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# Basic Electrical Engineering 

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## Dedication

This Book is
dedicated to my Parents,
Wife - Ritu Sahdev,
Son - Rohit Sahdev,
Daughter-in-Law - Robina Sahdev
\&
Grandson - Arnav Sahdev


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## Preface

Electrical energy, in the present scenario, has become an integral part of all the engineering applications. Therefore, a course on 'Basic Electrical Engineering' has been introduced by all the universities worldwide for the engineering disciplines.

In my teaching experience, I observed that most of the students, particularly the students belonging to the disciplines of civil, mechanical, computers, textile, etc. (other than electrical and ECE), have difficulty in understanding the contents from the available textbooks. They resort to memorize the statements and formulae of the basic principles. Keeping this in mind, every effort has been taken to make it a students' friendly book by using simple and lucid language. At the same time, a careful effort has been made to provide a comprehensive coverage of the course as per the need of various Indian and foreign universities.

The book is not only useful for B.Tech. students who are pursuing the courses in Basic Electrical Engineering but also useful for the practicing engineers and technicians. All efforts have been made to cover the revised syllabi of Basic Electrical Engineering taught at various universities. The book is also suitable for the students pursuing AMIE (The Institution of Engineers, India) and the students who are preparing for various other competitive examinations.

Each chapter contains neat and self-explanatory diagrams to understand the subject to a great extent. A large number of solved and unsolved examples have been added in various chapters to enable the students to attempt different types of questions asked in the examination, without any difficulty. Practice Exercises are added in all the chapters at regular intervals to keep the students proficient on the topic. At the end of each chapter summary, sufficient number of objective type questions, short answer questions, and test questions have been added to make the book complete and comprehensive in all respects.

Although every care has been taken to eliminate the errors, it is very difficult to claim perfection. I shall be very grateful to the readers (students and teachers) and users of this book if they point out any mistake that might have crept in. Suggestions for the improvement of the book shall be highly appreciated.

S. K. Sahdev

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We are never alone in doing any work and indebted to many people for their reviews, suggestions, and technical discussions. This generalisation is true for this book as well since I was never alone in my work on its script. Many people have contributed to make this book successful and I would like to acknowledge them here.

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I appreciate and thank all my colleagues, deans, faculties and students of various technical universities for their cooperation and support, either directly or indirectly in successful completion of this book. I am truly blessed to have such a fantastic team of experts directing, guiding, and assisting me.

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S. K. Sahdev


## MFARNING OBJFCTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is electricity?
* How electricity can be explained with the help of modern electron theory?
* How a body is electrically charged and what are the units of charge?
* What is electric potential and electric current?
*What are resistance, resistivity, conductance, and conductivity of a conductor and their units?
- How emf is different from potential difference?
* What are Ohm's law and its applications?
* How temperature affects the resistance of metals, alloys, semiconductors, and electrolytes?
* How electric power is different to electrical energy?
*What are the special units of energy when it is in the form of mechanical, electrical, and heat energies?
* What is the effective value of resistance when resistors are connected in series, parallel, or series-parallel combinations?
* What are the applications of series and parallel circuits?


### 1.1 INTRODUCTION

Electricity plays an important role in our day-to-day life, and hence, we should be aware of the various functions that electricity can perform. It is mainly used for lighting (lamps), heating
(heaters), cooling (refrigerators), entertainment (radio and TV), transportation (electric traction), calculations (calculators), etc. Nowadays, all our basic needs are completely dependent upon electricity. Hence, it is more important to increase the public awareness about electricity.

In this chapter, the readers will study the basic concepts of electricity, which enables them to understand the various applications of electricity in future.

### 1.2 ELECTRICITY

The invisible energy that constitutes the flow of electrons in a closed circuit to do work is called 'electricity'.

It is a form of energy that can be easily converted to any other form. Previously, it was thought that electricity is a matter which flows through the circuit to do work. However, now it has been established that electricity constitutes the flow of electrons in the circuit, and in this process, a work is done. It can be well explained by 'Modern Electron Theory'.

### 1.3 MODERN ELECTRON THEORY

Previously, to define electricity, several theories were developed through experiments and by observing its behaviour. Among them, the well-explained theory was 'Modern Electron Theory of Matter'. This theory was developed based on the research works of the famous scientists Sir William Crooks, J. J. Thomson, Robert A. Milliken, Sir Earnest Rutherford, and Neils Bohr.

According to modern electron theory, every matter (whether solid, liquid, or gas) consists of tiny divisible particles called 'molecules'. A molecule is further made up of very minute particles called 'atoms'.

Generally, an atom consists of two main parts, namely nucleus and extra nucleus.

1. Nucleus: The central part of an atom, which contains protons and neutrons, is called nucleus. A proton has positive charge ( $1.602 \times 10^{-19}$ coulombs), whereas a neutron has no charge. Therefore, the nucleus of an atom possesses positive charge. In the nucleus, the protons and neutrons are held together with tremendous force of attraction. The mass of proton is equal to the mass of neutron, and this total mass constitutes the entire mass of an atom.
2. Extra nucleus: The outer part of an atom, which contains only electrons, is called 'extra nucleus'. An electron has negative charge ( $1.602 \times 10^{-19}$ coulombs) equal to that of proton. The mass of an electron is nearly $1 / 1,840$ times to that of a proton and it is usually neglected. Therefore, the entire mass of an atom is constituted by the nucleus of an atom.

$$
\text { Atomic weight }=\text { Number of protons }+ \text { Number of neutrons in the nucleus }
$$

The electrons are not stationary particles; they move around the nucleus in different paths or orbits in a disciplined manner. The shape of an orbit is more or less elliptical, but for simplicity, circular orbits are shown in Figure 1.1. The number of electrons is same as that of protons in an atom. Therefore, an atom on the whole is neutral. The number of protons or electrons in an atom is called atomic number.

Atomic number $=$ Number of protons or electrons in an atom

To know the electrical behaviour of a matter, its simple atomic structure is required to be drawn. To draw simple atomic structure of an atom, the number of electrons in any orbit is determined by the following rules:

1. The number of electron in any orbit is given by relation $2 n^{2}$, where ' $n$ ' is the number of orbit counting from nucleus and going outwards.
For example, First orbit has $2 \times 1^{2}=2$ electrons
Second orbit has $2 \times 2^{2}=8$ electrons
Third orbit has $2 \times 3^{2}=18$ electrons and so on.
2. The last orbit cannot have more than eight electrons.
3. The last but one orbit cannot have more than 18 electrons.

Simple atomic structure of silver $(\mathrm{Ag})$, copper $(\mathrm{Cu})$, and aluminium $(\mathrm{Al})$ are shown in Figure 1.1.


Fig. 1.1 Atomic structure (Bohr's model) of various metals. (a) Silver (Ag), (b) Copper (Cu) (c) Aluminium (Al)

Figure 1.1(a) shows the atomic structure of silver ( Ag ), where
Atomic weight of silver $=108$
Atomic number of silver $=47$
Number of electrons $=$ Number of protons $=47$
Number of neutron $=$ Atomic weight - atomic number
Hence, number of neutron $=108-47=61$
Number of electrons in the first orbit $=2 \times 1^{2}=2$
Number of electrons in the second orbit $=2 \times 2^{2}=8$
Number of electrons in the third orbit $=2 \times 3^{2}=18$
Number of electrons in the fourth orbit $=18$
Number of electrons in the last orbit $=1$
The last orbit of an atom can have maximum eight electrons but silver atom has only one electron in the last (fifth) orbit. Hence, it is an incomplete orbit. This electron of the outermost orbit has poor binding forces with the nucleus and moves freely from one atom to the other at random. Therefore, it is known as 'free electron'.

Figure 1.1(b) shows the atomic structure of copper. The atomic weight and atomic number of copper are 64 and 29 , respectively. The 29 electrons are distributed in the first, second, third, and fourth orbits as $2,8,18$, and 1 , respectively. There is only one electron in the outermost (fourth) orbit, which moves at random from one atom to the other in the matter itself, and is known as free electron.

Figure 1.1(c) shows the atomic structure of aluminium. The atomic weight and atomic number of aluminium are 27 and 13 , respectively. The 13 electrons are distributed in first, second, and third orbits as 2,8 , and 3 , respectively. The last orbit of the atom can have maximum eight electrons, but the aluminium atom has only three electrons. Hence, it is an incomplete orbit. Of the three electrons of the outermost orbit, only one electron is free to move from one atom to the other and is called free electron.

The following conclusions were drawn from the discussion:

1. Every mater is electrical in nature since it contains the charged particles such as electrons and protons.
2. Under ordinary conditions, a body is electrically neutral since every atom of the body material is having the same number of protons and electrons.
3. Each matter differs from the other since they have different atomic number and structure.
4. An atom cannot have more than one free electron at the same instant. For example, in case of aluminium, there are three electrons in the outermost (third) orbit but only one of them is free at a time.
5. Silver is more conductive material than the other two (i.e., copper and aluminium), since in case of silver atomic structure, the free electron is in the fifth orbit and is more loosely attached. However, copper is more conductive than aluminium since the free electron in case of copper atomic structure is in the fourth orbit. The aluminium is the poorest conducting material of the three, since in its atomic structure the free electron is in the third orbit and is more rigidly attached to the nucleus.

### 1.4 NATURE OF ELECTRICITY

It has been observed that every matter is electrical in nature, since it contains charged particles such as protons and electrons. Therefore, the following points are observed:

1. A body is neutral as it contains same number of protons and electrons.
2. If some of the electrons are removed from the body, ${ }^{1}$ then a deficit of electrons occurs, and the body attains a positive charge.
3. If some of the electrons are supplied to the body, then the number of electrons will be more, and the body attains a negative charge.

### 1.5 CHARGED BODY

Every substance or body is electricity neutral as all the atoms of the body contain equal number of electrons and protons.

[^0]

Fig. 1.2 Charged bodies (a) Positively charged body (b) Negatively charged body

However, when some electrons are detached from the body, a deficit of electrons occurs. As a result, the body attains positive charge [Fig. 1.2(a)], whereas if the electrons are supplied to a neutral body, then the number of electrons will be more than its normal due share. Hence, the body attains negative charge [Fig. 1.2(b)]. Therefore, a body is said to be charged positive or negative if it has deficit or excess of electrons from its normal due share, respectively.

### 1.6 UNIT OF CHARGE

The charge on an electron is very small, and it is not convenient to take it as the unit of charge. Therefore, coulomb ${ }^{2}$ is used as the unit of charge in practice. Hence, the practical unit of charge is coulomb.

$$
1 \text { coulomb }=\text { charge on } 628 \times 10^{16} \text { electrons }
$$

If a body is said to have a negative charge of one coulomb, then it means that the body has an excess of $628 \times 10^{16}$ electrons from its normal due share.

### 1.7 FREE ELECTRONS

The valance electrons that are very loosely attached to the nucleus of an atom and can be easily detached are called free electrons. These free electrons are so loosely attached to the nucleus that they do not know the atom to which they belong originally. Thus, they move from one atom to the other at random in the metal itself. It has been observed that in the metals, all the valence electrons are not the free electrons at a time. In fact, one atom can provide only one free electron at a time. However, a small piece of metal has similar number of atoms and free electrons (i.e., billions of atoms and free electrons).

### 1.8 ELECTRIC POTENTIAL

The capacity of a charged body to do work is called electric potential. Obviously, the measure of electric potential is the work done to charge a body to one coulomb, that is,

$$
\text { Electric potential }=\frac{\text { Workdone }}{\text { Charge }} \text { or } \quad V=\frac{W}{Q}
$$

${ }^{2}$ Coulomb is the name of scientist. To honour him, his name is used for the unit of charge.

Unit: Since work done is measured in joule and charge in coulomb, the unit of electric potential is joule/coulomb or volt.

### 1.9 POTENTIAL DIFFERENCE

The difference in the electric potential of the two charged bodies is called potential difference. Unit: The unit of potential difference is volt.

### 1.10 ELECTRIC CURRENT



Fig. 1.3 Random movement of free electrons in metals


Fig. 1.4 Continuous drifting of free electrons constituting electric current

When an electric potential difference is applied across the metallic wire, the loosely attached free electrons, as shown in Figure 1.3, start drifting towards the positive terminal of the cell (see Fig. 1.4). This continuous drifting of electrons constitutes the electric current. Therefore, a continuous drifting of electrons in an electric circuit is called electric current.
The drifting of electrons in the wire is from B to A, that is, from negative terminal of the cell to the positive terminal through external circuit.

### 1.10.1 Conventional Direction of Flow of Current

Prior to electron theory, it was believed that some matter flows through the circuit when a potential difference is applied, which constitutes electric current. It was considered that this matter flows from higher potential to lower potential, that is, positive terminal to negative terminal of the cell through external circuit. This convention flow of current is so firmly established that it is still in use. Therefore, the conventional direction of flow of current is from A to B , that is, from positive terminal of the cell to the negative terminal through the external circuit.

The magnitude of flow of current at any section of the conductor is the rate of flow of electrons, that is, charge flowing per second. It can be expressed mathematically as follows:

Current,

$$
I=\frac{Q}{t}
$$

Unit: Since charge is measured in coulomb and time in second, the unit of electric current is coulomb/second (C/s) or ampere (A).

### 1.11 RESISTANCE

The opposition offered to the flow of electric current or free electrons, as shown in Figure 1.5, is called resistance.


Fig. 1.5 Opposition offered to electric current

Unit: Resistance is measured in ohm (or kilo ohm) and is denoted by symbol $\Omega$ or $\mathrm{k} \Omega$.
A wire is said to have a resistance of one ohm if one ampere current passing through it produces a heat of 0.24 calorie (or one joule).

### 1.11.1 Laws of Resistance

The resistance $(R)$ of a wire depends upon the following factors:

1. It is directly proportional to its length, $l$, that is, $R \propto l$.
2. It is inversely proportional to its area of cross section, $a$, that is,

$$
R \propto \frac{1}{a}
$$

3. It depends upon the nature (i.e., atomic structure) of the material of which the wire is made.
4. It also depends upon the temperature of the wire.

Neglecting the last factor for the time being $R \propto \frac{1}{a}$ or $R=\rho \frac{1}{a}$
where $\rho$ ('Rho' a Greek letter) is a constant of proportionality called resistivity of the wire material. Its value depends upon the nature (i.e., atomic structure) of the wire material representing the third factor earlier.

### 1.12 RESISTIVITY

The resistivity of a wire is given by the relation: $R=\rho \frac{1}{a}$

If $l=1 \mathrm{~m}$ and $a=1 \mathrm{~m}^{2}$ (Fig. 1.6(a)), then $R=\rho$

Hence, the resistance offered by onemetre length of wire of given material having an area of cross-section of one square metre is called the resistivity of the wire material.

In place of wire, if a cube of one metre side of a given material is taken as shown in Figure 1.6(b), then consider opposite two faces of the cube.


Fig. 1.6 Conductor size to determine specific resistance (a) Wire (b) Cube of 1 m side

## 8

$$
l=1 \mathrm{~m} ; a=1 \times 1=1 \mathrm{~m}^{2} \text { and } R=\rho
$$

Hence, the resistance offered between the opposite two faces of one-metre cube of the given material is called the resistivity of that material.
Unit: We know that $R=\rho \frac{l}{a}$ or $\rho=\frac{R a}{l}$
Substituting the units of various quantities as per SI units, we get

$$
\rho=\frac{\Omega \times \mathrm{m}^{2}}{\mathrm{~m}}=\Omega-\mathrm{m}
$$

Hence, the unit of resistivity is ohm metre in SI units.

### 1.13 SPECIFIC RESISTANCE

Specific resistance of a material is defined as the resistance of the material having specific dimensions, that is, one-metre length and one square metre as area of cross-section.

### 1.14 CONDUCTANCE

The ease to the flow of current is called conductance. It is generally denoted by letter $G$.
We know that the opposition to the flow of current is called resistance. Hence, conductance is just reciprocal of resistance, that is,

$$
G=\frac{1}{R}=\frac{1 \times a}{\rho l}=\sigma \frac{a}{l}
$$

Unit: The unit of conductance is mho (i.e., ohm spelt backward). The symbol for its unit is $\mho$.

### 1.14.1 Conductivity

From the expression given earlier, $\sigma$ ('Sigma' a Greek letter) is called the conductivity or specific conductance of the material. It is basically the property or nature (i.e., atomic structure) of the material due to which it allows the current to flow (conduct) through it.

$$
G=\sigma \frac{a}{l} \quad \text { or } \quad \sigma=G \frac{l}{a}
$$

Substituting the units of various quantities, we get

$$
\sigma=\frac{\mho \times \mathrm{m}}{\mathrm{~m}^{2}}=\mho / \mathrm{m}
$$

Hence, the unit of conductivity in SI units is mho/metre.

### 1.15 ELECTROMOTIVE FORCE

The electromotive force (emf) of a source, for example, a battery, is a measure of the energy that it gives to each coulomb of charge. Initially, emf implies that it is a force that causes the electrons
(the charged particles, i.e., current) to flow through the circuit. In fact, it is not a force but it is an energy supplied by some active source such as battery to one coulomb of charge.

### 1.16 EMF AND POTENTIAL DIFFERENCE

The amount of energy supplied by the source to each coulomb of charge is known as emf of the source, whereas the amount of energy used by one coulomb of charge in moving from one point to the other is known as potential difference between the two points.

For instant, consider a circuit as shown in Figure 1.7. If a battery has an emf of 12 V , it means that the battery supplies 12 joule of energy to each coulomb of charge continu-


Fig. 1.7 Electric circuit to represent emf and potential difference ously. When each coulomb of charge travels from positive terminal to negative terminal through external circuit, it gives up whole of the energy originally supplied by the battery.

The potential difference between any two points, say $A$ and $B$, is the energy used by one coulomb of charge in moving from one point (A) to the other (B). Therefore, potential difference between points $A$ and $B$ is 7 V .

### 1.17 OHM'S LAW

Ohm's law states that the current flowing between any two points of a conductor (or circuit) is directly proportional to the potential difference across them, as shown in Figure 1.8, provided physical conditions i.e. temperature etc. do not change.

Mathematically $I \propto V$
or

$$
\frac{V}{I}=\text { constant } \quad \text { or } \quad \frac{V_{1}}{I_{1}}=\frac{V_{2}}{I_{2}}=\cdots=\frac{V_{n}}{I_{n}}=\text { constant }
$$

In other words, Ohm's law can also be stated as follows:
The ratio of potential difference across any two points of a conductor to the current flowing between them is always constant, provided the physical conditions, that is, temperature, etc., do not change.

This constant is called the resistance $(R)$ of the conductor (or circuit).

$$
\therefore \quad \frac{V}{I}=R
$$

It can also be written as $V=I R$ and $I=\frac{V}{R}$.
In a circuit, when current flows through a resistor, the potential difference across the resistor is known as voltage drop across it, that is, $V=I R$.


Fig. 1.8 Potential difference (voltage) applied across a wire having resistance Rohm

### 1.17.1 Limitations of Ohm's Law

Ohm's law cannot be applied to the non-linear clients such as circuits containing electronic tubes or transistors and the circuits used to produce electric arc.

## Example 1.1

The specific resistance of platinum at $0^{\circ} \mathrm{C}$ is 10.5 microohm cm . What should be the length of platinum wire of No. 32 S.W.G. (diameter $=0.0274 \mathrm{~cm}$ ) to have a resistance of 4 ohms at $0^{\circ} \mathrm{C}$ ?

## Solution:

Resistance of a wire at $0^{\circ} \mathrm{C}$ is calculated as follows:

$$
R_{0}=\rho_{0} \frac{1}{a} \quad \text { or } \quad l=\frac{R_{0} a}{\rho_{0}}
$$

where $R_{0}=4 \Omega ; \rho_{0}=10.5 \times 10^{-6} \Omega \mathrm{~cm}=10.5 \times 10^{-8} \Omega \mathrm{~m}$

$$
\begin{array}{rl} 
& a=\frac{\pi}{4} d^{2}=\frac{\pi}{4}(0.0274)^{2} \mathrm{~cm}^{2}=\frac{\pi}{4}(0.0274) \times 10^{-4} \mathrm{~m}^{4} \\
\therefore \quad l & l=\frac{4 \times \pi \times(0.0274)^{2} \times 10^{-4}}{4 \times 10.5 \times 10^{-8}}=2.249 \mathrm{~m}(\text { Ans. })
\end{array}
$$

## Example 1.2

A current of 0.75 A is passed through a coil of nichrome wire which has a cross sectional area of $0.01 \mathrm{~cm}^{2}$. If the resistivity of the nichrome is $108 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$ and the potential difference across the ends of the coil is 81 V . What is the length of the wire? What is the conductivity and conductance of the wire?

## Solution:

Resistance,

$$
R=\rho \frac{1}{a}
$$

$$
R=\frac{V}{I}=\frac{81}{0.75}=108 \Omega ; a=0.01 \mathrm{~cm}^{2}=0.01 \times 10^{-4} \mathrm{~m}^{2}
$$

Where

$$
\rho=108 \times 10^{-6} \Omega \mathrm{~cm}=108 \times 10^{-8} \Omega \mathrm{~m}
$$

$$
\therefore \quad l=\frac{V}{I}=\frac{R a}{\rho}=\frac{108 \times 0.01 \times 10^{-4}}{108 \times 10^{-8}}=100 \mathrm{~m}(\text { Ans. })
$$

Conductivity,

$$
\sigma=\frac{1}{\rho}=\frac{1}{108 \times 10^{-8}}=92.59 \times 10^{4} \mathrm{mho} / \mathrm{m}(\text { Ans. })
$$

Conductance,

$$
G=\frac{1}{R}=\frac{1}{108}=9.259 \times 10^{-3} \mathrm{mho} \text { (Ans.) }
$$

### 1.18 EFFECT OF TEMPERATURE ON RESISTANCE

The electrical resistance generally changes with the change of temperature. The resistance does not only increase with the rise in temperature but it also decreases in some cases. In fact, the increase or decrease in resistance with the rise in temperature depends on the nature of the resistance material discussed as follows:

1. Pure metals: When the resistance is made of some pure metal (copper, aluminium, silver, etc.), its resistance increases with the increase in temperature. The increase is large and fairly uniform for normal range of temperature, and therefore, temperature-resistance graph is a straight line. Thus, pure metals have positive temperature coefficient of resistance.
2. Alloys: When the resistance is made of some alloy (e.g., Eureka, Manganin, Constantan, etc.), its resistance increases with the increase in temperature. But the increase is very small and irregular. In the case of above-mentioned alloys, the increase in resistance is almost negligible over a considerable range of temperature.
3. Semiconductors, insulators, and electrolytes: The resistance of semiconductors, insulators, and electrolytes (silicon, glass, varnish, etc.) decreases with the increase in temperature. The decrease is non-uniform. Thus, these materials have negative temperature co-efficient of resistance.

### 1.19 TEMPERATURE CO-EFFICIENT OF RESISTANCE

Consider a metallic resistor having a resistance of $R_{0}$ and $0^{\circ} \mathrm{C}$ and $R_{\mathrm{t}}$ at $t^{\circ} \mathrm{C}$. The increase in resistance $\left(R_{t}-R_{0}\right)$ is directly proportional to its initial resistance, that is,

$$
\left(R_{t}-R_{0}\right) \propto R_{0}
$$

( $R_{t}-R_{0}$ ) is directly proportional to rise in temperature, that is

$$
\left(R_{t}-R_{0}\right) \propto t
$$

( $R_{t}-R_{0}$ ) depends on the nature of its material.
Thus,

$$
\left(R_{t}-R_{0}\right) \propto R_{0} t
$$

or

$$
\begin{equation*}
\left(R_{t}-R_{0}\right)=a_{0} R_{0} t \tag{1.1}
\end{equation*}
$$

where $\alpha_{0}$ is a constant called temperature coefficient of resistance at $0^{\circ} \mathrm{C}$. Its value depends upon the nature of resistor material.

By rearranging the equation (1.1), we get
and

$$
\begin{equation*}
R_{\mathrm{t}}=R_{0}\left(1+\alpha_{0} t\right) \tag{1.2}
\end{equation*}
$$

$$
\begin{equation*}
\alpha_{0}=\frac{\left(R_{t}-R_{0}\right)}{R_{0} t} \tag{1.3}
\end{equation*}
$$

If $R_{0}=1 \mathrm{ohm} ; t=1^{\circ} \mathrm{C}$; then, $\alpha_{0}=\left(R_{t}-R_{0}\right)$
Hence, temperature coefficient of resistance at $0^{\circ} \mathrm{C}$ may be defined as the change in resistance per ohm original resistance per ${ }^{\circ} \mathrm{C}$ change in temperature.
Unit: We know that, $\alpha_{0}=\left(R_{t}-R_{0}\right) / R_{0} t$

Substituting the units of various quantities, we get,

$$
\text { Unit of } \alpha_{0}=\frac{\mathrm{ohm}}{\mathrm{ohm} \times{ }^{\circ} \mathrm{C}}=1{ }^{\circ} \mathrm{C}
$$

Hence, the unit of temperature coefficient is per ${ }^{\circ} \mathrm{C}$.

### 1.20 TEMPERATURE CO-EFFICIENT OF COPPER AT $0^{\circ} \mathrm{C}$



Fig. 1.9 Graph between temperature and resistance

It has been seen that $R_{t}=R_{0}\left(1+\alpha_{0} t\right)$
The above equation holds good for both rise and fall in temperature. The temperature verses resistance graph of copper material is a straight line as shown in Figure 1.9.

If this line is extended in the backward direction, it would cut the temperature axis at point A where temperature is $-234.5^{\circ} \mathrm{C}$.

Putting the value of $R_{t}=0$ and $t=-234.5^{\circ} \mathrm{C}$ in the above equation, we get,

$$
0=R_{0}\left[1+\alpha_{0}(-234.5)\right]
$$

$$
234.5 \alpha_{0}=1
$$

$$
\alpha_{0}=\frac{1}{234.5} /{ }^{\circ} \mathrm{C}
$$

where $\alpha_{0}$ is the temperature coefficient of resistance of copper at $0^{\circ} \mathrm{C}$.
However, in practice, the curve departs (at point B) from a straight line at very low temperature and the resistance never becomes zero.

### 1.21 EFFECT OF TEMPERATURE ON $\alpha$

The value of temperature coefficient of resistance $(\alpha)$ is not constant. Its value depends upon the initial temperature on which the increment in resistance is based. If the initial temperature is $0^{\circ} \mathrm{C}$, the value of $\alpha$ is $\alpha_{0}$. Similarly, if the initial temperature is $t_{1}{ }^{\circ} \mathrm{C}$, the value of $\alpha$ is $\alpha_{1}$.

Relation between $\alpha_{0}$ and $\alpha_{1}$.
Consider a conductor of resistance $R_{0}$ at $0^{\circ} \mathrm{C}$. When its temperature is raised to $t_{1}{ }^{\circ} \mathrm{C}$, its resistance increases to say $R_{1}$.

$$
\begin{array}{ll}
\therefore & \left(R_{1}-R_{0}\right)=R_{0} \alpha_{0} t_{1} \\
\text { or } & R_{1}=R_{0}\left(1+\alpha_{0} t_{1}\right)
\end{array}
$$

Let us suppose that the conductor of resistance $R_{1}$ at $t_{1}{ }^{\circ} \mathrm{C}$ be now cooled down to $0^{\circ} \mathrm{C}$ to give a resistance of final value $R_{0}$.

Then,

$$
\begin{aligned}
& R_{0}=R_{1}\left(1+\alpha_{1}\left(-t_{1}\right)\right)=R_{1}-t_{1} R_{1} \alpha_{1} \\
& \alpha_{1}=\frac{\left(R_{1}-R_{0}\right)}{t_{1} R_{1}}
\end{aligned}
$$

or
Substituting the value of $\left(R_{1}-R_{0}\right)$ from equation (1.4) and the value of $R_{1}$ from equation (1.5), we get,
or

$$
\begin{equation*}
\alpha_{1}=\frac{R_{0} \alpha_{0} t_{1}}{t_{1} R_{0}\left(1+\alpha_{0} t_{1}\right)} \quad \text { or } \quad \alpha_{1}=\frac{\alpha_{0}}{1+\alpha_{0} t_{1}} \tag{1.6}
\end{equation*}
$$

Relation between $\alpha_{1}$ and $\alpha_{2}$.
Rearranging the equation (1.6), we get,

$$
\begin{equation*}
\frac{1}{\alpha_{1}}=\frac{1+\alpha_{0} t_{1}}{\alpha_{0}} \tag{1.8}
\end{equation*}
$$

Similarly, if the initial temperature is $t_{2}{ }^{\circ} \mathrm{C}$ and, the value of $\alpha$ is $\alpha_{2}$, then

$$
\begin{equation*}
\frac{1}{\alpha_{2}}=\frac{1+\alpha_{0} t_{2}}{\alpha_{0}} \tag{1.9}
\end{equation*}
$$

Subtracting equation (1.7) from (1.8), we get,
or

$$
\frac{1}{\alpha_{2}}-\frac{1}{\alpha_{1}}=\frac{1+\alpha_{0} t_{2}-1-\alpha_{0} t_{1}}{\alpha_{0}}=\frac{\alpha_{0}\left(t_{2}-t_{1}\right)}{\alpha_{0}}
$$

$$
\begin{equation*}
\frac{1}{\alpha_{2}}=\frac{1}{\alpha_{1}}+\left(t_{2}-t_{1}\right) \quad \text { or } \quad \alpha_{2}=\frac{1}{\frac{1}{\alpha^{1}}+\left(t_{2}-t_{1}\right)} \tag{1.10}
\end{equation*}
$$

The following conclusions were drawn from the discussion

1. From equation (1.10), it is clear that with the rise in temperature, the value of $a$ decreases. Thus, $\alpha_{0}$ has the maximum value.
2. If the initial resistance at $0^{\circ} \mathrm{C}$ is $R_{0}$ and the final resistance at $t_{1}{ }^{\circ} \mathrm{C}$ is $R_{1}$, then

$$
R_{1}=R_{0}\left(1+\alpha_{0} t_{1}\right)
$$

3. If the initial resistance at $t_{1}{ }^{\circ} \mathrm{C}$ is $R_{1}$ and the final resistance at $t_{2}{ }^{\circ} \mathrm{C}$ is $R_{2}$, then

$$
R_{2}=R_{1}\left(1+\alpha_{1}\left(t_{2}-t_{1}\right)\right)
$$

4. If the value of $\alpha$ at $t_{1}{ }^{\circ} \mathrm{C}$ is $\alpha_{1}$, then its value at $t_{2}{ }^{\circ} \mathrm{C}$ will be

$$
a_{2}=\frac{1}{\frac{1}{a}+\left(t_{2}-t_{1}\right)}=\frac{a_{1}}{1+a_{1}\left(t_{2}-t_{1}\right)}
$$

### 1.22 EFFECT OF TEMPERATURE ON RESISTIVITY

It has been observed that resistivity or specific resistance of a conductor is the resistance of a unit length of conductor of unit cross-section. Thus, similarly, change in temperature effects the resistivity of a material as it effects the resistance. Over a significant range of temperature, the resistivity of metals varies linearly.

The relation between $\rho_{0}$ and $\rho_{t}$ is given by the equation

$$
\rho_{t}=\rho_{0}\left(1+\alpha_{0} t\right)
$$

where $\rho_{\mathrm{t}}=$ resistivity at $t^{\circ} \mathrm{C}, \rho_{0}=$ resistivity at $0^{\circ} \mathrm{C}$, and $\alpha_{0}=$ temperature coefficient at $0^{\circ} \mathrm{C}$.
However, if $\rho_{1}$ is the resistivity of metal at $t_{1}{ }^{\circ} \mathrm{C}$, then resistivity of metal at $t_{2}^{\circ} \mathrm{C}$ will be $\rho_{2}=$ $\rho_{1}\left(1+\alpha_{1}\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)\right)$
where $\alpha_{1}$ is the temperatures coefficient of resistance at $t_{1}{ }^{\circ} \mathrm{C}$.
It may be noted that the variation of resistivity both at very high and at very low temperatures may be non-linear.

## Example 1.3

A resistor has a resistance of 43.6 ohm at $20^{\circ} \mathrm{C}$ and 50.8 ohm at $60^{\circ} \mathrm{C}$. Determine the temperature coefficient of resistance of aluminium at $0^{\circ} \mathrm{C}$, its resistance at $0^{\circ} \mathrm{C}$ and at $40^{\circ} \mathrm{C}$.

## Solution:

$$
\begin{array}{lrl} 
& \begin{aligned}
R_{1} & =R_{0}\left(1+\alpha_{0} t_{1}\right)
\end{aligned} \text { and } R_{2}=R_{0}\left(1+\alpha_{0} t_{2}\right) \\
\therefore & \frac{R_{2}}{R_{1}} & =\frac{1+\alpha_{0} t_{2}}{1+\alpha_{0} t_{1}} \\
\text { where } R_{1}=43.6 \Omega ; R_{2}=50.8 \Omega \\
\frac{50.8}{43.6} & =\frac{1+\alpha_{0} \times 60}{1+\alpha_{0} \times 20} \\
\text { or } & 1600 \alpha_{0} & =7.2 \text { or } \alpha_{0}=0.0045 /{ }^{\circ} \mathrm{C} \text { (Ans.) } \\
\text { Now, } & R_{0} & =\frac{R_{1}}{1+a_{0} t_{1}}=\frac{43.6}{1+0.0045 \times 20}=40 \Omega \text { (Ans.) }
\end{array}
$$

Resistance at $40^{\circ} \mathrm{C}, R_{3}=R_{0}\left(1+a_{0} t_{3}\right)=40(1+0.0045 \times 40)=47.2 \Omega$ (Ans.)

## Example 1.4

A conductor has a cross section of $8 \mathrm{~cm}^{2}$ and specific resistance of $7.5 \mu \mathrm{ohm} \mathrm{cm}$ at $0^{\circ} \mathrm{C}$. What will be its resistance in ohm per kilometre when the temperature is $40^{\circ} \mathrm{C}$ ? Consider temperature coefficient of the material 0.005 per ${ }^{\circ} \mathrm{C}$.

Solution:
Resistance of conductor at $0^{\circ} ; R_{0}=\rho_{0} \frac{1}{a}$
Here, $\rho_{0}=7.5 \mu \Omega-\mathrm{cm}=7.5 \times 10^{-8} \Omega \mathrm{~m}$;

$$
\begin{array}{ll} 
& l=1 \mathrm{~km}=1000 \mathrm{~m} ; a=8 \mathrm{~cm}^{2}=8 \times 10^{-4} \mathrm{~m}^{2} \\
\therefore & R_{0}=\frac{7.5 \times 10^{-8} \times 1000}{8 \times 10^{-4}}=0.09375 \mathrm{ohm}
\end{array}
$$

Resistance of conductor at $40^{\circ} \mathrm{C}$;

$$
R_{\mathrm{t}}=R_{0}\left(1+\alpha_{0} t\right)=0.09375(1+0.005 \times 40)=0.1125 \text { ohm (Ans.) }
$$

## Example 1.5

A copper coil has a resistance of 12.8 ohm at $20^{\circ} \mathrm{C}$ and 14.3 ohm at $50^{\circ} \mathrm{C}$. Find (1) Temperature coefficient of resistance at $0^{\circ} \mathrm{C}$; (2) Resistance of coil at $0^{\circ} \mathrm{C}$; (3) Temperature coefficient of resistance at $90^{\circ} \mathrm{C}$; (4) Resistance of coil at $100^{\circ} \mathrm{C}$.

## Solution:

or

$$
\begin{aligned}
& R_{1}=R_{0}\left(1+\alpha_{0} t_{1}\right) \text { and } R_{2}=R_{0}\left(1+\alpha_{0} t_{2}\right) \\
& \frac{14.3}{12.8}=\frac{1+\alpha_{0} \times 50}{1+\alpha_{0} \times 20}
\end{aligned}
$$

or

$$
354 \alpha_{0}=1.5 \quad \text { or } \quad \alpha_{0}=\frac{1}{236} /{ }^{\circ} \mathrm{C} \text { (Ans.) }
$$

Now,

$$
14.3=R_{0}\left[1+\left(\frac{1}{236}\right) \times 50\right]
$$

or

$$
R_{0}=\frac{14.3 \times 236}{286}=11.8 \mathrm{ohm}(\text { Ans. })
$$

The temperature coefficient of resistance at $90^{\circ} \mathrm{C}$;

$$
a_{t}=\frac{1}{\frac{1}{a_{1}}+t}=\frac{1}{\frac{1}{1 / 236}+90}=\frac{1}{326} /{ }^{\circ} \mathrm{C} \text { (Ans.) }
$$

Resistance of coil at $100^{\circ} \mathrm{C}, R_{3}=R_{0}\left(1+\alpha_{0} t_{3}\right)=11.8\left(1+\frac{1}{236} \times 100\right)=16.8 \mathrm{ohm}$ (Ans.)

## Example 1.6

A semicircular ring of aluminium has an inner radius of 8 cm , radial thickness 6 cm , and axial thickness 4 cm . Calculate the resistance of the ring at $80^{\circ} \mathrm{C}$ between its two end faces. Specific resistance of aluminium at $20^{\circ} \mathrm{C}=3.1 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$. Resistance temperature coefficient of copper at $0^{\circ} \mathrm{C}$ is $0.005 /{ }^{\circ} \mathrm{C}$.

## Solution:

The semicircular ring is shown in Figure 1.10.
Cross sectional area of ring, $a=4 \times 6=24 \mathrm{~cm}^{2}=24 \times 10^{-4} \mathrm{~m}^{2}$


Fig. 1.10 Semicircular ring of radial thickness 6 cm and axial thickness at 4 cm

Mean radius of the ring, $r_{m}=\frac{8+14}{2}=11 \mathrm{~cm}$
Mean length between the end faces,

$$
\begin{aligned}
l_{m} & =\frac{2 \rho r_{m}}{2} \mathrm{~cm} \\
& =11 \pi \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

Resistance of the ring between the end faces at $20^{\circ} \mathrm{C}$

$$
\begin{aligned}
R_{1} & =\rho_{1} \frac{l_{m}}{a}=\frac{3.1 \times 10^{-6} \times 11 \pi \times 10^{-2}}{24 \times 10^{-4}} \\
& =4.463 \times 10^{-6} \mathrm{ohm}
\end{aligned}
$$

Now,

$$
\begin{aligned}
R_{2} & =R_{1}\left(\frac{1+a_{0} t_{2}}{1+a_{0} t_{1}}\right) \text { where } t_{2}=80^{\circ} \mathrm{C} \\
R_{2} & =4.463 \times 10^{-6}\left(\frac{1+0.005 \times 80}{1+0.005 \times 20}\right) \\
& =5.581 \times 10^{-6} \mathrm{ohm}(\text { Ans. })
\end{aligned}
$$

## Example 1.7

A motor winding has a resistance of 75 ohm at the room temperature of $20^{\circ} \mathrm{C}$ before switching ON to a 230 V . After a 4 h run the winding resistance is 90 ohm . Find the temperature rise if the material resistance temperature coefficient is $1 / 234.5 /{ }^{\circ} \mathrm{C}$.

## Solution:

$$
\alpha_{0}=1 / 234.5 /{ }^{\circ} \mathrm{C}=0.004264 /{ }^{\circ} \mathrm{C}
$$

Resistance of winding at $20^{\circ} \mathrm{C}, R_{1}=75$ ohm
Resistance of winding at $t_{2}{ }^{\circ} \mathrm{C}, R_{2}=90 \mathrm{ohm}$
Now,

$$
\frac{R_{2}}{R_{1}}=\frac{R_{0}\left(1+\alpha_{0} t_{2}\right)}{R_{0}\left(1+\alpha_{0} t_{1}\right)}
$$

or

$$
\frac{90}{75}=\frac{1+0.004264 \times t_{2}}{1+0.004264 \times 20} \quad \text { or } \quad t_{2}=70.9^{\circ} \mathrm{C}
$$

Rise in temperature

$$
t_{2}-t_{1}=70.9-20=50.9^{\circ} \mathrm{C} \text { (Ans.) }
$$

### 1.23 ELECTRICAL ENERGY

When a potential difference V (volt) is applied across a circuit, as shown in Figure 1.11, a current $I$ (ampere) flows through it for a particular period ( $t$ second). A work is said to be done by the moving stream of electrons (or charge) and is called electrical energy.

Thus, the total amount of work done in an electrical circuit is called electrical energy.

By definition, $\quad V=\frac{\text { Workdone }}{Q}$
Therefore, work done or electrical energy expanded

$$
\begin{aligned}
V Q & =V I t(\text { since } I=Q / t) \\
& =I^{2} R t=\frac{V^{2}}{R} t
\end{aligned}
$$

where

$$
\begin{aligned}
V & =\text { potential difference in volt; } \\
I & =\text { current in ampere; } \\
t & =\text { time in second; and } \\
R & =\text { resistance in ohm } .
\end{aligned}
$$

Units: The basic unit of electrical energy is joule (or Watt-second).
If, $V=1 \mathrm{~V}, I=1 \mathrm{~A}$, and $t=1 \mathrm{~second}$
Then, electrical energy $=1$ joule
Hence, the energy expanded in an electrical circuit is said to be one joule (or 1 watt-second) if one ampere current flows through the circuit for one second when a potential difference of 1 V is applied across it.

However, the practical or commercial unit of electrical energy is kilowatt-hour ( kWh ) which is also known as B.O.T. (Board of Trade) unit.

$$
1 \mathrm{kWh}=1000 \times 60 \times 60 \text { watt-second }=36 \times 10^{5} \mathrm{Ws} \text { or joule }
$$

Usually, 1 kWh is called one unit.

### 1.24 ELECTRICAL POWER

The rate at which work is being done in an electrical circuit is called electrical power.
Hence, electrical power $=\frac{\text { Work done in an electric circuit }}{\text { Time }}$

$$
P=\frac{V I t}{t}=V I=I^{2} R=\frac{V^{2}}{R}
$$

Unit: The unit of electrical power is watt (W).
If, $V=1$ volt and $I=1 \mathrm{~A}$. Then, $P=1 \mathrm{~W}$.
Thus, the power consumed in an electrical circuit is said to be 1 W if 1 A current flows through the circuit when a potential difference of $1 V$ is applied across it.

However, the bigger unit of electrical power is kilowatt $(\mathrm{kW})$, it is usually used in the power system.

$$
1 \mathrm{~kW}=1000 \mathrm{~W}
$$

### 1.25 MECHANICAL WORK

When a body, to which force is applied, moves in or opposite direction of the applied force, work is said to be done by or against the body.

Mathematically, Work $=$ Force $\times$ distance or $W=F \times d$

Unit: The unit of work is Newton metre (Nm) or joule.
If, $F=1 \mathrm{~N}$ and $d=1 \mathrm{~m}$; then, $W=1 \mathrm{Nm}$ or joule.
Thus, when a force of 1 N applied on the body moves it to a distance of 1 m , the work done on the body is said to be 1 Nm or joule.

### 1.26 MECHANICAL POWER

The rate of doing work or the amount of work done per unit time is called power, that is,

$$
\text { Power }=\frac{\text { Work done }}{\text { Time }}
$$

Unit: The unit of mechanical power is Newton metre per second (i.e., $\mathrm{Nm} / \mathrm{s}$ ) or joule/second (i.e., J/s).

However, the practical unit of mechanical power is horse power.
In fact, the rate of doing 75 kg m of work per second is known as one horse power.

### 1.27 HEAT ENERGY

The form of energy which produces a sensation of warmth is called heat.
Mathematically,
Heat, $\quad H=m S \theta$
Where $\quad m=$ mass of the body;
$S=$ specific heat of the body; and
$\theta=$ rise or fall in temperature.
Unit: The unit of heat is kilocalorie (kcal)
If, $m=1 \mathrm{~kg} ; \theta=1^{\circ} \mathrm{C}$, and $S=1$, that is, specific heat of water.
Then, $H=1 \mathrm{kcal}$
Hence, the amount of heat required to raise the temperature of 1 kg of water through $1^{\circ} \mathrm{C}$ is called one kilocalorie.

However, the smaller unit of heat energy is calorie.
One calorie is defined as the amount of heat required to raise the temperature of 1 gram of water through $1^{\circ} \mathrm{C}$.

$$
1 \text { kilocalorie }=1000 \text { calories }
$$

### 1.28 JOULES LAW OF ELECTRICAL HEATING

Joule (James Prescott Joule) established that there exists a definite relation between electrical energy expended and amount of heat produced. Thus, the relation is called Joule's law of electrical heating.

This law stated that the amount of heat produced $(\mathrm{H})$ is directly proportional to the electrical energy expended (W).

That is,

$$
\begin{equation*}
\left.H \propto W \quad \text { or } \quad \frac{W}{H}=\mathrm{J} \text { (constant }\right) \tag{1.11}
\end{equation*}
$$

Where J is a constant called mechanical equivalent of heat and its value is determined as 4.18 joule per calorie (i.e., 1 calorie $=4.18$ joule). It means that to produce one calorie of heat, 4.18 J of electrical energy is expended.

From equation (1.8), we get,

$$
H=\frac{W}{\mathrm{~J}} \quad \text { or } \quad H=\frac{I^{2} R t}{4.18} \text { calorie }
$$

where $I^{2} R t$ is the electrical energy in joule.

### 1.29 RELATION BETWEEN VARIOUS QUANTITIES

Some of the important relations between various electrical, mechanical, and thermal (heat) quantities are given below:

### 1.29.1 Relation between Horse Power and kW

By definition,

$$
\begin{aligned}
1 \text { H.P. } & =75 \mathrm{~kg} \mathrm{wt} \mathrm{~m} / \mathrm{s} \\
& =75 \times 9.81 \mathrm{Nm} / \mathrm{s} \text { or joule } / \mathrm{s} \text { or watt } \\
& =735.5 \mathrm{~W}, \text { that is, } 1 \mathrm{H} . \mathrm{P} .=0.7355 \mathrm{~kW}
\end{aligned}
$$

### 1.29.2 Relation between Horse Power and Torque

If a rotor of a radius $r \mathrm{~m}$ rotates at a speed of $N \mathrm{r} . \mathrm{p} . \mathrm{m}$. The force acting on the rotor tangential to its radius is $F$ newton, then

$$
\begin{aligned}
\text { Work done in one rotation } & =\text { Force } \times \text { distance covered } / \text { rev } \\
& =F \times 2 \pi r=2 \pi T \mathrm{Nm} \text { or joule }
\end{aligned}
$$

where $T$ is the torque, that is, moment acting on the rotor.
Work done/minute $=2 \pi N T$ (since $N$ revolutions are made in one minute)
Work done $/ \mathrm{sec}$ or power $=\frac{2 \pi N T}{60}$ joule $/ \mathrm{s}$ or watt
$\therefore$ H.P. $=\frac{2 \pi N T}{60 \times 735.5}$ (because 1 H.P. $\left.=735.5 \mathrm{~W}\right)$

### 1.29.3 Relation between kWh and kcal

Since,

$$
1 \mathrm{kWh}=1000 \times 60 \times 60 \mathrm{Ws} \text { or joule }
$$

$$
\begin{aligned}
& =\frac{36 \times 10^{5}}{4.18} \text { calorie }=\frac{36 \times 10^{5}}{4.8 \times 1000} \mathrm{kcal} \\
\therefore \quad 1 \mathrm{kWh} & =860 \mathrm{kcal}
\end{aligned}
$$

## Example 1.8

A building has (1) 12 light points of 60 W each burning $4 \mathrm{~h} /$ day, (2) a fan point of 75 W each running $10 \mathrm{~h} /$ day, (3) a plug point for a 750 W heater is used $1 \mathrm{~h} /$ day. (4) one radio 80 W used $6 \mathrm{~h} /$ day, and (5) a $1 / 2 \mathrm{H} . \mathrm{P}$. pump of 80 per cent efficiency running $2 \mathrm{~h} /$ day. Calculate the total
 and monthly bill. The supply is given at 230 V and energy costs $₹ 3.15$ per unit. The rent for a meter is ₹ 50 per month. Assume the month of 30 days.

## Solution:

| Load Points | Connected Load | Energy Consumed/Day |
| :---: | :---: | :---: |
| (1) 12 lights of 60 W each $4 \mathrm{~h} /$ day | $12 \times 60=720 \mathrm{~W}$ | $\frac{720 \times 4}{1000}=2.88 \mathrm{kWh}$ |
| (2) 4 fan points of 75 W each $10 \mathrm{~h} / \mathrm{day}$ | $4 \times 75=300 \mathrm{~W}$ | $\frac{300 \times 10}{1000}=3.00 \mathrm{kWh}$ |
| (3) 1 plug point of 750 W heater $-1 \mathrm{~h} / \mathrm{day}$ | $1 \times 750=750 \mathrm{~W}$ | $\frac{750 \times 1}{100}=0.75 \mathrm{kWh}$ |
| (4) 1 ratio of $80 \mathrm{~W}-6 \mathrm{~h} /$ day | $1 \times 80=80 \mathrm{~W}$ | $\frac{80 \times 6}{1000}=0.48 \mathrm{kWh}$ |
| (5) 1/2 H.P. Pump 80\% efficiency. - $2 \mathrm{~h} / \mathrm{day}$ | $\frac{1}{2} \times \frac{735.5}{80} \times 100=460 \mathrm{~W}$ | $\frac{460 \times 2}{1000}=0.92 \mathrm{kWh}$ |
| Total | 2310 W | 8.03 kWh |

Therefore, connected load,
Maximum possible current,
Energy consumption/day
Energy consumption/month
Rate of energy/month
Energy cost/month
Meter rent/month
Therefore, monthly bill

$$
\begin{aligned}
P & =2310 \mathrm{~W}=2.31 \mathrm{~kW} \text { (Ans.) } \\
P / V & =2310 / 230=10.043 \mathrm{~A} \text { (Ans.) } \\
& =8.03 \mathrm{kWh} \text { (Ans.) } \\
& =8.03 \times 30=240.9 \mathrm{kWh} \text { (Ans.) } \\
& =₹ 3.15 \\
& =3.15 \times 240.9=₹ 758.85 \\
& =₹ 50.00 \\
& =50+758.85=₹ 808.85 \text { (Ans.) }
\end{aligned}
$$

## Example 1.9

An electric kettle was marked $500 \mathrm{~W}, 230 \mathrm{~V}$ and was found to take 13 minute to bring 1 kg of water at $20^{\circ} \mathrm{C}$ to boiling point. Determine the heat efficiency of the kettle.

## Solution:

Heat absorbed by water, that is, output of kettle,

$$
\begin{array}{lll} 
& H & =m S \theta \\
\text { where } & m & =1 \mathrm{~kg}=1000 \mathrm{~g} ; S=1 ; \theta=t_{2}-t_{1}=100-20=80^{\circ} \mathrm{C} \\
\therefore & H & =1000 \times 1 \times 80=80000 \text { calorie } \\
\text { Energy input to kettle } & =\text { Power } \times \text { time } \\
& & =500 \times 13 \times 60=390000 \text { wattsec or joule } \\
& & =3900004.18 \text { calorie }=93301 \text { calorie }
\end{array}
$$

Heat efficiency of kettle $=\frac{\text { Heat utilized by water }}{\text { Heat produced by kettle }}=\frac{80000}{93301}=85.74 \%($ Ans. $)$

## Example 1.10

A geyser heater rated at 3 kW is used to heat its copper tank weighing 20 kg and holds 80 L of water. How long will it take to raise the temperature of water from $10^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$, if 20 per cent of energy supplied is wasted in heat losses?

Assuming specific heat of copper to be 0.095 and 4.2 joule to be equivalent to one calorie.

## Solution

$$
\begin{aligned}
& \text { Mass of water, } m_{1}=80 \mathrm{~kg} \text { (since } 1 \mathrm{~L} \text { of water weighs } 1 \mathrm{~kg} \text { ) } \\
& \text { Mass of tank, } m_{2}=20 \mathrm{~kg}
\end{aligned}
$$

Specific heat of copper, $S_{2}=0.095$
Change in temperature, $\theta=\left(t_{2}-t_{1}\right)=60-10=50^{\circ} \mathrm{C}$
Heat utilized to raise the temperature of water and tank or output

$$
\begin{aligned}
& =m_{1} S_{1} \theta+m_{2} S_{2} \theta=80 \times 1 \times 50+20 \times 0.095 \times 50 \\
& =4095 \mathrm{kcal}=4095 \times 10^{3} \times 4.2 \text { joule }
\end{aligned}
$$

Thermal efficiency, $\eta=80 \%$ (since loss $=20 \%$ )

$$
\begin{aligned}
\therefore \quad \text { Input energy } & =\frac{\text { Output }}{\eta}=\frac{4095 \times 10^{3} \times 4.2}{0.8} \text { joule } \\
& =\frac{4095 \times 10^{3} \times 4.2}{0 \times 8 \times 36 \times 10^{5}} \mathrm{kWh}=5.973 \mathrm{kWh}
\end{aligned}
$$

Time required to increase the temperature

$$
=\frac{\text { Input energy }}{\text { Power }}=\frac{5.973}{3}=1.991 \text { hour }(\text { Ans. })
$$

## Example 1.11

A diesel electric generating set supplies an output of 100 kW . The calorific value of the fuel oil used is $12500 \mathrm{kcal} / \mathrm{kg}$. If overall efficiency of the unit is 36 per cent, (1) calculate the mass of oil required per hour and (2) the electrical energy generated per tonne of the fuel.

## Solution:

Calorific value: The heat produced by the complete combustion of 1 kg of fuel is called the calorific value of the fuel.

Output $=100 \mathrm{~kW}$
Energy delivered/hour $=100 \times 1=100 \mathrm{kWh}$
Energy input $=\frac{\text { Output }}{\eta}=\frac{100 \times 100}{36}=277.78 \mathrm{kWh}$
Heat energy required $=277.78 \times 860 \mathrm{kcal}=238889 \mathrm{kcal}$
Fuel required $/$ hour $=\frac{\text { Heat produced }}{\text { Calorific value of the fuel }}=\frac{238889}{12500}=19.11 \mathrm{~kg}$ (Ans.)
Heat produced/tonne of fuel $=12500 \times 1000 \mathrm{kcal}$

$$
\begin{aligned}
\text { Electrical energy generated } & =\frac{\text { Heat produced }}{860} \times \eta=\frac{12500 \times 1000}{860} \times \frac{36}{100} \\
& =5232.56 \mathrm{kWh}(\text { Ans. })
\end{aligned}
$$

## Example 1.12

A hydroelectric power station operates at a mean heat of 25 m and is supplied from a reservoir having area of $6 \mathrm{sq} . \mathrm{km}$. Calculate the energy produced by the water if the water level in the reservoir decreases by 1 m . The overall efficiency of the power station may be considered 80 per cent.

## Solution:

$$
\begin{aligned}
\text { Area of reservoir } & =6 \mathrm{~km}^{2}=6 \times 10^{6} \mathrm{~m}^{2} \\
\text { Decrease in water level } & =1 \mathrm{~m} \\
\text { Volume of water used } & =6 \times 10^{6} \times 1=6 \times 10^{6} \mathrm{~m}^{3} \\
\text { Mass of water, } m & =6 \times 10^{6} \times 1000 \mathrm{~kg}\left(1 \mathrm{~m}^{3} \text { of water weighs } 1000 \mathrm{~kg}\right)
\end{aligned}
$$

Height of water fall or head, $H=25 \mathrm{~m}$

$$
\begin{aligned}
\text { Potential energy of water fall } & =m g H \\
& =6 \times 10^{9} \times 9.81 \times 25(g=9.81) \\
& =14.715 \times 10^{8} \mathrm{Nm}
\end{aligned}
$$

Energy utilized to generate electrical energy, that is,

$$
\begin{aligned}
\text { Output } & =\text { Input } \times \eta=\frac{14715 \times 10^{8} \times 80}{100} \mathrm{Nm} \text { or joules } \\
& =\frac{14715 \times 10^{8} \times 80}{100 \times 1000 \times 60 \times 60}=3,27,000 \mathrm{kWh}(\text { Ans. })
\end{aligned}
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. Define electric charged body.
2. What is the unit of charge? Define unit of charge.
3. What do you mean by electric potential and potential difference?
4. Define electric current and electric resistance.
5. What are the factors on which the resistance of a wire depends?
6. Define resistivity and conductance.
7. Define and explain Ohm's law. What are its limitations?
8. What is the effect of temperature on pure metals, alloys, dielectric materials, and semiconductors?
9. Define temperature coefficient of resistance. What are its units?
10. What is the effect of temperature on the temperature coefficient of resistance (a)?
11. Define electrical energy and electric power. What are their units?
12. Deduce the relation between electrical energy and heat energy as well as electric energy and mechanical work done?

## Test Questions

1. Define the following and mention their units:
(i) Electric charge, (ii) electric current, (iii) electric potential, (iv) emf, (v) electric resistance, (vi) conductance, (vii) resistivity, (viii) conductivity.
2. State and explain Ohm's law. What are its limitations?
3. How temperature affects the resistance of metals, alloys, and semiconductors? What is temperature coefficient of resistance and what are its units?
4. How temperature affects the value of temperature coefficient of resistance. Derive a relative

$$
\alpha_{2}=\frac{1}{\frac{1}{\alpha_{1}+\left(t_{2}-t_{1}\right)}}
$$

5. Derive a relation between (i) HP and watt, (ii) kWh and kCal , (iii) HP and torque.

## Numericals

1. What should be the cross sectional area of a conductor of 1 km length to transmit 200 A , so that the total voltage drop in the conductor may not exceed 12 V . The resistivity of conductor material is $3 \mu \Omega / \mathrm{cm}^{3}$.
(Ans. $5 \mathrm{~cm}^{2}$ )
2. A piece of $10 \mathrm{~cm}^{3}$ of copper having specific resistance of $1.7 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$ is (i) drawn into a wire of 200 m length, (ii) rolled into a square plate of 10 cm side. Determine the resistance of the wire and resistance between the opposite faces of the plate.
(Ans. $68 \Omega, 0.0017 \mu \Omega$ )
3. Find the resistance of a coil of mean diameter 4 cm containing 200 turns of manganin wire 0.05 cm in diameter. The resistivity of manganin is $42 \mu \mathrm{ohm} \mathrm{cm}$.
(Ans. $53.76 \Omega$ )
4. There are two wires $A$ and $B$ of the same material. $A$ is 20 times longer than $B$ and has one fifth of the cross-section as that of $B$. If the resistance of $A$ is 1 ohm , what is the resistance of $B$ ?
(Ans. 10 milliohm)
5. Find the resistance of 1500 m of a copper wire 25 sq . mm in cross section. The resistance of copper is $1 / 58 \mathrm{ohm}$ per metre length and $1 \mathrm{sq} . \mathrm{mm}$ cross section. What will be the resistance of another wire of the same material, three times as long and one half the cross-sectional area.
(Ans. $1.0345 \Omega, 6.207 \Omega$ )
6. Two wires having equal length and made of the same material have resistance of $25 \Omega$ and $81 \Omega$, respectively. Find the ratio of their diameter. If the length of first wire is 50 cm and resistivity is $1.72 \times 10^{-8} \Omega \mathrm{~m}$. Determine the diameter of each wire. (Ans. 1.8, $0.02093 \mathrm{~mm}, 0.01163 \mathrm{~mm}$ )
7. An aluminium resistor has a resistance of 43.6 ohm at $20^{\circ} \mathrm{C}$ and 47.2 ohm at $40^{\circ} \mathrm{C}$. Determine the temperature coefficient of resistance of aluminium at $0^{\circ} \mathrm{C}$.
(Ans. $0.0045 /{ }^{\circ} \mathrm{C}$ )
8. A conductor has a cross section of $10 \mathrm{~cm}^{2}$ and specific resistance of $7.5 \mu \mathrm{ohm} \mathrm{cm}$ at $0^{\circ} \mathrm{C}$. What will be its resistance in ohm per kilometre when the temperature is $40^{\circ} \mathrm{C}$ ? Take temperature coefficient of the material $0.005 \mathrm{per}^{\circ} \mathrm{C}$.
(Ans. 0.09 ohm)
9. A copper coil has a resistance of 12.7 ohm at $18^{\circ} \mathrm{C}$ and 14.3 ohm at $50^{\circ} \mathrm{C}$. Find (i) Temperature coefficient of resistance at $0^{\circ} \mathrm{C}$, (ii) Resistance of coil at $0^{\circ} \mathrm{C}$, (iii) Temperature coefficient of resistance at $80^{\circ} \mathrm{C}$, (iv) Resistance of coil at $100^{\circ} \mathrm{C}$. (Ans. $\frac{1}{236} /{ }^{\circ} \mathrm{C}, 11.8 \mathrm{ohm}, \frac{1}{316} /{ }^{\circ} \mathrm{C}, 16.8 \mathrm{ohm}$ )
10. A semicircular ring of copper has an inner radius of 8 cm , radial thickness 6 cm , and axial thickness 4 cm . Calculate the resistance of the ring at $80^{\circ} \mathrm{C}$ between its two end faces. Specific resistance of copper at $20^{\circ} \mathrm{C}=1.724 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$. Resistance temperature coefficient of copper at $0^{\circ} \mathrm{C}$ is $0.0043 /{ }^{\circ} \mathrm{C}$.
(Ans. $3.0716 \times 10^{-6} \mathrm{ohm}$ )
11. A motor winding has a resistance of 80 ohm at the room temperature of $20^{\circ} \mathrm{C}$ before switching ON to a 230 V . After a 4 h run the winding resistance is 100 ohm . Find the temperature rise if the material resistance temperature coefficient is $1 / 234.5 /{ }^{\circ} \mathrm{C}$.
(Ans. $63.63^{\circ} \mathrm{C}$ )
12. A wire 100 cm long and 0.05 cm diameter is kept at room temperature $20^{\circ} \mathrm{C}$. The specific resistance of the wire material is 3 micro-ohm per centimetre cube, while its temperature coefficient of resistance is 0.004 at $20^{\circ} \mathrm{C}$. Calculate the resistance of the wire at $20^{\circ} \mathrm{C}$. If the wire is heated to $80^{\circ} \mathrm{C}$, determine its increase in resistance.
(Ans. $0.1528 \Omega, 0.1894 \Omega$ )
13. A specimen of copper wire has a specific resistance of $1.72 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$ at $0^{\circ} \mathrm{C}$ and has a temperature coefficient $1 / 264.5$ at $30^{\circ} \mathrm{C}$. Find the temperature coefficient and specific resistance at $80^{\circ} \mathrm{C}$.

$$
\text { (Ans. } \frac{1}{3145} /{ }^{\circ} \mathrm{C}, 2.307 \times 10^{-6} \Omega \mathrm{~cm} \text { ) }
$$

14. An electric boiler draws 12 A at 115 V for a period of 6 h . (i) If electrical energy costs $₹ 3.00$ per kWh , determine the cost of boiler operation, (ii) Find the quantity of electricity in coulomb passing through the boiler.
(Ans. ₹ $24.84,259200 \mathrm{C}$ )
15. The demand for the lightning of a small village is 50 A at 200 V . This is supplied from a dynamo at a distant station. The generated voltage being 220 V . Find (i) the resistance of the leads from the dynamo to the village, (ii) the number of BOT units of energy ( kWh ) consumed in 10 h in the village, and (iii) number of BOT units of energy ( kWh ) wasted in the leads in the same time.
(Ans. $0.4 \Omega, 100$ BOT units, 10 BOT units)
16. The potential difference of 10 V is applied across a 2.5 ohm resistor. Calculate the current, the power dissipated, and energy consumed in heat units in 5 min . (Ans. $4 \mathrm{~A}, 40 \mathrm{~W}, 2870$ calorie)
17. An electric kettle was marked $500 \mathrm{~W}, 230 \mathrm{~V}$ and was found to take 15 min to bring 1 kg of water at $15^{\circ} \mathrm{C}$ to boiling point. Determine the heat efficiency of the kettle.
(Ans. 78.95 per cent)
18. An immersion heater rated at 3 kW is used to heat a copper tank weighing 20 kg and holds 120 L of water. How long it will take to raise the temperature of water from $10^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$, if 20 per cent of energy supplied is wasted in heat losses?
Assuming specific heat of copper to be 0.095 and 4.2 joule to be equivalent to one calorie.
(Ans. 2.963 h )
19. A diesel electrical generating set supplies an output of 50 kW . The calorific value of the fuel oil used is $12500 \mathrm{kcal} / \mathrm{kg}$. If overall efficiency of the unit is 36 per cent, (i) calculate the mass of oil required per hour, and (ii) the electrical energy generated per tonne of the fuel.
(Ans. $9.55 \mathrm{~kg}, 5232.56 \mathrm{kWh})$
20. A hydroelectric power station operates at a mean heat of 30 m and is supplied from a reservoir having area of $5 \mathrm{sq} . \mathrm{km}$. Calculate the energy produced by the water if the water level in the reservoir decreases by 1 m . The overall efficiency of the power station may be considered 80 per cent.
(Ans. 327000 kWh )

### 1.30 D.C. CIRCUITS



Fig. 1.13 Line diagram of dc circuit with measuring instruments

The closed path in which direct current flows is called d.c. circuit.

A simple d.c. circuit is shown in Figure 1.12 which contains a d.c. source (battery), a load (lamp), a switch, connecting leads and measuring instruments such as ammeter and voltmeter.

The simplified line diagram of the same d.c. circuit is shown in Figure 1.13. The load resistors are connected in series, parallel, or series-parallel combination as per the requirement.


Fig. 1.12 Pictorial view of dc circuit

### 1.31 SERIES CIRCUITS

In the circuit, a number of resistors are connected end to end so that same current flows through them is called series circuit.

Figure 1.14 shows a simple series circuit. In the circuit, three resistors $R_{1}, R_{2}$, and $R_{3}$ are connected in series across a supply voltage of $V$ volt. The same current ( $I$ ) is flowing through all the three resistors.

If $V_{1}, V_{2}$, and $V_{3}$ are the voltage drops across the three resistors $R_{1}, R_{2}$, and $R_{3}$, respectively, then


Fig. 1.14 Resistors connected in series

$$
V=V_{1}+V_{2}+V_{3}=I R_{1}+I R_{2}+I R_{3}(\text { Ohm's law })
$$

Let ' $R$ ' be the total resistance of the circuit, then

$$
I R=I R_{1}+I R_{2}+I R_{3} \quad \text { or } \quad R=R_{1}+R_{2}+R_{3}
$$

that is, Total resistance $=$ Sum of the individual resistances.
The common application of this circuit is in the marriages for decoration purposes where a number of low-voltage lamps are connected in series. In this circuit, all the lamps are controlled by a single switch, and they cannot be controlled individually. In domestic, commercial, and industrial wiring system, the main switch and fuses are connected in series to provide the necessary control and protection.

### 1.32 PARALLEL CIRCUITS

In this circuit, one end of all the resistors is joined to a common point and the other ends are also joined to another common point so that different current flows through them is called parallel circuit.

Figure 1.15 shows a simple parallel circuit. In this circuit, three resistors $R_{1}, R_{2}$, and $R_{3}$ are connected in parallel across a supply voltage of $V$ volt. The current flowing through them is $I_{1}, I_{2}$, and $I_{3}$, respectively.


Fig. 1.15 Resistors connected in parallel

The total current drawn by the circuit,

$$
I=I_{1}+I_{2}+I_{3}=\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}} \text { (according to Ohm's law) }
$$

Let ' $R$ ' be the total or effective resistance of the circuit, then

$$
\frac{V}{R}=\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}} \quad \text { or } \quad \frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}
$$

that is, Reciprocal of total resistance $=$ sum of reciprocal of the individual resistances.
All the appliances are operated at the same voltage, and therefore, all of them are connected in parallel. Each one of them can be controlled individually with the help of a separate switch.

### 1.33 SERIES-PARALLEL CIRCUITS

The circuit in which series and parallel circuits are connected in series is called series-parallel circuit.


Fig. 1.16 Resistors connected in series-parallel combination

Figure 1.16 shows a simple seriesparallel circuit. In this circuit, two resistors $R_{1}$ and $R_{2}$ are connected in parallel with each other across terminals AB. The other three resistors $R_{3}, R_{4}$, and $R_{5}$ are connected in parallel with each other across terminal BC. The two groups of resistors $R_{\mathrm{AB}}$ and $R_{\mathrm{BC}}$ are connected in series with each other across the supply voltage of $V$ volt.

The total or effective resistance of the whole circuit can be determined as given below:

$$
\frac{1}{R_{\mathrm{AB}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}=\frac{R_{1}+R_{2}}{R_{1} R_{2}} \text { or } R_{\mathrm{AB}}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}
$$

Similarly,

$$
\frac{1}{R_{\mathrm{BC}}}=\frac{1}{R_{3}}+\frac{1}{R_{4}}+\frac{1}{R_{5}}=\frac{R_{3} R_{4}+R_{4} R_{5}+R_{5} R_{3}}{R_{3} R_{4} R_{5}}
$$

or

$$
R_{\mathrm{BC}}=\frac{R_{3} R_{4} R_{5}}{R_{3} R_{4}+R_{4} R_{5}+R_{5} R_{3}}
$$

Total or effective resistance of the circuit, $R=R_{\mathrm{AB}}+R_{\mathrm{BC}}$

### 1.34 DIVISION OF CURRENT IN PARALLEL CIRCUITS

In parallel circuits, current is divided depending upon the value of resistors and the number of branches as discussed below.

### 1.34.1 When Two Resistors are Connected in Parallel

Figure 1.17 shows two resistors having resistance $R_{1}$ and $R_{2}$ connected in parallel across supply voltage of $V$ volt. Let the current in each branch be $I_{1}$ and $I_{2}$, respectively.

According to Ohm's law, $I_{1} R_{1}=I_{2} R_{2}=V$ or $\frac{I_{1}}{I_{2}}=\frac{R_{2}}{R_{1}}$
Hence, the current in each branch of a parallel circuit is inversely proportional to its resistance. The value of branch current can also be expressed in terms of total circuit current, that is,

$$
I_{1} R_{1}=I_{2} R_{2}=I R=V
$$

where $R$ is total or effective resistance of the circuit and $I$ is the total current.


Fig. 1.17 Division of current in two resistors connected in parallel

Now,

$$
R=\frac{R_{1} R_{2}}{R_{1}+R_{2}}
$$

$$
I_{1} R_{1}=I R=I \frac{R_{1} R_{2}}{R_{1}+R_{2}} \quad \text { or } \quad I_{1}=I \frac{R_{2}}{R_{1}+R_{2}}
$$

Similarly,

$$
I_{2}=I \frac{R_{1}}{R_{1}+R_{2}}
$$

### 1.34.2 When Three Resistors are Connected in Parallel

Figure 1.18 shows three resistors having resistance $R_{1}, R_{2}$, and $R_{3}$ connected in parallel across a supply voltage of $V$ volt. Let the current in each branch be $I_{1}, I_{2}$, and $I_{3}$, respectively.

According to Ohm's law,

$$
I_{1} R_{1}=I_{2} R_{2}=I_{3} R_{3}=I R=V
$$

Where $R$ is the total or effective resistance of the circuit and $I$ is the total current.


Fig. 1.18 Division of current in three resistors connected in parallel
or

$$
I_{1}=I \times \frac{R_{2} R_{3}}{R_{1} R_{2}+R_{2} R_{3}+R_{3} R_{1}}
$$

Similarly,

$$
I_{2}=I \times \frac{R_{1} R_{3}}{R_{1} R_{2}+R_{2} R_{3}+R_{3} R_{1}}
$$

and

$$
I_{3}=I \times \frac{R_{1} R_{2}}{R_{1} R_{2}+R_{2} R_{3}+R_{3} R_{1}}
$$

## Example 1.13

A resistor $R$ is connected in series with a parallel circuit comprising of two resistors having resistance of 12 and 8 ohm , respectively. The total power dissipated in the circuit is 96 W applied voltage is 24 V . Calculate the value of $R$.

## Solution:

Total power dissipated $P=96 \mathrm{~W}$; applied voltage, $V=24 \mathrm{~V}$
The circuit diagram is shown in Figure 1.19. Equivalent resistance of the two resistances connected in parallel is say $R_{P}$.

$\therefore \quad \frac{1}{R_{P}}=\frac{1}{12}+\frac{1}{8}=\frac{2+3}{24}=\frac{5}{24}$
or

$$
R_{P}=\frac{24}{5}=4.8 \mathrm{ohm}
$$

Current supplied to the circuit, $I=\frac{P}{V}=\frac{96}{24}=4 A$

## Fig. 1.19 Circuit diagram as per data

Now,

$$
R_{E F F}=R+R_{P}
$$

Effective resistance of the circuit,

$$
R_{e f f}=\frac{V}{I}=\frac{24}{4}=6 \mathrm{ohm}
$$

$$
\therefore \quad R=R_{E F F}-R_{P}=6-4.8=1.2 \text { ohm (Ans.) }
$$

## Example 1.14

A circuit consists of three resistances of $12 \mathrm{ohm}, 18 \mathrm{ohm}$, and 3 ohm , respectively, joined in parallel is connected in series with a fourth resistance. The whole circuit is supplied at 60 V and it is found that power dissipated in 12 ohm resistance is 36 W . Determine the value of fourth resistance and the total power dissipated in the group.

## Solution:

The circuit diagram is shown in Figure 1.20.
Power dissipated in 12 ohm resistor, $P_{1}=36 \mathrm{~W}$.
If the current in this resistor is $I_{1}$ ampere,


Fig. 1.20 Circuit diagram as per data
then,

$$
\begin{gathered}
I_{1}^{2} \times 12=P_{1} \\
I_{1}^{2}=\frac{36}{12}=3 \quad \text { or } \quad I_{1}=\sqrt{3}=1.732 \mathrm{~A}
\end{gathered}
$$

Voltage across parallel resistors, $V_{1}=I_{1} \times 12$

$$
=1.732 \times 12=20.785 \mathrm{~V}
$$

Current in 18 ohm resistor,

$$
I_{2}=\frac{V_{1}}{18}=\frac{20.785}{18}=1.155 \mathrm{~A}
$$

Current in 36 ohm resistor,

$$
I_{3}=\frac{V_{1}}{36}=\frac{20.785}{36}=0.577 \mathrm{~A}
$$

Current in resistor $R$,

$$
I_{1}+I_{2}+I_{3}=1.732+1.155+0.577=3.464 \mathrm{~A}
$$

Voltage across resistor $R$,

$$
V_{2}=60-V_{1}=60-20.785=39.215 \mathrm{~V}
$$

$\therefore$ Value of series resistor,

$$
R=\frac{V_{2}}{I}=\frac{39.215}{3.464}=11.32 \mathrm{ohm} \text { (Ans.) }
$$

## Example 1.15

Determine current I in the circuit shown in Figure 1.21, if all the resistors are given in ohms.


Fig. 1.21 Circuit diagram as per data

## Solution:

To solve this type of circuit, start from the far end of the supply. A simplified circuit is shown in Figure 1.22.

The far end resistors of value $2 \Omega$ and $4 \Omega$ are connected in series with each other. Let their effective value be $R_{1}$ ohms.

$$
\therefore \quad R_{1}=2+4=6 \Omega
$$

Then $12 \Omega$ resistor and $R_{1}$ are connected in parallel with each other (Figure 1.23). Let their effective value be $R_{\mathrm{CD}}$.

$$
\therefore \quad R_{\mathrm{CD}}=\frac{6 \times 12}{6+12}=4 \Omega
$$



Fig. 1.22 Simplified circuit


Fig. 1.23 Simplified circuit

This resistance $\left(R_{\mathrm{CD}}\right)$ is connected in series with $2 \Omega$ resistor as shown in Figure 1.24. Their effective value is say $R_{2}$.

$$
\therefore \quad R_{2}=2+4=6 \Omega
$$

This resistance $\left(R_{2}\right)$ is connected in parallel with the three resistors of $6 \Omega$ each already connected in parallel as shown in Figure 1.25. Their effective value is say $R_{\text {AB }}$.

$$
R_{\mathrm{AB}}=\frac{6}{4}=1.5 \Omega
$$

This resistance $\left(R_{\mathrm{AB}}\right)$ is connected in series with $2.5 \Omega$ resistor as shown in Figure 1.26. The total resistance of the circuit is say $R$ ohm.

Then,

$$
R=2.5+1.5=4 \Omega
$$

$$
\therefore \quad \text { Current, } I=\frac{V}{R}=\frac{100}{4}=25 \mathrm{~A} \text { (Ans.) }
$$

Alternatively, after analysing the circuit, the effective resistance across the supply is given by

$$
\begin{aligned}
R & =[[\{(2+4) \| 12\}+2]\|6\| 6]+2.5 \\
& =[\{(6 \| 4)+2\}\|6\| 6 \| 6]+2.5=\left[\left(\frac{6 \times 12}{6+12}+2\right)\|6\| 6 \| 6\right]+2.5 \\
& =[(4+2)\|6\| 6 \| 6]+2.5=\left(\frac{6}{4}\right)+2.5=1.5+2.5=4 \Omega \\
& \text { Current, } I=\frac{V}{R}=\frac{100}{4}=25 \text { A (Ans.) }
\end{aligned}
$$



Fig. 1.24 Simplified circuit


Fig. 1.25 Simplified circuit


Fig. 1.26 Simplified circuit

## Fer PRACTICE EXERCISES

## Short Answer Questions

1. When number of resistors are connected in series, show that their effective value increases.
2. When number of resistors are connected in parallel, show that their effective value decreases.
3. Give the applications of (i) series circuits, (ii) parallel circuits.

## Test Questions

1. In an electric circuit, the given number of resistors may be connected in series, parallel or seriesparallel combination, what is the significance of each connections?
2. When three resistors $R_{1}, R_{2}$, and $R_{3}$ are connected in parallel, show that the current flowing through one branch $R_{1}$ is given as, $I_{1}=\frac{R_{2} R_{3}}{R_{1} R_{2}+R_{2} R_{3}+R_{3} R_{1}}$

## Numericals

1. An oven takes 16 A at 220 V . It is desired to reduce the current to 12 A . Find (i) the resistance which must be connected in series and (ii) voltage across the resistor.
(Ans. 4.58 ohm, 54.96 V )
2. A $100 \mathrm{~W}, 250 \mathrm{~V}$ bulb is put in series with a $40 \mathrm{~W}, 250 \mathrm{~V}$ bulb across 500 V supply. What will be the current drawn, what will be the power consumed by each bulb and will such a combination work?
(Ans. 0.2286 A, 32.65 W, 81.63 W)
3. Two coils are connected in parallel and a voltage of 200 V is applied to the terminals. The total current taken is 25 A and the power dissipated in one of the coils is 1500 W . What is the resistance of each coil?
(Ans. $26.67 \Omega, 11.43 \Omega$ )
4. A cooking range has two coils of $500 \mathrm{~W}, 250 \mathrm{~V}$, and they can be run one at a time, both in series and in parallel. They are run in this way for 1 h each, that is, total 4 h per day. Cost of 1 unit of energy being $₹ 2.50$. Find the electric charges for a month of 31 days.
(Ans. ₹174.38)
5. A bulb rated $110 \mathrm{~V}, 60 \mathrm{~W}$ is connected in series with another bulb rated $110 \mathrm{~V}, 100 \mathrm{~W}$ across 220 V mains. Calculate the resistance which should be joined in parallel with the first bulb so that both the bulbs may take their rated power.
(Ans. 302.45 ohm)
6. A 100 A current is shared by three resistors connected in parallel. The resistor wires are of the same material and have their length in the ratio 2:3:4 and their cross sectional area in the ratio 1:2:3, determine the current in each resistor.
(Ans. 26.087 A, 34.782 A, 39.131 A)
7. An aluminium wire and a copper wire are connected in parallel. Their specific resistance are in the ratio of $30: 17$. The former carries 70 per cent current more than the later and the latter is 50 per cent longer than the former. Determine the ratio of their cross sectional area.
(Ans. 2:1)

## SUMMARY

1. Current: The magnitude of flow of current at any section of the conductor is defined as the rate of flow of charge (electrons) at that section, $I=\frac{Q}{t} \mathrm{C} / \mathrm{s}$ or ampere.
2. Laws of resistance: It is summed up as $R=\rho \frac{l}{a}$ ohm
3. Resistivity: The resistance offered between the two opposite faces of a metre cube of the material. $\rho=R$ when $l=1 \mathrm{~m}$ and $a=1 \mathrm{~m}^{2}$. Unit ohm m or ohm $/ \mathrm{m}^{3}$.
4. Conductance: Reciprocal of resistance, that is, $G=I / R \mathrm{mho}$.
5. Conductivity: Reciprocal of resistivity $\sigma=1 / \rho \mathrm{mho} / \mathrm{m}$.
6. Ohm's law: Ratio of $V$ to $I$ is constant for a circuit provided physical conditions remain the same $R=\frac{V}{I} ; V=I R$ or $I=\frac{V}{R}$
7. Temperature coefficient of resistance at $0^{\circ} \mathrm{C}: \alpha_{0}=\frac{\left(R_{t}-R_{0}\right)}{R_{0} t} /{ }^{\circ} \mathrm{C}$
8. Effect of temperature on resistance (for conductors): Resistance increases proportionately with the rise in temperature $R_{t}=R_{0}\left(1+a_{0} t\right) ; R_{t_{2}}=R_{r_{1}}\left[1+\alpha_{t_{1}}\left(t_{2}-t_{1}\right)\right]$

Temperature coefficient of resistance at any other temperatures.

$$
\alpha_{1}=\frac{1}{\frac{1}{\alpha_{0}}+t} /{ }^{\circ} \mathrm{C} ; \alpha_{t_{2}}=\frac{1}{\frac{1}{\alpha_{t_{1}}}+\left(t_{2}-t_{1}\right)} /{ }^{\circ} \mathrm{C}
$$

9. Effect of temperature on resistivity: Resistivity at any temperature

$$
\rho_{\mathrm{t}}=\rho_{0}\left(1+\alpha_{0} t\right) ; \rho_{2}=\rho_{1}\left[1+\alpha_{1}\left(t_{2}-t_{1}\right)\right]
$$

10. Electrical energy: An electrical work is being done, when current flows through a circuit.

Electrical energy: Volt joule or watt sec; commercial unit or BOT unit is kWh;
$1 \mathrm{kWh}=36 \times 10^{5}$ joule.
11. Electrical power: Electrical energy consumed per unit time is called electric power $(V I t / t=V I)$.
12. Relation between various quantities

1 Calorie $=4.18$ joule; $1 \mathrm{kcal}=1000$ calorie $=4186 \mathrm{~J} ; 1 \mathrm{kWh}=860 \mathrm{kcal}$
1 H.P. $=735.5 \mathrm{~W}=0.7335 \mathrm{~kW}$; H.P. $=\frac{2 \pi N T}{60 \times 735.5}(T$ is in Nm $)$
13. Effective value of the resistance $(R)$
(i) in series circuit, $R=R_{1}+R_{2}+\cdots+R_{n}$
(ii) in parallel circuit, $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots+\frac{1}{R_{n}}$

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. Resistance of a conductor is $\qquad$ proportional to its area of cross section.
2. The resistance of an insulator $\qquad$ with the increase in its temperature.
3. A $3 / 20$ S.W.G. wire can carry $\qquad$ current than 7/20 S.W.G. wire.
4. kWh is the unit of $\qquad$ _.
5. The unit of conductivity is $\qquad$ .
6. One unit of electrical energy means $\qquad$ .
7. The resistance of 40 W and 200 V lamp will be $\qquad$ than 100 Wand 200 V lamp.
8. The unit of specific resistance is $\qquad$ _.
9. An electric lamp of 40 W will consume one unit of energy in $\qquad$ hour.
10. The direction of flow of conventional current is $\qquad$ to the direction of flow of electrons.
11. The resistance of a conductor $\qquad$ with the decrease in temperature.
12. Ohm's law will hold good if the physical conditions particularly $\qquad$ remains constant.
13. The resistance of a conductor $\qquad$ with the decrease of its diameter.
14. The electric charge on an electron is $\qquad$ coulomb.
15. One coulomb of charge is equal to the charge on $\qquad$ electrons.
16. The resistivity of a wire depends upon the $\qquad$ of the material.
17. A man has seven resistors each of value $1 / 7 \mathrm{ohm}$. The maximum value which he can obtain by their combination will be $\qquad$ ohm.
18. A 100 W and 220 V lamp will take $\qquad$ ampere current at rated voltage.
19. Watt-hour is the unit of $\qquad$ _.
20. Energy consumed by a 750 W and 230 V electric iron in 8 h will be $\qquad$ unit.
21. Two resistors of 10 ohm each are connected in parallel, the combination is connected in series with another 5 ohm resistor, the effective resistance of the circuit will be $\qquad$ ohm.
22. All the domestic appliances are connected in $\qquad$ with the supply.
23. A $100 \mathrm{~W}, 200 \mathrm{~V}$ lamp is operated at 100 V , the power consumed by the lamp will be $\qquad$ watt.
24. A copper wire has a resistance of 10 ohm by drawing its length is increased to double. The new resistance of the wire will be $\qquad$ ohm.
25. A wire has a resistance of $x$ ohm and carries a current of $y$ ampere, the voltage across the wire will be
$\qquad$ volt.

## OBJECTIVE TYPE QUESTIONS

1. A flow of 10000 electrons per second constitutes a current of
(a) $2.6 \times 10^{-19} \mathrm{~A}$
(b) $1.6 \times 10^{-15} \mathrm{~A}$
(c) $1.602 \times 10^{-19} \mathrm{~A}$
(d) $628 \times 10^{-12} \mathrm{~A}$
2. Electrons which are loosely attached to the nucleus of an atom and can be easily detached are called
(a) free electrons
(b) valance electrons
(c) bonded electrons
(d) All (a), (b), and (c)
3. Ohm's law deals with the relation between
(a) charge and resistance
(b) charge and capacity
(c) current and potential difference
(d) capacity and potential difference
4. The specific resistance of a conductor depends upon
(a) length of conductor
(b) nature of conductor material
(c) diameter of conductor
(d) All (a), (b), and (c)
5. If the length of a conductor or wire is doubled and its area of cross section is also doubled, then the resistance will
(a) increase four times
(b) remains uncharged
(c) decrease to four times
(d) change at random
6. Unit of specific resistance of a conductor is
(a) $0 h \mathrm{~m} / \mathrm{cm}$
(b) ohm $/ \mathrm{cm}^{2}$
(c) ohm $/ \mathrm{ohm} / \mathrm{cm}^{2}$
(d) ohm cm
7. A certain piece of aluminium is to be shaped into a conductor of minimum resistance, its length and cross-sectional area shall be respectively.
(a) $L$ and $A$
(b) $2 L$ and $A / 2$
(c) $L / 2$ and $2 A$
(d) $L / 2$ and $A / 2$
8. A man has three resistances, each of value $1 / 3 \mathrm{ohm}$. What is the minimum resistance he can obtain by combining them?
(a) 1 ohm
(b) 3 ohm
(c) $1 / 6 \mathrm{ohm}$
(d) $1 / 9 \mathrm{ohm}$
9. If ' $n$ ' identical resistors are connected in series, their total value is K ohm. What will be their effective value if they are now connected in parallel?
(a) $K / n^{2}$
(b) $K^{2} / n$
(c) $K / n$
(d) $n^{2} / K$
10. A parallel arrangement of 3 ohm and 6 ohm resistors is placed in series with an 8 ohm resistor. If potential difference of 60 V is connected across the whole circuit, the current in the three ohm resistor is
(a) 6 A
(b) 4 A
(c) 2 A
(d) 10 A
11. The unit of electrical energy is
(a) watt-sec
(b) joule
(c) kWh
(d) All a, b, and c
12. A $100 \mathrm{~W}, 220 \mathrm{~V}$ lamp takes a current of
(a) $5 / 11 \mathrm{~A}$
(b) 2.2 A
(c) 0.22 A
(d) $5 / 22 \mathrm{~A}$
13. Four wires of equal length and of resistance 20 ohm each are connected in the form of a square. The equivalent resistance between the two opposite corners of the square is
(a) $10 \Omega$
(b) $20 \Omega$
(c) $40 \Omega$
(d) $20 / 4 \Omega$
14. Watt-hour is the unit of
(a) electric power
(b) electric capacity
(c) electric energy
(d) electric charge
15. Given three equal resistors. How many different combinations of those three can be made
(a) two
(b) four
(c) six
(d) eight
16. One kWh is equal to
(a) $3.6 \times 10^{8} \mathrm{~J}$
(b) $36 \times 10^{6} \mathrm{~J}$
(c) $6.6 \times 10^{8} \mathrm{~J}$
(d) $3.6 \times 10^{6} \mathrm{~J}$
17. An electric lamp is marked $100 \mathrm{~W}, 220 \mathrm{~V}$. The resistance of the filament will be
(a) 22 W
(b) 2200 W
(c) 484 W
(d) 4840 W
18. Two electric lamps each of $40 \mathrm{~W}, 230 \mathrm{~V}$ are connected in series across 230 V supply. The power consumed by each
(a) 20 W
(b) 10 W
(c) 40 W
(d) 80 W
19. A wire of resistance $R$ is drawn through a die so that its length is increased three times, its resistance is now
(a) $3 R$
(b) $R / 3$
(c) $R / 3 \sqrt{3}$
(d) $9 R$
20. Three 3 ohm resistors are connected in the form of an equilateral triangle. The total resistance between any two corners is
(a) 2 ohm
(b) 6 ohm
(c) 3 ohm
(d) $4 / 3 \mathrm{ohm}$
21. The rating of four electric appliances is as given below: Washing machine- 300 W and 230 V ; electric lamp- 100 W and 230 V ; room heater- 1500 W and 230 V ; electric fan- 80 W and 230 V . Which appliance would require the thickest lead wire?
(a) Washing machine
(b) Electric lamp
(c) Room heater
(d) Electric fan
22. Three electric lamps of 60 W each are connected in parallel. The power consumed by the combination will be
(a) 30 W
(b) 120 W
(c) 20 W
(d) 180 W
23. The rating of an electric lamp is 220 V and 100 W . If it is operated at 110 V , the power consumed by it will be
(a) 50 W
(b) 75 W
(c) 90 W
(d) 25 W

## NUMERICALS

1. Find out the number of electrons passing through a wire per second if the wire carries a current of 10 A . The charge on each electron is $1.6 \times 10^{-19} \mathrm{C}$.
(AMIE; W, 1987) (Ans. $6.25 \times 10^{19}$ )
2. A constantan wire of radius 2 mm and specific resistance $49 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$ is to be used for making a resistance of 1 ohm . Calculate the length of wire required. What will be the length of wire if the radius be doubled?
(Ans. $25.65 \mathrm{~m} ; 102.6 \mathrm{~m}$ )
3. A length of copper wire of mass 5.1 kg has a resistance of 2.679 ohm . Calculate the length and diameter of wire. Density of copper is $8.93 \mathrm{~g} \mathrm{~cm}^{-3}$ and specific resistance of copper is $1.7 \times 10^{-6} \mathrm{ohm} \mathrm{cm}$.
(Ans. $300 \mathrm{~m} ; 0.1557 \mathrm{~cm}$ )
4. A potential difference of 100 V exists between the ends of a wire of length 200 cm . If its diameter is 0.15 mm and the current through it is 2 A , calculate the specific resistance of the wire material.
(Ans. $44.175 \times 10^{-6} \mathrm{ohm} \mathrm{m}$ )
5. There are two wires $A$ and $B$ of the same material. $A$ is 20 times longer than $B$ and has one-third of the cross section as that of $B$. If the resistance of $A$ is one ohm, what is the resistance of $B$ ?
(Ans. 160 ohm)
6. A copper wire 150 m long has a diameter one-third of the diameter of a length of manganin wire. The resistivity of the manganin and copper are 0.42 and 0.017 micro-ohm-m, respectively. Calculate the length of the manganin wire if it has the same resistance as the copper wire.
(Ans. 54.645 m )
7. Determine the resistance of a metal tube in terms of external diameter $D$, the internal diameter $d$, the length $l$ and resistivity $\rho$. Hence, calculate the resistance of aluminium tube 0.5 cm thick and 2 m long. The external diameter is 15 cm and specific resistance is 3 micro ohm cm.
(Ans. $4 \rho 1 /\left[\pi\left(D^{2}-d^{2}\right] ; 26.34 \mu \Omega\right)$
8. An aluminium and a copper wire are connected together in parallel. The current flowing in the respective wires is in the ratio of $30: 40$. The aluminium wire is 45 per cent longer than the copper wire and the ratio of their specific resistance is $30: 17$. Find the ratio of their cross sectional areas.
(Ans. 1.919:1)
9. Five cubic centimetre of copper (i) are drawn into a wire 150 m long, (ii) rolled into a square sheet of 0.1 m side. Find the resistance of the wire and the resistance between opposite faces of the plate if the specific resistance of copper is $1.72 \mu \mathrm{ohm}-\mathrm{cm}$.
(Ans. 77.4 ohm; $8.6 \times 10^{-10}$ ohm)
10. A silver wire has a resistance of 1.28 ohm at $0^{\circ} \mathrm{C}$ and a temperature coefficient of resistance of 0.00375 per ${ }^{\circ} \mathrm{C}$. To what temperature must the wire be raised to double the resistance?
(Ans. $260^{\circ} \mathrm{C}$ )
11. The following are the details of the load on a circuit connected through a supply meter:
(i) Six lights of 60 W each working for $5 \mathrm{~h} /$ day.
(ii) Four fluorescent tubes of 40 W each working for $4 \mathrm{~h} /$ day.
(iii) Two heaters of 1000 W each working for $1 \mathrm{~h} /$ day.
(iv) One electric iron of 750 W working for $2 \mathrm{~h} /$ day.
(v) Six fans of 60 W each working for $18 \mathrm{~h} /$ day.

If each unit of energy costs ₹ 2.80 , what will be the total bill for the month of June. (Ans. ₹ 1043.28 )
12. Calculate the resistance between the points $A$ and $B$ in network shown in Figure 1.27. All the resistors are given in ohms.
(Ans. 2 ohm)


Fig. 1.27 Network containing resistors in series-parallel combination
13. Each resistance of the network shown in Figure 1.28 is equal to $R$. What is the resistance between the terminal A and B ?


Fig. 1.28 Given electrical network
14. Obtain the total power supplied by the 60 V source and the power absorbed in each resistor in the network shown in Figure 1.29.
(Ans. 360 W, 252 W, 27 W, 54 W, 15.75 W, 11.25 W)


Fig. 1.29 Given electrical network
15. A resistance of $10 \Omega$ is connected in series with two resistances each of $15 \Omega$ arranged in parallel. What resistance must be connected across the parallel combination that the total current taken shall be 1.5 A with 20 V applied.
(Ans. 6 ohm)
16. In the network shown in Figure 1.30, determine current and voltage drop across 40 ohm resistor.
(Ans. 1 A, 40 V)


Fig. 1.30 Given electrical network
17. A letter $A$ is constructed of a uniform wire of resistance $2 \mathrm{ohm} / \mathrm{cm}$. The sides of the letter are 60 cm long and the cross piece is 30 cm long, while the apex angle is $60^{\circ}$. Find the resistance of the letter between the two ends of the legs.
(Ans. 150 ohm)
18. A circuit consists of three resistances of $12 \Omega, 18 \Omega$, and $36 \Omega$, respectively, joined in parallel is connected in series with a fourth resistance. The whole circuit is supplied at 60 V , and it is found that power dissipated in $12 \Omega$ resistance is 48 W . Determine the value of fourth resistance and the total power dissipated in the circuit.
(Ans. $9 \Omega ; 240 \mathrm{~W}$ )
19. How many 60 W lamps may be safely run on a 230 V circuit fitted with a 5 ampere fuse.
(Ans. 19)
20. Determine the current supplied by the battery in circuit shown in Figure 1.31.
(U.P.T.U. Tut.) (Ans. 29.17 mA$)$


Fig. 1.31 Given electrical network
21. How many 60 W lamps may be safely operated on a 230 V mains fitted with a 5 A fuse?
(Ans. 19)

## VIVA VOCE OR REASONING QUESTIONS

1. The resistivity of a material is also called specific resistance, why?
2. The direction of flow of current is marked opposite to the direction of flow of electrons in a conductor, why?
3. While defining Ohm's law, the physical conditions are required to be kept constant, why?
4. Is emf different to potential difference?
5. Is there any difference between conductivity and conductance?
6. The temperature co-efficient of resistance of semiconductors is negative, why?
7. Whether fuse is connected in series or in parallel in the wiring system, state why?
8. All the appliances in the house hold wiring system are connected in parallel, why?

## SHORT ANSWER TYPE QUESTIONS

1. What is an electron?
2. What do you mean by nucleus of an atom?
3. What are the free electrons?
4. How will you define electric potential?
5. What do you mean by conventional direction of flow of current?
6. Distinguish between resistivity and conductivity.
7. Define conductance.
8. What are the limitations of Ohm's law?
9. The temperature co-efficient of resistance of copper is linear, what do you mean by this statement?
10. What is the effect of temperature on the temperature co-efficient of resistance of conductors?
11. What are SI and commercial units of electrical energy.
12. How electric power is related to electrical energy?
13. Derive a relation between horse power and watt.
14. In parallel connections, if the number of resistors are added further, whether the effective value increases or decreases, justify your answer.
15. What are the characteristics of series circuit?
16. What are the characteristics of parallel circuits?

## TEST QUESTIONS

1. What is electricity?
2. Distinguish between emf. and potential difference.
3. What do you understand by electric current? Give its unit. Explain why the flow of electron in an electric circuit is opposite to the direction of conventional current?
4. What are the factors affecting the resistance of a conductor? How they affect its value?
5. Write short note on resistance and conductance.
6. Define the following and mention their units: (i) Specific resistance, (ii) Conductivity.
7. State and explain Ohm's law. Give its limitations.
(U.P.T.U.Tut.)
8. What do you understand by temperature coefficient of resistance? Specify its units. Name five materials whose resistance decreases with rise in temperature.
9. When a $100 \mathrm{~W}, 230 \mathrm{~V}$ filament lamp is switched ON to rated supply, will it draw the same current at the start and under operating condition? Justify your answer.
10. Define the resistance temperature coefficient of a conductor. If $\mathrm{a}_{1}$ is the resistance temperature coefficient of a conductor at $t_{1}{ }^{\circ} \mathrm{C}$, show that the co-efficient at $t_{2}{ }^{\circ} \mathrm{C}$ is given by

$$
\alpha_{2}=\frac{1}{\left[\frac{1}{\alpha_{1}}+\left(t_{2}-t_{1}\right)\right]}
$$

11. What do you understand by electrical energy and electrical power? Give their practical units.
12. State and explain Joule's law of electric heating.
13. Establish relations between the following:
(i) Horse power and kW ; (ii) kWh and kilo calories; (iii) Horse power and torque.
14. What are the practical applications of (i) series circuits and (ii) parallel circuits?
15. Show that
(i) in series circuit, the effective value of resistance; $R=R_{1}+R_{2}+\cdots+R$.
(ii) in parallel circuit, the reciprocal of the effective value of resistance, $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots+\frac{1}{R}$,
16. Give reasons why all the equipment are connected in parallel to the supply.

## ANSWERS

## Fill in the Blanks

1. inversely
2. decreases
3. less
4. electric energy
5. $\mathrm{mho} / \mathrm{m}$
6. 1 kWh
7. more
8. ohm-m
9. 25
10. opposite
11. decreases
12. temperature
13. increases
14. $1.6 \times 10^{-19}$
15. $628 \times 10^{16}$
16. nature
17. one
18. 0.5
19. 10
20. parallel
21. 25
22. electric energy
23. 6
24. 40
25. $x y$

## Objective Type Questions

1. (b)
2. (a)
3. (c)
4. (b)
5. (b)
6. (d)
7. (c)
8. (d)
9. (a)
10. (c)
11. (d)
12. (a)
13. (b)
14. (c)
15. (b)
16. (d)
17. (c)
18. (b)
19. (d)
20. (a)
21. (d)
22. (d)


## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand:

* What is an electric network?
*What is the behaviour of different elements of an electric network?
* How electric or electronic circuits are analysed or designed? How the value of different components of an electric or electronic circuit are determined by using:
* Kirchhoff's law
- Loop or nodal analysis
* Delta-star or star-delta transformation
* Superposition theorem
* Thevenin's theorem
* Norton's theorem
* What is maximum power transfer theorem and its applications?


### 2.1 INTRODUCTION

The arrangement by which various electrical energy sources, resistance, and other parameters are connected together is called electrical circuit or electrical network. Various laws and
theorems have been developed to analyse these simple and complex electrical circuits. One of them, that is, Ohm's law, has already been discussed in the first chapter. In this chapter, we shall discuss some other important laws and theorems for the solution of different network problems.

### 2.2 ELECTRIC NETWORK



Fig. 2.1 An electric network

A simple electric network is shown in Figure 2.1. It contains two voltage sources $E_{1}$ and $E_{2}$ and three resistors $R_{1}, R_{2}$, and $R_{3}$. In fact, the interconnection of either passive elements or the interconnection of active and passive elements constitute an electric network.

### 2.2.1 Active elements

The elements that supply energy in an electric network are called active elements. In the circuit shown in Figure 2.1, $E_{1}$ and $E_{2}$ are the active elements.
Note: When a battery is delivering current from its positive terminal, it is under discharging condition. However, if it is receiving current at its positive terminal, then it is under charging condition. In both the cases, it will be considered as an active element.

### 2.2.2 Passive Elements

The elements that receive electrical energy and dispose the same in their own way of disposal are called passive elements. In the circuit shown in Figure 2.1, $R_{1}, R_{2}$, and $R_{3}$ are the passive elements. The other passive elements that are not used in this circuit are inductors and capacitors.

### 2.2.3 Network Terminology

Network theorems are applied to analyse the electrical network. While discussing these theorems, we come across the following terms:

1. Electric network: A combination of various electric elements connected in any manner is called an electric network.
2. Electric circuit: An electric circuit is a closed conducting path through which an electric current either flows or is intended to flow.
3. Parameters: The various elements of an electric circuit are called its parameters such as resistors, inductors, and capacitors.
4. Linear circuit: An electric circuit that contains parameters of constant value, that is, their value do not change with voltage or current is called linear circuit.
5. Non-linear circuit: An electric circuit that contains parameters whose value changes with voltage or current is called non-linear circuit.
6. Bilateral circuit: An electric circuit that possesses the same properties or characteristics in either direction is called bilateral circuit. A transmission line is bilateral because it can be made to perform its function equally well in either direction.
7. Unilateral circuit: An electric circuit whose properties or characteristics change with the direction of its operation is called unilateral circuit. A diode rectifier circuit is a unilateral circuit because it cannot perform similarly in both the directions.
8. Unilateral elements: The elements that conduct only in one direction, such as semiconductor diode, are called unidirectional elements.
9. Bilateral elements: The elements that conduct in both the directions similarly, such as a simple piece of wire (resistor), diac, and triac, are called bilateral elements.
10. Active network: An electric network that contains one or more sources of emf is called active network.
11. Passive network: An electric network that does not contain any source of emf is called passive network.
12. Node: A node is a point in the network where two or more circuit elements are joined. In Figure 2.1, A, B, C, and D are the nodes.
13. Junction: A junction is a point in the network where three or more circuit elements are joined. In fact, it is a point where current is divided. In Figure 2.1, B and D are junctions.
14. Branch: The part of a network that lies between two junction points is called branch. In Figure 2.1, DAB, BCD, and BD are the three branches.
15. Loop: The closed path of a network is called a loop. In Figure 2.1, ABDA, BCDB, and ABCDA are the three loops.
16. Mesh: The most elementary form of a loop that cannot be further divided is called a mesh. In Figure 2.1, ABDA and BCDB are the two meshes, but ABCDA is the loop.

### 2.3 VOLTAGE AND CURRENT SOURCES

To operate electrical or electronic circuits, a source of electrical power is required. The basic purpose of a source is to supply power to a load. Therefore, a load is connected to the source as shown in Figure 2.2. The source may be either a DC (direct current) source or an AC (alternating current) source.

