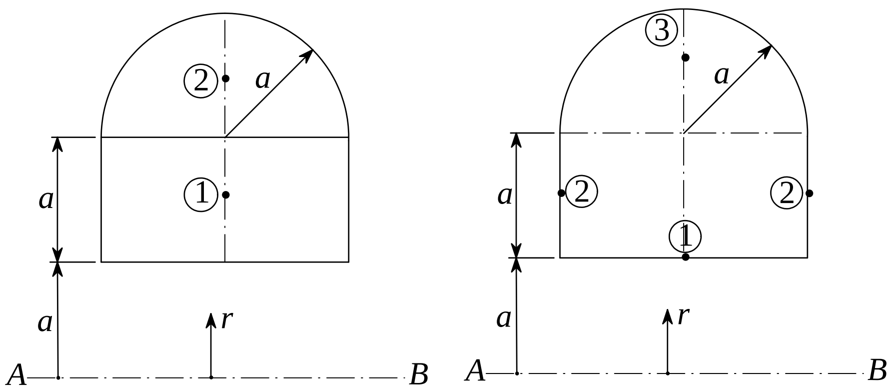
## 8.133




$$\begin{aligned}
W_1 &= \frac{2\pi}{3}R^3\gamma = \frac{2\pi}{3}(0.5^3)(0.055) = 0.01440 \text{ lb} \\
W_2 &= \pi R^2 L\gamma = \pi(0.5^2)(3.2)(0.055) = 0.138\ 23 \text{ lb} \\
W_3 &= -\pi(0.16^2)(1.2)(0.055) = -0.005\ 308 \text{ lb} \\
W_4 &= \pi(0.16^2)(7.2)(0.283) = 0.163\ 87 \text{ lb} \\
\bar{x}_1 &= -3.2 - \frac{3}{8}(0.5) = -3.3875 \text{ in.}
\end{aligned}$$

$$
\begin{aligned}
\bar{x} &= \frac{\Sigma W_i x_i}{\Sigma W_i} \\
&= \frac{0.01440(-3.3875) + 0.13823(-1.6) - 0.005308(-0.6) + 0.16387(2.4)}{0.01440 + 0.13823 - 0.005308 + 0.16387} \\
&= \frac{0.12652}{0.31119} = 0.407 \text{ in.} \quad \blacktriangleright
\end{aligned}
$$

## 8.134




Volume

$$
\begin{aligned}
Q_{AB} &= \Sigma A_i \bar{r}_i = (2a^2)\left(\frac{3}{2}a\right) + \left(\frac{\pi}{2}a^2\right)\left(2a + \frac{4}{3\pi}a\right) = 6.808a^3 \\
V &= 2\pi Q_{AB} = 2\pi(6.808a^3) = 42.8a^3 \quad \blacktriangleleft
\end{aligned}
$$

Surface area

$$
\begin{aligned}
Q_{AB} &= \Sigma L_i \bar{r}_i = (2a)(a) + (2a)\left(\frac{3}{2}a\right) + (\pi a)\left(2a + \frac{2}{\pi}a\right) = 13.283a^2 \\
A &= 2\pi Q_{AB} = 2\pi(13.283a^2) = 83.5a^2 \quad \blacktriangleleft
\end{aligned}
$$

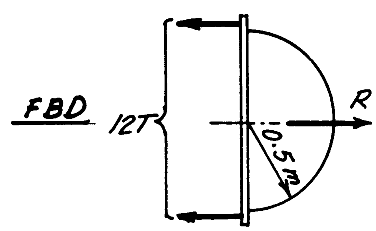
## 8.135

**The resultant of pressure actin on a hemisphere is**

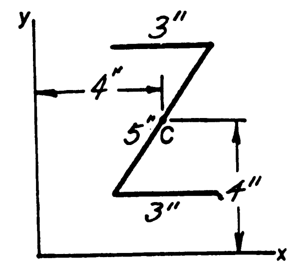
$$\mathbf{R = \pi p R^2 = \pi(300)(0.5)^2 = 235.6 \text{ kN}}$$

**The force in each bolt is**

$$\mathbf{T = \frac{R}{12} = \frac{235.6}{12} = 19.63 \text{ kN}} \quad \blacklozenge$$



## 8.136




$L = 3 + 5 + 3 = 11$ in

Centroid C of the curve can be located by inspection.

$\bar{x} = \bar{y} = 4$ in

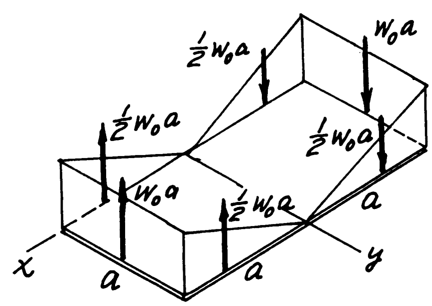
**(a)** $Q_x = L\bar{y} = (11)(4) = 44\ \text{in}^2$

$A = 2\pi Q_x = 2\pi(44) = 276\ \text{in}^2$ ◆

**(b)** Since $Q_x = Q_y$, we have from above

$A = 276\ \text{in}^2$ ◆

## 8.137



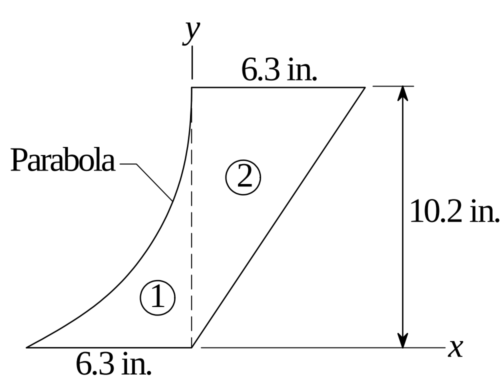

The resultant is the couple $\mathbf{C}^R = C^R\mathbf{j}$, where

$$C^R = \Sigma M_y = -2(w_0a)a - 4\left(\frac{1}{2}w_0a\right)\left(\frac{2a}{3}\right)$$

$$= -\frac{10}{3}w_0a = -\frac{10}{3}(36)(3) = -360\ \text{lb·ft}$$

$\therefore\ \mathbf{C}^R = -360\mathbf{j}$ lb·ft ◆

## 8.138



$$A_1 = \frac{1}{3}(6.3)(10.2) = 21.42 \text{ in}^2 \qquad A_2 = \frac{1}{2}(6.3)(10.2) = 32.13 \text{ in}^2$$
$$\bar{x}_1 = -\frac{3}{10}(6.3) = -1.89 \text{ in.} \qquad \bar{x}_2 = \frac{1}{3}(6.3) = 2.1 \text{ in.}$$
$$\bar{y}_1 = \frac{1}{4}(10.2) = 2.55 \text{ in.} \qquad \bar{y}_2 = \frac{2}{3}(10.2) = 6.8 \text{ in.}$$

$$A = \Sigma A_i = 21.42 + 32.13 = 53.55 \text{ in}^2$$

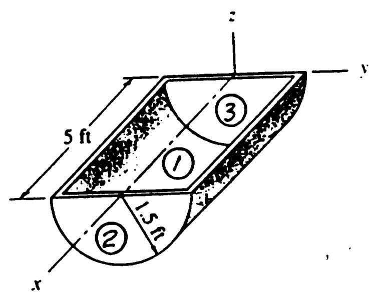
$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{A} = \frac{21.42(-1.89) + 32.13(2.1)}{53.55} = 0.504 \text{ in.} \blacktriangleleft$$
$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{21.42(2.55) + 32.13(6.8)}{53.55} = 5.10 \text{ in.} \blacktriangleleft$$

## 8.139

$A_1 = (1.5\pi)(5) = 23.56 \text{ ft}^2$ $\quad \bar{z}_1 = -\frac{2}{\pi}(1.5) = -0.9549 \text{ ft}$

$A_2 = A_3 = \frac{1}{2}\pi(1.5)^2 = 3.534 \text{ ft}^2$ $\quad \bar{z}_2 = \bar{z}_3 = -\frac{4}{3\pi}(1.5) = -0.6366 \text{ ft}$

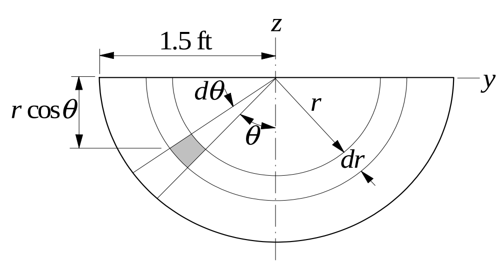
| Part | A ($ft^2$) | $\bar{z}$ (ft) | $A\bar{z}$ ($ft^3$) |
|---|---|---|---|
| 1 | 23.56 | −0.9549 | −22.50 |
| 2 | 3.53 | −0.6366 | −2.25 |
| 3 | 3.53 | −0.6366 | −2.25 |
| Sum | 30.62 | | −27.00 |




Due to symmetry: $\bar{x} = 2.5$ ft ◆ $\quad \bar{y} = 0$ ◆

$\bar{z} = -\frac{27.00}{30.62} = -0.882$ ft ◆

## 8.140



$$R = \int_A p\, dA \qquad p = \gamma h = 62.4\,(r\cos\theta) \qquad dA = (r\ d\theta)\, dr$$

$$R = 62.4 \int_{-\pi/2}^{\pi/2}\int_0^{1.5} r^2 \cos\theta\ dr\ d\theta = 62.4 \int_{-\pi/2}^{\pi/2} 1.125 \cos\theta\ d\theta$$

$$= 140.40 \text{ lb} \blacktriangleleft$$

$$R\bar{z} = \int_A pz\ dA = \int_A p\,(-r\cos\theta)\ dA = -62.4 \int_{-\pi/2}^{\pi/2}\int_0^{1.5} r^3 \cos^2\theta\ dr\ d\theta$$

$$= -62.4 \int_{-\pi/2}^{\pi/2} 1.2656 \cos^2\theta\ d\theta = -124.05 \text{ lb} \cdot \text{ft}$$

$$\bar{z} = \frac{-124.05}{140.40} = -0.884 \text{ ft} \blacktriangleleft \qquad \text{By symmetry } \bar{y} = 0 \blacktriangleleft$$

**8.141**

$W_1 = 0.2$ lb

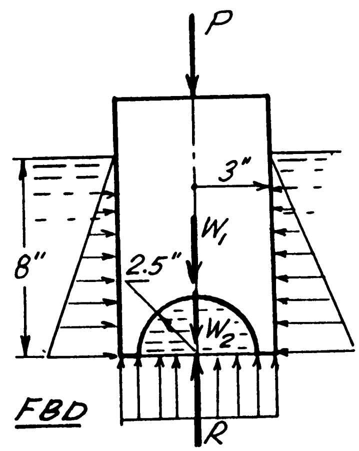
$W_2 = V_2\gamma = \frac{2\pi}{3}(2.5)^2(0.036) = 0.471$ lb

$R = A\gamma h = \pi(3)^2(0.036)(8) = 8.143$ lb

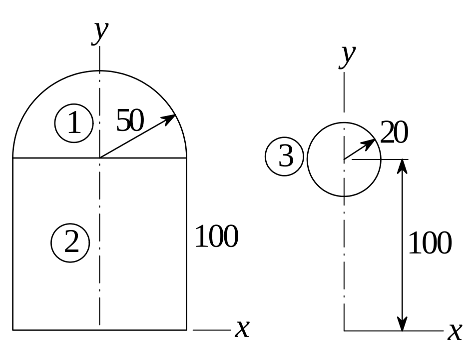
$\Sigma F = 0 \quad +\downarrow \quad P - R + W_1 + W_2 = 0$

$\therefore P = R - W_1 - W_2$

$= 8.143 - 0.2 - 0.471 = 7.47$ lb ♦




**8.142**

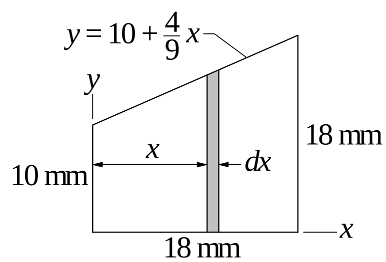


$$
\begin{aligned}
A_1 &= \frac{\pi}{2}(50^2) = 3927 \text{ mm}^2 \qquad \bar{y}_1 = 100 + \frac{4}{3\pi}(50) = 121.22 \text{ mm} \\
A_2 &= 100^2 = 10\ 000 \text{ mm}^2 \qquad \bar{y}_2 = 50 \text{ mm} \\
A_3 &= -\pi(20^2) = -1256.6 \text{ mm}^2 \qquad \bar{y}_3 = 100 \text{ mm}
\end{aligned}
$$

$$
\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{3927(121.22) + 10\ 000(50) - 1256.6(100)}{3927 + 10\ 000 - 1256.6} = 67.1 \text{ mm} \blacktriangleleft
$$

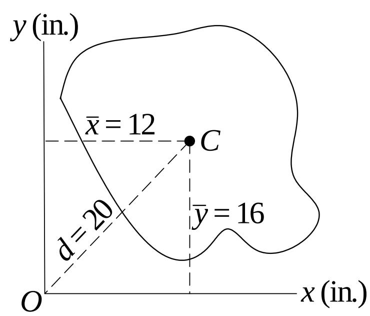
# Chapter 9

## 9.1




$$
\begin{aligned}
dA &= \left(10+\frac{4}{9}x\right)dx \\
I_y &= \int_{\mathcal{A}} x^2 dA = \int_0^{18} x^2\left(10+\frac{4}{9}x\right)dx = \left[10\frac{x^3}{3}+\frac{4}{9}\frac{x^4}{4}\right]_0^{18} \\
&= 31.1\times10^3\ \text{mm}^4 \blacktriangleleft
\end{aligned}
$$

## 9.2




$$
\begin{aligned}
J_O &= \bar{J}_C + Ad^2 = I_x + I_y \\
A &= \frac{I_x+I_y-\bar{J}_C}{d^2} == \frac{7000+4000-1000}{20^2} = 25.0\ \text{in}^2 \blacktriangleleft \\
\bar{I}_x &= I_x - A\bar{y}^2 = 7000 - 25.0(16^2) = 600\ \text{in}^4 \blacktriangleleft \\
\bar{I}_y &= I_y - A\bar{x}^2 = 4000 - 25.0(12^2) = 400\ \text{in}^4 \blacktriangleleft
\end{aligned}
$$

## 9.3

$$
\begin{aligned}
I_x - A\bar{y}^2 &= I_u - A(1.0-\bar{y})^2 \quad (=\bar{I}_x) \\
0.4 - 1.5\bar{y}^2 &= 0.6 - 1.5(1.0-\bar{y})^2 \qquad \bar{y} = 0.4333\ \text{ft} \blacktriangleleft
\end{aligned}
$$

$$
\bar{I}_x = 0.4 - 1.5\,(0.4333)^2 = 0.1184\ \text{ft}^4 \blacktriangleleft
$$

## 9.4

$$I_x = \bar{I}_x + A\bar{y}^2 \qquad \bar{y} = \sqrt{\frac{I_x - \bar{I}_x}{A}} = \sqrt{\frac{0.35 - 0.08}{1.5}} = 0.4243 \text{ ft} \blacktriangleleft$$

$$I_u = \bar{I}_x + A(1.0 - \bar{y})^2 = 0.08 + 1.5(1 - 0.4243)^2 = 0.577 \text{ ft}^4 \blacktriangleleft$$

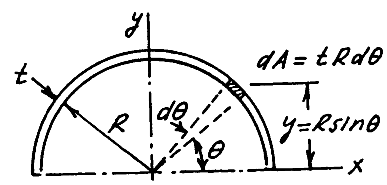
## 9.5

$$I_x = \int_0^\pi y^2\,dA = \int_0^\pi (R\sin\theta)^2\,(tR\,d\theta)$$

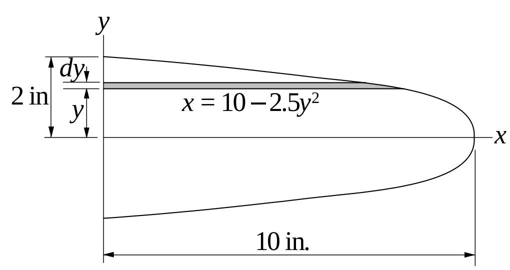
$$= tR^3\int_0^\pi \sin^2\theta\,d\theta = \frac{\pi}{2}tR^3$$

$$A = \int_0^\pi dA = \int_0^\pi tR\,d\theta = tR\int_0^\pi d\theta = \pi tR$$

$$k_x = \sqrt{I_x/A} = \sqrt{(\pi tR^3/2)/(\pi tR)} = R/\sqrt{2} \;\blacklozenge$$




## 9.6

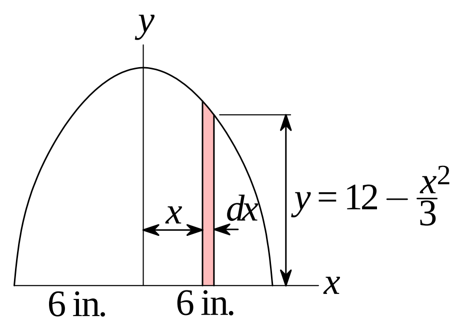



$$dA = (10 - 2.5y^2)dy$$

$$I_x = \int_{\mathcal{A}} y^2 dA = \int_{-2}^{2} y^2(10 - 2.5y^2)dy = 2\int_0^2 y^2(10 - 2.5y^2)dy$$

$$= 2\left[10\frac{y^3}{3} - 2.5\frac{y^5}{5}\right]_0^2 = 21.3 \text{ in}^4 \blacktriangleleft$$

**9.7**

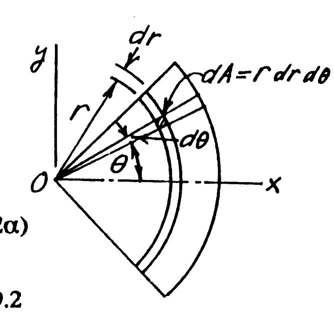



$$I_x = 2\int_0^6 \frac{y^3\ dx}{3} = \frac{2}{3}\int_0^6 \left(12 - \frac{x^2}{3}\right)^3 dx$$
$$= \frac{2}{3}\int_0^6 \left(-\frac{1}{27}x^6 + 4x^4 - 144x^2 + 1728\right) dx = 3160\ \text{in}^4 \ \blacktriangleleft$$

$$I_y = 2\int_0^6 x^2 y\ dx = 2\int_0^6 \left(12 - \frac{x^2}{3}\right) x^2\ dx$$
$$= 2\int_0^6 \left(12x^2 - \frac{x^4}{3}\right) dx = 691.2\ \text{in}^4 \ \blacktriangleleft$$

**9.8**

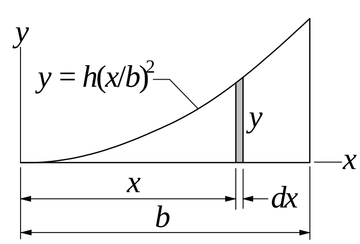
$$J_O = \int_A r^2\, dA = \int_{r=0}^{R}\int_{\theta=-\alpha}^{\alpha} r^2 (r\, dr\, d\theta)$$
$$= 2\alpha\int_0^R r^3\, dr = 2\alpha \frac{R^4}{4} = \frac{1}{2}R^4\alpha \ \blacklozenge$$




**Check:** $I_x = \frac{1}{8}R^4(2\alpha - \sin 2\alpha)$ $\quad I_y = \frac{1}{8}R^4(2\alpha + \sin 2\alpha)$

$\therefore J_O = I_x + I_y = \frac{1}{2}R^4\alpha$ **Checks with Table 9.2**

**9.9**



$$I_x = \int_{\mathcal{A}} dI_x = \int_0^b \frac{1}{3} y^3 dx = \frac{1}{3} \int_0^b h^3 \left(\frac{x}{b}\right)^6 dx = \frac{h^3}{3b^6} \left[\frac{x^7}{7}\right]_0^b = \frac{bh^3}{21} \blacktriangleleft$$

$$I_y = \int_{\mathcal{A}} x^2 dA = \int_0^b x^2 (y\ dx) = \int_0^b x^2 h \left(\frac{x}{b}\right)^2 dx = \frac{h}{b^2} \left[\frac{x^5}{5}\right]_0^b = \frac{b^3 h}{5} \blacktriangleleft$$

Results check with Table 9.2.

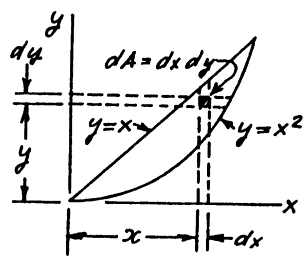
## 9.10

$$I_x = \int_A y^2\, dA = \int_{x=0}^{1} \int_{y=x^2}^{x} y^2\, dx\, dy = \int_0^1 \left[\frac{y^3}{3}\right]_{x^2}^{x} dx$$

$$= \frac{1}{3}\int_0^1 (x^3 - x^6)\, dx = \frac{1}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \frac{1}{28}\ \text{m}^4 \ \blacklozenge$$

$$I_y = \int_A x^2\, dA = \int_{x=0}^{1} \int_{y=x^2}^{x} x^2\, dx\, dy = \int_0^1 x^2 \big[y\big]_{x^2}^{x} dx$$

$$= \int_0^1 (x^3 - x^4)\, dx = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}\ \text{m}^4 \ \blacklozenge$$

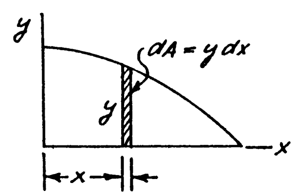



## 9.11

For the area element: $dI_x = \frac{1}{3} y^3\, dx$

Utilizing symmetry: $I_x = \int_A dI_x = 2\int_0^{20} \frac{1}{3} y^3\, dx$

$$= 2\int_0^{20} \frac{1}{3}\left(12 \cos \frac{\pi x}{40}\right)^3 dx$$




Let $\frac{\pi x}{40} = \xi \qquad \therefore\ dx = \frac{40}{\pi} d\xi \qquad \therefore\ I_x = \frac{2}{3}\frac{40}{\pi}(12^3)\int_0^{\pi/2} \cos^3\xi\, d\xi = \frac{2}{3}\frac{40}{\pi}(12^3)\frac{2}{3} = 9780\ \text{in}^4 \ \blacklozenge$

## 9.12

Utilizing symmetry: $I_y = \int_A x^2\,dA = 2\int_0^{20} x^2 y\,dx = 2(12)\int_0^{20} x^2 \cos\frac{\pi x}{40}\,dx$

From table of integrals: $\int x^2 \cos ax\,dx = \frac{2x}{a}\cos ax - \frac{2}{a^3}\sin ax + \frac{x^2}{a}\sin ax$

$$\therefore I_y = 2(12)\left[2x\left(\frac{40}{\pi}\right)\cos\frac{\pi x}{40} - 2\left(\frac{40}{\pi}\right)^3 \sin\frac{\pi x}{40} + x^2\left(\frac{40}{\pi}\right)\sin\frac{\pi x}{40}\right]_0^{20}$$

$$= 2(12)\left[-2\left(\frac{40}{\pi}\right)^3 \sin\frac{\pi x}{40} + x^2\left(\frac{40}{\pi}\right)\sin\frac{\pi x}{40}\right]_{x=20} = 23\ 150\ \text{in}^4\ \blacklozenge$$

## 9.13

For the rectangle: $\bar{I}_x = \frac{bh^3}{12} \quad \bar{I}_y = \frac{b^3h}{12}$

$\therefore \bar{I}_x\bar{I}_y = \frac{(bh)^4}{144} \quad \therefore A = bh = \sqrt[4]{144\,\bar{I}_x\bar{I}_y} = \sqrt[4]{144(272)(88.6)} = 43.16\ \text{in}^2 \approx 43.2\ \text{in}^2\ \blacklozenge$

$\bar{I}_x = \frac{(bh)h^2}{12} = \frac{Ah^2}{12} \quad \therefore h = \sqrt{(12\,\bar{I}_x)/A} = \sqrt{\frac{12(272)}{43.16}} = 8.696\ \text{in} \approx 8.70\ \text{in}\ \blacklozenge$

$b = \frac{A}{h} = \frac{43.16}{8.696} = 4.96\ \text{in}\ \blacklozenge$

## 9.14

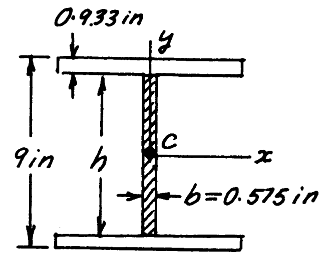
Column: $\bar{k}_x^2 = \bar{I}_x/A = \frac{272}{19.7} = 13.807\ \text{in}^2 \qquad \bar{k}_y^2 = \bar{I}_y/A = \frac{88.6}{19.7} = 4.497\ \text{in}^2$

Rectangle: $\bar{k}_x^2 = \frac{bh^3}{12}\frac{1}{bh} = \frac{h^2}{12} \qquad \bar{k}_y^2 = \frac{b^3h}{12}\frac{1}{bh} = \frac{b^2}{12}$

Equating the radii of gyration:

$\therefore \frac{h^2}{12} = 13.807 \quad \therefore h = 12.87\ \text{in}\ \blacklozenge \qquad \frac{b^2}{12} = 4.497 \quad \therefore b = 7.35\ \text{in}\ \blacklozenge$

## 9.15



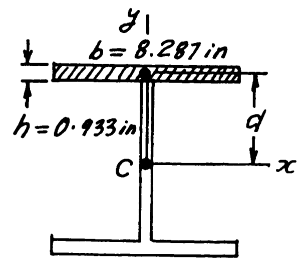

**Web:**

$h = 9 - 2(0.933) = 7.134$ in

$$I_x = \frac{bh^3}{12} = \frac{(0.575)(7.134^3)}{12} = 17.4 \text{ in}^4$$

$$I_y = \frac{b^3h}{12} = \frac{(0.575^3)(7.134)}{12} = 0.1 \text{ in}^4$$

**Flange:**




$A = bh = (8.287)(0.933) = 7.732 \text{ in}^2$

$$d = 4.5 - \frac{0.933}{2} = 4.0335 \text{ in}$$

$$I_x = \frac{bh^3}{12} + Ad^2 = \frac{(8.287)(0.933^3)}{12} + (7.732)(4.0335^2)$$

$$= 126.35 \text{ in}^4$$

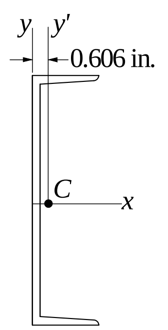
$$I_y = \frac{b^3h}{12} = \frac{(8.287^3)(0.933)}{12} = 44.25 \text{ in}^4$$

**Entire cross section:**

$\bar{I}_x = (I_x)_{web} + 2(I_x)_{flange} = 17.4 + 2(126.35) = 270.1 \text{ in}^4$ (272 $\text{in}^4$ in handbook) ◆

$\bar{I}_y = (I_y)_{web} + 2(I_y)_{flange} = 0.1 + 2(44.25) = 88.6 \text{ in}^4$ (88.6 $\text{in}^4$ in handbook) ◆

## 9.16



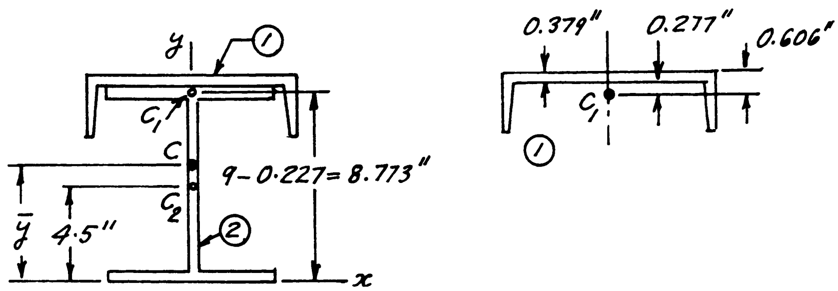

For one channel:

$$I_x = 78.9 \text{ in}^4 \qquad I_y = I_{y'} + Ad^2 = 2.81 + 5.88\,(0.606)^2 = 4.969 \text{ in}^4$$

For the assembly in Fig. (b):

$$\bar{I}_x = 2I_x = 2(78.9) = 157.8 \text{ in}^4 \blacktriangleleft$$
$$\bar{I}_y = 2I_y = 2(4.969) = 9.94 \text{ in}^4 \blacktriangleleft$$

## 9.17




For the composite areas:

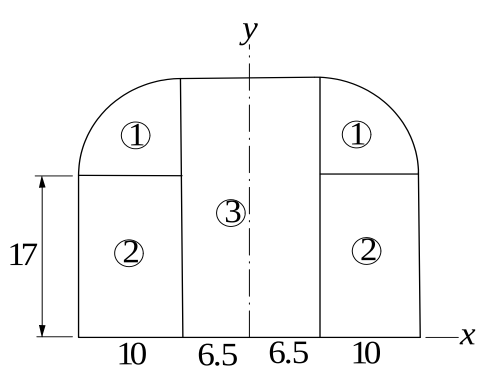
| Part | $\bar{I}_x$ (in$^4$) | A (in$^2$) | $\bar{y}$ (in) | A$\bar{y}$ (in$^3$) | $\bar{I}_x + A\bar{y}^2$ | $I_y$(in$^4$) |
|---|---|---|---|---|---|---|
| 1 | 2.81 | 5.88 | 8.773 | 51.59 | 455.4 | 78.9 |
| 2 | 272 | 19.70 | 4.50 | 88.65 | 670.9 | 88.6 |
| Sum | | 25.58 | | 140.24 | 1126.3 | 167.5 |

