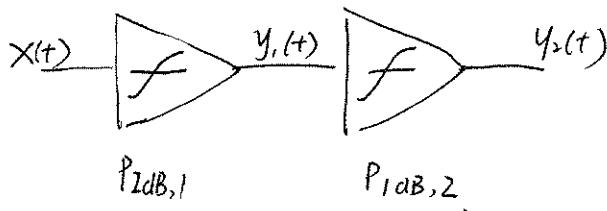


2.1 Solu:



$$Y_1(t) = \alpha_1 X(t) + \alpha_2 X^2(t) + \alpha_3 X^3(t)$$

$$Y_2(t) = \beta_1 Y_1(t) + \beta_2 Y_1^2(t) + \beta_3 Y_1^3(t)$$

then.

$$Y_2(t) = \beta_1 [\alpha_1 X(t) + \alpha_2 X^2(t) + \alpha_3 X^3(t)] + \beta_2 [\alpha_1 X(t) + \alpha_2 X^2(t) + \alpha_3 X^3(t)]^2 + \beta_3 [\alpha_1 X(t) + \alpha_2 X^2(t) + \alpha_3 X^3(t)]^3$$

Considering only the first - and third - order terms,

$$Y_2(t) = \alpha_1 \beta_1 X(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) X^3(t) + \dots \\ = [\alpha_1 \beta_1 + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3)] X(t) + \dots$$

$$P_{1dB,1} : A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|} ; P_{1dB,2} : A_{in,2,1dB} = \sqrt{0.145 \left| \frac{\beta_1}{\beta_3} \right|}$$

$$P_{1dB} \Rightarrow 20 \log \left| \alpha_1 \beta_1 + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) \cdot A_{in,1dB}^2 \right| = 20 \log |\alpha_1 \beta_1| - 1dB$$

$$A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

Represented by the  $P_{1dB}$  of first and second stage.

$$\frac{1}{A_{in,1dB}^2} = \frac{1}{0.145} \left| \frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2 \beta_3}{\beta_1} \right| \\ = \left| \frac{1}{A_{in,1dB}^2} + \frac{2}{0.145} \frac{\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2}{A_{in,1dB}^2} \right|$$

2.2 solve:

assuming  $-3 \text{ dBm}$   $A_1$  at  $2.42 \text{ G}$   
 $-35 \text{ dBm}$   $A_2$  at  $2.43 \text{ G}$ .

IM product :  $\frac{3}{4} \delta_3 A_1^2 \cdot A_2$

$$-3 \text{ dBm} \Rightarrow A_1 = \sqrt{2.50 \cdot 10^{-0.3} \times 10^{-3}} = 223.9 \text{ mV}$$

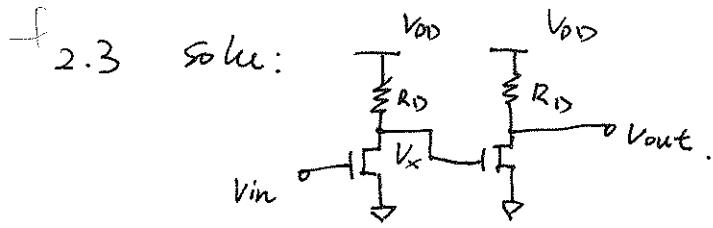
$$-35 \text{ dBm} \Rightarrow A_2 = \sqrt{2.50 \times 10^{-3.5} \times 10^{-3}} = 5.6 \text{ mV}.$$

We can write at LNA output :

$$20 \lg |\delta_1 \cdot A_{\text{sig}}| - 20 \text{ dB} = 20 \lg \left| \frac{3}{4} \delta_3 A_1^2 \cdot A_2 \right|.$$

$$\Rightarrow \lg |\delta_1 \cdot A_{\text{sig}}| = \lg \left| \frac{3}{4} \delta_3 A_1^2 \cdot A_2 \right|$$

$$\text{IIP}_3 = \sqrt{\frac{4}{3} \left| \frac{\delta_1}{\delta_3} \right|} = \sqrt{\frac{4}{3} \cdot \frac{30}{4} \cdot \frac{A_1^2 \cdot A_2}{A_{\text{sig}}}} = 9.43 \text{ V}_P \\ = 29.5 \text{ dBm}.$$



$$I_D = K \cdot (V_{GS} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D (V_x - V_T)^2$$

$$V_x = V_{DD} - K \cdot R_D (V_{in} - V_T)^2$$

$$\begin{aligned} V_{out} &= V_{DD} - K \cdot R_D [ V_{DD} - K \cdot R_D (V_{in} - V_T)^2 - V_T ]^2 \\ &= V_{DD} - K \cdot R_D [ (V_{DD} - V_T) - K \cdot R_D (V_{in} - V_T)^2 ]^2 \\ &= V_{DD} - K \cdot R_D [ (V_{DD} - V_T)^2 + K^2 R_D^2 (V_{in} - V_T)^4 - 2 K R_D (V_{DD} - V_T) (V_{in} - V_T)^2 ] \end{aligned}$$

1st order of  $V_{in}$

$$\Rightarrow [4 \cdot (V_{DD} - V_T) \cdot K \cdot R_D V_T - 4 K^2 R_D^2 V_T^3] \cdot V_{in}$$

3rd order of  $V_{in}$

$$\Rightarrow [-4 K^2 R_D^2 V_T] \cdot V_{in}^3$$

$$A_{IP3} = \sqrt{\frac{4}{3} \cdot \frac{4 \cdot (V_{DD} - V_T) K \cdot R_D V_T - 4 K^2 R_D^2 V_T^3}{4 K^2 R_D^2 V_T}}$$

2.4. Solu.:

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \alpha_4 x^4(t) + \alpha_5 x^5(t).$$

$$1^\circ \cos^3 wt = \frac{3}{4} \cos wt + \frac{1}{4} \cos 3wt.$$

$$2^\circ \cos^2 wt = \frac{1 + \cos 2wt}{2}$$

$$3^\circ \cos^4 wt = \frac{1 + \cos^2 2wt + 2 \cos 2wt}{2^2}$$

$$4^\circ \cos^5 wt = (\frac{3}{4} \cos wt + \frac{1}{4} \cos 3wt) \cdot (\frac{1 + \cos 2wt}{2})$$

$$= (\frac{3}{8} \cos wt + \frac{1}{8} \cos 3wt + \underbrace{\frac{3}{8} \cos wt \cdot \cos 2wt}_{\parallel})$$

$$\frac{3}{16} [\cos wt + \cos 3wt] + \underbrace{\frac{1}{8} \cos 3wt \cdot \cos 2wt}_{\parallel}$$

$$\frac{1}{16} [\cos wt + \cos 5wt]$$

$$1^{\text{st}} \text{ order} \Rightarrow \alpha_1 A + \frac{3}{4} \alpha_3 A^3 + (\frac{3}{8} + \frac{3}{16} + \frac{1}{16}) \cdot \alpha_5 A^5.$$

3rd order

$$\Rightarrow \frac{1}{4} \alpha_3 A^3 + (\frac{1}{8} + \frac{3}{16}) \alpha_5 A^5.$$

$$(1) P_{IDB} \Rightarrow 20 \lg |\alpha_1 + \frac{3}{4} \alpha_3 A^2 + \frac{5}{8} \alpha_5 A^4| = 20 \lg |\alpha_1| - 1 \text{ dB}.$$

$$\Rightarrow A_{in, 1 \text{ dB}} = \sqrt{\frac{0.8 \cdot (0.5625 \alpha_3^2 - 0.27175 \alpha_1 \cdot \alpha_5)^{\frac{1}{2}} - 0.6 \alpha_3}{\alpha_5}}$$

(2) II<sub>P3</sub> doesn't change.

$$A_{II P3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1}{\alpha_3} \right|}.$$

$$2.5(a) \text{ Soln: } A_{\text{sig}} = \frac{0.1mV}{0.7943} \Leftrightarrow -2 \text{ dB} = 10^{-0.1} = 0.7943$$

$$A_2 = 10mV \times 0.1413 \Leftrightarrow -17 \text{ dB} = 10^{-0.85} = 0.1413$$

$$A_3 = 10mV \times 0.0141 \Leftrightarrow -37 \text{ dB} = 10^{-1.85} = 0.0141$$

at the output of amplifier:

$$20 \lg |\alpha_1 \cdot A_{\text{sig}}| - 20 \text{ dB} = 20 \lg \left| \frac{3}{4} \alpha_3 \cdot A_2^3 \cdot A_3 \right|$$

$$\Rightarrow |\alpha_1 \cdot 0.07943_m| = \left| \frac{30}{4} \alpha_3 \cdot 1.413^2_m \cdot 0.141_m \right|$$

$$\therefore A_{\text{IP3}} = \sqrt{\frac{30}{4} \frac{1.413^2_m \cdot 0.141}{0.07943} \cdot \frac{4}{3}} = 5.95mV_p = -34.5 \text{ dBm},$$

Neglect the nonlinearity of BPF.

$$(b). \quad y_1(t) = \alpha_1 x(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_3 y_1^3(t)$$

Only considering the first and third order:

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + \alpha_1^3 \beta_3) x^3(t) + \dots$$

$$\therefore A_{\text{IP3}} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + \alpha_1^3 \beta_3} \right|}$$

$$\frac{1}{A_{\text{IP3}}^2} = \frac{1}{A_{\text{IP3},1}^2} + \frac{\alpha_1^2}{A_{\text{IP3},2}^2}$$

$$\Rightarrow \frac{1}{A_{\text{IP3}}^2} = \frac{1}{500m} + \frac{10}{5.95^2}$$

$$\Rightarrow A_{\text{IP3}} = 1.875mV_p$$

2.6 Solu:

Let  $x(t)$  be a random signal (wide-sense stationary process)

Auto correlation function:  $R_x(t) = E[x(t) \cdot x(t+\tau)]$

Let me prove that:  $S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f \tau} d\tau$

$$\text{Proof: } X_T(f) \triangleq \int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt.$$

$$S_T(f) \triangleq E\left[\frac{1}{T} |X_T(f)|^2\right]$$

$$S_x(f) = \lim_{T \rightarrow \infty} S_T(f).$$

$$\begin{aligned} E[|X_T(f)|^2] &= E\left[\left(\int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt\right)^2\right] \\ &= E\left[\int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt \cdot \int_{-T/2}^{T/2} x(\tau) e^{-j2\pi f \tau} d\tau\right] \\ &= E\left[\int_{-T/2}^{T/2} \int_{-T/2}^{T/2} x(t) x(\tau) e^{-j2\pi f (t-\tau)} dt d\tau\right] \\ &= \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[x(t) \cdot x(\tau)] e^{-j2\pi f (t-\tau)} dt d\tau \\ &= \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} R_x(t-\tau) e^{-j2\pi f (t-\tau)} dt d\tau \\ &= \int_{-T}^T (T - |\tau|) R_x(\tau) e^{-j2\pi f \tau} d\tau. \end{aligned}$$

$$E\left[\frac{1}{T} |X_T(f)|^2\right] = \int_{-T}^T (1 - \frac{|\tau|}{T}) R_x(\tau) e^{-j2\pi f \tau} d\tau.$$

$$\text{Therefore: } S_x(f) = \lim_{T \rightarrow \infty} E\left[\frac{1}{T} |X_T(f)|^2\right] = \int_{-\infty}^{+\infty} R_x(\tau) e^{-j2\pi f \tau} d\tau.$$

2.7. Solu:

Assume  $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$ .

$$x(t) = V_0 \cos \omega_0 t$$

$$\text{3rd harmonic} : \frac{\alpha_3 V_0^3}{4} = V_3$$

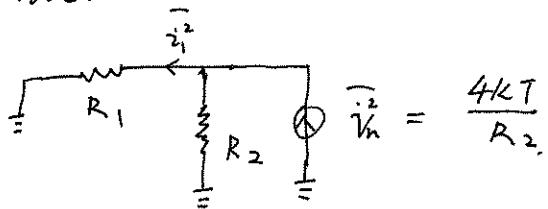
$$\Rightarrow \alpha_3 = \frac{4V_3}{V_0^3}$$

$$A_{in, 1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|}$$

$$= \sqrt{0.145 \cdot \left| \frac{1}{4} \frac{V_3}{V_0^3} \right|}$$

$$= \sqrt{\frac{0.145}{4} \left| \frac{\alpha_1 V_0^3}{V_3} \right|}$$

→ 2.8 soln:



$$\dot{V}_n = \frac{4kT}{R_2}$$

$$\begin{aligned} P_{R_1} &= \dot{i}_1^2 \cdot R_1 \\ &= \left( \sqrt{\frac{4kT}{R_2}} \cdot \frac{R_2}{R_1 + R_2} i^2 \right) \cdot R_1 \\ &= \frac{4kT}{(R_1 + R_2)^2} \cdot R_1 R_2. \end{aligned}$$

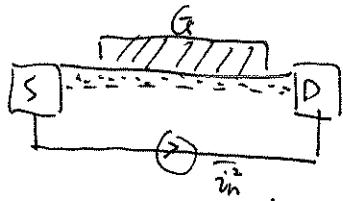
$$\therefore P_{R_1} = P_{R_2}$$

So it proves that the noise power delivered by  $R_1$  to  $R_2$  is equal to that delivered by  $R_2$  to  $R_1$  at the same temperature.

∴ If it is not the truth, the energy would not be conserved.

2.9 Solu:

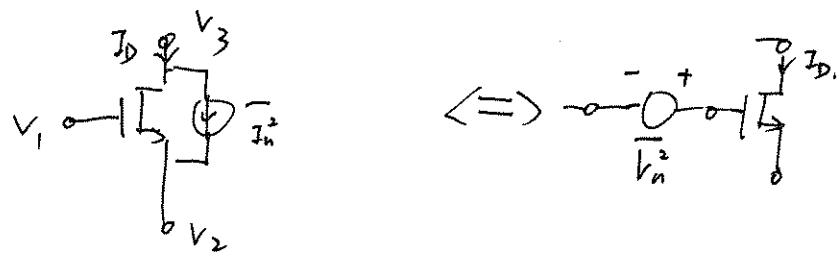
why the channel thermal noise of a MOSFET is model by a current source bw. S & D. rather than G & D.



Firstly, from the figure we can find the channel resistor is between source & drain. As a result, it is reasonable to model the noise by a current source between source and drain.

Secondly, MOSFET has the function of transconductance. It's easy to transfer the current source from between S & D to the voltage source at the gate.

2.10 Solu:



Proof: transconductance :  $g_m$ .

Assume the transistor is in saturation region.

For small-signal analysis,  $V_1 = 0$

$$I_D = \sqrt{I_n^2} = \sqrt{4KT\gamma g_m}$$

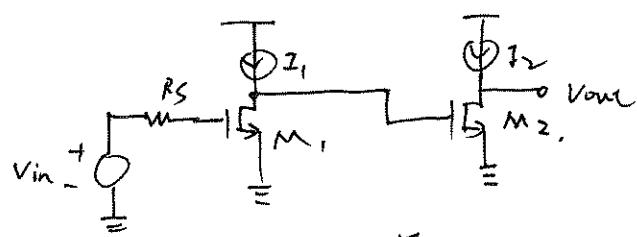
At the same time, for voltage source model.

$$I_D = V_{in} \cdot g_m = \sqrt{4KT\gamma \cdot g_m}$$

$$\Rightarrow V_{in} = \sqrt{\frac{4KT\gamma}{g_m}}$$

$$\Rightarrow V_n^2 = \frac{4KT\gamma}{g_m}$$

2.11 Soln:



$$NF_1 = 1 + \frac{f}{g_m R_s} \quad (2.122)$$

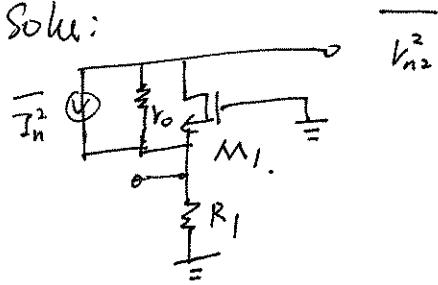
$$NF_2 = 1 + \frac{f}{g_m r_{o1}} ; \quad A_{p1} = \frac{P_{out,av,1}}{P_{in,av,1}} = \frac{V_{in}^2 \cdot A_{v1}^2 \cdot \frac{1}{4r_{o1}}}{V_{in}^2 \cdot \frac{1}{4R_s}}$$

$$\therefore NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} = g_m^2 \cdot r_{o1} \cdot R_s$$

$$= 1 + \frac{f}{g_m R_s} + \frac{f}{g_m r_{o1}} / g_m^2 r_{o1} R_s$$

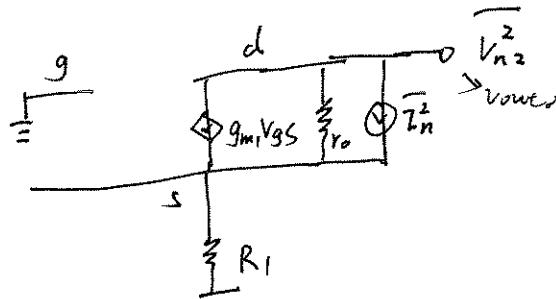
$$= 1 + \frac{f}{g_m R_s} + \frac{f}{g_m^2 r_{o1}^2 g_m R_s}$$

— | 2.12. Solu:



assume  $I_1$  is ideal, and neglect the noise of  $R_1$ .

Proof:



For small-signal analysis,

$$g_m(-V_S) + \frac{V_{out} - V_C}{r_o} + \overline{I_n} = \frac{V_S}{R_1}.$$

Because we cannot find any loop for the current through  $R_1$ ,

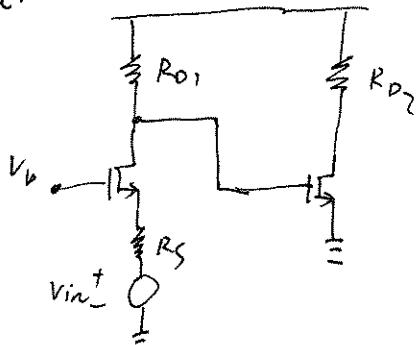
$$V_S = 0$$

$$\Rightarrow \frac{V_{out}}{r_o} = - \overline{I_n}$$

$$V_{out} = - \overline{I_n} \cdot r_o$$

$$\therefore \overline{V_{n2}^2} = \overline{I_{n2}^2} \cdot r_o^2$$

2.13. Soln:



Neglect:  
transistor cap.  
flicker noise.  
CLM.  
body effect.

For 1st stage:

$$R_{in1} = \frac{1}{g_m1}, R_{th2} = \infty,$$

$$\overline{V_{n1}^2} = 4KTR_{D1} + \frac{4KTF}{g_m1} \left( \frac{R_{D1}}{\frac{1}{g_m1} + R_S} \right)^2 \Leftrightarrow \frac{V_o}{\frac{4KTF}{g_m1} + R_S} : \text{unloaded output noise.}$$

For 2nd stage:

$$\overline{V_{n2}^2} = 4KTR_{D2} + 4KTF \cdot g_{m2} \cdot R_{D2}^2.$$

We now substitute these values in Eq. (2.126)

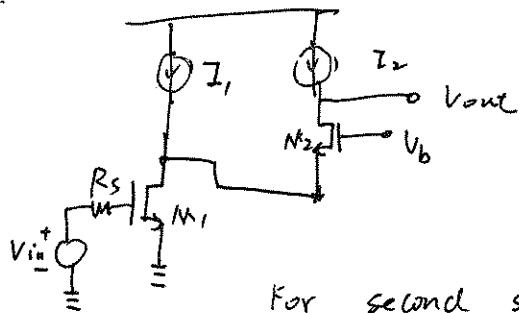
$$NF_{tot} = 1 + \frac{\frac{4KTR_{D1}}{g_m1} + \frac{4KTF}{g_m1} \left( \frac{R_{D1}}{\frac{1}{g_m1} + R_S} \right)^2}{\left( \frac{\frac{1}{g_m1}}{\frac{1}{g_m1} + R_S} \right)^2 \cdot (g_{m1} \cdot R_{D1})^2} \cdot \frac{1}{4KTR_S}.$$

$$+ \frac{\frac{4KTR_D}{g_m2} + 4KTF \cdot g_{m2} \cdot R_{D2}^2}{\left( \frac{\frac{1}{g_m1}}{\frac{1}{g_m1} + R_S} \right)^2 \cdot (g_{m1} \cdot R_{D1})^2 \cdot (g_{m2} \cdot R_{D2})^2} \cdot \frac{1}{4KTR_S}.$$

This result is different from the CS + CG configuration because the first stage's NF and input impedance are different, which affect the  $NF_{tot}$ .

2. 14. Solve:

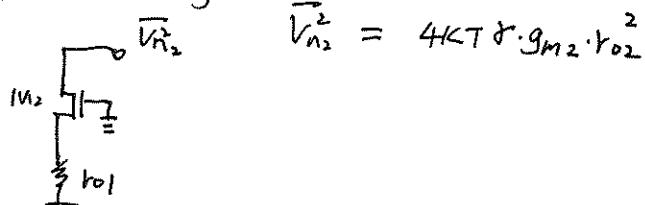
Consider CLM.



$$\overline{V_{n1}^2} = \frac{4KT\delta}{g_{m1}} \cdot (g_{m1}r_{o1})^2$$

$$= 4KT\delta \cdot g_{m1}V_{o1}^2$$

For second stage:

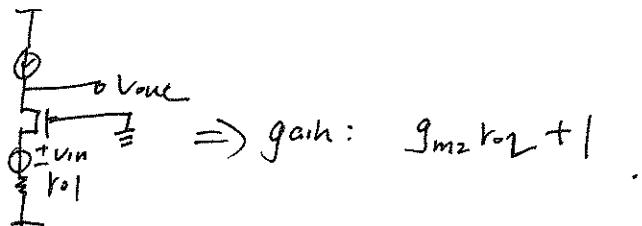


We now substitute these values in Eq. (2.26).

$$HF_{tot} = 1 + \frac{4KT\delta \cdot g_{m1}r_{o1}^2}{(g_{m1}r_{o1})^2} \cdot \frac{1}{4KTR_S}$$

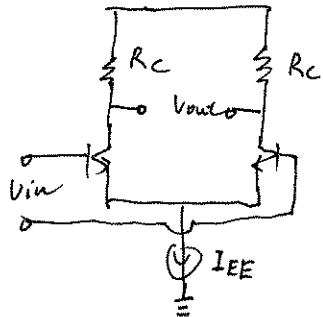
$$+ \frac{4KT\delta g_{m2}r_{o2}^2}{(g_{m1}r_{o1})^2 \cdot \left(\frac{1}{g_{m2}}\right)^2 \cdot (g_{m2}r_{o2}+1)^2} \cdot \frac{1}{4KTR_S}$$

Note :



2.15 solve:

IP<sub>3</sub>.



$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \dots$$

$$V_{out} = -2R_C I_{EE} \tanh \left[ \frac{V_{in}}{2V_T} \right].$$

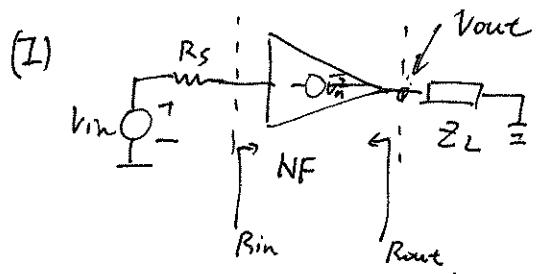
Only consider the first and third order.

$$V_{out} = -2R_C I_{EE} \left( \frac{V_{in}}{2V_T} - \frac{1}{3} \left( \frac{V_{in}}{2V_T} \right)^3 \right)$$

$$\begin{aligned} A_{in, IP_3} &= \sqrt{\frac{4}{3} \left| \frac{\partial x}{\partial z} \right|} \\ &= \sqrt{\frac{4}{3} \cdot \frac{\frac{1}{2V_T}}{\frac{1}{3} \left( \frac{1}{2V_T} \right)^3}} \end{aligned}$$

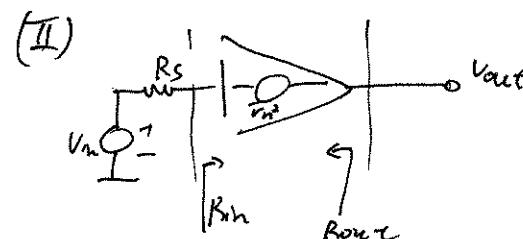
$$= 4V_T = 4 \frac{kT}{q} = 4 \times 26mV = 104mV$$

2.16 Soln:



$$\frac{V_{out}}{V_{in}} = A_o \frac{Z_L}{Z_L + R_{out}}$$

$$\overline{V_{n,out}^2} = \overline{V_n^2} \cdot \left( \frac{Z_L}{Z_L + R_{out}} \right)^2$$



$$NF = 1 + \frac{\overline{V_n^2}}{A_o^2} \times \frac{1}{4KTR_s} \quad (\text{unloaded gain})$$

$$A_o = \frac{V_{out}}{V_{in}}$$

Unloaded noise at output.

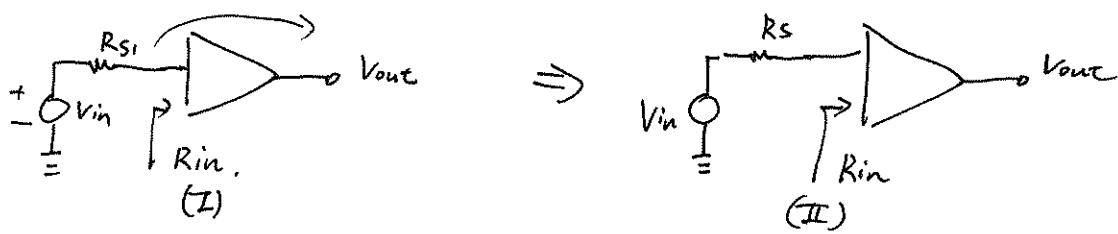
$$NF = 1 + \frac{\overline{V_{n,out}^2}}{\frac{V_{out}}{V_{in}}}.$$

$$= 1 + \frac{A_o \overline{V_n^2} \left( \frac{Z_L}{Z_L + R_{out}} \right)^2}{A_o^2 \left( \frac{Z_L}{Z_L + R_{out}} \right)^2} \cdot \frac{1}{4KTR_s}$$

$$= 1 + \frac{\overline{V_n^2}}{A_o^2} \cdot \frac{1}{4KTR_s}$$

Compared with (I) and (II), we find noise figures of two situations are the same. i.e. output load doesn't affect the circuits noise figure.

2.17 Soln:  $A_V$



$A_V$  is unloaded voltage gain from input to output of amplifier.

$\therefore$  In Figure (I),

$$NF_1 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_m}{R_{in}+R_S1} \cdot A_V\right)^2} \cdot \frac{1}{4kTR_S1}. \quad (1)$$

In Figure (II)

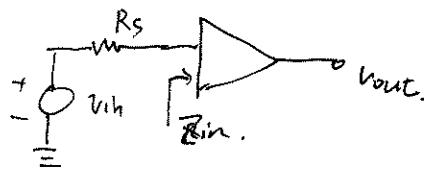
$$NF_2 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_m}{R_{in}+R_S2}\right)^2 \cdot A_V^2} \cdot \frac{1}{4kTR_S2}. \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{NF_1 - 1}{NF_2 - 1} = \frac{R_S2}{R_S1} \cdot \left( \frac{R_m + R_S2}{R_m + R_S1} \right)^2.$$

So if we know the input impedance and  $R_S1, R_S2$ , it's possible to compute the noise figure for another source impedance  $R_S2$ .

2.18 soln:

$$NF = \frac{1}{g_m R_s} + 1 \quad \text{Eq. (2.122),}$$



$$\alpha = \frac{Z_{in}}{Z_{in} + R_s}; \quad V_{RS}^2 = 4KTR_s$$

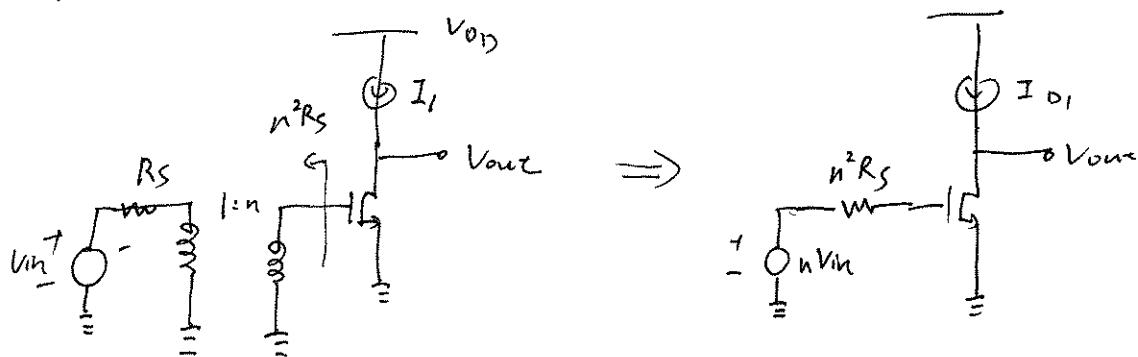
$$SNR_{in} = \frac{V_{in}^2}{V_{RS}^2}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{V_{RS}^2 |\alpha|^2 A_v^2 + V_n^2}$$

So. if  $R_s$  increases,  $SNR_{out}$  will fall.

It seems that the result is contradicted with. NF's falling, because  $SNR_{in}$  will also fall if  $R_s$  increases, the ratio of  $SNR_{in}$  and  $SNR_{out}$  is reasonable.

2.19 Soln:



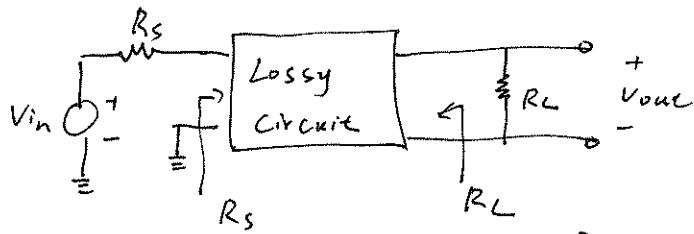
$$\overline{V_n^2} = 4kT \gamma \cdot g_m r_o^2$$

$$A = \frac{V_{out}}{V_{in}} = n \cdot g_m r_o$$

$$\begin{aligned}\therefore NF &= 1 + \frac{4kT + g_m r_o^2}{(n \cdot g_m r_o)^2} \cdot \frac{1}{4kT n^2 R_s} \\ &= 1 + \frac{\gamma}{n^4 g_m R_s}.\end{aligned}$$

From the result, we can find transformer improves the noise performance of Amplifier greatly.

2.20 Soln:



$$\text{Proof: } L = \frac{P_{in}}{P_{out}} = \frac{\frac{V_{in}^2}{4R_s}}{\frac{V_{out}^2}{R_L}} = \frac{V_{in}^2}{4R_s} \cdot \frac{R_L}{V_{out}^2} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2}$$

Theorem: For a passive (reciprocal) network, the PSD of thermal noise is given by  $\overline{V_n^2} = 4KT \operatorname{Re}\{Z_{out}\}$ .

$$\overline{V_{n,out}^2} = 4KT \cdot R_L \left(\frac{1}{2}\right)^2 - \text{all the noise} \quad ①$$

$$A_v = \frac{V_{out}}{V_{in}}. \quad ②$$

$$\therefore NF = \frac{①}{②^2} = \frac{\frac{4KT \cdot R_L \cdot \frac{1}{4}}{\frac{V_{out}^2}{V_{in}^2}}}{\frac{1}{4KTR_s}} \cdot \frac{1}{4KTR_s} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2}$$

$$= L.$$

2.21. Soln: Neglect CLM & Body effect.

$$(a). \overline{V_{n,out}^2} = \frac{4K\bar{T}\delta}{g_{m_1}} \left( \frac{g_{m_1}}{g_{m_2}} \right)^2 + 4K\bar{T}\delta \cdot g_{m_2}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m_1}}{g_{m_2}}$$

$$NF = 1 + \frac{\overline{V_{n,out}^2}}{\left( \frac{V_{out}}{V_{in}} \right)^2 \cdot \frac{1}{4K\bar{T}R_S}}$$

$$= 1 + 4K\bar{T}\delta \left( \frac{1}{g_{m_1}} + \frac{g_{m_2}^3}{g_{m_1}^2} \right) \cdot \frac{1}{4K\bar{T}R_S}$$

$$(d) \overline{V_{n,out}^2} = \frac{4K\bar{T}\delta}{g_{m_2}} \left( \frac{g_{m_2}}{g_{m_3}} \right)^2 + 4K\bar{T}\delta \cdot g_{m_3}$$

$$+ \frac{4K\bar{T}\delta}{g_{m_1}} \left( \frac{g_{m_1}}{g_{m_3}} \right)^2$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m_2}}{g_{m_3}}$$

$$NF = 1 + 4K\bar{T}\delta \left( \frac{1}{g_{m_2}} + \frac{g_{m_3}^3}{g_{m_2}^2} + \frac{1}{g_{m_1}} \cdot \frac{g_{m_1}^2}{g_{m_3}^2} \right) \cdot \frac{1}{4K\bar{T}R_S}$$

(b)

$$\overline{V_{n,out}^2} = \frac{4K\bar{T}\delta}{g_{m_1}} \cdot \left( \frac{g_{m_1}}{g_{m_3}} \right)^2 + 4K\bar{T}\delta \cdot g_{m_3}$$

$$+ \frac{4K\bar{T}\delta}{g_{m_2}} \cdot \left( \frac{g_{m_2}}{g_{m_3}} \right)^2$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m_1}}{g_{m_3}}$$

$$NF = 1 + 4K\bar{T}\delta \left( \frac{1}{g_{m_1}} + \frac{g_{m_3} \cdot g_{m_2}^2}{g_{m_3}^2} \right)$$

$$+ \frac{1}{g_{m_2}} \cdot \frac{g_{m_2}^2}{g_{m_3}^2} \cdot \frac{1}{4K\bar{T}R_S}$$

$$(e).$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m_1}}{g_{m_2}}$$

$$\overline{V_{n,out}^2} = \frac{4K\bar{T}\delta}{g_{m_1}} \left( \frac{g_{m_1}}{g_{m_2}} \right)^2 + 4K\bar{T}R_D$$

+  $\frac{4K\bar{T}\delta}{g_{m_2}} \Rightarrow M_2's \text{ contribution.}$

$$(c) \overline{V_{n,out}^2} = \frac{4K\bar{T}\delta}{g_{m_1}} \left( \frac{g_{m_1}}{g_{m_3}} \right)^2 + \frac{4K\bar{T}\delta}{g_{m_2}} \left( \frac{g_{m_2}}{g_{m_3}} \right)^2$$

$$+ 4K\bar{T}\delta \cdot g_{m_3}$$

$$\therefore NF = 1 + 4K\bar{T}\delta \left( \frac{1}{g_{m_1}} + \frac{R_D \cdot g_{m_2}^2}{\delta \cdot g_{m_1}^2} \right)$$

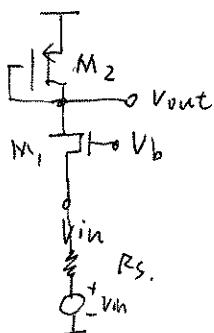
$$+ \frac{1}{g_{m_2}} \cdot \frac{g_{m_2}^2}{g_{m_1}^2} \cdot \frac{1}{4K\bar{T}R_S}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m_1}}{g_{m_3}}$$

$$NF = 1 + 4K\bar{T}\delta \left( \frac{1}{g_{m_1}} + \frac{1}{g_{m_2}} \frac{g_{m_2}^2}{g_{m_1}^2} + \frac{g_{m_3}^3}{g_{m_1}^2} \right) \cdot \frac{1}{4K\bar{T}R_S}$$

2.22 Part 1 Soln:

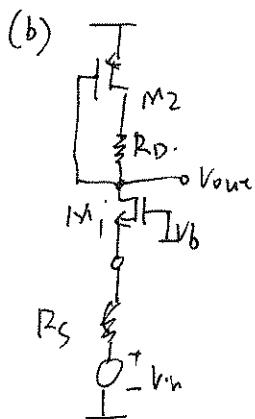
(a)



$$\overline{V_{n,out}^2} = 4KTR \cdot g_{m2} + \frac{4KTR}{g_{m1}} \left( \frac{1}{RS \cdot g_{m2}} \right)^2$$

$$|\frac{V_{out}}{V_{in}}| = \frac{1}{RS \cdot g_{m2}}$$

$$\begin{aligned} NF &= 1 + \frac{4KTR}{g_{m1}} \left( g_{m2} \cdot RS \cdot g_{m2}^2 + \frac{1}{g_{m1}} \right) \cdot \frac{1}{4KTR_S} \\ &= 1 + \frac{4KTR}{RS} \left( RS^2 g_{m2}^3 + \frac{1}{g_{m1}} \right) \end{aligned}$$



noise by  $M_2$ :

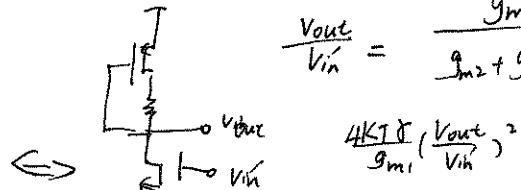
$$4KTR/g_{m2}$$

noise by  $R_D$ :

$$4KTR_D$$

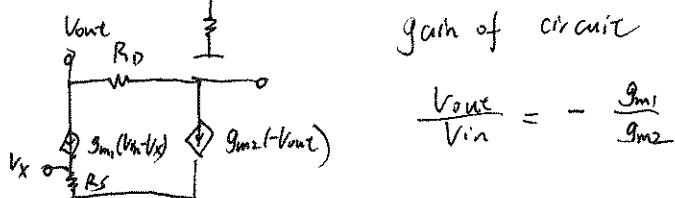
noise by  $M_1$ :

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m2} + g_{m1}g_{m2}R_S}$$



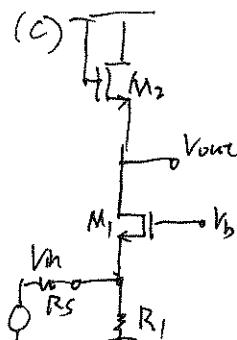
$$\frac{4KTR}{g_{m1}} \left( \frac{V_{out}}{V_{in}} \right)^2$$

gain of circuit



$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1}}{g_{m2}}$$

$$NF = 1 + 4KTR \left[ \frac{1}{g_{m2}} + R_D + \frac{\frac{4KTR}{g_{m1}}}{(g_{m2} + g_{m1}g_{m2}R_S)^2} \right] \times \frac{1}{4KTR_S} \times \frac{g_{m2}^2}{g_{m1}^2}$$



$$\text{noise of } M_2: \frac{4KTR}{g_{m2}}$$

$$\text{noise of } M_1: \frac{4KTR}{g_{m1}} \cdot \left( \frac{1}{R_S \cdot g_{m2}} \right)^2 \quad (2)$$

noise of  $R_S$ :

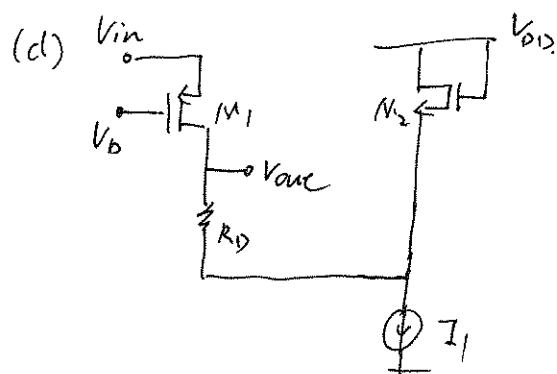
$$4KTR_S \left( \frac{g_{m1}^{-1}}{R_S + g_{m1}^{-1}} \right)^2 \left( \frac{1}{RS \cdot g_{m2}} \right)^2 \quad (3)$$

$$NF = 1 + \frac{(1) + (2) + (3)}{A_V} \frac{1}{4KTR_S}$$

Note:

$$A_V = \frac{R_1 || \frac{1}{g_{m1}}}{R_S + R_1 || \frac{1}{g_{m1}}} \cdot \frac{1}{R_1 || R_S \cdot g_{m2}}$$

2.22 Part 2. Soln:

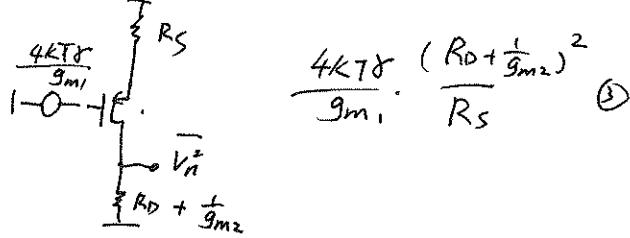


$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{R_D + \frac{1}{g_{m2}}}$$

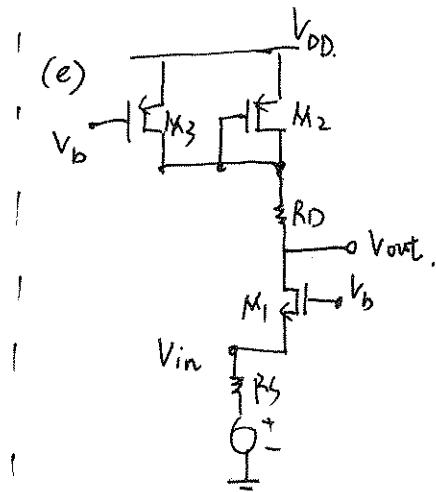
$$\text{noise of } R_D = 4KTR_D \quad (1)$$

$$\text{noise of } M_2 = \frac{4K\bar{T}\delta}{g_{m2}} \quad (2)$$

noise of  $M_1$ :



$$NF = 1 + \frac{(1) + (2) + (3)}{\left| \frac{g_{m1}}{R_D + \frac{1}{g_{m2}}} \right|^2} \times \frac{1}{4KTR_S}$$



$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R_D + \frac{1}{g_{m2}}}{R_S}$$

$$\text{noise of } R_D = 4KTR_D \quad (1)$$

Noise of  $M_1$ :

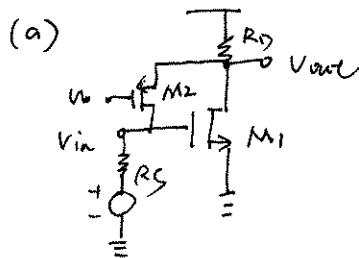
$$\frac{4K\bar{T}\delta}{g_m} \cdot \left( \frac{R_D + \frac{1}{g_{m2}}}{R_S} \right)^2 \quad (2)$$

$$\text{noise of } M_2 = \frac{4K\bar{T}\delta}{g_{m2}} \quad (3)$$

$$\text{noise of } M_3 = \frac{4K\bar{T}\delta}{g_{m3}} \left( \frac{g_3}{g_{m2}} \right)^2 \quad (4)$$

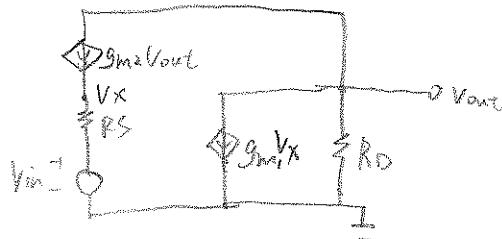
$$NF = 1 + \frac{(1) + (2) + (3) + (4)}{\left( \frac{R_D + \frac{1}{g_{m2}}}{R_S} \right)^2} \cdot \frac{1}{4KTR_S}$$

2.23 Part ①  
Solu:



$$NF = 1 + \frac{\frac{1}{(g_m + \frac{1}{R_D})^2} \cdot \frac{1}{4KTR_S}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_D}$$

Through small-signal analysis,

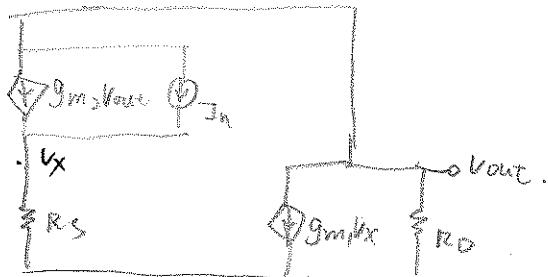


$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S}$$

noise from  $R_D = 4KTR_D$  ①

noise from  $M_1 : \frac{4KT\delta g_{m1}}{g_{m2} + g_{m1}g_{m2}R_S + \frac{1}{R_D}}$  ②