1. DC source: Any device that produces direct voltage output continuously is called a DC source. Some of the commonly used DC sources are batteries, generators, and DC power supplies (regulated power supplies).
(a) Battery: A battery is the most com-


Fig. 2.2 Transfer of energy from source to load mon DC voltage source used for the operation of electronic circuits. A battery is just a series, parallel, or series-parallel combination of primary or secondary cells. The secondary cells are rechargeable, whereas primary cells are not. The battery used in an automobile (car) contains number of secondary cells in series (see Fig. 2.3(a)), whereas the dry cells used in a
torch are primary cells (see Fig. 2.3(b)). Both of them are used for the operation of electrical or electronic circuits.


Fig. 2.3 (a) Secondary battery and (b) Primary cells

Solar cells (which convert light energy into electrical energy) are also used for the operation of electronic circuits, for example, in calculators, satellites, etc. Batteries or cells are used where small power (voltage and current) is required.
(b) Generator: A DC generator is an electrical machine that converts mechanical energy into electrical energy. When armature is rotated by a prime-mover (water turbine, steam turbine, diesel engine, etc.) in the stationary magnetic field, the required DC voltage appears across its terminals (see Fig. 2.4). DC Generators are used where large power (voltage and current) is required.
(c) Rectifier-type supply: It consists of a step-down transformer and a rectifier to obtain required DC output voltage. This DC supply is used most frequently in the electronic circuits. A rectifier-type supply is shown in Figure 2.5.


Fig. 2.4 DC Generator


Fig. 2.5 DC Power supply
2. AC Source: Any device that produces alternating voltage output continuously is called an AC source. Some of the commonly used AC sources are alternators and oscillators or signal generators.
(a) Alternators: An AC generator is known as an alternator. It is a similar machine as that of DC generator, the only difference is that its output is AC voltage instead of DC voltage. These machines are installed at the generating stations, where power is generated at 50 Hz . This power is applied to various electrical or electronic circuits.
(b) Oscillators or signal generators: An oscillator or signal generator is the equipment that supplies (output) AC voltages at different frequencies. Its output is used to test the working of various electrical or electronic circuits (e.g., amplifier, etc.). Some signal generators are capable of supplying different types of waveforms such as triangular wave and square wave in addition to the sinusoidal. A signal generator used in the laboratory is shown in Figure 2.6.


Fig. 2.6 Signal generator

### 2.3.1 Internal Resistance of a Source

The opposition to load current inside the DC source is called its internal resistance. All DC sources (battery, DC generator, or rectifier-type supply) have internal resistance and is represented by $R_{\mathrm{i}}$. The equivalent circuit of a DC source is the generated $\operatorname{emf} E$ is in series with internal resistance $R_{\mathrm{i}}$ of the source as shown in Figure 2.7.


Fig. 2.7 (a) Battery and
(b) Generator


Fig. 2.8 DC source on load

When load $\left(R_{\mathrm{L}}\right)$ is connected across the source (see Fig. 2.8).
Load current,

$$
I=\frac{E}{R_{\mathrm{L}}+R_{\mathrm{i}}}
$$

Terminal voltage, $V=E-I R_{\mathrm{i}}$ or $V=I R_{\mathrm{L}}=\frac{E}{R_{\mathrm{L}}+R_{\mathrm{i}}} \times R_{\mathrm{L}}=\frac{E}{I+R_{\mathrm{i}} / R_{\mathrm{L}}}$
The voltage across the load terminals is reduced because of the voltage drop in the internal resistance of the source. A source having smaller internal resistance will have smaller voltage drop.

### 2.3.2 Ideal Voltage Source

A voltage source that has zero internal resistance is called an ideal voltage source. In such cases, the terminal voltage remains the same, irrespective of the value of
 load resistance. As we know,
terminal voltage,

$$
V=\frac{E}{I+R_{\mathrm{i}} / R_{\mathrm{L}}}
$$

Since, $R_{\mathrm{i}}=0$,

$$
V=E
$$

An ideal voltage source is shown in Figure 2.9. An ideal voltage

Fig. 2.9 Ideal voltage source source cannot exist in nature, since all the voltage sources have some internal resistance (or impedance), although its value may be very small.

### 2.3.3 Real Voltage Source



Fig. 2.10 Real D.C. Voltage source

An ideal voltage source is not practically possible. If it would exist, it would supply an infinite current at short circuit, which is not possible. Therefore, all the voltage sources have some internal resistance that limits the current at short circuit. A real voltage source is shown in Figure 2.10.

However, it is always preferred to have a source that has very a small internal resistance, so that the terminal voltage of the source remains almost constant from no-load to full-load.

A voltage source that has very low internal resistance (or impedance) as compared to load resistance (or impedance) is known as a constant voltage source.

### 2.3.4 Current Source

A voltage source is a source that has low internal resistance, while a current source is a source that has high internal resistance.

### 2.3.5 Ideal Current Source

A voltage source that supplies a constant current no matter whatever is the load resistance (or impedance) is known as ideal current source.

A symbolic representation of such an ideal current source is shown in Figure 2.11(a). As abovementioned, the current supplied by the source should remain constant at all values of load resistance,


Fig. 2.11 (a) Ideal current source (b) Ideal current source on load (c) Relation between $I$ and $R_{L}$ and (d) Practical current source
as shown in Figure 2.11(b) and (c). It means when the load resistance is infinite $\left(R_{\mathrm{L}}=\infty\right)$, that is, terminal A and B are open, there should be some path (inside the source itself) through which the current $I_{\mathrm{S}}$ can flow as shown in Figure 2.11(d). This shows that in this type of source, there will be internal power loss even at no-load. Hence, ideal current source is merely an idea and not the real one.

### 2.3.6 Real Current Source

An ideal current source is not practically possible. If it would exist, it would supply a constant current even at no-load, which is not possible. A real current source is basically a voltage source that delivers almost the same current at all values of load resistance.


Fig. 2.12 (a) Real current source and (b) Load resistance and curve between I and $R_{L}$

A source that has very high internal resistance (or impedance) as compared to the load resistance (or impedance) is considered as a constant current source.

A real current source having an internal resistance of $10 \mathrm{M} \Omega$ with a load resistance $R_{\mathrm{L}}$ is shown in Figure 2.12(a). When load resistance varies from $1 \mathrm{~K} \Omega$ to $100 \mathrm{~K} \Omega$, the current varies from $1.19988 \mu \mathrm{~A}\left(I=E /\left(R_{\mathrm{i}}+R_{\mathrm{L}}\right)=12 / 10.001\right)$ to $1.1881 \mu \mathrm{~A}$.

This shows that the load current remains almost constant and the source behaves as a constant current source irrespective of the value of load resistance.

A current source is represented symbolically as shown in Figure 2.11(b).

### 2.3.7 Difference between Voltage Source and Current Source

Practically, there is no difference between voltage source and current source. In fact, a source can either be considered as a voltage source or as a current source. It basically depends upon its working conditions. When the value of load resistance (or impedance) is very large as compared to the internal resistance (or impedance) of the source, the source is treated as a voltage source. However, when the value of load resistance is very small as compared to the internal resistance of the source, the source is treated as a current source. Further, from the circuit point of view, it does not matter whether the source is treated as voltage source or current source.

### 2.4 SOURCE TRANSFORMATION (CONVERSION OF VOLTAGE SOURCE TO CURRENT SOURCE AND VICE VERSA)

In fact, a voltage source can be converted into current source and vice versa. Consider a DC source connected to a load resistance $R_{\mathrm{L}}$, as shown in Figure 2.13(a). The source can be treated as a voltage source, as shown in Figure 2.13(b), or as a current source as shown in Figure 2.13(c).


Fig. 2.13 (a) Load connected to source (b) Voltage source and (c) Current source

Both types of representations appear the same to the externally connected load resistance $R_{\mathrm{L}}$. They must give the same results.

The source is treated as voltage source as shown in Figure 2.13(b). If the load resistance $R_{\mathrm{L}}$ is reduced to zero as shown in Figure 2.14(a), (i.e., the terminal A and B are short circuited), the current supplied by the source is

$$
I_{\mathrm{L}}(\text { short circuit })=\frac{E}{R_{\mathrm{i} 1}}
$$

The source is treated as a current source, as shown in Figure 2.13(c). If the load resistance $R_{\mathrm{L}}$ is reduced to zero (the same resistance $R_{\mathrm{i} 2}$ connected in parallel with the short circuit is as good as not being present), as shown in Figure 2.14(b). However, the current obtained by shorting the terminals A and B simply source current $I_{\mathrm{S}}$. This current must be the same as supplied by the source when it is treated as voltage source.

$$
\begin{align*}
& I_{\mathrm{S}}=I_{\mathrm{L}}(\text { short circuit })=\frac{E}{R_{\mathrm{ii}}} \\
& E=I_{\mathrm{S}} R_{\mathrm{i} 1} \tag{2.1}
\end{align*}
$$

Again, the two representations of the source must give same terminal voltage when the load resistance $R_{\mathrm{L}}$ is disconnected from the source (i.e., when the terminals A and B are open circuited).

When the source is treated as voltage source (as shown in Fig. 2.14(c)), the terminal voltage,

$$
V=E
$$

When the source is treated as a current source (as shown in Fig. 2.14(d)), the terminal voltage,

$$
\begin{equation*}
V=I_{\mathrm{S}} R_{\mathrm{i} 2}=E \tag{2.2}
\end{equation*}
$$



Fig. 2.14 (a) Voltage source on short-circuit (b) Current source on short-circuit (c) Voltage source on open-circuit and
(d) Current source on open-circuit

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From Equation (2.1) and (2.2), we get

$$
\begin{gathered}
I_{\mathrm{s}} R_{\mathrm{i} 1}=I_{\mathrm{s}} R_{\mathrm{i} 2} \\
R_{\mathrm{i} 1}=R_{\mathrm{i} 2}=R_{\mathrm{i}}(\mathrm{say})
\end{gathered}
$$

Both Equation (2.1) and (2.2) reduce to

$$
\begin{equation*}
E=I_{\mathrm{s}} R_{\mathrm{i}} \tag{2.3}
\end{equation*}
$$

Hence, it is seen that in both the representations of the source, the source impedance as faced by the load resistance at the terminal AB is the same (i.e., $R_{\mathrm{i}}$ ). Thus, it establishes the equivalence between the voltage source representation and the current source representation under short circuit and open circuit conditions.

We can also test the equivalence at a given load resistance $R_{\mathrm{L}}$. In the case of voltage source representation, as shown in Figure 2.14(b), the current through the load resistance

$$
I_{1}=\frac{E}{R_{\mathrm{i}}+R_{\mathrm{L}}}
$$

In the case of current source representation, as shown in Figure 2.14(c), the current $I_{\mathrm{S}}$ is divided into two branches. Current through the load resistance,

$$
\begin{equation*}
I_{2}=I_{\mathrm{S}} \times \frac{R_{\mathrm{i}}}{R_{\mathrm{i}}+R_{\mathrm{L}}}=\frac{I_{\mathrm{S}} R_{\mathrm{i}}}{R_{\mathrm{i}}+R_{\mathrm{L}}}=\frac{E}{R_{\mathrm{i}}+R_{\mathrm{L}}} \tag{2.4}
\end{equation*}
$$

The two currents $I_{1}$ and $I_{2}$ are exactly the same. Thus, we conclude that a given voltage source can be converted into its equivalent current source and vice versa by using Equation (2.3).

## Example 2.1

Figure 2.15 shows a DC voltage source having an open-circuit voltage of 6 V and an internal resistance of $1 \Omega$. Obtain its equivalent current source representation.


Fig. 2.15 Voltage source


Fig. 2.16 Equivalent current source

## Solution:

If the terminals A and B of voltage source are short circuited, the current supplied by the source,

$$
I_{\mathrm{S}}=\frac{E}{R_{\mathrm{i}}}=\frac{6}{1}=6 \mathrm{~A}
$$

In the equivalent current source representation, the current source is of 6 A . The internal resistance of the source is represented in parallel with the current source as shown in Figure 2.16.

## Example 2.2

Figure 2.17 shows a DC current source. Obtain its equivalent voltage source representation.


Fig. 2.17 Current source


Fig. 2.18 Equivant voltage source

## Solution:

The open-circuit voltage of the current source across the terminals A and B.

$$
E=I_{\mathrm{S}} R_{\mathrm{i}}=2 \times 100=200 \mathrm{~V}
$$

This will be the generated emf or ideal voltage of the equivalent voltage source representation. The internal impedance $\left(R_{\mathrm{i}}=100 \Omega\right)$ is connected in series with the ideal voltage source, as shown in Figure 2.18.

This gives the equivalent voltage source representation of the given current source.

### 2.5 KICHHOFF'S LAWS

Gustav Kirchhoff, a German scientist, summed up his findings in a set of two laws known as Kirchhoff's laws.

### 2.5.1 Kirchhoff's First Law

Since this law relates the currents following through the circuit, it is also known as Kirchhoff's Current Law (KCL). This law states that the algebraic sum of all the currents meeting at a point or junction is zero.

Mathematically,

$$
\sum I=0
$$

To determine algebraic sum in Figure 2.19, let us consider the incoming currents as positive and outgoing currents as negative. By applying KCL to junction O in Figure 2.19, we get
or

$$
I_{1}-I_{2}-I_{3}-I_{4}+I_{5}=0
$$

$$
I_{1}+I_{5}=I_{2}+I_{3}+I_{4}
$$



Fig. 2.19 Current of various branches meeting at a junction
that is, sum of incoming currents = sum of outgoing currents.
Hence, KCL can also be stated as the sum of incoming currents are equal to the sum of outgoing currents at a point or junction in an electrical network.

### 2.5.2 Kirchhoff's Second Law

Since this law relates the voltages in a closed circuit of an electrical network, it is also known as Kirchhoff's voltage law (KVL) or Kirchhoff's mesh law. This law states that in a closed circuit or mesh, the algebraic sum of all the emfs and the algebraic sum of all the voltage drops (i.e., product of current and resistances) is zero.

Therefore, algebraic sum of all the emfs + algebraic sum of all the voltage drops $=0$
Mathematically,

$$
\sum E+\sum V=0 \text { (algebraic values) }
$$

As algebraic sum of emf and voltage drops to be taken, we have to determine their signs while tracing a circuit. For this, we always consider a rise in potential as positive and a fall in potential as negative.

(a)

(b)

Fig. 2.20 (a) emf of a
cell (b) Voltage drop in a resistor

Consider a branch AB containing only one source of emf $(E)$, as shown in Figure 2.20(a). While tracing this branch from A to B, there is fall in potential, and therefore, $E$ is negative. However, if we trace it from B to A, there is rise in potential and $E$ will be positive. In short,

If tracing branch A to B , then $E$ is negative (i.e., $-E$ ).
If tracing branch B to A , then $E$ is positive (i.e., $+E$ ).
Note: The direction of flow of current is not considered since it has no effect.

To determine the sign for the voltage drop $V(=I \mathrm{R})$, consider a branch containing resistor of resistance $R$ in which current $I$ flows from A to B, as shown in Figure 2.20(b). Therefore, A is at high potential w.r.t. terminal B .
Thus, while tracing branch from A to $\mathrm{B}, V$ is negative (i.e., $-V$ ) as there is a fall in potential and we are moving in the direction of flow of current. However, while the tracing branch is from B to $\mathrm{A}, V$ is positive (i.e., $+V$ ) as there is rise in potential and we are moving opposite to the direction of flow of current.
Note: Only the direction of flow of current determines the sign of $V$.
Illustration: Consider a network shown in Figure 2.21. In this circuit, we can apply KVL for the closed circuit BCDA and form an equation as

$$
-I_{1} R_{1}-E_{2}+I_{2} R_{3}=0
$$



Fig. 2.21 Electric network
where $I_{1} R_{1}$ (voltage drop) is considered as negative, since we are tracing the circuit along the direction of flow of current. $E_{2}$ (emf of battery) is taken as negative as we move from positive to negative terminal at the battery and there is fall in potential.
$I_{2} R_{3}$ (voltage drop) in taken as positive as we move opposite to the flow of current.

Similarly, applying KVL to the closed circuit ABDA, we get $-\left(I_{1}+I_{2}\right) R_{1}-I_{2} R_{3}+E_{1}=0$

### 2.5.3 Solution of Network by Kirchhoff's Laws

When a network is to be solved by applying Kirchhoff's laws, the following steps are taken:

1. Mark the assumed direction of flow of current in various branches of the network according to KCL.
2. Choose as many number of closed circuits as the number of unknown quantities.
3. Apply KVL to the chosen closed circuits and prepare the equations.
4. After solving the equations, determine the unknown values.

Note: If the determined current carries the negative, sign, it shows that the actual direction of flow of current is opposite to that of the assumed direction of flow of current in the given branch.

### 2.6 WHEATSTONE BRIDGE

For the first time, Wheatstone (an English telegraph engineer) proposed this bridge for measuring the value of an unknown resistance. This bridge consists of four arms $\mathrm{AB}, \mathrm{BC}, \mathrm{AD}$, and DC having resistances $P, Q, X$, and $R$, respectively (see Fig. 2.22). Resistance $P$ and $Q$ are the known (fixed value) resistances and are called ratio arms. While resistance $R$ is a variable resistance of known value and $X$ is an unknown resistance whose value is to be determined.

To determine the value of $X$, connect a battery $E$ across $A$ and $C$ and a galvanometer $G$ across $B$ and $D$ through key $K$. The bridge is said to be balanced, when galvanometer $G$ gives zero defection on closing key $K$. The balance is obtained by selecting the values of resistors $P$ and $Q$ suitably, and finally, adjusting the value of $R$.

At balance,
Current flowing through galvanometer $=0$
Current flowing through resistor $P$ and $Q=I_{1}$
Current flowing through resistor $X$ and $R=I_{2}$
Voltage drop across $\mathrm{AB}=$ Voltage drop across AD

$$
\begin{equation*}
I_{1} P=I_{2} X \tag{2.5}
\end{equation*}
$$

Voltage drop across $\mathrm{BC}=$ Voltage drop across DC

$$
\begin{equation*}
I_{1} Q=I_{2} R \tag{2.6}
\end{equation*}
$$



Fig. 2.22 Wheatstone Bridge

Dividing Equation (2.5) by (2.6), we get

$$
\frac{P}{Q}=\frac{X}{R} \quad \text { or } \quad X Q=P R
$$

Product of opposite arms $=$ Product of other opposite arms

$$
X=\frac{P}{Q} \times R
$$

## Example 2.3

Figure 2.23 shows two batteries connected in parallel, each represented by an emf along with its internal resistance. A load resistance of $6 \Omega$ is connected across the ends of the batteries. Calculate the current through each battery and the load.
(U.P.T.U. July 2002)

## Solution:

Let the current flowing through various branches be as marked in Figure 2.24.
By applying KVL to mesh ABEFA, we get

$$
-2 I_{1}+4 I_{2}-44+40=0
$$

or

$$
\begin{equation*}
2 I_{1}-4 I_{2}=40-44 \text { or } 2 I_{2}-I_{1}=2 \tag{2.7}
\end{equation*}
$$



Fig. 2.24 Assumed direction of flow of current in various branches

By applying KVL to the mesh BCDEB, we get
or

$$
\begin{gather*}
-4 I_{2}-6\left(I_{1}+I_{2}\right)+44=0 \\
4 I_{2}+6\left(I_{1}+I_{2}\right)=44 \quad \text { or } \quad 5 I_{2}+3 I_{1}=22 \tag{2.8}
\end{gather*}
$$

Multiplying Equation (2.7) by 3 and adding with Equation (2.8), we get

$$
I_{2}=\frac{28}{11} \mathrm{~A}
$$

Substituting the value of $I_{2}$ in Equation (2.7), we obtain

$$
I_{1}=\frac{34}{11} \mathrm{~A}
$$

Current through load, $I_{\mathrm{L}}=I_{1}+I_{2}=\frac{34}{11}=\frac{28}{11}=\frac{62}{11} \mathrm{~A}=5.636 \mathrm{~A}$

## Example 2.4

Two batteries A and B are connected in parallel and a load of $10 \Omega$ is connected across their terminals. A has an emf of 12 V and an internal resistance of $2 \Omega$ and B has an emf of 8 V and an internal resistance of $1 \Omega$. Use Kirchhoff's laws to determine the magnitude of currents and also the directions in each of the batteries. Further, determine the potential difference across external resistance.
(U.P.T.U. Tut.)

## Solution:

Let the current flowing through various branches be as marked in Figure 2.25.
By applying KVL to mesh ABCDA, we get

$$
\begin{align*}
& 2 I_{1}-I_{2}+8-12 & =0 \\
\text { or } & 2 I_{1}-I_{2} & =12-8 \\
\text { or } & 2 I_{1}-I_{2} & =4
\end{align*}
$$

By applying KVL to mesh ADCEA, we get


Fig. 2.25 Circuit as per given data
or

$$
\begin{equation*}
I_{2}+10\left(I_{1}+I_{2}\right)-8=0 \tag{2.10}
\end{equation*}
$$

Solving Equations (2.9) and (2.10), we get
and

$$
\begin{aligned}
& I_{1}=1.625 \mathrm{~A} \\
& I_{2}=-0.75 \mathrm{~A}
\end{aligned}
$$

The negative sign with current $I_{2}$ shows that current $I_{2}$ is flowing opposite to the assumed direction.

Current flowing through the load resistance of $10 \Omega=I_{1}+I_{2}=1.625+(-0.75)=0.875 \mathrm{~A}$ Potential difference across load resistance $=\left(I_{1}+I_{2}\right) R=0.875 \times 10=8.75 \mathrm{~V}$

## Example 2.5

By using Kirchhoff's laws, find the current in XY in the circuit shown in Figure 2.26.

## Solution:

The assumed direction of flow of current in various sections is marked in Figure 2.27.


Fig. 2.26 Given network


Fig. 2.27 Assumed direction of flow of current in various branches

By applying Kirchhoff's second law, we get
Circuit XYAX
or

$$
\begin{align*}
-0.05 I_{1}-0.05\left(I_{1}-40\right)+0.1 I_{2} & =0 \\
-0.05 I_{1}-0.05 I_{1}+2+0.1 I_{2} & =0 \\
I_{1}-I_{2} & =20 \tag{2.11}
\end{align*}
$$

## Circuit XYBX

or

$$
\begin{align*}
-0.1 I_{2}-0.1\left(I_{1}+I_{2}-110\right)-0.05\left(I_{1}+I_{2}-160\right) & =0 \\
-0.1 I_{2}-0.1 I_{1}-0.1 I_{2}+11-0.05 I_{1}-0.05 I_{2}+8 & =0 \\
-0.15 I_{1}-0.25 I_{2}+19 & =0 \\
3 I_{1}+5 I_{2} & =380 \tag{2.12}
\end{align*}
$$

or

Multiplying Equation (2.11) by 3 and subtracting from (2.12), we get
Current in section XY, $I_{2}=\frac{320}{8}=40 \mathrm{~A}$

$$
8 I_{2}=320
$$

## Example 2.6

In Figure 2.28 , the potential of point A is -30 V . Using Kirchhoff's laws, find (a) value of $V$ and (b) power dissipated by $5 \Omega$ resistance.
(U.P.T.U. Tut.)

## Solution:

The potential at point A is -30 V and potential at point G is zero, being grounded. Potential difference across $12-\Omega$ resistor $=30 \mathrm{~V}$

Current through $12-\Omega$ resistor $=\frac{30}{12}=2.5 \mathrm{~A}$
A simplified circuit of the network and assumed current distributed is shown in Figure 2.29. By applying KVL to mesh CDEC, we get
or

$$
\begin{align*}
-3\left(I_{1}-I_{2}\right)+6 I_{2} & =0 \\
-3 I_{1}+3 I_{2}+6 I_{2}=0 \quad \text { or } \quad I_{1} & =3 I_{2} \tag{2.13}
\end{align*}
$$



Fig. 2.28 Given network


Fig. 2.29 Assumed direction of flow of current in various branches

By applying KVL to mesh AGEFA, we get
or

$$
\begin{aligned}
12 \times 2.5-4\left(I_{1}-2.5\right) & =0 \\
30-4 I_{1}+10=0 & \text { or } \quad I_{1}
\end{aligned}=10 \mathrm{~A} .
$$

From Equation (2.13),

$$
I_{2}=\frac{I_{1}}{3}=\frac{10}{3} \mathrm{~A}
$$

By applying KVL to mesh ABCGA, we get
or

$$
\begin{aligned}
V-5 I_{1}-6 I_{2}-2.5 \times 12 & =0 \\
V-5 \times 10-6 \times \frac{10}{3}-30=0 \quad \text { or } \quad V & =100 \mathrm{~V}
\end{aligned}
$$

Power dissipated in 5- $\Omega$ resistor $=I_{1}^{2} \times 5=(10)^{2} \times 5=500 \mathrm{~W}$

## Example 2.7

In the circuit shown in Figure 2.30, find the value of $I_{\mathrm{S}}$ for $I=0$.

## Solution:

By applying KCL at junction B , we get
Current through branch BE (i.e., $2-\Omega$ resistor)

$$
=I+I_{\mathrm{S}}=0+I_{\mathrm{S}}=I_{\mathrm{S}}(I=0)
$$

By applying KVL to mesh ABEFA, we get

$$
-3 I_{1}-2 I_{\mathrm{S}}+4=0
$$

or

$$
\begin{aligned}
3 I+2 I_{\mathrm{S}} & =4 \\
I_{\mathrm{S}} & =\frac{4}{2}=2 \mathrm{~A}(I=0)
\end{aligned}
$$



Fig. 2.30 Given network

## Example 2.8

Find the value of $R$ and the current through it in the circuit shown in Figure 2.31, when the current is zero in branch OA.

## Solution:

Since current flowing through branch OA is zero, the same current $I_{1}$ flows through branch BA and AC. The assumed direction of flow of current in other branches is also marked in Figure 2.32, according to Kirchhoff's first law.


Fig. 2.31 Given network


Fig. 2.32 Assumed direction of flow of current in various branches

By applying Kirchhoff's second law to the following meshes, we get Mesh BAOB

$$
\begin{align*}
-I_{1} \pm 0+4 I_{2} & =0 \\
I_{1} & =4 I_{2} \tag{2.14}
\end{align*}
$$

Mesh BACB

$$
\begin{aligned}
-I_{1}-1.51_{1}-2\left(I_{1}+I_{2}\right)+10 & =0 \\
4.5 I_{1}+2 I_{2} & =10
\end{aligned}
$$

Substituting the value of $I_{1}$ from Equation (2.14), we get

$$
4.5\left(4 I_{2}\right)+2 I_{2}=10
$$

or
and

$$
I_{2}=0.5 \mathrm{~A}
$$

$$
I_{1}=4 \times 0.5=2 \mathrm{~A}
$$

Mesh BOCB
or
$-4 \times 0.5-R \times 0.5-2(2+0.5)+10=0$

$$
R=6 \Omega
$$

## Example 2.9

Determine the current in the $4-\Omega$ resistance of circuit shown in Figure 2.33.
(U.P.T.U. Tut.)

## Solution:

A simplified circuit is redrawn as shown in Figure

Fig. 2.33 Given network
 2.34. Let the current flowing through various branches be as marked in Figure 2.34.

By applying KVL to various meshes, that is, BCDHB, DEFHD, and BHFGAB, we get

$$
\begin{align*}
& -2\left(I_{1}-I_{2}\right)-10 I_{3}+1 \times\left(I_{2}-6\right)=0 ; \\
& \quad-2\left(I_{1}-I_{2}+6-I_{3}\right)+3\left(I_{2}-6+I_{3}\right)+10 I_{3}=+10 \tag{2.16}
\end{align*}
$$

and $-1 \times\left(I_{2}-6\right)-3\left(I_{2}-6+I_{3}\right)-4 I_{1}=-24$
From Equation (2.15), $2 I_{1}-3 I_{2}+10 I_{3}=-6$
From Equation (2.16), $2 I_{1}-5 I_{2}-15 I_{3}=-40$
From Equation (2.17), $4 I_{1}+4 I_{2}+3 I_{3}=48$

## Solving equations by matrices:

The abovementioned equations can be given in the matrix form as

$$
\left[\begin{array}{ccc}
2 & -3 & +10 \\
2 & -5 & -15 \\
4 & 4 & +3
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right]=\left[\begin{array}{l}
-6 \\
-40 \\
48
\end{array}\right]
$$



Fig. 2.34 Assumed direction of flow of current in various branches

The common determinant is given as

$$
\begin{aligned}
\Delta & =\left|\begin{array}{ccc}
2 & -3 & +10 \\
2 & -5 & -15 \\
4 & 4 & +3
\end{array}\right| \\
& =2[(-5) \times 3-4 \times(-15)]-2[(-3) \times 3-4 \times 10]+4[(-3) \times(-15)-(-5) \times 10] \\
& =2[-15+60]-2[-9-40]+[45+50] \\
& =90+98+380=568
\end{aligned}
$$

The determinant for $I_{1}$ is given as

$$
\begin{aligned}
\Delta_{1} & =\left|\begin{array}{ccc}
-6 & -3 & +10 \\
-40 & -5 & -15 \\
48 & 4 & +3
\end{array}\right| \\
& =-6[(-5) \times 3-4 \times(-15)]+40[(-3) \times 3-4 \times 10]+48[(-3) \times(-15)-(-5) \times 10] \\
& =-6[-15+60]+40[-9-40]+48[45+50] \\
& =-270-1,960+4,560=2,330
\end{aligned}
$$

As per Cramer's rule, $I_{1}=\frac{\Delta_{1}}{\Delta}=\frac{2330}{568}=4.1 \mathrm{~A}$

## Example 2.10

What is the difference of potential between X and Y in the network shown in Figure 2.35 .


Fig. 2.35 Given network

## Solution:

Refer to Figure 2.36. Let the current in branch AX be $I_{1}$ and in branch BY be $I_{2}$.


Fig. 2.36 Current in different loops

$$
\begin{aligned}
& \text { Current, } I_{1}=\frac{2}{2+3}=0.4 \mathrm{~A}(\text { from A to X) } \\
& \text { Current, } I_{2}=\frac{4}{3+5}=0.5 \mathrm{~A}(\text { from B to } \mathrm{Y})
\end{aligned}
$$

Rise in potential from X to Y is given as

$$
V_{\mathrm{XY}}=3 \times 0.4+5-3 \times 0.5=4.7 \mathrm{~V}
$$

## Example 2.11

Find the total power delivered to the circuit by two sources in Figure 2.37.
(U.P.T.U. Tut.)

## Solution:

Let the current flowing through different branches be as marked in Figure 2.38.


Fig. 2.37 Given network


Fig. 2.38 Assumed direction of flow of current in various branches

By applying KVL to mesh ABCFEA, we get

$$
-2 I_{1}-\left(I_{1}+6\right) \times 3-5=0 \quad \text { or } \quad I_{1}=\frac{-13}{5}=-2.6 \mathrm{~A}
$$

By applying KVL to mesh ADCFEA, we get

$$
-I_{2}-4\left(I_{2}-6\right)-5=0 \quad \text { or } \quad I_{2}=\frac{29}{5}=5.8 \mathrm{~A}
$$

Total power delivered by the two sources

$$
\begin{aligned}
& =\text { Total power absorbed by various resistors } \\
& =I_{1}^{2} \times 2+\left(I_{1}+6\right)^{2} \times 3+\left(I_{2}-6\right)^{2} \times 4+I_{2}^{2} \times 1 \\
& =(-2.6)^{2} \times 2+(-2.6+6)^{2} \times 3+(5.8-6)^{2} \times 4+(5.8)^{2} \times 1 \\
& =82 \mathrm{~W}
\end{aligned}
$$

## Example 2.12

In the circuit shown in Figure 2.39, determine the value of $E_{2}$ that will reduce the galvanometer current to zero. The galvanometer resistance is $10 \Omega$.

## Solution:

By applying KCL, the direction of flow of current in various branches is marked in Figure 2.40. Since galvanometer does not carry any current, the current in branch BC is the same as that in branch AB , that is, $I_{1}$. Similarly, current flowing through branch DC is the same as that in branch AD , that is, $I_{2}$.

By applying KVL to various meshes, we get:


Fig. 2.39 Given network

Mesh ABCDA

$$
-11 I_{1}+9 I_{2}+E_{2}=0
$$

or

$$
\begin{equation*}
11 I_{1}-9 I_{2}=E_{2} \tag{2.21}
\end{equation*}
$$

Mesh $\mathrm{ABC} E_{1} \mathrm{~A}$

$$
-11 I_{1}-2\left(I_{1}+I_{2}\right)+2=0
$$

or

$$
\begin{equation*}
13 I_{1}+2 I_{2}=2 \tag{2.22}
\end{equation*}
$$

Mesh BCDB

$$
-5 I_{1}+4 I_{2}=0
$$

or

$$
\begin{equation*}
I_{1}=0.8 I_{2} \tag{2.23}
\end{equation*}
$$



Fig. 2.40 Assumed direction of flow of current in various branches
or

$$
\begin{aligned}
& 13\left(0.8 I_{2}\right)+2 I_{2}=2 \\
& I_{2}=\frac{2}{12.4}=\frac{5}{31} \mathrm{~A}
\end{aligned}
$$

$$
I_{1}=\frac{0.8 \times 2}{12.4}=\frac{4}{31} \mathrm{~A}
$$

Substituting the value of $I_{1}$ and $I_{2}$ in Equation (2.21), we get
or

$$
\begin{aligned}
11 \times(4 / 31)-9(5 / 31) & =E_{2} \\
E_{2} & =\frac{-1}{31} \mathrm{~V}
\end{aligned}=-0.032258 \mathrm{~V} .
$$

Note: The negative sign shows that the cell $E_{2}$ should be connected in reverse direction.

## Example 2.13



Fig. 2.41 Given network


Fig. 2.42 Assumed direction of flow of current in various branches

Determine the current $I$ in $4-\Omega$ resistance in the circuit shown in Figure 2.41.

## Solution:

A simplified circuit is shown in Figure 2.42. By applying KCL, different currents are marked in various sections.

By applying KVL to various loops, we get Loop ABHGA:
$10 I_{2}+\left(I-I_{1}-6\right)-2 I_{1}=0 \quad$ or $\quad I-3 I_{1}+10 I_{2}=6$

Loop BCDHB:

$$
\begin{gather*}
-2\left(I_{1}+I_{2}+6\right)-10+3\left(I-I_{1}-I_{2}-6\right)-10 I_{2}=0 \\
3 I-5 I_{1}-15 I_{2}=40 \tag{2.25}
\end{gather*}
$$

Loop GHDEFG:

$$
\begin{align*}
& -\left(I-I_{1}-6\right)-3\left(I-I_{1}-I_{2}-6\right)-4 I+24=0 \\
& \text { or } \quad 8 I-4 I_{1}-3 I_{2}=48
\end{align*}
$$

Eliminating $I_{1}$ from Equations (2.24) and (2.25), we get

$$
\begin{equation*}
4 I-95 I_{2}=90 \tag{2.27}
\end{equation*}
$$

Eliminating $I_{1}$ from Equations (2.25) and (2.26), we get

$$
\begin{equation*}
28 I+45 I_{2}=80 \tag{2.28}
\end{equation*}
$$

Eliminating $I_{2}$ from Equations (2.27) and (2.28), we get

$$
568 I=2330
$$

Therefore, current in $4-\Omega$ resistor, $I=4.102 \mathrm{~A}$ Alternatively, the three equation are as follows:

$$
\begin{gathered}
I-3 I_{1}+10 I_{2}=6 \\
3 I-5 I_{1}-15 I_{2}=40 \\
8 I-4 I_{1}-3 I_{2}=48
\end{gathered}
$$

The three equations can be solved by the method of determinants, that is, by applying Cramer's rule. The matrix from of the abovementioned equation is

$$
\begin{aligned}
&\left(\begin{array}{ccc}
1 & -3 & 10 \\
3 & -5 & -15 \\
8 & -4 & -3
\end{array}\right)\left(\begin{array}{l}
I \\
I_{1} \\
I_{2}
\end{array}\right)=\left(\begin{array}{l}
6 \\
40 \\
48
\end{array}\right) \\
& D_{0}=\left(\begin{array}{ccc}
1 & -3 & 10 \\
3 & -5 & -15 \\
8 & -4 & -3
\end{array}\right)=1(15-60)+3(-9+120)+10(-12+40)=568 \\
& D=\left(\begin{array}{ccc}
6 & -3 & 10 \\
40 & -5 & -15 \\
48 & -4 & -3
\end{array}\right)=6(15-60)+3(-120+720)+10(-160+240)=2,330 \\
& I=\frac{D}{D_{0}}=\frac{2330}{568}=4.102 \mathrm{~A}
\end{aligned}
$$

### 2.7 MAXWELL'S MESH CURRENT METHOD (LOOP ANALYSIS)

In this method, mesh or loop currents are taken instead of branch currents (as in Kirchhoff's laws). The following steps are taken while solving a network by this method:

1. The whole network is divided into number of meshes. Each mesh is assigned a current having continuous path (current is not split at a junction). These mesh currents are preferably drawn in clockwise direction. The common branch carries the algebraic sum of the mesh currents flowing through it.
2. Write KVL equation for each mesh using the same signs as applied to Kirchhoff's laws.
3. Number of equations must be equal to the number of unknown quantities. Solve the equations and determine the mesh currents.

## Example 2.14

Using loop current method, find the current $I_{1}$ and $I_{2}$ as shown in Figure 2.43.
(U.P.T.U. 2005-06)

## Solution:

Let the current flowing through the two loops be $I_{1}$ and $I_{2}$, as shown in Figure 2.44.

By applying KVL to different loops, we get Loop ABEFA


Fig. 2.43 Given network

$$
\begin{aligned}
-2 I_{1}-6\left(I_{1}-I_{2}\right)-6+10 & =0 \\
8 I_{1}-6 I_{2} & =4
\end{aligned}
$$



Fig. 2.44 Loop currents in various sections

$$
\begin{equation*}
4 I_{1}-3 I_{2}=2 \tag{2.29}
\end{equation*}
$$

Loop BCDEB

$$
\begin{array}{r}
-3 I_{2}-2+6-6\left(I_{2}-I_{1}\right)=0 \\
-6 I_{1}+9 I_{2}=4 \tag{2.30}
\end{array}
$$

Multiplying Equation (2.29) by 3 and Equation (2.30) by 2 , we get

$$
\begin{array}{r}
12 I_{1}-9 I_{2}=6 \\
-12 I_{1}+18 I_{2}=8 \tag{2.32}
\end{array}
$$

Adding Equations (2.31) and (2.32), we get

$$
I_{1}=\frac{5}{3}=1.667 \mathrm{~A}
$$

## Example 2.15



Fig.2.45 Given network


Fig. 2.46 Loop currents in various sections

Using mesh equation method, find current in the resistance $R_{1}$ of the network shown in Figure 2.45.
(U.P.T.U. 2004-05)

## Solution:

Converting current source of 1 A and internal resistance $5 \Omega$ into voltage source,
emf of voltage source, $V=I \times R=1 \times 5$ $=5 \mathrm{~V}$

Internal resistance of voltage source, $R=R$ $=5 \Omega$
The circuit is shown in Figure 2.46.
In mesh ABEFA, $I_{1}(5+5)+10\left(I_{1}+I_{2}\right)=5$ or $20 I_{1}+10 I_{2}=5$

$$
\begin{equation*}
4 I_{1}+2 I_{2}=1 \tag{2.33}
\end{equation*}
$$

In mesh BCDEB, $5 I_{2}+10\left(I_{1}+I_{2}\right)=10$ or $10 I_{1}+15 I_{2}=10$

$$
\begin{equation*}
2 I_{1}+3 I_{2}=2 \tag{2.34}
\end{equation*}
$$

Solving Equations (2.33) and (2.34), we get

$$
I_{2}=\frac{3}{4} \mathrm{~A} \quad \text { and } \quad I_{1}=-\frac{5}{40} \mathrm{~A}
$$

Current through $R_{1}=I_{1}+I_{2}=-\frac{5}{40}+\frac{30}{40}=\frac{25}{40} \mathrm{~A}$

$$
=0.625 \mathrm{~A}(\text { from } \mathrm{B} \text { to } \mathrm{E})
$$

## Example 2.16

Using mesh current method, determine current $I_{\mathrm{x}}$ in the circuit shown in Figure 2.47.
(U.P.T.U. 2005-06)

## Solution:

Let the circuit be as shown in Figure 2.48. Suppose voltage across 2 A current source is $V_{x}$,

By applying KVL in mesh $1 ; 3 I_{1}+\left(I_{1}-I_{2}\right)=2$

$$
\begin{equation*}
4 I_{1}-I_{2}=2 \tag{2.35}
\end{equation*}
$$



Fig. 2.47 Given network


Fig. 2.48 Loop currents in various sections

By applying KVL in mesh 2; $\left(I_{2}-I_{1}\right)+V_{\mathrm{x}}=0$

$$
\begin{equation*}
I_{1}-I_{2}=V_{\mathrm{x}} \tag{2.36}
\end{equation*}
$$

By applying KVL in mesh $3 ; 2 I_{3}=5+V_{\mathrm{x}}$
Further,

$$
\begin{equation*}
I_{3}-I_{2}=2 \tag{2.37}
\end{equation*}
$$

From Equations (2.36) and (2.37) $2 I_{3}=5+\left(I_{1}-I_{2}\right)$
or

$$
\begin{equation*}
-I_{1}+I_{2}+2 I_{3}=5 \tag{2.39}
\end{equation*}
$$

From Equations (2.35), (2.38), and (2.39),

$$
\begin{gathered}
\text { In matrix form }\left[\begin{array}{ccc}
4 & -1 & 0 \\
0 & -1 & 1 \\
-1 & 1 & 2
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right]=\left[\begin{array}{l}
2 \\
2 \\
5
\end{array}\right]=4(-2-1)+1(0+1)=-11 \\
\Delta \\
\Delta\left|\begin{array}{ccc}
4 & -1 & 0 \\
0 & -1 & 1 \\
-1 & 1 & 2
\end{array}\right|=4(-2-1)+1(0+1)=-11 \\
\Delta
\end{gathered} \begin{aligned}
\Delta & -1
\end{aligned}\left|\begin{array}{lll}
2 & -1 & 1 \\
5 & 1 & 2
\end{array}\right|=2(-2-1)-(-1)(4-5)=-79
$$

$$
\begin{aligned}
\Delta_{12} & =\left|\begin{array}{lll}
4 & 2 & 0 \\
0 & 2 & 1 \\
-1 & 5 & 2
\end{array}\right|=4(4-5)-(2)(0+1)=-4-2=-6 \\
I_{1} & =\frac{\Delta_{11}}{\Delta}=\frac{-7}{-11}=\frac{7}{11} \mathrm{~A} \\
I_{2} & =\frac{\Delta_{12}}{\Delta}=\frac{-6}{-11}=\frac{6}{11} \mathrm{~A}
\end{aligned}
$$

Current, $I_{\mathrm{x}}=I_{1}-I_{2}=\frac{7}{11}-\frac{6}{11}=\frac{1}{11} \mathrm{~A}$

### 2.8 NODAL ANALYSIS



Fig. 2.49 Network with Node B and D

In this method, one of the nodes is taken as the reference node and the other as independent nodes. The voltages at the different independent nodes are assumed and the equations are written for each node as per KCL. After solving these equations, the node voltages are determined. Then, the branch currents are determined.

Consider a circuit shown in Figure 2.49, where D and B are the two independents nodes. Let D be the reference node and the voltage of node B be $V_{\mathrm{B}}$.

According to KCL,

$$
\begin{equation*}
I_{1}+I_{2}=I_{3} \tag{2.40}
\end{equation*}
$$

In mesh ABDA , the potential difference across $R_{1}$ is $E_{1}-V_{\mathrm{B}}$

$$
I_{1}=\frac{E_{1}-V_{B}}{R_{1}}
$$

In mesh BCDB , the potential difference across $R_{2}$ is $E_{2}-V_{\mathrm{B}}$

$$
I_{2}=\frac{E_{2}-V_{\mathrm{B}}}{R_{2}}
$$

Further, current, $I_{3}=\frac{V_{\mathrm{B}}}{R_{3}}$
Substituting these values in Equation (2.40), we get

$$
\frac{E_{1}-V_{\mathrm{B}}}{R_{1}}=\frac{E_{1}-V_{\mathrm{B}}}{R_{2}}=\frac{V_{\mathrm{B}}}{R_{3}}
$$

Rearranging the terms,

$$
V_{\mathrm{B}}\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}\right)-\frac{E_{1}}{R_{1}}-\frac{E_{2}}{R_{2}}=0
$$

Since all other value are known, except $V_{\mathrm{B}}$, calculate the value of $V_{\mathrm{B}}$. Then, determine the value of $I_{1}, I_{2}$, and $I_{3}$. This method is faster as the result are obtained by solving lesser number of equations.

## Example 2.17

Find the current $I_{1}$ and $I_{2}$ in the passive elements of the network shown in Figure 2.50.
(U.P.T.U. Tut.)

## Solution:

The independent nodes are $\mathrm{A}, \mathrm{B}$, and C . Let C be the reference node and $V_{\mathrm{A}}$ and $V_{\mathrm{B}}$ be the voltages at node A and B, respectively. Let us assume the direction of flow of current as in Figure 2.51 .

For node $\mathrm{A}, I_{1}=\frac{15-V_{\mathrm{A}}}{1} ; I_{4}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{0.5}$ and $I_{3}=\frac{V_{\mathrm{A}}}{1}$, assuming $V_{\mathrm{A}}>V_{\mathrm{B}}$

Similarly for node $\mathrm{B}, I_{2}=\frac{20-V_{\mathrm{B}}}{1}$ and $I_{5}=\frac{V_{\mathrm{B}}}{2}$

Now, by applying KCL to node A, we get


Fig. 2.50 Given network


Fig. 2.51 Assumed direction of flow of current in various branches
or

$$
\begin{gather*}
I_{1}=I_{4}+I_{3} \\
\frac{15-V_{\mathrm{A}}}{1}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{0.5}+\frac{V_{\mathrm{A}}}{1} \text { or } 4 V_{\mathrm{A}}-2 V_{\mathrm{B}}=15 \tag{2.41}
\end{gather*}
$$

By applying KCL at node B , we get
or

$$
\begin{gather*}
I_{2}+I_{4}=I_{5} \\
\frac{200-V_{\mathrm{B}}}{1}+\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{0.5}=\frac{V_{\mathrm{B}}}{2} \text { or } 4 V_{\mathrm{A}}-7 V_{\mathrm{B}}=-40 \tag{2.42}
\end{gather*}
$$

Solving Equations (2.41) and (2.42), we get

$$
V_{\mathrm{A}}=9.25 \mathrm{~V} \text { and } V_{\mathrm{B}}=11 \mathrm{~V}
$$

Current $I_{1}=\frac{15-V_{\mathrm{A}}}{1}=15-9.25=5.75 \mathrm{~A}$

$$
I_{2}=\frac{20-V_{\mathrm{B}}}{1}=20-11=9 \mathrm{~A}
$$

## Example 2.18

Two batteries A and B are connected in parallel to a load of $10 \Omega$. Battery A has an emf of 12 V and an internal resistance of $2 \Omega$ and battery B has an emf of 10 V and internal resistance of $1 \Omega$. Using nodal analysis, determine the currents supplied by each battery and load current.
(U.P.T.U. Dec. 2003)

## Solution:

Considering node Z as reference node and the potentials of nodes X and Y be $V_{\mathrm{X}}$ and $V_{\mathrm{Y}}$, respectively. The assumed current distribution is shown in Figure 2.52.


Fig. 2.52 Given network

For node $\mathrm{X}, \quad \quad I_{1}=\frac{12-V_{\mathrm{X}}}{2}$
For node $\mathrm{Y}, \quad I_{2}=\frac{10-V_{\mathrm{Y}}}{1}$
and

$$
I_{3}=\frac{V_{\mathrm{Y}}}{10}
$$

By applying KCL to node B , we get

$$
\begin{equation*}
I_{1}+I_{2}=I_{3} \quad \text { or } \quad \frac{12-V_{\mathrm{X}}}{2}+\frac{10-V_{\mathrm{Y}}}{1}=\frac{V_{\mathrm{Y}}}{10} \tag{2.43}
\end{equation*}
$$

Moreover,

$$
\begin{equation*}
V_{\mathrm{X}}=V_{\mathrm{Y}} \tag{2.44}
\end{equation*}
$$

Solving Equations (2.43) and (2.44), we get

$$
V_{\mathrm{X}}=V_{\mathrm{Y}}=10 \mathrm{~V}
$$

Thus, current supplied by battery $\mathrm{A}=I_{1}=\frac{12-V_{\mathrm{X}}}{2}=\frac{10-10}{2}=1 \mathrm{~A}$
Current supplied by battery $\mathrm{B}=I_{2}=\frac{10-V_{\mathrm{Y}}}{1}=\frac{10-10}{2}=0$
Load current, $I_{3}=\frac{V_{\mathrm{Y}}}{10}=\frac{10}{10}=1 \mathrm{~A}$


Fig. 2.53 Given network

## Example 2.19

Using nodal analysis, find current $I$ through $10-\Omega$ resistor in Figure 2.53 .

## Solution:

The independent nodes are A, B, and C. Let C be the reference node and $V_{\mathrm{A}}$ and $V_{\mathrm{B}}$ be the voltages at node A and B, respectively. Let us
assume the direction of flow of current is as marked in Figure 2.54. By applying KCL at node A, we get
or

$$
\begin{gathered}
I_{1}+I_{2}=I \\
\frac{0-V_{A}}{4}+\frac{15-V_{A}}{5}=\frac{V_{A}-V_{B}}{10} \\
-5 V_{\mathrm{A}}+60-4 V_{\mathrm{A}}=2 V_{\mathrm{A}}-2 V_{\mathrm{B}} \\
11 V_{\mathrm{A}}-2 V_{\mathrm{B}}=60
\end{gathered}
$$



Fig.2.54 Assumed direction of flow of current in various branches

By applying KCL at node B , we get

$$
\begin{align*}
I & =I_{4}+I_{3} \\
\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10} & =\frac{V_{\mathrm{B}}-30}{4}+\frac{V_{\mathrm{B}}}{6} \\
12 V_{\mathrm{A}}-12 V_{\mathrm{B}} & =30 V_{\mathrm{B}}-900+20 V_{\mathrm{B}} \\
12 V_{\mathrm{A}}-62 V_{\mathrm{B}} & =-900 \tag{2.46}
\end{align*}
$$

or
or
Solving Equation (2.45) and (2.46), we get

Current,

$$
V_{\mathrm{A}}=8.39 \mathrm{~V} \text { and } V_{\mathrm{B}}=16.14 \mathrm{~V}
$$

$$
\begin{aligned}
& I=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}=\frac{8.39-16.14}{10}=\frac{-7.75}{10}=-0.775 \mathrm{~A} \\
& I=0.775 \mathrm{~A}(\text { from B to A) }
\end{aligned}
$$

## Example 2.20

Calculate currents in all the resistors of the circuit shown in Figure 2.55 using node analysis method.
(U.P.T.U. 2006-07)

## Solution:

The independent nodes are $\mathrm{A}, \mathrm{B}$, and C . Let C be the reference node and $V_{\mathrm{A}}$ and $V_{\mathrm{B}}$ be the voltages at node A and B, respectively. Let us assume the direction of flow of current is marked as in


Fig. 2.55 Given network Figure 2.56.

Here,

$$
V_{\mathrm{A}}=6 \mathrm{~V}
$$

By applying KCL at node B , we get

$$
\begin{array}{lll}
I_{1}+I_{2}=4 & \text { or } & \frac{V_{\mathrm{B}}+V_{\mathrm{A}}}{2}+\frac{V_{\mathrm{B}}}{12}=4 \\
\frac{V_{\mathrm{B}}-6}{2}+\frac{V_{\mathrm{B}}}{12}=4 & \text { or } & \frac{V_{\mathrm{B}}}{2}-3+\frac{V_{\mathrm{B}}}{12}=4
\end{array}
$$



Fig. 2.56 Assumed direction of flow of current in various branches

$$
V_{\mathrm{B}}\left(\frac{1}{2}+\frac{1}{12}\right)=3+4 \quad \text { or } \quad V_{\mathrm{B}}=\frac{7 \times 12}{7}=12 \mathrm{~V}
$$

Current in $12-\Omega$ resistor, $I_{2}=\frac{V_{\mathrm{B}}}{12}=\frac{12}{12}=1 \mathrm{~A}($ from B to C)

Current in $2-\Omega$ resistor, $I_{1}=\frac{V_{\mathrm{B}}-V_{\mathrm{A}}}{2}=\frac{12-6}{2}=3 \mathrm{~A}($ from A to B$)$

Current in $3-\Omega$ resistor, $I_{3}=\frac{V_{\mathrm{A}}}{3}=\frac{6}{3}=2 \mathrm{~A}$ (from A to C)

## Example 2.21

Use nodal analysis to find the current in various resistors of the circuit shown in Figure 2.57.
(U.P.T.U. 2005-06)


Fig. 2.57 Given network


Fig. 2.58 Assumed direction of flow of current in various branches

## Solution:

The independent nodes are A, B, C, and D. Let D be the reference node and $V_{\mathrm{A}}, V_{\mathrm{B}}$, and $V_{\mathrm{C}}$ be the voltages at nodes $\mathrm{A}, \mathrm{B}$, and C , respectively, The current flowing through various branches are marked in Figure 2.58.

By applying KCL at different nodes, different node voltage equations are obtained as follows:

Node A

$$
\begin{gather*}
I_{1}+I_{2}+I_{3}=I \\
\frac{V_{\mathrm{A}}}{2}+\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{3}+\frac{V_{\mathrm{A}}-V_{\mathrm{C}}}{5}=10 \\
15 V_{\mathrm{A}}+10\left(V_{\mathrm{A}}-V_{\mathrm{B}}\right)+6\left(V_{\mathrm{A}}-V_{\mathrm{C}}\right)=300  \tag{2.47}\\
\text { or } 31 V_{\mathrm{A}}-10 V_{\mathrm{B}}-6 V_{\mathrm{C}}=300
\end{gather*}
$$

Node B

$$
\begin{gathered}
I_{2}-I_{4}-I_{5}=0 \\
\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{3}-\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{1}-\frac{V_{\mathrm{B}}}{5}=0
\end{gathered}
$$

$$
5\left(V_{\mathrm{A}}-V_{\mathrm{B}}\right)-15\left(V_{\mathrm{B}}-V_{\mathrm{C}}\right)-3 V_{\mathrm{B}}=0
$$

$$
\begin{equation*}
\text { or } \quad 5 V_{\mathrm{A}}-23 V_{\mathrm{B}}+15 V_{\mathrm{C}}=0 \tag{2.48}
\end{equation*}
$$

Node C

$$
\begin{gather*}
I_{3}+I_{4}-I_{6}-I_{7}=0 \\
\frac{V_{\mathrm{A}}-V_{\mathrm{C}}}{5}+\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{1}-\frac{V_{\mathrm{C}}}{4}-2=0 \\
4\left(V_{\mathrm{A}}-V_{\mathrm{C}}\right)+20\left(V_{\mathrm{B}}-V_{\mathrm{C}}\right)-5 V_{\mathrm{C}}-40=0 \text { or } 4 V_{\mathrm{A}}+20 V_{\mathrm{B}}-29 V_{\mathrm{C}}=40 \tag{2.49}
\end{gather*}
$$

The three equations in matrices form are:

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
31 & -10 & -6 \\
5 & -23 & 15 \\
4 & 20 & -29
\end{array}\right]\left[\begin{array}{l}
V_{\mathrm{A}} \\
V_{\mathrm{B}} \\
V_{\mathrm{C}}
\end{array}\right]=\left[\begin{array}{c}
300 \\
0 \\
40
\end{array}\right] } \\
& D_{0}=\left[\begin{array}{ccc}
31 & -10 & -6 \\
5 & -23 & 15 \\
4 & 20 & -29
\end{array}\right]=31(667-300)+10(-145-60)-6(100+92) \\
&=11,377-2,050-1,152=8,175 \\
& D_{1}=\left[\begin{array}{ccc}
300 & -10 & -6 \\
0 & -23 & 15 \\
40 & 20 & -29
\end{array}\right]=300(667-300)+10(-600)-6(+920) \\
& D_{2}=\left[\begin{array}{ccc}
31 & 300 & -6 \\
5 & 0 & 15 \\
4 & 40 & -29
\end{array}\right]=31(0-600)-300(-145-60)-6(200) \\
&=-18,600+61,500-1,200=41,700 \\
& D_{3}=\left[\begin{array}{ccc}
31 & -10 & 300 \\
5 & -23 & 0 \\
4 & 20 & 40
\end{array}\right]=31(-920-0)+10(200-0)+300(100+92) \\
&=-28,520+2,000+57,600=31,080 \\
& V_{\mathrm{A}}=\frac{D_{1}}{D_{0}}=\frac{98,580}{8,175}=12.06 ; V_{\mathrm{B}}=\frac{D_{2}}{D_{0}}=\frac{41,700}{8,175}=5.1 \mathrm{~V} \\
& V_{\mathrm{C}}=\frac{D_{3}}{D_{0}}=\frac{31,080}{8,175}=3.802 \mathrm{~V} \\
& \hline
\end{aligned}
$$

Current in various resistors:

$$
\begin{aligned}
& I_{1}=\frac{V_{\mathrm{A}}}{2}=\frac{12.06}{2}=6.03 \mathrm{~A} ; \quad I_{2}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{3}=\frac{12.06-5.1}{3}=2.32 \mathrm{~A} ; \\
& I_{3}=\frac{V_{\mathrm{A}}-V_{\mathrm{C}}}{5}=\frac{12.06-3.802}{5}=1.652 \mathrm{~A} ; \quad I_{4}=\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{1}=\frac{5.1-3.802}{1}=1.298 \mathrm{~A} ; \\
& I_{5}=\frac{V_{\mathrm{B}}}{5}=\frac{5.1}{5}=1.02 \mathrm{~A} ; \quad I_{6}=\frac{V_{\mathrm{C}}}{4}=\frac{3.802}{4}=0.95 \mathrm{~A}
\end{aligned}
$$

## Example 2.22

Using nodal analysis, determine current in each branch of the network as shown in Figure 2.59. Further, find total power loss in the network.
(U.P.T.U. Feb. 2002)


Fig. 2.59 Given network

## Solution:

Redraw the circuit and mark the arbitrary assumed values of currents in various branches as shown in Figure 2.60. Let G (or D) be the reference node.


Fig. 2.60 Assumed direction of flow of current in various branches

By applying KCL at different nodes, we get
At node A
or

$$
I_{1}=I_{2}+I_{3}
$$

At node B

$$
I_{3}=I_{4}+I_{5} \quad \text { or } \quad \frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}=\frac{V_{\mathrm{B}}+10}{20}+\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{20}
$$

or

$$
\begin{equation*}
2 V_{\mathrm{A}}-4 V_{\mathrm{B}}+V_{\mathrm{C}}=10 \tag{2.51}
\end{equation*}
$$

At node C
or

$$
\begin{gather*}
I_{5}+I_{7}=I_{\mathrm{C}} \\
\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{20}+0.5=\frac{V_{\mathrm{C}}}{20} \text { or } 2 V_{\mathrm{C}}-V_{\mathrm{B}}=10 \tag{2.52}
\end{gather*}
$$

Solving Equations (2.50), (2.51), and (2.52), we get

$$
V_{\mathrm{A}}=6 \mathrm{~V}, V_{\mathrm{B}}=2 \mathrm{~V} \text {, and } V_{\mathrm{C}}=6 \mathrm{~V}
$$

Current through different branches:
Current through current source, $I_{1}=1 \mathrm{~A}$
Current through branch AG (10- $\Omega$ resistor), $I_{2}=\frac{V_{\mathrm{A}}}{10}=\frac{6}{10}=0.6 \mathrm{~A}$
Current through branch $\mathrm{AB}(10-\Omega$ resistor $), I_{3}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}=\frac{6-2}{10}=0.4 \mathrm{~A}$
Current through branch BG $(20-\Omega$ resistor $), I_{4}=\frac{V_{\mathrm{B}}+10}{20}=\frac{2+10}{20}=0.6 \mathrm{~A}$
Current through branch $\mathrm{BC}(20-\Omega$ resistor $), I_{5}=\frac{V_{\mathrm{B}}-V_{\mathrm{C}}}{20}=\frac{2-6}{20}=0.2 \mathrm{~A}($ from C to B)
Current through branch CG (20- $\Omega$ resistor), $I_{6}=\frac{V_{\mathrm{C}}}{20}=\frac{6}{20}=0.3 \mathrm{~A}$
Current through current source, $I_{7}=0.5 \mathrm{~A}$
Total power loss $=(0.6)^{2} \times 10+(0.4)^{2} \times 10+(0.6)^{2} \times 20+(0.2)^{2} \times 20+(0.3)^{2} \times 20$

$$
=3.6+1.6+7.2+0.8+1.8=15 \mathrm{~W}
$$

## Example 2.23

Use the node voltage method to solve the mesh currents in the network shown in Figure 2.61.
(U.P.T.U. June 2001)

## Solution:

Redraw the circuit and mark the arbitrary assumed values of currents in various branches, as shown in Figure 2.62. Let C be the reference node.

By applying KCL at different nodes, we get


Fig. 2.61 Given network

At node A: $\frac{-V_{\mathrm{A}}}{2}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}+\frac{V_{\mathrm{A}}-25}{5}$ considering $V_{\mathrm{A}}>V_{\mathrm{B}}$
or

$$
\begin{equation*}
8 V_{\mathrm{A}}-V_{\mathrm{B}}=50 \tag{2.53}
\end{equation*}
$$

At node $\mathrm{B}: \frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}=\frac{V_{\mathrm{B}}}{4}+\frac{V_{\mathrm{B}}-50}{2}$
or $\quad 2 V_{\mathrm{A}}-17 V_{\mathrm{B}}=-500$
Multiplying Equation (2.54) by 4 and subtracting it from Equation (2.53), we get


Fig. 2.62 Assumed direction of flow of current in various branches

$$
V_{\mathrm{A}}=10.0746 \mathrm{~V}
$$

Substituting the value of $V_{\mathrm{A}}$ in Equation (2.53), we get

$$
V_{\mathrm{B}}=30.597 \mathrm{~V}
$$

Various currents of the network
and

$$
\begin{aligned}
& I_{1}=\frac{-V_{\mathrm{A}}}{2}=\frac{-10.0746}{2}=5.0373 \\
& I_{2}=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{10}=\frac{10.0746-30.597}{10}=-2.0522 \mathrm{~A} \\
& I_{3}=\frac{V_{\mathrm{B}}-50}{2}=\frac{30.597-50}{2}=-9.7015 \mathrm{~A}
\end{aligned}
$$

## Example 2.24



Fig. 2.63 Given network


Fig. 2.64 Assumed direction of flow of current in various branches

By applying KCL , determine current $I_{\mathrm{s}}$ in the electric circuit at Figure 2.63. Take $V_{0}=16 \mathrm{~V}$
(U.P.T.U. Feb. 2001)

## Solution:

Let the current flowing through the various branches of the circuit be as shown in Figure 2.64 .

By applying KCL to node B , we get

$$
\begin{equation*}
I_{2}+I_{\mathrm{S}}=I_{1} \tag{2.55}
\end{equation*}
$$

By applying KCL to node C , we get

$$
\begin{equation*}
I_{2}+I_{3}=\frac{V_{1}}{4} \tag{2.56}
\end{equation*}
$$

Voltage at node $\mathrm{C}=V_{0}=16 \mathrm{~V}$

$$
\begin{equation*}
4 I_{2}+V_{1}=16 \tag{2.57}
\end{equation*}
$$

$$
\begin{equation*}
\text { In branch BG, } I_{1}=\frac{V_{1}}{6} \quad \text { or } \quad V_{1}=6 I_{1} \tag{2.58}
\end{equation*}
$$

In branch $\mathrm{DE}, I_{3}=\frac{V_{0}}{8}=\frac{16}{8}=2 \mathrm{~A}$
Substituting the value of $I_{3}$ and $V_{1}$ in Equation (2.56), we get

$$
\begin{equation*}
I_{2}+2=\frac{6 I_{1}}{4} \quad \text { or } \quad 3 I_{1}-2 I_{2}=4 \tag{2.59}
\end{equation*}
$$

Substituting the value of $V_{1}=6 I_{1}$, we get

$$
\begin{equation*}
4 I_{2}+6 I_{1}=16 \text { or } 3 I_{1}+2 I_{2}=8 \tag{2.60}
\end{equation*}
$$

Solving Equations (2.59) and (2.60), we get

$$
6 I_{1}=12 \text { or } I_{1}=2 \mathrm{~A} \text { and } I_{2}=1 \mathrm{~A}
$$

From Equation (2.55), we get $I_{\mathrm{S}}=I_{1}-I_{2}=2-1=1 \mathrm{~A}$
whereas $V_{1}=6 I_{1}=6 \times 2=12 \mathrm{~V}$

## 国豕 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by electric network?
2. What do you understand by active and passive elements of an electric network?
3. What do you mean by unidirectional and bidirectional elements?
4. What do you mean by linear and non-linear circuits?
5. How will you differentiate between a loop and mesh?
6. State Kirchhoff's first and second law.

## Test Questions

1. Draw and explain an electric network. Draw a network and show a node, junction, loop, mesh, and a branch.
2. How will you define the following terms:
(i) bilateral circuit, (ii) unilateral circuit, (iii) active network, (iv) passive network, (v) linear circuit, and (vi) non-linear circuit
3. Define and explain Kirchhoff's laws of electric circuit.
4. Explain Maxwell's mesh current method (loop analysis) of solving an electric network.
5. Explain nodal method (or nodal analysis) of solving an electric network.

## Numericals

1. Two batteries A and B connected in parallel and load of $10 \Omega$ is connected across their terminals. A has an emf of 12 V and an internal resistance of $2 \Omega$; B has an emf of 8 V and an internal resistance of $1 \Omega$. Using Kirchhoff's laws, determine the value and direction of flow of current in each battery and in the external resistance. Further, determine the potential difference across the external resistance.
(Ans. 1.625 A charging; 0.75 A discharging; $0.875 \mathrm{~A} ; 8.75 \mathrm{~V}$ )
2. A Wheatstone bridge $A B C D$ is arranged as follows:

Resistance between A-B, B-C, C-D, D-A, and B-D are 10, 2, 8,4 , and $5 \Omega$, respectively. A $100-\mathrm{V}$ supply is connected between terminals A and C. Determine the currents in branches AB and AD of the circuit and total current taken from the supply.
(Ans. 7.44 A; 11.905 A; 19.345 A)
3. Using Maxwell's mesh current method, find the current in all the branches of the circuit shown in Figure 2.65.
(Ans. $4.4915 \mathrm{~A}, 2.2881 \mathrm{~A}, 2.2034 \mathrm{~A})$
4. For the circuit shown in Figure 2.66, find the current in $7.5-\Omega$ resistor by nodal analysis.
(Ans. 0.671 A )


Fig. 2.65 Given network


Fig. 2.66 Given network

### 2.9 DELTA-STAR AND STAR-DELTA TRANSFORMATION

To apply simple series-parallel circuit technique for the solution of network, something is required to transform the resistors connected in delta to star or vice versa. At such places, just to simplify the network delta-star or star-delta transformations applied.

### 2.9.1 Delta-Star Transformation

Consider a circuit shown in Figure 2.67(a), where three resistors $R_{\mathrm{AB}}, R_{\mathrm{BC}}$, and $R_{\mathrm{CA}}$ are connected in delta. This circuit is converted to a star-connection circuit, as shown in Figure 2.67(b). Let the resistance be $R_{A}, R_{\mathrm{B}}$, and $R_{\mathrm{C}}$.

(a)

(b)

Fig. 2.67 (a) Delta connections (b) Equivalent star connection

For the two circuits to be equivalent, the resistance measured between any two of the terminals $\mathrm{A}, \mathrm{B}$, and C must be the same in the two cases.
$R_{\mathrm{AB}}$ for star connections $=R_{\mathrm{AB}}$ for delta connections.

$$
\begin{equation*}
R_{\mathrm{A}}+R_{\mathrm{B}}=R_{\mathrm{AB}} \|\left(R_{\mathrm{BC}}+R_{\mathrm{CA}}\right) \quad \text { or } \quad R_{\mathrm{A}}+R_{\mathrm{B}}=\frac{R_{\mathrm{AB}}\left(R_{\mathrm{BC}}+R_{\mathrm{CA}}\right)}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}} \tag{2.61}
\end{equation*}
$$

Similarly, $\quad R_{\mathrm{B}}+R_{\mathrm{C}}=\frac{R_{\mathrm{BC}}\left(R_{\mathrm{CA}}+R_{\mathrm{AB}}\right)}{R_{\mathrm{BC}}+R_{\mathrm{CA}}+R_{\mathrm{AB}}}$
and

$$
\begin{equation*}
R_{\mathrm{C}}+R_{\mathrm{A}}=\frac{R_{\mathrm{CA}}\left(R_{\mathrm{AB}}+R_{\mathrm{BC}}\right)}{R_{\mathrm{CA}}+R_{\mathrm{AB}}+R_{\mathrm{BC}}} \tag{2.63}
\end{equation*}
$$

Subtracting Equation (2.62) from Equation (2.61) and adding the result to Equation (2.63), we get

$$
\begin{align*}
R_{\mathrm{A}} & =\frac{R_{\mathrm{AB}} \times R_{\mathrm{CA}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}  \tag{2.64}\\
R_{\mathrm{B}} & =\frac{R_{\mathrm{BC}} \times R_{\mathrm{AB}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}  \tag{2.65}\\
R_{\mathrm{C}} & =\frac{R_{\mathrm{CA}} \times R_{\mathrm{BC}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}} \tag{2.66}
\end{align*}
$$

To remember this relation, we should refer to Figure 2.68. According to Equations (2.64), (2.65), and (2.66),

$$
\text { Resistance of any arm of } Y=\frac{\text { Product to two adjacent arms of } \Delta}{\text { Sum of arms of } \Delta}
$$

### 2.9.2 Star-Delta Transformation

For the replacement of star connected network, proceed further as follows:
Dividing Equation (2.66) by Equation (2.65), we get

$$
\begin{equation*}
\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}=\frac{R_{\mathrm{CA}}}{R_{\mathrm{BC}}} \quad \text { or } \quad R_{\mathrm{CA}}=\frac{R_{\mathrm{A}} \times R_{\mathrm{BC}}}{R_{\mathrm{B}}} \tag{2.67}
\end{equation*}
$$

Similarly,

$$
\begin{equation*}
\frac{R_{\mathrm{A}}}{R_{\mathrm{C}}}=\frac{R_{\mathrm{AB}}}{R_{\mathrm{BC}}} \quad \text { or } \quad R_{\mathrm{AB}}=\frac{R_{\mathrm{A}} \times R_{\mathrm{BC}}}{R_{\mathrm{C}}} \tag{2.68}
\end{equation*}
$$

Substituting the value of $R_{\mathrm{CA}}$ and $R_{\mathrm{AB}}$ in Equation (2.66) and reshuffling the quantities, we get

Similarly,

$$
\begin{equation*}
R_{\mathrm{BC}}=R_{\mathrm{B}}+R_{\mathrm{C}}+\frac{R_{\mathrm{B}} R_{\mathrm{C}}}{R_{\mathrm{A}}} \tag{2.69}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{\mathrm{CA}}=R_{\mathrm{C}}+R_{\mathrm{A}}+\frac{R_{\mathrm{C}} R_{\mathrm{A}}}{R_{\mathrm{B}}} \tag{2.70}
\end{equation*}
$$

To remember this relation, one should refer Figure 2.68. According to Equations (2.69), (2.70), and (2.71),

Resistance of an arm of delta $=$ Sum of star resistances connected
Fig. 2.68 Equivalent Delta connection across that arm and product of the same two resistances divided by the third.

## Example 2.25

Three resistances $r, 2 r$, and $3 r$ are connected in delta. Determine the resistance for an equivalent star connections.
(U.P.T.U. Jan. 2003)

## Solution:

The three resistors connected in delta are shown in Figure 2.69(a). The equivalent star connected resistors shown in Figure 2.69(b) are worked out as follows:


Fig. 2.69 Delta to star connection

$$
\begin{aligned}
R_{\mathrm{A}} & =\frac{R_{\mathrm{AB}} \cdot R_{\mathrm{CA}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}=\frac{r \times 3 r}{r+2 r+3 r}=\frac{r}{2} \\
R_{\mathrm{B}} & =\frac{R_{\mathrm{BC}} \cdot R_{\mathrm{AB}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}} \\
& =\frac{2 r \times r}{r+2 r+3 r}=\frac{r}{3} \\
R_{\mathrm{C}} & =\frac{R_{\mathrm{BC}} \cdot R_{\mathrm{BC}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}=\frac{3 r \times 2 r}{r+2 r+3 r}=r
\end{aligned}
$$

## Example 2.26

Find the resistance between terminals XY of the bridge circuit shown in Figure 2.70 by using delta-star transformations.
(U.P.T.U. July 2002)


Fig. 2.70 Given network

## Solution:

As illustrated in Figure 2.71(a), delta ABC has been reduced to its equivalent star circuit.

$$
\begin{aligned}
& R_{1}=\frac{R_{\mathrm{AB}}+R_{\mathrm{CA}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}=\frac{4 \times 6}{4+2+6}=2 \Omega \\
& R_{2}=\frac{R_{\mathrm{AB}}+R_{\mathrm{BC}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}=\frac{4 \times 2}{4+2+6}=\frac{2}{3} \Omega \\
& R_{3}=\frac{R_{\mathrm{CA}} R_{\mathrm{BC}}}{R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}}=\frac{6 \times 2}{4+2+6}=1 \Omega
\end{aligned}
$$

Hence, the given circuit is reduced to the circuit shown in Figure 2.71(b). As seen, there are two parallel paths between terminals N and D , one of resistance $1+14=15 \Omega$ and the other is of resistance

$$
\frac{2}{3}+10 \text {, i.e., } \frac{32}{3} \Omega
$$



Fig. 2.71 (a) Delta to star conversion (b) Simplified equivalent circuit

Total resistance of the network across XY, $R_{\mathrm{XY}}=2+\left(15 \| \frac{32}{3}\right)$.

$$
2+\frac{15 \times \frac{32}{3}}{15+\frac{32}{3}}=2+\frac{480}{77}=\frac{634}{77}=8.234 \Omega
$$

## Example 2.27

Find the resistance between AB of the circuit shown in Figure 2.72 using star-delta transformation.
(U.P.T.U. Feb. 2001)

## Solution:

Mark the terminals as shown in Figure 2.73(a). Transforming delta connections CDE into star connections, as shown in Figure 2.73(b), we get

$$
\begin{aligned}
& R_{1}=\frac{4 \times 6}{4+6+2}=2 \Omega \\
& R_{2}=\frac{4 \times 2}{4+6+2}=\frac{2}{3} \Omega \\
& R_{3}=\frac{2 \times 6}{4+6+2}=1 \Omega
\end{aligned}
$$



Fig. 2.72 Given network

Resistance across AB or CF, $R_{\mathrm{AB}}=2+\left(\frac{2}{3}+3\right) \|(1+5)+2=2+\left(\frac{11}{3} \| 6\right)+2$

$$
=2+\frac{\frac{11}{3} \times 6}{\frac{11}{3}+6}+2=2+\frac{66}{29}+2=\frac{182}{29}=0.6276 \Omega
$$


(a)

(b)

Fig. 2.73 (a) Delta to star conversion (b) Equivalent circuit

## Example 2.28

Using delta to star transformation, determine the resistance between terminals a and b and the total power drawn from the supply in the circuit shown in Figure 2.74.
(U.P.T.U. 2006-07)

## Solution:



Fig. 2.74 Given network


Fig. 2.75 Delta to star conversion

$$
\begin{aligned}
& R_{1}=\frac{8 \times 3}{8+3+7}=\frac{4}{3} \Omega \\
& R_{2}=\frac{8 \times 7}{8+3+7}=\frac{28}{9} \Omega \\
& R_{3}=\frac{3 \times 7}{8+3+7}=\frac{7}{6} \Omega
\end{aligned}
$$

$$
\text { Resistance across ab, } \begin{aligned}
R_{\mathrm{ab}} & =\frac{4}{3}+\left(\frac{28}{9}+10\right) \|\left(\frac{7}{6}+4\right) \\
& =\frac{4}{3}+\left(\frac{118}{9}\right) \|\left(\frac{31}{6}\right) \\
& =\frac{4}{3}+\frac{\frac{118}{9} \times \frac{31}{6}}{\frac{118}{9}+\frac{31}{6}} \\
& =\frac{4}{3}+\frac{1829}{27} \times \frac{18}{329} \\
& =1.333+3.706 \\
& =5.039 \Omega
\end{aligned}
$$

Power drawn from the supply, $P=\frac{V^{2}}{R_{\mathrm{ab}}}=\frac{10 \times 10}{5.039}=19.843 \mathrm{~W}$

## Example 2.29

Find the resistance at the $\mathrm{A}-\mathrm{B}$ terminals in the electric circuit of Figure 2.76 using $\Delta-\mathrm{Y}$ transformation.
(U.P.T.U. Feb. 2001)

## Solution:

Mark the terminals as shown in Figure 2.77(a). By transforming delta connections CDE into star connections, we get

$$
\begin{aligned}
& R_{1}=\frac{20 \times 30}{20+30+50}=6 \Omega \\
& R_{2}=\frac{20 \times 50}{20+30+50}=10 \Omega \\
& R_{3}=\frac{50 \times 30}{20+30+50}=15 \Omega
\end{aligned}
$$



Fig 2.76 Given network

Resistance across AB or CF, $R_{\mathrm{AB}}=6+(15+45) \|(10+50)$
or

$$
R_{\mathrm{AB}}=R_{\mathrm{CF}}=6+\frac{(15+45)(10+50)}{(15+45)+(10+50)}=6+\frac{60 \times 60}{120}=6+30=36 \Omega
$$


(a)

(b)

Fig. 2.77 (a) Delta to star conversion (b) Equivalent circuit

## Example 2.30

In the network shown in Figure 2.78, determine the resistance between A and B. The numbers represent the respective resistances in ohms.

## Solution:

The inner delta DEF can be reduced to star as shown in Figure 2.79(b), where


Fig. 2.78 Given network


Fig. 2.80 Equivalent circuit

(a)

(b)

Fig. 2.79 (a) Delta conversion (b) Equivalent star connection

$$
\begin{aligned}
& R_{1}=\frac{12 \times 8}{12+8+4}=4 \Omega ; \\
& R_{2}=\frac{12 \times 4}{12+4+8}=2 \Omega ; \\
& R_{3}=\frac{8 \times 4}{8+4+12}=\frac{4}{3} \Omega
\end{aligned}
$$

The original network is reduced to the shape shown in Figure 2.80. The resultant internal star is shown in Figure 2.81(a), which can be further reduced to delta connections shown in Figure 2.81(b).

(a)

(b)

Fig. 2.81 (a) Equivalent star connection (b) Equivalent delta

$$
R_{\mathrm{AB}}=16+16+\frac{16 \times 16}{8}=64 \Omega ; R_{2}=16+8+\frac{16 \times 8}{16}=32 \Omega ; R_{\mathrm{CA}}=8+6+\frac{8 \times 16}{16}=32 \Omega
$$

The network is further reduced to the shape shown in Figure 2.82(a), which is further simplified to the network shown in Figure 2.82(b).

Effective resistance between terminals A and $\mathrm{B}=\frac{(16+16) 64 / 3}{16+16+(64 / 3)}=12.8 \Omega$

(a)

(b)

Fig. 2.82 (a) Final delta connection (b) Final equivalent circuit

## Example 2.31

Find current $I$ in the network shown in Figure 2.83 using star-delta transformation.
(U.P.T.U. Dec. 2003)

## Solution:

Converting star connection of Figure 2.84(a) into its equivalent delta, we get

$$
\begin{aligned}
& R_{\mathrm{AB}}=R_{\mathrm{A}}+R_{\mathrm{B}}+\frac{R_{\mathrm{A}} R_{\mathrm{B}}}{R_{\mathrm{C}}}=8+6+\frac{8 \times 6}{6}=22 \Omega \\
& R_{\mathrm{AB}}=R_{\mathrm{B}}+R_{\mathrm{C}}+\frac{R_{\mathrm{B}} R_{\mathrm{C}}}{R_{\mathrm{A}}}=8+6+\frac{6 \times 6}{8}=16.5 \Omega \\
& R_{\mathrm{CA}}=R_{\mathrm{C}}+R_{\mathrm{A}}+\frac{R_{\mathrm{C}} R_{\mathrm{A}}}{R_{\mathrm{B}}}=6+8+\frac{6 \times 8}{8}=22 \Omega
\end{aligned}
$$



Fig. 2.83 Given network

The equivalent circuit reduces to as shown in Figure 2.84(b), which when further simplified to that shown in Figure 2.84(c), 2.84(d), and 2.84(e).
Total equivalent resistance, $R_{\text {eq }}=[\{(16.5 \| 4)+(22 \| 5)\}| | 22]+4$

$$
=\left[\left(\frac{16.5 \times 4}{16.5+4}+\frac{22 \times 5}{22+5}\right) \| 22\right]+4=\left[\left(\frac{132}{41}+\frac{110}{27}\right) \| 22\right]+4
$$


(a)

(b)

(c)

$(110 / 27) \Omega+(132 / 41) \Omega$

$$
=7.3 \Omega
$$

(d)

$7.3|\mid 22 \Omega=5.478 \Omega$
(e)

Fig. 2.84 (a) Star connections (b) Conversion of star to delta (c) Simplified circuit (d) Simplified equivalent circuit (e) Final equivalent circuit

$$
=(7.3 \| 22)+4=\frac{7.3 \times 22}{7.3+22}+4=5.478+4=9.478
$$

Current, $I=\frac{10}{9.478}=1.055 \mathrm{~A}$

## Example 2.32

Using a suitable theorem, obtain current in $13-\Omega$ resistor in the network shown in Figure 2.85.
(U.P.T.U. 2007-08)


Fig. 2.85 Given network

## Solution:

Let us solve this network by using Thevenin's theorem. To determine Thevenin voltage, that is, open-circuit voltage across terminal AB , consider Figure 2.86(a).

By applying KVL to loop CEFC, we get

$$
\begin{align*}
& 15\left(I_{1}+I_{2}\right)+20 I_{1}
\end{align*}=200 \quad \text { or } \quad 35 I_{1}+15 I_{2}=200
$$

By applying KVL to loop EDFE, we get

$$
\begin{align*}
15\left(I_{2}+I_{1}\right)+12 I_{2} & =80 \text { or } 37 I_{2}+15 I_{1}=80 \\
15 I_{1}+37 I_{2} & =80 \tag{2.73}
\end{align*}
$$

Solving Equations (2.72) and (2.73), we get

$$
I_{1}=5.794 \mathrm{~A} \quad \text { and } \quad I_{2}=-0.187 \mathrm{~A}
$$

Thevenin voltage, $E_{\mathrm{th}}=V_{\mathrm{AB}}=-50-20 I_{1}+20 I_{2}$

$$
\begin{aligned}
& =-50-20 \times 5.794+20(-0.187) \\
& =-167.75 \mathrm{~V}
\end{aligned}
$$

To determine Thevenin resistance, replace the voltage sources by their internal resistance, that is, short circuit, as shown in Figure 2.86(b). Convert delta connected network EDF into star as shown in Figure 2.86(c) and 2.86(d).

$$
\begin{aligned}
R_{1}=\frac{15 \times 12}{15+12+10} & =4.865 \Omega ; \quad R_{2}=\frac{15 \times 10}{15+12+10}=4.054 \Omega ; \quad R_{3}=\frac{10 \times 12}{15+12+10}=3.243 \Omega \\
R_{\mathrm{th}} & =(20+4.054) \| 4.865+3.243=(24.054 \| 4.865)+3.243 \\
& =\frac{24.054 \times 4.865}{24.054+4.865}+3.243=7.29 \Omega
\end{aligned}
$$


(a)

(b)

(d)

(c)

(e)

Fig. 2.86 Circuit simplified step by step (a) Loop currents in various sections (b) Circuit to determine Thevenin resistance (c) Conversion of delta DEF to star (d) Equivalent circuit (e) Simplified equivalent circuit

Thevenin equivalent circuit is shown in Figure 2.86(e). Current through 13- $\Omega$ resistor is given as

$$
\begin{aligned}
I_{\mathrm{L}} & =\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{-167.75}{7.29+13} \\
& =-8.268 \mathrm{~A}, \text { that is, } 8.268 \mathrm{~A}(\text { from B to A) }
\end{aligned}
$$

### 2.10 SUPERPOSITION THEOREM

According to this theorem, if there are two or more sources of emfs acting simultaneously in a linear bilateral network, the current flowing through any section is the algebraic sum of all the currents that should flow in that section if each source of emf were considered separately and all other sources are replaced, for the time being, by their internal resistances.


Fig. 2.87 (a) Given network (b) Considering one source only (c) Considering other source only

For instant, considering a circuit shown in Figure 2.87(a). Let the current flowing through various branches be as marked in Figure 2.87(a). According to superposition theorem,

1. Consider only one source $E_{1}$ and replace the other source $E_{2}$ by its internal resistance. As its internal resistance is not given, it is taken as zero (short circuit). Draw the circuit as shown in Figure 2.87(b) and determine currents in various sections as $I_{1}^{\prime}, I_{2}^{\prime}$, and $I_{3}^{\prime}$, respectively.
2. Then, consider the other source $E_{2}$ and replace the source $E_{1}$ by its internal resistance $r_{1}$, as shown in Figure 2.87(c). Determine the currents in various sections as $I_{1}^{\prime \prime}, I_{2}^{\prime \prime}$, and $I_{3}^{\prime \prime}$, respectively.

Actual flow of current in various sections is

$$
I_{1}=I_{1}^{\prime}-I_{1}^{\prime \prime} ; I_{2}=I_{2}^{\prime}-I_{2}^{\prime \prime} ; I_{3}=I_{3}^{\prime}-I_{3}^{\prime \prime}
$$



Fig. 2.88 Given network

## Example 2.33

By using superposition theorem, find the current in resistance $R$ shown in Figure 2.88. $R_{1}=$ $0.05 \Omega, R_{2}=0.04 \Omega, R=1 \Omega, E_{1}=2.05 \mathrm{~V}, E_{2}=$ 2.15 V . Internal resistance of cells are negligible.
(Allahabad Univ. 1992)

## Solution:

Let the current flowing through different branches of the network be as marked in Figure 2.89(a).

According to superposition theorem, replace source $E_{2}$ by its internal resistance ( $0 \Omega$ ), as shown in Figure 2.89(b).

Equivalent resistance of the circuit,

$$
R^{\prime}=R_{1}+\frac{R_{2} R}{R_{2}+R}=0.05+\frac{0.04 \times 1}{0.04+1}=0.0885 \Omega
$$

Current supplied by battery $E_{1}$,

$$
I_{1}=\frac{E_{1}}{R^{\prime}}=\frac{2.05}{0.0885}=23.2 \mathrm{~A}
$$

Current in $R \quad \Omega$ resistor, $I_{3}^{\prime}=I_{1}^{\prime} \times \frac{R_{2}}{R+R_{2}}=$
$23.2 \times 0.04$

(a)

Fig. 2.89 (a) Assumed direction of flow of current in various branches $\frac{23.2 \times 0.04}{1.04}=0.892 \mathrm{~A}$, that is, from E to F

(b)

(c)

Fig. 2.89 (b) Considering one source only (c) Considering other source only

Again repeating source $E_{1}$ by its internal resistance (i.e., $0 \Omega$ ) as shown in Figure 2.89(c). Equivalent resistance of the circuits,

$$
R^{\prime \prime}=R_{2}+\frac{R_{1} R}{R_{1}+R}=0.04+\frac{0.05 \times 1}{0.05+1}=0.0876 \Omega
$$

Current supplied by battery $E_{2}, I_{2}^{\prime \prime}=\frac{E_{2}}{R^{\prime \prime}}=\frac{2.15}{0.0876}=24.54 \mathrm{~A}$

Current in $R \Omega$ resistor, $I_{3}^{\prime}=I_{2}^{\prime \prime} \times \frac{R_{1}}{R+R_{1}}=\frac{24.54 \times 0.05}{1.05}=1.169 \mathrm{~A}$, that is, from E to F
Total current through 1- $\Omega$ resistor when both batteries are present

$$
=I_{3}^{\prime}+I_{3}^{\prime \prime}=0.892+1.169=2.06 \mathrm{~A}
$$

## Example 2.34



Fig. 2.90 Given network

(a)

Fig. 2.91 (a) Assumed direction of flow of current in various branches

Determine the branch currents by superposition theorem in the network shown in Figure 2.90.

## Solution:

The simplified circuit is shown in Figure 2.91(a). Let the current flowing through different branches be $I_{1}, I_{2}$, and $I_{3}$ as marked in Figure 2.91(a).

According to superposition theorem, replacing 32 V battery by its internal resistance as shown in Figure 2.91(b).
Total resistance across $20-\mathrm{V}$ source

$$
=14.5+0.5+\frac{10 \times(9+1)}{10+(9+1)}=20 \Omega
$$

Current supplied by the source, $I_{1}^{\prime}=\frac{20}{20}=1 \mathrm{~A}$
Current in branch BCDE, $I_{2}^{\prime}=1 \times \frac{10}{10+10}=0.5 \mathrm{~A}$
Current in branch BE, $I_{3}^{\prime}=1 \times \frac{10}{10+10}=0.5 \mathrm{~A}$
Now, replace 20-V battery by its internal resistance as shown in Figure 2.91(c).
Total resistance across $32-\mathrm{V}$ source, $=9+1 \frac{15 \times 10}{15+10}=16 \Omega$
Current supplied by the source,

$$
I_{2}^{\prime \prime}=\frac{32}{16}=2 \mathrm{~A}
$$

Current in branch BAFE, $I_{1}^{\prime \prime}=2 \times \frac{10}{10+15}=0.8 \mathrm{~A}$
Current in branch BE, $I_{3}^{\prime \prime}=2 \times \frac{15}{10+15}=1.2 \mathrm{~A}$
The actual flow of current in various branches is obtained by superimposition of the sets of currents, that is,
$I_{1}=I_{1}^{\prime}-I_{1}^{\prime \prime}=1-0.8=0.2 \mathrm{~A}$ from A to B
$I_{2}=I_{2}^{\prime}-I_{2}^{\prime \prime}=0.5-2.0=-1.5 \mathrm{~A}$ from C to B
$I_{3}=I_{3}^{\prime}-I_{3}^{\prime \prime}=0.5+1.2=1.7 \mathrm{~A}$ from B to E


## Example 2.35

Determine the current through $8-\Omega$ resistor in the following network (see Fig. 2.92) using superposition theorem.
(U.P.T.U. June 2003)

Solution:
Replacing voltage source by its internal resistance (i.e., short circuit), a simplified circuit is shown in Figure 2.93(a).

Using the current divider rule, we get


Fig. 2.92 Given network

Current through $8-\Omega$ resistor, $I_{1}=2 \times \frac{2}{2+8}$

$$
=0.4 \mathrm{~A}(\text { from } \mathrm{B} \text { to } \mathrm{A})
$$

Again replacing the current source by open circuit and reducing the circuit as shown in Figure 2.93(b).

Current through 8- $\Omega$ resistor, $I_{2}=\frac{20}{2+8}=2 \mathrm{~A}$ (from A to B)
The resultant current in $8-\Omega$ resistor, $I=I_{2}-I_{1}$

$$
=2-0.4=1.6 \mathrm{~A}(\text { from } \mathrm{A} \text { to } \mathrm{B})
$$


(a)

(b)

Fig. 2.93 (a) Considering one source only (b) Considering second source only

## Example 2.36

Using superposition theorem, determine currents in all the resistances of the network shown in Figure 2.94.
(U.P.T.U. 2005-06)

## Solution:

Let the current through various branches be $I_{1}, I_{2}$, and $I_{3}$, as shown in Figure 2.95(a).
Case I: Considering 2 A current source and short circuiting the voltage source of 10 V as shown in Figure 2.95(b).

Since $10-\Omega$ resistor is directly short circuited, and therefore,
$I_{3}^{\prime}=0$. Further, by current divider rule
$I_{1}^{\prime}=I_{2}^{\prime}=1 \mathrm{~A}$ (both the $5-\Omega$ resistors are parallel with each other)


Fig. 2.95 (b) Considering one source only (c) Considering second source only

Case II: Considering only 10 V voltage source and open circuiting the current source of 2 A , as shown in Figure 2.95(c).

$$
\begin{aligned}
& I_{3}^{\prime \prime}=\frac{10}{10}=1 \mathrm{~A} ; I_{1}^{\prime \prime}=\frac{10}{10}=1 \mathrm{~A} \\
& I_{2}^{\prime \prime}=-I_{1}^{\prime}=-1 \mathrm{~A}
\end{aligned}
$$

By superposition theorem,
and

$$
\begin{aligned}
& I_{1}=I_{1}^{\prime}+I_{1}^{\prime \prime}=1+1=2 \mathrm{~A} \\
& I_{2}=I_{2}^{\prime}+I_{2}^{\prime \prime}=1+(-1)=0 \mathrm{~A} \\
& I_{3}=I_{3}^{\prime}+I_{3}^{\prime \prime}=0+1=1 \mathrm{~A}
\end{aligned}
$$

## Example 2.37

Find the current in the circuit given in Figure 2.96.
(U.P.T.U. 2006-07)

## Solution:

By applying superposition theorem, we get
Case I: Considering 24 V source only and denoting 4 A source by its internal resistance, that is, open circuit, as shown in Figure 2.97(a).

Current supplied by 24 V battery,


Fig. 2.96 Given network

$$
I_{1}=\frac{24}{6+5}=2.18 \mathrm{~A}(\text { from } \mathrm{A} \text { to } \mathrm{B})
$$


(a)

(b)

Fig. 2.97 (a) Considering one source only (b) Considering the other source only

Case II: Considering 4 A current source only and replacing 24 V voltage battery by its internal resistance, that is, short circuit, as shown in Figure 2.97(b).

Current is divided in two paths: current in $5-\Omega$ resistor,

$$
I_{2}=\frac{6}{5+6} \times 4=2.18 \mathrm{~A}(\text { from B to } \mathrm{A})
$$

Therefore, resultant current flowing through $5-\Omega$ resistor

$$
I=2.18-2.18=0 \mathrm{~A}
$$

## Example 2.38

For the circuit given in Figure 2.98, find $I$ using superposition theorem. (U.P.T.U. Sep. 2001)

## Solution:

Replacing 64 V source by its internal resistance (i.e., short circuit), the circuit is reduced to a circuit shown in Figure 2.99(a).

Equivalent resistance $R_{\text {eq }}^{\prime}$


Fig. 2.98 Given network

$$
\begin{aligned}
& =\left[\left\{\left(R_{4} \| R_{5}\right)+R_{2}\right\} \| R_{3}\right]+R_{1}=[\{(4 \| 12)+5\} \| 20]+5 \\
& =\left[\left\{\frac{4 \times 12}{4+12}+5\right\} \| 20\right]+5=\left[\frac{8 \times 20}{8+20}+5\right]=\frac{75}{7} \Omega
\end{aligned}
$$



Fig． 2.99 （a）Considering one source only（b）Considering the other source only

Current through branch $\mathrm{AB}=\frac{V_{1}}{R_{\text {eq }}}=\frac{75}{75 / 7}=7 \mathrm{~A}$
Current through branch BC，$I^{\prime}=I_{\mathrm{AB}} \times \frac{R_{\mathrm{BG}}}{R_{\mathrm{BG}}+R_{\mathrm{BF}}}=7 \times \frac{20}{20+8}=5 \mathrm{~A}$（from B to C）
Replacing 75 V source by its internal resistance（i．e．，short circuit），the circuit is reduced to a circuit shown in Figure 2．99（b）．

Equivalent resistance，

$$
\begin{aligned}
R_{\mathrm{eq}}^{\prime \prime} & =\left[\left\{\left(R_{1} \| R_{3}\right)+R_{2}\right\} \| R_{4}\right]+R_{5}=[\{(5 \| 20)+5\} \| 12]+4 \\
& =\left[\left\{\frac{5 \times 20}{5+20}+5\right\} \| 12\right]+4=\left[\frac{9 \times 12}{9+12}+4\right]=\frac{64}{7} \Omega
\end{aligned}
$$

Current through branch DC $=\frac{V_{2}}{R_{\text {eq }}^{\prime \prime}}=\frac{64}{64 / 7}=7 \mathrm{~A}$
Current through branch $\mathrm{CB}, I^{\prime \prime}=I_{\mathrm{DC}} \times \frac{R_{\mathrm{CF}}}{R_{\mathrm{CG}}+R_{\mathrm{CF}}}=7 \times \frac{12}{9+12}=4 \mathrm{~A}$（from C to B）
Current，$I=I^{\prime}-I^{\prime \prime}=5-4=1 \mathrm{~A}$

## ⿴囗玉ㄹㄹㄹ PRACTICE EXERCISES

## Short Answer Questions

1．When three resistors $R_{\mathrm{AB}}, R_{\mathrm{BC}}$ ，and $R_{\mathrm{CA}}$ are connected in delta，what will be their effective value when these are to be represented in star connections？
2．When three resistors $R_{\mathrm{A}}, R_{\mathrm{B}}$ ，and $R_{\mathrm{C}}$ are connected in star，what will be their effective value when these are to be represented in delta？
3．State superposition theorem．

## Test Questions

1．If three resistors $R_{\mathrm{AB}}, R_{\mathrm{BC}}$ ，and $R_{\mathrm{CA}}$ are connected in delta，what will be their effective value when representing them in a star connection？Derive the relation．
2．If three resistors，$R_{\mathrm{A}}, R_{\mathrm{B}}$ ，and $R_{\mathrm{C}}$ are connected in star，derive a relation for the resistors to be trans－ formed in delta connections．
3．State and explain superposition theorem．

## Numericals

1. Using superposition theorem, determine the current is $5-\Omega$ resistor of the network shown in Figure 2.100.
(U.P.T.U. Tut.)
(Ans. 0.433 A )
2. Using delta-star transformation, determine current supplied by the source in the Wheatstone bridge given in problem number 2 of Practice Exercises-2.1 (page 73).
(Ans. 19.345 A)


Fig. 2.100 Given network

### 2.11 THEVENIN'S THEOREM

Thevenin's theorem, named after French physicist M. Leon Thevenin, who proposed it in 1883, states that the current flowing through a resistor connected across any two terminals of a network can be determined by replacing the remaining network by an equivalent circuit having a voltage source $E_{\mathrm{th}}$ in series with a resistor $R_{\mathrm{th}}$.
where $E_{\text {th }}=$ the open-circuit voltage between the required two terminals called Thevenin voltage and $R_{\mathrm{th}}=$ the equivalent resistance of the network as seen from the terminals with all other sources replaced by their internal resistances called Thevenin resistance.

To visualise the application of this theorem, consider a simple circuit shown in Figure 2.101. To


Fig. 2.101 Given network determine the current through load resistance $R_{\mathrm{l}}$, proceed with the following steps:

1. Remove the resistance $R_{\mathrm{L}}$ in which current is to be determined, thus creating an open circuit between terminals A and B as shown in Figure 2.102(a).
2. Determine the open-circuit voltage (Thevenin voltage $E_{\text {th }}$ ) between the terminals A and B, that is, voltage across

$$
\begin{aligned}
& R_{2}=I_{2} R_{2} \\
& E_{\mathrm{th}}=\left(\frac{E}{r+R_{1}+R_{2}}\right) R_{2}
\end{aligned}
$$



Fig. 2.102 (a) Circuit to determine Thevenin voltage
3. Replace the source (battery) by its internal resistance and determine the resistance $R_{\mathrm{th}}$ (Thevenin resistance) of the network as seen from the terminals A and B as shown in Figure 2.102(b).

$$
R_{\mathrm{th}}=\frac{\left(r+R_{1}\right) R_{2}}{\left(r+R_{1}\right)+R_{2}}+R_{3}
$$

4. Replace the entire network by a single Thevenin voltage source having an emf $E_{\mathrm{th}}$ and internal resistance $R_{\mathrm{th}}$ as shown in Figure 2.102(c).


Fig. 2.102 (b) Circuit to determine Thevenin resistance (c) Equivalent Thevenin circuit
5. Connect the load resistance $R_{\mathrm{L}}$ back to its terminals A and B from where it was removed.
6. Determine the current flowing through the load resistance $R_{\mathrm{I}}$ by applying Ohm's law, that is,

$$
I=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}
$$

Applications: Sometimes, it is required to study the variation of current in a particular branch when the resistance of that branch is varied while the remaining network remains the same, for example, designing of electronic circuits. Thevenin theorem is most suitable at such places.

## Example 2.39



Fig. 2.103 Given network

Find Thevenin equivalent of the circuit shown in Figure 2.103 .
(U.P.T.U. 2004-05)

## Solution:

To determine $E_{\text {th }}$, consider Figure 2.104(a). By applying KVL in the loop,

Current, $I=\frac{60-30}{5+5}=3 \mathrm{~A}$

Note that no current is flowing through $10-\Omega$ resistor. Voltage across terminals AB is equal to VCD ,

(a)

(b)

Fig. 2.104 (a) Circuit to determine Thevenin voltage (b) Circuit to determine Thevenin resistance

Therefore,

$$
E_{\mathrm{th}}=V_{\mathrm{CD}}=60-5 \times 3=45 \mathrm{~V}
$$


(c)

Fig. 2.104 (c) Equivalent Thevenin circuit


Fig. 2.105 Given network

(a)

Fig. 2.106 (a) Circuit to determine Thevenin voltage

(b)

(c)

Fig. 2.106 (b) Circuit to determine Thevenin resistance (c) Equivalent Thevenin

## Example 2.41



Fig. 2.107 Given network

(a)

Fig. 2.108 (a) Circuit to determine Thevenin voltage

For a network shown in Figure 2.107, determine the current flowing through $R_{\mathrm{L}}$ when the value of load resistance is (1) $3 \Omega$, (2) $6 \Omega$, and (3) $9 \Omega$.

## Solution:

To determine voltage $E_{\mathrm{th}}$, load resistance $R_{\mathrm{L}}$ is removed as shown in Figure 2.108(a).

Effective resistance across 54 V battery.

$$
R=18+\frac{(9+9) \times 18}{(9+9)+18}=27 \Omega
$$

Current supplied by the battery $=\frac{54}{27}=2 \mathrm{~A}$
Current flowing through circuit ECDF or CD,

$$
=2 \times \frac{(9+9)}{(9+9)+18}=1 \mathrm{~A}
$$

Therefore, Thevenin voltage or voltage across AB or CD is

$$
E_{\mathrm{th}}=1 \times 9=9 \mathrm{~V}
$$

To determine Thevenin resistance $R_{\mathrm{th}}$, the source (battery) is replaced by its internal resistance (zero in this case) as shown in Figure 2.108(b).

(b)

(c)

Fig. 2.108 (b) Circuit to determine Thevenin resistance (c) Equivalent Thevenin circuit

From terminals A and B, Thevenin resistance can be given as

$$
R_{\mathrm{th}}=\frac{\left(\frac{18 \times 18}{18+18}+9\right) \times 9}{\left(\frac{18 \times 18}{18+18}+9\right)+9}+3=\frac{(9+9) \times 9}{(9+9)+9}+3=\frac{18 \times 9}{27}+3=9 \Omega
$$

The circuit is reduced to a Thevenin source of emf $E_{\mathrm{th}}(=9 \mathrm{~V})$ and internal resistance $R_{\mathrm{th}}(=9 \Omega)$ as shown in Figure 2.108(c).