For the assembly:

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{140.24}{25.58} = 5.482 \text{ in} \qquad I_x = \Sigma(\bar{I}_x - A\bar{y}^2) = 1126.3 \text{ in}^4 \qquad I_y = \Sigma I_y = 167.5 \text{ in}^4$$

$$\bar{I}_x = I_x - A\bar{y}^2 = 1126.3 - (25.58)(5.482^2) = 358 \text{ in}^4 \blacklozenge \qquad \bar{I}_y = I_y = 167.5 \text{ in}^4 \blacklozenge$$

## 9.18



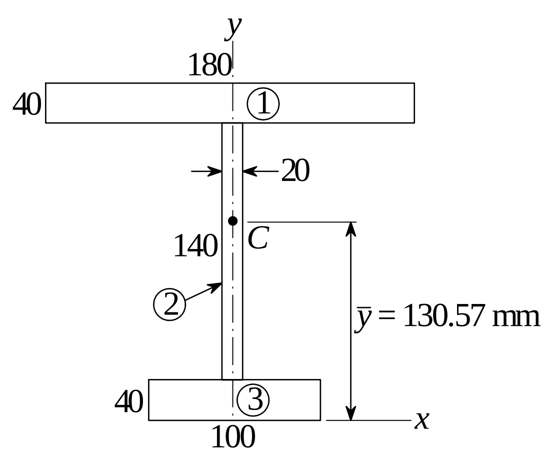
$$
\begin{aligned}
(I_x)_1 &= (\bar{I}_x)_1 + A_1\bar{y}_1^2 = 0.05488(10^4) + \frac{\pi(10^2)}{4}\left[17 + \frac{4(10)}{3\pi}\right]^2 \\
&= 35.99 \times 10^3 \text{ in}^4 \\
(I_x)_2 &= \frac{bh^3}{3} = \frac{10(17^3)}{3} = 16.377 \times 10^3 \text{ in}^4 \\
(I_x)_3 &= \frac{bh^3}{3} = \frac{13(27^3)}{3} = 85.29 \times 10^3 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
I_x &= 2(I_x)_1 + 2(I_x)_2 + (I_x)_3 = [2(35.99) + 2(16.377) + 85.29]\,10^3 \\
&= 190.0 \times 10^3 \text{ in}^4 \ \blacktriangleleft
\end{aligned}
$$

$$
\begin{aligned}
(I_y)_1 &= (\bar{I}_y)_1 + A_1\bar{x}_1^2 = 0.05488(10^4) + \frac{\pi(10^2)}{4}\left[6.5 + \frac{4}{3\pi}(10)\right]^2 \\
&= 9.615 \times 10^3 \text{ in}^4 \\
(I_y)_2 &= \frac{bh^3}{12} + A_2\bar{x}_2^2 = \frac{17(10^3)}{12} + (10 \times 17)(11.5^2) \\
&= 23.90 \times 10^3 \text{ in}^4 \\
(I_y)_3 &= \frac{bh^3}{12} = \frac{27(13^3)}{12} = 4.943 \times 10^3 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
I_y &= 2(I_y)_1 + 2(I_y)_3 + (I_y)_3 = [2(9.615) + 2(23.90) + 4.943] \times 10^3 \\
&= 72.0 \times 10^3 \text{ in}^4 \ \blacktriangleleft
\end{aligned}
$$

**9.19**

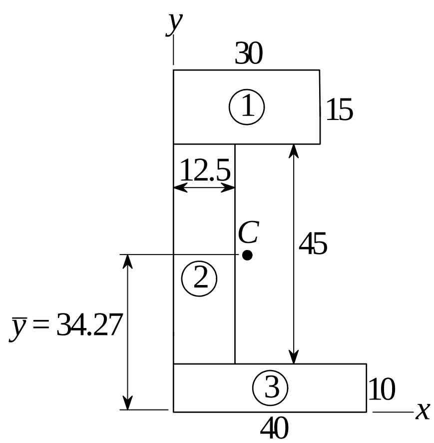



$$
\begin{aligned}
\bar{y} &= \frac{\Sigma A_i\bar{y}_i}{\Sigma A_i} = \frac{(180 \times 40)(200) + (140 \times 20)(110) + (100 \times 40)(20)}{(180 \times 40) + (140 \times 20) + (100 \times 40)} \\
&= 130.57\,\text{mm}
\end{aligned}
$$

$$\bar{I}_y = \frac{40(180^3)}{12} + \frac{140(20^3)}{12} + \frac{40(100^3)}{12} = 22.9 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

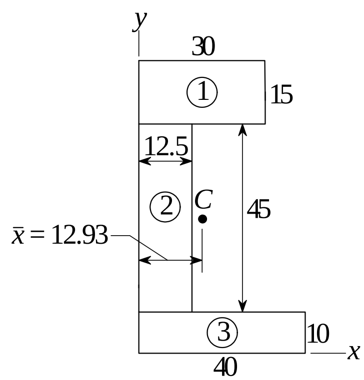
$$\begin{aligned}\bar{I}_x &= \frac{180(40^3)}{12} + (180 \times 40)(200 - 130.57)^2 + \frac{20(140^3)}{12} \\ &\quad +(20 \times 140)(110 - 130.57)^2 \\ &\quad + \frac{100(40^3)}{12} + (40 \times 100)(20 - 130.57)^2 \\ &= 90.9 \times 10^6 \text{ mm}^4 \blacktriangleleft\end{aligned}$$

**9.20**



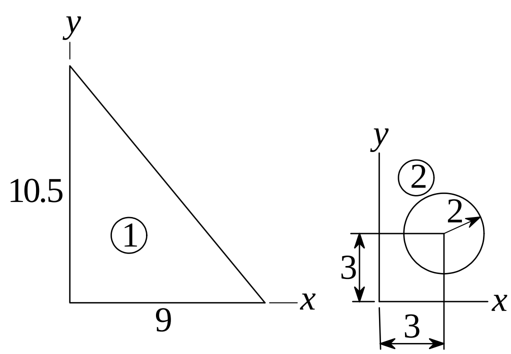

$$\begin{aligned}\bar{I}_x &= \frac{30(15^3)}{12} + (30 \times 15)(62.5 - 34.27)^2 \\ &\quad + \frac{12.5(45^3)}{12} + (12.5 \times 45)(32.5 - 34.27)^2 \\ &\quad + \frac{40(10^3)}{12} + (40 \times 10)(5 - 34.27)^2 \\ &= (367.1 + 96.7 + 346.0)\, 10^3 = 810 \times 10^3 \text{ mm}^4 \blacktriangleleft\end{aligned}$$

**9.21**




$$\bar{I}_y = \frac{15(30^3)}{12} + (30 \times 15)(15 - 12.93)^2 + \frac{45(12.5^3)}{12} + (12.5 \times 45)(6.25 - 12.93)^2 + \frac{10(40^3)}{12} + (40 \times 10)(20 - 12.93)^2$$
$$= (35.68 + 32.42 + 73.33)\,10^3 = 141.4 \times 10^3 \text{ mm}^4 \blacktriangleleft$$

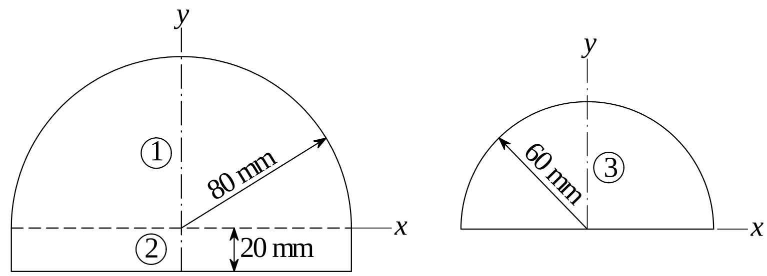
**9.22**




$$I_x = (I_x)_1 - (I_x)_2 = \frac{9(10.5^3)}{3} - \frac{\pi(2^4)}{4} - \pi(2^2)(3^2) = 3347 \text{ in}^4 \blacktriangleleft$$

$$A = \frac{1}{2}(9)(10.5) - \pi(2^2) = 34.68 \text{ in}^2$$
$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{\frac{1}{2}(9)(10.5)(3.5) - \pi(2^2)(3)}{34.68} = 3.681 \text{ in.}$$
$$\bar{I}_x = I_x - A\bar{y}^2 = 3347 - 34.68(3.681^2) = 2880 \text{ in}^4 \blacktriangleleft$$

**9.23**




$$I_x = \Sigma(I_x)_i = \frac{\pi(80^4)}{8} + \frac{160(20^3)}{3} - \frac{\pi(60^4)}{8} = 11.422 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y = \Sigma(I_y)_i = \frac{\pi(80^4)}{8} + \frac{20(160^3)}{12} - \frac{\pi(60^4)}{8} = 17.82 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$A = \Sigma A_i = \frac{\pi(80^2)}{2} + 160(20) - \frac{\pi(60^2)}{2} = 7598 \text{ mm}^2$$

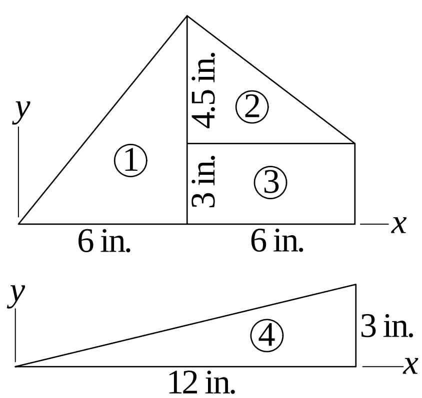
$$\Sigma A_i \bar{y}_i = \frac{\pi(80^2)}{2}\frac{4(80)}{3\pi} + (160 \times 20)(-10) - \frac{\pi(60^2)}{2}\frac{4(60)}{3\pi}$$

$$= 0.16533 \times 10^6 \text{ mm}^3$$

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{A} = \frac{0.16533 \times 10^6}{7598} = 21.76 \text{ mm}$$

$$\bar{I}_x = I_x - A\bar{y}^2 = 11.422 \times 10^6 - 7598(21.76^2) = 7.82 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

**9.24**



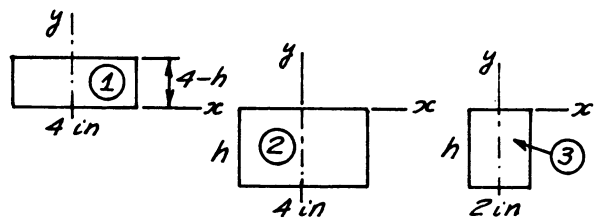

| Part | $A$ (in$^2$) | $\bar{y}$ (in.) | $A\bar{y}$ (in$^3$) |
|---|---|---|---|
| 1 | 22.5 | 2.5 | 56.25 |
| 2 | 13.5 | 4.5 | 60.75 |
| 3 | 18.0 | 1.5 | 27.00 |
| 4 | −18.0 | 1.0 | −18.00 |
| Sum | 36.0 | | 126.00 |

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{126.0}{36.0} = 3.50 \text{ in.}$$

$$I_x = \frac{6(7.5^3)}{12} + \left[\frac{6(4.5^3)}{36} + 13.5(4.5^2)\right] + \frac{6(3^3)}{3} - \frac{12(3^3)}{12} = 526.5 \text{ in}^4$$

$$\bar{I}_x = I_x - A\bar{y}^2 = 526.5 - 36(3.50^2) = 85.5 \text{ in}^4 \blacktriangleleft$$

## 9.25




$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3 = \frac{4(4-h)^3}{3} + \frac{4h^3}{3} - \frac{2h^3}{3} = \frac{4(4-h)^3 + 2h^3}{3}$$

$I_x$ is minimized with respect to h if $\dfrac{\partial I_x}{\partial h} = \dfrac{-(4)(3)(4-h)^2 + (2)(3)h^2}{3} = 0$

$\therefore h^2 - 16h + 32 = 0 \qquad \therefore h = \dfrac{16 \pm \sqrt{16^2 - 4(32)}}{2} = \dfrac{16 \pm 11.314}{2}$

The smaller root (h < 4 ft) is h = 2.34 ft ◆

## 9.26

For circle with hole:

$$I_x = (I_x)_{circle} - (I_x)_{hole} = \frac{\pi R^4}{4} - \left[\frac{\pi (R/2)^4}{4} + \pi (R/2)^2 d^2\right] = \frac{15\pi R^4}{64} - \frac{\pi R^2 d^2}{4}$$

$A = \pi R^2 - \pi\left(\dfrac{R}{2}\right)^2 = \dfrac{3\pi R^2}{4} \qquad \therefore k_x^2 = \dfrac{I_x}{A} = \dfrac{5R^2}{16} - \dfrac{d^2}{3}$ .................... (a)

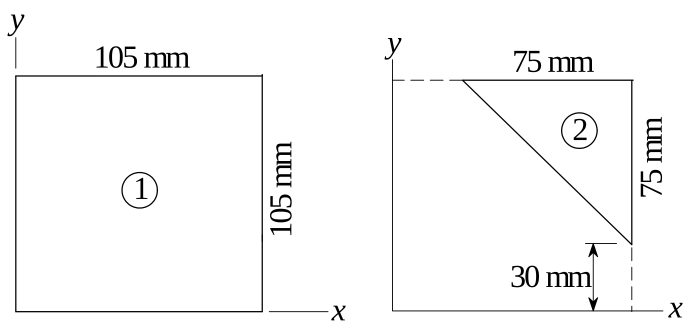
For solid circle:

$I_x = \dfrac{\pi R^4}{4} \qquad A = \pi R^2 \qquad \therefore k_x^2 = \dfrac{I_x}{A} = \dfrac{R^2}{4}$ ......................... (b)

Equating the right-hand sides of Eqs. (a) and (b):

$\dfrac{5R^2}{16} - \dfrac{d^2}{3} = \dfrac{R^2}{4} \qquad \therefore d = \sqrt{\dfrac{3}{16}}\, R = 0.433\ R$ ◆

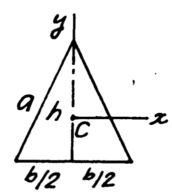
## 9.27




$$A_1 = 105^2 = 11\ 025 \text{ mm}^2 \qquad \bar{y}_1 = 52.5 \text{ mm}$$
$$A_2 = -\frac{1}{2}(75^2) = -2813 \text{ mm}^2 \qquad \bar{y}_2 = 80 \text{ mm}$$
$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{11\ 025\,(52.5) - 2813(80)}{11\ 025 - 2813} = 43.08 \text{ mm}$$

$$\begin{aligned}\bar{I}_x &= \Sigma\left[\left(\bar{I}_x\right)_i + A_i(\bar{y} - \bar{y}_i)^2\right] \\ &= \frac{105^4}{12} + 11\ 025(52.5 - 43.08)^2 - \frac{75^4}{36} - 2813(80 - 43.08)^2 \\ &= 6.39 \times 10^6 \text{ mm}^4 \ \blacktriangleleft\end{aligned}$$

## 9.28

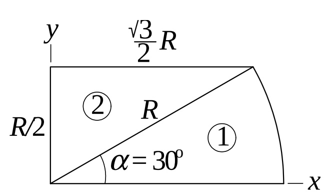



$\bar{I}_x = \frac{1}{36} bh^3 \qquad \bar{I}_y = \frac{1}{48} b^3h$

$\therefore\ \bar{I}_x = \bar{I}_y$ gives $\frac{1}{36} bh^3 = \frac{1}{48} b^3h \qquad \therefore\ h^2 = \frac{3}{4} b^2$

From geomerty: $h^2 = a^2 - (b/2)^2 \qquad \therefore\ a^2 - \frac{1}{4} b^2 = \frac{3}{4} b^2 \qquad \therefore\ \frac{a}{b} = 1$ ◆

## 9.29



Consider a quarter of the cross section:

$$\begin{aligned}(I_x)_1 &= \frac{R^4}{16}(2\alpha - \sin 2\alpha) = \frac{R^4}{16}\left(\frac{\pi}{3} - \sin 60^\circ\right) = 0.011323R^4 \\ (I_x)_2 &= \frac{bh^3}{36} + A\bar{y}^2 = \frac{1}{36}\left(\frac{\sqrt{3}}{2}R\right)\left(\frac{1}{2}R\right)^3 + \left(\frac{\sqrt{3}}{8}R^2\right)\left(\frac{R}{3}\right)^2 \\ &= 0.02706R^4\end{aligned}$$

For the full cross section:

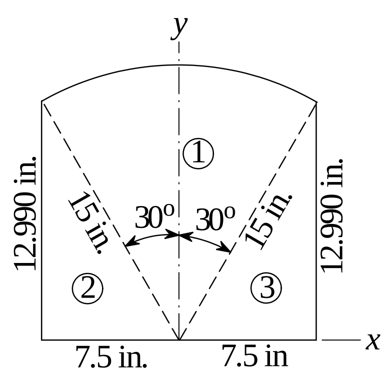
$$I_x = 4\left[(I_x)_1 + (I_x)_2\right] = 4(0.011323 + 0.02706)R^4 = 0.1535R^4 \blacktriangleleft$$

Before sawing

$$I_x = \frac{\pi R^4}{4} = 0.7854 \text{ in}^4$$

Percent reduction in $I_x$ is $\dfrac{0.7854 - 0.1535}{0.7854} \times 100\% = 80.5\% \blacktriangleleft$

**9.30**

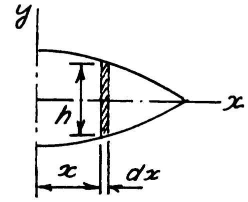



$$\begin{aligned}(I_x)_1 &= \frac{R^4}{8}(2\alpha + \sin 2\alpha) = \frac{15^4}{8}\left(2\frac{\pi}{6} + \sin 60^\circ\right) = 12\ 107 \text{ in}^4 \\ (I_x)_2 &= (I_x)_3 = \frac{7.5(12.990)^3}{12} = 1370 \text{ in}^4 \\ I_x &= \Sigma(I_x)_i = 12\ 107 + 2(1370) = 14\ 850 \text{ in}^4 \blacktriangleleft\end{aligned}$$

$$\begin{aligned}(I_y)_1 &= \frac{R^4}{8}(2\alpha - \sin 2\alpha) = \frac{15^4}{8}\left(2\frac{\pi}{6} - \sin 60^\circ\right) = 1146.5 \text{ in}^4 \\ (I_y)_2 &= (I_y)_3 = \left(\bar{I}_y\right)_2 + A_2 d^2 = \frac{12.990(7.5)^3}{36} + \frac{1}{2}(7.5)(12.990)(5^2) \\ &= 1370 \text{ in}^4 \\ I_y &= \Sigma(I_y)_i = 1146.5 + 2(1370) = 3890 \text{ in}^4 \blacktriangleleft\end{aligned}$$

## 9.31

Consider the <u>right half</u> of the region only. For the area element shown:




$$dA = h\,dx \qquad dI_x = \frac{1}{12}h^3\,dx \qquad dI_y = x^2 dA = hx^2\,dx$$

$$I_x = \int_A dI_x = \int_0^{24\text{ in}} \frac{h^3}{12}dx \qquad I_y = \int_A dI_y = \int_0^{24\text{ in}} hx^2\,dx$$

Using Simpson's rule to evaluate the integrals:

$$I_x \approx \frac{1}{12}\left(\sum_i W_i h_i^3\right)\frac{\Delta x}{3} = \frac{1}{12}\left[24^3 + 4(22.16^3) + 2(16.97^3) + 4(9.18^3) + 0\right]\frac{6}{3} = 11\,703\text{ in}^4$$

$$I_y \approx \left(\sum_i W_i h_i x_i^2\right)\frac{\Delta x}{3} = \left[0 + 4(22.16)(6^2) + 2(16.97)((12^2) + 4(9.18)(18^2) + 0\right]\frac{6}{3}$$

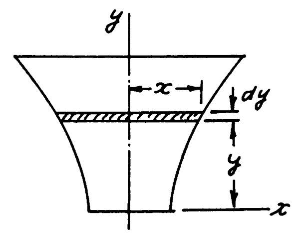
$$= 39\,951\text{ in}^4$$

<u>For the entire region</u>: $I_x = 2(11\,703) = 23.4 \times 10^3\text{ in}^4$ ♦

$I_y = 2(39\,951) = 79.9 \times 10^3\text{ in}^4$ ♦

## 9.32

For the area element shown: $dA = 2x\,dy$




$$dI_x = y^2\,dA = 2xy^2\,dy \qquad dI_y = \frac{1}{12}(2x)^3\,dx = \frac{2}{3}x^3\,dx$$

$$\therefore I_x = \int_A dI_x = 2\int_0^{120\text{ mm}} xy^2\,dy$$

$$\therefore I_y = \int_A dI_y = \frac{2}{3}\int_0^{120\text{ mm}} x^3\,dx$$

Using Simpson's rule to evaluate the integrals:

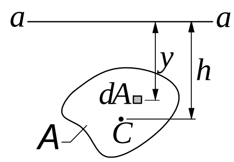
$$I_x \approx 2\left(\sum_i W_i x_i y_i^2\right)\frac{\Delta y}{3} = 2\left[0 + 4(22)(20^2) + 2(27)(40^2) + 4(35)(60^2) + 2(46)(80^2)\right.$$

$$\left. + 4(60)(100^2) + (77)(120^2)\right]\frac{20}{3} = 62.98 \times 10^6\text{ mm}^4 \;♦$$

$$I_y \approx \frac{2}{3}\left(\sum_i W_i x_i^3\right)\frac{\Delta y}{3} = \frac{2}{3}\left[20^3 + 4(22^3) + 2(27^3) + 4(35^3) + 2(46^3) + 4(60^3) + 77^3\right]\frac{20}{3}$$

$$= 7.90 \times 10^6\text{ mm}^4 \;♦$$

**9.33**

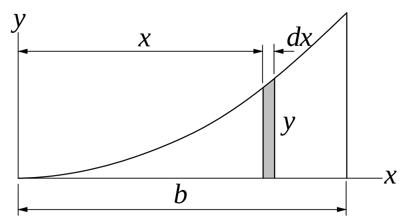



$$p = \gamma y$$

$$R = \int_A p \; dA = \gamma \int_A y \; dA = \gamma Q_a \quad \text{Q.E.D.}$$

$$\begin{aligned} Rh &= \int_A py \; dA = \gamma \int_A y^2 \; dA = \gamma I_a \\ h &= \frac{\gamma I_a}{R} = \frac{\gamma I_a}{\gamma Q_a} = \frac{I_a}{Q_a} \quad \text{Q.E.D.} \end{aligned}$$

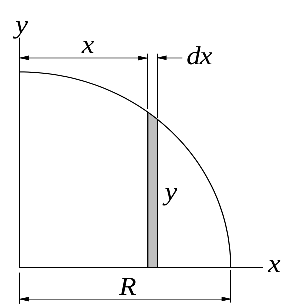
**9.34**




$$y = h\left(\frac{x}{b}\right)^2$$

$$\begin{aligned} dI_{xy} &= d\bar{I}_{xy} + \bar{x}_{\text{el}}\bar{y}_{\text{el}} \; dA = 0 + x\left(\frac{y}{2}\right)(y \; dx) = \frac{1}{2}xh^2\left(\frac{x}{b}\right)^4 \\ I_{xy} &= \int_0^b dI_{xy} = \frac{h^2}{2b^4}\int_0^b x^5 dx = \frac{b^2h^2}{12} \quad \blacktriangleleft \text{ Checks} \end{aligned}$$

**9.35**



**(a)**

$$x^2 + y^2 = R^2 \qquad y = \sqrt{R^2 - x^2}$$

$$dI_{xy} = d\bar{I}_{xy} + \bar{x}_{\text{el}}\bar{y}_{\text{el}}dA = 0 + x\left(\frac{y}{2}\right)(y\ dx) = 0 + \frac{1}{2}x(R^2 - x^2)dx$$

$$I_{xy} = \frac{1}{2}\int_0^R x\left(R^2 - x^2\right)dx = \frac{R^4}{8} \quad \blacktriangleleft \text{ Checks}$$

**(b)**

$$\bar{I}_{xy} = I_{xy} - A\bar{x}\bar{y} = \frac{R^4}{8} - \frac{\pi R^2}{4}\left(\frac{4R}{3\pi}\right)\left(\frac{4R}{3\pi}\right)$$

$$= R^4\left(\frac{1}{8} - \frac{4}{9\pi}\right) = -0.01647R^4 \quad \blacktriangleleft \text{ Checks}$$

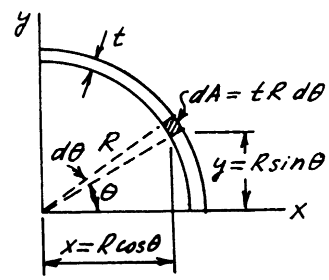
## 9.36

$$dI_{xy} = xy\ dA = (R\cos\theta)(R\sin\theta)(tR\ d\theta)$$

$$= tR^3 \sin\theta\cos\theta\ d\theta = \frac{1}{2}tR^3 \sin 2\theta\ d\theta$$

$$\therefore I_{xy} = \int_A dI_{xy} = \frac{1}{2}tR^3\int_0^{\pi/2} \sin 2\theta\ d\theta$$

$$= \frac{1}{2}tR^3\left[-\frac{1}{2}\cos 2\theta\right]_0^{\pi/2} = \frac{1}{2}tR^3 \quad \blacklozenge$$




## 9.37

The terms in the above equation have the same magnitudes for the four triangles, but differ in signs. The sign of $I_{xy}$ is determined by the quadrant in which the triangle lies. The sign of $A\bar{x}\bar{y}$ is determined by the signs of $\bar{x}$ and $\bar{y}$.

| Triangle | $I_{xy}$ | $\bar{x}$ | $\bar{y}$ | $\bar{I}_{xy}$ | |
|---|---|---|---|---|---|
| a | neg. | neg. | pos. | $-b^2h^2/72$ | |
| b | pos. | pos. | pos. | $b^2h^2/72$ | ◀ |
| c | pos. | neg. | neg. | $b^2h^2/72$ | |
| d | neg. | pos. | neg. | $-b^2h^2/72$ | |

## 9.38

$$\bar{I}_{xy} = I_{xy} - A\bar{x}\bar{y} = 520 \times 10^3 - (400)(30)(40) = 40 \times 10^3\ \text{mm}^4$$

$$I_{uv} = \bar{I}_{xy} + A\bar{u}\bar{v} = \bar{I}_{xy} + A(\bar{x} - d)\bar{v} = 40 \times 10^3 + (400)(30 - 40)(40) = -120 \times 10^3\ \text{mm}^4 \quad \blacklozenge$$

## 9.39

$$\bar{I}_{xy} = I_{xy} - A\bar{x}\bar{y} = 320 \times 10^3 - (400)\bar{x}(40) = (320 - 16\bar{x}) \times 10^3 \; \ldots\ldots\ldots \; \text{(a)}$$

$$\bar{I}_{xy} = I_{uv} - A\bar{u}\bar{v} = 0 - (400)(\bar{x} - d)(40) = 16(d - \bar{x}) \times 10^3 \; \ldots\ldots\ldots\ldots \; \text{(b)}$$

Equating the right-hand sides of Eqs. (a) and (b):

$$(320 - 16\bar{x}) = 16(d - \bar{x}) \qquad \therefore\ 320 = 16d \qquad \therefore\ d = 20.0 \text{ mm} \ \blacklozenge$$

## 9.40

Outer rectangle:

$$A_1 = 120(140) = 16\ 800 \text{ mm}^2 \qquad \bar{x}_1 = 60 \text{ mm} \qquad \bar{y}_1 = 70 \text{ mm}$$

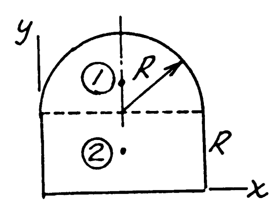
Inner rectangle:

$$A_2 = -60(80) = -4800 \text{ mm}^2 \qquad \bar{x}_2 = 70 \text{ mm} \qquad \bar{y}_2 = 80 \text{ mm}$$

$$\begin{aligned} A &= \Sigma A_i = 16\ 800 - 4800 = 12\ 000 \text{ mm}^2 \\ \bar{x} &= \frac{\Sigma A_i \bar{x}_i}{A} = \frac{16\ 800(60) - 4800(70)}{12\ 000} = 56.0 \text{ mm} \\ \bar{y} &= \frac{\Sigma A_i \bar{y}_i}{A} = \frac{16\ 800(70) - 4800(80)}{12\ 000} = 66.0 \text{ mm} \end{aligned}$$

$$\begin{aligned} \bar{I}_{xy} &= \Sigma A_i(\bar{x}_i - \bar{x})(\bar{y}_i - \bar{y}) \\ &= 16\ 800(60 - 56)(70 - 66) - 4800(70 - 56)(80 - 66) \\ &= -672\ 000 \text{ mm}^4 \ \blacktriangleleft \end{aligned}$$