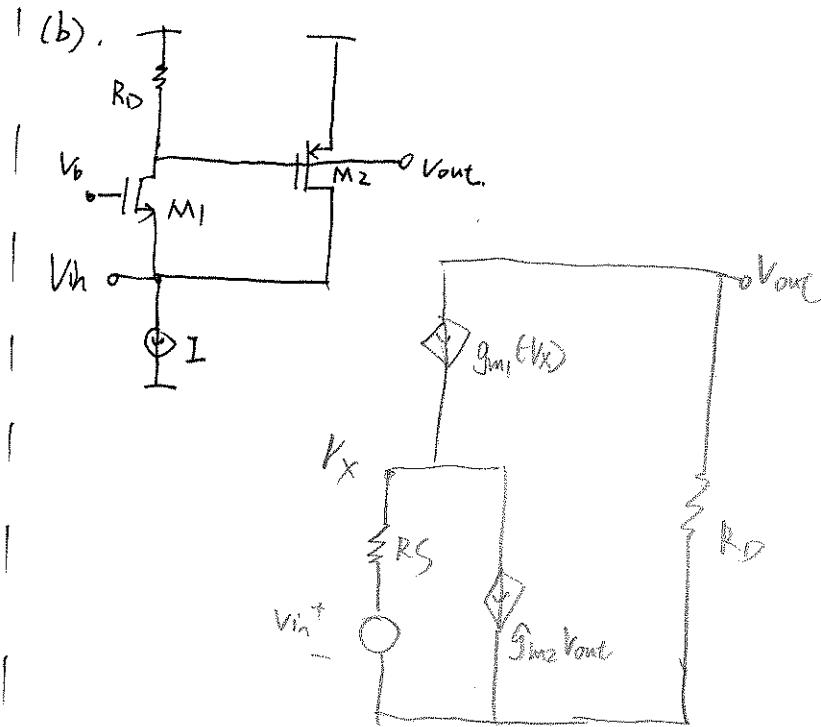
noise from  $M_2 :$



$$g_{m1}b_X + \frac{V_{out}}{R_D} = -\frac{b_X}{R_X} = -(I_n + g_{m2}V_{out})$$

$$\Rightarrow | \frac{V_{out}}{I_n} | = \frac{g_{m1}R_D + 1}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S}$$

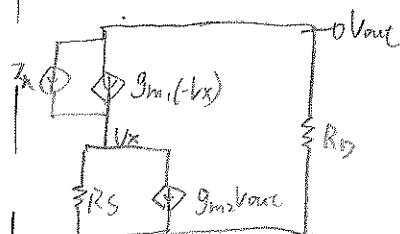
$$\Rightarrow | \frac{V_{out}}{I_n} |^2 \cdot 4KT\delta g_{m2} \quad ③$$



$$\frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + R_S}$$

noise from  $R_D : 4KTR_D$  ①

noise from  $M_1 :$



$$4KT\delta g_{m1} \cdot \left[ \frac{1}{g_{m1}R_S} \left( g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_D R_S} \right)^2 \right]$$

noise from  $M_2 :$

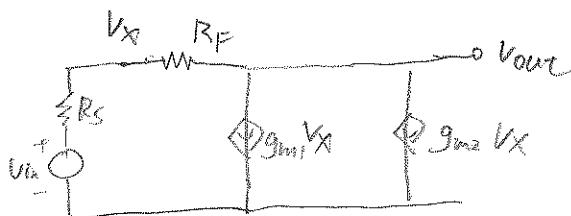
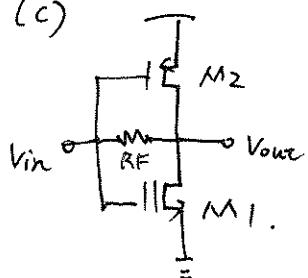
$$4KT\delta g_{m2} \left[ g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_D R_S} \right]^2 \quad ③$$

$$NF = 1 + \frac{(① + ② + ③)}{\left( \frac{R_D}{\frac{1}{g_{m1}} + R_S} \right)^2} \cdot \frac{1}{4KTR_S}$$

2.23 Part ②

Solu:

(c)



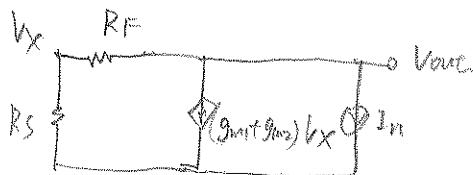
∴

$$\frac{V_{out}}{V_{in}} = \frac{(g_{m1} + g_{m2}) R_F - 1}{(g_{m1} + g_{m2}) R_S + 1}$$

noise from M<sub>1</sub>

⇒

$$4K T f \cdot g_{m1} \cdot \left( \frac{R_S + R_S}{(g_{m1} + g_{m2}) R_S + 1} \right)^2 \quad (1)$$



noise from M<sub>2</sub>

⇒

$$4K T f \cdot g_{m2} \cdot \left( \frac{R_S + R_L}{(g_{m1} + g_{m2}) R_S + 1} \right)^2. \quad (2)$$

noise from RF ⇒ 4K T f R<sub>F</sub>. (3)

$$\therefore NF = 1 + \frac{(1) + (2) + (3)}{\left( \frac{(g_{m1} + g_{m2}) R_F - 1}{(g_{m1} + g_{m2}) R_S + 1} \right)^2} \cdot \frac{1}{4K T R_S}$$

3.1 Soln:

$$X_{16QAM}(t) = \alpha_1 A_c \cos(\omega_c t + \Delta\theta) - \alpha_2 A_c (1 + \varepsilon) \sin \omega_c t.$$

(a).  $\Delta\theta \neq 0, \varepsilon = 0$

$$\begin{aligned} X_{16QAM}(t) &= \alpha_1 A_c \cos(\omega_c t + \Delta\theta) - \alpha_2 A_c \sin \omega_c t \\ &= \alpha_1 A_c [\cos \omega_c t \cos \Delta\theta - \sin \omega_c t \sin \Delta\theta] - \alpha_2 A_c \sin \omega_c t \\ &= \alpha_1 A_c \cos \Delta\theta \cos \omega_c t - (\alpha_1 A_c \sin \Delta\theta + \alpha_2 A_c) \cdot \sin \omega_c t \end{aligned}$$

normalized coefficient:  $\alpha_1 \cos \Delta\theta, -(\alpha_1 \sin \Delta\theta + \alpha_2)$

$$\beta_1 = \cos \Delta\theta, \beta_2 = -\sin \Delta\theta + 1 ; \quad \beta_1 = \cos \Delta\theta, \beta_2 = -\sin \Delta\theta + 2 ;$$

$$\beta_1 = \cos \Delta\theta, \beta_2 = -\sin \Delta\theta - 1 ; \quad \beta_1 = \cos \Delta\theta, \beta_2 = -\sin \Delta\theta - 2 ;$$

$$\beta_1 = -\cos \Delta\theta, \beta_2 = \sin \Delta\theta + 1 ; \quad \beta_1 = -\cos \Delta\theta, \beta_2 = \sin \Delta\theta + 2 ;$$

$$\beta_1 = -\cos \Delta\theta, \beta_2 = \sin \Delta\theta - 1 ; \quad \beta_1 = -\cos \Delta\theta, \beta_2 = \sin \Delta\theta - 2 ;$$

$$\beta_1 = 2\cos \Delta\theta, \beta_2 = -2\sin \Delta\theta + 1 ; \quad \beta_1 = 2\cos \Delta\theta, \beta_2 = -2\sin \Delta\theta + 2 ;$$

$$\beta_1 = 2\cos \Delta\theta, \beta_2 = -2\sin \Delta\theta - 1 ; \quad \beta_1 = 2\cos \Delta\theta, \beta_2 = -2\sin \Delta\theta - 2 ;$$

$$\beta_1 = -2\cos \Delta\theta, \beta_2 = 2\sin \Delta\theta + 1 ; \quad \beta_1 = -2\cos \Delta\theta, \beta_2 = 2\sin \Delta\theta + 2 ;$$

$$\beta_1 = -2\cos \Delta\theta, \beta_2 = 2\sin \Delta\theta - 1 ; \quad \beta_1 = -2\cos \Delta\theta, \beta_2 = 2\sin \Delta\theta - 2 ;$$

(b).  $\Delta\theta = 0, \varepsilon \neq 0$ .

$$X_{16QAM}(t) = \alpha_1 A_c \cos \omega_c t - \alpha_2 (A_c (1 + \varepsilon)) \sin \omega_c t.$$

normalized coefficient:  $(\alpha_1, -\alpha_2 (1 + \varepsilon))$

Similar to (a), there are 16 different combinations

3.2 Soln:

If  $NF < 10 \text{ dB}$ .

$$NF = \frac{\text{Noise, out}}{A_o^2 \cdot P_{RS}}$$

$$\frac{\text{Noise, out}}{A_o^2} = \text{Noise, in} = NF \cdot P_{RS}$$

$$\begin{aligned}\therefore \text{Noise, in} \cdot B &= NF/\text{dB} - 174 \text{ dB/Hz} + 10 \lg B \\ &= (10 - 174 + 53) \text{ dBm} \\ &= -111 \text{ dBm.}\end{aligned}$$

$$\text{If } NF < 10 \text{ dB} \Rightarrow \text{Noise, in} \cdot B < -111 \text{ dBm.}$$

$$\text{IIP}_3 = P_{in} + \frac{P_{in} - P_{IM, \text{max}}}{2}$$

Since the maximum tolerable noise is  $-108 \text{ dBm}$ ,  
the IM can contribute more than 3 dB.

$$\text{So } P_{IM, \text{in}} > -111 \text{ dBm}$$

$$\Rightarrow \text{IIP}_3 < -18 \text{ dBm.}$$

It demonstrates that if the RX contributes less noise,  
the receiver's linearity requirement can be loose.

3.3 solve:

For WCDMA.

Receiver sensitivity : -104 dBm.

$\beta$  : 5 MHz.

Assume NF = 3 dB

$$\text{Noise}_{\text{in},B} = -174 \text{ dBm} + 3 + 10 \log(384 \text{ kHz}) \\ = -115 \text{ dBm.}$$

For an acceptable BER, SNR of 9 dB is required.  
i.e. the total noise in the desired channel must remain below -113 dBm.

$\Rightarrow$  The intermodulation can contribute at most 2 dB  
i.e.  $P_{\text{IM}, m} = -117 \text{ dBm.}$

$$\text{IIP}_3 = \frac{-46 \text{ dBm} - (-117 \text{ dBm})}{2} + (-46 \text{ dBm}) \\ = -10.5 \text{ dBm.}$$

3.4 Soln:

IMX2000. maximum tolerable relative noise floor.

In DCS 1800 RX Band  $1805 \sim 1880$  MHz.

TX Power remain below  $-71$  dBm. in  $100$ -kHz bandwidth.  
of DCS 1800.

$$-71 \text{ dBm} - 10 \lg(10 \text{ kHz}) = -121 \text{ dBm / Hz}.$$

Tx ouptower :  $24$  dBm.

$\Rightarrow$  The Max Tolerable relative noise floor :

$$-145 \text{ dBc / Hz}.$$

3.5

Solu:

This problem is the same as Problem 3.3.

→ 3.6 Soln:

L

$$SNR = 17 \text{ dB}$$

That means the total noise should remain below  $-81 \text{ dBm}$ .

Let me assume  $NF = 10 \text{ dB}$ .

$$B = 1 \text{ MHz.}$$

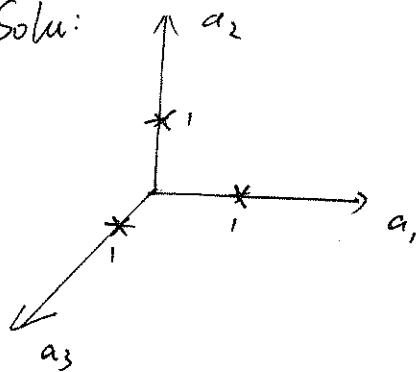
$$\begin{aligned} \text{Noise by Rx} &= -174 \text{ dBm/Hz} + 10 \lg B + NF \\ &= -104 \text{ dBm.} \end{aligned}$$

So IM can contribute maximum 23 dB.

$$\Rightarrow -81.02 \text{ dBm.}$$

$$\begin{aligned} IIP_3 &= \frac{(-39) - (-81.02)}{2} \text{ dB} + (-39 \text{ dBm}) \\ &= -18 \text{ dBm.} \end{aligned}$$

3.7 Solu:

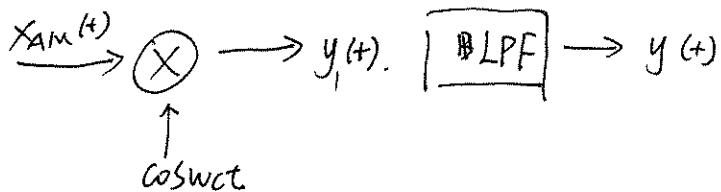


the constellation of  $X_{FSK}(t) = a_1 \cos \omega_1 t + a_2 \cos \omega_2 t + a_3 \cos \omega_3 t$ .

### 3.8 SDRU:

De-modulation of AM

$$x_{AM}(t) = A_c [1 + m x_{BB}(t)] \cdot \cos w_c t$$



$$y_1(t) = A_c [1 + m x_{BB}(t)] \cos^2 w_c t$$

$$= A_c [1 + m x_{BB}(t)] \frac{1 + \cos 2\theta}{2}$$

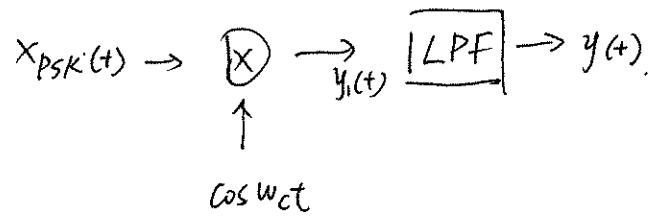
$$\begin{aligned} y(t) &= LBF[y_1(t)] \\ &= \frac{1}{2} A_c [1 + m x_{BB}(t)] \end{aligned}$$

From the equation of  $y(t)$ , we can easily find the original information  $x_{BB}(t)$ .

3.9 solve:

Demodulation of PSK

$$x_{PSK}(t) = a_n \cos w_c t$$



$$y_i(t) = a_n \cos w_c t \cdot \cos w_c t$$

$$= \frac{1}{2} a_n (1 + \cos 2w_c t)$$

∴

$$y(t) = LPF[y_i(t)]$$

$$= \frac{1}{2} a_n.$$

From the result of  $y(t)$ , we can easily find  
the original binary sequence  $a_n$ .

3.10 Soln:

$$x_{BPSK}(t) = a_n \cos \omega_c t$$

Proof:

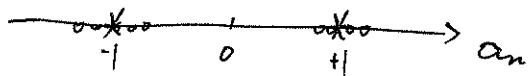
$$y_1(t) = x_{BPSK} \cdot \cos(\omega_c t + \omega_w t) = \underline{\underline{a_n \cos \omega_c t \cdot \cos(\omega_c t + \omega_w t)}}$$

$$= a_n \cos \omega_c t \cdot \cos(\omega_c t + \omega_w t)$$

$$= \frac{1}{2} a_n [\cos(2\omega_c + \omega_w)t + \cos \omega_w t]$$

$$y(t) = \overset{LPF}{[y_1(t)]}$$

$$= \frac{1}{2} a_n \cos \omega_w t$$



4.1 Soln:

(a). If input RF range is from  $f_1$  to  $f_2$ .

$$\therefore \left[ \frac{4}{5}f_1, \frac{4}{5}f_2 \right] - \text{LO freq. range.}$$

Suppose input band is partitioned into  $N$  channels

$$\frac{f_2 - f_1}{N} = \Delta f.$$

$$\text{The first channel} \Rightarrow f_{\text{LO}} = \frac{4}{5}(f_1 + \frac{\Delta f}{2})$$

$$\text{The Second channel} \Rightarrow f_{\text{LO}} = \frac{4}{5}(f_1 + \frac{3}{2}\Delta f).$$

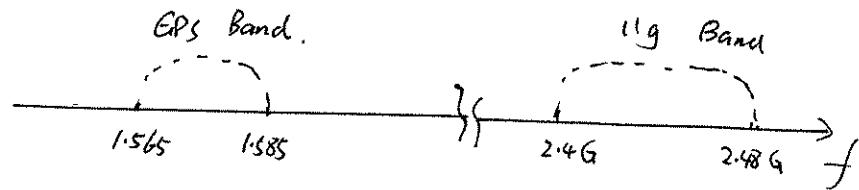
$\therefore$  LO increments in steps of  $\frac{4}{5}\Delta f$ .

(b). Image Range.

For  $f_{\text{LO}} = \frac{4}{5}f_1$ , the image lies at  $2f_{\text{LO}} - f_{\text{in}} = \frac{3}{5}f_1$

$$\Rightarrow \text{Image Freq. Range} = \left[ \frac{3}{5}f_1, \frac{3}{5}f_2 \right]$$

4.2 Soln :



$$f_{LO1} = \frac{2}{3} f_{in}$$

$$\begin{aligned}\text{Image} &= 2f_{LO1} - f_{in} \\ &= \frac{1}{3} f_{in} \\ &\in [0.8 \text{ GHz}, 0.827 \text{ GHz}]\end{aligned}$$

Image Range : 27 MHz.

GPS Band : 20 MHz.

So. it's not possible to design an L1g receiver whose image is confined to GPS Band.

4.3 Soln:

$$f_{L01} = \frac{2}{3} f_{in}$$

$$f_{L02} = \frac{1}{3} f_{in}$$

$$f_{in} - \frac{2}{3} f_{in} - \frac{1}{3} f_{in} = 0$$

$$W_{int} \pm m\omega_{L01} \pm n\omega_{L02} = 0$$

$$W_{int} = \pm m\omega_{L0} \pm n\omega_{L02}$$

i.e.  $f_{int} = \pm m f_{L01} \pm n f_{L02}$   
 $= (\pm 2m \pm n) \frac{f_{in}}{3}$ .

$\Rightarrow$  mixing spurs. ( $m, n$  are integers).

4.4. Soln:

(a) assume the second IF is zero.

$$f_{\text{in}} - \frac{1}{2}f_{\text{LO}} - f_{\text{LO}} = 0.$$

$$f_{\text{LO}} = \frac{2}{3}f_{\text{in}}.$$

If the input RF range is  $[f_1, f_2]$ , the LO freq.

$$\text{Range is } \left[ \frac{2}{3}f_1, \frac{2}{3}f_2 \right]$$

(b)  $2 \cdot \left( \frac{1}{2}f_{\text{LO}} \right) - f_{\text{in}} = f_{\text{image}}$ .

$$\Rightarrow f_{\text{image}} = -\frac{1}{3}f_{\text{in}}.$$

The Range of image freq. is  $\left[ -\frac{1}{3}f_2, -\frac{1}{3}f_1 \right]$ .

(c) No.

Because the image freq. range doesn't change.

And. the I. Q mixer's operate at much higher frequency  $\Rightarrow$  difficult to design.

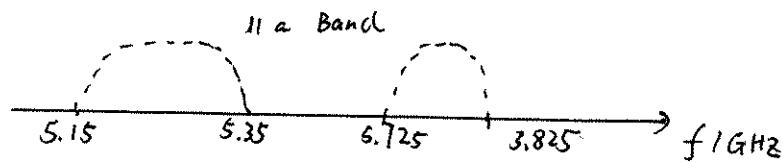
Solu<sup>4.5</sup>:

Mixing spurs:

$$\begin{aligned}f_{\text{mix}} &= \pm m f_{L01} \pm n f_{L02} \\&= (\pm \frac{m}{2} \pm n) \cdot \frac{2}{3} f_{\text{in}}\end{aligned}$$

4.6.

(a).



assume the second IF is zero

$$f_{\text{int}} - f_{\text{LO1}} - \frac{1}{8} f_{\text{LO1}} = 0$$

$$f_{\text{LO1}} = \frac{8}{9} f_{\text{int}}$$

$$\begin{aligned} f_{\text{image}} &= 2f_{\text{LO1}} - f_{\text{int}} \\ &= \frac{7}{9} f_{\text{int}} \Rightarrow \text{Image band:} \end{aligned}$$

$$[4.01 \text{ GHz}, 4.16 \text{ GHz}] \cup [4.45 \text{ GHz}, 4.53 \text{ GHz}]$$

(b). Mixed with 3rd harmonic of the first LO.

$$f_{\text{int}} - 3 \cdot f_{\text{LO1}} - m \frac{1}{8} f_{\text{LO1}} = 0$$

$$f_{\text{int}} = 3f_{\text{LO1}} + \frac{m}{8} f_{\text{LO1}} = \left(3 + \frac{m}{8}\right) f_{\text{LO1}}$$

Mixed with 3rd harmonic of the second LO

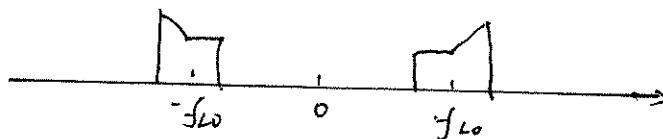
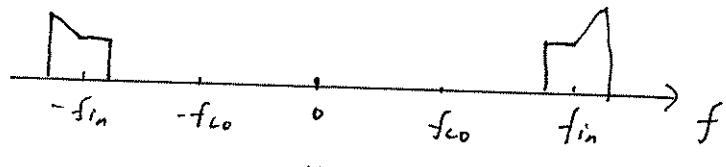
$$f_{\text{int}} - 3n \cdot f_{\text{LO1}} - \frac{3}{8} f_{\text{LO1}} = 0$$

$$f_{\text{int}} = n \cdot f_{\text{LO1}} - \frac{3}{8} f_{\text{LO1}} = \left(n - \frac{3}{8}\right) f_{\text{LO1}}$$

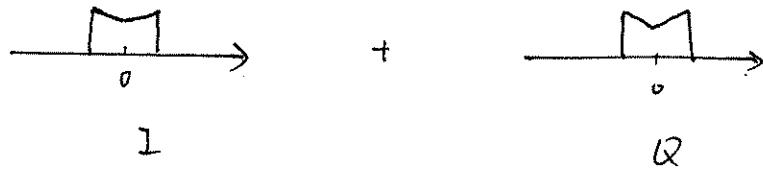
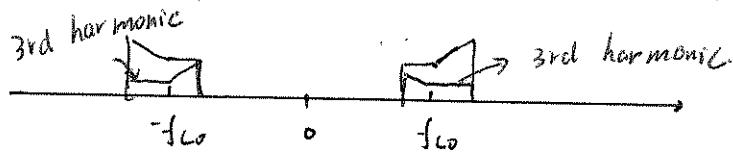
4.7. soln:

$$f_{\text{LO}} = \frac{f_{\text{in}}}{z}$$

(a)



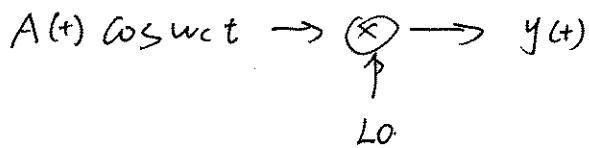
(b)



(c)

Because the second IF is zero, while the flicker noise has a huge component at low frequency.

4.8 Soln:



(a)  $A(t) \cdot \cos \omega_c t \cdot \cos \omega_c t.$

$$= A(t) \frac{(1 + \cos 2\omega_c t)}{2}$$

After LPF, baseband signal :  $\frac{A(t)}{2}$ .

(b)  $A(t) \cdot \cos \omega_c t \cdot \sin(\omega_c t).$

$$= \frac{1}{2} A(t) \sin(2\omega_c t).$$

After LPF, baseband signal is nothing.

A signal modulated by  $\cos \omega_c t$  should be demodulated by  $\cos \omega_c t$ .

Vice Versa. If a signal modulated by  $\cos(\omega_c t + \phi)$ ,  $\phi$  is phase mismatch, the quadrature downconversion is necessary.

4.9 Solu:

$$V_o \cos w_{lo} t + V_{int(t)} \cos w_{int} t$$

(a). Components near carrier.

assume LNA :  $y(t) = \alpha_1 X(t) + \alpha_2 X^2(t) + \alpha_3 X^3(t)$ .

$$\begin{aligned} & \frac{3}{4} \alpha_3 V_o^2 V_{int(t)} \cos(2w_{lo} - w_{int}) \\ & + \frac{3}{4} \alpha_3 V_{int(t)}^2 V_o \cos(2w_{int} - w_{lo}). \end{aligned}$$

(b). baseband component.

$$(\alpha_1 V_o + \frac{3}{4} \alpha_3 V_o^3 + \frac{3}{2} \alpha_3 V_o \cdot V_{int(t)}^2) \cdot \cos w_{lo} t,$$

They will corrupt the desired signal surely.

4.10 Soln:

A higher front-end gain directly arise Sth. in Fig. 4.44.

$$\frac{P_{n1}}{P_{n2}} = \frac{8.2 f_c}{100 \text{ kHz}},$$

$$\frac{\delta}{f_c} = \text{Sth.}$$

$$f_{c \text{ new}} = \frac{\delta}{\text{Sth.}} \cdot \frac{1}{A} = f_c \cdot \frac{1}{A}.$$

$$\therefore \frac{P_{n1}}{P_{n2}} = \frac{8.2}{100k} \cdot \frac{200k}{A} < 10$$

$$\Rightarrow A > 1.64.$$

So there the penalty remains below 1 dB.

4.11 Soln:

$$\frac{P_{n1}}{P_{n1}} = \frac{8.2 f_c}{100K} < 10$$

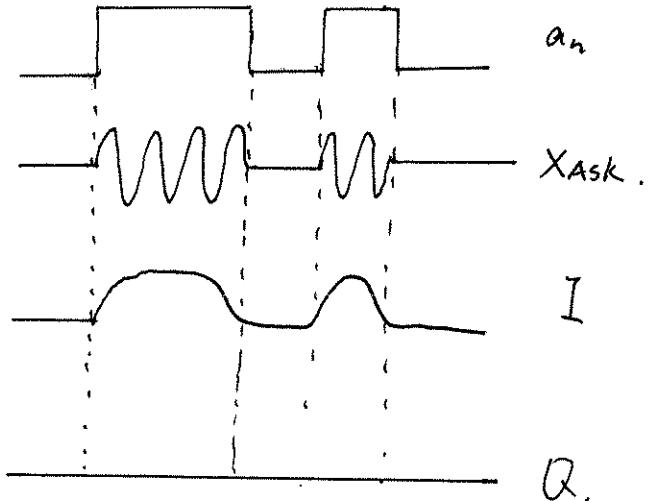
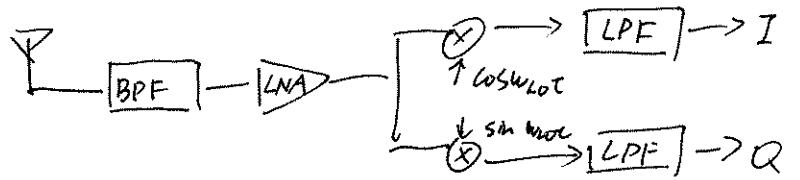
$$f_c < 122 \text{ kHz}.$$

If the penalty must remain below 1 dB,  
the flicker noise corner frequency should  
be smaller than 122 kHz.

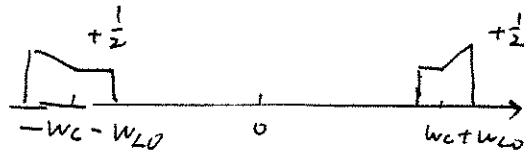
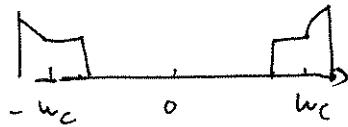
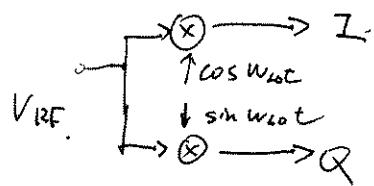
4.12 Soln:

$$X_{ASK}(t) = a_n \cos \omega_c t$$

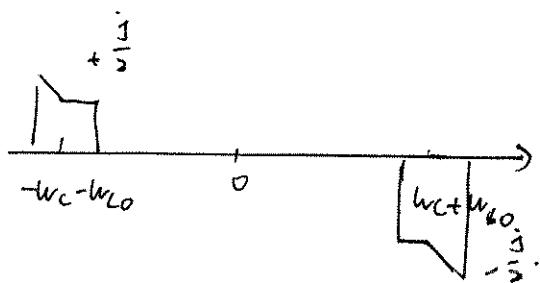
$(a_n = 1 \text{ or } 0)$



4.13 Solu:



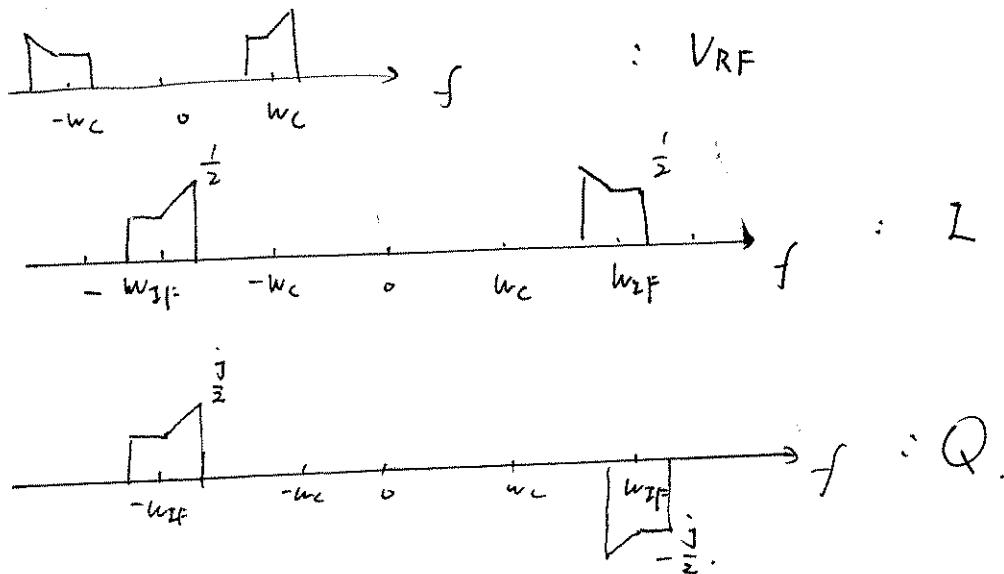
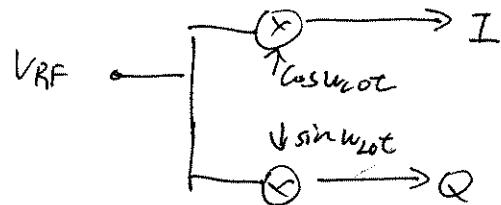
I : up converted output



Q : up converted output.

Fig 4.59(a) performs a Hilbert Transform when if the up converted are considered.

4. 14. Soln: If  $w_{RF} > w_c$ .



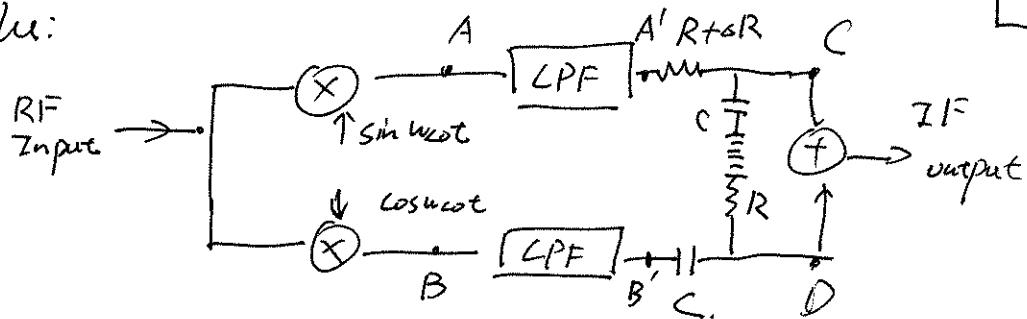
The result is the same as original analysis.

4.15 Soln:

Yes. Hartley architecture can cancel the image if the ZF Low-pass filters are replaced with high-pass filters.

As the analysis of problem 4.13, the upconverted components can be used successfully, just like the downconverted components.

4.16 Solu:



$$X_{A(t)} = -\frac{A_{sig}}{2} \sin((\omega_c - \omega_{L0})t + \phi_{sig}) - \frac{A_{lim}}{2} \sin((\omega_{lim} - \omega_{L0})t + \phi_{lim})$$

$$X_B'(t) = \frac{A_{sig}}{2} \cos((\omega_c - \omega_{L0})t + \phi_{sig}) + \frac{A_{lim}}{2} \cos((\omega_{lim} - \omega_{L0})t + \phi_{lim})$$

$$\arctan(RC \cdot \frac{1}{RC}) = 45^\circ$$

$$\arctan((R + \alpha R) \cdot C - \frac{1}{RC}) = \arctan(1 + \frac{\alpha R}{R})$$

$$\Delta\theta = \arctan(1 + \frac{\alpha R}{R}) - \arctan(1)$$

$$\begin{aligned} &= \arctan\left(\frac{\alpha R}{1 + 1 + \frac{\alpha R}{R}}\right) && \therefore \cos(\arctan(x)) \\ &= \arctan\left(\frac{\alpha R}{2 + \frac{\alpha R}{R}}\right). && = \frac{1}{\sqrt{x^2 + 1}} \end{aligned}$$

$$\begin{aligned} IRR &= \frac{2 + 2 \cos \Delta\theta}{2 - 2 \cos \Delta\theta} = \frac{1 + \cos \Delta\theta}{1 - \cos \Delta\theta} \\ &= \frac{1 + \frac{1}{\sqrt{\left(\frac{2}{2 + \frac{1}{R}} + \frac{1}{R}\right)^2 + 1}}}{1 - \frac{1}{\sqrt{\left(\frac{1}{2 + \frac{1}{R}}\right)^2 + 1}}} \\ &= \frac{2 \sqrt{\left(\frac{\alpha R + R}{2R + \alpha R}\right)^2 + 1} + 2}{\left(\frac{\alpha R \cdot R}{2R + \alpha R}\right)^2 + 1} + 1. \end{aligned}$$

4.17 Solve:

$$w_{IF} = (R, C)^T.$$

$$\Delta\theta = 2 \arctan(R_i G (w_{IF} + \omega)) - \frac{\pi}{2}.$$

$$IRR = \frac{1 + \cos\theta}{1 - \cos\theta}$$

$$= \frac{1 + \sin(2 \arctan(1 + \frac{\omega}{w_{IF}}))}{1 - \sin(2 \arctan(1 + \frac{\omega}{w_{IF}}))}$$

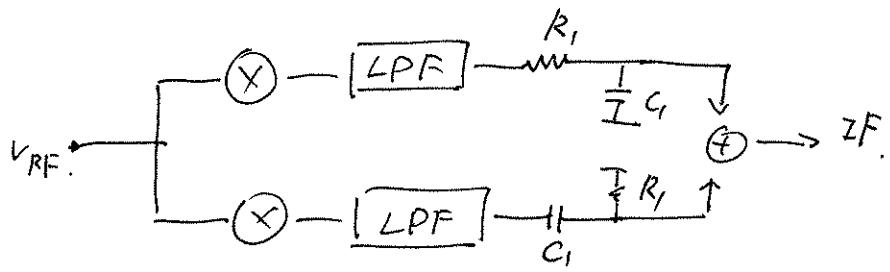
$$= \left( \frac{\frac{\omega}{w_{IF}} + 1}{\frac{\omega}{w_{IF}}} \right)^2$$

$$\therefore \omega \ll w_{IF}$$

$$\therefore IRR \approx \left( \frac{w_{IF}}{\omega} \right)^2$$

$$\begin{aligned} & \sin(\arctan(x)) \\ &= \frac{x}{\sqrt{x^2 + 1}} \end{aligned}$$

4.18. Soln:



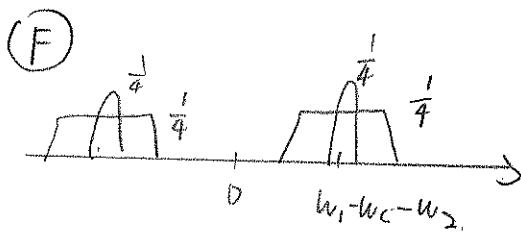
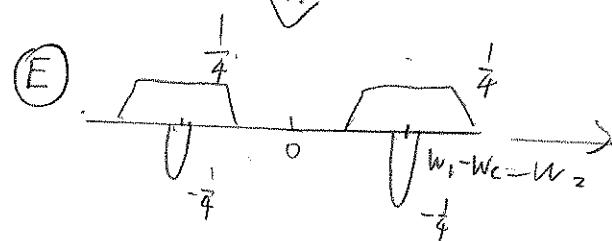
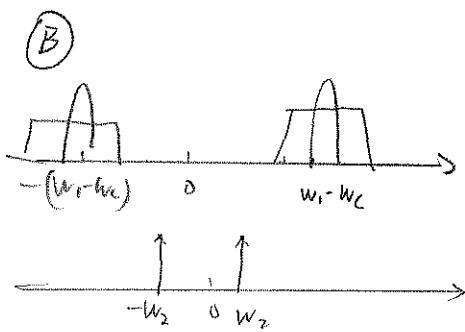
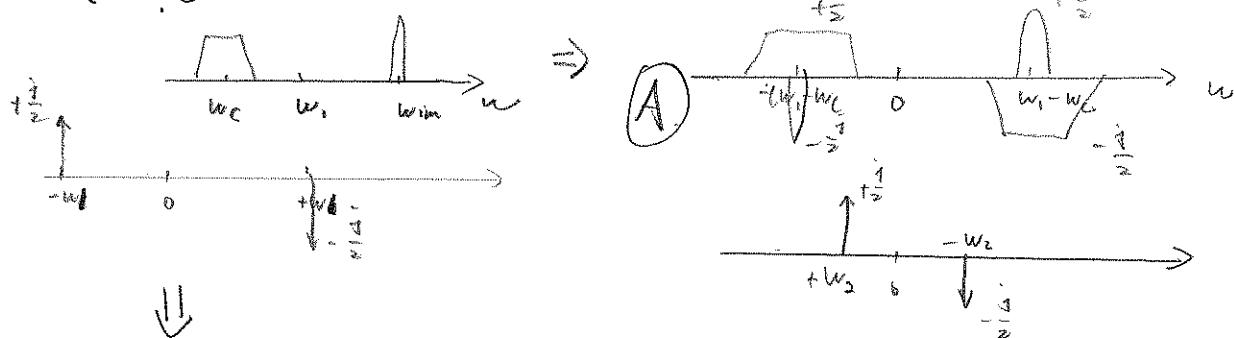
assume mixers, and LPF and adder  
are free of noise.

$$NF = 1 + \frac{2 \cdot 4kT \cdot R_1}{A_1^2} \cdot \frac{1}{4kT \cdot R_D}$$

$$= 1 + \frac{2R_1}{R_D} \cdot \frac{1}{A_1^2}$$

4.19 Soln:

(I) high-side, low-side.



(E) + (F)



For high-side, low-side configuration,

E should be added to F. In order to

cancel the image component.

(II) high-side, high-side

similar analysis to (I).

(III) low-side, high-side.

4.20 Soln:

(a). It cannot reject the image.

LO are not quadrature.

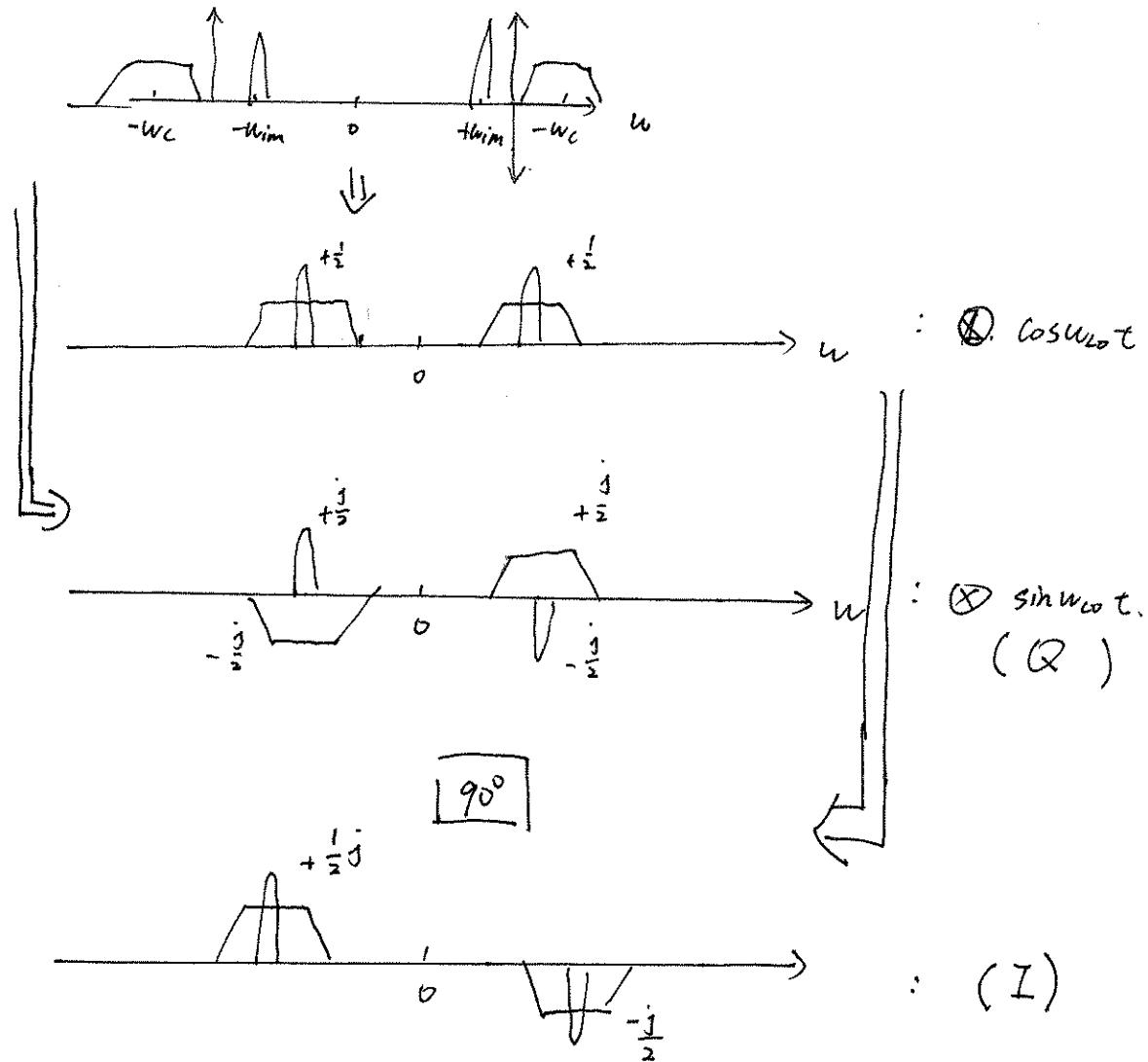
(b). It can reject the image.

The structure is similar to the original one.

(c) It cannot reject the image.

LO are not quadrature. So it is not possible to provide  $\pm j$  factor, which can cancel the image by proper operations.

4.21. Soln:

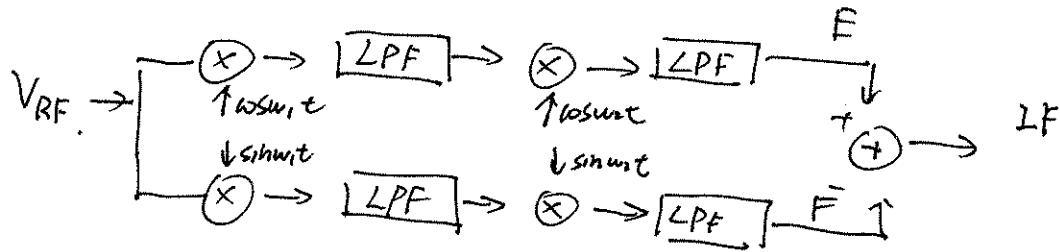


$I - Q$  operation can cancel the image.

So the answer is Yes,  $\sin \omega t$  &  $\cos \omega t$  can be swapped.  
 The only thing needs to be considered is that  $I$  should be subtracted by  $Q$ .

4.22 Solu:

In Weaver architecture.



The answer is the same as the previous

problem 4.21. Yes. It can cancel the image.

which need to be paid more attention is that

the last operation on  $E$  and  $F$ .