Current flowing through load resistance $R_{\mathrm{L}}, I=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}$

1. When $R_{\mathrm{L}}=3 \Omega ; I_{1}=9 /(9+3)=0.75 \mathrm{~A}$
2. When $R_{\mathrm{L}}=6 \Omega ; I_{2}=9 /(9+6)=0.6 \mathrm{~A}$
3. When $R_{\mathrm{L}}=9 \Omega ; I_{3}=9 /(9+6)=0.5 \mathrm{~A}$

## Example 2.42

In the circuit given in Figure 2.109, find the branch current $I_{2}$ that flows through $R_{2}$, when $R_{2}$ has the following values: $5 \Omega, 15 \Omega$, and $50 \Omega$.
(U.P.T.U. Sep. 2001)

## Solution:

Thevenin resistance of the network through the terminals A and B replaces voltage sources by their internal resistances (short circuited in this case), as shown in Figure 2.110(a).

$$
R_{\mathrm{th}}=30 \| 70=\frac{30 \times 70}{30+70}=\frac{30 \times 70}{100}=21 \Omega
$$



Fig. 2.109 Given network


Fig. 2.110 (a) Circuit to determine Thevenin resistance

Thevenin equivalent circuit is shown in Figure 2.110(c).


Fig. 2.110 (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

When $R_{2}=5 \Omega$, current $I_{\mathrm{L} 1}=\frac{123.5}{21+5}=4.75$
When $R_{2}=15 \Omega$, current $I_{\mathrm{L} 2}=\frac{123.5}{21+15}=3.43$
When $R_{2}=50 \Omega$, current $I_{\mathrm{L} 3}=\frac{123.5}{21+50}=1.74$

## Example 2.43



Fig. 2.Ill Given network

Using Thevenin's theorem, determine current and voltage in $2-\Omega$ resistor in the circuit shown in Figure 2.111. (U.P.T.U. 2006-07) Solution:
To determine Thevenin equivalent resistance of the given circuit with reference to terminals AB , replace current source with open circuit and voltage source with short circuit as shown in Figure 2.112(a).

$$
R_{\mathrm{th}}=5+(10 \| 6)=5+\frac{10 \times 6}{10+6}=8.75 \Omega
$$

To determine open-circuit voltage $\left(E_{\mathrm{th}}\right)$ across the terminals AB , redraw the circuit as shown in Figure 2.112(b).


Fig. 2.112 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage

(c)

Voltage at A with respect to C, $V_{\mathrm{AC}}=7 \times 5=35 \mathrm{~V}$
Voltage at B with respect to C, $V_{\mathrm{BC}}=\frac{12}{10+6} \times 10=7.5 \mathrm{~V}$
Open-circuit voltage, $E_{\mathrm{th}}=V_{\mathrm{AB}}=V_{\mathrm{AC}}-V_{\mathrm{BC}}=35-7.5=27.5 \mathrm{~V}$
Thevenin equivalent circuit is shown in Figure 2.112(c).
Fig. 2.112 (c) Equivalent Thevenin circuit

Current through 2- $\Omega$ resistor, $I_{\mathrm{L}}=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{27.5}{8.75+2}=2.56 \mathrm{~A}$

## Example 2.44

Obtain Thevenin's equivalent circuit at AB , as shown in Figure 2.113.
(U.P.T.U. Tut.)

## Solution:

To determine Thevenin equivalent resistance of the given network with reference to terminals AB , replace the voltage sources by their internal resistances (here, $0 \Omega$ ), as shown in Figure 2.114(a).

$$
R_{\mathrm{th}}=3 \|[8+(4 \| 5)]=\frac{276}{119}=2.32 \Omega
$$

To determine voltage across terminals AB , consider the circuit shown in Figure 2.114(b).
By applying KVL to mesh I and II, keeping terminals AB open circuited, we get
Mesh I: $5 I_{1}+4\left(I_{1}-I_{2}\right)=80$
or

$$
\begin{equation*}
9 I_{1}-4 I_{2}=80 \tag{2.74}
\end{equation*}
$$

Mesh II: $\quad 11 I_{2}-4\left(I_{1}-I_{2}\right)=20$
or

$$
\begin{equation*}
4 I_{1}-15 I_{2}=-20 \tag{2.75}
\end{equation*}
$$

Solving Equations (2.74) and (2.75), we get

$$
I_{1}=\frac{1280}{119} \mathrm{~A} \quad \text { and } \quad I_{2}=\frac{500}{119} \mathrm{~A}
$$



Fig. 2.113 Given network

Open-circuit voltage across AB, $E_{\mathrm{th}}=80-5 I_{1}-8 I_{2}$

$$
=80-5 \times \frac{1,280}{119}-8 \times \frac{500}{119}=-\frac{880}{119}=-7.395 \mathrm{~V}
$$

Thevenin's equivalent circuit is shown in Figure 2.114(c).


Fig 2.114 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

## Example 2.45

Draw the Thevenin's equivalent circuit across AB shown in Figure 2.115.
(U.P.T.U. Tut.)

## Solution:

To determine Thevenin equivalent resistance of the given circuit with reference to terminals AB , replace voltage sources by short circuit as shown in Figure 2.116(a).


Fig. 2.115 Given network

$$
\begin{aligned}
R_{\mathrm{th}} & =\left[\{(1+1) \| 1\}\left\|2=\left(\frac{2 \times 1}{2+1}+1\right)\right\| 2\right. \\
& =\frac{5}{3} \| 2=\frac{\frac{5}{3} \times 2}{\frac{5}{3}+2}=\frac{10}{3} \times \frac{3}{11}=\frac{10}{11}=0.91 \Omega .
\end{aligned}
$$

To determine voltage across terminals AB , let us apply KVL to mesh I and II in the network shown in Figure 2.116(b), and therefore, we get

## Mesh I:

$$
(1+1) I_{1}+\left(I_{1}-I_{2}\right) \times 1=10
$$

or

$$
\begin{equation*}
3 I_{1}-I_{2}=10 \tag{2.76}
\end{equation*}
$$

Mesh II:

$$
I_{2}(1+2)-1\left(I_{1}-I_{2}\right)=-5
$$

or

$$
\begin{equation*}
I_{1}-4 I_{2}=5 \tag{2.77}
\end{equation*}
$$

Solving Equations (2.76) and (2.77), we get

$$
I_{1}=\frac{35}{11} \mathrm{~A} \quad \text { and } \quad I_{2}=\frac{-5}{11} \mathrm{~A}
$$

Open-circuit voltage across terminals AB is

$$
E_{\mathrm{th}}=5+2 I_{2}=5-2 \times \frac{5}{11}=\frac{45}{11}=4.091 \mathrm{~V}
$$


(a)


Fig. 2.116 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

When $2-\Omega$ resistor is connected across $A B$, then
Load current, $I_{\mathrm{L}}=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{4.091}{2+0.91}=1.4063 \mathrm{~A}$
Thevenin equivalent circuit is shown in Figure 2.116(c).

## Example 2.46

Using Thevenin's Theorem, find current $I$ in the circuit shown in Figure 2.117. (U.P.T.U. Jan. 2003)

## Solution:

To determine $R_{\mathrm{th}}$, replace voltage source by its internal resistance (i.e., short circuit) as shown in Figure 2.118(a).

$$
R_{\mathrm{th}}=(6 \| 12)+(3 \| 10)=\frac{6 \times 12}{6+12}=\frac{3 \times 10}{3+10}=\frac{12}{3}+\frac{30}{13}=\frac{82}{13}=6.3077 \Omega
$$

To determine open-circuit voltage (i.e., Thevenin voltage) across BD , remove the resistance connected across BD as shown in Figure 2.118(b).

Now, in circuit shown in Figure 2.118(b), potential at terminal B is


Fig. 2.117 Given network

$$
V_{\mathrm{B}}=10-\frac{10}{6+12} \times 6=6.667 \mathrm{~V}
$$

Potential at terminal D, $V_{\mathrm{D}}=10-\frac{10}{6+12} \times 3=8.333 \mathrm{~V}$

(a)

(b)


Fig. 2.118 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

Potential difference between terminals B and D ,

$$
V_{\mathrm{BD}}=E_{\mathrm{th}}=V_{\mathrm{B}}-V_{\mathrm{C}}=6.667-8.333=-1.666
$$

Current,

$$
\begin{aligned}
I & =\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{BD}}}=\frac{-1.666}{6.3077+10} \\
& =-0.1022 \mathrm{~A} \text { (i.e., from D to B) }
\end{aligned}
$$

The magnitude of flow of current in $10-\Omega$ resistor connected across BD is 0.1022 A and it flows from D to B as shown in Figure 2.118(c).

## Example 2.47

Find the magnitude and direction of the current in the $2-\Omega$ resistor by using Thevenin's theorem for the circuit shown in Figure 2.119.
(U.P.T.U. Tut.)

## Solution:

The circuit is redrawn in a simplified manner as shown in Figure 2.120(a). To determine open-circuit voltage across BD, remove the resistor connected across BD as shown in Figure 2.120(b).


Fig. 2.119 Given network

Voltage at $\mathrm{B}, V_{\mathrm{B}}=4.5-\frac{4.5}{6+3} \times 6=1.5 \mathrm{~V}$
Voltage at $\mathrm{D}, V_{\mathrm{D}}=4.5-\frac{4.5}{6+3} \times 3=3.0 \mathrm{~V}$
Open-circuit voltage across BD, $V_{\mathrm{BD}}=E_{\mathrm{th}}=V_{\mathrm{B}}-$ $V_{\mathrm{D}}=1.5-3.0=-1.5 \mathrm{~V}$. To determine $R_{\mathrm{th}}$, replace voltage source by its internal resistance (i.e., $0 \Omega$ or short circuit) as shown in Figure 2.120(c).

$$
R_{\mathrm{th}}=(6 \| 3)+(3 \| 6)
$$



Fig. 2.120 (a) Simplified equivalent circuit (b) Circuit to determine Thevenin voltage
(c) Circuit to determine Thevenin resistance

$$
=\frac{6 \times 3}{6+3}+\frac{3 \times 6}{3+6}=2+2=4 \Omega
$$

Current in $2-\Omega$ resistor,

$$
I=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{-1.5}{4+2}=-0.25 \mathrm{~A}
$$

The negative sign in the answer shows that the actual direction of flow of current is opposite to that of the assumed direction, that is, actual flow of current is from D to B.

## Example 2.48

Calculate current in $2-\Omega$ resistor in the network shown in Figure 2.121 using Thevenin's theorem.

## Solution:

To determine voltage $E_{\mathrm{th}}$, the $2-\Omega$ resistor is removed as shown in Figure 2.122(a). For simplicity, the circuit is further redrawn as shown in Figure 2.122(b). Let the current in the circuit be as marked in Figure 2.122(c).

By applying KVL to circuit abcdefa, we get
or

$$
\begin{aligned}
& -10 I_{1}-30-5 I_{1}-15+50=0 \\
& 15 I_{1}=5 \quad \text { or } \quad I_{1}=\frac{5}{15}=0.3333 \mathrm{~A}
\end{aligned}
$$



Fig. 2.121 Given network

Potential difference across $\mathrm{AB}=3 \times 12+(0.3333+3) \times 10=69.33 \mathrm{~V}$


(c)

(d)

(e)

Fig. 2.122 (a) Circuit to determine Thevenin voltage (b) Equivalent circuit (c) Assumed direction of current in various branches (d) Circuit to determine Thevenin resistance (e) Simplified equivalent circuit

Thevenin voltage, $E_{\mathrm{th}}=69.33 \mathrm{~V}$
To determine Thevenin resistance $R_{\mathrm{th}}$, the voltage source is replaced by its internal resistance (zero as short circuited) and the current source is replaced by its internal resistance (infinity, that is, open circuited) as shown in Figure 2.122(d). From the open circuit terminals A and B, the circuit is further simplified as shown in Figure 2.122(e).

Thevenin resistance, $R_{\mathrm{th}}=12+\frac{10 \times 5}{10+5}=15.33 \Omega$
Current thr.ough 2- $\Omega$ resistor, $R=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{69.33}{15.33+2}=4 \mathrm{~A}$

## Example 2.49



Fig. 2.123 Given network

Determine the current flowing through $5-\Omega$ resistor in the network shown in Figure 2.123 using Thevenin's theorem.
(U.P.T.U. Jun. 2003)

## Solution:

To determine Thevenin equivalent resistance $R_{\mathrm{th}}$, the voltage sources are replaced by short circuit and current sources by open circuit as shown in Figure 2.124(a).

$$
R_{\mathrm{th}}=3+(4 \| 2)=3+\frac{4 \times 2}{4+2}=3+\frac{4}{3}=4.33 \Omega
$$

Converting the current source of 6 A connected across $2-\Omega$ resistor into an equivalent voltage source of voltage, $E=2 \times 6=12$ with internal resistance of $2 \Omega$, as shown in Figure 2.124(b).

The current flowing through the mesh formed by voltage sources and resistors of $4 \Omega$ and 2 $\Omega$ is given as

$$
I=\frac{15-12}{4+2}=0.5 \mathrm{~A}
$$

Open-circuit voltage across terminals A and B, $V_{\mathrm{AB}}=E_{\mathrm{th}}=15-4 \times 0.5=13 \mathrm{~V}$
The equivalent Thevenin circuit of the network is shown in Figure 2.124(c).
Current flowing through resistor of $5 \Omega, I_{\mathrm{L}}=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{3}{4.333+5}=1.3928 \mathrm{~A}$


Fig. 2.124 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

## Example 2.50

Find Thevenin's equivalent circuit across AB shown in Figure 2.125.
(U.P.T.U. 2005-06)

## Solution:

For determining the Thevenin's equivalent resistance $R_{\mathrm{th}}$ of the circuit with reference to terminals AB , the voltage source is short circuited and current source is open circuited, as shown in Figure 2.126(a).

$$
R_{\mathrm{th}}=3+(20 \| 5)+2=3+\frac{20 \times 5}{20+5}+2=9 \Omega
$$

Converting current source of 4 A connected across $3-\Omega$ resistor into the equivalent voltage source, that is, $4 \times 3=12 \mathrm{~V}$ with series resistance of $3 \Omega$, as shown in Figure 2.126(b).

Since there is no current through $3-\Omega$ and $2-\Omega$ resistors, (hence, no voltage drop occurs across them),

Voltage across $5 \Omega=$ resistor $=\frac{30}{20+5} \times 5=6 \mathrm{~V}$
Thevenin voltage across $\mathrm{AB}, E_{\mathrm{th}}=-12+6=-6 \mathrm{~V}$


Fig. 2.125 Given network

Thevenin's equivalent circuit is shown in Figure 2.126(c).


Fig. 2.126 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

### 2.12 NORTON'S THEOREM

Norton's theorem was named after E.L. Norton (an engineer employed at Bell Laboratory, USA) who developed it for the first time. This theorem states that the current flowing through a resistance connected across any two terminals of a network can be determined by replacing the whole network by an equivalent circuit of a current source having a current output of $I_{\mathrm{N}}$ in parallel with a resistance $R_{\mathrm{N}}$.
where $I_{\mathrm{N}}$ = the short circuit current supplied by the source that would flow between the two selected terminals when they are short circuited. It is generally called Norton's current.
$R_{\mathrm{N}}=$ the equivalent resistance of the network as seen from the two terminals with all other emf sources replaced by their internal resistances and current sources replaced by open circuit. It is generally called Norton's resistance.
Steps to determine Norton's equivalent circuit.

1. Short circuit the terminals across which the load resistor is connected and calculate the current that would flow between them. This is the Norton current $I_{\mathrm{N}}$.
2. Redraw the network by replacing each voltage source by a short circuit in series with its internal resistance if any and each current source by an open circuit in parallel with its internal resistance.
3. Determine the resistance $R_{\mathrm{N}}$ of the network as seen from the network terminals. (Its value is the same as that of $R_{\mathrm{th}}$ ).

### 2.13 CONVERSION OF THEVENIN'S EQUIVALENT INTO NORTON'S EQUIVALENT AND VICE VERSA

A Thevenin's equivalent can be converted into its Norton's equivalent and vice versa. According to the statement, Norton's current source equals the current $I_{\mathrm{SC}}$ or $I_{\mathrm{N}}$ that flows across the terminals A and B when they are short circuited.

Hence,

$$
\begin{equation*}
I_{\mathrm{SC}}=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}} \tag{2.78}
\end{equation*}
$$

Likewise, a Norton's circuit can be converted into its Thevenin's equivalent. The Thevenin's equivalent source $V_{\mathrm{OC}}$ or $E_{\mathrm{th}}$ is the voltage on open circuit and is given as

$$
\begin{equation*}
V_{\mathrm{oc}} \quad \text { or } \quad E_{\mathrm{th}}=I_{\mathrm{sc}} R_{\mathrm{th}} \tag{2.79}
\end{equation*}
$$

Each theorem is dual of the other.

## Example 2.51

For the circuit shown in Figure 2.127, obtain Norton's current and equivalent resistance seen from AB .
(U.P.T.U. Dec. 2003)

## Solution:



Fig. 2.127 Given circuit

Equivalent resistance of the network with reference to terminals A and B, as shown in Figure 2.128(a).

$$
R_{\mathrm{N}}=(15 \| 10)+4=\frac{15 \times 10}{15+10}+4=6+4=10 \Omega
$$

Current supplied by 30 V source,

$$
I=\frac{30}{15+\frac{4 \times 10}{4+10}}=\frac{42}{25} \mathrm{~A}
$$

Norton's current, $I_{\mathrm{N}}$, that is, the current from terminal A to B when they are short circuited as shown in Figure 2.128(b).

$$
I_{\mathrm{N}}=\frac{10}{10+4} \times I=\frac{10}{14} \times \frac{42}{25}=1.2 \mathrm{~A}
$$

The Norton's equivalent circuit is shown in Figure 2.128(c).


Fig. 2.128 (a) Circuit to determine Norton resistance (b) Circuit to determine Norton current (c) Equivalent Norton circuit

## Example 2.52

Draw the Norton's equivalent circuit across AB, and determine current flowing through $12-\Omega$ resistor form the network shown in Figure 2.129. (U.P.T.U. Jun. 2004)

## Solution:

To determine Norton's current or short circuit current, remove $12-\Omega$ resistor connected across AB and short circuit the terminals as shown in Figure 2.130(a).

By using superposition theorem, the following conditions are explained.

1. When only current source is present (see Fig. 2.130(b))

In this case, 40 V battery is replaced by a short circuit. The 20 A current divides at point C between parallel combination of $8 \Omega$ and $5 \Omega$. However, current will not flow through $4-\Omega$ resistor being short circuited at terminals A and B .

$$
I_{\mathrm{SC}}=20 \times \frac{8}{8+5}=\frac{160}{3}=12.31 \mathrm{~A}
$$

2. When only voltage source is present (see Fig. 2.130(c))

In this case, current source is replaced by an open circuit. Voltage across terminals EF is equal to voltage across terminals A and B , that is, zero. Therefore, short circuit current is limited by only $4-\Omega$ resistor.

$$
I_{\mathrm{SC}}=\frac{40}{4}=10 \mathrm{~A}
$$

$$
\text { Norton's current } I_{\mathrm{N}}=I_{\mathrm{SC} 1} I_{\mathrm{SC} 2}=12.31+10=22.31 \mathrm{~A}
$$

The determine the Norton's resistance of the network, remove $12-\Omega$ resistor connected across AB and short circuit the voltage source and open circuit the current source as shown in Figure 2.130(d).

$$
R_{\mathrm{N}}=4\|(5+8)=4\| 13=\frac{4 \times 13}{4+13}=\frac{52}{17}=3.06 \Omega
$$



Fig. 2.130 (a) Circuit to determine Norton current (b) Simplified circuit (c) Simplified circuit (d) Circuit to determine Norton resistance (e) Equivalent Norton circuit

Figure 2.130 (e) shows the Norton's equivalent circuit with the load resistor of $12 \Omega$.

$$
\text { Load current, } I_{\mathrm{L}}=I_{\mathrm{N}} \times \frac{R_{\mathrm{N}}}{R_{\mathrm{N}}+R_{\mathrm{L}}}=22.31 \times \frac{3.06}{3.06 \times 12}=4.533 \mathrm{~A}
$$

## Example 2.53

Determine current in $10-\Omega$ resistor using Norton's theorem in the circuit shown in Figure 2.131.

## Solution:

To determine Norton's current, we can use mesh analysis method. Redraw the circuit as shown in Figure 2.132(a).

Let $I_{1}$ and $I_{2}$ be the current in loop 1 to 2 , respectively. By applying KVL to loop 1, we get
or

$$
\begin{align*}
8 I_{1}+6\left(I_{1}-I_{2}\right) & =15-30 \\
14 I_{1}-6 I_{2} & =-15 \tag{2.80}
\end{align*}
$$

By applying KVL to loop 2, we get


Fig. 2.131 (a) Given network

$$
4 I_{2}+6\left(I_{2}-I_{1}\right)=30 \text { or }-6 I_{1}+10 I_{2}=30
$$

or

$$
\begin{equation*}
-3 I_{1}+5 I_{2}=15 \tag{2.81}
\end{equation*}
$$

By multiplying Equation (2.80) by 5 and Equation (2.81) by 6 and adding, we get

$$
I_{1}=\frac{15}{52} \mathrm{~A}=0.288 \mathrm{~A}
$$



(b)

(c)

Fig. 2.132 (a) Circuit to determine Norton current (b) Circuit to determine Norton resistance (c) Equivalent Norton circuit

Substituting the value of $I_{1}$ in Equation (2.80), we get

$$
I_{2}=\frac{165}{52}=3.173 \mathrm{~A}
$$

Norton's current $I_{\mathrm{N}}=I_{2}=3.173 \mathrm{~A}$
To determine Norton's resistance, replace the voltage source by their internal resistance, that is, short circuit as shown in Figure 2.132(b).

$$
R_{\mathrm{N}}=(8 \| 6)+4=\frac{8 \times 6}{8+6}+4=\frac{52}{7}=7.428 \Omega
$$

Current through $10-\Omega$ resistor,

$$
I_{\mathrm{L}}=\frac{R_{\mathrm{N}}}{R_{\mathrm{N}}+R_{\mathrm{L}}} \times I_{\mathrm{N}}=\frac{7.428}{7.428+10} \times 3.173=1.352 \mathrm{~A}
$$

## Example 2.54

Using Norton's theorem, find the current that would flow in a $25-\Omega$ resistor connected between points N and O in Figure 2.133.
(U.P.T.U. Tut.)


Fig. 2.133 Given network

## Solution:

Equivalent resistance of network when viewed through terminals N and O , keeping all the voltage sources short circuited, as shown in Figure 2.134(a). Its simplified circuit is shown in Figure 2.134(b).

$$
R_{\mathrm{N}}=5\|20\| 20=\frac{1}{\frac{1}{5}+\frac{1}{20}+\frac{1}{20}}=\frac{20}{7} \Omega
$$

Short circuit current, that is, the current when the terminals O and $\mathrm{N}_{\mathrm{m}}$ are short circuited as shown in Figure 2.134(c).

$$
I_{\mathrm{SC}}=I_{1}+I_{2}+I_{3}=\frac{10}{5}+\frac{30}{20}+\frac{20}{10}=5.5 \mathrm{~A}
$$

Current through a resistance of $25 \Omega$ when connected between terminal O and N ,

$$
I=\frac{I_{\mathrm{N}}}{R_{\mathrm{N}}+R_{\mathrm{L}}} \times R_{\mathrm{N}}=\frac{5.5}{\frac{20}{7}+25} \times \frac{20}{7}=\frac{22}{39} \mathrm{~A}=0.564 \mathrm{~A}
$$



Fig. 2.134 (a) Circuit to determine Norton resistance (b) Simplified equivalent circuit (c) Circuit to determine Norton current (d) Equivalent Norton circuit

## Example 2.55

Define Norton's theorem. In the network shown in Figure 2.135, find the Norton's equivalent circuit at terminals A-B and the maximum power that can be provided to a resistor $R$ connected to terminals A-B.
(U.P.T.U. Feb. 2002)

## Solution:

In Figure 2.135, the three resistors $2 \Omega, 4 \Omega$, and $8 \Omega$ are connected in star. Converting them into their equivalent delta connections as shown in Figure 2.136(a), we get

$$
\begin{aligned}
& R_{12}=R_{1}+R_{2}+\frac{R_{1}+R_{2}}{R_{3}}=4+8+\frac{4 \times 8}{2}=28 \Omega \\
& R_{23}=R_{2}+R_{3}+\frac{R_{2}+R_{3}}{R_{1}}=8+2+\frac{8 \times 2}{4}=14 \Omega \\
& R_{31}=R_{3}+R_{1}+\frac{R_{3}+R_{1}}{R_{2}}=2+4+\frac{2 \times 4}{8}=7 \Omega
\end{aligned}
$$



Fig. 2.135 Given network

Considering Figure 2.136(a) effective value of resistance across the terminals are determined as shown in Figure 2.136(b).

$$
\begin{aligned}
& R_{12}^{\prime}=28 \| 4=\frac{28 \times 4}{28+4}=3.5 \Omega \\
& R_{31}^{\prime}=3 \| 7=\frac{3 \times 7}{7+3}=2.1 \Omega
\end{aligned}
$$

Converting current source of 48 A shunted by a resistor of $2.1 \Omega$ into an equivalent voltage source and redrawing the circuit as shown in Figure 2.136(c)

Voltage of equivalent voltage source,

$$
V=48 \times 2.1=100.8 \mathrm{~V} \text { with series resistance of } 2.1 \Omega
$$



Fig. 2.136 (a) Given network (b) Simplified circuit (c) Simplified equivalent circuit (d) Circuit to determine Norton resistance (e) Circuit to determine Norton current (f) Equivalent Norton circuit

Equivalent resistance of the network when viewed through terminals A and B after removing resistor $R$ from the circuit and short circuiting the voltage source (see Fig. 2.136(d)).

$$
R_{\mathrm{N}}=(2.1+3.5) \| 14=\frac{5.6 \times 14}{5.6+14}=4 \Omega
$$

Short circuit current when terminals A and B are short circuited, as shown in Figure 2.136(e).

$$
I_{\mathrm{N}}=I_{\mathrm{SC}}=\frac{100.8}{2.1+3.5}=18 \mathrm{~A}(14-\Omega \text { resistor is circuited })
$$

Norton's equivalent circuit at terminal A-B is shown in Figure 2.136(f). The power delivered will be maximum when $R_{\mathrm{L}}=R_{\mathrm{N}}=4 \Omega$. In that case, source current of 18 A will be equally divided among $R_{\mathrm{N}}$ and $R_{\mathrm{L}}$, that is, current flowing through

$$
R_{\mathrm{L}}=\frac{18}{2}=9 \mathrm{~A}
$$

Maximum power delivered, $P_{\text {max }}=I^{2}{ }_{\text {max }} R_{\mathrm{L}}=(9)^{2} \times 4=324 \mathrm{~W}$

### 2.14 MAXIMUM POWER TRANSFER THEOREM

This theorem states that the output obtained from a network is maximum when the load resistance $R_{\mathrm{L}}$ is equal to the internal resistance of the network as seen from the terminals of the load. According to Thevenin theorem, every network can be represented by a single voltage source (Thevenin source) having an effective internal resistance $R_{\mathrm{th}}$ as shown in Figure 2.137. Let us determine the value of load resistor $R_{\mathrm{L}}$ so that source delivers maximum power to it.

Output power, $P=I^{2} R_{\mathrm{L}}=\left(\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}\right)^{2} R_{\mathrm{L}}$

Power will be maximum if $\frac{d}{d R_{\mathrm{L}}} P=0$


Fig. 2.137 Given network

$$
\frac{d}{d R_{\mathrm{L}}}\left(\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}\right)^{2} R_{\mathrm{L}}=0
$$

or

$$
E_{\mathrm{th}}^{2}\left(\frac{\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)^{2}-2 R_{\mathrm{L}}\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)}{\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)^{4}}\right)=0
$$

or

$$
\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)^{2}-2 R_{\mathrm{L}}\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)=0
$$

or

$$
\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)\left(R_{\mathrm{th}}-R_{\mathrm{L}}\right)=0
$$

Since, $\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)$ cannot be zero, and therefore,
or

$$
\begin{aligned}
R_{\mathrm{th}}-R_{\mathrm{L}} & =0 \\
R_{\mathrm{L}} & =R_{\mathrm{th}}
\end{aligned}
$$

Load resistance $=$ Internal resistance

Thus, for maximum power transfer, the load resistance $R_{\mathrm{L}}$ must be made equal to the internal resistance of the source (or of the whole network) $R_{\mathrm{th}}$. In this case, the efficiency will by very poor only $50 \%$, but the low efficiency is of no importance because the aim is to transfer the maximum power at the output. The adjustment of load resistance for maximum power transfer is called load matching. It is generally done in case of radio transistors, TV aerial lead, starter motor of cars with battery, etc.

## Example 2.56

Find the value of load resistance $R_{\mathrm{L}}$ for maximum power flow through it in the circuit shown in


Fig. 2. 138 Given network


Fig. 2.139 Circuit to determine Thevenin resistance

Figure 2.138.
(U.P.T.U. 2006-07)

## Solution:

The power drawn by the load resistor $R_{\mathrm{L}}$ will be maximum when its value is equal to the Thevenin equivalent resistance of the network. To determine $R_{\text {th }}$ across terminal AB (load), remove $R_{\mathrm{L}}$ and replace the sources by their internal resistances, that is, short circuit the voltage source and open circuit the current source, as shown in Figure 2.139. Then,

$$
\begin{aligned}
R_{\mathrm{th}} & =[(3+2)| | 2]+1=\frac{5 \times 2}{5+2}+1=\frac{10}{7}+1=\frac{17}{7} \\
& =2.43 \Omega
\end{aligned}
$$

The power transferred to the load by the source will be maximum when

$$
R_{\mathrm{L}}=R_{\mathrm{th}}=2.43 \Omega
$$

## Example 2.57

Consider the electric circuit shown in Figure 2.140. Determine the value of $R$ so that load of $20 \Omega$ should draw the maximum power and the value of the maximum power drawn by the load.

## Solution:

The power delivered to the load will be maximum when the internal resistance of the network from the output terminal AB (i.e., $R_{\mathrm{th}}$ ), see Figure 2.141(a), is equal to that of


Fig. 2.140 Given network the load resistance.

(U.P.T.U. Feb. 2001)

Fig. 2.141 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

$$
\frac{R \times 60}{R+60}=20 \quad \text { or } \quad R+60=\frac{R \times 60}{20} \quad \text { or } \quad R=30 \Omega
$$

Open-circuit voltage across AB, see Figure 2.141(b), that is,

$$
E_{\mathrm{th}}=I \times 60=\frac{180}{30+60} \times 60=120 \mathrm{~V}
$$

Maximum value of current supplied to the load,

$$
I_{\max }=\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}=\frac{120}{20+20}=3 \mathrm{~A}
$$

Maximum power drawn by the load,

$$
P_{\max }=I_{\max }^{2} R_{\mathrm{L}}=(3)^{2} \times 20=180 \mathrm{~W}
$$

## Example 2.58

In the network shown in Figure 2.142, determine the value of load resistance $R_{\mathrm{L}}$ to give maximum power transfer and the power delivered to the load.
(U.P.T.U. Jun. 2003)


Fig. 2.142 Given network

## Solution:

The equivalent resistance of the network when viewed from terminals A and B after removing resistor $R_{\mathrm{L}}$ from the circuit and short circuiting the voltage source (see Fig. 2.143(a)), we get

$$
R_{\mathrm{th}}=4+(10 \| 10)=4+\frac{10 \times 10}{10+10}=9 \Omega
$$

Maximum power will be transferred to the load, when it is equal to Thevenin's equivalent resistance $R_{\mathrm{th}}$,

$$
R_{\mathrm{L}}=R_{\mathrm{th}}=9 \Omega
$$

When the terminals A and B are open (i.e., load resistor $R_{\mathrm{L}}$ is removed), the current flowing through the mesh ECDFE, as shown in Figure 2.143(b), is

$$
I=\frac{20}{10+10}=1 \mathrm{~A}
$$

Open-circuit voltage across $\mathrm{AB}, V_{\mathrm{AB}}=V_{\mathrm{CD}}=E_{\mathrm{th}}=10 \times 1=10 \mathrm{~V}$
(since no current is flowing through $4-\Omega$ resistor).
Maximum power delivered to the load,

$$
P_{\max }=I_{\max }^{2} R_{\mathrm{L}}=\left(\frac{E_{\mathrm{th}}}{R_{\mathrm{th}}+R_{\mathrm{L}}}\right)^{2} \times R_{\mathrm{L}}=\left(\frac{10}{9+9}\right)^{2} \times 9=2.778 \mathrm{~W}
$$



Fig. 2.143 (a) Circuit to determine Thevenin resistance (b) Circuit to determine Thevenin voltage (c) Equivalent Thevenin circuit

## Example 2.59

A generator develops 200 V and an internal resistance of $100 \Omega$. Find the power delivered to a load of $100 \Omega$ and $400 \Omega$. Comment on the results.

## Solution:

Generated emf, $E=200 \mathrm{~V}$
Internal resistance, $R_{\mathrm{i}}=100 \mathrm{~W}$
When the load resistance, $R_{\mathrm{L}}=100 \Omega$
Load current, $I=\frac{E}{R_{\mathrm{i}}+R_{\mathrm{L}}}=\frac{200}{100+100}=1 \mathrm{~A}$

Power delivered, $P_{0}=I^{2} R_{\mathrm{L}}=(1)^{2} \times 100=100 \mathrm{~W}$
Power generated, $P=I^{2}\left(R_{\mathrm{i}}+R_{\mathrm{L}}\right)=200 \mathrm{~W}$
Efficiency, $\eta=\frac{p_{0}}{p} \times 100=\frac{100}{200} \times 100=50 \%$
When the load resistance, $R_{\mathrm{L}}=400 \Omega$
Load current, $I=\frac{200}{100+100}=0.4 \mathrm{~A}$
Power delivered, $P_{0}=(0.4)^{2} \times 400=64 \mathrm{~W}$
Power generated, $P=(0.4)^{2} \times(100+400)=80 \mathrm{~W}$

$$
\eta=\frac{64}{80} \times 100=80 \%
$$

Comments: Although the efficiency of the system is poor, but the power delivered by the source is maximum when $R_{\mathrm{L}}=R_{\mathrm{i}}=100 \Omega$

## Example 2.60

Find the value of resistance ' $R$ ' to have maximum power transfer in the circuit as shown in Figure 2.144. Further, obtain the amount of maximum power.
(U.P.T.U. Jun. 2004)

## Solution:

Let the current source of 2 A shunted by $15-\Omega$ resistor be converted into its equivalent voltage source of $2 \times 15=30 \mathrm{~V}$ connected in series with a resistance of $15 \Omega$, as shown in Figure 2.145(a). To determine Thevenin equiv-


Fig. 2.144 Given network alent resistance of the network, remove $R$ and replace voltage sources with short circuit as shown in Figure 2.145(b).

$$
R_{\mathrm{th}}=(6+15)\|3=21\| 3=\frac{21 \times 3}{21+3}=\frac{21 \times 3}{24}=\frac{21}{8}=2.625 \Omega
$$

Current flowing through a resistance of $3 \Omega$ in a mesh ECDFE as shown in Figure 2.145(c)

$$
I=\frac{30-6}{15+6+3}=1 \mathrm{~A}
$$

Open-circuit voltage, $E_{\text {th }}=$ Voltage across $3-\Omega$ resistor $+8 \mathrm{~V}=1 \times 3+8=11 \mathrm{~V}$
According to maximum power transfer theorem, load resistor $R$ will absorb maximum power when it is equal to $R_{\mathrm{th}}$, that is,

$$
R=R_{\mathrm{th}}=2.625 \Omega
$$

Maximum power transferred, $P_{\max }=\frac{E_{\mathrm{th}}^{2}}{4 R_{\mathrm{L}}}=\frac{11^{2}}{4 \times 2.625}=11.524 \mathrm{~W}$


Fig. 2.145 (a) Equivalent circuit (b) Circuit to determine Thevenin resistance (c) Circuit to determine Thevenin voltage

Example 2.61


Fig. 2.146 Given network


Fig.2.147 Circuit to determine Thevenin resistance

Find the value of $R$ in the circuit of Figure 2.146 such that the maximum power transfer takes place.
(U.P.T.U. Sept. 2001)

## Solution:

The equivalent resistance of the network (Thevenin resistance) when viewed through the terminals A and B after removing the given resistor $R$ and replacing the voltage sources by their internal resistance (short circuited in this case as their internal resistance is zero), as shown in Figure 2.147.

$$
R_{\mathrm{th}}=[(2 \| 1)+3]\left\|2=\left[\frac{2 \times 1}{2+1}+3\right]\right\| 2=\frac{11}{3} \| 2
$$

$$
=\frac{\frac{11}{3} \times 2}{\frac{11}{3}+2}=\frac{22}{3} \times \frac{3}{17}=\frac{22}{17}=1.294 \Omega
$$

The power transferred will be maximum when the value of load resistance is equal to the total resistance of network.

$$
R=R_{\mathrm{th}}=1.294 \Omega
$$

## Example 2.62

In the circuit shown in Figure 2.148, find the condition for maximum power transfer to $R_{\mathrm{L}}$. Hence, determine the maximum power transferred.
(U.P.T.U. Tut.)


Fig. 2.149 (a) Equivalent circuit (b) Circuit to determine Thevenin resistance

## Solution:

The current source of 1 A in parallel with a resistance of $10 \Omega$ is converted into its equivalent voltage source of $1 \times 10=10 \mathrm{~V}$ in series with a resistance of $10 \Omega$. After removing $R_{\mathrm{L}}$, the circuit is redrawn as shown in Figure 2.149(a). Thevenin equivalent resistance of the network, when two voltage sources are short circuited and viewed from terminals A and B (see Fig. 2.149(b)) is given as

$$
\begin{aligned}
R_{\mathrm{th}} & =[\{(10 \| 10)+2\} \| 3]+5 \\
& =\left[\left\{\frac{10 \times 10}{10+10}+2\right\} \| 3\right]+5 \\
& =\left[\left(\frac{7 \times 3}{7+3}\right)+5\right]=7.1 \Omega
\end{aligned}
$$


(c)

Fig. 2.149 (c) Circuit to determine Thevenin voltage

Let the current distribution be as shown in the circuit diagram (see Fig. 2.149(c)).
By applying KVL to various meshes:
Mesh FGDEF: $10 I_{1}-10 I_{2}=5-10$
or

$$
\begin{equation*}
2 I_{1}-2 I_{2}=-1 \tag{2.82}
\end{equation*}
$$

Mesh DGHCD:

$$
\begin{align*}
10 I_{2}+2\left(I_{1}+I_{2}\right)+3\left(I_{1}+I_{2}\right) & =10 \\
I_{1}+3 I_{2} & =2 \tag{2.83}
\end{align*}
$$

By solving Equations (2.82) and (2.83), we get

$$
I_{1}=0.125 \mathrm{~A} \text { and } I_{2}=0.625 \mathrm{~A}
$$

Open-circuit voltage across terminals AB , that is, Thevenin voltage,

$$
E_{\mathrm{th}}=3\left(I_{1}+I_{2}\right)=3 \times(0.125+0.625)=2.25 \mathrm{~V}
$$

The resistance required for transfer of maximum power,

$$
R_{\mathrm{L}}=R_{\mathrm{th}}=7.1 \Omega
$$

Maximum power transferred, $P_{\max }=\frac{E_{\mathrm{th}}^{2}}{4 R_{\mathrm{L}}}=\frac{(2.25)^{2}}{4 \times 7.1}$

$$
=0.178 \mathrm{~W}
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. State Thevenin's theorem.
2. State Norton's theorem.
3. State the most important application of Thevenin's theorem.
4. How Thevenin's equivalent can be converted into Norton's equivalent?
5. State maximum power transfer theorem.

## Test Questions

1. State and explain Thevenin's theorem.
2. State and explain Norton's theorem.
3. State and explain maximum power transfer theorem.
4. Show that when a machine (or device) transfers maximum power, its efficiency is only $50 \%$.

## Numericals

1. Using Thevenin's theorem, determine the current flowing through $10-\Omega$ resistor in the network shown in Figure 2.150. If this resistor is replaced by another $15-\Omega$ and then $20-\Omega$ resistor what will be the current flowing through them?
(Ans. 1.887 A; 1.527 A; 1.282 A)


Fig. 2.150 Given network
2. Using Norton's theorem, determine current in $7.5-\Omega$ resistor of the network shown in Figure 2.66 (see Test Your Learning 1).
(P.T.U.) (Ans. 0.671 A$)$

### 2.15 RECIPROCITY THEOREM

This theorem allows the transfer of a source from one branch in the circuit to another branch. It may be stated as in any linear bilateral network, if a source of emf $E$ acting in any branch ' X ' produces a current $I$ in any other branch ' Y ', then the same emf $E$ when shifted to branch Y would produce the same current $I$ in the first branch X. In fact, this theorem states that $E$ and $I$ are interchangeable. The ratio of $E / I$ is constant and is known as transfer resistance.
Note: While shifting the source of emf $E$ from one branch to the other, its internal resistance is not shifted. Internal resistance of the source has to be kept in its original branch.

## Example 2.63

For the two circuits shown in Figure 2.151(a) and 2.151(b), calculate the current $A$ and draw the conclusion.

## Solution:

Short circuiting the terminals CD, the equivalent resistance of the network is, as shown in Figure 2.151(c),


Fig. 2.151 (a) Given network (b) Given network (c) Equivalent circuit (d) Equivalent circuit

$$
R_{\mathrm{eq}}^{\prime}=5+(10 \| 8)=5+\frac{10 \times 8}{10+8}=\frac{85}{9} \Omega
$$

Current supplied by battery, $I_{1}=\frac{25}{85 / 9}=\frac{45}{17} \mathrm{~A}$
Ammeter current, $I^{\prime}=I_{1} \times \frac{10}{10+8}=\frac{45}{17} \times \frac{10}{18}=\frac{25}{17} \mathrm{~A}$

Short circuiting the terminals EF, the equivalent resistance of the network is, as shown in Figure 2.151(d),

$$
R_{\mathrm{eq}}^{\prime \prime}=8+(5 \| 10)=8+\frac{5 \times 10}{5+10}=\frac{34}{3} \Omega
$$

Current supplied by battery, $I_{2}=\frac{25}{34 / 3}=\frac{75}{34} \mathrm{~A}$
Ammeter current, $I^{\prime \prime}=I_{2} \times \frac{10}{10+5}=\frac{75}{34} \times \frac{10}{15}=\frac{25}{17} \mathrm{~A}$
The results signify the reciprocity theorem.

## Example 2.64



Fig. 2.152 Given network


Fig. 2.153 Equivalent circuit

Verify the reciprocity theorem for the network shown in Figure 2.152.

## Solution:

Refer Figure 2.152.
Total circuit resistance $=8+4+\frac{24(6+2)}{24+6+2}=18 \Omega$
Current supplied by the source $=\frac{144}{18}=8 \mathrm{~A}$
Current in $2-\Omega$ resistor, $I=8 \times \frac{24}{32}=6 \mathrm{~A}$
When source of emf is transferred to branch $A B$ as shown in Figure 2.153, then

Total circuit resistance $=2+6+\frac{24(8+4)}{24+8+4}=16 \Omega$
Current supplied by the source $=\frac{144}{16}=9 \mathrm{~A}$
Current is $8-\Omega$ resistor, $I=9 \times \frac{24}{36}=6 \mathrm{~A}$
This shows that $E$ and $I$ are interchangeable that verifies the reciprocity theorem.
Transfer resistance $=\frac{E}{I}=\frac{144}{6}=24 \Omega$

## SUMMARY

1. Active elements: The elements that supply energy in an electric network are called active elements.
2. Passive elements: The elements that receive electrical energy and dispose the same in their own way of disposal are called passive elements.
3. Unilateral elements: The elements that conduct only in one direction, such as semi-conductor diode, are called unilateral elements.
4. Bilateral elements: The elements that conduct in both the directions similarly, such as a simple piece of wire (resistor), diac, and triac, are called bilateral elements.
5. Unilateral circuit: An electric circuit whose properties or characteristics change with the direction of its operation is called unilateral circuit.
6. Bilateral circuit: An electric circuit that possesses the same properties or characteristics in either direction is called bilateral circuit.
7. Voltage and current sources: To operate electrical or electronic circuits, a source of electrical power is required. The basic purpose of a source is to supply power to a load.
8. Ideal voltage source: A voltage source that has zero internal resistance is called an ideal voltage source.
9. Real voltage source: Voltage source that has very low internal resistance (or impedance) as compared to load resistance (or impedance) is known as a constant voltage source.
10. Ideal current source: A voltage source that supplies a constant current no matter what is the load resistance (or impedance) is known as ideal current source.
11. Real current source: A source that has very high internal resistance (or impedance) as compared to the load resistance (or impedance) is considered as a constant current source.
12. Kirchhoff's laws: KCL states that $\sum I=0$ at a junction. KVL states that $\sum E+\sum I R=0$ in a loop. While applying KCL, incoming currents are taken as positive and outgoing currents as negative.
13. Wheatstone bridge: At balance, $X / R=P / Q ; P / Q$ are the ratio arms.
14. Maxwell's mesh current method: In this case, independent mesh currents are taken and network is solved by framing equations according to KVL.
15. Nodal analysis: In this method, independent nodes are considered and voltages are assumed at these anodes w.r.t. one reference anode. Then, equations are framed according to KCL that reveal the required result after their solution.
16. Delta-star transformation: $R_{\mathrm{A}}=R_{\mathrm{AB}} R_{\mathrm{CA}} /\left(R_{\mathrm{AB}}+R_{\mathrm{BC}}+R_{\mathrm{CA}}\right)$
17. Star-delta transformation: $R_{\mathrm{AB}}=R_{\mathrm{A}}+R_{\mathrm{B}}+\left(R_{\mathrm{A}}+R_{\mathrm{B}} / R_{\mathrm{C}}\right)$
18. Superposition theorem: In this method, if there are two or more sources, current is determined in the required branch by considering each source separately and then resultant value is determined by superimposing them.
19. Thevenin's theorem: According to this theorem, to determine current in a resistor, the resistor is removed and an open-circuit voltage across the two terminals is determined called Thevenin voltage $E_{\mathrm{th}}$. Then, resistance of the whole network is determined across the terminals called Thevenin resistance $R_{\mathrm{th}}$ replacing all the voltage sources by their internal resistances, $I=E_{\mathrm{th}} /\left(R_{\mathrm{th}}+R_{\mathrm{L}}\right)$.
20. Norton's theorem: It is similar to Thevenin's theorem, but in this case, the network is reduced to a current source having a current output of $I_{\mathrm{N}}$ when the given terminals are short circuited in parallel with a resistance $R_{\mathrm{N}}$ similar to $R_{\mathrm{th}}$. Then, $I=I_{\mathrm{N}} R_{\mathrm{N}} /\left(R_{\mathrm{N}}+R_{\mathrm{L}}\right)$.
21. Maximum power transfer theorem: According to this theorem, the power output will be maximum when $R_{\mathrm{L}}=R_{\mathrm{th}}$ or $R_{\mathrm{L}}=R_{\mathrm{i}}$.
22. Reciprocity theorem: This theorem states that if a source of emf $E$ acting in any branch ' $X$ ' produces a current $I$ in any other branch ' Y '. Then, the same emf $E$ when shifted to branch Y would produce the same current $I$ in the first branch X , that is, $E$ and $I$ are interchangeable.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. According to KCL, at any junction of an electrical network, the sum of $\qquad$ currents $=$ sum of outgoing currents.
2. The voltage drop in a resistor will be taken as $\qquad$ if we are tracing the circuit in the direction of flow of current.
3. Three resistors each of $R \Omega$ are connected in delta. If they are transformed into star connections, the resistance of each resistor will be $\qquad$ $\Omega$.
4. To obtain maximum output power from a source, the load resistance must be equal to the $\qquad$ of the source.

## OBJECTIVE TYPE QUESTIONS

1. Kirchhoff's first law states that at a junction in an electric network
(a) $\Sigma E=0$
(b) $\Sigma I=0$
(c) $\sum V=0$
(d) $\Sigma E+\sum V=0$
2. KVL states that in a closed circuit of an electric network,
(a) $\Sigma E=0$
(b) $\Sigma I=0$
(c) $\sum V=0$
(d) $\Sigma \mathrm{E}+\sum V=0$
3. An ideal voltage source should have
(a) zero internal resistance
(b) infinite internal resistance
(c) large value of emf
(d) low value of current
4. The polarity of the voltage drop across a resistor is determined by
(a) value of resistor
(b) value of current
(c) direction of current
(d) All (a), (b), and (c)
5. An ideal current source should have
(a) zero internal resistance
(b) infinite internal resistance
(c) large value of emf
(d) None of the above
6. Thevenin's equivalent circuit can be used to calculate the power loss in the original circuit.
(a) True
(b) False
7. An electrical network with six independent nodes will have
(a) 10 loop equations
(b) 3 loop equations
(c) 5 loop equations
(d) 7 loop equations
8. While determining $R_{\mathrm{th}}$ of a network
(a) the voltage and current source should be short circuited.
(b) all current and voltage sources must be open circuited.
(c) the voltage sources should be open circuit and current sources should be short circuited.
(d) all sources should be replaced by their source resistance.
9. Wheatstone bridge is used to measure
(a) voltage
(b) resistance
(c) current
(d) power
10. A network that does not have either voltage sources or current sources is called
(a) active network
(b) passive network
(c) resistive network
(d) dummy network
11. In the circuit shown in Figure 2.154, according to Kirchhoff's first law, which of the following relation is correct?
(a) $I_{1}+I_{2}=I_{3}$
(b) $I_{1}+I_{3}=I_{2}$
(c) $I_{1}-I_{2}=I_{3}$
(d) None of the above


Fig. 2.154 Given network
12. A voltage divider and its Thevenin's equivalent circuit is shown in Figure 2.155. What will be the value of $E_{\mathrm{th}}$ and $R_{\mathrm{th}}$ ?


Fig. 2.155 Given network
(a) $8 \mathrm{~V}, 48 \Omega$
(b) $4 \mathrm{~V}, 48 \Omega$
(c) $4 \mathrm{~V}, 24 \Omega$
(d) $8 \mathrm{~V}, 96 \Omega$
13. A Norton's equivalent is
(a) parallel circuit
(b) series circuit
(c) series-parallel circuit
(d) None of the above
14. If an electrical network with one or more voltage and current source is transformed into equivalent electrical network with a single voltage source with internal resistance of the network with all sources replaced by their internal resistance, then this statement is called
(a) Norton's theorem
(b) Reciprocity theorem
(c) Thevenin's theorem
(d) Superposition theorem
15. The three resistors each of $R \Omega$ are connected in star. When they are transformed into delta connections, the resistance of each arm will be
(a) $2 R \Omega$
(b) $3 R \Omega$
(c) $4 R \Omega$
(d) $R / 2 \Omega$
16. In the circuit shown in Figure 2.156, the power consumed in $2-\Omega$ resistor connected across A-B is


Fig. 2.156 Given network
(a) 16 W
(b) 0 W
(c) 32 W
(d) 8 W
(AMIE; Model Question Papers)
17. When the given battery is connected across terminals $C$ and $D$ in the circuit shown in Figure 2.157, the current flowing through $6-\Omega$ resistor connected across terminals $\mathrm{A}-\mathrm{B}$ will be


Fig. 2.157 Given network
(a) 2 A
(b) 3 A
(c) 4 A
(d) 6 A
18. When a source is delivering maximum power to the load, the efficiency will be
(a) maximum
(b) below $50 \%$
(c) above $59 \%$
(d) $50 \%$
19. A resistor of $5 \Omega$ is connected in one branch of a complex network. The current in this branch is 5 A . If this $5-\Omega$ resistor is replaced by $10-\Omega$ resistor, the current in this branch will be
(a) 10 A
(b) 2.5 A
(c) 5 A
(d) less than 5 A
20. The current flowing through $6-\Omega$ resistor connected across terminal $B$ and $D$ shown in Figure 2.167 will be
(a) 2 A
(b) 1 A
(c) 3 A
(d) 4 A

## 曲 NUMERICALS

1. In the circuit shown in Figure 2.158, determine the current supplied by the DC generator of 140 V .
(Ans. 4 A)


Fig. 2. 158 Given network
2. By using Kirchhoff's laws, determine current in $X Y$ in the circuit shown in Figure 2.159. (P.T.U.)
(Ans. 40 A )


Fig. 2.159 Given network
3. Determine current supplied by the source in the network shown in Figure 2.160 using Maxwell's mesh current method.
(U.PT.U.) (Ans. 72 mA )


Fig. 2. 160 Given network
4. Using method of nodal analysis, determine the current in various resistors of the network shown in Figure 2.161.
(Ans. $18 \mathrm{~A}, 8 \mathrm{~A}, 2 \mathrm{~A}, 4 \mathrm{~A}, 4 \mathrm{~A}$ )


Fig. 2.161 Given network
5. Using superposition theorem, determine potential difference across $120-\Omega$ resistor in the network shown in Figure 2.162.
(Ans. 87.3 V)


Fig. 2.162 Given network
6. Using delta-star transformation, determine current in $60-\Omega$ resistor of the network shown in Figure 2.163.


Fig. 2. 163 Given network
7. Using star-delta transformation, determine the resistance between terminals $A$ and $B$ of the network shown in Figure 2.164.
(Ans. $64.71 \Omega$ )


Fig. 2. 164 Given network
8. Using Thevenin's theorem, find the current in section XY of distribution network shown in Figure 2.165.
(P.T.U.) (Ans. 60 A)


Fig. 2.165 Given network
9. Determine current in branch BD of a Wheatstone bridge given in Question 2 of Practise Exercises-2.1 by using Thevenin's theorem.
(Ans. 5.357 A from D to B)
10. Using Norton's theorem, determine current in $10-\Omega$ resistor of the network shown in Figure 2.150 .
(Ans. 1.887 A )


Fig. 2. 166 Given network
11. Find the current in $20-\Omega$ resistor in the network shown in Figure 2.166 . If 360 V generator is removed from the branch ACB and a battery of 45 V is introduced in branch BDA, determine the current in $40-\Omega$ resistor using reciprocity theorem.
(Ans. 0.5 A )


Fig. 2.167 Given network
12. Find the current in $30-\Omega$ resistor connected across AB in the circuit shown in Figure 2.167 using:
(i) Maxwell's mesh current method
(ii) nodal voltage method
(iii) superposition theorem
(iv) Kirchhoff's laws.
(Ans. 1.2 A)

## VIVA VOCE OR REASONING QUESTIONS

1. In a network, loop and mesh are different, how?
2. Wheatstone bridge is called a bridge, why?
3. Power transformed by a source is maximum when $R_{\mathrm{L}}=R_{\mathrm{i}}$, but efficiency is as low as $50 \%$, why?
4. Algebraic additions are different to numerical additions, how?
5. To solve the network, various theorems are studied, why?

## SHORT ANSWER TYPE QUESTIONS

1. What do you mean by active and passive elements?
2. What do you mean by constant voltage source? Name some of them.
3. What do you mean by a constant current source? How it differs from constant voltage source?
4. Give the symbolic notation of voltage source and current source.
5. Distinguish between node and junction.
6. State Kirchhoff's current and voltage laws.
7. If $R_{12}, R_{23}$, and $R_{31}$, are the three resistors connected in delta, what will be the effective value of three resistors $R_{1}, R_{2}$, and $R_{3}$, respectively, when connected in star according to delta-star transformation.
8. State superposition theorem.
9. State Thevenin's theorem.
10. State Norton's theorem.
11. State maximum power transfer theorem.
12. State reciprocity theorem.

## TEST QUESTIONS

1. Distinguish between
(i) active and passive elements
(ii) node and junction
(iii) loop and mesh
2. State and explain Kirchhoff's current and voltage law.
(U.P.T.U. Tut.)
3. While applying KVL to a loop, how signs are applied to the emf and voltage drop?
4. Describe the working of a Wheatstone bridge.
5. What steps are taken while solving a network by Maxwell's mesh current method?
6. Explain the nodal voltage method for solving a network. How are the nodal equations written?
7. State and explain superposition theorem. How is it applied for solving a network?
8. State Thevenin's theorem. Illustrate the application of the theorem with reference to an appropriate electric network.
(P.T.U.)
9. State and explain Norton's theorem. Show that this theorem is just the converse of Thevenin's theorem.
(P.T.U.)
10. State the maximum power transfer theorem. Show that for maximum power transfer $R_{\mathrm{L}}=R_{\text {th }}$ and explain its importance.
(U.P.T.U. 2003-04)
11. Derive the necessary equations for converting a delta network into an equivalent star network.
12. When a star connected network is transformed to delta connected network, show that the resistance of an arm of delta is equal to the sum of star resistances connected across that arm and product of the same two resistances divided by the third.
(U.P.T.U. 2003-04)
13. Discuss the reciprocity theorem.
14. Write short notes on the following:
(i) superposition theorem
(ii) Thevenin's theorem
(iii) Norton's theorem
(iv) maximum power transfer theorem
(v) star-delta transformation

## ANSWERS

Fill in the Blanks

1. incoming
2. negative
3. $R / 3$
4. internal resistance
5. three

Objective Type Questions

1. (b)
2. (d)
3. (a)
4. (c)
5. (b)
6. (a)
7. (c)
8. (d)
9. (b)
10. (b)
11. (c)
12. (a)
13. (a)
14. (c)
15. (b)
16. (b)
17. (a)
18. (d)
19. (d)
20. (b)


## LFARNING OBJFGTIVES

After the completion of this chapter, the students or readers will be able to understand:
*What is electrostatics and how Coulomb's laws related to it?

* How terms such as, electric field, electric density, electric intensity at a point, potential, and potential difference are related to electrostatics?
* What is Gauss theorem of electrostatics?
* How to determine electric potential at a point due to charged sphere?
* What is potential gradient and breakdown potential?
* What is a capacitor and how it is charged and discharged?
- What are various types of capacitors?
* What is the necessity of grouping of capacitors?
* How electric energy is stored in a capacitor?


### 3.1 INTRODUCTION

The branch of engineering or science that deals with the static (i.e., stationary) charges is called electrostatics.

When a glass rod is rubbed with a silk cloth, the former attains positive charge and the latter attains negative charge of equal magnitude. Since both the glass rod and the silk cloth are insulators, they retain these charges. These charges are known as static charges and the process comes under the general heading of electrostatics. These charged bodies are considered to be point charges when they are very small as compared to the distance between them. It is the same process due to which huge charges ${ }^{1}$ are developed over the clouds. Consequently, a potential difference of millions of volts is developed across the clouds having opposite charges. The static
charges can be stored in a device called capacitor. In this chapter, we shall deal with various aspects of electrostatics and electric capacitors.

### 3.2 COULOMB'S LAWS OF ELECTROSTATICS

Charles Coulomb, a French scientist, performed a number of experiments to see the effect of placing small charges near each other. From his experimental observations, he drew some conclusions and summed them up into two laws, known as Coulomb's Laws of Electrostatics:

1. First law relates the nature of force acting on the two charged bodies when placed near each other. It may be stated as like charges repel each other, whereas unlike charges attract each other.
2. Second law tells us about the magnitude of the force exerted between two bodies when placed near to each other. It may be stated as the force exerted between two ${ }^{1}$ point charges:
(a) is directly proportional to the product of their strength;
(b) is inversely proportional to the square of the distance between them;
(c) depends upon the nature of medium in which the charges are placed.

Mathematically,


Fig. 3.1 Charges kept in a medium

$$
\begin{equation*}
F \propto \frac{Q_{1} Q_{2}}{d^{2}} \quad \text { or } \quad F=\mathrm{k} \frac{Q_{1} Q_{2}}{d^{2}} \tag{3.1}
\end{equation*}
$$

where
$Q_{1}$ and $Q_{2}=$ strength of two point charges or strength of the charge on two bodies in coulombs (see Fig. 3.1)
$d=$ distance between the two charge points in metre, and
$\mathrm{k}=\mathrm{a}$ constant whose value depends upon the medium in which the charges are placed and the system of units employed. In SI units, the value of k is given by

$$
\mathrm{k}=\frac{1}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}}}
$$

where $\varepsilon_{0}=$ absolute permittivity of vacuum or air in SI units and its value is $8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$.
$\varepsilon_{\mathrm{r}}=$ relative permittivity of the medium in which the charges are placed. Its value depends upon the type of medium; for vacuum or air, its value is 1 .

Now, the equation may be written as

$$
\begin{equation*}
F=\frac{Q_{1} Q_{2}}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \tag{3.2}
\end{equation*}
$$

The unit of force is Newton.
Here,

$$
\begin{equation*}
\frac{1}{4 \pi \varepsilon_{0}}=\frac{1}{4 \pi \times 8.854 \times 10^{-12}}=9 \times 10^{9} \tag{3.3}
\end{equation*}
$$

${ }^{1}$ The charged bodies are considered to be point charges when they are very small as compared to the distance between them.

$$
F=9 \times 10^{9} \frac{Q_{1} Q_{2}}{\varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} \text { (in a medium) }
$$

Therefore,

$$
\begin{equation*}
F=9 \times 10^{9} \frac{Q_{1} Q_{2}}{d^{2}} \mathrm{~N}(\text { in air }) \tag{3.4}
\end{equation*}
$$

### 3.2.1 Unit Charge

In Equation (3.4), if we assume $Q_{1}=Q_{2}=Q, d=1 \mathrm{~m}$, and $F=9 \times 10^{9} \mathrm{~N}$, then

$$
9 \times 10^{9}=9 \times 10^{9} \frac{Q^{2}}{(1)^{2}}
$$

or

$$
Q^{2}=1
$$

or

$$
Q= \pm 1 \mathrm{C}
$$

Hence, 1 coulomb is that charge which when placed in air (strictly vacuum) at a distance of 1 m form an equal and similar charge that repels it with a force of $9 \times 10^{9} \mathrm{~N}$.

### 3.3 ABSOLUTE AND RELATIVE PERMITTIVITY

While discussing electrostatic phenomenon, a certain property of the medium called permittivity plays an important role. In fact, permittivity is the property of a medium that affects the magnitude of force exerted between two point charges. The greater the permittivity of a medium placed between the charged bodies, the lesser the force between them.

The absolute (or actual) permittivity of air or vacuum $\varepsilon_{0}$ (Greek letter epsilon) is minimum and its value is $8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$. The value of absolute (or actual) permittivity $\varepsilon$ of all other insulating materials is more than $\varepsilon_{0}$ and the ratio between $\varepsilon$ and $\varepsilon_{0}$ is called the relative permittivity of that material and is denoted by $\varepsilon_{\mathrm{r}}$.

Therefore,

$$
\varepsilon_{\mathrm{r}}=\frac{\varepsilon}{\varepsilon_{0}}
$$

where $\varepsilon=$ absolute (or actual) permittivity of material;
$\varepsilon_{0}=$ absolute (or actual) permittivity of air or vacuum (i.e., $8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ );
$\varepsilon_{\mathrm{r}}=$ relative permittivity of material;


## Example 3.1

If 108 electrons are added to a body, determine the charge on the body.

## Solution:

Charge on the body, $Q=n e$,
where

$$
\begin{aligned}
n & =10^{8} \\
e & =1.6 \times 10^{-19} \mathrm{C} \\
Q & =10^{8} \times 1.6 \times 10^{-19}=1.6 \times 10^{-11} \mathrm{C}
\end{aligned}
$$

## Example 3.2

How many electrons are shifted to charge a body to 5 C ?

## Solution:

or

$$
\begin{aligned}
& Q=n e \\
& n=\frac{Q}{e}=\frac{5}{1.6 \times 10^{-19}}=3.125 \times 10^{19} \text { electrons }
\end{aligned}
$$

## Example 3.3

One-coulomb point charge is placed at a distance of 1 m from an equal but opposite charge in air. Find the force acting on the charge and whether it is a force of attraction or repulsion?

## Solution:

Here, $Q=1 \mathrm{C}, Q_{2}=-1 \mathrm{C}$, and $d=1 \mathrm{~m}$

$$
F=\frac{Q_{1} Q_{2}}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}}=\frac{1 \times(-1)}{4 \pi \times 8.854 \times 10^{-12} \times(1)^{2}}=-9 \times 10^{9} \mathrm{~N}
$$

The negative sign indicates that it is a force of attractor.

## Example 3.4

Find the force of interaction between two charges spaced 10 cm apart in a vacuum. The charges are $8 \times 10^{-8}$ and $6 \times 10^{-5}$ coulomb, respectively. If the same charges are separated by the same distance in kerosene ( $\varepsilon_{\mathrm{r}}=2$ ), what is the corresponding force of interaction?

## Solution:

Charge $Q_{1}=8 \times 10^{-8} \mathrm{C}$; charge $Q_{2}=6 \times 10^{-5} \mathrm{C}$
Distance between the two, $d=10 \mathrm{~cm}=0.1 \mathrm{~m}$

$$
\text { Electrostatic force } F=\frac{Q_{1} Q_{2}}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N}
$$

When charges are placed in vacuum, $\varepsilon_{\mathrm{r}}=1$

$$
\therefore \quad F=\frac{8 \times 10^{-8} \times 6 \times 10^{-5}}{4 \pi \times 8.854 \times 10^{-12} \times 0.1 \times 0.1}=4.314 \mathrm{~N}
$$

When charges are placed in kerosene, $\varepsilon_{\mathrm{r}}=2$

$$
F=\frac{4.314}{2}=2.157 \mathrm{~N}
$$

## Example 3.5

A small sphere is given a charge of $40 \mu \mathrm{C}$ and a second sphere of equal diameter is given a charge of $-10 \mu \mathrm{C}$. The two spheres are allowed to touch each other and they are spaced 5 cm apart. What force exists between them? Here, air is assumed as the medium.

## Solution:

Charge on one sphere, $Q_{1}=40 \mu \mathrm{C}$; charge on second sphere, $Q_{2}=-10 \mu \mathrm{C}$
When the two sphere are connected together, the total charge on the two spheres can be given as

$$
Q_{1}+Q_{2}=40+(-10)=30 \mu \mathrm{C}
$$

When they are separated, charge of each sphere is $Q_{1}=Q_{2}=\frac{30}{2}=15 \mu \mathrm{C}$
Therefore, force between the two, $F=9 \times 10^{9} \times \frac{Q_{1} Q_{2}}{d^{2}}$

$$
\begin{aligned}
& =\frac{9 \times 10^{9} \times 15 \times 10^{-6} \times 15 \times 10^{-6}}{(0.05)^{2}} \\
& =810 \mathrm{~N} \text { repulsive }
\end{aligned}
$$

## Example 3.6

Determine the force of attraction between the electron and nucleus of the hydrogen atom, which are spaced $5.28 \times 10^{-11} \mathrm{~m}$ apart. The hydrogen atom possesses one electron and the nucleus has a charge equal but opposite in sign to that of the electron. The charge on the electron is 1.603 $\times 10^{-19} \mathrm{C}$.

## Solution:

Charges on an electron, $Q_{1}=1.603 \times 10^{-19} \mathrm{C}$ (negative)
Charges on the nucleus, $Q_{2}=1.603 \times 10^{-19} \mathrm{C}$ (positive)
Distance between the two, $d=5.28 \times 10^{-11} \mathrm{~m}$
Force of attraction between the electron and the nucleus,

$$
\begin{aligned}
F & =\frac{Q_{1} Q_{2}}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \quad\left(\varepsilon_{\mathrm{r}}=1\right) \\
F & =\frac{1.603 \times 10^{-19} \times 1.603 \times 10^{-19}}{4 \pi \times 8.854 \times 10^{-12} \times 1 \times\left(5.28 \times 10^{-11}\right)^{2}} \\
& =8.284 \mathrm{~N}
\end{aligned}
$$

## Example 3.7

If the magnitude of two charges is doubled and the distance between them is also doubled, what will the effect on the force acting on them?

Solution:
We know $F \propto \frac{Q_{1} Q_{2}}{d^{2}}$ and after change $F^{\prime} \propto \frac{Q_{1}^{\prime} Q_{2}^{\prime}}{\left(d^{\prime}\right)^{2}}$
where

$$
\begin{aligned}
Q_{1}^{\prime} & =2 Q ; \\
Q_{2}^{\prime} & =2 Q_{2} ; \\
d^{\prime} & =2 d
\end{aligned}
$$

Therefore,

$$
F^{\prime} \propto \frac{2 Q_{1} \times 2 Q_{2}}{(2 d)^{2}} \propto \frac{Q_{1} Q_{2}}{d^{2}}=F
$$

Hence, force acting on them remains the same.

## Example 3.8

Three point charges, each of +10 C , are placed at the vertices of an equilateral triangle that has sides 15 cm long. Find the force on each charge.


Fig. 3.2 Charges located at the vertices of an equilateral triangle

## Solution:

Three charges, each of $10 \mu \mathrm{C}$, placed at the vertices of an equilateral triangle are shown in Figure 3.2. Consider the charge placed at corner $B$. It is being repelled by charges placed at A and C along ABD and CBE , respectively. These two forces are equal in magnitude and are given as

$$
F=9 \times 10^{9} \times \frac{10 \times 10^{-6} \times 10 \times 10^{-6}}{\left(15 \times 10^{-2}\right)^{2}}=40 \mathrm{~N}
$$

Angle between the two forces $=60^{\circ}$
Therefore, resultant force $=2 F \cos 30^{\circ}$

$$
=2 \times 40 \times \frac{\sqrt{3}}{2}=69.28 \mathrm{~N}
$$

The force acting on the other two charges placed at the corners A and C will also be the same.

### 3.4 ELECTRIC FIELD

When a charged body is kept at some place, a region surrounding this body comes under stress and strain. According to Coulomb's Law, if a charge (positive or negative) is brought into this stressed region, a force of repulsion or attraction is experienced by it. This stressed region around a charged body is called electric field.

Thus, a region or space around a charged body in which a charge experiences a force of attraction or repulsion is called an electric field or electrostatic field.

If electric field is the force acting on a unit positive charge, then mathematically,

$$
\begin{equation*}
\vec{E}=\frac{\vec{F}}{q_{0}} \tag{3.5}
\end{equation*}
$$

SI unit of electric field is newton per coulomb (N/C)
Equation (3.5) can also be written as

$$
\begin{equation*}
\vec{F}=q_{0} \vec{E} \tag{3.6}
\end{equation*}
$$

Both the electric field $(\vec{E})$ and force $(\vec{F})$ are the vector quantities.

### 3.4.1 Electric Lines of Force ${ }^{2}$

The path traced by a unit positive charge when placed in an electric field is called electric line of force. The electric field around a charged body is represented by imaginary lines called electric lines of force (see Fig. 3.3). The direction of these lines of force at any point is determined by

[^1]

Fig. 3.3 Representation of electric lines of force
the direction along which a unit positive charge placed at that point would move or tend to move. According to this convention, the electric lines of force are supposed to originate at the surface of a positively charged body and terminate at the surface of a negatively charged body, as shown in Figure 3.4.


Fig. 3.4 Electric field produced by opposite charges

## Properties of electric lines of force

Following are the important properties of electric lines of force:

1. Electric lines of force emanate from the positively charged body and terminate at the negatively charged body (see Fig. 3.4).
2. Electric lines of force emanate or terminate on the charged body surface normally.
3. Two electric lines of force never intersect each other.
4. The electric lines of force in the same direction always repel each other (see Fig. 3.5), whereas the electric lines of force in opposite


Fig. 3.5 Electric field produced by similar charges


Fig. 3.6 Effect of conducting body when placed in an electric field
direction attract each other. This is property due to which similar charges repel each other and the dissimilar charges attract each other.
5. The tendency of electric lines of force is to take an easy path.

When a hollow cylinder of some conducting material is placed near a charged body, as shown in Figure 3.6, the electric lines of force try to pass through the conducting material and not through the hollow space. It is because conduction material provides an easy path for the electric lines of force.


Fig. 3.7 Electric flux produced by two parallel plates.

### 3.5 ELECTRIC FLUX

It has already been seen that a positively charged body emanates electric lines of force, whereas these lines of force are terminated at the negatively charged body. The total electric lines of force emanated from a positive charge is called electric flux.

Electric flux, in fact, is a measure of the overall size of electric field. It is measured in coulomb $(\mathrm{C})$ and is generally represented by $\psi(\mathrm{psi}$ a Greek letter). Two parallel plates separated by some dielectric medium are shown in Figure 3.7. When one plate is charged with $+Q$ coulomb and the other with $-Q$ coulomb, an electric field is established between them. This field is distributed uniformly. The total electric lines of force or electric flux established between them will be $\psi=Q$ coulomb.

### 3.6 ELECTRIC FLUX DENSITY (D)

The electric flux crossing per unit area at a given section in an electric field is known as electric flux density at that section. It is generally represented by letter $\sigma$ (or $D$ ).

Mathematically,

$$
\text { Electric flux density } \sigma=\frac{\psi}{A} \mathrm{C} / \mathrm{m}^{2}
$$

where $\psi=$ electric flux in coulombs passing normally through an area $A \mathrm{~m}^{2}$. The unit of electric flux density is coulomb per square metre ( $\mathrm{C} / \mathrm{m}^{2}$ ).

### 3.7 ELECTRIC INTENSITY OR FIELD STRENGTH (E)

It has been already discussed that electric field is represented by electric lines of force. The strength of the electric field is different at different points and is called field strength or field intensity or electric intensity. The strength or intensity of an electric field at a point depends upon the concentration of electric lines of force at that point, that is, the point where the electric lines of force are spaced closer and the field strength is higher at that point, and vice versa.

Mathematically, field strength at a point in an electric field is determined by the force acting on a unit positive charge placed at that point. Obviously, this force decreases as we go away from a charged body and ultimately diminishes to zero. Hence, electric intensity or field strength at a point in an electric field is the force acting on a unit positive charge placed at that point.

$$
\text { Electric intensity at a point, } E=\frac{F}{+Q} \mathrm{~N} / \mathrm{C}
$$

where $Q=$ charge in coulomb placed at the given point.
$F=$ force in newton acting on $Q$ coulombs of charge.
Consider a body, charged at $+Q$ coulomb of charge. It is required to determine electric intensity at point $P$ at a distance of $d$ metre away from the charged body. Place a unit positive charges $(+1 \mathrm{C})$ at point $P$, as shown in Figure 3.8. Then, the force acting on this unit positive charge will be the electric intensity at


Fig. 3.8 Electric intensity at a point near a charged body that point.

$$
\begin{aligned}
E & =\frac{Q \times 1}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} / \mathrm{C} \\
& =9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} / \mathrm{C}(\text { in medium }) \\
& =9 \times 10^{9} \frac{Q}{d^{2}} \mathrm{~N} / \mathrm{C} \text { (in air) }
\end{aligned}
$$

The direction of electric intensity is determined by Coulomb's first law. In this case, it is radially away from the charged body. However, for negative charge (i.e., $-Q$ ), its direction would have been radially towards the charged body.

### 3.8 RELATION BETWEEN $\sigma$ AND E

Consider a charge of $+Q$ coulomb placed in a medium of relative permittivity $\varepsilon_{\mathrm{r}}$, as shown in Figure 3.9. To determine electric flux density at point $P$, consider an imaginary sphere passing through point $P$ having radius $d$.

The electric flux emanated by the charge in all directions $=Q$ coulomb

The surface area of imaginary sphere $=4 \pi d^{2}$
Therefore, flux density at point $P$,

$$
\sigma=\frac{\text { flux }}{\text { area }}=\frac{Q}{4 \pi d^{2}}
$$



Fig. 3.9 Electric flux density at a point near a charged body

Electric intensity at point $P$, that is, the force acting on a unit positive charge when placed at point $P$,
or

$$
E=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}}=\frac{Q}{4 \pi d^{2}} \times \frac{1}{\varepsilon_{0} \varepsilon_{\mathrm{r}}}=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}} \quad\left(\text { since } \sigma=\frac{Q}{4 \pi d^{2}}\right)
$$

$$
\sigma=\varepsilon_{0} \varepsilon_{\mathrm{r}} E
$$

Hence, flux density $(\sigma)$ at any point in an electric field is $\varepsilon_{0} \varepsilon_{\mathrm{r}}$ times the electric intensity (E) at that point.

## Example 3.9

A charge of $6 \mu \mathrm{C}$ placed in an electric field experiences a force of 0.24 N . What is the magnitude of electric intensity?

## Solution:

Charge $\mathrm{Q}=6 \mu \mathrm{C}=6 \times 10^{-6} \mathrm{C}$; force acting on the charge, $F=0.24 \mathrm{~N}$
Therefore, electric intensity, $E=\frac{F}{Q}=\frac{0.24}{6 \times 10^{-6}}=4 \times 104 \mathrm{~N} / \mathrm{C}$.

## Example 3.10

A charge of $0.5 \mu \mathrm{C}$ is placed in an electric field where intensity is $80 \times 105 \mathrm{~N} / \mathrm{C}$. What is the magnitude of force acting on the charge?

## Solution:

Force acting on the charge, $F=E Q$

$$
\begin{array}{ll}
\text { Here, } & E=80 \times 10^{5} \mathrm{~N} / \mathrm{C} \text { and } Q=0.5 \times 10^{-6} \mathrm{C} \\
\therefore & F=80 \times 10^{5} \times 0.5 \times 10^{-6}=4 \mathrm{~N}
\end{array}
$$

## Example 3.11

Two equal and opposite charges of magnitude $5 \times 10^{-6} \mathrm{C}$ are placed 20 cm apart. What is the magnitude and direction of electric intensity $(E)$ at a point mid-way between the charges? What force would act on a proton (charge $=+1.6 \times 10^{-9} \mathrm{C}$ ) placed there?

If the two charges are similar of given magnitude and placed 20 cm apart, what will be the electric intensity at mid-way and the force acting on a proton when placed there?

## Solution:

Let two equal and opposite charges $\left(5 \times 10^{-6} \mathrm{C}\right)$ separated by a distance of 20 cm (i.e., 0.2 m$)$ be placed at point $A$ and $B$, as shown in Figure 3.10. To determine the field intensity at mid-way point M, consider a unit positive charge placed at point M .


Fig. 3.10 Electric intensity at the mid-point of two charged bodies

Electric intensity at point M due to charge $+5 \times 10^{-6} \mathrm{C}$ is given as

$$
E_{1}=9 \times 10^{9} \times \frac{5 \times 10^{-6}}{(0.1)^{2}}
$$

$$
=4.5 \times 10^{6} \mathrm{~N} / \mathrm{C} \text { along } \mathrm{AM}
$$

Electric intensity at point M due to change $-5 \times 10^{-6} \mathrm{C}$ is

$$
\begin{aligned}
E_{2} & =9 \times 10^{9} \times \frac{5 \times 10^{-6}}{(0.1)^{2}} \\
& =4.5 \times 10^{6} \mathrm{~N} / \mathrm{C} \text { along } \mathrm{MB}
\end{aligned}
$$

Both the electric intensities are acting in the same direction.
Therefore, resistant intensity at point M,

$$
\begin{aligned}
E & =E_{1}+E_{2} \\
& =4.5 \times 10^{6}+4.5 \times 10^{6}=9 \times 10^{6} \mathrm{~N} / \mathrm{C} \text { along AB }
\end{aligned}
$$

When a proton carrying a charge of $+1.6 \times 10^{-19} \mathrm{C}$ is placed at point M , force acting on the proton can be written as

$$
\begin{aligned}
F & =E Q=9 \times 10^{6} \times 1.6 \times 10^{-19} \\
& =1.44 \times 10^{-12} \mathrm{~N} \text { along } \mathrm{AB}
\end{aligned}
$$

When the two charges are similar, that is, positive of given magnitude $\left(5 \times 10^{-6} \mathrm{C}\right)$, an equal and opposite force is exerted on a unit positive charge when placed at mid-way.

Therefore, resultant electric intensity at $\mathrm{M}=0$ and force acting on a proton $=E \times Q=0$

## PRACTICE EXERCISES

## Short Answer Questions

1. How will you define a unit charge?
2. Two identical metallic spheres are of equal mass. If one of them is charged to $+q$ coulomb and the other one is charged to $-q$ coulomb. What will be the effect on their masses?
3. Would electrons move from higher potential to lower potential or vice versa?
4. How will you distinguish between absolute and relative permittivity?
5. An electron and a proton are free to move when placed at a point in an electric field, will the electric force on them be equal? Will their acceleration be equal? Justify your answer.
6. Why electric lines of force never intersect each other?
7. Differentiate between electric field and electric lines of force.
8. State two properties of electric lines of force.

## Test Questions

1. State and explain Coulomb's laws of Electrostatics.
2. What is electric field? Describe its properties.
3. How will you differentiate between electric flux density and electric intensity? What is the relation between them?

## Numericals

1. If $10^{6}$ electrons are added to a body, what will be the total charge on the body?(Ans. $\left.1.6 \times 10^{-13} \mathrm{C}\right)$
2. A proton and an electron are placed $10^{-9} \mathrm{~m}$ apart in a free space. Compute the force between them.
(Ans. $23 \times 10^{-11} \mathrm{~N}$ )
3. Two similar charges each of 1 C are placed at a distance of 1 m in free space. Find the force acting on the charge and also state whether it is a force of repulsion or attraction. (Ans. $9 \times 10^{9} \mathrm{~N}$, repulsive)
4. When a body is charged to 1 C , a body is of deficit or excess of how many electrons?
(Ans. $6.25 \times 10^{18}$ electrons)
5. If the magnitude of two charges is doubled and the distance between them is reduced to half, what will be the Coulomb's force between them?
(Ans. increases to 16 times.)
6. A charge of $0.5 \mu \mathrm{C}$ is placed in an electric field of intensity $40 \times 10^{5} \mathrm{~N} / \mathrm{C}$. Determine the force acting on the charge.
(Ans. 2 N )

### 3.9 AREA VECTOR



Fig. 3.11 Representation of area vector

Although area is a scalar quantity and when its magnitude is multiplied by a unit vector $\hat{n}$ perpendicular to the area, it becomes the area vector. If $d S$ is small area and unit vector $\hat{n}$ perpendicular to area, then area vector is written as

$$
\begin{equation*}
\overrightarrow{d S}=\hat{n} d S \tag{3.7}
\end{equation*}
$$

The direction of area vector is always perpendicular to area, that is, along the direction of unit vector $\hat{n}$.

### 3.10 ELECTRIC FLUX THROUGH AN AREA



Fig. 3.12 Electric flux through an area

Electric flux through an area is defined as the number of electric lines of force passing perpendicularly through that area. It is a scalar quantity and is denoted by $\psi$.

Consider an area $S$ placed in an electric field $\vec{E}$ and $\overrightarrow{d S}$ is a small area vector element, then small flux $(d \psi)$ passing through small area $(d S)$ is given by

$$
\begin{align*}
d \psi & =\vec{E} \cdot \overrightarrow{d S}  \tag{3.8}\\
d \psi & =E \cdot d s \cos \theta \tag{3.9}
\end{align*}
$$

where $\theta$ is angle between $\vec{E}$ and $\overrightarrow{d S}$
When $\theta=0^{\circ}$, that is, $\vec{E} \| \overrightarrow{d S}$, flux will be maximum and when $\theta=90^{\circ}$, that is, $\vec{E} \perp \overrightarrow{d S}$, then the flux will be zero.
To find total flux through the whole closed surface $S$, we integrate Equations (3.8) or (3.9) as

$$
\psi=\int_{S} \vec{E} \cdot \overrightarrow{d S}=\int_{S} E d S \cos \theta
$$

The SI unit of electric flux is $\mathrm{Nm}^{2} \mathrm{C}^{-1}$ or $\mathrm{JmC}^{-1}$. Electric flux is a scalar quantity as it is dot product of two vector quantities $\vec{E}$ and $\overrightarrow{d S}$.

## 3.ll DIFFERENT WAYS OF CHARGE DISTRIBUTION

On a conductor or body, charge can be distributed in three ways:

### 3.11.1 Linear Charge Distribution

When the charge is uniformly distributed over a line (straight or circular), the distribution is called linear charge distribution. If a charge $Q$ is distributed over the length $l$ of the conductor, then its linear charge density $(\lambda)$ is given by the relation:

$$
\lambda=\frac{Q}{l}=\frac{\text { Charge }}{\text { Length }} \Rightarrow Q=\lambda l
$$

Hence, linear charge density $\lambda$ can be defined as the charge per unit length of conductor and its SI unit is $\mathrm{Cm}^{-1}$.

### 3.11.2 Surface Charge Distribution

When the charge is uniformly distributed over the surface of a body, the distribution is called surface charge distribution. If $Q$ is charge distributed over a surface area $S$, then its surface charge density $(\sigma)$ is given by the relation:

$$
\begin{aligned}
& \sigma=\frac{Q}{S}=\frac{\text { Charge }}{\text { Area }} \\
& Q=\sigma S
\end{aligned}
$$

Hence, surface charge density $(\sigma)$ is defined as the charge distributed per unit area of the surface and its SI unit is $\mathrm{Cm}^{-2}$.

### 3.11.3 Volume Charge Distribution

When the charge is distributed uniformly over the volume of a body, the distribution is called volume charge distribution. If $Q$ is charge distributed over volume $V$, then its volume charge density $(\rho)$ is given by the relation:

$$
\begin{aligned}
\rho=\frac{Q}{V} & =\frac{\text { Charge }}{\text { Volume }} \\
Q & =\rho V
\end{aligned}
$$

Hence, volume charge density $(\rho)$ is defined as the charge distributed per unit volume of the conductor and its SI unit in $\mathrm{Cm}^{-3}$.

### 3.12 GAUSS THEOREM OF ELECTROSTATICS

This law states that the total electric flux through any closed surface is always equal to $\frac{1}{\epsilon_{0}}$ times
the total charge enclosed by the surface. the total charge enclosed by the surface.

$$
\begin{equation*}
\psi=\int_{S} \vec{E} \cdot \overrightarrow{d S}=\frac{Q}{\epsilon_{0}} \tag{3.10}
\end{equation*}
$$

where $Q=$ total charge enclosed by the surface $S$.

### 3.12.1 Proof of Gauss Theorem



Fig. 3.13 Sphere having charge at its centre

Consider a positive charge $+Q$ coulomb placed at point $O$, as shown in Figure 3.13. Draw a surface of radius ' $d$ ' around charge $Q$. This surface is called as Gaussian surface. Let its total surface area be $S, d S$ be small area, $\hat{n}$ is unit vector so that $\overrightarrow{d S}$ is area vector, and $\vec{E}$ is electric field due to charge $Q$, as shown in Figure 3.13.

Electric field at point $P$ due to charge $Q$ at a distance $d$ is given as

$$
\begin{equation*}
\vec{E}=\frac{Q}{4 \pi \epsilon_{0} d^{2}} \hat{d} \tag{3.11}
\end{equation*}
$$

Small electric flux $d \psi$ through small area $d S$ is given as

$$
\begin{equation*}
d \psi=\vec{E} \cdot \overrightarrow{d S} \tag{3.12}
\end{equation*}
$$

Substituting the value of E in Equation (3.12), we get

$$
\begin{equation*}
d \psi=\frac{Q}{4 \pi \in_{0} d^{2}}(\hat{d} \cdot \overrightarrow{d S}) \tag{3.13}
\end{equation*}
$$

The direction of $\hat{d}$ and $\overrightarrow{d S}$ is same, and therefore, angle between them is $0^{\circ}$; hence,

$$
d \psi=\frac{Q d S}{4 \pi \in_{0} d^{2}}
$$

(magnitude of $\hat{d}=1$ and $\cos 0^{\circ}=1$ )
Total flux through whole surface $S$ is given as

$$
\begin{aligned}
& \psi=\int d \psi=\int \frac{Q d S}{4 \pi \in_{0} d^{2}} ; \quad \psi=\frac{Q}{4 \pi \in_{0} d^{2}} \int d S \\
& \psi=\frac{Q}{4 \pi \in_{0} d^{2}} \times 4 \pi d^{2}
\end{aligned}
$$

(Therefore, $\int d S=4 \pi d^{2}=$ surface area of sphere)
or

$$
\psi=\frac{Q}{\epsilon_{0}}
$$

This proves Gauss law in electrostatics.

### 3.13 DEDUCTION OF COULOMB'S LAW FROM GAUSS'S LAW

Consider two point charges $Q_{1}$ and $Q_{2}$ separated by a distance $d$, as shown in Figure 3.14. Let $\vec{E}$ be the electric field due to charge $+Q_{1}$ at point $P$ and $\overrightarrow{d S}$ be area vector that is parallel to $\vec{E}$, as shown in Figure 3.14.

Then,

$$
\vec{E} \cdot \overrightarrow{d S}=E d S \cos 0^{\circ}=E d S
$$

According to Gauss law,

$$
\begin{equation*}
\int \vec{E} \cdot \overrightarrow{d S}=\frac{Q_{1}}{\epsilon_{0}} \tag{3.14}
\end{equation*}
$$

Since $E$ is constant, the equation can be written as
or

$$
E \int d S=\frac{Q_{1}}{\epsilon_{0}}
$$

$$
E\left(4 \pi d^{2}\right)=\frac{Q_{1}}{\epsilon_{0}}
$$

$\left(\int d S=4 \pi d^{2}=\right.$ surface area of a sphere $)$


Fig. 3.14 Two point charges $Q_{1}$ and $\mathrm{O}_{2}$ separated by a distance d
or

$$
\begin{equation*}
E=\frac{Q_{1}}{4 \pi \in_{0} d^{2}} \tag{3.15}
\end{equation*}
$$

$$
\begin{equation*}
E=\frac{F}{Q_{2}} \tag{3.16}
\end{equation*}
$$

From Equations (3.15) and (3.16), we get

$$
\begin{equation*}
\frac{F}{Q_{2}}=\frac{Q_{1}}{4 \pi \epsilon_{0} d^{2}} \quad \text { or } \quad F=\frac{Q_{1} Q_{2}}{4 \pi \epsilon_{0} d^{2}} \tag{3.17}
\end{equation*}
$$

### 3.14 ELECTRIC INTENSITY DUE TO A CHARGED SPHERE

Consider a uniformly charged sphere at $Q$ coulomb is placed in a medium of relative permittivity $\varepsilon_{\mathrm{r}}$. It is desired to determine the electric intensity at various points due to this charged sphere. There can be two cases:

### 3.14.1 Point P Is Outside the Sphere

Let point $P$ be an external point at a distance of $d$ metre from the centre of the sphere. Consider a sphere passing through point $P$ concentric with the charged sphere, as shown in Figure 3.15.

Therefore, by Gauss theorem, electric flux crossing the sphere passing through point:

$$
\begin{equation*}
P=Q \text { coulomb } \tag{3.18}
\end{equation*}
$$

Further, electric flux crossing the sphere at $P$

$$
\begin{align*}
& =\text { flux density } \times \text { area }=D \times 4 \pi d^{2} \\
& =\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 4 \pi d^{2}\left(\text { since } D=\varepsilon_{0} \varepsilon_{\mathrm{r}} E\right) \tag{3.19}
\end{align*}
$$

where $E$ is the electric intensity at point $P$.
From Equations (3.18) and (3.19), we get


Fig. 3.15 Electric intensity at a point outside the charged sphere

$$
\begin{gathered}
\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 4 \pi d^{2}=Q \text { or } \\
E=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} / \mathrm{C}
\end{gathered}
$$

Thus, the electric intensity at a point outside the sphere is the same as if the charge on the sphere was concentrated at the centre of the sphere.

### 3.14.2 Point P Is Inside the Sphere

Let point $P$ be an internal point at a distance of $d_{1}$ metre from the centre of


Fig. 3.16 Electric intensity at a point inside a charged sphere the sphere. Consider a sphere passing through point $P$ concentric with the charged sphere, as shown in Figure 3.16.

According to Gauss theorem, electric flux crossing the sphere passing through point

$$
\begin{equation*}
P=0 \tag{3.20}
\end{equation*}
$$

Its value is zero, since there is no charge inside the dotted sphere.
Further, electric flux crossing the sphere at

$$
\begin{equation*}
P=\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 4 \pi d_{1}^{2} \tag{3.21}
\end{equation*}
$$

From Equations (3.20) and (3.21), we get

$$
\begin{aligned}
& E=0 \quad \varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 4 \pi d_{1}^{2}=0 \\
& E=0
\end{aligned}
$$

Thus, electric intensity at a point inside the charged sphere is always zero.

## Example 3.12

A hallow sphere is charged to $10 \mu \mathrm{C}$ and is placed in air. If the sphere radius is 15 cm , determine

1. flux emanated;
2. electric intensity at a distance of 10 cm from the centre of sphere.
3. electric intensity at a distance of 5 cm from the sphere surface.

What will be the effect on the abovementioned values if the charged sphere is placed in oil of relative permittivity 3 ?

## Solution:

Charge on the sphere, $Q=10 \mu \mathrm{C}=10 \times 10^{-6} \mathrm{C}$
Radius of sphere, $r=15 \mathrm{~cm}=0.15 \mathrm{~m}$

1. By Gauss theorem, flux emanated $\psi=Q=10 \mu \mathrm{C}$
2. Since the point is 10 cm away from the centre which is inside the sphere. Therefore, electric intensity at this point,

$$
E=0
$$

3. Distance of the point from the centre of the sphere,

$$
d=15+5=20 \mathrm{~cm}=0.2 \mathrm{~m}
$$

Therefore, electric intensity at this point, $E=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}}=9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}} d^{2}}$

$$
=9 \times 10^{9} \times \frac{10 \times 10^{-6}}{1 \times(0.2)^{2}}=2.25 \times 106 \mathrm{~N} / \mathrm{C}
$$

When the sphere is placed in oil of relative permittivity 3 (i.e., $\varepsilon_{r}=3$ ), the first two parts of the problem will not be affected. However, the electric intensity outside the sphere will change to

$$
E=9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}} d^{2}}=9 \times 10^{9} \times \frac{10 \times 10^{-6}}{3 \times(0.2)^{2}}=0.75 \times 106 \mathrm{~N} / \mathrm{C}
$$

## Example 3.13

A charge of $20 \mu \mathrm{C}$ is uniformly distributed on the surface of a hollow sphere of radius 10 cm . Find electric intensity at distance of

1. 5 cm from the centre of sphere
2. 20 cm from the centre of sphere

## Solution:

1. As the point is 5 cm from the centre and is inside the sphere, electric intensity is always zero.
Electric intensity at 5 cm from the centre $=0$
2. The point is 20 cm from the centre of sphere, $d=20 \mathrm{~cm}=0.2 \mathrm{~m}$

Electric intensity,

$$
\begin{aligned}
E & =\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}}, \\
\varepsilon_{\mathrm{r}} & =1 \\
& =9 \times 10^{9} \times \frac{20 \times 10^{-6}}{(0.2)^{2}}=45 \times 105 \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

where

### 3.15 ELECTRIC INTENSITY DUE TO A LONG CHARGED CONDUCTOR

Consider a long straight conductor carrying a charge of $Q$ coulombs per metre length, as shown in Figure 3.17. It is desired to determine the electric intensity at point $P$ at a distance of $d$ metre outside the conductor. Consider a concentric (dotted) cylinder of length $l$ metre passing through point $P$.

Charge inside the dotted cylinder $=Q l$ coulomb The dotted cylinder is enclosing a charge of Ql coulomb, and therefore, according to Gauss theorem,
flux crossing the dotted cylinder $=Q l$ coulomb


Fig. 3.17 Electric intensity at a point due to a long charged conductor

Further, flux crossing the dotted cylinder $=$ flux density $\times$ surface area of dotted cylinder.

$$
\begin{align*}
& =\sigma \times 2 \pi d l \\
& =\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 2 \pi d l \tag{3.23}
\end{align*}
$$

From Equations (3.22) and (3.23), we get

$$
\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 2 \pi d l=Q l \quad \text { or } \quad E=\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d} \mathrm{~N} / \mathrm{C}
$$

If point $P$ is inside the charged conductor, the dotted cylinder passing through that point will not enclose any charge. Hence, $\varepsilon_{0} \varepsilon_{\mathrm{r}} E \times 2 \pi d l=0$ or $E=0$, that is, electric intensity at such a point is zero.

### 3.16 ELECTRIC POTENTIAL

We know that when a body is raised above the ground level, it possesses mechanical potential because of the gravitational pull of the earth. The amount of potential energy gained by the body is equal to the amount of work done in raising the body to that point against the gravitational pull. The greater the height to which the body is raised, the greater will be its potential energy. Thus, the potential energy of a body will be zero on the earth's surface. ${ }^{3}$

Every charge (or charged body) has electric field that theoretically extends up to infinity. Figure 3.18 shows an isolated charge of $+Q$


Fig. 3.18 Electric potential at a point due to a charged body coulombs fixed in space. The force exerted on a unit positive charge placed at infinity is zero. ${ }^{4}$ When the unit positive charge is moved towards $+Q$ charge fixed in space, a work is done against the repulsive force acting on the unit positive charge (a force of repulsion acts between the like charges). Hence, the unit positive charge attains some electric potential energy (commonly known as electric potential) when it is moved from $\infty$ to point $P$. The greater the distance moved by the unit positive charge from infinity towards the $+Q$ charge, the greater will be the electric potential attained by it. In other words, the closer the point $P$ to the charge $(+Q)$, the higher will be electric potential at that point. Obviously, in an electric field, infinity is chosen as the point of zero potential. ${ }^{5}$

Hence, the amount of work done in bringing a unit positive charge (i.e., +1 C ) from infinity to a given point in an electric field is called electric potential at that point.
Mathematically,

$$
\text { Electric potential }=\frac{\text { Work }}{\text { Charge }}=\frac{W}{Q} \text { Joule/coulomb }
$$

[^2]where $W=$ work done to bring a charge of $Q$ coulombs from infinity to the given point.
The SI unit of electric potential is volt. ${ }^{6}$ It may be defined as follows:
If one joule of work is done in bringing a unit positive charge from infinity to a given point in an electric field, the potential at the point is said to be one volt.

### 3.16.1 Potential at a Point

Consider a positive charge of $Q$ coulomb placed in a medium of relative permittivity $\varepsilon_{\mathrm{r}}$. It is desired to determine the electric potential at point $P$ due to this charge that is kept at a distance of $d$ metre from the charge, as shown in Figure 3.19.

Let a unit positive charge (i.e., +1 C) be placed at point A at a distance of $x$ metre from the given charge. Then, the force acting on this unit positive charge (or


Fig. 3.19 Electric potential at a point due to a charged body electric field intensity) will be

$$
F=E=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} x^{2}}
$$

If this unit positive charge at A is moved through a distance of $d x$ towards the charge, then work done

$$
d W=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} x^{2}} \times(-d x,(\text { since work }=\text { force } \times \text { distance })
$$

Total work done in bringing a unit positive charge from infinity to point $P$, which is at a distance of $d$ metre from the charge, will be

$$
W=\int_{\infty}^{d} \frac{-Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} 2^{2}} d x=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d} \text { joule }
$$

However, by definition, the work done in bringing a unit positive charge from infinity to a given point in the electric field of a charge is the potential at that point. Therefore, potential at point $P$

$$
V_{\mathrm{p}}=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d} \text { volt }=9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}} d} \text { volt }(\text { in medium })=9 \times 10^{9} \frac{Q}{d} \text { volt (in air) }
$$

From the expression, it is clear that the potential at a point varies inversely proportional to distance $d$ from the point charge. As the distance from the charge decreases, potential increases.

### 3.16.2 Potential at a Point Due to Number of Charges

Since potential is a scalar quantity, it possesses magnitude only. The potential at a point due to number of charges is equal to the algebraic sum of the potential due to each charge at that point.

[^3]Thus, to determine potential at a point due to a number of charges:

1. Calculate the potential (with sign) at the given point due to each charge as if other charges were not present.
2. Then, take the algebraic sum of all these potentials.

### 3.17 ELECTRIC POTENTIAL DIFFERENCE



Fig. 3.20 Potential difference between ' A ’ and 'B'

The work done in moving a unit positive charge from the point of lower potential to the point of higher potential in an electric field is called the electric potential difference between the two points.

Consider two points A and B in the electric field of charge $+Q$ as shown in Figure 3.20. It is clear that point A is at higher potential than B . The work done in bringing a unit positive charge from infinity to point B is, say, $V_{1}$ joules. Then, potential at point B ,

$$
V_{\mathrm{B}}=V_{1}
$$

If the extra work done to bring the unit positive charge from point B to A is $W$ joules. Then, potential at point A,

$$
V_{\mathrm{A}}=V_{\mathrm{B}}+W=V_{1}+W
$$

Therefore,

$$
V_{\mathrm{A}}-V_{\mathrm{B}}=\left(V_{1}+W\right)-V_{1} \quad \text { or } \quad V_{\mathrm{A}}-V_{\mathrm{B}}=W
$$

or $V_{\mathrm{A}}-V_{\mathrm{B}}=$ work done in moving a unit positive charge from point B to A .
Thus, it shows that the potential difference between two points is just the work done in moving a unit positive charge from a point of lower potential to a point of higher potential. The unit of potential difference is volt. If one joule of work is done in bringing a unit positive charge (i.e., +1 C ) from the point of lower potential to the point of higher potential, the potential difference between the points is said to be 1 V .


Fig. 3.21 Electric lines of force produced by a sphere

### 3.18 POTENTIAL DUE TO CHARGED SPHERE

Consider an isolated sphere of radius $r$ metre charged uniformly with $Q$ coulomb and is placed in medium of relative permittivity $\varepsilon_{\mathrm{r}}$. To determine potential due to this charged sphere, the following three cases can be considered:

### 3.18.1 Potential at the Sphere Surface

Figure 3.21 shows the electric lines of force produced by a charged sphere. The lines of force are uniformly distributed and it looks that as if they coming from a point charge placed at the
centre of the sphere. Thus, if a sphere is charged to $Q$ coulomb, then it can be supposed that point charge of $Q$ coulomb is placed at the centre of the sphere.
$\begin{aligned} \text { Therefore, potential at the sphere surface } & =\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} r} \text { volt } \\ & =9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}} r} \mathrm{~V}(\text { in medium })=9 \times 10^{9} \frac{Q}{r} \mathrm{~V} \text { (in air) }\end{aligned}$

### 3.18.2 Potential Inside the Sphere

The electric flux, in fact, is emanated from the surface of the sphere, and there is no electric flux inside the sphere. Therefore, electric intensity inside the sphere is zero.

Now,
or

$$
\begin{aligned}
\text { Electric intensity } & =\frac{\text { Change in potential }}{r} \\
0 & =\text { Change in potential }
\end{aligned}
$$

Hence, there is no change in potential inside the sphere as compared to the potential at the surface of the sphere. In other words, all the points inside the sphere are at the same potential as the points on its surface.

### 3.18.3 Potential Outside the Sphere

Consider a point $P$ outside the sphere at a distance of $d$ metre from its surface, as shown in Figure 3.22.

Potential at point $P=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}}(d+r)}$ volts

$$
=9 \times 10^{9} \frac{Q}{\varepsilon_{\mathrm{r}}(d+r)} \mathrm{V}=9 \times 10^{9} \frac{Q}{(d+v)} \mathrm{V} \text { (in air) }
$$



Fig. 3.22 Electric potential at a point outside the charged sphere

## Example 3.14

In a closed surface, if the number of lines entering it is 30,000 and emerging out is 80,000 , find the resultant charge enclosed by it.

## Solution:

Electric lines of force entering the surface, $\phi_{\text {in }}=30,000$
Electric lines of force emerging out of it, $\phi_{\text {out }}=80,000$
Net flux, $\phi=\phi_{\text {out }}-\phi_{\text {in }}=80,000-30,000=50,000$
Resultant charge, $Q=\varepsilon_{0} \times \phi=8.854 \times 10^{-12} \times 50,000$

$$
=44.27 \times 10^{-8} \mathrm{C}
$$

## Example 3.15

A surface has an area $\vec{S}=200 \hat{k}$. Find the electric flux crossing through it if

$$
\vec{E}=2 \hat{i}+5 \hat{i}+8 \hat{k}
$$

## Solution:

Here,

$$
\vec{E}=2 \hat{i}+5 \hat{j}+8 \hat{k} \quad \text { and } \quad \vec{S}=200 \hat{k}
$$

Electric flux,

$$
\begin{aligned}
\phi & =\vec{E} \cdot \vec{S}=(2 \hat{i}+5 \hat{i}+8 \hat{k}) \times(200 \hat{k}) \\
& =2 \times 100(\hat{i} \times \hat{k})+5 \times 200(\hat{i} \cdot \hat{k})+8 \times 200(\hat{k} \cdot \hat{k}) \\
& =0+0+1600=1600 \mathrm{Nm}^{2} \mathrm{C}^{-1}
\end{aligned}
$$

(since $\hat{i} \cdot \hat{k}=\hat{j} \cdot \hat{k}=0 ; \hat{k} \cdot \hat{k}=1$ )

## Example 3.16

A cube of side 5 cm contains a charge of $106.25 \mu \mathrm{C}$ located at its centre. Determine the total flux emanated through the cube and the flux emanated by each of its surface.