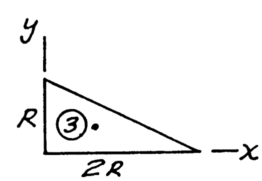
## 9.41




**Part 1**: $I_{xy} = \bar{I}_{xy} + A\bar{x}\bar{y}$

$$= 0 + \left(\frac{\pi}{2}R^2\right)(R)\left(R + \frac{4R}{3\pi}\right) = \left(\frac{\pi}{2} + \frac{2}{3}\right)R^4$$

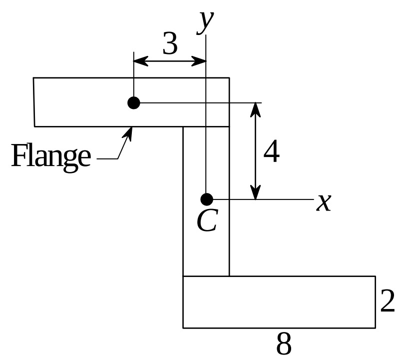
**Part 2**: $I_{xy} = \dfrac{b^2h^2}{4} = \dfrac{(2R)^2(R)^2}{4} = R^4$




**Part 3**: (to be subtracted) $I_{xy} = \dfrac{b^2h^2}{24} = \dfrac{(2R)^2(R)^2}{24} = \dfrac{R^4}{6}$

**Assembly**: $I_{xy} = \left(\dfrac{\pi}{2} + \dfrac{2}{3} + 1 - \dfrac{1}{6}\right)R^4 = 3.07R^4 \ \blacklozenge$

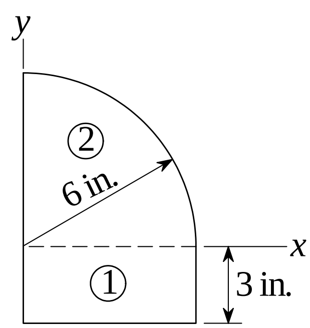
**9.42**




Centroid $C$ of the region can be found by inspection.

$$\begin{aligned}\bar{I}_{xy} &= \Sigma\left[(\bar{I}_{xy})_i + A_i\bar{x}_i\bar{y}_i\right] = 2(A\bar{x}\bar{y})_{\text{flange}} \\ &= 2(8 \times 2)(-3)(4) = -384 \text{ in}^4 \quad \blacktriangleleft\end{aligned}$$

**9.43**

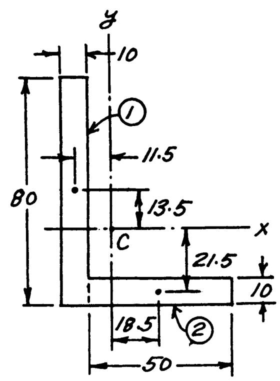



$$I_{xy} = A_1\bar{x}_1\bar{y}_1 + \frac{R_2^4}{8} = (6 \times 3)(3)(-1.5) + \frac{6^4}{8} = 81 \text{ in}^4 \quad \blacktriangleleft$$

## 9.44

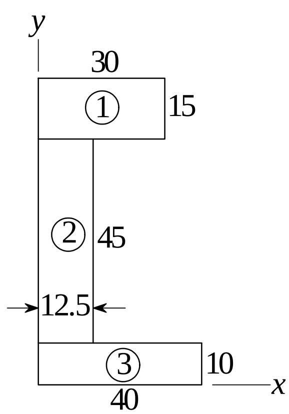
$$\bar{I}_{xy} = \sum_i \left[ (\bar{I}_{xy})_i + A_i \bar{x}_i \bar{y}_i \right] = 0 + \sum_i A_i \bar{x}_i \bar{y}_i$$

| Part | A (mm$^2$) | $\bar{x}$ (mm) | $\bar{y}$ (mm) | $A\bar{x}\bar{y}$ (mm$^4$) |
|---|---|---|---|---|
| 1 | 800 | −11.5 | 13.5 | $-124.2 \times 10^3$ |
| 2 | 500 | 18.5 | −21.5 | $-198.9 \times 10^3$ |
| Sum | | | | $-323.1 \times 10^3$ |




$\therefore I_{xy} = -323 \times 10^3 \text{ mm}^4$ ♦

## 9.45



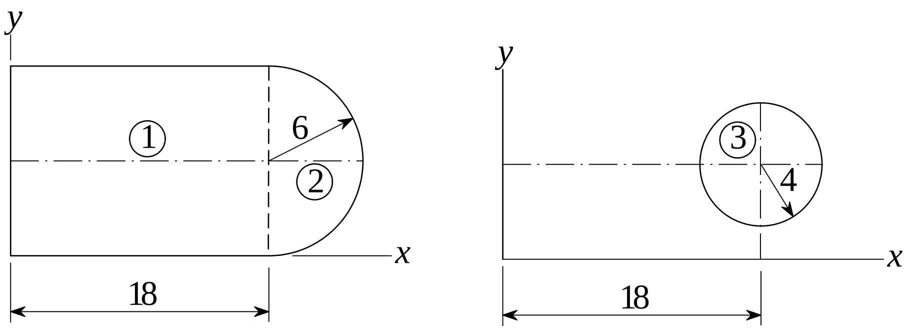

$$I_{xy} = \Sigma \left[ (\bar{I}_{xy})_i + A_i \bar{x}_i \bar{y}_i \right] = 0 + \Sigma A_i \bar{x}_i \bar{y}_i$$

| Part | $A$ (mm$^2$) | $\bar{x}$ (mm) | $\bar{y}$ (mm) | $A\bar{x}\bar{y}$ (mm$^4$) |
|---|---|---|---|---|
| 1 | 450.0 | 15.00 | 62.5 | $421.9 \times 10^3$ |
| 2 | 562.5 | 6.25 | 32.5 | $114.3 \times 10^3$ |
| 3 | 400.0 | 20.00 | 5.0 | $40.0 \times 10^3$ |
| Sum | 1412.5 | | | $576.2 \times 10^3$ |

$\bar{I}_{xy} = I_{xy} - A\bar{x}\bar{y} = 576.2 \times 10^3 - 1412.5(12.93)(34.27) = -49.7 \times 10^3 \text{ mm}^4$ ◄

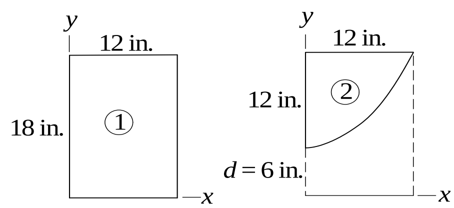
## 9.46

Due to symmetry $\bar{I}_{xy} = 0$ ◄




$$
\begin{aligned}
I_{xy} &= \Sigma A_i \bar{x}_i \bar{y}_i \\
&= (18 \times 12)(9)(6) + \frac{\pi(6^2)}{2}\left[18 + \frac{4(6)}{3\pi}\right](6) - \pi(4^2)(18)(6) \\
&= 13\ 210\ \text{in}^4 \ \blacktriangleleft
\end{aligned}
$$

**9.47**




Find the centroid first. From Table 8.1:

$$
\begin{aligned}
A_2 &= -\frac{2}{3}(12)^2 = -96\ \text{in}^2 \\
\bar{x}_2 &= \frac{3}{8}(12) = 4.5\ \text{in.} \qquad \bar{y}_2 = 6 + \frac{3}{5}(12) = 13.2\ \text{in.}
\end{aligned}
$$

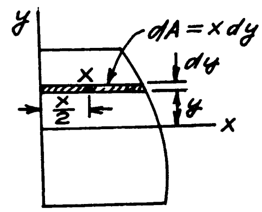
| Part | $A$ (in$^2$) | $\bar{x}$ (in.) | $\bar{y}$ (in.) | $A\bar{x}$ (in$^3$) | $A\bar{y}$ (in$^3$) |
|---|---|---|---|---|---|
| 1 | 216 | 6.0 | 9.0 | 1296 | 1944.0 |
| 2 | −96 | 4.5 | 13.2 | −432 | −1267.2 |
| Sum | 120 | | | 864 | 676.8 |

$$
\bar{x} = \frac{864}{120} = 7.200\ \text{in.} \qquad \bar{y} = \frac{676.8}{120} = 5.640\ \text{in.}
$$

$$
\begin{aligned}
(I_{xy})_1 &= \frac{b^2h^2}{4} = \frac{(12)^2\,(18)^2}{4} = 11\ 664\ \text{in}^4 \\
(I_{xy})_2 &= (\bar{I}_{xy})_2 + A_2\bar{x}_2\bar{y}_2 = \frac{b^2h^2}{60} + \frac{2bh}{3}\left(\frac{3h}{8}\right)\left(d + \frac{3h}{5}\right) \\
&= \frac{(12)^2(12)^2}{60} + \frac{2(12)(12)}{3}\left[\frac{3(12)}{8}\right]\left[6 + \frac{3(12)}{5}\right] = 6048\ \text{in}^4
\end{aligned}
$$

$$I_{xy} = (I_{xy})_1 - (I_{xy})_2 = 11\ 664 - 6048 = 5620\ \text{in}^4$$
$$\bar{I}_{xy} = I_{xy} - \bar{x}\bar{y}A = 6048 - (7.200)(5.640)(120) = 1175\ \text{in}^4 \blacktriangleleft$$

**9.48**




$$dI_{xy} = \bar{x}_{el}\bar{y}_{el}\, dA = \frac{x}{2} y\,(x\,dy) = \frac{1}{2}x^2 y\, dy$$

$$I_{xy} = \int_A dI_{xy} = \frac{1}{2}\int_A x^2 y\, dy$$

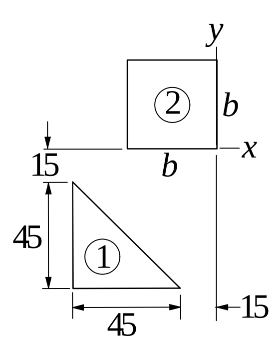
Using Simpson's rule:

$$I_{xy} \approx \frac{1}{2}\left(\frac{\Delta y}{3}\sum_{i=1}^{5} W_i x_i^2 y_i\right) = \frac{1}{2}\frac{185}{3}\sum_{i=1}^{5} W_i x_i^2 y_i$$

| $x_i$ (mm) | $y_i$ (mm) | $W_i$ | $W_i x_i^2 y_i$ (mm$^3$) |
|---|---|---|---|
| 406 | −370 | 1 | $-60.99 \times 10^6$ |
| 420 | −185 | 4 | $-130.54 \times 10^6$ |
| 392 | 0 | 2 | $0.00 \times 10^6$ |
| 338 | 185 | 4 | $84.54 \times 10^6$ |
| 218 | 370 | 1 | $17.58 \times 10^6$ |
| Sum | | | $-89.41 \times 10^6$ |

$$\therefore I_{xy} = \frac{1}{2}\frac{185}{3}(-89.41 \times 10^6) = -2.76 \times 10^9\ \text{mm}^4 \ \blacklozenge$$

**9.49**



Only the two cutouts contribute to $I_{xy}$. Therefore, $I_{xy} = 0$ if $(I_{xy})_1 + (I_{xy})_2 = 0$.

$$A_1 = \frac{1}{2}(45^2) = 1012.5 \text{ mm}^2 \qquad \bar{x}_1 = \bar{y}_1 = -15 - \frac{2}{3}(45) = -45 \text{ mm}$$

$$\begin{aligned} (I_{xy})_1 &= (\bar{I}_{xy})_1 + A_1\bar{x}_1\bar{y}_1 = -\frac{(45^2)(45^2)}{72} + 1012.5(-45)^2 \\ &= 1.9934 \times 10^6 \text{ mm}^4 \\ (I_{xy})_2 &= (\bar{I}_{xy})_2 + A_2\bar{x}_2\bar{y}_2 = 0 + b^2\left(-\frac{b}{2}\right)\left(\frac{b}{2}\right) = -\frac{b^4}{4} \end{aligned}$$

$$(I_{xy})_1 + (I_{xy})_2 = 0 \qquad 1.9934 \times 10^6 - \frac{b^4}{4} = 0 \qquad b = 53.1 \text{ mm} \blacktriangleleft$$

## 9.50

**(a)** Due to symmetry, the $xy$-axes are the principal axes at $C$.

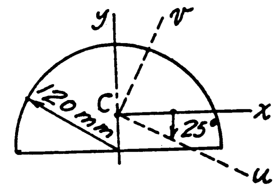
$$\begin{aligned} I_1 &= I_x = \frac{bh^3}{12} = \frac{12(16^3)}{12} = 4096 \text{ in}^4 \blacktriangleleft \\ I_2 &= I_y = \frac{b^3h}{12} = \frac{(12^3)(16)}{12} = 2304 \text{ in}^4 \blacktriangleleft \end{aligned}$$

**(b)**

$$\begin{aligned} b &= \frac{I_x + I_y}{2} = \frac{4096 + 2304}{2} = 3200 \text{ in}^4 \\ R &= \frac{I_x - I_y}{2} = \frac{4096 - 2304}{2} = 896 \text{ in}^4 \\ I_u &= b + R\cos 2\theta = 3200 + 896\cos 60^\circ = 3650 \text{ in}^4 \blacktriangleleft \\ I_v &= b - R\cos 2\theta = 3200 - 896\cos 60^\circ = 2750 \text{ in}^4 \blacktriangleleft \\ I_{uv} &= R\sin 2\theta = 896\sin 60^\circ = 776 \text{ in}^4 \blacktriangleleft \end{aligned}$$

## 9.51

(a) Due to symmetry, the x and y-axes are the principal axes at C. ◆




$I_1 = I_y = \frac{\pi}{8}R^4 = \frac{\pi}{8}(120)^4 = 81.43 \times 10^6 \text{ mm}^4$ ◆

$I_2 = I_x = 0.1098R^4 = 0.1098(120)^4 = 22.77 \times 10^6 \text{ mm}^4$ ◆

(b) $\frac{1}{2}(I_x + I_y) = \frac{1}{2}(22.77 + 81.43) \times 10^6 = 52.10 \times 10^6 \text{ mm}^4$

$\frac{1}{2}(I_x - I_y) = \frac{1}{2}(22.77 - 81.43) \times 10^6 = -29.33 \times 10^6 \text{ mm}^4 \quad \theta = -25°$

$I_u = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta = \left[52.10 - 29.33 \cos(-50°) - 0\right] \times 10^6$

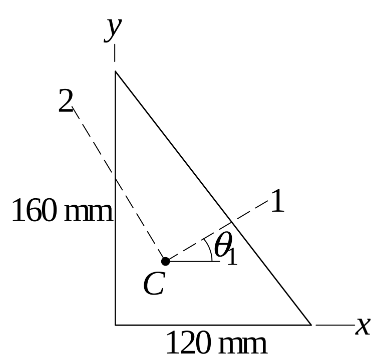
$= 33.2 \times 10^6 \text{ mm}^4$ ◆

$I_v = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta = \left[52.10 + 29.33 \cos(-50°) + 0\right] \times 10^6$

$= 71.0 \times 10^6 \text{ mm}^4$ ◆

$I_{uv} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta = \left[-29.33 \sin(-50°) + 0\right] \times 10^6 = 22.5 \times 10^6 \text{ mm}^4$ ◆

## 9.52




$$\bar{I}_x = \frac{bh^3}{36} = \frac{120(160^3)}{36} = 13.653 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = \frac{b^3h}{36} = \frac{(120^3)(160)}{36} = 7.680 \times 10^6 \text{ mm}^4$$
$$\bar{I}_{xy} = -\frac{b^2h^2}{72} = -\frac{(120^2)(160^2)}{72} = -5.120 \times 10^6 \text{mm}^4$$

$$b = \frac{\bar{I}_x + \bar{I}_y}{2} = \frac{13.653 + 7.680}{2} \times 10^6 = 10.667 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left(\frac{13.653 - 7.680}{2}\right)^2 + (-5.120)^2} \times 10^6$$

$$= 5.927 \times 10^6 \text{ mm}^4$$

$$I_1 = b + R = (10.667 + 5.927) \times 10^6 = 16.59 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

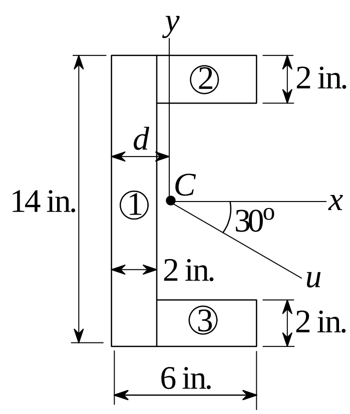
$$I_2 = b - R = (10.667 - 5.927) \times 10^6 = 4.74 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\sin 2\theta_1 = -\frac{\bar{I}_{xy}}{R} = -\frac{(-5.120)}{5.927} = 0.8638 \qquad \cos 2\theta_1 = \frac{\bar{I}_x - \bar{I}_y}{2R} > 0$$

$\therefore$ $2\theta_1$ lies in the first quadrant $\qquad 2\theta_1 = \sin^{-1}(0.8638) = 59.75^\circ$

$$\theta_1 = 29.9^\circ \blacktriangleleft \qquad \theta_2 = \theta_1 + 90^\circ = 119.9^\circ \blacktriangleleft$$

**9.53**




$$d = \frac{28(1.0) + 2\left[8(4)\right]}{28 + 2(8)} = 2.091 \text{ in.}$$

$$I_x = \frac{2(14)^3}{12} + 2\left[\frac{4(2)^3}{12} + 8(6)^2\right] = 1038.7 \text{ in}^4$$

$$I_y = \frac{14(2)^3}{12} + 28(1.0 - 2.091)^2 + 2\left[\frac{2(4)^3}{12} + (2 \times 4)(4 - 2.091)^2\right]$$

$$= 122.30 \text{ in}^4$$

$$I_{xy} = 0 \text{ due to symmetry}$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{1038.7 + 122.3}{2} + \frac{1038.7 - 122.3}{2}\cos(-60^\circ) = 810 \text{ in}^4 \blacktriangleleft$$

## 9.54

$$\frac{I_x + I_y}{2} = \frac{3000 + 2000}{2} = 2500 \text{ in}^4$$
$$\frac{I_x - I_y}{2} = \frac{3000 - 2000}{2} = 500 \text{ in}^4$$

$$I_{u,v} = \frac{I_x + I_y}{2} \pm \frac{I_x - I_y}{2}\cos 2\theta \mp I_{xy}\sin 2\theta$$
$$I_u = 2500 + 500\cos 240^\circ - (-500)\sin 240^\circ = 1817 \text{ in}^4 \blacktriangleleft$$
$$I_v = 2500 - 500\cos 240^\circ + (-500)\sin 240^\circ = 3180 \text{ in}^4 \blacktriangleleft$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$
$$= 500\sin 240^\circ + (-500)\cos 240^\circ = -183.0 \text{ in}^4 \blacktriangleleft$$

## 9.55

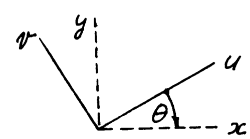
$$\frac{I_x + I_y}{2} = \frac{10 + 20}{2} \times 10^6 = 15 \times 10^6 \text{ mm}^4 \qquad \frac{I_x - I_y}{2} = \frac{10 - 20}{2} \times 10^6 = -5 \times 10^6 \text{ mm}^4$$

$$I_u = (15 - 5\cos 67.4^\circ - 12\sin 67.4^\circ) \times 10^6 = 2.00 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

$$I_v = (15 + 5\cos 67.4^\circ + 12\sin 67.4^\circ) \times 10^6 = 28.00 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

$$I_{uv} = (-5\sin 67.4^\circ + 12\cos 67.4^\circ) \times 10^6 = 0 \ \blacklozenge$$

## 9.56




To transform the moments of inertia from (u, v) to (x, y) coordinates, we can use Eqs. (9.17)–(9.19), but we must interchange the roles of (x, y) and (u, v). In addition, the direction of θ must be reversed, so that it points from u-axis to the x-axis. Thus Eqs. (19.17)–(19.19) become

$$I_x = \frac{I_u + I_v}{2} + \frac{I_u - I_v}{2}\cos 2\theta = \frac{7600 + 5000}{2} + \frac{7600 - 5000}{2}\cos 67.4^\circ = 6800 \text{ in}^4 \ \blacklozenge$$

$$I_y = \frac{I_u + I_v}{2} - \frac{I_u - I_v}{2}\cos 2\theta = \frac{7600 + 5000}{2} - \frac{7600 - 5000}{2}\cos 67.4^\circ = 5800 \text{ in}^4 \ \blacklozenge$$

$$I_{xy} = \frac{I_u - I_v}{2}\sin(-2\theta) = \frac{7600 - 5000}{2}\sin(-67.4^\circ) = -1200 \text{ in}^4 \ \blacklozenge$$

## 9.57

(a) $I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \frac{I_x - I_y}{2}\sin 2\theta + 0$

$I_{uv}$ is maximized when $\sin 2\theta = 1$, i.e. when $\theta = 45°$ ◆

(b) $\frac{I_x + I_y}{2} = \frac{6+2}{2} \times 10^6 = 4 \times 10^6 \text{ mm}^4 \quad \frac{I_x - I_y}{2} = \frac{6-2}{2} \times 10^6 = 2 \times 10^6 \text{ mm}^4$

$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta = (4 + 2\cos 90° - 0) \times 10^6 = 4 \times 10^6 \text{ mm}^4$ ◆

$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta = (4 - 2\cos 90° + 0) \times 10^6 = 4 \times 10^6 \text{ mm}^4$ ◆

$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = 2 \times 10^6 \sin 90° = 2 \times 10^6 \text{ mm}^4$ ◆

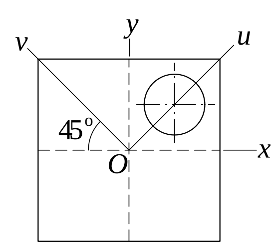
## 9.58

$\sin 2\theta_1 = -\frac{I_{xy}}{R} \qquad \therefore R = -\frac{I_{xy}}{\sin 2\theta_1} = -\frac{(-30 \times 10^6)}{\sin 36.88°} = 50.0 \times 10^6 \text{ mm}^4$

$I_1 = I_u = \frac{I_x + I_y}{2} + R \qquad \therefore I_x + I_y = 2(I_u - R) = 2(160 - 50) \times 10^6 = 220 \times 10^6 \text{ mm}^4$

But $I_x + I_y = I_u + I_v \qquad \therefore I_v = (I_x + I_y) - I_u = (220 - 160) \times 10^6 = 60 \times 10^6 \text{ mm}^4$ ◆

## 9.59




Due to symmetry, $u$ and $v$ are the principal axes at $O$.

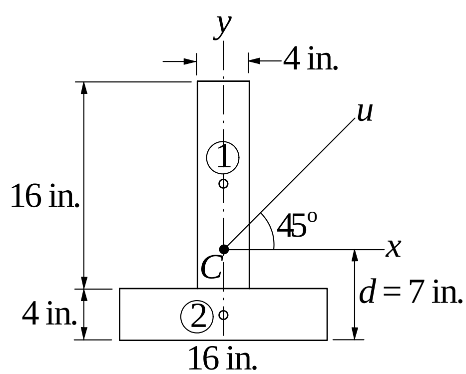
$$I_{u,v} = \frac{I_x + I_y}{2} \pm \frac{I_x - I_y}{2}\cos 2\theta \mp I_{xy}\sin 2\theta$$

With $I_x - I_y = 0$ and $2\theta = 90°$, this becomes

$$I_{u,v} = [16.023 \mp (-1.1310)] \times 10^6$$

$$I_u = I_1 = (16.023 + 1.1310) \times 10^6 = 17.15 \times 10^6 \text{ mm}^4 \blacktriangleleft$$
$$I_v = I_2 = (16.023 - 1.1310) \times 10^6 = 14.89 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

**9.60**

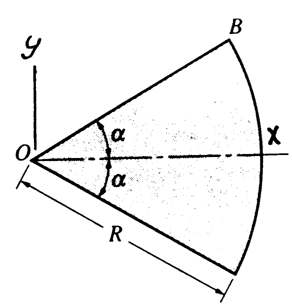



$$d = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{64(12) + 64(2)}{2(64)} = 7.0 \text{ in.}$$
$$I_x = \frac{4(16^3)}{12} + 64(5^2) + \frac{16(4^3)}{12} + 64(5^2) = 4651 \text{ in}^4$$
$$I_y = \frac{16(4^3)}{12} + \frac{4(16^3)}{12} = 1451 \text{ in}^4$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$
$$= \frac{4651 + 1451}{2} + \frac{4651 - 1451}{2}\cos 90^\circ - 0 = 3050 \text{ in}^4 \blacktriangleleft$$

**9.61**

$I_x = \frac{R^4}{8}(2\alpha - \sin 2\alpha)$ $\quad I_y = \frac{R^4}{8}(2\alpha + \sin 2\alpha)$ $\quad I_{xy} = 0$



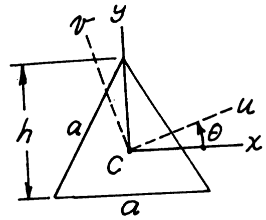

$\frac{I_x + I_y}{2} = \frac{R^4}{8}(2\alpha)$ $\quad \frac{I_x - I_y}{2} = -\frac{R^4}{8}\sin 2\alpha$

$$I_{OB} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\alpha - I_{xy}\sin 2\alpha$$
$$= \frac{R^4}{8}(2\alpha) - \frac{R^4}{8}\sin 2\alpha \cos 2\alpha - 0 = \frac{R^4}{16}(4\alpha - \sin 4\alpha) \blacklozenge$$

When $\alpha = 45°$, i.e., $4\alpha = \pi$ rad:

$I_{OB} = \frac{\pi R^4}{16}$ which agrees with $I_x$ of quarter circle in Table 9.2.