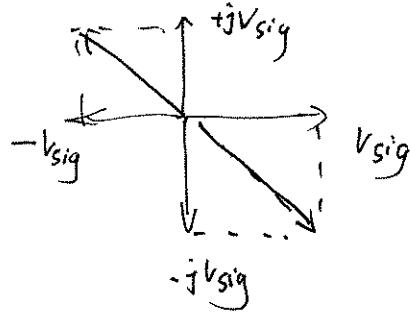
4.23. Soln:

IRR at the output is.

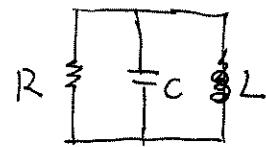
$$\frac{P_{im}}{P_{sig}} = \frac{|V_{im}|^2 \cdot \frac{(RC\omega)^2}{2}}{|V_{sig}|^2 (2\sqrt{2})^2}$$

assume  $|V_{im}| = |V_{sig}|$

$$IRR = \frac{(RC\omega)^2}{16}$$



4.24.  
Solve:



$$1^{\circ} \quad Z_{W_0} = \frac{1}{\sqrt{LC}}$$

$$2^{\circ} \quad Q = R \cdot C \cdot Z_{W_0}$$

Proof:  $Y_{in} = \frac{1}{R} + \frac{1}{jwL} + jwC$ .

where  $w = w_0$

$$Y_{in, w_0} = \frac{1}{R} + \frac{1}{jw_0 L} + jw_0 C$$

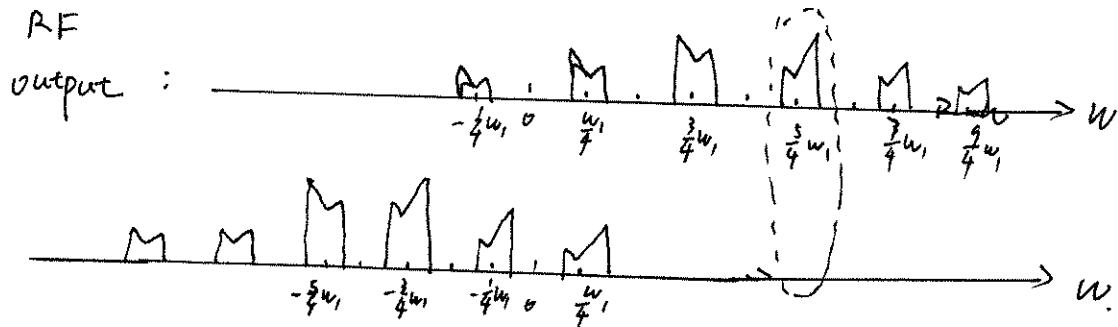
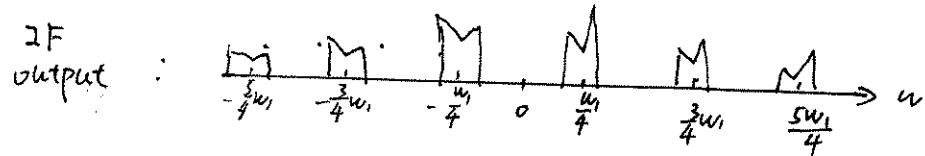
$$= \frac{1}{R} + j(w_0 C - \frac{1}{w_0 L})$$

$$Y_{in, 3w_0} = \frac{1}{R}$$

$$\begin{aligned} \frac{|Y_{in, w_0}|}{|Y_{in, 3w_0}|} &= \sqrt{\left(\frac{1}{R}\right)^2 + \left(w_0 C - \frac{1}{w_0 L}\right)^2} \cdot R \\ &= \sqrt{1 + w_0^2 C^2 \cdot R^2 + \frac{R^2}{w_0^2 L^2} - 2 \cdot \frac{C}{L} \cdot R^2} \\ &= \sqrt{1 + \frac{Q^2}{9} + 9Q^2 - 2Q^2} \\ &= \sqrt{1 + \frac{64}{9}Q^2} \quad (\text{assume } Q \gg 1) \\ &\approx \frac{8}{3}Q \end{aligned}$$

4.25 Solve:  $\frac{1}{4}w_1$  and  $w_1$ . Output  $\frac{5}{4}w_1$

1° Consider the first LO.



The unwanted signal at  $\frac{1}{4}w_1$ ,  $\frac{3}{4}w_1$ ,  $\frac{7}{4}w_1$ ,  $\frac{9}{4}w_1$  must be suppressed by RF bandpass filter.

2° Consider the second LO.

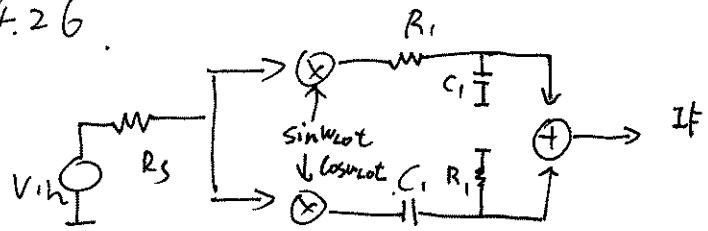
IF output should be mixed not only  $w_1$ , but  $3w_1$  and  $5w_1$ .

$\otimes 3w_1 \Rightarrow -\frac{5}{4}w_1$  will be translated to  $\frac{7}{4}w_1$   
 $-\frac{3}{4}w_1$  will be translated to  $\frac{9}{4}w_1$

$\otimes 5w_1 \Rightarrow -\frac{5}{4}w_1$  will be translated to  $\frac{15}{4}w_1$ .

So in the band of  $\pm \frac{5}{4}w_1$ , the wanted spectrum is alone.

4.26.



$$1^\circ \text{ Noise, out} = 4kT R_1 \cdot 2.$$

2° find the gain.

$$\text{only up-branch} \Rightarrow A_{mix} \times \frac{1}{2} \times \frac{1}{2} \quad \left. \right\} \times 2 \Rightarrow \frac{1}{2} A_{mix},$$

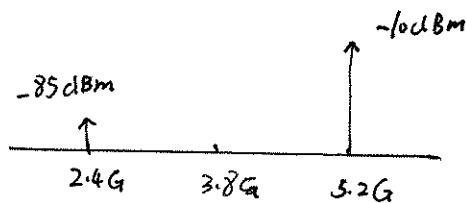
$$\text{only up-branch} \Rightarrow A_m \times \frac{1}{2} \times \frac{1}{2}$$

$$3^\circ \text{ NF} = 1 + \frac{\cancel{4kT} \cdot R_1 \cdot 2}{\left(\frac{1}{2} \cdot A_{mix}\right)^2} \cdot \frac{1}{\cancel{4kT} R_S} =$$

$$= 1 + \frac{8 \cdot R_1}{R_S} \cdot \frac{1}{A_{mix}^2}.$$

4.2) Soln:

(a).



$$\text{IRR} = 45 \text{ dB}$$

Neglect the noise and nonlinearity of Rx.

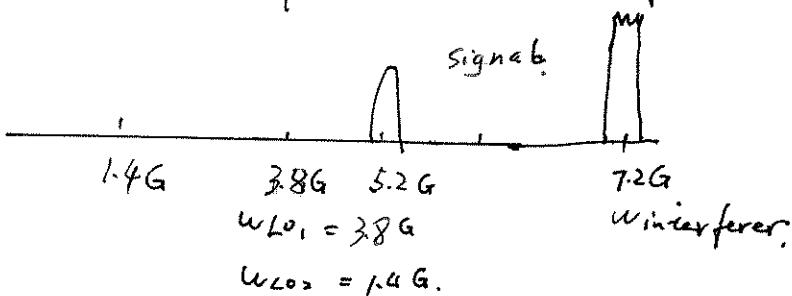
$$\text{SNR} = 20 \text{ dB} \quad V_{\text{RF}} = -85 \text{ dBm}$$

$$\Rightarrow \text{Noise, in } \leftarrow -65 \text{ dBm.}$$

$$-10 \text{ dBm} - 45 \text{ dB} = -55 \text{ dBm.}$$

So BPF need provide 10 dB rejection at 5.2 GHz.

(b).



$$3w_{L0_1} = 11.4 \text{ G.}$$

$$3w_{L0_2} = 4.2 \text{ G}$$

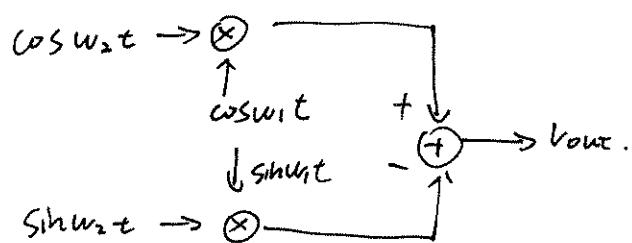
$$W_{\text{interferer}} - 3w_{L0_1} - 3w_{L0_2} = 0. \text{ baseband.}$$

Yes. The Weaver Architecture can prohibit this phenomenon.

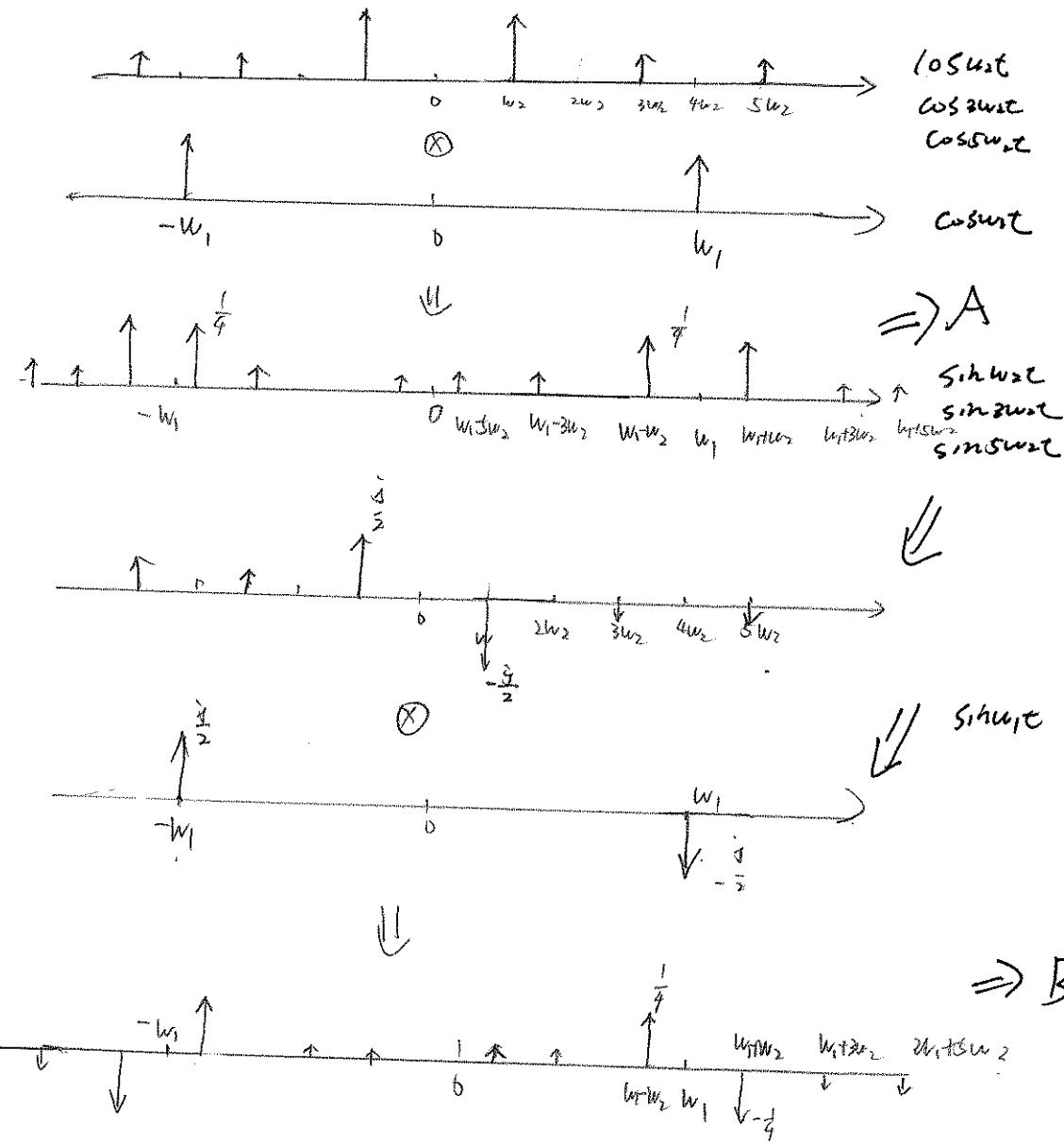
Because the 5.2 GHz band circuit is design for high-side injection. And the 7.2 G Interferer can be mixed to baseband. This only happens for low-side injection.

Part ①.

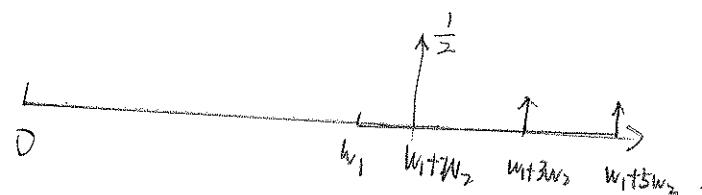
4.28. Soln:



$$(a) \omega_1 > 3\omega_2$$

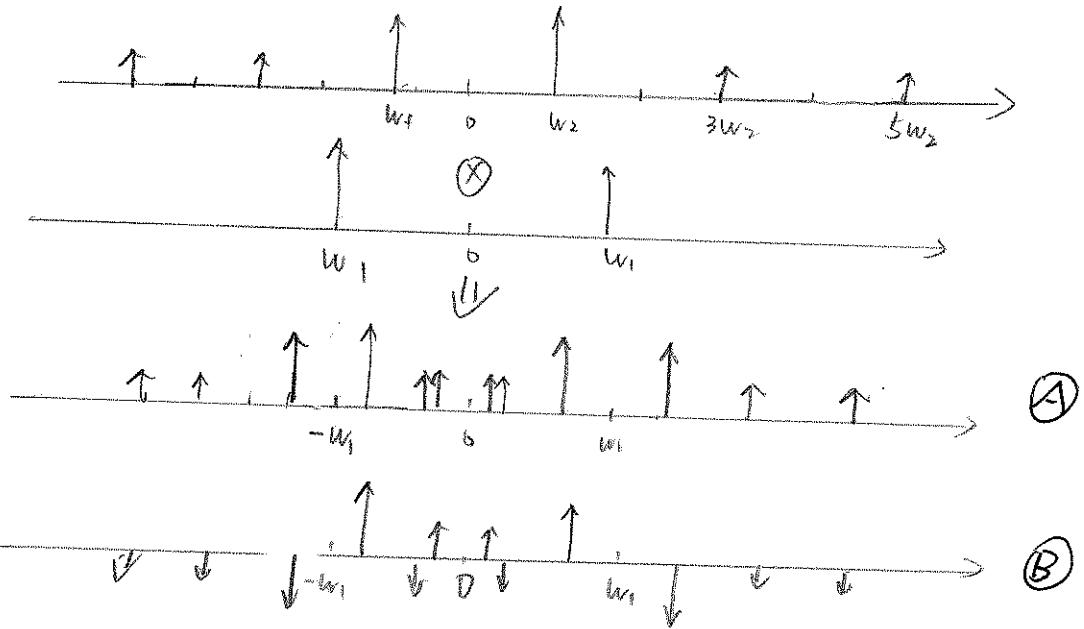


$$\textcircled{A} - \textcircled{B}$$



4.28 part (2)

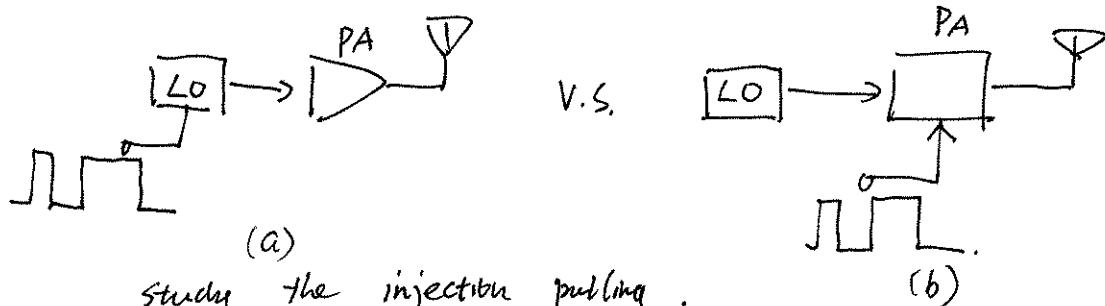
(b)  $w_1 < 3w_2$ .



A - B.



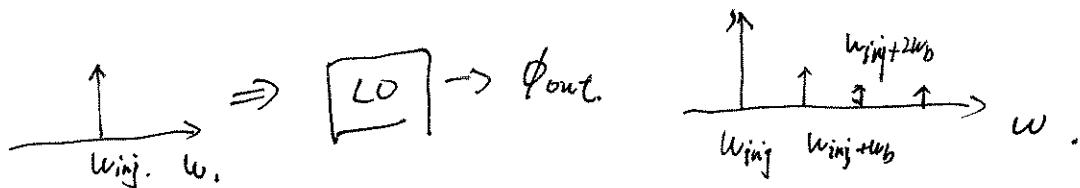
4.29 Solu:



study the injection pulling .

In The architecture (b), PA is controlled to turn on & off.

The output spectrum of LO changes a lot in this situation.



5.1 Sol:

$$P = \left| \frac{x+jy - R_s}{x+jy + R_s} \right|^2$$

$$P = \frac{(x-R_s)^2 + y^2}{(x+R_s)^2 + y^2}$$

$$P(x^2 + R_s^2 + 2R_s x) + P y^2 = (x + R_s^2 - 2R_s x) + y^2$$

$$(P-1)x^2 + 2R_s(P+1)x + (P-1)R_s^2 + (P-1)y^2 = 0.$$

$$x^2 + 2R_s \frac{P+1}{P-1} x + R_s^2 + y^2 = 0.$$

$$x^2 + 2R_s \frac{P+1}{P-1} x + \left(R_s \cdot \frac{P+1}{P-1}\right)^2 + y^2 = -R_s^2 + R_s^2 \left(\frac{P+1}{P-1}\right)^2$$

$$\left(x - \frac{1+P}{1-P} R_s\right)^2 + y^2 = 4P \frac{R_s^2}{(P-1)^2}$$

$\Rightarrow$  This is a circle with  $(\frac{1+P}{1-P} R_s, 0)$  as center,

with  $\sqrt{4P \frac{R_s^2}{(P-1)^2}}$  as radius.

5.2 Solu:

Eq. (5.18)

$$NF = 1 + \frac{R_s}{R_p} + \frac{g_m R_s}{g_m^2 (R_s/R_p)^2} + \left| \frac{\frac{R_s}{g_m^2 (R_s/R_p)^2 R_D}}{\text{neglect}} \right|$$

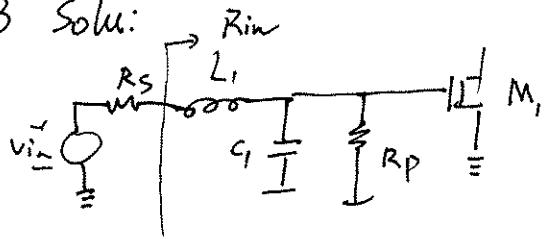
If  $R_s = R_p$   $g_m R_s \approx 1$ .

$$NF = 1 + 1 + \frac{8}{g_m \cdot 4} = 2 + \frac{4\gamma}{g_m} = 3.5 \text{ dB}$$

$$\Rightarrow 2 + \frac{4\gamma}{g_m} = 10^{0.35}$$

$$\Rightarrow g_m = 16.76 \text{ S}$$

5.3 Solu:



$$R_{in} = j\omega L_1 + \left( R_P \parallel \frac{1}{j\omega C_1} \right)$$

$$= j\omega L_1 + \frac{R_P}{j\omega R_P C_1 + 1}.$$

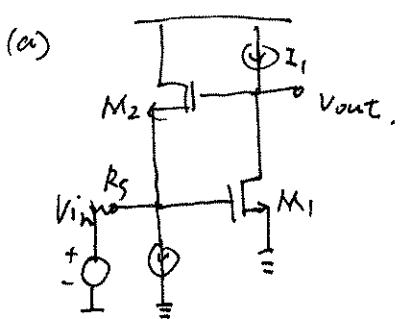
$$= \underbrace{\frac{R_P}{1 + (R_P C_1 \omega)^2}}_{= R_S} + j \underbrace{\left( \omega L_1 - \frac{R_P C_1 \omega}{1 + (R_P C_1 \omega)^2} \right)}_{= 0}.$$

$R_P$  cannot choose arbitrary because  $\omega$  &  $C_1$  depend on system requirements and technology.

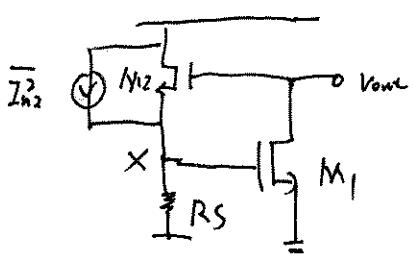
So this topology is not different from Fig. 5.9 (a).

As a result, it's impossible to achieve a noise figure less than 3 dB.

→ 5.4. Soln - Part ①.



1° noise of  $M_2$



For  $M_1$ , there is no ac pass

$$\Rightarrow V_x = 0.$$

$$g_{m2} \cdot V_{out} = I_{n2}$$

$$V_{out} = \frac{I_{n2}}{g_{m2}}$$

$$\overline{V_{n, M_2}^2} = \frac{4K\Gamma\delta}{g_{m2}}$$

noise of  $M_1$

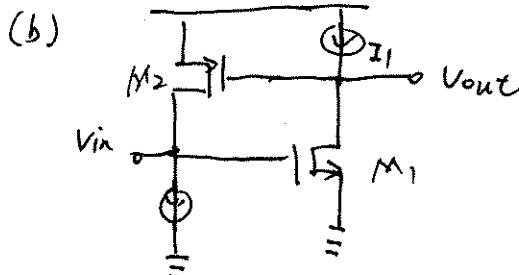
$$\overline{V_{n, M_1}^2} = 4K\Gamma\delta \cdot g_{m1} \cdot \left( \frac{g_{m1} \cdot g_{m2}}{\frac{1}{R_S} + g_{m2}} \right)^2$$

$$2^{\circ} \frac{V_{out}}{V_{in}}$$

$$= - \frac{1}{R_S \cdot g_{m2}}$$

$$3^{\circ} NF = 1 + \frac{\frac{1}{g_{m2}} + \frac{1}{g_{m1}} \left( \frac{g_{m1} \cdot g_{m2}}{\frac{1}{R_S} + g_{m2}} \right)^2}{\frac{1}{R_S^2 g_{m2}^2}} \cdot \frac{1}{R_S}$$

$$= 1 + \frac{1 \cdot g_{m2} R_S + \left( g_{m1} R_S g_{m2} \right)^2 \left( \frac{g_{m1} \cdot g_{m2}}{\frac{1}{R_S} + g_{m2}} \right)^2}{R_S^2 g_{m2}^2}$$



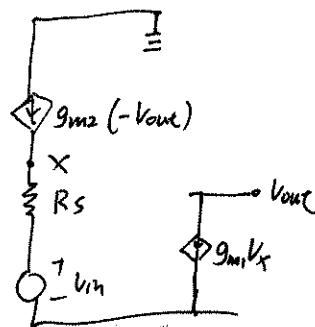
1° noise of  $M_2$

$$\overline{V_{n, M_2}^2} = \frac{4K\Gamma\delta}{g_{m2}}$$

noise of  $M_1$

$$\overline{V_{n, M_1}^2} = \frac{4K\Gamma\delta \cdot g_{m1}}{g_{m1}^2 R_S^2 g_{m2}^2}$$

$$2^{\circ} \frac{V_{out}}{V_{in}} = ?$$



$$g_{m2}(-V_{out}) = \frac{V_x - V_{in}}{R_S}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{g_{m2} R_S}$$

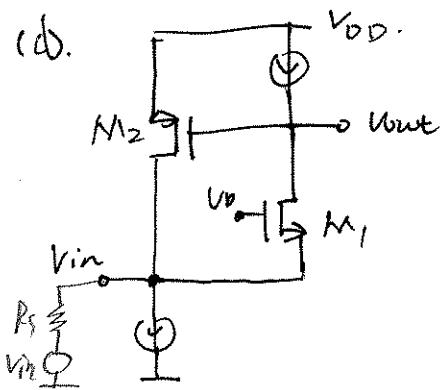
3° NF

$$= 1 + \frac{\frac{1}{g_{m2}} + \frac{\frac{1}{g_{m1}} R_S^2 g_{m2}^2}{\frac{1}{R_S^2} + g_{m2}^2} \cdot \frac{1}{R_S}}{\frac{1}{g_{m2}^2 R_S^2}} \cdot \frac{1}{R_S}$$

$$= 1 + \frac{1 \cdot g_{m2} R_S + \frac{1}{g_{m1} R_S}}{R_S^2 g_{m2}^2}$$

5.4 Solve: Part ②

(d)



1° noise of  $M_2$

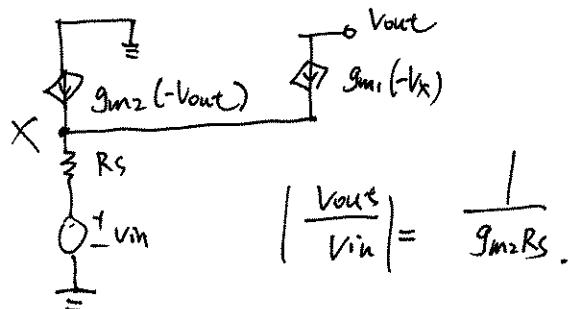
$$g_{m2}(-V_{out}) = I_{n2}$$

$$\overline{V_{n, M_2}^2} = \frac{4KTR}{g_{m2}}$$

noise of  $M_1$

$$\overline{V_{n, M_1}^2} = \frac{4KTR}{g_{m1} g_{m2} R_s^2}$$

2°  $\frac{V_{out}}{V_{in}} = ?$

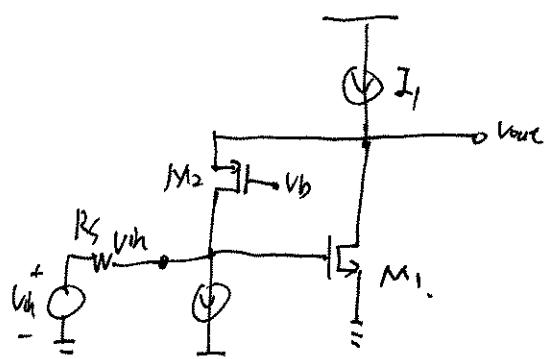


3° HF

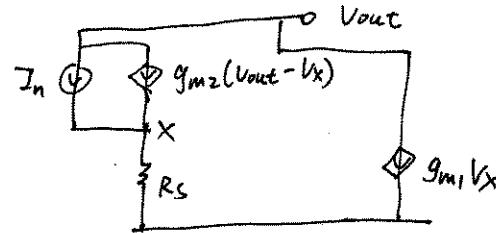
$$= 1 + \frac{\frac{1}{g_{m2}} + \frac{1}{g_{m1} g_{m2} R_s^2}}{\frac{1}{g_{m2}^2 R_s^2}} \cdot \frac{1}{R_s}$$

$$= 1 + \frac{g_{m2} R_s}{g_{m1} R_s} + \frac{1}{g_{m1} R_s}$$

(c)

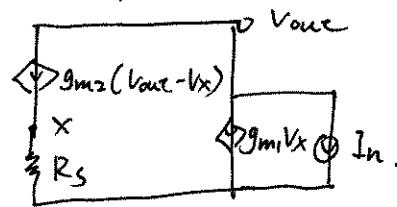


1° noise of  $M_2$



$$\overline{V_{n, M_2}^2} = \frac{4KTR}{g_{m2}} \quad (1)$$

noise of  $M_1$



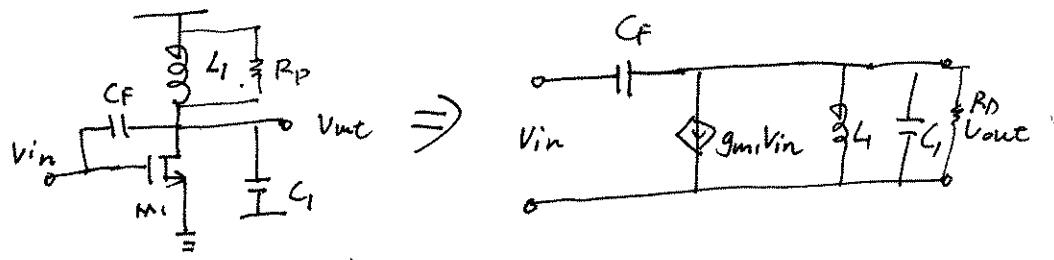
$$\left\{ \begin{array}{l} g_{m1} V_x + I_n + g_{m2} (V_{out} - V_x) = 0 \\ g_{m2} (V_{out} - V_x) = \frac{V_x}{R_s} \end{array} \right.$$

$$\Rightarrow \overline{V_{n, M_1}^2} = 4KTR \cdot g_{m1} \left[ \frac{\frac{1}{R_s}}{g_{m2}(g_{m1} + \frac{1}{R_s})} \right]^2 \quad (2)$$

$$2° \left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{g_{m2} R_s - (g_{m2} R_s + 1) \cdot \frac{g_{m2}}{g_{m2} + g_{m1}}}$$

$$3° NF = 1 + \frac{(1 + \frac{1}{\overline{|V_{out}|^2}}) \cdot \frac{1}{4KTRs}}{1 + \frac{1}{\overline{|V_{out}|^2}} \cdot \frac{1}{g_{m2} R_s}}$$

5.5 solu:



$$\text{at } \omega_0 = \frac{1}{\sqrt{L_1(C_I + C_F)}} \quad |jC_I\omega_0| \ll g_m.$$

$$\frac{V_{in} - V_{out}}{C_F s} = g_{m1}V_{in} + \frac{V_{out}}{Ls} + \frac{V_{out}}{\frac{1}{C_I s}} + \frac{V_{out}}{R_P}$$

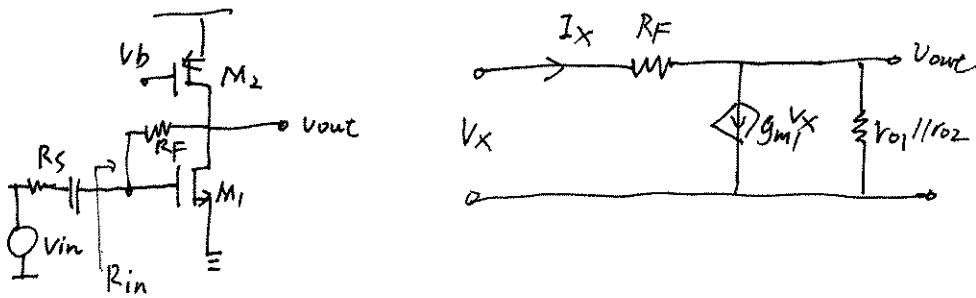
$$\frac{V_{out}}{V_{in}} = \frac{C_F s - g_{m1}}{(C_I + C_F)s + \frac{1}{Ls} + R_P}$$

( $R_P$  is the model of loss).

at  $\omega_0$

$$\frac{V_{out}}{V_{in}}(\omega_0) = \frac{j\omega_0 C_F - g_{m1}}{R_P} \approx -\frac{g_{m1}}{R_P}$$

5.6 Soln:



1° R<sub>in</sub> = ?

$$\frac{V_x - I_x \cdot R_F}{R_{O1} // R_{O2}} + g_{m1} V_x = I_x$$

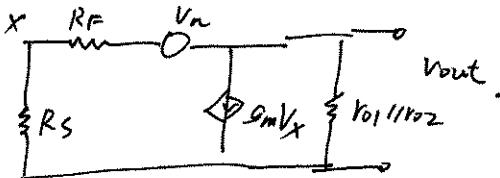
$$\Rightarrow \frac{V_x}{I_x} = \frac{R_F + R_{O1} // R_{O2}}{1 + g_{m1} R_{O1} // R_{O2}} = R_S$$

2°

$$\begin{aligned} \frac{V_{out}}{V_x} &= 1 - \frac{1 + g_{m1} (R_{O1} // R_{O2})}{R_F + R_{O1} // R_{O2}} \cdot R_F \\ &= 1 - \frac{R_F}{R_S}. \end{aligned}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \left( 1 - \frac{R_F}{R_S} \right).$$

3° noise of R<sub>F</sub>.



$$\overline{V_n^2}_{RF} = 4kT R_F \left( \frac{1 - g_m R_F}{1 - g_m R_S - \frac{R_F R_S}{R_o}} \right)^2 \approx 4kT R_F.$$

4° noise of M<sub>1</sub>, M<sub>2</sub>

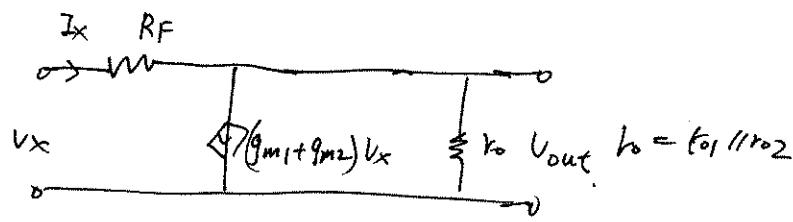
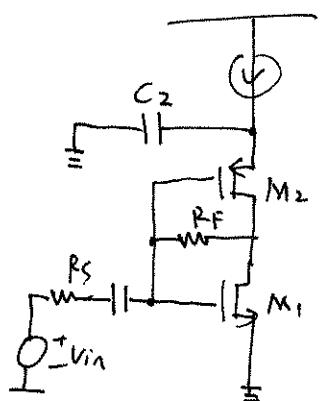
$$R_{out} = \frac{R_F + R_S}{\frac{R_F + R_S}{R_o} + R_F g_{m1}} \quad (R_o = R_{O1} // R_{O2})$$

$$\overline{V_n^2}_{M_1 \& M_2} = 4kT \gamma (g_{m1} + g_{m2}) R_{out}^2.$$

5° NF

$$= 1 + \frac{\frac{4kTR_F}{R_o} + 4kT R_{out}^2 (g_{m1} + g_{m2})}{\left(\frac{1}{2} \left(1 - \frac{R_F}{R_S}\right)\right)^2} \cdot \frac{1}{4kTR_S}.$$

S.7 Solu:



1°  $R_{in} = ?$

$$I_x = (g_m1 + g_m2)V_x + \frac{V_x - I_x \cdot R_F}{r_o}$$

$$\Rightarrow \frac{V_x}{I_x} = R_{in} = \frac{R_F + r_o}{(g_m1 + g_m2)r_o + 1} = R_S.$$

$$2^{\circ} \quad \frac{V_{out}}{V_x} = 1 - \frac{R_F}{R_S}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \left( 1 - \frac{R_F}{R_S} \right)$$

3° noise of RF.

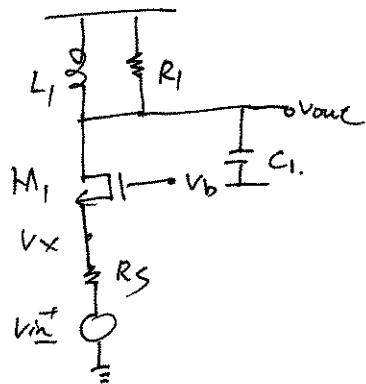
$$\overline{V_{n,RF}^2} = 4KTR_F \left( \frac{1 - (g_m1 + g_m2)R_F}{1 - (g_m1 + g_m2)R_F - \frac{R_F + R_S}{r_o}} \right)^2$$

4° noise of M<sub>1</sub>, M<sub>2</sub>

$$\overline{V_{n,M_1 \& M_2}^2} = 4KT \gamma (g_m1 + g_m2) \cdot \frac{R_F + R_S}{\frac{R_F + R_S}{r_o} + R_F(g_m1 + g_m2) + 1}$$

$$5^{\circ} \quad NF = 1 + \frac{\overline{V_{n,RF}^2} + \overline{V_{n,M_1 \& M_2}^2}}{\left( \frac{1}{2} \left( 1 - \frac{R_F}{R_S} \right) \right)^2} \cdot \frac{1}{4KTR_S}$$

5.8 Soln:



$$1^{\circ} \frac{V_{out}}{V_x} = g_m \cdot R_1$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{\frac{1}{g_m}}{R_S + \frac{1}{g_m}} \cdot g_m \cdot R_1 \\ &= \frac{R_1}{R_S + \frac{1}{g_m}} \end{aligned}$$

2<sup>o</sup> noise of M<sub>1</sub>

$$\overline{v_{n,M_1}^2} = \frac{4kT\delta}{g_m} \left( \frac{R_1}{R_S + \frac{1}{g_m}} \right)^2.$$

3<sup>o</sup> noise of R<sub>1</sub>

$$\overline{f_{n,R_1}^2} = 4kTR_1.$$

$$4^{\circ} NF = 1 + \frac{k}{g_m R_S} + \frac{R_1}{R_S} \cdot \left( \frac{R_S + \frac{1}{g_m}}{R_1} \right)^2$$

$$= 1 + \frac{k}{g_m R_S} + \frac{1}{R_1 R_S} (R_S + \frac{1}{g_m})^2.$$

$$= 1 + \frac{k}{g_m R_S} + \frac{R_S}{R_1} \left( 1 + \frac{1}{g_m R_S} \right)^2$$

$$NF < 1 + \frac{k}{g_m R_S} + 4 \frac{R_S}{R_1}$$

$$\Rightarrow g_m \cdot R_S > 1$$

$$\text{i.e. } g_m > \frac{1}{R_S}.$$

5.9 Soln:

$$(1) \quad NF = 1 + \frac{\overline{V_n^2}_{out}}{A_o^2} \cdot \frac{1}{4kT_{Rs}}$$

where  $\overline{V_n^2}_{out}$  is not due to  $R_s$ .

$$\therefore NF = 1 + \frac{\overline{V_n^2}_{in}}{4kT_{Rs}} = 3 \text{ dB.}$$

$$\therefore \overline{V_n^2}_{in} = 4kT_{Rs}$$

$\Rightarrow 50\%$

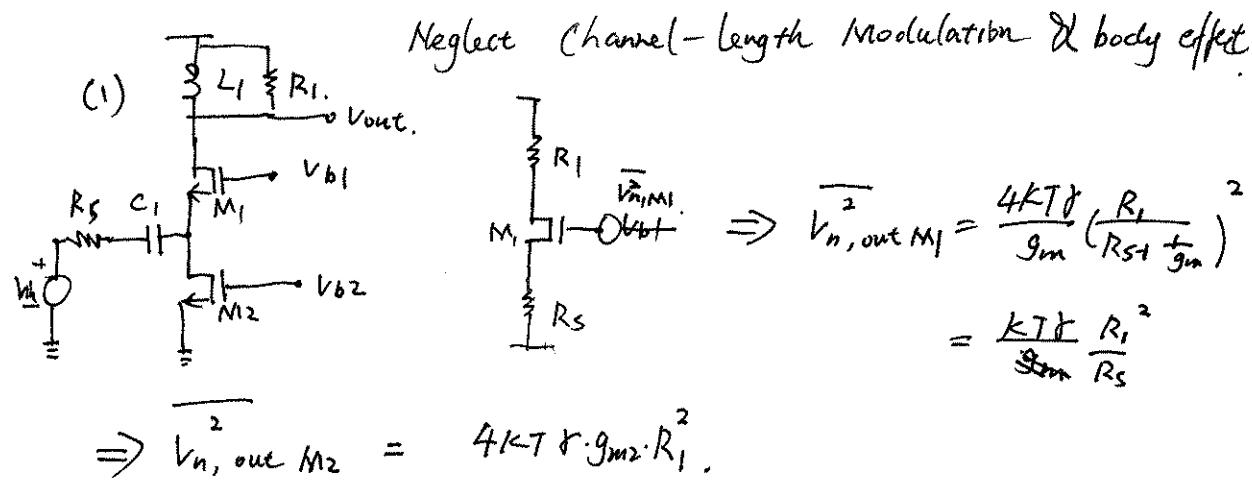
$$(2) \quad NF = 1 \text{ dB} = 10^{0.1} = 1.26.$$

$$NF = 1 + \frac{\overline{V_n^2}_{in}}{4kT_{Rs}} = 1.26$$

$$\overline{V_n^2}_{in} = 0.26 \cdot 4kT_{Rs}$$

$$\Rightarrow 1 - \frac{0.26}{1.26} = 20.6\% = 79.4\%$$

.. S.10 Soln:



$$\begin{aligned} M_1 &\text{---} \overline{V_{n,M_1}^2} \\ &\text{---} R_1 \\ &\text{---} R_S \end{aligned} \Rightarrow \overline{V_{n,\text{out},M_1}^2} = \frac{4kT}{g_m} \left( \frac{R_1}{R_S + g_m} \right)^2$$

$$= \frac{kT}{g_m} \frac{R_1^2}{R_S}$$

$$\Rightarrow \overline{V_{n,\text{out},R_1}^2} = 4kT R_1.$$

If input is matched.  $\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_1}{2R_S}$ .

$$NF = 1 + \frac{1}{g_m R_S} + 4 \frac{R_S}{R_1} + 4 R_S \cdot g_{m2}.$$

(2).

If.  $M_2$  is replaced by  $R_B$

$$\overline{V_{n,\text{out},R_B}^2} = \frac{4kT}{R_B} \cdot R_1^2.$$

$$NF = 1 + \frac{1}{g_m R_S} + 4 \frac{R_S}{R_1} + 4 \frac{R_S}{R_B}.$$

5.11 Soln:

$$(R_1 C_x)^{-1} \ll \omega \ll \frac{g_{m2}}{C_{GS2} + C_x}$$

$$\frac{1}{R_1 C_x} \ll \frac{g_{m2}}{C_{GS2} + C_x}$$

$$\Rightarrow \frac{1}{R_1 C_x} \ll \frac{g_{m2}}{2 C_x}$$

$$\Rightarrow g_{m2} R_1 \gg 2$$

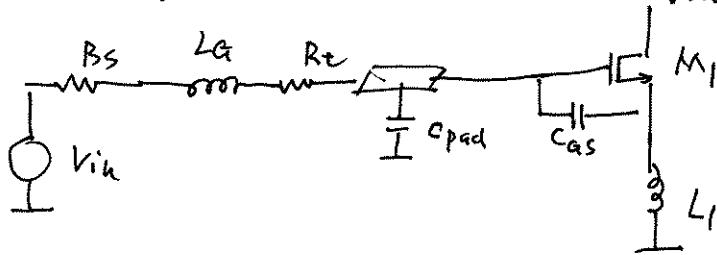
$$\Rightarrow g_{m1} R_1 \gg 2$$

gain of LNA is often  $15 \sim 20 \text{ dB}$

i.e.  $g_{m1} R_1 = 32 \sim 100 \gg 2$ .

$\therefore$  Such a frequency range does exist.

Solu: 5.12. neglect. CLM, body-effect,  $C_{GD}$  &  $C_{pad}$ .