## Solution:

Electric flux produced by the charge $=$ total flux emanated through the cube

$$
=\frac{Q}{\varepsilon_{0}}=\frac{106.25 \times 10^{-6}}{8.854 \times 10^{-12}}=12 \times 10^{6} \mathrm{Nm}^{2} \mathrm{C}^{-1}
$$

A cube has six similar surfaces. Therefore, electric flux emanated by each surface

$$
=\frac{12 \times 10^{6}}{6}=2 \times 10^{6} \mathrm{Nm}^{2} \mathrm{C}^{-1}
$$

## Example 3.17

A charge of $3,000 \mu \mathrm{C}$ is uniformly distributed over a sheet of surface area $500 \mathrm{~m}^{2}$. What will be the electric field intensity at a distance of 25 cm from the sheet?

## Solution:

Surface density of charge, $\sigma=\frac{3000 \times 10^{-6}}{500}=6 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}$
Electric intensity at a point due to a charged plane sheet,

$$
\mathrm{E}=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}}=\frac{6 \times 10^{-6}}{8.854 \times 10^{-12} \times 1}=6.76 \times 10^{4} \mathrm{~N} / \mathrm{C}
$$

## Example 3.18

The electric field at a point 25 cm away from a point charge is $4 \mathrm{~N} / \mathrm{C}$. Determine the magnitude of the point charge.

## Solution:

$$
\begin{aligned}
& E=\frac{Q}{4 \pi \varepsilon_{0} d^{2}}=9 \times 10^{9} \times \frac{Q}{d^{2}} \\
& Q=\frac{E d^{2}}{9 \times 10^{9}}=\frac{4 \times(0.25)^{2}}{9 \times 10^{9}}=2.78 \times 10^{-11} \mathrm{C}
\end{aligned}
$$

## Example 3.19

Find the electric potential at the surface of the nucleus of a hydrogen atom having diameter $10^{-13} \mathrm{~m}$ when the electron has been removed from the atom.

## Solution:

Charge at the surface, $Q=n e=1 \times 1.6 \times 10^{-19} \mathrm{C}$
Radius of the nucleus, $r=\frac{d}{2}=\frac{10^{-13}}{2}=0.5 \times 10^{-13} \mathrm{~m}$
Electric potential at the surface, $V=9 \times 10^{9} \times \frac{Q}{r}$

$$
=9 \times 10^{9} \times \frac{1.6 \times 10^{-19}}{0.5 \times 10^{-13}}=28.8 \times 10^{3} \mathrm{~V}
$$

## Example 3.20

Find the electric potential at the surface of an atomic nucleus having $Z=50$ and radius $r=9 \times 10^{-15} \mathrm{~m}$.

## Solution:

Charge at the surface, $Q=n e=50 \times 1.6 \times 10^{-19}=80 \times 10^{-19} \mathrm{C}$
Electric potential, $V=\frac{Q}{4 \pi \varepsilon_{0} r}=9 \times 10^{9} \times \frac{80 \times 10^{-19}}{9 \times 10^{-15}}=8 \times 10^{6} \mathrm{~V}$

## Example 3.21

An electric potential of 200 V is obtained at a point which is 10 cm away from an isolated positive point charge. Determine the magnitude of point charge.

## Solution:

Here, $V=200 \mathrm{~V} ; d=10 \mathrm{~cm}=0.1 \mathrm{~m}$
We know electric potential, $V=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d}=9 \times 10^{9} \times \frac{Q}{d} \quad$ where $\varepsilon_{\mathrm{r}}=1$
Therefore,

$$
Q=\frac{V \times d}{9 \times 10^{9}}=\frac{200 \times 0.1}{9 \times 10^{9}}=2.2 \times 10^{-9} \mathrm{C}
$$

## Example 3.22

A long straight conductor has a charge of $0.02 \mu \mathrm{C} / \mathrm{m}$ length. What will be the electric intensity at a point 10 cm from the conductor?

## Solution:

Electric intensity due to long, charged straight conductor,

$$
E=\frac{Q}{2 \pi d \varepsilon_{0} \varepsilon_{\mathrm{r}}} \mathrm{~N} / \mathrm{C}
$$

where $Q=0.02 \mu \mathrm{C}=0.02 \times 10^{-6} \mathrm{C} ; d=10 \mathrm{~cm}=0.1 \mathrm{~m}$
Therefore,

$$
E=\frac{0.02 \times 10^{-6}}{2 \pi \times 0.1 \times 8.854 \times 10^{-12} \times 1}=3,600 \mathrm{~N} / \mathrm{C}
$$

## Example 3.23



Fig. 3.23 Point charges placed at the comers of a square

Potential at point $P$

Here,

Four charges are placed in air at the corners of a square, each side of the square is 1 m , as shown in Figure 3.23.

$$
\begin{aligned}
& q_{1}=1.0 \times 10^{-8} \mathrm{C} ; q_{2}=-2.0 \times 10^{-8} \mathrm{C} \\
& q_{3}=3.0 \times 10^{-8} \mathrm{C} ; q_{4}=2.0 \times 10^{-8} \mathrm{C}
\end{aligned}
$$

Find the potential at the centre of the square.

## Solution:

Distance of each charge from the centre point $P$ of the square

$$
\begin{aligned}
d & =\frac{\sqrt{(1)^{2}+(1)^{2}}}{2}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}} \mathrm{~m} \\
& =\sum 9 \times 10^{9} \times \frac{Q}{d} \\
& =9 \times 10^{9}\left[\frac{q_{1}}{d_{1}}+\frac{q_{2}}{d_{2}}+\frac{q_{3}}{d_{3}}+\frac{q_{4}}{d_{4}}\right]
\end{aligned}
$$

$$
d_{1}=d_{2}=d_{3}=d_{4}=d=\frac{1}{\sqrt{2}}
$$

Therefore, potential at point $P$

$$
\begin{aligned}
& =9 \times 10^{9} \times \sqrt{2}\left[1 \times 10^{-8}-2 \times 10^{-8}+3 \times 10^{-8}+2 \times 10^{-8}\right] \\
& =9 \times 10^{9} \times \sqrt{2} \times 10^{-8} \times 4 \\
& =509.12 \mathrm{~V}
\end{aligned}
$$

### 3.19 POTENTIAL GRADIENT

The change in potential per unit distance in an electric field is called potential gradient.
Let $V_{\mathrm{A}}=$ Potential at point A in an electric field


Fig. 3.24 Potential gradient between point A and B
$V_{\mathrm{B}}=$ Potential at point B in an electric field $S=$ Distance between the two points If $V_{\mathrm{A}}>V_{\mathrm{B}}$

$$
\text { Potential gradient }=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{S}
$$

Obviously, the unit of potential gradient is volt $/ \mathrm{m}$.
Consider a charge of $+Q$ coulomb and let there be two points A and B situated $S$ metre apart in its electric field, as shown in Figure 3.24. Since point A is nearer to the charge, the potential at point A will be more than that at point $B$. If distance
$S$ is small, then electric intensity E will be approximately same in this small distance. In other words, the force exerted on a unit positive charge will be $E$ newton when placed anywhere between A and B. If a unit positive charge is moved from B to A, some work will be done, which is given as

$$
\text { Work done }=E \times S \text { joule }
$$

However, by definition, work done in bringing unit positive charge from B to A is the potential difference $\left(V_{\mathrm{A}}-V_{\mathrm{B}}\right)$ between points A and B .

$$
\begin{aligned}
& \text { Therefore, } \\
& \text { or } \begin{array}{rl}
E \times S & =V_{\mathrm{A}}-V_{\mathrm{B}} \\
\text { or } & E=\frac{V_{A}-V_{B}}{S}=\text { Potential gradient } \\
\text { In differential form, }{ }^{7} & E
\end{array} \quad-\frac{d V}{d S}
\end{aligned}
$$

Hence, electric intensity at a point is numerically equal to potential gradient at that point. The unit of electric intensity will be V/m. However, it can be shown that $1 \mathrm{~V} / \mathrm{m}=1 \mathrm{~N} / \mathrm{C}$, that is,

$$
1 \mathrm{~V} / \mathrm{m}=\frac{1 \text { joule } / \text { coulomb }}{\text { metre }}=\frac{1 \text { newton } \times \text { metre }}{\text { metre } \times \text { coulomb }}=1 \mathrm{~N} / \mathrm{C}
$$

### 3.20 BREAKDOWN POTENTIAL OR DIELECTRIC STRENGTH

When a potential difference is applied across an insulating material or dielectric, no current flows since the valance electrons of such a material are tightly bound and no free electrons are available for current conduction. However, when the voltage applied across the dielectric is gradually increased, the medium comes under stress and strain and ultimately a point is reached when the electrons are torn away. Then, a large current (much larger than usual leakage current) flows in the form of a spark or arc through the dielectric and the medium is ruptured. ${ }^{8}$

The maximum voltage at which a unit thickness of dielectric can withstand without being punctured by the spark is called its breakdown potential or dielectric strength of the dielectric.

The dielectric strength is generally measured in $\mathrm{kV} / \mathrm{cm}$ or $\mathrm{kV} / \mathrm{mm}$. The students should not be confused with dielectric constant and dielectric strength. The dielectric constant is just the relative permittivity of the insulating material, whereas the dielectric strength is the breakdown potential of the insulating material.

The dielectric constant and dielectric strength of some of the common insulating materials or dielectrics are given in Table 3.1.

[^4]Table 3.1 Dielectric Constant and Dielectric Strength of Some of the Common Dielectrics

| S. No. | Dielectric | Dielectric Constant $\left(\varepsilon_{\mathrm{r}}\right)$ | Dielectric Strength in $\mathbf{~ k V / c m}$ |
| :--- | :--- | :--- | :---: |
| 1. | Air | 1 | 30 |
| 2. | Impregnated paper | 2 | 400 |
| 3. | Paraffin | 2.25 | 350 |
| 4. | Mica | 6 | 500 |
| 5. | Glass | 8 | 1,000 |

## Example 3.24

A parallel-plate capacitor has plates 0.15 mm apart, a plate area of $1,000 \mathrm{~cm}^{2}$ and a dielectric with a relative permittivity of 3 . Find the electric flux density, electric intensity, and the voltage between the plates. If the capacitor has a charge of $0.5 \mu \mathrm{C}$.

## Solution:

Distance between the plates, $d=0.15 \mathrm{~mm}=1.5 \times 10^{-4} \mathrm{~m}$
Area of plates, $A=1,000 \mathrm{~cm}^{2}=0.1 \mathrm{~m}^{2}$; relative permittivity, $\varepsilon_{\mathrm{r}}=3$
Charge on plate, $Q=0.5 \mu \mathrm{C}=0.5 \times 10^{-6} \mathrm{C}$
Therefore, electric flux density, $\sigma=\frac{\psi}{A}=\frac{Q}{A}=\frac{0.5 \times 10^{-6}}{0.1}=5 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}$
Electric field intensity, $E=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}}=\frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3}=1.88 \times 10^{5} \mathrm{~V} / \mathrm{m}$
Voltage between the plates, $V=E \times d$

$$
=1.88 \times 10^{5} \times 1.5 \times 10^{-4}=28.2 \mathrm{~V}
$$

## Example 3.25

In a parallel-plate capacitor with solid dielectric, the plates are 0.015 cm apart and when charged, the surface charge density is $10^{-9}$ coulomb $/ \mathrm{cm}^{2}$. Calculate the electric flux density. If the specific inductive capacity of the solid dielectric is 6 , calculate the potential difference between the plates.

## Solution:

Distance between the plates, $d=0.015 \mathrm{~cm}=1.5 \times 10^{-4} \mathrm{~m}$; surface charge density, $\sigma=10^{-9} \mathrm{C} / \mathrm{cm}^{2}=10^{-5} \mathrm{C} / \mathrm{m}^{2}$; specific inductive capacity, $\varepsilon_{\mathrm{r}}=6$
Now, electric flux density, $\quad \sigma=\frac{\psi}{A}=\frac{Q}{A}=\sigma=10^{-5} \mathrm{C} / \mathrm{m}^{2}$

$$
\text { Further, } \quad \sigma=\varepsilon_{0} \varepsilon_{\mathrm{r}} E \quad \text { or } \quad E=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}}
$$

Potential difference between the charged plates,

$$
\begin{aligned}
V & =E \times d=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}} \times d=\frac{10^{-5}}{8.854 \times 10^{-12} \times 6} \times 1.5 \times 10^{-4} \\
& =28.238 \mathrm{~V}
\end{aligned}
$$

## 目改 PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by area vector?
2. State Gauss law of electrostatics.
3. Define electric potential.
4. Can a metal sphere of radius 1 cm hold a charge of 1 C ?
5. Two equipotential surfaces never intersect, justify.
6. The electric field at the outer surface of a hollow charged sphere is always normal to the surface, why?
7. Can the whole charge of a conductor be transferred to another isolated conductor? If yes, how?
8. There is a hollow sphere and a solid sphere of same radius that can hold more charge, justify your answer.

## Test Questions

1. Define and explain Gauss theorem of electrolysis.
2. How will you determine the electric intensity due to charged sphere at a point (a) when it lies outside the sphere (b) when it lies inside the sphere?
3. Determine electric intensity at a point placed $d$ metre away from centre of a long current-carrying conductor.
4. Determine electric potential at a point outside a charged sphere placed at a distance of $d$ metre from the centre of sphere.
5. Define and explain breakdown potential.

## Numericals

1. If 20,000 electric lines of force are entering a surface and 50,000 are emerging out of it, find the total charge enclosed by the surface.
(Ans. $0.266 \mu \mathrm{C}$ )
2. A charge $79.68 \mu \mathrm{C}$ is located at the centre of a cube of side 4 cm . Find the total electric flux through cube and through each face of cube.
(Ans. $9 \times 10^{6} \mathrm{Nm}^{2} \mathrm{C}^{-1} ; 1.5 \times 10^{6} \mathrm{Nm}^{2} \mathrm{C}^{-1}$ )
3. A charge of $5,000 \mu \mathrm{C}$ is uniformly distributed over a sheet of surface area $500 \mathrm{~m}^{2}$. What will be the electric field intensity at a distance of 20 cm from the sheet?
(Ans. $1.13 \times 10^{6} \mathrm{~N} / \mathrm{C}$ )
4. The electric field at a point 20 cm away from a point charge is $9 \mathrm{~N} / \mathrm{C}$. What will be the magnitude of the point charge?
(Ans. $4 \times 10^{-11} \mathrm{C}$ )
5. What will be the electric potential at the surface of the nucleus of an hydrogen atom having diameter $10^{-13} \mathrm{~m}$ when the electron has been removed from the atom?
(Ans. 28.8 mega-volt)
6. Find the electric potential at the surface of an atomic nucleus having $Z=45$ and radius $r=8 \times 10^{-14}$ m.
(Ans. $81 \times 10^{4} \mathrm{~V}$ )
7. An electric potential of 500 V is obtained at a point, which is 9 cm from an isolated positive point charge. Find the magnitude of point charge.
(Ans. $5 \times 10^{-9} \mathrm{C}$ )
8. A charge of $50 \mu \mathrm{C}$ is uniformly distributed on the surface of a hollow sphere of radius 10 cm . Find electric intensity at a distance
(i) 5 cm from the centre of sphere and (ii) 5 cm away from the surface of sphere
(Ans. zero; $20 \times 10^{6} \mathrm{~N} / \mathrm{C}$ )
9. A long, straight conductor has a charge of $0.05 \mu \mathrm{C} / \mathrm{m}$ length. What will be the electric intensity at a point 20 cm from the conductor?
(Ans. 4,500 N/C)

### 3.21 CAPACITOR

Two surfaces separated by an insulating material or dielectric ${ }^{9}$ is called a capacitor or condenser.

Since this arrangement has the capacity to store electricity, it is named as capacitor. It is also known as condenser due to the fact that when potential difference is applied across the conducting plates, the electric lines of force are condensed in the small space


Fig. 3.25 Symbol of a capacitor between them. A capacitor is generally named after the dielectric used, that is, air capacitor, paper capacitor, mica capacitor, ceramic capacitor, etc. Depending upon the shapes of the conducting surfaces or plates, the capacitor may be in the form of parallel plates, concentric cylinders, spheres, etc. The symbol of a capacitor is shown in Figure 3.25.

### 3.21.1 Types of Capacitors

The capacitors are generally classified according to the dielectric used. The most commonly used capacitors are paper, mica, ceramic, electrolytic, and air capacitors. These may be either fixed or variable type.

## Paper capacitors

Paper capacitors are the most common of all capacitors. For the construction of these capacitors, two metal (aluminium or tin) foils separated by


Tubular capacitor
Fig. 3.26 Tubular type paper capacitor


Internal construction of mica capacitors.
Fig. 3.27 Internal construction of mica capacitors paper impregnated with a dielectric material such as wax, plastic, or oil are rolled into a compact cylinder. Connecting leads are attached to the two metal plates. The entire cylinder is generally placed in a cardboard container coated with wax or encased in plastic. A tubular-type paper capacitor is shown in Figure 3.26.

Paper capacitors are available in a wide range of capacitance values and voltage ratings. Typical capacitance values ranging from $0.0001 \mu \mathrm{~F}$ to $1.0 \mu \mathrm{~F}$ having tolerance of $\pm 10 \%$ usually with voltage ranging from 200 to 10,000 volts and more are available.

## Mica capacitor

These capacitors consist of alternate thin sheets of metal (aluminium or tin) foils separated by thin mica sheets, as shown in Figure 3.27. Alternate metal sheets are connected together and brought out as one terminal for one set of plates, while the opposite terminal connects to the

[^5]other set of plates. The entire unit is generally encased in a plastic housing or moulded in a Bakelite case. Mica capacitors are often used for small capacitance values ranging 50 to 500 pF with voltage ratings ranging from 200 to $1,000 \mathrm{~V}$.

## Ceramic capacitors

The ceramic is a dielectric material made from earth fired under extreme heat. Titanium oxide or several other types of silicates are used to obtain very high value of dielectric constant of ceramic material. The ceramic capacitors may be of disc type, as shown in Figure 3.28, or tabular type. These capacitors are also available in other shapes. Disc ceramic capacitors are commonly available in capacitance values ranging from 47 pF to $0.05 \mu \mathrm{~F}$, with voltage ratings ranging from 200 to $1,000 \mathrm{~V}$.

In tubular ceramic capacitors, the inner and outer surfaces of a hollow ceramic tube are coated with silver and form the two plates of the capacitor. These capacitors are available in capacitance values ranging from 1 to 500. Special low-voltage and high capacitance ceramic capacitors are also available for use in transistor circuits.


Fig. 3.28 Disc-type ceramic capacitor

## Electrolytic capacitors

An electrolytic capacitor contains two aluminium electrodes, as shown in Figure 3.29(a). Between the two electrodes, absorbent gauze soaks up electrolyte (borax, phosphate, or carbonate) to provide the required electrolysis that produces an oxide film (a molecular-thin layer of aluminium oxide) at the positive electrode when d.c. voltage is applied. The oxide film acts as an insulator and forms a capacitance between the positive aluminium electrode and the electrolyte in the gauze separator. The negative aluminium electrode simply provides a connection to the electrolyte. Usually, the metal itself acts as a negative terminal of the capacitor. Tubular electrolytic capacitors are shown in Figure 3.29(b).

Electrolytic capacitors have a high capacitance-to-size ratio since the aluminium oxide layer is molecular thin. These capacitors are available in capacitance values ranging from less than $1 \mu \mathrm{~F}$ to $10,000 \mu \mathrm{~F}$ or more. Common voltage ratings range from 1 to 700 volt.

These capacitors must be connected in the circuit as per polarity marked on the capacitor. If they are connected in opposite polarity, the reversed electrolysis forms gas in the capacitor. It becomes hot and may explode. Non-polarized electrolytic capacitors are also available that are applied in the starting winding of a.c. single-phase motors. Basically, a non-polarized electrolytic capacitor contains two capacitors, connected internally in series with opposing polarity. The capacitance is one-half of either capacitor, but the oxide film is maintained.


Fig. 3.29 Electrolytic capacitor (a) Internal constructional features of an electrolytic capacitor (b) Tubular capacitor

## Variable capacitors

The electronic circuits in which frequency is to be changed as per requirement such as tuning circuits, and hence, variable capacitors are used. Air-gang capacitor is the most common variable capacitor. These capacitors are available in single-, dual- and triple-section units. A dual-section unit of air-gang capacitor is shown in Figure 3.30.

Another type of variable capacitor is called trimmer (sometimes also called paddler). These capacitors are used in the electronic circuits where the variation of capacitance is not frequent. Once a setting is obtained to match the circuit, the capacitance is not to be changed after that. This capacitor consists of two or more metal (aluminium) sheets separated by sheets of mica or ceramic, as shown in Figure 3.30. The metal sheets are under spring tension so that as they are squeezed together by the tuning of an adjusting screw, their physical spacing and hence, trimmer capacitance is varied.


Fig. 3.30 Variable capacitor (a) Air gap variable capacitor (b) Mica variable capacitor

### 3.21.2 Capacitor Action

When a d.c. supply is connected across a capacitor, it stores charge. The manner in which it stores charge is illustrated in Figure 3.31. A parallel-plate capacitor having plates A and B is
connected across a battery of voltage $V$ volt through a key $K$.

1. When key $K$ is open as shown in Figure 3.31(a), the capacitor plates are neutral, that is, the plates are having no charge.
2. When key $K$ is closed as shown in Figure 3.31 (b), the electrons from plate A are attracted by the positive terminal of the battery and reach at plate $B$ through the battery. Thus, the electrons detached from plate A start piling ${ }^{10}$ up on plate B. As a result, plate A attains more and more positive charge and plate $B$ attains more and more negative charge. This process is called charging of capacitor.
3. This flow of electrons continues from


Fig. 3.31 Capacitor action plate A to B through battery till the capacitor is charged to $V$ volt (that is equal to the supply voltage). Once the capacitor is charged to $V$ volts, the flow of electrons ceases, as shown in Figure 3.31(c).
4. Now, an electrostatic field is established between the plates in the dielectric.

### 3.22 CAPACITANCE

The capability of a capacitor to store charge is called its capacitance. It has been previously explained that charge $Q$ stored by a capacitor is directly proportional to the potential difference $V$ applied across it.

$$
Q \propto V \quad \text { or } \quad \frac{Q}{V}=\text { constant }=\mathrm{C}
$$

where $\mathrm{C}=$ constant called capacitance of a capacitor.
Hence, the charge on the capacitor plates per unit potential difference across the plate is called the capacitance of the capacitor.

The unit of capacitor is farad (F). ${ }^{11}$
If $Q=1$ coulomb and $V=1$ volt, then $\mathrm{C}=1 \mathrm{~F}$
Hence, a capacitor is said to have a capacitance of 1 farad if a charge of 1 coulomb accumulates on each plate when a potential difference of 1 volt is applied across the plates. Farad is an extremely large unit of capacitance. Practically, the capacitors are manufactured having the capacitance of the order of microfarad $(\mu \mathrm{F})$ and micro-microfarad $(\mu \mu \mathrm{F})$ or picofarad $(\mathrm{pF})$

$$
1 \mu \mathrm{~F}=10^{-6} \mathrm{~F} \quad \text { and } \quad 1 \mu \mu \mathrm{~F}=1 \mathrm{pF}=10^{-12} \mathrm{~F}
$$

[^6]
### 3.22.1 Dielectric Constant or Relative Permittivity

A capacitor consists of two conducting surfaces (plates) separated by an insulating material called dielectric. When the capacitor is charged, the electrostatic field is established between the conducting plates. The concentration of electric lines of force increases with the presence of dielectric. Actually, the degree of concentration of electric lines of force between the plates depends upon the nature of dielectric.

The ability of a dielectric material to concentrate electric lines of force between plates of the capacitor is called dielectric constant or relative permittivity of the material. Air has been assigned a reference value of dielectric constant (or relative permittivity) as one. The dielectric constant of all other insulating material is more than one.

Let us assume
$V=$ potential difference applied across the capacitor;
$Q=$ charge on the capacitor plates with air as the insulating medium;
Then,

$$
C_{\mathrm{air}}=\frac{Q}{V}
$$

Now, if mica is used as an insulating medium in the same capacitor, it can hold a charge of $6 Q$ coulomb with the same voltage $V$.

Therefore,

$$
\begin{aligned}
& C_{\text {mica }}=\frac{6 Q}{V}=6 \frac{Q}{V}=6 C_{\text {air }} \\
& \frac{C_{\text {mica }}}{C_{\text {air }}}=6 \text { (i.e., dielectric constant of mica) }
\end{aligned}
$$

Hence, dielectric constant or relative permittivity of a material may be defined as a ratio of capacitance of capacitor with that material filling the space between the plates to the capacitance of the same capacitor having air as the filling material.

### 3.22.2 Capacitance of Parallel-plate Capacitor



Figure 3.32 Parallel plate capacitor
Although there are many forms of capacitor, the most important arrangement is the paral-lel-plate capacitor, as shown in Figure 3.32(a). Consider a capacitor having two parallel plates of area $A \mathrm{~m}^{2}$ separated by a dielectric of thickness $d$ metre with relative permittivity $\varepsilon_{\mathrm{r}}$, as shown in Figure 3.32(b). When a potential difference of $V$ volt is applied across the plates, it establishes a charge of $+Q$ and $-Q$ coulomb on the two plates, respectively. Since the electric field between the plates is uniform, electric flux density between the plates, $\sigma=Q / A$ coulomb $/ \mathrm{m}^{2}$.

Electric intensity between the plates is

$$
E=\frac{V}{d} \mathrm{volt} / \mathrm{m}
$$

We know that $\sigma=\varepsilon_{0} \varepsilon_{\mathrm{r}} E$

Therefore,

$$
\frac{Q}{A}=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} V}{d}
$$

or

$$
\frac{Q}{V}=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}
$$

Now, the charge per unit potential difference (i.e., $Q / V$ ) is the capacitance of the capacitor.

$$
\begin{aligned}
C & =\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d} \text { farads (in medium) } \\
& =\frac{\varepsilon_{0} A}{d} \text { farads (in air) }
\end{aligned}
$$

### 3.22.3 Factors Affecting Capacitance

The capacitance (that is, ability to store charge) of a capacitor depends upon the following factors:

1. Area of plates: The larger the area of capacitor plates, the greater is the capacitance of the capacitor and vice versa. This is because the larger plates can hold greater charge for a given potential difference that increases the capacitance of the capacitor.
2. Thickness of dielectric: The capacitance of a capacitor is inversely proportional to the thickness of the dielectric, that is, distance between the plates. In other words, the smaller the thickness of dielectric, the greater the capacitance and vice versa. This is because the electrostatic field is intensified when the plates are brought closer, and hence increases the capacitance.
3. Relative permittivity of dielectric: Capacitance of a capacitor depends upon the type of insulating material (or dielectric) placed between the plates. The greater the value of relative permittivity of the insulating material, the greater will be the capacitance of the capacitor and vice versa. It is because the insulating materials with higher dielectric constant allow more electric lines of force to establish between the plates that increase the capacitance of the capacitor.

### 3.22.4 Dielectric and Its Effect on Capacitance

The insulating material or electrolyte filled between the conducting plates of a capacitor is called dielectric. The dielectric may be air, paper, mica, electrolyte, ceramic, etc.

## Effect of dielectric on capacitance of a capacitor

The capacitance of a capacitor depends upon the permittivity of the dielectric used. The larger the permittivity, the larger the capacitance. This is very clear from the following mathematical relation $C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$ or $C \propto \varepsilon_{\mathrm{r}}$

### 3.23 PARALLEL-PLATE CAPACITOR WITH COMPOSITE MEDIUM

Consider a parallel-plate capacitor consisting of two plates each of area $A \mathrm{~m}^{2}$ separated by the dielectric of thickness $d_{1}, d_{2}$, and $d_{3}$ metre and relative permittivity of $\varepsilon_{\mathrm{r} 1}, \varepsilon_{\mathrm{r} 2}$, and $\varepsilon_{\mathrm{r} 3}$, respectively, as shown in Figure 3.33. The charge on each plate of the capac-


Fig. 3.33 Parallel plate capacitor with composite medium itor is $Q$ coulomb.

The electric flux density $D$ in all the three dielectrics remain the same and is equal to $Q / A$ coulomb per square metre. However, the electric intensities in the three dielectrics will be different and are given as

$$
E_{1}=\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 1}} ; E_{2}=\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 2}} ; E_{3}=\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 3}}
$$

Now, the potential difference across the capacitor $V$ is the sum of the potential difference across the three dielectrics, respectively, that is,

$$
\begin{aligned}
V & =V_{1}+V_{2}+V_{3} \\
& =\mathrm{E}_{1} d_{1}+\mathrm{E}_{2} d_{2}+\mathrm{E}_{3} d_{3} \\
& =\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 1}} d_{1}+\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 2}} d_{2}+\frac{D}{\varepsilon_{0} \varepsilon_{\mathrm{r} 3}} d_{3}
\end{aligned}
$$

or

$$
=\frac{D}{\varepsilon_{0}}\left(\frac{d_{1}}{\varepsilon_{r 1}}+\frac{d_{2}}{\varepsilon_{r 2}}+\frac{d_{3}}{\varepsilon_{r 3}}\right)=\frac{Q}{\varepsilon_{0} A}\left(\frac{d_{1}}{\varepsilon_{r 1}}+\frac{d_{2}}{\varepsilon_{r 2}}+\frac{d_{3}}{\varepsilon_{r 3}}\right)
$$

or

$$
\frac{Q}{V}=\frac{\varepsilon_{0} A}{\left(\frac{d_{1}}{\varepsilon_{r 1}}+\frac{d_{2}}{\varepsilon_{r 2}}+\frac{d_{3}}{\varepsilon_{r 3}}\right)}
$$

Therefore,

$$
C=\frac{\varepsilon_{0} A}{\left(\frac{d_{1}}{\varepsilon_{r 1}}+\frac{d_{2}}{\varepsilon_{r 2}}+\frac{d_{3}}{\varepsilon_{r 3}}\right)} \text { farad. }
$$

In general,

$$
C=\frac{\varepsilon_{0} A}{\sum \frac{d}{\varepsilon_{r}}} \mathrm{farad}
$$

Other cases are as follows.

### 3.23.1 Medium Partly Air

A parallel-plate capacitor having medium partly air between the plates is shown in Figure 3.34. Let the two plates of the capacitor be $d$ metre apart and a dielectric of thickness $t$ metre having relative permittivity of $\varepsilon_{\mathrm{r} 2}$ be introduced between them.

Thickness of air $=d-t$
Using the relation, $C=\frac{\varepsilon_{0} A}{\frac{d-t}{1}+\frac{t}{\varepsilon_{r 2}}}=\frac{\varepsilon_{0} A}{d-\left(t-\frac{t}{\varepsilon_{r 2}}\right)}$ farad

### 3.23.2 Slab of Dielectric Is Introduced

When a slab of some dielectric of thickness $t$ metres having relative permittivity $\varepsilon_{\mathrm{r} 2}$ is introduced between the parallel-plate air capacitor, as shown in Figure 3.35, where the plates of the capacitor are $d$ metre apart, this will have the same capacitance as in the above-mentioned expression:

$$
C=\frac{\varepsilon_{0} A}{d-\left(t-\frac{t}{\varepsilon_{r 2}}\right)}
$$

### 3.24 MULTI-PLATE CAPACITORS

A large value of capacitance of a parallel-plate capacitor can be achieved by using larger plate area. However, the larger plate area may increases the size of the capacitor enormously. Therefore, to obtain a larger area of plate surface without using too large capacitor, multiplate construction is employed.

Figure 3.36 shows a simple schematic view of a multi-plate ( 9 plate) capacitor. The odd-numbered metal plates are connected to one terminal $T_{1}$ and the even-numbered metal plates are connected to another terminal $T_{2}$. This arrangement (a capacitor having 9 plates) is equivalent to 8 capacitors in parallel.

Therefore, the total capacitance of $n$-plate capacitor, $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{r} A}{d}$
Mostly, multi-plate capacitors are variable capacitors, as shown in Figure 3.37. In these capacitors, there are two sets to plates: stationary and movable. By changing the position of movable plates, the mesh area between the movable plates and the fixed plates is changed, which in turn changes the capacitance of the capacitor. The capacitance of such variable capacitors can be changed from 0 to about $4,000 \mathrm{pF}$.


Fig. 3.36 Multi-plate capacitor


Fig. 3.37 Multiplate (gang) capacitor

## Example 3.26

What will be the capacitance of a capacitor that contains a charge of 0.05 coulomb when a potential difference of 500 V is applied across it?

## Solution:

Here,

$$
\begin{aligned}
& Q=0.05 \mathrm{C} ; V=500 \mathrm{~V} \\
& C=\frac{Q}{V}=\frac{0.05}{500}=100 \mu \mathrm{~F}
\end{aligned}
$$

## Example 3.27

Two metal plates each of area $25 \mathrm{~m}^{2}$ are separated by a dielectric of thickness 1 mm having relative permittivity of 5 . Calculate the capacitance between the plates.

## Solution:

Here, $A=25 \mathrm{~m}^{2} ; d=1 \mathrm{~mm}=1 \times 10^{-3} \mathrm{~m} ; \varepsilon_{\mathrm{r}}=5$

$$
C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}=\frac{8.854 \times 10^{-12} \times 5 \times 25}{1 \times 10^{-3}}=1.106 \mu \mathrm{~F}
$$

## Example 3.28

A parallel-plate capacitor is made of circular plates of radius 10 cm that are separated through a distance of 1 mm . If 120 V is applied across the plates, what will be the capacitance of the capacitor and charge on the plates when medium between the plates is air?

## Solution:

Radius of each plate, $r=10 \mathrm{~cm}=0.1 \mathrm{~m}$

Distance between plates, $d=1 \mathrm{~mm}=1 \times 10^{-3} \mathrm{~m}$

$$
\begin{array}{r}
\text { Area of plates, } A=\pi r^{2}=\pi \times(0.1)^{2}=0.01 \times \pi m^{2} \\
C=\frac{\varepsilon_{0} \varepsilon_{r} A}{d}=\frac{8.854 \times 10^{-12} \times 1 \times 0.01 \times \pi}{1 \times 10^{-3}}=278 \mathrm{pF}
\end{array}
$$

Charge on the plates, $Q=C V=278 \times 10^{-12} \times 120=33.36 \times 10^{-9} \mathrm{C}$

## Example 3.29

A potential difference of 400 V is maintained across a capacitor of value $25 \mu \mathrm{~F}$. Calculate

1. The charge
2. The electric field strength
3. The electric flux density in dielectric, if distance between the plates of capacitor is 0.5 mm and area of the plates is $1.2 \mathrm{~m}^{2}$ (given $\varepsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ ).

## Solution:

Potential difference across the capacitor, $V=400 \mathrm{~V}$
Capacitance of the capacitor, $C=25 \mu \mathrm{~F}=22 \times 10^{-6} \mathrm{~F}$
Distance between the plates, $d=0.5 \mathrm{~mm}=5 \times 10^{-4} \mathrm{~m}$
Area of the plates, $A=1.2 \mathrm{~m}^{2}$

1. Charge on the plates, $Q=C V=25 \times 10^{-6} \times 400=0.01 \mathrm{C}$
2. Electric field strength, $E=\frac{V}{d}=\frac{400}{5 \times 10^{-4}}=8 \times 10^{5} \mathrm{~V} / \mathrm{m}$
3. Electric flux density, $\sigma=\frac{Q}{A}=\frac{0.01}{1.2}=8.33 \times 10^{-3} \mathrm{C} / \mathrm{m}^{2}$

## Example 3.30

Determine the capacitance of a parallel-plate capacitor of tin-foil sheet ( $25 \mathrm{~cm}^{2}$ ) separated by a glass dielectric 0.5 cm of thickness with relative permittivity 6 .

## Solution:

Each side of the square plate $=25 \mathrm{~cm}$; thickness of dielectric, $d=0.5 \mathrm{~cm}=0.005 \mathrm{~m}$;
relative permittivity, $\varepsilon_{\mathrm{r}}=6$; and area of plate, $A=25 \times 25=625 \mathrm{~cm}^{2}=0.0625 \mathrm{~m}^{2}$
Capacitance of parallel-plate capacitor

$$
C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}=\frac{8.854 \times 10^{-12} \times 6 \times 0.0625}{0.005}=664.05 \mathrm{pF}
$$

## Example 3.31

A parallel-plate capacitor has plates of area $2 \mathrm{~m}^{2}$ spaced by the three slabs of different dielectric materials. The relative permittivities are 2,3 , and 6 and the thicknesses are $0.4,0.6$, and 1.2 mm , respectively. Calculate the combined capacitance and the electric stress in each material when the applied voltage is $1,000 \mathrm{~V}$.

## Solution:

Area of the plates, $A=2 \mathrm{~m}^{2}$
Relative permittivity of three dielectric materials, $\varepsilon_{\mathrm{r} 1}=2 ; \varepsilon_{\mathrm{r} 2}=3 ; \varepsilon_{\mathrm{r} 3}=6$
Thickness of each dielectric material,

$$
d_{1}=0.4 \mathrm{~mm}=0.0004 \mathrm{~m}, d_{2}=0.6 \mathrm{~mm}=0.0006 \mathrm{~m}, d_{3}=1.2 \mathrm{~mm}=0.0012 \mathrm{~m}
$$

Combined capacitance of capacitor,

$$
\begin{aligned}
C & =\frac{\varepsilon_{0} A}{\sum \frac{d}{\varepsilon_{\mathrm{r}}}}=\frac{\varepsilon_{0} A}{\frac{d_{1}}{\varepsilon_{\mathrm{r} 1}}+\frac{d_{2}}{\varepsilon_{\mathrm{r} 2}}+\frac{d_{3}}{\varepsilon_{\mathrm{r} 3}}} \\
& =\frac{8.854 \times 10^{-12} \times 2}{\frac{0.0004}{2}+\frac{0.006}{3}+\frac{0.0012}{6}}=0.0295 \mu \mathrm{~F}
\end{aligned}
$$

Charge on the plates $=Q=C V=0.295 \times 10^{-6} \times 1,000$

$$
=29.5 \times 10^{-6} \mathrm{C}
$$

Electric flux density,

$$
\sigma=\frac{Q}{A}=\frac{29.5 \times 10^{-6}}{2}=14.75 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}
$$

Electric intensity of first layer

$$
E_{1}=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r}}}=\frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 2}=832.956 \mathrm{kV} / \mathrm{m}
$$

Electric intensity of second layer, $\quad E_{2}=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r} 2}}=\frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 2}=555.304 \mathrm{kV} / \mathrm{m}$
Electric intensity of third layer, $\quad E_{3}=\frac{\sigma}{\varepsilon_{0} \varepsilon_{\mathrm{r} 3}}=\frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 6}=277.652 \mathrm{kV} / \mathrm{m}$

## Example 3.32

A parallel-plate paper capacitor has 17 plates, each having an effective area of $5 \mathrm{~cm}^{2}$ and each separated by a paper sheet with thickness of 0.005 mm . Find the capacitance by considering the relative permittivity of paper as 4 .

## Solution:

Number of parallel plates, $n=17$
Area of each plate, $A=5 \mathrm{~cm}^{2}=5 \times 10^{-4} \mathrm{~m}^{2}$
Thickness of paper sheet, $d=0.005 \mathrm{~mm}=5 \times 10^{-6} \mathrm{~m}$
Relative permittivity of paper, $\varepsilon_{\mathrm{r}}=4$
Capacitance of multi-plate capacitor, $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{r} A}{d}$

$$
\begin{aligned}
& =(17-1) \frac{8.854 \times 10^{-12} \times 4 \times 5 \times 10^{-4}}{5 \times 10^{-6}} \\
& =56.66 \times 10^{-9} \mathrm{~F}
\end{aligned}
$$

## Example 3.33

A variable air capacitor has 15 movable plates and 16 fixed plates. The area of each plate is $15 \mathrm{~cm}^{2}$ and separation between the opposite plates is 0.2 mm . Determine:
(a) the maximum capacitance of this variable capacitor;
(b) the capacitance when $1 / 4$ th area of movable plates is overlapping the fixed plates.

## Solution:

Number of movable plates, $n_{1}=15$
Number of fixed plates, $n_{2}=16$
Area of each plate, $A=15 \mathrm{~cm}^{2}=15 \times 10^{-4} \mathrm{~m}^{2}$
Distance between opposite plates, $d=0.2 \mathrm{~mm}=2 \times 10^{-4} \mathrm{~m}$
Total number of plates $=n=n_{1}+n_{2}=15+16=3$
Capacitance of multi-plate capacitor, $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$
(a) Capacitance will be maximum when movable plates are rotated so that two sets of plates are completely overlapping each other.

$$
C_{\mathrm{m}}=(31-1) \frac{8.854 \times 10^{-12} \times 1 \times 15 \times 10^{-4}}{2 \times 10^{-4}}=1.992 \times 10^{-9} \mathrm{~F}
$$

(b) Capacitance of capacitor, when $1 / 4^{\text {th }}$ area of movable plates is overlapping the fixed plates.

$$
C_{1}=(31-1) \frac{8.854 \times 10^{-12} \times 1 \times 15 \times 10^{-4}}{2 \times 10^{-4} \times 4}=4.98 \times 10^{-10} \mathrm{~F}
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. How do you distinguish between capacitor and capacitance?
2. What will happen if the plates of a charged capacitor are connected through a copper wire?
3. A hollow sphere and a solid sphere are charged through same potential, which will have more charge, justify.
4. A charged capacitor is required to be handled carefully, why?
5. Can we give any desired charge to a capacitor?
6. Why mercury cannot be used as a dielectric in a capacitor?
7. Define capacitor, capacitance, and farad.
8. The radius of a planet is $6,400 \mathrm{~km}$, determine its capacitance.

## Test Questions

1. What is a capacitor? Classify the capacitor on the basis of electrolyte used. Explain construction of a paper capacitor.
2. Explain the construction of an electrolyte capacitor with the help of neat sketches. Why polarity is marked on the terminals of an electrolytic capacitor? If the terminals of this capacitor are connected in opposite polarity, what will happen?
3. Explain capacitor action. What are the factors on which the capacitance of a parallel-plate capacitor depends?
4. Derive a relation, $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{0} A}{d}$ for a multi-plate capacitor.

## Numericals

1. What must be the capacitance of a capacitor that can store a charge of 0.015 C when a potential difference of 250 V is applied across it?
(Ans. $60 \mu \mathrm{~F}$ )
2. Calculate the capacitance of a capacitor having two metal plates of area $20 \mathrm{~m}^{2}$ and separated by a dielectric of thickness 1.5 mm and of relative permittivity 5 .
(Ans. $590 \times 10^{9} \mathrm{~F}$ )
3. Find the capacitance in microfarad of a parallel-plate capacitor consisting of two plates $100 \mathrm{~m}^{2}$ separated by 1 -mm-thick mica plate of dielectric 6 . What will be the charge on the plates when 100 V is applied across it?
(Ans. $5.3 \mu \mathrm{~F}, 530 \mu \mathrm{C}$ )
4. A parallel-plate capacitor has circular plates of 12 cm radius and 1 mm separation of air. What will be the charge on the plates if potential difference of 100 V is applied?
(Ans. $400 \mu \mathrm{C}$ )

### 3.25 GROUPING OF CAPACITORS

The capacitors of different capacitance rated at different voltages are available in the market. To obtain the required value of the capacitance, the capacitors may be connected in parallel (for larger value) in series (for smaller value) or in series-parallel grouping.

### 3.25.1 Capacitors in Series

Consider three capacitors having capacitances $C_{1}, C_{2}$, and $C_{3}$ farad, respectively, connected in series, as shown in Figure 3.38. When a potential difference of $V$ volts is applied across the grouping, momentarily a charging current flows through the circuit that develops the same charge $Q$ on each capacitor at different potential differences $V_{1}, V_{2}$, and $V_{3}$, respectively.

Now, supply voltage $=$ sum of potential differences


Fig. 3.38 Capacitors connected in series
across each capacitor

$$
\begin{aligned}
V & =V_{1}+V_{2}+V_{3}=\frac{Q}{C_{1}}+\frac{Q}{C_{2}}+\frac{Q}{C_{3}} \\
& =Q\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}\right) \\
\frac{V}{Q} & =\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}
\end{aligned}
$$

However, $Q / V$ is the total or effective capacitance $C_{\mathrm{T}}$ of the combination, as shown in Figure 3.39.

$$
\frac{1}{C_{\mathrm{T}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}
$$

Hence, when a number of capacitors are connected in series, the reciprocal of total capacitance is equal to the sum of reciprocal of their individual capacitances.

### 3.25.2 Capacitors in Parallel



Fig. 3.39 Circuit representing effective capacitor


Fig. 3.40 Capacitors connected in parallel


Fig. 3.41 Circuit representing effective capacitor 3.42(a). In this type of circuits, first, the effective value of each parallel grouping is determined and the circuit is reduced to series circuit, as shown in Figure 3.42(b). Then, capacitance of the whole circuit is determined as

$$
\begin{aligned}
C_{\mathrm{p} 1} & =C_{2}+C_{3} ; C_{\mathrm{p} 2} \\
& =C_{4}+C_{5}+C_{6} \\
\frac{1}{C_{\mathrm{T}}} & =\frac{1}{C_{1}}+\frac{1}{C_{\mathrm{p} 1}}+\frac{1}{C_{\mathrm{p} 2}}
\end{aligned}
$$

where $C_{\mathrm{T}}$ is the total or effective capacitance of the whole circuit, as shown in Figure 3.42(c).


Fig. 3.42 Effective capacitance of capacitors connected in series-parallel combination

### 3.26 ENERGY STORED IN A CAPACITOR

When a potential difference is applied across a capacitor, the electrons are transferred from one plate to the other and the capacitor is charged. This involves expenditure of energy because electrons are moved against the opposing faces. This energy is stored in the electrostatic field set up between the plates of the capacitor in the dielectric medium. When the capacitor discharges, the field collapses and the stored energy is released.

Consider a capacitor of capacitance $C$ farads connected across a source of $V$ volt for charging. During charging, work is being done in shifting the charge (electrons) from one plate to other, as shown in Figure 3.43. Let, at any instance, the charge


Fig. 3.43 A capacitor connected across a dc supply of V volt on the capacitor is $q$ coulomb and potential difference across it is $v$ volts.
Then, $C=q / v$
If further charged, the charge $d q$ is shifted and then amount of work done is
or

$$
d w=v d q
$$

$$
d w=C v d v\left(\begin{array}{lc}
\Theta & q=C v \\
\therefore d q=C d v
\end{array}\right)
$$

Therefore, the total work done in raising the potential of uncharged capacitor 0 volt to $V$ volt will be

$$
W=\int_{0}^{v} C v d v=C \int_{0}^{v} v d v=C\left(\frac{v^{2}}{2}\right)_{0}^{v}=\frac{1}{2} C V^{2} \text { joule }
$$

This work done or energy is stored in the electrostatic field set-up in the dielectric. Therefore, the energy store in the capacitor or electrostatic field is

$$
W=\frac{1}{2} C V^{2}=\frac{1}{2} Q V=\frac{Q^{2}}{2 C}(\text { since } C=Q / V \text { or } V=Q / C)
$$

## Example 3.34

If five capacitors, each of capacitance $1 \mu \mathrm{~F}$, are joined in series, what will be the capacitance of the combination?

## Solution:

Capacitance of each capacitor, $C=1 \mu \mathrm{~F}$
Let the total capacitance of combination be $C_{\mathrm{S}}$.
Then,

$$
\begin{aligned}
\frac{1}{C_{\mathrm{S}}} & =\frac{1}{C_{1}}+\frac{1}{C_{2}}+\cdots+\frac{1}{C_{\mathrm{n}}}=\frac{1}{1}+\frac{1}{1}+\frac{1}{1} \cdots 5 \text { times } \\
& =\frac{1+1+1+1+1}{1}=\frac{5}{1}
\end{aligned}
$$

$$
\therefore \quad C_{\mathrm{S}}=\frac{1}{5} \mu \mathrm{~F}
$$

## Example 3.35

If 10 capacitors, each of capacitance $1 \mu \mathrm{~F}$, are joined in parallel, what will be the capacitance of the combination?

## Solution:

Capacitance of each capacitor, $C=1 \mu \mathrm{~F}$
Let the total capacitance of the combination be $C_{\mathrm{p}}$.
Then,

$$
\begin{aligned}
& C_{\mathrm{p}}=C_{1}+C_{2}+C_{3}+\cdots+C_{\mathrm{n}}=1+1+1+\cdots 10 \text { times } \\
& C_{\mathrm{p}}=10 \mu \mathrm{~F}
\end{aligned}
$$

## Example 3.36

Capacitors of 4,6 , and $12 \mu \mathrm{~F}$ are connected first in parallel, and then, in series. Compare the effective capacitance in the two combinations.

## Solution:

When connected in parallel, let the capacitance be $C_{\mathrm{p}}$
Therefore, $C_{\mathrm{p}}=4+6+12=22 \mu \mathrm{~F}$

When connected in series, let the capacitance be $C_{\mathrm{s}}$

$$
\begin{array}{ll}
\therefore & \frac{1}{C_{\mathrm{S}}}=\frac{1}{4}+\frac{1}{6}+\frac{1}{12}=\frac{3+2+1}{12}=\frac{6}{12}=\frac{1}{2} \\
\therefore & C_{\mathrm{s}}=2 \mu \mathrm{~F} \\
\text { Ratio, } & \frac{C_{\mathrm{p}}}{C_{\mathrm{s}}}=\frac{22}{2}=11
\end{array}
$$

## Example 3.37

The four capacitors connected in series-parallel combination are shown in Figure 3.44 (a) and their simplified circuit is shown in Figure 3.44(b).

## Solution:

$$
\begin{aligned}
C_{\mathrm{p}} & =C_{1}+C_{2}+C_{3} \\
& =2+2+2=6 \mu \mathrm{~F} \\
\frac{1}{C_{\mathrm{T}}} & =\frac{1}{6}+\frac{1}{6}=\frac{1+1}{6}=\frac{1}{3} \\
C_{\mathrm{T}} & =3 \mu \mathrm{~F}
\end{aligned}
$$



Fig. 3.44 Circuit diagram

## Example 3.38

Four identical capacitors each of capacitance $C$ farad are connected in series. One more similar capacitor is connected across this combination. Determine the capacitance of the whole combination.

## Solution:

Let the capacitance of series combination be $C_{\mathrm{S}}$.
Then,

$$
\begin{aligned}
\frac{1}{C_{\mathrm{s}}} & =\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\frac{1}{C}=\frac{4}{C} \\
C_{\mathrm{S}} & =\frac{C}{4}
\end{aligned}
$$

or

$$
C_{\mathrm{AB}}=C_{\mathrm{S}}+C=\frac{C}{4}+C=\frac{1+4}{4}=\frac{5}{4} C=1.25 \mathrm{C} \text { farad }
$$

## Example 3.39

The capacitance of two capacitors is $25 \mu \mathrm{~F}$ when connected in parallel and $6 \mu \mathrm{~F}$ when connected in series. Determine the capacitance of each capacitor.

## Solution:

Let $C_{1}$ and $C_{2}$ be the capacitance of the two capacitors.
When connected in parallel,

$$
\begin{equation*}
C_{1}+C_{2}=25 \tag{3.24}
\end{equation*}
$$

When connected in series,

$$
\frac{C_{1} C_{2}}{C_{1}+C_{2}}=6
$$

or

$$
\begin{equation*}
\frac{C_{1} C_{2}}{25}=6 \quad \text { or } \quad C_{1} C_{2}=150 \tag{3.25}
\end{equation*}
$$

Now,

$$
C_{1}-C_{2}=\sqrt{\left(C_{1}+C_{2}\right)^{2}-4 C_{1} C_{2}}=\sqrt{(25)^{2}-4 \times 150}=\sqrt{625-600}
$$

$$
\begin{equation*}
C_{1}-C_{2}=5 \tag{3.26}
\end{equation*}
$$

Adding Equations (3.24) and (3.26), we get

$$
2 C_{1}=30 \text { or } C_{1}=15 \mu \mathrm{~F} \text { and } C_{2}=10 \mu \mathrm{~F}
$$

## Example 3.40

Two capacitors of capacity $2 \mu \mathrm{~F}$ and $4 \mu \mathrm{~F}$, respectively, are connected in series. A potential different of 900 V is applied between the extreme terminals. Find the potential difference across each capacitor.

## Solution:

The circuit diagram is shown in Figure 3.45

$$
C_{2}=4 \mu \mathrm{~F}
$$

Voltage across the combination, $V=900 \mathrm{~V}$

Total capacitance,

$$
C=\frac{2 \times 4}{2+4}=\frac{4}{3} \mu \mathrm{~F}
$$

Charge on each capacitor,

$$
Q=C V=\frac{4}{3} \times 900=1200 \mu \mathrm{C}
$$

Potential difference across $2-\mu \mathrm{F}$ capacitor, $V_{1}=\frac{Q}{C_{1}}$

$$
=\frac{1200 \times 10^{-6}}{2 \times 10^{-6}}=600 \mathrm{~V}
$$



Potential difference across $4-\mu \mathrm{F}$ capacitor,

$$
V_{2}=\frac{Q}{C_{2}}=\frac{1200 \times 10^{-6}}{4 \times 10^{-6}}=300 \mathrm{~V}
$$

Fig. 3.45 Circuit diagram

## Example 3.41



Fig. 3.46 Circuit diagram

As the bridge is balance, the capacitor connected across branch PQ is neglected. The circuit is further simplified as shown in Figure 3.47(b) and 3.47(c).

## Solution:

As the bridge is balance, the capacitor connected across branch PQ is neglected. Therefore,

$$
\begin{aligned}
& \frac{1}{C_{\mathrm{APB}}}=\frac{1}{C_{\mathrm{AQB}}}=\frac{1}{C}+\frac{1}{C}=\frac{2}{C} \quad \text { or } \quad C_{\mathrm{APB}}=C_{\mathrm{AQB}}=\frac{C}{2} \\
& C_{\mathrm{AB}}=C_{\mathrm{APB}}+C_{\mathrm{AQB}}=\frac{C}{2}+\frac{C}{2}=\mathrm{C} \text { farad }
\end{aligned}
$$


(a)

(b)

(c)

Fig. 3.47 Simplified equivalent circuit

## Example 3.42

Determine the capacitance of the condenser combination between terminals shown in Figure 3.48. The capacitances shown are in microfarads.

## Solution:

From Figure 3.49, we can understand that branches $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$, and DA are forming the four arms of a wheat-stone bridge.

Now,

$$
\frac{C_{\mathrm{AB}}}{C_{\mathrm{BC}}}=\frac{15}{30}=0.5 \quad \text { and } \quad \frac{C_{\mathrm{AD}}}{C_{\mathrm{DC}}}=\frac{10}{20}=0.5
$$

The ratio arms

$$
\frac{C_{\mathrm{AB}}}{C_{\mathrm{BC}}}=\frac{C_{\mathrm{AD}}}{C_{\mathrm{DC}}}
$$



Fig. 3.48 Circuit diagram


Fig. 3.49 Equivalent circuit

The bridge is balanced and no current would flow through branch BD when some potential difference is applied across the terminals XY. Thus, branch BD can be removed and the circuit is reduced, as shown in Figure 3.49.

Now, in one branch, $15-\mu \mathrm{F}$ and $30-\mu \mathrm{F}$ capacitors are connected in series with each other, and in second branch, $10-\mu \mathrm{F}$ and $20-\mu \mathrm{F}$ capacitors are connected in series with each other. The two branches are connected in parallel. Therefore, the capacitance between the terminal X and Y is

$$
C=\frac{15 \times 30}{15+30}+\frac{10 \times 20}{10+20}=10+\frac{20}{3}=\frac{50}{3}=16.67 \mu \mathrm{~F}
$$

## Example 3.43

Two capacitors of capacitance $8 \mu \mathrm{~F}$ and $2 \mu \mathrm{~F}$ are connected in series across a $100-\mathrm{V}$ d.c. supply. Now, if the supply voltage is removed and the capacitors are then connected in parallel, what will be the final charge on each capacitor?

## Solution:

Capacitance of first capacitor, $C_{1}=8 \mu \mathrm{~F}$
Capacitance of second capacitor, $C_{2}=2 \mu \mathrm{~F}$
There are two capacitors connected in series across $100-\mathrm{V}$ d.c. supply as shown in Figure 3.50(a)

Then, total capacitance, $\quad C=\frac{C_{1} \times C_{2}}{C_{1}+C_{2}}=\frac{8 \times 2}{8+2}=1.6 \mu \mathrm{~F}$
Total charge,

$$
Q=C V=1.6 \times 100=160 \mu \mathrm{C}
$$

Now, the capacitors are connected in parallel after removing from the supply mains, as shown in Figure 3.50(b). Let the charge on two capacitors be $Q_{1}$ and $Q_{2}$, respectively, and the voltage across each of them is $V$ volts.

Total charge on the two capacitors remains the same, that is,


Fig. 3.50 (a) Two capacitors connected in series


Fig. 3.50 (b) Two capacitors connected in parallel

$$
\begin{equation*}
Q_{1}+Q_{2}=160 \mu \mathrm{C} \tag{3.27}
\end{equation*}
$$

Voltage across each capacitor, $V=V=\frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}}$
or

$$
\begin{equation*}
\frac{Q_{1}}{8}=\frac{Q_{2}}{2} \quad \text { or } \quad Q_{1}=4 Q_{2} \tag{3.28}
\end{equation*}
$$

Substituting the value of $Q_{1}$ in Equation (3.27), we get

$$
\begin{aligned}
4 Q_{2}+Q_{2} & =160 \\
Q_{2} & =32 \mu \mathrm{C} \\
Q_{1} & =4 \times 32=128 \mu \mathrm{C}
\end{aligned}
$$

and

## PRACTICE EXERCISES

## Short Answer Questions

1. In parallel circuit, show that effective value of capacitance is equal to sum of the individual capacitances.
2. There are $n$ identical capacitors connected in parallel and are charged to a potential of $V$ volt. Now, they are separated and connected in series with additive polarities. What potential difference will be obtained across the combination?
3. Can we charge a capacitor beyond its capacity, justify.

## Test Questions

1. Determine the value of capacitance of a grouping when a number of capacitors are connected parallel and a number of capacitors are connected in series.
2. Determine the effective value of capacitor's bank where three groups are connected in parallel and each group contains 2,3 , and 4 capacitors are connected in series.
3. Derive an expression for the energy stored in a capacitor. If the distance between the plates of a capacitor is decreased, whether its capacity to store energy will increase or decrease? Justify.

## Numericals

1. Ten capacitors each of capacitance $1 \mu \mathrm{~F}$ are joined in parallel and series. Find the resultant capacitance of each combination.
(Ans. $10 \mu \mathrm{~F}, 0.1 \mu \mathrm{~F}$ )
2. If capacitors with capacitance of 2,3 , and $6 \mu \mathrm{~F}$ are first connected in series and then in parallel, compare the effective capacitances of the two combinations.
(Ans. $\frac{1}{11}$ )
3. When two capacitors are connected in parallel, their capacitance is $18 \mu \mathrm{~F}$, whereas when connected in series, their effective value becomes $4 \mu \mathrm{~F}$. Find the capacitance of each capacitor.
(Ans. $12 \mu \mathrm{~F}, 6 \mu \mathrm{~F}$ )
4. Five capacitors each of capacitance $2 \mu \mathrm{~F}$ are connected to form a wheat-stone bridge across terminals A and B. What will be the effective capacitance of the combination across AB? (Ans. $2 \mu \mathrm{~F}$ )
5. Three identical capacitors each of capacitance 3 C farad are connected in series. This combination is connected in parallel with one more similar capacitor. Find the capacitance of whole combination.
(Ans. 4 C)

## SUMMARY

1. Coulomb's laws of Electrostatics.

First Law: Like charges repel each other, whereas unlike charges attract each other.
Second Law: The force exerted between two point charges
(a) is directly proportional to the product of their strength
(b) is inversely proportional to the square of the distance between them
(c) depends upon the nature of medium in which the charges are placed Mathematically,

$$
F \propto \frac{Q_{1} Q_{2}}{d^{2}} \quad \text { or } \quad F=k \frac{Q_{1} Q_{2}}{d^{2}}
$$

2. Absolute permittivity of a dielectric: The ability of dielectric material or electrolyte to hold electric lines of force is called its absolute permittivity.
3. Absolute permittivity of air: The absolute or actual permittivity of air or vacuum is minimum, and its value is $8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$.
4. Relative permittivity of a dielectric: The ratio of actual permittivity of a dielectric to the actual permittivity of vacuum is called relative permittivity of that dielectric, that is,

$$
\varepsilon_{\mathrm{r}}=\frac{\varepsilon}{\varepsilon_{0}} \quad \text { or } \quad \varepsilon_{\mathrm{r}}=\varepsilon_{0} \varepsilon_{\mathrm{r}}
$$

5. Electric field: A region or space around a charged body in which a charge experiences a force of attraction or repulsion is called an electric field or electrostatic field.
6. Electric lines of force: The path traced by a unit positive charge when placed in an electric field is called electric lines of force.
7. Electric flux: The total electric lines of force emanated from a positive charge are called electric flux.
8. Electric flux density (D): The electric flux crossing per unit area at a given section in an electric field is known as electric flux density at that section. It is generally represented by letter $\sigma($ or $D)$.
9. Electric intensity or field strength (E): Electric intensity or field strength at a point in an electric field is the force acting on a unit positive charge placed at that point.
10. Gauss theorem of electrostatics: It states that the total electric flux through any closed surface is always equal to $\frac{1}{\epsilon_{0}}$ times the total charge enclosed by the surface.

$$
y=\int_{S} \vec{E} \cdot \overrightarrow{d S}=\frac{Q}{\epsilon_{0}}
$$

11. Electric intensity due to a charged sphere:
(a) When point $P$ is outside the sphere $E=\frac{Q}{4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} / \mathrm{C}$
(b) When point $P$ is inside the sphere $E=0$

Thus, electric intensity at a point inside the charged sphere is always zero.
12. Electric intensity due to a long charged conductor: $E=\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d^{2}} \mathrm{~N} / \mathrm{C}$
13. Electric potential: The amount of work done in bringing a unit positive charge (i.e., +1 C ) from infinity to a given point in an electric field is called electric potential at that point. Mathematically,

$$
\text { Electric potential }=\frac{\text { Work }}{\text { Charge }}=\frac{W}{Q} \text { Joule } / \text { coulomb }
$$

14. Electric potential difference: The work done in moving a unit positive charge from the point of lower potential to the point of higher potential in an electric field is called the electric potential difference between the two points.
15. Political gradient: The change in potential per unit distance in an electric field is called potential gradient.

$$
\text { Potential gradient }=\frac{V_{\mathrm{A}}-V_{\mathrm{B}}}{S}
$$

16. Breakdown potential or dielectric strength: The maximum voltage in which a unit thickness of dielectric can withstand without being punctured by the spark is called its breakdown potential or dielectric strength of the dielectric.
17. Capacitor: Two surfaces separated by an insulating material or dielectric are called a capacitor or condenser.
18. Types of capacitors: Paper capacitors, mica capacitor, ceramic capacitors, and electrolytic capacitors.
19. Capacitance: The capability of a capacitor to store charge is called its capacitance.
20. Capacitance of parallel-plate capacitor:

Therefore, $C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$ farads (in medium) $=\frac{\varepsilon_{0} A}{d}$ farads (in air)
21. Factors affecting capacitance: Area of plates, thickness of dielectric, and relative permittivity of dielectric.
22. Parallel-plate capacitor with composite medium: $C=\frac{\varepsilon_{0} A}{\sum \frac{d}{\varepsilon_{\mathrm{r}}}}$ farad
23. Multi-plate capacitors: Total capacitance of $n$-plate capacitor, $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$.
24. Capacitors in series: When a number of capacitors are connected in series, the reciprocal of total capacitance is equal to the sum of reciprocal of their individual capacitances, that is, $\frac{1}{C_{\mathrm{T}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}$.
25. Capacitors in parallel: When a number of capacitors are connected in parallel, the total capacitance is equal to the sum of their individual capacitances, that is, $C_{\mathrm{T}}=C_{1}+C_{2}+C_{3}$.
26. Energy stored in a capacitor: Energy store in the capacitor or electrostatic field is

$$
W=\frac{1}{2} C V^{2}=\frac{1}{2} Q V=\frac{Q^{2}}{2 C} W=(\text { since } C=Q / V \text { or } V=Q / C)
$$

## TEST YOUR PREPARATTON

## 7 FILL IN THE BLANKS

1. Coulomb first law states that like charges $\qquad$ each other.
2. The two unlike charges of same magnitude are placed at a distance of 1 cm , they experience a force of repulsion of 4 N . When they are brought closer by 0.5 cm , the force will be $\qquad$ _.
3. The ratio of absolute permittivity of some insulating material to the absolute permittivity of air is called $\qquad$ .
4. Two similar charges placed at a distance of 1 cm experience a force of $9 \times 10^{9} \mathrm{~N}$, and therefore, the magnitude of the two charges will be each $\qquad$ _.
5. The unit of electric flux density is $\qquad$ .
6. The force acting on a unit positive charge when placed in an electric field is called $\qquad$ .
7. The relation between s and E is given by the equation $\qquad$ .
8. The electric potential inside a charged sphere is $\qquad$ .
9. The electric potential at which unit thickness of a dielectric medium punctures is called $\qquad$ -.
10. The unit of electric intensity is $\qquad$ .
11. The two conducting surfaces separated by a dielectric medium are called a $\qquad$ .
12. The charge on the capacitor plates per unit potential difference across the plate is called $\qquad$ .
13. 1 pF capacitance $=$ $\qquad$ $\mu \mathrm{F}$.
14. The capacitance of parallel-plate capacitor is inversely proportional to $\qquad$ between the plates.
15. The capacitance of parallel-plate capacitor $\qquad$ by inserting some dielectric medium between the plates.
16. The capacitance of a multi-plate (parallel-plate) having $n$ number of plates is given by the relation
$\qquad$ .
17. If a number of capacitors are connected in series, their effective value is $\qquad$ than the individual capacitance.

## OBJECTIVE TYPE QUESTIONS

1. The charge on a body is measured in
(a) volt
(b) farad
(c) eV
(d) coulomb
2. The electric intensity at a point is defined as
(a) force exerted on a unit positive charge when placed at that point.
(b) the electric flux passing per unit area at that point.
(c) the work done in bringing a unit positive charge from infinity to that point.
(d) All the above.
3. The relation between $\sigma$ and $E$ is given by the expression
(a) $\sigma=\varepsilon_{0} \varepsilon_{\mathrm{r}} E \mathrm{~s}$
(b) $E=\sigma / \varepsilon_{0} \varepsilon_{\mathrm{r}}$
(c) $E=\varepsilon_{0} \varepsilon_{\mathrm{r}} \sigma$
(d) Both (a) and (b)
4. The Energy expanded per unit charge is measured in
(a) volt
(b) joule
(c) watt
(d) coulomb
5. The potential at a point due to a charge of $Q$ coulomb placed at a distance of $d$ metre is given by
(a) $Q / 4 \pi \varepsilon_{0} \varepsilon_{\mathrm{r}} d$
(b) $9 \times 10^{9} \mathrm{Q} / \mathrm{d}$
(c) Both (a) and (b)
(d) $Q / \varepsilon_{\mathrm{r}} d^{2}$
6. The unit of electric intensity is
(a) V/C
(b) N/C
(c) $\mathrm{V} / \mathrm{m}$
(d) Both (a) and (b)
7. The maximum voltage which a unit thickness of a dielectric can withstand without being punctured is called
(a) breakdown potential
(b) breaking strength
(c) dielectric strength
(d) Both (a) and (c)
8. A positive and a negative charge are initially 4 cm apart. When they are moved closer together so that they are now only 1 cm apart, the force between them will be
(a) 4 times smaller than before
(b) 4 time larger than before
(c) 16 times larger than before
(d) 8 times larger than before
9. If one of the two given charges is doubled and the distance between them is reduced to half, the force between them will be
(a) 4 times smaller than before
(b) 4 time larger than before
(c) 16 times larger than before
(d) 8 times larger than before
10. The electric intensity at a place is $E \mathrm{~N} / \mathrm{C}$. If a charge of $q$ units placed at that place, then the force on this charge will be
(a) $E / q$
(b) $E \times q$
(c) $q / E$
(d) $E^{2} / q$
11. To move a unit positive charge from one point to another point on an equipotential surface,
(a) work is done by charge
(b) work is done on the charge
(c) no work is done
(d) work done is a constant
12. Potential inside a hollow charged sphere is
(a) same as on its surface
(b) greater than on its surface
(c) less than on its surface
(d) zero
13. A capacitor that has the ability to store more charge at a particular supply voltage is said to have more capacitance
(a) True
(b) False
14. In a parallel-plate capacitor, the capacitance is given by the relation
(a) $C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} d}{A}$
(b) $C=\frac{d}{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}$
(c) $\frac{1}{C}=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} d}{A}$
(d) $\frac{1}{C}=\frac{d}{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}$
15. A capacitor has a capacitance of 10 pF , which is equal to
(a) $10 \times 10^{-6} \mathrm{~F}$
(b) $10 \times 10^{-12} \mathrm{~F}$
(c) $10 \times 10^{12} \mathrm{~F}$
(d) $10 \times 10^{-9} \mathrm{~F}$
16. The capacitance of an air capacitor decreases when air is replaced by some dielectric material
(a) True
(b) False
17. The capacitance of a multi-plate capacitor having ' $n$ ' number of plates is given by the relation
(a) $C=n \frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$
(b) $C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{n d}$
(c) $C=(n-1) \frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$
(d) $C=\frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{(n-1) d}$
18. One $\mu \mu \mathrm{F}$ is equal to
(a) $10^{6}$ farad
(b) $10^{-6}$ farad
(c) $10^{12}$ farad
(d) $10^{-12}$ farad
19. When air is replaced by a dielectric medium of dielectric constant $K$, the maximum capacity of the condenser
(a) decreases K times
(b) increases $K$ times
(c) remains unchanged
(d) increases by $\mathrm{K}^{2}$ times
20. When two capacitors of capacitance $C_{1}$ and $C_{2}$ are connected in series, their effective value will be
(a) more than $C_{1}$ and $C_{2}$ separately
(b) less than $C_{1}$ and $C_{2}$ separately
(c) equal to $C_{1}+C_{2}$
(d) more than $C_{1}$ only
21. When two capacitors of $9 \mu \mathrm{~F}$ and $18 \mu \mathrm{~F}$ are connected in series, their effective value will be
(a) $27 \mu \mathrm{~F}$
(b) $12 \mu \mathrm{~F}$
(c) $6 \mu \mathrm{~F}$
(d) $21 \mu \mathrm{~F}$
22. If two capacitors are charged and then connected in parallel after removing them from the batteries, their common potential is given by
(a) $V=V_{1}+V_{2}$
(b) $V=V_{1}-V_{2}$
(c) $V=$ total charge/total capacitance
(d) $V=$ total capacitance/total charge
23. In a capacitor, generally
(a) both the plates are earthed
(b) one plate is earthed
(c) no plate is earthed
(d) both the plates are short circuited
24. Capacitance of a parallel-plate capacitor decreases by
(a) increasing the area of plates.
(b) increasing the distance between the plates.
(c) placing a dielectric between the plates.
(d) decreasing the distance between the plates.
25. When four capacitors of capacitance $1 / 4 \mu \mathrm{~F}$ are connected in series, the resultant capacitance will be
(a) $1 \mu \mathrm{~F}$
(b) $\frac{1}{8} \mu \mathrm{~F}$
(c) $\frac{1}{16} \mu \mathrm{~F}$
(d) $4 \mu \mathrm{~F}$
26. Three identical capacitors of capacitance $C \mathrm{mF}$ each are connected in series and such three combinations are connected in parallel. The capacitance of the whole combination will be
(a) $\mathrm{C} / 3 \mu \mathrm{~F}$
(b) $3 \mathrm{C} \mu \mathrm{F}$
(c) $1 / 3 \mathrm{C} \mu \mathrm{F}$
(d) $\mathrm{C} \mu \mathrm{F}$
27. The number of $1-\mu \mathrm{F}$ capacitor needed to be connected in parallel in order to store a charge of 1 C with a potential difference of 500 V across the capacitors is
(a) 500
(b) 2,000
(c) 4,000
(d) None of these
28. A parallel-plate capacitor is made from a stack of $n$ equally spaced plates of same size connected alternately if the capacitance between any two plates is $C$, then the resultant capacitance is
(a) $C / n$
(b) $n C$
(c) $(n+1) C$
(d) $(n-1) C$
29. Three identical capacitors of capacitance $C$ each are connected in series and this combination is connected in parallel with one more identical capacitor. The capacitance of the whole combination will be
(a) $C / 3$
(b) $3 / C$
(c) $4 C / 3$
(d) $3 C / 4$
30. The capacitances of the two capacitors are in the ratio of $1: 2$. When they are connected in parallel across supply voltage $V$ volt, their charges will be in the ratio of
(a) $1: 2$
(b) $2: 1$
(c) $3: 2$
(d) $2: 3$

## NUMERICALS

1. The equal but opposite charges experience a force of $25 \times 10^{5} \mathrm{~N}$ when they are placed 2 cm apart in a medium of relative permittivity 5 . Determine the charge.
(Ans. $745 \mu C$ )
2. Three identical charges, each of $Q$ coulombs, are placed at the vertices of an equilateral triangle 20 cm apart. Determine force on each charge.
(Ans. $2.25 \times 10^{11} \times \sqrt{3} Q^{2} \mathrm{~N}$ )
3. Two charges each of $Q$ coulombs are placed at two opposite corners of a square. What additional charges $Q_{1}$ be placed at each of the two corners so that the resultant electric force acting on each of the charges $Q$ be reduced to zero.
(Ans. $-Q / 2 \sqrt{2}$ coulomb)
4. Calculate the force on a unit positive charge placed at point $P(x=0, y=0)$ metre due to point charges $Q_{1}$ and $Q_{2}$. The charge $Q_{1}=+10^{-12} \mathrm{C}$ at $(x=0.5, y=0)$ metre and $Q_{2}=-10^{-11} \mathrm{C}$ at $(x=-0.5, y=0)$ metre.
(Ans. 0.396 N towards charge $Q_{2}$ )
5. A point charge of $3.5 \times 10^{-9} \mathrm{C}$ is placed in a medium of relative permittivity of 4 . Calculate the electric intensity at a point 15 cm from the point charge.
(Ans. $350 \mathrm{~N} / \mathrm{C}$ )
6. A charge of $4 \mu \mathrm{C}$ is placed in an electric field of intensity $50 \times 10^{5} \mathrm{~N} / \mathrm{C}$. What is the magnitude of force acting on the charge?
(Ans. 20 N )
7. Two equal and opposite charges of magnitude $10 \mu \mathrm{C}$ are placed 10 cm apart. What will be the magnitude and direction of force acting on an electron $\left(-1.6 \times 10^{-19} \mathrm{C}\right)$ when placed at the centre of two charges?
(Ans. $11.52 \times 10^{-12} \mathrm{~N}$ towards positive charge)
8. Three-point charges of $+9 \times 10^{-12} \mathrm{C},-72 \times 10^{-12} \mathrm{C}$, and $+27 \times 10^{-12} \mathrm{C}$ are placed at the corners $\mathrm{A}, \mathrm{B}$, and C of square ABCD having each side 9 cm . Find the electric intensity at the corner D. Assume air as a medium.
(Ans. 18.36 N/C)
9. Point charges in air are located as follows: $+5 \times 10^{-8} \mathrm{C}$ at $(0,0) \mathrm{m},+4 \times 10^{-8} \mathrm{C}$ at $(3,0) \mathrm{m}$, and $-6 \times$ $10^{-8} \mathrm{C}$ at $(0,4) \mathrm{m}$. Find electric field intensity at $(3,4) \mathrm{m}$. (Ans. $61.5 \mathrm{~N} / \mathrm{C} ; \phi=36.9^{\circ}$ )
10. Three charges are placed in air at the three corners of a square having each side of $3 \mathrm{~m} . Q_{1}=2 \times 10^{-9} \mathrm{C}$; $Q_{2}=-4 \times 10^{-9} \mathrm{C} ; Q_{3}=3 \times 10^{-9} \mathrm{C}$.
Determine the potential at the fourth corner of the square.
(Ans. 6.515 V )
11. Two parallel plates each measuring $8 \mathrm{~cm} \times 8 \mathrm{~cm}$ are separated by a dielectric of 1 mm thickness. If the potential difference between the plates is 20 kV and charge on each plate is $5 \mu \mathrm{C}$, determine electric flux density between plates and relative permittivity of the dielectric. (Ans. $7.8 \times 10^{-4} \mathrm{C} / \mathrm{m}^{2} ; 4.412$ )
12. A parallel-plate capacitor has plates 1.0 mm apart, and a dielectric with relative permittivity of 4.5 . Find electric intensity and the voltage between the plates if the surface charge is $4 \times 10^{-4} \mathrm{C} / \mathrm{m}^{2}$.
(Ans. $100.394 \times 10^{5} \mathrm{~V} / \mathrm{m} ; 10.039 \mathrm{kV}$ )
13. A parallel-plate capacitor has plates 0.1 mm apart, a plate area of $1 \times 10^{5} \mathrm{~mm}^{2}$, and a dielectric with a relative permittivity of 5. Find the electric flux density, electric field intensity, and the voltage between the plates, if the capacitor has a charge of $5 \mu \mathrm{C}$.
(Ans. $5 \times 10^{-5} \mathrm{C} / \mathrm{m}^{2} ; 11.29 \times 10^{5} \mathrm{~V} / \mathrm{m} ; 112.94 \mathrm{~V}$ )
14. Determine the capacitance of a parallel-plate capacitor composed of thin sheet $\left(50 \mathrm{~cm}^{2}\right)$ separated by a mica dielectric of 2-mm thickness with relative permittivity 6. Further, determine the charge on the plates when a potential difference of 500 V is applied across them.
(Ans. $1.328 \times 10^{-10} \mathrm{~F} ; 6.6410^{-8} \mathrm{C}$ )
15. A mica dielectric parallel-plate capacitor has 25 plates each having an effective area of $8 \mathrm{~cm}^{2}$ and each separated by a gap of 0.01 mm . Find the capacitance, take relative permittivity of mica as 6 .
(Ans. $0.102 \mu \mathrm{~F}$ )
16. Two capacitors having capacitances of $4 \mu \mathrm{~F}$ and $6 \mu \mathrm{~F}$ are connected in series across a $120-\mathrm{V}$ d.c. supply. Calculate the potential difference across each capacitor and the energy stored on each capacitor.
(Ans. $72 \mathrm{~V}, 48 \mathrm{~V}, 0.010368 \mathrm{~J}, 0.0069 \mathrm{~J})$
17. Two capacitors have capacitance of $6 \mu \mathrm{~F}$ and $10 \mu \mathrm{~F}$, respectively.
(a) Find the total capacitance when they are connected in parallel and series.
(b) When the two capacitors are connected in series across a $200-\mathrm{V}$ supply, find the potential difference across each capacitor and the charge on each capacitor.
(Ans. $3.75 \mu \mathrm{~F}, 16 \mu \mathrm{~F} ; 750 \mu \mathrm{C}, 125 \mathrm{~V}, 75 \mathrm{~V}$ )
18. Three capacitors of 10,20 , and $40 \mu \mathrm{~F}$ are placed in series across a $350-\mathrm{V}$ source. Determine
(a) Charge on each capacitor
(b) Voltage drop across each capacitor
(Ans. $2 \times 10^{-3} \mathrm{C} ; 200 \mathrm{~V}, 100 \mathrm{~V}, 50 \mathrm{~V}$ )
19. Given some capacitor of $0.1 \mu \mathrm{~F}$ capable of withstanding up to 15 V each, calculate the number of capacitors needed if it is desired to obtain a capacitance of $0.1 \mu \mathrm{~F}$ for use in circuit involving 60 V .
(Ans. 16, that is, 4 in series and 4 rows in parallel)
20. Three capacitors A, B, and C are connected in series across a $240-\mathrm{V}$ d.c. supply. The potential difference across the three capacitors are $60 \mathrm{~V}, 80 \mathrm{~V}$, and 100 V , respectively. If the capacitance of A is $12 \mu \mathrm{~F}$, what are capacitances of B and C ?
(Ans. $9 \mu \mathrm{~F}, 7.2 \mu \mathrm{~F}$ )
21. When three capacitors $\mathrm{A}, \mathrm{B}$, and C are connected in parallel their effective value is $60 \mu \mathrm{~F}$ and when they are connected in series their effective value is $60 / 11 \mu \mathrm{~F}$. If the capacitance of A is $10 \mu \mathrm{~F}$, determine the capacitances of $B$ and $C$.
(Ans. $20 \mu \mathrm{~F}, 30 \mu \mathrm{~F}$ )
22. Three capacitors of equal dimensions and having dielectrics of relative permittivities 2,3 , and 4 , respectively. Determine the voltage across each capacitor given that a potential difference of 1.3 kV is applied across the whole arrangement.
(Ans. $600 \mathrm{~V}, 400 \mathrm{~V}, 300 \mathrm{~V}$ )

## VIVA VOCE OR REASONING QUESTIONS

1. The charge accumulated on the moving clouds is called a static charge, why?
2. Each material is constituted by atoms that contain charge particles, namely protons and electrons, but it does not exhibit electricity, why?
3. The unit of electric intensity is $\mathrm{V} / \mathrm{m}$ or $\mathrm{N} / \mathrm{C}$; however, both these units are one and the same, how?
4. Similar charges repel each other, why?
5. The force of attraction or repulsion between the two charges for the same distances is greatest in air, why?
6. A capacitor is also called a condenser, why?
7. A capacitor does not allow the electric current to pass through it, still it is used in the electric circuits, why?
8. Multi-plate capacitors are preferred over single-plate capacitors, why?
9. The capacitance of a capacitor increases when air is replaced by mica between the plates of the capacitor, why?
10. When a charged capacitor is short circuited, a spark and noise is produced, why?
11. When a capacitor is connected in parallel with an existing capacitor, the overall capacitance increases, why?
12. A capacitor can store more energy if the size of the capacitor plates is larger but the distance between them is smaller, why?
13. A multi-plate variable capacitor using air as dielectric is called a gang capacitor, why?

## SHORT ANSWER TYPE QUESTIONS

1. Define electrostatics.
2. State Coulomb's first law of electrostatics.
3. What is the unit of electric charge? Define it.
4. What do you mean by permittivity of a dielectric?
5. Define electric field.
6. State Coulomb's laws of electrostatics.
7. What are electric lines of force?
8. State three properties of electric lines of force.
9. Define electric flux density.
10. What do you mean by electric field intensity?
11. What do you mean by area vector?
12. What do you mean by surface charge distribution?
13. State Gauss law.
14. Show that electric field intensity inside a hollow charged sphere is zero.
15. What is electric potential?
16. Define unit of electric potential.
17. What do you mean by potential gradient?
18. Define breakdown potential.
19. What is dielectric strength?
20. How will you define a capacitor?
21. How will you differentiate between capacitor and capacitance?
22. Define dielectric constant.
23. What factors affect the capacitance of a parallel-plate capacitor?
24. What happens if air is replaced by mica in a parallel-plate capacitor?
25. Define pollution.
26. How will you define pollutant?
27. How will you differentiate between natural and man-made sources of pollution?

## TEST QUESTIONS

1. What is electrostatics? Mention some practical applications of static electricity.
2. State and explain Coulomb's laws of electrostatics.
or
State and explain the laws relating to the force between two point charges.
3. Electric intensity is measured in $\mathrm{N} / \mathrm{C}$ or $\mathrm{V} / \mathrm{m}$. Show that both the units are same.
4. Define and explain the following terms:
(a) Electric stress
(b) Electric displacement
(d) Electric potential
(e) Potential difference
(c) Electric field strength
5. Derive an expression for the potential of a charged metal sphere of radius $r$ having a charge $q$ coulomb.
6. Explain and define the potentials at a point in an electric field. Derive the potential at any point in a field due to a unit charge.
7. Define electric potential, field intensity, and flux density and give the inter-relationship between them.
8. State Gauss Theorem.
9. What do you understand by potential gradient and breakdown potential?
10. Define and explain permittivity or dielectric constant of an insulating material.
11. What is a capacitor? Explain its action when connected in an electric circuit.
12. Derive the expression of capacitance of parallel-plate condenser.
13. State the factors on which the capacitance of a capacitor would depend.
14. A parallel-plate capacitor with plate separation ' $d$ ' has a capacitance $C_{1}$. Find its new capacitance when an insulated material slab of thickness ' $t$ ' is placed between the plates.
15. Derive an expression for the equivalent capacitance for a number of capacitors connected in series and parallel.
16. Derive an expression for the potential energy stored in a charged condenser.
17. Derive an expression for the energy stored in an electric field.
18. A d.c. voltage $V$ is applied across a circuit consisting of resistance $R$ ohms in series with a capacitance $C$ farads. Derive expression for variation of voltage across $C$ with time.

## q. ANSWERS

## Fill in the Blanks

1. repel
2. 16 N
3. relative permittivity
4. 1 C
5. $\mathrm{C} / \mathrm{m}^{2}$
6. electric intensity
7. $D=\varepsilon_{0} \varepsilon_{\mathrm{r}} E$
8. zero
9. breakdown potential
10. $\mathrm{N} / \mathrm{C}$ or $\mathrm{V} / \mathrm{m}$
11. Capacitor
12. Capacitance
13. $10^{-6}$
14. distance
15. increases
16. $(n-1) \frac{\varepsilon_{0} \varepsilon_{\mathrm{r}} A}{d}$
17. less

## Objective Type Questions

1. (d)
2. (a)
3. (a)
4. (c)
5. (a)
6. (a)
7. (b)
8. (b)
9. (a)
10. (d)
11. (b)
12. (d)
13. (a)
14. (d)
15. (d)
16. (b)
17. (b)
18. (c)
19. (b)
20. (b)
21. (b)
22. (c)
23. (c)
24. (d)
25. (b)
26. (c)
27. (d)
28. (b)
29. (c)
30. (a)


## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is an electric cell and how it is formed?

What are the primary and secondary cells?

* How the cells are connected in series, parallel, and in series-parallel combination and what we achieved out of these combinations?
*What is a battery?
*What are the constructional features of a lead-acid battery?
- How to check the charged condition of a battery?
- How a battery is charged?

What is the construction of a small cadmium cell and solar cell?

### 4.1 INTRODUCTION

Current not only flows through metallic conductors but also flows though some liquids called electrolytes. ${ }^{1}$ These liquids provide oppositely charged ions, and the conduction is due to the movement of these ions (not due to the movement of free electrons). The flow of current through electrolyte leads to chemical changes. Therefore, in this process, electrical energy is converted into chemical energy and the converse of this is also true. The chemical effect of electric current has many applications, for example, extraction of pure metals from ores, electrodeposition, electroplating, electrotyping, production of oxygen, storage batteries, and so on.
${ }^{1}$ Electrolytes are the liquids that break up into oppositely charged atoms called ions, for example, acids $\left(\mathrm{H}_{2} \mathrm{SO}_{4}, \mathrm{HCl}\right.$, etc.), solution of inorganic compounds $\left(\mathrm{NaCl}, \mathrm{CuSO}_{4}, \mathrm{AgNO}_{3}\right.$, etc.), hydroxides of metals $(\mathrm{KOH}, \mathrm{NaOH}$, etc.), etc.

One of the major applications of chemical effect of electric current is storage batteries. Battery is the only device in which electrical energy can be stored in the form of chemical energy. The important applications of batteries are as follows:

1. In automobiles
2. Used for lighting purposes in trains
3. To operate protective devices at substations
4. At telephone exchanges
5. Used in emergency lights at hospitals, theatres, and other places

In this chapter, storage batteries are discussed in detail.

### 4.2 ELECTRIC CELL

An electric device, such as battery, that converts chemical energy into electrical energy is known as electric cell.

### 4.2.1 Forming of a Cell

An electric cell essentially consists of the following:

1. Two metal plates (electrodes) of different materials so that different potentials are built-up when chemical action takes place on them.
2. A suitable solution (electrolyte) such as an acid, alkali, or salt solution-the solution must be capable to react chemically with the two electrodes.

When the two electrodes are immersed in a suitable electrolyte, different chemical actions take place on them and a potential difference is established between them.

### 4.2.2 EMF Developed in a Cell

The magnitude of emf of a cell depends upon the nature or material of the plates used as electrodes and the nature or type of electrolyte used.

### 4.3 TYPES OF CELLS

Broadly, electric cells may be divided into following two categories:

1. Primary cells: The cells in which chemical action is not reversible are called primary cells, for example, voltaic cell, Daniell cell, Leclanche cell, dry cell, etc.
In these cells, during discharging, one of the plates (usually negative plate) is consumed, which cannot be recovered by reversing the direction of flow of current through the cell. Therefore, chemical action in this case is not reversible and the cells cannot be recharged. This fact makes the primary cells rather an expensive source of electrical energy and rarely used in commercial applications.
2. Secondary cells: The cells in which chemical action is reversible are called secondary cells, for example, lead-acid cell, nickel-iron alkaline cell (or Edison cell), nickel-cadmium-alkaline cell, etc.

In these cells, no electrode is consumed during discharging; however, the chemical composition of the plates is changed. When the direction of flow of current through the cell is reversed (charging), the plates and the electrolyte regain their original composition. Therefore, chemical action in this case is reversible, which is stored in the cell itself. For this reason, these cells are called storage cells or accumulators and are used in almost all commercial applications.

### 4.4 IMPORTANT TERMS RELATING TO AN ELECTRIC CELL

The following are the important terms related to an electric cell:

1. Electromotive force (emf) of a cell: The energy supplied by a cell to one coulomb of charge is called emf of a cell. In general, it is defined as a potential difference (p.d.) between the two electrodes (positive and negative) of a cell on open circuit. It is represented by letter ' $E$ '.
2. Internal resistance of a cell: The opposition offered to the flow of current by the internal composition of the cell itself is called its internal resistance. It is generally represented by letter ' $r$ '. Figure 4.1(a) shows a cell with its internal resistance.
3. Terminal voltage: The potential

(a)

(b)

Fig. 4.1 Terminal voltage of a cell of no-lead and at load (a) Cell with internal resistance of no-load (b) Cell at load difference across the terminals of a cell at load is known as its terminal voltage. It is generally represented by letter ' $V$ '.

When load is applied, it delivers current $I$ to the external (load) resistor $R$ as shown in Figure 4.1(b). The potential difference across the terminals reduces to $V$ volts because of the voltage drop $(I \times r)$ in the internal resistance of the cell.

Therefore,

$$
V=E-I \times r
$$

### 4.5 GROUPING OF CELLS

A single cell can supply a very low current at low voltage. Generally, to operate electrical and electronic circuits, a large current at high voltage is required. Therefore, a number of cells are connected in series, parallel, and series-parallel grouping.

### 4.5.1 Series Grouping

When a number of cells are connected in such a way that the negative terminal of one is connected to positive terminal of the other and so on, as shown in Figure 4.2, the cells are said to be connected in series grouping.
Where
$n=$ No. of cell connected in series;
$E=\mathrm{emf}$ of each cell;


Fig. 4.2 Cells connected in series
$r=$ Internal resistance of each cell;
$R=$ Load resistance.
Total emf of the grouping $=n \times E$
Total internal resistance of the grouping $=n \times r$
Total resistance of the circuit $=R+n \times r$
Current delivered to the load, $I=\frac{n \times E}{R+n \times r}$
This grouping is used where higher voltages are required.

### 4.5.2 Parallel Grouping

When a number of cells are connected in such a way that the positive terminals of all the cells are connected together and negative terminals are connected together separately, as shown in Figure 4.3 , the cells are said to be connected in parallel grouping.

Where


Fig. 4.3 Cells connected in parallel

$$
\begin{aligned}
m & =\text { No. of cells connected in parallel; } \\
E & =\text { emf of each cell; } \\
r & =\text { Internal resistance of each cell; } \\
R & =\text { Load resistance. }
\end{aligned}
$$

Total emf of the grouping $=E$
Total resistance of the grouping $=r / m$
Total resistance of the circuit $=R+(r / m)$
Current delivered to the load, $I=\frac{E}{R+(r / m)}$
Current supplied by each cell $=I / m$
This grouping is used where higher currents are required.

### 4.5.3 Series-Parallel Grouping



Fig. 4.4 Cells connect in series-parallel

The grouping in which a number of cells are connected in series in one row and a number of such rows are connected in parallel is called a seriesparallel grouping of cells.

Figure 4.4 shows a series-parallel grouping of cells.
Where,
$n=$ No. of cells connected in series in each row;
$E=$ e.m.f. of each cell;
$r=$ Internal resistance of each cell;
$m=$ No. of rows connected in parallel;
$R=$ Load resistance.
Total emf of each row or grouping $=n \times E$
Total internal resistance of each row $=n \times r$

Total internal resistance of grouping $=n \times r / m$
Total resistance of the circuit $=R+(n \times r / m)$
Current delivered to the load, $I=\frac{n \times E}{R+(n \times r / m)}$
Current supplied by each row $=\frac{I}{m}$
The current supplied by a series-parallel grouping will be maximum when
Load resistance $=$ Internal resistance of the grouping
i.e.,

$$
R=\frac{n \times r}{m}
$$

This grouping used where both higher currents and higher voltages are required.

## Example 4.1

How many cells, each having an emf of 1.5 V and internal resistance of $0.25 \Omega$ would be required (connected in series) to pass a current of 1.5 A through a resistance of $15 \Omega$.

## Solution:

Let $n$ be the number of cells connected in series:
Current delivered to the load, $I=\frac{n \times E}{R+n \times r}$
Here, $E=1.5 \mathrm{~V} ; R=15 \Omega ; r=0.25 \Omega$, and $I=1.5 \mathrm{~A}$

$$
\therefore \quad 1.5=\frac{n \times 1.5}{15+n \times 0.25} \quad \text { or } \quad n=\frac{22.5}{1.125}=20
$$

## Example 4.2

Four dry cells each of which has an emf of 1.5 V and an internal resistance of $0.06 \Omega$ are connected in parallel. Determine current and power dissipated in the external load resistance of $2.985 \Omega$.

## Solution:

In parallel circuit
Current delivered to load resistance, $I=\frac{E}{R+(r / m)}$
Here, $E=1.5 \mathrm{~V} ; R=2.985 \Omega ; r=0.06 \Omega$, and $m=4$

$$
I=\frac{1.5}{2.985+(0.06 / 4)}=0.5 \mathrm{~A}
$$

Power dissipated in external load resistor

$$
=I^{2} \times R=(0.5)^{2} \times 2.985=0.74625 \mathrm{~W}
$$

## Example 4.3

Four batteries are connected in parallel to supply current to a resistive load for heating. Each battery consists of 12 cells connected in series. The emf of each cell is 2 V and internal resistance $0.15 \Omega$. What should be the resistance of the load so that power consumed in the load is 307.2 W ?

## Solution:

Current supplied by series - parallel grouping

$$
\begin{equation*}
I=\frac{n \times E}{R+n \times r / m} \tag{4.1}
\end{equation*}
$$

Here, $n=12 ; E=2 \mathrm{~V} ; m=5 ; r=0.15$

$$
I=\frac{12 \times 2}{R+(0.15 \times 12 / 5)} \quad \text { or } \quad I=\frac{24}{R+0.36}
$$

Power consumed in the load resistor, $P=I^{2} \times R$

$$
\begin{aligned}
& 307.2=\left(\frac{24}{R+0.36}\right)^{2} \times R \\
& \text { or } \quad R^{2}+0.72 R+0.1296=1.875 R \text { or } R^{2}-1.155 R+0.1296=0 \\
& \text { or } \quad R=\frac{1.55 \pm \sqrt{1.344-0.5184}}{2}=\frac{1.155 \pm 0.90312}{2} ; R=1.029 \Omega \text { or } 0.12594 \Omega
\end{aligned}
$$

Substituting the value of $R$ in Equation (4.1), we get

$$
I=17.28 \mathrm{~A} \text { or } 49.39 \mathrm{~A}
$$

### 4.6 BATTERY

A series, parallel, or series-parallel grouping of cells is called a battery.
Generally, a cell can deliver a small current at low voltage. For a circuit, if higher voltage is required, a battery containing number of cells connected in series is applied; if higher current is required, a battery containing number of cells connected in parallel is applied; if larger current at higher voltage is required, a battery containing number of cells in series further connected in parallel is applied.

Usually, a number of cells connected in series placed in a single container are called a battery.

### 4.6.1 Lead-Acid Battery

Figure 4.5 (a) shows the cut-away view of 6 V commercial lead-acid battery. The following are the important parts of the battery.

1. Container: It is the outer body of the battery. It is made of a hard rubber or plastic material and is sealed at the top to prevent spilling of the electrolyte. A large space is left at the bottom of the container so that the sediments that drop from the plates are collected here and may not short circuit the positive and negative plates.
2. Plates: Generally, alloy of lead-antimony sheets covered with lead peroxide and spongy lead forming positive and negative plates, respectively, are used as electrodes. To increase the capacity of the battery, we use a large number of plates in each cell instead of only two plates. The number of positive ${ }^{2}$ and negative plates (i.e., $11,13,15$, or 17 ) of each cell are alternatively placed and sandwiched with an insulator called separator as shown in Figure 4.5(b). One group of positive and negative plates forms a cell which develops an emf of 2.0 V . A separate compartment is provided for each cell in the container of the battery.

[^7]

Fig. 4.5 Constructional features of a lead-acid battery (a) Sectional view of a lead-acid battery (b) Battery plates (c) Separator
3. Separator: To reduce the internal resistance of the cell and to save the space, the plates are placed very close to each other. To prevent the plates touching each other if they wrap or buckle, they are separated by a rubber sheet (non-conducting material) having large number of small holes called separator (Fig. 4.5(c)).
4. Electrolyte: Dilute sulphuric acid $\left(\mathrm{H}_{2} \mathrm{SO}_{4}\right)$ is used as an electrolyte in lead-acid batteries. Sulphuric acid is added to water in such a proportion that with a fully charged battery, its specific gravity is about 1.28 to 1.29 .
5. Battery cover: Each cell compartment is covered usually with a moulded hard rubber and the joints between covers and container are sealed with an acid-resistant material. In each cell cover, openings are provided-two for positive and negative terminals, and third for a vent. The whole container is fitted with a leak proof cover.
6. Vent caps: The vent cap has a vent hole that allows free exit of the gases formed in the cell during charging. The vent caps can be easily removed to add water. The vent cap is also removed to insert the nozzle of hydrometer for checking the specific gravity of electrolyte used to analyse the battery charge condition.
7. Inter-cell connector: The cells, placed in the same container, are connected in series with a lead alloy link called intercell connector.
8. Cell terminals: Each cell has two terminals that are generally made of lead as it does not corrode due to the electrolyte. The positive terminal of the battery is marked with a red colour or by a large positive (+) sign.

### 4.6.2 Working Principle of Lead-Acid Cell

When a lead-acid cell is ready for use, its positive plate is made of load peroxide $\left(\mathrm{PbO}_{2}\right)$ chocolate brown in colour and the negative plate is of spongy lead $(\mathrm{Pb})$ grey in colour. Both the plates are immersed in a dilute sulphuric acid $\left(\mathrm{H}_{2} \mathrm{SO}_{4}\right)$ of specific gravity about 1.28 . When a load is connected across the terminals of the cell, it starts delivering current to the load, and the process is called discharging of cell. In this process, the chemical energy stored in the cell is converted into electrical energy which is delivered to the load (external circuit).

## Chemical action during discharging

The sulphuric acid when dissolved in water, its molecules are dissociated into hydrogen ions $\left(2 \mathrm{H}^{+}\right)$and sulphate ions $\left(\mathrm{SO}_{4}^{--}\right)$which move freely (at random) in the electrolyte. When the load (resistor) is connected across the terminals as shows in Figure 4.6(a), the sulphate ions move towards the plate made of lead and hydrogen ions towards the plate made of lead peroxide. The following chemical action takes place in the two electrodes.

## At lead plate

$$
\mathrm{SO}_{4}^{--}-2 \mathrm{e} \ldots \longrightarrow \mathrm{SO}_{4}(\text { radical })
$$

The electrons travel through external load and after doing work reach at other electrode. The sulphate radical, chemically reacts with the electrode material $(\mathrm{Pb})$ and forms lead sulphate $\left(\mathrm{PbSO}_{4}\right)$.


Fig. 4.6 Working of lead-acid cell (a) Working during discharging (b) Working during recharging

$$
\underset{\text { (Grey in colour) }}{\mathrm{Pb}+\mathrm{SO}_{4} \xrightarrow[\text { (whitish in colour) }]{\longrightarrow}}
$$

## At $\mathrm{PbO}_{2}$ plate

Each hydrogen ion $\left(\mathrm{H}^{+}\right)$on reaching the $\mathrm{PbO}_{2}$ plate takes one electron (the same electron which is given up by the sulphate ion at Pb state and has come to the other plate via external load resistor) from it to become hydrogen gas:

$$
\underset{\text { (ion) }}{2 \mathrm{H}^{+}+2 \mathrm{e}} \longrightarrow \underset{\text { (gas) }}{2 \mathrm{H}}
$$

The hydrogen gas liberated at this electrode acts chemically with electrode material $\left(\mathrm{PbO}_{2}\right)$ and reduces it to PbO which further reacts with $\mathrm{H}_{2} \mathrm{SO}_{4}$ forming $\mathrm{PbSO}_{4}$.

$$
\underset{\text { (Chocolate brown in colour) }}{\mathrm{PbO}_{2}+2 \mathrm{H}}
$$

$$
\underset{\substack{\text { (Whitish in colour) }}}{\mathrm{PbO}}+\mathrm{HbSO}_{4} \mathrm{SO}_{4} \longrightarrow \mathrm{H}_{2} \mathrm{O}
$$

Therefore, during discharging;

1. Both the plates are converted to lead sulphate $\left(\mathrm{PbSO}_{4}\right)$ that is whitish in colour.
2. Sulphuric acid is consumed and water is formed which reduces the specific gravity of $\mathrm{H}_{2} \mathrm{SO}_{4}$ from 1.28 to 1.15 .
3. Terminal voltage per cell falls to 1.8 V form 2.0 V .
4. Chemical energy is converted into electrical energy which is delivered to the load.