**9.62**




$h = \sqrt{a^2 - (a/2)^2} = (\sqrt{3}/2)a$

$\bar{I}_x = \dfrac{bh^3}{36} = \dfrac{(a)\left[(\sqrt{3}/2)a\right]^3}{36} = \dfrac{\sqrt{3}a^4}{96}$

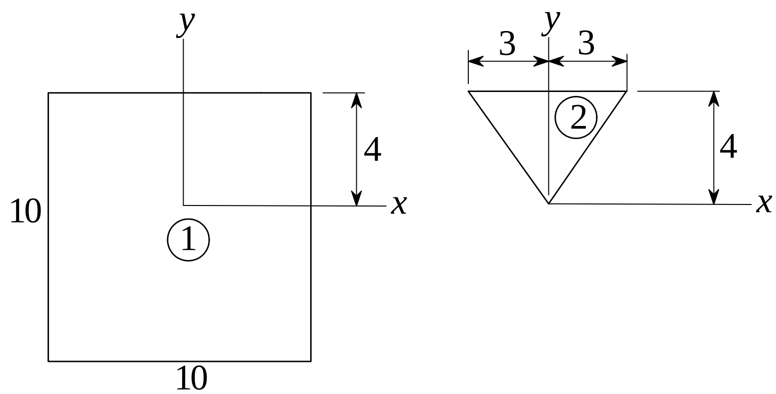
$\bar{I}_y = \dfrac{b^3h}{48} = \dfrac{(a^3)(\sqrt{3}/2)a}{48} = \dfrac{\sqrt{3}a^4}{96}$ $\qquad \bar{I}_{xy} = 0$

Since $\bar{I}_x = \bar{I}_y$ and $\bar{I}_{xy} = 0$, we conclude from Eqs. (9.17)–(9.19) that

$I_u = I_v = \dfrac{\sqrt{3}a^4}{96}$ and $I_{uv} = 0$ for all $\theta$

$\therefore$ Every axis passing through C is a principal axis ($I_{uv} = 0$), with the moment of inertia $\dfrac{\sqrt{3}a^4}{96}$

**9.63**




$$
\begin{aligned}
I_x &= \left(\bar{I}_x\right)_1 + A_1 d_1^2 - (\bar{I}_x)_2 - A_2 d_2^2 = \frac{10^4}{12} + 100(1.0^2) - \frac{6(4)^3}{36} - 12\left(\frac{2}{3}4\right)^2 = 837.3 \text{ in}^4 \\
I_y &= (\bar{I}_y)_1 - (\bar{I}_y)_2 = \frac{10^4}{12} - \frac{6^3(4)}{48} = 815.3 \text{ in}^4 \\
I_{xy} &= 0 \text{ due to symmetry}
\end{aligned}
$$

$$
\begin{aligned}
I_u &= \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \\
&= \frac{837.3 + 815.3}{2} + \frac{837.3 - 815.3}{2}\cos(90^\circ) = 826 \text{ in}^4 \quad \blacktriangleleft \\
I_v &= \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \\
&= \frac{837.3 + 815.3}{2} - \frac{837.3 - 815.3}{2}\cos(90^\circ) = 826 \text{ in}^4 \quad \blacktriangleleft \\
I_{xy} &= \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta \\
&= \frac{837.3 - 815.3}{2}\sin 90^\circ = 11.0 \text{ in}^4 \quad \blacktriangleleft
\end{aligned}
$$

## 9.64

**(a)** $I_x + I_y = I_1 + I_2 = (0.808 + 0.388) \times 10^6 = 1.196 \times 10^6 \text{ mm}^4$

$\therefore I_1 = I_x + I_y - I_2 = (1.196 - 0.213) \times 10^6 = 0.983 \times 10^6 \text{ mm}^4$ ♦

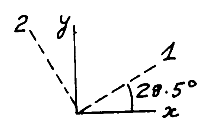
**(b)** $I_1 = \frac{1}{2}(I_x + I_y) + R$

$\therefore R = I_1 - \frac{1}{2}(I_x + I_y) = \left(0.983 - \frac{1.196}{2}\right) \times 10^6 = 0.385 \times 10^6 \text{ mm}^4$

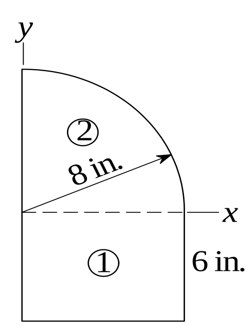
$\cos 2\theta_1 = \frac{I_x - I_y}{2R} = \frac{0.808 - 0.388}{2(0.385)} = 0.5455 \qquad \therefore 2\theta_1 = \pm 56.94^\circ$

$\sin 2\theta_1 = -\frac{I_{xy}}{R} > 0$ (it was stated that $I_{xy} < 0$) $\qquad \therefore 2\theta_1 = 56.94^\circ$

$\therefore \theta_1 = 28.5^\circ$ ♦ $\qquad \therefore \theta_2 = 28.5^\circ + 90^\circ = 118.5^\circ$ ♦




## 9.65



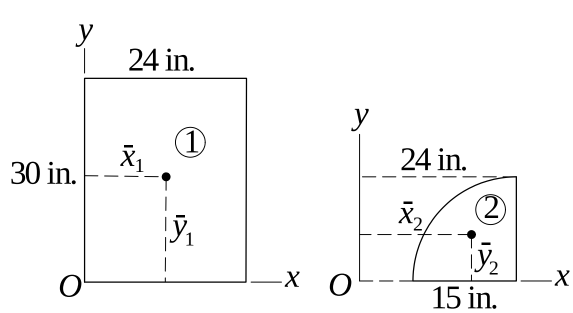
$$
\begin{aligned}
A_1 &= 48 \text{ in}^2 \qquad \bar{x}_1 = 4 \text{ in.} \qquad \bar{y}_1 = -3 \text{ in.} \\
A_2 &= \frac{\pi(8^2)}{4} = 50.27 \text{ in}^2 \qquad \bar{x}_2 = \bar{y}_2 = \frac{4(8)}{3\pi} = 3.395 \text{ in} \\
A &= \Sigma A_i = 48 + 50.27 = 98.27 \text{ in}^2 \\
\bar{x} &= \frac{\Sigma A_i \bar{x}_i}{A} = \frac{48(4) + 50.27(3.395)}{98.27} = 3.691 \text{ in.} \\
\bar{y} &= \frac{\Sigma A_i \bar{y}_i}{A} = \frac{48(-3) + 50.27(3.395)}{98.27} = 0.2714 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
I_x &= \frac{8(6^3)}{3} + \frac{\pi(8^4)}{16} = 1380.2 \text{ in}^4 \\
I_y &= \frac{6(8^3)}{3} + \frac{\pi(8^4)}{16} = 1828.2 \text{in}^4 \\
I_{xy} &= 48(4)(-3) + \frac{8^4}{8} = -64.0 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
\bar{I}_x &= I_x - A\bar{y}^2 = 1380.2 - 98.27(0.2714)^2 = 1373.0 \text{ in}^4 \\
\bar{I}_y &= I_y - A\bar{x}^2 = 1828.2 - 98.27(3.691)^2 = 489.4 \text{ in}^4 \\
\bar{I}_{xy} &= I_{xy} - A\bar{x}\bar{y} = -64.0 - 98.27(3.961)(0.2714) = -169.64 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
R &= \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left(\frac{1373.0 - 489.4}{2}\right)^2 + (-169.64)^2} \\
&= 473.25 \text{ in}^4 \\
I_1 &= \frac{\bar{I}_x + \bar{I}_y}{2} + R = \frac{1373.0 + 489.4}{2} + 473.25 = 1405 \text{ in}^4 \quad \blacktriangleleft \\
I_2 &= \frac{\bar{I}_x + \bar{I}_y}{2} - R = \frac{1373.0 + 489.4}{2} - 473.25 = 458 \text{ in}^4 \quad \blacktriangleleft
\end{aligned}
$$

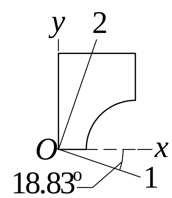
**9.66**



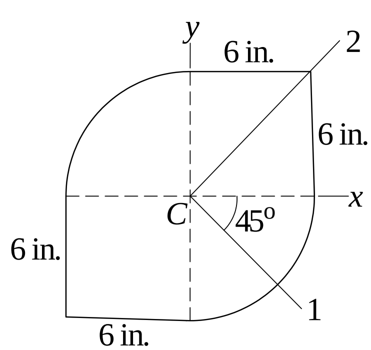
$$
\begin{aligned}
A_1 &= (24)(30) = 720 \text{ in}^2 \qquad \bar{x}_1 = 12 \text{ in.} \qquad \bar{y}_1 = 15 \text{ in.} \\
A_2 &= \frac{\pi(15^2)}{4} = 176.71 \text{ in}^2 \qquad \bar{x}_2 = 24 - \frac{4(15)}{3\pi} = 17.634 \text{ in.} \\
& \qquad \bar{y}_2 = \frac{4(15)}{3\pi} = 6.366 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
I_x &= \frac{24(30^3)}{3} - \frac{\pi(15^4)}{16} = 206.1 \times 10^6 \text{ in}^4 \\
I_y &= \frac{30(24^3)}{3} - \left[0.05488(15^4) + 176.71(17.634^2)\right] = 80.51 \times 10^3 \text{ in}^4 \\
I_{xy} &= 720(12)(15) - \left[-0.01647(15^4) + 176.71(17.634)(6.366)\right] \\
&= 110.60 \times 10^3 \text{ in}^4
\end{aligned}
$$

$$
\begin{aligned}
R &= \sqrt{\left(\frac{I_x + I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left(\frac{206.1 + 80.51}{2}\right)^2 + 110.60^2} \times 10^3 \\
&= 181.02 \times 10^3 \text{ in}^4 \\
\sin 2\theta_1 &= -\frac{I_{xy}}{R} = -\frac{110.60}{181.02} = -0.6110 \qquad \cos 2\theta_1 = \frac{I_x - I_y}{2R} > 0 \\
&\therefore \; 2\theta_1 \text{ lies in the 4th quadrant} \\
2\theta_1 &= \sin^{-1}(-0.6110) = -37.66^\circ \qquad \theta_1 = -18.83^\circ \;\blacktriangleleft \\
\theta_2 &= \theta_1 + 90^\circ = -18.83^\circ + 90^\circ = 71.2^\circ \;\blacktriangleleft
\end{aligned}
$$




**9.67**



The principal axes can be located by inspection—they are the axis of symmetry of the region:

$$\theta_1 = -45^\circ \quad \blacktriangleleft \qquad \theta_2 = 45^\circ \quad \blacktriangleleft$$

$$\bar{I}_x = \bar{I}_y = 2\left[\frac{6^4}{3} + \frac{\pi(6)^4}{16}\right] = 1372.9 \text{ in}^4$$
$$\bar{I}_{xy} = 2\left(\frac{6^4}{4} - \frac{6^4}{8}\right) = 324.0 \text{ in}^4$$

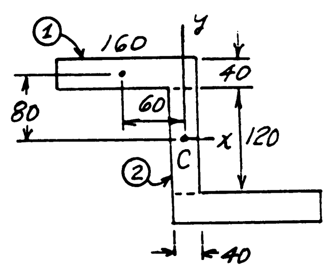
$$I_1 = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta_1 - \bar{I}_{xy}\sin 2\theta_1$$
$$= 1372.9 + 0 - 324.0\sin(-90^\circ) = 1697 \text{ in}^4 \quad \blacktriangleleft$$
$$I_2 = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta_2 - \bar{I}_{xy}\sin 2\theta_2$$
$$= 1372.9 + 0 - 324.0\sin 90^\circ = 1049 \text{ in}^4 \quad \blacktriangleleft$$

## 9.68

<u>Part 1</u>: $I_x = \frac{bh^3}{12} + A\bar{y}^2 = \frac{(160)(40)^3}{12} + (6400)(80)^2 = 41.81 \times 10^6 \text{mm}^4$

$I_y = \frac{b^3h}{12} + A\bar{x}^2 = \frac{(160)^3(40)}{12} + (6400)(-60)^2 = 36.69 \times 10^6 \text{ mm}^4$

$I_{xy} = A\bar{x}\bar{y} = (6400)(-60)(80) = -30.72 \times 10^6 \text{ mm}^4$




<u>Part 2</u>: $I_x = \frac{bh^3}{12} = \frac{(40)(120)^3}{12} = 5.76 \times 10^6 \text{ mm}^4$

$I_y = \frac{b^3h}{12} = \frac{(40)^3(120)}{12} = 0.64 \times 10^6 \text{ mm}^4 \qquad I_{xy} = 0$

<u>Composite area</u> ($I = 2I_1 + I_2$):

$I_x = \left[2(41.81) + 5.76\right] \times 10^6 = 89.38 \times 10^6 \text{ mm}^4$

$I_y = \left[2(36.69) + 0.64\right] \times 10^6 = 74.02 \times 10^6 \text{ mm}^4$

$I_{xy} = 2(-30.72) \times 10^6 + 0 = -61.44 \times 10^6 \text{ mm}^4$

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = 10^6\sqrt{\left(\frac{89.39 - 74.03}{2}\right)^2 + (-61.44)^2} = 61.92 \times 10^6 \text{ mm}^4$$

$$b = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{89.39 + 74.03}{2} \times 10^6 = 81.71 \times 10^6 \text{ mm}^4$$

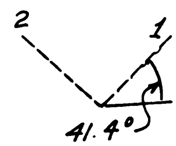
$$I_1 = b + R = (81.71 + 61.92) \times 10^6 = 143.6 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

$$I_2 = b - R = (81.71 - 61.92) \times 10^6 = 19.8 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

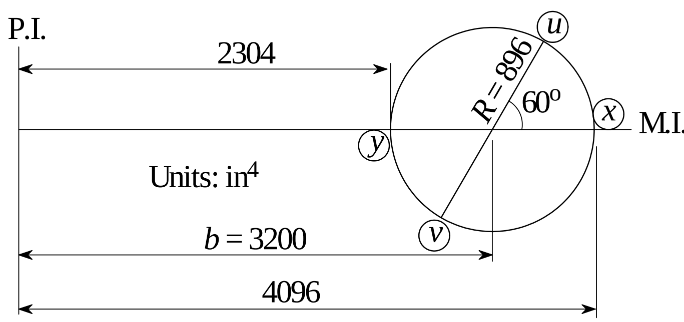
$$\sin 2\theta_1 = -\frac{\bar{I}_{xy}}{R} = -\frac{(-61.44)}{61.92} = 0.9922 \qquad \therefore 2\theta_1 = 82.86° \text{ or } 97.14°$$

$$\cos 2\theta_1 = \frac{\bar{I}_x - \bar{I}_y}{R} = \frac{89.39 - 74.03}{2(61.92)} > 0 \qquad \therefore 2\theta_1 = 82.86°$$

$$\therefore \theta_1 = 41.4° \ \blacklozenge \qquad \therefore \theta_2 = 41.4° + 90° = 131.4° \ \blacklozenge$$




**9.69**




$$I_x = \frac{12(16^3)}{12} = 4096 \text{ in}^4 \qquad I_y = \frac{16(12^3)}{12} = 2304 \text{ in}^4 \qquad I_{xy} = 0$$

$$b = \frac{4096 + 2304}{2} = 3200 \text{ in}^4 \qquad R = \frac{4096 - 2304}{2} = 896 \text{ in}^4$$

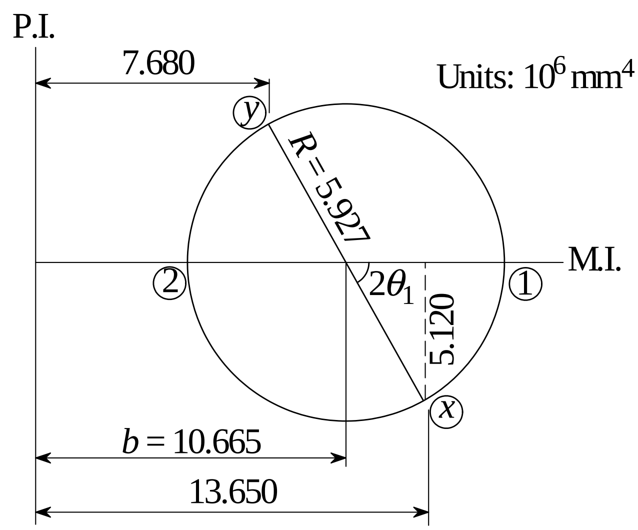
$$I_u = 3200 + 896\cos 60° = 3650 \text{ in}^4 \ \blacktriangleleft$$

$$I_v = 3200 - 896\cos 60° = 2750 \text{ in}^4 \ \blacktriangleleft$$

$$I_{uv} = 896\sin 60° = 776 \text{ in}^4 \ \blacktriangleleft$$

**9.70**

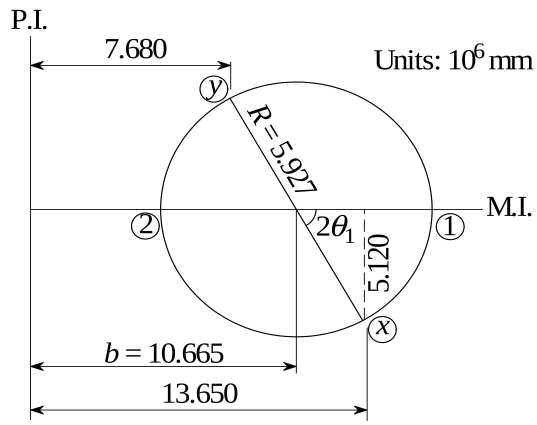
$$\bar{I}_x = \frac{8bh^3}{175} = \frac{8(120)(90^3)}{175} = 3.999 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = \frac{19b^3h}{480} = \frac{19(120^3)(90)}{480} = 6.156 \times 10^6 \text{ mm}^4$$
$$\bar{I}_{xy} = \frac{b^2h^2}{60} = \frac{(120^2)(90^2)}{60} = 1.9440 \times 10^6 \text{ mm}^4$$




$$b = \frac{3.999 + 6.156}{2} \times 10^6 = 5.078 \times 10^6 \text{ mm}^4$$
$$R = \sqrt{\left(\frac{6.156 - 3.999}{2}\right)^2 + 1.9440^2} \times 10^6 = 2.223 \times 10^6 \text{ mm}^4$$
$$\beta = \sin^{-1}\frac{1.944}{2.223} = 60.99^\circ \qquad \alpha = 90^\circ - \beta = 29.01^\circ$$

$$I_u = b - R\cos\alpha = (5.078 - 2.223\cos 29.01^\circ)\,10^6 = 3.13 \times 10^6 \text{ mm}^4 \blacktriangleleft$$
$$I_v = b + R\cos\alpha = (5.078 + 2.223\cos 29.01^\circ)\,10^6 = 7.02 \times 10^6 \text{ mm}^4 \blacktriangleleft$$
$$I_{uv} = -R\sin\alpha = -(2.223\sin 29.01^\circ)\,10^6 = -1.078 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

**9.71**



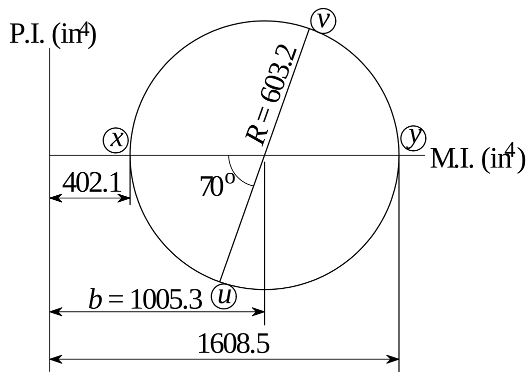

$$\bar{I}_x = \frac{120(160^3)}{36} = 13.650 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = \frac{160(120^3)}{36} = 7.680 \times 10^6 \text{ mm}^4$$
$$\bar{I}_{xy} = -\frac{(120^2)(160^2)}{72} = 5.120 \times 10^6 \text{ mm}^4$$

$$b = \frac{13.650 + 7.680}{2} \times 10^6 = 10.665 \times 10^6 \text{ mm}^4$$
$$R = \sqrt{\left(\frac{13.650 - 7.680}{2}\right)^2 + 5.120^2} \times 10^6 = 5.927 \times 10^6 \text{ mm}^4$$
$$I_1 = (10.665 + 5.927) \times 10^6 = 16.59 \times 10^6 \text{ mm}^4 \blacktriangleleft$$
$$I_2 = (10.665 - 5.927) \times 10^6 = 4.74 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$2\theta_1 = \sin^{-1}\frac{5.120}{5.927} = 59.75^\circ \qquad \therefore \theta_1 = 29.9^\circ \blacktriangleleft$$
$$\therefore \quad \theta_2 = 29.9^\circ + 90^\circ = 119.9^\circ \blacktriangleleft$$

**9.72**

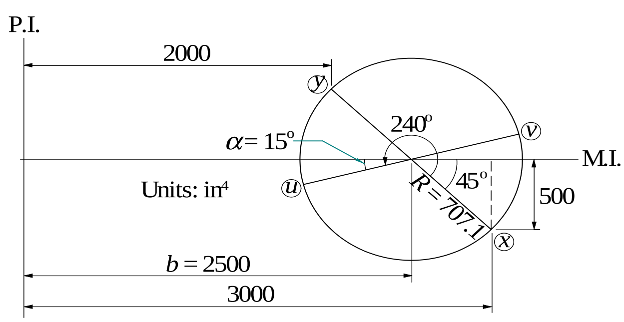
$$I_x = \frac{\pi a b^3}{4} = \frac{\pi(8)(4^3)}{4} = 402.1 \text{ in}^4$$
$$I_y = \frac{\pi a^3 b}{4} = \frac{\pi(8^3)(4)}{4} = 1608.5 \text{ in}^4 \qquad I_{xy} = 0$$




$$b = \frac{I_x + I_y}{2} = \frac{402.1 + 1608.5}{2} = 1005.3 \text{ in}^4$$
$$R = \frac{I_y - I_x}{2} = \frac{1608.5 - 402.1}{2} = 603.2 \text{ in}^4$$

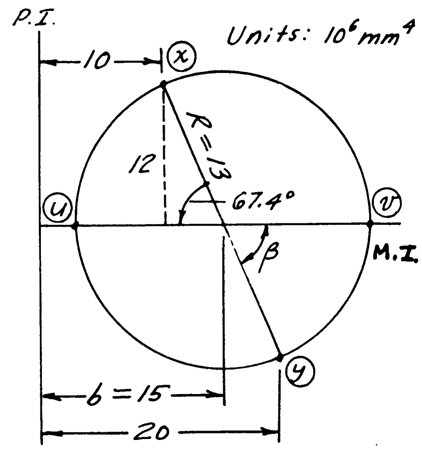
$$I_u = b - R\cos 70^\circ = 1005.3 - 603.2\cos 70^\circ = 799 \text{ in}^4 \blacktriangleleft$$
$$I_v = b + R\cos 70^\circ = 1005.3 + 603.2\cos 70^\circ = 1212 \text{ in}^4 \blacktriangleleft$$
$$I_{uv} = -R\sin 70^\circ = -603.2\sin 70^\circ = -567 \text{ in}^4 \blacktriangleleft$$

**9.73**




$$I_u = b - R\cos\alpha = 2500 - 707.1\cos 15^\circ = 1817 \text{ in}^4 \blacktriangleleft$$
$$I_v = b + R\cos\alpha = 2500 + 707.1\cos 15^\circ = 3180 \text{ in}^4 \blacktriangleleft$$
$$I_{uv} = -R\sin\alpha = -707.1\sin 15^\circ = -183.0 \text{ in}^4 \blacktriangleleft$$

## 9.74




$b = \frac{20 + 10}{2} \times 10^6 = 15 \times 10^6 \text{ mm}^4$

$R = 10^6\sqrt{5^2 + 12^2} = 13 \times 10^6 \text{ mm}^4$

$\beta = \tan^{-1}\frac{12}{5} = 67.4°$

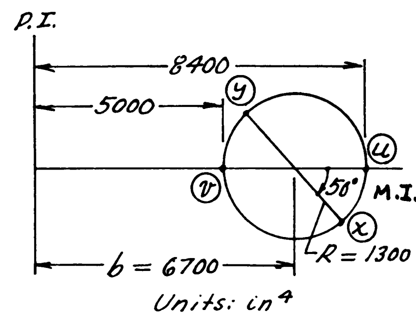
$2\theta = 2(33.7°) = 67.4°$

Since $\beta = 2\theta$, the u and v-axes are the principal directions.

$I_u = b - R = (15 - 13) \times 10^6 = 2 \times 10^6 \text{ mm}^4$ ◆

$I_v = b + R = (15 + 13) \times 10^6 = 28 \times 10^6 \text{ mm}^4$ ◆ $I_{uv} = 0$ ◆

## 9.75




$b = \frac{8400 + 5000}{2} = 6700 \text{ in}^4$

$R = \frac{8400 - 5000}{2} = 1700 \text{ in}^4$

$I_x = 6700 + 1700 \cos 50° = 7790 \text{ in}^4$ ◆

$I_y = 6700 - 1700 \cos 50° = 5610 \text{ in}^4$ ◆

$I_{xy} = -1700 \sin 50° = -1302 \text{ in}^4$ ◆

## 9.76

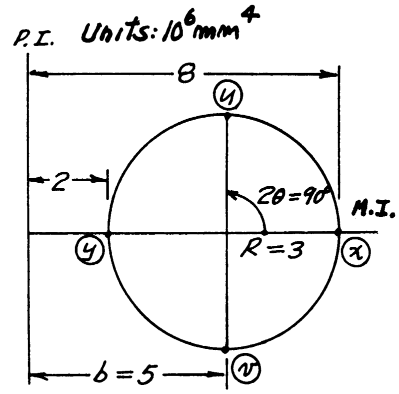
$$b = \frac{8 + 2}{2} \times 10^6 = 5 \times 10^6 \text{ mm}^4$$

$$R = \frac{8 - 2}{2} \times 10^6 = 3 \times 10^6 \text{ mm}^4$$

**(a)** $I_{uv}$ is maximized at $2\theta = 90°$, or $\theta = 45°$ ◆

**(b)** $|I_{uv}| = 3 \times 10^6 \text{ mm}^4$ ◆

$I_u = I_v = 5 \times 10^6 \text{ mm}^4$ ◆

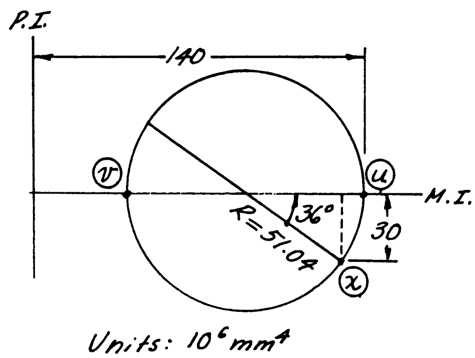



## 9.77

$$R = \frac{30 \times 10^6}{\sin 36°} = 51.04 \times 10^6 \text{ mm}^4$$

$$I_v = I_u - 2R = \left[140 - 2\,(51.40)\right] \times 10^6$$

$$= 37.9 \times 10^6 \text{ mm}^4$$ ◆




## 9.78

$$a = \frac{I_x - I_y}{2} = \frac{0.808 - 0.388}{2} \times 10^6$$

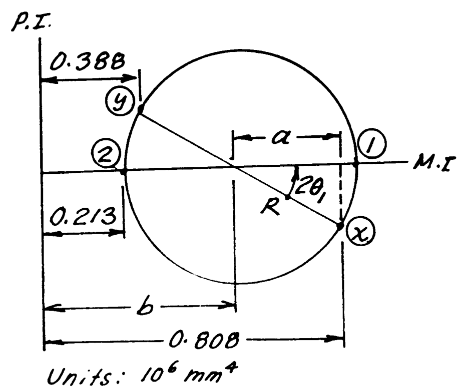
$$= 0.210 \times 10^6 \text{ mm}^4$$

$$b = \frac{I_x + I_y}{2} = \frac{0.808 + 0.388}{2} \times 10^6$$

$$= 0.598 \times 10^6 \text{ mm}^4$$

$$R = b - I_2 = (0.598 - 0.213) \times 10^6$$

$$= 0.385 \times 10^6 \text{ mm}^4$$



**(a)** $I_1 = b + R = (0.598 + 0.385) \times 10^6 = 0.983 \times 10^6 \text{ mm}^4$ ♦

**(b)** $2\theta_1 = \cos^{-1}\dfrac{a}{R} = \cos^{-1}\dfrac{0.210}{0.385} = 56.94°$ $\quad \therefore \theta_1 = 28.5°$ ♦ $\quad \theta_2 = 28.5° + 90° = 118.5°$ ♦

## 9.79

$$I_x = \frac{18(12^3)}{12} = 2592 \text{ in}^4 \qquad I_y = \frac{12(18^3)}{12} = 5832 \text{ in}^4 \qquad I_{xy} = 0$$

$$\theta = -\tan^{-1}\frac{6}{9} = -33.69° \qquad 2\theta = -67.38°$$

$$\frac{I_x + I_y}{2} = \frac{2592 + 5832}{2} = 4212 \text{ in}^4$$

$$\frac{I_x - I_y}{2} = \frac{2592 - 5832}{2} = -1620 \text{ in}^4$$

$$I_u = 4212 + (-1620)\cos(-67.38°) = 3590 \text{ in}^4 \blacktriangleleft$$

$$I_v = 4212 - (-1620)\cos(-67.38°) = 4840 \text{ in}^4 \blacktriangleleft$$

$$I_{uv} = -1620\sin(-67.38°) = 1495 \text{ in}^4 \blacktriangleleft$$

## 9.80

**(a)** $I_1 = \dfrac{I_x + I_y}{2} + R \qquad I_2 = \dfrac{I_x + I_y}{2} - R$

$$\therefore R = \frac{I_1 - I_2}{2} = \frac{60 - 30}{2} \times 10^6 = 15 \times 10^6 \text{ mm}^4$$

$$\therefore \frac{I_x + I_y}{2} = \frac{I_1 + I_2}{2} = \frac{60 + 30}{2} \times 10^6 = 45 \times 10^6 \text{ mm}^4 \ldots\ldots\ldots\ldots\ldots\text{(a)}$$

$$R^2 = \left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2$$

$$\therefore \frac{I_x - I_y}{2} = \sqrt{R^2 - I_{xy}^2} = 10^6\sqrt{15^2 - 10^2} = 11.18 \times 10^6 \text{ mm}^4 \ldots\ldots\text{(b)}$$

**Solution of Eqs. (a) and (b) is** $I_x = 56.2 \times 10^6 \text{ mm}^4$ ♦ $\qquad I_y = 33.8 \times 10^6 \text{ mm}^4$ ♦

(b) $I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

$= (45 + 11.18\cos 100° - 10\sin 100°) \times 10^6 = 33.2 \times 10^6\ \text{mm}^4$ ◆

$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$

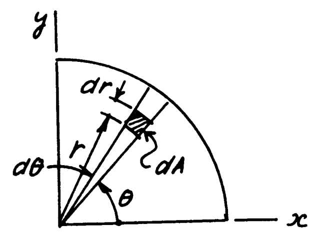
$= (45 - 11.18\cos 100° + 10\sin 100°) \times 10^6 = 56.8 \times 10^6\ \text{mm}^4$ ◆

**9.81**

$dA = dr(r\,d\theta) \qquad x = r\cos\theta \qquad y = r\sin\theta$

$$I_{xy} = \int_A xy\,dA = \int_0^{\pi/2}\int_0^R r^3 \sin\theta\cos\theta\,dr\,d\theta$$

$$= \frac{R^4}{4}\int_0^{\pi/2} \frac{1}{2}\sin 2\theta\,d\theta = \frac{R^4}{8}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{R^4}{8}$$ ◆