From. (5.89) Eq.

$$\Rightarrow V_{in} = I_{out} \left( jL_1 w_o + \frac{j(R_s + R_t)}{g_m} \right) - I_{in} \frac{j(R_s C_{AS} w_o)}{g_m}$$

1° Transconductance gain:

$$\left| \frac{I_{out}}{V_{in}} \right| = \frac{1}{w_o \left( L_1 + \frac{(R_s + R_t) C_{GS}}{g_m} \right)} = \frac{w_T}{(2R_s + R_t) w_o}$$

$$\left| I_{n,out} \right|_{m_p} = |I_{in}| \frac{(R_s + R_t) C_{GS}}{g_m L_1 + (R_s + R_t) C_{GS}} \approx \frac{|I_{in}|}{2 + \frac{R_t}{R_s}}$$

$$2^{\circ} \left| \overline{I_{n,out}^2} \right|_{M_1} = 4KTR_t g_m \cdot \frac{1}{\left( 2 + \frac{R_t}{R_s} \right)^2}$$

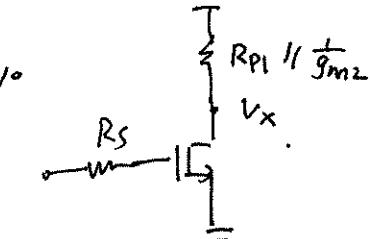
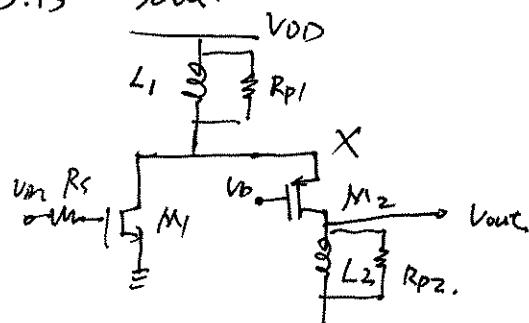
3° noise of  $R_t$ .

$$\left| \overline{I_{n,out}^2} \right|_{R_t} = 4KTR_t \left[ \frac{w_T}{(2R_s + R_t) w_o} \right]^2$$

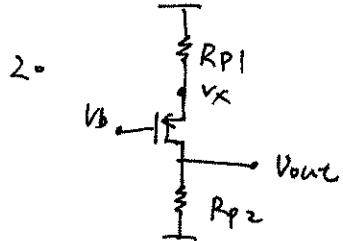
$$4^{\circ} NF = 1 + \frac{\left| \overline{I_{n,out}^2} \right|_{M_1} + \left| \overline{I_{n,out}^2} \right|_{R_t}}{\left| \overline{\frac{I_{out}}{V_{in}}} \right|^2} \cdot \frac{1}{4KTR_s}$$

$$= 1 + \frac{R_t}{R_s} + \frac{f \cdot g_m R_s \left( \frac{w_o}{w_T} \right)^2}{}$$

5.13 Soln:



$$\frac{v_x}{v_{in}} = g_m1 \cdot (R_P1 // \frac{1}{g_m2})$$



$$\frac{v_{out}}{v_x} = \frac{g_m2}{1 + g_m2 \cdot R_P1} \cdot R_P2.$$

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_m1 \cdot g_m2 \cdot R_P1 \cdot R_P2}{(1 + g_m2 \cdot R_P1)^2}$$

3°.  $\overline{V_{n,M1}^2} = \frac{4KTR}{g_m1} \cdot \left| \frac{v_{out}}{v_{in}} \right|^2$

4° noise of M<sub>2</sub>

$$\overline{V_{n,M2}^2} = 4KTR g_m2 \cdot R_P2^2$$

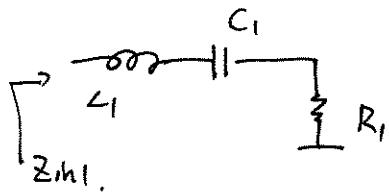
5° noise of R<sub>P1</sub>.

$$\overline{V_{n,RP1}^2} = 4KTR P1 \cdot \left| \frac{v_{out}}{v_x} \right|^2$$

6° NF =  $1 + \frac{\frac{4KTR}{g_m1} \left| \frac{v_{out}}{v_{in}} \right|^2 + 4KTR g_m2 R_P2^2 + 4KTR P1 \cdot \left| \frac{v_{out}}{v_x} \right|^2}{\left| \frac{v_{out}}{v_{in}} \right|^2} \cdot \frac{1}{4KTR_S}$

$$= 1 + \frac{1}{g_m1 R_S} + \frac{R_P1}{R_S} \left( \frac{1 + g_m2 R_P1}{g_m1 R_P1} \right)^2 + \frac{g_m2 R_P2^2}{R_S} \cdot \left| \frac{v_{out}}{v_x} \right|^2$$

5.14 Solu:



$$\operatorname{Re}\{Z_{h1}\} = R_1$$

$$\operatorname{Im}\{Z_{h1}\} = \frac{L_1 C_1 w^2 - 1}{C_1 w}$$

$$\approx 2L_1 \Delta w \frac{L_1 \Delta w}{w_0}$$

$$S_{11} = \frac{j 2L_1 \Delta w \frac{L_1 \Delta w}{w_0}}{2R_1 + j 2L_1 \Delta w \frac{L_1 \Delta w}{w_0}}$$

$$\frac{2L_1 \Delta w \frac{L_1 \Delta w}{w_0}}{\sqrt{4R_1^2 + (2L_1 \Delta w \frac{L_1 \Delta w}{w_0})^2}} \leq 0.1$$

$\Rightarrow$  solve  $\Delta w_{\max}$

$$\operatorname{Re}\{Z_{h2}\} \approx R_1$$

$$\operatorname{Im}\{Z_{h2}\} = (L_2 - R_1^2 C_2) w$$

$$\approx (L_2 - R_1^2 C_2) (w_0 + \Delta w)$$

$$S_{11} = \frac{j (L_2 - R_1^2 C_2) (w_0 + \Delta w)}{2R_1 + j (L_2 - R_1^2 C_2) (w_0 + \Delta w)}$$

$$\frac{(L_2 - R_1^2 C_2) (w_0 + \Delta w)}{\sqrt{4R_1^2 + (L_2 - R_1^2 C_2) (w_0 + \Delta w)}} \leq 0.1$$

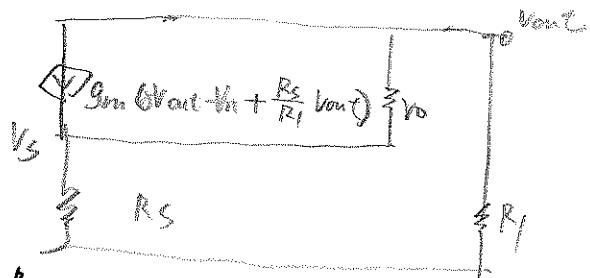
$\Rightarrow$  solve  $\Delta w_{\max}$

5. 15 Solu:

$$1^{\circ} \quad Z_{in} = \frac{\frac{g_m}{R_s} // R_o}{(1 + g_m(R_i // R_o)\delta)} = R_s \\ = \left(\frac{1}{g_m}\right) \cdot (1 + g_m(R_i // R_o)\delta) \\ = \frac{1}{g_m} // R_o + \frac{R_o g_m^2 \delta}{1 + R_o g_m} (R_i // R_o)$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \cdot \frac{g_m(R_i // R_o)}{1 + \delta g_m(R_i // R_o)}$$

2° noise of  $M_1$ .



$$\frac{V_{out}}{R_1} + \frac{V_{out} + \frac{R_s}{R_1} V_{out}}{R_o} + g_m (\delta V_{out} - V_{n1} + \frac{R_s}{R_1} V_{out}) = 0$$

$$\Rightarrow \frac{V_{out}}{V_{n1}/I_{M1}} = \frac{g_m}{\frac{1}{R_1} + g_m(\delta + \frac{R_s}{R_1}) + \frac{1}{R_o} + \frac{R_s}{R_1 R_o}}$$

$$\overline{V_{n, M1}^2} = \frac{4kT\delta}{g_m} \cdot \left| \frac{V_{out}}{V_{n1}/I_{M1}} \right|^2.$$

3° noise of  $R_1$ .

$$R_{out} = R_i // R_o // (1 + g_m R_s / (\delta g_m))$$

$$\overline{V_{n, R1}^2} = \frac{4kT \cdot R^2}{R_1} R_{out}.$$

$$4^{\circ} \quad NF = 1 + \frac{\overline{V_{n, M1}^2} + \overline{V_{n, R1}^2}}{\left| \frac{V_{out}}{V_{in}} \right|^2} \cdot \frac{1}{4kTR_s}.$$

5.16 Soln:

From 4.15 Problem, we can find the gain from the gate of M to  $v_{out}$  is  $\left| \frac{v_{out}}{v_{in}} |_M \right|^2$ . So

is  $\overline{V_n^2 F}$ .

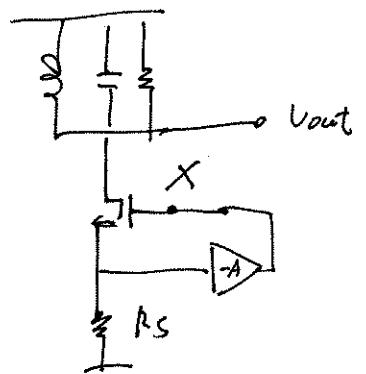
$$\overline{V_n^2 F} |_{out} = \overline{V_n^2 F} \cdot \left| \frac{v_{out}}{v_{in}} |_M \right|^2$$

$$\therefore NF = 1 + \frac{\overline{V_n^2 F} + \overline{V_{n,R1}^2} + \overline{V_{n,M1}^2}}{\left| \frac{v_{out}}{v_{in}} \right|^2} \cdot \frac{1}{4KTR_S}$$

$$= NF_{\text{Problem 4.15}} + \frac{\overline{V_{n,F}^2}}{\left| \frac{v_{out}}{v_{in}} \right|^2} \cdot \frac{1}{4KTR_S}$$

5.17. Soln:

Proof:



$$\frac{V_{out}}{V_x} = -\frac{g_m R_i}{(1+A) g_m R_s + 1} \quad (\text{Eq. 5.122}).$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} (1+A) g_m R_i \quad (\text{matched}).$$

The fourth term in Eq. (5.124)

$$= \frac{\overline{V_n^2} \cdot A^2 \cdot \left(\frac{V_{out}}{V_x}\right)^2}{\left|\frac{V_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4KTR_s}$$

$$= \frac{\overline{V_n^2} \cdot A^2 \cdot \frac{g_m^2 R_i^2}{((1+A) g_m R_s + 1)^2}}{\frac{1}{4} ((1+A) g_m R_s)^2} \cdot \frac{1}{4KTR_s}.$$

$R_s = R_{in}$  (matched)

$$\approx \frac{\overline{V_n^2} \cdot A^2 \cdot \frac{g_m^2 R_i^2}{4}}{\frac{1}{4} ((1+A) g_m R_i)^2} \cdot \frac{1}{4KTR_s}$$

$$= \frac{\overline{V_n^2} A \cdot \frac{A^2}{(1+A)^2}}{4KTR_s}$$

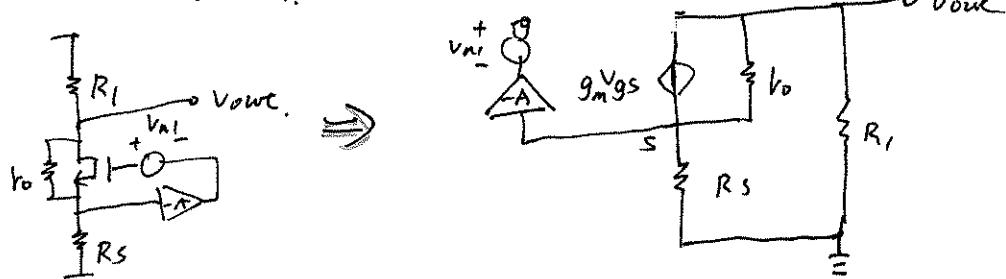
5.18 Soln:

$$1^{\circ} \quad R_{in\text{-original}} = \frac{R_1 + r_o}{1 + g_m r_o}$$

$$R_{in} = \frac{R_1 + r_o}{1 + g_m(1+A)r_o} = R_s \quad (\text{matched})$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{1}{1 + g_m r_o} \cdot \frac{(1+A)g_m r_o + 1}{r_o + R_s + (1+A)g_m r_o + R_s} \cdot R_1. \quad (\text{matched})$$

$$2^{\circ} \text{ noise of } M_1. \quad = \frac{R_1}{2R_s}$$



$$\left\{ \frac{V_{out}}{R_1} + \frac{V_{out} - V_S}{r_o} + g_m (-AV_S + V_{n1} - V_S) = 0 \right.$$

$$\left. \frac{V_{out}}{R_1} + \frac{V_{out}}{R_s} = 0 \right.$$

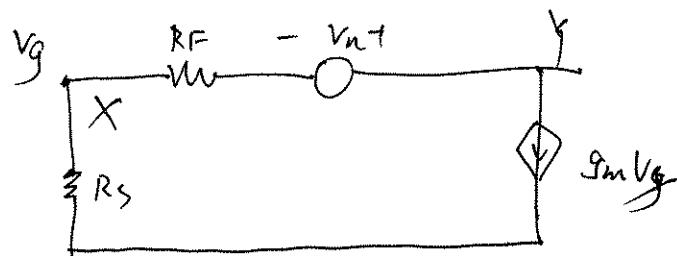
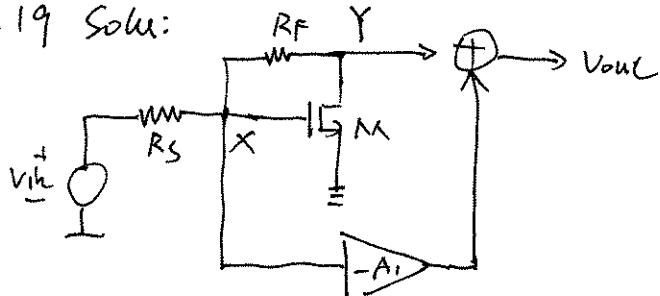
$$\Rightarrow \frac{V_{out\text{,out, } M_1}}{V_{n1}} = \frac{-g_m D_{n1}}{\left( \frac{1}{R_1} + \frac{1}{r_o} + \frac{R_s}{R_o} + g_m(A+1)\frac{R_s}{R_1} \right)} \\ (\text{matched}) = \frac{-\frac{R_1 r_o g_m}{2(r_o + R_1)}}{}$$

3<sup>o</sup>. noise of  $R_1$

$$R_{out} = \frac{2R_1 r_o + R_1^2}{2(r_o + R_1)} \quad \overline{V_{n,R_1}^2} = \frac{4kT}{R_1} \cdot R_{out}.$$

$$4^{\circ} \quad NF = 1 + \frac{\frac{4kT}{R_1} R_{out}^2 + \frac{4kT}{g_m} \left( \frac{R_1 r_o g_m}{2(r_o + R_1)} \right)^2}{\left( \frac{R_1}{2R_s} \right)^2} \cdot \frac{1}{4kTR_s}.$$

5.19 Soln:



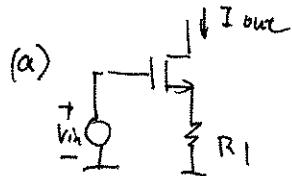
$$\frac{V_Y - V_n}{R_F + R_S} = - g_m V_g = \frac{V_Y - V_X - V_n}{R_F}$$

$$\Rightarrow \begin{cases} V_Y = V_n \\ V_X = 0 \end{cases}$$

The noise voltage of  $R_F$  produces  $V_n$  at node  $Y$ , but produces zero at node  $X$ .

$\Rightarrow$  So the architecture cannot cancel the noise of  $R_F$ .

5.20 Soln:



$$I_{out} = K (V_{in} - I_{out} \cdot R_1 - V_{th})^2$$

$$\frac{\partial I_{out}}{\partial V_{in}} = K \cdot 2 \cdot (V_{in} - I_{out} \cdot R_1 - V_{th}) \left( 1 - R_1 \frac{\partial I_{out}}{\partial V_{in}} \right)$$

$$\Rightarrow \delta_1 = \left. \frac{\partial I_{out}}{\partial V_{in}} \right|_{\frac{V_o}{I_o}} = \frac{2K (V_o - I_o R_1 - V_{th})}{1 + R_1 K (V_o - I_o R_1 - V_{th})} = \frac{g_m}{1 + g_m R_1}$$

(b).

$$\frac{\partial^2 I_{out}}{\partial V_{in}^2} = 2K \left( 1 - R_1 \frac{\partial I_{out}}{\partial V_{in}} \right)^2 + 2K (V_{in} - R_1 I_{out} - V_{th}) \times \left( -R_s \frac{\partial^2 I_D}{\partial V_{in}^2} \right)$$

$$\Rightarrow \left. \frac{\partial^2 I_{out}}{\partial V_{in}^2} \right|_{\frac{V_o}{I_o}} = 2 \delta_2 = \frac{2K}{(1 + g_m R_1)^3} = \frac{g_m^2}{2 I_o (1 + g_m R_1)^3}$$

(c).

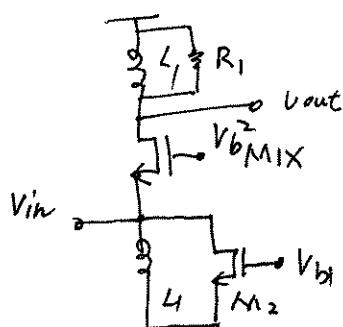
$$\begin{aligned} y &= a_1 x + \delta_2 x^2 \\ &= a_1 \cos \omega t + \delta_2 \frac{1 + 2 \cos 2\omega t}{2} \\ &= a_1 \cos \omega t + \frac{1}{2} \delta_2 \cos 2\omega t + \frac{\delta_2}{2}. \end{aligned}$$

$$|\alpha_1 A_{IP_2}| = \left| \frac{1}{2} \delta_2 \cdot A_{IP_2}^2 \right|$$

$$\begin{aligned} \therefore A_{IP_2} &= 2 \cdot \frac{\delta_1}{\delta_2} = 2 \cdot \frac{\frac{g_m}{1 + g_m R_1}}{\frac{g_m^2}{2 I_o (1 + g_m R_1)^3}} \\ &= \frac{2 \cdot 2 I_o (1 + g_m R_1)^2}{g_m} \end{aligned}$$

$$= \frac{8 I_o (1 + g_m R_1)^2}{g_m}$$

5.21. Soln:



$$\therefore g_m = \text{min} \max \frac{w}{L} (V_{GS} - V_{th}) \approx w.$$

$$g_{mix} = \frac{g_m}{\sqrt{2}}$$

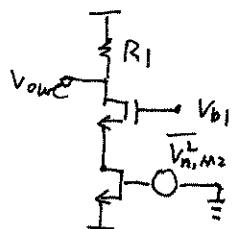
$$R_{in,2} = \frac{\sqrt{2}}{\sqrt{2}-1} R_s \quad (\text{input matched}).$$

$$1^{\circ} \quad \frac{V_{out}}{V_{in}} = \frac{g_m R_1}{\sqrt{2}}$$

2<sup>o</sup> noise of mix.

$$\overline{V_{n,out,mix}^2} = \frac{4kT g_m}{\sqrt{2}} R_1^2$$

3<sup>o</sup> noise of M2



$$\overline{V_{n,out,M2}^2} = \frac{4kT \cdot (g_{m2} \cdot \frac{\sqrt{2}}{g_m})^2}{g_{m2}}$$

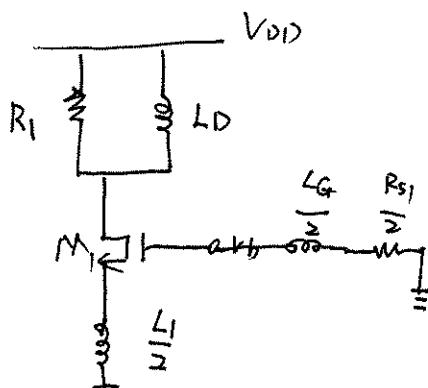
4<sup>o</sup> noise of R1

$$\overline{V_{n,out,R1}^2} = 4kT \cdot R_1$$

$$5^{\circ} \quad NF = 1 + \frac{\sqrt{2}}{g_m R_s} + \frac{4(\frac{g_{m2}^2}{g_m})^2}{(g_m R_1)^2 R_s} + \frac{2 R_1}{(g_m R_1)^2 R_s}$$

5.22 Soln:

(a).



If the input is matched.

$$\frac{L_1}{2} \cdot \frac{g_{m1}}{C_{GS1}} = \frac{R_{S1}}{2}$$

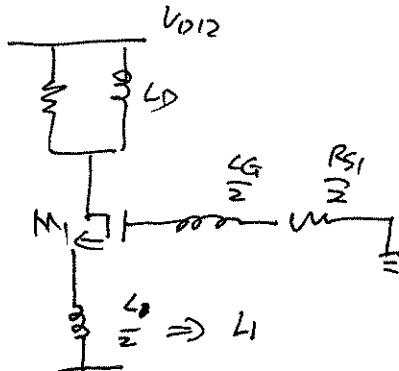
$$g_{m1} = \frac{R_{S1} \cdot C_{GS1}}{L_1} = \frac{2I_D}{V_{GS}-V_{TH}}$$

$$= \sqrt{2I_D \ln(6x) \frac{W}{L}} / K$$

$$\text{Power-diff} = \frac{g_{m1}^2}{2K} \cdot V_{DD} \cdot 2 = \frac{V_{DD} \cdot g_{m1}^2}{K}$$

$$\text{Power-sing} = \frac{g_{m1}^2}{2K} \cdot V_{DD} = \frac{1}{2} \text{Power-diff.}$$

(b)



If we need NF is the same as signal-ended one.

$$A_{v2} = \frac{W_1}{W_2} \cdot \frac{R_1}{2 \cdot R_{S1}} \text{ unchanged}$$

If only consider the noise of

MOS.

$$V_{n,M1}^2 = 2 \times K T \delta g_{m1} \cdot R_1$$

So if  $g_{m1}$  changes to  $g_{m1}/2$ ,

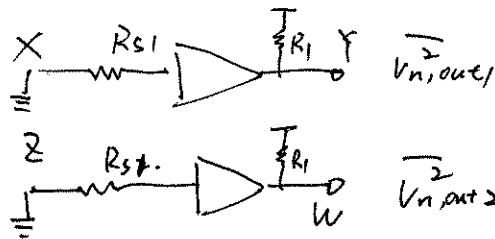
the M1's contribution in noise figure is unchanged.

$$\text{Power-diff} = \frac{(g_{m1})^2}{2K} \cdot V_{DD} \cdot 2 = \frac{1}{4} \frac{V_{DD} \cdot g_{m1}^2}{K}$$

$$\text{Power-sing} = \frac{g_{m1}^2}{2K} \cdot V_{DD} = 2 \cdot \text{Power-diff.}$$

5.23 Soln:

If 1 to 2 balun is used,



gain from X to Y

$$\overline{V_{n,out1}^2} = 4kTR_1 \quad \text{is} \quad \frac{R_1}{2R_{S1}}$$

$$+ 4kTR_{S1} \left( \frac{R_1}{2R_{S1}} \right)^2$$

$$+ kT \gamma \cdot \frac{R_1^2}{R_{S1}} \quad A_V = \frac{R_1}{2R_{S1}} \cdot 2$$

$$\therefore NF = \frac{\overline{2V_{n,out}^2}}{2^2 \left( \frac{R_1}{2R_{S1}} \right)^2} \cdot \frac{1}{4kTR_{S1}}$$

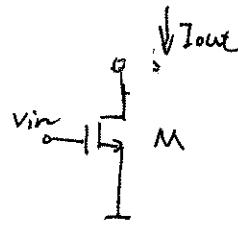
$$= \frac{2}{4} \left( \frac{R_1}{R_{S1}} \frac{4R_{S1}}{R_1^2} + 1 + \frac{kT}{4} \right)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{2R_S}{R_1}$$

From the result compared with 1 to 1 balun, NF is much smaller.

5.24 Soln:

$$(a) \quad I_{\text{out}} = K (V_{GS} - V_{th})^2 \\ = K (V_{in} - V_{th})^2.$$



$$\frac{\partial I_{\text{out}}}{\partial V_{in}} = 2K(V_{in} - V_{th}) = \delta_1$$

$$\frac{\partial^2 I_{\text{out}}}{\partial V_{in}^2} = 2K = 2\delta_2 \Rightarrow \delta_3 = 0$$

$$\frac{\partial^3 I_{\text{out}}}{\partial V_{in}^3} = 0 = 6\delta_3 \Rightarrow \begin{cases} P_{1dB} = \infty \\ 2P_3 = \infty \end{cases}$$

(b).

$$I_{\text{out}} = \frac{1}{2} M_0 C_{ox} \frac{W}{L} \frac{(V_{in} - V_{th})^2}{1 + \left(\frac{M_0}{2V_{sat}L} + \theta\right)(V_{in} - V_{th})}.$$

From (5.186) (5.187) Eq.

$$\delta_1 = K [2 - 3a(V_{GSO} - V_{th})] (V_{GSO} - V_{th})$$

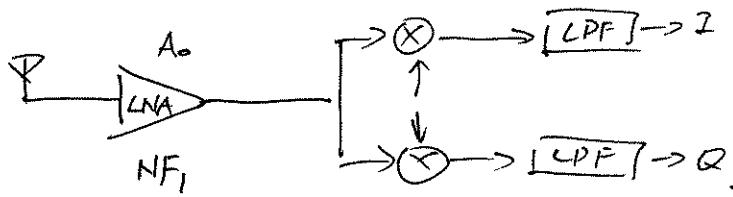
$$\delta_3 = -ka$$

$$\text{where } k = \frac{1}{2} M_0 C_{ox} \frac{W}{L}, \quad a = M_0 / (2V_{sat}L) + \theta.$$

$$A_{IIP_3} = \sqrt{\frac{4}{3} \times \frac{2-3a(V_{GSO}-V_{th})}{a} (V_{GSO} - V_{th})}.$$

$$A_{1dB} = \sqrt{0.145 \cdot \frac{2-3a(V_{GS}-V_{th})}{a} (V_{GS} - V_{th})}.$$

6.1 Soln:



assuming the conversion gain of mixers is unity.

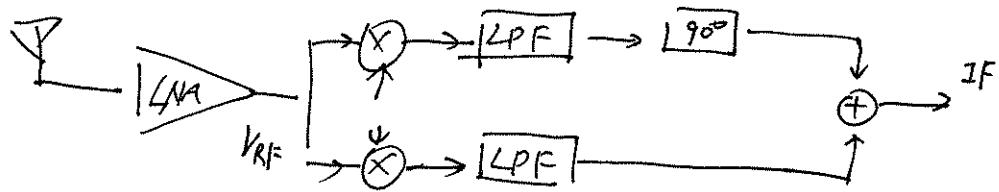
$$\therefore NF_1 = 1 + \frac{\overline{V_{n,out,LNA}^2}}{A_o^2} \cdot \frac{1}{4kTR_s}$$

$$\Rightarrow \overline{V_{n,out,LNA}^2} = (NF_1 - 1) \cdot A_o^2 \cdot 4kTR_s.$$

$$\overline{V_{n,out}^2} = \overline{V_{n,out,LNA}^2} + 1 \cdot \overline{V_{n,in,mixerI or Q}^2} \cdot 2$$

$$\Rightarrow NF_{tot} = 1 + \frac{(NF_1 - 1)A_o^2 \cdot 4kTR_s + 2\overline{V_{n,in,mixerI or Q}^2}}{A_o^2} \cdot \frac{1}{4kTR_s}$$

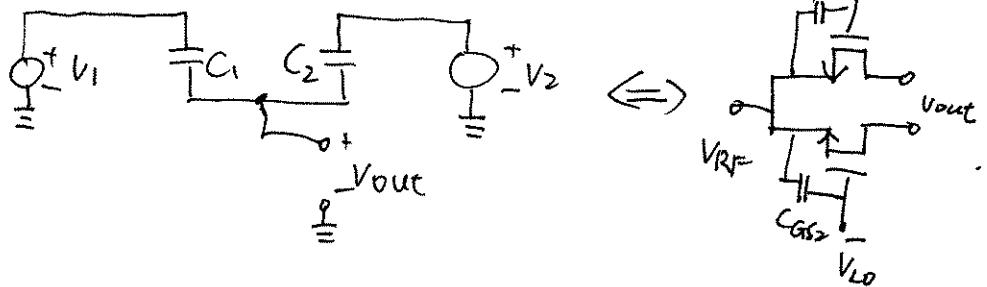
6.2 Solu:



the same as the 6.1's analysis.

$$NF_{tot} = 1 + \frac{1}{4kT_{RS}} \frac{(NF_{LNA} - 1) A_o^2 \cdot 4kT_{RS} + 2 \overline{V_{n,m,max}^2 I \text{ or } Q}}{A_o^2}$$

6.3 Solu:



$$(a) \quad C_1 = C_2 = C_0(1 + \delta_1 V)$$

$V_{out}$  @ RF input port

$$= V_1 \cdot \frac{C_1}{C_1 + C_2} + V_2 \cdot \frac{C_2}{C_1 + C_2}$$

$$= V_0 \cos \omega_0 t \frac{C_1 - C_2}{C_1 + C_2}$$

$$= 0$$

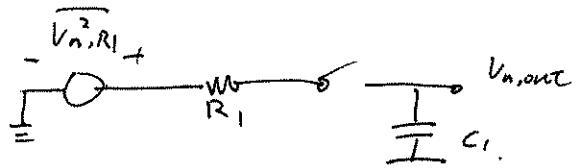
$\therefore$  So for single-balanced mixer like Fig 6.5(b),  
the LO-RF feedthrough at  $\omega_0$  vanishes if.

the circuit is symmetric.

(b). the result is the same as (a)

because of symmetry.

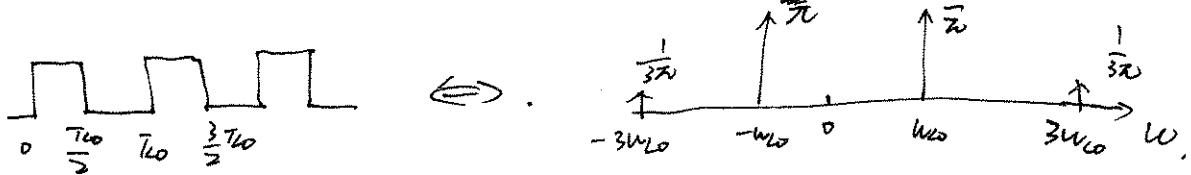
Solu: 6.4



the  $V_{n,1}(t)$  is the product of  $V_{n,LPF}(t)$  and a square wave between 0 and 1

Assume  $\frac{1}{R_1 C_1} \ll 3\omega_{LO}$ .

$$\overline{V_{n,LPF}^2} = \overline{V_{n,R_1}^2} \frac{1}{1 + (R_1 C_1 \omega)^2}.$$



$$\begin{aligned}
 V_{n,1}(f) &= V_{n,LPF}(f) * \text{Square}(f) \\
 &= V_{n,LPF}(f) * \left[ \frac{1}{j\pi} (1 - e^{-j\pi f T_0/2}) \frac{1}{T_0} \sum_{k=-3}^{+3} \delta(f - \frac{k}{T_0}) \right] \\
 &= V_{n,LPF}(f) * \left[ \frac{1}{j\pi} \delta(f - \frac{1}{T_0}) + \frac{1}{j\pi} \delta(f + \frac{1}{T_0}) \right. \\
 &\quad \left. + \frac{1}{j3\pi} \delta(f - \frac{3}{T_0}) + \frac{1}{j3\pi} \delta(f + \frac{3}{T_0}) \right] \\
 &= V_{n,LPF}(f - \frac{1}{T_0}) \cdot \frac{1}{j\pi} + V_{n,LPF}(f) (f + \frac{1}{T_0}) \cdot \frac{1}{j\pi} \\
 &\quad + V_{n,LPF}(f - \frac{3}{T_0}) \cdot \frac{1}{j3\pi} + V_{n,LPF}(f) (f + \frac{3}{T_0}) \cdot \frac{1}{j3\pi}. \\
 \therefore \overline{V_{n,1}^2(f)} &= 2 \times \left( \frac{1}{\pi^2} + \frac{1}{9\pi^2} \right) \frac{2kTR_1}{1 + (2\pi R_1 C_1 f)^2}.
 \end{aligned}$$

6.5 Solu:

Eg (654)

$$I_{IF}(t) = \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot I_{RF_0} \cos \omega_{IF} t.$$

$$V_{IF}(t) = \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot I_{RF_0} \cos \omega_{IF} t \times 2 \times Z_{BB}$$

by differential  
output.

So  $\Rightarrow$  voltage conversion gain

$$= \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot 2 \cdot$$

where  $d$  stands for duty cycle.

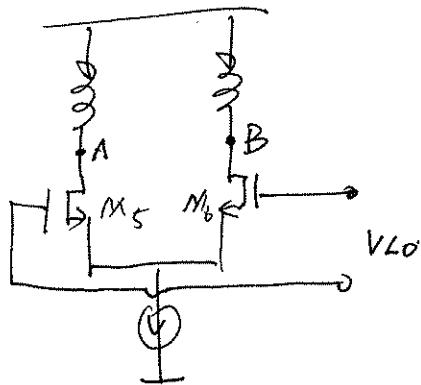
$$\lim_{d \rightarrow 0} \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot 2 = 2.$$

So voltage conversion gain

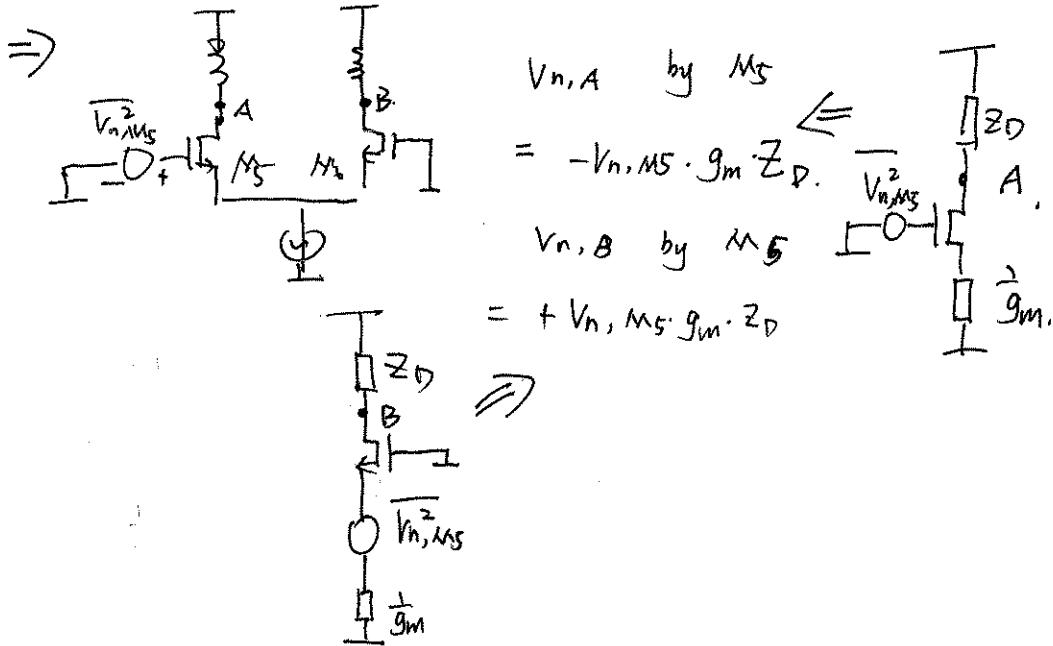
$$= 20 \log_{10} (2)$$

$$= 6 \text{ dB}.$$

6.6 Soln:



Assume the buffer's MOS are in saturation region.  
and the  $M_5$  and  $M_6$  are the same.

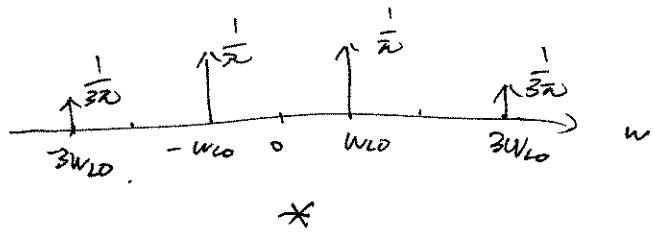


the situation of  $M_6$ 's noise is the same.

So we can easily say that. the noise of  $M_5$  &  $M_6$  appears differentially at nodes A & B.

6.7 Soln:

LO: 50% duty cycle.



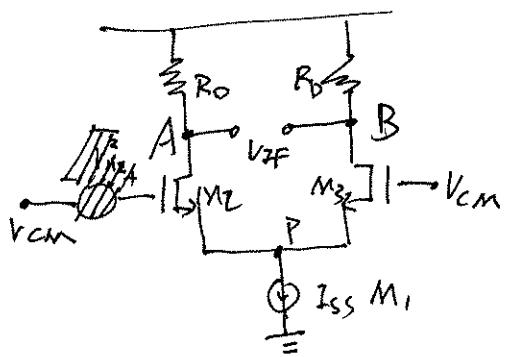
$$\frac{\frac{4kTf/g_m}{2}}{w} \quad (\text{two-side})$$

↓

$$\begin{aligned} & \frac{4kTf/g_m}{2} \left[ \frac{1}{\pi^2} + \frac{1}{(3\pi)^2} + \frac{1}{(5\pi)^2} + \dots \right] \times 2 \\ &= 4kTf/g_m \cdot \frac{1}{\pi^2} \cdot \frac{\pi^2}{8} \quad (\text{two side}) \end{aligned}$$

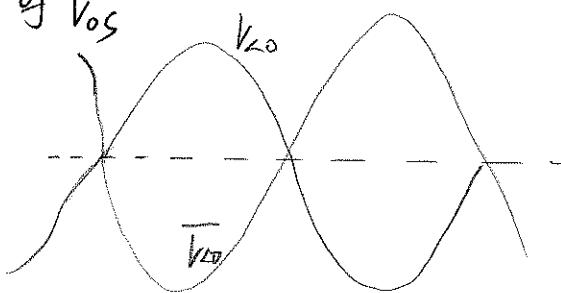
$$\Rightarrow \frac{4kTf}{g_m} \cdot \frac{1}{\pi^2} \cdot \left[ \frac{\pi^2}{4} \right]$$

6.8 Soln:



consider a threshold mismatch

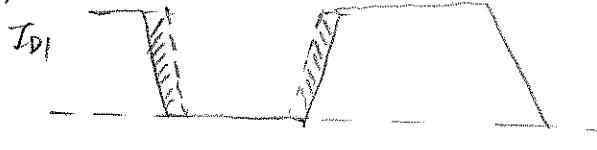
of  $V_{OS}$



From the figures on the right,  
we find that the mismatch

change the zero crossing

of  $M_2, M_3$  current.



$$v_{cm} + V_{p,20} \sin \omega_{20} t + V_{os} = v_{cm} - V_{p,20} \sin \omega_{20} t.$$

$$2V_{p,20} \sin \omega_{20} t = -V_{os}.$$

In the vicinity of  $t=0$ .

$$\Rightarrow 2V_{p,20} \cdot \omega_{20} t \approx -V_{os}$$

$$\Rightarrow |\Delta T| = \frac{|V_{os}|}{2V_{p,20} \omega_{20}}$$

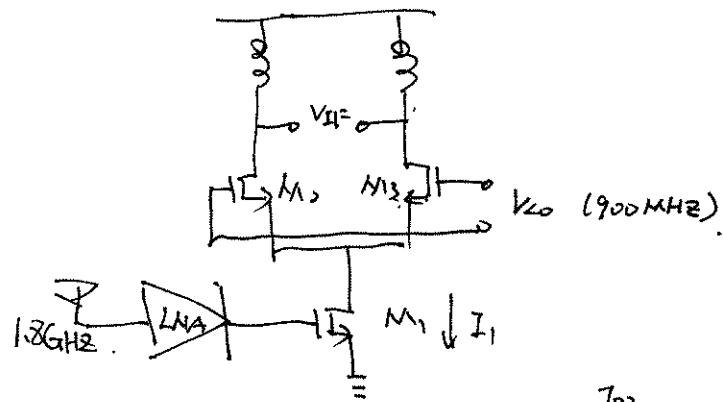
$$\textcircled{1} \quad B \Rightarrow \overline{V_{n,B}^2} = \frac{1}{2} \overline{I_{n,M_1}^2} R_D^2$$

$$\textcircled{2} \quad A \Rightarrow \overline{V_{n,A}^2} = \overline{I_{n,M_1}^2} \left( \frac{1}{2} - \frac{2\Delta T}{T_{20}} \right) R_D^2 = \overline{I_{n,M_1}^2} \left( \frac{1}{2} - \frac{|V_{os}|}{2T_{20} \cdot V_{p,20}} \right) R_D^2$$

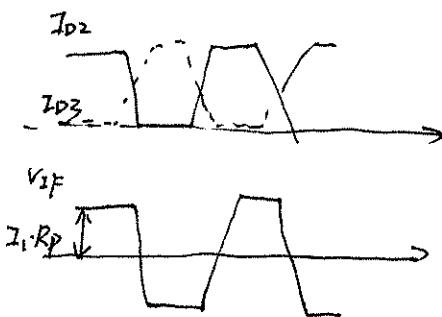
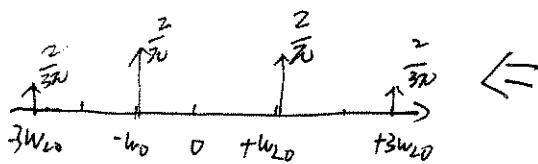
Output noise due to  $I_{ss}$ 's flicker noise

$$= \frac{|V_{os}| R_D^2}{2\pi V_{p,20}} \cdot \underbrace{\overline{I_{n,M_1}^2}}_{\text{flicker noise}}$$

6.9 Soln:



(a) If  $V_{RF}$  is zero,



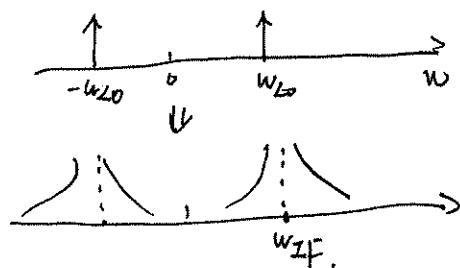
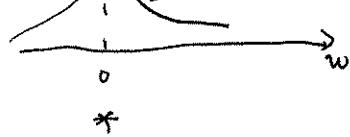
LO-IF feedthrough

$$= I_1 \cdot R_P \cdot \frac{4}{\pi} \quad \left( \frac{R_P}{w_0 L_1} = Q \right)$$

$$= I_1 w_0 L_1 Q \cdot \frac{4}{\pi}.$$

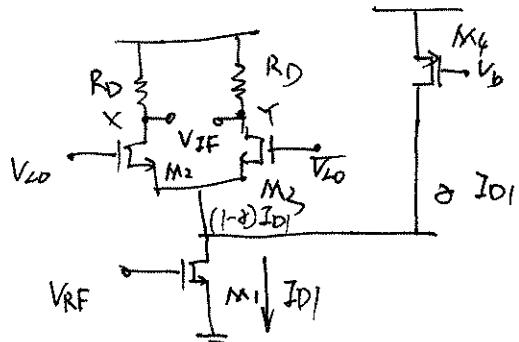
(b). flicker noise of  $M_1$  is critical.

because: flicker noise of  $M_1$



$\Rightarrow$  we notice that flicker noise is transferred to IF band.

6.10 Soln:



$$I_2 = I_3 = \frac{1-\alpha}{2} I_{D1} = \frac{1}{2} I'_3, \text{org} = \frac{1}{2} I'_3, \text{org}$$

$$\Rightarrow \alpha = \frac{1}{2}$$

$$V_{n,out,org}(f) = \left( \frac{I_{D1} \cdot R_D}{2 k_B T_0} \right) \cdot V_{n2}(f)$$

$\Rightarrow R_D$  & gain hence can be doubled.

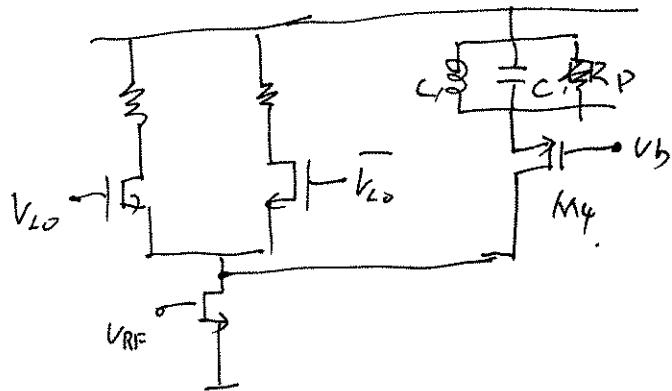
$$\therefore \overline{V_{n,out}^2} = \frac{1}{4} \overline{V_{n,out,org}^2}$$

$$\text{gain} = 2 \text{ gain}_{\text{org}}$$

$$\therefore \overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{\text{gain}} = \frac{1}{16} \cdot \overline{V_{n,in,org}^2}$$

So. the input-referred flicker noise falls to sixteenth. of original one.

6.11 Soln:



Let me analyse the noise current we saw from

$V_{RF}$  MOS drain.