## Chemical action during recharging

For recharging, position plate is connected to positive terminal of the source and negative plate is connected to the negative terminal as shown in Figure 4.6(b). During recharging, hydrogen ions $\left(2 \mathrm{H}^{+}\right)$move towards the negative plate and sulphate ions $\left(\mathrm{SO}_{4}^{--}\right)$towards the positive plate. The following chemical actions take place at the two plates:

## At positive plate

$$
\begin{aligned}
& \mathrm{SO}_{4}^{--}-2 \mathrm{e} \longrightarrow \mathrm{SO}_{4}(\text { radical }) \\
& \mathrm{SO}_{4}+\mathrm{H}_{2} \mathrm{O} \longrightarrow \mathrm{H}_{2} \mathrm{SO}_{4}+\mathrm{O}
\end{aligned}
$$

The oxygen (in atomic state) reacts chemically with this plate material $\left(\mathrm{PbSO}_{4}\right)$ :

$$
\mathrm{PbSO}_{4}+\mathrm{O}+\mathrm{H}_{2} \mathrm{O} \longrightarrow \mathrm{PbO}_{2}+\mathrm{H}_{2} \mathrm{SO}_{4}
$$

(Whitish in colour) (Chocolate brown in colour)

## At negative plate

Each hydrogen ion $\left(\mathrm{H}^{+}\right)$on reaching the negative plate takes one electron (the same electron which is given up by the sulphate ion at positive plate and has come to negative plate via external circuit (i.e., source) from to become hydrogen gas;

$$
2 \mathrm{H}^{+}+2 \mathrm{e} \longrightarrow 2 \mathrm{H}
$$

The hydrogen gas liberated at negative plate acts chemically with its material $\left(\mathrm{PbSO}_{4}\right)$;

$$
\underset{\substack{\mathrm{PbSO}_{4} \\ \text { (Whitish in colours) } \\(\text { (Grey in colour) }}}{\mathrm{Pb}}+\mathrm{H}_{2} \mathrm{SO}_{4}
$$

Therefore, during recharging:

1. Both the plates regain their original composition, that is, anode is converted back into lead peroxide $\left(\mathrm{PbO}_{2}\right)$ and cathode into spongy lead $(\mathrm{Pb})$.
2. Water is consumed and sulphuric acid is formed which improves the specific gravity of $\mathrm{H}_{2} \mathrm{SO}_{4}$ from 1.15 to 1.28 .
3. Terminal voltage per cell increases from 1.8 V to $2.2 / 2.5 \mathrm{~V}$.
4. Electrical energy is converted into chemical energy which is stored in the cell.

## 目冒 PRACTICE EXERCISES

## Short Answer Questions

1. What is an electric cell?
2. What are the requirements to form a cell?
3. How will you categorize the electric cells?
4. What is the necessity of connecting the cells in series, parallel, or series-parallel combination?
5. How will you differentiate an electric cell and battery?
6. Name at least five major parts of a lead-acid battery.
7. Write down the chemical reactions occurring at positive and negative plate of a lead-acid cell during discharging.
8. Write down the chemical reactions occurring at positive and negative plate of a lead-acid cell during recharging.

## Test Questions

1. How will you differentiate primary and secondary cells? Define emf, terminal voltage, and internal resistance of an electric cell.
2. What is the necessity of grouping of electric cells? Derive a relation for the current delivered by a series-parallel grouping of cells. Show that the current delivered by a series-parallel grouping will be maximum when load resistance is equal to the internal resistance of the grouping.
3. Explain with neat diagram, the constructional details of a lead-acid battery.
4. Explain the working of a lead-acid battery with the help of chemical reactions.

## Numericals

1. Six lead-acid cells each having an emf of 2 V and internal resistance $0.05 \Omega$ are connected in series. How much current will be delivered by the combination to a load resistance of $3.7 \Omega$.
(Ans. 3 A)
2. Six dry cells each of 1.5 V and an internal resistance of $0.06 \Omega$ are connected in parallel and a resistance of $0.24 \Omega$ is connected across it. Determine (i) current delivered to the load resistor, (ii) current delivered by each cell, (iii) voltage across the load resistor, and (iv) terminal voltage across each cell.
(Ans. $6 \mathrm{~A}, 1 \mathrm{~A}, 1.44 \mathrm{~V}, 1.44 \mathrm{~V}$ )
3. Two lead-acid batteries each of 12 V are connected in parallel to supply a load resistor of $1.2 \Omega$. Each battery consists of six cells each having $2.0 \mathrm{~V}, 0.4 \Omega$ internal resistance, determine the current supplied by each battery.
(Ans. 2.5 A )

### 4.7 CAPACITY OF A BATTERY

The quantity of electricity that a battery can delivery during single discharge until its terminal voltage falls to $1.8 \mathrm{~V} /$ cell is called the capacity of a battery.

The capacity of a battery or cell is commercially expressed in ampere-hour and is generally denoted by A-H.

Capacity of a battery or cell $=I_{\mathrm{d}} T_{\mathrm{d}}$ ampere-hour
Where
$I_{\mathrm{d}}=$ discharging current in ampere
$T_{\mathrm{d}}=$ discharging time of battery or cell in hour.

### 4.8 EFFICIENCY OF A BATTERY

The efficiency of a battery (or cell) can be defined in the following two ways:

1. Quantity or A-H efficiency: The ratio of output ampere-hour during discharging to the input ampere-hour during charging of the battery is called quantity or ampere-efficiency of the battery.

$$
\text { Mathematically, } \eta_{\mathrm{AH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}}}
$$

Where

$$
\begin{array}{ll}
I_{\mathrm{d}}=\text { discharging current in ampere; } & T_{\mathrm{d}}=\text { discharging time in hour; } \\
I_{\mathrm{c}}=\text { charging current in ampere; } & T_{\mathrm{c}}=\text { charging time in hour. }
\end{array}
$$

2. Energy or W-H efficiency: The ratio of output watt-hour during discharging to the input watt-hour during charging of the battery is called energy or watt-hour efficiency of the battery.

$$
\text { Mathematically, } \eta_{\mathrm{wH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}} V_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}} V_{\mathrm{c}}}
$$

Where,
$V_{\mathrm{d}}=$ Average terminal voltage during discharging,
$V_{\mathrm{c}}=$ Average terminal voltage during charging

## Example 4.4

An alkaline cell is discharged at a steady current of 4 A for 12 hour, the average terminal voltage being 1.2 V to restore it to its original state of voltage, a steady current of 3 A for 20 hour is required, the average terminal voltage being 1.44 V . Calculate the ampere-hour and watt-hour efficiencies in this particular case.

## Solution:

Where

$$
\text { Ampere-hour efficiency, } \eta_{\mathrm{AH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}}} \times 100
$$

$I_{\mathrm{d}}=4 \mathrm{~A} ; T_{\mathrm{d}}=12$ hour; $I_{\mathrm{c}}=3 \mathrm{~A} ; T_{\mathrm{c}}=20$ hour;

$$
\eta_{\mathrm{AH}}=\frac{4 \times 12}{3 \times 20} \times 100=80 \%
$$

$$
\text { Watt-hour efficiency, } \eta_{\mathrm{WH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}} V_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}} V_{\mathrm{c}}} \times 100
$$

Where, $V_{\mathrm{d}}=1.2 \mathrm{~V} ; V_{\mathrm{c}}=1.44 \mathrm{~V} ; \quad \eta_{\mathrm{WH}}=\frac{4 \times 12 \times 1.2}{3 \times 20 \times 1.44} \times 100=66.67 \%$

## Example 4.5

A lead-acid cell has 17 plates, each $30 \mathrm{~cm} \times 20 \mathrm{~cm}$. The clearance between the neighbouring plates is 1 mm . If the resistivity of the acid is 1.6 ohm cm , find the internal resistance of the cell.

## Solution:

Since there are 17 plates, the arrangement constitutes 16 tiny cells in parallel.
Resistance of each tiny cell, $R=\rho \frac{l}{a}$
Where $\rho=1.6 \Omega \mathrm{~cm} ; l=0.1 \mathrm{~cm} ; a=30 \times 20=600 \mathrm{~cm}^{2}$
$\therefore \quad R=1.6 \times \frac{0.1}{600}=266.67 \times 10^{-6}$
Total internal resistance of the cell, $r=\frac{R}{16}=\frac{266.67 \times 10^{-6}}{16}=16.67 \mu \Omega$

## Example 4.6

A discharged battery is put on charge at 5 A for 4 hour at a mean charging voltage of 13.5 V . It is then discharged in 6 hour at a constant terminal voltage of 12 V through a resistance of $R$ ohm. Calculate the value of $R$ for an ampere-hour efficiency of $85 \%$.

## Solution:

Ampere-hour efficiency, $\eta_{\mathrm{AH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}}}$
Where $\eta_{\mathrm{AH}}=85 \% ; T_{\mathrm{d}}=6$ hour; $I_{\mathrm{c}}=5 \mathrm{~A} ; T_{\mathrm{c}}=3.5$ hour

$$
\therefore \quad 85=\frac{I_{\mathrm{d}} \times 6}{5 \times 3.5} \times 100 \quad \text { or } \quad I_{\mathrm{d}}=2.48 \quad \text { and } \quad R=\frac{V_{\mathrm{d}}}{I_{\mathrm{d}}}=\frac{12}{2.48}=4.84 \mathrm{ohm}
$$

### 4.9 CHARGE INDICATIONS OF A LEAD-ACID BATTERY OR CELL

In batteries, the charge condition is determined by checking the specific gravity of the electrolyte $\left(\mathrm{H}_{2} \mathrm{SO}_{4}\right)$. For a fully charged battery, the specific gravity of $\mathrm{H}_{2} \mathrm{SO}_{4}$ is 1.28 to 1.29. However, when the specific gravity falls below 1.15 , the battery is fully discharged. In fact, to increase the life of battery, it should be recharged when the specific gravity of the electrolyte is found to be less than 1.18. The values of specific gravity for different condition of charge are given in Table 4.1.
Table 4.1 Specific Gravity V/s Charged Condition of a Lead-Acid Cell

| Specific Gravity | Condition |
| :--- | :--- |
| 1.280 to 1.290 | $100 \%$ charged |
| 1.230 to 1.250 | $75 \%$ charged |
| 1.190 to 1.200 | $50 \%$ charged |
| 1.150 to 1.160 | $25 \%$ charged |
| below 1.130 | fully discharged |

To check the specific gravity of $\mathrm{H}_{2} \mathrm{SO}_{4}$, an instrument called hydrometer is used (Fig. 4.7) which works on Archimedes principle. Commonly, the decimal point is omitted from the value of specific gravity, that is, 1.280 specific gravity, is spoken as 1280 and so on.

However, the state of a battery can also be determined by checking the following:

1. Voltage: When the terminal voltage of the battery on load is 2.1 to 2.5 V per cell, the battery is said to be fully charged. Whereas when the voltage of battery falls to below 1.8 V per cell, the battery is considered to be fully discharged and it is immediately put on charging.
2. Colour of plates: When a lead-acid cell or battery is fully charged, its anode is $\mathrm{PbO}_{2}$ which is chocolate brown in colour and cathode is of Pb which is grey in colour. However, when the battery is fully discharged, both the plates attain $\mathrm{PbSO}_{4}$ as active material which is whitish in colour.

### 4.10 CHARGING OF LEAD-ACID BATTERY

Whenever terminal voltage of a battery falls below 1.8 V per cell, it is put under recharging. The following points must be kept in mind while charging a battery:

1. Only a DC voltage source is applied for recharging.
2. Ensure that positive terminal of the source is connected with positive terminal of the battery and negative with negative.
3. The charging voltage of the source should be approximately 2.5 V per cell.
4. The charging current should be maintained approximately 1 A per positive plate of the battery per cell. For instant, if a cell of the battery contains 17 plates, it will have eight positive plates. Thus, the charging current for such a battery will be 8 A .

Note: An over current may produce excessive heat and damage the battery.
Generally, the following two methods are employed for charging a battery:

1. Constant current method: In this method, the charging current supplied to the battery is kept constant throughout the charging period by adjusting the value of variable resistor $R$ form $R_{1}$ to $R_{2}$ as shown in Figure 4.8(a).
At the beginning of the charging, current supplied to the battery,

Where

$$
I=\frac{V-n \times E_{1}}{R_{1}+n \times r}
$$

$$
\begin{aligned}
n & =\text { No. of cells in series; } & r & =\text { internal resistance of each cell } \\
E_{1} & =\text { emf of each cell in the beginning; } & R_{1} & =\text { initial value of series resistor. }
\end{aligned}
$$

$$
R_{1}=\frac{V-n \times E_{1}}{I}-n \times r
$$

As charging starts, the emf developed in each cell starts increasing which decreases the circuit current. To maintain the current constant, the value of series resistor is decreased gradually. Finally, when the battery is fully charged, let the emf developed per cell be $E_{2}$ and the value of series resistance be $R_{2}$.

Then,

$$
R_{2}=\frac{V-n \times E_{2}}{I}-n \times r
$$



Fig. 4.8 Recharging of a battery (a) Constant current method (b) Constant voltage method
2. Constant voltage method: In this method, the supply voltage for charging the battery is kept constant. A fixed resistance $R$ is connected in series with the battery, as shown in Figure 4.8(b) to limit the current supplied to the battery in the beginning. In this case, the battery draws heavy current $I_{1}$ in the beginning which reduces to $I_{2}$ finally when the battery is fully charged.

$$
I_{1}=\frac{V-n \times E_{1}}{R+n \times r} ; \quad I_{2}=\frac{V-n \times E_{2}}{R+n \times r}
$$

This method of battery charging is applied commercially as total time required for complete charging is far less (nearly half) than that for constant current method. Moreover, no attendant is required to vary the series resistance.

## Example 4.7

A battery of 50 cells in series is charged through a resistance of $4 \Omega$ from a 230 V supply. If the terminal voltage per cell is 2 V and 2.7 V , respectively, at the beginning and at the end of the charge, calculate the charging current at the beginning and at the end of the charge.

## Solution:

Charging current at the beginning,

$$
I_{1}=\frac{V-n \times E_{1}}{R+n \times r}=\frac{V-n \times E_{1}}{R}(\text { since } r=0, \text { not given })
$$

Where $V=230 \mathrm{~V}, n=50 ; E_{1}=2 \mathrm{~V} ; R=4 \Omega$;

$$
\therefore \quad I_{1}=\frac{230-50 \times 2}{4}=\frac{130}{4}=32.5 \mathrm{~A}
$$

Charging current at the end,
or

$$
\begin{aligned}
& I_{2}=\frac{V-n \times E_{2}}{R}=\frac{230-50 \times 2.7}{4}\left(\mathrm{Q} E_{2}=2.7 \mathrm{~V}\right) \\
& I_{2}=95 / 4=23.75 \mathrm{~A}
\end{aligned}
$$

Learning outcome: The charging current in the beginning comes out to be 32.5 A . It will be most appropriate charging if each cell of the battery contains 63 plates. If the cells contain less than 63 plates, they will be overheated in the beginning while changing and the life of the battery reduces.

## Example 4.8

A secondary battery consists of six cells in series, each having an emf of 1.8 V at discharge and 2.5 V when fully charged. The internal resistance of each cell is $0.1 \Omega$. If a charging supply of 24 V is available, calculate the value of variable series resistance required to maintain the charging current at a constant value of 10 A throughout the charging period.

## Solution:

Value of series resistance in the beginning.

$$
R_{1}=\frac{V-n \times E_{1}}{I}-n \times r
$$

Where $V=24 \mathrm{~V} ; n=6 ; E_{1}=1.8 \mathrm{~V} ; I=10 \mathrm{~A} ; r=0.1 \Omega$

$$
R_{1}=\frac{24-6 \times 1.8}{10}-6 \times 0.1=0.72 \Omega
$$

Value of series resistance when the battery is fully charged.

$$
\begin{aligned}
R_{2} & =\frac{V-n \times E_{2}}{I}-n \times r ; \quad\left(\text { Where } E_{2}=2.5 \mathrm{~V}\right) \\
& =\frac{24-6 \times 2.5}{10}-6 \times 0.1=0.3 \Omega
\end{aligned}
$$

Learning outcome: The value of series resistor has to be regulated from $0.72 \Omega$ to $0.3 \Omega$ to maintain the charging current at 10 A .

## Example 4.9

A battery of emf 80 V having an internal resistance of $2 \Omega$ is to be charged from 200 V mains. What is the value of series resistance for a charging current of 5 A ? If the battery is charged for 10 hours, calculate the cost of energy consumed if the rate of energy is $₹ 4.75$ per unit.

Solution:
The value of series resistance $R=\frac{V-n \times E}{I}-n \times r$
Where $V=200 \mathrm{~V} ; I=5 \mathrm{~A} ; n \times E=80 \mathrm{~V} ; n \times r=2 \Omega$;

$$
\therefore \quad R=\frac{200-80}{5}-2=22 \Omega
$$

Energy supplied during charging $=V \times I \times t / 1000=200 \times 5 \times 10 / 1000=10 \mathrm{kWh}$
Energy charges $=₹ 4.75 \times 10=₹ 47.50$

### 4.11 CARE AND MAINTENANCE OF LEAD-ACID BATTERIES

The average life of a lead-acid battery is two to four years depending upon its manufacturing qualities and technique. However, to obtain longer life and efficient service, the following points must be kept in view:

1. The battery should not be allowed to use when the emf of the battery falls to 1.8 V per cell. Otherwise, the lead sulphate of the plates partly changes to non-active lead sulphate and reduces the life of the battery.
2. The specific gravity to the electrolyte should not be allowed to falls below 1.15 .
3. The battery should never be left standing in a discharged condition; otherwise, sulphation will occur, and the battery cells are permanently damaged.
4. When not in use, the battery must be fully charged and stored in a cool and dry place.
5. Great care should be taken that the acid used as electrolyte should not contain any substantial impurity. It should be colourless when viewed through a 12 cm column.
6. The electrodes must remain completely immersed in the electrolyte, preferably the level of electrolyte should always be about 10 mm above the electrodes.
7. Whenever the level of the electrolyte decreases due to evaporation or gassing, distilled water should be added so as to keep the same concentration of electrolyte.
8. The battery should be charged and discharged at low rate so that its temperature may not rise above $45^{\circ} \mathrm{C}$. The high temperature may buckle the plates and damage the separators and the battery may be totally damaged.
9. The battery terminals should never be short circuited.
10. While charging the polarity must be checked carefully.
11. The room where the batteries are charged should be well ventilated as the atmosphere near the batteries would be charged with corresponding acid fumes.
12. The flames must be kept away from the vent of the battery; otherwise, hydrogen and oxygen produced within the battery cells may get fire.
13. The battery terminals should always be kept clean and periodically greased with Vaseline to prevent corrosion.

### 4.12 APPLICATIONS OF LEAD-ACID BATTERIES

Lead-acid batteries have innumerable commercial applications. Some of the important applications are as follows:

1. Used in automobiles for starting and lighting.
2. For lighting on steam and diesel railway trains.
3. Used at generating stations and substations for the operation of protective devices and for emergency lighting.
4. Used at telephone exchanges.
5. Used for emergency lighting at important places, including hospitals, theatres, banks, etc.
6. Used for lighting purposes in remote rural areas.

### 4.13 NICKEL-IRON ALKALINE CELL

It is also known as Edison cell as it was developed by an American scientist Thomson A Edison in 1909.

### 4.13.1 Construction

It contains two plates, that is, a positive plate (cathode) and a negative plate (anode). The active material of cathode is $\mathrm{Ni}(\mathrm{OH})_{4}$ and of anode is iron $(\mathrm{Fe})$ when fully charged. The two plates are immersed in the electrolyte, a solution of potassium hydroxide $(\mathrm{KOH})$. The specific gravity of the electrolyte is 1.2 . In this case, the container is made of nickel-plated iron to which negative plates are connected. This cell is quit compact as small quantity of electrolyte is used.

### 4.13.2 Working

When the cell is fully charged, its positive plate is of $\mathrm{Ni}(\mathrm{OH})_{4}$ and its negative plate is of iron $(\mathrm{Fe})$. The electrolyte is potassium hydroxide $(\mathrm{KOH})$ of specific gravity 1.2 . When dissolved in water, the KOH is dissociated into potassium $\left(\mathrm{K}^{+}\right)$and hydroxyl $\left(\mathrm{OH}^{-}\right)$ions.

### 4.13.3 Discharging

When a load (resistor) is connected across the terminals of the cell, the hydroxyl ions go to anode and potassium ions go to cathode. The following chemical action takes places during discharging:

At anode: $\quad \mathrm{Fe}+2 \mathrm{OH} \longrightarrow \mathrm{Fe}(\mathrm{OH})_{2}$
At cathode $\mathrm{Ni}(\mathrm{OH})_{4}+2 \mathrm{~K} \longrightarrow 2 \mathrm{KOH}+\mathrm{Ni}(\mathrm{OH})_{2}$
Therefore, cathode is converted from $\mathrm{Ni}(\mathrm{OH})_{4}$ to $\mathrm{Ni}(\mathrm{OH})_{2}$ and anode is converted from iron $(\mathrm{Fe})$ to iron hydroxide $\mathrm{Fe}(\mathrm{OH})_{2}$. However, the strength of the electrolyte remains the same.

### 4.13.4 Recharging

When the cell is put on charging, the hydroxyl $\left(\mathrm{OH}^{-}\right)$ions move towards cathode and potassium $\left(\mathrm{K}^{+}\right)$ions move towards anode. The following chemical actions take place during recharging:

At anode: $\quad \mathrm{Ni}(\mathrm{OH})_{2}+2 \mathrm{OH} \longrightarrow \mathrm{Ni}(\mathrm{OH})_{4}$
At cathode: $\quad \mathrm{Fe}(\mathrm{OH})_{2}+2 \mathrm{~K} \longrightarrow \mathrm{Fe}+2 \mathrm{KOH}$
Therefore, both the electrodes regain their original chemical composition without changing the strength of the electrolyte.

### 4.13.5 Electrical Characteristics

1. The emf of a fully charged cell is 1.4 V which decreases to 1.3 V rapidly. However, the average emf of the cell is 1.2 V which decreases to 1.0 V when fully discharged.
2. The internal resistance of this cell is quite high nearly 5 times to that of a lead-acid cell.
3. The A-H efficiency of this cell is nearly $80 \%$, whereas the W-H efficiency is $60 \%$.

### 4.13.6 Advantages

It has the following advantages in comparison to that of a lead-acid cell:

1. Longer life-about 5 year
2. Its electrolyte $(\mathrm{KOH})$ is not harmful if spilled away.
3. The specific gravity of its electrolyte does not change when discharged; therefore, it can be left in a fully discharged condition for a considerable period of time without damage.
4. Lower weight - nearly half to that of lead-acid cell.
5. It can be discharged and recharged at higher rate for longer period without damage.
6. It can withstand higher temperature.
7. It is more rugged and can withstand more mechanical and electrical stresses.

### 4.13.7 Disadvantages

1. Higher cost-nearly double.
2. As the emf developed per cell is less (only 1.2 V ), more number of cells are required for a particular voltage.
3. Higher internal resistance-nearly 5 times. Therefore, it cannot provide large current and is unsuitable for automobile starting.
4. Lower efficiency.

### 4.14 COMPARISON BETWEEN LEAD-ACID AND NICKEL-IRON ALKALINE CELL

The two cells can be compared on the basis of the following particulars (Table 4.2):
Table 4.2 Comparison of Lead-Acid and Nickel-Iron Cell

| S.No. | Particulars | Lead-Acid cell | Nickel-Iron cell |
| :---: | :---: | :---: | :---: |
| 1 | Active material of electrodes |  |  |
|  | (i) When charged |  |  |
|  | (a) Anode | $\mathrm{PbO}_{2}$ | $\mathrm{Ni}(\mathrm{OH})_{4}$ |
|  | (b) Cathode | Pb | Fe |
|  | (ii) When discharged |  |  |
|  | (a) Anode | $\mathrm{PbSO}_{4}$ | $\mathrm{Ni}(\mathrm{OH})_{2}$ |
|  | (b) Cathode | $\mathrm{PbSO}_{4}$ | $\mathrm{Fe}(\mathrm{OH})_{2}$ |
| 2 | Electrolyte | $\mathrm{H}_{2} \mathrm{SO}_{4}$ | KOH |
| 3 | Specific gravity of electrolyte |  |  |
|  | (i) Charged | 1.29 (nearly) | 1.20 |
|  | (ii) Discharged | 1.15 (nearly) | 1.20 |
| 4 | EMF of each cell |  |  |
|  | (i) Average | 2.0 V | 1.2 V |
|  | (ii) Charged | 2.5 V | 1.4 V |
|  | (iii) Discharged | 1.8 V | 1.0 V |
| 5 | Internal resistance | Low—about $0.1 \Omega$ | High—nearly 5 times to that of lead-acid cell |

## Table 4.2 (Continued)

| 6 | Efficiency |  |  |
| :---: | :---: | :---: | :---: |
|  | (i) Ampere-hour | 85 to 90\% | 75 to 80\% |
|  | (ii) Watt-hour | 70 to 80\% | 60 to 65\% |
| 7 | Life | About 1200 charge and discharge | About 5 year |
| 8 | Cost |  |  |
|  | (i) Initial cost | low | High—nearly double |
|  | (ii) Maintenance cost | High | Low |
| 9 | Mechanical strength | Low | High |
| 10 | Rate of charge or discharge | Low | High |
| 11 | Discharging period | Smaller | Longer |
| 12 | Chemical effects of electrolyte | Harmful | Not harmful |
| 13 | Storage condition | Can be stored in fully charged condition | Can be stored in any condition |
| 14 | Weight for same capacity | Heavier-nearly double | Lighter |

### 4.15 NICKEL-CADMIUM CELL

It was developed by a Swedish scientist Waldemar Jungner in 1899.

### 4.15.1 Construction

Cathode $\quad-\quad \mathrm{Ni}(\mathrm{OH})_{4}$
Anode - Cd (Cadmium)
Electrolyte - KOH (Potassium hydroxide) of specific gravity 1.2.
Its construction is similar to a nickel-iron cell with the difference that its extreme plates are positive instead of negative as in case of nickel-iron cell. Moreover, in this case, the positive plates are electrically connected to the container.

### 4.15.2 Chemical Action during Discharging

At anode $\quad-\quad \mathrm{Cd}+2 \mathrm{OH} \longrightarrow \mathrm{Cd}(\mathrm{OH})_{2}$
At cathode $-\quad \mathrm{Ni}(\mathrm{OH})_{4}+2 \mathrm{~K} \longrightarrow 2 \mathrm{KOH}+\mathrm{Ni}(\mathrm{OH})_{2}$
No change in the specific gravity of the electrolyte.

### 4.15.3 Chemical Action during Recharging

At cathode $-\quad \mathrm{Ni}(\mathrm{OH})_{2}+2 \mathrm{OH} \longrightarrow \mathrm{Ni}(\mathrm{OH})_{4}$
At anode $\quad-\quad \mathrm{Cd}(\mathrm{OH})_{2}+2 \mathrm{~K} \longrightarrow \mathrm{Cd}+2 \mathrm{KOH}$
The two electrodes regain their original chemical composition.

### 4.15.4 Electrical Characteristics

1. EMF—fully charged cell—1.4 V which decreases to 1.3 V rapidly. Average value 1.2 V which decreases to 1.0 V when fully discharged
2. Internal resistance-very low, less than even lead-acid cell
3. Efficiency-Ampere-hour: about $80 \%$; watt-hour: about $65 \%$

### 4.15.5 Advantages

1. Very long active life nearly 20 years
2. Can be stored in any condition, as there is no change in the specific gravity of the electrolyte.
3. These cells can be charged in a short period (one hour)

### 4.15.6 Disadvantages

1. It is very costly
2. Low average emf and therefore, more cells are required for a particular voltage

### 4.16 SMALL NICKEL-CADMIUM CELLS

The various advantages of nickel-cadmium cells such as longer life, low maintenance cost, low internal resistance, etc. prompted the scientists to develop these cells in small sizes. Since the emf developed by these cells is 1.2 V , which is very near to the emf developed by dry cells $(1.5 \mathrm{~V})$, these cells have been developed by scientists of the same size as that of small carbon zinc or dry primary cells (called pencil cells). Today, these small-sized nickel-cadmium cells are used in cordless electric appliances, such as electric shavers, hearing aids, photography equipment, radios, tape recorders, and in space exploration.

The ingredients of a small nickel-cadmium cell are similar to that of a larger type. The plates are woven in the form of a screen, and a paste of active material is pressed into the spaces within the screen. A separator is placed between the positive and negative plate and then rolled in the form of a cylinder. The complete assembly is placed into a small can for protection. The negative plate is connected with the body of the can, which forms the negative terminal. An insulated metal button is placed at the top to which positive plate is connected and forms the positive terminal.

Although the initial cost of a nickel-cadmium cell is very high as compared to a carbon zinc primary cell (i.e., dry cell), but it is less expensive in the long run. This is because nickelcadmium cell can be recharged and has very long life, whereas the dry cell (primary cell) cannot be recharged and has to be discarded when it is discharged.

### 4.16.1 Silver Button Cell

The constructional details of a silver oxide cell are shown in Figure 4.9. It contains cathode of silver oxide and anode of powdered zinc with an electrolyte of alkaline potassium hydroxide. These type of cells are generally made in button size (with typical dimensions of diameter 0.76 to 1.27 cm and thickness 0.2 to 1.5 cm ). The working potential of this cell is 1.5 V .

Chemical reaction during discharging
$\mathrm{AgO}+\mathrm{Zn}+\mathrm{H}_{2} \mathrm{O} \longrightarrow \mathrm{Ag}+\mathrm{Zn}(\mathrm{OH})_{2}$
These cells are leak-proof (sealed), have very small internal resistance, deliver current at a constant voltage of 1.5 V , very handy, and occupy small space.

These cells are best suited for hearing aids, cameras, electronics watches, small electronic toys, and other electronic circuits.


Fig. 4.9 Silver button cell

### 4.17 SOLAR CELLS

A device that converts light energy (e.g., sun light) direct into electrical energy is called solar cell.

The construction of a simple solar cell is shown in Figure 4.10(a). A pure silicon (semiconductor) wafer is doped (the process of adding a suitable impurity to a pure semiconductor is called doping) with a specific amount of arsenic (donor impurity which has 5 electrons in its outermost orbit). This makes it an N-type semiconductor that contains excess of free electrons. The wafer is coated at its top with a very thin layer of silicon doped with appropriate amount of boron (acceptor impurity which has 3 electrons in its outermost orbit). This makes the top layer as a P-type semiconductor. Therefore, a contact surface becomes a $\mathrm{P}-\mathrm{N}$ junction. A spot on the P-type layer and bottom of wafer (N-type material) are tinned for connecting the leads.


Fig. 4.10 Solar cell and Module (a) Solar cell (b) Solar module

When light (sun rays) falls on the top of P-type layer and penetrates into the N-type material just below it the free electrons in N-type material are activated and move across the $\mathrm{P}-\mathrm{N}$ junction into P -type semiconductor. This continuous movement of charge carries (i.e., free electrons from N -type and holes from P-type) constitute electric current.

The operating voltage of one solar cell is about 0.39 V and the current varies between 30 to 40 mA . The power developed by a solar cell depends upon the exposed area and the intensity of light falling on its surface. Since it is difficult to have large silicon crystals heavy power cannot be developed. The maximum output power delivered by a solar cell, with sun light falling directly on a clear day is about 8 to $9 \mathrm{~mW} / \mathrm{cm}^{2}$. To obtain higher voltages and current, a number of solar cells are connected in series parallel combination. A typical solar module having

52, 90 mm diameter silicon cells is shown in Figure 4.10(b). It is interesting to note that an $1100 \times 42.5 \mathrm{~cm}$ module produces 31 W of power to charge a 12 V battery at 13.8 V and 2.25 A . The operating efficiency of a solar cell is very low (about $10 \%$ ). Since they do not deteriorate when not in use, they have very long life (estimated to be thousands of years).

### 4.17.1 Applications

The major applications of solar cells are as follows:

1. To charge nickel-cadmium batteries in satellites
2. To provide power for calculators radio transistors, clocks, etc.
3. To provide power to control devices, such as aperture control for movie cameras, microwave relay stations, etc.

## PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by capacity of a battery? What are its units?
2. Define ampere-hour efficiency of a battery.
3. What are the charge indications of a lead-acid battery?
4. What the precautions are to be taken while charging a battery?
5. Mention five important applications of a lead-acid battery.
6. Name the important components of a Nickel-cadmium alkaline cell.
7. What are the electrical characteristics of a nickel-cadmium cell?
8. If a nickel-cadmium cell rechargeable, show with chemical reactions.
9. What is a solar cell and solar panel (or solar module)?
10. What are the major advantages and applications of solar cells?

## Test Questions

1. How hydrometer is used to determine the charged condition of a lead-acid cell? Explain.
2. Explain the commercial method of charging a lead-acid battery.
3. Write a note on care and maintenance of a lead-acid battery.
4. Explain the working, characteristics, advantages, and applications of a nickel-iron alkaline cell.
5. Write a short note on silver button cell.
6. Write a short note on solar cell.

## Numericals

1. An alkaline cell is discharged at a steady current of 5 A for 12 hour, the average terminal voltage being 1.2 V to restore it to its original state of voltage, a steady current of 4 A for 20 hour is required, the average terminal voltage being 1.44 V . Calculate the ampere-hour and watt-hour efficiencies in this particular case.
(Ans. 75\%, 62.5\%)
2. A lead-acid cell has 23 plates, each $30 \mathrm{~cm} \times 20 \mathrm{~cm}$. The clearance between the neighbouring plates is 1 mm . If the resistivity of the acid is 1.6 ohm cm , find the internal resistance of the cell.
(Ans. $12.12 \mu \Omega$ )
3. A discharged battery is put on charge at 6 A for 4 hour at a mean charging voltage of 13.5 V . It is then discharged in 6 hour at a constant terminal voltage of 12 V through a resistance of $R$ ohm. Calculate the value of $R$ for an ampere-hour efficiency of $85 \%$.
(Ans. $3.53 \Omega$ )
4. A battery of 60 cells in series is charged through a resistance of $4 \Omega$ from a 230 V supply. If the terminal voltage per cell is 2 V and 2.7 V , respectively, at the beginning and at the end of the charge, calculate the charging current at the beginning and at the end of the charge. (Ans. 27.5 A, 17 A)
5. A secondary battery consists of 6 cells in series, each having an emf of 1.8 V at discharge and 2.4 V when fully charged. The internal resistance of each cell is $0.1 \Omega$. If a charging supply of 24 V is available, calculate the value of variable series resistance required to maintain the charging current at a constant value of 10 A throughout the charging period.
(Ans. $0.72 \Omega, 0.36 \Omega$ )

## SUMMARY

1. Electric cell: A device in which chemical energy is converted into electrical energy.
(a) Primary cell: The cells in which chemical action is not reversible.
(b) Secondary cell: The cells in which chemical action is reversible.
2. Battery: A number of cells connected in series, placed in single container are called a battery.
3. Lead-acid cell (Working) - Chemical action
(a) During discharging:

At negative plates - $\mathrm{SO}_{4}^{--}-2 \mathrm{e} \longrightarrow \mathrm{SO}_{4}$ (radical)
At positive plates -

$$
\mathrm{Pb}+\mathrm{SO}_{4} \longrightarrow \mathrm{PbSO}_{4}
$$

$$
2 \mathrm{H}^{+}+2 \mathrm{e}^{4} \longrightarrow 2 \mathrm{H}
$$

$$
\mathrm{PbO}_{2}+2 \mathrm{H} \longrightarrow \mathrm{PbO}+\mathrm{H}_{2} \mathrm{O}
$$

(b) During recharging:

$$
\mathrm{PbO}+\mathrm{H}_{2} \mathrm{SO}_{4} \longrightarrow \mathrm{PbSO}_{4}+\mathrm{H}_{2} \mathrm{O}
$$

At positive plates -

$$
\begin{aligned}
\mathrm{SO}_{4}^{--}-2 \mathrm{e} & \longrightarrow \mathrm{SO}_{4} \text { (radical) } \\
\mathrm{SO}_{4}+\mathrm{H}_{2} \mathrm{O} & \longrightarrow \mathrm{H}_{2} \mathrm{SO}_{4}+\mathrm{O} \\
\mathrm{PbSO}_{4}+\mathrm{O}+\mathrm{H}_{2} \mathrm{O} & \longrightarrow \mathrm{PbO}_{2}+\mathrm{H}_{2} \mathrm{SO}_{4} \\
2 \mathrm{H}^{+}+2 \mathrm{e} & \longrightarrow 2 \mathrm{H} \\
\mathrm{PbSO}_{4}+2 \mathrm{H} & \longrightarrow \mathrm{~Pb}+\mathrm{H}_{2} \mathrm{SO}_{4}
\end{aligned}
$$

At negative plates -
4. Capacity of battery: The quantity of electricity (in Ah) that a battery can deliver in single discharge is called its capacity.
5. Efficiency of a cell or battery:

$$
\eta_{\mathrm{AH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}}} ; \eta_{\mathrm{WH}}=\frac{I_{\mathrm{d}} T_{\mathrm{d}} V_{\mathrm{d}}}{I_{\mathrm{c}} T_{\mathrm{c}} V_{\mathrm{c}}}
$$

6. Charge indications of a lead-acid cell or battery:
(a) Specific gravity of electrolyte: 1.280 to $1.290-100 \%$ charged
(b) Voltage: more than 2 V - fully charged; below 1.8 V - fully discharged.
(c) Colour of plates: anode - chocolate brown and cathode - grey fully charged; both whitish - fully discharged.
7. Nickel-iron alkaline cell: Anode $-\mathrm{Ni}(\mathrm{OH})_{4}$; cathode -Fe ; electrolyte -KOH ; working - chemical action during.
(a) Discharging - At cathode -

$$
2 \mathrm{OH}^{-}-2 \mathrm{e} \longrightarrow 2 \mathrm{OH}
$$

$$
\mathrm{Fe}+2 \mathrm{OH} \longrightarrow \mathrm{Fe}(\mathrm{OH})_{2}
$$

At anode -

$$
2 \mathrm{~K}^{+}+2 \mathrm{e} \longrightarrow 2 \mathrm{~K}
$$

$$
\mathrm{Ni}(\mathrm{OH})_{4}+2 \mathrm{~K} \longrightarrow 2 \mathrm{KOH}+\mathrm{Ni}(\mathrm{OH})_{2}
$$

(b) Charging - At anode $2 \mathrm{OH}^{-}-2 \mathrm{e} \longrightarrow 2 \mathrm{OH}$

At cathode -

$$
\mathrm{Ni}(\mathrm{OH})_{2}+2 \mathrm{OH} \longrightarrow \mathrm{Ni}(\mathrm{OH})_{4}
$$

$$
2 \mathrm{~K}^{+}+2 \mathrm{e} \longrightarrow 2 \mathrm{~K}
$$

$$
\mathrm{Fe}(\mathrm{OH})_{2}+2 \mathrm{~K} \longrightarrow \mathrm{Fe}+2 \mathrm{KOH}
$$

7. Electrical characteristics: (a) EMF of a fully charged cell is 1.4 V which decreases rapidly to 1.3 V . Average emf of the cell is 1.2 V which decreases to 1.0 V when fully discharged.
(b) High internal resistance -5 times to lead-acid cell.
(c) $\eta_{\mathrm{AH}}-80 \%$ (App); $\eta_{\mathrm{wH}}-60 \%$ (App.)

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. The device which converts chemical energy into electrical energy is called $\qquad$ .
2. The internal resistance of a lead-acid cell is $\qquad$ than the internal resistance of a nickel-iron alkaline cell.
3. For electroplating, $\qquad$ supply is required.
4. The specific gravity of the electrolyte is measured to check the charge condition of a $\qquad$ battery.
5. The active material of the positive and negative plates of a charged nickel-cadmium cell is $\qquad$ and $\qquad$ , respectively.
6. When a lead-acid cell is fully charged, the specific gravity of the electrolyte will be $\qquad$ .
7. When a lead-acid cell is fully discharged, the colour of its positive plates will be $\qquad$ .
8. The electrolyte used in a nickel-cadmium cell is $\qquad$ _.
9. The capacity of a battery is measured in $\qquad$ .
10. For battery charging, $\qquad$ supply is required.
11. $\qquad$ is used to measure the specific gravity of electrolyte of a lead-acid battery.
12. When a nickel-iron alkaline cell is discharged, the specific gravity of the electrolyte $\qquad$ .
13. The number of negative plates in a lead-acid cell is one $\qquad$ than the positive plates.
14. A lead-acid cell should not be discharged beyond $\qquad$ volt.
15. The average voltage of one nickel-cadmium cell is $\qquad$ volt.

## OBJECTIVE TYPE QUESTIONS

1. The cell in which chemical action is not reversible is known as
(a) secondary cell
(b) primary cell
(c) voltaic cell
(d) Edison cell
2. The magnitude of emf developed in a cell depends upon
(a) nature of plate's material and electrolyte;
(b) size of plates and quantity of electrolyte;
(c) Both (a) and (b);
(d) None of above.
3. A 2.2 lead-acid cell
(a) is a primary cell
(b) is a secondary cell
(c) is a dry cell
(d) has maximum current rating of about 150 mA
4. The Edison cell
(a) is primary cell
(b) uses $\mathrm{H}_{2} \mathrm{SO}_{4}$ as electrolyte
(c) has 2.9 V output
(d) contains nickel and iron electrodes.
5. The capacity of a cell depends upon
(a) nature of plate's material and electrolyte
(b) size of plates and quantity of electrolyte
(c) Both (a) and (b)
(d) None of above
6. A 12 V lead-acid battery used in a car contains
(a) 10 cells connected is series
(b) 10 cells connected in parallel
(c) 6 cells connected are parallel
(d) 6 cells connected in series.
7. To check the specific gravity of an electrolyte, name of the instrument used is
(a) hydrometer
(b) lactometer
(c) barometer
(d) voltameter
8. The number of negative plates in a lead-acid cell is
(a) one less than positive plates
(b) one more than positive plates
(c) equal to the positive plates
(d) there is no such restriction.
9. The electrolyte used in a nickel-iron battery is
(a) $\mathrm{H}_{2} \mathrm{SO}_{4}$
(b) $\mathrm{K}(\mathrm{OH})_{2}$
(c) NaCl
(d) None of above
10. When a lead-acid battery is put under charging, the vent caps are removed otherwise
(a) the battery will not be charged
(b) the battery may be overheated
(c) heavy pressure may develop in the battery due to gassing which may crack the container
(d) the electrolyte may start boiling.
11. When lead-acid battery is fully charged.
(a) The colour of positive plates will be chocolate brown
(b) The colour of negative plates will be whitish
(c) the specific gravity of electrolyte will be 1.15 .
(d) All above
12. The number of negative plates in a nickel-cadmium cell is
(a) one less than positive plates
(b) one more than positive plates
(c) equal to the positive plates
(d) there is no such restriction.
13. When a nickel-iron cell is fully charged
(a) the active material of positive and negative plates will be $\mathrm{Ni}(\mathrm{OH})_{4}$ and Fe , respectively.
(b) the potential difference across the terminals of the cell will be 1.4 V
(c) the specific gravity of the electrolyte $(\mathrm{KOH})$ will be 1.2 .
(d) All above.

## NUMERICALS

1. A lead-acid cell maintains a constant current of 2 A for 15 hour before its terminal voltage falls to 1.8 V . What is the capacity of the cell?
(Ans. 30 Ah )
2. A lead-acid cell has 13 plates, each $25 \mathrm{~cm} \times 30 \mathrm{~cm}$. The clearance between the neighbouring plates is 1 mm . If the specific resistance of the acid is 1.6 ohm cm , determine the internal resistance of the cell.
(Ans. $17.78 \times 10^{-6} \Omega$ )
3. Calculate the ampere-hour and watt-hour efficiencies of a secondary cell having 20 hour charge rate of 10 A and delivery rate of 5 A for 36 hour with mean terminal voltage of 1.96 V . The terminal voltage during charging has a mean value of 2.35 V .
(Ans. 90\%; 75.06\%)
4. Thirty-five lead-acid cells, each of discharging capacity 100 Ah at 10 -hour rate are to be fully charged at constant current for 8 hour. The DC supply is 120 V , the ampere-hour efficiency is $80 \%$ and the emf of each cell at the beginning and end of charge is 1.9 V and 2.6 V , respectively. Calculate the maximum and minimum value of the necessary resistance. Ignore internal resistance of the cells.
(Ans. $3.42 \Omega ; 1.86 \Omega$ )
5. Fifty cells of 1.8 V each, having an internal resistance of 0.01 ohm , are charged form 120 V DC supply. Determine the charging current and power drawn from the supply mains at the beginning if the external resistance the circuit is 1.25 ohms. What will be the current drawn from the supply mains when the cells are charged to 2.4 V each.
(Ans. 17.143 A, 2057 W: zero)
6. Four lamps in parallel, each requiring 0.5 A at 8 V are to be lighted with a 12 V battery of internal resistance $0.5 \Omega$. Calculate the value of additional resistor required and draw the circuit diagram showing the method of connection.
(Ans. $1.5 \Omega$ )

## VIVA-VOCE/REASONING QUESTIONS

1. Usually, a large space is left between the plates and the bottom of the battery container. Why?
2. Large number of holes are provided in the separator (rubber sheet) placed between the positive and negative plates of lead acid battery. Why?
3. While checking the conditions of battery, it is seen that the float dips at different levels at different specific gravity of electrolyte. Why?
4. Nickel-cadmium cells or batteries are preferred over lead-acid cells or batteries. Why?

## SHORT ANSWER TYPE QUESTIONS

1. How will you define an electric cell?
2. What are the essentials for forming an electric cell?
3. How will you differentiate between primary and secondary cells?
4. What do you mean by emf and terminal voltage at a cell?
5. What is the necessity of grouping of electric cells?
6. Name the parts of a lead-acid battery. What is the function of container?
7. Why many holes are provided in the separator?
8. Write down the chemical reactions that took place in the lead-acid battery during discharging.
9. Write down the chemical reactions that took place in a lead-acid battery during recharging.
10. What do you mean by capacity of a battery, what are the factors on which it depends?
11. Mention the charge indications of a lead-acid battery.
12. Why and how do we recharge the lead-acid battery?
13. How do we care a lead-acid battery? Write five major points.
14. What are the applications of lead-acid batteries?
15. Explain the construction and working of a small nickel-cadmium cell.
16. What is solar cell? Give its major applications.

## TEST QUESTIONS

1. What are the essentials of an electric cell? How the cells are classified?
2. Describe the construction and action of a lead-acid cell giving the nature of the chemical changes that occur at the electrodes both during charging and discharging.
3. Explain the construction and working of a lead-acid storage battery.
4. How can you estimate the efficiency of lead-acid cell when the charging the discharging characteristics are available?
5. State the precautions to be observed in the use of a lead-acid cell.
6. Explain the constructional feature of a typical nickel-iron alkaline cell.
7. Explain the constructional details of a nickel alkaline cell and compare its performance with that of lead-acid cell.
8. Explain constant current and constant voltage method of charging a battery. Which method is employed commercially and why?
9. Explain each term of specification of a lead-acid battery given below:
$8 \mathrm{~A} \mathrm{H} ; 20$ Hour; 1.8 V per cell; $15^{\circ} \mathrm{C}$
10. Give any five important application of a lead-acid battery.
11. Explain construction and working of a nickel-cadmium accumulator and mention its electrical characteristics.
12. Give comparison between nickel-cadmium and lead-acid cell.

## answers

## Fill in the Blanks

1. an electrical cell
2. less
3. $\mathrm{Ni}(\mathrm{OH})_{4} ; \mathrm{Cd}$
4. 1.28 to 1.29
5. A-H
6. DC
7. DC
8. lead-acid
9. more
10. Whitish
11. KOH
12. remain the same
13. Hydrometer
14. 1.8
15. 1.2

Objective Type Questions

1. (b)
2. (a)
3. (b)
4. (d)
5. (b)
6. (d)
7. (a)
8. (b)
9. (d)
10. (c)
11. (a)
12. (a)
13. (d)


## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What are magnetic circuits?
* How magnetic circuits are similar to electric circuits?

What is magnetic hysteresis and hysteresis losses?
*What is the meaning of electromagnetic induction?
What is dynamically and statically induced emf?
*What is self and mutual inductor?
What are eddy current losses and how they can be reduced?

### 5.1 INTRODUCTION

The operation of all electrical machines such as DC machines, transformers, synchronous machines, and induction motors rely upon their magnetic circuits. The closed path followed by the magnetic lines of force is called a magnetic circuit. The operation of all the electrical devices (e.g., transformers, generators, motors, etc.) depends upon the magnetism produced by their magnetic circuits. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully. In this chapter, we shall focus our attention on the basic fundamentals of magnetic circuits and their solution.

### 5.2 MAGNETIC FIELD AND ITS SIGNIFICANCE

The region around the magnet where its poles exhibit a force of attraction or repulsion is called magnetic field. The existence of the magnetic field at a point around the magnet can also be determined
by placing a magnetic needle at that point, as shown in Figure 5.1. Although magnetic lines of force have no real existence and are purely imaginary, yet their concept is very useful to understand various magnetic effects. It is assumed (because of their effects) that the magnetic lines of force possess the following important properties:

1. The direction of magnetic lines of force is from N-pole to the S-pole outside the magnet. However, inside the magnet, their direction is from S-pole to N-pole.
2. They form a closed loop.
3. Their tendency is to follow the least reluctance path.
4. They act like stretched cords, always trying to shorten themselves.
5. They never intersect each other.

Fig. 5.1 Magnetic lines of force around a bar magnet

6. They repel each other when they are parallel and are in the same direction.
7. They remain unaffected by non-magnetic materials.

### 5.3 MAGNETIC CIRCUIT AND ITS ANALYSIS

The closed path followed by magnetic flux is called a magnetic circuit. A magnetic circuit usually consists of magnetic materials having high permeability (e.g., iron, soft steel, etc.). In this circuit, magnetic flux starts from a point and finishes at the same point after completing its path.

Figure 5.2 shows a solenoid having $N$ turns wound on an iron core (ring). When current $I$ ampere is passed through the solenoid, magnetic flux $\phi \mathrm{Wb}$ is set up in the core.

Let $l=$ mean length of magnetic circuit in m ;


Fig. 5.2 Magnetic circuit
$a=$ area of cross-section of core in $\mathrm{m}^{2}$;
$\mu_{\mathrm{r}}=$ relative permeability of core material.
Flux density in the core material, $B=\frac{\phi}{a} \mathrm{~Wb} / \mathrm{m}^{2}$
Magnetising force in the core material

$$
H=\frac{B}{\mu_{0} \mu_{\mathrm{r}}}=\frac{\phi}{a \mu_{0} \mu_{\mathrm{r}}} \mathrm{AT} / \mathrm{m}
$$

According to work law, the work done in moving a unit pole once round the magnetic circuit (or path) is equal to the AT enclosed by the magnetic circuit.

Therefore,

$$
H l=N I \quad \text { or } \quad \frac{\phi}{a \mu_{0} \mu_{\mathrm{r}}} \times l=N I \quad \text { or } \quad \phi=\frac{N I}{\left(l / a \mu_{0} \mu_{\mathrm{r}}\right)} \mathrm{Wb}
$$

This expression reveals that the amount of flux set up in the core is

1. directly proportional to $N$ and $I$, that is, $N I$ called magneto-motive force (mmf). It shows that the flux increases if either of the two increases and vice versa.
2. inversely proportional to $\left(l / a \mu_{0} \mu_{\mathrm{r}}\right)$ called reluctance of the magnetic path. In fact, reluctance is the opposition offered to the magnetic flux by the magnetic path. The lower is the reluctance, the higher will be the flux and vice versa.

Thus,

$$
\text { Flux }=\frac{\mathrm{mmf}}{\text { Reluctance }}
$$

It may be noted that the abovementioned expression has a strong resemblance to Ohm's law for electric current ( $I=\mathrm{emf} /$ resistance). The mmf is analogous to emf in electric circuit, reluctance is analogous to resistance, and flux is analogous to current. Because of this similarity, the abovementioned expression is sometimes referred to as Ohm's law of magnetic circuits.

### 5.4 IMPORTANT TERMS

Generally, while studying magnetic circuits, we come across the following terms:

1. Magnetic field: The region around a magnet where its poles exhibit a force of attraction or repulsion is called magnetic field.
2. Magnetic flux ( $\phi$ ): The amount of magnetic lines of force set up in a magnetic circuit is called magnetic flux. Its unit is weber (Wb). It is analogous to electric current, $I$, in electric circuit.

The magnetic flux density at a point is the flux per unit area at right angles to the flux at that point.

Generally, it is represented by letter ' $B$ '. Its unit is $\mathrm{Wb} / \mathrm{m}^{2}$ or Tesla, that is,

$$
B=\frac{\phi}{A} \mathrm{~Wb} / \mathrm{m}^{2} \text { or } \mathrm{T}\left(1 \mathrm{~Wb} / \mathrm{m}^{2}=1 \times 10^{4} \mathrm{~Wb} / \mathrm{cm}^{2}\right)
$$

3. Permeability: The ability of a material to conduct magnetic lines of force through it is called the permeability of that material.

It is generally represented by $\mu(\mathrm{mu}$, a Greek letter). The greater the permeability of a material, the greater is its conductivity for the magnetic lines of force and vice versa. The permeability of air or vacuum is the poorest and is represented as $\mu_{0}$ (where $\mu_{0}=4$ $\left.\pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)$.
4. Relative permeability: The absolute (or actual) permeability $\mu$ of a magnetic material is much greater than absolute permeability of air $\mu_{0}$. The relative permeability of a magnetic material is given in comparison with air or vacuum.

Hence, the ratio of the permeability of material $\mu$ to the permeability of air or vacuum $\mu_{0}$ is called the relative permeability $\mu_{\mathrm{r}}$ of the material.

Therefore,

$$
\mu_{\mathrm{r}}=\frac{\mu}{\mu_{0}} \quad \text { or } \quad \mu=\mu_{0} \mu_{\mathrm{r}}
$$

Obviously, the relative permeability of air would be $\mu_{0} \mu_{\mathrm{r}}=1$. The value of relative permeability of all the non-magnetic materials is also 1 . However, its value is as high as 8,000 for soft iron, whereas its value for Mu metal (iron $22 \%$ and nickel $78 \%$ ) is as high as 120,000 .
5. Magnetic field intensity: The force acting on a unit north pole ( 1 Wb ) when placed at a
point in the magnetic field is called the magnetic intensity of the field at that point. It is denoted by $H$. In magnetic circuits, it is defined as mmf per unit length of the magnetic path. It is denoted by $H$, and mathematically, it can be given as

$$
H=\frac{\mathrm{mmf}}{\text { Length of magnetic path }}=\frac{N I}{l} \mathrm{AT} / \mathrm{m}
$$

6. Magneto-motive force (mmf): The magnetic pressure that sets up or tends to set up magnetic flux in a magnetic circuit is called magneto-motive force. As per work law, it may be defined as the work done in moving a unit magnetic pole ( 1 Wb ) once round the magnetic circuit is called mmf. In general,

$$
\mathrm{mmf}=N I \text { ampere turns }(\text { or AT })
$$

It is analogous to emf in an electric circuit.
7. Reluctance ( $\boldsymbol{S}$ ): The opposition offered to the magnetic flux by a magnetic circuit is called its reluctance.

It depends upon length $(l)$, area of cross section $(a)$, and permeability $\left(\mu=\mu_{0} \mu_{\mathrm{r}}\right)$ of the material that makes up the magnetic circuit. It is measured in AT/Wb.

Reluctance,

$$
S=\frac{l}{a \mu_{0} \mu_{\mathrm{r}}}
$$

It is analogous to resistance in an electric circuit.
8. Permeance: It is a measure of the ease with which flux can be set up in the material. It is just reciprocal of reluctance of the material and is measured in $\mathrm{Wb} / \mathrm{AT}$ or Henry.

$$
\text { Permeance }=\frac{1}{\text { Reluctance }}=\frac{a \mu_{0} \mu_{\mathrm{r}}}{l} \mathrm{~Wb} / \mathrm{AT} \text { or } \mathrm{H}
$$

It is analogous to conductance in an electric circuit.
9. Reluctivity: It is specific reluctance and analogous to resistivity in an electric circuit.

### 5.5 COMPARISON BETWEEN MAGNETIC AND ELECTRIC CIRCUITS

Although magnetic and electric circuits have many points of similarity, but still they are not analogous in all respects. A comparison of the two circuits is given in Table 5.1.

Table 5.1 Comparison between Magnetic and Electric Circuits
S.No. Magnetic circuits Electrical Circuits


Fig. 5.3 Magnetic circuit


Fig. 5.4 Electric circuit

## Similarities

1. The closed path for magnetic flux is called magnetic circuit.
2. Flux $=m m f /$ reluctance
3. Flux, $\phi$ in Wb
4. mmf in AT
5. Reluctance, $S=\frac{1}{a \mu}=\frac{l}{a \mu_{0} \mu_{\mathrm{r}}}$ AT/WB
6. $\quad$ Permeance $=1$ reluctance
7. Permeability, $\mu$
8. Reluctivity
9. Flux density, $B=\frac{\phi}{a} \mathrm{~Wb} / \mathrm{m}^{2}$
10. Magnetic intensity, $H=$ NI/I

The closed path for electric current is called electric circuit.
Current $=$ emf/resistance
Current, I in ampere
emf in $V$
Resistance, $R=\rho \frac{l}{a} \Omega$ or $R=\frac{1}{\sigma} \frac{l}{a} \Omega$
Conductance $=1 /$ resistance
Conductivity, $\sigma=1 / \rho$
Resistivity
Current density, $J=\frac{1}{a} \mathrm{~A} / \mathrm{m}^{2}$
Electric intensity, $E=V / d$

## Dissimilarities

1. In fact, the magnetic flux does not flow but it sets up in the magnetic circuit (basically, molecular poles are aligned).
2. For magnetic flux, there is no perfect insulator. It can be set up even in the non-magnetic materials such as air, rubber, and glass with reasonable mmf
3. The reluctance $(S)$ of a magnetic circuit is not constant rather it varies with the value of $B$. It is because the value of $\mu_{\mathrm{r}}$ changes considerably with the change in $B$.
4. Once the magnetic flux is set up in a magnetic circuit, no energy is expanded. However, a small amount of energy is required at the start to create flux in the circuit.

The electric current (electrons) actually flows in an electric circuit.

For electric current, there are large number of perfect insulators such as glass, air, and rubber that do not allow it to follow through them under normal conditions.

The resistance ( $R$ ) of an electric circuit is almost constant as its value depends upon the value of $r$, which is almost constant. However, the value of $r$ and $R$ may vary slightly if temperature changes.
Energy is expanded continuously, so long as the current flows through an electric circuit. This energy is dissipated in the form of heat.

### 5.6 AMPERE TURNS CALCULATIONS

In a magnetic circuit, flux produced is given as
or

$$
\phi=\frac{\mathrm{mmf}}{\text { Reluctance }}=\frac{N I}{l / a \mu_{0} \mu_{\mathrm{r}}}
$$

$$
\text { AT required, } N I=\frac{\phi}{a \mu_{0} \mu_{\mathrm{r}}}=\frac{B}{\mu_{0} \mu_{\mathrm{r}}} l=H I
$$

### 5.7 SERIES MAGNETIC CIRCUITS

A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a series magnetic circuit. Such series magnetic circuit (composite circuit) is shown in Figure 5.5.

Total reluctance of the magnetic circuit,

$$
\begin{aligned}
& S=S_{1}+S_{2}+S_{3}+S_{\mathrm{g}} \\
= & \frac{l_{1}}{a_{1} \mu_{0} \mu_{\mathrm{r} 1}}+\frac{l_{2}}{a_{2} \mu_{0} \mu_{\mathrm{r} 2}}+\frac{l_{3}}{a_{3} \mu_{0} \mu_{\mathrm{r} 3}}+\frac{l_{\mathrm{g}}}{a_{\mathrm{g}} \mu_{0}}
\end{aligned}
$$

Total $\mathrm{mmf}=\phi S$

$$
\begin{aligned}
& \quad=\phi\left(\frac{l_{1}}{a_{1} \mu_{0} \mu_{\mathrm{r} 1}}+\frac{l_{2}}{a_{2} \mu_{0} \mu_{\mathrm{r} 2}}+\frac{l_{3}}{a_{3} \mu_{0} \mu_{\mathrm{r} 3}}+\frac{l_{\mathrm{g}}}{a_{\mathrm{g}} \mu_{0}}\right) \\
& \quad=\frac{B_{1} l_{1}}{\mu_{0} \mu_{\mathrm{r} 1}}+\frac{B_{2} l_{2}}{\mu_{0} \mu_{\mathrm{r} 2}}+\frac{B_{3} l_{3}}{\mu_{0} \mu_{\mathrm{r} 3}}+\frac{B_{\mathrm{g}} l_{\mathrm{g}}}{\mu_{0}} \\
& =H_{1} l_{1}+H_{2} l_{2}+H_{3} l_{3}+H_{\mathrm{g}} l_{\mathrm{g}}
\end{aligned}
$$



Fig. 5.5 Series magnetic circuit

### 5.8 PARALLEL MAGNETIC CIRCUITS

A magnetic circuit that has two or more than two paths for the magnetic flux is called a parallel magnetic circuit. Its behaviour can be just compared to a parallel electric circuit.

Figure 5.6 shows a parallel magnetic circuit. A current-carrying coil is wound on the central limb AB. This coil sets up a magnetic flux $\phi_{1}$ in the central limb that is further divided into two paths, that is, path ADCB that carries flux $\phi_{2}$ and path AFEB that carries flux $\phi_{3}$.

It is clear that $\phi_{1}=\phi_{2}+\phi_{3}$
The two magnetic paths ADCB and


Fig. 5.6 Parallel magnetic circuit AFEB are in parallel. The AT required for this parallel circuit is equal to the AT required for any one of the paths. If

$$
\begin{aligned}
& S_{1}=\text { reluctance of path BA, that is, } l_{1} / a_{1} \mu_{0} \mu_{\mathrm{r} 1} \\
& S_{2}=\text { reluctance of path ADCB, that is, } l_{2} / a_{2} \mu_{0} \mu_{\mathrm{r} 2} \\
& S_{3}=\text { reluctance of path AFEB, that is, } l_{3} / a_{3} \mu_{0} \mu_{\mathrm{r} 3}
\end{aligned}
$$

Therefore, total mmf required $=\mathrm{mmf}$ required for path $\mathrm{BA}+\mathrm{mmf}$ required path ADCB or path AFEB

$$
\text { Total } \mathrm{mmf} \text { or AT }=\phi_{1} S_{1}+\phi_{2} S_{2}=\phi_{1} S_{1}+\phi_{3} S_{3}
$$

### 5.9 LEAKAGE FLUX



Fig. 5.7 Magnetic circuit with leakage flux

The magnetic flux that does not follow the intended path in a magnetic circuit is called leakage flux.

When some current is passed through a solenoid, as shown in Figure 5.7, magnetic flux is produced by it. Most of this flux is set up in the magnetic core and passes through the air gap (an intended path). This flux is known as useful flux $\phi_{\mathrm{u}}$. However, some of the flux is just set up around the coil and is not utilised for any work. This flux is called leakage flux $\phi_{1}$.

Total flux produced by the solenoid.

$$
\phi=\phi_{\mathrm{u}}+\phi_{1}
$$

Leakage co-efficient or leakage factor: the ratio of total flux $(\phi)$ produced by the solenoid to the useful flux $\left(\phi_{\mathrm{u}}\right)$ set up in the air gap is known as leakage co-efficient. It is generally represented by letter ' $\lambda$ '.

Therefore, leakage co-efficient, $\lambda=\frac{\phi}{\phi_{\mathrm{u}}}$

### 5.9.1 Fringing

It may be seen in Figure 5.7 that the useful flux when sets up in the air gap, it tends to bulge outwards at $b$ and $b^{\prime}$ since the magnetic lines set up in the same direction repel each other. This increases the effective area in the air gap and decreases the flux density. This effect is known as fringing. The fringing is directly proportional to the length of the air gap.

## Example 5.1

An iron ring of 400 cm mean circumference is made from round iron of cross section $20 \mathrm{~cm}^{2}$. Its permeability is 500 . If it is wound with 400 turns, what current would be required to produce a flux of 0.001 Wb ?

## Solution:



Fig. 5.8 Given magnetic circuit

The magnetic circuit is shown in Figure 5.8.
Mean length of magnetic path, $l_{\mathrm{m}}=400 \mathrm{~cm}=4 \mathrm{~m}$ Area of $X$-section of iron ring, $a=20 \times 10^{-4} \mathrm{~m}^{2}$
Absolute permeability, $\mu_{0}=4 \pi \times 10^{-7}$
Now, mmf $=$ flux $\times$ reluctance

$$
\begin{aligned}
N I & =\phi \times \frac{l_{\mathrm{m}}}{a \mu_{0} \mu_{\mathrm{r}}} \\
400 \times I & =0.001 \times \frac{4}{20 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 500} \\
\therefore \quad \text { Current, } I & =\frac{0.001 \times 4}{20 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 500 \times 400} \\
\text { Current, } I & =7.958
\end{aligned}
$$

## Example 5.2

An electromagnet has an air gap of 4 mm and flux density in the gap is $1.3 \mathrm{~Wb} / \mathrm{m}^{2}$. Determine the AT for the gap.
(UPTU 2006-2007)

## Solution:

Here, $\quad l_{\mathrm{g}}=4 \mathrm{~mm}=0.4 \mathrm{~cm}=4 \times 10^{-3} \mathrm{~m} ; \quad B_{\mathrm{g}}=1.3 \mathrm{~Wb} / \mathrm{m}^{2}$
Ampere turns required for the gap

$$
=H_{\mathrm{g}} \times I_{\mathrm{g}}=\frac{B_{\mathrm{g}}}{\mu_{0}} \times l_{\mathrm{g}}=\frac{1.3}{4 \pi \times 10^{-7}} \times 4 \pi \times 10^{-3}=4,136.83 \mathrm{AT}
$$

## Example 5.3

A coil of insulated wire of 500 turns and of resistance $4 \Omega$ is closely wound on an iron ring. The ring has a mean diameter of 0.25 m and a uniform cross-sectional area of $700 \mathrm{~mm}^{2}$. Calculate the total flux in the ring when a DC supply of 6 V is applied to the ends of the winding. Assume a relative permeability of 550 .
(UPTU July 2002)

## Solution:

Mean length of the iron ring,

$$
l=\pi D=\pi \times 0.25=0.25 \pi \mathrm{~m}
$$

Area of cross section,

$$
\alpha=700 \mathrm{~mm}^{2}=700 \times 10^{-6} \mathrm{~m}^{2}
$$

Current flowing through the coil,

$$
\begin{aligned}
I & =\frac{\text { Voltage applied across coil }}{\text { Resistance of coil }} \\
& =\frac{6}{5}=1.5 \mathrm{~A}
\end{aligned}
$$

Total flux in the ring, $\phi=\frac{N I}{l / a \mu_{0} \mu_{\mathrm{r}}}=\frac{N I \times a \mu_{0} \mu_{\mathrm{r}}}{l}$


Fig. 5.9 Given magnetic circuit

$$
=\frac{500 \times 1.5 \times 700 \times 10^{-6} \times 4 \pi \times 10^{-7} \times 550}{0.25 \pi}=0.462 \mathrm{mWb}
$$

## Example 5.4

What are the similarities between electrical circuits and magnetic circuits? An iron ring of mean length 50 cm and relative permeability 300 has an air gap of 1 mm . If the ring is provided with winding of 200 turns and a current of 1 A is allowed to flow through, find the flux density across the air gap.
(UPTU June 2001)

## Solution:

Here, $l_{\mathrm{i}}=50 \mathrm{~cm}=0.5 \mathrm{~m} ; \mu_{\mathrm{r}}=300 ; l_{\mathrm{g}}=1 \mathrm{~mm}=0.001 \mathrm{~m} ; N=200$ turns; $I=1 \mathrm{~A}$
Ampere turns required for air gap $=\frac{B}{\mu_{\mathrm{o}}} l_{\mathrm{g}}$
Ampere turns required for iron ring $=\frac{B}{\mu_{0} \mu_{\mathrm{r}}} l_{\mathrm{i}}$
or

$$
\begin{equation*}
\text { total AT required }=\frac{B}{\mu_{\mathrm{o}}} l_{\mathrm{g}}+\frac{B}{\mu_{\mathrm{o}} \mu_{\mathrm{r}}} l_{\mathrm{i}} \tag{5.1}
\end{equation*}
$$

Ampere turns provided by the coil $=N I=200 \times 1=200$
Equating Equations (5.1) and (5.2), we get
or

$$
\begin{aligned}
200 & =\frac{B}{\mu_{\mathrm{o}}}\left(l_{\mathrm{g}}+\frac{l_{\mathrm{i}}}{\mu_{\mathrm{r}}}\right)=\frac{B}{\mu_{\mathrm{o}}}\left(0.01+\frac{0.5}{300}\right) \\
& =\frac{B}{\mu_{\mathrm{o}}}(0.001+0.00167)=\frac{B}{\mu_{\mathrm{o}}} \times 0.00267
\end{aligned}
$$

or

$$
\text { Flux density, } B=\frac{200 \times \mu_{0}}{0.00267}=\frac{200 \times 4 \pi \times 10^{-7}}{0.00267}=0.09425 \mathrm{~T}
$$

## Example 5.5

A coil of 1,000 turns is wound on a laminated core of steel having a cross section of $5 \mathrm{~cm}^{2}$. The core has an air gap of 2 mm cut at right angle. What value of current is required to have an airgap flux density of 0.5 T ? The permeability of steel may be taken as infinity. Determine the coil inductance.
(UPTU December 2003)

## Solution:

Here, $N=1,000$ turns; $a=5 \mathrm{~cm}^{2}=5 \times 10^{-4} \mathrm{~m}^{2}$;

$$
l_{\mathrm{g}}=2 \mathrm{~mm}=2 \times 10^{-3} \mathrm{~m} ; B=0.5 \mathrm{~T} ; \mu_{\mathrm{r}}=\infty
$$

Total AT required, $\quad \mathrm{AT}=\frac{B}{\mu_{0}} l_{\mathrm{g}}+\frac{B}{\mu_{0} \mu_{\mathrm{r}}} l_{\mathrm{i}}=\frac{0.5}{4 \pi \times 10^{-7}} \times 2 \times 10^{-3}+0=796$

$$
\left(\text { As } \mu_{\mathrm{r}}=\infty ; \frac{B}{\mu_{0} \mu_{\mathrm{r}}} \times l_{\mathrm{i}}=0\right)
$$

Current required, $\quad I=\frac{\mathrm{AT}}{I}=\frac{796}{1,000}=0.796 \mathrm{~A}$
Inductance of coil, $\quad L=\frac{N \phi}{I}=\frac{N \times B \times \alpha}{I}=\frac{1,000 \times 0.5 \times 5 \times 10^{-4}}{0.796}$

$$
=0.314 \mathrm{H}
$$

## Example 5.6

A flux density of $1.2 \mathrm{~Wb} / \mathrm{m}^{2}$ is required in 2-mm air gap of an electromagnet having an iron path 1 m long. Calculate the magnetising force and current required if the electromagnet has 1,273 turns. Assume relative permeability of iron to be 1,500.
(PTU)

## Solution:

Flux density,

$$
B=1.2 \mathrm{~Wb} / \mathrm{m}^{2}
$$

Relative permeability of iron, $\mu_{\mathrm{r}}=1,500$
Number of turns,
$N=1,273$
Length of iron path,
$l_{\mathrm{i}}=1 \mathrm{~m}$
Length of air gap,
$l_{\mathrm{g}}^{1}=2 \mathrm{~mm}=0.002 \mathrm{~m}$

Magnetising force for iron $\quad H_{\mathrm{i}}=\frac{B}{\mu_{0} \mu_{\mathrm{r}}}=\frac{1.2}{4 \pi \times 10^{-7} \times 1,500}=636.6 \mathrm{AT} / \mathrm{m}$
Magnetising force for air gap, $H_{\mathrm{g}}=\frac{B}{\mu_{0}}=\frac{1.2}{4 \pi \times 10^{-7}}=954,900 \mathrm{AT} / \mathrm{m}$
AT required for iron path
$=H_{\mathrm{i}} l_{\mathrm{i}}=636.6 \times 1=636.6$
AT required for air gap
$=H_{\mathrm{g}} l_{\mathrm{g}}=954,900 \times 0.002=1,909.8$
Total AT

$$
=636.6+1,909.8=2,546.4
$$

Current required,

$$
I=\frac{\operatorname{Total} \mathrm{AT}}{N}=\frac{2,546.4}{1.273}=2 \mathrm{~A}
$$

## Example 5.7

Estimate the number of AT necessary to produce a flux of $1,00,000$ lines round an iron ring of $6 \mathrm{~cm}^{2}$ cross section and 20 cm mean diameter having an air gap 2 mm wide across it. The permeability of the iron may be taken as 1,200 . Neglect the leakage flux outside the $2-\mathrm{mm}$ air gap.
(PTU)

## Solution:

The magnetic circuit is shown in Figure 5.10.
Area of cross section of the ring, $a=6 \mathrm{~cm}^{2}=6 \times 10^{-4} \mathrm{~m}^{2}$
Mean diameter of the ring, $D_{\mathrm{m}}=20 \mathrm{~cm}=0.2 \mathrm{~m}$
Length of air gap, $l_{\mathrm{g}}=2 \mathrm{~mm}=2 \times 10^{-3} \mathrm{~m}$
Flux set up in the ring, $\phi=100,000$ lines

$$
=100,000 \times 10^{-8}=0.001 \mathrm{~Wb}
$$

Relative permeability of iron, $\mu_{\mathrm{r}}=1,200$
Mean length of ring, $l_{\mathrm{m}}=\pi D=\pi \times 0.2$

$$
=0.6283 \mathrm{~m}
$$

Length of air gap, $\quad l_{\mathrm{g}}=0.002 \mathrm{~m}$
Length of iron path, $l_{\mathrm{i}}=0.6283-0.002$

$$
=0.6263 \mathrm{~m}
$$

Now, $\mathrm{mmf}=$ flux $\times$ reluctance
Ampere turns required for iron path,

$$
\begin{aligned}
\mathrm{AT}_{\mathrm{i}}=\phi \times \frac{l_{\mathrm{i}}}{a \mu_{0} \mu_{\mathrm{r}}} & =0.001 \times \frac{0.6263}{6 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 1,200} \\
& =692.21 \mathrm{AT}
\end{aligned}
$$

Ampere turns required for air gap,

$$
\mathrm{AT}_{\mathrm{g}}=\phi \times \frac{l_{\mathrm{g}}}{a \mu_{0}}=0.001 \times \frac{0.002}{6 \times 10^{-4} \times 4 \pi \times 10^{-7}}=2,652.58 \mathrm{AT}
$$

Total AT required to produce the given flux

$$
=\mathrm{AT}_{\mathrm{i}}+\mathrm{AT}_{\mathrm{g}}=692.21+2,652.58=3,344.79 \mathrm{AT}
$$

## Example 5.8

A wrought iron bar 30 cm long and 2 cm in diameter is bent into a circular shape, as given in Figure 5.11. It is then wound with 500 turns of wire. Calculate the current required to produce a flux of 0.5 mWb in magnetic circuit with an air gap of $1 \mathrm{~mm} ; \mu_{\mathrm{r}}$ (iron) $=4,000$ (assume constant).
(UPTU 2004-2005)

## Solution:



Fig. 5.11 Magnetic circuit with air-gap

Here, $I_{\mathrm{i}}=30 \mathrm{~cm}=0.3 \mathrm{~m}$;
Diameter, $\quad d=2 \mathrm{~cm}$
$\therefore \quad$ Area, $\alpha=\frac{\pi}{4} d^{2}=\frac{\pi(2)^{2}}{4} \times 10^{-4} \mathrm{~m}^{2}$

$$
=\pi \times 10^{-4} \mathrm{~m}^{2}
$$

$$
\phi=0.5 \mathrm{mWb}=0.5 \times 10^{-3} \mathrm{~Wb}
$$

$$
N=500 \text { turns }
$$

$$
N I=\frac{\phi}{\alpha}\left[\frac{l_{\mathrm{i}}}{\mu_{0} \mu_{\mathrm{r}}}+\frac{l_{\mathrm{g}}}{\mu_{0}}\right]
$$

$$
I=\frac{0.5 \times 10^{-3}}{500 \times \pi \times 10^{-4}}\left[\frac{0.3}{4 \pi \times 10^{-7} \times 400}+\frac{0.001}{4 \pi \times 10^{-7}}\right]=4.433 \mathrm{~A}
$$

## Example 5.9

A circular ring 20 cm in diameter has an air gap 1 mm wide cut in it. The area of a cross section of the ring is $3.6 \mathrm{~cm}^{2}$. Calculate the value of DC needed in a coil of 1,000 turns uniformly wound round the ring to create a flux of 0.5 mWb in the air gap. Neglect fringing and assume relative permeability for iron as 650 .
(UPTU 2006-2007)

## Solution:

Here, area of cross section of the ring, $a=3.6 \mathrm{~cm}^{2}=3.6 \times 10^{-4} \mathrm{~m}^{2}$
Number of turns of the coil,
Flux set up,
Relative permeability of iron,

$$
N=1,000
$$

$\phi=0.5 \mathrm{mWb}=0.5 \times 10^{-3} \mathrm{~Wb}$
Length of air gap,

$$
\mu_{\mathrm{r}}=650
$$

$l_{\mathrm{g}}=1 \mathrm{~mm}=1 \times 10^{-3} \mathrm{~m}$
Mean diameter of ring $=20 \mathrm{~cm}=20 \times 10^{-2} \mathrm{~m}$
$\therefore \quad$ Length of iron path $l_{\mathrm{i}}=\rho D=\rho \times 20 \times 10^{-2} \mathrm{~m}=62.83 \times 10^{-4} \mathrm{~m}$
Reluctance of iron path $=\frac{l_{\mathrm{i}}}{\mu_{0} \mu_{\mathrm{r}} a}$

$$
=\frac{62.83 \times 10^{-2}}{4 \pi \times 10^{-7} \times 650 \times 3.6 \times 10^{-4}}=213,669 \mathrm{AT} / \mathrm{Wb}
$$

AT required for iron path $=0.5 \times 10^{-3} \times 213,669=1,068.3 \mathrm{AT}$
Reluctance of air gap $\quad=\frac{l_{g}}{\mu_{0} a}=\frac{1 \times 10^{-3}}{4 \pi \times 10^{-7} \times 3.6 \times 10^{-4}}=2,210,485 \mathrm{AT} / \mathrm{Wb}$

AT required for air gap $=0.5 \times 10^{-3} \times 2,210,485=1,105.2 \mathrm{AT}$
Total AT $=(\mathrm{AT})_{\mathrm{i}}+(\mathrm{AT})_{\text {gap }}=1,068.3+1,105.2=2,173.5 \mathrm{AT}$
Current $I$

$$
=\frac{\text { Total AT }}{N}=\frac{2,173.5}{1,000}=2.1735 \mathrm{~A}
$$

## Example 5.10

A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean diameter of 20 cm . The steel ring is made of bar having circular cross section of diameter 2 cm . If the coil has a resistance of $50 \Omega$ and is connected to 250 V DC supply, calculate the mmf of the coil, the field intensity in the ring, reluctance of the magnetic path, total flux, and permeance of the ring.

## Solution:

The magnetic circuit is shown in Figure 5.12.
Current through the coil, $I=\frac{V}{R}=\frac{250}{50}=5 \mathrm{~A}$
mmf of the coil $=N I=300 \times 5=1,500 \mathrm{AT}$
Field intensity $H=\frac{N I}{l}$


Fig. 5.12 Given magnetic circuit
where

$$
l=\pi D=0.2 \pi \mathrm{~m}
$$

$$
H=\frac{1,500}{0.2 \pi}=2,387.3 \mathrm{AT} / \mathrm{m}
$$

Reluctance of the magnetic path, $S=\frac{1}{a \mu_{0} \mu_{\mathrm{r}}}$
where

$$
\begin{aligned}
& a=\pi d^{2}=\frac{\pi}{4} \times(0.02)^{2}=\pi \times 10^{-4} \mathrm{~m}^{2} ; \mu_{\mathrm{r}}=900 \\
& S=0.2 \pi / \pi \times 10^{-4} \times 4 \pi \times 10^{-7} \times 900=17.684 \times 10^{5} \mathrm{AT} / \mathrm{Wb}
\end{aligned}
$$

Total flux,

$$
\phi=\frac{\mathrm{mmf}}{S}=\frac{1,500}{17.684 \times 10^{5}}=0.848 \mathrm{mWb}
$$

Permeance $=1 / S=1 / 17.684 \times 10^{5}=5.655 \times 10^{-7} \mathrm{~Wb} /$ AT

## Example 5.11

Calculate the relative permeability of an iron ring when the exciting current taken by the 600 turn coil is 1.2 A and the total flux produced is 1 mWb . The mean circumference of the ring is 0.5 m and the area of cross section is $10 \mathrm{~cm}^{2}$.

## Solution:

$$
N I=\frac{\phi \times l}{a \mu_{0} \mu_{\mathrm{r}}} \quad \therefore \quad \mu_{\mathrm{r}}=\frac{\phi \times l}{a \mu_{0} N I}
$$

where $N=600$ turns; $I=1.2 \mathrm{~A} ; \phi=1 \mathrm{mWb}=1 \times 10^{-3} \mathrm{~Wb} ; l=0.5 \mathrm{~m}$;

$$
a=10 \mathrm{~cm}^{2}=10 \times 10^{-4} \mathrm{~m}^{2}
$$

Therefore, $\mu_{r}=\frac{1 \times 10^{-3} \times 0.5}{10 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 600 \times 1.2}=552.6$

## Example 5.12

An iron ring of mean length 1 m has an air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 500 when a current of 1 A flows through the coil, find the flux density.
(PTU)


Fig. 5.13 Magnetic circuit with air-gap

## Solution:

The magnetic circuit is shown in Figure 5.13.
Now, $\mathrm{mmf}=$ flux $\times$ reluctance

$$
\begin{aligned}
& N I=\phi\left(\frac{l_{\mathrm{i}}}{a \mu_{0} \mu_{\mathrm{r}}}+\frac{l_{\mathrm{g}}}{a \mu_{0}}\right) \\
& N I=B\left(\frac{l_{\mathrm{i}}}{\mu_{0} \mu_{\mathrm{r}}}+\frac{l_{\mathrm{g}}}{\mu_{0}}\right)
\end{aligned}
$$

where $N=200$ turns; $I=1 \mathrm{~A} ; \mu_{\mathrm{r}}=500$

$$
\begin{aligned}
& l_{\mathrm{g}}=1 \mathrm{~mm}=0.001 \mathrm{~m} \\
& l_{\mathrm{i}}=(1-0.001)=0.999 \mathrm{~m}
\end{aligned}
$$

Therefore, $200 \times 1=B\left(\frac{0.999}{4 \pi \times 10^{-7} \times 500}+\frac{0.001}{4 \pi \times 10^{-7}}\right)$
or

$$
B=\frac{200}{2,385.73}=0.0838 \mathrm{~Wb} / \mathrm{m}^{2}
$$

## Example 5.13

A rectangular a magnetic core shown in Figure 5.14 (a) has square cross section of area $16 \mathrm{~cm}^{2}$.
An air gap of 2 mm is cut across one of its limbs. Find the exciting current needed in the coil having 1,000 turns wound on the core to create an air-gap flux of 4 mWb . The relative permeability of the core is 2,000 .
(UPTU February 2002)

## Solution:

Here, area of $x$-section, $a=16 \mathrm{~cm}^{2}=16 \times 10^{-4} \mathrm{~m}^{2} ; l_{\mathrm{g}}=2 \mathrm{~mm}=2 \times 10^{-3} \mathrm{~m}$
Number of turns, $N=1,000$; flux, $\phi=4 \mathrm{mWb}=4 \times 10^{-3} \mathrm{~Wb} ; \mu_{\mathrm{r}}=2,000$
Flux density required, $\quad B=\frac{\phi}{a}=\frac{4 \times 10^{-3}}{16 \times 10^{-4}}=2.5 \mathrm{~T}$
Each side of the cross section $=\sqrt{16}=4 \mathrm{~cm}$
Length of iron path,

$$
l_{\mathrm{i}}=\left(25-2 \times \frac{4}{2}+20-2 \times \frac{4}{2}\right) \times 2-0.2
$$



Fig. 5.14 Given magnetic circuit

$$
=73.8 \mathrm{~cm}=0.738 \mathrm{~m}
$$

Total AT required

$$
\begin{aligned}
& =\frac{B}{\mu_{0}} l_{\mathrm{g}}+\frac{B}{\mu_{0} \mu_{\mathrm{r}}} l_{\mathrm{i}}=\frac{2.5 \times 2 \times 10^{-3}}{4 \mu \times 10^{-7}}+\frac{2.5 \times 0.738}{4 \pi \times 10^{-7} \times 2,000} \\
& =3,979+734=4,713
\end{aligned}
$$

Exciting current required, $I=\frac{\text { Total AT }}{N}=\frac{4,713}{1,000}=4.713 \mathrm{~A}$

## Example 5.14

An iron ring of $10 \mathrm{~cm}^{2}$ area has a mean circumference of 100 cm . It has a saw cut of 0.2 cm wide. A flux of 1 mWb is required in the air gap. The leakage factor is 1.2 . The flux density of iron for relative permeability 400 is $1.2 \mathrm{~Wb} / \mathrm{m}^{2}$. Calculate the number of AT required.

## Solution:

Flux density in air gap, $B_{\mathrm{g}}=\frac{\phi_{\mathrm{g}}}{a}=\frac{1 \times 10^{-3}}{10 \times 10^{-4}}=1 \mathrm{~Wb} / \mathrm{m}^{2}$
Flux in iron ring,

$$
\begin{aligned}
\phi_{\mathrm{i}} & =l \times \phi_{\mathrm{g}}(\text { where } \lambda=\text { leakage factor }) \\
& =1.2 \times 1 \times 10^{-3}=1.2 \times 10^{-3} \mathrm{~Wb}
\end{aligned}
$$

Flux density in iron ring, $B=\frac{\phi}{a}=\frac{1.2 \times 10^{-3}}{10 \times 10^{-4}}=1.2 \mathrm{~Wb} / \mathrm{m}^{2}$
Total AT required

$$
\begin{aligned}
& =H_{\mathrm{g}} l_{\mathrm{g}}+H_{\mathrm{i}} l_{\mathrm{i}}=\frac{B_{\mathrm{g}}}{\mu_{0}} l_{\mathrm{g}}+\frac{B_{\mathrm{i}}}{\mu_{0} \mu_{\mathrm{r}}} l_{\mathrm{i}} \\
& =\frac{1}{4 \pi \times 10^{-7}} \times 0.2 \times 10^{-2}+\frac{1.2}{4 \pi \times 10^{-7} \times 400} \times 1=3,978.87
\end{aligned}
$$

## Example 5.15

A steel ring with a mean radius of 10 cm and of circular cross section 1 cm in radius has an air gap of 1 mm length. It is wound uniformly with 500 turns of wire carrying current of 3 A . Neglect magnetic leakage. The air gap takes $60 \%$ of the total mmf Find the total reluctance.

## Solution:

Total $\mathrm{mmf}=N I=500 \times 3=1,500 \mathrm{AT}$
mmf for air gap $=60 \%$ of total $\mathrm{mmf}=0.6 \times 1,500=900 \mathrm{AT}$
Reluctance of air gap, $S_{\mathrm{g}}=\frac{l_{\mathrm{g}}}{a \mu_{0}}$
where $l_{\mathrm{g}}=1 \mathrm{~mm}=1 \times 10^{-3} \mathrm{~m} ; a=\rho \times(0.01)^{2}=\rho \times 10^{-4} \mathrm{~m}^{2}$;
Therefore,

$$
S_{\mathrm{g}}=\frac{1 \times 10^{-3}}{\rho \times 10^{-4} \times 4 \rho \times 10^{-7}} \frac{10^{8}}{4 \rho^{2}} \mathrm{ATs} / \mathrm{Wb}
$$

Flux in the air gap, $\quad \phi_{\mathrm{g}}=\frac{\mathrm{mmf} \text { of air gap }}{\text { Reluctance }}=\frac{900}{10^{8}} \times 4 \pi^{2}=36 \pi^{2} \times 10^{-6} \mathrm{~Wb}$
mmf for iron $=$ total $\mathrm{mmf}-$ air $\mathrm{mmf}=1,500-900=600 \mathrm{AT}$
Flux in the iron ring, $\quad \phi_{\mathrm{i}}=\phi_{\mathrm{g}}=36 \pi^{2} \times 10^{-6} \mathrm{~Wb}$ (since there is no magnetic leakage)
Reluctance of iron ring, $S_{\mathrm{i}}=\frac{\mathrm{mmf} \text { of iron }}{\phi_{\mathrm{i}}}=\frac{600}{36 \pi^{2} \times 10^{-6}}=\frac{10^{8}}{6 \pi^{2}} \mathrm{AT} / \mathrm{Wb}$

Total reluctance,

$$
\begin{aligned}
S & =S_{\mathrm{g}}+S_{\mathrm{i}}=\frac{10^{8}}{4 \pi^{2}}+\frac{10^{8}}{6 \pi^{2}}=\frac{10^{8}}{\pi^{2}}\left(\frac{1}{4}+\frac{1}{6}\right) \\
& =4.22 \times 10^{6} \mathrm{AT} / \mathrm{Wb}
\end{aligned}
$$

## Example 5.16

Determine mmf , magnetic flux, reluctance, and flux density if a steel ring of 30 cm mean diameter and a circular cross section 2 cm in diameter has an air gap 1 mm long. It is wound uniformly with 600 turns of wire carrying a current of 2.5 A . Neglect magnetic leakage. The iron path takes $40 \%$ of the total mmf.

## Solution:

mmf of the magnetic circuit $=N I=600 \times 2.5=1,500 \mathrm{AT}$
As iron path takes $40 \%$ of the total mmf, the reluctance of iron is $40 \%$ and the rest of the reluctance $(60 \%)$ is of air path.

Reluctance of air path, $S_{\mathrm{a}}=\frac{l_{\mathrm{a}}}{a \mu_{0}}$
where $l_{\mathrm{a}}=1 \times 10^{-3} \mathrm{~m} ; a=\frac{\pi}{4}(2)^{2} \times 10^{-4}=\pi \times 10^{-4} \mathrm{~m}^{2}$;
Therefore, $S_{\mathrm{a}}=\frac{1 \times 10^{-4}}{\pi \times 10^{-4} \times 4 \pi \times 10^{-7}}=2.533 \times 10^{6} \mathrm{AT} / \mathrm{Wb}$
Reluctance of iron path, $S_{\mathrm{i}}=\frac{S_{\mathrm{a}}}{1.5}=\frac{2.533 \times 10^{6}}{1.5}=1.688 \times 10^{6} \mathrm{AT} / \mathrm{Wb}$

Total reluctance $=S_{\mathrm{a}}+S_{\mathrm{i}}=(2.533+1.688) \times 10^{6} \mathrm{AT} / \mathrm{Wb}$

$$
=4.221 \times 10^{6} \mathrm{AT} / \mathrm{Wb}
$$

Magnetic flux, $\phi=\frac{\mathrm{mmf}}{\text { Reluctance }}=\frac{1,500}{4.221 \times 10^{6}}=0.3554 \mathrm{mWb}$
Flux density, $B=\frac{\phi}{a}=\frac{0.3554 \times 10^{-3}}{\pi \times 10^{-4}}=1.131 \mathrm{~Wb} / \mathrm{m}^{2}$

## Example 5.17

An iron ring is made up of three parts: $l_{1}=10$ $\mathrm{cm}, a_{1}=5 \mathrm{~cm}^{2} ; l_{2}=8 \mathrm{~cm}, a_{2}=3 \mathrm{~cm}^{2} ; l_{3}=6 \mathrm{~cm}$, $a_{3}=2.5 \mathrm{~cm}^{2}$. It is wound with a 250 -turn coil. Calculate current required to produce flux of 0.4 mWb . $\mu_{1}=2,670, \mu_{2}=1,050$, and $\mu_{3}=600$
(UPTU 2007-2008)

## Solution:

Flux density $B_{1}=\frac{\phi}{a_{1}}=\frac{0.4 \times 10^{-3}}{5 \times 10^{-4}}=0.8 \mathrm{~Wb} / \mathrm{m}^{2}$

$$
\begin{aligned}
B_{2} & =\frac{\phi}{a_{2}}=\frac{0.4 \times 10^{-3}}{3 \times 10^{-4}} \\
& =1.33 \mathrm{~Wb} / \mathrm{m}^{2}
\end{aligned}
$$



Fig. 5.15 Composite magnetic circuit

$$
\begin{aligned}
B_{3} & =\frac{\phi}{a_{3}}=\frac{0.4 \times 10^{-3}}{2.5 \times 10^{-4}} \\
& =1.6 \mathrm{~Wb} / \mathrm{m}^{2}
\end{aligned}
$$

Total AT required

$$
\begin{aligned}
\mathrm{AT} & =\frac{B_{1}}{\mu_{0} \mu_{1}} l_{1}+\frac{B_{2}}{\mu_{0} \mu_{2}} l_{2}+\frac{B_{3}}{\mu_{0} \mu_{3}} l_{3} \\
& =\frac{1}{4 \pi \times 10^{-7}}\left[\frac{0.8}{2670} \times 0.10+\frac{1.33}{1050} \times 0.08+\frac{1.6}{600} \times 0.06\right] \\
& =231.92
\end{aligned}
$$

Current required to produce given flux,

$$
I=\frac{\mathrm{AT}}{N}=\frac{231.92}{250}=0.928 \mathrm{~A}
$$

## Example 5.18

The ring-shaped core shown in Figure 5.16 is made of a material having a relative permeability of 1,000 . The flux density in the smallest area of cross section is 2 T . If the current through the coil is not to exceed 1.5 A , compute the number of turns of the coil.


Fig. 5.16 Given composite magnetic circuit

## Solution:

Flux in the core,

$$
\phi=B \times A=2 \times 2 \times 10^{-4}=4 \times 10^{-4} \mathrm{~Wb}
$$

Total reluctance of the magnetic path,

$$
\begin{aligned}
S & =S_{1}+S_{2}+S_{3} \\
& =\frac{l_{1}}{a_{1} \mu_{0} \mu_{\mathrm{r}}}+\frac{l_{2}}{a_{2} \mu_{0} \mu_{\mathrm{r}}}+\frac{l_{3}}{a_{3} \mu_{0} \mu_{\mathrm{r}}} \\
& =\frac{1}{\mu_{0} \mu_{\mathrm{r}}}\left(\frac{l_{1}}{a_{1}}+\frac{l_{2}}{a_{2}}+\frac{l_{3}}{a_{3}}\right)
\end{aligned}
$$

$$
=\frac{1}{4 \pi \times 10^{-7} \times 1,000}\left(\frac{0.1}{4 \times 10^{-4}}+\frac{0.15}{3 \times 10^{-4}}+\frac{0.2}{2 \times 10^{-4}}\right)=13.926 \times 10^{5} \mathrm{AT} / \mathrm{Wb}
$$

Total mmf required, $N I=\phi S$
or

$$
N \times 1.5=4 \times 10^{-4} \times 13.926 \times 10^{5}
$$

Therefore, number of turns, $N=371.36$

## Example 5.19

The magnetic frame shown in Figure 5.17 is built-up of iron of square cross section of 3 cm side. Each air gap is 2 mm wide. Each of the coil is wound with 1,000 turns and the exciting current is 1.0 A . The relative permeability of path A


Fig. 5.17 Given magnetic circuit and path B may be taken as 1,000 and 1,200 , respectively.
Calculate

1. reluctance of path A ;
2. reluctance of path $B$;
3. reluctance of two air gaps;
4. total reluctance of the complete magnetic circuit;
5. mmf produced;
6. flux set up in the circuit.

## Solution:

1. Reluctance of path $\mathrm{A}, S_{\mathrm{A}}=\frac{l_{\mathrm{A}}}{a \mu_{0} \mu_{\mathrm{rA}}}$ where $l_{\mathrm{A}}=20-(1.5+1.5)+(1.5+1.5)=20 \mathrm{~cm}=0.2 \mathrm{~m}$ $a=3 \times 3=9 \mathrm{~cm}^{2}=9 \times 10^{-4} \mathrm{~m}^{2} ; \mu_{\mathrm{rA}}=1,000 ;$
Therefore, $S_{\mathrm{A}}=\frac{0.2}{9 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 1,000}=176,839 \mathrm{AT} / \mathrm{Wb}$
2. Reluctance of path $\mathrm{B}, S_{\mathrm{B}}=\frac{l_{\mathrm{B}}}{a \mu_{0} \mu_{\mathrm{rB}}}$
where $l_{\mathrm{B}}=(20-1.5-1.5)+2(10-1.5)=34 \mathrm{~cm}=0.34 \mathrm{~m} ; \mu_{\mathrm{rB}}=1,200$

$$
S_{\mathrm{B}}=\frac{0.34}{9 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 1,200}=250,521 \mathrm{AT} / \mathrm{Wb}
$$

3. Reluctance of two air gaps, $S_{\mathrm{g}}=\frac{l_{g}}{a \mu_{0}}$ where $l_{\mathrm{g}}=2+2=4 \mathrm{~mm}=4 \times 10^{-3} \mathrm{~m}$

$$
S_{\mathrm{g}}=4 \times 10^{-3} / 9 \times 10^{-4} \times 4 \pi \times 10^{-7}=3,536,776 \mathrm{AT} / \mathrm{Wb}
$$

4. Total reluctance of the composite magnetic circuit,

$$
S=S_{\mathrm{A}}+S_{\mathrm{B}}+S_{\mathrm{g}}=176,839+250,521+3,536,776=3,964,136 \mathrm{AT} / \mathrm{Wb}
$$

5. Total $\mathrm{mmf}=N I=(2 \times 1,000) \times 1=2,000 \mathrm{AT}$
6. Flux set up in the circuit, $\phi=\frac{\mathrm{mmf}}{\text { Reluctance }}=\frac{2,000}{3,964,136}=0.5045 \mathrm{mWb}$

## Example 5.20

A magnetic core made of annealed sheet steel has the dimensions as shown in Figure 5.18. The cross section everywhere is $25 \mathrm{~cm}^{2}$. The flux in branches A and B is $3,500 \mu \mathrm{~Wb}$, but that in branch C is zero. Find the required AT for coil A and for coil C. Relative permeability of sheet steel is 1,000 .


Fig. 5.18 Given magnetic circuit


Fig. 5.19 Parallel magnetic circuit

## Solution:

The given magnetic circuit is a parallel circuit. To determine the AT for coil ' A ', the flux distribution is shown in Figure 5.19.

Since, path ' $B$ ' and ' $C$ ' are in parallel with each other w.r.t. path 'A',
Therefore, mmf for path ' B ' $=\mathrm{mmf}$ for path C ,

$$
\begin{gathered}
\phi_{1} S_{1}=\phi_{2} S_{2} \\
\frac{3,500 \times 10^{-6} \times 30 \times 10^{-2}}{a \mu_{0} \mu_{\mathrm{r}}}=\phi_{2} \times \frac{80 \times 10^{-2}}{a \mu_{0} \mu_{\mathrm{r}}}
\end{gathered}
$$

Therefore, $\phi_{2}=1,312.5 \times 10^{-6} \mathrm{~Wb}$
Total flux in the path 'A', $\phi=\phi_{1}+\phi_{2}$

$$
=(3,500+1,312.5) \times 10^{-6}=4,812.5 \times 10^{-6} \mathrm{~Wb}
$$

Actual (resultant) flux in path ' A ' $=\phi+\phi_{2}=3,500 \times 10^{-6} \mathrm{~Wb}$.
Therefore, AT required for coil ' A ' = AT for path ' A ' +AT for path ' B ' or ' C '

$$
=\frac{3,500 \times 10^{-6}}{4 \pi \times 10^{-7} \times 1,000 \times 25 \times 10^{-4}}(0.8+0.3)=1,225.5
$$

To neutralise the flux in section ' C ', the coil produces flux of $1,312.5 \mu \mathrm{~Wb}$ in opposite direction. Therefore, AT required for coil ' C ' $=\mathrm{AT}$ for path ' C ' only

$$
=\frac{1,312.5 \times 10^{-6} \times 0.8}{4 \pi \times 10^{-7} \times 1,000 \times 25 \times 10^{-4}}=334.22
$$

### 5.10 MAGNETISATION OR B-H CURVE

The graph plotted between flux density $B$ and magnetising force $H$ of a material is called the magnetisation or $B-H$ curve of that material.

The general shape of the $B-H$ curve of a magnetic material is shown in Figure 5.20. The shape of the curve is non-linear. This indicates that the relative permeability $\left(\mu_{\mathrm{r}}=B / \mu_{0} H\right)$ of a magnetic material is not constant but it varies. The value of $\mu_{\mathrm{r}}$ largely depends upon the value of flux density. Its shape is shown in Figure 5.21 (for cast steel).


Fig. 5.20 Magnetisation curve


Fig. 5.21 Curve between $\mathbf{B}$ and $\mu_{\mathrm{r}}$

The $B-H$ curves of some of the common magnetic materials are shown in Figure 5.22. The $B-H$ curve for a non-magnetic material is shown in Figure 5.23. It is a straight line curve since $B=\mu_{0} H$ or $B \propto H$ as the value of $\mu_{0}$ is constant.


Fig 5.22 Magnetisation curve for sheet steel, cast steel and cast iron


Fig 5.23 Magnetisation curve for nonmagnetic materials

### 5.11 MAGNETIC HYSTERESIS

When a magnetic material is magnetised first in one direction and then in the other (i.e., one cycle of magnetisation), it is found that flux density $B$ in the material lags behind the applied magnetising force $H$. This phenomenon is known as magnetic hysteresis. Hence, the phenomenon of flux density $B$ lagging behind the magnetising force $H$ in a magnetic material is called magnetic hysteresis.
'Hysteresis' is the term derived from the Greek word hysterein, meaning to lag behind. To understand the complete phenomenon of magnetic hysteresis, consider a ring of magnetic material on which a solenoid is wound uniformly as shown in Figure 5.24 (a). The solenoid is connected to a DC source through a double pole double throw reversing switch (position ' 1 ').


Fig. 5.24 Magnetic Hysteresis (a) Electrical circuit to draw hysteresis loop
(b) Hysteresis loop

When the field intensity $H$ is increased gradually by increasing current in the solenoid (by decreasing the value of $R$ ), the flux density $B$ also increases until saturation point $a$ is reached and curve so obtained is $o a$. Now, if the magnetising force is gradually reduced to zero by decreasing current in the solenoid to zero, the flux density does not become zero and the curve so obtained is $a b$, as shown in Figure 5.24 (b). When magnetising force $H$ is zero, the flux density still has value $o b$.

### 5.11.1 Residual Magnetism and Retentivity

This value of flux density ' $o b$ ' retained by the magnetic material is called residual magnetism and the power of retaining this residual magnetism is called retentivity of the material. To demagnetise the magnetic ring, the magnetising force $H$ is reversed by reversing the direction of flow of current in the solenoid. This is achieved by changing the position of double pole, double throw switch (i.e., position ' 2 '). When $H$ is increased in reverse direction, the flux density starts decreasing and becomes zero and curve follows the path bc. Thus, residual magnetism of the magnetic material is removed by applying magnetising force oc in opposite direction.

### 5.11.2 Coercive Force

This value of magnetising force $o c$ required to remove the residual magnetism is called coercive force. To complete the loop, the magnetising force $H$ is increased further in reverse direction till saturation reaches (point ' $d$ ') and the curve follows the path $c d$. Again $H$ is reduced to zero and the curve follows the path $d e$, where $o e$ represents the residual magnetism. Then, $H$ is increased in the positive direction by changing the position of reversible switch to position ' 1 ' and increasing the flow of current in the solenoid. The curve follows the path of efa and the loop is completed. Again of is the magnetising force utilised to remove the residual magnetism oe.

Hence, $c f$ is the total coercive force required in one cycle of magnetisation to remove the residual magnetism. Since the meaning of hysteresis is lagging behind, and in this case, flux density $B$ always lags behind the magnetising force, $H$. Therefore, loop ( $a b c d e f a$ ) so obtained is called hysteresis loop.

### 5.12 HYSTERESIS LOSS

When a magnetising force is applied, the magnetic material is magnetised and the molecular magnets are lined up in a particular direction. However, when the magnetising force in a magnetic material is reversed, the internal friction of the molecular magnets opposes the reversal of magnetism, resulting in hysteresis. To overcome this internal friction of the molecular magnets (or to remove the residual magnetism), a part of the magnetising force is used. The work done by the magnetising force against this internal friction of molecular magnets produces heat. This energy, which is wasted in the form of heat due to hysteresis, is called hysteresis loss.