**9.82**

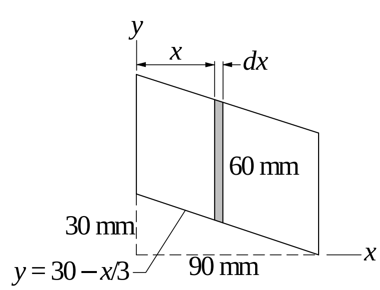
$$I_x = (I_x)_{\text{ellipse}} - (I_x)_{\text{circle}} = \frac{\pi a b^3}{8} - \frac{\pi R^4}{8}$$

$$= \frac{\pi(200)(105^3)}{8} - \frac{\pi(75^4)}{8} = 78.5 \times 10^6\ \text{mm}^4$$ ◄

$$I_y = (I_y)_{\text{ellipse}} - (I_y)_{\text{circle}} = \frac{\pi a^3 b}{8} - \frac{\pi R^4}{8}$$

$$= \frac{\pi(200^3)(105)}{8} - \frac{\pi(75^4)}{8} = 317 \times 10^6\ \text{mm}^4$$ ◄

**9.83**




$$\bar{x}_{\text{el}} = x \qquad \bar{y}_{\text{el}} = 60 - \frac{x}{3} \qquad dA = 60\ dx$$

$$dI_x = d\bar{I}_x + \bar{y}_{\rm el}^2\ dA = \frac{60^3\ dx}{12} + \left(60 - \frac{x}{3}\right)^2 (60\ dx)$$
$$= \left(\frac{20}{3}x^2 - 2400x + 234\ 000\right) dx$$
$$I_x = \int_0^{90} \left(\frac{20}{3}x^2 - 2400x + 234\ 000\right) dx = 12.96 \times 10^6 \text{mm}^4 \blacktriangleleft$$

$$dI_y = \bar{x}_{\rm el}^2\ dA = 60x^2\ dx \qquad I_y = \int_0^{90} 60x^2\ dx = 14.58 \times 10^6 \text{mm}^4 \blacktriangleleft$$

**9.84**

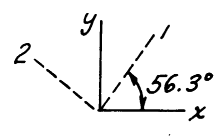
$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = 10^6\sqrt{\left(\frac{200-300}{2}\right)^2 + (-120)^2} = 130 \times 10^6 \text{ mm}^4$$

$$b = \frac{I_x + I_y}{2} = \frac{200+300}{2} \times 10^6 = 250 \times 10^6 \text{ mm}^4$$

$$I_1 = b + R = (250 + 130) \times 10^6 = 380 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

$$I_2 = b - R = (250 - 130) \times 10^6 = 120 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

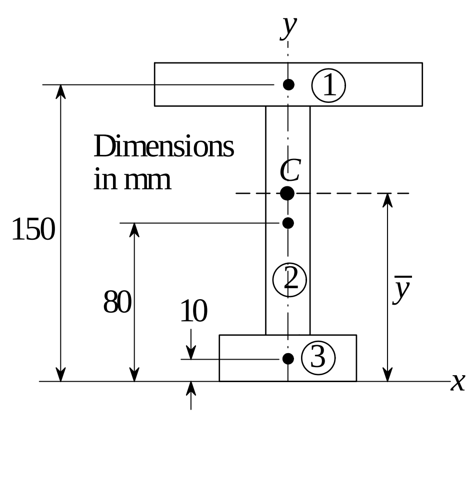
$$\sin 2\theta_1 = -\frac{I_{xy}}{R} = -\frac{(-120)}{130} = 0.9231 \qquad \therefore 2\theta_1 = 67.38° \text{ or } 112.62°$$




$$\cos 2\theta_1 = \frac{I_x - I_y}{2R} = \frac{200-300}{2(130)} < 0 \qquad \therefore 2\theta_1 = 112.62° \qquad \therefore \theta_1 = 56.3° \ \blacklozenge$$

$$\therefore \theta_2 = 56.3° + 90° = 146.3° \ \blacklozenge$$

**9.85**



| Part | $A$ (mm$^2$) | $\bar{y}$ (mm) | $A\bar{y}$ (mm$^3$) |
|---|---|---|---|
| 1 | 2400 | 150 | 360 000 |
| 2 | 2400 | 80 | 192 000 |
| 3 | 1200 | 10 | 12 000 |
| Sum | 6000 | | 564 000 |

$$\bar{y} = \frac{564\ 000}{6000} = 94.0 \text{ mm}$$

$$I_x = \frac{120(20^3)}{12} + 2400(150^2) + \frac{20(120^3)}{12} + 2400(80^2) + \frac{60(20^3)}{12} + 1200(10^2) = 72.48 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{20(120^3)}{12} + \frac{120(20^3)}{12} + \frac{20(60^3)}{12} = 3.32 \times 10^6 \text{ mm}^4$$

$$\bar{I}_x = I_x - A\bar{y}^2 = 72.48 \times 10^6 - 6000(94.0^2) = 19.46 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_y = I_y = 3.32 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

## 9.86

**Solid plate:** $(J_O)_1 = \frac{\pi R^4}{2} = \frac{\pi (5)^4}{2} = 981.7 \text{ in}^4$

**10 holes:** $(J_O)_2 = 10\left[\frac{\pi (0.25)^4}{2} + \pi (0.25)^2(4)^2\right] = 31.48 \text{ in}^4$

**% reduction** $= (J_O)_2/(J_O)_1 = \frac{31.48}{981.7} \times 100\% = 3.21\%$ ♦

## 9.87

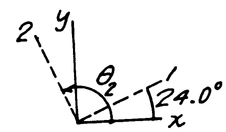
**(a)** $\bar{I}_2 = A\bar{k}_2^2 = (2400)(21.9)^2 = 1.151 \times 10^6 \text{ mm}^4$

$\bar{I}_1 + \bar{I}_2 = \bar{I}_x + \bar{I}_y$

$\therefore \bar{I}_1 = \bar{I}_x + \bar{I}_y - \bar{I}_2 = (5.58 + 2.03 - 1.151) \times 10^6 = 6.459 \times 10^6 \text{ mm}^4$ ♦

(b) $\bar{I}_1 = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + R$

$$\therefore R = \bar{I}_1 - \frac{\bar{I}_x + \bar{I}_y}{2} = \left(6.459 - \frac{5.58 + 2.03}{2}\right) \times 10^6 = 2.654 \times 10^6 \text{ mm}^4$$

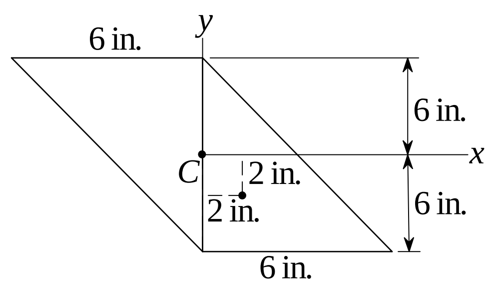



$\sin 2\theta_2 = \bar{I}_{xy}/R \quad \therefore \bar{I}_{xy} = R \sin 2\theta_2$

But $\theta_2 = \theta_1 + 90° = 24.0° + 90° = 114.0° \quad \therefore 2\theta_2 = 228.0°$

$\therefore \bar{I}_{xy} = (2.654 \times 10^6)\sin 228.0° = -1.972 \times 10^6 \text{ mm}^4$ ◆

## 9.88




For one triangle:

$$I_x = \bar{I}_x + A\bar{y}^2 = \frac{6(12^3)}{36} + \frac{6(12)}{2}(2^2) = 432.0 \text{ in}^4$$

$$I_y = \frac{12(6^3)}{12} = 216.0 \text{ in}^4$$

$$I_{xy} = \bar{I}_{xy} + A\bar{x}\bar{y} = -\frac{(6^2)(12^2)}{72} + \frac{6(12)}{2}(2)(-2) = -216.0 \text{ in}^4$$

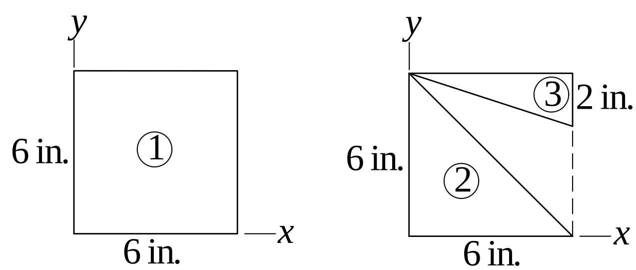
For the two triangles:

$$I_x = 2(432.0) = 864 \text{ in}^4 \blacktriangleleft$$

$$I_y = 2(216.0) = 432 \text{ in}^4 \blacktriangleleft$$

$$I_{xy} = 2(-216.0) = -432 \text{ in}^4 \blacktriangleleft$$

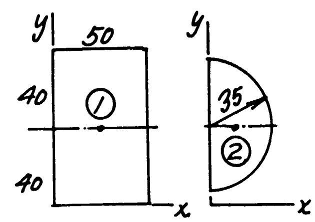
**9.89**




$$I_x = \frac{6^4}{3} - \frac{6^4}{12} - \left[\frac{6(2^3)}{36} + \frac{2\times 6}{2}\left(6-\frac{2}{3}\right)^2\right] = 152.0 \text{ in}^4 \blacktriangleleft$$

$$I_{xy} = \frac{6^4}{4} - \left[-\frac{6^4}{72} + \frac{6^2}{2}(2)(2)\right] - \left[-\frac{(6^2)(2^2)}{72} + \frac{6\times 2}{2}(4)\left(6-\frac{2}{3}\right)\right]$$
$$= 144.0 \text{ in}^4 \blacktriangleleft$$

**9.90**




$$A_1 = (50)(80) = 4000 \text{ mm}^2$$

$$A_2 = \frac{\pi}{2}(35)^2 = 1924 \text{ mm}^2$$

$$A = A_1 - A_2 = 4000 - 1924 = 2076 \text{ mm}^2$$

$$\bar{x}_1 = 25 \text{ mm} \qquad \bar{x}_2 = \frac{4R}{3\pi} = \frac{4(35)}{3\pi} = 14.854 \text{ mm}$$

$$\bar{x} = \frac{A_1\bar{x}_1 - A_2\bar{x}_2}{A} = \frac{(4000)(25) - (1924.2)(14.854)}{2076} = 34.40 \text{ mm}$$

$$\bar{I}_x = \frac{bh^3}{12} - \frac{\pi R^4}{8} = \frac{(50)(80)^3}{12} - \frac{\pi(35)^4}{8} = 1.544 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

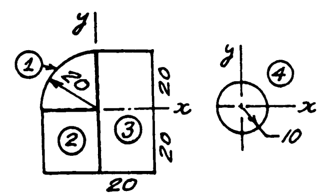
$$I_y = \frac{b^3h}{3} - \frac{\pi R^4}{8} = \frac{(50)^3(80)}{3} - \frac{\pi(35)^4}{8} = 2.744 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = I_y - A\bar{x}^2 = 2.744 \times 10^6 - (2076)(34.40)^2 = 0.287 \times 10^6 \text{ mm}^4 \ \blacklozenge$$

Due to symmetry $\bar{I}_{xy} = 0$ ◀

## 9.91

**(a)** $I_x = \Sigma(I_x)_i = \dfrac{\pi(20)^4}{16} + \dfrac{(20)(20)^3}{3} + \dfrac{(20)(40)^3}{12} - \dfrac{\pi(10)^4}{4}$




$= 183.57 \times 10^3\ \text{mm}^4$

Due to symmetry: $I_y = I_x = 183.57 \times 10^6\ \text{mm}^4$ ♦

**(b)**

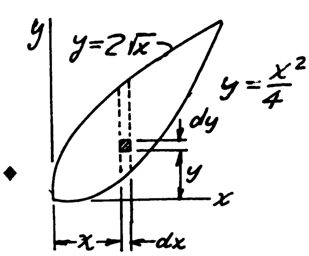
| Part | A (mm$^2$) | $\bar{y}$ (mm) | $A\bar{y}$ (mm$^3$) |
|---|---|---|---|
| 1 | $\pi(20)^2/4 = 314.2$ | $4(20)/(3\pi) = 8.488$ | 2667 |
| 2 | $(20)(20) = 400.0$ | −10 | −4000 |
| 3 | $(20)(40) = 800.0$ | 0 | 0 |
| 4 | $-\pi(10)^2 = -314.2$ | 0 | 0 |
| Sum | 1200.0 | | −1333 |

$\bar{y} = \dfrac{-1333}{1200} = -1.111\ \text{mm}$

$\bar{I}_x = \bar{I}_y = I_x - A\bar{x}^2 = 180.57 \times 10^3 - (1200)(1.111)^2 = 182.1 \times 10^3\ \text{mm}^4$ ♦

## 9.92

Use double integration.




$I_x = \int_A y^2\, dA = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} y^2\, dy\, dx = \int_0^4 \left[\dfrac{y^3}{3}\right]_{x^2/4}^{2\sqrt{x}} dx$

$= \dfrac{1}{3}\int_0^4 \left(8x^{3/2} - \dfrac{x^6}{64}\right) dx = \dfrac{1}{3}\left[\dfrac{16x^{5/2}}{5} - \dfrac{x^7}{448}\right]_0^4 = 21.9\ \text{in}^4$ ♦

$I_y = \int_A x^2\, dA = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} x^2\, dy\, dx = \int_0^4 x^2 \left[y\right]_{x^2/4}^{2\sqrt{x}} dx$

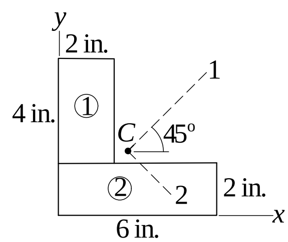
$= \int_0^4 \left(2x^{5/2} - \dfrac{x^4}{4}\right) dx = \left[\dfrac{4}{7}x^{7/2} - \dfrac{x^5}{20}\right]_0^4 = 21.9\ \text{in}^4$ ♦

Note that $I_x = I_y$, which was expected due to symmetry.

$$I_{xy} = \int_A xy\,dA = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy\,dy\,dx = \int_0^4 x\left[\frac{y^2}{2}\right]_{x^2/4}^{2\sqrt{x}} dx = \int_0^4 \left(2x^2 - \frac{x^5}{32}\right) dx$$

$$= \left[\frac{2}{3}x^3 - \frac{x^6}{192}\right]_0^4 = 21.3 \text{ in}^4 \ \blacklozenge$$

**9.93**




$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{\Sigma A_i} = \frac{8(1.0) + 12(3)}{8 + 12} = 2.20 \text{ in.} \qquad \text{By symmetry } \bar{y} = \bar{x} = 2.20 \text{ in.}$$

$$I_x = (\bar{I}_x)_1 + A_1\bar{y}_1^2 + (I_x)_2 = \frac{2(4)^3}{12} + 8(4)^2 + \frac{6(2)^3}{3} = 154.67 \text{ in}^4$$

By symmetry $I_y = I_x = 154.67 \text{ in}^4$

$$I_{xy} = A_1\bar{x}_1\bar{y}_1 + A_2\bar{x}_2\bar{y}_2 = 8(1.0)(4) + 12(3)(1.0) = 68.0 \text{ in}^4$$

$$\bar{I}_x = I_x - A\bar{y}^2 = 154.67 - 20(2.20)^2 = 57.87 \text{ in}^4$$

By symmetry $\bar{I}_y = \bar{I}_x = 57.87 \text{ in}^4$

$$\bar{I}_{xy} = I_{xy} - A\bar{x}\bar{y} = 68.0 - 20(2.20)^2 = -28.80 \text{ in}^4$$

Due to symmetry, "1" and "2" are the principal axes.

$$\frac{\bar{I}_x - \bar{I}_y}{2} = 0 \qquad \frac{\bar{I}_x + \bar{I}_y}{2} = 57.87 \text{ in}^4 \qquad R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = 28.80 \text{ in}^4$$

$$I_1 = \frac{\bar{I}_x + \bar{I}_y}{2} + R = 57.87 + 28.20 = 86.1 \text{ in}^4 \ \blacktriangleleft$$

$$I_2 = \frac{\bar{I}_x + \bar{I}_y}{2} - R = 57.87 - 28.20 = 29.7 \text{ in}^4 \ \blacktriangleleft$$

**9.94**

Equation (9.17) is

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

Substituting

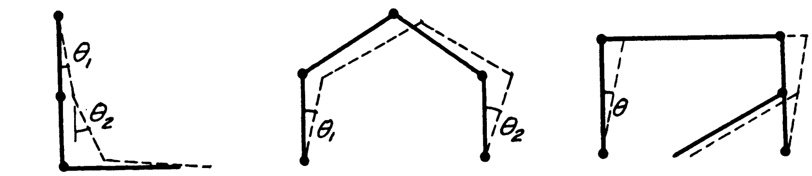
$$\theta = 90^\circ + 28.5^\circ = 118.5^\circ \qquad 2\theta = 237^\circ$$

we get

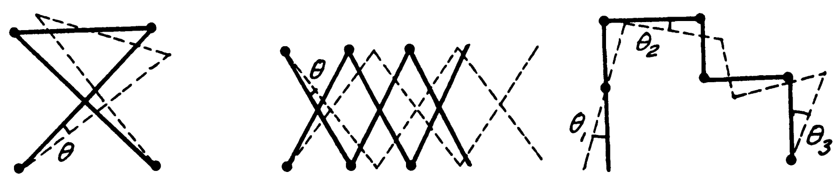
$$\begin{aligned} 21.3 &= \frac{80.9 + 38.8}{2} + \frac{80.9 - 38.8}{2}\cos 237^\circ - I_{xy}\sin 237^\circ \\ 21.3 &= 48.39 + 0.8387 I_{xy} \qquad I_{xy} = -32.3 \text{ in}^4 \ \blacktriangleleft \end{aligned}$$

# Chapter 10

## 10.1




(a) Two DOF (b) Two DOF (c) One DOF




(d) One DOF (e) One DOF (f) Three DOF

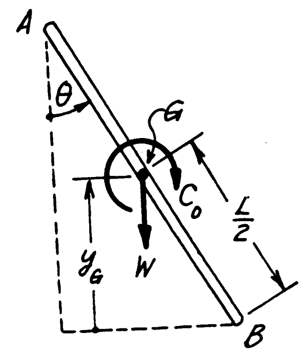
## 10.2

$$y_G = \frac{L}{2}\cos\theta \quad \therefore \ \delta y_G = -\frac{L}{2}\sin\theta\ \delta\theta$$

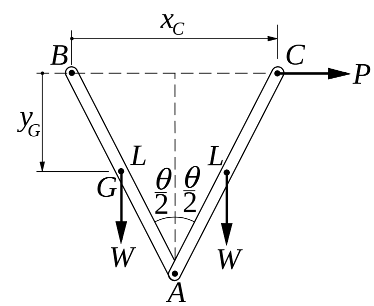
$$\delta U = -C_0\delta\theta - W\delta y_G = \left[-C_0 - W\left(-\frac{L}{2}\sin\theta\right)\right]\delta\theta$$

$$\delta U = 0: \quad -C_0 + \frac{WL}{2}\sin\theta = 0$$

$$\therefore \ C_0 = \frac{WL}{2}\sin\theta \ \blacklozenge$$



**10.3**




$$\delta U = 2W\ \delta y_G + P\ \delta x_C$$

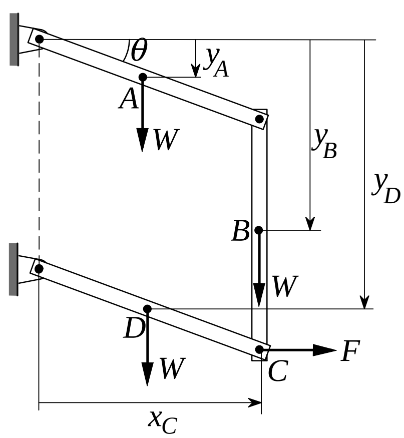
$$x_C = 2L\sin\frac{\theta}{2} \qquad \delta x_C = L\cos\frac{\theta}{2}\ \delta\theta$$

$$y_G = \frac{L}{2}\cos\frac{\theta}{2} \qquad \delta y_G = -\frac{L}{4}\sin\frac{\theta}{2}\ \delta\theta$$

$$\delta U = \left[2W\left(-\frac{L}{4}\sin\frac{\theta}{2}\right) + PL\cos\frac{\theta}{2}\right]\delta\theta = 0$$

$$P = \frac{W}{2}\tan\frac{\theta}{2} \blacktriangleleft$$

**10.4**




$$y_A = \frac{a}{2}\sin\theta \qquad \delta y_A = \frac{a}{2}\cos\theta\ \delta\theta$$

$$y_B = a\sin\theta + \frac{a}{2} \qquad \delta y_B = a\cos\theta\ \delta\theta$$

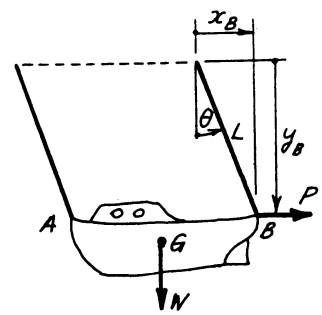
$$y_D = \frac{a}{2}\sin\theta + a \qquad \delta y_D = \frac{a}{2}\cos\theta\ \delta\theta$$

$$x_C = a\cos\theta \qquad \delta x_C = -a\sin\theta\ \delta\theta$$

$$\delta U = W(\delta y_A + \delta y_B + \delta y_D) + F\delta x_C = 0$$
$$(2Wa\cos\theta - Fa\sin\theta)\,\delta\theta = 0 \qquad F = \frac{2W}{\tan\theta} \blacktriangleleft$$

## 10.5

**The boat translates only. Thus all points on the boat have the same virtual displacement.**




$x_B = L\sin\theta \quad \therefore\ \delta x_B = L\cos\theta\,\delta\theta$

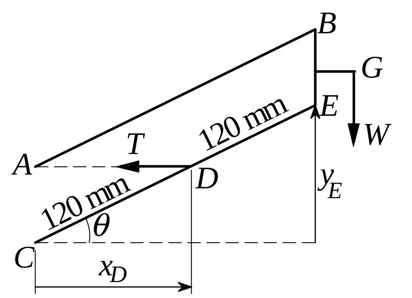
$y_B = L\cos\theta \quad \therefore\ \delta y_B = \delta y_G = -L\sin\theta\,\delta\theta$

$\delta U = P\,\delta x_B + W\,\delta y_G = [PL\cos\theta + W(-L\sin\theta)]\delta\theta$

$\delta U = 0$: $PL\cos\theta - WL\sin\theta = 0$

$\therefore\ P = W\tan\theta = 17.658\tan 20^\circ = 6.43$ kN ◆

## 10.6

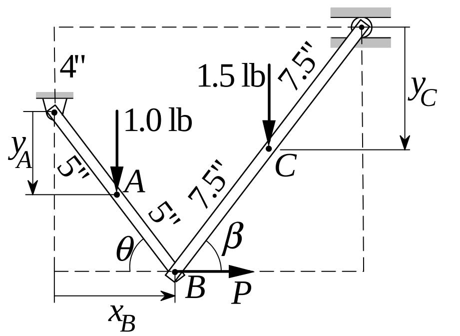



Note that $BE$ remains vertical.

$$x_D = 120\cos\theta \qquad \delta x_B = -120\sin\theta\ \delta\theta$$
$$y_E = 240\sin\theta \qquad \delta y_E = \delta y_G = 240\cos\theta\ \delta\theta$$

$$\delta U = -T\ \delta x_D - W\ \delta y_G = [T(120\sin\theta) - (2.4)(9.81)(240\cos\theta)]\,\delta\theta$$
$$= (120T\sin\theta - 5651\cos\theta)\,\delta\theta = (120T\sin 30^\circ - 5651\cos 30^\circ)\delta\theta$$
$$\delta U = 0 \qquad T = \frac{5651}{120}\cot 30^\circ = 81.6\ \text{N} \blacktriangleleft$$

## 10.7




Geometry:

$$\sin\theta = \frac{8}{10} = 0.8 \qquad \cos\theta = \frac{6}{10} = 0.6$$
$$\sin\beta = \frac{12}{15} = 0.8 \qquad \cos\beta = \frac{9}{15} = 0.6$$

$$y_A = 5\sin\theta \qquad \delta y_A = 5\cos\theta\ \delta\theta = 5(0.6)\delta\theta = 3\delta\theta$$
$$x_B = 10\cos\theta \qquad \delta x_B = -10\sin\theta\ \delta\theta = -10(0.8)\delta\theta = -8\delta\theta$$
$$y_C = 7.5\sin\beta \qquad \delta y_C = 7.5\cos\beta\ \delta\beta = 7.5(0.6)\delta\beta = 4.5\delta\beta$$

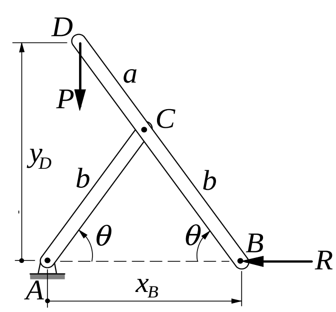
Constraint:

$$15\sin\beta - 10\sin\theta = 4 \qquad 15\cos\beta\ \delta\beta - 10\cos\theta\ \delta\theta = 0$$
$$15(0.6)\delta\beta - 10(0.6)\delta\theta = 0 \qquad \delta\beta = 0.6667\delta\theta$$

Virtual work:

$$\delta U = 1.0\delta y_A + 1.5\delta y_C + P\delta x_B$$
$$= [1.0(3) + 1.5(4.5)(0.6667) + P(-8)]\,\delta\theta = 0$$
$$P = 0.938 \text{ lb} \blacktriangleleft$$

## 10.8

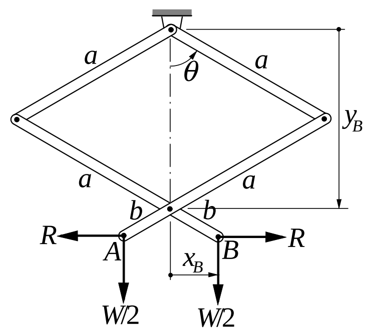


$$\delta U = -P\ \delta y_D - R\ \delta x_B$$

$$\begin{aligned} y_D &= (a+b)\sin\theta \qquad \delta y_D = (a+b)\cos\theta\ \delta\theta \\ x_B &= 2b\cos\theta \qquad \delta x_B = -2b\sin\theta\ \delta\theta \end{aligned}$$

$$\begin{aligned} \delta U &= [-P(a+b)\cos\theta - R(-2b\sin\theta)]\,\delta\theta = 0 \\ R &= \frac{P(a+b)}{2b}\cot\theta \ \blacktriangleleft \end{aligned}$$

**10.9**

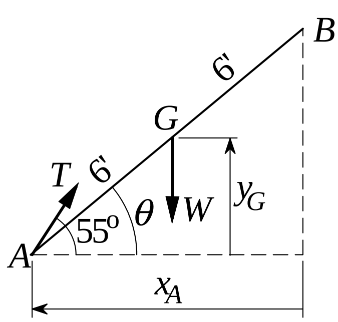



$$\delta U = 2\left(\frac{W}{2}\ \delta y_b + R\ \delta x_b\right)$$

$$\begin{aligned} x_B &= b\sin\theta \qquad \delta x_B = b\cos\theta\ \delta\theta \\ y_B &= (2a+b)\cos\theta \qquad \delta y_B = -(2a+b)\sin\theta\ \delta\theta \end{aligned}$$

$$\begin{aligned} \delta U &= [-W(2a+b)\sin\theta + 2Rb\cos\theta]\,\delta\theta = 0 \\ R &= \frac{2a+b}{2b}W\tan\theta \ \blacktriangleleft \end{aligned}$$

**10.10**



Release the cable at $A$, so that the cable tension $T$ becomes an active force.

$$
\begin{aligned}
y_G &= 6\sin\theta \qquad \delta y_G = 6\cos\theta\ \delta\theta = 6\cos 40^\circ\ \delta\theta = 4.596\ \delta\theta \\
x_A &= 12\cos\theta \qquad \delta x_A = -12\sin\theta\ \delta\theta = -12\sin 40^\circ\ \delta\theta = -7.713\ \delta\theta
\end{aligned}
$$

$$
\begin{aligned}
\delta U &= -T\cos 55^\circ\ \delta x_A - W\ \delta y_G = -0.5736T\ \delta x_A - 280\ \delta y_G \\
&= -0.5736T(-7.713\ \delta\theta) - 280(4.596\ \delta\theta) = 0 \\
T &= \frac{280(4.596)}{0.5736(7.713)} = 291 \text{ lb} \ \blacktriangleleft
\end{aligned}
$$

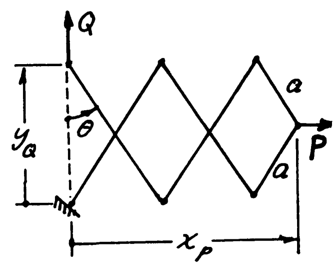
## 10.11

$$y_Q = 2a\cos\theta \qquad \therefore\ \delta y_Q = -2a\sin\theta\ \delta\theta$$

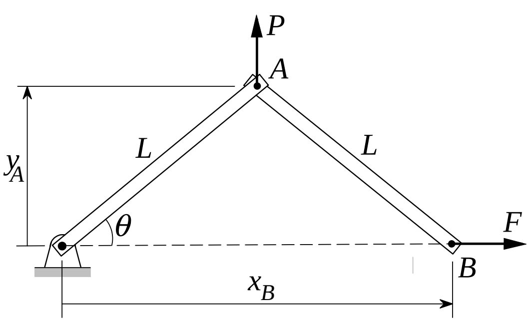
$$x_P = 5a\sin\theta \qquad \therefore\ \delta x_P = 5a\cos\theta\ \delta\theta$$

$$\delta U = P\ \delta x_P + Q\ \delta y_Q = \left[P(5\cos\theta) + Q(-2\sin\theta)\right]a\ \delta\theta$$

$$\delta U = 0:\ \ P(5\cos\theta) - Q(2\sin\theta) = 0 \qquad \therefore\ \frac{P}{Q} = 0.4\tan\theta\ \blacklozenge$$




## 10.12




$$
\begin{aligned}
y_A &= L\sin\theta \qquad \delta y_A = L\cos\theta\ \delta\theta \\
x_B &= 2L\cos\theta \qquad \delta x_B = -2L\sin\theta\ \delta\theta
\end{aligned}
$$

$$
\begin{aligned}
\delta U &= P\ \delta y_A + F\ \delta x_B = L(P\cos\theta - 2F\sin\theta)\delta\theta = 0 \\
P &= 2F\tan\theta
\end{aligned}
$$

$$
\begin{aligned}
F &= k\delta_{\text{spring}} = 2kL(1-\cos\theta) \\
P &= 4kL\tan\theta(1-\cos\theta) = 4(3000)(0.2)(\tan 40^\circ)(1-\cos 40^\circ) \\
&= 471 \text{ N} \ \blacktriangleleft
\end{aligned}
$$

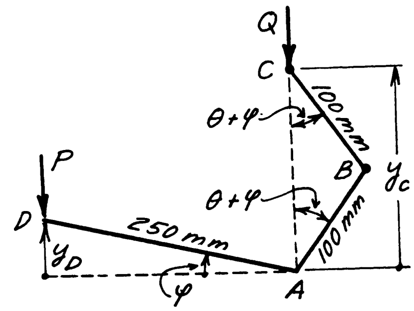
## 10.13

Let φ be the kinematically independent coordinate, measured from the equilibrium position.