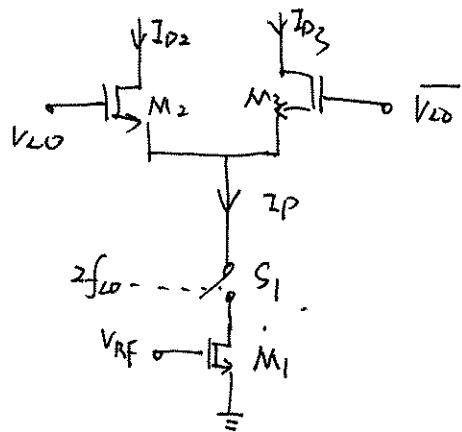
$$\frac{g_{m4}}{1+g_{m4}R_P} \approx \frac{1}{R_P} \quad (g_{m4}R_P \gg 1)$$

$$\begin{aligned} \text{Circuit diagram: } & \text{A drain terminal with noise voltage } V_{n,RP} + \frac{V_{n,M4}}{R_P} \text{ is connected to the drain of } M_4 \text{ through a resistor } R_P. \\ \Rightarrow & \overline{I_{n,out}^2} = \frac{4kT\gamma}{g_{m4}} \left( \frac{1}{R_P} \right)^2 + 4kT R_P \left( \frac{1}{R_P} \right)^2 \\ & = \frac{4kT\gamma}{g_{m4}R_P^2} + \frac{4kT}{R_P}. \end{aligned}$$

So Eq. (6.116) should be re-written as

$$\begin{aligned} & \overline{I_{n,M1}^2} + \overline{I_{n,M4}^2} + \overline{I_{n,RP}^2} \\ & = 4kT \frac{2Z_D}{(V_{GS-V_{th}})_1} + \frac{4kT\gamma}{R_P^2} \frac{|V_{GS-V_{th}}|_4}{2Z_D I_D} + \frac{4kT}{R_P}, \end{aligned}$$

6.12 Soln:



Yes, we can view this scheme.

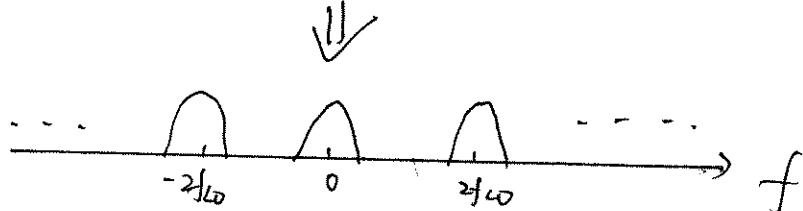
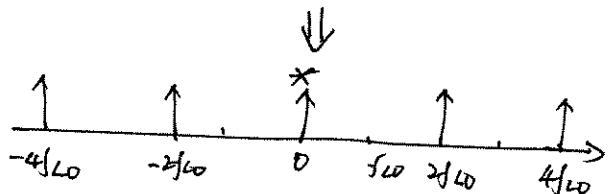
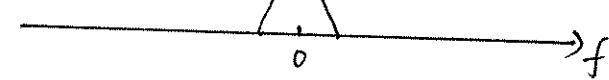
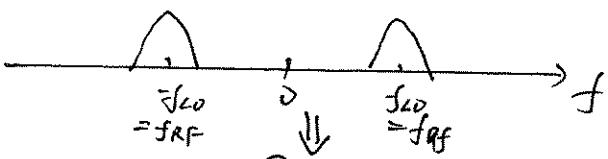
as a differential pair whose tail current is modulated at a rate of  $2f_{L0}$ .

$$I_{D2} - I_{D3}(t) = \sum_{k=0}^{+\infty} g_m \cdot V_{RF}(t) \cos(\omega_{RF}t) \cdot \delta(t - k \cdot \frac{T_{L0}}{2})$$

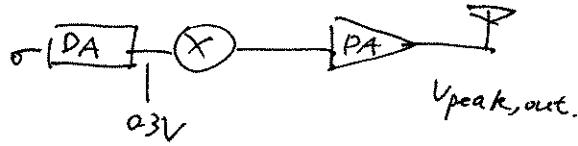
F.T.

$$I_{D2} - I_{D3}(f) = \sum_{k=-\infty}^{\infty} g_m V_{RF}(f) * \delta(f - f_{L0}) * \delta(f + 2kf_{L0})$$

We can appreciate from spectrum.



6.13 Soln:



1<sup>o</sup> gain

assume 50Ω antenna system.

$$V_{\text{peak,out}} = \sqrt{1 \text{W} \cdot 50\Omega \cdot 2}$$
$$= 10 \text{V.}$$

$$\text{gain} = \frac{V_{\text{peak,out}}}{V_{\text{peak,in}}} = \frac{10}{0.3} = 33.3.$$

2<sup>o</sup>. noise power = -155 dBm.

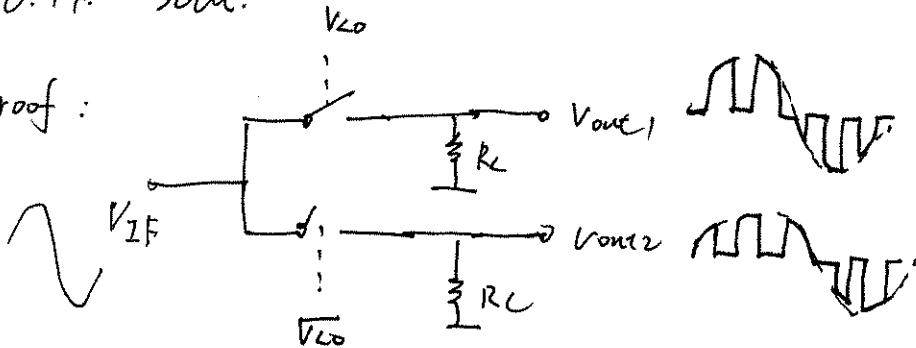
$$\Rightarrow 10^{-15.5} \text{ mW} = 3.16 \times 10^{-16} \text{ mW.}$$

$$V_{\text{noise,rms,out}} = 3.98 \text{ nV.}$$

$$3^{\circ} \Rightarrow \overline{V_{\text{noise,rms,in}}^2} = \frac{(3.98 \text{ nV})^2}{\text{gain}^2} = 1.43 \times 10^{-20} \text{ V}^2/\text{Hz.}$$

6.14. Soln:

Proof:



$$V_{out1}(+) = V_{IF}(+) \cdot \text{Square}(+)$$

$$V_{out2}(+) = V_{IF}(+) \cdot \text{Square}\left(t - \frac{T_{LO}}{2}\right).$$

only consider the fundamental freq.

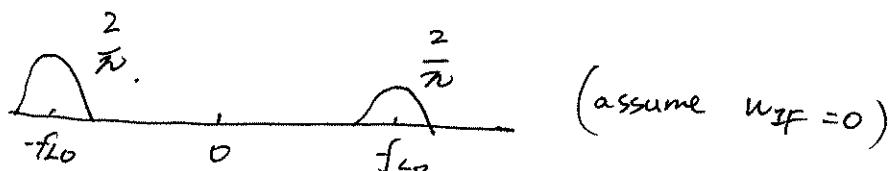
$$V_{out1}(+) \approx V_{IF}(+) \cdot \frac{2}{\pi} \cos \omega_{LO} t.$$

$$V_{out2}(+) \approx -V_{IF}(+) \cdot \frac{2}{\pi} \cos \omega_{LO} t.$$

$$\therefore V_{out}(t) = (V_{out1} - V_{out2})(+)$$

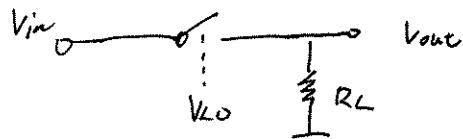
$$F.T. = V_{IF}(+) \frac{4}{\pi} \cdot \cos \omega_{LO}(+).$$

$$V_{out}(f) = \frac{2}{\pi} V_{IF}(f) * [\delta(f + f_{LO}) + \delta(f - f_{LO})]$$



$\therefore$  Voltage conversion gain of a single-balanced return-to-zero mixer is equal to  $\frac{2}{\pi}$ .

6.15 Soln:



Let's study the effect on the spectrum of  $V_{LO}$  about duty cycle's distortion.

$$V_{LO}(t) = \sum_{k=-\infty}^{+\infty} rec(\alpha t) \cdot \delta(t - kT_{LO})$$

where  $rec(\alpha t) = \begin{cases} 1 & \text{if } 0 < \alpha t < 1 \\ 0 & \text{otherwise} \end{cases}$ .

$$\text{So duty cycle} = \frac{1}{T_{LO}}$$

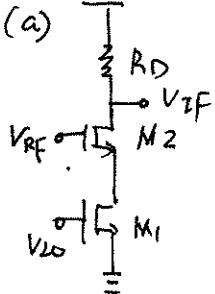
That means, if  $T_{LO}$  is unchangeable, duty cycle distortion is equivalent to  $\alpha$ 's distortion.

$$V_{LO}(f) = \frac{1}{T_{LO}} \cdot \sum_{k=-\infty}^{+\infty} \sin\left(\frac{f}{\alpha}\right) \cdot \frac{1}{|\alpha|} * \delta(f - kf_{LO})$$

From the above equation, we can find. duty cycle only affect the gain of mixer, but doesn't produce any feedthrough at the output.

6.16 Soln:

$$(a) \quad V_{out}(t) = I_{RF(+)} \cdot R_D \frac{2}{\pi} \cos \omega_0 t + \dots$$



$$I_{RF(+)} = \frac{g_m 2}{1 + g_m 2 R_{on1}} R_D V_{RF} \cdot \cos \omega_{RF} t$$

$$V_{ZF(t)} = \frac{1}{\pi} \cdot \frac{g_m 2}{1 + g_m 2 R_{on1}} R_D V_{RF} \cdot \cos(\omega_{RF} - \omega_0)t$$

$$\therefore \frac{V_{ZF} \cdot p}{V_{RF} \cdot p} = \frac{1}{\pi} \cdot \frac{g_m 2}{1 + g_m 2 R_{on1}} \cdot R_D$$

$$(b). \quad \text{If } R_{on1} \ll \frac{1}{g_m 1}$$

$$\therefore \text{gain} = \frac{1}{\pi} \cdot g_m 2 \cdot R_D$$

$$V_{RF} = V_m \cos \omega_1 t + V_n \cos \omega_2 t + V_{aso}$$

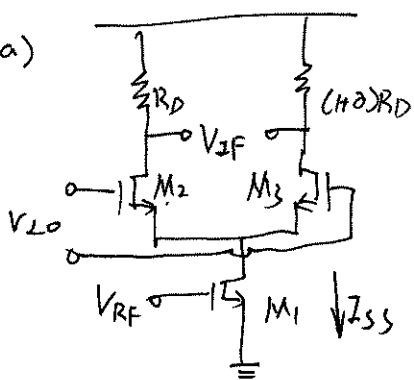
$$I_{IM2} = \frac{1}{2} \mu_n \cos \frac{\omega}{L} V_m^2 \cdot \cos(\omega_1 - \omega_2)t$$

$$\frac{1}{2} \mu_n \cos \frac{\omega}{L} V_{ZP2}^2 R_D = \frac{1}{\pi} \cdot g_m 2 \cdot R_D \cdot V_{ZP2}$$

$$\Rightarrow V_{ZP2} = \frac{2}{\pi} (V_{aso} - V_{th})_2$$

6.17 solu:

(a)



$$V_{RF} = 0.$$

$$I_2 = I_{ss} \cdot S(t)$$

$$I_2 = I_{ss} \cdot S(t - \frac{T_{LO}}{2})$$

$$V_{out} = I_2 R_D (1+\alpha) - I_2 R_D$$

output offset.

$$\Rightarrow I_{ss} R_D \cdot \frac{2}{\pi}$$

= output offset.

$$(b). \quad V_{RF} = V_m \cos \omega_1 t + V_m \cos \omega_2 t + V_{aso}$$

$$I_{M2} = \frac{1}{2} \mu_n C_{ox} \frac{w}{L} V_m^2 \cos(\omega_1 - \omega_2)t$$

$$V_{out, M2} = \frac{1}{2} \mu_n C_{ox} \frac{w}{L} V_m^2 \cos(\omega_1 - \omega_2)t R_D \cdot \frac{2}{\pi}$$

$$\frac{1}{2} \mu_n C_{ox} \frac{w}{L} \underline{I_{ZP2}^2 R_D} \cdot \frac{2}{\pi} = \frac{2}{\pi} \cdot g_m, R_D \cdot V_{ZP2}$$

$$= \frac{2}{\pi} \cdot \underline{\mu_n C_{ox} \frac{w}{L} (V_{GS} - V_{th})_1} \cdot \underline{R_D V_{ZP2}}$$

$$\Rightarrow V_{ZP2} = \frac{2 (V_{GS} - V_{th})_1}{\underline{\partial}}$$

7.1 Solu:

$N$ -turn spirals.

$$L_{\text{tot}} = L_1 + L_2 + \cdots + L_N$$

$$\left. \begin{aligned} &+ M_{12} + M_{13} + M_{14} + \cdots + M_{1N} \\ &+ M_{23} + M_{24} + \cdots + M_{2N} \\ &+ M_{34} + \cdots \\ &\vdots \\ &+ M_{N-1N} \end{aligned} \right\} C_N^2 \cdot M_{ij}.$$

$$C_N^2 = \frac{N(N-1)}{2} \quad \text{terms for mutual inductance.}$$

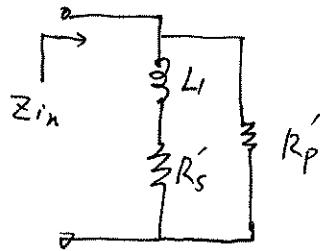
Total terms Number

$$= N + \frac{N(N-1)}{2}$$

$$= \frac{N^2 - N + 2N}{2} = \frac{N(N+1)}{2}.$$

## 7.2 Solu:

Proof:



$$Z_{in}(s) = (sL_1 + R_s') // R_p'$$

$$= \frac{sL_1 R_p' + R_s' R_p'}{sL_1 + R_s' + R_p'}$$

$$Z_{in}(j\omega) = \frac{R_s' R_p' + j\omega L_1 R_p'}{R_s' + R_p' + j\omega L_1} = \frac{(R_s' R_p' + j\omega L_1 R_p')(R_s' + R_p' - j\omega L_1)}{(R_s' + R_p')^2 - \omega^2 L_1^2}$$

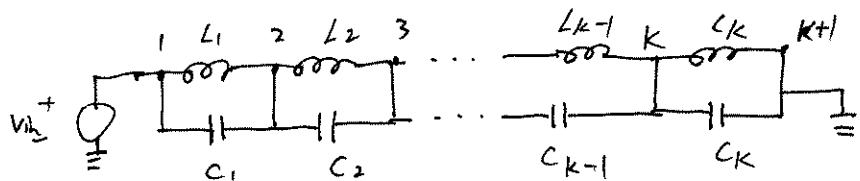
$$Q \triangleq \frac{\text{Im}(Z_{in})}{\text{Re}(Z_{in})} = \frac{\omega L_1 R_p'(R_s' + R_p') - \omega L_1 (R_s' R_p')}{R_s'^2 R_p' + R_p'^2 R_s' + \omega^2 L_1^2 R_p'}$$

$$= \frac{\omega L_1 R_p'^2}{L_1^2 \omega^2 R_p' + R_s' R_p'(R_s' + R_p')}$$

$$= \frac{L_1 \omega}{L_1^2 \omega^2 + R_s' (R_s' + R_p)}$$

### 7.3 Soln:

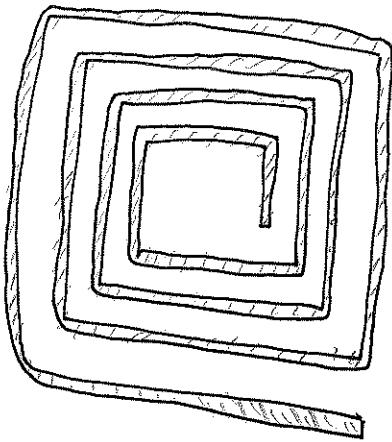
Proof. model of interwinding capacitance



For N-turn spiral inductor.

the equivalent interwinding capacitance

is  $C_{eq} = \frac{c_1 + c_2 + \dots + c_{N-1}}{(N-1)^2}$



For example: 4-turn spiral inductor.

$$K=4(N-1)$$

$$C_{eq} = \frac{c_1 + c_2 + \dots + c_{4(N-1)}}{(4(N-1))^2}$$

7.4 Soln:

$$\text{Eq. (7.62)} \quad Q = \frac{L_1 w R_p'}{L_1^2 w^2 + R_s' (R_s' + R_p')} \leftarrow \begin{array}{c} L_1 \\ | \\ R_s' \\ | \\ R_p' \end{array}$$

For Fig. 7.37 (b).  $Q_1 = \frac{L w R_1}{L w^2 + R_s (R_s + R_1)}$

For Fig. 7.37 (d)

$$Q_2 = \frac{\frac{L}{2} w \cdot R_1}{\frac{L}{2} w^2 + \frac{R_s}{2} (\frac{R_s}{2} + R_1)}$$

$$= \frac{L w R_1}{L w^2 + R_s (\frac{R_s}{2} + R_1)}.$$

We can find the differences between  $Q_1$  &  $Q_2$ .

the denominator of  $Q_2$  is smaller than  
that of  $Q_1$ .

7.5 Soln:

For Fig. 7.41 (a).

using the right-hand rule, we observe that  
the magnetic field due to  $L_1$  points into page.

so does the magnetic field due to  $L_2$ ,  
(at far-from point)

On the other hand, for fig. 7.41(b)

using the right-hand rule, we also observe  
that the magnetic field due to  $L_1$  points out of the  
page but that due to  $L_2$  points into the page  
(at far-from point).

So Fig 7.41(b)'s topology can cancel the magnetic field  
at a point far from the circuit. and has  
the less net magnetic field.

7.6 soln:

$$L = 5\text{mH}, \quad W = 5\text{mm}, \quad S = 0.5\text{mm}. \quad N = 4.$$

$$R_o = 22\text{m}\Omega/\square$$

$$\begin{aligned} f_{\text{crit}} &\approx \frac{3.1}{2\pi M} \cdot \frac{W+S}{W^2} \cdot R_o \\ &= \frac{3.1}{2\pi \cdot 4\pi \times 10^{-7}} \cdot \frac{5\text{m} + 0.5\text{m}}{(5\text{m})^2} \cdot 22\text{m}\Omega/\square \\ &= 1.9 \text{GHz}. \end{aligned}$$

$$R_{\text{eff}} = R_o \left[ 1 + \frac{1}{10} \int \left( \frac{900\text{m}}{1.9\text{G}} \right)^2 \right] = 1.0224 R_o = 16\mu\Omega.$$

(For  $R_o = 15.7\mu\Omega \Leftarrow L = 5\text{mH}$ ).

From Eq. (7.15) we can calculate the length.

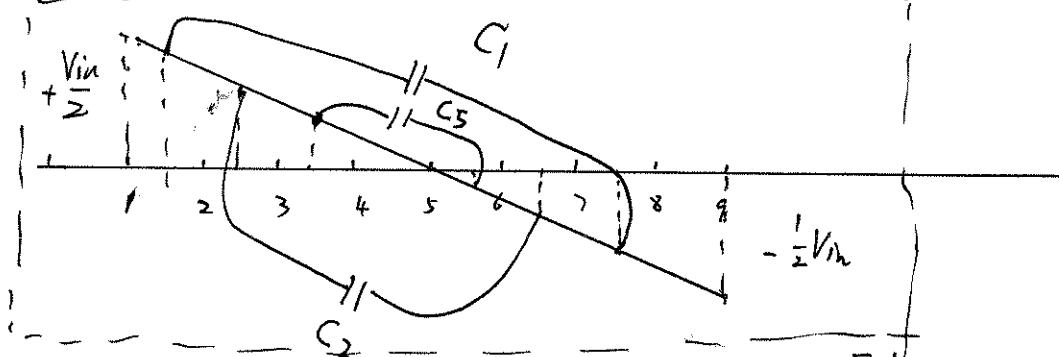
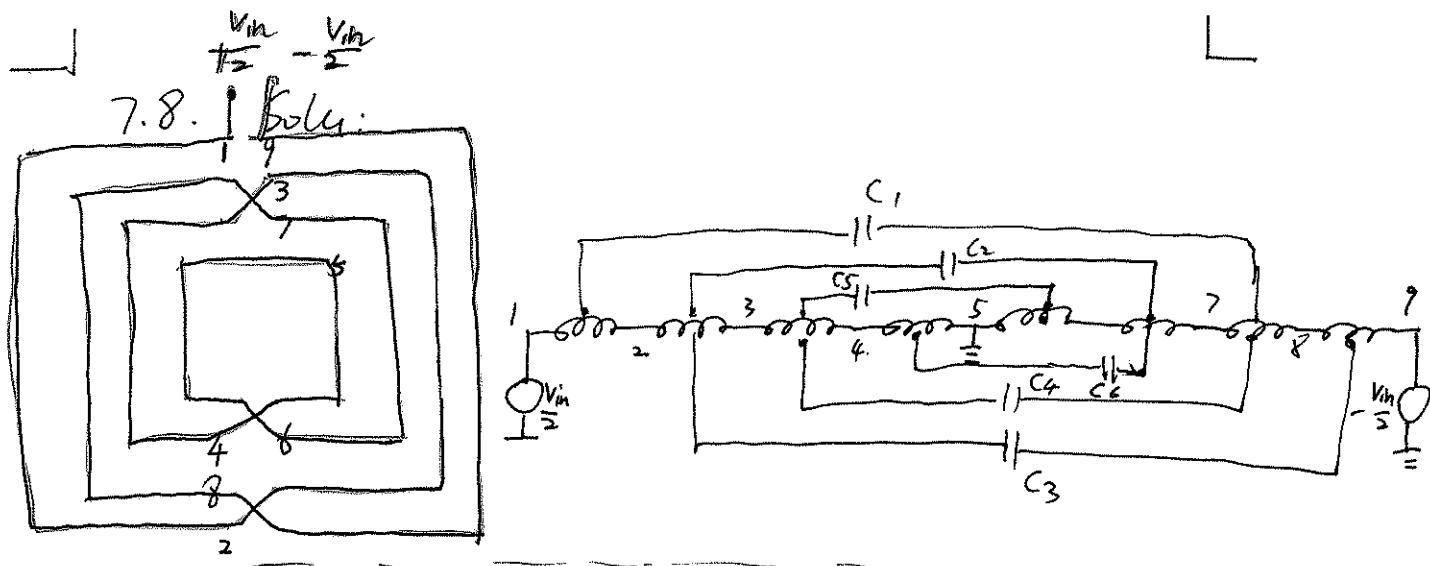
So with length, width, space and no. of turn,  
the outer diameter is determinate.

7.7 Solu:

$$Y_{11}(s) = \frac{I_{1h}(s)}{V_{1h}(s)} = \frac{R_{sub} + L_2 s}{L_1 R_{sub}s + (M^2 - L_1 L_2)s^2}$$

$$\begin{aligned} Y_{11}(jw) &= \frac{R_{sub} + jL_2 w}{jL_1 R_{sub} w + (M^2 - L_1 L_2) w^2} \\ &= \frac{(R_{sub} + jL_2 w)[(M^2 - L_1 L_2)w^2 - jL_1 R_{sub} w]}{[(M^2 - L_1 L_2)w^2]^2 + L_1^2 R_{sub}^2 w^2} \\ &= \underbrace{\frac{R(M^2 - L_1 L_2)w^2 + L_1 L_2 R_{sub}}{(M^2 - L_1 L_2)w^2 + L_1^2 R_{sub}^2}}_{\frac{1}{R_P}} + \underbrace{\frac{1}{jw} \left( \frac{L_1 R_{sub}^2 - L_2 (M^2 - L_1 L_2)w^2}{L_1^2 R_{sub}^2 + (M^2 - L_1 L_2)^2 w^2} \right)}_{\frac{1}{jwL'}} \end{aligned}$$

We can find that the result is not the same  
as that shown in Eq. (7.55).



$C_1 \& C_3$  sustains  $\frac{6}{8} V_{in}$

$C_2 \& C_4$  sustain  $\frac{4}{8} V_{in}$

$C_5 \& C_6$  sustain  $\frac{2}{8} V_{in}$

$$E_{tot} = 2 \cdot \left[ \frac{1}{2} C \left( \frac{6}{8} V_{in} \right)^2 + \frac{1}{2} C \cdot \left( \frac{4}{8} V_{in} \right)^2 + \frac{1}{2} C \left( \frac{2}{8} V_{in} \right)^2 \right]$$

$$= \frac{7}{8} \cdot C \cdot V_{in}^2$$

$$\therefore C_1 + C_2 + \dots + C_6 = C_{tot} \Rightarrow C = \frac{C_{tot}}{6}$$

$$\therefore E_{tot} = \frac{7}{8} \cdot \frac{C_{tot}}{6} \cdot V_{in}^2$$

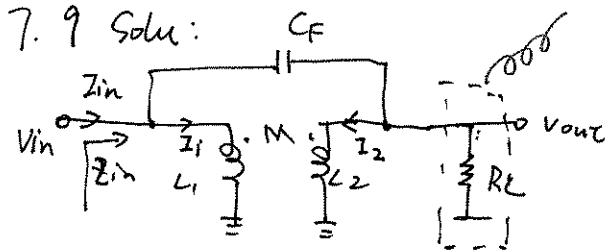
$$\Rightarrow C_{eq} = \frac{7}{24} C_{tot}$$

$$\text{For } N\text{-turn} \Rightarrow E_{tot} = 2 \cdot \frac{1}{2} \cdot \frac{C_{tot}}{2(N-1)} V_{in}^2 \cdot \left[ \left( \frac{N-1}{N} \right)^2 + \left( \frac{N-2}{N} \right)^2 + \left( \frac{N-3}{N} \right)^2 + \dots + \left( \frac{1}{N} \right)^2 \right]$$

$$C_{eq} = \frac{2N-1}{6N} C_{tot}$$

$$= \frac{C_{tot}}{2(N-1)} \cdot V_{in}^2 \cdot \frac{1}{N^2} \cdot \frac{(N-1)N(2N-1)}{6}$$

$$= \frac{C_{tot}}{12} \cdot V_{in}^2 \cdot \frac{2N-1}{N}$$



$$\left\{ \begin{array}{l} V_{in} = L_1 s I_1 + M s I_2 \\ V_{out} = L_2 s I_2 + M s I_1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} I_1 = \frac{V_{in} - V_{out}}{L_1 s} \\ I_2 = \frac{V_{in} \cdot S_F (1 - \frac{M}{L_1})}{1 + L_2 s^2 C_F - \frac{M^2}{L_1} s^2 C_F} \end{array} \right.$$

$$Z_{in} = I_1 + (V_{in} - V_{out}) S_C F = I_2.$$

$$\Rightarrow Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = \frac{L_1 s I_1 + M s I_2}{I_1 + (V_{in} - L_2 s I_2 - M s I_1) \cdot S_C F}$$

Substitute  $I_1, I_2$  into  $Z_{in}(s)$ . then we can get

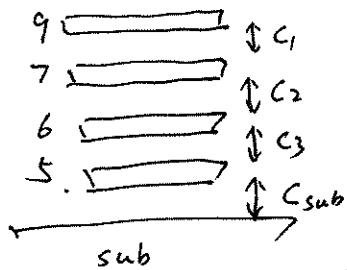
the result.

$$Z_{in}(s) = \frac{L_1 s - \frac{M \cdot C_F (L - M) s^3}{1 + s^2 C_F (L_2 - M)}}{1 + s^2 C_F (L_1 - M) - \frac{s^4 C_F^2 (L_2 - M)(L_1 - M)}{1 + s^2 C_F (L_2 - M)}}$$

Solu: 7.10.

Assume we choose to use Metal 9, metal 7,

Metal 6 and metal 5.



each spiral must provide  
an inductance of  $\frac{4.96 \text{nH}}{16} = 0.3 \text{nH}$ .

choose  $N = 3$ ,  $w = 4 \text{ um}$ ,  $s = 0.5 \text{ um}$ .

From Eq. (7.15), yields

$$l_{\text{tot}} \approx 167 \text{ um}$$

$$D_{\text{out}} = \frac{l_{\text{tot}}}{4N} + w + (N-1)(w+s)$$

~~20~~  
each spiral has an area of  $167 \text{ um} \times 4 \text{ um} = 667 \text{ um}^2$

$$\therefore C_1 = 16 \times 667 \times 10^{-18} = 10.7 \text{ fF}$$

$$C_2 = 88 \times 667 \times 10^{-18} = 58.7 \text{ fF} \quad C_{\text{sub}} = 5.7 \text{ fF}$$

$$C_3 = 88 \times 667 \times 10^{-18} = 58.7 \text{ fF}.$$

$$\therefore C_{\text{eq}} = \frac{4 \cdot (C_1 + C_2 + C_3) + C_{\text{sub}}}{3 \cdot 4^2} = 10.8 \text{ fF.}$$

7. 11. Soln:

for pn-junction varactor.



p-sub

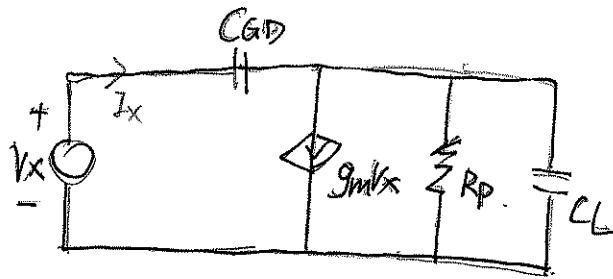
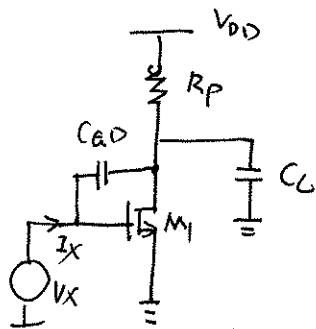
$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_D}{V_0}\right)^m}$$

Range of control voltage:

$$V_D \in [0, V_{DD}]$$

the output swing should be as much as the supply voltage, however the actual output swing depends on the LC VCO design.

8.1. Soln:



$$I_x = g_m V_x + \frac{V_x - I_x \cdot \frac{1}{sC_{GD}}}{R_p \parallel \frac{1}{sC_L}}$$

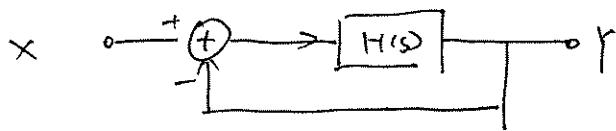
$$\begin{aligned} \Rightarrow \frac{I_x}{V_x} &= Y_h = \frac{g_m + \frac{1}{R_p \parallel \frac{1}{sC_L}}}{1 + \frac{1}{sC_{GD}} \cdot \frac{1}{R_p \parallel \frac{1}{sC_L}}} \\ &= \frac{(R_p \parallel \frac{1}{sC_L}) g_m + 1}{(R_p \parallel \frac{1}{sC_L}) + \frac{1}{sC_{GD}}} \\ &= \frac{s^2 C_L (G_D R_p + (R_p g_m + 1) C_{GD} \cdot s)}{1 + s R_p (C_{GD} + C_L)} \end{aligned}$$

$$s = j\omega$$

$$Y_h(j\omega) = \frac{-\omega^2 C_L C_{GD} R_p + j\omega C_{GD} (R_p g_m + 1)}{1 + j\omega R_p (C_{GD} + C_L)}$$

$$\begin{aligned} \therefore \operatorname{Re}[Y_h(j\omega)] &= \frac{\omega^2 C_{GD} R_p (R_p g_m + 1) (C_{GD} + C_L) - \omega^2 C_L C_{GD} R_p}{1 + \omega^2 R_p^2 (C_{GD} + C_L)^2} \\ &= \frac{\omega^2 R_p C_{GD} [(1 + R_p g_m) (C_{GD} + R_p g_m C_L)]}{1 + \omega^2 R_p^2 (C_{GD} + C_L)^2}. \end{aligned}$$

8.2 Soln:



$$|H(j\omega_1)| = 1$$

$$\angle H(j\omega_1) = 170^\circ$$

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$

$$\left| \frac{Y}{X}(j\omega_1) \right| = \frac{1}{|1 + e^{j\frac{170}{180}\pi}|} = 1/0.174 = 5.73$$

$$\begin{aligned} \angle \frac{Y}{X}(j\omega_1) &= \frac{170}{180}\pi - \angle(1 + H(j\omega_1)) \\ &= \frac{17}{18}\pi - 1.4835 = 1.4835 \Rightarrow 85^\circ \end{aligned}$$

So if  $x(t)$  is a sinusoidal signal at  $\omega_1$ ,

the amplitude of output will be multiplied by  $1/0.174 = 5.73$  and the phase

will change by  $85^\circ$ .

8.3 Soln:

$$|H(j\omega_1)| = A > 1$$

$$\angle H(j\omega_1) = 180^\circ$$

$$\therefore H(j\omega_1) = A \cdot e^{j\pi} = -A.$$

$$\left| \frac{Y}{X}(j\omega_1) \right| = \frac{A}{-(1-A)} = \frac{A}{A-1} > 1.$$

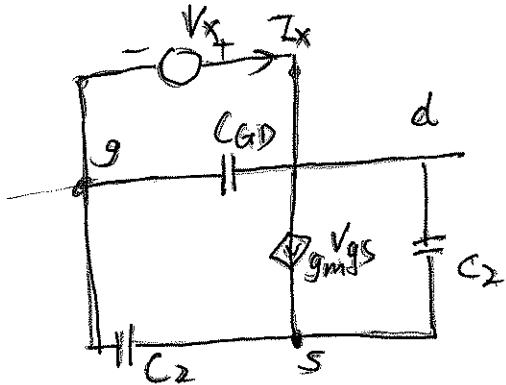
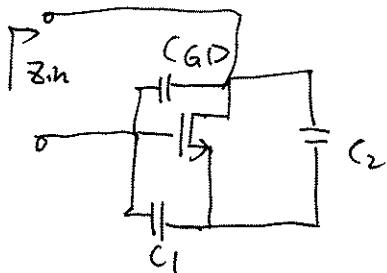
$$\angle \frac{Y}{X}(j\omega_1) = \pi - \pi = 0.$$

So if the input of the system is a sinusoidal signal at  $\omega_1$ ,

the amplitude of output will be multiplied by  $\frac{A}{A-1}$  and

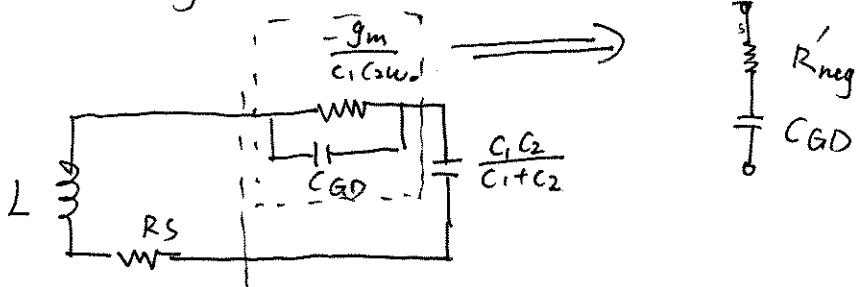
the phase of output will not change, compared with input.

8.4. Solu:



$$Z_{in} = \frac{1}{sC_{GD}} \parallel -\frac{g_m}{C_1 C_2 w^2}$$

So the Fig. 8.15 will change to



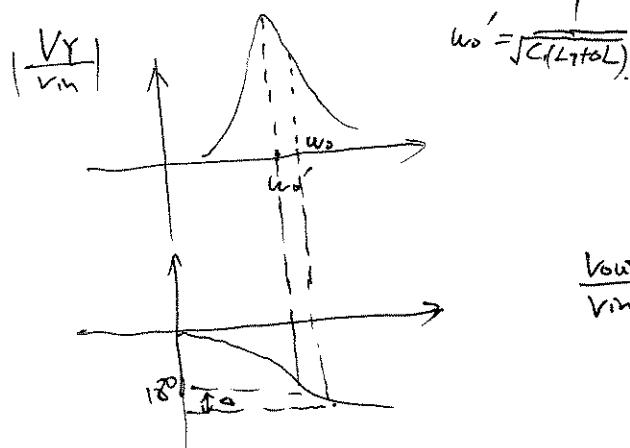
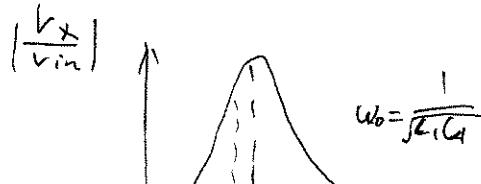
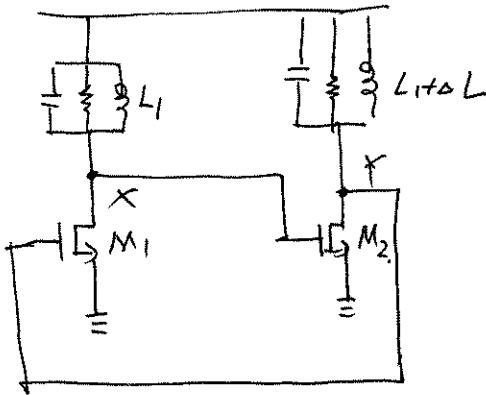
$$\text{So the } \omega_{osc} = \frac{1}{\sqrt{L_1 \cdot \frac{C_1 C_2 C_{GD}}{C_1 + C_2} / \left( \frac{C_1 C_2}{C_1 + C_2} + C_{GD} \right)}}$$

8.5 Soln:

Yes. Any feedback oscillator that employs a lossy resonator be viewed as one-pole system of 8.13(c).  
Figure

Only in this way, the feedback system can satisfy  
the Barkhausen's criteria  $|H(s=j\omega)| = 1$  at resonance  
 $\angle H(s=j\omega) = -180^\circ$ . freq.

8.6 solve:



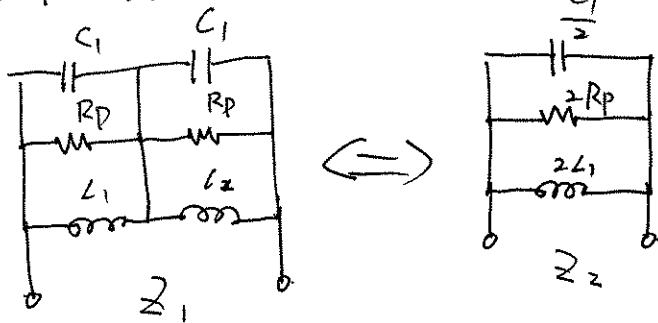
$$\begin{aligned} \frac{V_{out}}{V_{in}} &= (-g_m) \left( R_p \parallel \frac{1}{C_{1S}} \parallel S L_1 \right) \cdot (-g_m) \left[ R_p \parallel \frac{1}{C_{1S}} \parallel S(L_1 + \Delta L) \right] \\ &= g_m^2 \underbrace{\left( R_p \parallel \frac{1}{C_{1S}} \parallel S L_1 \right)}_{\textcircled{1}} \underbrace{\left( R_p \parallel \frac{1}{C_{1S}} \parallel S(L_1 + \Delta L) \right)}_{\textcircled{2}} \end{aligned}$$

phase contribution by ① & ②

$$\pi - \left[ \arctan \frac{w L_1}{R_p (1 - L_1 C_1 w^2)} + \arctan \frac{w C (L_1 + \Delta L)}{R_p (1 - w^2 (L_1 + \Delta L) C_1)} \right] = 0$$

$\Rightarrow$  Solve the w.

8.7 Soln:



$$\text{Proof: } Z_1 = 2 \cdot \frac{1}{C_1 s / R_p + L_1 s}$$

$$= 2 \cdot \frac{L_1 R_p s}{L_1 s + R_p (1 + s^2 L_1 C_1)}$$

$$\text{For } Z_2 = \frac{1}{s C_1} / 2R_p \parallel \cancel{2L_1 s}$$

$$= \frac{2 L_1^2 R_p s}{2 L_1 s + 2 R_p (1 + s^2 L_1 C_1)}$$

$$= \frac{2 L_1 R_p s}{L_1 s + R_p (1 + s^2 L_1 C_1)}$$

$$\therefore Z_1 = Z_2.$$

8.8 Soln:

If  $C_b$  is placed at node P & Q.

$$1/C_{var} = 1/C_{var} + 1/C_b$$

$$C_{var} = \frac{C_{var} C_b}{C_{var} + C_b}$$

Without  $C_s$  &  $C_b$ , Eq. (8.73) shows

$$\Delta w_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{0.5 C_{max}}{2 C_1}$$

For this range to reach 60% of centre freq.

$$C_{max} = \frac{2}{5} C_1$$

With effect of  $C_s$  &  $C_b$ . From Eq. 8.69,

$$\begin{aligned} \Delta w_{osc} &= \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{1}{2 C_1} \cdot \frac{C_s^2 \left( \frac{C_{max} C_b}{C_{max} + C_b} - \frac{C_{min} C_b}{C_{min} + C_b} \right)}{\left( s + \frac{C_{max}(n)}{C_{max} + C_b} \right) \left( s + \frac{C_{min} C_b}{C_{min} + C_b} \right)} \\ &\stackrel{(s=10C_{max})}{=} \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{100 C_{max} \left( \frac{0.5 C_{max}^2}{1.5 C_{max}} - \frac{0.25 C_{max}^2}{1.2 C_{max}} \right)}{\left( 10 C_{max} + \frac{1}{3} C_{max} \right) \left( 70 C_{max} - \frac{1}{4} C_{max} \right)} \cdot \frac{1}{2 C_1} \\ &= \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{100}{31.392} \cdot \frac{C_{max}}{C_1} \end{aligned}$$

$$= \frac{100}{31.392} \cdot \frac{2}{5} \cdot w_0$$

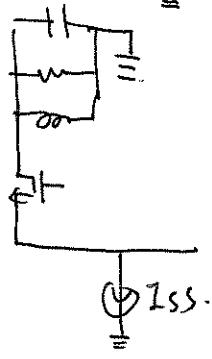
$$= 1.65 \% w_0$$

So the turning range there falls to 1.65% around  $(L_1 C_1)^{1/2}$ .

8.9. Soln:



Why do the PMOS devices in Fig 8.36  
carry a current of  $I_{SS}$ ?



Because we know the ground in  
the middle is only ac ground.

The dc path at this time is the  
right PMOS & the left NMOS.

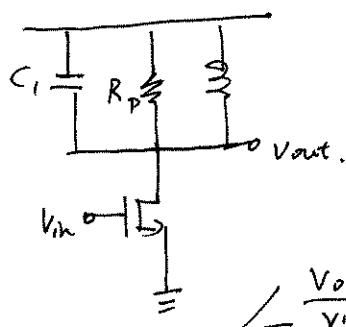
That's why PMOS carries  $I_{SS}$ .

Solu: 8.10.