Hysteresis loss occurs in all the magnetic parts of electrical machines where there is reversal of magnetisation. This loss results in wastage of energy in the form of heat. Consequently, it increases the temperature of the machine, which is undesirable. Therefore, a suitable magnetic material is selected for the construction of such parts, for example, silicon steel is most suitable in which hysteresis loss is minimum.

### 5.13 IMPORTANCE OF HYSTERESIS LOOP

The shape and size of hysteresis loop of a magnetic material largely depend upon its nature. For a particular location, the choice of the magnetic material depends upon the shape and size (i.e.,
area) of its hysteresis loop. The hysteresis loops of some of the common magnetic materials are shown in Figure 5.25.

1. Hard steel: The hysteresis loop for hard steel is shown in Figure 5.25 (a). This loop has larger area that indicates that this material will have more hysteresis loss. Therefore, it is never used for the construction of machine


Fig. 5.25 Hysteresis loop for different magnetic materials (a) Hardsteel (b) Silicon steel (c) Wrought iron parts. However, its loop shows that the material has high retentivity and coercivity. Therefore, it is more suitable for making permanent magnets.
2. Silicon steel: The hysteresis loop for silicon steel is shown in Figure 5.25 (b). This loop has the smallest area that indicates that this material will have small hysteresis loss. Therefore, it is most suitable for the construction of those parts of electrical machines in which reversal of magnetisation is very quick, for example, armature of DC machines, transformer core, starter of induction motors, etc.
3. Wrought iron: Figure 5.25 (c) shows the hysteresis loop for wrought iron. This loop shows that this material has fairly good residual magnetism and coercivity. Therefore, it is best suited for making cores of electromagnets.

## 國 PRACTICE EXERCISES

## Short Answer Questions

1. Define magnetic field.
2. What do you mean by magnetic flux?
3. Define magnetic flux density.
4. What is permeability in magnetic circuits?
5. How will you define mmf?
6. Define reluctance.
7. How will you define magnetic flux in a magnetic circuits?
8. What is the relation between mmf , reluctance, and flux?
9. Define permeance.
10. How will you define reluctivity?
11. How will you calculate AT required to produce a field intensity in a magnetic circuit?
12. What do you mean by series magnetic circuit? What will be the resultant reluctance of the circuit?
13. How will you represent a parallel magnetic circuit?
14. What do you mean by leakage flux?
15. Define leakage factor for a magnetic circuit.
16. Define and draw a $B-H$ curve.
17. What do you mean by magnetic hysteresis?
18. Define and represent residual magnetism and retentivity on a $B-H$ curve.
19. Define coercive force.
20. What are hysteresis losses?
21. What are the factors on which hysteresis losses depend?

## Test Questions

1. Analyse a magnetic circuit and derive a relation between mmf, reluctance, and magnetic flux.
2. What are the similarities between electric and magnetic circuits?
3. What are the dissimilarities between magnetic and electric circuits?
4. How will you determine (calculate) the AT required to set up a desired flux in a series magnetic circuit?
5. What is magnetic hysteresis? Explain it with the help of a neat sketch. Further, represent residual magnetism and coercive force on the hysteresis loop.
6. What are hysteresis losses? Can these be minimised in a given material?

## Numericals

1. Estimate the total mmf required to produce of a flux of 1 mWb round an iron ring of $6 \mathrm{~cm}^{2}$ cross section and 90 cm mean diameter having an air gap of 9 mm wide across it. The relative permeability of iron is 1.200 .
(Ans. 15,052 AT)(PU)
2. A steel ring 15 cm mean radius and of circular section 1 cm in radius has an air gap of 1 mm length. It is wound uniformly with 500 turns of wire carrying a current of 3 A . Neglect magnetic leakage. The air gap takes $20 \%$ of the total mmf. Find reluctance.
(Ans. $12.6 \times 10^{6} \mathrm{AT} / \mathrm{Wb}$ )
3. An iron ring has cross section of $3 \mathrm{~cm}^{2}$ and a mean diameter of 25 cm . An air gap of 0.4 mm has been made by saw cut across the section. The ring is wound with 200 turns through which a current of 2 A is passed. If the total flux is $21 \times 10^{-5} \mathrm{~Wb}$, find $\mu$ for iron assuming no leakage.
(Ans. 2,470) (PU)
4. A circular iron ring has a mean circumference of 1.5 m and a cross-sectional area of $100 \mathrm{~cm}^{2}$. A saw cut of 0.4 cm wide is made in the ring. Calculate the magnetising current required to produce a flux of 0.8 mWb in the air gap if the ring is wound with a coil of 350 turns. Assuming relative permeability of iron as 400 and leakage factor as 1.25 .
(Ans. 1.58 A$)$
5. A steel ring 50 cm mean diameter and of circular section 3 cm in diameter has an air gap 2 mm long. It is wound uniformly with 1,000 turns of wire carrying a current of 4 A . Calculate mmf, magnetic flux, reluctance, and flux density. Neglect magnetic leakage. Iron path takes about $40 \%$ of total mmf.
(Ans. $4,000 \mathrm{AT} ; 1.066 \mathrm{mWb} ; 3.753 \times 10^{6} \mathrm{AT} / \mathrm{Wb} ; 1.508 \mathrm{~T}$ )

### 5.14 ELECTROMAGNETIC INDUCTION

The phenomenon by which an emf is induced in a circuit (and hence current flows when the circuit is closed) when magnetic flux linking with it changes is called electromagnetic induction.

For illustration, consider a coil having a large number of turns to which galvanometer is connected. When a permanent bar magnet is taken nearer to the coil or away from the coil, as shown in Figure. 5.26 (a), a deflection occurs in the needle of the galvanometer. Although, the deflection in the needle is opposite in two cases.

On the other hand, if the bar magnet is kept stationary and the coil is brought near to the magnet or away from the magnet, as shown in Figure 5.26 (b), again a deflection occurs in the needle of the galvanometer. The deflection in the needle is opposite in the two cases. However, if both the magnet and the coil are kept stationary, no matter how much flux is linking with the coil, there is no deflection in the galvanometer needle.


Fig. 5.26 Electromagnetic induction (a) Magnetic bar taken nearer to the coil or away from it (b) Coil taken nearer to the magnet or away from it

The following points are worth noting:

1. The deflection in the galvanometer needle shows that emf is induced in the coil. This condition occurs only when flux linking with the circuit changes, that is, either magnet or coil is in motion.
2. The direction of induced emf in the coil depends upon the direction of magnetic field and the direction of motion of coil.

### 5.15 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Michael Faraday summed up conclusions of his experiments regarding electromagnetic induction into two laws, known as Faraday's laws of electromagnetic induction.

### 5.15.1 First Law

This law states that 'Whenever a conductor cuts across the magnetic field, an emf is induced in the conductor'.
or 'Whenever the magnetic flux linking with any circuit (or coil) changes, an emf is induced in the circuit'.

Figure 5.27 shows a conductor placed in the magnetic field of a permanent magnet to which a galvanometer is connected. Whenever the conductor is moved upward or downward, that is across the field, there is deflection in the galvanometer needle that indicates that an emf is induced in the conductor. If the conductor is moved along (parallel) the field, there is no deflection in the needle that indicates that no emf is induced in the conductor.


Fig. 5.27 Phenomenon for Faraday's laws of electromagnetic induction

For the second statement, consider a coil placed near a bar magnet and a galvanometer connected across the coil, as shown in Figure 5.28. When the bar magnet (N-pole) is taken near to the coil (see Fig. 5.28 (a)), there is deflection in the needle of the galvanometer. Now, if the bar magnet ( $N$-pole) is taken away from the coil (see Fig. 5.28 (b)), again there is deflection in the needle of galvanometer but in opposite direction. The deflection in the needle of galvanometer indicates that emf is induced in the coil.


Fig. 5.28 Electromagnetic induction (a) Magnetic bar taken nearer to the coil (b) Magnetic bar taken away to the coil

### 5.15.2 Second Law

This law states that 'The magnitude of induced emf in a coil is directly proportional to the rate of change of flux linkages.'

Rate of change of flux linkages $=\frac{N\left(\phi_{2}-\phi_{1}\right)}{t} \mathrm{~Wb}$-turns $/ \mathrm{s}$
where $N=$ number of turns of the coil; $\left(\phi_{2}-\phi_{1}\right)=$ change of flux in Wb
$t=$ time in seconds for the change
According to Faraday's second law of electromagnetic induction,

$$
\begin{aligned}
& \text { Induced emf, } e \propto \frac{N\left(\phi_{2}-\phi_{1}\right)}{t} \\
& \qquad e=\frac{N\left(\phi_{2}-\phi_{1}\right)}{t} \text { (taking proportionality constant as unity) }
\end{aligned}
$$

In differential form, $e=N \frac{d \phi}{d t} \mathrm{~V}$
Usually, a minus sign is given to the right-hand side expression that indicates that emf is induced in such a direction that opposes the cause (i.e., change in flux) that produces it (according to Lenz's law).

$$
e=-N \frac{d \phi}{d t} \mathrm{~V}
$$

### 5.16 DIRECTION OF INDUCED EMF

The direction of induced emf and hence current in a conductor or coil can be determined by either of the following two methods:

1. Fleming's Right-hand Rule: This rule is applied to determine the direction of induced emf in a conductor moving across the field and is stated as
'Stretch first finger, second finger, and thumb of your right-hand mutually perpendicular to each other. If first finger indicates the direction of magnetic field and thumb indicates the direction of motion of conductor, then second finger will indicate the direction of induced emf in the conductor.'
Its illustration is shown in Figure 5.27.
2. Lenz's Law: This law is more suitably applied to determine the direction of induced emf in a coil or circuit when flux linking with it changes. It is stated as
'In effect, electromagnetically induced emf and hence current flows in a coil or circuit in such a direction that the magnetic field set up by it always opposes the very cause which produces it.'

When $N$-pole of a bar magnet is taken near to the coil, as shown in Figure 5.28 (a), an emf is induced in the coil, and hence, current flows through it in such a direction that side ' $B$ ' of the coil attains North polarity that opposes the movement of the bar magnet. While $N$-pole of the bar magnet is taken away from the coil, as shown in Figure 5.28 (b), the direction of emf induced in the coil is reversed and side ' B ' of the coil attains South polarity that again opposes the movement of the bar magnet.

### 5.17 INDUCED EMF

When flux linking with a conductor (or coil) changes, an emf is induced in it. This change in flux linkages can be obtained in the following two ways:

1. By either moving the conductor and keeping the magnetic field system stationary or moving the magnetic field system and keeping the conductor stationary; in such a way, the conductor cuts across the magnetic field (as in the case of DC and AC generators). The emf induced in this way is called dynamically induced emf.
2. By changing the flux linking with the coil (or conductor) without moving either coil or field system. However, the change of flux produced by the field system linking with the coil is obtained by changing the current in the field system (solenoid), as in transformers. The emf induced in this way is called statically induced emf.

### 5.18 DYNAMICALLY INDUCED EMF

By either moving the conductor and keeping the magnetic field system stationary, or moving the field system and keeping the conductor stationary so that flux is cut by the conductor, the emf thus induced in the conductor is called dynamically induced emf.

### 5.18.1 Mathematical Expression

Considering a conductor of length $l \mathrm{~m}$ placed in the magnetic field of flux density $B \mathrm{~Wb} / \mathrm{m}^{2}$ is moving at right angle to the field at a velocity $v \mathrm{~m} / \mathrm{s}$, as shown in Figure 5.29(a). Let the conductor be moved through a small distance $d x \mathrm{~m}$ in time $d t \mathrm{~s}$, as shown in Figure 5.29 (b).


Fig. 5.29 Dynamically induced emf (a) Conductor moves perpendicular to the magnetic field (b) Distance covered (dx) (c) Area swept (d) Conductor moves at an angle $\theta$ with the direction of magnetic field

Area swept by the conductor, $A=l \times d x$
Flux cut by the conductor, $\phi=B \times A=B l d x$
According to Faraday's Law of electromagnetic induction;
Induced emf, $e=\frac{\text { flux cut }}{\text { time }}=\frac{\phi}{d t}=\frac{B l d x}{d t}=B l v$ (since $d x / d t=v$ (velocity))
Now, if the conductor is moved at an angle $\theta$ with the direction of magnetic field at a velocity $v \mathrm{~m} / \mathrm{s}$, as shown in Figure 5.29(d). A small distance covered by the conductor in that direction is $d x$ in time $d t$ s. Then, the component of distance perpendicular to the magnetic field, which produces emf, is $d x \sin \theta$.

Therefore, area swept by the conductor, $A=l \times d x \sin \theta$
Flux cut by the conductor, $\phi=B \times A=B l d x \sin \theta$
Induced emf, $e=\frac{B l d x \sin \theta}{d t}=B l v \sin \theta$

## Example 5.21

A coil of 500 turns is linked with a flux of 2 mWb . If this flux is reversed in 4 ms , calculate the average emf induced in the coil.

## Solution:

Average induced emf, $e=N \frac{d \phi}{d t}$
where $N=500$ turns; $d \phi=2-(-2)=4 \mathrm{mWb} ; d t=4 \times 10^{3} \mathrm{~s}$

$$
e=500 \times \frac{4 \times 10^{-3}}{4 \times 10^{-3}}=500 \mathrm{~V}
$$

## Example 5.22

A coil of 250 turns is wound on a magnetic circuit of reluctance $100,000 \mathrm{AT} / \mathrm{Wb}$. If a current of 2 A flowing in the coil is reversed in 5 ms , find the average emf induced in the coil.

## Solution:

$\phi=\mathrm{mmf}$ /reluctance, that is, $\phi=N I / S$
where $N=250, I=2 \mathrm{~A}$, and $S=100,000 \mathrm{AT} / \mathrm{Wb}$

$$
\therefore \quad \phi=\frac{250 \times 2}{100,000}=5 \mathrm{mWb}
$$

Average induced emf, $e=N \frac{d \phi}{d t}$
where $d \phi=5-(-5)=10 \mathrm{mWb}$ (since current is reversed)

$$
e=250 \times \frac{10 \times 10^{-3}}{5 \times 10^{-3}}=500 \mathrm{~V}
$$

### 5.19 STATICALLY INDUCED EMF

When both the coil and magnetic field system are stationary but the magnetic field linking with the coil changes (by changing the current producing the field), the emf thus induced in the coil is called statically induced emf. The statically induced emf are of two types: self-induced emf and mutually induced emf.

### 5.19.1 Self-induced Emf

The emf induced in a coil due to the change of flux produced by it linking with its own turns is called self-induced emf. The direction of this induced emf is such that it opposes the cause that produces it (Lenz's law), that is, change of current in the coil.

Since the rate of change of flux linking with the coil depends upon the rate of change of current in the coil. Therefore, the magnitude of self-induced emf is directly proportional to the rate of change of current in the coil. Therefore, the magnitude of self-induced emf is directly proportional to the rate of change of current in the coil, that is,

$$
e \propto \frac{d I}{d t} \quad \text { or } \quad e=L \frac{d I}{d t}
$$

where $L=$ a constant of proportionality and is called self-inductance or co-efficient of self-inductance or inductance of the coil.

### 5.19.2 Mutually induced Emf

The emf induced in a coil due to the change of flux produced by another (neighbouring) coil linking with it is called mutually induced emf.

Since the rate of change of flux linking with coil ' $B$ ' depends upon the rate of change of current in coil ' $A$ '. Therefore, the magnitude of mutually induced emf is directly proportional to the rate of change of current in coil ' $A$ ', that is,


Fig. 5.30 Circuit for self induced emf


Fig. 5.31 Circuit for mutually induced emf

$$
e_{\mathrm{m}} \propto \frac{d I_{1}}{d t} \quad \text { or } \quad e_{\mathrm{m}}=M \frac{d I_{1}}{d t}
$$

where $M=$ a constant of proportionality and is called mutual inductance or co-efficient of mutual inductance.

### 5.20 SELF-INDUCTANCE

The property of a coil due to which it opposes the change of current flowing through itself is called self-inductance or inductance of the coil. This property (i.e., inductance) is attained by a coil due to self-induced emf produced in the coil itself by the changing current flowing through it. If the current in the coil is increasing (by the change in circuit conditions), the self-induced emf is produced in the coil in such a direction so as to oppose the rise of current, that is, the direction of self-induced emf is opposite to that of the applied voltage. On the other hand, if the current in the coil is decreasing, the self-induced emf is produced in the coil in such direction so as to oppose the fall of current. In other words, the direction of self-induced emf is in the same direction as that of the applied voltage. In fact, self-inductance does not prevent the change of current, but it delays the change of current flowing through a coil.

It may be noted that this property of the coil only opposes the changing current (i.e., AC). However, it does not affect the steady (i.e., DC) current when flows through it. In other words, the self-inductance of the coil (by virtue of its geometrical and magnetic properties) will exhibit its presence to the AC , but it will not exhibit its presence to the DC .

### 5.20.1 Expressions for Self-inductance

$$
\begin{aligned}
L & =\frac{e}{d I / d t}\left(\text { since } e=L \frac{d I}{d t}\right) \\
& =\frac{N \phi}{I}\left(\text { since } e=N \frac{d \phi}{d t}=L \frac{d I}{d t}\right)=\frac{N^{2}}{l / a \mu_{0} \mu_{\mathrm{r}}}\left(\text { since } \phi=\frac{N I}{l / a \mu_{0} \mu_{\mathrm{r}}}\right)
\end{aligned}
$$

### 5.21 MUTUAL INDUCTANCE

The property of one coil due to which it opposes the change of current in the other (neighbouring) coil is called mutual inductance between the two coils. This property (i.e., mutual inductance) is attained by a coil due to mutually induced emf in the coil, while current in the neighbouring coil is changing.

### 5.21.1 Expression for Mutual Inductance

$$
\begin{aligned}
M & =\frac{e_{\mathrm{m}}}{d I_{1} / d t}\left(\text { since } e_{\mathrm{m}}=M \frac{d I_{1}}{d t}\right) \\
& =\frac{N_{2} \phi_{12}}{I_{1}}\left(\text { since } e_{\mathrm{m}}=N_{2} \frac{d \phi_{12}}{d t}=M \frac{d I_{1}}{d t}\right)=\frac{N_{1} N_{2}}{l / a \mu_{0} \mu_{\mathrm{r}}}\left(\text { since } \phi_{12}=\frac{N_{1} I_{1}}{l / a \mu_{0} \mu_{\mathrm{r}}}\right)
\end{aligned}
$$

### 5.22 CO-EFFICIENT OF COUPLING

When current flows through one coil, it produces flux $\left(f_{1}\right)$. The whole of this flux may not be linking with the other coil coupled to it, as shown in Figure 5.32. It may be reduced because of leakage flux $\mathrm{f}_{l}$ by a fraction $k$ known as co-efficient of coupling.

Thus, the fraction of magnetic flux produced by the current in one coil that links with the other is known as co-efficient of coupling ( $k$ ) between the two coils. If the flux produced by one coil completely links with the other, then the value of $k$ is $l$ and the coils are said to be magnetically tightly coupled. However, if the flux produced by one coil does not link at all with the other, then the value of $k$ is zero and the coils are said to be magnetically isolated.


Fig. 5.32 Magnetic circuit for co-efficient of coupling

### 5.22.1 Mathematical Expression

Consider the magnetic circuit shown in Figure 5.32. When current $I_{1}$ flows through coil 1,

$$
\begin{gather*}
L_{1}=\frac{N_{1} \phi_{1}}{I_{1}} \quad \text { and } \quad M=\frac{N_{2} \phi_{12}}{I_{1}}=\frac{N_{2} k \phi_{1}}{I_{1}}  \tag{5.3}\\
\left(\Theta \phi_{12}=k \phi_{1}\right)
\end{gather*}
$$

Now, considering coil 2 carrying current $I_{2}$;

$$
\begin{gather*}
L_{2}=\frac{N_{2} \phi_{2}}{I_{2}} \quad \text { and } \quad M=\frac{N_{1} \phi_{21}}{I_{2}}=\frac{N_{1} k \phi_{2}}{I_{2}}  \tag{5.4}\\
\left(\Theta \phi_{21}=k \phi_{2}\right)
\end{gather*}
$$

Multiplying Equations (5.3) and (5.4), we get
or

$$
M \times M=\frac{N_{2} k \phi_{1}}{I_{1}} \times \frac{N_{1} k \phi_{2}}{I_{2}}
$$

$$
\begin{align*}
M^{2} & =k^{2} \frac{N_{1} \phi_{1}}{I_{1}} \times \frac{N_{2} \phi_{2}}{I_{2}}=k^{2} L_{1} L_{2} \\
M & =k \sqrt{L_{1} L_{2}} \tag{5.5}
\end{align*}
$$

The abovementioned expression gives a relation between mutual inductance between the two coils and their respective self-inductances.
Expression (5.5) can also be written as

$$
k=\frac{M}{\sqrt{L_{1} L_{2}}}
$$

### 5.23 INDUCTANCES IN SERIES AND PARALLEL

Consider two coils magnetically coupled having self-inductance of $L_{1}$ and $L_{2}$, respectively, and a mutual inductance of $M \mathrm{H}$. The two coils, in an electrical circuit, may be connected in different ways giving different values of resultant inductance as the following.


Fig. 5.33 Inductances in series, fields are additive


Fig. 5.34 Inductances in series, fields are subtractive

### 5.23.1 Inductances in Series

The two coils may be connected in series in two ways: when their fields (or mmfs) are additive, that is, their fluxes are set up in the same direction as shown in Figure 5.33. In this case, the inductance of each coil is increased by $M$, that is, inductance, $L_{T}=\left(L_{1}+M\right)+\left(L_{2}+M\right)=L_{1}+L_{2}+2 M$ When their fields (or mmfs) are subtractive, that is, their fluxes are set up in opposite direction, as shown in Figure 5.34. In this case, the inductance of each coil is decreased by $M$, that is,
Total inductance, $L_{T}=\left(L_{1}-M\right)+\left(L_{2}-M\right)=L_{1}+L_{2}-2 \mathrm{M}$ Note: It may be noted that direction of field produced by a coil is denoted by a dot placing it at the side of which the current enters (or flux enters the core) (see Figs. 5.33 and 5.34).

### 5.23.2 Inductances in Parallel

The two coils may be connected in parallel in two ways: when the fields (or mmfs) produced by them are in the same direction, as shown in Figure 5.35, then Total inductance, $L_{T}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}-2 M}$
When the fields (or mmfs) produced by them are in the opposite direction, as shown Figure 5.36:
Total inductance, $L_{T}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}+2 M}$


Fig. 5.35 Inductance connected in parallel with addition fields


Fig. 5.36 Inductance connected in parallel with subtractive fields

## Example 5.23

A coil has 1,500 turns. A current of 4 A causes a flux of 8 mWb to link the coil. Find the self-inductance of the coil.

## Solution:

Inductance of the coil, $L=\frac{N \phi}{I}$
where $N=1,500 ; \phi=8 \times 10^{-3} \mathrm{~Wb}$ and $I=4 \mathrm{~A}$.

$$
\therefore \quad L=\frac{1500 \times 8 \times 10^{-3}}{4}=3 \mathrm{H}
$$

## Example 5.24

Calculate the value of emf induced in circuit having an inductance of $700 \mu \mathrm{H}$ if the current flowing through it varies at a rate of $5,000 \mathrm{~A} / \mathrm{s}$.

## Solution:

Inductance of the coil, $L=700 \times 10^{-6} \mathrm{H}$
Rate of change of current, $\frac{d I}{d t}=5,000 \mathrm{~A} / \mathrm{s}$
Magnitude of emf induced in the coil,

$$
e=L \frac{d I}{d t}=700 \times 10^{-6} \times 5,000=3.5 \mathrm{~V}
$$

## Example 5.25

An air-cored solenoid has 300 turns; its length is 25 cm and its cross section is $3 \mathrm{~cm}^{2}$. Calculate the self-inductance in Henry.

## Solution:

Number of turns of the solenoid, $N=300$
Length of solenoid, $l=25 \mathrm{~cm}=0.25 \mathrm{~m}$
Area of cross section, $a=3 \mathrm{~cm}^{2}=3 \times 10^{-4} \mathrm{~m}^{2}$
For air core, $\mu_{\mathrm{r}}=1$
Inductance of the solenoid, $L=\frac{N^{2}}{l / a \mu_{0} \mu_{\mathrm{r}}}=\frac{N^{2}}{l} a \mu_{0}$

$$
=\frac{300 \times 300}{0.25} \times 3 \times 10^{-4} \times 4 \pi \times 10^{-7}=0.1375 \mathrm{mH}
$$

## Example 5.26

Calculate the inductance of toroid, 25 cm mean diameter and $6.25 \mathrm{~cm}^{2}$ circular cross section wound uniformly with 1,000 turns of wire. Hence, calculate the emf induced when current in it increases at the rate of $100 \mathrm{~A} / \mathrm{s}$.

## Solution:

Inductance of the toroid, $L=\frac{N^{2}}{l} \times a \mu_{0} \mu_{\mathrm{r}}$
where number of turns, $N=1,000$ turns
Mean length $l=\pi D=0.25 \pi \mathrm{~m}$;
Area of $x$-section, $a=6.25 \times 10^{-4} \mathrm{~m}^{2}$ and
Relative permeability, $\mu_{\mathrm{r}}=1$

$$
L=(1,000)^{2} \times 6.25 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 1 / 0.25 \pi=1 \mathrm{mH}
$$

Induced emf, $e=L \frac{d I}{d t}=1 \times 10^{-3} \times 100=0.1 \mathrm{~V}$

## Example 5.27

The iron core of a choke has mean length 25 cm with an air gap of 1 mm . The choke is designed for an inductance of 15 H when operating at a flux density of $1 \mathrm{~Wb} / \mathrm{m}^{2}$. The iron core has a relative permeability of 3,000 and $8 \mathrm{~cm}^{2}$ area of cross section. Determine the required number of turns of the coil.

## Solution:

Inductance of the coil, $L=N^{2} / S_{\mathrm{T}}$
where $S_{\mathrm{T}}$ = total reluctance of the magnetic circuit

$$
\begin{aligned}
S_{\mathrm{T}} & =\frac{l_{\mathrm{i}}}{a \mu_{0} \mu_{\mathrm{r}}}=\frac{l_{\mathrm{g}}}{a \mu_{0}} \\
& =\frac{0 \cdot 25}{8 \times 10^{-4} \times 4 \pi \times 10^{-7} \times 3,000}+\frac{1 \times 10^{-3}}{8 \times 10^{-4} \times 4 \pi \times 10^{-7}}=1,077,612 \mathrm{AT} / \mathrm{Wb}
\end{aligned}
$$

Now, $\quad N=\sqrt{L S_{\mathrm{T}}}=\sqrt{15 \times 1,077,612}=4,020.5$ turns

## Example 5.28

Two coils have a mutual inductance of 0.6 H . If current in one coil is varied from 4 A to 1 A in 0.2 s , calculate the average emf induced in the other coil and the change of flux linking the latter, assuming that it is wound with 150 turns.

## Solution:

Mutually induced emf, $e_{\mathrm{m}}=e_{\mathrm{m}}=M \frac{d I_{1}}{d t}$
where $M=0.6 \mathrm{H} ; d I_{1}=4-1=3 \mathrm{~A}$ and $d t=0.2 \mathrm{~s}$

$$
e_{\mathrm{m}}=0.6 \times 3 / 0.2=9 \mathrm{~V}
$$

Now,

$$
e_{\mathrm{m}}=N_{2} \frac{d \phi_{12}}{d t}
$$

Therefore, change of flux with second coil,

$$
d \phi_{12}=\frac{e_{\mathrm{m}} \times d t}{N_{2}}=\frac{9 \times 0.2}{150}=12 \mathrm{mWb}
$$

## Example 5.29

Two coils having 100 and 50 turns, respectively, are wound on a core with $\mu=4,000 \mu_{0}$. Effective core length $=60 \mathrm{~cm}$ and core area $=9 \mathrm{~cm}^{2}$. Find the mutual inductance between the coils.
(UPTU 2004-2005)

## Solution:

We know that mutual Inductance

$$
M=\frac{N_{1} N_{2} \mu a}{l}
$$

where $N_{1}=100 ; N_{2}=50 ; \mu=4,000 \mu_{0} ; l=60 \mathrm{~cm}=60 \times 10^{-2} \mathrm{~m} ; a=9 \mathrm{~cm}^{2}=9 \times 10^{-4} \mathrm{~m}^{2}$

$$
\begin{aligned}
M & =\frac{100 \times 20 \times 4,000 \mu_{0} \times 9 \times 10^{-4}}{60 \times 10^{-2}} \\
& =\frac{100 \times 20 \times 4000 \times 4 \pi \times 10^{-7} \times 9 \times 10^{-4}}{60 \times 10^{-2}}=37.7 \mathrm{mh}
\end{aligned}
$$

## Example 5.30

A wooden ring has a mean diameter of 150 mm and a cross-sectional area of $250 \mathrm{~mm}^{2}$. It is wound with 1,500 turns of insulated wire. A second coil of 900 turns is wound on the top of the first. Assuming that all flux produced by the first coil links with the second, calculate the mutual inductance.
(UPTU)

## Solution:

Mutual inductance,

$$
M=\frac{N_{1} N_{2}}{l} a \mu_{0} \mu_{\mathrm{r}}
$$

where $N_{1}=1,500 ; N_{2}=900 ; l=\pi D=0.15 \pi \mathrm{~m} ; a=250 \times 10^{-6} \mathrm{~m}^{2} ; \mu_{\mathrm{r}}=1$

$$
M=\frac{1,500 \times 900}{0.15 \pi} \times 250 \times 10^{-6} \times 4 \pi \times 10^{-7} \times 1=0.9 \mathrm{mH}
$$

## Example 5.31

Two coils A and B of 600 and 1,000 turns, respectively, are connected in series on the same magnetic circuit of reluctance $2 \times 10^{6} \mathrm{AT} / \mathrm{Wb}$. Assuming that there is no flux leakage, calculate self-inductance of each coil and mutual inductance between the two coils. What would be the mutual inductance if the co-efficient of coupling is $75 \%$ ?

## Solution:

Self-inductance of coil $A$, where

Similarly,

$$
\begin{aligned}
& L_{1}=N_{1}^{2} / s \\
& N_{1}=600 \text { and } S=2 \times 10^{6} \mathrm{AT} / \mathrm{Wb} \\
& L_{1}=(600)^{2} / 2 \times 10^{6}=0.18 \mathrm{H} \\
& L_{2}=(1,000)^{2} / 2 \times 10^{6}=0.5 \mathrm{H}
\end{aligned}
$$

Mutual inductance,
When $k=1 ; M=1 \sqrt{0.18 \times 0.5}=0.3 \mathrm{H}$
and $k=0.75 ; M=0.75 \sqrt{0 \cdot 18 \times 0 \cdot 5}=0.225 \mathrm{H}$

## Example 5.32

Two air-cored coils are placed close to each other so that $80 \%$ of the flux of one coil links with the other. Each coil has mean diameter of 2 cm and a mean length of 50 cm . If there are 1,800 turns of wire on one coil, calculate the number of turns on the other coil to give a mutual inductance of 15 mH .

## Solution:

Reluctance,

$$
S=\frac{1}{a \mu_{0} \mu_{\mathrm{r}}}=\frac{0.5}{\frac{\pi}{4} \times(0.02)^{2} \times 4 \pi \times 10^{-7} \times 1}=1.2665 \times 10^{9} \mathrm{AT} / \mathrm{Wb}
$$

Now,

$$
\begin{gathered}
L_{1}=N_{1}^{2} / S \quad \text { and } \quad L_{2}=N_{2}^{2} / S \\
\sqrt{L_{1} L_{2}}=N_{1} N_{2} / S
\end{gathered}
$$

Further, $M=k \sqrt{L_{1} L_{2}}=k N_{1} N_{2} / S$
where $M=15 \times 10^{-3} \mathrm{H} ; N_{1}=1,800 ; k=0.8 ;$
or

$$
\begin{aligned}
& 15 \times 10^{-3}=0.8 \times 1,800 \times N_{2} / 1.2665 \times 10^{9} \\
& N_{2}=\frac{15 \times 10^{-3} \times 1.2665 \times 10^{9}}{0.8 \times 1,800}=13,193 \text { turns }
\end{aligned}
$$

## Example 5.33

Two coils with negligible resistance and of self-inductance of 0.2 H and 0.1 H , respectively, are connected in series. If their mutual inductance is 0.1 H , determine the effective inductance of the combination.

## Solution:

Total inductance of the two coils when connected in series;

$$
L=L_{1}+L_{2} \pm 2 M=0.2+0.1 \pm 2 \times 0.1=0.5 \mathrm{H} \text { or } 0.1 \mathrm{H}
$$

## Example 5.34

The combined inductance of two coils connected in series is 0.6 H and 0.1 H depending upon the relative direction of currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H , calculate the mutual inductance of the coils and the self-inductance of the other coil.

## Solution:

The combined inductance of the two coils when connected in series having their

$$
\begin{align*}
\text { field additive } & =L_{1}+L_{2}+2 M=0.6  \tag{5.6}\\
\text { fields subtractive } & =L_{1}+L_{2}-2 M=0.1 \tag{5.7}
\end{align*}
$$

Subtracting equation (5.7) from (5.6), we get,

$$
4 M=0.5 \quad \text { or } \quad M=0.125 \mathrm{H}
$$

From equation (5.7), $L_{1}+L_{2}-2 \times 0.125=0.1$ or $L_{1}+L_{2}=0.35 \mathrm{H}$
Self-inductance of one coil, $L_{1}=0.2 \mathrm{H}$
Therefore, self-inductance of second coil, $L_{2}=0.25-0.2=0.15 \mathrm{H}$

## Example 5.35

Two coils of self-inductance 120 mH and 250 mH and mutual inductance of 100 mH are connected in parallel. Determine the equivalent inductance of combination if mutual flux helps the individual fluxes and mutual flux opposes the individual fluxes.

## Solution:

When mutual flux helps the individual fluxes:

$$
L_{\mathrm{T}}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}-2 M}=\frac{120 \times 250-(100)^{2}}{120+250-2 \times 100}=117.65 \mathrm{mH}
$$

When mutual flux opposes the individual fluxes:

$$
L_{\mathrm{T}}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}+2 M}=\frac{120 \times 250-(100)^{2}}{120+250+2 \times 100}=35.088 \mathrm{mH}
$$

### 5.24 ENERGY STORED IN A MAGNETIC FIELD

When some electrical energy is supplied to a coil, it is spent in two ways:

1. A part of it is spent to meet $I^{2} R$ loss that is dissipated in the form of heat and cannot be recovered.
2. The remaining part is used to create magnetic field around the coil and is stored in the magnetic field. When this field collapses, the stored energy is released by the coil and is returned to the circuit.

$$
\text { The energy stored in the magnetic field is } \frac{1}{2} L I^{2}
$$

## Example 5.36

A solenoid of 1 m in length and 10 cm in diameter has 5,000 turns. Calculate the energy in the magnetic field when a current of 2 A flows in the solenoid.

## Solution:

Inductance of the solenoid, $L=\frac{N^{2}}{l} \times a \mu_{0} \mu_{\mathrm{r}}$
where $N=5,000 ; a=\pi d^{2} / 4=25 \pi \times 10^{-4} \mathrm{~m}^{2} ; l=1 \mathrm{~m} ; \mu_{\mathrm{r}}=1$

$$
L=(5,000)^{2} \times 25 \pi \times 10^{-4} \times 4 \pi \times 10^{-7} \times 1 / 1=0.2467 \mathrm{H}
$$

Energy stored $=\frac{1}{2} L I^{2}=\frac{1}{2} \times 0.2467 \times(2)^{2}=0.4934 \mathrm{~J}$

### 5.25 AC EXCITATION IN MAGNETIC CIRCUITS

To magnetise the magnetic circuits of electrical devices such as transformers, AC machines, electromagnetic relays, etc., AC supply is used. The magnetisation of magnetic circuits is called their excitation. The magnetic circuits are never excited by DC supply. This is because in the case of DC excitation, the steady-state current is determined by the impressed voltage and resistance of the circuit. The inductance of the coil comes into picture only during transient period, that is, when the current is building-up or decaying during switching (ON or OFF) instants. The magnetic flux in the magnetic circuit adjusts itself in accordance with this steady value of current so that the relationship imposed by magnetisation $(B-H)$ curve in satisfied. However, with AC excitation, inductance comes into picture even at steady-state condition. As a result, for most of the magnetic circuits (not for all), the flux is determined by the impressed voltage and frequency. Then, the magnetisation current adjusts itself in accordance with this flux so that the relationship imposed by the magnetisation $(B-H)$ curve is satisfied.

Usually, for economic reasons, the normal working flux density in a magnetic circuit is kept beyond the linear portion of the magnetisation curve, and thus, accurate analysis cannot be predicted for determining self-inductance. However, for all practical purposes, the parameters of the magnetic circuit are considered to be constant.
The reactive effect of the alternatively flux set up by the exciting current can readily be shown as per Faraday's laws, that is,

$$
e=N \frac{d \phi}{d t}
$$



Fig. 5.37 Excitation of magnetic circuit

Consider a magnetic core that is excited by a coil (winding) having $N$ turns and carrying a current of $i \mathrm{~A}$, as shown in Figure 5.37. A magnetic flux $\phi$ is set up by the exciting current $i$. Let the magnetic flux $\phi$ varies sinusoidally with respect to time $t$. Then, its instantaneous value is given by the relation:

$$
\begin{equation*}
\phi=\phi_{\mathrm{m}} \sin w t=\phi_{\mathrm{m}} \sin 2 \pi f t \tag{5.8}
\end{equation*}
$$

where $\phi_{m}=$ maximum value of alternating flux $f=$ frequency of supply impressed across the coil
The induced emf in the coil

$$
\begin{align*}
e & =N \frac{d \phi}{d t}=N \frac{d}{d t}\left(\phi_{\mathrm{m}} \sin 2 \pi f t\right) \\
& =2 \pi f N \phi_{\mathrm{m}} \cos 2 \pi f t=2 \pi f N \phi_{\mathrm{m}} \sin \left(2 \pi f t+\frac{\pi}{2}\right) \tag{5.9}
\end{align*}
$$

The value of induced emf will be maximum, when $\cos 2 \pi f t=1$, and therefore

$$
E_{\mathrm{m}}=2 \pi f N \phi_{\mathrm{m}}
$$

Its effective or rms value,

$$
E_{\mathrm{rms}}=\frac{E_{m}}{\sqrt{2}}=\frac{2 \pi f N \phi_{m}}{\sqrt{2}}=4.44 f N \phi_{m}
$$

Equations (5.8) and (5.9) reveal that the induced emf leads the flux, and hence, the exciting current by $\frac{\pi}{2}$ radian or $90^{\circ}$. This induced emf and the coil resistance drop oppose the applied voltage. In the case of electrical machines, usually the drop in resistance is only a few percent of applied voltage, and therefore, neglected for close approximation. Thus, the induced emf $E$ and applied voltage $V$ may be considered equal in magnitude.

## Example 5.37

For the AC excited magnetic circuit shown in Figure 5.38, calculate the excitation current and induced emf of the coil to produce a core flux of $0.6 \sin 314 t \mathrm{mWb}$.
(UPTU 2001)

## Solution:

Here, $\phi=\phi_{\mathrm{m}} \sin w t=0.6 \sin 314 t \mathrm{mWb}$
Maximum value of flux, $\phi_{\mathrm{m}}=0.6 \mathrm{mWb}=6 \times 10^{-4} \mathrm{~Wb}$
Area of $x$-section, $a=3 \times 3=9 \mathrm{~cm}^{2}=9 \times 10^{-4} \mathrm{~m}^{2}$
Flux density,

$$
B_{\mathrm{m}}=\frac{\phi_{\mathrm{m}}}{\alpha}=\frac{6 \times 10^{-4}}{9 \times 10^{-4}}=0.667 \mathrm{~T}
$$



Fig. 5.38 Magnetic circuit with air-gap

Length of air gap, $l_{\mathrm{g}}=1.5 \mathrm{~mm}=1.5 \times 10^{-3} \mathrm{~m}$

Length of iron path,

$$
\begin{aligned}
l_{\mathrm{i}} & =\left(25-2 \times \frac{3}{2}+35-2 \times \frac{3}{2}\right) \times 2-0.15=107.85 \mathrm{~cm} \\
& =1.0785 \mathrm{~m}
\end{aligned}
$$

Total AT required, $\mathrm{AT}_{\mathrm{m}}=\frac{B_{\mathrm{m}}}{\mu_{0}} l_{\mathrm{g}}+\frac{B_{\mathrm{m}}}{\mu_{0} \mu_{\mathrm{r}}} l_{\mathrm{i}}$

$$
\begin{aligned}
& =\frac{0.667 \times 1.5 \times 10^{-3}}{4 \pi \times 10^{-7}}+\frac{0.667 \times 1.0785}{4 \pi \times 10^{-7} \times 3,775} \\
& =796.3+151.7=948
\end{aligned}
$$

The maximum value of excitation current required,

$$
I_{\mathrm{m}}=\frac{\mathrm{AT}_{\mathrm{m}}}{N}=\frac{948}{500}=1.896 \mathrm{~A}
$$

The rms value of excitation current,

$$
I_{\mathrm{rms}}=\frac{I_{\mathrm{m}}}{\sqrt{2}}=\frac{1.896}{1.414}=1.34 \mathrm{~A}
$$

The rms value of induced emf in the coil, $E_{\mathrm{rms}}=4.44 f N \phi_{\mathrm{m}}$

$$
\begin{aligned}
& =4.44 \times \frac{314}{2 \pi} \times 500 \times 0.6 \times 10^{-3} \\
& =66.57 \mathrm{~V}
\end{aligned}
$$

### 5.26 EDDY CURRENT LOSS

When a magnetic material is subjected to a changing (or alternating) magnetic field, an emf is induced in the magnetic material itself according to Faraday's laws of electromagnetic induction. Since the magnetic material is also a conducting material, these emfs. circulate currents within the body of the magnetic material. These circulating currents are known as eddy currents. As these currents are not used for doing any useful work, and therefore, these currents produce a loss $\left(i^{2} R\right.$ loss $)$ in the magnetic material called eddy current loss. Like hysteresis loss, this loss also increases the temperature of the magnetic material. The hysteresis and eddy current losses in a magnetic material are called iron losses or core losses or magnetic losses.


Fig. 5.39 Eddy currents (a) Solid core (b) Laminated core

A magnetic core subjected to a changing flux is shown in Figure 5.39. For simplicity, a sectional view of the core is shown. When changing flux links with the core itself, an emf is induced in the core that set up circulating (eddy) currents (i) in the core, as shown in Figure 5.39 (a). These currents produce eddy current loss $\left(i^{2} R\right)$, where $i$ is the value of eddy currents and $R$ is resistance of eddy current path. As the core is a continuous iron block of large cross section, the magnitude of $i$ will be very large and hence great eddy current loss will result.

To reduce the eddy current loss, the obvious method is to reduce magnitude of eddy currents. This can be achieved by splitting the solid core into thin sheets (called laminations) in the planes parallel to the magnetic field, as shown in Figure 5.39 (b). Each lamination is insulated from the other by a fine layer of insulation (varnish or oxide film). This arrangement reduces the area of each section, and hence, the induced emf. It also increases the resistance of eddy currents path, since the area through which the currents can pass is smaller. This loss can further be reduced by using a magnetic material having higher value of resistivity (like silicon steel).

### 5.26.1 Useful Applications of Eddy Currents

It has been seen that when the effects of eddy currents (production on heat) are not utilised, the power or energy consumed by these currents is known as eddy current loss. However, there are the places where eddy currents are used to do some useful work, for example, in the case of induction heating. In this case, an iron shaft is placed as a core of an inductive coil. When high frequency current is passed through the coil, a large amount of heat is produced at the outermost periphery of the shaft by eddy currents. The amount of heat reduces considerably when we move towards the centre of the shaft. This is because outer periphery of the shaft offers low resistance path to eddy currents. This process is used for surface hardening of heavy shafts like axils of automobiles. Eddy current effects are also used in electrical instrument, for example, providing damping torque in permanent magnet moving coil instruments and braking torque in the case of induction-type energy meters.

### 5.26.2 Mathematical Expression for Eddy Current Loss

Although it is difficult to determine the eddy-current power loss from the current and resistance values. However, experiments reveal that the eddy-current power loss in a magnetic material can be expressed as

$$
P_{\mathrm{e}}=K_{\mathrm{e}} B_{\mathrm{m}}^{2} t^{2} f^{2} V W
$$

where $K_{e}=$ co-efficient of eddy current, its value depends upon the nature of magnetic material
$B_{m}=$ maximum value of flux density in $\mathrm{Wb} / \mathrm{m}^{2}$
$t=$ thickness of lamination in m
$f=$ frequency of reversal of magnetic field in Hz
$V=$ volume of magnetic material in $\mathrm{m}^{3}$

PRACTICE EXERCISES

## Short Answer Questions

1. What do mean by electromagnetic induction?
2. Define Faraday's laws of electromagnetic induction.
3. State Fleming's right-hand rule.
4. State Lenz's law.
5. What do you mean by dynamically induced emf?
6. What do you mean by statically induced emf? How self-induced emf is different to mutually induced emf?
7. What do understand by self-inductance?
8. What is mutual inductance?
9. Define co-efficient of coupling.
10. What will be the total inductance, if two inductances are connected in series?
11. What will be the total inductance, if two inductances are connected in parallel?
12. How can you determine the energy stored in a magnetic field?

## Test Questions

1. State and explain Faraday's laws of electromagnetic induction.
2. State and explain Fleming's right-hand rule and Lenz's law. How these are useful to determine the direction of electromagnetically induced emf?
3. Show that dynamically induced emf $e=B l v \sin \theta$.
4. Define and explain statically induced emf. How self-induced emf is different to mutually induced emf?
5. Define and explain self and mutual inductance.
6. What do you mean by co-efficient of coupling and derive the relation $k=\frac{M}{\sqrt{L_{1} L_{2}}}$

## Numericals

1. A coil of 500 turns is wound on a magnetic circuit of reluctance $2 \times 10^{5} \mathrm{AT} / \mathrm{Wb}$. If a current of 2 A flowing through the coil is reversed in 10 ms , find the average emf induced in the coil.(Ans. 300 V )
2. An iron ring or toroid 0.2 m in diameter and $10 \mathrm{~cm}^{2}$ cross-sectional area of the core is wound with 250 turns of wire. If the flux density in the core is to be $1 \mathrm{~Wb} / \mathrm{m}^{2}$ and relative permeability of iron is 500 , what is the exciting current required to be passed in the winding? Further, determine the self-inductance.
(Ans. $4 \mathrm{~A} ; 62.5 \mathrm{mH}$ ) (UPTU)
3. Two identical coils A and B each having 1,500 turns lie in parallel planes such that $60 \%$ of the flux produced by one coil links with the other. A current of 10 A in coil A produces a flux of 1 mWb in it. If the current in coil A changes from +15 A to -15 A in 0.02 s , then (a) what would be the mutual inductance and magnitude of the emf induced in coil B? (b) Calculate the self-inductance of each coil.
(Ans. $0.09 \mathrm{H}, 135 \mathrm{~V} ; 0.15 \mathrm{H}$ )
4. Two coils $A$ and $B$ have self-inductance of $240 \mu \mathrm{H}$ and $600 \mu \mathrm{H}$, respectively. When a current of 3 A through coil A is reversed, the deflection on the flux meter connected across $B$ is $1,200 \mu \mathrm{~Wb}$-turns. Calculate (a) the mutual inductance between the coils, (b) the average emf induced in coil $B$ if the flux is reversed in 0.2 s , and (c) co-efficient of coupling.
(Ans. $200 \mu \mathrm{H} ; 6 \mathrm{mV} ; 0.527$ )
5. When two coils are connected in series, their effective inductance is found to be 14 H . When the connections of one coil are reversed, the effective inductance is 8 H . If the co-efficient of coupling is 0.8 , calculate the self-inductance of each coil and the mutual inductance. (Ans. $10.67 \mathrm{H} ; 0.33 \mathrm{H}, 1.5 \mathrm{H}$ )

## SUMMARY

1. Magnetic field: The region around a magnet where its poles exhibit a force is called magnetic field.
2. Magnetic flux: The magnitude of magnetic lines of force set up in a magnetic circuit is called magnetic flux.
3. Magnetic flux density: The flux per unit area at right angles to the flux at a point is called magnetic flux density at that point.
4. Permeability: The ability of a material to conduct magnetic lines of force through it is called the permeability of that material.
5. mmf : The work done in moving a unit magnetic pole once round the magnetic circuit is called mmf; $\mathrm{mmf}=N I \mathrm{AT}$.
6. Reluctance ( $S$ ): The opposition offered to the magnetic flux by a magnetic circuit is called its reluctance.

$$
S=\frac{l}{a \mu_{0} \mu_{\mathrm{r}}} \mathrm{AT} / \mathrm{Wb}
$$

7. Magnetic flux ( $\phi$ ): The amount of magnetic lines of force set up in a magnetic circuit is called magnetic flux.
8. Relation between mmf, reluctance, and flux:

$$
\phi=\frac{\mathrm{mmf}}{\text { Reluctance }} \quad \text { or } \quad \phi=\frac{N I}{l} a \mu_{0} \mu_{\mathrm{r}}
$$

9. Permeance: It is reciprocal to reluctance.
10. Reluctivity: It is specific reluctance.
11. Calculation of AT: AT, that is, $N I=H l$.
12. Series magnetic circuit: The magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called series magnetic circuit. In these circuits,
total reluctance, $S=S_{1}+S_{2}+\ldots \ldots .+S_{\mathrm{n}}$;
total $\mathrm{mmf}=H_{1} l_{1}+H_{2} l_{2}+\ldots \ldots .+H_{\mathrm{n}} l_{\mathrm{n}}$
13. Parallel magnetic circuit: The magnetic circuit in which the magnetic flux is divided is called parallel magnetic circuit. While calculating AT in this case, only one parallel path is considered.
14. Leakage flux: The flux that does not follow the intended path in a magnetic circuit is called leakage flux.
15. Leakage factor: The ratio of total flux to useful flux is called leakage factor $\left(\lambda=\frac{\phi}{\phi_{u}}\right)$.
16. $B-H$ curve: The graph between $B$ and $H$ of a material is called $B-H$ curve of that material. It shows the properties of the material.
17. Magnetic hysteresis: The phenomenon of $B$ lagging behind $H$ in a magnetic material is called magnetic hysteresis.
18. Residual magnetism and retentivity: The value of flux density retained by a magnetic material when no magnetising force is existing is called residual magnetism and the power of retaining this residual magnetism is called retentivity of the material.
19. Coercive force: The magnetising force required to remove the residual magnetism is called coercive force.
20. Hysteresis loss: To magnetise the magnetic material from one direction to the other and vice versa, some energy is wasted called hysteresis loss. Its value depends upon the area of hysteresis loop of the magnetic material. The larger the area, the more is the loss and vice versa.
21. Magnitude of hysteresis loss: Hysteresis energy loss, $W_{\mathrm{h}}=\left(\right.$ area of hysteresis loop in $\left.\mathrm{cm}^{2}\right) \times$ scale used for $B$ and $H \mathrm{~J} / \mathrm{m}^{3} / \mathrm{cycle}$
Hysteresis power loss, $P_{\mathrm{h}}=\left(\right.$ energy loss in $\left.\mathrm{J} / \mathrm{m}^{3} / \mathrm{cycle}\right) \times$ volume $\times$ frequency
According to Steinmetz, hysteresis power loss, $P_{\mathrm{h}}=\eta B_{\max }^{1.6} V f$ joule
22. Electromagnetic induction: The phenomenon by which an emf is induced in a coil or circuit when magnetic flux linking with it changes is called electromagnetic induction.
23. Faraday's laws of electromagnetic induction:

First law: When even magnetic flux linking with any circuit changes, an emf is induced in it.
Second law: The magnitude of induced emf is directly proportional to the rate of change of flux linkages.

$$
e \propto N \frac{d \phi}{d t} ; e=N \frac{d \phi}{d t} ; e=N\left(\frac{\phi_{2}-\phi_{1}}{t}\right)
$$

24. Fleming's right-hand rule: Stretch first finger, second finger, and thumb of right-hand mutually perpendicular to each other. If the first finger represents magnetic field and the thumb represents the direction of motion of conductor w.r.t. magnetic field, then the second finger will represent the direction of induced emf in the conductor.
25. Lenz's law: In effect, electromagnetically induced emf always opposes the cause that produces it.
26. Dynamically induced emf: When either conductor or field system or both of them are moving so that flux is cut by the conductor, the emf induced in the conductor by this way is called dynamically induced emf $e=B l v \sin \theta$.
27. Statically induced emf: When both circuit (or coil) and field system are stationary but the flux produced by the field system linking with circuit changes, the emf thus induced is called statically induced emf
(a) Self-induced emf: The emf induced in a coil due to change of flux produced by it linking with its own turns is called self-induced emf.

$$
e=L \frac{d I}{d t}
$$

(b) Mutually induced emf: The emf induced in a coil due to change of flux produced by the other (neighbouring) coil linking with it is called mutually induced emf.

$$
e_{\mathrm{m}}=M \frac{d I_{1}}{d t}
$$

28. Self-inductance: The property of a coil due to which it opposes the change of current flowing through itself is called self-inductance of the coil.

$$
L=\frac{e}{d I / d t} ; L=\frac{N \phi}{I} ; L=\frac{N^{2}}{l} a \mu_{0} \mu_{\mathrm{r}}
$$

29. Mutual inductance: The property of one coil due to which it opposes the change of current flowing through the other (neighbouring) coil is called mutual inductance between the two coils.

$$
\begin{aligned}
M & =\frac{e_{\mathrm{m}}}{d I_{1} d t} \\
M & =\frac{N_{2} \phi_{12}}{I_{1}} \\
M & =\frac{N_{1} N_{2}}{l} a \mu_{0} \mu_{\mathrm{r}}
\end{aligned}
$$

30. Co-efficient of coupling: The fraction of magnetic flux produced by one coil linking with the other is known as co-efficient of coupling between the two coils.

$$
K=M / \sqrt{L_{1} L_{2}}
$$

31. Inductances in series:
$L_{\mathrm{T}}=L_{1}+L_{2}+2 M \ldots$ two fields are in same direction
$L_{\mathrm{T}}=L_{1}+L_{2}-2 M \ldots$ two fields are in opposite direction
32. Inductances in parallel:

$$
\begin{aligned}
& L_{\mathrm{T}}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}-2 M} \ldots \text { two fields are in same direction } \\
& L_{\mathrm{T}}=\frac{L_{1} L_{2}-M^{2}}{L_{1}+L_{2}+2 M} \ldots \text { two fields are in opposite direction }
\end{aligned}
$$

33. Energy stored in the magnetic field: $W$ or $E=\frac{1}{2} L I^{2} \mathrm{~J}$

## TEST YOUR PREPARATHON

## 7 FILL IN THE BLANKS

1. Magnetic flux in the magnetic circuit corresponds to $\qquad$ in electric circuit.
2. The resistance in an electric circuit is analogous to $\qquad$ in magnetic circuit.
3. In a composite magnetic circuit, the total reluctance will be $\qquad$ of individual parts.
4. During reversal of magnetic field in a magnetic material $B$ lags behind $H$, this phenomenon is known as $\qquad$ .
5. The conductance in an electric circuit corresponds to $\qquad$ in magnetic circuit.
6. According to Steinmetz's law, the hysteresis loss in a magnetic material is given by the relation
$\qquad$ —.
7. The best suited magnetic material for the construction of core of a transformer is $\qquad$ .
8. Permeability in a magnetic material corresponds to $\qquad$ in conducting material.
9. The magnetic flux set up by a solenoid that does not follow the intended path in a magnetic circuit is called $\qquad$ -.
10. The magnitude of hysteresis loss depends upon area of hysteresis loop, volume of magnetic material, and $\qquad$ .
11. The magnitude of electromagnetically induced emf is directly proportional to the rate of change of
$\qquad$ .
12. The co-efficient of coupling is the $\qquad$ of magnetic flux produced by one coil linking with the other.
13. The mutual inductance is given by the relation $M=\frac{N_{1} \ldots}{\ldots}$
14. The unit of co-efficient of coupling is $\qquad$ .
15. The property of a coil due to which it opposes the change of current in the neighbouring coil is called
$\qquad$ _.
16. The magnitude of dynamically induced emf is given by $e=$ $\qquad$
17. The energy stored in the magnetic field is $\qquad$ $\mathrm{w}-\mathrm{s}$.
18. The direction of electromagnetically induced emf is determined by $\qquad$ or $\qquad$ .
19. The property of a coil due to which it opposes the change of current through it is called $\qquad$ .
20. The unit of self-inductance is $\qquad$ _.
21. If the two coils are magnetically coupled, each having inductance of 4 and 6 H , respectively, and mutual inductance of 3 H , then the co-efficient of coupling between them is $\qquad$ .
22. An inductance of 20 H carrying a current of 600 A will store $\qquad$ kWh energy.

## OBJECTIVE TYPE QUESTIONS

1. The perfect magnetic insulator is
(a) copper
(b) iron
(c) rubber
(d) None of above
2. The permeance in a magnetic circuit corresponds to
(a) resistance in an electric circuit
(b) emf in an electric circuit
(b) conductivity in an electric circuit
(d) conductance in an electric circuit
3. The AT are
(a) the product of the number of turns and current of the coil
(b) the number of turns of a coil through which current is flowing
(c) the currents of all turns of the coil
(d) the turns of transformer winding
4. The magnetic field can penetrate empty space
(a) True
(b) 2.5
(c) 4.0
(d) 0.4
5. What will be the current passing through the ring shaped air-cored coil when number of turns is 800 and AT are 3,200 .
(a) 0.25
(b) 2.5
(c) 4.0
(d) 0.4
6. How many AT are required to produce a field intensity of $4,000 \mathrm{AT} / \mathrm{m}$ in a magnetic circuit of length 4 mm ?
(a) $4,000 \mathrm{AT}$
(b) 16 AT
(c) 4 AT
(d) $1,000 \mathrm{AT}$
7. The magnetic reluctance of a material
(a) increases with increasing cross-sectional area of material
(b) does not vary with increasing the cross-sectional area of material
(c) decreases with increasing cross-sectional area of material
(d) decreases with increasing the length of material
8. The magnetic field strength $H$ and flux density $B$ are independent of each other.
(a) True
(b) False
9. mmf is analogous to
(a) electric current in electric circuit
(b) current density in conductor
(c) electromotive force
(d) voltage
10. The correct expression in the following is
(a) $\phi=\frac{N}{l / a \mu_{0} \mu_{\mathrm{r}}}$
(b) $N I=\frac{B}{\mu_{0} \mu_{\mathrm{r}}} \times l$
(c) $N=H \times l$
(d) $N I=\frac{\phi}{\mu_{0} \mu_{\mathrm{r}}} \times l$
11. The magnetic strength of an electromagnet can be increased by
(a) increasing current in the solenoid
(b) increasing the number of turns of the solenoid
(c) Both (a) and (b)
(b) None of above
12. The reluctance of a material is defined as
(a) opposition offered to the magnetic field by it
(b) its ability to conduct magnetic flux
(c) opposition offered to the flow of current through the solenoid
(d) None of above
13. Hysteresis loss in a magnetic material depends upon
(a) area of hysteresis loop
(b) frequency of reversed of field
(c) volume of magnetic material
(d) All the above
14. The best suited magnetic material for the construction of transformer core is
(a) silicon steel, since it has small hysteresis loop area
(b) hard steel, since it has small hysteresis loop area
(c) silicon steel, since it has large hysteresis loop area
(d) wrought iron, since it has large hysteresis loop area
15. The Steinmetz expression to determine the hysteresis loss in the magnetic material is
(a) $P_{\mathrm{h}}=\eta B_{\text {max }}^{1.6}$.
(b) $P_{\mathrm{h}}=\eta B_{\max }^{1.6} f$.
(c) $P_{\mathrm{h}}=\eta B_{\max }^{1.6} f V$.
(d) All the above
16. The emf is induced in a coil
(a) less than the period during which flux changes through it
(b) only for the period during which flux changes through it
(c) more than the period during which flux changes through it
(d) None of the above
17. The direction of electromagnetically induced emf is determined by
(a) Flemings' right-hand rule
(b) Lenz's law
(c) Right hand thumb rule
(d) Both (a) and (b)
18. When the current flowing through a circuit is switched off, then
(a) induced current flows in the same direction as that of the main current
(b) induced current flows in opposite as that of the main current
(c) no induced current will flow
(d) None of the above
19. The energy store in the magnetic field is
(a) given by the relation $L I^{2} / 2$
(b) directly proportional to the square of current flowing through the coil
(c) directly proportional to the inductance of the coil
(d) All the above
20. The co-efficient of coupling of two coils is proportional to
(a) $L_{1} L_{2}$
(b) $\sqrt{L_{1} L_{2}}$
(c) $1 / \sqrt{L_{1} L_{2}}$
(d) $1 / L_{1} L_{2}$
21. The self-inductance of a solenoid of $N$-turns is proportional to
(a) $N$
(b) $N^{2}$
(c) $1 / N$
(d) $1 / N^{2}$
22. The self-induced emf in a 0.2 H coil when a current in it is changing at the rate of $100 \mathrm{~A} \mathrm{~s}^{-1}$ is
(a) 20 V
(b) 200 V
(c) $2 \times 10^{-3} \mathrm{~V}$
(d) $2 \times 10^{-2} \mathrm{~V}$
23. To reduce eddy current loss in the core of magnetic material
(a) the core is laminated
(b) the magnetic material used should have high resistivity
(c) Both (a) and (b)
(d) None of the above
24. A 1 H inductance carrying a current of 3 A will store energy of
(a) 3 W
(b) 9 Ws
(c) 3 J
(d) 9 W
25. If two coils having self-inductances $L_{1}$ and $L_{2}$ and a mutual inductance M are connected in series with opposite polarity, then the total inductance of the combination will be
(a) $L_{1}+L_{2}+2 M$
(b) $L_{1}-L_{2}-2 M$
(c) $L_{1}-L_{2}+2 M$
(d) $L_{1}+L_{2}-2 M$
26. If the length, number of turns, and area of coil are doubled, the inductance of the coil is
(a) the same
(b) doubled
(c) quadrupled
(d) one-fourth
27. An inductive coil of 10 H develops a counter voltage of 50 V . What should be the rate of change of current in the coil
(a) $5 \mathrm{~A} / \mathrm{s}$
(b) $0.2 \mathrm{~A} / \mathrm{s}$
(c) $1 \mathrm{~A} / \mathrm{s}$
(d) $500 \mathrm{~A} / \mathrm{s}$
28. A coil of 0.02 mH is carrying a current of 1 A . If this current is reversed in 0.02 s , the induced emf in the coil will be
(a) 0.08 V
(b) 0.004 V
(c) 0.002 V
(d) 0.02 V

## NUMERICALS

1. An iron ring has a cross-sectional area of $400 \mathrm{~mm}^{2}$ and a mean diameter of 25 cm . It is wound with 500 turns. If the value of relative permeability is 500 , find the total flux set up in the ring. The coil resistance is $400 \Omega$ and the supply voltage is 200 V .
(Ans. 0.08 mWb )
2. An iron ring of mean diameter 22 cm and cross section $10 \mathrm{~cm}^{2}$ has an air gap 1 mm wide. The ring is wound uniformly with 200 turns of wire. The permeability of ring material is 1,000 . A flux of 0.16 mWb is required in the gap. What current should be passed through the wire?
(Ans. 1.076 A) (PU April 1988)
3. An iron ring has a mean circumferential length of 60 cm with an air gap of 1 mm and a uniform winding of 300 turns. When a current of 1 A flows through the coil, find the flux density. The relative permeability of iron is 300 . Assume $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$.
(Ans. 0.1256 T) (Delhi Univ.)
4. In the magnetic circuit shown in Figure 5.40, a coil of 500 turns is wound on the central limb. The magnetic path from A to B by way of outer limbs have a mean length of 100 cm each and an effective cross-sectional area of $2.5 \mathrm{~cm}^{2}$. The central limb is 25 cm long and $5 \mathrm{~cm}^{2}$ cross-sectional area. The air gap is 0.8 cm long. Calculate the current flowing through the coil to produce a flux of 0.3 mWb in the air gap. The relative permeability of the core material is 800 (neglect leakage and fringing).
(Ans. 9.13 A$)$


Fig. 5.40 Magnetic circuit


Fig. 5.41 Magnetic structure
5. A steel magnetic structure made of a bar of section $2 \mathrm{~cm} \times 2 \mathrm{~cm}$ is shown in Figure 5.41. Determine the current that the 600 -turn magnetising coil on the left limb should carry so that a flux of 3 mWb is produced in the right limb. Take $\mu_{\mathrm{r}}=800$ and neglect leakage.
$\left(\right.$ Hints: $\left.\phi_{1} S_{1}=\phi_{2} S_{2}\right)$
(Ans. 13.056 A)(Pune Univ.)
6. A current of 8 A through a coil of 300 turns produces a flux of 4 mWb . If this current is reduced to 2 A in 0.1 s , calculate average emf induced in the coil, assuming flux to be proportional to current.
(Ans. 9 V)
7. Estimate the inductance of a solenoid of 2,500 turns wound uniformly over a length of 0.5 m on a cylindrical paper tube 4 cm in diameter. The medium is air.
(Ans. 19.74 mH )
8. Calculate the inductance of a toroid 25 cm mean diameter and $6.25 \mathrm{~cm}^{2}$ circular cross section wound uniformly with 1,000 turns of wire. Further, determine the emf induced when a current increasing at the rate of $200 \mathrm{~A} / \mathrm{s}$ flows in the winding.
(Ans. $1 \mathrm{mH} ; 0.2 \mathrm{~V}$ )
9. Two coils having turns 100 and 1,000 , respectively, are wound side by side on a closed iron circuit of cross-sectional area $8 \mathrm{~cm}^{2}$ and mean length 80 cm . The relative permeability of iron is 900 . Calculate the mutual inductance between the coils. What will be the induced emf in the second coil if current in the first coil is increased uniformly from 0 to 10 A in 0.2 s ?
(Ans. 0.113 H; 5.65 V) (PTU)
10. Two identical coils, when connected in series have total inductance of 24 H and 8 H depending upon their method or connection. Find self-inductance of each coil and mutual inductance between the coils.
(Ans. $8 \mathrm{H} ; 4 \mathrm{H}$ )
11. Two coils of self-inductance 100 mH and 150 mH and mutual inductance 80 mH are connected in parallel. Determine the equivalent inductance of combination if mutual flux helps the individual fluxes and mutual flux opposes the individual fluxes.
(Ans. $95.56 \mathrm{mH} ; 20.97 \mathrm{mH}$ )
12. A toroid having 25 cm mean diameter and $6.25 \mathrm{~cm}^{2}$ circular area of cross section is wound uniformly with 1,000 turns of wire. Determine inductance of the toroid and emf induced in the coil when current in it is increasing at the rate of $200 \mathrm{~A} / \mathrm{s}$, and energy stored in its magnetic field when coil carries a current of 10 A .
(Ans. $1 \mathrm{mH} ; 0.2 \mathrm{~V} ; 0.05 \mathrm{~J}$ )
13. A current of 20 A is passed through a coil of self-inductance 800 mH . Find the magnetic energy stored. If the current is reduced to half, find the new value of energy stored and the energy released back to the electrical circuit.
(Ans. $160 \mathrm{~J}, 40 \mathrm{~J}, 120 \mathrm{~J})$

## VIVA VOCE OR REASONING QUESTIONS

1. Fringing decreases the flux density in the air gap. How?
2. Air-gap flux is termed as useful flux. Why?
3. The relative permeability of a magnetic material changes with flux density. Why?
4. Does a magnetic circuit consume energy? Why?
5. In a magnetic material, hysteresis loss cannot be reduced to zero. Why?
6. To set up a required flux in a magnetic circuit having an air gap, the number of AT required are more than the iron path. Explain.
7. In all the magnetic circuits, leakage flux exists. Why?
8. Inductance appears in the circuit only when AC flows through it, but it disappears when DC flows through it. Why?
9. An arc appears across the opening contacts when an inductive circuit is opened, but it does not appear when the contacts of the same circuit are closed. Why?
10. Electromagnets are used as jaws of cranes. How?
11. To reduce eddy current losses, the magnetic core in laminated. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What is a magnet?
2. What do you understand by magnetism?
3. What are permanent and temporary magnets?
4. Give the advantages and practical applications of temporary and permanent magnets.
5. What are magnetic poles?
6. What are magnetic lines of force?
7. What do you understand by magnetic field?
8. Define magnetic flux and give its unit.
9. What do you mean by magnetic axis?
10. Do magnetic lines of force are purely imaginary?
11. Define and explain a magnetic circuit.
12. Define magnetic induction.
13. Mention at least four properties of magnetic lines of force.
14. Define magnetic flux density.
15. Explain the term mmf.
16. Define permeability or absolute permeability of a medium.
17. Define relative permeability.
18. Define reluctance in a magnetic circuit and give its formula.
19. Define permeance of magnetic circuit.
20. Does a magnetic circuit consume energy?
21. Give similarities of electric and magnetic circuits.
(UPTU June 2001)
22. Give the dissimilarities between magnetic and electric circuits.
23. What is a composite magnetic circuit?
24. How to calculate the total reluctance in a composite magnetic circuit?
25. State 'Ohm's law' of a magnetic circuit.
26. Discuss the concept of leakage flux. What is its effect?
27. Define leakage factor.
28. Why is air gap necessary in practical magnetic circuits?
29. Why is it necessary to keep air gaps in magnetic circuits as small as possible?
30. Why is air-gap flux termed as useful flux?
31. Why does leakage occur in a magnetic circuit?
32. There are a number of electric insulators, but there is no magnetic insulator. Explain.
33. What is magnetic fringing?
(MU April 1995; Mano April 1994)
34. Define fringing flux.
(MU April 1994; Mano April 1994)
35. What is hysteresis in a magnetic material?
36. State Kirchhoff's laws of magnetism.
37. Give the units of mmf, reluctance, flux, and give the relation between them.
(MU April 1996, April 1997; Mano November 1994)
38. What do you understand by electromagnetic induction?
39. Which material would you select for making electromagnets?
40. Define Faraday's laws of electromagnetic induction.
(MU April 94, April 1995, October 1997, October 1998; BU April 1999)
41. State Lenz's law.
(MU October 1996; BDN November 1999; BU November 1999, April 1999)
42. State Fleming right-hand rule as well as Fleming left-hand rule.
(MU November 1994, April 1997, April 1998, April 1999; BDN April 1999; BU November 1999)
43. State the right-hand rule that is used to find the direction of magnetic field and current.
44. State right-hand cork screw rule.
45. What happens when a current carrying conductor is placed in a magnetic field?
46. What do you mean by dynamically induced emf?
47. What do you understand by statically induced emf?
48. Define self-inductance and give its unit.
(MU October 1997, April 1995; BU April 1999)
49. Define mutual inductance and give its unit. (MU October 1997, April 1995; BU April 1999)
50. Distinguish between self-induced and mutually induced emf.
(BU April 1995)
51. Define the co-efficient of mutual inductance.
(BU November 1999)
52. Does inductance play any role in DC circuit?
53. What is the effect of mutual inductance on the coils between which it exists?
54. What is a closed circuit?
55. What is an open circuit?
56. What is a short circuit?
57. What do you call the ability of a magnetic substance to retain its magnetism after the magnetising force is removed?
(MKU November 1999)
58. What type of energy being stored in a capacitor
(BU April 1999)
59. What is the energy store in an inductor?
(BU November 1999)

## TEST QUESTIONS

1. Define the terms mmf, magnetic flux, and magnetic reluctance, and establish the relation that holds between these quantities for a magnetic circuit.
(UPTU)
2. Define the terms permeance, reluctivity, and leakage flux.
3. Explain the terms: permeability, reluctance, and permeance.
(UPTU Model Question Paper)
4. Define the following terms as applied to magnetic circuits:
(a) mmf, (b) flux density, (c) reluctance, (d) permeability, and (e) relative permeability.
(UPTU Elect. Engg. Second Semester 2002-2003)
5. Make comparison between magnetic and electric circuits.
6. Deduce analogy between magnetic circuit and electric circuit. What are the major points of differences between them?
(UPTU Elect. Engg. February 2001)
7. What are the similarities between electrical circuits and magnetic circuits?
(UPTU Elect. Engg. June 2001)
8. Draw a magnetisation curve and define the hysteresis and eddy current losses.
(UPTU Tut. Question Bank)
9. How is $B-H$ curve of ferromagnetic material different from that of non-magnetic material? Name all the salient regions of $B-H$ curve of magnetic material.
(UPTU Elect. Engg. First Semester 2003-2004)
10. Explain hysteresis, residual magnetism, and coercive force.
11. Explain hysteresis loss.
12. Using dot convention, discuss the mutual coupling in a simple magnetic circuit.
(UPTU Tut. Question Bank)
13. State and explain Faraday's laws of electromagnetic induction.
14. State and explain Fleming's right-hand rule and Lenz's law.
15. Write down the expression for dynamically induced emf developed in a conductor of length $l \mathrm{~m}$ moving with a velocity of $v \mathrm{~m} / \mathrm{s}$ in a uniform magnetic flux density of $B \mathrm{~Wb} / \mathrm{m}^{2}$.
16. What do you understand by self-induced emf and mutually induced emf?
17. Explain the terms self- and mutual inductance.
18. Distinguish between self- and mutual inductance.
19. Define co-efficient of coupling and show that $k=M / \sqrt{L_{1} L_{2}}$
20. Explain the statement: inductance in the electric circuit is analogous to inertia in mechanics.
21. Derive the expression for the equivalent inductance when two coupled coils are connected in series and parallel.
(UPTU Tut. Question Bank)
22. Differentiate between statically and dynamically induced emfs and electric and magnetic circuits.
(UPTU Elect. Engg. Second Semester 2003-2004)
23. Explain eddy currents.
24. What are the eddy currents? Explain their nature. Suggest a remedy for reducing such currents. Give a brief account of some of the useful applications of the eddy current effects.
(AMIE; W, 1983)

## ANSWERS

## Fill in the Blanks

1. electric current
2. reluctance
3. sum of the reluctances
4. hysteresis
5. permeance
6. $\eta B_{\max }^{1.6} V f$
7. silicon steel
8. conductivity
9. leakage flux
10. frequency of reversal of magnetic field
11. fraction
12. $\frac{N_{2}}{l / a \mu_{0} \mu_{\mathrm{r}}}$
13. mutual inductance
14. $B l v \sin \theta$
15. flux linkages
16. Fleming's right-hand rule; Lenz's law
17. Henry
18. 0.6124
19. no unit
20. $L R^{\prime} / 2$
21. self-inductance
22. one

## Objective Type Questions

| 1. (d) | 2. (d) | 3. (a) | 4. (a) | 5. (c) |
| ---: | ---: | ---: | ---: | ---: |
| 6. (b) | 7. (c) | 8. (b) | 9. (c) | 10. (b) |
| 11. (c) | 12. (a) | 13. (d) | 14. (a) | 15. (c) |
| 16. (b) | 17. (d) | 18. (a) | 19. (d) | 20. (c) |
| 21. (b) | 22. (a) | 23. (c) | 24. (b) | 25. (d) |
| 26. (c) | 27. (a) | 28. (c) |  |  |



## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is an alternating voltage and current and how it is differed from direct current (DC)?
* How sinusoidal alternating current is produced?
- Why alternating voltages are represented by phasors?
* What is frequency, time period, cycle, and alternation?
* What are instantaneous value, peak value, average value, and root-mean-square (rms) value of an alternating current?
* What is phase and phase difference?
* How alternating quantities are added or subtracted?


### 6.1 INTRODUCTION

As discussed earlier, the flow of current in the circuits was steady and in only one direction, that is, direct current (DC). The use of DC is limited to few applications, for example electroplating, charging of batteries, electric traction, electronic circuits, etc. However, for large-scale power generation, transmission, distribution, and utilization, an alternative current (AC) system is invariably adopted. In AC system, voltage acting in the circuit changes polarity and magnitude at regular interval of time, and hence, the flow of current in the circuits will reverse the direction periodically.

The most important advantages of AC system over DC system are the following:

1. An alternating voltage can be stepped up and stepped down efficiently by means of transformer. To transmit huge power over a long distance, the voltages are stepped up
(up to 400 kV ) for economical reasons at the generating stations, whereas they are stepped down to a very low level $(400 / 230 \mathrm{~V})$ for the utilization of electrical energy from safety point of view.
2. The AC motors (i.e., induction motors) are cheaper in cost, simple in construction, more efficient and robust as compared to DC motors.
3. The switchgear (e.g., switches, circuit breakers, etc.) for AC system is simpler than DC system.

Thus, AC system is universally adopted for generation, transmission, distribution, and utilization of electrical energy. In this chapter, we shall confine over attention to the fundamentals of alternating currents.

### 6.2 ALTERNATING VOLTAGE AND CURRENT

A voltage that changes its polarity and magnitude at regular intervals of time is called an 'alternating voltage'.

When an alternating voltage source is connected across a load resistor $R$ as shown in Figure 6.1, the current flows through it in one direction and then in opposite direction when the polarity is reversed.

Figure 6.1(c) shows the wave shape of the source voltage (representing the variation of voltage with respect to time) and current flowing through the circuit (i.e., load resistor $R$ ).


Fig. 6.1 Alternating voltage and current (a) AC voltage applied across resistor, flow of current during first half cycle (b) AC voltage applied across resistor, flow of current during next half cycle

### 6.2.1 Wave Form

As shown in graph (Fig. 6.2), an alternating voltage or current changes with respect to time is known as 'wave form or wave shape'. While plotting a graph, usually the instantaneous values of the alternating quantities are taken along Y-axis and time along X-axis. The alternating voltage or current may vary in different manner, as shown in Figure 6.2, and accordingly, their wave shapes are named in different ways such as irregular wave, triangular wave, square wave, periodic wave, sawtooth wave, sine wave, etc.


Fig. 6.2 AC wave shapes (a) General ac wave (b) Triangular wave (c) Square wave (d) Periodic wave (e) Triangular/saw tooth wave (f) Sinusoidal wave

### 6.3 DIFFERENCE BETWEEN AC AND DC

## AC

(1) An alternating current reverses periodically and its magnitude changes.
(2) Amplitude and polarities are varying continuously.
(3) It has a particular frequency.
(4) AC can be generated at higher voltages.
(5) In case of $A C$, the cost of generation is less.
(6) Alternating voltage can be increased (stepped up or decreased (stepped down) easily with the help of a transformer.
(7) AC motors are of less cost, more robust, and durable.
(8) The maintenance cost of AC equipment and appliances is less.
(9) AC cannot be used directly for electroplating.
(10) The speed of $A C$ motors cannot be controlled easily.

DC
(1) Direct current flows only in one direction and remains unaltered.
(2) Amplitude and polarities are fixed.
(3) It is independent of frequency.
(4) DC cannot be generated at high voltages because of commutation difficulties.
(5) In case of DC, the cost of generation is more.
(6) Direction voltage cannot be increased or decreased easily.
(7) DC motors are costly and less durable.
(8) The maintenance cost of DC equipment and appliances is more.
(9) Only DC can be used directly for electroplating.
(10) The speed control of DC motor is very easy and economical.

### 6.4 SINUSOIDAL ALTERNATING QUANTITY

An alternating quantity (i.e., voltage or current) that varies according to sine of angle $\theta(\theta=\omega t)$ is known as 'sinusoidal alternating quantity'. Its wave shape is shown in Figure 6.2(f). For the generation of electric power, sinusoidal voltages and currents are selected all over the world due to the following reasons:

1. The sinusoidal voltages and currents cause low iron and copper losses in AC rotating machines and transformers. This improves the efficiency of AC machines.
2. The sinusoidal voltages and currents offer less interference to nearby communication system (e.g., telephone lines)
3. They produce least disturbance in the electrical circuits.

Whenever the word 'alternating voltage or current' is used in this text, it means sinusoidal alternating voltage or current unless stated otherwise.

### 6.5 GENERATION OF ALTERNATING VOLTAGE AND CURRENT

An alternating voltage can be generated either by rotating a coil in a uniform magnetic field at constant speed as shown in Figure 6.3 or by rotating a uniform magnetic field within a stationary coil at a constant speed as shown in Figure 6.4.


Fig. 6.3 Production of ac voltage (rotating coil, field stationary)


Fig. 6.4 Production of ac voltage (rotating field, coil stationary)

The first method is generally applied in small AC generators, whereas second method is applied in large AC generators due to economical considerations. In both cases, magnetic field is cut by the conductors (or coil sides) and an emf is induced in them. The direction and magnitude of the induced emf in the conductors depend upon the position of the conductors explained as follows:

For simplicity, consider a coil placed in a uniform magnetic field to which a load (LM) is connected through brushes and slip rings as shown in Figure 6.5. When it is rotated in anticlockwise
direction at a constant angular velocity of $\omega$ radians per second, an emf is induced in the coil sides. The cross-sectional view of the coil and its different positions at different instants are shown in Figure 6.5.


Fig. 6.5 Position of coil at various instants. Wave shape of generated voltage

The magnitude of induced emf depends upon the rate at which the flux is cut by the conductors. At (i), (iii), and (v) instants, induced emf in the conductors A and B is zero as they are moving parallel to the magnetic lines of force and the rate of flux cut is zero, whereas the magnitude of emf induced in the conductors A and B is maximum at instant (ii) and (iv) as the conductors are moving perpendicular to the magnetic lines of force and the rate of flux cut is maximum.

The direction of emf induced in the conductors is determined by applying Fleming's righthand rule. At instant (ii), the direction of emf induced in conductor A is outward, whereas at instant (iv), the direction of induced emf in the conductor A is inward (i.e., the direction of induced emf at this instant is opposite to that of the direction of induced emf at instant (ii).

The wave shape of the emf induced in the coil is also shown in Figure 6.5.

### 6.6 EQUATION OF ALTERNATING EMF AND CURRENT

Consider a coil having ' $N$ ' turns rotating in a uniform magnetic field of density $B \mathrm{~Wb} / \mathrm{m}^{2}$ in the counterclockwise direction at an angular velocity of $\omega$ radians per second as shown in Figure 6.6.

At the instant, as shown in Figure 6.6(b), maximum flux $\phi_{\mathrm{m}}$ is linking with the coil. After $t$ seconds, the coil is rotated through an angle $\theta=\omega t$ radians. The component of flux linking with


Fig. 6.6 Multi-turn coil rotating in a constant magnetic field (a) Multi-turn coil (b) Position of coil at an instant (c) Position of coil after $t$ second
the coil at this instant is $\phi_{\mathrm{m}} \cos \omega t$, whereas the other component $\phi_{\mathrm{m}} \sin \omega t$ is parallel to the plane of the coil.

According to Faraday's laws of electromagnetic induction, the magnitude of emf induced in the coil at this instant, that is,

Instantaneous value of emf induced in the coil,
$e=-N \frac{d f}{d t}$ (negative sign indicates that an induced emf is opposite to the very cause which produces it)
or

$$
e=-N \frac{d}{d t} \phi_{\mathrm{m}} \cos t \omega t\left(\phi=\phi_{\mathrm{m}} \cos \omega t\right)
$$

or $\quad e=-N \phi_{\mathrm{m}}(-\omega \sin \omega t)$
or $\quad e=\omega N \phi_{\mathrm{m}} \sin \omega t$
The value of an induced emf will be maximum when angle q
or

$$
\begin{align*}
& \left.\omega t=90^{\circ} \text { (i.e., } \sin \omega t=1\right) \\
& E_{\mathrm{m}}=\omega N \phi_{\mathrm{m}} \tag{6.2}
\end{align*}
$$

Putting this value in equation (6.1), we get,

$$
e=E_{\mathrm{m}} \sin \omega t=E_{\mathrm{m}} \sin \theta
$$

From the above equation, it is clear that the magnitude of the induced emf varies according to sine of angle $\theta$. The wave shape of an induced emf is shown in Figure 6.7. This wave form is called sinusoidal wave.

If this voltage is applied across resistor, an alternating current will flow through it varying sinusoidally, that is, following a sine law and its wave shape will be same as shown in Figure 6.7.


Fig. 6.7 Wave shape of a sinusoidal voltage or current

This alternating current is given by the following equation:

$$
i=I_{\mathrm{m}} \sin \omega t=I_{\mathrm{m}} \sin \theta
$$

### 6.7 IMPORTANT TERMS

An alternating voltage or current changes its magnitude and direction at regular intervals of time. A sinusoidal voltage or current varies as a sine function of time $t$ or angle $\theta(=\omega t)$. The following important terms are generally used in alternating quantities:

1. Wave form: The shape of the curve obtained by plotting the instantaneous values of alternating quantity (voltage or current) along Y-axis and time or angle $(\theta=\omega t)$ along X -axis is called 'wave form or wave shape'. Figure 6.7 shows the waveform of an alternating quantity varying sinusoidally.
2. Instantaneous value: The value of an alternating quantity, that is, voltage or current at any instant is called its instantaneous value and is represented by ' $e$ ' or ' $i$ ', respectively.
3. Cycle: When an alternating quantity goes through a complete set of positive and negative values or goes through 360 electrical degrees, it is said to have completed one cycle.
4. Alternation: One half-cycle is called 'alternation'. An alternation spans 180 electrical degrees.
5. Time period: The time taken in seconds to complete one cycle by an alternating quantity is called time period. It is generally denoted by ' $T$ '.
6. Frequency: The number of cycles made per second by an alternating quantity is called 'frequency'. It is measured in cycles per second (c/s) or hertz ( Hz ) and is denoted by ' $f$ '.
7. Amplitude: The maximum value (positive or negative) attained by an alternating quantity in one cycle is called its 'amplitude or peak value or maximum value'. The maximum value of voltage and current is generally denoted by $E_{\mathrm{m}}\left(\right.$ or $\left.V_{\mathrm{m}}\right)$ and $I_{\mathrm{m}}$, respectively.

### 6.8 IMPORTANT RELATIONS

Some of the terms used in AC terminology have definite relations among themselves as given below:

1. Relation between frequency and time period: Consider an alternating quantity having a frequency of $f \mathrm{c} / \mathrm{s}$. Then, time taken to complete $f$ cycle $=1 \mathrm{~s}$
Time taken to complete 1 cycle $=1 / f \mathrm{~s}$
Hence, time period, $T=1 / f \mathrm{~s}$ or $f=1 / T \mathrm{c} / \mathrm{s}$
2. Relation between frequency and angular velocity: Consider an alternating quantity having a frequency of $f \mathrm{c} / \mathrm{s}$.
Angular distance covered in one cycle $=2 \pi$ radian
$\therefore$ Angular distance covered per second in $f$ cycles $=2 \pi$ radian
Hence, $\omega=2 \pi f$ radian $/ \mathrm{s}$

### 6.9 DIFFERENT FORMS OF ALTERNATING VOLTAGE EQUATION

The alternating voltage is given by the following standard equation:
or

$$
\begin{aligned}
& e=E_{\mathrm{m}} \sin \theta \text { or } e=E_{\mathrm{m}} \sin \omega t \\
& e=E_{\mathrm{m}} \sin 2 \pi f t(\sin \omega=2 \pi f)=E_{\mathrm{m}} \sin 2 \pi t / T(\text { since } f=1 / \mathrm{T})
\end{aligned}
$$

Which form of the above equation is to be applied will depend upon the data given?
To determine the various values, for example, maximum value, frequency, time period, angular velocity, etc., the given equation is compared with the standard equation of any one of the form given above. For instant,

Maximum value of voltage, $E_{\mathrm{m}}=$ Coefficient of sine of time angle
Frequency,

$$
f=\frac{\text { Coefficient of time in the angle }}{2 \pi}
$$

### 6.10 VALUES OF ALTERNATING VOLTAGE AND CURRENT

The voltage and current in DC system are constant so that there is no problem of specifying their magnitudes, whereas in AC system, the alternating voltage and current vary from time to time. Hence, it is necessary to explain the ways to express the magnitude of alternating voltage and current. The following three ways are adopted to express the magnitude of these quantities:

1. Peak value
2. Average value or Mean value
3. Effective value or rms value

The rms value of an alternating quantity (voltage or current) represents the real magnitude, whereas the peak and average values are important in some of the engineering applications.

### 6.11 PEAK VALUE

The maximum value attained by an alternating quantity during one cycle is called 'peak value'. This is also called 'maximum value or crest value or amplitude'. A sinusoidal alternating quantity obtains its maximum value at $90^{\circ}$ as shown in Figure 6.7. The peak of an alternating voltage and current is represented by $E_{\mathrm{m}}$ and $I_{\mathrm{m}}$. The knowledge of peak value is important in case of testing dielectric strength of insulating materials.

### 6.12 AVERAGE VALUE

The arithmetic average of all the instantaneous values considered an alternating quantity (current or voltage) over one cycle is called average value.

In case of symmetrical waves (such as sinusoidal current or voltage wave), the positive half is exactly equal to the negative half; therefore, the average value over a complete cycle is zero. Since work is being done by the current in both the positive and the negative half cycle, average value is determined regardless of signs. Hence, to determine average value of alternating


Fig. 6.8 Positive half cycle divided into $n$ equal parts

### 6.13 AVERAGE VALUE OF SINUSOIDAL CURRENT



Fig. 6.9 Current varying sinusoidally
quantities having symmetrical waves, only (positive half) cycle is considered.

Divide the positive half cycle into ' $n$ ' number of equal parts as shown in Figure 6.8. Let $i_{1}, i_{2}, i_{3}, \ldots, i_{\mathrm{n}}$ be the mid-ordinates.

Average value of current, $I_{\mathrm{av}}=$ mean of mid-ordinates.

$$
\begin{aligned}
& =\frac{i_{1}+i_{2}+i_{3}+\ldots+i_{\mathrm{n}}}{n} \\
& =\frac{\text { Area of alternation }}{\text { Base }}
\end{aligned}
$$

The alternating current varying sinusoidally, as shown in Figure 6.9 , is given by the equation:

$$
i=I_{\mathrm{m}} \sin \theta
$$

Consider an elementary strip of thickness $d \theta$ in the positive half cycle, $i$ be its mid-ordinate. Then,

Area of strip $=i d \theta$

$$
\begin{aligned}
& \text { Area of half cycle } \int_{0}^{\pi} i d \theta=\int_{0}^{\pi} I_{\mathrm{m}} \sin \theta d \theta \\
& \qquad \begin{array}{l}
=I_{\mathrm{m}}(-\cos \theta)_{0}^{\pi}=I_{\mathrm{m}}(-(\cos \pi-\cos 0)) \\
=
\end{array} I_{\mathrm{m}}[-1(-1-1)]=2 I_{\mathrm{m}}
\end{aligned}
$$

$$
\text { Base }=0 \text { to } \pi=\pi-0=\pi
$$

$\therefore$ Average value,

$$
I_{a v}=\frac{\text { Area of alternation }}{\text { base }}=\frac{2 I_{\mathrm{m}}}{\pi}=\frac{I_{\mathrm{m}}}{\pi / 2}=0.637 I_{\mathrm{m}}
$$

### 6.14 EFFECTIVE OR RMS VALUE



Fig. 6.10 Positive half cycle divided into $n$ equal parts

The steady current when flows through a resistor of known resistance for a given time produces the same amount of heat as produced by an alternating current when flows through the same resistor for the same time is called effective or rms value of an alternating current.

Let $i$ be an alternating current flowing through a resistor of resistance $R$ for time $t$ seconds which produces the same amount of heat as produced by $I_{\text {eff }}$ (direct current). The base of one alternation is divided into $n$ equal parts, as shown in Figure 6.10, so that interval is of $\frac{t}{n}$ second. Let $i_{1}, i_{2}, i_{3}, \ldots, i_{\mathrm{n}}$ be the mid-ordinate.

Then, heat produced in the
First interval $=i_{1}^{2} R t / J n$ calorie
Second interval $=i_{2}^{2} R t / J n$ calorie
Third interval $=i_{3}^{2} R t / J n$ calorie
$n$th interval $=i_{\mathrm{n}}^{2} R t / J n$ calorie
Total heat produced $=\frac{R t}{J}\left(\frac{i_{1}^{2}+i_{2}^{2}+i_{3}^{2}+\ldots \ldots i_{\mathrm{n}}^{2}}{n}\right)$ calorie
Since $I_{\text {eff }}$ is considered as the effective value of this current.
Then, total heat produced by this current $=$ calorie
Equating equation (6.3) and (6.4), we get,

$$
\frac{I_{e f f}^{2} R t}{J}=\frac{R t}{J}\left(\frac{i_{1}^{2}+i_{2}^{2}+i_{3}^{2}+\ldots \ldots i_{\mathrm{n}}^{2}}{n}\right)
$$

or
or

$$
I_{\mathrm{eff}}=\sqrt{\frac{i_{1}^{2}+i_{2}^{2}+i_{3}^{2}+\cdots+i_{\mathrm{n}}^{2}}{n}}=\sqrt{\text { mean of squares of instantaneous values }}
$$

$$
I_{\text {eff }}=\text { Square root of mean of squares of instantaneous values }
$$

$$
=\text { root-mean-square }(\mathrm{rms}) \text { value }
$$

It is the actual value of an alternating quantity which tells us the energy transfer capability of an AC source. For example, if we say that 5 A AC is flowing through a circuit, it means the rms value of an AC which flows through the circuit is 5 A . It transfers the same amount of energy as is transferred by 5 A DC.

The ammeters and voltmeters record the rms values of alternating currents and voltages, respectively. The domestic single-phase AC supply is $230 \mathrm{~V}, 50 \mathrm{~Hz}$. Where 230 V is the rms value of an alternating voltage.

### 6.15 RMS VALUE OF SINUSOIDAL CURRENT

An alternating current varying sinusoidally is given by the following equation:

$$
i=I_{\mathrm{m}} \sin \theta
$$

To determine the rms value, the squared wave of the alternating current is drawn as shown in Figure 6.11.

Considering an elementary strip of thickness $\mathrm{d} \theta$ in the first half-cycle of the squared wave, let $\mathrm{i}^{2}$ be its mid-ordinate. Then,

$$
\text { Area of strip }=i^{2} d \theta
$$

Area of first half cycle of squared wave


Fig. 6.11 Squared-wave shape of a sine wave

$$
\begin{aligned}
& =\int_{0}^{\pi} i^{2} d \theta=\int_{0}^{\pi}\left(I_{\mathrm{m}} \sin \theta\right)^{2} d \theta \\
& =I_{\mathrm{m}}^{2} \int_{0}^{\pi} \sin ^{2} \theta d \theta=I_{\mathrm{m}}^{2} \int_{0}^{\pi} \frac{1-\cos 2 \theta}{2} d \theta \\
& =\frac{I_{\mathrm{m}}^{2}}{2} \int_{0}^{\pi}(1-\cos 2 \theta) d \theta=\frac{I_{\mathrm{m}}^{2}}{2}\left(\theta-\frac{\sin 2 \theta}{2}\right)_{0}^{\pi} \\
& =\frac{I_{\mathrm{m}}^{2}}{2}\left((\pi-0)-\frac{\sin 2 \pi-\sin 0}{2}\right) \\
& =\frac{I_{\mathrm{m}}^{2}}{2}\left[(\pi-0)-(0-0)=\frac{\pi I_{\mathrm{m}}^{2}}{2}\right.
\end{aligned}
$$

Base $=0$ to $\pi=\pi-0=\pi$
Effective or rms value,

$$
I_{\mathrm{rms}}=\sqrt{\frac{\text { Area of first half of squared wave }}{\text { Base }}}=\sqrt{\frac{\pi I_{\mathrm{m}}^{2}}{2 \pi}}=\sqrt{\frac{I_{\mathrm{m}}^{2}}{2}}=\frac{I_{\mathrm{m}}}{\sqrt{2}}=0.707 I_{\mathrm{m}}
$$

Usually, rms value of an AC is simply represented by $I$ instead of $I_{\mathrm{rms}}$. Similarly, rms value of an alternating voltage is represented by $E$ or $V$.

### 6.16 FORM FACTOR AND PEAK FACTOR

There exists a definite relation among the average value, rms value, and peak value of an alternating quantity. The relationship is expressed by the two factors, namely form factor and peak factor.

1. Form factor: The ratio of rms value to average value of an alternating quantity is called form factor.
Mathematically, form factor $=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}$ or $\frac{E_{\mathrm{rms}}}{E_{\mathrm{av}}}$
For the current varying sinusoidally
Form factor $=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}=\frac{I_{\mathrm{m}} / \sqrt{2}}{2 I_{\mathrm{m}} / \pi}=\frac{\pi I_{\mathrm{m}}}{2 \sqrt{2} I_{\mathrm{m}}}=1.11$
2. Peak factor: The ratio of maximum value to rms value of an alternating quantity is called peak factor.
Mathematically, peak factor $=\frac{I_{\mathrm{m}}}{I_{\mathrm{rms}}}$ or $\frac{E_{\mathrm{m}}}{E_{\mathrm{rms}}}$
For current varying sinusoidally
Peak factor $=\frac{I_{\mathrm{m}}}{I_{\mathrm{rms}}}=\frac{I_{\mathrm{m}}}{I_{\mathrm{m}} / \sqrt{2}}=\sqrt{2}=1.4142$

## Example 6.1

The equation of an alternating current is $i=42.42 \sin 628 t$
Determine (i) its maximum value; (ii) Frequency; (iii) rms value; (iv) Average value; and (v) Form factor.
(U.P.T.U. 2005-2006)

## Solution:

Given equation is

$$
i=42.42 \sin 628 t
$$

Comparing above equation with the standard equation, $i=I_{\mathrm{m}} \sin \omega t$

$$
i_{\mathrm{m}}=42.42 \mathrm{~A} \text { and } \omega=628 \mathrm{rad} / \mathrm{sec}
$$

(i) Maximum value, $I_{\mathrm{m}}=42.42 \mathrm{~A}$ (Ans.)
(ii) Here, $\omega=628 \mathrm{rad} / \mathrm{sec}$ or $2 \pi f=628$

$$
\text { Frequency, } f=\frac{628}{2 \pi}=100 \mathrm{~Hz} \text { (Ans.) }
$$

(iii) RMS value, $I_{\mathrm{rms}}=\frac{I_{\mathrm{m}}}{\sqrt{2}}=\frac{42.42}{\sqrt{2}}=30 \mathrm{~A}$ (Ans.)
(iv) Average value, $I_{\mathrm{av}}=\frac{2 I_{\mathrm{m}}}{\pi}=\frac{2 \times 42.42}{\pi}=27 \mathrm{~A}$ (Ans.)
(v) Form factor $=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}=\frac{30}{27}=1.11$ (Ans.)

## Example 6.2

A supply voltage of $230 \mathrm{~V}, 50 \mathrm{~Hz}$ is fed to a residential building. Write down its equation for instantaneous value.

## Solution:

$$
v=V_{\mathrm{m}} \sin \omega t
$$

where

$$
V_{\mathrm{m}}=\sqrt{2} V_{\mathrm{rms}}=\sqrt{2} \times 230=325.27 \mathrm{~V}
$$

$$
\omega=2 \times \pi \times f=2 \times \pi \times 50=314.16 \text { radians }
$$

$$
\therefore \quad v=325.27 \sin 314.16 t \text { (Ans.) }
$$

## Example 6.3

An alternating voltage is given by $v=141.1 \sin 314 t$. Find the following:
(i) Frequency
(ii) RMS value
(iii) Average value
(iv) The instantaneous value of voltage when ' $t$ ' is 3 msec
(v) The time taken for voltage to reach 100 V for the first time after passing through zero value.
(U.P.T.U. 2006-2007)

## Solution:

(i) Given that,

$$
v=141.14 \sin 314 t
$$

Comparing above equation with
$v=V_{\text {in }} \sin \omega t$
$\omega=314 \mathrm{rad} / \mathrm{sec}$.
$\therefore$ Frequency,

$$
f=\frac{314}{2 \pi}=\frac{314}{2 \times 3.14}=50 \mathrm{~Hz} \text { (Ans.) }
$$

(ii) RMS value of the voltage, $V_{\mathrm{rms}}=\frac{V_{\mathrm{m}}}{\sqrt{2}}=\frac{141.4}{1.414}=100 \mathrm{~V}$ (Ans.)
(iii) Average value, $V_{\mathrm{av}}=\frac{2 V_{\mathrm{m}}}{\pi}=\frac{2 \times 141.4}{\pi}=90 \mathrm{~V}$ (Ans.)
(iv) $v=141.1 \sin 314 t$.

At $t=3 \mathrm{~ms}=3 \times 10^{-3} \mathrm{~s}$

$$
v=141.4 \sin 314 \times 3 \times 10^{-3}=114.4 \mathrm{~V} \text { (Ans.) }
$$

(v) $v=141.4 \sin 314 t$
or

$$
t=\frac{1}{314} \sin ^{-1}\left(\frac{100}{141.4}\right)=2.5 \times 10^{-3} \mathrm{~s}=2.5 \mathrm{~m} \mathrm{~s} \text { (Ans.) }
$$

## Example 6.4

A sinusoidally alternating current of frequency 60 Hz has a maximum value of 15 A .
(i) Write down the equation for instantaneous value.
(ii) Find the value of current after $1 / 200$ second.
(iii) Find the time taken to reach 10 A for the first time and
(iv) Find its average value.
(U.P.T.U., July 2003)

## Solution:

Here, $I_{\mathrm{m}}=15 \mathrm{~A} f=10 \mathrm{~Hz}$
(i) The equation for instantaneous value of sinusoidal alternating current,

$$
i=I_{\mathrm{m}} \sin 2 \pi f t=15 \sin 2 \pi \times 60 t=15 \sin 120 \pi t \text { (Ans.) }
$$

(ii) When, $t=\frac{1}{200}$ second.

$$
i=15 \sin 120 \pi \times \frac{1}{200}=15 \sin 0.6 \pi=15 \times 0.951=14.27 \mathrm{~A}(\text { Ans. })
$$

(iii) When the current is zero and becoming positive, $i=15 \sin 120 \pi t$

Where $i=10 \mathrm{~A}$

$$
\begin{aligned}
& \therefore \\
& \text { or } \quad t=\frac{1}{120 \pi} \sin ^{-1} \frac{10}{15}=\frac{1}{120 \pi} \sin ^{-1} 0.6667=0.001936 \text { second }(\text { Ans. })
\end{aligned}
$$

(iv) Average value, $I_{\mathrm{av}}=\frac{2}{\pi} I_{\mathrm{m}}=\frac{2}{\pi} \times 15=9.55 \mathrm{~A}$ (Ans.)

## Example 6.5

Find the rms value, average value, and form factor of the voltage waveform shown in Figure 6.12.
(U.P.T.U. 2002-2003)

## Solution:

Average value of voltage over one cycle

$$
\begin{aligned}
& =\frac{2 \int_{0}^{\pi} V_{\mathrm{m}} \sin \theta d \theta}{2 \pi}=\frac{V_{\mathrm{m}}}{\pi} \int_{0}^{\pi} \sin \theta d \theta \\
& =\frac{2 V_{\mathrm{m}}}{\pi}=\frac{2}{\pi} \times 100=63.66 \mathrm{~V} \text { (Ans.) }
\end{aligned}
$$



Fig. 6.12 Full rectified wave

RMS value of voltage over one cycle

$$
\begin{aligned}
=\sqrt{\frac{2 \int_{2}^{\pi} V_{\mathrm{m}}^{2} \sin ^{2} \theta d \theta}{2 \pi}} & =\sqrt{\frac{V_{\mathrm{m}}^{2}}{2 \pi} \int_{0}^{\pi} 2 \sin ^{2} \theta d \theta}=\sqrt{\frac{V_{\mathrm{m}}^{2}}{2 \pi} \int_{0}^{\pi}(1-\cos 2 \theta) d \theta} \\
& =\sqrt{\frac{V_{\mathrm{m}}^{2}}{2 \pi} \times \pi}=\frac{V_{\mathrm{m}}}{\sqrt{2}}=\frac{100}{\sqrt{2}}=70.71 \mathrm{~V} \text { (Ans.) }
\end{aligned}
$$

Form factor $=\frac{\text { RMS value }}{\text { Average value }}=\frac{70.71}{63.66}=1.11$ (Ans.)