$y_C = 2[100\cos(\theta + \varphi)]$

$\therefore\ \delta y_C = -200\sin(\theta + \varphi)\,\delta\varphi$

$y_D = 250\sin\varphi \quad \therefore\ \delta y_D = 250\cos\varphi\,\delta\varphi$




<u>When φ = 0</u>:

$\delta y_C = -200\sin\theta\,\delta\varphi \quad \delta y_D = 250\,\delta\varphi$

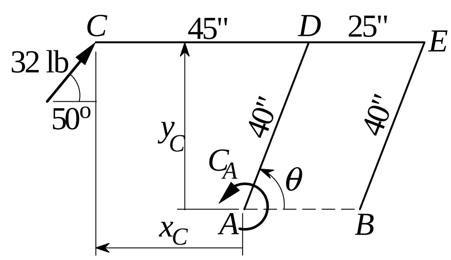
$\delta U = -P\,\delta y_D - Q\,\delta y_C$

$= [-P(250) - Q(-200\sin\theta)]\delta\varphi$

$\delta U = 0\text{:}\ -250\,P + 200\,Q\sin\theta = 0$

$\therefore\ \sin\theta = \dfrac{250}{200}\dfrac{P}{Q} = \dfrac{250}{200}\dfrac{1}{4} = 0.3125 \qquad \therefore\ \theta = 18.21°$ ◆

## 10.14



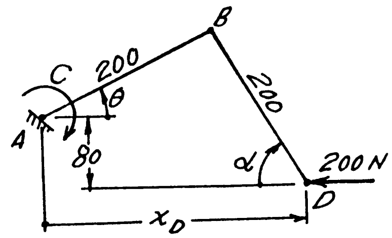

$$\begin{aligned} x_C &= 45 - 40\cos\theta \qquad \delta x_C = 40\sin\theta\ \delta\theta \\ y_C &= 40\sin\theta \qquad \delta y_C = 40\cos\theta\ \delta\theta \end{aligned}$$

When $\theta = 70°$:

$$\begin{aligned} \delta x_C &= 40\sin 70°\delta\theta = 37.59\ \delta\theta \qquad \delta y_C = 40\cos 70°\ \delta\theta = 13.681\ \delta\theta \\ \delta U &= 32\sin 50°\ \delta y_C - 32\cos 50°\ \delta x_C + C_A\ \delta\theta \\ &= [(32\sin 50°)(13.681) - (32\cos 50°)(37.59) + C_A]\ \delta\theta \\ &= (-437.8 + C_A)\,\delta\theta = 0 \qquad C_A = 438\ \text{lb}\cdot\text{in} \end{aligned}$$ ◄

## 10.15

The mechanism has a single degree of freedom. Choose $\theta$ as the kinematically independent variable.




$\theta$ and $\alpha$ are related by the geometric constraint

$$200\sin\alpha - 200\sin\theta = 80$$

$$\therefore \sin\alpha = 0.4 + \sin\theta = 0.4 + \sin 25^\circ = 0.8226$$

$$\therefore \alpha = 55.35^\circ$$

The geometric constraint also yields

$$200\cos\alpha\,\delta\alpha - 200\cos\theta\,\delta\theta = 0 \qquad \therefore \delta\alpha = \frac{\cos\theta}{\cos\alpha}\delta\theta = \frac{\cos 25^\circ}{\cos 55.35^\circ}\delta\theta = 1.5940\,\delta\theta$$

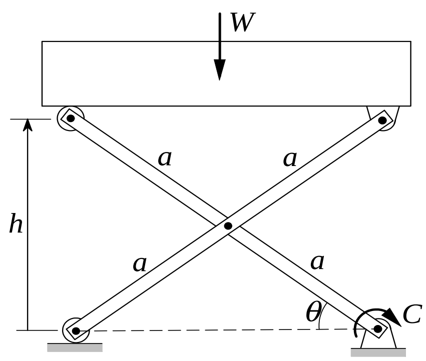
$$x_D = 200\cos\theta + 200\cos\alpha \text{ mm}$$

$$\therefore \delta x_D = -200(\sin\theta\,\delta\theta + \sin\alpha\,\delta\alpha) = -200\left[\sin 25^\circ + \sin 55.35^\circ(1.5940)\right]\delta\theta = -346.8\,\delta\theta \text{ mm}$$

$$\delta U = -C\,\delta\theta - 200\,\delta x_D = -C\,\delta\theta - 200(-346.8)\delta\theta = (-C + 69\,360)\delta\theta$$

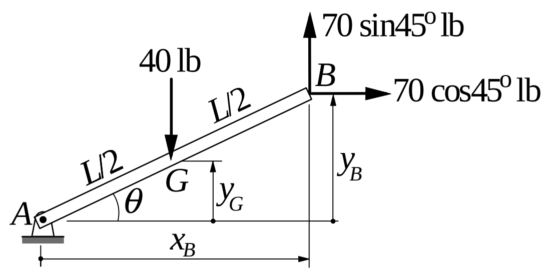
$\delta U = 0$: $C = 69\,400$ N·mm = 69.4 N·m ♦

## 10.16




$$\begin{aligned} h &= 2a\sin\theta \qquad \delta h = 2a\cos\theta\;\delta\theta \\ \delta U &= -W\;\delta h + C\;\delta\theta = (-2aW\cos\theta + C)\,\delta\theta = 0 \\ C &= 2aW\cos\theta \;\blacktriangleleft \end{aligned}$$

## 10.17




$$
\begin{aligned}
\delta U &= -40\ \delta y_G + 70\cos 45^\circ\ \delta x_B + 70\sin 45^\circ\ \delta y_B \\
&= -40\ \delta y_G + 49.50(\delta x_B + \delta y_B)
\end{aligned}
$$

$$
\begin{aligned}
y_G &= \frac{L}{2}\sin\theta \qquad \delta y_G = \frac{L}{2}\cos\theta\ \delta\theta \\
x_B &= L\cos\theta \qquad \delta x_B = -L\sin\theta\ \delta\theta \\
y_B &= L\sin\theta \qquad \delta y_B = L\cos\theta\ \delta\theta
\end{aligned}
$$

$$
\begin{aligned}
\delta U &= \left[-40\frac{L}{2}\cos\theta + 49.50L(-\sin\theta+\cos\theta)\right]\delta\theta = 0 \\
& \quad 29.50\cos\theta - 49.50\sin\theta = 0 \\
\theta &= \tan^{-1}\frac{29.50}{49.50} = 30.8^\circ \blacktriangleleft
\end{aligned}
$$

## 10.18

**Consider kinematics of half of the mechanism. If screw turned though 2π radians, D moves <u>to the left</u> by**

$$s = \frac{1}{2}(2.5) = 1.25 \text{ mm}$$

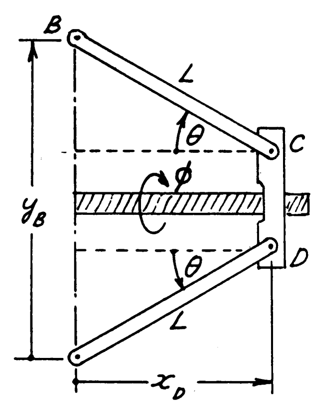
$$\therefore\ \delta x_D = -\frac{s}{2\pi}\delta\phi = -\frac{1.25}{2\pi}\delta\phi$$

$$= -0.19894\ \delta\phi \ldots\ldots\ldots\ldots \text{(a)}$$

**From geomerty:**

$$x_D = L\cos\theta \quad \therefore\ \delta x_D = -L\sin\theta\ \delta\theta \quad \text{(b)}$$

$$y_B = 2L\sin\theta + \overline{CD} \quad \therefore\ \delta y_B = 2L\cos\theta\ \delta\theta$$



Equating (a) and (b): $\delta\theta = \dfrac{0.19894\,\delta\phi}{L\sin\theta}$

$\therefore\ \delta y_B = 2L\cos\theta\,\dfrac{0.19894\,\delta\phi}{L\sin\theta} = 0.3979\cot\theta\,\delta\phi = 0.3979\cot 30°\,\delta\phi = 0.6892\,\delta\phi$

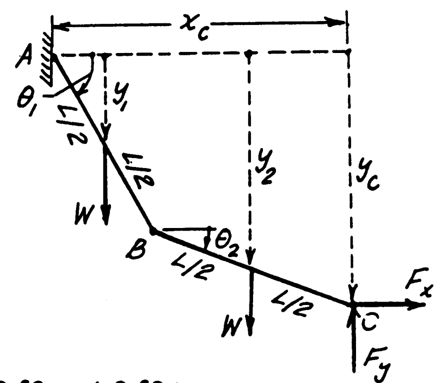
Consider the virtual work on the whole mechanism.

$\delta U = C_0\,\delta\phi - P\,\delta y_B = [C_0 - (3)(0.6892)]\delta\phi = (C_0 - 2.068)\delta\phi$

$\delta U = 0:\ C_0 - 2.068 = 0\quad \therefore\ C_0 = 2.07\ \text{kN}\cdot\text{m}$ ◆

10.19

The linkage has two DOF.




$y_1 = \dfrac{L}{2}\sin\theta_1\quad \therefore\ \delta y_1 = \dfrac{L}{2}\cos\theta_1\delta\theta_1$

$y_2 = L\sin\theta_1 + \dfrac{L}{2}\sin\theta_2$

$\therefore\ \delta y_2 = L\left(\cos\theta_1\delta\theta_1 + \dfrac{1}{2}\cos\theta_2\delta\theta_2\right)$

$y_C = L\sin\theta_1 + L\sin\theta_2$

$\therefore\ \delta y_C = L(\cos\theta_1\delta\theta_1 + \cos\theta_2\delta\theta_2)$

$x_C = L\cos\theta_1 + L\cos\theta_2\quad \therefore\ \delta x_C = -L(\sin\theta_1\delta\theta_1 + \sin\theta_2\delta\theta_2)$

$\delta U = W(\delta y_1 + \delta y_2) + F_x\delta x_C - F_y\delta y_C$

$= WL\left(\dfrac{3}{2}\cos\theta_1\delta\theta_1 + \dfrac{1}{2}\cos\theta_2\delta\theta_2\right) - F_xL(\sin\theta_1\delta\theta_1 + \sin\theta_2\delta\theta_2)$

$- F_yL(\cos\theta_1\delta\theta_1 + \cos\theta_2\delta\theta_2)$

$= \left(\dfrac{3}{2}W\cos\theta_1 - F_x\sin\theta_1 - F_y\cos\theta_1\right)L\,\delta\theta_1 + \left(\dfrac{1}{2}W\cos\theta_2 - F_x\sin\theta_2 - F_y\cos\theta_2\right)L\,\delta\theta_2$

$\delta U = 0:\ \dfrac{3}{2}W\cos\theta_1 - F_x\sin\theta_1 - F_y\cos\theta_1 = 0$

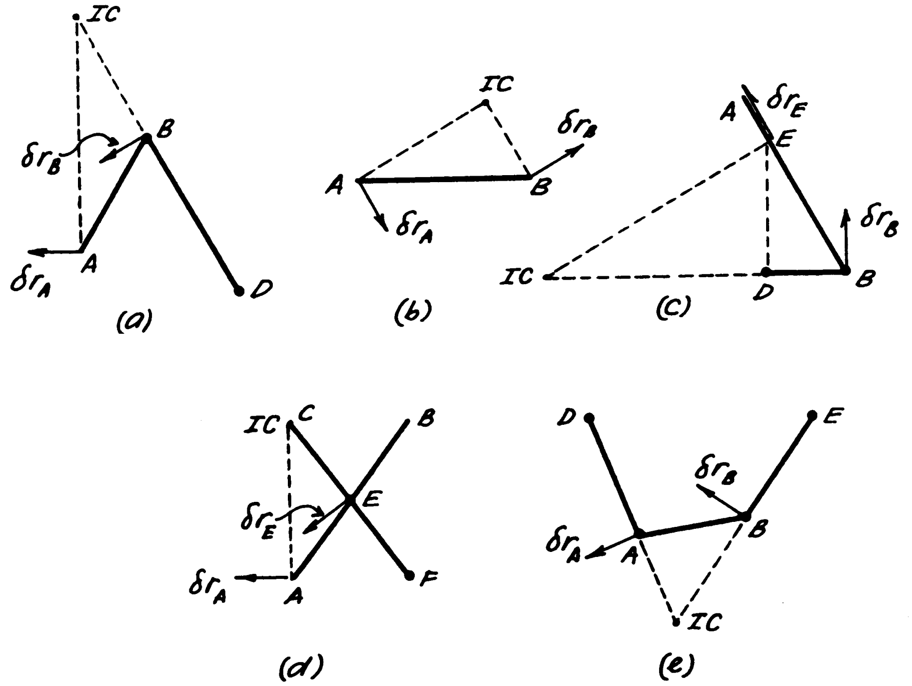
$\dfrac{1}{2}W\cos\theta_2 - F_x\sin\theta_2 - F_y\cos\theta_2 = 0$

Substituting $\theta_1 = 60°$ and $\theta_2 = 15°$, we get

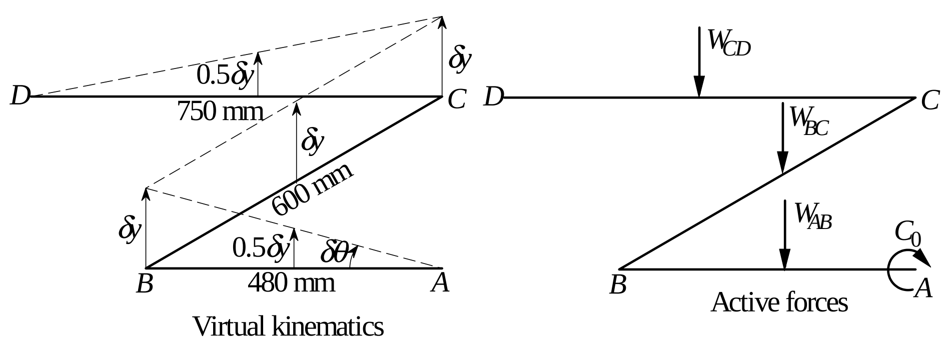
$$\left.\begin{aligned} 0.8660F_x + 0.5F_y &= 0.75W \\ 0.2588F_x + 0.9659F_y &= 0.4830W \end{aligned}\right\}\ \text{Solution is: } F_x = 0.6830W\quad F_y = 0.3171W$$

$\therefore\ F = W\sqrt{0.6830^2 + 0.3171^2} = 0.753W$ ◆ $\quad \alpha = \tan^{-1}\dfrac{0.3171}{0.6830} = 24.9°$ ◆

**10.20**




**10.21**



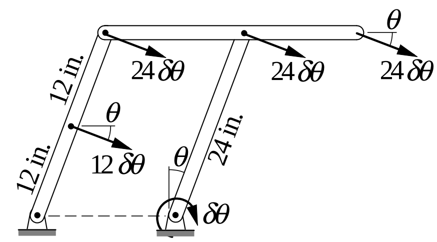

$$W_{AB} = (18 \times 9.81)(0.480) = 84.76 \text{ N}$$
$$W_{BC} = (18 \times 9.81)(0.600) = 105.95 \text{ N}$$
$$W_{CD} = (18 \times 9.81)(0.750) = 132.44 \text{ N}$$

Bar $AB$ rotates about $A$ : $\delta y = 0.480\ \delta\theta$ m

Bar $BC$ translates: $\delta y_B = \delta y_C = \delta y$

$$\begin{aligned}\delta U &= C_0\ \delta\theta - W_{AB}(0.5\ \delta y) - W_{BC}\ \delta y - W_{CD}(0.5\ \delta y)\\ &= \{C_0 - 0.480\,[84.76(0.5) + 105.95 + 132.44(0.5)]\}\,\delta\theta\\ &= (C_0 - 102.98)\,\delta\theta = 0 \qquad C_0 = 103.0\ \text{N}\cdot\text{m}\ \blacktriangleleft\end{aligned}$$

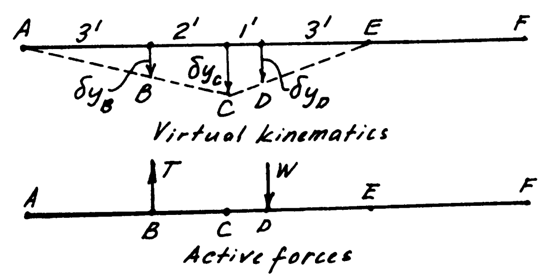
## 10.22




$$\begin{aligned}\delta U &= -P(12\ \delta\theta)\cos\theta + 60(24\ \delta\theta)\sin\theta = 0\\ P &= 120\tan\theta\ \text{lb}\ \blacktriangleleft\end{aligned}$$

## 10.23

**Release the cable, so that cable tension T becomes an active force.**




$$\delta y_B = \frac{3}{5}\delta y_C \quad \delta y_D = \frac{3}{4}\delta y_C$$

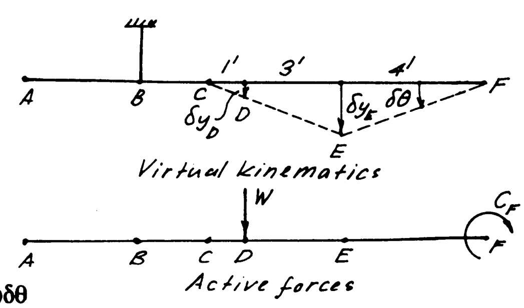
$$\delta U = -T\ \delta y_B + W\ \delta y_D$$

$$= \left[-\frac{3}{5}T + \frac{3}{4}(400)\right]\delta y_C$$

$$\delta U = 0:\ -\frac{3}{5}T + \frac{3}{4}(400) = 0 \qquad \therefore\ T = 500\ \text{lb}\ \blacklozenge$$

## 10.24

**Release rotational constraint at F, so that the restraining couple $C_F$ becomes active.**




$$\delta y_E = 4\ \delta\theta\ \text{ft}$$

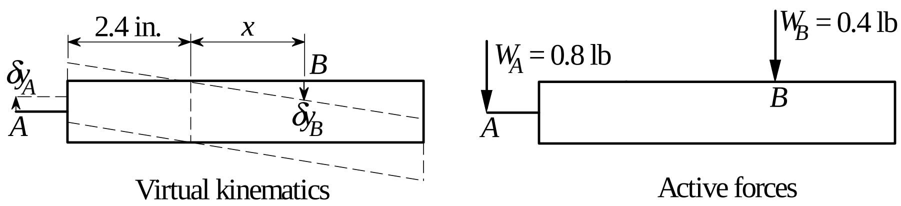
$$\delta y_D = \frac{1}{4}\delta y_E = 1.0\ \delta\theta\ \text{ft}$$

$$\delta U = W\ \delta y_D - C_F\delta\theta$$

$$= 400(1.0\ \delta\theta) - C_F\delta\theta = (400 - C_F)\delta\theta$$

$$\delta U = 0:\ C_F = 400\ \text{lb}\cdot\text{ft}\ \blacklozenge$$

## 10.25

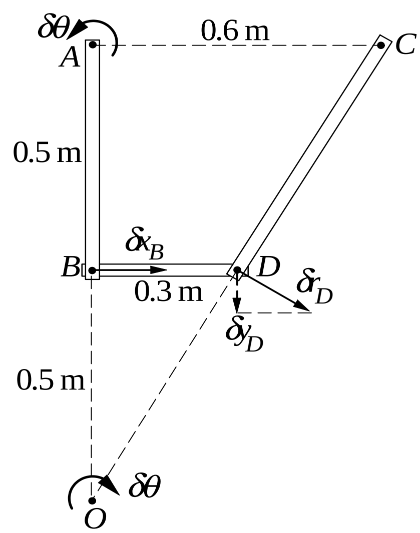



The scales form a parallelogram linkage.

$$\frac{\delta y_A}{2.4} = \frac{\delta y_B}{x} \qquad \delta y_B = 0.4167x\ \delta y_A$$

$$\begin{aligned}
\delta U &= -W_A\ \delta y_A + W_B\ \delta y_B \\
&= -0.8\ \delta y_A + 0.4(0.4167x\ \delta y_A) = (-0.8 + 0.16668x)\delta y_A = 0 \\
x &= \frac{0.8}{0.16668} = 4.80 \text{ in.} \quad \blacktriangleleft
\end{aligned}$$

## 10.26

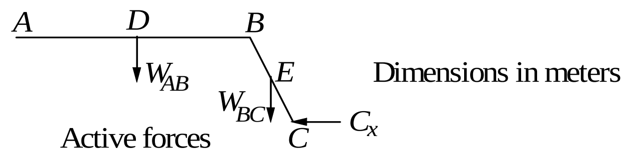


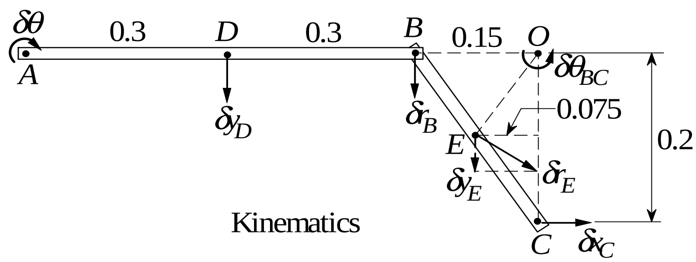

The I.C. of bar $BD$ is located at $O$, determined by the known directions of $\delta x_B$ and $\delta r_D$.

$$\begin{aligned}
\text{Bar } AB \text{ rotates about } A &: \quad \delta x_B = 0.5\delta\theta \text{ m} \\
\text{Bar } BD \text{ rotates about } O &: \quad \delta y_D = 0.3\delta\theta \text{ m}
\end{aligned}$$

$$\delta U = -P\ \delta x_B + Q\ \delta y_D = (-0.5P + 0.3Q)\,\delta\theta = 0$$
$$\frac{P}{Q} = \frac{0.3}{0.5} = 0.6 \blacktriangleleft$$

## 10.27




Release the horizontal constraint at $C$ so that $C_x$ becomes an active force.

The I.C. of bar $BC$ is located at $O$, determined by the known directions of $\delta r_B$ and $\delta x_C$.

Bar $AB$ rotates about $A$ : $\delta y_D = 0.3\ \delta\theta$ m $\quad \delta r_B = 0.6\ \delta\theta$ m

Bar $BC$ rotates about $O$ : $\delta r_B = 0.15\ \delta\theta_{BC}$ m $\quad \delta x_C = 0.2\ \delta\theta_{BC}$ m

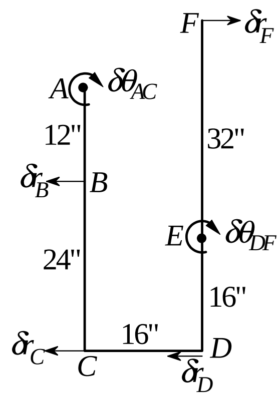
$$\delta y_E = 0.075\ \delta\theta_{BC} \text{ m}$$

Equating the expressions for $\delta r_B$: $0.6\ \delta\theta = 0.15\ \delta\theta_{BC}$ m $\quad \delta\theta_{BC} = 4\ \delta\theta$ m

$$\delta x_C = 0.2(4\ \delta\theta) = 0.8\ \delta\theta \text{ m} \qquad \delta y_E = 0.075(4\ \delta\theta) = 0.3\ \delta\theta \text{ m}$$

$$\delta U = W_{AB}\delta y_D + W_{BC}\delta y_E - C_x\delta x_C = (0.3W_{AB} + 0.3W_{BC} - 0.8C_x)\,\delta\theta = 0$$
$$C_x = \frac{0.3W}{0.8} = \frac{0.3(20\times 9.81)}{0.8} = 73.6 \text{ N} \blacktriangleleft$$

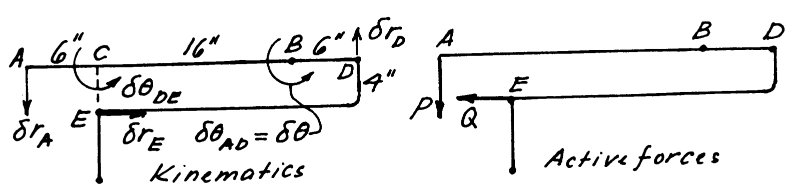
## 10.28




$$\begin{aligned}
\text{Bar } ABC \text{ rotates about } A &: \quad \delta r_B = 12\ \delta\theta_{AC} \qquad \delta r_C = 36\ \delta\theta_{AC} \\
\text{Bar } CD \text{ translates:} &\quad \delta r_D = \delta r_C = 36\ \delta\theta_{AC} \\
\text{Bar } DEF \text{ rotates about } E &: \quad \delta\theta_{DF} = \frac{\delta r_D}{16} = \frac{36}{16}\delta\theta_{AC} = 2.25\ \delta\theta_{AC} \\
\delta r_F &= 32\ \delta\theta_{DF} = 32(2.25\ \delta\theta_{AC}) = 72.0\ \delta\theta_{AC}
\end{aligned}$$

$$\begin{aligned}
\delta U &= P\ \delta r_B - 4000\ \delta r_F = [P(12) - 4000(72)]\,\delta\theta_{AC} = 0 \\
P &= 24\ 000 \text{ lb} \blacktriangleleft
\end{aligned}$$

## 10.29




**The I.C. of bar DE is at C, determined by the known directions of $\delta r_D$ and $\delta r_E$.**

**From motion of AD (rotates about B): $\delta r_A = 22\ \delta\theta$ in, $\delta r_D = 6\ \delta\theta$ in**

**From motion of DE (rotates about C): $\delta r_D = 22\ \delta\theta_{DE}$ in, $\delta r_E = 4\ \delta\theta_{DE}$ in**

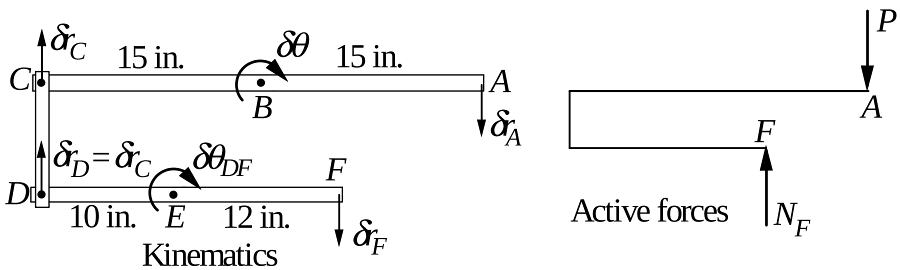
Equating the two expressions for $\delta r_D$: $6\,\delta\theta = 22\,\delta\theta_{DE}$ $\therefore\ \delta\theta_{DE} = \frac{6}{22}\delta\theta = \frac{3}{11}\delta\theta$

$$\therefore\ \delta r_E = 4\left(\frac{3}{11}\delta\theta\right) = \frac{12}{11}\delta\theta \text{ in}$$

$$\delta U = P\,\delta r_A - Q\,\delta r_E = \left[P(22) - Q\frac{12}{11}\right]\delta\theta$$

$$\delta U = 0:\quad 22P - \frac{12}{11}Q = 0 \quad \therefore\ Q = \frac{(22)(11)}{12}P = \frac{(22)(11)}{12}(30) = 605 \text{ lb} \blacklozenge$$

## 10.30




Remove the roller at $F$ so that $N_F$ becomes an active force.