Proof: For a CS stage loaded by a

Second-order parallel RLC tank, prove that.

$$\frac{R_p}{Lw_0} = \frac{w_0}{2} \left| \frac{d\phi}{dw} \right|.$$



$$\frac{V_{out}}{V_{in}}(jw) = -\frac{jg_m R_p L_1 w}{R_p(1-L_1 C_1 w^2) + jL_1 w}$$

$$\angle \frac{V_{out}}{V_{in}}(jw) = \left[ -\frac{\pi}{2} - \tan^{-1} \frac{L_1 w}{R_p(1-L_1 C_1 w^2)} \right]$$

$$\frac{d \angle \frac{V_{out}}{V_{in}}(jw)}{dw} = -\frac{1}{1 + \left[ \frac{C_1 w}{R_p(1-L_1 C_1 w^2)} \right]^2} \left[ \frac{L_1}{R_p(1-L_1 C_1 w^2)} + \frac{L_1 w R_p(-1) \cdot (-L_1 C_1 \cdot 2w)}{R_p^2(1-L_1 C_1 w^2)^2} \right]$$

$$= -\frac{1}{1 + \left[ \frac{C_1 w}{R_p(1-L_1 C_1 w^2)} \right]^2} \left[ \frac{L_1}{R_p(1-L_1 C_1 w^2)} + \frac{2L_1^2 C_1 w^2 R_p}{R_p^2(1-L_1 C_1 w^2)^2} \right]$$

$$= -\frac{L_1 R_p(1-L_1 C_1 w^2) + 2L_1^2 C_1 w^2 R_p}{[R_p(1-L_1 C_1 w^2)]^2 + (L_1 w)^2}$$

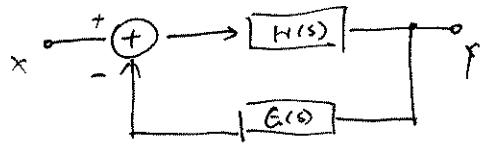
$$\left| \frac{w_0}{2} \frac{d \angle \frac{V_{out}}{V_{in}}(jw)}{dw} \right|_{w=w_0} = \frac{w_0^2}{2} \frac{2C_1 R_p \cdot w_0}{w_0^2} = w_0^2 C_1 R_p$$

$$= \frac{1}{\sqrt{C_1 L_1}} G R_p$$

$$= \frac{R_p}{L_1 w_0}$$

$$So \cdot \frac{R_p}{Lw_0} = \frac{w_0}{2} \left| \frac{d\phi}{dw} \right|.$$

8.11 Soln:



$$(X(s) - Y(G(s))) \cdot H(s) = Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s) \cdot H(s)}$$

$$H(s) \Rightarrow H(j(\omega_0 + \Delta\omega)) \approx H(j\omega_0) + \Delta\omega \frac{dH}{d\omega}$$

$$G(s) \Rightarrow G(j(\omega_0 + \Delta\omega)) \approx G(j\omega_0) + \Delta\omega \frac{dG}{d\omega}$$

$$\begin{aligned} \frac{Y}{X}(j\omega_0 + j\Delta\omega) &\approx \frac{-1 + \Delta\omega \frac{dH}{d\omega}}{1 + G(j\omega_0)H(j\omega_0)} \\ &\approx \frac{-1}{1 + G(j\omega_0)H(j\omega_0) + \Delta\omega \frac{dGH}{d\omega}} \\ &\approx -\frac{1}{\Delta\omega \frac{dGH}{d\omega}} \quad (\text{assume } G(j\omega_0)H(j\omega_0) \approx -1) \end{aligned}$$

$$\left| \frac{dGH}{d\omega} \right|^2 = \left| \frac{dGH}{d\omega} \right|^2 + \left| d \frac{\phi}{d\omega} \right|^2 |GH|^2$$

$$\begin{aligned} \therefore \left| \frac{Y}{X}(j\omega_0 + j\Delta\omega) \right|^2 &= \frac{1}{\Delta\omega^2} \cdot \frac{1}{\left| \frac{dGH}{d\omega} \right|^2 + |G(j\omega_0)|^2} \\ &= \frac{1}{\frac{w_0^2}{4} \left| \frac{dGH}{d\omega} \right|^2} \cdot \frac{4 \left( \frac{w_0}{2} \right)^2}{\Delta\omega^2} \cdot \frac{1}{|G(j\omega_0)|^2} \end{aligned}$$

$$= \frac{1}{4 \cdot Q^2} \cdot \left( \frac{w_0}{\Delta\omega} \right)^2 \cdot \frac{1}{|G(j\omega_0)|^2}$$

$$\text{where } Q = \frac{w_0}{2} \cdot \left| \frac{dGH}{d\omega} \right|$$

8.12 Soln:

$$x(t) = A \cos \omega_0 t + n_1(t) \cos \omega_0 t - n_2(t) \sin \omega_0 t.$$

$$x(t) = \sqrt{[A+n_1(t)]^2 + n_2^2(t)} \cos [\omega_0 t + \tan^{-1} \frac{n_2(t)}{A+n_1(t)}]$$

$$\approx \sqrt{[A+n_1(t)]^2 + n_2^2(t)} \cos [\omega_0 t + \frac{n_2(t)}{A}]$$

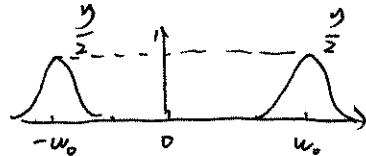
For PM,

$$x_1(t) = A \cos (\omega_0 t + \frac{n_2(t)}{A})$$

$$\approx A \cos \omega_0 t - n_2(t) \cdot \sin \omega_0 t.$$

Noise Power in PM sidebands

$$= n_2^2(t) \cdot \sin \omega_0 t.$$



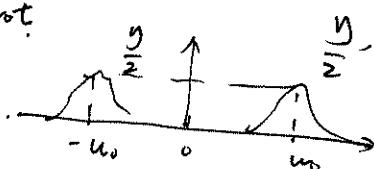
For AM,

$$x_2(t) = \sqrt{(A+n_1(t))^2 + n_2^2(t)} \cos \omega_0 t.$$

Noise Power in AM sidebands

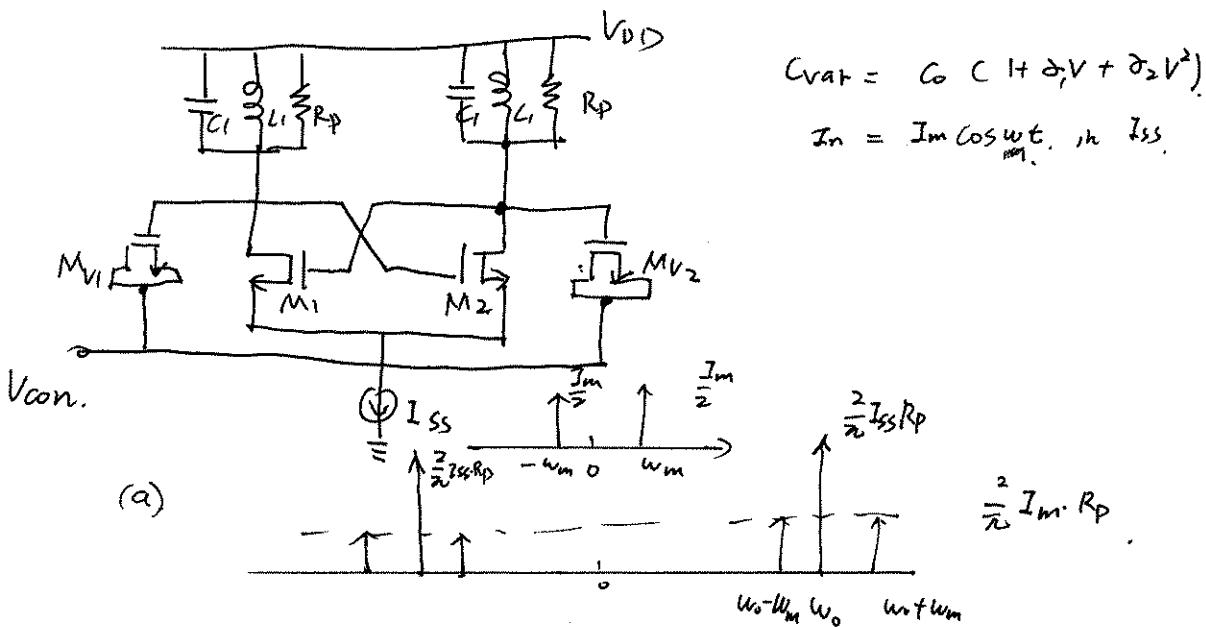
$$x_2(t) \approx A [1 + \frac{1}{2A} (n_1(t) + n_2(t))] \cdot \cos \omega_0 t$$

$$\Rightarrow n_1(t) \cdot \cos \omega_0 t.$$



So, the power carried by AM sidebands is equal to that carried by the PM sidebands and equal to the half power of n(t).

8.13 Soln: part (2)



$$\therefore V_{n,out} \Big|_{\text{near } w_0} = \frac{2}{\pi} I_m \cdot R_p \cdot [\cos(w_0 - w_m)t + \omega_s (w_0 + w_m)]$$

$$= \frac{4}{\pi} I_m R_p \cdot \cos w_{mt} \cdot \cos w_{st} \quad (\text{only near the } w_0).$$

(b)  $C_{var} = C_0 (1 + \partial_1 V + \partial_2 V^2)$

( $1 + \partial_1 V$ ) don't bring non-zero component, so only consider  $\partial_2 V^2$  term.

$$V_{out, total} \Big|_{\text{near } w_0} = \frac{4}{\pi} I_m R_p \cdot \cos w_{mt} \cdot \cos w_{st} + \frac{4}{\pi} I_{ss} R_p \cdot \cos w_{st}$$

$$V_{out, total}^2 = \left( \frac{4}{\pi} I_m R_p \right)^2 (\cos w_{mt} \cdot \cos w_{st})^2 + \left( \frac{4}{\pi} I_{ss} R_p \right)^2 \cos^2 w_{st} + \left( \frac{4}{\pi} R_p \right)^2 2 I_m I_{ss} \cdot \cos w_{mt} \cdot \cos w_{st}$$

take the DC part:

$$\begin{aligned} V_{out, total} \Big|_{\text{DC part}} &= \left( \frac{4}{\pi} I_{ss} R_p \right)^2 \cdot \frac{1}{2} + \frac{1}{4} \cdot \left( \frac{4}{\pi} I_m R_p \right)^2 \\ &= \left( \frac{4}{\pi} R_p \right)^2 \left( \frac{I_{ss}^2}{2} + \frac{I_m^2}{4} \right). \end{aligned}$$

$$\therefore C_{avg} = C_0 \partial_2 \left( \frac{4}{\pi} R_p \right)^2 \left( \frac{I_{ss}^2}{2} + \frac{I_m^2}{4} \right).$$

8.13 Soln: Part II

(b), (continue). If the definition of  $C_{avg}$  is above mentioned, the result is meaningless. That require us, to compute the average value in the  $\frac{2\pi}{\omega_0}$ .

So,  $C_{avg}$  is revised to

$$\Rightarrow C_{avg} = C_0 + \left(\frac{4}{\pi} R_p\right)^2 [I_{ss} \cdot I_m \cos \omega_m t + \frac{1}{4} I_m^2 \cdot \cos 2\omega_m t]$$

(c). Compute the <sup>PM.</sup> noise because of tank freq. modulation.

From the reference paper [13],

the conversion coefficient

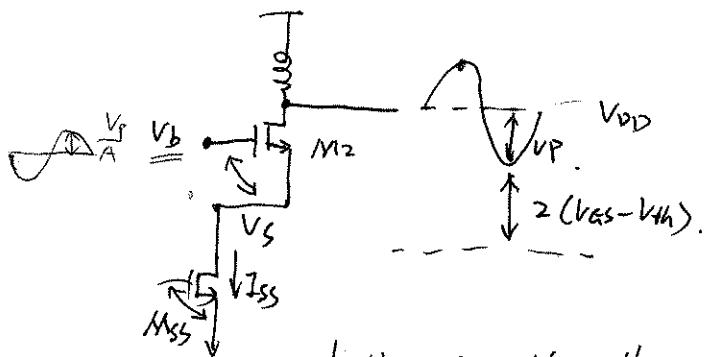
$$K_{AM/PM} = \left| \frac{\partial w}{\partial A} \right| \frac{A}{\omega}$$

$\left\{ \begin{array}{l} A \text{ is amplitude} \Rightarrow \frac{4}{\pi} I_{ss} R_p; \\ \omega \text{ is } \omega_m; \\ w \text{ is oscillation freq.;} \end{array} \right.$

8.14 Soln:-

In Fig. 8.86 (b).

the peak drain voltage swing  $V_{D_{max}}$ .



Let's consider the critical point, which is when  $M_{SS}$  &  $M_2$  are on the edge of triode region.

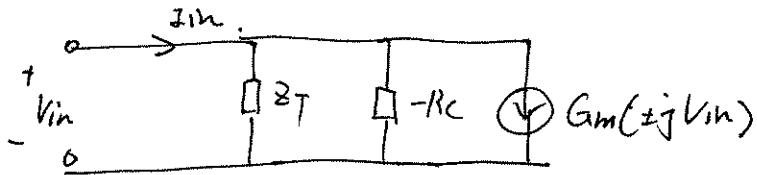
In this situation, the peak drain voltage swing is maximized as  $V_{DD} - 2(V_{GS} - V_{th})$ .

In this situation,  $V_b$ , the gate voltage, cannot change too much, that's because when the  $M_2$  is on the edge of triode region,  $V_{DS2} = V_b - V_s - V_{th}$ .

$$V_{DS1} = V_b - V_s - V_{th}, \quad (V_s = V_{GS} - V_{th}) \quad \text{where } V_b = V_b + \frac{V_p}{A}$$

If  $\frac{V_p}{A}$  is too large, the stress for  $M_{SS}$  will be too large.

8.5 Solve:



$$Z_T = (L, s) \parallel (C_1, s)^{-1} \parallel R_p.$$

$$\begin{cases} G_{m1} = G_{m2} = G_m, \\ V_x = \pm v_x \end{cases}$$

$$V_{in} = [ \dots [ Z_T \parallel (-R_C) ] \dots [ i_{in} \rightarrow G_m (\pm j V_{in}) ] ]$$

$$V_{in} (1 + G_m (\pm j) (-R_C) \parallel Z_T) = i_{in} (Z_T \parallel (-R_C))$$

$$\frac{i_{in}}{V_{in}} = \frac{1 \pm j G_m (Z_T \parallel (-R_C))}{Z_T \parallel (-R_C)}$$

$$= \frac{1}{Z_T \parallel (-R_C)} + j G_m.$$

$$= \frac{1}{j w_0} + j w_0 C_1 + \frac{1}{R_p} - \frac{1}{R_C} \pm G_m.$$

So.  $\frac{i_{in}}{V_{in}}$  can be zero even if  $\frac{1}{R_p} - \frac{1}{R_C} \neq 0$ .

$\Rightarrow$  Because of each oscillator receiving energy from each other  
the start-up condition need not to be as stringent  
as  $Z_T (s=w_0) = R_C$ .

8.16 Solu:

$$\Delta W = \frac{w_0}{2Q\tau_{\text{ank}}} \tan^{-1} \frac{g_{m3}}{g_{m1}}$$

$$g_{m3} = \sqrt{2m_n G_\infty \left( \frac{w}{L} \right)_3 \cdot (I_{T1} + I_n) / 2}$$

assume  $K = m_n G_\infty \left( \frac{w}{L} \right)_3$

$$g_{m3} = \sqrt{K(I_{T1} + I_n)} \approx \sqrt{KI_{T1}} + 2\sqrt{\frac{K}{I_{T1}}} \cdot I_n$$

$$\tan^{-1} \frac{g_{m1}}{g_{m3}} \approx \tan^{-1} \left( \sqrt{KI_{T1}} + 2\sqrt{\frac{K}{I_{T1}}} \cdot I_n \right)$$

$$\approx \tan^{-1}(\sqrt{KI_{T1}}) + \frac{2\sqrt{\frac{K}{I_{T1}}}}{1+KI_{T1}} \cdot I_n$$

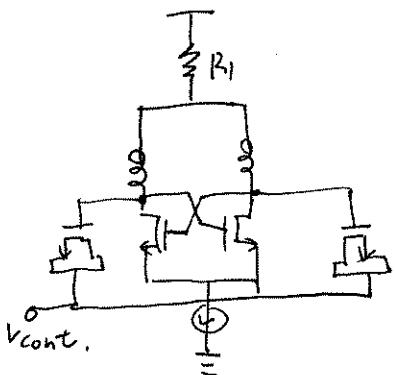
The approximation is done according to Taylor

Seriers, we choose the first and second terms.

Note:

$$f(x) \approx f(a) + f'(a) \cdot (x-a)$$

8.17 Soln:



$$(a) \quad w_{out} = \frac{1}{\sqrt{L_1 C_0 (1 + \delta \cdot V_{var})}}$$

$V_{var}$  is the average voltage across the varactor.

$$V_{var} = V_{DD} - R_1 \cdot I_{SS} - V_{cont.}$$

$$\begin{aligned} w_{out} &= \frac{1}{\sqrt{L_1 C_0 [1 + \delta (V_{DD} - R_1 \cdot I_{SS} - V_{cont.})]}} \\ &\approx \frac{1}{\sqrt{L_1 C_0 [1 + \delta (V_{DD} - V_{cont.})]}} + \frac{-L_1 C_0 \delta R_1}{2 \sqrt{L_1 C_0 [1 + \delta (V_{DD} - V_{cont.})]}} \cdot I_{SS}. \end{aligned}$$

$\Rightarrow$  the "gain" from  $I_{SS}$  to  $w_{out}$

$$= - \frac{L_1 C_0 \delta R_1}{2 \sqrt{L_1 C_0 [1 + \delta (V_{DD} - V_{cont.})]}}$$

$$(b) \quad w_{out} = w_0 + \frac{L_1 C_0 \delta R_1}{2 \sqrt{L_1 C_0 [1 + \delta (V_{DD} - V_{cont.})]}} \cdot I_{SS} \cos \omega_m t.$$

The frequency is modulated by  $I_{SS} \cos \omega_m t$ .

Using narrowband FM approximation,

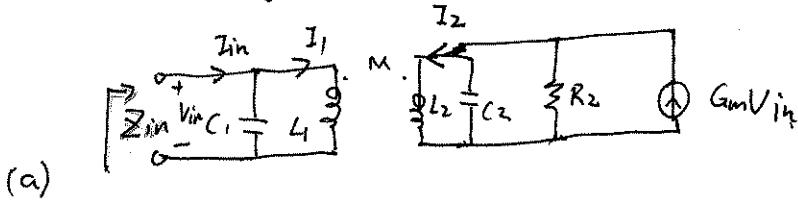
$$\Rightarrow x(t) = A \cos (w_0 t + B(t)) \quad B(t) = I_{SS} \cos \omega_m t$$

$$\approx A \cos w_0 t - A \cdot B(t) \cdot \sin w_0 t.$$

$$\approx A \cos w_0 t - A I_{SS} \cos \omega_m t \sin w_0 t.$$

So the sidebands relative magnitude is  $\frac{I_{SS}}{2}$ .

8.18 Solu:



(a)

$$\left\{ \begin{array}{l} V_{in} = I_1 L_1 s + I_2 M s \\ I_1 = I_{in} - V_{in} \cdot s C_1 \\ I_2 + \frac{(I_2 L_2 s + I_1 M s)}{R_2} = G_m V_{in} \end{array} \right. \quad \begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array}$$

$$V_{in} = (I_{in} - V_{in} \cdot s C_1) L_1 s + I_2 M s$$

$$\Rightarrow I_2 = \frac{V_{in} (1 + s^2 L_1 C_1) - I_{in} L_1 s}{M s} \quad \textcircled{4}$$

substitute  $I_1$ ,  $I_2$  into  $\textcircled{3}$ .

$$V_{in} \frac{1 + s^2 L_1 C_1}{M s} - I_{in} \frac{L_1}{M} + \left[ V_{in} \frac{L_2 (1 + s^2 L_1 C_1)}{M} - I_{in} \frac{L_1 L_2 s}{M} + I_{in} \cdot M s - V_{in} s^2 C_1 M \right] \frac{S R_2 C_2 + 1}{R_2} = G_m V_{in}$$

$$V_{in} \left( \frac{1 + s^2 L_1 C_1}{M} + \frac{L_2 (1 + s^2 L_1 C_1)}{M} \cdot \frac{S R_2 C_2 + 1}{R_2} - s^2 C_1 M \frac{S R_2 C_2 + 1}{R_2} - G_m \right)$$

$$= I_{in} \left( \frac{L_1}{M} + \frac{L_1 L_2 s}{M} - M s \right)$$

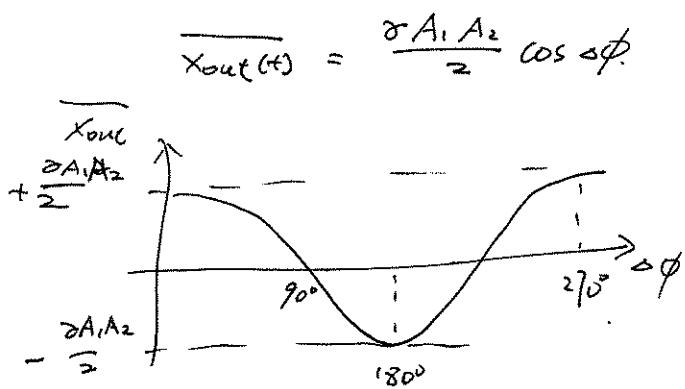
$$Z_{in} = \frac{V_{in}}{I_{in}}$$

$$(b). \text{ when } \frac{1 + s^2 L_1 C_1}{M} + \frac{L_2 (1 + s^2 L_1 C_1)}{M} \cdot \frac{S R_2 C_2 + 1}{R_2} - s^2 C_1 M \frac{S R_2 C_2 + 1}{R_2} - G_m = 0 \quad s = j\omega$$

$$\Rightarrow \omega_1 = \sqrt{\frac{R_2 + L_2 - G_m M R_2}{L_1 C_1 R_2 + L_1 L_2 C_1 - C_1 M^2}}$$

$$\omega_2 = \sqrt{\frac{L_1}{L_1 L_2 C_1 - C_1 M^2}}$$

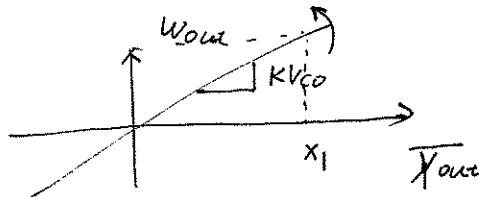
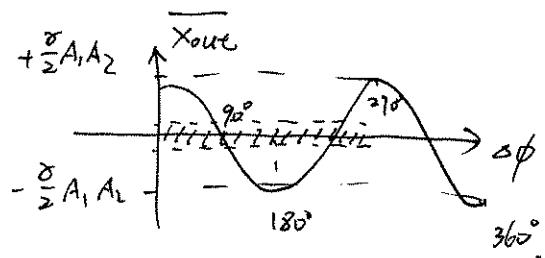
9.1 Solu:



The "gain" should be defined carefully in Phase Detector.

The zero gain at  $\Delta\phi = 180^\circ \& 0^\circ$ , only means when  $\Delta\phi$  is very near to  $0^\circ$  or  $180^\circ$ , the average of  $x_{out}$  will not change or change very slowly. However, with the accumulation of  $\Delta\phi$ , the gain cannot remain zero.

9.2 Soln:



$$w_{out} = w_0 + KVCO \cdot \overline{x}_{out}$$

$\therefore KVCO$  is very high  
 $\therefore \overline{x}_{out}$  is very small.

From the Figure above,

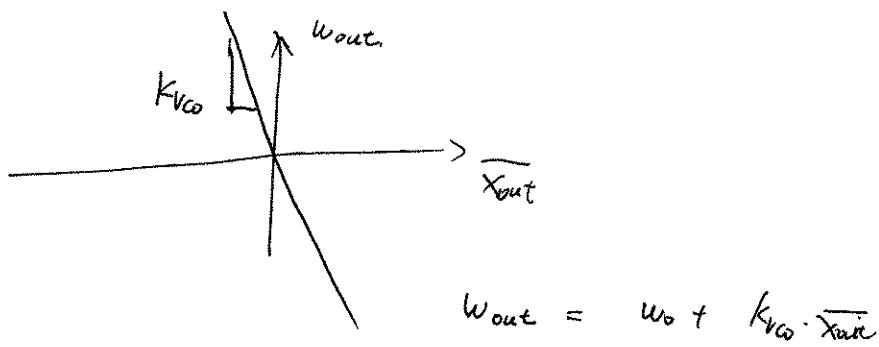
we can find  $\omega\phi$  should be near  $90^\circ + 180^\circ k$  ( $k=0, 1, 2, \dots$ )

$$\Rightarrow \omega\phi \approx 90^\circ + 180^\circ k \\ (k=0, 1, 2, \dots)$$

9.3 soln:

For Problem 2, if the  $k_{vo}$ 's sign is change.

the  $\bar{v}_{out}$  v.s.  $\bar{x}_{out}$  diagram should be



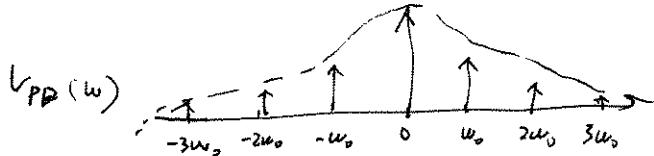
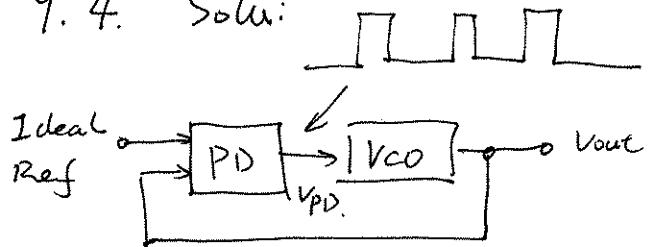
$\bar{x}_{out}$  will be also changing its sign.

However, our result won't change.

$$\Delta\phi \approx 90^\circ + 180^\circ \cdot k$$

$$(k=0, 1, 2, \dots)$$

9.4. Soln:



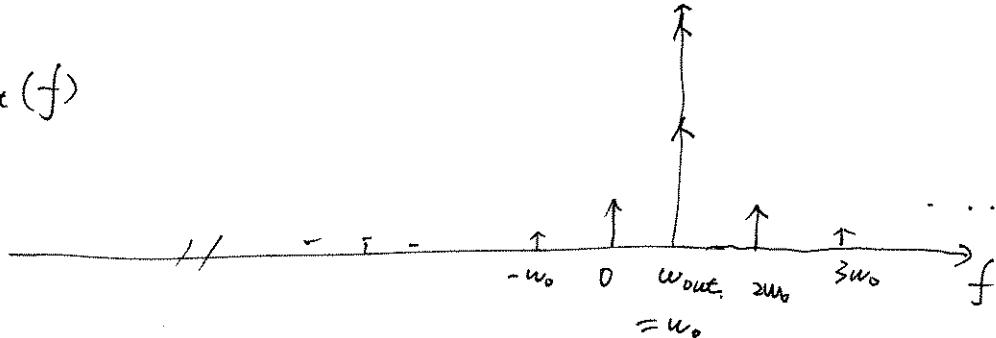
$V_{pd} \Rightarrow V_{out}$

$$\omega_{out} = \omega_0 + v_{pd} k_{vco}$$

$$V_{out} = V_0 \cos [ \omega_0 t + K_{vco} \int v_{pd}(t) dt ]$$

$$\approx V_0 \cos \omega_0 t - \underline{V_0 \cdot \int v_{pd}(t) dt \cdot k_{vco} \cdot \sin \omega_0 t}$$

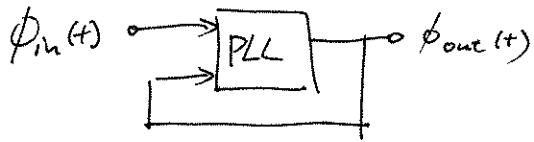
$V_{out}(f)$



$\Rightarrow$  So,

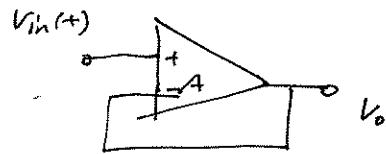
Output sidebands are located as the figure above.

9.5. Soln:



$$\therefore \frac{d\phi_{out}}{dt} = \frac{d\phi_{in}}{dt}$$

$$\Rightarrow \Delta\phi_{out} = \Delta\phi_{in}.$$



~~$\Delta V_{out} \approx V_{in}$~~

$$\Delta V_{out} = \frac{\Delta V_{in}}{A_{out}}$$

The statement of <sup>unity</sup> buffer is not correct.

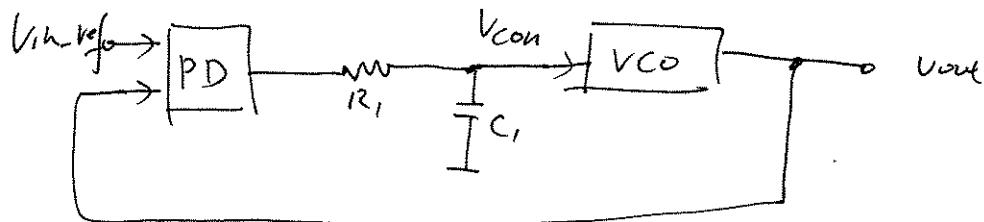
We can compute the transfer function of unity-gain buffer.

$$H(s) = \frac{A(s)}{1+A(s)}$$

$$s \approx 0 \text{ 时 } H(s) = \frac{A_0}{1+A_0} \approx 1$$

$$\text{So } \Delta V_{in} \Rightarrow \Delta V_{out} = \Delta V_{in}.$$

9.6 Soln:



(I) VCO noiseless

If we break the  $R_1$ ,

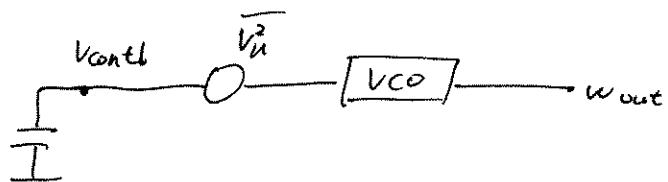
$V_{con}$  won't change because the charge is conserved on capacitor  $C_1$ .

So the wave will also be stable.

(II) VCO noisy.

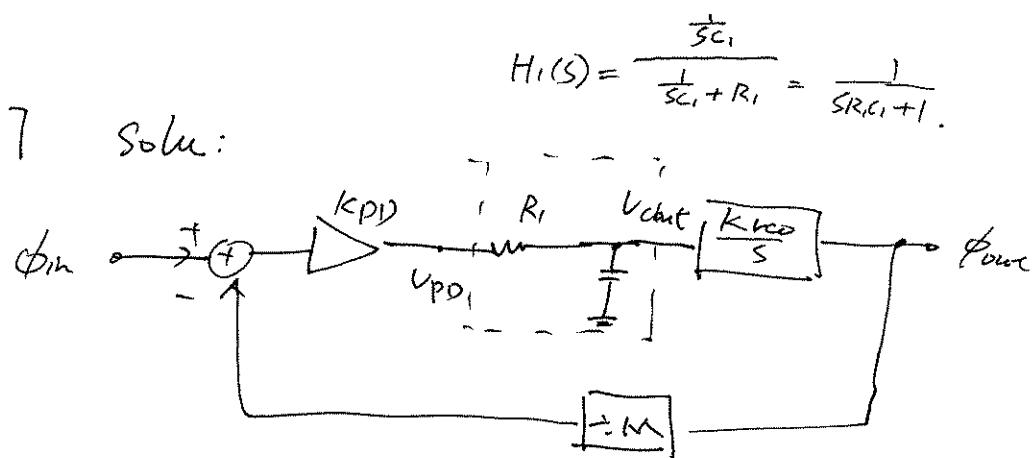
If we break the  $R_1$ ,

$V_{con}$  on  $C_1$  will be in series with a input-referred noise source.



So, the  $w_{out}$  will be modulated by  $\sqrt{V_n^2}$ .

9.7 Solve:



$$(\phi_{in} - \frac{\phi_{out}}{M}) \cdot K_{PD} \cdot \frac{1}{sC_1R_1 + 1} \cdot \frac{K_{VCO}}{s} = \phi_{out}$$

$$\phi_{in} \cdot \frac{K_{PD} \cdot K_{VCO}}{s(SR_1C_1 + 1)} = \phi_{out} \left[ 1 + \frac{K_{PD} \cdot K_{VCO}}{s(SR_1C_1 + 1)M} \right]$$

$$\frac{\phi_{out}}{\phi_{in}}(s) = \frac{\frac{K_{PD} \cdot K_{VCO} M}{s(SR_1C_1 + 1)M}}{s(SR_1C_1 + 1) + K_{PD} \cdot K_{VCO}}$$

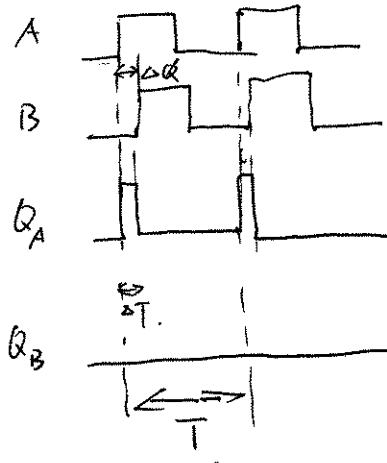
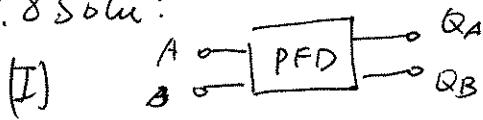
$$= \frac{\frac{K_{PD} \cdot K_{VCO} \cdot M}{s^2 R_1 C_1 M + s M + K_{PD} \cdot K_{VCO}}}{s^2 R_1 C_1 M + s M + K_{PD} \cdot K_{VCO}}$$

$$= \frac{\frac{K_{PD} \cdot K_{VCO} \cdot M}{R_1 C_1 M}}{s^2 + \frac{M}{R_1 C_1 M} \cdot s + \frac{K_{PD} \cdot K_{VCO}}{R_1 C_1 M}}$$

$$\omega_n = \sqrt{\frac{K_{PD} \cdot K_{VCO} \cdot \omega_{LPF}}{M}} \quad (\omega_{LPF} = \frac{1}{R_1 C_1})$$

$$\zeta = \frac{1}{2} \sqrt{\frac{\omega_{LPF} M}{K_{PD} \cdot K_{VCO}}}$$

9.8 Soln:

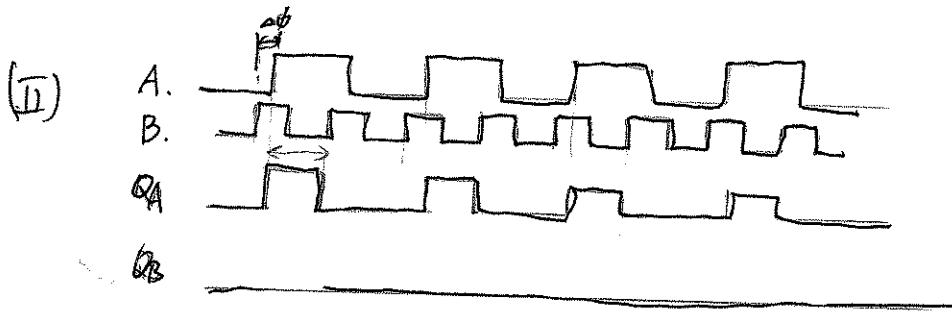


assume the magnitude of  
all signals  
will be 1.

$$\frac{\Delta\phi_{AB}}{2\pi} \cdot T = \Delta T$$

$$\overline{Q_A - Q_B} = \frac{\Delta T \cdot 1}{T} = \frac{\Delta\phi_{AB}}{2\pi}$$

$\Rightarrow \overline{Q_A - Q_B}$  is a linear function of input phase error.



assume  $f_B = 2f_A$ ,  $Q_A = f_A$ .

$$\overline{Q_A - Q_B} = \frac{\pi - \Delta\phi}{2\pi} \cdot \frac{1}{f_A} \cdot \frac{1}{f_A} = \frac{\pi - \Delta\phi}{2\pi}$$

$\Rightarrow \overline{Q_A - Q_B}$  is a linear function of input freq. difference.  
(It's a special case to demonstrate, but not  
a strict proof).

9.9. Soln.

Eg. (9.19)

$$H(s) = \frac{\frac{I_p \cdot K_{VCO}}{2\pi s} \cdot (R_1 C_1 s + 1)}{s^2 + \frac{I_p}{2\pi} K_{VCO} R_1 s + \frac{I_p}{2\pi} K_{VCO}}$$

$$= \frac{\frac{I_p \cdot K_{VCO} R_1}{2\pi} (s + \frac{w_n}{2s})}{s^2 + 2s w_n s + w_n^2}$$

peak is at  $w_n$ .

$$\begin{aligned} |H(jw_n)| &= \left| \frac{I_p \cdot K_{VCO} \cdot R_1}{2\pi} \right| \sqrt{\frac{jw_n + \frac{w_n}{2s}}{-w_n^2 + w_n^2 + 2s w_n^2}} \\ &= 2s w_n \sqrt{\frac{\sqrt{4s^2 + \frac{w_n^2}{4s^2}}}{2s w_n^2}} \\ &= 2s \sqrt{1 + \frac{1}{4s^2}} \\ &= \sqrt{4s^2 + 1} \end{aligned}$$

9. 10 SoLü:

$$\therefore f = \frac{R_1}{2} \sqrt{\frac{I_p C_1 K_{VCO}}{2\pi}}$$

$$w_n = \sqrt{\frac{I_p K_{VCO}}{2\pi C_1}}$$

$$f = 1$$

time constant

$$= 1/f \cdot w_n = \frac{25}{w_{in}}$$

$$w_n = \frac{w_{in}}{25}$$

$$K_{VCO} (V_{con, max} - V_{con, min}) = 10\% \cdot w_{in}$$

$$K_{VCO} \cdot V_{DD} = 0.1 \cdot w_{in}$$

$$\Rightarrow w_n = \frac{R_1}{2} \cdot \frac{I_p \cdot K_{VCO}}{2\pi}$$

$$\Rightarrow w_n = \frac{R_1}{2} \cdot \frac{I_p}{2\pi} \cdot \frac{0.1 \cdot w_{in}}{V_{DD}}$$

$$\Rightarrow \frac{R_1 \cdot I_p}{25} = R_1 \cdot I_p \cdot \frac{0.1}{4\pi} \frac{w_{in}}{V_{DD}}$$

$$\Rightarrow R_1 \cdot I_p = \frac{1}{4\pi} \cdot \frac{w_{in}}{25} \cdot V_{DD}$$

$$= 1.6 \pi V_{DD}$$

9. 11. Soln:

$$M_1 = 1000$$

AA

(a) If output freq. remain

$$M_2 = 500$$

$$\frac{A_{\text{side}}}{A_{\text{carrier}}} = \frac{1}{2\pi} \cdot \frac{\Delta T \cdot I_p}{C_2} f_{\text{res}} \cdot K_{VCO}$$

the ration doesn't change.

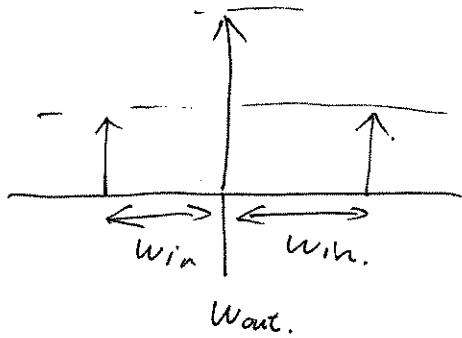
(b) If output freq. doubled,  $K_{VCO}$  doubled.

$$M_2 = 1000$$

$$\frac{A_{\text{side}}}{A_{\text{carrier}}} = \frac{1}{2\pi} \cdot \frac{\Delta T \cdot I_p}{C_2} f_{\text{res}} \cdot K_{VCO} \cdot 2$$

The ration also doubles.

9.12 Soln:

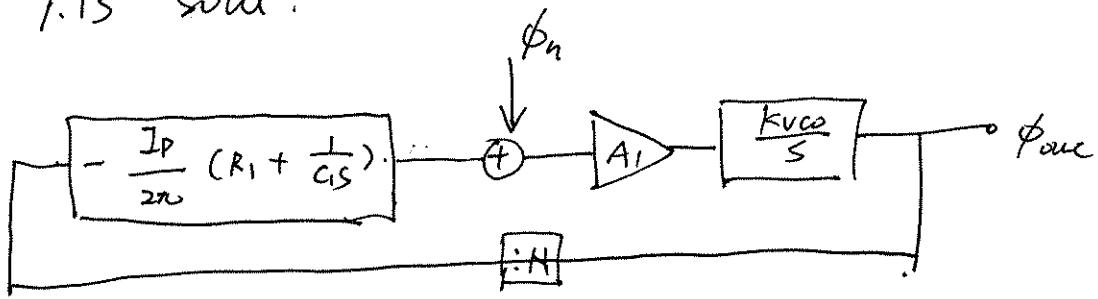


loop bandwidth  $\ll w_{in}$  to ensure the  
continuous-time approximation's validity.

So, the sidebands is located at  $w_{in}$  away from  
 $w_{out}$ . PLL cannot suppress high-frequency noise.

$\Rightarrow$  That's why PLL suppress VCO phase noise  
but not the sidebands due to ripple.

9.13 Solu:



$$\left[ \frac{\phi_{out}}{N} \left( -\frac{I_p}{2\pi} (R_1 + \frac{1}{C_1 s}) \right) + \phi_n \right] \cdot A_1 \cdot \frac{K_{VCO}}{s} = \phi_{out}.$$

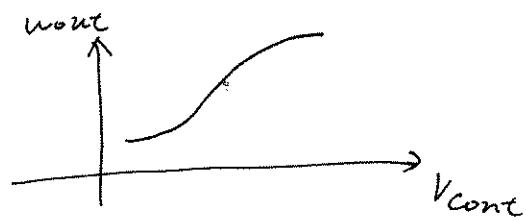
$$\left( 1 + \frac{I_p}{2\pi} (R_1 + \frac{1}{C_1 s}) \frac{A_1 K_{VCO}}{N s} \right) \phi_{out} = A_1 \cdot \frac{K_{VCO}}{s} \cdot \phi_n.$$

$$\phi_{out} = \frac{\frac{A_1 \cdot K_{VCO}}{s}}{1 + \frac{I_p}{2\pi} (R_1 + \frac{1}{C_1 s}) \frac{A_1 K_{VCO}}{N s}} \phi_n.$$

$$\phi_{out,n}^2 = \frac{(A_1 K_{VCO})^2}{\sqrt{\left( \frac{I_p A_1 K_{VCO}}{2\pi N C_1} - \omega^2 \right)^2 + \left( \frac{I_p A_1 K_{VCO} R_1}{2\pi} \right)^2}} \cdot \frac{\sigma^2}{f^2}$$

9.14. Solve:

14.



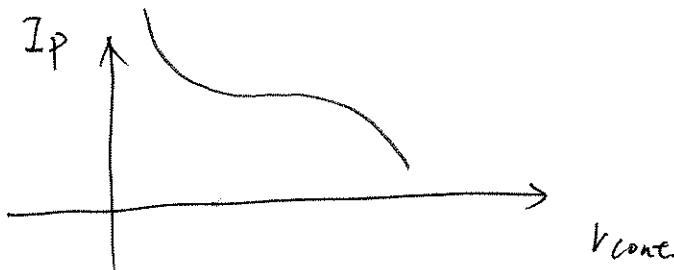
$V_{CO}$  characteristics

Characteristic equation :

$$: s^2 + \frac{I_p}{2a} K_{VCO} R_1 s + \frac{I_p}{2aC} K_{VCO} = 0.$$

$I_p \cdot K_{VCO}$  should be constant.

$$\Rightarrow I_p \propto \frac{1}{K_{VCO}}, \quad (K_{VCO} = \frac{dV_{out}}{dV_{cone}})$$



9.15 Soln:

(a) PFD now make half as many phase comparisons per second, pumping half as much as charge into the loop filter. Thus, loop is less stable.

Not correct.

Because of unchanged all loop parameters, the stability is unchanged.

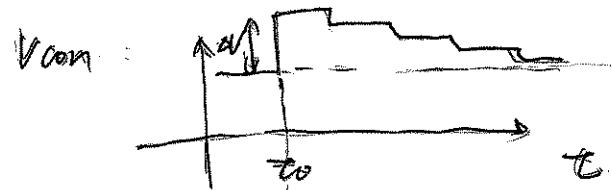
(b). Equation  $s = \frac{R_p}{2} \sqrt{\frac{J_p K_v C_p}{2\pi}}$  indicates that  $\gamma$  remains constant and the loop is as stable as before.

Correct.

$$H(s) = \frac{2swn \left( s + \frac{wn}{2} \right)}{s^2 + 2swns + wn^2} \text{ is unchanged.}$$

So  $\gamma$  is poles & zero. That means the Phase Margin of this loop is unchanged. Only operation point is different.

9.16 soln:



assume non-idealities  
are all neglected.



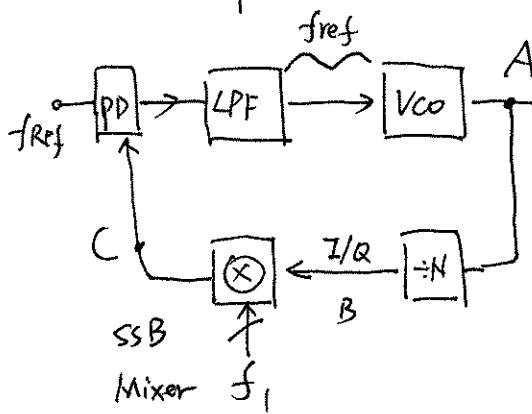
$V_{com}$  total change should be zero.

$V_{LPF}$  total change should be  $-\Delta V$ .

output frequency won't change. when phase-locked.