## Example 6.6

Find the average and rms values of the current, $I(t)=10+10 \sin 314 t$.

## Solution:

The wave diagram of given current $i(t)=10+10 \sin 314 t$ is shown in Figure 6.13. Average value of the given current,
$I_{\mathrm{av}}=I_{\mathrm{dc}}+I_{\mathrm{av}}$ of ac component.

$$
=10+\frac{2 I_{\mathrm{m}}}{\pi}=10+\frac{2 \times 10}{\pi}=10+6.366=16.366 \mathrm{~A}(\text { Ans. })
$$

RMS value of the given current

$$
\begin{aligned}
I_{\mathrm{rms}} & =I_{\mathrm{dc}}+I_{\mathrm{rms}} \text { of AC component } \\
& =10+\frac{I_{\mathrm{m}}}{\sqrt{2}}=10+\frac{10}{\sqrt{2}}=10+7.071=17.071 \mathrm{~A} \text { (Ans.) }
\end{aligned}
$$



Fig. 6.13 Wave shape as per data

## Example 6.7

A transmission line carries a DC voltage of 50 V and a half-wave sinusoidal voltage as shown in Figure 6.14. Calculate the following:
(i) rms value
(ii) the average value, and
(iii) form factor

## Solution:

Peak value of half rectified wave (neglected DC).

$$
\begin{gathered}
V_{\mathrm{m}}=100 \mathrm{~V} \\
\text { RMS value, } V_{\mathrm{rms}}=\frac{V_{\mathrm{m}}}{2}=\frac{100}{2}=50 \mathrm{~V}
\end{gathered}
$$

RMS value of resultant wave,

$$
V_{\mathrm{rms}}=50+50=100 \mathrm{~V} \text { (Ans.) }
$$

Average value of half rectified wave, reject-


Fig. 6.14 Wave shape as per data ing DC,

$$
V_{\mathrm{av}}=\frac{V_{\mathrm{m}}}{\pi}=\frac{100}{\pi}=16 \mathrm{~V}
$$

Average value of resultant wave,

$$
\begin{gathered}
\qquad V_{\mathrm{av}}=50+16=66 \mathrm{~V}(\text { Ans. }) \\
\text { Form factor }=\frac{\text { RMS value }}{\text { Average value }}=\frac{100}{60}=1.54(\text { Ans. })
\end{gathered}
$$

## Example 6.8

An alternating current of frequency 60 Hz has a maximum value of 120 A . Write down the equation for its instantaneous value. Reckoning time from the instant, the current is zero and is becoming positive, find (i) the instantaneous value after $1 / 360$ seconds and (ii) the time taken to reach 96 A for the first time.

## Solution:

Instantaneous value of current is given by the following equation.

$$
i=I_{\mathrm{m}} \sin \omega t=I_{\mathrm{m}} \sin 2 \pi f t=120 \sin 2 \pi \times 60 t=120 \sin 377 t \text { (Ans.) }
$$

When the reckoning time is taken from the instant, the current is zero and becoming positive, equation for current is given as follows:

$$
i=120 \sin 377 t
$$

(i) At $t=1 / 360 \mathrm{~s} ; i=120 \sin 2 \pi \times 60 \times \frac{1}{360}=120 \sin \pi / 3=120 \times 0.866=103.92 \mathrm{~A}$ (Ans.)
(ii) Let $t$ seconds be the time taken to reach the current to 96 A for the first time. Then,

$$
96=120 \sin 2 \times \pi \times 60 \times t \text { or } \sin 120 \times \pi \times t=\frac{96}{120}
$$

or

$$
\sin 120 \times 180^{\circ} \times t=0.8
$$

or

$$
120 \times 180^{\circ} \times t=\sin ^{-1} 0.8
$$

or

$$
120 \times 180^{\circ} \times t=53.13^{\circ} ; t=\frac{53.13}{120 \times 180}=0.00246 \mathrm{~s} \text { (Ans.) }
$$

Note: On both the sides, the angle must be in the same units, that is, either in degrees or in radians.

## Example 6.9

Calculate the average value, rms value, form factor, and peak factor of a periodic current wave having values for equal time interval changing suddenly from one value to next: $0,30,45,70$, $90,70,45,30,0,-30,-45,-70$, etc. in ampere. What would be the average and the rms value of a sine wave having the same peak value?

## Solution:

The periodic wave form of the alternating current is shown in Figure 6.15.
Average value of current,

$$
\begin{aligned}
I_{\mathrm{av}} & =\frac{i_{1}+i_{2}+\cdots+i_{\mathrm{n}}}{n} \quad(\text { where } n=8) \\
& =\frac{0+30+45+70+90+70+45+30}{8} \\
& =47.5 \mathrm{~A} \text { (Ans.) }
\end{aligned}
$$

RMS value of current,

$$
I_{\mathrm{rms}}=\sqrt{\frac{i_{1}^{2}+i_{2}^{2}+\ldots i_{\mathrm{n}}^{2}}{n}}=\sqrt{\frac{0+(30)^{2}+(45)^{2}+(70)^{2}+(90)^{2}+(70)^{2}+(45)^{2}+(30)^{2}}{8}}
$$

$=\sqrt{\frac{23750}{8}}=54.486 \mathrm{~A}$ (Ans.)
Form factor $=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}=\frac{54.486}{47.5}=1.147$ (Ans.);
Peak factor $=\frac{I_{\mathrm{m}}}{I_{\mathrm{rms}}}=\frac{90}{54.486}=1.652($ Ans. $)$
Average value of sinusoidal AC having peak value of 90 A ;

$$
I_{\mathrm{av}}=0.637 \times I_{\mathrm{m}}=0.637 \times 90=57.33 \mathrm{~A}
$$

(Ans.)
RMS value,

$$
I_{\mathrm{rms}}=\frac{I_{\mathrm{m}}}{\sqrt{2}}=\frac{90}{\sqrt{2}}=63.64 \mathrm{~A}(\text { Ans. })
$$



Fig. 6.15 Periodic wave as per data

## Example 6.10

An alternating voltage $e=200 \sin 314 t$ is applied to a device which offers an ohmic resistance of $20 \Omega$ to the flow of current in one direction, while preventing the flow of current in opposite direction. Calculate rms value, average value, and form factor for the current over one cycle.
(Nagpur University, 1992)


Fig. 6.16 (a) Give sine wave (b) Half rectified wave

## Solution:

The wave diagram of an ac supply voltage is shown in Figure 6.16(a). The wave diagram of the rectified current that flows in the circuit is shown in Figure 6.16(b). The instantaneous value of applied

$$
e=200 \sin 314 t
$$

Maximum value of applied voltage, $E_{\mathrm{m}}=200 \mathrm{~V}$
Resistance of rectifying device, $R=20 \Omega$
Maximum value of half-wave rectified alternating current,

$$
I_{\mathrm{m}}=\frac{E_{\mathrm{m}}}{R}=\frac{200}{20}=10 \mathrm{~A}
$$

RMS value of half-wave rectified alternating current,

$$
I_{\mathrm{rms}}=\frac{I_{\mathrm{m}}}{2}=\frac{10}{2}=5 \mathrm{~A}(\text { Ans. })
$$

Average value of the half-wave rectified alternating current,

$$
I_{\mathrm{av}}=\frac{I_{\mathrm{m}}}{\pi}=\frac{10}{\pi}=3.18 \mathrm{~A} \text { (Ans.) }
$$

Form factor of the half-wave rectified alternating current

$$
=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}=\frac{5}{3.18}=1.57 \text { (Ans.) }
$$

## Example 6.11

Find the average and effective values of voltage for sinusoidal waveform shown in Figure 6.17.
(Allahabad University, 1991)

## Solution:

The equation of the given sinusoidal waveform is $v=100 \sin \theta$.

$$
\begin{aligned}
& \begin{aligned}
V_{\mathrm{av}}=\frac{1}{2 \pi} \int_{\pi / 4}^{\pi} 100 \sin \theta d \theta & =\frac{100}{2 \pi}[-\cos \theta]_{\pi / 4}^{\pi} \\
& =27.17 \mathrm{~V}(\text { Ans. }) \\
V_{\mathrm{rms}}^{2}= & \frac{1}{2 \pi} \int_{\pi / 4}^{\pi} 100^{2} \sin ^{2} \theta d \theta
\end{aligned}=\frac{100^{2}}{2 \times 2 \pi} \int_{\pi / 4}^{\pi} 2 \sin ^{2} \theta d \theta \\
&=\frac{1000}{4 \pi} \int_{\pi / 4}^{\pi}(1-\cos 2 \theta) d \theta=\frac{2500}{\pi}\left[\theta-\frac{\sin 20}{2}\right]_{\pi / 4}^{\pi}=\frac{2500}{\pi}\left[\pi-\frac{\pi}{4}+\frac{1}{2}\right]=2273
\end{aligned}
$$



## Example 6.12

Find the average value of the periodic function shown in Figure 6.18.

## Solution:

Average value,

$$
V_{\mathrm{av}}=\frac{\text { Area of alternation }}{\text { Base }}
$$

Area of alternation $=\frac{1}{2} V_{\mathrm{m}} \times \frac{\pi}{3}+V_{\mathrm{m}}\left(\frac{2 \pi}{3}-\frac{\pi}{3}\right)+\frac{1}{2} V_{\mathrm{m}} \times \frac{\pi}{3}$


Fig. 6.18 Given wave shape

$$
=\frac{\pi V_{\mathrm{m}}}{6}+\frac{\pi V_{\mathrm{m}}}{3}+\frac{\pi V_{\mathrm{m}}}{6}=\frac{2 \pi V_{\mathrm{m}}}{3}
$$

Base $=\pi$
Average value,

$$
V_{\mathrm{av}}=\frac{2 \pi V_{\mathrm{m}}}{3 \pi}=0.667 V_{\mathrm{m}} \text { (Ans.) }
$$

## Example 6.13

For a half-wave rectified alternating current, find (i) average value, (ii) rms value, (iii) form factor, and (iv) peak factor.

## Solution:

(i) A half-wave rectified AC is shown in Figure 6.19.

The negative half cycle is suppressed in this case, that is, current flows only during positive half cycle.

The wave is unsymmetrical, and therefore, one complete cycle is to be considered.


Fig. 6.19 Half wave rectified ac

$$
I_{\mathrm{av}}=\frac{\text { Area of complete cycle }}{\text { Base }}
$$

$$
\text { Area of complete cycle }=\text { Area of half cycle }
$$

$$
=\int_{0}^{\pi} i d \theta=\int_{0}^{\pi} I_{\mathrm{m}} \sin \theta d \theta=I_{\mathrm{m}}[-\cos \theta]_{0}^{\pi}=2 I_{\mathrm{m}}
$$

$$
\text { Base }=2 \pi
$$

$$
\therefore \quad I_{\mathrm{av}}=\frac{2 I_{\mathrm{m}}}{2 \pi}=\frac{I_{\mathrm{m}}}{\pi}=0.3183 I_{\mathrm{m}} \text { (Ans.) }
$$

(ii) The squared wave of a half wave rectified wave is shown in Figure 6.19.
RMS value,

$$
I_{r m s}=\sqrt{\frac{\text { Area of squared wave over a complete cycle }}{\text { Base }}}
$$

Area of squared wave over a complete cycle

$$
=\text { Area of squared wave in half cycle }
$$

$$
\begin{aligned}
\int_{0}^{\pi} i^{2} d \theta & =\int_{0}^{\pi} I_{\mathrm{m}}^{2} \sin ^{2} \theta d \theta \text { (as negative half cycle is suppressed) } \\
& =I_{\mathrm{m}}^{2} \int_{0}^{\pi}\left(\frac{1-\cos 2 \theta}{2}\right) d \theta=\frac{\pi I_{\mathrm{m}}^{2}}{2}
\end{aligned}
$$

Base $=2 \pi ; I_{\mathrm{rms}}=\sqrt{\frac{\pi I_{\mathrm{m}}^{2}}{2 \times 2 \pi}}=\frac{I_{\mathrm{m}}}{2}$ (Ans.)
(iii) Form factor $=\frac{I_{\mathrm{rms}}}{I_{\mathrm{av}}}=\frac{I_{\mathrm{m}} \times \pi}{2 \times I_{\mathrm{m}}}=\frac{\pi}{2}=1.57$ (Ans.)
(iv) Peak factor $=\frac{I_{\mathrm{m}}}{I_{\mathrm{rms}}}=\frac{I_{\mathrm{m}}}{I_{\mathrm{m}} / 2}=2$ (Ans.)

## Example 6.14

Determine the form factor of voltage waveform shown in Figure 6.20. (U.P.T.U. 2004-2005)

## Solution:

As per the wave diagram shown in Figure 6.21, the point $\mathrm{A}(1,10)$ shows, $t=1$ and $V=10$.

$$
\begin{aligned}
& \theta(t)=m \times t+0=10 \times t \\
& V_{\mathrm{av}}=\frac{1}{T} \int_{o}^{T} v(t) d t=\frac{1}{T} \int_{o}^{T} 10 t d t=\left[\frac{10 t^{2}}{2}\right]_{o}^{1}=5 \mathrm{~V} \\
& V_{\mathrm{rms}}=\sqrt{\frac{1}{T} \int_{o}^{T}(10 t)^{2} d t}=\sqrt{\left[\frac{100 t^{3}}{3}\right]_{o}^{1}}=\sqrt{\frac{100}{3}}=\frac{10}{\sqrt{3}}
\end{aligned}
$$

Form factor $=\frac{V_{\mathrm{rms}}}{V_{\mathrm{ar}}}=\frac{10 / \sqrt{3}}{5}=\frac{2}{\sqrt{3}}=1.1547$ (Ans.)


Fig. 6.20 Given voltage wave


Fig. 6.21 Voltage wave as per data

## 目旡 PRACTICE EXERCISES

## Short Answer Questions

1. Define an alternating quantity.
2. What do you mean by sinusoidal alternating quantity?
3. How the instantaneous value of a sinusoidal quantity is represented mathematically?
4. Define wave form of an alternating quantity.
5. Define a cycle of an alternating quantity.
6. Define alternation, time period, and frequency of an alternating quantity.
7. Give the relation between (i) frequency and time period; (ii) frequency and angular velocity.
8. Define instantaneous and peak value of an alternating quantity.
9. What do you mean by an average value of an alternating quantity?
10. How will you define rms value of an alternating quantity?
11. Define form factor and peak factor.

## Test Questions

1. Derive a relation between average value and maximum value for a sinusoidal quantity.
2. Derive a relation between rms value and maximum value for a sinusoidal quantity.
3. Derive a relation between rms and average value of a sinusoidal quantity.
4. Derive a relation for peak factor of a sinusoidal quantity.

## Numericals

1. A current is given by $i=22.62 \sin 377 \times t$, find (i) maximum value of current; (ii) rms value of current; (iii) frequency of current; (iv) radians through which its phasor has advanced after 0.01 s ; (v) number of degrees in (iv) and (vi) value of current at instant mentioned in part (iv).
(Ans. 22.62 A; 16 A; $60 \mathrm{~Hz} ; 3.77 \mathrm{rad} ; 216 ;-13.3 \mathrm{~A}$ )
2. A sinusoidal voltage of 50 Hz has a maximum value of $200 \sqrt{2} \mathrm{~V}$. At what time measured from a positive maximum value will the instantaneous voltage be 141.4 V ?
(Ans. 3.33 ms )
3. Calculate the rms value of a periodic voltage having the following values for equal intervals changing suddenly from one value to the next: $0,10,30,60,30,10,0,-10,-30,-60,-30,-10$, etc.
(Ans. 30.55 V )

### 6.17 PHASOR REPRESENTATION OF SINUSOIDAL QUANTITY

It has been seen that an alternating quantity (varying sinusoidally) can be represented in the form of wave and equation. The wave form shows the graphical representation, whereas the equation represents the mathematical expression of the instantaneous value of an alternating quantity.


Fig. 6.22 Phasor representation and wave diagram

The same alternating quantity can also be represented by a line of definite length (representing its maximum value) rotating in counterclockwise direction at a constant velocity ( $\omega$ radians/second). Such a rotating line is called a 'phasor'.

Thus, an alternating quantity can be represented by a phasor that shows its magnitude and direction at that instant.

For instant, consider an alternating quantity (current) represented by the equation $i=I_{\mathrm{m}} \sin \omega \times t$. Take a line OA to represent the maximum value of current $I_{\mathrm{m}}$ to scale. Imagine this line is rotating in counterclockwise direction at an angular velocity of $\omega$ radian/s about point O . After $t \mathrm{~s}$, the line is rotated through an angle $\theta(\theta=\omega \times t)$, from its horizontal position as shown in Figure 6.22. The projection of line OA on the Y -axis is OB .

$$
\begin{aligned}
\mathrm{OB} & =\mathrm{OA} \sin \theta=I_{\mathrm{m}} \sin \omega t \\
& =i \text { (the value of current at that instant) }
\end{aligned}
$$

Hence, the projection of the phasor OA on the Y -axis (i.e., OB ) at any instant gives the value of current at that instant.

Thus, a sinusoidal alternating quantity is represented by a phasor (vector) of length to scale equal to its maximum value rotated through an angle $\theta$ with the axis of reference (i.e., X -axis).

The phasor representation of an alternating quantity enables us to understand its magnitude and position on the axis. The alternating quantities can be added and subtracted with a fair degree of ease by representing them vectorially (phasor diagram).

### 6.18 PHASE AND PHASE DIFFERENCE

The phase of an alternating quantity (current or voltage) at an instant is defined as the fractional part of a cycle through which the quantity has advanced from a selected origin (Fig. 6.23). In actual practice, we are more concerned with the phase difference between the two alternating quantities rather than their absolute phase.

The two alternating quantities having same frequency, when attain their zero value at different instants, the quantities are said to have a phase difference. This angle between zero points (and are becoming positive) of two alternating quantities is called angle of phase difference.

In Figure 6.24, two alternating currents of magnitude $I_{\mathrm{m} 1}$ and $I_{\mathrm{m} 2}$ are shown vectorially. Both the vectors are rotating at same angular velocity of $\omega$ radian per second. The zero values are obtained by the two currents at different instants. Therefore, they are said to have a phase difference of angle $\phi$.

In other words, the phase difference may be defined as the angular displacement between the maximum positive value of the two alternating quantities having the same frequency.

The quantity that attains its positive maximum value prior to the other is called a 'leading quantity', whereas the quantity that


Fig. 6.23 Phasor representation and its instantaneous value on the wave diagram


Fig. 6.24 Phasor and wave diagram of two ac quantities with phase difference attains its positive maximum value after the other is called a 'lagging quantity'. In this case, current $I_{\mathrm{m} 1}$ is leading current with respect to $I_{\mathrm{m} 2}$ or is other words current $I_{\mathrm{m} 2}$ is the lagging current with respect to $I_{\mathrm{m} 1}$.

### 6.19 ADDITION AND SUBTRACTION OF ALTERNATING QUANTITIES

In $A C$ circuits, it is required to add or subtract the alternating quantities. In such cases, it should be proceed as follows:

### 6.19.1 Addition of Alternating Quantities

The given alternating quantities are represented as phasor, and then, they are added in the same manner as forces are added. Only phasors of the similar quantities are added, that is, either all the currents are added or all the voltages are added. Voltages and currents are never added with each other. For addition, the following methods are accomplished:

1. Parallelogram method
2. Method of components


Fig. 6.25 AC current in parallel circuits


Fig. 6.26 Phasor representation (for finding resultant by parallelogram method)

1. Parallelogram Method: This method is applied for the addition of two phasors at a time. The two quantities are represented in magnitude and direction (phasors) as the adjacent sides of a parallelogram. The diagonal of this parallelogram represents the magnitude of the resultant.

Consider an AC parallel circuit having two branches, as shown in Figure 6.25, carrying a current of $i_{1}$ and $i_{2}$, respectively. Let the two currents be represented as follows:

$$
i_{1}=I_{\mathrm{m} 1} \sin \omega t \text { and } i_{2}=I_{\mathrm{m} 2} \sin (\omega t+\theta)
$$

The maximum values of the two currents $I_{\mathrm{m} 1}$ and $I_{\mathrm{m} 2}$ are represented as the two adjacent sides OA and OB, respectively, of a parallelogram OACB as shown in Figure 6.26. The current $I_{\mathrm{m} 2}$ leads the current $I_{\mathrm{m} 1}$ by an angle $\theta^{\circ}$. The maximum value of the resultant is say $I_{\mathrm{mr}}$ represented by the diagonal OC of the parallelogram and leads the phasor $I_{\mathrm{m} 1}$ by an angle $\phi$.

$$
\begin{aligned}
\mathrm{OC} & =\sqrt{(\mathrm{OD})^{2}+(\mathrm{DC})^{2}}=\sqrt{(\mathrm{OA}+\mathrm{AD})^{2}+(\mathrm{DC})^{2}} \\
I_{\mathrm{m} \mathrm{r}} & =\sqrt{\left(I_{\mathrm{m} 1}+I_{\mathrm{m} 2} \cos \theta\right)^{2}+\left(I_{\mathrm{m} 2} \sin \theta\right)^{2}} \\
& =\sqrt{I_{\mathrm{m} 1}^{2}+I_{\mathrm{m} 2}^{2}\left(\cos ^{2} \theta+\sin ^{2} \theta\right)+2 I_{\mathrm{m} 1} I_{\mathrm{m} 2} \cos \theta} \\
& =\sqrt{I_{\mathrm{m} 1}^{2}+I_{\mathrm{m} 2}^{2}+2 I_{\mathrm{m} 1} I_{\mathrm{m} 2} \cos \theta}\left(\sin ^{2} \theta+\cos ^{2} \theta=1\right)
\end{aligned}
$$

Phase angle, $\phi=\tan ^{-1} \frac{\mathrm{CD}}{\mathrm{OD}}=\tan ^{-1} \frac{I_{\mathrm{m} 2} \sin \theta}{I_{\mathrm{m} 1}+I_{\mathrm{m} 2} \cos \theta}$
The instantaneous value of the resultant current is given by the relation;

$$
i_{\mathrm{r}}=I_{\mathrm{mr}} \sin (\omega \times t+\phi) \text { as } \phi \text { is positive. }
$$

2. Method of components: In this method, each phasor is resolved into horizontal and vertical components. The horizontal components are added algebraically to obtain the resultant horizontal component $I_{\mathrm{xx}}$. Similarly, ver-


Fig. 6.27 AC current in parallel circuit tical components are summed up algebraically to obtain the resultant vertical component $I_{\mathrm{YY}}$.

Consider an AC parallel circuit consisting of three branches each carrying a current of $i_{1}, i_{2}$, and $i_{3}$, respectively, as shown in Figure 6.27. Let the three currents be represented as follows:

$$
\begin{aligned}
& i_{1}=I_{\mathrm{m} 1} \sin \left(\omega t+\theta_{1}\right) \\
& i_{2}=I_{\mathrm{m} 2} \sin \omega t \\
& i_{3}=I_{\mathrm{m} 3} \sin \left(\omega t-\theta_{2}\right)
\end{aligned}
$$

and

The maximum values of the three currents $I_{\mathrm{m} 1}, I_{\mathrm{m} 2}$, and $I_{\mathrm{m} 3}$ are represented by the phasors as shown in Figure 6.28(a). The components are resolved horizontally and vertically.

Algebraic sum of horizontal components

$$
I_{\mathrm{xx}}=I_{\mathrm{m} 1} \cos \theta_{1}+I_{\mathrm{m} 2}+I_{\mathrm{m} 3} \cos \theta_{2}
$$

Algebraic sum of vertical components

$$
I_{\mathrm{YY}}=I_{\mathrm{m} 1} \sin \theta_{1}+0-I_{\mathrm{m} 3} \sin \theta_{2}
$$

Maximum value of resultant components

$$
I_{\mathrm{mr}}=\sqrt{\left(I_{\mathrm{XX}}\right)^{2}+\left(I_{\mathrm{YY}}\right)^{2}}
$$

If $\phi$ is the phase difference (leading) between resultant current and horizontal axis as shown in Figure 6.28(b). Then,

$$
\phi=\tan ^{-1} \frac{I_{\mathrm{YY}}}{I_{\mathrm{XX}}}
$$

The instantaneous value of the resultant current is given by the relation;

$$
i_{\mathrm{r}}=I_{\mathrm{mr}} \sin (\omega t+\phi)
$$

However, if $I_{\mathrm{YY}}$ comes out to be negative, the angle of phase difference will be lagging (i.e., $-\phi$ ). Then, the instantaneous value of the resultant current will be given by the relation

$$
i_{\mathrm{r}}=I_{\mathrm{mr}} \sin (\omega t-\phi)
$$


(a)

(b)

Fig. 6.28 (a) Phasor representation of three current (b) Finding resultant by component method

### 6.19.2 Subtraction of Alternating Quantities

The methods explained above (i.e., parallelogram method and method of components) are also applied for the subtraction of an alternating quantity. The only difference is that in this case, the phasor of the alternating quantity which is to be subtracted is reversed or represented $180^{\circ}$ out of phase. Then it is added with the other alternating quantity (or quantities) as usual.

## Example 6.15

Calculate (i) the maximum value and (ii) the root-mean-square value of the following quantities:
(i) $40 \sin \omega t$
(ii) $\mathrm{B} \sin (\omega t-\pi / 2)$
(iii) $10 \sin \omega t-17.3 \cos \omega t$

Draw the vectors showing the phase difference with respect to $\mathrm{A} \sin (\omega t-\pi / 6)$.

## Solution:

The instantaneous value of an alternating quantity is given by the relation

$$
i=I_{\mathrm{m}} \sin \omega t
$$

(i) The given alternating quantity is $40 \sin \omega t$
$\therefore$ Maximum value $=40$ (Ans.)
RMS value $=$ Max. value $/ \sqrt{2}=40 / \sqrt{2}=28.284$ (Ans.)


Fig. 6.29 (a) Phasor diagram as per data

(b)

Fig. 6.29 (b) Resultant phasor diagram

The vector lies on the horizontal axis as shown in Figure 6.29(a).
(ii) The given alternating quantity is $\mathrm{B} \sin (\omega t-\pi / 2)$
$\therefore$ Maximum value $=\mathrm{B}$ (Ans.)

$$
\text { RMS value }=B / \sqrt{2} \text { (Ans.) }
$$

The vector lags behind the horizontal axis by $90^{\circ}$ as shown in Figure 6.29(b).
(iii) The given alternating quantity is $10 \sin \omega t-17.3$ $\cos \omega t$. In fact, this quantity has two components displaced from each other by $90^{\circ}$ as shown in Figure 6.29(a).

Resultant maximum value $=\sqrt{(10)^{2}+(17.3)^{2}}=20$ Let the phase angle of the resultant with the horizontal be $\theta^{\circ}$.

$$
\begin{aligned}
\therefore & \tan \theta & =17.3 / 10=1.73 \\
\text { or } & \theta & =\tan ^{-1} 1.73=60^{\circ}
\end{aligned}
$$

Hence, this vector lags behind the horizontal axis by $60^{\circ}$ as shown in Figure 6.29(b). This vector is also represented in Figure 6.29(a).

The quantity $\mathrm{A} \sin (\omega t-\pi / 6)$ makes as angle of lag, $\theta_{1}=\pi / 6=30^{\circ}$ with the horizontal as shown in Figure 6.29(a).

Considering phasor diagram shown in Figure 6.29(a);
The phase difference between first quantity (i.e., 40) and $\mathrm{A}=30^{\circ}$ (Ans.)
The phase difference between second quantity (i.e., B) and $\mathrm{A}=60^{\circ}$ (Ans.)
The phase difference between third quantity (i.e., 20) and $\mathrm{A}=60^{\circ}-30^{\circ}=30^{\circ}$ (Ans.)

## Example 6.16

Two AC voltages are represented as follows:

$$
\begin{aligned}
& v_{1}(t)=30 \sin \left(314 t+45^{\circ}\right) \\
& v_{2}(t)=60 \sin \left(314 t+60^{\circ}\right)
\end{aligned}
$$

Calculate the resultant $v(t)$ and express in the form:

$$
v(t)=V_{\mathrm{m}} \sin (314 t \pm \phi)
$$

(U.P.T.U. 2003-2004)

## Solution:

$$
\theta(t)=m \times t+0=1 \times t
$$

The phasor representation of the two voltages is shown in Figure 6.30.

$$
v_{1}(t)=30 \sin \left(314 t+45^{\circ}\right) ; v_{2}(t)=60 \sin \left(314 t+60^{\circ}\right)
$$

Resolving the vectors $V_{1 \mathrm{~m}}$ and $V_{2 \mathrm{~m}}$ horizontally and vertically, we get,

$$
\begin{aligned}
& \Sigma V_{\mathrm{xx}}=30 \cos 45^{\circ}+60 \cos 60^{\circ} \\
& \quad=30 \times \frac{1}{\sqrt{2}}+60 \times \frac{1}{2}=51.21 \\
& \Sigma V_{\mathrm{yy}}=30 \sin 45^{\circ}+60 \sin 60^{\circ} \\
& =30 \times \frac{1}{\sqrt{2}}+60 \times \frac{\sqrt{3}}{2}=73.17
\end{aligned}
$$

Maximum value of resultant voltage,

$$
\begin{aligned}
V_{\mathrm{r}(\mathrm{~m})}= & \sqrt{\left(\sum V_{\mathrm{xx}}\right)^{2}+\left(\sum V_{\mathrm{yy}}\right)^{2}} \\
& =\sqrt{(51.21)^{2}+(73.17)^{2}}=89.31
\end{aligned}
$$



Fig. 6.30 Phasor diagram as per data

Phase angle, $\phi_{\mathrm{r}}=\tan ^{-1} \frac{\Sigma V_{\mathrm{yy}}}{\Sigma V_{\mathrm{xx}}}=\tan ^{-1} \frac{73.17}{51.21}=55^{\circ}$
Resultant voltage, $v(t)=V_{\mathrm{r}(\mathrm{m})} \sin \left(\omega t+\phi_{\mathrm{r}}\right)=89.31 \sin \left(314 t+55^{\circ}\right)$ (Ans.)

## Example 6.17

Draw a phasor diagram showing the following voltages.

$$
\begin{array}{ll}
v_{1}=100 \sin 500 t ; & v_{2}=200 \sin (500 t+\pi / 3) \\
v_{3}=-50 \cos 500 t ; & v_{4}=150 \sin (500 t-\pi / 4)
\end{array}
$$

Find RMS value of resultant voltage.
(U.P.T.U. 2005-2006)

## Solution:

$$
\begin{gathered}
v_{1}=100 \sin 500 t ; \\
v_{2}=200 \sin \left(500 t+\frac{\pi}{3}\right) \\
v_{3}=-50 \cos 500 t=-50 \sin \left(\frac{\pi}{2}-500 t\right) \\
=50 \sin (500 t-\pi / 2) \\
v_{4}=150 \sin (150 t-\pi / 4)
\end{gathered}
$$

All the four voltages are shown vectorially in Figure 6.31.


Fig. 6.31 Phasor diagram as per data

Resolving the phasors in horizontal axis

$$
\begin{aligned}
V_{\mathrm{xx}} & =V_{1} \cos 0+V_{2} \cos \frac{\pi}{3}+V_{3} \cos \frac{\pi}{2}+V_{4} \cos \frac{\pi}{4} \\
& =100 \times 1+200 \times 0.5+50 \times 0+150 \times 0.707 \\
& =306.05 \mathrm{~V}
\end{aligned}
$$

Resolving the phasors in vertical axis

$$
\begin{aligned}
V_{\mathrm{yy}} & =V_{1} \sin 0+V_{2} \sin \frac{\pi}{3}-V_{3} \sin \frac{\pi}{2}-V_{4} \sin \frac{\pi}{4} \\
& =100 \times 0+200 \times 0.866-50 \times 1-150 \times 0.707 \\
& =17.15 \mathrm{~V}
\end{aligned}
$$

Maximum value of resultant voltage

$$
V_{\mathrm{mr}}=\sqrt{V_{\mathrm{xx}}^{2}+V_{\mathrm{yy}}^{2}}=\sqrt{(306.05)^{2}+(17.15)^{2}}=306.53 \mathrm{~V}
$$

RMS value of resultant voltage, $V_{\mathrm{rms(r)}}=\frac{V_{\mathrm{mr}}}{\sqrt{2}}=\frac{306.53}{\sqrt{2}}=216.75 \mathrm{~V}$ (Ans.)

## Example 6.18

Three sinusoidal voltages acting in series are given by

$$
v_{1}=10 \sin 440 t ; v_{2}=10 \sqrt{2} \sin \left(440 t-45^{\circ}\right) ; v_{3}=20 \cos 440 t
$$

(i) an expression for the resultant voltage,
(ii) the frequency and rms value of the resultant voltage.
(U.P.T.U., February 2001)

## Solution:

Here,

$$
\begin{aligned}
& v_{1}=10 \sin 440 t \\
& v_{2}=10 \sqrt{2} \sin \left(440 t-45^{\circ}\right) \\
& v_{3}=20 \cos 440 t=20 \sin \left(440 t+\frac{\pi}{2}\right)
\end{aligned}
$$

All the three voltages are shown vectorially in Figure 6.32.
Resolving voltage along X -axis and Y -axis, we get,


Fig. 6.32 Phasor diagram as per data

$$
\begin{aligned}
V_{\mathrm{xx}} & =10 \cos 0^{\circ}+10 \sqrt{2} \cos \left(-45^{\circ}\right)+20 \cos 90^{\circ} \\
& =10+10 \sqrt{2} \times \frac{1}{\sqrt{2}}+0=20 \\
V_{\mathrm{yy}} & =10 \sin 0^{\circ}+10 \sqrt{2} \sin \left(-45^{\circ}\right)+20 \sin 90^{\circ} \\
& =0-10 \sqrt{2} \times \frac{1}{\sqrt{2}}+20=10
\end{aligned}
$$

Maximum value of resultant voltage,

$$
V_{\mathrm{r}(\mathrm{~m})}=\sqrt{\left(V_{\mathrm{xx}}\right)^{2}+\left(V_{\mathrm{yy}}\right)^{2}}=\sqrt{20^{2}+10^{2}}=22.36 \mathrm{~V}
$$

$$
\text { Phase angle, } \phi_{\mathrm{r}}=\tan ^{-1} \frac{V_{\mathrm{yy}}}{V_{\mathrm{xx}}}=\tan ^{-1} \frac{10}{20}=26.56^{\circ} \text { or } 0.1476 \pi \text { radian }
$$

(i) Expression for resultant voltage, $v_{\mathrm{r}}=22.36 \sin (440 t+0.1476 \pi)$ (Ans.)
(ii) Frequency, $f=\frac{440}{2 \pi}=70 \mathrm{~Hz}$ (Ans.)

RMS value of resultant voltage, $V_{\mathrm{rms}(\mathrm{r})}=\frac{V_{\mathrm{m}}}{\sqrt{2}}=\frac{22.36}{\sqrt{2}}=15.81 \mathrm{~V}$ (Ans.)

## Example 6.19

Three voltages represented by $e_{1}=10 \sin w t, e_{2}=15 \sin (\omega t+\pi / 4)$, and $e_{3}=20 \cos (\omega t-\pi / 6)$ act together in a circuit. Find an expression for the resulting voltage.

## Solution:

The three voltages are represented vectorially in Figure 6.33. The third voltage ( 20 V ) makes an angle of lag of $30^{\circ}$ with the vertical.

Resolving the components along X -axis

$$
\begin{aligned}
E_{\mathrm{xx}} & =10+15 \cos 45^{\circ}+20 \cos 60^{\circ} \\
& =10+15 \times 0.707+20 \times 0.5=30.607 \mathrm{~V}
\end{aligned}
$$

Resolving the components along Y -axis,

$$
\begin{aligned}
E_{\mathrm{YY}} & =0+15 \sin 45^{\circ}+20 \sin 60^{\circ} \\
& =0+15 \times 0.707+20 \times 0.866=27.927 \mathrm{~V}
\end{aligned}
$$

Maximum value of resultant voltage,
$E_{\mathrm{mr}}=\sqrt{E_{\mathrm{XX}}^{2}+E_{\mathrm{YY}}^{2}}=\sqrt{(30.607)^{2}+(27.927)^{2}}=41.433 \mathrm{~V}$
Let $\phi$ be the angle which the resultant voltage makes with X -axis

$$
\tan \phi=\frac{E_{\mathrm{YY}}}{E_{\mathrm{XX}}}=\frac{27.927}{30.607}=0.9124
$$



Fig. 6.33 Phasor diagram as per data
$\therefore \quad \phi=\tan ^{-1} 10.9124=42.38^{\circ}$
$\therefore \quad$ Expression for the resultant voltage,

$$
e_{\mathrm{r}}=E_{\mathrm{mr}} \sin (\omega t+\phi)=41.433 \sin \left(\omega t+42.38^{\circ}\right)(\text { Ans. })
$$

## Example 6.20

Two currents $i_{1}$ and $i_{2}$ are given by the expressions: (i) $40 \sin (314 t+\pi / 6)$ (ii) $20 \sin$ (314 $t-\pi / 3$ )

Find $i_{1}-i_{2}$ and express the answer in the same form as individual currents. Find also the rms value and frequency of the resultant current.

## Solution:

The two currents are represented by phasors as shown in Figure 6.34. Since current $i_{2}$ is to be subtracted from $i_{1}$, reverse the phasor $I_{\mathrm{m} 2}$ (dotted phasor) and find its vector sum with $I_{\mathrm{m} 1}$.


Fig. 6.34 Phasor diagram as per data

Resolving the components along X-axis;

$$
\begin{aligned}
I_{\mathrm{xx}} & =40 \cos 30^{\circ}-20 \cos 60^{\circ} \\
& =40 \times 0.866-20 \times 0.5=24.64 \mathrm{~A}
\end{aligned}
$$

Resolving the components along Y-axis;

$$
\begin{aligned}
I_{\mathrm{YY}} & =40 \sin 30^{\circ}+20 \sin 60^{\circ} \\
& =40 \times 0.5+20 \times 0.866=37.32 \mathrm{~A}
\end{aligned}
$$

Maximum value of resultant current,

$$
\begin{aligned}
I_{\mathrm{mr}}=\sqrt{\left(I_{\mathrm{XX}}\right)^{2}\left(I_{\mathrm{YY}}\right)^{2}} & =\sqrt{(24.64)^{2}+(37.32)^{2}} \\
& =44.72 \mathrm{~A}
\end{aligned}
$$

Let $\phi$ be the angle which resultant current makes with X -axis

$$
\tan \phi=\frac{I_{\mathrm{YY}}}{I_{\mathrm{Xx}}}=\frac{37.32}{24.64}=1.5146 \quad \therefore \phi=\tan ^{-1} 1.5146=56.56^{\circ}
$$

$\therefore \quad$ Expression for the resultant current,

$$
i_{\mathrm{r}}=I_{\mathrm{mr}} \sin (\omega t+\phi)=44.72 \sin \left(314 t+56.56^{\circ}\right)(\text { Ans. })
$$

or

$$
i_{\mathrm{r}}=44.72 \sin (314 \times t+0.9872) \text { [where } \phi \text { is in radians] }
$$

RMS value of resultant current,

$$
I_{\mathrm{r}}=\frac{44.72}{\sqrt{2}}=31.62(\text { Ans. })
$$

From the equation, $\omega=314$ or $2 \pi f=314$
$\therefore \quad$ Frequency, $f=314 / 2 \pi=50 \mathrm{~Hz}$ (Ans.)

## Example 6.21

The instantaneous values of two alternating voltages are represented by $v_{1}=60 \sin \theta$ and $v_{2}=40$ $\sin (\theta-\pi / 3)$. Derive expression for the instantaneous values of (i) the sum and (ii) the difference of these voltages.
(U.P.T.U. July, 2002)

## Solution:

Here,

$$
v_{1}=60 \sin \theta \quad \text { and } \quad v_{2}=40 \sin \left(\theta-\frac{\pi}{3}\right)
$$

The two voltages are shown vectorially in Figure 6.35(a)
(i) When the two voltages are to be added,

$$
\begin{aligned}
V_{\mathrm{xx}} & =V_{1} \cos 0^{\circ}+V_{2} \cos \left(-60^{\circ}\right)=60 \times 1+40 \times 0.5=80 \mathrm{~V} \\
V_{\mathrm{yy}} & =V_{1} \sin 0^{\circ}+40 \sin \left(-60^{\circ}\right)=60 \times 0+40 \times(-0.866)=-34.64 \mathrm{~V} \\
V_{\mathrm{r}(\mathrm{~m})} & =\sqrt{V_{\mathrm{xx}}^{2}+V_{\mathrm{yy}}^{2}}=\sqrt{(80)^{2}+(-34.64)^{2}}=87.2 \mathrm{~V}
\end{aligned}
$$

Phase angle, $\phi_{\mathrm{r}}=\tan ^{-1} \frac{V_{\mathrm{yy}}}{V_{\mathrm{xx}}}=\tan ^{-1} \frac{-34.64}{80}=-23.41^{\circ}=-0.13 \pi$ radian
Expression for the instantaneous value of the resultant voltage,

$$
V_{\mathrm{r}}=V_{\mathrm{r}(\mathrm{~m})} \sin \left(\theta-\phi_{\mathrm{r}}\right)=87.2 \sin (\theta-0.13 \pi) \text { volt (Ans.) }
$$

(ii) When one of the voltage (say $V_{2}$ ) is to be subtracted from the other (say $V_{1}$ )

The vector representing $V_{2}$ is reversed as shown in Figure $6.35(\mathrm{~b})$ and then added to $V_{1}$.

$$
\begin{aligned}
V_{\mathrm{xx}} & =V_{1} \cos \theta^{\circ}-V_{2} \sin \left(-60^{\circ}\right)=60 \times 1-40 \times 0.5=40 \mathrm{~V} \\
V_{\mathrm{yy}} & =V_{1} \sin \theta^{\circ}-V_{2} \sin \left(-60^{\circ}\right)=60 \times 0-40 \times(-0.866)=34.64 \mathrm{~V} \\
V_{\mathrm{r}(\mathrm{~m})} & =\sqrt{V_{\mathrm{xx}}^{2}+V_{\mathrm{yy}}^{2}}=\sqrt{(40)^{2}+(34.64)^{2}}=52.9 \mathrm{~V}
\end{aligned}
$$

Phase angle, $\phi_{\mathrm{r}}=\tan ^{-1} \frac{V_{\mathrm{yy}}}{V_{\mathrm{xx}}}=\tan ^{-1} \frac{34.64}{40}=40.89=0.227 \pi$ radians
Expression for the instantaneous value of resultant voltage,

$$
V_{\mathrm{r}}=V_{\mathrm{r}(m)} \sin \left(\theta+\phi_{\mathrm{r}}\right)=52.9 \sin (\theta+0.227 \pi)(\text { Ans. })
$$


(a)

(b)

Fig. 6.35 (a) Phasor diagram for addition (b) Phasor diagram for subtraction

### 6.20 PHASOR DIAGRAMS USING RMS VALUES

For all practical purposes, the rms values of voltages and currents are mentioned for circuit or system. Moreover, the voltmeters and ammeters are calibrated to read the rms values. Therefore, it is much more convenient to draw the phasor diagram using rms values rather than maximum values. While drawing phasor diagrams using rms values, the phase angles do alter. However, these phasor diagrams will not generate the sine waves of proper amplitude unless the lengths of the phasor are increased by times.

The phasor diagrams using maximum values and rms values are shown in Figure 6.36(a) and 6.36(b), respectively.


Fig. 6.36 (a) Phasor diagram with maximum values
(b) Phasor diagram with rms values

## 园根 PRACTICE EXERCISES

## Short Answer Questions

1. How will you define phase of an alternating quantity?
2. What do you mean by phase difference?
3. What do you mean by a leading quantity in an AC system?
4. What do you mean by a lagging quantity in an AC system?
5. How are the alternating quantities added or subtracted?

## Test Questions

1. What is phase and phase difference? Explain with the help of phasor and wave diagrams.
2. Explain the method of components used for the addition and subtraction of alternating quantities.

## Numericals

1. Represent the following alternating currents as phasors on the same axes and determine phase angle of all other currents with $I_{1}: \quad i_{1}=10 \sin (\omega \times t+\pi / 3) ; \quad i_{2}=20 \cos (\omega \times t+\pi / 6)$; $i_{3}=15 \cos (\omega \times t-\pi / 3) ; i_{4}=12 \sin (\omega \times t+\pi / 6)$
(Ans. $60^{\circ} ; 30^{\circ} ; 90^{\circ}$ )
2 Two currents $i_{1}$ and $i_{2}$ are given by the expression: $i_{1}=15 \sin (\omega \times t+\pi / 3)$ and $i_{2}=5 \sin (\omega \times t-\pi / 3)$
Find $i_{1}-i_{2}$ and express the answer in the same form as individual currents.
(P.T.U.) (Ans. $\left.18.028 \sin \left(\omega \times t+73.89^{\circ}\right)\right)$

## SUMMARY

1. Alternating voltage: A voltage that changes its polarity and magnitude at regular intervals of time is called an alternating voltage.
2. Sinusoidal alternating quantity: An alternating quantity that varies according to sine of angle $\theta$ is known as sinusoidal alternating quantity.
3. Equation of an alternating quantity

$$
e=E_{\mathrm{m}} \sin \theta=E_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin \theta=I_{\mathrm{m}} \sin \omega t
$$

4. Wave form: The shape of the curve obtained by plotting the instantaneous values along Y-axis and time or angle $\theta(=\omega t)$ along X -axis is called wave form or wave shape.
5. Cycle: When an alternating quantity goes through a complete set of positive and negative value or goes through 360 electrical degrees, it is said to have completed a cycle.
6. Alternation: One half of cycle is called an alternation.
7. Time period $(T)$ : Time taken to complete one cycle per second is called time period.
8. Frequency $(f)$ : The number of cycles made per second by an alternating quantity is called its frequency. It is measured in $\mathrm{c} / \mathrm{s}$ or Hz (Hertz).
9. Relations: $f=1 / T$ and $\omega=2 \times \pi \times f$
10. Instantaneous value ( $v$ or $i$ ): The value of an alternating quantity at any instant is called its instantaneous value.
11. Amplitude ( $V_{\mathrm{m}}$ or $I_{\mathrm{m}}$ ): The maximum value obtained by an alternating quantity during a cycle is called its amplitude or maximum value or peak value or crest value.
12. Average value ( $V_{\mathrm{av}}$ or $I_{\mathrm{av}}$ ) : The arithmetic average of all instantaneous values considered of an alternating quantity over one cycle is called its average value.

$$
I_{\mathrm{av}}=\frac{i_{1}+i_{2}+i_{3}+\ldots i_{\mathrm{n}}}{n}
$$

For sinusoidal current, $I_{\mathrm{av}}=2 \times I_{\mathrm{m}} / \pi=0.637 \times I_{\mathrm{m}}$.
13. Effective or rms value ( $V_{\mathrm{rms}}$ or $I_{\mathrm{rms}}$ ): The steady current which when flows through a resistor of known resistance for a given time produces the same amount of heat as produced by the alternating current when flows through the same resistor for the same time is called effective or rms. value of the alternating current.

For sinusoidal current,

$$
I_{\mathrm{rms}}=\sqrt{\frac{i_{1}^{2}+i_{2}^{2}+i_{3}^{2}+\ldots+i_{\mathrm{n}}^{2}}{n}}
$$

$$
I_{\mathrm{rms}}=I_{\mathrm{m}} / \sqrt{2}=0.707 \times I_{\mathrm{m}}
$$

14. Form factor: $I_{\mathrm{ms}} / I_{\mathrm{av}}$; for sinusoidal quantities, its value is 1.11 .
15. Peak factor: $I_{\mathrm{m}} / I_{\mathrm{rms}}$; for sinusoidal quantities, its value is 1.414 .
16. Phase: The phase of an alternating quantity at an instant is defined as the fractional part of a cycle through which the quantity has advanced from a selected origin. It has less importance in practice.
17. Phase difference: The angular displacement between the maximum positive values of two alternating quantities having the same frequency is called the phase difference between them.

Mathematically, If $i_{1}=I_{\mathrm{m} 1} \sin \omega t$, then $i_{2}=I_{\mathrm{m} 2} \sin (\omega t \pm \phi)$
18. Leading quantity: An alternating quantity that attains its positive maximum value prior to the other is called leading quantity.
19. Lagging quantity: An alternating quantity that attains its positive maximum value after the other is called lagging quantity.
20. Phasor representation using rms values: Phasors can be represented by using rms values but they do not generate the sine waves of proper amplitude unless their lengths are increased by $\sqrt{2}$ times.

## TEST YOUR PREPARATION

## 7 fill in the blanks

1. Peak factor is defined as the ratio of $\qquad$ and $\qquad$ .
2. In an AC circuit, an ammeter measures $\qquad$ value of current.
3. The number of cycles made per second by an alternating quantity is called $\qquad$ .
4. The relation between angular velocity and frequency of an alternating quantity is given as $\omega=$ $\qquad$ _.
5. In case of sinusoidal voltage $V_{\mathrm{mms}}=V_{\mathrm{m}} /$ $\qquad$ .
6. Form factor is defined as the ratio of $\qquad$ and $\qquad$ .
7. One half cycle of an alternating quantity is called $\qquad$ .
8. The angular displacement between the positive maximum values of two alternating quantities having same frequency is called $\qquad$ _.
9. Average value of a sinusoidally varying alternating current is given by the relation $I_{\mathrm{av}}=I_{\mathrm{m}} /$
$\qquad$ —.
10. If the frequency of an alternating current is 200 kHz , its time period will be $\qquad$ _.

## OBJECTIVE TYPE QUESTIONS

1. The rms value of an alternating current is given by steady current that flows through a circuit for specific period of time produces
(a) the same amount of heat as produced by AC when flowing through the same circuit
(b) more heat than produced by AC when flowing through the same circuit
(c) less heat than produced by AC when flowing through the same circuit
(d) 144 Calories
2. The average value of sinusoidal quantity is given by the relation
(a) $I_{\mathrm{m}} / \sqrt{2}$
(b) $0.707 \times I_{\mathrm{m}}$
(c) $2 \times I_{\mathrm{m}} / \pi$
(d) None of above
3. In case of an unsymmetrical alternating quantity, the average value must always be taken over
(a) the half cycle
(b) the whole cycle
(c) the quarter cycle
(d) any fraction of the cycle
4. The rms value of sinusoidal alternating current is given by the relation
(a) $I_{\mathrm{m}} / 2$
(b) $0.637 \times I_{\mathrm{m}}$
(c) $2 \times I_{\mathrm{m}} / \pi$
(d) $I_{\mathrm{m}} / \sqrt{2}$
5. The average value of an alternating quantity is more than the rms value
(a) true
(b) false
6. The amplitude factor of sinusoidal current is
(a) 1.11
(b) 1.57
(c) 1.414
(d) 0.637
7. If the rms value is a fraction of maximum value of sinusoidal current, the value of the fraction is
(a) $1 / 1.414$
(b) 0.637
(c) $\sqrt{2}$
(d) None of above
8. The rms value of an AC signal is 10 V . The peak-to-peak value will be
(a) 6.37 V
(b) 14.14 V
(c) 141 V
(d) 28.28 V
9. Two waves have the frequency of 600 Hz and one is set at its maximum value, whereas the other at zero, the phase angle between them will be
(a) $0^{\circ}$
(b) $90^{\circ}$
(c) $90 \times 600 / 50^{\circ}$
(d) $90 \times 50 / 600^{\circ}$
10. A wave completes one cycle in 10 m sec , its frequency will be
(a) $10 \mu \mathrm{~Hz}$
(b) 50 Hz
(c) 100 kHz
(d) 10 kHz
11. An alternating quantity which attains its positive maximum value prior to other is called
(a) in phase quantity
(b) lagging quantity
(c) leading quantity
(d) None of above
12. If the effective value of the sinusoidal voltage is 111 V . Its average value will be
(a) 100 V
(b) 123.21 V
(c) 156.51
(d) None of above
13. If the frequency of power supply is 60 Hz , the time period of one cycle will be
(a) 0.02 s
(b) 20 ms
(c) 16.67 ms
(d) 0.1667 s
14. If a sinusoidal wave has frequency of 50 Hz with 15 A rms value which of the following equation represents this wave
(a) $15 \times \sin 50 \times t$
(b) $30 \times \sin 25 \times t$
(c) $42.42 \times \sin 100 \times t$
(d) $21.21 \times \sin 314 \times t$
15. Peak factor of an alternating current is given by the relation
(a) $I_{\mathrm{rms}} / I_{\mathrm{av}}$
(b) $I_{\mathrm{m}} / I_{\mathrm{rms}}$
(c) $I_{\mathrm{av}} / I_{\mathrm{rms}}$
(d) $I_{\mathrm{rms}} / I_{\mathrm{m}}$

## NUMERICALS

1. An alternating voltage is given as $v=400 \times \sin 314 \times t$. Determine its (i) maximum value; (ii) effective value; (iii) form factor; (iv) value of voltage after 0.0025 s taking reckoning time from the instant when voltage is zero and becoming positive; (v) time after which voltage attains 200 V for the first time.
(Ans. $400 \mathrm{~V} ; 282.84 \mathrm{~V} ; 50 \mathrm{~Hz} ; 1.11 ; 282.84 \mathrm{~V} ; 1.67 \mathrm{~ms})$
2. An alternating current of frequency 60 Hz has an rms value of 84.853 A . (i) Write down the equation for its instantaneous value; (ii) Find the instantaneous value of current 0.002083 s after passing through zero and becoming positive; (iii) Find the instantaneous value of current 1.389 ms after passing through positive maximum value and (iv) Time taken to reach 96 A for the first time after passing through positive maximum value. (P.T.U.) (Ans. $120 \times \sin 377 \times t ; 84.85 \mathrm{~A} ; 103.92 \mathrm{~A} ; 1.7 \mathrm{~ms}$ )
3. For a full wave rectified alternating current, determine (i) average value; (ii) rms value; (iii) form factor and (iv) peak factor.
(Ans. $0.637 \times I_{\mathrm{m}} ; 0.707 \times I_{\mathrm{m}} ; 1.11 ; 1.4142$ )
4. An electromotive force (emf) $e_{1}=50 \times \sin \omega \times t$ and the other $e_{2}=30 \times \sin (\omega t-\pi / 6)$ act together in the same circuit. Find the resultant emf (i) analytically and (ii) graphically.
(Ans. $\left.77.45 \sin \left(\omega \times t+11.17^{\circ}\right)\right)$
5. Three voltages represented by $e_{1}=20 \sin (\omega \times t+\pi / 3) ; e_{2}=30 \cos (\omega \times t+\pi / 6)$, and $e_{3}=40 \sin (\omega \times$ $t-\pi / 6)$ act together in a circuit. Find an expression for the resultant voltage.
(U.P.T.U.) (Ans. $\left.37.7 \sin \left(\omega t+38.17^{\circ}\right)\right)$

## VIVA-VOCE OR REASONING QUESTIONS

1. When a coil is rotated in a magnetic field at a constant speed, the wave shape of an induced emf is sinusoidal (not square or triangular). Why?
2. The effective value of current is named as rms value. Why?
3. The rms value is also called effective value. Why?
4. Is the magnitude of voltage supplied to our residences remains only 230 V ? Why?
5. Why the rms value of an alternating current or voltage is used to denote its magnitude?
6. How are alternating quantities added?
7. Alternating currents are not added arithmetically. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What do you understand by alternating current?
2. What is difference the between AC and DC ?
3. Why sinusoidal voltages and currents are used while generating, transmitting, and utilizing AC electric power?
(P.T.U., Jan. 2000)
4. What is instantaneous value?
5. What is peak value of an alternating quantity?
6. What are scalar and vector quantities?
7. What is meant by phase?
8. What is an in-phase condition?
9. What is an out-of-phase condition?
10. Under what circumstances does a generator produce an alternating induced voltage?
11. List the various factors governing the emf generation.
12. Define average value of an alternating quantity.
13. What is rms value of an alternating quantity?
(P.T.U., Dec. 2003)
14. Why are alternating voltages and currents expressed in rms values instead of average values?
15. Define form factor.
16. Define peak factor of an alternating quantity.
17. What is the significance of form factor?
18. What is the significance of peak factor?
19. Difference between form factor and peak factor.
20. What does the form factor of a wave other than sinusoidal wave indicate?
21. Our houses are supplied with alternating voltage whose instantaneous value is given by the equation $e=325 \sin \omega \times t$, but we always say that AC voltage at home is 230 V . How do you explain this difference?
22. How will you calculate average value for symmetrical waves?
23. What is a phasor?
24. What is the significance of phasor representation of an alternating quantity?
25. List the important points to be considered while representing an alternating quantity by a vector.
26. What is meant by phase difference?
27. What are leading and lagging quantities?
28. A phasor is always assumed to rotate with a constant angular velocity in anticlockwise direction. Why?

## TEST QUESTIONS

1. Explain how a sinusoidal emf is generated.
2. Derive an expression for the instantaneous value of an alternating voltage varying sinusoidally.
(P.T.U.)
3. Explain the terms amplitude, cycle, time period, frequency, and phase difference as applied to alternating wave forms.
(U.P.T.U.)
4. Define rms value and average value of an alternating quantity.
5. Show that the rms value of a sinusoidal AC voltage of amplitude $V_{\mathrm{m}}$ is $V_{\mathrm{m}} / \sqrt{2}$.
6. Derive the expression for average and rms value of a sinusoidally varying quantity.
7. Show that the form factor of the sinusoidal wave form is 1.11 .
8. Derive the values of form factor and peak factor of a sinusoidally varying quantity.
9. Explain how sinusoidal quantities can be represented by vectors.
10. Explain the component method for the addition of alternating quantities.
(U.P.T.U.)
11. Distinguish between
(i) Time period and frequency
(ii) Cycle and alternation
(iii) Peak factor and form factor
(iv) Average and rms value of an alternating current.

## q. aNSWERS

## Fill in the Blanks

1. max. value; rms value
2. rms
3. frequency
4. $2 \times \pi \times f$
5. alternation
6. $5 \mu \mathrm{~s}$
7. $\sqrt{2}$
8. phase difference
9. rms value; average value
10. $\pi / 2$

## Objective Type Questions

1. (a)
2. (c)
3. (b)
4. (d)
5. (b)
6. (c)
7. (a)
8. (d)
9. (b)
10. (c)
11. (c)
12. (a)
13. (c)
14. (d)
15. (b)


## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is pure resistive, pure inductive, and pure capacitive circuit?
* How the components R, L, and C behave when placed in AC circuits?
* What is the relation between the voltage and the current in pure resistive, pure inductive, and pure capacitive circuits?
* What power is consumed by these components?
* What is the behaviour of these components when connected in series, that is, what is the behaviour of $\mathrm{R}-\mathrm{L}, \mathrm{R}-\mathrm{C}$, or $\mathrm{R}-\mathrm{L}-\mathrm{C}$ circuits when connected across single-phase AC supply?
* What is the behaviour of these components when connected in parallel?
* How AC series and parallel circuits (or networks) are solved?
* How J-method and polar methods are used for the solution of AC circuits?
* What is AC series and parallel resonance and their applications?
* How capacitors are used to improve the power factor of induction motors (or inductive loads)?


### 7.1 INTRODUCTION

Alternating supply is invariably used for domestic and industrial applications. The path for the flow of alternating current is called an AC circuit. In DC circuits, the opposition to the flow of current is only the resistance of the circuit. While in AC circuits, the opposition to the flow
of current is due to resistance $(R)$, inductive reactance ( $X_{\mathrm{L}}=2 \pi f \mathrm{~L}$ ) and capacitive reactance $\left(X_{C}=1 / 2 \pi f C\right)$ of the circuit. In AC circuits, frequency plays an important role. In these circuits, the currents and voltages are represented with magnitude and direction (phasors). The voltage and current may or may not be in phase with each other depending upon the parameters ( $\mathrm{R}, \mathrm{L}$, and C) of the circuit. Moreover, in AC circuits, the currents as well as voltages are added and subtracted vectorially instead of arithmetically as in DC circuits. Hence, AC circuits need more attention for their solution. In this chapter, we shall confine our attention to the fundamentals related with various types of AC circuits.

### 7.2 AC CIRCUIT CONTAINING RESISTANCE ONLY

The circuit containing a pure resistance of $R \Omega$ is shown in Figure 7.1. Let the alternating voltage applied across the circuit be given by the equation;


Fig. 7.1 Circuit containing resistance only

$$
\begin{equation*}
v=V_{\mathrm{m}} \sin \omega t \tag{7.1}
\end{equation*}
$$

Then, the instantaneous value of current flowing through the resistor will be;

$$
\begin{equation*}
i=\frac{v}{R}=\frac{V_{\mathrm{m}}}{R} \sin \omega t \tag{7.2}
\end{equation*}
$$

The value of current will be maximum when

$$
\begin{gathered}
\omega t=90^{\circ} \quad \text { or } \quad \sin \omega t=1 \\
I_{\mathrm{m}}=V_{\mathrm{m}} / R
\end{gathered}
$$

Substituting this value in equation (7.2), we get

$$
\begin{equation*}
i=I_{\mathrm{m}} \sin \omega t \tag{7.3}
\end{equation*}
$$

### 7.2.1 Phase Angle

From Equations (7.1) and (7.3), it is clear that there is no phase difference between the applied voltage and the current flowing through pure


Fig. 7.2 (a) Phasor diagram (b) Wave diagram for voltage, current and power resistive circuit, that is, phase angle between the voltage and the current is zero. The phasor diagram and wave diagram are shown in Figure 7.2(a) and (b), respectively.

Hence, in an AC circuit containing pure resistance, current is in phase with the voltage.

### 7.2.2 Power

Instantaneous power, $p=v i=\left(V_{\mathrm{m}} \sin \omega t\right)$ $\left(I_{\mathrm{m}} \sin \omega t\right)$

$$
=\frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} 2 \sin ^{2} \omega t
$$

$$
=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}}(1-\cos 2 \omega t)=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}}-\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos 2 \omega t
$$

Average power consumed in the circuit over a complete cycle:

$$
P=\text { average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \text { - average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \omega t
$$

or

$$
\mathrm{P}=V_{\mathrm{rms}} I_{\mathrm{rms}}-0 \text { or } P=V I
$$

### 7.2.3 Power Curve

Figure 7.2(b) shows the power curve for a pure resistive circuit. Points on the power curve are obtained from the product of the corresponding instantaneous values of voltage and current.

### 7.3 AC CIRCUIT CONTAINING PURE INDUCTANCE ONLY

The circuit containing a pure inductance of $L$ Henry is shown in Figure 7.3.
Let the alternating voltage applied across the circuit be given by the equation;

$$
\begin{equation*}
v=V_{\mathrm{m}} \sin \omega t \tag{7.4}
\end{equation*}
$$

As a result, an AC $i$ flows through the inductance that induces an emf in it, given by the relation;

$$
e=-L \frac{d i}{d t}
$$

This induced emf is equal and opposite to the applied

$$
\begin{aligned}
& \text { voltage. } \\
& \therefore
\end{aligned} \quad v=-e=-\left(-L \frac{d i}{d t}\right)
$$



Fig. 7.3 Circuit containing pure inductance only
or

$$
V_{\mathrm{m}} \sin \omega t=L \frac{d i}{d t} \quad \text { or } \quad d i=\frac{V_{\mathrm{m}}}{L} \sin \omega t d t
$$

Integrating both sides
or

$$
\begin{aligned}
\int d i & =\int \frac{V_{\mathrm{m}}}{L} \sin \omega t d t \quad \text { or } \quad i=\frac{V_{\mathrm{m}}}{\omega L}(-\cos \omega t) \\
i & =\frac{V_{\mathrm{m}}}{\omega L} \sin (\omega t-\pi / 2)=\frac{V_{\mathrm{m}}}{X_{\mathrm{L}}} \sin (\omega t-\pi / 2)
\end{aligned}
$$

where $X_{\mathrm{L}}=\omega L$ is the opposition offered to the flow of AC by a pure inductance and it is called inductive reactance.

The value of current will be maximum when $\sin (\omega t-\pi / 2)=1$; i.e., $I_{\mathrm{m}}=\frac{V_{\mathrm{m}}}{X_{\mathrm{L}}}$

$$
\begin{equation*}
\therefore \quad i=I_{\mathrm{m}} \sin (\omega t-\pi / 2) \tag{7.5}
\end{equation*}
$$

### 7.3.1 Phase Angle

From Equations (7.4) and (7.5), it is clear that current flowing through a pure inductive circuit lags behind the applied voltage $v$ by $90^{\circ}$. The phasor diagram is shown in Figure 7.4(a) and 7.4(b), respectively.

Hence, in an AC circuit containing pure inductance, current lags behind the voltage by $90^{\circ}$.


Fig. 7.4 (a) Phasor diagram (b) Wave diagram for voltage, current and power

### 7.3.2 Power

Instantaneous power, $p=v i=V_{\mathrm{m}} \sin \omega t \times I_{\mathrm{m}} \sin (\omega t-\pi / 2)$

$$
\begin{aligned}
& =V_{\mathrm{m}} I_{\mathrm{m}} \sin \omega t \cos \omega t=\frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} 2 \sin \omega t \cos \omega t \\
& =\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \sin 2 \omega t
\end{aligned}
$$

Average power consumed in the circuit over a complete cycle,

$$
P=\text { average } \frac{V_{\mathrm{m}}}{\sqrt{2}} \cdot \frac{I_{\mathrm{m}}}{\sqrt{2}} \sin 2 \omega t=0
$$

Hence, average power consumed in a pure inductive circuit is zero.

### 7.3.3 Power Curve

The power curve for a pure inductive circuit is shown in Figure 7.4(b). It is very clear that average power in a half cycle (one alternation) is zero, as the negative and positive loop area under the power curve is the same.

It is interesting to note that during the first quarter cycle, whatever power (or energy) is supplied by the source to the inductance (or coil) is stored in the magnetic field set-up around it. However, in the next quarter cycle, the magnetic field collapses and the power (or energy) stored in the field is returned to the source. This process is repeated in each and every alternation. Hence, no power or energy is consumed in this circuit.

### 7.4 AC CIRCUIT CONTAINING PURE CAPACITOR ONLY

The circuit containing a pure capacitor of capacitance $C$ Farad is shown in Figure 7.5. Let the alternating voltage applied across the circuit be given as

$$
\begin{equation*}
v=V_{\mathrm{m}} \sin \omega t \tag{7.6}
\end{equation*}
$$

Charge on the capacitor at any instant,

$$
q=C v
$$

Current flowing through the circuit,

$$
\begin{aligned}
& i=\frac{d}{d t} q=\frac{d}{d t}(C v) \\
& i=\frac{d}{d t} C V_{\mathrm{m}} \sin \omega t=C V_{\mathrm{m}} \frac{d}{d t} \sin \omega t
\end{aligned}
$$

or
or

$$
\begin{equation*}
i=\omega C V_{\mathrm{m}} \cos \omega t=\frac{V_{\mathrm{m}}}{1 / \omega C} \sin (\omega t+\pi / 2)=\frac{V_{\mathrm{m}}}{X_{\mathrm{C}}} \sin (\omega t+\pi / 2) \tag{7.7}
\end{equation*}
$$

where $X_{\mathrm{C}}=1 / \omega C$ is the opposition offered to the flow of AC by a pure capacitor and is called capacitive reactance.

The value of current will be maximum when $\sin (\omega t+\pi / 2)=1$
i.e.,

$$
I_{\mathrm{m}}=V_{\mathrm{m}} / X_{\mathrm{C}}
$$

Substituting this value is Equation (7.7), we get

$$
\begin{equation*}
i=I_{\mathrm{m}} \sin (\omega t+\pi / 2) \tag{7.8}
\end{equation*}
$$

### 7.4.1 Phase Angle

From Equations (7.6) and (7.8), it is clear that the current flowing through pure capacitive circuit leads the applied voltage by $90^{\circ}$. The phasor diagram and wave diagram are shown in Figure 7.6(a) and (b), respectively. Hence, in an AC circuit containing pure capacitance current leads the voltage by $90^{\circ}$.


Fig. 7.6 (a) Phasor diagram (b) Wave diagram for voltage, current and power

### 7.4.2 Power

Instantaneous power, $p=v i=V_{\mathrm{m}} \sin \omega t \times I_{\mathrm{m}} \sin (\omega t+\pi / 2)$

$$
=V_{\mathrm{m}} I_{\mathrm{m}} \sin \omega t \cos \omega t=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \sin 2 \omega t
$$

or average power over a complete cycle,

$$
P=0
$$

Hence, average power consumed in a pure capacitive circuit is zero.

### 7.4.3 Power Curve

The power curve for a pure capacitive circuit is shown in Figure 7.6(b). It is very clear from the curve that average power in a half cycle (one alternation) is zero since the positive and negative loop area under power curve is the same.

It is interesting to note that during the first quarter cycle, whatever power (or energy) is supplied by the source to the capacitor is stored in the electric field set-up between the capacitor plates. In the next quarter cycle, the electric field collapses and the power (or energy) stored in the field is returned to the source. This process is repeated in each alternation. Hence, no power is consumed by this circuit.

## Example 7.1

An AC circuit consists of a pure resistance of $10 \Omega$ and is connected across an AC supply of $230 \mathrm{~V}, 50 \mathrm{~Hz}$. Calculate (i) current, and (ii) power consumed; further, (iii) write down the equation for voltage and current.

## Solution:

(i) Current in the circuit, $I=\frac{V}{R}=\frac{230}{10}=23 \mathrm{~A}$
(ii) Power consumed, $P=V I=230 \times 23=5,290 \mathrm{~W}$
(iii) Maximum value of applied voltage, $V_{\mathrm{m}}=\sqrt{2} V=\sqrt{2} \times 230=325.27 \mathrm{~V}$

Maximum value of current, $I_{\mathrm{m}}=\sqrt{2} \times 23=32.53 \mathrm{~A}$
Angular velocity, $\omega=2 \pi f=2 \pi \times 50=314.16 \mathrm{rad} / \mathrm{s}$
Equation for applied voltage;

$$
v=V_{\mathrm{m}} \sin \omega t=325.27 \sin 314.16 t
$$

As in a pure resistive circuit, voltage and current are in phase with each other, and therefore, current is given by the equation;

$$
i=I_{\mathrm{m}} \sin \omega t=32.53 \sin 314.16 t
$$

## Example 7.2

An inductive coil having negligible resistance and 0.1 H inductance is connected across 200 V, 50 Hz supply. Find (i) the inductive reactance, (ii) rms value of current, (iii) power, and (iv) equations for voltage and current.

## Solution:

Inductive reactance,
Current,
$X_{\mathrm{L}}=2 \pi L=2 \pi \times 50 \times 0.1=31.416 \Omega$
Power,
$I=V / X_{\mathrm{L}}=200 / 31.416=6.366 \mathrm{~A}$
$P=0$
Now,

$$
V_{\mathrm{m}}=\sqrt{2} V=\sqrt{2} \times 200=282.84 \mathrm{~V}
$$

$$
I_{m}=\sqrt{2} I=\sqrt{2} \times 6.366=9 \mathrm{~A}
$$

and

$$
\omega=2 \pi f=314 \mathrm{rad} / \mathrm{s}
$$

$\therefore \quad v=V_{\mathrm{m}} \sin \omega t=282.84 \sin 314 t$
In pure inductive circuit, current lags behind voltage by $\pi / 2$ radian.

$$
\therefore \quad i=I_{\mathrm{m}} \sin (\omega t-\pi / 2)=9 \sin (314 t-\pi / 2)
$$

## Example 7.3

A capacitor has a capacitance of $30 \mu \mathrm{~F}$. Find its capacitive reactance for frequencies of 25 and 50 Hz . Find in each case the current if the supply voltage is 440 V .

## Solution:

Capacitance of the capacitor, $C=30 \times 10^{-6} \mathrm{~F}$
Supply voltage, $V=440 \mathrm{~V}$
When supply frequency, $f_{1}=25 \mathrm{~Hz}$
Capacitive reactance, $X_{\mathrm{Cl}}=\frac{1}{\omega_{1} C}=\frac{1}{2 \pi f_{1} C}=\frac{1}{2 \pi \times 25 \times 30 \times 10^{-6}}=212.2 \Omega$
Current in the circuit, $I_{1}=\frac{V}{X_{\mathrm{C} 1}}=\frac{440}{212.2}=2.073 \mathrm{~A}$
When supply frequency $f_{2}=50 \mathrm{~Hz}$,
capacitive reactance, $X_{\mathrm{C} 2}=\frac{1}{\omega_{2} C}=\frac{1}{2 \pi f_{2} C}=\frac{1}{2 \pi \times 50 \times 30 \times 10^{-6}}=106.1 \Omega$
Current in the circuit, $I_{2}=\frac{V}{X_{\mathrm{C} 2}}=\frac{440}{106.1}=4.146 \mathrm{~A}$

## Example 7.4

A $100 \mu \mathrm{~F}$ capacitor is connected across a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Determine (i) the maximum instantaneous charge on the capacitor and (ii) the maximum instantaneous energy stored in the capacitor.

## Solution:

(i) Maximum instantaneous charge on the capacitor

$$
=C V_{\mathrm{m}}=\left(100 \times 10^{-6}\right) \times(230 \times \sqrt{2})=32.527 \times 10^{-3} \mathrm{C}
$$

(ii) Maximum instantaneous energy stored in the capacitor

$$
=\frac{1}{2} C V_{\mathrm{m}}^{2}=\frac{1}{2}\left(100 \times 10^{-6}\right)(230 \times \sqrt{2})^{2}=5.29 \mathrm{~J}
$$

## 둘

## PRACTICE EXERCISES

## Short Answer Questions

1. What is the phase difference between voltage and current when sinusoidal $A C$ voltage is applied across a pure resistor?
2. What is the phase difference between $V$ and $I$ in pure inductive circuit?
3. What is the power consumption in a pure capacitive circuit?

## Test Questions

1. When sinusoidal AC voltage is applied across a pure inductor, show that power consumed in the circuit is zero. Further, draw the phasor and wave diagram for voltage and current.
2. Show that in a pure capacitive AC circuit, power consumption is zero.

## Numericals

1. An AC circuit consists of a pure resistance of $20 \Omega$ and is connected across an AC supply of 240 V , 50 Hz . Calculate (i) current and (ii) power consumed; further (iii) write down the equation for voltage and current.
(Ans. $12 \mathrm{~A}, 2880 \mathrm{~W}, v=339.4 \sin 314 t, i=16.97 \sin 314 t$ )
2. An inductive coil having negligible resistance and 0.1 H inductance is connected across 230 V , 50 Hz supply. Find (i) the inductive reactance, (ii) rms value of current, (iii) power, and (iv) equations for voltage and current.
(Ans. $31.416 \Omega, 7.32 \mathrm{~A}$, zero, $v=325.27 \sin 314 t$, $i=10.35 \sin (314 t-\pi / 2))$
3. A capacitor has a capacitance of $20 \mu \mathrm{~F}$. Find its capacitive reactance for frequencies of 50 Hz . Find the capacitive reactance and current if the supply voltage is 230 V .
(Ans. $159.15 \Omega, 1.445 \mathrm{~A})$

### 7.5 AC SERIES CIRCUITS

So far, we have dealt with simple AC circuits containing pure components such as resistance, inductance, and capacitance. However, in actual practice, AC circuits contain two or more than two such components connected in series or parallel. A series circuit is a circuit in which each component carries the same current. An AC series circuit may be (i) R-L series circuit (ii) R-C series circuit, and (iii) $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit.

### 7.6 R-L SERIES CIRCUIT

A circuit that contains a pure resistance $R \Omega$ connected in series with a coil having pure inductance of $L$ Henry is known as $\mathrm{R}-\mathrm{L}$ series circuit. This is the most general case that we come across in practice.


Fig. 7.7 Circuit containing resistance and inductance in series

An $\mathrm{R}-\mathrm{L}$ series circuit and its phasor diagram are shown in Figures 7.7 and 7.8, respectively. To draw the phasor diagram, current $I$ (rms value) is taken as the reference vector. Voltage drop in resistance $V_{\mathrm{R}}(=I R)$ is taken in phase with current vector, whereas voltage drop in inductive reactance $V_{\mathrm{L}}\left(=I X_{\mathrm{L}}\right)$ is taken $90^{\circ}$ ahead of the current vector (since current lags behind the voltage by $90^{\circ}$ in pure inductive circuit). The vector sum of these two voltages (drops) is equal to the applied voltage $V$ (rms value).

Now, $V_{\mathrm{R}}=I R$ and $V_{\mathrm{L}}=I X_{\mathrm{L}}\left(\right.$ where $\left.X_{\mathrm{L}}=2 \pi f L\right)$
In right-angled triangle OAB ,

$$
\begin{array}{r}
V=\sqrt{\left(V_{\mathrm{R}}\right)^{2}+\left(V_{\mathrm{L}}\right)^{2}}=\sqrt{(I R)^{2}+\left(I X_{\mathrm{L}}\right)^{2}}=I \sqrt{R^{2}+X_{\mathrm{L}}^{2}} \\
I=\frac{V}{\sqrt{R^{2}+X_{\mathrm{L}}^{2}}}=\frac{V}{Z}
\end{array}
$$

or


Fig. 7.8 Phasor diagram
where $Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}$ is the total opposition offered to the flow of AC
by an $\mathrm{R}-\mathrm{L}$ series circuit and is called impedance of the circuit. It is measured in ohms.

### 7.6.1 Phase Angle

From the phasor diagram shown in Figure 7.8, it is clear that current in this circuit lags behind the applied voltage by an angle $\phi$ called phase angle.

From phasor diagram, $\tan \phi=\frac{V_{\mathrm{L}}}{V_{\mathrm{R}}}=\frac{I X_{\mathrm{L}}}{I R}=\frac{X_{\mathrm{L}}}{R}$ or $\phi=\tan ^{-1} X_{\mathrm{L}} / R$

### 7.6.2 Power

If the alternating voltage applied across the circuit is given by the equation.

Then,

$$
\begin{aligned}
& v=V_{\mathrm{m}} \sin \omega t \\
& i=I_{\mathrm{m}} \sin (\omega t-\phi)
\end{aligned}
$$

$\therefore$ Instantaneous power, $p=v i$

$$
\begin{aligned}
& =V_{\mathrm{m}} \sin \omega t I_{\mathrm{m}} \sin (\omega t-\phi)=\frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} 2 \sin \omega t \sin (\omega t-\phi) \\
& =\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}}[\cos \phi-\cos (2 \omega t-\phi)] \\
& =\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t-\phi)
\end{aligned}
$$

Average power consumed in the circuit over a complete cycle,
or

$$
\begin{aligned}
& P=\text { average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\text { average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t-\phi) \\
& P=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\text { zero }=V_{\mathrm{rms}} I_{\mathrm{rms}} \cos \phi=V I \cos \phi
\end{aligned}
$$

where $\cos \phi$ is called power factor of the circuit.
From phasor diagram, $\cos \phi=\frac{V_{\mathrm{R}}}{V}=\frac{I R}{I Z}=\frac{R}{Z}$
Therefore, power factor is defined as the cosine of the angle between the voltage and the current in an AC circuit. It may also be defined as the ratio of resistance to impedance of an AC circuit.

Alternatively, power, $P=V I \cos \phi=I Z \cdot I \cdot \frac{R}{Z}=I^{2} R$
This shows that power is actually consumed in resistance only; inductance does not consume any power.

### 7.6.3 Power Curve

The phasor diagram and wave diagram for voltage and current are shown in Figure 7.9(a) and (b), respectively, where applied voltage ( $v=V_{\mathrm{m}} \sin \omega t$ ) is taken as reference quantity. The power curve for R-L series circuit is also shown in Figure 7.9(b). The points on the power curve are obtained from the product of the corresponding instantaneous values of voltage and current. It is clear that power is negative between angle 0 and $\phi$ and between $180^{\circ}$ and $(180+\phi)$. During rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, the net power over a complete cycle is positive. Hence, a definite quantity of power is utilised or consumed by this circuit.


Fig. 7.9 (a) Phasor diagram (b) Wave diagram for voltage, current and power

### 7.7 IMPEDANCE TRIANGLE

The simplified phasor diagram of $\mathrm{R}-\mathrm{L}$ series circuit is shown in Figure 7.10. When each side of this phasor diagram is divided by a common factor $I$, we get another right-angled triangle, as shown


Fig. 7.10 Phasor diagram for R-L series circuit in Figure 7.11, whose sides represent $R, X_{\mathrm{L}}$, and $Z$. Such a triangle is known as impedance triangle.

Therefore, a right-angled triangle whose base represents circuit resistance, perpendicular represents circuit reactance, and hypotenuse represents circuit impedance is called an impedance triangle. The concept of impedance triangle is useful since it enables us to calculate:
(i) the impedance of the circuit,

$$
Z=\sqrt{R^{2}+X^{2}}
$$

(ii) the power factor of the circuit,

$$
\cos \phi=R / Z
$$

(iii) phase angle, $\phi=\tan ^{-1} X_{\mathrm{L}} / R$

### 7.8 TRUE POWER AND REACTIVE POWER

The power that is actually consumed or utilised in an AC circuit


Fig. 7.ll Impedance triangle is called true power or active power or real power. It has already been seen that power is consumed only in resistance. A pure inductor and a pure capacitor do not consume any power, since in a half cycle whatever power is received from the source by these components, the same is returned to the source. This power that flows back and forth (i.e., in both directions in the circuit) or reacts upon itself is called reactive power. It does not do any useful work in the circuit. It has been seen that in pure resistive circuit, current is in phase with the applied voltage, whereas in pure inductive and capacitive circuit, current is $90^{\circ}$ out of phase. Therefore, it is concluded that the current in phase with the voltage produces true or active power, whereas the current $90^{\circ}$ out of the phase with the voltage contributes to reactive power. Hence,

$$
\begin{aligned}
\text { true power } & =\text { voltage } \times \text { current in phase with voltage } \\
\text { reactive power } & =\text { voltage } \times \text { current } 90^{\circ} \text { out of phase with voltage. }
\end{aligned}
$$

The phasor diagram for an inductive circuit is shown in Figure 7.12, where current $I$ lags behind the voltage $V$ by an angle $\phi^{\circ}$. Current $I$ can be resolved into two rectangular components, that is, (i) $I \cos \phi$, which is in phase with voltage $V$ and (ii) $I \sin \phi$, which is $90^{\circ}$ out of phase with voltage $V$.

$$
\begin{aligned}
\therefore \quad \text { True power, } P & =V \times I \cos \phi=V I \cos \phi \mathrm{~W} \\
\text { Reactive power, } P_{\mathrm{r}} & =V \times I \sin \phi=V I \sin \phi \mathrm{VAR}
\end{aligned}
$$

Apparent power, $P_{\mathrm{a}}=V \times I=V I \mathrm{VA}$


Fig. 7.12 Phasor diagram representing active, reactive and apparent current

The bigger units of true power, reactive power, and apparent power are kW (or MW), kVAR (or MVAR), and kVA (or MVA), respectively.

### 7.8.1 Active Component of Current

The current component that is in phase with circuit voltage (i.e., $I \cos \phi$ ) and contributes to active or true power of the circuit is called active component or watt-full component or in-phase component of current.

### 7.8.2 Reactive Component of Current

The current component that is in quadrature ( or $90^{\circ}$ out of phase) to circuit voltage (i.e., $I \sin \phi$ ) and contributes to reactive power of the circuit is called reactive component of current.

### 7.8.3 Power Triangle

When each component of current, in Figure 7.12, is multiplied by voltage $V$, a power triangle is obtained, as shown in Figure 7.13. This


Fig. 7.13 Power triangle right-angled triangle indicates the relation among true power, reactive power, and apparent power.

In the abovementioned discussion, the following points are worth noting:

1. When an active component of current is multiplied with circuit voltage, it results in active or true power. It is this power that produces torque in motors, heat in heaters, light in lamps, etc. Further, wattmeter indicates this power.
2. When the reactive component of current is multiplied with circuit voltage, it results in
reactive power. It is this power that merely flows back and forth without doing any work. This power determines the power factor of the circuit.
3. When the circuit current is multiplied with circuit voltage, it results in apparent power. It is so called because it appears that product of voltage and current is power. However, in AC circuits (except pure resistive circuit), there is usually phase difference between voltage and current so that $V I$ does not give real power. To avoid confusion, it is measured in volt-ampere.
4. From power triangle shown in Figure 7.13, the power factor may also be determined by taking ratio of true power to apparent power, that is, power factor, $\cos \phi=$ true power/ apparent power.

### 7.9 POWER FACTOR AND ITS IMPORTANCE

In AC circuits, the power factor may be expressed as $\mathrm{pf}=\cos \phi=R / Z=$ true power/apparent power

In the case of pure resistive circuit, current is in phase with circuit voltage, that is, $\phi=0$. Therefore, power factor of the circuit, $\cos \phi=1$. While in the case of pure inductive or capacitive circuit, current is $90^{\circ}$ out of phase with circuit voltage, that is, $\phi=90^{\circ}$. Therefore, power factor of the circuit $\cos \phi=0$. For circuits having resistance-inductance, resistance-capacitance or resistance-inductance, and capacitance, the power factor lies between 0 and 1 . It may be noted that the value of pf can never be more than one.

Usually, the word lagging or leading is attached with the numerical value of pf to signify whether the current lags or leads the voltage. In inductive circuits, current always lags behind the voltage and their power factors are mentioned as lagging pf While for capacitive circuits, the power factor is mentioned as leading pf , since in these cases, current always leads the voltage vector.

### 7.9.1 Importance of Power Factor

The power factor of an AC circuit plays an important role in the power system. Since power of an AC circuit is given by

$$
P=V I \cos \phi=\text { or } I=\frac{P}{V \cos \phi}
$$

From this relation, it is clear that for fixed power at constant voltage, the current drawn by the circuit increases with a decrease in pf. Therefore, at low pf, AC circuits draw more current from their mains and results in the following disadvantages:

1. Greater conductor size: At low pf, the conductors are to carry more current for the same power, and therefore, they require large area of cross-section.
2. Poor efficiency: At low power factors, the conductors have to carry larger current that increases copper losses $\left(I^{2} R\right)$ and results in poor efficiency.
3. Larger voltage drop: At low power factors, the conductors have to carry large current that increases voltage drop $(I R)$ in the system and results in poor regulation.
4. Larger kVA rating of equipment: The kVA rating of electrical machines and equipment connected in the power system such as alternators, transformers, and switch gears will be more at low power factors since it is inversely proportional to power factor (i.e., $\mathrm{kVA}=\mathrm{kW} / \cos \phi$ ).

To improve the power factor of an AC circuit, a capacitor is connected across the circuit, that is, parallel to the circuit.

### 7.10 Q-FACTOR OF A COIL

Reciprocal of power factor of a coil is known as its $Q$-factor. It is also called quality factor or figure of merit of the coil.

Mathematically,

$$
Q \text {-factor }=\frac{1}{\mathrm{p} . \mathrm{f}}=\frac{1}{\cos \phi}=\frac{Z}{R}
$$

If the value of $R$ is very small in comparison to its inductive reactance $X_{\mathrm{L}}$, then

Further,

$$
\begin{gathered}
Q \text {-factor }=\frac{X_{\mathrm{L}}}{R}=\frac{\omega L}{R} \\
Q=2 \pi \times \frac{\text { Maximum energy stored }}{\text { Energy dissipated per cycle }}
\end{gathered}
$$

## Example 7.5

A coil having a resistance of $12 \Omega$ and an inductance of 0.1 H is connected across a $100 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate the (i) reactance and impedance of the coil, (ii) current, (iii) phase difference between the current and the applied voltage, and (iv) power factor. Draw also the phasor diagram showing voltage and current.

## Solution:

The circuit is shown in Figure 7.14.
(i) Reactance, $X_{\mathrm{L}}=2 \pi f L=2 \pi 50 \times 0.1=31.416 \Omega$


Fig. 7.14 Circuit as per data


Fig. 7.15 Phasor diagram

Impedance, $Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(12)^{2}+(31.416)^{2}}=33.63 \Omega$
(ii) Current, $I=\frac{V}{Z}=\frac{100}{33.67}=2.97 \mathrm{~A}$
(iii) Phase difference,

$$
\phi=\tan ^{1} \frac{X_{\mathrm{L}}}{R}=\tan ^{-1} \frac{31.416}{12}=\tan ^{-1} 2.618=69.1^{\circ}
$$

(iv) Power factor, $\cos \phi=0.3568 \mathrm{lag}$

The phasor diagram for the circuit is shown in Figure 7.15.

## Example 7.6

The voltage and current through a circuit element are

$$
\begin{aligned}
& v=50 \sin \left(314 t+55^{\circ}\right) \mathrm{V} \\
& i=10 \sin \left(314 t+325^{\circ}\right) \mathrm{A}
\end{aligned}
$$

Find the value of power drawn by the element.
(U.P.T.U. 2006-07)

## Solution:



Fig. 7.16 Phasor diagram as per data

Given

$$
v=50 \sin \left(314 t+55^{\circ}\right) \mathrm{V}
$$

$$
i=10 \sin \left(314 t+325^{\circ}\right) \mathrm{A}
$$

$$
\text { or } \quad i=10 \sin \left(314 t-35^{\circ}\right) \mathrm{A}
$$

Now, their phasor representation is shown in Figure 7.16.
$\therefore$ Phase difference between the voltage and the current is $90^{\circ}$.
Now, power drawn by the circuit, $P=V I \cos \phi$

$$
\begin{aligned}
& =\frac{50}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 90^{\circ} \\
& =\frac{500}{4} \times 0^{\circ}=0 \mathrm{~W}
\end{aligned}
$$

This result indicates that the element is pure inductive.

## Example 7.7

A coil connected to 100 V DC supply draws 10 and the same coil when connected to $100 \mathrm{~V}, \mathrm{AC}$ voltage of frequency 50 Hz draws 5 A . Calculate the parameters of the coil and power factor.
(U.P.T.U. 2004-05)

## Solution:

Let the resistance and inductance of the coil be $R \Omega$ and $L$ Henry, respectively. When coil in connected to DC supply, the opposition is only resistance of the coil,
$\therefore$ Resistance of the coil,

$$
R=\frac{V_{\mathrm{DC}}}{I_{\mathrm{DC}}}=\frac{100}{10}=10 \Omega
$$

When coil is connected across AC supply of $100 \mathrm{~V}, 50 \mathrm{~Hz}$, the opposition is impedance of the coil.
$\therefore$ Impedance of the coil,

$$
Z=\frac{V_{\mathrm{AC}}}{I_{\mathrm{AC}}}=\frac{100}{5}=20 \Omega
$$

Now, $Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}} \quad$ or $\quad Z^{2}=R^{2}+X_{\mathrm{L}}^{2}$
or

$$
X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(20)^{2}-(10)^{2}}=\sqrt{300}
$$

and

$$
L=\frac{X_{\mathrm{L}}}{2 \pi f}=\frac{\sqrt{300}}{2 \pi \times 50}=55.13 \mathrm{mH}
$$

$\therefore$ Parameters are $R=10 \Omega$ and $L=55.13 \mathrm{~mA}$

Power factor,

$$
\cos \phi=\frac{R}{Z}=\frac{10}{20}=0.5 \text { lagging }
$$

## Example 7.8

A non-inductive resistance of $10 \Omega$ is connected in series with an inductive coil across $200 \mathrm{~V}, 50 \mathrm{~Hz} \mathrm{AC}$ supply. The current drawn by the series combination is 10 A . The resistance of the coil is $2 \Omega$. Determine (i) inductance of the coil, (ii) power factor, and (iii) voltage across the coil.
(U.P.T.U. 2005-06)

## Solution:

The circuit as per data is shown in Figure 7.17


Fig. 7.17 Circuit as per data

Total impedance of the circuit, $Z=\frac{V}{I}=\frac{200}{10}=20 \Omega$
Total resistance of the circuit, $R=10+2=12 \Omega$
Inductive reactance of the coil,

$$
X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(20)^{2}-(12)^{3}}=16 \Omega
$$

or

$$
2 \pi f L=16 \quad \text { or } \quad L=\frac{16}{100 \pi}=50.93 \mathrm{mH}
$$

$\therefore$ Inductance of the coil,

$$
L=50.93 \mathrm{mH}
$$

(ii) Power factor of the circuit, $\cos \phi=\frac{R}{Z}=\frac{12}{20}=0.6 \mathrm{lag}$
(iii) Impedance of the coil, $Z_{\mathrm{c}}=\sqrt{R_{\mathrm{c}}^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(2)^{2}+(16)^{2}}=16.124 \Omega$

Voltage across the coil, $V_{\mathrm{c}}=I Z_{\mathrm{c}}=10 \times 16.124=161.24 \mathrm{~V}$
Power factor of the coil, $\cos \phi_{\mathrm{c}}=\frac{R_{\mathrm{c}}}{Z_{\mathrm{c}}}=\frac{2}{16.124}=0.124$ lagging

## Example 7.9

An inductive load is connected in series with a non-inductive resistance of $8 \Omega$. The combination is connected across an AC supply of $100 \mathrm{~V}, 50 \mathrm{~Hz}$. A voltmeter connected across the
non-inductive resistor and then across the inductive load gives the reading of 64 V and 48 V , respectively. Calculate the following: (i) impedance of the load, (ii) impedance of the combination, (iii) power absorbed by the load, (iv) power absorbed by the resistor, (v) total power taken from the supply, (vi) power factor of the load, and (vii) power factor of the whole circuit.

## Solution:

The circuit diagram and phasor diagram for the circuit are shown in Figures 7.18 and 7.19, respectively.

Current in the circuit $=$ Current in $8 \Omega$ resistor

$$
I=\frac{V_{1}}{R_{1}}=\frac{64}{8}=8 \mathrm{~A}
$$



Fig. 7.18 Circuit as per data


Fig. 7.19 Phasor diagram
(i) Load impedance, $Z_{\mathrm{L}}=\frac{V_{\mathrm{L}}}{I}=\frac{48}{8}=6 \Omega$
(ii) Impedance of the combination, $Z=\frac{V}{I}=\frac{100}{8}=12.5 \Omega$

From the phasor diagram shown in Figure 7.19, we get

$$
(\mathrm{OB})^{2}=(\mathrm{OA})^{2}+(\mathrm{AB})^{2}+2(\mathrm{OA})(\mathrm{AB}) \cos \phi_{\mathrm{L}}
$$

$\therefore$ Power factor of the load,

$$
\cos \phi_{\mathrm{L}}=\frac{(\mathrm{OB})^{2}-(\mathrm{OA})^{2}-(\mathrm{AB})^{2}}{2(\mathrm{OA})(\mathrm{AB})}=\frac{(100)^{2}-(64)^{2}-(48)^{2}}{2 \times 64 \times 48}=0.586 \mathrm{lag}
$$

Load resistance, $R_{\mathrm{L}}=Z_{\mathrm{L}} \cos \phi_{\mathrm{L}}=6 \times 0.586=3.516 \Omega$
(iii) Power absorbed by the load, $P_{\mathrm{L}}=V_{\mathrm{L}} I \cos \phi_{\mathrm{L}}=48 \times 8 \times 0.586=225 \mathrm{~W}$
(iv) Power absorbed by the resistor $R_{1}, P_{\mathrm{R} 1}=I^{2} R_{1}=8 \times 8 \times 8=512 \mathrm{~W}$
(v) Total power taken from the supply, $P=P_{\mathrm{R} 1}+P_{\mathrm{L}}=512+225=737 \mathrm{~W}$
(vi) Power factor of the load, $\cos \phi_{\mathrm{L}}=0.586 \mathrm{lag}$
(vii) Power factor of the whole circuit, $\cos \phi=\frac{R_{1}+R_{\mathrm{L}}}{Z}=\frac{8+3.516}{12.5}=0.92$ lag

## Example 7.10

A coil of resistance $1.5 \Omega$ and impedance $6 \Omega$ is placed in series with a second coil of resistance $2 \Omega$. When a voltage of $230 \mathrm{~V}, 50 \mathrm{~Hz}$ is applied to the circuit, the current flowing through the circuit is 7 A . Find the inductance of the second coil.

## Solution:

The circuit is shown in Figure 7.20.


Fig. 7.20 Circuit as per data
Impedance of whole circuit,

$$
Z=V / I=230 / 7=32.86 \Omega
$$

Resistance of whole circuit,

$$
R=R_{1}+R_{2}=1.5+2=3.5 \Omega
$$

Inductive reactance,

$$
X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(32.86)^{2}-(3.5)^{2}}=32.67 \Omega
$$

Inductive reactance of coil I, $X_{\mathrm{L} 1}=\sqrt{Z_{1}^{2}-R_{1}^{2}}=\sqrt{(6)^{2}-(1.5)^{2}}=5.81 \Omega$
Inductive reactance of coil II, $X_{\mathrm{L} 2}=X_{1}-X_{\mathrm{L} 1}=32.67-5.81=26.86 \Omega$
$\therefore$ Inductance, $L_{2}=X_{\mathrm{L} 2} / 2 \pi f=26.86 / 2 \pi \times 50=85.5 \mathrm{mH}$

## Example 7.11

An arc lamp (which may be regarded as being non-inductive) takes 10 A at 50 V . Calculate the impedance of choke of $1 \Omega$ resistance to be placed in series with it in order that it may be operated at $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Find also the total power used and the power factor.

Solution:
Resistance of the arc lamp, $R_{1}=\frac{V_{1}}{I}$
where

$$
V_{1}=50 \mathrm{~V} \quad \text { and } \quad I=10 \mathrm{~A}
$$

$\therefore \quad R_{1}=\frac{50}{10}=5 \Omega$
Choke resistance, $R_{2}=1 \Omega$
When choke is connected, impedance of the whole circuit, $Z=\frac{V}{I}=\frac{200}{10}=20 \Omega$
Total resistance of the circuit, $R=R_{1}+R_{2}=5+1=6 \Omega$
Inductive reactance of the choke, $X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(20)^{2}-(6)^{2}}=19.08 \Omega$

Choke impedance, $Z_{2}=\sqrt{R_{\mathrm{L}}^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(1)^{2}-(19.08)^{2}}=19.105 \Omega$
Power factor of the circuit, $\cos \phi=\frac{R}{Z}=\frac{6}{20}=0.3 \mathrm{lag}$
Total power used, $P=V I \cos \phi=200 \times 10 \times 0.3=600 \mathrm{~W}$

## 园曺 PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by impedance of an $\mathrm{R}-\mathrm{L}$ series circuit?
2. Draw an impedance triangle for an $\mathrm{R}-\mathrm{L}$ series circuit and mention its sides.
3. Define true power, apparent power, and reactive power.
4. Draw power triangle for an $\mathrm{R}-\mathrm{L}$ series circuit.
5. Define power factor for an $\mathrm{R}-\mathrm{L}$ series circuit.
6. State the disadvantages of poor power factor.
7. What do you mean by $Q$-factor of a coil?

## Test Questions

1. Show that current lags behind the voltage vector in an R-L series circuit. Further, show that in $\mathrm{R}-\mathrm{L}$ series circuit, power $P=V I \cos \phi$ where $V$ and $I$ are the rms values of voltage and current, respectively.
2. Draw the phasor and wave diagram for voltage and current in an $\mathrm{R}-\mathrm{L}$ series circuit.
3. What do you mean by active and reactive component of AC ?
4. What is power factor? Give its significance.
5. Why it is suggested to the industries to install a power factor improvement equipment in their premises?

## Numericals

1. A coil having a resistance of $24 \Omega$ and an inductance of 0.2 H is connected across a $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate the (i) reactance and impedance of the coil, (ii) current, (iii) phase difference between the current and the applied voltage, and (iv) power factor. Draw also the phasor diagram showing voltage and current.
(Ans. $\left.67.26 \Omega, 2.97 \mathrm{~A}, 69.1^{\circ}, 0.3568 \mathrm{lag}\right)$
2. The voltage and current through a circuit element are

$$
\begin{aligned}
& v=120 \sin \left(314 t+45^{\circ}\right) \mathrm{V} \\
& i=12 \sin \left(314 t+315^{\circ}\right) \mathrm{A}
\end{aligned}
$$

Find the value of power drawn by the element.
(Ans. 0) (U.P.T.U. Type)
3. A coil connected to 120 V DC supply draws 12 A and the same coil when connected to $100 \mathrm{~V}, \mathrm{AC}$ voltage of frequency 50 Hz draws 6 A . Calculate the parameters of the coil and power factor.
(Ans. $10 \Omega, 45.45 \mathrm{mH}, 0.6$ lag) (U.P.T.U. Type)
4. A voltage $e=250 \sin 100 \pi t$ is applied to a coil having $R=200 \Omega$ and $L=638 \mathrm{mH}$. Find the expression for the current and also determine the power taken by the coil.
(Ans. $0.8829 \sin \left(100 \pi t-45.06^{\circ}\right), 78 \mathrm{~W}$ )
5. A non-inductive resistance of $10 \Omega$ is connected in series with an inductive coil across 240 V , 50 Hz AC supply. The current drawn by the series combination is 12 A . The resistance of the coil is $2 \Omega$. Determine (i) inductance of the coil (ii) power factor and (iii) voltage across the coil.
(Ans. $50.93 \mathrm{mH}, 0.6$ lag, 193.5 V ) (U.P.T.U. Type)
6. A coil when connected to 250 V DC supply dissipates $3,125 \mathrm{~W}$ of power. When connected across 250 V AC supply of frequency 50 c.p.s., it dissipates $1,250 \mathrm{~W}$ of power. Calculate the value of resistance and inductance of the coil.
(Ans. $20 \Omega, 0.078 \mathrm{H}$ )
7. An inductive load is connected in series with a non-inductive resistance of $16 \Omega$. The combination is connected across an AC supply of $200 \mathrm{~V}, 50 \mathrm{~Hz}$. A voltmeter connected across the non-inductive resistor and then across the inductive load gives the reading of 128 V and 96 V , respectively. Calculate the following:
(i) impedance of the load, (ii) impedance of the combination, (iii) power absorbed by the load, (iv) power absorbed by the resistor, (v) total power taken from the supply, (vi) power factor of the load, and (vii) power factor of the whole circuit.
(Ans. $12 \Omega, 25 \Omega, 450 \mathrm{~W}, 1,024 \mathrm{~W}, 1,474 \mathrm{~W}, 0.586$ lag 0.9213 lag )
8. A coil of resistance $6 \Omega$ and impedance $10 \Omega$ is placed in series with a second coil of resistance $9 \Omega$. When a voltage of $250 \mathrm{~V}, 50 \mathrm{~Hz}$ is applied to the circuit, the current flowing through the circuit is 10 A . Find the inductance of the second coil.
(Ans. 38.2 mH )

## 7.II R-C SERIES CIRCUIT

A circuit that contains a pure resistance $R \Omega$ connected in series with a pure capacitor of capacitance $C$ Farad is known as $\mathrm{R}-\mathrm{C}$ series circuit.

An R-C series circuit and its phasor diagram is shown in Figures 7.21 and 7.22, respectively. To draw the phasor diagram, current $I$ (rms value) is taken as the reference vector. Voltage drop in resistance $V_{\mathrm{R}}(=I R)$ is taken in phase with current vector, whereas voltage drop in capacitive reactance $V_{\mathrm{C}}\left(=I X_{\mathrm{C}}\right)$ is taken $90^{\circ}$ behind the current vector (since current leads the voltage by $90^{\circ}$ in pure capacitive circuit). The vector sum of these two voltage drops is equal to the applied voltage $V$ (rms value).