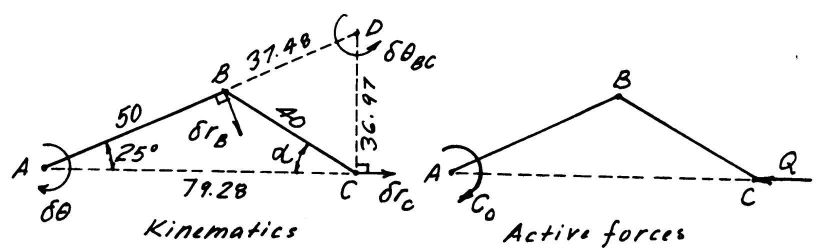
Bar $AC$ rotates about $B$ : $\delta r_C = 15\ \delta\theta$ in. $\delta r_A = 15\ \delta\theta$ in.

Bar $DF$ rotates about $E$ : $\delta r_D = 10\ \delta\theta_{DF}$ in. $\delta r_F = 12\ \delta\theta_{DF}$ in.

$$\delta r_D = \delta r_C \qquad 10\ \delta\theta_{DF} = 15\ \delta\theta \qquad \delta\theta_{DF} = 1.5\ \delta\theta$$

$$\begin{aligned}\delta U &= P\ \delta r_A - N_F\ \delta r_F = P(15\ \delta\theta) - N_F(12)(1.5\ \delta\theta)\\ &= (15P - 18N_F)\,\delta\theta = 0 \qquad N_F = \frac{15}{18}P = \frac{15}{18}(60) = 50.0 \text{ lb} \blacktriangleleft\end{aligned}$$

## 10.31




The I.C. of bar BC is located at D, as determined by the known directions of $\delta r_B$ and $\delta r_C$.

Geometry: $\frac{\sin\alpha}{50} = \frac{\sin 25°}{40}$ $\quad\therefore\ \alpha = \sin^{-1}\frac{50\sin 25°}{40} = 31.89°$

$\overline{AC} = 50 \cos 25° + 40 \cos\alpha = 50 \cos 25° + 40 \cos 31.89° = 79.28$ mm

$\overline{CD} = \overline{AC} \tan 25° = 79.28 \tan 25° = 36.97$ mm

$\overline{AD} = \overline{AC}/\cos 25° = 79.28/\cos 25° = 87.48$ mm

$\overline{BD} = \overline{AD} - \overline{AB} = 87.48 - 50 = 37.48$ mm

From motion of bar AB (rotates about A): $\delta r_B = 50\ \delta\theta$ mm

From motion of bar BC (rotates about D): $\delta r_B = 37.48\ \delta\theta_{BC}$ mm, $\delta r_C = 36.97\ \delta\theta_{BC}$ mm

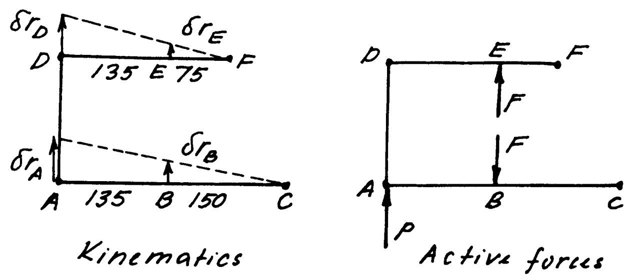
Equating the expressions for $\delta r_B$: $50\ \delta\theta = 37.48\ \delta\theta_{BC}$ $\quad\therefore\ \delta\theta_{BC} = 1.3340\ \delta\theta$

$\therefore\ \delta r_C = 36.97(1.3340\ \delta\theta) = 49.32\ \delta\theta$ mm

$\delta U = C_0\delta\theta - Q\ \delta r_C = C_0\delta\theta - 200(49.32\ \delta\theta) = (C_0 - 9864)\delta\theta$

$\delta U = 0$: $C_0 - 9864 = 0 \quad \therefore\ C_0 = 9864$ N·mm = 9.86 N·m ♦

10.32




Choose $\delta r_A$ as the kinemátically independent variable.

From motion of bar AC (rotates about C): $\delta r_B = \dfrac{150}{285}\delta r_A = 0.5263\ \delta r_A$

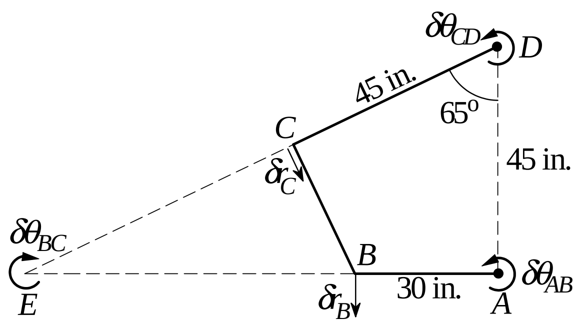
From motion of bar AD (translates): $\delta r_D = \delta r_A$

From motion of bar DF (rotates about F): $\delta r_E = \dfrac{75}{210}\delta r_D = 0.3571\ \delta r_D = 0.3571\ \delta r_A$

$\delta U = P\ \delta r_A - F(\delta r_B - \delta r_E) = \left[P - F(0.5263 - 0.3571)\right]\delta r_A = (P - 0.1692\ F)\delta r_A$

$\delta U = 0$: $P - 0.1692\ F = 0 \quad \therefore\ F = \dfrac{P}{0.1692} = \dfrac{2600}{0.1692} = 15\ 370$ N = 15.37 kN ♦

## 10.33



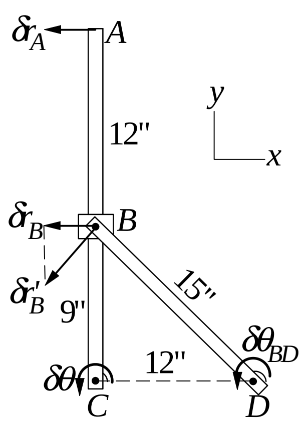

Point $E$ is the I.C. of bar $BC$.

$$\begin{aligned}\overline{BE} &= \overline{AE} - 30 = 45\tan 65^\circ - 30 = 66.50 \text{ in.}\\ \overline{CE} &= \overline{DE} - 45 = 45\sec 65^\circ - 45 = 61.48 \text{ in.}\end{aligned}$$

$$\begin{aligned}\text{Bar } AB \text{ rotates about } A \quad &: \quad \delta r_B = 30\ \delta\theta_{AB}\\ \text{Bar } BC \text{ rotates about } E \quad &: \quad \delta\theta_{BC} = \frac{\delta r_B}{\overline{BE}} = \frac{30\ \delta\theta_{AB}}{66.50} = 0.4511\ \delta\theta_{AB}\\ \delta r_C \quad &= \quad \overline{CE}\ \delta\theta_{BC} = 61.48(0.4511\ \delta\theta_{AB}) = 27.73\ \delta\theta_{AB}\\ \text{Bar } CD \text{ rotates about } D \quad &: \quad \delta\theta_{CD} = \frac{\delta r_C}{45} = \frac{27.73\ \delta\theta_{AB}}{45} = 0.6162\ \delta\theta_{AB}\end{aligned}$$

$$\begin{aligned}\delta U &= C_1\ \delta\theta_{AB} - C_2\ \delta\theta_{CD} = [C_1 - C_2(0.6162)]\,\delta\theta_{AB} = 0\\ \frac{C_1}{C_2} &= 0.616 \ \blacktriangleleft\end{aligned}$$

## 10.34



$$\begin{aligned}\delta r_B &= \text{virtual displacement of point } B \text{ on bar} AC \\ \delta r'_B &= \text{virtual displacement of point } B \text{ on sliding collar}\end{aligned}$$

Bar $AC$ rotates about $C$ : $\delta r_A = 21\ \delta\theta$ in. $\delta r_B = 9\ \delta\theta$ in.
Bar $BD$ rotates about $D$ : $\delta r'_B = 15\ \delta\theta_{BD}$ in.

Constraint: $(\delta r'_B)_x = \delta r_B$ $\quad \frac{3}{5}\delta r'_B = \delta r_B$ $\quad \frac{3}{5}(15\ \delta\theta_{BD}) = 9\ \delta\theta$

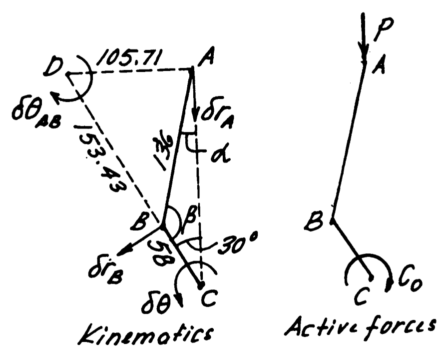
$\therefore\ \delta\theta_{BD} = \delta\theta$

$$\begin{aligned}\delta U &= P\ \delta r_A - C_D\ \delta\theta_{BD} = (21P\ - C_D)\ \delta\theta = 0 \\ C_D &= 21P = 21(25) = 525 \text{ lb} \cdot \text{in.} \blacktriangleleft\end{aligned}$$

## 10.35

**The I.C. of rod AB is located at D,.determined by the directions of $\delta r_A$ and $\delta r_B$.**

**Geometry:**




$\dfrac{\sin 30^\circ}{136} = \dfrac{\sin \alpha}{58}$ $\quad \therefore \alpha = 12.31^\circ$

$\beta = 180^\circ - (30^\circ + 12.31^\circ) = 137.69^\circ$

$\dfrac{\sin \beta}{\overline{AC}} = \dfrac{\sin 30^\circ}{136}$ $\quad \therefore \overline{AC} = 183.09$ mm

$\overline{AD} = \overline{AC} \tan 30^\circ = = 105.71$ mm

$\overline{BD} = \overline{AC}/\cos 30^\circ - 58 = 153.43$ mm

From motion of BC (rotates about C): $\delta r_B = 58\ \delta\theta$ mm

From motion of AB (rotates about D): $\delta r_B = 153.43\ \delta\theta_{AB}$ mm, $\delta r_A = 105.71\ \delta\theta_{AB}$ mm

Equating the expressions for $\delta r_B$: $58\ \delta\theta = 153.43\ \delta\theta_{AB}$ $\quad \therefore \delta\theta_{AB} = 0.3780\ \delta\theta$

$\therefore \delta r_A = 105.71(0.3780\ \delta\theta) = 39.96\ \delta\theta$ mm

$\delta U = P\ \delta r_A - C_0\ \delta\theta = (39.96\ P - C_0)\delta\theta$

$\delta U = 0$: $39.96\ P - C_0 = 0$ $\quad \therefore C_0 = 39.96\ P = (39.96)(1600) = 63\ 900$ N·mm $= 63.9$ N·m ♦

## 10.36

The I.C. of member CDF is G, found from the directions of $\delta r_C$ and $\delta r_D$.

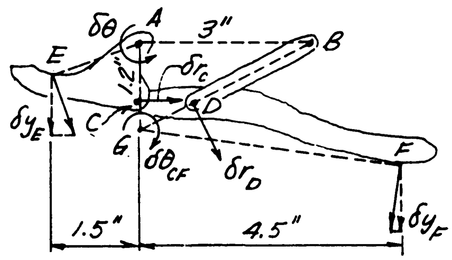
From similar triangles ABG and CDG:

$$\frac{1.2 + \overline{CG}}{3} = \frac{\overline{CG}}{1.0} \quad \therefore \overline{CG} = 0.6 \text{ in}$$

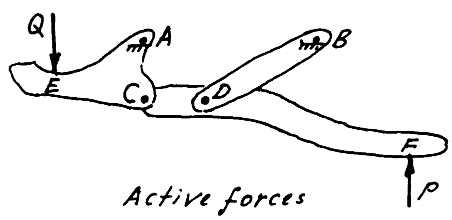
From motion of member CDF (rotates about G):

$$\delta r_C = \overline{CG}\,\delta\theta_{CF} = 0.6\,\delta\theta_{CF} \text{ in}$$

$$\delta y_F = 4.5\,\delta\theta_{CF} \text{ in}$$




Kinematics




Active forces

From motion of member ACE (rotates about A):

$$\delta r_C = 1.2\,\delta\theta \text{ in}, \quad \delta y_E = 1.5\,\delta\theta \text{ in}$$

Equating the expressions for $\delta r_C$:

$$0.6\,\delta\theta_{CF} = 1.2\,\delta\theta \quad \therefore \delta\theta_{CF} = 2\,\delta\theta$$

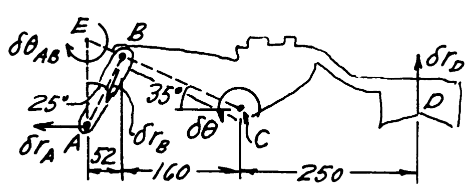
$$\therefore \delta y_F = 4.5(2\,\delta\theta) = 9\,\delta\theta \text{ in}$$

$$\delta U = Q\,\delta y_E - P\,\delta y_F = (1.5Q - 9P)\delta\theta$$

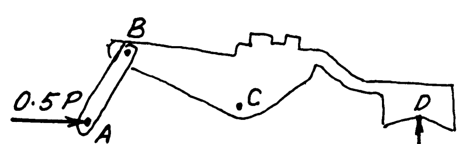
$$\delta U = 0: \; 1.5Q - 9.0P = 0 \quad \therefore Q = 6.0\,P \; ♦$$

## 10.37

Consider half of the mechanism. The I.C. of link AB is located at E, determined by the directions of $\delta r_A$ and $\delta r_B$.




Kinematics




Active forces

Geometry:

$$\overline{BC} = \frac{160}{\cos 35^\circ} = 195.32 \text{ mm}$$

$$\overline{EB} = \frac{52}{\cos 35^\circ} = 63.48 \text{ mm}$$

$$\overline{EA} = \frac{52}{\tan 25^\circ} + 52 \tan 35^\circ$$

$$= 147.93 \text{ mm}$$

From motion of link AB (rotates about E):

$\delta r_B = \overline{EB}\ \delta\theta_{AB} = 63.48\ \delta\theta_{AB}$ mm, $\delta r_A = \overline{EA}\ \delta\theta_{AB} = 147.93\ \delta\theta_{AB}$ mm

From motion of part BCD (rotates about C):

$\delta r_B = \overline{BC}\ \delta\theta = 195.32\ \delta\theta$ mm, $\delta r_D = 250\ \delta\theta$ mm

Equating the expressions for $\delta r_B$: $63.48\ \delta\theta_{AB} = 195.32\ \delta\theta \quad \therefore \delta\theta_{AB} = 3.077\ \delta\theta$

$\therefore \delta r_A = 147.93(3.077\ \delta\theta) = 455.2\ \delta\theta$ mm

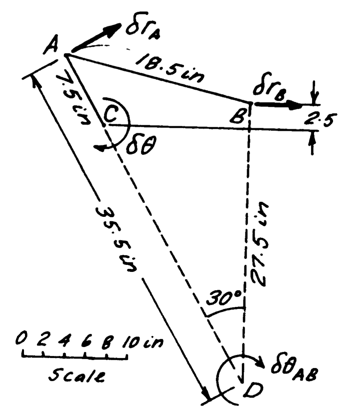
$\delta U = -0.5P\ \delta r_A + F\ \delta r_D = \left[-0.5P(455.2) + F(250)\right]\delta\theta$

$\delta U = 0$: $-0.5P(455.2) + 250F = 0 \quad \therefore F = \dfrac{0.5(455.2)}{250}P = \dfrac{0.5(455.2)}{250}(120) = 109.2$ N ♦

## 10.38

(a) The I.C. of rod AB, located at D, was determined from the known directions of $\delta r_A$ and $\delta r_B$.

By measuring the drawing, we get $\overline{AD} = 35.5$ in and $\overline{BD} = 27.5$ in.




Virtual kinematics

(b)

From motion of flywheel radius AC (rotates about C):

$\delta r_A = 7.5\ \delta\theta$ in

From motion of connecting rod AB (rotates about D):

$\delta r_A = 35.5\ \delta\theta_{AB}$ in, $\delta r_B = 27.5\ \delta\theta_{AB}$ in

Equating the expressions for $\delta r_A$: $7.5\ \delta\theta = 35.5\ \delta\theta_{AB}$

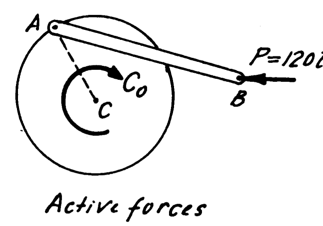
$\therefore \delta\theta_{AB} = 0.211\ \delta\theta$

$\therefore \delta r_B = 27.5\ (0.211\ \delta\theta) = 5.80\ \delta\theta$

$\delta U = C_0\ \delta\theta - P\ \delta r_B = (C_0 - 5.80\ P)\delta\theta$

$\delta U = 0$: $C_0 - 5.80\ P = 0$

$\therefore C_0 = 5.80\ P = (5.80)(120) = 696$ lb·in ≈ 700 lb·in ♦



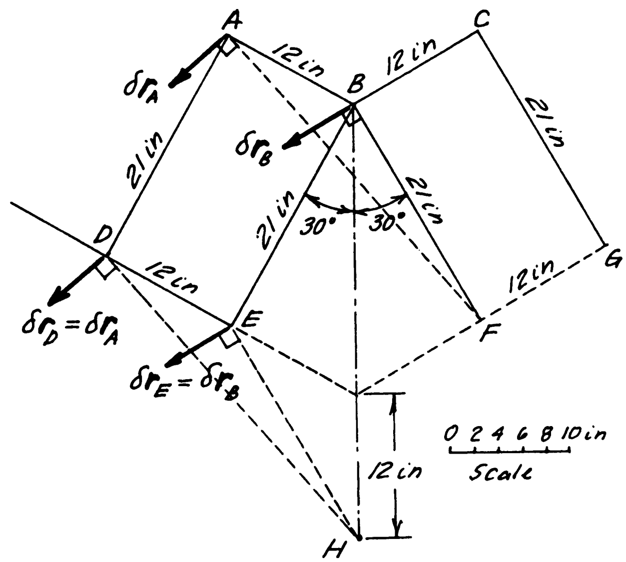

Active forces

## 10.39

**1) Member ABF rotates about F, which determines the directions of $\delta\mathbf{r}_A$ and $\delta\mathbf{r}_B$.**

**2) Since ABDE is a parallelogram linkage, $\delta\mathbf{r}_D = \delta\mathbf{r}_A$ and $\delta\mathbf{r}_E = \delta\mathbf{r}_B$.**

**3) The directions of $\delta\mathbf{r}_D$ and $\delta\mathbf{r}_E$ determine the location of H, the I.C. of the door.**

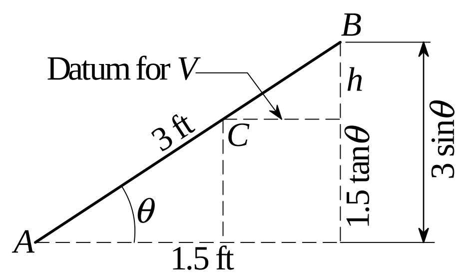



## 10.40

Choose the $\theta = 0$ position as the datum for gravitational potential energy.

$$
\begin{aligned}
V &= WL\sin\theta + \frac{1}{2}k(L\sec\theta - 1.5L)^2 \\
\frac{dV}{d\theta} &= WL\cos\theta + kL^2(\sec\theta - 1.5)\sec\theta\tan\theta = 0 \\
& \quad W\cos 30^\circ + kL(\sec 30^\circ - 1.5)\sec 30^\circ \tan 30^\circ = 0 \\
& \quad 0.8660W - 0.2302kL = 0 \\
W &= \frac{0.2302}{0.8660}kL = \frac{0.2302}{0.8660}(1.5)(20) = 7.97 \text{ lb} \blacktriangleleft
\end{aligned}
$$

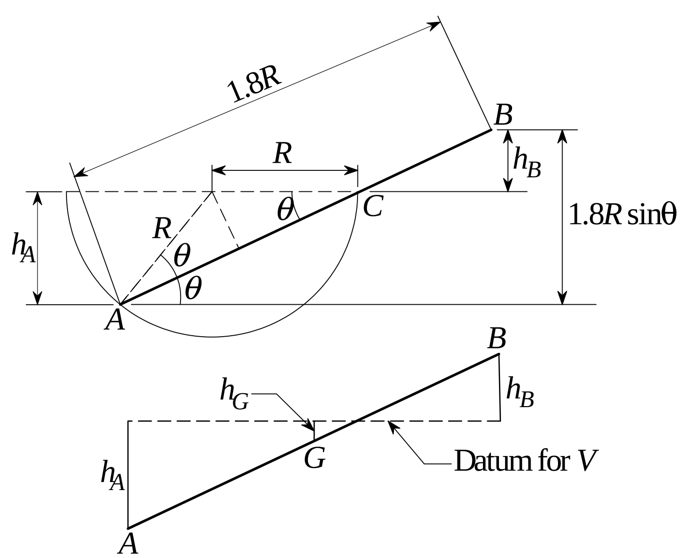
**10.41**




$$\begin{aligned} V &= Wh = W(3\sin\theta - 1.5\tan\theta) \\ \frac{dV}{d\theta} &= W(3\cos\theta - 1.5\sec^2\theta) \\ \frac{dV}{d\theta} &= 0: \qquad 3 - \frac{1.5}{\cos^3\theta} = 0 \qquad \cos^3\theta = 0.5 \qquad \theta = 37.5^\circ \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \frac{d^2V}{d\theta^2} &= W(-3\sin\theta - 3\sec^2\theta\tan\theta) \\ \left.\frac{d^2V}{d\theta^2}\right|_{\theta=37.5^\circ} &= W(-3\sin 37.5^\circ - 3\sec^2 37.5^\circ \tan 37.5^\circ) \\ &= -5.48W < 0 \quad \therefore \text{Unstable} \blacktriangleleft \end{aligned}$$

**10.42**



$$\begin{aligned} h_A &= \overline{AC}\sin\theta = (2R\cos\theta)\sin\theta \\ h_B &= \overline{BD} - h_A = 1.8R\sin\theta - 2R\sin\theta\cos\theta = R\sin\theta(1.8 - 2\cos\theta) \\ h_G &= \frac{1}{2}(h_A - h_B) = \frac{1}{2}\left[2R\cos\theta\sin\theta - (R\sin\theta)(1.8 - 2\cos\theta)\right] \\ &= R\sin\theta(2\cos\theta - 0.9) \end{aligned}$$

$$\begin{aligned} V &= -Wh_G = -WR\sin\theta(2\cos\theta - 0.9) \\ \frac{dV}{d\theta} &= -WR\left[(\cos\theta)(2\cos\theta - 0.9) + (\sin\theta)(-2\sin\theta)\right] \\ &= -WR\left(4\cos^2\theta - 0.9\cos\theta - 2\right) \\ \frac{dV}{d\theta} &= 0: \quad 4\cos^2\theta - 0.9\cos\theta - 2 = 0 \\ & \cos\theta = 0.8285 \qquad \theta = 34.1^\circ \ \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \frac{d^2V}{d\theta^2} &= -WR(-8\cos\theta\sin\theta + 0.9\sin\theta) \\ \left.\frac{d^2V}{d\theta^2}\right|_{\theta=34.1^\circ} &= -WR(-8\cos 34.1^\circ\sin 34.1^\circ + 0.9\sin 34.1^\circ) = 3.21WR > 0 \\ \therefore & \ \text{Equilibrium is stable} \ \blacktriangleleft \end{aligned}$$

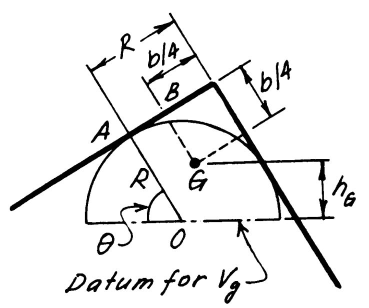
## 10.43

**The center of gravity G of the bar can be found by inspection.**

$$h_G = \overline{OA}\sin\theta + \overline{AB}\cos\theta - \overline{BG}\sin\theta$$

$$= R\sin\theta + \left(R - \frac{b}{4}\right)\cos\theta - \frac{b}{4}\sin\theta$$

$$= \left(R - \frac{b}{4}\right)(\sin\theta + \cos\theta)$$




$$V_g = Wh_G = W\left(R - \frac{b}{4}\right)(\sin\theta + \cos\theta)$$

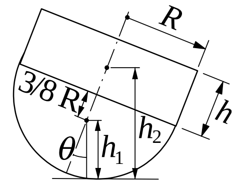
$$\therefore \frac{dV_g}{d\theta} = W\left(R - \frac{b}{4}\right)(\cos\theta - \sin\theta)$$

$$\frac{d^2V_g}{d\theta^2} = W\left(R - \frac{b}{4}\right)(-\sin\theta - \cos\theta)$$

**<u>When θ = 45°</u>:** $\frac{dV_g}{d\theta} = 0$ **and** $\frac{d^2V_g}{d\theta^2} = W\left(R - \frac{b}{4}\right)(-\sqrt{2})$

**Equilibrium position θ = 45° is stable when $d^2V_g/d\theta^2 > 0$, i.e. b/4 > R, or b/R > 4 ♦**

## 10.44




$$\text{Hemisphere:} \qquad W_1 = \frac{2\pi}{3}R^3\gamma \qquad h_1 = R - \frac{3}{8}R\cos\theta$$

$$\text{Cylinder:} \qquad W_2 = \pi R^2 h\gamma \qquad h_2 = R + \frac{h}{2}\cos\theta$$

$$\begin{aligned} V &= W_1h_1 + W_2h_2 \\ &= \frac{2\pi}{3}R^3\gamma\left(R - \frac{3}{8}R\cos\theta\right) + \pi R^2h\gamma\left(R + \frac{h}{2}\cos\theta\right) \end{aligned}$$

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{\pi}{4}R^4\gamma\sin\theta - \frac{\pi}{2}R^2h^2\sin\theta \\ \frac{d^2V}{d\theta^2} &= \frac{\pi}{4}R^4\gamma\cos\theta - \frac{\pi}{2}R^2h^2\cos\theta \end{aligned}$$

$$\text{At } \theta = 0: \qquad \frac{d^2V}{d\theta^2} = \frac{\pi}{4}R^4\gamma - \frac{\pi}{2}R^2h^2$$

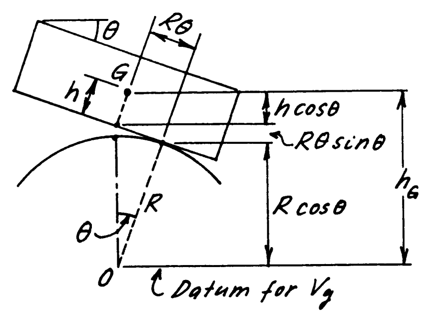
Stability requirement is

$$\frac{d^2V}{d\theta^2} > 0 \qquad \frac{h}{R} < \frac{1}{\sqrt{2}} \blacktriangleleft$$

## 10.45

$h_G = R\cos\theta + R\theta\sin\theta + h\cos\theta$

$= (R + h)\cos\theta + R\theta\sin\theta$




$V_g = Wh_G = W\left[(R + h)\cos\theta + R\theta\sin\theta\right]$

$\therefore \dfrac{dV_g}{d\theta} = W\left[-(R + h)\sin\theta + R\sin\theta + R\theta\cos\theta\right]$

$= W(-h\sin\theta + R\theta\cos\theta)$

$\therefore \dfrac{d^2V_g}{d\theta^2} = W(-h\cos\theta + R\cos\theta - R\theta\sin\theta)$

<u>At $\theta = 0$:</u> $\dfrac{d^2V_g}{d\theta^2} = W(-h + R)$ $\quad\therefore$ **The equilibrium position is stable if $R > h$** ◆

## 10.46

Choose the $\theta = 0$ position of rod $AB$ as the datum.

$$V_g = W\frac{L}{2}\sin\theta \qquad V_e = \frac{1}{2}k\left(L\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = W\frac{L}{2}\cos\theta + kL^2\sin\theta\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -W\frac{L}{2}\sin\theta + kL^2(\cos^2\theta - \sin^2\theta)$$

Equilibrium position:

$$\frac{dV}{d\theta} = 0 \qquad \sin\theta = \frac{W}{2kL} \qquad \theta = \sin^{-1}\frac{W}{2kL} \blacktriangleleft$$

Stability of the equilibrium position:

$$\frac{d^2V}{d\theta^2} = -W\frac{L}{2}\left(\frac{W}{2kL}\right) + kL^2\left[1 - \left(\frac{W}{2kL}\right)^2 - \left(\frac{W}{2kL}\right)^2\right] = \frac{4L^2k^2 - 3W^2}{4k}$$

$$\text{Stable if } \frac{d^2V}{d\theta^2} > 0 \qquad \frac{Lk}{W} > \frac{\sqrt{3}}{2} \blacktriangleleft$$