The input & output phase difference:

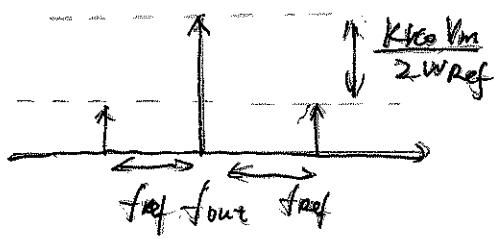
9.17 Soln:



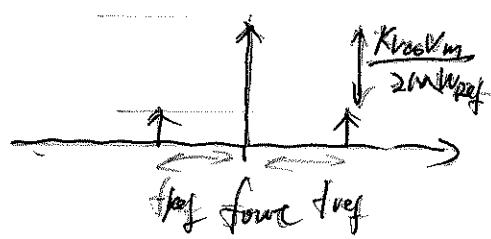
$$(a) \frac{f_{out}}{N} + f_1 = f_{ref}$$

$$f_{out} = N(f_{ref} - f_1)$$

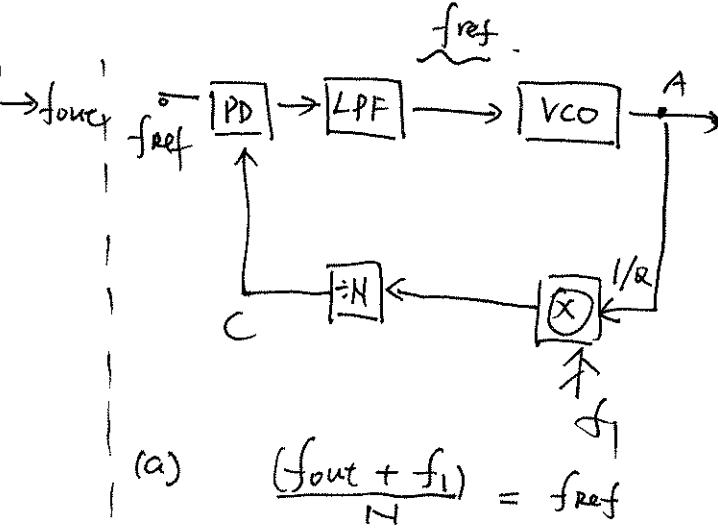
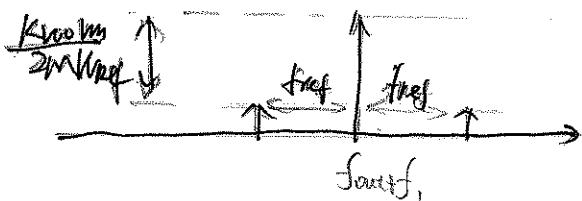
(b)



(c) ② B mode



③ C mode

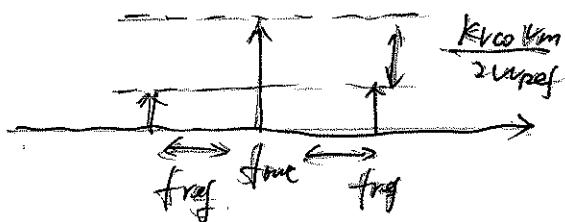


$$(a) \frac{(f_{out} + f_1)}{N} = f_{ref}$$

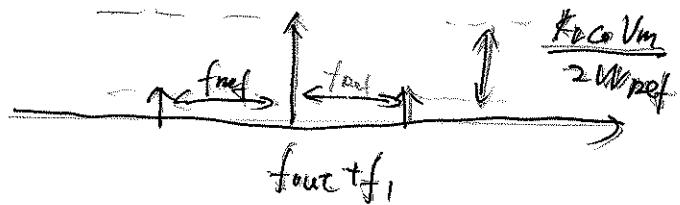
$$f_{out} = N \cdot f_{ref} - f_1$$

\* ( $V_m$  is the magnitude of ripple).

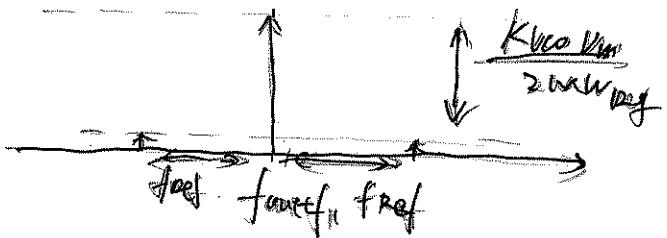
(b)



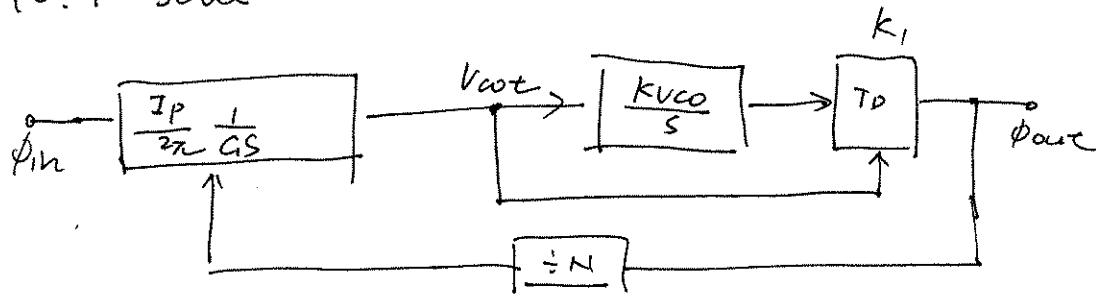
(c) ② B mode



③ C mode



10.1. Solutie:

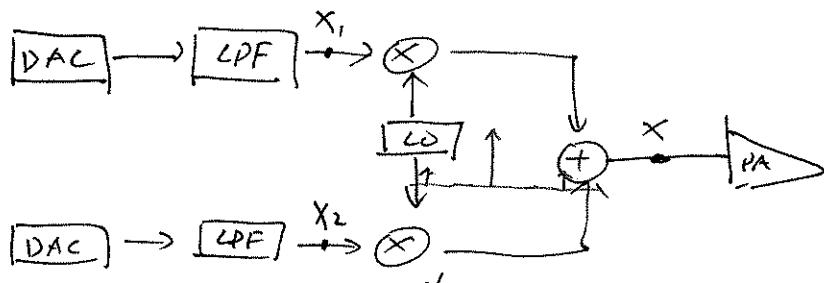


$$H_{open}(s) = \frac{I_p}{2\pi C_1 s} \left( \frac{KVCO}{s} + K_1 \right)$$

$$\begin{aligned} H_{close}(s) &= \frac{H_{open}(s)}{1 + 1/N \cdot H_{open}(s)} \\ &= \frac{\frac{I_p}{2\pi C_1 s} \left( \frac{KVCO}{s} + K_1 \right)}{1 + \frac{1/N I_p}{2\pi C_1 s} \left( \frac{KVCO}{s} + K_1 \right)} \\ &= \frac{\frac{I_p}{2\pi C_1} \frac{KVCO}{s} + \frac{I_p K_1}{2\pi C_1} \cdot s}{s^2 + \frac{1/N I_p K_1}{2\pi C_1} \cdot s + \frac{1/N I_p \cdot KVCO}{2\pi C_1}}. \end{aligned}$$

$$\left\{ \begin{array}{l} \frac{1/N I_p K_1}{2\pi C_1} = 2 \} w_n \\ w_n^2 = \frac{1/N I_p \cdot KVCO}{2\pi C_1} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} w_n = \sqrt{\frac{I_p \cdot KVCO}{2\pi C_1 N}} \\ ? = \frac{K_1}{2} \sqrt{\frac{I_p}{2\pi C_1 \cdot KVCO \cdot N}} \end{array} \right.$$

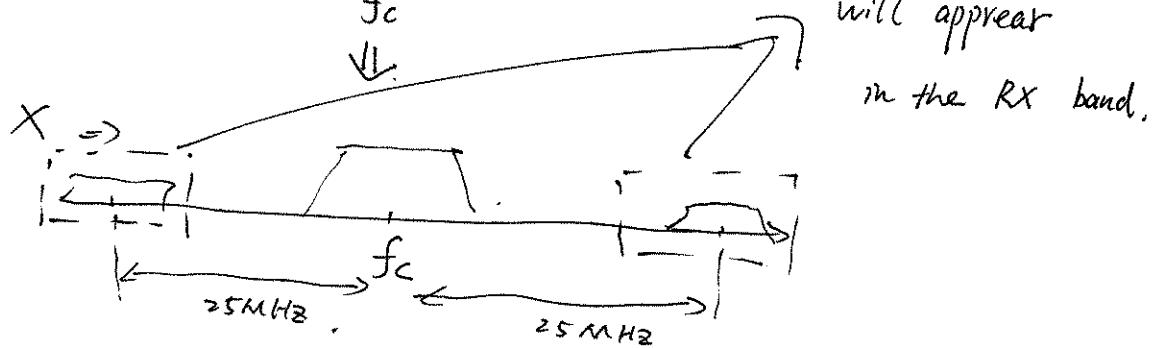
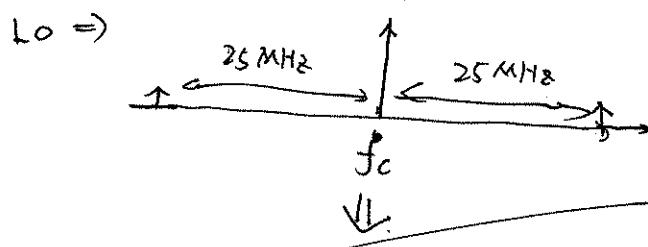
10.2 Soln:



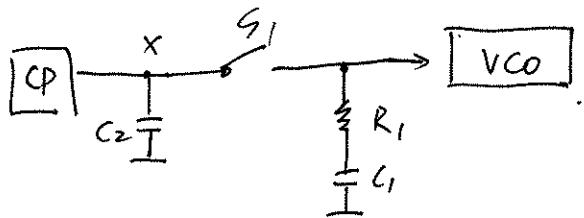
Prove that far-out noise of LO also appears as noise in RX band.

Determine the phase noise  $\textcircled{2}$  25 MHz offset for GSM.

Model the phase noise of LO as impulse.  
far-out



10.3 Solu:

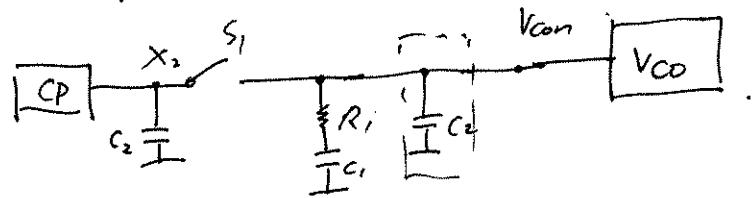


No. the sampling filter cannot remove the effect of the mismatch between the up & down current.

When there is an  $\Delta I = I_{\text{up}} - I_{\text{down}}$ , duration times  $\Delta T$ .  
~~At~~  $Q = \Delta I \cdot \Delta T$ , this amount of charge will be stored in capacitor  $C_2$  when  $S_1$  is off.

when  $S_1$  is on,  $Q = \Delta I \cdot \Delta T$  will share between  $C_1$  and  $C_2$ , which will affect the  $V_{\text{con}}$ .

Solu 10.4.

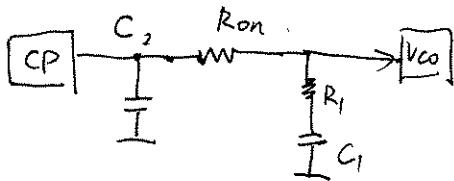


We still need a  $C_2$  at the node of  $V_{con}$ .

because the switch  $S_1$  truly helps us to remove the noise of ripple by PFD/CP circuit, but it also brings charge injection and clock feed through to the node of  $V_{con}$ .

The purpose of  $C_2$  is tied to  $V_{con}$  is to suppress the charge injection and clock feedthrough of  $S_1$ .

10.5 soln:



This  $R_{on}$  suppresses the ripple at node  $V_{cont}$ ,  
However, it degrades the phase Margin of the  
loop.

From Eq. (9.42) and Appendix (I).

we know,

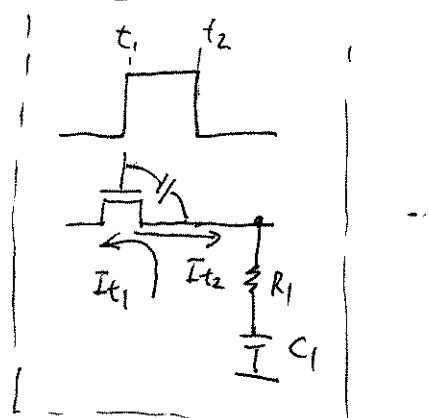
$$PM \approx \tan^{-1}(4\zeta^2) - \tan^{-1}\left(4\zeta^2 \frac{R_{on}C_2}{R_1C_1}\right)$$

which means  $(R_{on}C_2)^{-1}$  must remain 5 to 10 times  
higher than  $\omega_2$ . So  $R_{on}$  cannot be too large.

10.6 Soln:

Sure. Even when the PLL is Locked, the charge injection and clock feedthrough of  $s$ , still produce ripple on the  $V_{cont}$ . (Even Neglect the CP/PFD nonideality)

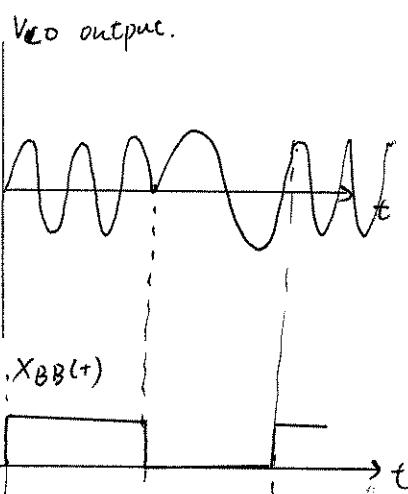
Because. when the switch is on i.e, discharges and  $I_{t2}$  charges the  $C_1$ , which will affect the  $V_{cont}$ .



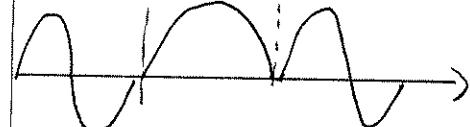
10. 7. Solu:

(I) The base band bit period  
much shorter than  
loop time constant.

(a). output of VCO.

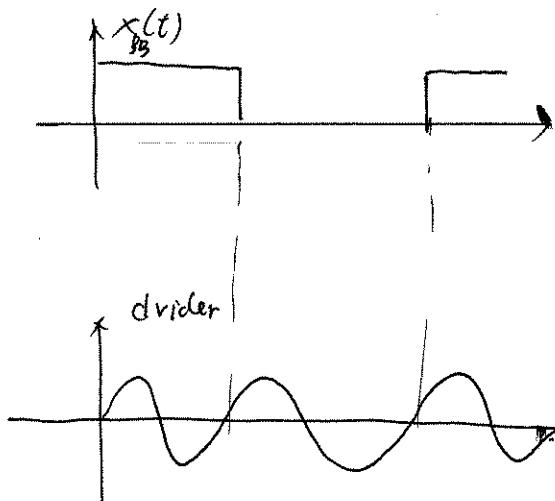
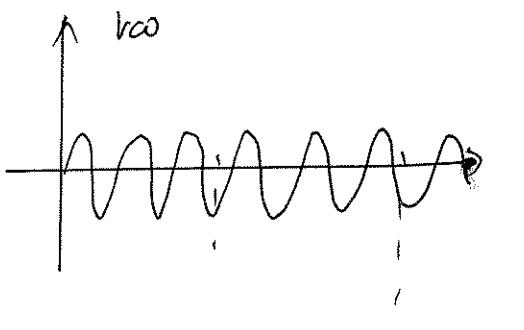


(b)



(II) The base band bit period  
much longer than  
the loop time constant

(a) output of VCO



10.8 Soln:

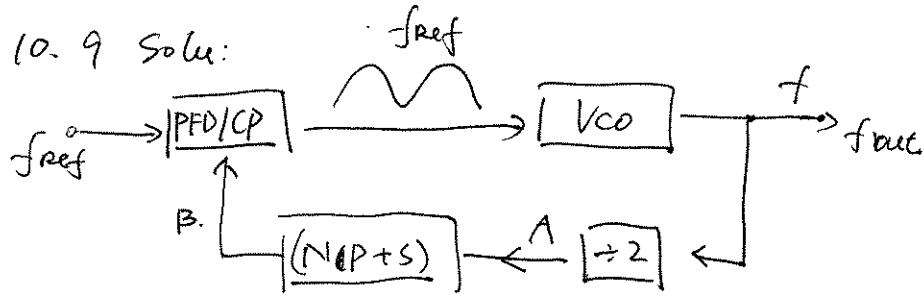
If the modulus control of the prescaler assume the change happens at prescaler is  $m (< N)$  at the beginning.

Ideally, the modulus should be  $N+1$ , now it changes to  $N$ .

$$\begin{aligned} N &\leq + (N+1)(P-S) \\ &= (N+1)P. \end{aligned}$$

So the result is that the modulus of the overall <sup>swallow</sup> divider is changed to  $(N+1)P$ .  
<sup>pulse</sup>

10. 9 Solu:



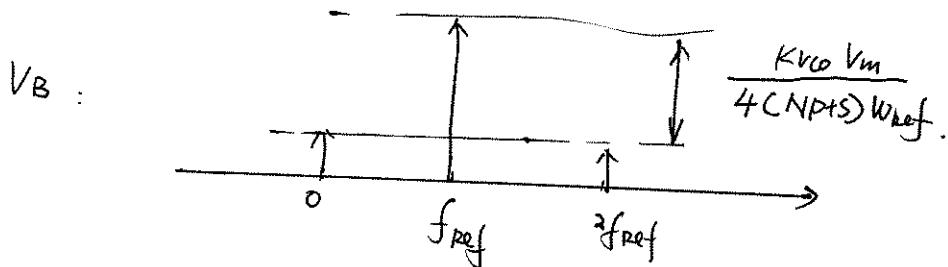
$$f_{out} = 2 \cdot (N+P+S) f_{ref.} + K_{VCO} V_m \sin f_{ref.} \pi t.$$

② mode A.

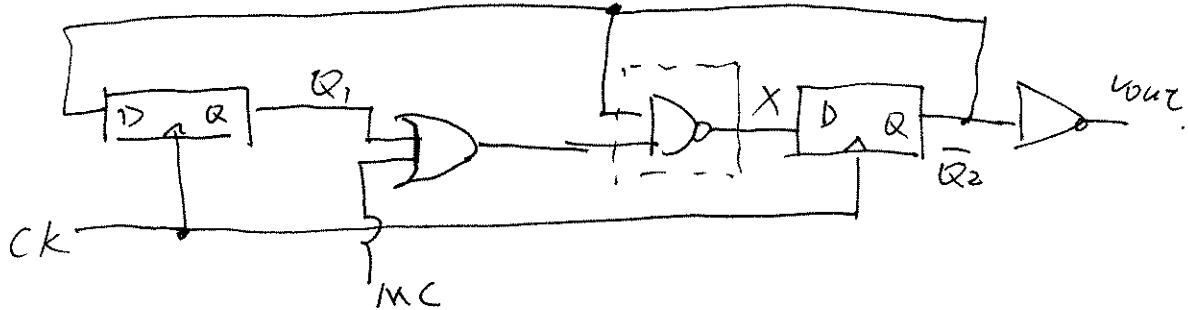
$$V_A = V_o \cos \left[ \frac{1}{2} f_{out} \pi t + \frac{K_{VCO}}{2} \int V_m \sin f_{ref.} \pi t \right]$$

$$\approx V_o \cos \left[ 2\pi \frac{f_{out}}{2} t \right] - \frac{K_{VCO} V_m}{2 \cdot 2 \cdot w_{in}} V_o \cos \left( \frac{w_{out}}{2} + w_{in} \right) t \quad \frac{K_{VCO} V_m}{2 M \cdot w_{in}} V_o \cos \left( \frac{w_{out}}{2} - w_{in} \right)$$

② mode B.



10. 10. Soln:



If  $G_1$  is the NAND Gate,

when  $MC = 1$ ,  $X$  is always 0.

So the  $\div 2$  divider cannot work correctly.

When  $MC = 0$ ,

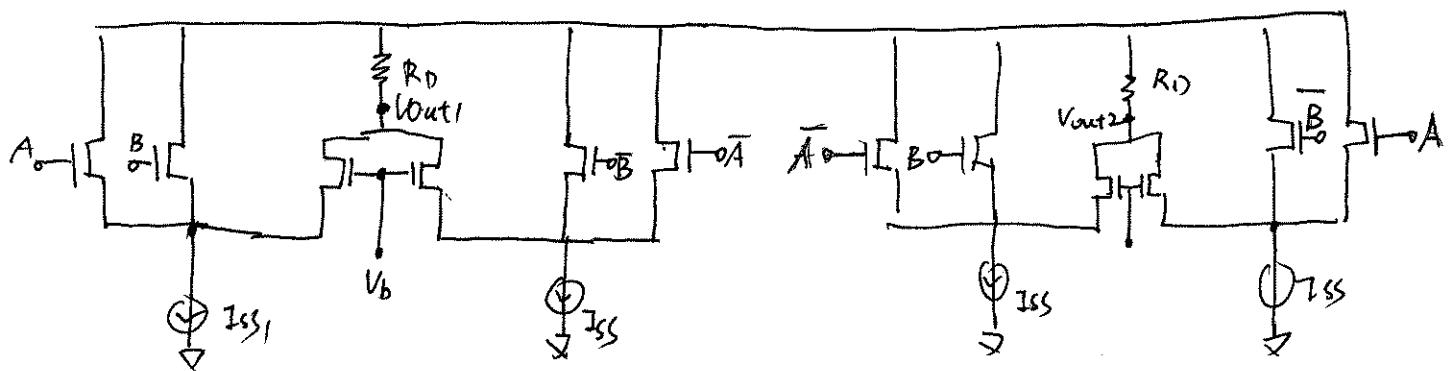
$Q_1$	$Q_2$	$\bar{Q}_2$	$X$	$V_{out}$
0	0	1	0	0
1	0	1	0	0
1	0	0	1	0

$X$  is always 0 also.

So the  $\div 3$  divider can't work correctly too.

10.11. Soln:

Modify Fig. 10.42 to provide differential outputs.



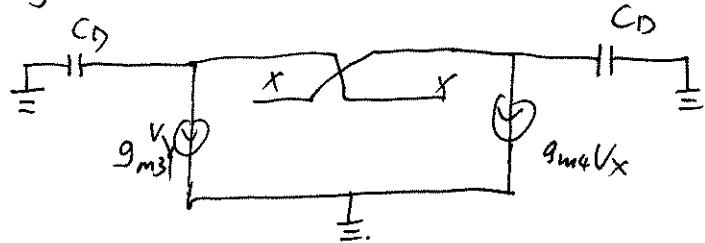
$$\therefore V_{out1} = \bar{A}B + A\bar{B} ;$$

$$\begin{aligned} \overline{V_{out1}} &= \overline{\bar{A}B + A\bar{B}} = (A + \bar{B}) \cdot (\bar{A} + B) \\ &= A \cdot B + \bar{B} \cdot \bar{A}. \end{aligned}$$

$$\therefore \underline{V_{out2} = A \cdot B + \bar{A} \cdot \bar{B}}$$

10. 12 Solu:

If Fig 10.44 (a) changes into



$$C_D \cdot \frac{dV_X}{dt} + g_{m3,4} V_Y = 0$$

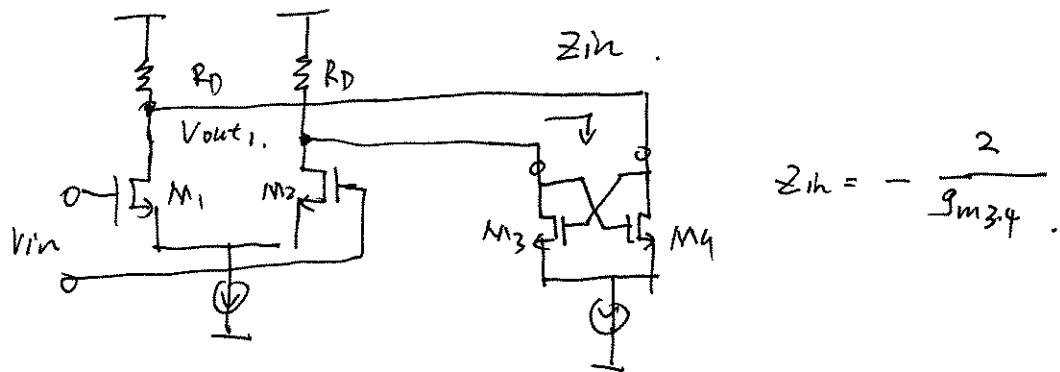
$$C_D \cdot \frac{dV_Y}{dt} + g_{m3,4} V_X = 0$$

$$C_D \left( \frac{d(V_X - V_Y)}{dt} \right) = - (g_{m3,4}) (V_X - V_Y)$$

$$\Rightarrow V_{XY} = V_{XY_0} \exp \left( - \frac{g_{m3,4}}{C_D} t \right)$$

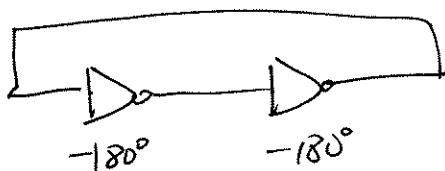
$$\Rightarrow T = \frac{C_D}{g_{m3,4}}$$

10.13 Soln:



From stage 1 operation, we can find that there are  $180^\circ$  phase shift., and also produces some gain.

So for two-stage consideration, we can model this as



so Barkhausen condition is satisfied.

10.14. Soln:

From example 10.18.

$$T_{reg} = \frac{R_D C_D}{g_{m3,4} R_D - 1} \quad \text{Equation (10.44)}$$

$$\text{If } g_{m3,4} R_D \gg 1 \Rightarrow T_{reg} = \frac{C_D}{g_{m3,4}}$$

independent of  $R_D$ .

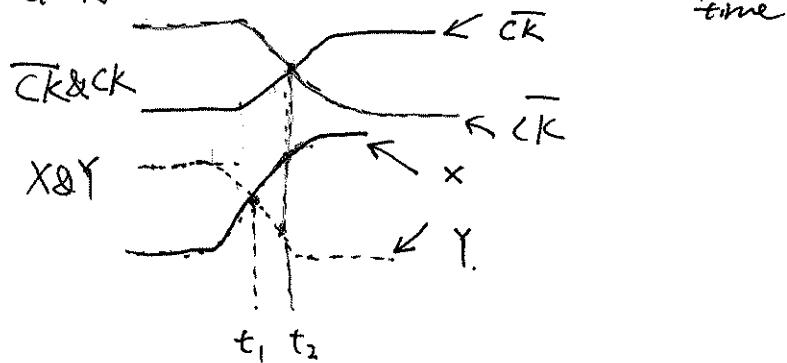
When  $R_D$  is very large, the current generated by  $M_3$  &  $M_4$  will mainly charge the capacitor  $C_D$ . That means the current flowing into  $R_D$  is neglected.

So  $T_{reg}$  is determined by the  $C_D$  and  $g_{m3,4}$ .

10.15 Solu:

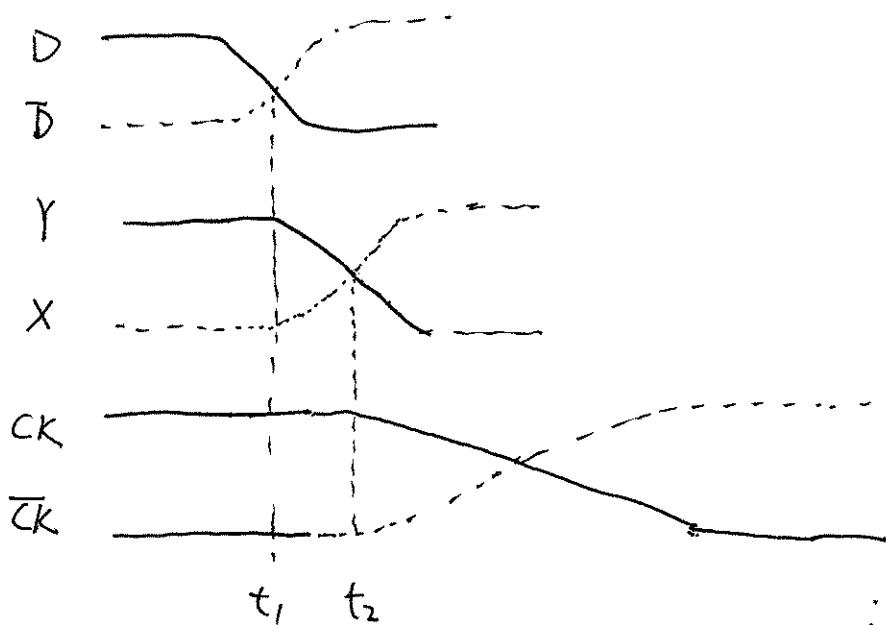
In Fig. 10.43.

(a) clock transition time on the  $X \cdot Y$  <sup>order of</sup> <sub>time</sub> constant.



From the above figure, when clock transition time is on the order of  $X \cdot Y$  come constant, the D latch can be working correctly.

(b) clock transition time is much longer.



From the above, figure, we know that even the clock transition is much longer, the operation will be still correct.

10.16. Solu:

Fig. 10.68.

loop gain = mixer conversion gain \* amplif.

$$= \frac{\pi^2}{4} \cdot g_{m5,6} \cdot \left| R_p \parallel \left( -\frac{2}{g_{m7,8}} \right) \right|$$

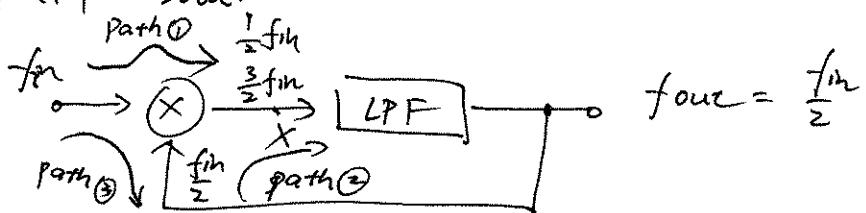
$$= \frac{\pi^2}{4} \cdot g_{m5,6} \cdot \left| \frac{R_p \cdot \frac{2}{g_{m7,8}}}{\frac{2}{g_{m7,8}} - R_p} \right|$$

$$= \frac{84}{\pi} \frac{g_{m5,6}}{g_{m7,8}} \cdot \left| \frac{g_{m7,8} \cdot R_p}{2 - R_p \cdot g_{m7,8}} \right|$$

$$= \frac{4 g_{m5,6} R_p}{\pi |2 - R_p \cdot g_{m7,8}|}$$

10.17

Solu:



Path ①  $f_{in}$  to  $X$  node, will be attenuated by LPF.

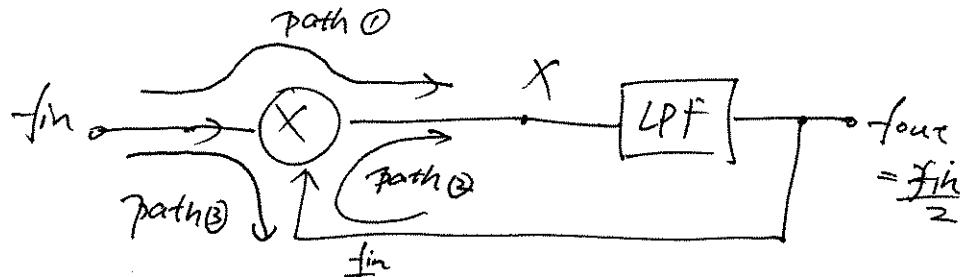
path ②. the frequency is  $f_{out}$  itself, which doesn't contribute spurs.

Path ③.

Feed through  $f_{in}$  signal with mixing with the input  $f_{in}$  signal, which results at output is just DC signal.

In summary, even if the mixer suffers from port to port feed throughs, there are not spurs at output.

10.18 Soln:



Assume node nonlinearity by  $y(x) = ax + bx^2 + cx^3$ ;

path ① at  $X$  node.

$\Rightarrow f_{in}, 2f_{in}, 3f_{in}$  components.

path ② at  $X$  node

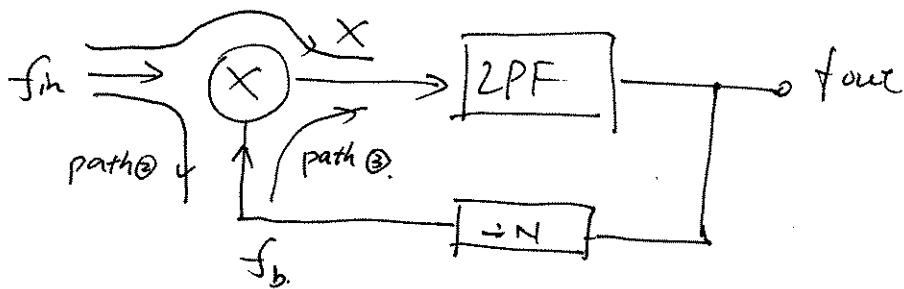
$\Rightarrow \frac{f_{in}}{2}, f_{in}, \frac{3}{2}f_{in}$  components.

path ③ at  $X$  node.

$\Rightarrow 0, f_{in}, 2f_{in}$  components.

In summary, if the Lowpass filter has enough orders to attenuate the component at  $f_{in}$ , then the results are the same with the previous problem.

10. 19 Soln: path①



$$f_{out} = \frac{N}{N+1} f_{in}, \quad f_b = \frac{1}{N+1} f_{in}.$$

(a) port-to-port feedthrough.

path ①  $\Rightarrow$  ② X node.

$$f_{in}, \quad f_{in} + \frac{1}{N+1} f_{in}, \quad f_{in} - \frac{1}{N+1} f_{in}$$

path ③  $\Rightarrow$  ② X node.

DC,

path ④  $\Rightarrow$  ② X node

$$\frac{1}{N+1} f_{in}$$

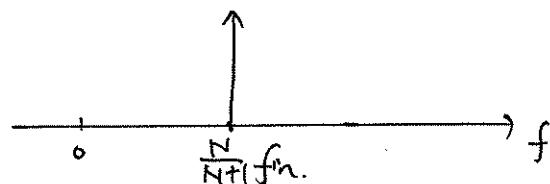
In sum, there are three spurs.

(b) port nonlinearity

② X node.

$$f_{in} + \frac{1}{N+1} f_{in}, \quad \frac{N}{N+1} f_{in}, \quad \frac{2N}{N+1} f_{in}, \quad \dots$$

output spectrum.



port nonlinearity can produce high-order harmonics of  $\frac{N}{N+1} f_{in}$ . However, they can be filtered by LPF.

10.20 Soln:

Which mixer topologies are suited to the Miller Divider?

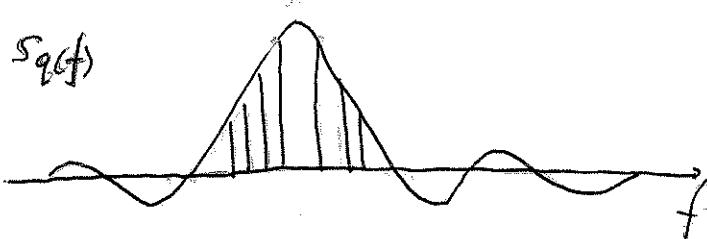
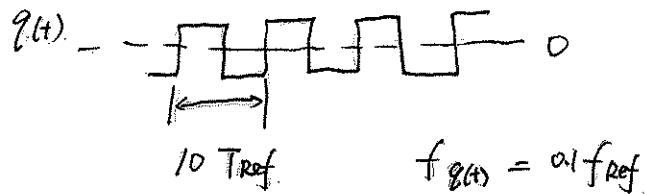
In order to achieve more gain, I'd like to choose active mixer. Because, <sup>before</sup> steady state the loop need enough loop gain to startup.

If passive mixer must be used, some gain enhancement technique should be used like Fig. 10.68. a coupled pair M7 & M8.

11.1 Soln:

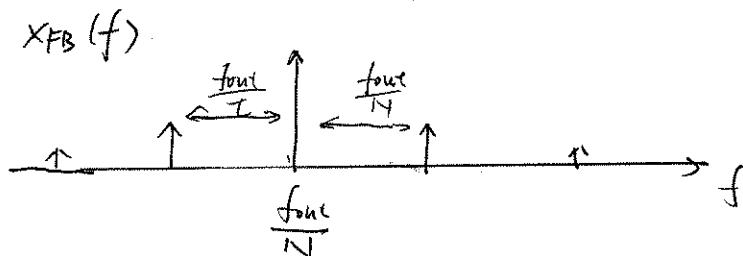
$$-f_{FB}(t) = \frac{f_{out}}{N + b(t)} \approx \frac{f_{out}}{N} \left(1 - \frac{b(t)}{N}\right)$$

$$b(t) = \alpha + g(t), \quad \alpha = 0.1, \text{ periodic.}$$



$$X_{FB}(t) = V_0 \cos \left( \frac{f_{out}}{N} \left(1 - \frac{b(t)}{N}\right) t \right)$$

With narrowband FM approximation



11.2 Soln:

$$f_{FB}(t) \approx \frac{f_{out}}{N} \left(1 - \frac{b(t)}{N}\right)$$

$$\begin{aligned}\phi_{out}(t) &= \frac{f_{out}}{N} t + \phi_0 \\ &= \frac{f_{BB} N}{1 - \frac{b(t)}{N}} t + \phi_0\end{aligned}$$

$$= \frac{f_{FB}(t) N^2}{N - b(t)} \cdot t + \phi_0.$$

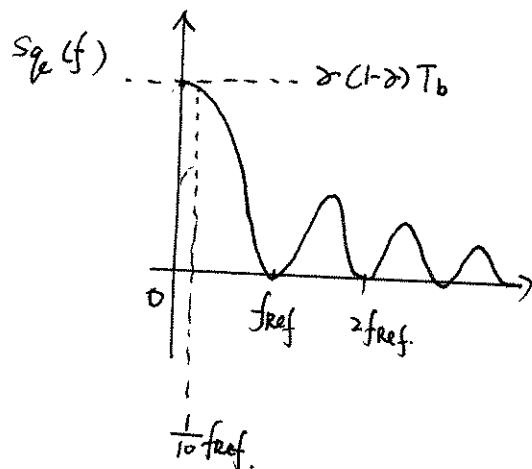
From the equation above, we can conclude that  $f_{FB}(t)$  and  $b(t)$  are periodic  $\Rightarrow \frac{f_{FB}(t) N^2}{N - b(t)}$  are also periodic.

$$\phi_{out}(t) = A(t) \cdot t + \phi_0.$$

$$\frac{d\phi_{out}(t)}{dt} = A(t) + A'(t)(t).$$

Solu 11.3.

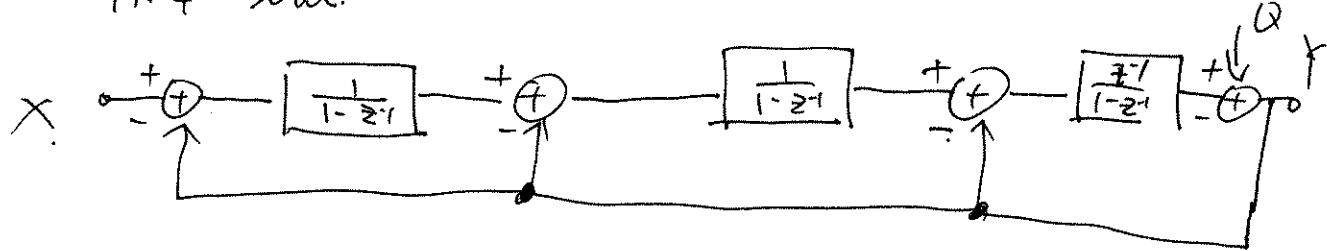
If  $T_b = T_{ref}$ .



It implies that the low-frequency part ( $f < \frac{1}{10} f_{ref}$ ) of  $S_g(f)$  can be suppressed by PLL. The components with higher frequencies will be the critical part of  $S_g(f)$ .

$$\begin{aligned}
 S_g\left(\frac{1}{10}f_{ref}\right) &= \delta(1-\delta) \cdot f_{ref} \left( \frac{\sin \pi \frac{f}{f_{ref}}}{\pi f} \right) \quad |f = \frac{1}{10}f_{ref} \\
 &= \delta(1-\delta) f_{ref} \frac{\sin \pi \frac{1}{10}}{\pi \cdot \frac{1}{10} f_{ref}} \\
 &= 0.98 \delta(1-\delta).
 \end{aligned}$$

11.4 Solvi:



assume  $X = 0$ .

$$\left\{ \left[ (0 - Y(z)) \frac{1}{1-z^{-1}} - Y(z) \right] \frac{1}{1-z^{-1}} - Y(z) \right\} \frac{z^{-1}}{1-z^{-1}} + Q = Y(z)$$

$$\Rightarrow \frac{Y(z)}{Q(z)} = (1-z^{-1})^3.$$

$$\Rightarrow S_Q(f) = S_g(f) |2 \sin(\pi f T_{CK})|^6.$$

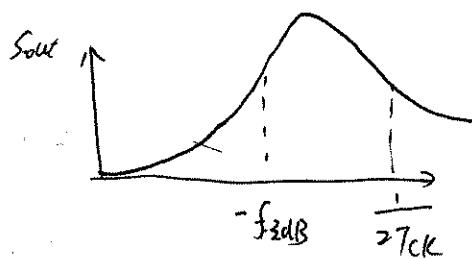
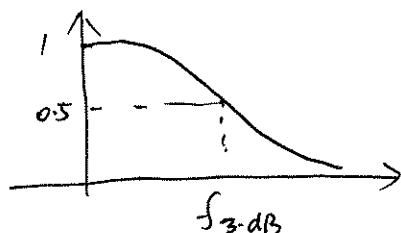
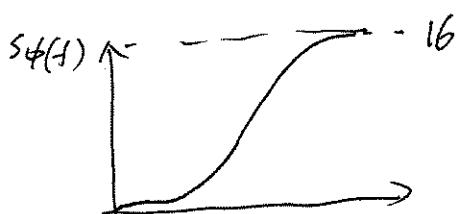
$$\because \phi(z) = Y(z) / (1-z^{-1})$$

$$\therefore \phi(z) = (1-z^{-1})^2 Q(z)$$

$$\therefore S_\phi(f) = |1-z^{-1}|^4 S_g(f)$$

$$= |2 \sin \pi f T_{CK}|^4 S_g(f)$$

$$\therefore S_{out}(f) = S_\phi(f) \cdot N^2 \frac{4 \zeta^2 w_n^2 \omega^2 + w_n^2}{(\omega^2 - w_n^2)^2 + 4 \zeta^2 w_n^2 \omega^2}$$



11.5 soln:

For a second-order:

$$S_y(f) = S_g(f) |2 \sin(\pi f T_{ck})|^4$$

For a fourth-order:

$$S_y(f) = S_g(f) |2 \sin(\pi f T_{ck})|^8$$

$|H_{PLL}(f)|^2$  is the same.

For 2nd order

$$S_{out}(f)_{2nd} = S_g(f) |2 \sin(\pi f T_{ck})|^2 |H_{PLL}(f)|^2$$

For 4th order

$$S_{out}(f)_{4th} = S_g(f) |2 \sin(\pi f T_{ck})|^6 |H_{PLL}(f)|^2$$
$$\frac{f_{ref.}}{10} \approx f < f_{ck}$$

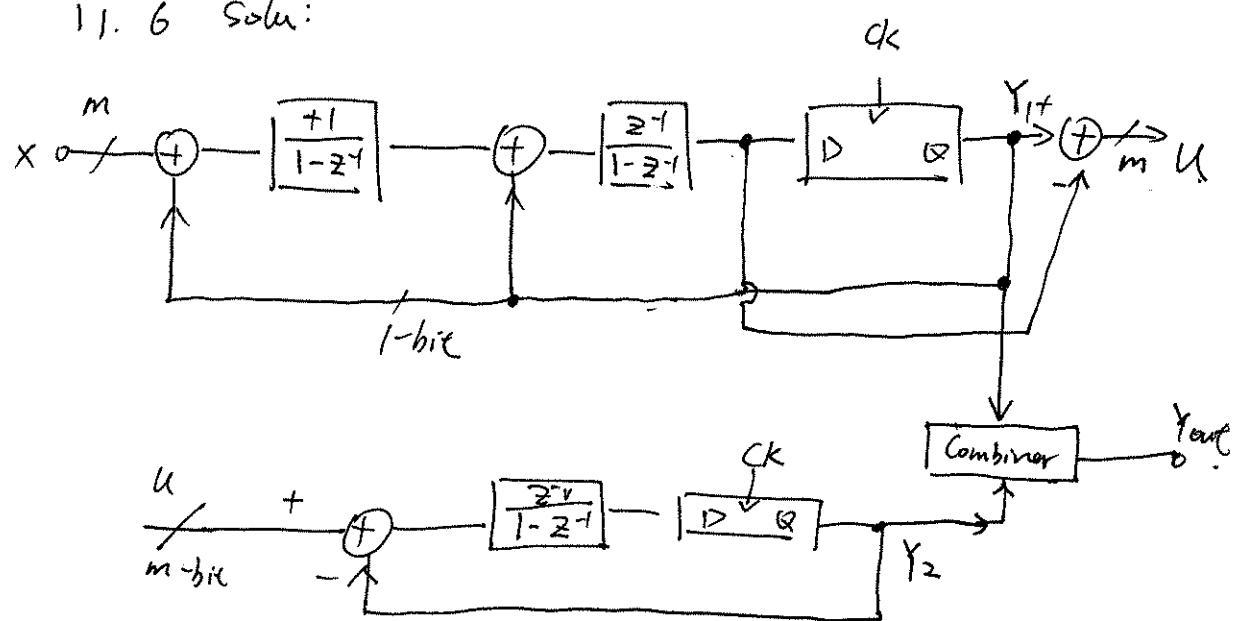
$$\therefore \frac{S_{out}(f)_{4th}}{S_{out}(f)_{2nd}} = |2 \sin(\pi f T_{ck})|^4 \approx 16 f^4 (\pi T_{ck})^4$$

assume  $f = \frac{f_{ck}}{10}$

$$10 \lg \left( 16 - \frac{f_{ck}^4}{10^4} \cdot \pi^4 \right)^4 = 10 \log_{10} \left( \frac{16 \pi^4}{10^4} \right)^{-1}$$

$$= 8.1 \text{ dB.}$$

11. 6 Soln:



$$Y_1(z) = (1 - z^{-1})^2 Q(z)$$

$$Y_2(z) = z^{-1} Q(z) + (1 - z^{-1}) Q'(z)$$

$$Y_{out}(z) = Y_1(z) \cdot z^{-1} - (1 - z^{-1})^2 Y_2(z)$$

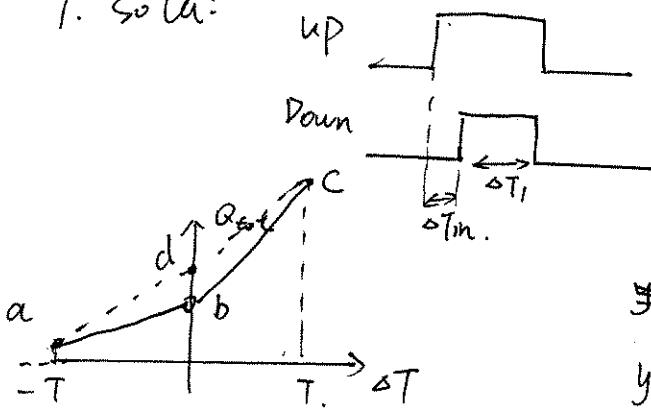
$$= z^{-1} (1 - z^{-1})^2 Q(z) - z^{-1} (1 - z^{-1})^2 Q(z) - (1 - z^{-1})^3 Q'(z)$$

$$= -(1 - z^{-1})^3 Q'(z)$$

↓

This is what combiner should do.

7. Soln:



$$\text{assume } (I_1 - I_2) \Delta T_1 = A$$

$$b \leftarrow (0, (I_1 - I_2) \Delta T_1)$$

$$a \leftarrow (-T, -I_2 T + (I_1 - I_2) \Delta T_1)$$

$$c \leftarrow (+T, +I_1 T + (I_1 - I_2) \Delta T_1)$$

$$\frac{y - I_1 T - (I_1 - I_2) \Delta T_1}{x - T} = \frac{I_1 + I_2}{2}$$

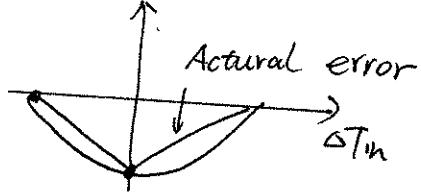
$$y = \frac{I_1 + I_2}{2} x + \frac{I_1 - I_2}{2} \cdot T + A.$$

$$-b + \frac{I_1 - I_2}{2} \cdot T + A = A$$

$$\Rightarrow b = \frac{I_1 - I_2}{2} \cdot T$$

$$a \Delta T_{in}^2 - b = 0$$

$$\Rightarrow \Delta T_{in} = \pm \sqrt{\frac{a}{b}}$$



Actual error:

$$\text{Error} = \frac{I_1 - I_2}{2} \Delta T_{in} - \frac{I_1 - I_2}{2} T = 0$$

$$\Rightarrow \Delta T_{in} = T$$

$$\therefore \sqrt{\frac{a}{b}} = T$$

$$\Rightarrow a = T^2 b = T^3 \frac{I_1 - I_2}{2} = 2.5\% T^3$$

11.8 Solu:

(a) unequal Up and Down pulsewidths.

Unequal up and down pulse widths are equivalent to the  $I_1$  and  $I_2$  mismatch in CP. So it's noise folding behavior can be moderate as the same as Fig. 11.30.

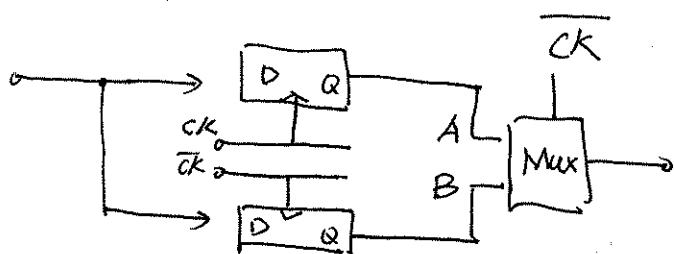
(b) charge injection mismatch between up & down switch in CP.

For charge injection mismatch, it contribute to the  $(I_1 - I_2)$  mismatch every cycle. That means,

In equation  $Q_{tot} \propto I_{av} \Delta T_m + a \Delta T_m^2 - b$ ,  
a will be larger.

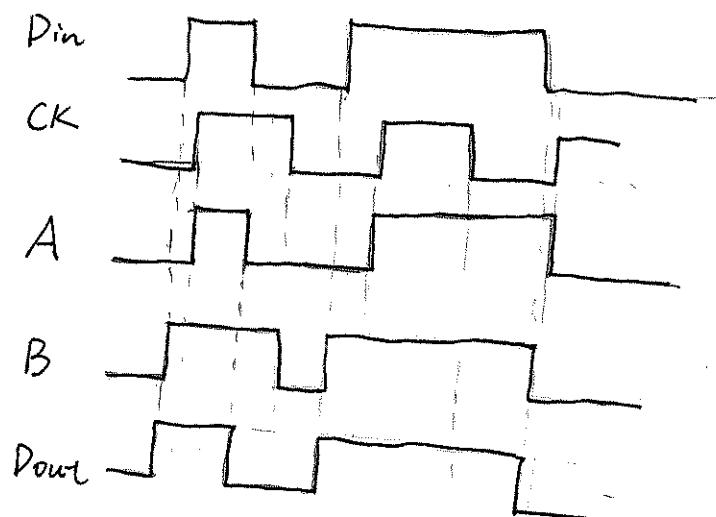
So the mismatch makes noise folding worse.

11.9 Soln:



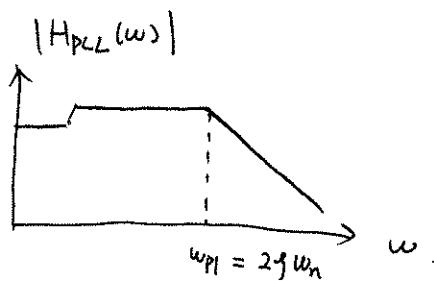
Previously, when  $CK$  is high  $B$  is selected.

Now, when  $CK$  is high.  $A$  is selected.



11.10 Solu:

PLL



Model as one-pole system.

$$H_{PLL}(s) = \frac{2.3w_n}{s + 2.3w_n}$$

$$= \frac{0.1 \cdot \frac{2\pi}{T_1}}{s + 0.1 \frac{2\pi}{T_1}}$$

$$\therefore |H_{PLL}(j\omega)| = \left| \frac{0.1}{j\frac{10f}{f_1} + 0.1} \right| \quad (f_1 = \frac{1}{T_1})$$

$$= \frac{0.1}{\sqrt{0.1^2 + \frac{10f}{f_1}^2}}$$

$$|H_{PLL}(j2\pi \frac{f_1}{2})| = \frac{0.1}{\sqrt{0.1^2 + 5^2}} = 0.02$$

$$= -16.9 \text{ dB}.$$

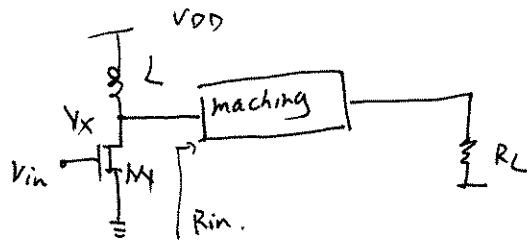
If  $f_{out} = N \cdot f_{in}$ . (assume  $w_n, s$  remain the same).

$$s = \frac{R_1}{2} \sqrt{\frac{Z_p G K_{VCO}}{2\pi M}} ; w_n = \sqrt{\frac{Z_p K_{VCO}}{2\pi M}}$$

If  $s, w_n$  is the same, then  $Z_p \cdot K_{VCO}$  will be change to  $M$  times larger.

So the attenuation is not changed, but the circuit design is more difficult.

12.1 Soln:



$$V_x(t) = V_{DD} (1 + \cos \omega t)$$

$$i_d(t) = \frac{V_{DD}}{R_m} (1 + \cos \omega t)$$

$P_{FET}$

$$= \frac{1}{2T} \int_0^T V_x(t) i_d(t) dt$$

$$= \frac{1}{2T} \int_0^T \frac{V_{DD}^2}{R_m} (1 + 2\cos \omega t + \cos^2 \omega t) dt$$

$$= \frac{V_{DD}^2}{2R_m}$$

assume accurate bias

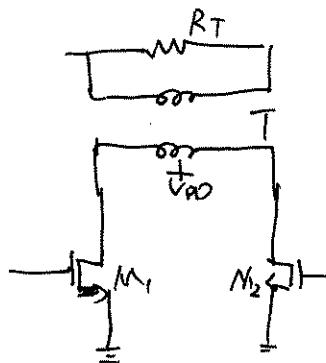
$$= \frac{V_{DD}}{R_m}$$

only on this condition

g = 50%

$\therefore$  The other 50% of supply power is dissipated by M1 itself.

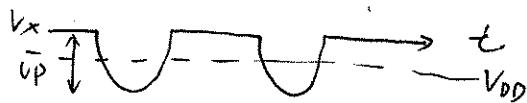
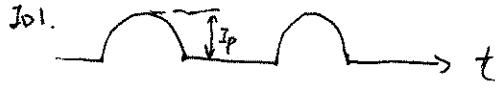
12.2 solu:



This class B amplifier seems very symmetric like differential structure. However, when it works,  $M_1, M_2$  is ~~spea~~ separately operating just like single-end structure.

So it's still sensitive to bond wire inductance in series with  $V_{DD}$ .

12.3 Soln:



$$I_{D1} = I_p \sin \omega_0 t \quad (0 < t < \frac{\pi}{\omega_0})$$

$$\frac{1}{T} \left( \int_0^{\frac{T}{2}} (V_X - V_p \sin \omega_0 t) dt + \int_0^{\frac{T}{2}} V_X dt \right) = V_{DD}$$

$$\Rightarrow V_X = V_{DD} + \frac{V_p}{\pi}$$

The voltage swing above  $V_{DD}$  is:

$$V_X - V_{DD} = \frac{V_p}{\pi} \approx 0.32 V_p$$

The voltage swing below  $V_{DD}$  is:

$$V_p - \frac{V_p}{\pi} = V_p (1 - \frac{1}{\pi}) \approx 0.68 V_p$$

So the swing above  $V_{DD}$  is approx. half that below  $V_{DD}$ .

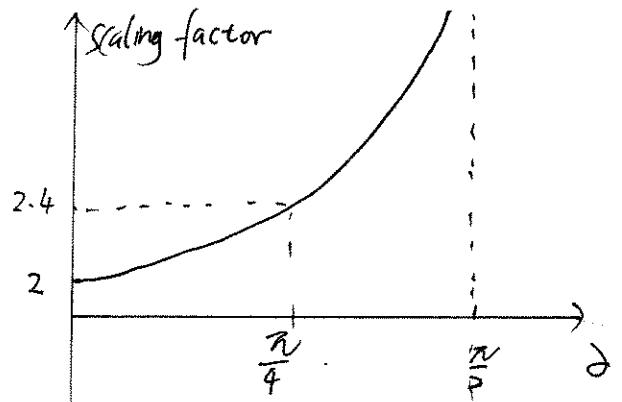
12.4 solve:

From Eq. 12.39 & Eq. 12.40.

$$\alpha_1 = \frac{I_p}{2\pi} \frac{\pi - 2\delta}{2\pi} + \frac{I_p}{2\pi} \sin 2\delta, \\ (\delta \in (0, \frac{\pi}{2}))$$

scaling factor

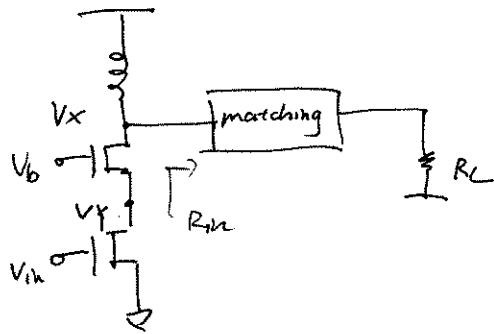
$$= \frac{1}{\alpha_1 I_p} = \frac{1}{\frac{\pi - 2\delta}{2\pi} + \frac{\sin 2\delta}{2\pi}}$$



From the figure above, we can conclude that.

only when the transistor is infinitely large, the efficiency  
can be 100% on the condition of providing  
an comparable output power to that of A class.

12.5 soln:



when  $V_x$  reaches to  $2V_{DD}$  and nearly zero.

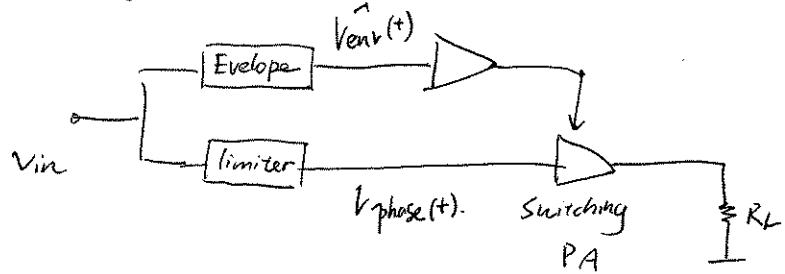
$$P_L = \left(\frac{2V_{DD}}{2}\right)^2 / 2R_{in}.$$

$$\text{The } I_D = \frac{V_{DD}}{R_{in}}.$$

$$\therefore \eta_{max} = \frac{P_L}{I_D \cdot V_{DD}} = \frac{V_{DD}^2 / 2R_{in}}{V_{DD}^2 / R_{in}} = 50\%$$

In sum, the efficiency is the same  
as class A PA.

12-6 Soln:



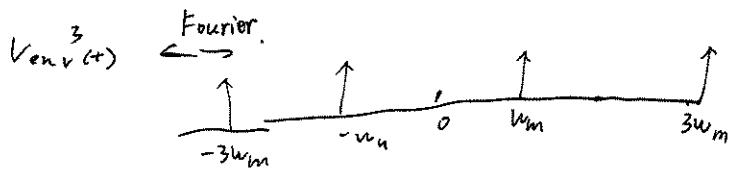
$$\cdot V_{in} = \hat{V}_{env}(t) \cos(\omega_0 t + \phi(t))$$

$$\left\{ \begin{array}{l} \hat{V}_{env}(t) = V_0 \cos(\omega_0 t + \phi(t)) \\ V_{phase}(t) = V_0 \cos(\omega_0 t + \phi(t)) \end{array} \right.$$

$$V_{out} = \hat{V}_{env}(t) \cdot V_{phase}(t)$$

$$= V_0 \hat{V}_{env}(t) \cdot \cos(\omega_0 t + \phi(t)) + \frac{1}{3} V_0 \partial \hat{V}_{env}^3(t) \cos(\omega_0 t + \phi(t)).$$

Assume



This part of spectrum will also convert to the vicinity of  $w_0$ .

So the output spectrum exhibits growth in the adjacent channels.

12.7 Soln:

Assuming the phase signal experiences a delay mismatch of  $\Delta T$ .

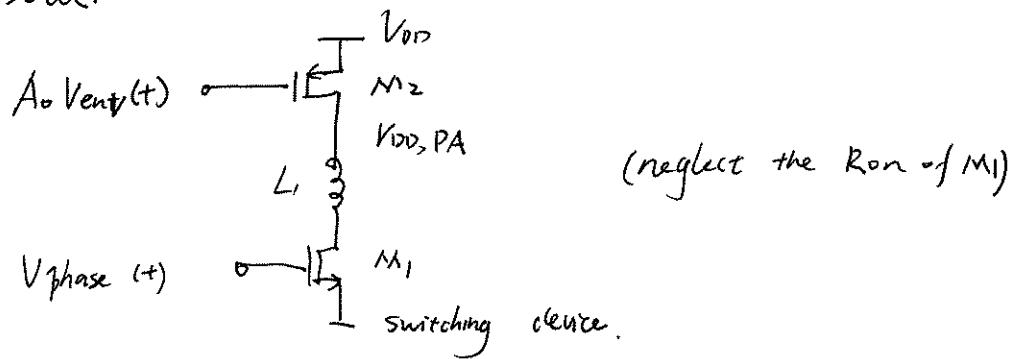
$$\begin{aligned} V_{out} &= A_0 V_{env}(t) \cos [w_0(t - \Delta T) + \phi(t - \Delta T)] \\ &\approx A_0 V_{env}(t) \cos [w_0 t - w_0 \Delta T + \phi(t) - \Delta T \frac{d\phi(t)}{dt}] \\ &\approx A_0 V_{env}(t) \cos [w_0 t + \phi(t)] \cos [(w_0 + \frac{d\phi(t)}{dt}) \Delta T] \\ &\quad + A_0 V_{env}(t) \sin [w_0 t + \phi(t)] \sin [(w_0 + \frac{d\phi(t)}{dt}) \Delta T] \end{aligned}$$

assume  $\Delta T \ll \frac{1}{w_0 + \frac{d\phi(t)}{dt}}$

$$V_{out} \approx A_0 V_{env}(t) \cos [w_0 t + \phi(t)] + \underbrace{\Delta T (w_0 + \frac{d\phi(t)}{dt}) A_0 V_{env}(t) \sin [w_0 t + \phi(t)]}_{\downarrow}$$

From the second term, we can also conclude that this mismatch  $\Delta T$  leads to substantial spectral regrowth.

12. 8 Soln:



In this stage,  $I_L$  does not consume power, neither  $M_1$ .  
So the only dissipated power is consumed by  $M_2$ .

$$\text{efficiency} = 1 - \frac{I_0 V_D}{I_0 \cdot V_{DD}}$$

$$= 1 - \frac{V_D}{V_{DD}}.$$

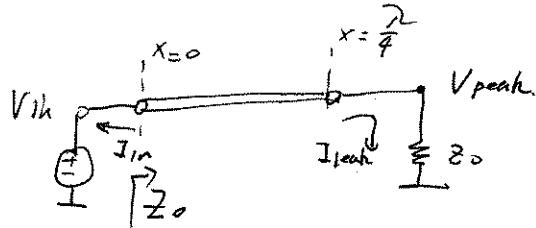
12. 9 Solu:

$v_1(t) \quad v_2(t)$  are defined by Eq (12.109) (12.110)

$$\begin{aligned}
 & v_1(t) + v_2(t) \\
 &= \left( \frac{V_0}{2} + \Delta V \right) \sin [w_0 t + \phi(t) + \theta(t) + \Delta \theta] - \frac{V_0}{2} \sin [w_0 t + \phi(t) - \theta(t)] \\
 &= \frac{V_0}{2} \left\{ \sin [w_0 t + \phi(t) + \theta(t) + \Delta \theta] - \sin [w_0 t + \phi(t) - \theta(t)] \right\} \\
 &\quad + \Delta V \sin [w_0 t + \phi(t) + \theta(t) + \Delta \theta] \\
 &= \frac{V_0}{2} \left\{ \sin [w_0 t + \phi(t) + \theta(t)] \cdot \cos \Delta \theta - \sin [w_0 t + \phi(t) - \theta(t)] \right\} \\
 &\quad + \frac{V_0}{2} \sin \Delta \theta \cos [w_0 t + \phi(t) + \theta(t)] + \Delta V \left\{ \sin [w_0 t + \phi(t) + \theta(t)] \cos \Delta \theta \right. \\
 &\quad \left. + \cos [w_0 t + \phi(t) + \theta(t)] \cdot \sin \Delta \theta \right\} \\
 &\approx \frac{V_0}{2} \cdot 2 \cdot \cos [w_0 t + \phi(t)] \sin \theta(t) + \Delta V \sin [w_0 t + \phi(t) + \theta(t)] \\
 &\quad + \left( \frac{V_0}{2} + 1 \right) \Delta \theta \cos [w_0 t + \phi(t) + \theta(t)] \\
 &\approx \frac{V_0}{V_a} V_{env}(t) \cos [w_0 t + \phi(t)] + \Delta V \sin [w_0 t + \phi(t) + \theta(t)] \\
 &\quad + \left( \frac{V_0}{2} + 1 \right) \Delta \theta \cos [w_0 t + \phi(t) + \theta(t)]
 \end{aligned}$$

12.10 Soln:

If the input is driven by an ideal voltage source.



$$V(t, x) = V^+ \cos(\omega_0 t - \beta x) + V^- \cos(\omega_0 t + \beta x)$$

$$I(t, x) = \frac{V^+}{Z_0} \cos(\omega_0 t - \beta x) - \frac{V^-}{Z_0} \cos(\omega_0 t + \beta x).$$

$$V(t, 0) = (V^+ + V^-) \cos \omega_0 t = V_{in}$$

$$I(t, 0) = \left( \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \right) \cos \omega_0 t = I_{in}$$

$$V(t, \frac{\lambda}{4}) = (-V^+ + V^-) \sin \omega_0 t = V_{peak}$$

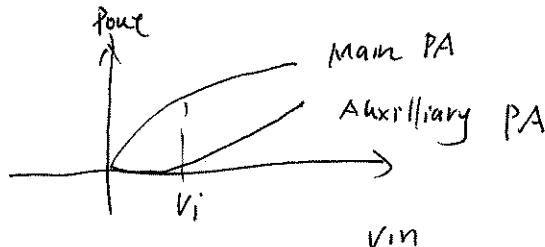
$$I(t, \frac{\lambda}{4}) = \left( -\frac{V^+}{Z_0} - \frac{V^-}{Z_0} \right) \sin \omega_0 t = I_{peak}$$

$$\frac{V_{peak}}{Z_0} = I(t, \frac{\lambda}{4})$$

$$\frac{(-V^+ + V^-) \sin \omega_0 t}{Z_0} = \left( -\frac{V^+}{Z_0} - \frac{V^-}{Z_0} \right) \sin \omega_0 t \Rightarrow -V^+ + V^- = -\left( V^+ + V^- \right)$$

$$\frac{V_{ih}}{I_{in}} = \frac{V^+ + V^-}{V^+ - V^-} \cdot Z_0 = Z_0$$

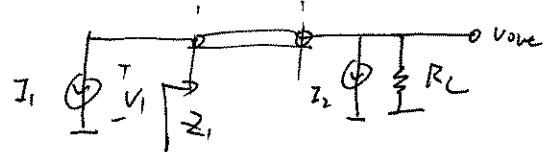
The inputs of peaking PA and carrier PA are the exactly same. So it cannot satisfy the figure below.



12.11. Solve:

$\delta = 0.5$ . The waveform at  $x=0$  &  $x=\pi/4$ .  
 $Z_0 = R_L$ .

Output network:



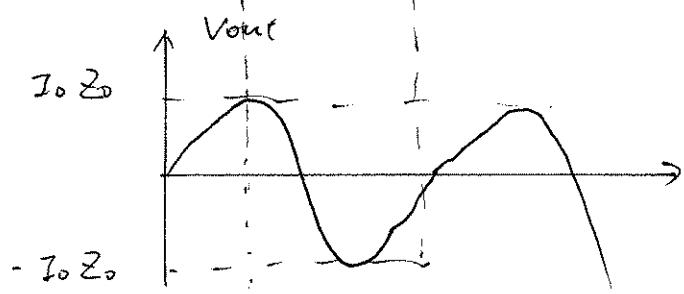
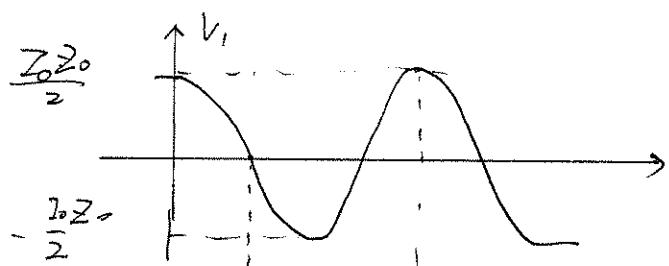
$$Z_1 = Z_0 \left( \frac{Z_0}{R_L} - \delta \right) = \frac{Z_0}{2}$$

assume  $I_1 = -I_0 \cos \omega t$ .

$$V_1 = I_0 \cos \omega t - \frac{Z_0}{2}$$

$$v_{out} = 2 \cdot I_0 \cos(\omega t + \frac{\pi}{2}) \cdot \frac{Z_0}{2}. \quad I(t, \frac{\pi}{4}) = -I_0 \frac{Z_0}{2} \sin \omega t$$

$$V(t, \frac{\pi}{4}) = I_0 Z_0 \sin \omega t$$



13.1. Solve:

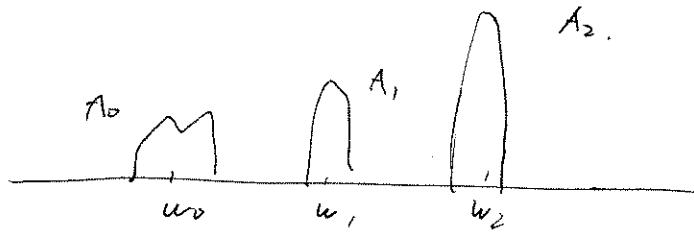
$$\text{data rate} = 54 \text{ Mb/s}$$

sensitivity of  $-65 \text{ dBm}$ .

desire signal :  $A_0 \cos \omega_0 t$

adjacent :  $= A_1 \cos \omega_1 t$

alternate :  $= A_2 \cos \omega_2 t$



$$20 \log A_0 = -62 \text{ dBm}$$

$$20 \log A_1 = -46 \text{ dBm}$$

$$20 \log A_2 = -30 \text{ dBm}.$$

$$\begin{aligned} 20 \log \left| \frac{3a_3}{4a_1} \right| &= -15 \text{ dB} - 40 \log A_1 - 20 \log A_2 + 20 \log A_0 \\ &= (-15 + 92 + 60 - \frac{124}{2}) \text{ dBm} \\ &= +75 \text{ dBm} \end{aligned}$$

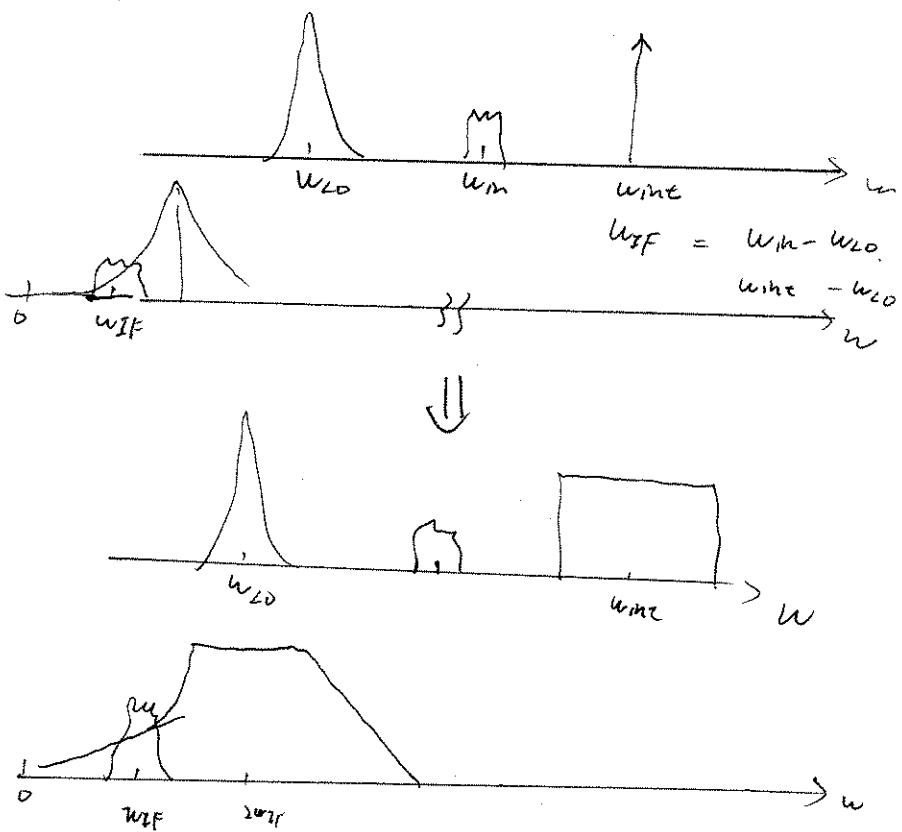
$$\text{IIP}_3 \text{ dBm} = 20 \log \sqrt{\frac{4a_1}{3a_3}} = -37.5 \text{ dBm}$$

13.2 Soln:

If interferences are not approximated by narrow-band signals,

The corruption due to reciprocal mixing is

larger.



13.3 Soles:

desired input = -65 dBm

~~SNR~~ SNR = -35 dB.

desired input = -62 dBm

adjacent signal = -46 dBm

alternative : = -30 dBm.

$$\frac{P_{PN,tot}}{P_{sig}} = a_1 \delta \left( \frac{1}{f_1} - \frac{1}{f_2} \right) + a_2 \delta \left( \frac{1}{f_3} - \frac{1}{f_4} \right)$$
$$= 39.8 \delta \left( \frac{1}{10M} - \frac{1}{30M} \right) + 1585 \delta \left( \frac{1}{30M} - \frac{1}{50M} \right)$$

$$= -35 \text{ dB} \quad \boxed{-35 = 10 \log_{10} \frac{P_{N,tot}}{P_{sig}}}$$

$$\Rightarrow 10^{ \frac{-35}{20} } \cdot 3787 \delta = 10$$

$$\delta \approx 13.3$$

$$\therefore S_n(f) = \frac{13.3}{f^2}$$

13.4 soln:

from Eq (6. 51)

$$Z_{ih, SB} = \frac{1}{2} \left[ R_i + \frac{1}{j C_i u} \right] \quad u \approx u_0$$

$\left\{ \begin{array}{l} R_i : \text{on resistance of switch} \\ C_i : \text{the load of the switch} \\ \text{capacitance.} \end{array} \right.$

In the Fig. 13. 19.

$$C_i \approx \frac{2}{3} w L C_{ox} = 130 \text{ fF}$$

$$R_i \approx 100 \Omega.$$

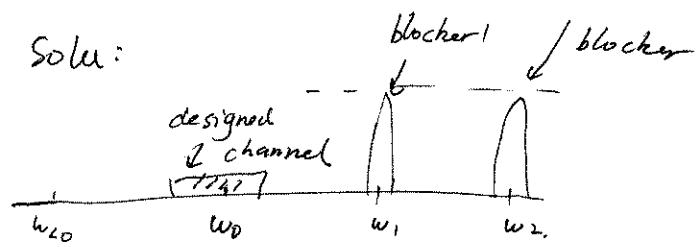
④ 2.4 G

$$\begin{aligned} Z_{ih, SB} &= \frac{1}{2} \left( 100 + \frac{1}{48G \cdot 130f + j \frac{\pi \cdot 2 \cdot 7.6G \cdot 130f}{2}} \right) \\ &= \frac{1}{2} \left( 100 + \frac{1}{48 \cdot 130 \times 10^{-6} + j \pi \cdot 24 \cdot 130 \cdot 10^{-6}} \right) \\ &= 459 e^{-j 0.91} \end{aligned}$$

⑤ 6 G

$$Z_{ih, SB} = 203.4 e^{-j 0.795}.$$

13.5 Solu:



$$S_n(f) = \frac{\sigma}{f^2} \quad \text{assume } -100 \text{ dBc / Hz } \textcircled{D} 1 \text{ MHz}$$

$$10 \log \frac{\sigma}{f^2} = -100 \quad \frac{\sigma}{10^{12}} = 10^{-5 \times 2} \Rightarrow \sigma = 100 \text{ dB}$$

$$\frac{P_{NN, \text{tot}}}{P_{\text{sig}}} = a \cdot 100 \left( \frac{1}{f_1} - \frac{1}{f_2} \right) + a \cdot 100 \left( \frac{1}{f_3} - \frac{1}{f_4} \right)$$

$$100a \left( \frac{1}{10M} - \frac{1}{30M} + \frac{1}{30M} - \frac{1}{50M} \right)$$

$$= \frac{8a}{10^6}$$

$$\therefore 10 \log \frac{8a}{10^6} = -30$$

$$\frac{8a}{10^6} = 10^{-3}$$

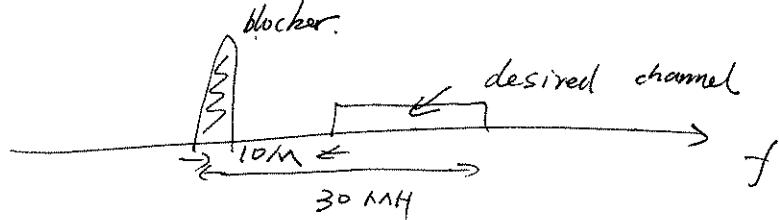
$$a = 125$$

$$10 \log a = 21 \text{ dB}$$

So the highest blocker can be stronger than designed signal than 21 dB.

13.6 Solu:

If only one blocker is located in the adjacent channel.  $\delta = 100$ . (from previous problem)



$$\frac{P_{PN}}{P_{sig}} = \alpha \int_{f_1}^{f_2} \frac{\alpha}{f^2} df \\ = \alpha \delta \left( \frac{1}{f_1} - \frac{1}{f_2} \right) = -30 \text{ dB}$$

$$= 100 \alpha \cdot \left( \frac{1}{10 \text{MHz}} - \frac{1}{30 \text{MHz}} \right) = 10^{-3}$$

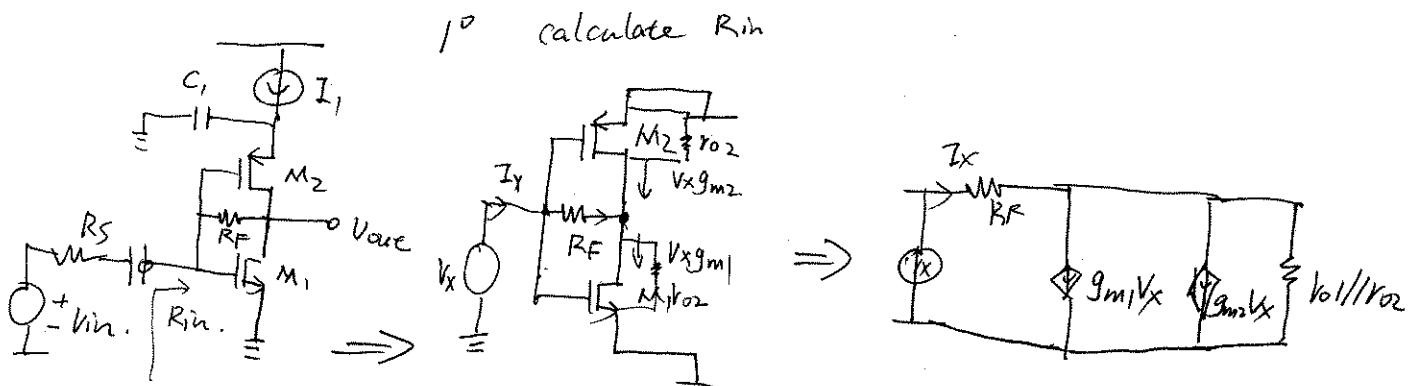
$$6.67 \alpha \times 10^{-6} = 10^{-3}$$

$$\alpha = 150$$

$$\Rightarrow 10 \log_{10} \alpha = 21.76 \text{ dB}$$

Compared with the result of Problem 5, 21 dB,  
we can conclude that the main effect is  
because of the blocker in the adjacent channel.

13.7 Solu:



$$\frac{V_x - I_x R_F}{r_o1 // r_o2} + g_{m1} V_x + g_{m2} V_x - I_x = 0$$

$$\Rightarrow R_{in} = \frac{V_x}{I_x} = \boxed{\frac{r_o1 // r_o2 + R_F}{1 + (g_{m1} + g_{m2})(r_o1 // r_o2)}}$$

2<sup>o</sup> calculate  $\frac{V_{out}}{V_{in}}$

$$\frac{V_{out}}{V_x} = \frac{(r_o1 // r_o2) [1 - (g_{m1} + g_{m2}) R_F]}{R_F + (r_o1 // r_o2)}$$

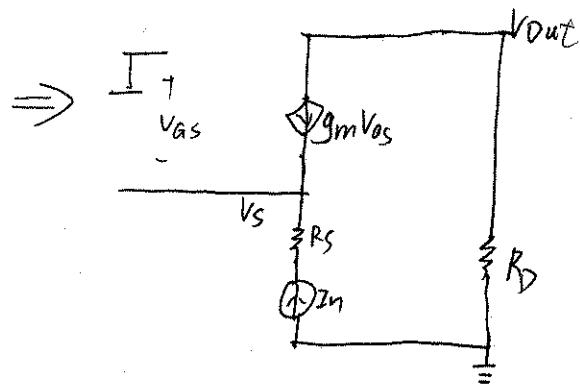
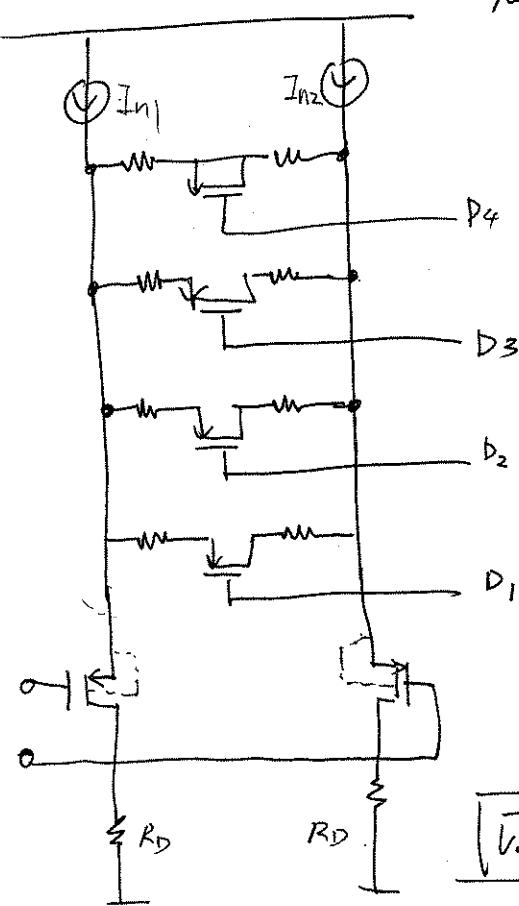
$$\frac{R_{in}}{R_s + R_{in}} = \frac{(r_o1 // r_o2) + R_F}{[1 + (g_{m1} + g_{m2})(r_o1 // r_o2)] R_s + r_o1 // r_o2 + R_F}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_{in}}{R_s + R_{in}} \cdot \frac{V_{out}}{V_x}$$

$$= \boxed{\frac{[1 - (g_{m1} + g_{m2}) R_F] \cdot (r_o1 // r_o2)}{[1 + (g_{m1} + g_{m2}) R_s] (r_o1 // r_o2) + R_F + R_s}}$$

13.8 Solu:

neglect on-resistance of switch,  
channel-length modulation,  
body effect.



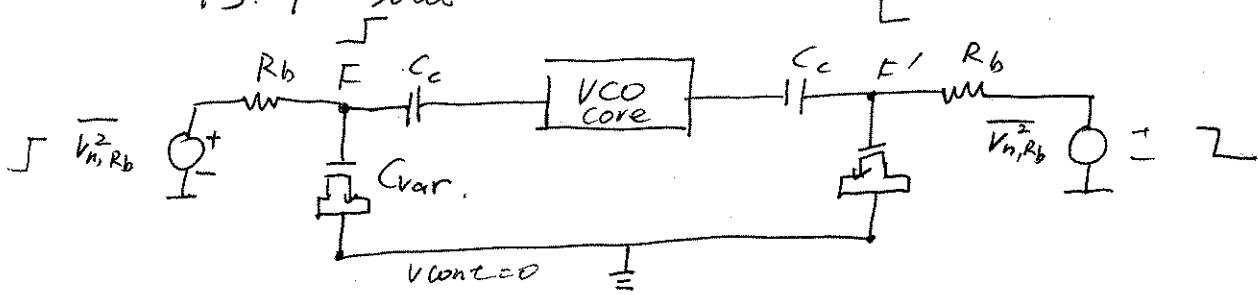
$$\frac{V_{out}}{R_D} = I_{in}$$

$$V_{out} = I_{in} \cdot R_D$$

$$\boxed{\overline{V_{out}^2} = (\overline{I_{n1}^2} + \overline{I_{n2}^2}) \cdot R_D^2}$$

So the  $I_{n1}$  and  $I_{n2}$  contribute the output directly,  
which is very bad.

13.9 Solu:



$$\Delta V_{cont} \cdot k_{VCO} = \Delta w.$$

The noise of  $R_b$  directly modulates the vector, as if it were in series with  $V_{cont}$ , offset freq. below  $\omega_{-3dB} = \frac{1}{R_b C_c}$

$$C_c \gg C_{var}, \quad \omega \approx \frac{1}{R_b C_c}$$

$$\begin{aligned}\Delta V_F &= \frac{\frac{1}{4\pi r_s}}{R_b + \frac{1}{C_{var} \cdot S}} \cdot V_n, R_b \\ &= \frac{1}{R_b \cdot C_{var} \cdot S + 1} \cdot V_n, R_b\end{aligned}$$

$$\approx V_n, R_b$$

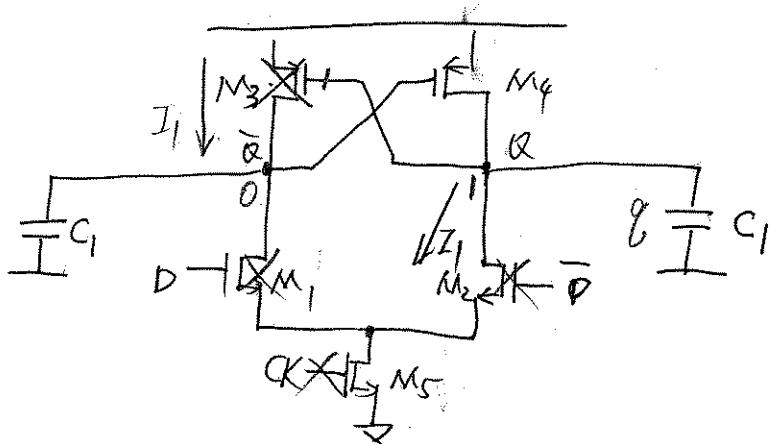
$$\Delta V_{F'} \approx V_n, R_b.$$

$$\Delta V_{cont} \approx \Delta V_F = V_n, R_b.$$

$$\therefore \Delta w = V_n, R_b \cdot k_{VCO}.$$

$\Rightarrow$  the gain from the noise voltage of each resistor to VCO output frequency is equal to  $k_{VCO}$ .

13.10 Soln:



Assume last state is  $Q = 1$ ,  $M_3$  is off,  $M_4$  is on,  
 $C_1$  in the left hand have  $q$  charge in it. ( $M_3$  is off).

And  $\bar{Q}$  node has a leakage from  $M_3$ , which can charge the  $C_1$  in the left hand.  $Q$  node has a leakage  $I_1$  through  $M_3$  &  $M_5$ .

when the voltage  $V_Q < V_{\bar{Q}}$ , the state will be corrupted.

$$\text{Estimate Time} \Rightarrow \frac{I_1 \cdot t}{C_1} \rightarrow \frac{q - I_1 t}{C_1}$$

$$\text{Critical Time} \Rightarrow t = \frac{q}{2I_1}$$

$$q = V_{DD} \cdot C_1$$

$$\Rightarrow t = \frac{V_{DD} \cdot C_1}{2I_1}$$