## 10.47

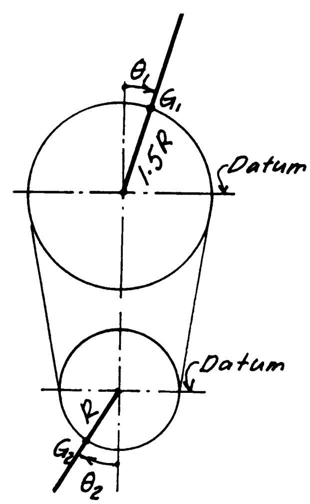
**Kinematics:** $1.5R\theta_1 = R\theta_2 \quad \therefore \theta_2 = 1.5\theta_1$

$V_g = W_1(1.5R\cos\theta_1) - W_2(R\cos\theta_2)$

$= R\left[1.5W_1\cos\theta_1 - W_2\cos(1.5\theta_1)\right]$

$\therefore dV_g/d\theta_1 = R\left[-1.5W_1\sin\theta_1 + 1.5W_2\sin(1.5\theta_1)\right]$

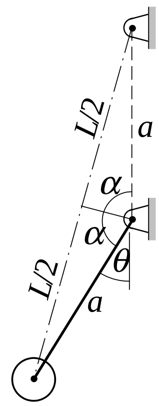
$\therefore d^2V_g/d\theta_1^2 = 1.5R\left[-W_1\cos\theta_1 + 1.5W_2\cos(1.5\theta_1)\right]$




**At $\theta_1 = 0$:** $d^2V_g/d\theta_1^2 = 1.5R(-W_1 + 1.5W_2)$

$\therefore$ **Equilibrium is stable if $W_1/W_2 < 1.5$** ◆

**10.48**




Geometry:

$$\begin{aligned} \theta + 2\alpha &= 180^\circ \qquad \alpha = 90^\circ - \frac{\theta}{2} \\ \frac{L}{2} &= a\sin\alpha = a\sin\left(90^\circ - \frac{\theta}{2}\right) = a\cos\frac{\theta}{2} \\ L_0 &= \sqrt{2}a \quad \text{(free length of spring)} \end{aligned}$$

Choosing the $\theta = 90^\circ$ position as the datum:

$$V = -Wa\cos\theta + \frac{1}{2}k(L - L_0)^2 = -Wa\cos\theta + \frac{1}{2}k\left(2a\cos\frac{\theta}{2} - \sqrt{2}a\right)^2$$

$$\begin{aligned} \frac{dV}{d\theta} &= Wa\sin\theta + k\left(2a\cos\frac{\theta}{2} - \sqrt{2}a\right)(-a\sin\frac{\theta}{2}) \\ &= Wa\sin\theta + ka^2\left(-\sin\theta + \sqrt{2}\sin\frac{\theta}{2}\right) \end{aligned}$$

$$\begin{aligned} \frac{d^2V}{d\theta^2} &= Wa\cos\theta + ka^2\left(-\cos\theta + \frac{\sqrt{2}}{2}\cos\frac{\theta}{2}\right) \\ \left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} &= Wa + ka^2\left(-1 + \frac{\sqrt{2}}{2}\right) = Wa - 0.2929ka^2 \end{aligned}$$

For stability we need

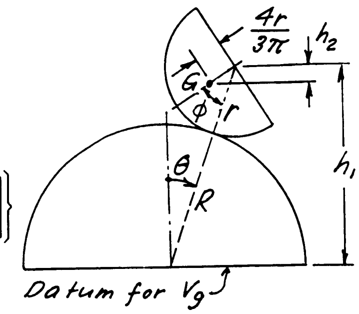
$$Wa > 0.2929ka^2 \qquad \frac{W}{ka} > 0.2929 \blacktriangleleft$$

## 10.49

Kinematics: $R\theta = r\phi \quad \therefore \phi = \frac{R}{r}\theta$

Geometry: $h_1 = (R + r)\cos\theta$

$$h_2 = \frac{4r}{3\pi}\cos(\theta + \phi) = \frac{4r}{3\pi}\cos\left[\left(1 + \frac{R}{r}\right)\theta\right]$$




$$V_g = W(h_1 - h_2) = W\left\{(R + r)\cos\theta - \frac{4r}{3\pi}\cos\left[\left(1 + \frac{R}{r}\right)\theta\right]\right\}$$

$$= Wr\left\{\left(1 + \frac{R}{r}\right)\cos\theta - \frac{4}{3\pi}\cos\left[\left(1 + \frac{R}{r}\right)\theta\right]\right\}$$

$$\therefore \frac{dV_g}{d\theta} = Wr\left\{-\left(1 + \frac{R}{r}\right)\sin\theta + \frac{4}{3\pi}\left(1 + \frac{R}{r}\right)\sin\left[\left(1 + \frac{R}{r}\right)\theta\right]\right\}$$

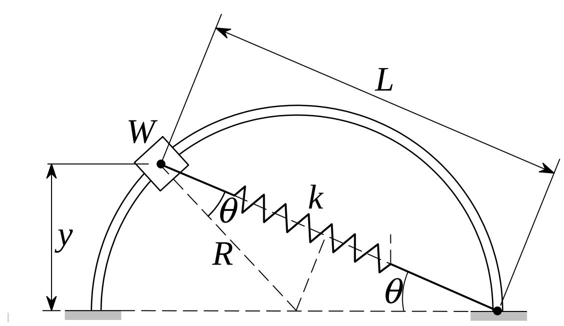
$$= Wr\left(1 + \frac{R}{r}\right)\left\{-\sin\theta + \frac{4}{3\pi}\sin\left[\left(1 + \frac{R}{r}\right)\theta\right]\right\}$$

$$\therefore \frac{d^2V_g}{d\theta^2} = Wr\left(1 + \frac{R}{r}\right)\left\{-\cos\theta + \frac{4}{3\pi}\left(1 + \frac{R}{r}\right)\cos\left[\left(1 + \frac{R}{r}\right)\theta\right]\right\}$$

<u>When $\theta = 0$</u>: $\frac{d^2V_g}{d\theta^2} = Wr\left(1 + \frac{R}{r}\right)\left[-1 + \frac{4}{3\pi}\left(1 + \frac{R}{r}\right)\right]$

$\therefore$ Equilibrium is stable if $-1 + \frac{4}{3\pi}\left(1 + \frac{R}{r}\right) > 0$, or $\frac{R}{r} > \frac{3\pi}{4} - 1 = 1.356$ ◆

## 10.50




$$L = 2R\cos\theta \qquad y = L\sin\theta = 2R\cos\theta\sin\theta = R\sin 2\theta$$

$$V = Wy + \frac{1}{2}k(L - L_0)^2 = WR\sin 2\theta + \frac{1}{2}k(2R\cos\theta - R)^2$$
$$= WR\sin 2\theta + \frac{1}{2}kR^2(2\cos\theta - 1)^2$$

$$\frac{dV}{d\theta} = 2RW\cos 2\theta + R^2k(2\cos\theta - 1)(-2\sin\theta)$$
$$= 2RW\cos 2\theta + 2R^2k(-\sin 2\theta + \sin\theta)$$
$$= 2R^2k\left(\frac{W}{kR}\cos 2\theta - \sin 2\theta + \sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = 2R^2k\left(-2\frac{W}{kR}\sin 2\theta - 2\cos 2\theta + \cos\theta\right)$$

With $W/(kR) = 0.5$ and $\theta = 23.91^\circ$ we get

$$\frac{dV}{d\theta} = 2R^2k(0.5\cos 47.82^\circ - \sin 47.82^\circ + \sin 23.91^\circ) \approx 0 \quad \text{In equilibrium} \blacktriangleleft$$

$$\frac{d^2V}{d\theta^2} = 2R^2k\left(-\sin 47.82^\circ - 2\cos 47.82^\circ + \cos 23.91^\circ\right)$$
$$= 2R^2k(-1.170) < 0 \quad \text{Unstable} \blacktriangleleft$$

## 10.51

$$V = W\left(\frac{a}{2}\cos\theta\right) + \frac{1}{2}k\left(b\sin\theta\right)^2$$
$$\frac{dV}{d\theta} = -\frac{Wa}{2}\sin\theta + kb^2\sin\theta\cos\theta = -\frac{Wa}{2}\sin\theta + \frac{kb^2}{2}\sin 2\theta$$
$$\frac{d^2V}{d\theta^2} = -\frac{Wa}{2}\cos\theta + kb^2\cos 2\theta \qquad \left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} = -\frac{Wa}{2} + kb^2$$

Stability exists if

$$\frac{Wa}{2} < kb^2 \qquad k > \frac{Wa}{2b^2} = \frac{10(24)}{2(6^2)} = 3.33 \text{ lb/in.} \blacktriangleleft$$

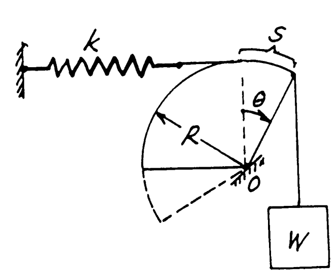
## 10.52

**Let the $\theta = 0$ position be the datum position.**

$$V_g = -WR(1 - \cos\theta) \qquad V_e = \frac{1}{2}ks^2 = \frac{1}{2}k(R\theta)^2$$

$$V = V_g + V_e = -WR(1 - \cos\theta) + \frac{1}{2}kR^2\theta^2$$

$$\therefore \frac{dV}{d\theta} = -WR\sin\theta + kR^2\theta$$

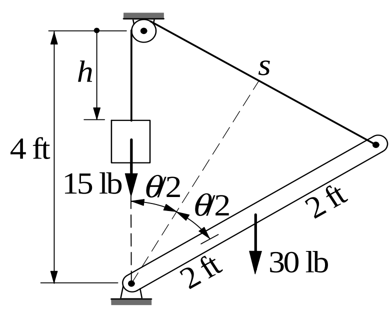


$$\frac{dV}{d\theta} = 0 \text{ when } \theta = 30° \ (\pi/6 \text{ rad}): \quad -WR\sin 30° + kR^2\frac{\pi}{6} = 0 \qquad \therefore \frac{kR}{W} = \frac{6\sin 30°}{\pi} = 0.9549$$

$$\therefore \frac{d^2V}{d\theta^2} = -WR\cos\theta + kR^2 = WR\left(-\cos\theta + \frac{kR}{W}\right) = WR(-\cos\theta + 0.9549)$$

<u>When $\theta = 30°$</u>: $\dfrac{d^2V}{d\theta^2} = WR(-\cos 30° + 0.9549) = 0.0889WR > 0 \quad \therefore$ **Equilibrium is stable** ◆

***10.53**




$$V = -15h + 30(2\cos\theta - 4) \qquad \frac{dV}{d\theta} = -15\frac{dh}{d\theta} - 60\sin\theta$$

$$h + s = \text{const (length of rope)}$$

$$h + 2(4\sin\frac{\theta}{2}) = \text{const} \qquad \frac{dh}{d\theta} + 4\cos\frac{\theta}{2} = 0 \qquad \frac{dh}{d\theta} = -4\cos\frac{\theta}{2}$$

$$\frac{dV}{d\theta} = -15\left(-4\cos\frac{\theta}{2}\right) - 60\sin\theta \qquad \frac{d^2V}{d\theta^2} = -30\sin\frac{\theta}{2} - 60\cos\theta$$

Equilibrium:

$$\frac{dV}{d\theta} = 0 \qquad \cos\frac{\theta}{2} - \sin\theta = 0 \qquad \cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$\cos\frac{\theta}{2} = 0 \quad \text{or} \quad 1 - 2\sin\frac{\theta}{2} = 0$$

$$\theta = 180° \ ◀ \quad \text{or} \quad \theta = 60° \ ◀$$

Stability at $\theta = 180°$:

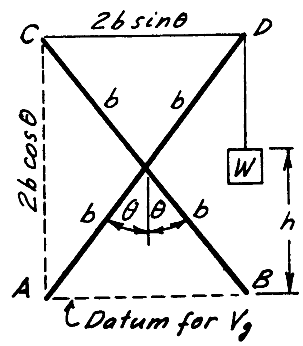
$$\frac{d^2V}{d\theta^2} = -30\sin 90° - 60\cos 180° = 30 > 0 \quad \text{Stable} \ ◀$$

Stability at $\theta = 60°$:

$$\frac{d^2V}{d\theta^2} = -30\sin 30° - 60\cos 60° = -30 - 60 = -45 < 0 \quad \text{Unstable} \ ◀$$

## 10.54

Let L be the length of the rope CDE.




$\therefore L = \overline{CD} + \overline{DE} = 2b\sin\theta + (2b\cos\theta - h)$

$\therefore h = 2b(\sin\theta + \cos\theta) - L$

$V_g = Wh = W\left[2b(\sin\theta + \cos\theta) - L\right]$

$$\frac{dV_g}{d\theta} = 2Wb(\cos\theta - \sin\theta)$$

$$\frac{d^2V_g}{d\theta^2} = 2Wb(-\sin\theta - \cos\theta)$$

$$\frac{dV_g}{d\theta} = 0: \quad \cos\theta - \sin\theta = 0 \quad \therefore \tan\theta = 1 \quad \therefore \theta = 45^\circ \blacklozenge$$

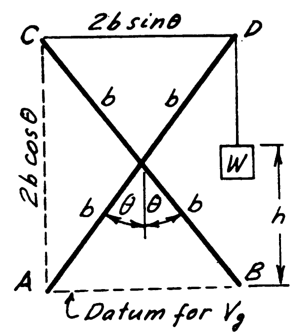
When $\theta = 45^\circ$, $\dfrac{d^2V_g}{d\theta^2} = -2\sqrt{2}\,Wb < 0$ $\therefore$ Equilibrium is <u>unstable</u>

## 10.55

From solution of Prob. 10.54 we have

$h = 2b(\sin\theta + \cos\theta) - L$

where L is the length of the rope CDE.




Deformation of the spring AB is

$s = \overline{AB} - b = 2b\sin\theta - b$

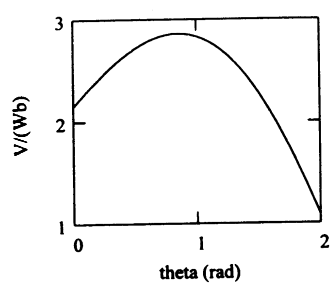
$$V = V_g + V_e = Wh + \frac{1}{2}ks^2$$

$$= W\left[2b(\sin\theta + \cos\theta) - L\right] + \frac{1}{2}\left(0.3\frac{W}{b}\right)(2b\sin\theta - b)^2$$

$$= Wb(0.15 + 1.4\sin\theta + 2\cos\theta + 0.6\sin^2\theta) - WL$$

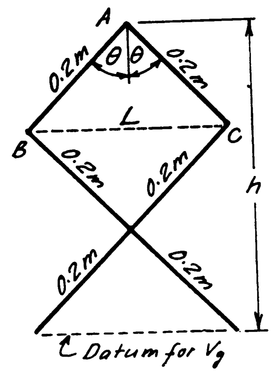
$$\frac{dV}{d\theta} = Wb(1.4\cos\theta - 2\sin\theta + 1.2\sin\theta\cos\theta) = Wb(1.4\cos\theta - 2\sin\theta + 0.6\sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = Wb(-1.4\sin\theta - 2\cos\theta + 1.2\cos 2\theta)$$




Plot of V/(Wb) indicates an <u>unstable</u> equilibrium position at $\theta \approx 0.9$ rad (the plot neglects the constant term WL). A more precise value is obtained by solving $dV/d\theta = 0$ by Newton's method: $\theta \leftarrow \theta - \dfrac{dV/d\theta}{d^2V/d\theta^2}$. Starting with $\theta = 0.9$, Newton's method converges after only two iterations to $\theta = 0.8565$ rad $= 49.1^\circ$ (unstable equilibrium) $\blacklozenge$

## 10.56




**Deformation of spring:**

$$s = L - L_0 = 2(0.2 \sin\theta) - 2(0.2 \sin 20^o)$$

$$= 0.4(\sin\theta - \sin 20^o)$$

$$h = 3(0.2 \cos\theta) = 0.6 \cos\theta$$

$$V = V_g + V_e = mgh + \frac{1}{2}ks^2$$

$$= m(9.81)(0.6 \cos\theta) + \frac{1}{2}(250)(0.4)^2 (\sin\theta - \sin 20^o)^2$$

$$= 5.886m \cos\theta + 20(\sin\theta - 0.3420)^2 \quad N{\cdot}m$$

$$\therefore \frac{dV}{d\theta} = -5.886m \sin\theta + 40(\sin\theta - 0.3420) \cos\theta \quad N{\cdot}m$$

$$\therefore \frac{d^2V}{d\theta^2} = -5.886m \cos\theta - 40(\sin\theta - 0.3420) \sin\theta + 40 \cos^2\theta \quad N{\cdot}m$$

<u>At $\theta = 60^o$</u>

$$\frac{dV}{d\theta} = 0: \; -5.886m\,(0.8660) + 40(0.8660 - 0.3420)(0.5) = 0$$

$$\therefore -5.097m + 10.480 = 0 \quad \therefore m = 2.06 \text{ kg} \; \blacklozenge$$

$$\frac{d^2V}{d\theta^2} = -5.886(2.06)(0.5) - 40(0.8660 - 0.3420)(0.8660) + 40\,(0.5)^2 = -14.2 \;\; N{\cdot}m < 0$$

∴ The equilibrium is <u>unstable</u>. ♦

## 10.57

**From the solution of Prob. 10.56:**

$$V = 5.886m \cos\theta + 20(\sin\theta - 0.3420)^2 = 5.886(2.06) \cos\theta + 20(\sin\theta - 0.3420)^2$$

$$= 12.125 \cos\theta + 20(\sin\theta - 0.3420)^2 \quad N{\bullet}m$$

$$\therefore \frac{dV}{d\theta} = -\,12.125 \sin\theta + 40(\sin\theta - 0.3420) \cos\theta \quad N{\cdot}m$$

$$\therefore \frac{d^2V}{d\theta^2} = -12.125 \sin\theta - 40(\sin\theta - 0.3420) \sin\theta + 40 \cos^2\theta$$

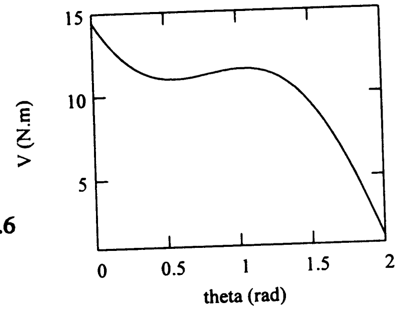
$$= 40(\cos^2\theta - \sin^2\theta) + 1.555 \sin\theta \quad N{\cdot}m$$

From plot of V vs. θ we see that there is a stable equilibrium position at $\theta \approx 0.6$ rad. An accurate value of θ can be computed by Newton's method:

$$\theta \leftarrow \theta - \frac{dV/d\theta}{d^2V/d\theta^2}$$

This yields after two iterations, starting with $\theta = 0.6$

$$\theta = 0.5620 \text{ rad} = 32.2^\circ \blacklozenge$$

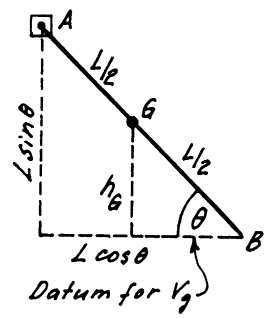



10.58

Geometry: $h_G = \frac{L}{2}\sin\theta$

Let $\theta = \theta_0$ when the spring is undeformed. Thus the deformation of the spring is

$$s = L(\cos\theta - \cos\theta_0)$$




$$V = V_g + V_e = Wh_G + \frac{1}{2}ks^2$$

$$= (kL)\left(\frac{L}{2}\sin\theta\right) + \frac{1}{2}kL^2(\cos\theta - \cos\theta_0)^2$$

$$= \frac{1}{2}kL^2\left[\sin\theta + (\cos\theta - \cos\theta_0)^2\right]$$

$$\therefore \frac{dV}{d\theta} = \frac{1}{2}kL^2\left[\cos\theta - 2(\cos\theta - \cos\theta_0)\sin\theta\right]$$

$$\therefore \frac{d^2V}{d\theta^2} = \frac{1}{2}kL^2\left[-\sin\theta - 2(\cos\theta - \cos\theta_0)\cos\theta + 2\sin^2\theta\right]$$

<u>When $\theta = 65^\circ$</u>:

$$\frac{dV}{d\theta} = 0: \ \cos 65^\circ - 2(\cos 65^\circ - \cos\theta_0)\sin 65^\circ = 0$$

$$\therefore -0.3434 + 1.8126\cos\theta_0 = 0 \quad \therefore \theta_0 = 79.1^\circ \blacklozenge$$

$$\frac{d^2V}{d\theta^2} = \frac{1}{2}kL^2\left[-\sin 65^\circ - 2(\cos 65^\circ - \cos 79.1^\circ)\cos 65^\circ + 2\sin^2 65^\circ\right]$$

$$= \frac{1}{2}kL^2(0.539) > 0 \quad \therefore \text{Equilibrium is } \underline{\text{stable}} \ \therefore$$

## 10.59

From solution of Prob. 10.58 we get, after substituting $\theta_0 = 80^\circ$:

$$V = \frac{1}{2}kL^2\left[\sin\theta + (\cos\theta - \cos 80^\circ)^2\right] \qquad \frac{dV}{d\theta} = \frac{1}{2}kL^2\left[\cos\theta - 2(\cos\theta - \cos 80^\circ)\sin\theta\right]$$

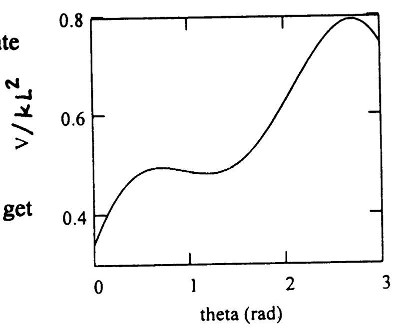
$$\frac{d^2V}{d\theta^2} = \frac{1}{2}kL^2\left[-\sin\theta - 2(\cos\theta - \cos 80^\circ)\cos\theta + 2\sin^2\theta\right]$$

The plot of V vs. θ shows that there is a stable equilibrium position at $\theta \approx 1.2$ rad. A more accurate result can be obtained by solving $dV/d\theta = 0$ by Newton's method using the iterative formula

$$\theta \leftarrow \theta - \frac{dV/d\theta}{d^2V/d\theta^2}$$

After two iterations, starting from θ = 1.2 rad, we get

$\theta = 1.184 \text{ rad} = 67.8^\circ$ ♦




## 10.60

Geometry: $h = 10 \sin\theta$ in

Deformation of spring: $s = L - L_0 = 5\cos\theta - 2$ in

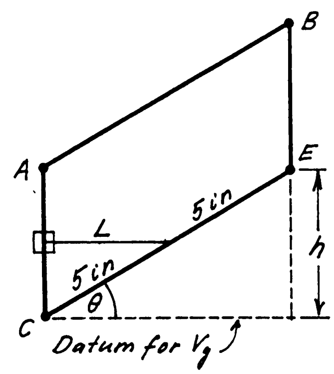
$$V = V_g + V_e = Wh + \frac{1}{2}ks^2$$

$$= (5)(10\sin\theta) + \frac{1}{2}(7.5)(5\cos\theta - 2)^2$$

$$= 50\sin\theta + 3.75(5\cos\theta - 2)^2 \text{ lb·in}$$

$$\therefore \frac{dV}{d\theta} = 50\cos\theta + 7.5(5\cos\theta - 2)(-5\sin\theta)$$

$$= 50\cos\theta + 75\sin\theta - 187.5\sin\theta\cos\theta \text{ lb·in}$$



$$\therefore \frac{d^2V}{d\theta^2} = -50\sin\theta + 75\cos\theta + 187.5(\sin^2\theta - \cos^2\theta)\ \text{lb}\bullet\text{in}$$

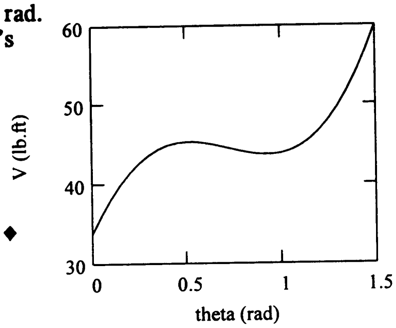
The plot of V vs. θ shows an unstable equilibrium position at $\theta \approx 0.5$ rad, and a stable one at $\theta \approx 0.9$ rad. Using these angles as the starting values, Newton's method

$$\theta \leftarrow \theta - \frac{dV/d\theta}{d^2V/d\theta^2}$$

yields after a few iterations:

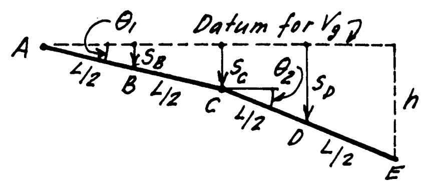
$\theta = 0.5165$ rad $= 29.6^\circ$ (unstable equlibrium) ♦

$\theta = 0.9273$ rad $= 53.1^\circ$ (stable equlibrium) ♦




## 10.61

Geometry: $h = L(\sin\theta_1 + \sin\theta_2)$




$$s_B = \frac{L}{2}\sin\theta_1 \quad s_C = L\sin\theta_1$$

$$s_D = L\sin\theta_1 + \frac{L}{2}\sin\theta_2$$

$$V = V_g + V_e = -Wh + \frac{k}{2}(s_B^2 + s_C^2 + s_D^2)$$

$$= -\left(\frac{kL}{10}\right)L(\sin\theta_1 + \sin\theta_2) + \frac{kL^2}{2}\left[\frac{1}{4}\sin^2\theta_1 + \sin^2\theta_1 + \left(\sin\theta_1 + \frac{1}{2}\sin\theta_2\right)^2\right]$$

$$= kL^2\left[-\frac{1}{10}(\sin\theta_1 + \sin\theta_2) + \frac{9}{8}\sin^2\theta_1 + \frac{1}{8}\sin^2\theta_2 + \frac{1}{2}\sin\theta_1\sin\theta_2\right]$$

$$\therefore \frac{dV}{d\theta_1} = kL^2\left(-\frac{1}{10}\cos\theta_1 + \frac{9}{4}\sin\theta_1\cos\theta_1 + \frac{1}{2}\cos\theta_1\sin\theta_2\right)$$

$$\therefore \frac{dV}{d\theta_2} = kL^2\left(-\frac{1}{10}\cos\theta_2 + \frac{1}{4}\sin\theta_2\cos\theta_2 + \frac{1}{2}\sin\theta_1\cos\theta_2\right)$$

Discarding the obvious equilibrium position $\cos\theta_1 = \cos\theta_2 = 0$ ($\theta_1 = \theta_2 = 90^\circ$), we get

$$\frac{dV}{d\theta_1} = 0:\ \frac{9}{4}\sin\theta_1 + \frac{1}{2}\sin\theta_2 = \frac{1}{10} \qquad \frac{dV}{d\theta_2} = 0:\ \frac{1}{2}\sin\theta_1 + \frac{1}{4}\sin\theta_2 = \frac{1}{10}$$

The solution is: $\sin\theta_1 = -0.0800$, $\sin\theta_2 = 0.5600$ $\therefore \theta_1 = -4.6^\circ$, $\theta_2 = 34.1^\circ$ ♦

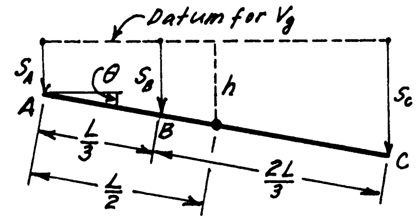
## 10.62

Choose $s_A$ and $\theta$ as the kinematically independent variables.

$\therefore s_B = s_A + \frac{L}{3}\sin\theta$

$\therefore s_C = s_A + L\sin\theta$

$\therefore h = s_A + \frac{L}{2}\sin\theta$




$$V = V_g + V_e = -Wh + \frac{k}{2}(s_A^2 + s_B^2 + s_C^2)$$

$$= -(kL)\left(s_A + \frac{L}{2}\sin\theta\right) + \frac{k}{2}\left[s_A^2 + \left(s_A + \frac{L}{3}\sin\theta\right)^2 + (s_A + L\sin\theta)^2\right]$$

$$= k\left(-Ls_A - \frac{1}{2}L^2\sin\theta + \frac{3}{2}s_A^2 + \frac{4}{3}Ls_A\sin\theta + \frac{5}{9}L^2\sin^2\theta\right)$$

$$\therefore \frac{dV}{d\theta} = k\left(-\frac{1}{2}L^2\cos\theta + \frac{4}{3}Ls_A\cos\theta + \frac{10}{9}L^2\sin\theta\cos\theta\right)$$

$$\therefore \frac{dV}{ds_A} = k\left(-L + 3s_A + \frac{4}{3}L\sin\theta\right)$$

Discarding the $\cos\theta = 0$ ($\theta = 90^\circ$) equilibrium position, the equilibrium equations are

$$\frac{dV}{d\theta} = 0:\ \frac{4}{3}s_A + \frac{10}{9}L\sin\theta = \frac{1}{2}L \qquad \frac{dV}{ds_A} = 0:\ 3s_A + \frac{4}{3}L\sin\theta = L$$

The solution is: $s_A = \frac{2}{7}L,\ \sin\theta = \frac{3}{28}\ \ \therefore \theta = 6.15^\circ$ ♦