Fig. 7.21 Circuit containing resistance and capacitance in series


Fig. 7.22 Phasor diagram

Now,

$$
V_{\mathrm{R}}=I R \text { and } V_{\mathrm{C}}=I X_{\mathrm{C}}\left(\text { where } X_{\mathrm{C}}=1 / 2 \pi f C\right)
$$

In right-angled triangle OAB
or

$$
\begin{gathered}
V=\sqrt{\left(V_{\mathrm{R}}\right)^{2}+\left(V_{\mathrm{C}}\right)^{2}}=\sqrt{(I R)^{2}+\left(I X_{\mathrm{C}}\right)^{2}}=\sqrt{R^{2}+X_{\mathrm{C}}^{2}} \\
I=\frac{V}{\sqrt{R^{2}+X_{\mathrm{C}}^{2}}}=\frac{V}{Z}
\end{gathered}
$$

where $Z=\sqrt{R^{2}+X_{\mathrm{C}}^{2}}$ is the total opposition offered to the flow of AC by an $\mathrm{R}-\mathrm{C}$ series circuit and is called impedance of the circuit. It is measured in ohm.

### 7.11.1 Phase Angle

From the phasor diagram, it is clear that current in this circuit leads the applied voltage by an angle $\phi$ called phase angle.

From the phasor diagram shown in Figure 7.22,

$$
\tan \phi=\frac{V_{\mathrm{C}}}{V_{\mathrm{R}}}=\frac{I X_{\mathrm{C}}}{I R}=\frac{X_{\mathrm{C}}}{R} \quad \text { or } \quad \phi=\tan ^{-1} X_{\mathrm{C}} / R
$$

### 7.11.2 Power

If the alternating voltage applied across the circuit is given by the equation:

$$
\begin{align*}
v & =V_{\mathrm{m}} \sin \omega t  \tag{7.13}\\
i & =I_{\mathrm{m}} \sin (\omega t+\phi) \tag{7.14}
\end{align*}
$$

Then,
$\therefore$ Instantaneous power,

$$
\begin{aligned}
p & =v i=V_{\mathrm{m}} \sin \omega t I_{\mathrm{m}} \sin (\omega t+\phi)=\frac{V_{\mathrm{m}} I_{\mathrm{m}}}{2} 2 \sin (\omega t+\phi) \sin \omega t \\
& =\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}}[\cos \phi-\cos (2 \omega t+\phi)]=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi+\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t+\phi)
\end{aligned}
$$

Average power consumed in the circuit over a complete cycle,

$$
\begin{aligned}
& P=\text { average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\text { average of } \frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t+\phi) \\
& P=\frac{V_{\mathrm{m}}}{\sqrt{2}} \cdot \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi=V_{\mathrm{rms}} I_{\mathrm{rms}} \cos \phi=V I \cos \phi
\end{aligned}
$$

where $\cos \phi$ is called power factor of the circuit.
From phasor diagram,

$$
\cos \phi=\frac{V_{\mathrm{R}}}{V}=\frac{I R}{I Z}=\frac{R}{Z} \text { same as in } \mathrm{R}-\mathrm{L} \text { series circuit }
$$

Alternatively, power

$$
P=V I \cos \phi=I Z \cdot I \cdot \frac{R}{Z}=I^{2} R
$$

This shows that power is actually consumed in resistance only; capacitor does not consume any power.

### 7.11.3 Power Curve

The phasor diagram and wave diagram for voltage and current are shown in Figure 7.23(a) and 7.23(b), respectively, where applied voltage ( $v=V_{\mathrm{m}} \sin \omega t$ ) is taken as reference quantity. The power curve for $\mathrm{R}-\mathrm{C}$ circuit is also shown in 7.23 (b). The points on the power curve are obtained from the product of the corresponding instantaneous values of voltage and current. It is clear that power is negative between angle $\left(180^{\circ}-\phi\right)$ and $180^{\circ}$ and between $\left(360^{\circ}-\phi\right)$ and $360^{\circ}$. During rest of the cycle, the power is positive. Since the area under the positive loops is greater than
that under the negative loops, the net power over a complete cycle is positive. Hence, a definite quantity of power is utilised or consumed by this circuit.


Fig. 7.23 (a) Phasor diagram (b) Wave diagram for voltage, current and power

### 7.11.4 Impedance Triangle

When each side of the simplified phasor diagram shown in Figure 7.24 is divided by a common factor $I$, we get another right-angled triangle (shown in Fig. 7.25) known as impedance triangle.


Fig. 7.24 Phasor diagram for R-C series circuit


Fig. 7.25 Impedance triangle

## Example 7.12

A resistance of $15 \Omega$ and capacitor of $150 \mu \mathrm{~F}$ capacitance are connected in series across a 230 V, 50 Hz supply. Calculate (i) impedance of the circuit, (ii) current, (iii) power factor and phase angle, and (iv) power consumed in the circuit.

## Solution:

The circuit is shown in Figure 7.26.


Fig. 7.26 Circuit as per data

Impedance, $\quad Z=\sqrt{R^{2}+X_{\mathrm{C}}^{2}}$
where $\quad X_{\mathrm{C}}=1 / 2 \pi \phi C=1 / 2 \pi \times 50 \times 150 \times 10^{-6}$

$$
=21.22 \Omega
$$

$$
R=15 \Omega
$$

(i) $\therefore Z=\sqrt{(15)^{2}+(21.22)^{2}}=25.987 \Omega$
(ii) Current, $I=\frac{V}{Z}=\frac{230}{25.987}=8.85 \mathrm{~A}$
(iii) Power factor, $\cos \phi=\frac{R}{Z}=\frac{15}{25.987}=0.577$ leading

Phase angle, $\phi=\cos ^{-1} 0.577=54.75^{\circ}$
(iv) Power, $P=V I \cos \phi=230 \times 8.85 \times 0.577=1,174.9 \mathrm{~W}$

## Example 7.13

A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a condenser. The current in the circuit is 2.2 A . The power loss in the resistor is 96.8 W and that in the condenser is negligible. Calculate the resistance and the capacitance.

## Solution:

Applied voltage, $V=125 \mathrm{~V}$; supply frequency, $f=50 \mathrm{~Hz}$
Current in the circuit, $I=2.2 \mathrm{~A}$; power loss in the resistor, $P=96.8 \mathrm{~W}$
Power loss in resistor, $P=I^{2} R$
$\therefore$ Circuit resistance,

$$
R=\frac{P}{I^{2}}=\frac{96.8}{2.2 \times 2.2}=20 \Omega
$$

Circuit impedance,

$$
Z=\frac{V}{V}=\frac{125}{2.2}=56.82 \Omega
$$

Capacitive reactance,

$$
X_{\mathrm{C}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(56.82)^{2}-(20)^{2}}=53.18 \Omega
$$

Capacitance of capacitor, $\quad C=\frac{1}{2 \pi f X_{\mathrm{C}}}=\frac{1}{2 \pi \times 50 \times 53.18}=59.85 \mu \mathrm{~F}$

## Example 7.14

A $120 \mathrm{~V}, 60 \mathrm{~W}$ lamp is to be operated on $220 \mathrm{~V}, 50 \mathrm{~Hz}$ supply mains. For the lamp to operate in correct voltage, calculate the value of (i) non-inductive resistance and (ii) pure inductance
(U.P.T.U. 2005-06)

## Solution:

Lamp's rating: $120 \mathrm{~V}, 60 \mathrm{~W}$
Supply voltage, $V_{\mathrm{S}}=220 \mathrm{~V}$ and Frequency, $f=50 \mathrm{~Hz}$
Resistance of the lamp,

$$
R_{\mathrm{L}}=\frac{(120)^{2}}{60}=240 \Omega
$$

Operating current,

$$
I=\frac{60}{120}=0.5 \mathrm{~A}
$$



Fig. 7.27 (a) Circuit as per data (b) Circuit as per data
(i) For operating the lamp using non-inductive resistance, as shown in Figure 7.27(a). Let the value of resistance be $R \Omega$.

$$
\begin{array}{lrl}
\therefore & I\left(R+R_{\mathrm{L}}\right) & =V \\
\text { or } & R+R_{\mathrm{L}} & =\frac{220}{0.5}=440 \\
\text { or } & R & =440-R_{\mathrm{L}}=440-240=200 \Omega
\end{array}
$$

(ii) For operating the lamp using pure inductance as shown in Figure 7.27(b). Let the value of inductance be $L$ heavy and $X_{\mathrm{L}}=2 \pi f L$,

$$
\begin{array}{ll}
\therefore & I Z=V \quad \text { or } \quad Z=\frac{V}{I}=\frac{220}{0.5}=440 \Omega \\
\text { or } & \sqrt{R_{\mathrm{L}}^{2}+X_{\mathrm{L}}^{2}}=440 \\
R_{\mathrm{L}}^{2}+X_{\mathrm{L}}^{2}=(440)^{2} & \text { or } X_{\mathrm{L}}^{2}=440^{2}-240^{2} \\
\text { or } & X_{\mathrm{L}}=\sqrt{440^{2}-240^{2}}=368.78 \Omega \\
\text { or } & 2 \pi f L=368.78 \text { or } L=\frac{368.78}{2 \pi \times 50}=1.174 \mathrm{H}
\end{array}
$$

### 7.12 R-L-C SERIES CIRCUIT

A circuit that contains a pure resistance of $R$ $\Omega$, a pure inductance of $L$ Henry, and a pure capacitor of capacitance $C$ Farad; all connected in series is known as $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit.

An $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit is shown in Figure 7.28.

Here, $X_{\mathrm{L}}=2 \pi f L$ and $X_{\mathrm{C}}=1 / 2 \pi f C$


Fig. 7.28 Circuit containing resistance, inductance and capacitance in series


Fig. 7.29 Phasor diagram

When a resulting current $I$ (rms value) flows through the circuit, the voltage across each component will be
$V_{\mathrm{R}}=I R$, that is, voltage across $R \ldots . . . . . .$. in phase with $I$;
$V_{\mathrm{L}}=I X_{\mathrm{L}}$, that is, voltage across $L . . . . . . . . .$. leads $I$ by $90^{\circ}$;
$V_{\mathrm{C}}=I X_{\mathrm{C}}$, that is, voltage across C.......... lags $I$ by $90^{\circ}$; The phasor diagram is shown in Figure 7.29, where current is taken as the reference phasor. Since voltage across inductance $V_{\mathrm{L}}$ leads the current vector $I$ by $90^{\circ}$ and voltage across capacitance $V_{\mathrm{C}}$ lags the current vector $I$ by $90^{\circ}$, they act opposite to each other. If $V_{\mathrm{L}}>V_{\mathrm{C}}$, in effect, the circuit behaves as an inductive circuit; however, when $V_{\mathrm{L}}<V_{\mathrm{C}}$, the circuit behaves as a capacitive circuit. Here, the phasor diagram is drawn for an inductive circuit (i.e., when $V_{\mathrm{L}}>V_{\mathrm{C}}$ ).

$$
V=\sqrt{\left(V_{\mathrm{R}}\right)^{2}+\left(V_{\mathrm{L}}-V_{\mathrm{C}}\right)^{2}}=\sqrt{(I R)^{2}+\left(I X_{\mathrm{L}}-I X_{\mathrm{C}}\right)^{2}}
$$

or

$$
V=I \sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}
$$

or

$$
I=\frac{V}{\sqrt{(R)^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}}=\frac{V}{Z}
$$

where $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}$ is the total opposition offered to the flow of AC by an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit and is called impedance of the circuit.

### 7.12.1 Phase Angle

From phasor diagram: $\tan \phi=\frac{V_{\mathrm{L}}-V_{\mathrm{C}}}{V_{\mathrm{R}}}=\frac{X_{\mathrm{L}}-X_{\mathrm{C}}}{R}$
or

$$
\phi=\tan ^{-1} \frac{X_{\mathrm{L}}-X_{\mathrm{C}}}{R}
$$

### 7.12.2 Power

Average power, $P=V I \cos \phi=I^{2} R$
Power factor, $\cos \phi=\frac{V_{\mathrm{R}}}{V}=\frac{R}{Z}$
The alternating voltage applied across the circuit is given by

$$
v=V_{\mathrm{m}} \sin \omega t
$$

Therefore, the circuit current is represented by the equation as per the constants or parameters, as explained in the following three cases of $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit:

1. When $X_{\mathrm{L}}>X_{\mathrm{C}}$, the phase angle $\phi$ is positive. In effect, the circuit behaves as an $\mathrm{R}-\mathrm{L}$ series circuit. The circuit current lags behind the applied voltage and pf is lagging. The current is given by the equation.

$$
i=I_{\mathrm{m}} \sin (\omega t-\phi)
$$

2. When $X_{\mathrm{L}}<X_{\mathrm{C}}$, the phase angle $\phi$ is negative. In effect, the circuit behaves as an $\mathrm{R}-\mathrm{C}$ series circuit. The circuit current leads the applied voltage and pf is leading. The current is given by the equation.

$$
i=I_{\mathrm{m}} \sin (\omega t+\phi)
$$

3. When $X_{\mathrm{L}}=X_{\mathrm{C}}$, the phase angle $\phi$ is zero. In effect, the circuit behaves like a pure resistive circuit. The circuit current is in phase with applied voltage and pf is unity. The current is given by the equation.

$$
i=I_{\mathrm{m}} \sin \omega t
$$

### 7.12.3 Impedance Triangle

Figure 7.30(a) shows the impedance triangle of the circuit when $X_{\mathrm{L}}>X_{\mathrm{C}}$, while Figure 7.30(b) shows the impedance triangle of the circuit when $X_{\mathrm{L}}<X_{\mathrm{C}}$.


Fig. 7.30 (a) Impedance triangle $\left(X_{L}>X_{c}\right)$ (b) Impedance triangle $\left(X_{c}>X_{L}\right)$

### 7.13 SERIES RESONANCE

In an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit, when circuit current is in phase with the applied voltage, the circuit is said to be in series resonance. This condition is obtained in an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ circuit shown in Figure 7.31,
when

$$
X_{\mathrm{L}}=X_{\mathrm{C}}\left(\text { or } X_{\mathrm{L}}-X_{\mathrm{C}}=0\right)
$$

At resonance, $\quad X_{\mathrm{L}}-X_{\mathrm{C}}=0$ or $X_{\mathrm{L}}=X_{\mathrm{C}}$
Impedance, $Z_{\mathrm{r}}=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}=R$

Current,

$$
I_{\mathrm{r}}=\frac{V}{Z_{\mathrm{r}}}=\frac{V}{R}
$$



Fig. 7.31 R-L-C series circuit

Since at resonance, the opposition to the flow of current is only resistance $(R)$ of the circuit, the circuit draws maximum current under this condition.

### 7.13.1 Resonant Frequency



Fig. 7.32 Phasor diagram for series resonant circuit

The value of $X_{\mathrm{L}}(=2 \pi f L)$ and $X C(=1 / 2 \pi f C)$ can be changed by changing the supply frequency. When frequency increases, the value of $X_{\mathrm{L}}$ increases, whereas the value of $X_{\mathrm{C}}$ decreases, and vice versa. Therefore, to obtain series resonance, the frequency is adjusted to $f_{\mathrm{r}}$, so that $X_{\mathrm{L}}=X_{\mathrm{C}}$, the condition at point $P$ shown in Figure 7.32.

At series resonance, $X_{\mathrm{L}}=X_{\mathrm{C}}$

$$
2 \phi f_{\mathrm{r}} L=\frac{1}{2 \pi f_{\mathrm{r}} C} \quad \text { or } \quad f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}
$$

where $f_{\mathrm{r}}$ is the resonant frequency in Hz when $L$ and $C$ are measured in Henry and Farad, respectively.

### 7.13.2 Effects of Series Resonance

The following are the main effects of series resonance:
(i) At resonance $X_{\mathrm{L}}=X_{\mathrm{C}}$, and therefore, the impedance of the circuit is minimum and is reduced to the resistance of the circuit only, that is,

$$
Z_{\mathrm{r}}=R
$$

(ii) Since, impedance is minimum, the circuit current is maximum at resonance, that is

$$
I_{\mathrm{r}}=V / Z_{\mathrm{r}}=V / R
$$

(iii) Power taken by the circuit is maximum, as $I_{\mathrm{r}}$ is maximum,

$$
P_{r}=I_{r}^{2} R
$$

(iv) As the current drawn by the circuit, at resonance, is very large (maximum), the voltage drop across $L$ (i.e., $V_{\mathrm{L}}=I X_{\mathrm{L}}=I \times 2 \pi f_{\mathrm{r}} L$ ) and (i.e., $V_{\mathrm{C}}=I X_{\mathrm{C}}=I \times 1 / 2 \pi f_{\mathrm{r}} C$ ) are also very large. In power system, at resonance, the excessive voltage built up across the inductive and capacitive components (such as circuit breakers and reactors) may cause damage. Therefore, series resonance should be avoided in power system. However, in some of the electronic devices (such as antenna circuit of radio and TV receiver and tuning circuits), the principle of series resonance is used to increase the signal voltage and current at a desired frequency $\left(f_{\mathrm{r}}\right)$. Since a series resonant circuit has the capability to draw heavy current and power from the mains, it is often regarded as acceptor circuit.

### 7.14 RESONANCE CURVE

The curve obtained by plotting a graph between the current and the frequency is known as resonance curve. A resonance curve of a typical R-L-C series circuit is shown in Figure 7.33.

It may be noted that current reaches its maximum value at the resonant frequency $\left(f_{\mathrm{r}}\right)$, falling off rapidly on either side of that point. It is because when the value of frequency is lower than resonance frequency, $X_{\mathrm{C}}>X_{\mathrm{L}}$ and when the value of frequency is higher than $f_{\mathrm{r}}, X_{\mathrm{C}}<X_{\mathrm{L}}$. In both the cases, impedance of the circuit increases $\left(Z>Z_{\mathrm{r}}\right)$ and the value of current decreases.

Note that resistance of the circuit also plays its own role. The smaller the resistance, the greater is the current at resonance.

### 7.14.1 Bandwidth

The range of frequency over which circuit current is equal to or more than $70.7 \%$ of maximum value (i.e., $I_{\mathrm{r}}$, current at resonance) is known as the bandwidth of a series resonant circuit.

Figure 7.34 shows a resonance curve of a typical $\mathrm{R}-\mathrm{L}-\mathrm{C}$ circuit where the circuit current is equal to or greater than $70.7 \%$ of maximum current (i.e., $I_{\mathrm{r}}=V / R$ ) between frequency range $f_{1}$ to $f_{2}$.

Bandwidth,

$$
B W=f_{2}-f_{1}
$$

Here, the frequency $f_{1}$ is called the lower cutoff frequency and the frequency $f_{2}$ is called the upper cut-off frequency. The bandwidth represents the frequency range at which the circuit offers low impedance to circuit current.

The following points may be noted here:

1. If the resonant frequency is not located at the centre of upper and lower cut-off fre-


Fig. 7.34 Graph representing band width in R-L-C series circuit quency, then

$$
F_{\mathrm{r}}=\sqrt{f_{1} f_{2}}
$$

2. When the resonant frequency is located sufficiently near to the centre of the two cut-off frequencies and $Q$ of the circuit is $\geq 10$, then

$$
f_{1}=f_{\mathrm{r}}-\frac{B W}{2} \quad \text { and } \quad f_{2}=f_{\mathrm{r}}+\frac{B W}{2}
$$

### 7.14.2 Selectivity

From the resonance curve, it is clear that for smaller resistance the resonance curve is sharp and flat for the larger resistance. A sharper resonance curve provides smaller band of frequencies to give reasonable response, and hence, provides better selectivity. It also shows that selectivity is reciprocal of bandwidth.

### 7.15 Q-FACTOR OF SERIES RESONANT CIRCUIT

We have seen that at series resonance, the circuit draws the largest current from the mains, this produces a heavy voltage across $L$ or $C$. The factor by which the potential difference across $L$ or $C$ increases to that of the applied voltage is called the $Q$-factor of the series resonant circuit.
where

$$
\begin{aligned}
Q \text {-factor } & =\frac{\text { Voltage across } L \text { or } C}{\text { Applied voltage }}=\frac{I_{\mathrm{r}} X_{\mathrm{L}}}{I_{\mathrm{R}}}=\frac{X_{\mathrm{L}}}{R}=\frac{\omega_{\mathrm{r}} L}{R} \\
\omega_{\mathrm{r}} & =2 \pi f_{\mathrm{r}}=2 \pi \frac{1}{2 \pi \sqrt{L C}}=\frac{1}{\sqrt{L C}} \\
Q \text {-factor } & =\frac{L}{R} \times \frac{1}{\sqrt{L C}}=\frac{1}{R} \sqrt{\frac{L}{C}}
\end{aligned}
$$

The value of $Q$-factor depends entirely upon the design of coil (i.e., $\mathrm{R}-\mathrm{L}$, which is a part of R-L-C circuit).

## Example 7.15

A coil resistance $10 \mu$ and inductance 0.14 H is connected in series with a capacitor of $150 \mu \mathrm{~F}$ across a $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate (i) inductive reactance, (ii) capacitive reactance, (iii) impedance, (iv) current, and (v) voltage across coil and capacitor.
(U.P.T.U. Tut.)

## Solution:

The circuit is shown in Figure 7.35.
Let us consider $R=10 \Omega ; L=0.14 \mathrm{H} ; C=150 \mu \mathrm{~F}=150 \times 10^{-6} \mathrm{~F} ; V=200 \mathrm{~V} ; f=50 \mathrm{~Hz}$


Fig. 7.35 Circuit as per data
(i) Inductive reactance,

$$
X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.14=44 \Omega
$$

(ii) Capacitive reactance,

$$
\begin{aligned}
X_{\mathrm{C}} & =\frac{1}{2 \pi f C} \\
& =\frac{1}{2 \pi \times 50 \times 150 \times 10^{-6}}=21.22 \Omega
\end{aligned}
$$

(iii) Impedance, $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}=\sqrt{10^{2}+(44-21.22)^{2}}$

$$
=24.88 \Omega
$$

(iv) Current, $I=\frac{V}{Z}=\frac{200}{24.88}=8.04 \mathrm{~A}$
(v) Voltage across coil, $V_{1}=I Z_{\text {Coil }}=I \sqrt{R^{2}+X_{\mathrm{L}}^{2}}$

$$
=8.04 \sqrt{10^{2}+44^{2}}=362.8 \mathrm{~V}
$$

Voltage across capacitor, $V_{2}=I X_{\mathrm{C}}=8.04 \times 21.22=170.6 \mathrm{~V}$

## Example 7.16

A coil of resistance $8 \Omega$ and inductance 0.12 H is connected in series with a condenser of capacitance $140 \mu \mathrm{~F}$ across a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Determine
(i) impedance of the entire circuit, (ii) current flowing through the condenser, (iii) power factor of the circuit, and (iv) voltage across the condenser.
(U.P.T.U., 2002-03)

## Solution:

The circuit is shown in Figure 7.36
Resistance of the coil, $R=8 \Omega$
Inductive reactance of the coil,

$$
X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.12=37.7 \Omega
$$

Capacitive reactance of the condenser,

$$
X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 140 \times 10^{-6}}=22.74 \Omega
$$



Fig. 7.36 Circuit as per data
(i) Impedance of the whole circuit, $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}=\sqrt{8^{2}+(37.7-22.74)^{2}}$

$$
=17 \Omega
$$

(ii) Current flowing through the condenser,

$$
I_{\mathrm{C}}=\text { Circuit current, } I=\frac{V}{Z}=\frac{230}{17}=13.53 \mathrm{~A}
$$

(iii) Power factor of the circuit, $\cos \phi=\frac{R}{Z}=\frac{8}{17}=0.471$ (lagging)

$$
X_{\mathrm{L}}>X_{\mathrm{C}}
$$

(iv) Voltage across condenser, $V_{\mathrm{C}}=I_{\mathrm{C}} X_{\mathrm{C}}=13.53 \times 22.74=307.7 \mathrm{~V}$

## Example 7.17

Find applied voltage and power loss in the circuit shown in Figure 7.37.
(U.P.T.U. 2007-08)

## Solution:

Here, $C=20 \mu \mathrm{~F} ; I=0.345 \mathrm{~A}$
Applied voltage,

$$
V=\sqrt{25^{2}+(40-55)^{2}}=29.155 \mathrm{~V}
$$

Power loss, $P=V_{\mathrm{R}} \times I=25 \times 0.345=8.625 \Omega$


Fig. 7.37 Given circuit

## Example 7.18

A coil of resistance $12 \Omega$ and inductance 0.05 H , a non-inductive resistance of $20 \Omega$ resistance and a loss-free $40 \mu \mathrm{~F}$ capacitor are connected across a $240 \mathrm{~V}, 50 \mathrm{~Hz}$ sinusoidal supply. Calculate (i) the current and (ii) the power factor of the circuit.
(U.P.T.U., Sep. 2001)

## Solution:

The circuit is shown in Figure 7.38.


Fig. 7.38 Circuit as per data

Here, $\quad R_{1}=12 \Omega ; L=0.05 \mathrm{H} ; R_{2}=20 \Omega$;

$$
C=40 \mu \mathrm{~F}=40 \times 10^{-6} \mathrm{~F} ; V=240 \mathrm{~V} ; f=50 \mathrm{~Hz}
$$

Resistance of the whole circuit, $R=R_{1}+R_{2}$

$$
=12+20=32 \Omega
$$

Inductive reactance, $X_{\mathrm{L}}=2 \pi f L$

$$
\begin{aligned}
& =2 \pi \times 50 \times 0.05 \\
& =15.7 \Omega
\end{aligned}
$$

Capacitive reactance, $\quad X_{\mathrm{C}}=\frac{1}{2 \pi f C}$

$$
=\frac{1}{2 \pi \times 50 \times 40 \times 10^{-6}}=79.58 \Omega
$$

Impedance,

$$
\begin{aligned}
Z & =\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}} \\
& =\sqrt{32^{2}+(15.7-79.58)^{2}}=71.45 \Omega
\end{aligned}
$$

(i) Circuit current, $I=\frac{V}{Z}=\frac{240}{71.45}=3.36 \mathrm{~A}$
(ii) Power factor of the circuit, $\cos \phi=\frac{R}{Z}=\frac{32}{71.45}=0.448$ (leading)

## Example 7.19

A series R-L-C circuit has $100 \Omega$ resistor, 0.318 H inductance and $\mathrm{C}, v=230 \times \sqrt{2} \sin \omega t \mathrm{~V}$, $i=2.3 \times \sqrt{2} \sin \omega t \mathrm{~A}$, find (i) $C$, (ii) $V_{\mathrm{L}}$, (iii) Power taken $\omega=314.15 \mathrm{rad} / \mathrm{s}$.
(U.P.T.U. 2007-08)

## Solution:

Here,

$$
R=100 \Omega ; L=0.318 \mathrm{H}
$$

$$
\begin{aligned}
& v=230 \sqrt{2} \sin \omega t ; \quad i=23 \sqrt{2} \sin \omega t ; \quad \omega=314.15 \\
& \therefore \quad V=\frac{V_{\mathrm{m}}}{\sqrt{2}}=\frac{230 \sqrt{2}}{\sqrt{2}}=230 \mathrm{~V} \\
& I=\frac{I_{\mathrm{m}}}{\sqrt{2}}=\frac{2.3 \sqrt{2}}{\sqrt{2}}=2.3 \mathrm{~A} \\
& \omega=2 \pi f \\
& \text { or } \\
& f=\frac{\omega}{2 \pi}=\frac{314.15}{2 \pi}=50 \mathrm{~Hz}
\end{aligned}
$$

Total Impedance of the circuit,

$$
Z=\frac{V}{I}=\frac{230}{2.3}=100 \Omega
$$

The $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit is shown in Figure 7.39
From the equation of $v$ and $i$, it is clear that current is in phase with voltage vector.

$$
\therefore \quad Z=R=100 \Omega
$$

and $\quad X_{\mathrm{L}}=X_{\mathrm{C}} \quad$ or $\quad 2 \pi f L=\frac{1}{2 \pi f C}$
or


Fig. 7.39 Circuit diagram

$$
C=\frac{1}{(2 \pi f)^{2} \times L}=\frac{1}{(314.5)^{2} \times 0.318}=31.86 \mu \mathrm{~F}
$$

Voltage across inductor, $V_{\mathrm{L}}=I X_{\mathrm{L}}=2.3 \times 314.15 \times 0.318=230 \mathrm{~V}$
Real power, $P=I^{2} R=(2.3)^{2} \times 100=529 \mathrm{~W}$

## Example 7.20

A load having impedance of $(1+j 1) \Omega$ is connected to an AC voltage represented as $v=20 \sqrt{2}$ $\cos \left(\omega t+10^{\circ}\right) \mathrm{V}$
(i) Find the current in load expressed in the form of $i=I_{\mathrm{m}} \sin (\omega t+\phi)$ A.
(ii) Find the real power consumed by the load.
(U.P.T.U. 2004-05)

## Solution:

The phasor diagram is shown in Figure 7.40.
Here, impedance,

$$
\begin{gathered}
\bar{Z}=(1+j 1) \Omega=\sqrt{2} \angle 45^{\circ} \Omega \\
v=20 \sqrt{2} \cos \left(\omega t+10^{\circ}\right)=20 \sqrt{2} \sin \left(\pi / 2-\left(\omega t+10^{\circ}\right)\right) \\
=-20 \sqrt{2} \sin \left(\omega t-90^{\circ}+10^{\circ}\right)=-20 \sqrt{2} \sin \left(\omega t-80^{\circ}\right)
\end{gathered}
$$

$$
\bar{V}=-20 \sqrt{2} \angle-80^{\circ}(\text { in polar form })
$$

(i) $\bar{I}=\frac{\bar{V}}{\bar{Z}}=\frac{-20 \sqrt{2} \angle-80^{\circ}}{\sqrt{2} \angle 45^{\circ}}=-20 \angle-125^{\circ}$

$$
\therefore \quad i=-20 \sin \left(\omega t-125^{\circ}\right)
$$

(ii) $V_{\mathrm{rms}}=V=\frac{20 \sqrt{2}}{\sqrt{2}} 20 \mathrm{~V}$
and $\quad I_{\mathrm{rms}}=I=\frac{20}{\sqrt{2}} \mathrm{~A}$
Phase angle, $\phi=\left(-80+125^{\circ}\right)=45^{\circ}$
$\therefore$ Real power consumed by load,

$$
\begin{aligned}
P & =V I \cos \phi \\
& =20 \times \frac{20}{\sqrt{2}} \times \cos 45^{\circ}=200 \mathrm{~W} .
\end{aligned}
$$



Fig. 7.40 Phasor diagram as per data

## Example 7.21

A series R-L-C circuit consisting of a resistance of $20 \Omega$, inductance 0.2 H and capacitance of $150 \mu \mathrm{~F}$ is connected across a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ source. Calculate (i) the impedance, (ii) the current, (iii) the magnitude and nature of the power factor, and (iv) the frequency of supply to be adjusted to make power factor unity.
(U.P.T.U., Feb. 2002)


Fig. 7.41 Circuit as per data

## Solution:

The circuit is shown in Figure 7.41
Here, $R=20 \Omega ; L=0.2 \mathrm{H} ; C=150 \mu \mathrm{~F}=150 \times 10^{-6} \mathrm{~F}$
Inductive reactance, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.2$

$$
=62.83 \Omega
$$

Capacitance reactance,

$$
X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 150 \times 10^{-6}}=21.22 \Omega
$$

(i) Impedance, $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}$

$$
\begin{aligned}
& =\sqrt{20^{2}+(62.83-21.22)^{2}} \\
& =46.17 \Omega
\end{aligned}
$$

(ii) Circuit current, $I=\frac{V}{Z}=\frac{230}{46.17}=4.98 \mathrm{~A}$
(iii) Power factor $=\cos \phi=\frac{R}{Z}=\frac{20}{46.167}=0.433$

The power factor is lagging because inductive reactance is more than capacitive reactance, that is, the circuit behaves as an inductive circuit. Power factor will be unity when
or

$$
\begin{gathered}
X_{\mathrm{L}}=X_{\mathrm{C}} \quad \text { or } \quad 2 \pi f_{\mathrm{r}} L=\frac{1}{2 \pi f_{\mathrm{r}} C} \\
f_{\mathrm{r}}=\frac{1}{2 \pi} \frac{1}{\sqrt{L C}}=\frac{1}{2 \pi \sqrt{0.2 \times 150 \times 10^{-6}}}=29.06 \mathrm{~Hz}
\end{gathered}
$$

## Example 7.22

A coil of pf 0.8 is in series with a $200 \mu \mathrm{~F}$ capacitor. When connected to a 50 Hz supply, the voltage across the capacitor is equal to the voltage across the coil. Find the resistance and inductance of the coil.

## Solution:

The circuit is shown in Figure 7.42.
Power factor of the coil, $\cos \phi=0.8$
$\therefore \quad \tan \phi=\tan \cos ^{-1} 0.8=0.75$
For a coil,

$$
\tan \phi=\frac{\omega L}{R}
$$

or

$$
R=\omega L / 0.75
$$

Impedance of coil,

$$
Z_{1}=\sqrt{R^{2}+(\omega L)^{2}}
$$

Impedance of capacitor,

$$
Z_{2}=1 / \omega C
$$

Given that voltage across the coil $=$ voltage across the capacitor
i.e., $\quad I Z_{1}=I Z_{2}$ or $Z_{1}=Z_{2}$
or $\quad R^{2}+(\mathrm{w} L)^{2}=(1 / \omega C)^{2}$
or
$\left(\frac{\omega L}{0.75}\right)^{2}+(\omega L)^{2}=\frac{1}{(\omega C)^{2}} \quad$ or $\quad \frac{5}{3} \omega L=\frac{1}{\omega C}$


Fig. 7.42 Circuit as per data
or $L=\frac{3}{5 \omega^{2} C}=\frac{3}{5(2 \pi \times 50)^{2} \times 100 \times 10^{-6}}=0.0608 \mathrm{H}$
and

$$
R=2 \pi \times 50 \times 0.0608 / 0.75=25.46 \Omega
$$

## Example 7.23

An inductive coil takes 10 A and dissipates $1,000 \mathrm{~W}$ when connected to a $250 \mathrm{~V}, 25 \mathrm{~Hz}$ supply. Calculate the (i) impedance, (ii) effective resistance, (iii) reactance, (iv) value of the capacitance required to be connected in series with coil to make the power factor of the circuit unity, and (vi) current taken by the coil. Further, draw the phasor diagram of the two cases.

## Solution:

The circuit is shown in Figure 7.43.
Power dissipated in the coil,
$r^{2} R=1,000 \mathrm{~W}$
$\therefore$ Resistance of the coil, $\quad R=1,000 /(10)^{2}=10 \Omega$
Impedance of the coil,

$$
Z=V / I=250 / 10=25 \Omega
$$

Inductive reactance of the coil, $X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(25)^{2}-(10)^{2}}$ $=22.91 \Omega$
Power factor, $\cos \phi=R / Z=10 / 25=0.4$ lag
The pf of the circuit will be unity when, $X_{\mathrm{C}}=X_{\mathrm{L}}$
or

$$
1 / 2 \pi f C=22.91
$$



Fig. 7.43 Circuit as per data
or

$$
C=\frac{1}{2 \pi \times 25 \times 22.91}=277.84 \mu \mathrm{~F}
$$


(a)

(b)

Fig. 7.44 (a) Phasor diagram for the circuit without capacitor (b) Phasor diagram of the circuit with capacitor

Now, current $I=\frac{V}{R}=\frac{250}{10}=25 \mathrm{~A}$
The phasor diagram for the two cases is drawn in Figure 7.44(a) and 7.44(b), respectively.

## Example 7.24

Voltages across resistance, inductance, and capacitance connected in series are $3 \mathrm{~V}, 4 \mathrm{~V}$, and 5 V , respectively. If supply voltage has 50 Hz frequency, calculate the magnitude of supply voltage


Fig. 7.45 Circuit as per data and the resonant frequency of this series $\mathrm{R}-\mathrm{L}-\mathrm{C}$ circuit.
(U.P.T.U. 2004-05)

## Solution:

The circuit diagram is shown in Figure 7.45.
Let supply voltage be $V \mathrm{~V}$.

$$
\begin{aligned}
V=\sqrt{V_{\mathrm{R}}^{2}+\left(V_{\mathrm{L}}-V_{\mathrm{C}}\right)^{2}} & =\sqrt{(3)^{2}+(4-5)^{2}} \\
& =\sqrt{9+1}=\sqrt{10} \mathrm{~V}
\end{aligned}
$$

and

$$
\begin{aligned}
V_{\mathrm{L}} & =I \times X_{\mathrm{L}}=I \times 2 \pi f L \\
V_{\mathrm{C}} & =I \times X_{\mathrm{C}}=\frac{I}{2 \pi f C} \\
\frac{V_{\mathrm{L}}}{V_{\mathrm{c}}} & =\frac{4}{5}=\frac{2 \pi f L \times I}{I} \times 2 \pi f C \\
L C & =\frac{4}{5(2 \pi f)^{2}}
\end{aligned}
$$

Resonant frequency,

$$
\begin{aligned}
f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}} & =\frac{1}{2 \pi \sqrt{\frac{4}{5 \times(2 \pi f)^{2}}}} \\
& =\frac{1 \times 2 \pi f \sqrt{5}}{2 \pi \times \sqrt{4}}=\frac{50 \sqrt{5}}{2}=25 \sqrt{5} \mathrm{~Hz}
\end{aligned}
$$

## Example 7.25

A choke coil is connected series with a $200 \mu \mathrm{~F}$ capacitor. With a constant supply voltage of 250 V , it is found that the circuit takes its maximum current of 50 A when the supply frequency is 100 Hz . Determine (i) resistance and inductance of the choke coil, (ii) voltage across the capacitor, and (iii) $Q$-factor of the circuit.

## Solution:

At resonance, current $I_{\mathrm{r}}=V / R$
or

$$
R=V / I_{\mathrm{r}}=250 / 50=5 \Omega
$$

Further,

$$
2 \pi f_{\mathrm{r}} L=1 / 2 \pi f_{\mathrm{r}} C
$$

or

$$
L=\frac{1}{\left(2 \pi f_{\mathrm{r}}\right)^{2} C}=\frac{1}{(2 \pi \times 100)^{2} \times 200 \times 10^{-6}}=12.66 \mathrm{mH}
$$

Voltage across capacitor,

$$
V_{\mathrm{C}}=I_{\mathrm{r}} X_{\mathrm{C}}=50 \times 1 / 2 \pi \times 100 \times 200 \times 10^{-6}=397.88 \mathrm{~V}
$$

## Example 7.26

A series resonant circuit has a $Q$-factor of 150 ; an inductance of 0.1 H and a capacitance of $0.1 \mu \mathrm{~F}$. Calculate the bandwidth of the circuit.

## Solution:

At resonance frequency

Now,

$$
f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}=\frac{1}{2 \pi \sqrt{0.1 \times 0.1 \times 10^{-6}}}=\frac{10^{4}}{2 \pi} \mathrm{~Hz}
$$

$$
Q=\frac{f_{\mathrm{r}}}{B W}
$$

$\therefore$ Bandwidth,

$$
\begin{aligned}
B W & =\frac{f_{\mathrm{r}}}{Q}=\frac{10^{4}}{2 \pi \times 150}=10.61 \mathrm{~Hz} \\
Q \text {-factor } & =V_{\mathrm{C}} / V=397.88 / 250=1.59
\end{aligned}
$$

## Example 7.27

A series R-L-C circuit with $R=10 \Omega ; L=0.02 \mathrm{H}, C=2 \mu \mathrm{~F}$ is connected to 100 V variable frequency source. Find the frequency for which the current is maximum.
(U.P.T.U. 2004-05)

## Solution:

In $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit, the current will be maximum when impedance is minimum, that is, when $Z=R$ or $X_{\mathrm{L}}=X_{\mathrm{C}}$ (at resonance)
or
or

$$
\begin{aligned}
2 \pi f_{\mathrm{r}} L & =\frac{1}{2 \pi f_{\mathrm{r}} C} \\
f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}} & =\frac{1}{2 \pi \sqrt{0.02 \times 2 \times 10^{-6}}}=795.77 \mathrm{~Hz}
\end{aligned}
$$

## Example 7.28

A coil of resistance $40 \Omega$ and inductance 0.75 H are in a series circuit with a capacitor C. The resonant frequency is 55 Hz . If supply is $250 \mathrm{~V}, 50 \mathrm{~Hz}$ find (i) line current, (ii) power factor, and (iii) power consumed.
(U.P.T.U. 2006-07)

## Solution:

The circuit is shown in Figure 7.46(a) and (b).
Here, $R=40 \Omega ; L=0.75 \mathrm{H} ; f_{\mathrm{r}}=55 \mathrm{~Hz} ; V=250 \mathrm{~V} ; f=50 \mathrm{~Hz}$
At resonance,

$$
\begin{gathered}
X_{\mathrm{L}}=X_{\mathrm{C}} \quad \text { or } \quad 2 \pi f_{\mathrm{r}} L=\frac{1}{2 \pi f_{\mathrm{r}} C} \\
C=\frac{1}{\left(2 \pi f_{\mathrm{r}}\right)^{2} L}=\frac{1}{(2 \pi \times 55)^{2} \times 0.75}=11.165 \mu \mathrm{~F}
\end{gathered}
$$

or
At $50 \mathrm{~Hz}, X_{\mathrm{L}}=2 \pi f \mathrm{~L}=2 \pi \times 50 \times 0.75=235.6 \Omega$

$$
X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 11.165 \times 10^{-6}}=285 \Omega
$$



Fig. 7.46 (a) Circuit under resonance (b) Circuit as per data

Impedance, $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}=\sqrt{(40)^{2}+(235.6-285)^{2}}=63.56 \Omega$
(i) Line current, $I=\frac{V}{Z}=\frac{250}{6356}=3.93 \mathrm{~A}$
(ii) Power factor, $\cos \phi=\frac{R}{Z}=\frac{40}{63.56}=0.6293$ leading
(iii) Power consumed, $P=V I \cos \phi=250 \times 3.93 \times 0.6293=618.3 \mathrm{~W}$

## Example 7.29

Determine the parameters of an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit that will resonate at $1,000 \mathrm{~Hz}$, has a bandwidth of 100 Hz , and draws 16 W from a 200 V generator operating at the resonant frequency of the circuit.

## Solution:

At resonance: voltage across resistor, $V_{\mathrm{R}}=$ supply voltage $=200 \mathrm{~V}$

$$
\begin{array}{ll}
\therefore & R=\frac{V_{\mathrm{R}}^{2}}{P}=\frac{(200)^{2}}{16}=2,500 \Omega \\
\text { However, } & Q \text {-factor }=\frac{f_{\mathrm{r}}}{B W}=\frac{1,000}{100}=10 \\
\therefore & Q=\frac{V_{\mathrm{L}}}{B W}=\frac{I X_{\mathrm{L}}}{R}=\frac{2 \pi f_{\mathrm{r}} L}{R} \\
\therefore & L=\frac{Q R}{2 \pi f_{\mathrm{r}}}=\frac{10 \times 2,500}{2 \pi \times 1,000}=3.98 \mathrm{H}
\end{array}
$$

Now,

$$
2 \pi f_{\mathrm{r}} L=\frac{1}{2 \pi f_{\mathrm{r}} C}
$$

$$
\therefore \quad C=\frac{1}{\left(2 \pi f_{\mathrm{r}}\right)^{2} \times L}=\frac{1}{(2 \pi \times 1,000)^{2} \times 3.98}=6.3 \times 10^{-9} \mathrm{~F}
$$

## 国豕 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by impedance of an AC circuit?
2. Draw an impedance triangle for $\mathrm{R}-\mathrm{L}, \mathrm{R}-\mathrm{C}$, and $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit and label it.
3. Define power factor of an AC circuit.
4. In an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit, show that power is consumed only by resistor.
5. What do you mean by series resonance?
6. What do you mean by bandwidth in a series resonant circuit?
7. Define $Q$-factor of a series resonant circuit.
8. When does an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit called as inductive circuit or capacitive circuit?

## Test Questions

1. Show that current vector leads the voltage vector in an $\mathrm{R}-\mathrm{C}$ series circuit. Further, show that in an R-C series circuit, power $P=V I \cos f$ where $V$ and $I$ are the rms values of voltage and current, respectively.
2. Draw the phasor and wave diagram for voltage and current in an $\mathrm{R}-\mathrm{C}$ series circuit.
3. What is power factor in an $\mathrm{R}-\mathrm{C}$ series circuit and explain how it is different to $\mathrm{R}-\mathrm{L}$ series circuit?

## Numericals

1. A coil of power factor 0.6 is in series with a $100 \mu \mathrm{~F}$ capacitor. When connected to 50 Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.
(Ans. $19.098 \Omega ; 81.06 \mathrm{mH}$ )
2. A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a condenser. The current in the circuit is 2.2 A . The power loss in the resistor is 96.8 W and that is the condenser is negligible. Calculate the resistance and the capacitance.
(P.T.U.) (Ans. $20 \Omega ; 59.85 \mu \mathrm{~F}$ )
3. A metal filament lamp, rated at $750 \mathrm{~W}, 100 \mathrm{~V}$ is to be connected in series with a capacitor across a $230 \mathrm{~V}, 60 \mathrm{~Hz}$ supply. Calculate (i) the capacitance required and (ii) the power factor.
(AMIE; W, 1981) (Ans. $115.26 \mu \mathrm{~F} ; 0.4346$ leading)
4. An AC circuit having a resistance of $10 \Omega$, an inductance of 0.2 H and a capacitance of $100 \mu \mathrm{~F}$ in series, is connected across a single-phase $110 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate (i) resultant reactance, (ii) impedance, (iii) current, (iv) voltages across R, L, and C, and (v) phase difference between the current and the supply voltage; further, draw the phasor diagram of the circuit.
(Ans. $31 \Omega ; 32.573 \Omega ; 3.377 \mathrm{~A}, 33.77 \mathrm{~V}, 212.18 \mathrm{~V}, 107.49 \mathrm{~V} ; 72.12^{\circ}$ )
5. A coil of pf 0.6 is in series with a $80 \mu \mathrm{~F}$ capacitor. When connected to a 60 Hz supply, the voltage across the capacitor is equal to the voltage across the coil. Find the resistance and inductance of the coil.
(Ans. $19.89 \Omega ; 70.36 \mathrm{mH}$ )

### 7.16 AC PARALLEL CIRCUITS

The AC circuits in which number of branches are connected in such a manner so that voltage across each branch is the same, but current flowing through them is different are called AC parallel circuits. The parallel circuits are used more frequently in AC system because of the following reasons:

1. Almost all the electrical appliances (or devices) of different ratings are operated at the same supply voltage and are connected in parallel.
2. Each device is required to be operated independently (with a switch) without disturbing the operation of other devices. Hence, they are connected in parallel.

### 7.17 METHODS OF SOLVING PARALLEL AC CIRCUITS

In parallel circuits, a number of branches are connected in parallel. Each branch, generally, contains number of components such as resistance, inductance, and capacitance forming series circuits. Therefore, each branch is analysed separately as a series circuit, and then, the effects of separate branches are combined together. While carrying out circuit calculations, the magnitudes and phase angles of voltages and currents are taken into account. The following methods may be applied for solving AC parallel circuits.

1. Phasor (or vector) method
2. Admittance method

Method of phasor algebra (or symbolic method or J-method).
The method to be applied for the solution depends upon the conditions of the problem. However, in general, the method that yields quick results is applied.

### 7.18 PHASOR (OR VECTOR) METHOD

To solve parallel AC circuits by this method, we proceed as follows:
Step I: Draw the circuit as per the given problem, as shown in Figure 7.47(a) (here, for illustration, we have considered two branches connected in parallel. One branch contains resistance and inductance in series, whereas second branch contains resistance and capacitance in series. The supply voltage is $V \mathrm{~V}$ ).
Step II: Find the impedance of each branch of the circuit separately.

$$
\begin{aligned}
Z_{1}=\sqrt{R_{1}^{2}+X_{\mathrm{L} 1}^{2}}, & \text { where } X_{\mathrm{L} 1}=2 \pi f L_{1} \\
Z_{2}=\sqrt{R_{2}^{2}+X_{\mathrm{C} 2}^{2}}, & \text { where } X_{\mathrm{C} 2}=1 / 2 \pi f C_{2}
\end{aligned}
$$


(a)

(b)

Fig. 7.47 (a) Circuit diagram (b) Phasor diagram

Step III: Determine the magnitude of current and phase angle with the voltage in each branch.

$$
\begin{array}{ll}
I_{1}=\frac{V}{Z_{1}} ; \quad \phi_{1}=\tan ^{-1} \frac{X_{\mathrm{L} 1}}{R_{1}} ; \text { (lagging) (for inductive branch) } \\
I_{2}=\frac{V}{Z_{2}} ; \quad \phi_{2}=\tan ^{-1} \frac{X_{\mathrm{C} 2}}{R_{2}} ; \text { (lagging) (for capacitive branch) }
\end{array}
$$

Step IV: Draw the phasor diagram by considering voltage as the reference phasor. Represent the branch currents on it as shown in Figure 7.47(b).
Step V: Find the phasor sum of branch currents by the method of components.

$$
\begin{aligned}
I_{\mathrm{XX}} & =I_{1} \cos \phi_{1}+I_{2} \cos \phi_{2} \\
I_{\mathrm{YY}} & =-I_{1} \sin \phi_{1}+I_{2} \sin \phi_{2} \text { (negative) } \\
I & =\sqrt{\left(I_{\mathrm{XX}}\right)^{2}+\left(I_{\mathrm{YY}}\right)^{2}}
\end{aligned}
$$

Step VI: Find the phase angle $\phi$ between the total current $I$ and circuit voltage $V$.

$$
\phi=\tan ^{-1} \frac{I_{\mathrm{YY}}}{I_{\mathrm{XX}}} \text { lagging (since } I_{\mathrm{YY}} \text { is negative) }
$$

Power factor of the circuit $=\cos \phi$ (lagging)
or

$$
\text { power factor }=\frac{I_{\mathrm{xx}}}{I} \text { (lagging) }
$$

## Example 7.30

A coil of resistance $15 \Omega$ and inductance 0.05 H is connected in parallel with a non-inductive resistance of $20 \Omega$. Find (i) current in each branch of the circuit, (ii) total current supplied, (iii) phase angle and pf of combination when a voltage of 200 V at 50 Hz is applied, and (iv) power consumed in the circuit.

## Solution:

The circuit is shown in Figure 7.48(a).
Applied voltage, $V=200 \mathrm{~V}$; supply frequency, $f=50 \mathrm{~Hz}$


Fig. 7.48 (a) Circuit as per data (b) Phasor diagram

## Branch I

$$
R_{2}=15 \Omega ; X_{\mathrm{L} 1}=2 \pi f L_{1}=2 \pi \times 50 \times 0.05=15.7 \Omega
$$

Impedance, $\quad Z_{1}=\sqrt{R_{1}^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(15)^{2}+(15.7)^{2}}=21.72 \Omega$
Current in the coil, $I_{1}=\frac{V}{Z_{1}}=\frac{200}{21.72}=9.2 \mathrm{~A}$
Phase angle, $\quad \phi_{1}=\tan ^{-1} \frac{X_{\mathrm{L} 1}}{R}=\tan ^{-1} \frac{15.7}{15}=46.3^{\circ}$ lagging

## Branch II

Resistance,

$$
R_{2}=20 \Omega
$$

Branch current,

$$
I_{2}=\frac{V}{R_{2}}=\frac{200}{20}=10 \mathrm{~A}
$$

Phase angle,

$$
\phi_{2}=0\left(I_{2} \text { is in phase with } V\right)
$$

The two currents are shown vectorially in Figure 7.48(b). Resolving the currents horizontally and vertically,

$$
\begin{aligned}
& I_{\mathrm{XX}}=I_{2}+I_{1} \cos \phi_{1}=10+9.2 \cos 46.3^{\circ}=10+9.2 \times 6,909=16.356 \mathrm{~A} \\
& I_{\mathrm{YY}}=0-I_{1} \sin \phi_{1}=0-9.2 \sin 46.3^{\circ}=-9.2 \times 0.723=-6.65 \mathrm{~A}
\end{aligned}
$$

Total current supplied,

$$
I=\sqrt{I_{\mathrm{XX}}^{2}+I_{\mathrm{YY}}^{2}}=\sqrt{(16.356)^{2}+(-6.65)^{2}}=17.656 \mathrm{~A}
$$

Phase angle,

$$
\phi=\tan ^{-1} \frac{I_{\mathrm{YY}}}{I_{\mathrm{XX}}}=\tan ^{-1}\left(\frac{-6.65}{16.356}\right)=-22.126^{\circ}
$$

Power factor of the circuit, $\cos \phi=\cos \left(-22.126^{\circ}\right)=0.9264$ (lagging)
Power,

$$
P=V I \cos \phi=200 \times 17.656 \times 0.9264=3,271.3 \mathrm{~W}
$$

## Example 7.31

A series AC circuit has a resistance of $15 \Omega$ and inductive reactance of $10 \Omega$. Calculate the value of capacitor that is connected across this series combination so that system has unity power factor. The frequency of AC supply is 50 Hz .
(U.P.T.U. 2005-06)

## Solution:

The circuit is shown in Figure 7.49(a). Let the supply voltage be $V \mathrm{~V}$.

(a)

(b)

Fig. 7.49 (a) Circuit as per data (b) Phasor diagram

## Branch I

Impedance, $Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(15)^{2}+(10)^{2}}=18.03 \Omega$
Current, $I_{1}=\frac{V}{Z}=\frac{V}{18.03} \mathrm{~A}$
Power factor, $\cos \phi=\frac{R}{Z}=\frac{15}{18.03}=0.832 \mathrm{lag}$
Phase angle, $\phi=\cos ^{-1} 0.832=33.7^{\circ} \mathrm{lag}$
Reactive component of current, $I_{r_{1}}=I_{1} \sin \phi=\frac{V}{18.03} \times \sin 33.7^{\circ}$

$$
\begin{equation*}
=\frac{0.5548 \mathrm{~V}}{18.03} \tag{7.15}
\end{equation*}
$$

## Branch II

Current drawn by the capacitor, $I_{\mathrm{c}}=\frac{V}{X_{\mathrm{C}}}$
Power factor of the circuit will be unity when

$$
\begin{gathered}
I_{\mathrm{c}}=I_{r_{1}} \\
\frac{V}{X_{\mathrm{C}}}=\frac{0.5548 \mathrm{~V}}{18.03} \quad \text { or } \quad X_{\mathrm{C}}=\frac{18.03}{0.5548}=32.5 \Omega \\
X_{\mathrm{C}}=\frac{1}{2 \pi f C} \quad \text { or } \quad C=\frac{1}{2 \pi f X_{\mathrm{C}}}
\end{gathered}
$$

## Example 7.32

The parallel circuit shown in Figure 7.50(a) is connected across a single phase $100 \mathrm{~V}, 50 \mathrm{~Hz}$ AC supply. Calculate the (i) branch currents, (ii) total current, (iii) supply power factor, and (iv) active and reactive power supplied by the supply.


Fig. 7.50 (a) Given circuit diagram (b) Circuit as per data

## Solution:

$$
V=100 \mathrm{~V}, f=50 \mathrm{~Hz}
$$

(i) Impedance of Branch I, $Z_{1}=8+j 6=10 \angle 36.87^{\circ} \Omega$

Impedance of Branch II, $Z_{2}=6-j 8=10 \angle-53.13^{\circ} \Omega$
Current through Branch I, $I_{1}=\frac{V}{Z_{1}}=\frac{100}{10 \angle 36.87^{\circ}}=10 \angle-36.87^{\circ} \mathrm{A}$
Current through Branch II, $I_{2}=\frac{V}{Z_{2}}=\frac{100}{10 \angle-53.13^{\circ}}=10 \angle 53.13^{\circ} \mathrm{A}$
(ii) Total current $I=I_{1}+I_{2}=10 \angle-36.87^{\circ}+10 \angle 53.13^{\circ}$

$$
=(8-j 6)+(6+j 8)=(14+j 2) \mathrm{A}=14.14 \angle 8.13^{\circ} \mathrm{A}
$$

(iii) Supply power factor $=\cos \phi=\cos 8.13^{\circ}=0.989$ leading
(iv) Active power supplied by the supply,

$$
P=V I \cos \phi=100 \times 14.14 \times 0.989=1,400 \mathrm{~W}
$$

Reactive power supplied by the supply

$$
P_{\mathrm{r}}=V I \sin \phi=100 \times 14.14 \sin 8.13^{\circ}=200 \mathrm{VAR}
$$

## Example 7.33

Find the active and reactive components of current taken by a series circuit consisting of a coil of inductance 0.1 H , resistance $8 \Omega$, and a capacitor of $120 \mu \mathrm{~F}$ connected to a $240 \mathrm{~V}, 50 \mathrm{~Hz}$ supply mains. Find the value of the capacitor that has to be connected in parallel with the abovementioned series circuit so that the pf of the entire circuit is unity.

## Solution:

The circuit is shown in Figure 7.51(a)


Fig. 7.51 (a) Circuit as per data (b) Circuit when $\mathrm{C}_{1}$ is connected across the given circuit

Here, $R=8 \Omega ; L=0.1 \mathrm{H} ; C=120 \mu \mathrm{~F}$
Inductive reactance, $X_{\mathrm{L}}=2 \pi f L$

$$
=2 \pi \times 50 \times 0.1=31.416 \Omega
$$

Capacitive reactance, $X_{\mathrm{C}}=\frac{1}{2 \pi f C}$

$$
=\frac{1}{2 \pi \times 50 \times 120 \times 10^{-6}}=26.526 \Omega
$$

Impedance, $Z=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}$

$$
\begin{aligned}
& =\sqrt{8^{2}+(31.416-26.526)^{2}} \\
& =9.376 \Omega
\end{aligned}
$$

Circuit current, $I=\frac{V}{Z}=\frac{240}{9.376}=25.6 \mathrm{~A}$
Phase angle, $\phi=\tan ^{-1} \frac{X_{\mathrm{L}}-X_{\mathrm{C}}}{R}=\tan ^{-1} \frac{31.416-26.526}{8}=31.435^{\circ}$ lagging
Active component of current, $I_{\mathrm{a}}=I \cos \phi=25.6 \cos 31.435^{\circ}=21.84 \mathrm{~A}$
Reactive component of current, $I_{\mathrm{r}}=I \sin \phi=25.6 \sin 31.435^{\circ}=13.35$ A (lagging)
The power factor of the whole circuit will become unity if the current drawn by the capacitor $C_{1}$, connected across the series circuit, as shown in Figure 7.51(b), is made equal to the reactive (lagging) component of current of the series circuit.
that is, when

$$
I_{\mathrm{C}}=I_{\mathrm{r}} \quad \text { or } \quad \frac{V}{X_{\mathrm{C}_{1}}}=I \sin \phi
$$

or

$$
2 \pi f C_{1} V=13.35 \quad \text { or } \quad C_{1}=\frac{13.35}{2 \pi \times 50 \times 240}=177 \mu \mathrm{~F}
$$

## Example 7.34

An AC circuit includes two sections AB and BC in series. The section AB consists of two branches in parallel. The first of these is formed of resistance of $60 \Omega$ in series with a capacitor of $50 \mu \mathrm{~F}$, while the second consists of a resistance of $60 \Omega$ having an inductance of 250 mH . The section BC consists of a resistance of $100 \Omega$ having an inductance of 300 mH . The frequency of the current is 50 Hz . The voltage across section AB is 500 V . What is the voltage across the section BC ?

## Solution:

The circuit for the given problem is shown in Figure 7.52.


Fig. 7.52 Circuit as per data

Let us consider section AB.

## Branch I

Capacitive reactance, $X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{2}{2 \pi \times 50 \times 50 \times 10^{-6}}=63.66 \Omega$
Impedance, $Z_{1}=\sqrt{R_{1}^{2}+X_{\mathrm{C} 1}^{2}}=\sqrt{(60)^{2}+(63.66)^{2}}=87.48 \Omega$

Current, $I=\frac{V_{\mathrm{AB}}}{Z_{1}}=\frac{500}{87.48}=5.72 \mathrm{~A}$
Phase angle, $\phi_{1}=\tan ^{-1} \frac{X_{\mathrm{C} 1}}{R_{1}}=\tan ^{-1} \frac{63.66}{60}=46.7^{\circ}$ (leading)

## Branch II



Fig. 7.53 Phasor diagram

Inductive reactance, $X_{\mathrm{L} 2}=2 \pi f L=2 \pi \times 50 \times 250 \times 10^{-3}=$ $78.52 \Omega$

Impedance, $Z_{2}=\sqrt{R_{2}^{2}+X_{\mathrm{L} 2}^{2}}=\sqrt{(60)^{2}+(78.54)^{2}}=98.84 \Omega$
Current, $I_{2}=\frac{V_{\mathrm{AB}}}{Z_{2}}=\frac{500}{98.84}=5.06 \mathrm{~A}$
Phase angle, $\phi_{2}=\tan ^{-1} \frac{X_{\mathrm{L} 2}}{R_{2}}=\tan ^{-1} \frac{78.54}{60}=52.62^{\circ}$ (lagging)
The two currents $I_{1}$ and $I_{2}$ are shown vectorially in Figure 7.53.

Resolving the currents horizontally and vertically:

$$
\begin{aligned}
& I_{\mathrm{XX}}=I_{1} \cos \phi_{1}+I_{2} \cos \phi_{2}=5.72 \times \cos 46.7^{\circ}+5.06 \times \cos 52.62^{\circ}=6.9948 \mathrm{~A} \\
& I_{\mathrm{YY}}=I_{1} \sin \phi_{1}-I_{2} \sin \phi_{2}=5.72 \times \sin 46.7^{\circ}-5.06 \times \sin 52.62^{\circ}=0.142 \mathrm{~A}
\end{aligned}
$$

Total current, $I=\sqrt{I_{\mathrm{XX}}{ }^{2}+{I_{\mathrm{YY}}}^{2}}=\sqrt{(6.9948)^{2}+(0.142)^{2}}=6.996 \mathrm{~A}$
Let us consider section BC.
Inductive reactance, $X_{\mathrm{L} 3}=2 \pi f L_{3}=2 \pi \times 50 \times 300 \times 10^{-3}=94.25 \Omega$
Impedance, $Z_{3}=\sqrt{R_{3}^{2}+X_{\mathrm{L} 3}^{2}}=\sqrt{(100)^{2}+(94.25)^{2}}=137.41 \Omega$
Current, $I=6.996 \mathrm{~A}$
$\therefore$ Voltage across BC, $V_{B C}=I Z_{3}=6.996 \times 137.41=961.35 \mathrm{~V}$

## Example 7.35

A single-phase motor takes 5 A current at $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply at a pf 0.707 lagging. It is required to improve the pf of the motor to 0.9 by connecting a capacitor in parallel with it. Determine the capacitance of capacitor.


Fig. 7.54 Circuit as per data

## Solution:

The circuit is shown in Figure 7.54.
Active component of current, $I_{\mathrm{a}}=I_{\mathrm{m}} \cos \phi_{1}=5 \times 0.707=3.535 \mathrm{~A}$ Since load on the motor remains the same, the active component of current drawn by the motor remains the same.

Now, $\cos \phi_{1}=0.707 ; \tan \phi_{1}=\tan \cos ^{-1} 0.707=1$
Reactive component of current at pf 0.707;
$I_{\mathrm{r} 1}=I_{\mathrm{a}} \tan \phi_{1}=3.535 \times 1=3.535 \mathrm{~A}$
When $\mathrm{pf}=\cos \phi_{2}=0.9 ; \tan \phi_{2}=\tan \cos ^{-1} 0.9=0.4843$

Reactive component of current at pf 0.9;
$I_{\mathrm{r} 2}=I_{\mathrm{a}} \tan \phi_{2}=3.535 \times 0.4843=1.712 \mathrm{~A}$
From phasor diagram shown in Figure 7.55, we get

Current drawn by capacitor,
$I_{\mathrm{C}}=I_{\mathrm{r} 1}-I_{\mathrm{r} 2}=3.535-1.712=1.823 \mathrm{~A}$
Now, $I_{\mathrm{C}}=\frac{V}{X_{\mathrm{C}}}=\frac{V}{1 / 2 \pi f C}=2 \pi f C V$
or $C=\frac{I_{\mathrm{C}}}{2 \pi f V}=\frac{1.823}{2 \pi \times 50 \times 230}=25.23 \mu \mathrm{~F}$


Fig. 7.55 Phasor diagram

## Example 7.36

A single-phase motor takes 50 A at a pf of 0.6 lagging from $250 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. What value of capacitance must a shunt capacitor have to increase the overall power factor to 0.9 ?
(U.P.T.U. Tut.)

## Solution:

The circuit is shown in Figure 7.56.
Active component of current drawn by motor, $I_{\mathrm{a}}=I \cos \phi_{1}$ $=50 \times 0.6=30 \mathrm{~A}$
Initial power factor, $\cos \phi_{1}=\cos ^{-1} 0.6$ lagging

$$
\tan \phi_{1}=\tan \cos ^{-1} 0.6=1.333
$$

Improved power factor, $\cos \phi_{2}=\cos ^{-1} 0.9$ lagging

$$
\tan \phi_{2}=\tan \cos ^{-1} 0.9=0.4843
$$

The reactive current drawn by capacitor,


Fig. 7.56 Circuit as per data

$$
\begin{aligned}
& I_{\mathrm{C}}=I_{\mathrm{r}_{1}}-I_{\mathrm{r}_{2}} I_{\mathrm{a}} \tan \phi_{1}-I_{\mathrm{a}} \tan \phi_{2} \\
& I_{\mathrm{C}}=I_{\mathrm{a}}\left(\tan \phi_{1}-\tan \phi_{2}\right)=30(1.333-0.4843)=25.47 \mathrm{~A}
\end{aligned}
$$

The value of capacitance required, $C=\frac{I_{\mathrm{C}}}{2 \pi f V}=\frac{25.47}{2 \pi \times 50 \times 250}=324.3 \mu \mathrm{~F}$

## Example 7.37

A capacitor is placed with two inductive loads, one of 20 A at $30^{\circ}$ lag and other of 40 A at $60^{\circ}$ lag. What must be the current in the capacitor so that the current from the external source shall be at unity power factor?
(U.P.T.U. Tut.)

## Solution:

Reactive component of current drawn by 20 A inductive load,

$$
I_{\mathrm{r}_{\mathrm{i}}}=I_{1} \sin \phi_{1}=20 \sin 30^{\circ}=10 \mathrm{~A} \text { (lagging) }
$$

Reactive component of current drawn by 40 A inductive load,

$$
I_{\mathrm{r}_{2}}=I_{2} \sin \phi_{2}=40 \sin 60^{\circ}=34.64 \mathrm{~A} \text { (lagging) }
$$

The resultant power factor becomes unity only when the current drawn by the shunt capacitor becomes equal to the sum of reactive components (lagging) of current drawn by the two inductive loads.
i.e.,

$$
I_{\mathrm{c}}=I_{\mathrm{r}_{\mathrm{i}}}+I_{\mathrm{r}_{2}}=10+34.64=44.64 \mathrm{~A}
$$

### 7.19 ADMITTANCE METHOD

Before applying this method for the solution of parallel AC circuits, the reader should be familiar with the following important terms:

### 7.19.1 Admittance

The reciprocal of impedance of an AC circuit is called admittance of the circuit. Since impedance is the total opposition to the flow of AC in an AC circuit, the admittance is the effective ability of the circuit due to which it allows the AC to flow through it. It is represented by letter ' $Y$ '.

Now,

$$
Z=\frac{V}{I} \therefore Y=\frac{I}{V} \quad\left(\text { since } Y=\frac{1}{Z}\right)
$$

The unit of admittance is mho (i.e., ohm spelled backward and its symbol is $\Psi$ ).

### 7.19.2 Admittance Triangle

Admittance can also be represented by a triangle similar to that of impedance.
Impedance $Z$ of the circuit has two rectangular components resistance $R$ and reactance $X$. Similarly, admittance $Y$ also has two rectangular components conductance $g$ and susceptance $b$, as shown in Figure 7.57.


Fig. 7.57 (a) Admittance triangle for inductive circuit (b) Admittance triangle for capacitive circuit

### 7.19.3 Conductance

The base of an admittance triangle is representing conductance as shown in Figure 7.57.
Conductance,

$$
g=Y \cos \phi=\frac{1}{Z} \cdot \frac{X}{Z}=\frac{X}{Z^{2}}=\frac{X}{R^{2}+X^{2}}
$$

Conductance is always positive irrespective of the circuit parameters. The unit of conductance is $m h o$.

### 7.19.4 Susceptance

The perpendicular or an admittance triangle is representing susceptance, as shown in Figure 7.57.
Susceptance,

$$
g=Y \sin \phi=\frac{1}{Z} \cdot \frac{X}{Z}=\frac{X}{Z^{2}}=\frac{X}{R^{2}+X^{2}}
$$

Susceptance is positive for capacitive reactance (see Fig. 7.57(b)) and negative for inductive reactance (see Fig. 7.57(a)). The unit of susceptance is mho.

### 7.19.5 Solution of Parallel AC Circuits by Admittance Method

Consider a parallel AC circuit shown in Figure 7.58. For its solution, we shall proceed as follows:
Step I: Draw the circuit as per the given problem as shown in Figure 7.58.
Step II: Find impedance and phase angle of each branch.

$$
\begin{array}{ll}
Z_{1}=\sqrt{R_{1}^{2}+X_{\mathrm{C} 1}^{2}} ; \quad & \phi_{1}=\tan ^{-1} \frac{X_{\mathrm{C} 1}}{R_{1}} \\
Z_{2}=\sqrt{R_{2}^{2}+X_{\mathrm{L} 2}^{2}} ; & \phi_{2}=\tan ^{-1} \frac{X_{\mathrm{L} 2}}{R_{\mathrm{L}}}
\end{array}
$$

Step III: Find conductance, susceptance, and admittance of each branch.

$$
\begin{array}{ll}
g_{1}=\frac{R_{1}}{Z_{1}^{2}} ; \quad b_{1}=\frac{X_{\mathrm{C} 1}}{Z_{1}^{2}}(\text { positive }) ; \quad Y_{1}=\sqrt{g_{1}^{2}+b_{1}^{2}} \\
g_{2}=\frac{R_{2}}{Z_{2}^{2}} ; \quad b_{2}=\frac{X_{\mathrm{L} 2}}{Z_{2}^{2}}(\text { negative }) ; \quad Y_{2}=\sqrt{g_{2}^{2}+b_{2}^{2}}
\end{array}
$$



Fig. 7.58 AC circuits connected in parallel

Step IV: Find the algebraic sum of conductance and susceptance.

$$
G=g_{1}+g_{2} ; B=b_{1}-b_{2}
$$

Step V: Find total admittance of the circuit.

$$
Y=\sqrt{G^{2}+B^{2}}
$$

Step VI: Find branch currents and total current.

$$
I_{1}=V Y_{1} ; I_{2}=V Y_{2} ; I=V Y
$$

Step VII: Find the phase angle and the pf of the whole circuit.

$$
\begin{aligned}
\phi & =\tan ^{-1} \frac{B}{G} \text { (lagging if } B \text { is negative) } \\
\mathrm{pf} & =\cos \phi=\frac{G}{Y}
\end{aligned}
$$

## Example 7.38

The active and lagging reactive components of current taken by an AC circuit from a $250-\mathrm{V}$ supply are 50 A and 25 A , respectively. Calculate the conductance, susceptance, admittance, and power factor of the circuit.
(U.P.T.U. Tut.)

## Solution:

Active component of current, $I_{\mathrm{a}}=I \cos \phi=50 \mathrm{~A}$
Leading reactive component of current, $I_{\mathrm{r}}=I \sin \phi=25 \mathrm{~A}$
Circuit current, $I=\sqrt{\left(I_{\mathrm{a}}\right)^{2}+\left(I_{\mathrm{r}}\right)^{2}}=\sqrt{50^{2}+25^{2}}=55.9 \mathrm{~A}$
Admittance of the circuit, $Y=\frac{I}{V}=\frac{55.9}{250}=0.224 \mathrm{mho}$
Power factor of the circuit, $\cos \phi=\frac{I_{\mathrm{a}}}{I}=\frac{50}{55.9}=0.8944$ (lagging)
Conductance of the circuit, $G=Y \cos \phi=0.224 \times 0.8944=0.2 \mathrm{mho}$
Susceptance of the circuit, $B=\sqrt{Y^{2}-G^{2}}=\sqrt{(0.224)^{2}-(0.2)^{2}}$

$$
=0.1 \mathrm{mho} \text { (inductive) }
$$

## Example 7.39

A parallel circuit has two branches. One branch contains a resistance of $8 \Omega$ and inductance of 19.1 mH in series and the other contains a resistance of $6 \Omega$ and capacitor of capacitance 601.55 F in series. This parallel circuit is connected across a supply voltage of $240 \mathrm{~V}, 50 \mathrm{~Hz}$. Determine (i) current drawn by each branch, (ii) total current drawn from the mains, and (iii) pf of the whole circuit.

## Solution:

The circuit is shown in Figure 7.59.

## Branch I



Fig. 7.59 Circuit as per data

$$
\begin{aligned}
X_{\mathrm{L} 1} & =2 \phi f L_{1}=2 p \times 50 \times 0.0191=6 \Omega \\
Z_{1}^{2} & =R_{1}^{2}+X_{\mathrm{L} 1}^{2}=(8)^{2}+(6)^{2}=100 \\
g_{1} & =\frac{R_{1}}{Z_{1}^{2}}=\frac{8}{100}=0.08 \mathrm{mho} \\
b_{1} & =\frac{X_{\mathrm{L} 1}}{Z_{1}^{2}}=\frac{6}{100}=0.06 \text { mho (negative) }
\end{aligned}
$$

$$
Y_{1}=\sqrt{g_{1}^{2}+b_{1}^{2}}=\sqrt{(0.08)^{2}+(0.06)^{2}}=0.1 \mathrm{mho}
$$

$$
I_{1}=V Y_{1}=240 \times 0.1=24 \mathrm{~A}
$$

## Branch II

$$
\begin{aligned}
X_{\mathrm{C} 2}=\frac{1}{2 \pi f C_{2}} & =\frac{1}{2 \pi \times 50 \times 601.55 \times 10^{-6}} \\
& =5.291 \Omega \\
\therefore \quad Z_{2}^{2} & =R_{2}^{2}+X_{\mathrm{C} 2}^{2}=(6)^{2}+(5.291)^{2}=64 ; \\
g_{2} & =\frac{R_{2}}{Z_{2}^{2}}=\frac{6}{64}=0.09375 \mathrm{mho}
\end{aligned}
$$

$$
\begin{aligned}
& b_{2}=\frac{X_{\mathrm{C} 2}}{Z_{2}^{2}}=\frac{5.291}{64}=0.08268 \mathrm{mho}(\text { positive }) \\
& Y_{2}=\sqrt{g_{2}^{2}+b_{2}^{2}}=\sqrt{(0.09375)^{2}+(0.08268)^{2}}=0.125 \mathrm{mho} \\
& I_{2}=V Y_{2}=240 \times 0.125=30 \mathrm{~A}
\end{aligned}
$$

Total conductance, $G=g_{1}+g_{2}=0.08+0.09375=0.17375 \mathrm{mho}$
Total susceptance, $B=-b_{1}+b_{2}=-0.06+0.08268=0.02268 \mathrm{mho}$ (positive)
Total admittance, $Y=\sqrt{G^{2}+B^{2}}=\sqrt{(0.17375)^{2}+(0.02268)^{2}}=0.1752 \mathrm{mho}$
Total current, $I=V Y=240 \times 0.1752=42.05 \mathrm{~A}$
Power factor, $\cos \phi=G / Y=0.17375 / 0.1752=0.9917$ leading

## Example 7.40

Calculate the impedance and admittance of the Branch AB and CD of the circuit shown in Figure 7.60. Further, find the resistant impedance and admittance of the circuit.

## Solution:

## Branch AB

$$
R_{1}=10 \Omega ; L_{1}=0.01 \mathrm{H}
$$

Inductive reactance, $X_{\mathrm{L} 1}=2 \pi f L_{1}=2 \pi \times 50 \times 0.01=3.1416 \Omega$
Impedance, $Z_{1}=Z_{2}=\sqrt{R_{2}^{2}+X_{\mathrm{L} 1}^{2}}=\sqrt{(10)^{2}+(3.1416)^{2}}=10.48 \Omega$

$$
Z_{1}^{2}=(10.48)^{2}=109.87
$$



Fig. 7.60 Give circuit

Conductance, $g_{1}=\frac{R_{1}}{Z_{1}^{2}}=\frac{10}{109.87}=0.091 \mathrm{mho}$
Susceptance, $b_{1}=\frac{-X_{\mathrm{L} 1}}{Z_{1}^{2}}=\frac{-3.1416}{109.87}=-0.0286 \mathrm{mho}$ (negative, being inductive)
Admittance, $Y_{1}=\sqrt{g_{1}^{2}+b_{1}^{2}}=\sqrt{(0.091)^{2}+(0.0286)^{2}}=0.0954 \mathrm{mho}$

## Branch CD

$$
R_{2}=20 \Omega ; L_{2}=0.05 \mathrm{H} ; X_{\mathrm{L} 2}=2 \pi f L_{2}=2 \pi \times 50 \times 0.05=15.7 \Omega
$$

Impedance, $Z_{2}=\sqrt{R_{2}^{2}+X_{\mathrm{L} 2}^{2}}=\sqrt{(20)^{2}+(15.7)^{2}}=25.43 \Omega$

$$
Z_{2}^{2}=(25.43)^{2}=646.5
$$

Conductance, $g_{2}=\frac{R_{2}}{Z_{2}^{2}}=\frac{20}{646.5}=0.031 \mathrm{mho}$
Susceptance, $b_{2}=\frac{-X_{\mathrm{L} 2}}{Z_{2}}=\frac{-15.7}{646.5}=-0.0243 \mathrm{mho}$ (negative, being inductive)

Admittance, $Y_{2}=\sqrt{g_{2}^{2}+b_{2}^{2}}=\sqrt{(0.031)^{2}+(0.0243)^{2}}=0.03938 \mathrm{mho}$
Total conductance, $G=g_{1}+g_{2}=0.091+0.031=0.122 \mathrm{mho}$
Total susceptance, $B=b_{1}+b_{2}=-0.086-0.0243=-0.0529 \mathrm{mho}$
Total admittance, $Y=\sqrt{G^{2}+B^{2}}=\sqrt{(0.122)^{2}+(-0.0529)^{2}}=0.133 \mathrm{mho}$
Impedance of the whole circuit, $Z=\frac{1}{Y}=\frac{1}{0.133}=7.52 \Omega$

### 7.20 METHOD OF PHASOR ALGEBRA OR SYMBOLIC METHOD OR J-METHOD

Before applying the method of phasor algebra for solving parallel AC circuits, let us have an idea of important topics of phasor algebra. A technique, developed by engineers, to represent a phasor in an algebraic (i.e., mathematical) form is known as phasor algebra or complex algebra.

This technique has provided a relatively simple but powerful tool for obtaining quick solution of AC circuits. It simplifies the mathematical manipulation of phasors to a great extent.

### 7.21 j-NOTATION OF PHASOR ON RECTANGULAR CO-ORDINATE AXES



Fig. 7.61 Vector representation by $j$-notation

Consider a phasor $\bar{V}$ lying along OX-axis as shown in Figure 7.61. The phasor is reversed when it is multiplied by -1 , that is, the phasor is rotated through $180^{\circ}$ in counter clockwise (CCW) direction and attains the position along $\mathrm{OX}^{\prime}$-axis. Let us consider $j$ as a factor which when multiplied by the phasor $\bar{V}$, the phasor is rotated through $90^{\circ}$ in CCW direction. This means that multiplying the phasor by $j^{2}$ is the same as multiplying by -1 . Therefore, it follows that

$$
j^{2}=-1 \quad \text { or } \quad 1 j=\sqrt{-1}
$$

Therefore, it is concluded that $j$ is just an operator that is when multiplied with a phasor, it shows that the phasor is rotated through $90^{\circ}$ in CCW direction. Each successive multiplication of $j$, rotates the phasor further by $90^{\circ}$ as

$$
\begin{aligned}
& j=\sqrt{-1} \ldots \ldots \ldots \ldots . \ldots \ldots . . . . . .0^{\circ} \mathrm{CCW} \text { rotation from OX-axis } \\
& j^{2}=-1 . . . . . . . . . . . . . . . . . . . . .180^{\circ} \mathrm{CCW} \text { rotation from OX-axis } \\
& j^{3}=j^{2} j=-\sqrt{-1} \ldots \ldots \ldots . .270^{\circ} \mathrm{CCW} \text { rotation from OX-axis } \\
& j^{4}=j^{2} j^{2}=1 \ldots \ldots \ldots \ldots \ldots . . . . . .360^{\circ} \mathrm{CCW} \text { rotation from OX-axis }
\end{aligned}
$$

[^8]The symbol $j$ is used to represent the vertical (quadrature) components of phasor quantities. For instance, consider a phasor $V$ rotated through $\theta^{\circ}$ counter clockwise from OX-axis as shown in Figure 7.62(a). The phasor has two rectangular components (i) the horizontal component ' $a$ ' along X-axis and (ii) the vertical component ' $b$ ' rotated through $90^{\circ}$ in CCW direction from OX-axis and is expressed as ' $j b$ '. Therefore, in rectangular form, the phasor $\bar{V}$ is represented as:

$$
\bar{V}=a+j b \text { having magnitude } V=\sqrt{a^{2}+b^{2}} \text { and angle } \theta=\tan ^{-1}(b / a) \text { [positive] }
$$



Fig. 7.62 (a) Position of vector V at an instant (b) Position of vector V at an instant

If the phasor $\bar{V}$ is displaced through an angle $\theta^{\circ}$ in clockwise direction as shown in Figure 7.62(b), the vertical component will be expressed as ' $-j b$ '. Therefore, in rectangular form, the phasor is represented as

$$
\bar{V}=a-j b \text { having magnitude } V=\sqrt{a^{2}+b^{2}} \text { and angle } \theta^{\circ}=\tan ^{-1}(-b / a) \text { [negative] }
$$

### 7.21.1 Mathematical Representation of Phasors

In the mathematical form, a phasor can be represented in (i) rectangular form, (ii) trigonometric form, and (iii) polar form. Consider a voltage phasor $\bar{V}$ displaced $\theta^{\circ} \mathrm{CCW}$ from the reference axis (i.e., OX-axis) as shown in Figure 7.62(a). Let us see how this phasor is represented in different forms.

## Rectangular form

This method is also known as symbolic notation. In this method, the phasor is resolved into horizontal and vertical components and expressed in the complex form, i.e.,

Magnitude of phasor,

$$
\bar{V}=a+j b
$$

Its angle with OX-axis, $\theta=\tan ^{-1}(b / a)$
If angle $\theta$ would have been negative as shown in Figure 7.62(b), the vertical component would be negative. Then, the phasor $\bar{V}$ would have been represented as

$$
\bar{V}=a-j b
$$

## Trigonometric form

In this case, the horizontal and vertical components of the phasor are expressed in the trigonometric form. For example, in Figure 7.62(a), we get horizontal component, $a=V \cos \theta$ and vertical component, $b=V \sin \theta$
$\therefore \quad \bar{V}=V \cos \theta+j V \sin \theta \quad$ or $\quad \bar{V}=V(\cos \theta+j \sin \theta)$
If angle $\theta$ is negative, as shown in Figure 7.62(b), then

$$
\bar{V}=V(\cos \theta-j \sin \theta)
$$

## Polar form

The short form of trigonometric representation of a phasor is called polar form.

$$
\bar{V}=V \angle \theta^{\circ}
$$

where $V=$ the magnitude of the phasor and $\theta=$ phase angle measured in CCW direction from the reference axis, that is, OX-axis. There is no mathematical explanation for this form.

If angle $\theta$ is negative, as shown in Figure 7.62(b), then

$$
\bar{V}=V \angle-\theta^{\circ}
$$

In fact, all the above mentioned three mathematical forms of representing a phasor convey the same information, that is, magnitude of the phasor and its direction with the horizontal axis. Therefore, one form is converted into the other form rapidly as per the requirement to speed up the calculations.

### 7.22 ADDITION AND SUBTRACTION OF PHASOR QUANTITIES

The rectangular form is the best suited for addition and subtraction of phasor quantities. Therefore, if the phasor quantities are given in polar form, they are first converted into rectangular form and then added or subtracted.

Consider two voltage phasors represented as

$$
\bar{V}_{1}=a_{1}+j b_{1} \quad \text { and } \quad \bar{V}_{2}=a_{2}-j b_{2}
$$

### 7.22.1 Addition

In this case, the in-phase components of the quantities are added together, that is, horizontal components are added separately and the vertical components are added separately as

Resultant voltage, $\bar{V}=\bar{V}_{1}+\bar{V}_{2}=\left(a_{1}+j b_{1}\right)+\left(a_{2}-j b_{2}\right)=\left(a_{1}+a_{2}\right)+j\left(b_{1}-b_{2}\right)$
Magnitude of resultant, $V=\sqrt{\left(a_{1}+a_{2}\right)^{2}+\left(b_{1}-b_{2}\right)^{2}}$
Its angle with OX -axis, $\theta=\tan ^{-1} \frac{\left(b_{1}-b_{2}\right)^{2}}{\left(a_{1}+a_{2}\right)^{2}}$

### 7.22.2 Subtraction

Similar to addition, ordinary rules of phasor algebra are followed while subtracting the phasor quantities. Let phasor $\bar{V}_{2}$ be subtracted from phasor $\bar{V}_{1}$.
$\therefore$ Resultant voltage $\bar{V}_{1}=\bar{V}_{1}-\bar{V}_{2}=\left(a_{1}+j b_{1}\right)-\left(a_{2}-j b_{2}\right)=\left(a_{1}-a_{2}\right)+j\left(b_{1}+b_{2}\right)$
Magnitude of resultant, $V=\sqrt{\left(a_{1}+a_{2}\right)^{2}+\left(b_{1}-b_{2}\right)^{2}}$
Its angle with OX-axis, $\theta=\tan ^{-1} \frac{\left(b_{1}+b_{2}\right)}{\left(a_{1}-a_{2}\right)}$

### 7.23 MULTIPLICATION AND DIVISION OF PHASORS

The polar form is the best suited for multiplication and division of phasor quantities. Consider two voltage phasors represented as:

$$
\bar{V}_{1}=a_{1}+j b_{1}=V_{1} \angle \theta_{1} \text { and } \bar{V}_{2}=a_{2}-j b_{2}=V_{2} \angle-\theta_{2}
$$

### 7.23.1 Multiplication

While multiplying the phasor quantities when they are represented in polar form, their magnitudes are multiplied and their angles are added (algebraically).

$$
\bar{V}_{1} \times \bar{V}_{2}=V_{1} \angle \theta_{1} \times V_{2} \angle-\theta_{2}=V_{1} V_{2} \angle\left(\theta_{1}+\left(-\theta_{2}\right)\right)=V_{1} V_{2} \angle\left(\theta_{1}-\theta_{2}\right)
$$

### 7.23.2 Division

While dividing the phasor quantities when they are represented in polar form, their magnitudes are divided and their angles are subtracted (algebraically).

$$
\frac{\bar{V}_{1}}{\bar{V}_{2}}=\frac{V_{1} \angle \theta_{1}}{V_{2} \angle-\theta_{2}}=\frac{V_{1}}{V_{2}} \angle\left(\theta_{1}-\left(-\theta_{2}\right)\right)=\frac{V_{1}}{V_{2}} \angle\left(\theta_{1}+\theta_{2}\right)
$$

### 7.24 CONJUGATE OF A COMPLEX NUMBER

The two complex numbers (or phasors) are said to be conjugate if they differ only in the algebraic sign of their quadrature components. For example, consider a complex number $\bar{N}=a+j b=N \angle \theta$. Its conjugate will be $* \bar{N}=a-j b=N$ $\angle-\theta$. The two conjugate numbers are shown graphically in Figure 7.63. The following are the important properties of two conjugate number.

### 7.24.1 Addition

$$
\bar{N}+* \bar{N}=(a+j b)+(a-j b)=2 a
$$

The resultant is the sum of two horizontal components only.


Fig. 7.63 Position of vector $V$ and its conjugate

### 7.24.2 Subtraction

$$
\bar{N}-* \bar{N}=(a+j b)-(a-j b)=j 2 b
$$

The resultant is the sum of two vertical components only.

### 7.24.3 Multiplication

$$
\bar{N}-* \bar{N}=(a+j b)(a-j b)=a^{2}-j^{2} b^{2}=a^{2}+b^{2}
$$

The resultant contains no quadrature component. The conjugate of a complex number is used in determining the apparent power of an AC circuit in complex form.

### 7.25 POWERS AND ROOTS OF PHASORS

Powers and roots of a phasor quantity can be found very conveniently in polar form. If the phasor is not in polar form, convert it into polar form first, and then carry out the algebraic operation as follows:

1. Powers: Consider a phasor quantity represented in polar form as $\bar{A}=A \angle \theta$ Then, $(\bar{A})^{n}=A^{n} \angle n \times \theta$
2. Roots: Consider a phasor quantity represented in polar form as $\bar{A}=A \angle \theta$ Then, $(\bar{A})^{1 / n}=(a)^{1 / n} \angle \theta / n$

### 7.26 SOLUTION OF SERIES AND PARALLEL AC CIRCUITS BY PHASOR ALGEBRA

While solving series and parallel AC circuits, the alternating quantities are represented in complex form. The quantities are changed from rectangular to polar form or vice versa frequently as per requirement to obtain a quick solution. The readers are strongly advised to use polar form for multiplication and division of complex quantities. However, rectangular form should be used for addition and subtraction. Further, rectangular form is not used for multiplication and division.

## Example 7.41

An alternating voltage of $(160+j 120)$ is applied to a circuit and the current in the circuit is given by $(6+j 8)$ A. Find (i) the values of elements of the circuit, (ii) the power factor of the circuit, and (iii) power consumed.

## Solution:

Supply voltage, $\quad \bar{V}=(160+j 120)=200 \angle 36.87^{\circ} \mathrm{V}$
Circuit current, $\quad \bar{I}=(6+j 8)=10 \angle 53.13^{\circ} \mathrm{A}$
Circuit impedance, $\bar{Z}=\frac{\bar{V}}{\bar{I}}=\frac{200 \angle 36.87^{\circ}}{\angle 53.13^{\circ}}=20 \angle-16.26^{\circ} \Omega$

$$
=20\left(\cos 16.26^{\circ}-j \sin 16.26^{\circ}\right)=(19.2-j 5.6) \Omega
$$

(i) Resistance, $R=19.2 \Omega$

Capacitive reactance, $X_{\mathrm{C}}=5.6 \Omega ; C=1 / 2 \pi f X_{\mathrm{C}}=1 / 2 \pi \times 50 \times 5.6=568.4 \mu \mathrm{~F}$
(ii) Power factor, $\cos \phi=R / Z=19.2 / 20=0.96$ leading
(iii) Apparent power, $\bar{S}=\bar{V} * \bar{I}=200 \angle 36.87^{\circ} \times 10 \angle-53.13^{\circ}$

$$
=2,000 \angle-16.26^{\circ}=(1,920-j 560) \mathrm{VA}
$$

True power, $P=1,920 \mathrm{~W}$
Alternatively, $P=V I \cos \phi=200 \times 10 \times 0.96=1,920 \mathrm{~W}$

## Example 7.42

$Z_{1}=(150-j 157) \Omega, Z_{2}=(100-j 100) \Omega$ are connected in parallel to $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Find (i) $I_{1}, I_{2}$, (ii) $I$ (total current), (iii) total power, and (iv) pf Further, draw phasor diagram.
(U.P.T.U. 2007-08)

## Solution:

As per data, the circuit is shown in Figure 7.64

$$
\begin{aligned}
& \bar{Z}_{1}=150-j 157=217.14 \mid-46.31 \Omega \\
& \bar{Z}_{2}=100-j 110=148.66-47.73 \Omega
\end{aligned}
$$



Fig. 7.64 Circuit as per data


Fig. 7.65 Phasor diagram
(i) Current, $\bar{I}_{1}=\frac{\bar{V}}{\bar{Z}_{1}}=\frac{200 \underline{0^{\circ}}}{217.14-46.31}=0.92 \underline{46.31^{\circ} \mathrm{A}}$

Current, $\bar{I}_{2}=\frac{\bar{V}}{\bar{Z}_{2}}=\frac{20000^{\circ}}{148.66-47.73^{\circ}}=1.34547 .73^{\circ} \mathrm{A}$
(ii) Total current, $\bar{I}=\bar{I}_{1}+\bar{I}_{2}=0.92\left\lfloor 46.31^{\circ}+1.345 \Delta 47.73^{\circ}\right.$

$$
\begin{aligned}
& =(0.6355+j 0.6652)+(0.9047+j 0.9953) \\
& =(1.5402+j 1.6605)=2.265 \angle 47.15^{\circ}
\end{aligned}
$$

(iii) Total apparent power, $P_{\mathrm{a}}=V \bar{I}=200 \underline{0^{\circ}} \times 2.264 \underline{47.15^{\circ}}=45347.15^{\circ} \mathrm{VA}$

Real power, $P=P_{\mathrm{a}} \cos \phi=453 \times \cos 47.15=308 \mathrm{~W}$
(iv) Power factor $=\cos \phi=\cos 47.15^{\circ}=0.68$ leading

Phasor diagram is shown in Figure 7.65.

## Example 7.43

An inductive circuit draws 10 A and 1 kW from a 200 V , 50 Hz AC supply. Determine the (i) impedance in Cartesian from $(a+j b)$, (ii) impedance in polar form $Z$ $\angle \theta$, (iii) power factor, (iv) reactive power, and (v) apparent power.
(U.P.T.U.)

## Solution:

As per data, the circuit is shown in Figure 7.66
Here, $I=10 \mathrm{~A} ; P=1 \mathrm{~kW}=1,000 \mathrm{~W} ; V=200 \mathrm{~V} ; \phi=50 \mathrm{~Hz}$


Fig. 7.66 Circuit as per data

Impedance,

$$
Z=\frac{V}{I}=\frac{200}{10}=20 \Omega
$$

Resistance,

$$
R=\frac{P}{I^{2}}=\frac{1 \times 1000}{10^{2}}=10 \Omega
$$

Inductive reactance,

$$
\begin{aligned}
X_{\mathrm{L}} & =\sqrt{Z^{2}-R^{2}} \\
& =\sqrt{20^{2}-10^{2}}=17.32 \Omega
\end{aligned}
$$

Phase angle,

$$
\phi=\tan ^{-1} \frac{X_{L}}{R}=\tan ^{-1} \frac{17.32}{10}=60^{\circ}
$$

(i) Impedance in Cartesian form, $\bar{Z}=\left(R+j X_{\mathrm{L}}\right)=(10+j 17.32) \Omega$
(ii) Impedance in polar form, $\bar{Z}=Z \angle \phi=\sqrt{10^{2}+17.32^{2}} \angle \tan ^{-1} \frac{17.32}{10}$

$$
=20 \angle 60^{\circ} \Omega
$$

(iii) Power factor of the circuit, $\cos \phi=\cos 60^{\circ}=0.5$ lagging
(iv) Reactive power, $P_{\mathrm{r}}=V I \sin \phi=200 \times 10 \sin 60^{\circ}=1,732$ VAR
(v) Apparent power, $\stackrel{P}{\mathrm{r}}_{\mathrm{a}}=V I=200 \times 10=2,000 \mathrm{VA}$

## Example 7.44

Express in rectangular notation the admittance of the circuit have the following impedance: (i) $(4+j 6) \Omega$ and (ii) $20 \angle 30^{\circ} \Omega$

## Solution:

(i) Impedance, $\bar{Z}=4+j 6=\sqrt{(4)^{2}+(6)^{2}} \angle \tan ^{-1} 6 / 4=7.21 \angle 56.31^{\circ} \Omega$

Admittance, $\bar{Y}=\frac{1}{\bar{Z}}=\frac{1}{7.21 \angle 56.31^{\circ}}=0.1387 \angle-56.31^{\circ} \mathrm{mho}$
Admittance in rectangular form,

$$
\bar{Y}=0.1387\left(\cos 56.31^{\circ}-j \sin 56.31^{\circ}\right)=(0.0769-j 0.1154) \text { mho }
$$

i.e., conductance $=0.0769 \mathrm{mho}$; susceptance $=0.1154 \mathrm{mho}$ (inductive)
(ii) Impedance, $\bar{Z}=20 \angle 30^{\circ} \Omega$

Admittance, $\bar{Y}=\frac{1}{\bar{Z}}=\frac{1}{20 \angle 30^{\circ}}=0.05 \angle-30^{\circ} \mathrm{mho}$
Admittance in rectangular form:
$\bar{Y}=0.05\left(\cos 30^{\circ}-j \sin 30^{\circ}\right)=(0.0433+j 0.025)$ mho
i.e., conductance $=0.0433 \mathrm{mho}$; susceptance $=0.025 \mathrm{mho}$ (inductive)


Fig. 7.67 Circuit as per data

## Example 7.45

For the circuit shown in Figure 7.67, find the current and power drawn from the source.
(U.P.T.U. 2004-05)

Solution:

$$
\begin{aligned}
& \bar{Z}_{1}=3+j 4=553.13^{\circ} \\
& \bar{Z}_{2}=6+j 8=1053.13^{\circ}
\end{aligned}
$$

$$
\bar{Z}_{1}+\bar{Z}_{2}=(3+j 4)+(6+j 8)=9+j 12=1553.13^{\circ}
$$

Total impedance, $\quad \bar{Z}=\frac{\bar{Z}_{1} \times \bar{Z}_{2}}{\bar{Z}_{1}+\bar{Z}_{2}}$

$$
=\frac{553.13^{\circ} \times 10 \sqsubseteq 53.13^{\circ}}{155^{53.13^{\circ}}}=3.33353 .13^{\circ}
$$

Current drawn from the mains

Now,

$$
\bar{I}=\frac{\bar{V}}{\bar{Z}}=\frac{230 \underline{0^{\circ}}}{3.333553 .13^{\circ}}=69-53.13^{\circ}
$$

Power drawn,

$$
V=230 \mathrm{~V} ; I=69 \mathrm{~A} ; \phi=53.13^{\circ} \mathrm{lag}
$$

$$
P=V I \cos \phi=230 \times 69 \times \cos 53.13^{\circ}=9,522 \mathrm{~W}
$$

## Example 7.46

A coil parallel with a 200 mF capacitor is connected across a $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. The coil takes a current of 8 A and loss in the coil is 960 W . Calculate the (i) resistance of the coil, (ii) inductance of the coil, and (iii) power factor of the entire circuit.

## Solution:

The circuit is shown in Figure 7.68.
Power loss in the coil,

$$
P=I_{1}^{2} R=960 \mathrm{~W}
$$

$\therefore$ Resistance of the coil, $R=960 /(8)^{2}=15 \Omega$


Fig. 7.68 Circuit as per data

Impedance of the coil, $Z=V / I_{1}=200 / 8=25 \Omega$
Inductive reactance,

$$
X_{\mathrm{L}}=\sqrt{Z^{2}-R^{2}}=\sqrt{(25)^{2}-(15)^{2}}=20 \Omega
$$

Inductance,

$$
L=\frac{X_{\mathrm{L}}}{2 \pi f}=\frac{20}{2 \pi \times 50}=63.66 \mathrm{mH}
$$

Power factor of the coil,

$$
\cos \phi_{1}=\frac{R}{Z}=\frac{15}{25}=0.6 \mathrm{lag}
$$

and

$$
\sin \phi_{1}=\sin \cos ^{-1} 0.6=0.8
$$

$\therefore$ Current, $\quad \bar{I}_{1}=I_{1}\left(\cos \phi_{1}-j \sin \phi_{1}\right)=8(0.6-j 0.8)=(4.8-j 6.4) \mathrm{A}$
Capacitive reactance of capacitor in other branch,

$$
X_{\mathrm{C}}=\frac{1}{\omega C}=\frac{1}{2 \pi \times 50 \times 200 \times 10^{-6}}=15.915 \Omega
$$

Current in this branch,

$$
I_{2}=\frac{V}{X_{\mathrm{C}}}=\frac{200}{15.915}=12.566 \mathrm{~A}
$$

or

$$
\bar{I}_{2}=j 12.566 \mathrm{~A}\left(\_ \text {current leads the voltage by } 90^{\circ}\right)
$$

Total current,

$$
\bar{I}=\bar{I}_{1}+\bar{I}_{2}=4.8-j 6.4+j 12.566=(4.8+j 6.166) \mathrm{A}
$$

Power factor angle of whole circuit, $\phi=\tan ^{-1} 6.166 / 4.8=52.1^{\circ}$
$\therefore$ Power factor of entire circuit $=\cos \phi=\cos =52.1^{\circ}=0.6142$ leading

## Example 7.47

Consider an electric circuit shown in Figure 7.69. Determine (i) the current and power consumed in each branch and (ii) the supply current and power factor.
(U.P.T.U. 2001)


Fig. 7.69 Circuit as per data

Solution:
Branch I
Impedance, $\quad \bar{Z}_{1}=R \pm j 0$
$=10 \pm j 0=10 \angle 0^{\circ} \Omega$
$\bar{I}_{1}=\frac{\bar{V}}{\bar{Z}_{1}}=\frac{100 \angle 45^{\circ}}{10 \angle 0^{\circ}}$

$$
=10 \angle 45^{\circ}
$$

## Branch II

Impedance,

$$
\bar{Z}_{2}=R_{2}+j X_{\mathrm{L}_{2}}=5+j 5 \sqrt{3}=10 \angle 60^{\circ} \Omega
$$

Current,

$$
\bar{I}_{2}=\frac{\bar{V}}{\bar{Z}_{2}}=\frac{100 \angle 45^{\circ}}{10 \angle 60^{\circ}}=10 \angle-15^{\circ} \mathrm{A}
$$

## Branch III

Impedance,

$$
\bar{Z}_{3}=R_{3}-j X_{C_{3}}=5-j 5 \sqrt{3}=10 \angle-60^{\circ} \Omega
$$

Current,

$$
\bar{I}_{3}=\frac{\bar{V}_{3}}{\bar{Z}_{3}}=\frac{100 \angle 45^{\circ}}{10 \angle-60^{\circ}}=10 \angle 105^{\circ} \mathrm{A}
$$

Power consumed in Branch I, $P_{1}=I_{1}^{2} R_{1}=10^{2} \times 10=1,000 \mathrm{~W}$
Power consumed in Branch II, $P_{2}=I_{2}^{2} R_{2}=10^{2} \times 5=500 \mathrm{~W}$
Power consumed in Branch III, $P_{3}=I_{3}^{2} R_{3}=10^{2} \times 5=500 \mathrm{~W}$
(ii) Supply current, $\bar{I}=\bar{I}_{1}+\bar{I}_{2}+\bar{I}_{3}=10 \angle 45^{\circ}+10 \angle-15^{\circ}+10 \angle 105^{\circ}$

$$
\begin{aligned}
& =(7.071+j 7.071)+(9.659-j 2.588)+(-2.588+j 9.659) \\
& =14.142+j 14.142=20 \angle 45^{\circ} \mathrm{A}
\end{aligned}
$$

Power factor, $\cos \phi=\cos \left(45^{\circ}-45^{\circ}\right)=\cos 0^{\circ}=1.0$

## Example 7.48

Differentiate among real, reactive, and apparent power. Obtain the power factor of a two-branch parallel circuit where the first branch has $Z_{1}=(2+j 4)$ and second $Z_{2}=(6+j 0)$. To what value must the $6 \Omega$ resistor be changed to result in the overall power factor 0.9 lagging?
(U.P.T.U. June 2001)

## Solution:

The circuit is shown in Figure 7.70.
Admittance of Branch I, $\bar{Y}_{1}=\frac{1}{\bar{Z}_{1}}=\frac{1}{2+j 4}=\frac{2-j 4}{2^{2}+4^{2}}=(0.1-j 0.2) \mathrm{mho}$
Admittance of Branch II, $\bar{Y}_{2}=\frac{1}{\bar{Z}_{2}}=\frac{1}{6+j 0}=(0.167+j 0) \mathrm{mho}$
Equivalent admittance of the circuit, $\bar{Y}=\bar{Y}_{1}+\bar{Y}_{2}=(0.1-j 0.2)+(0.167+j 0)$

$$
=(0.267-j 0.2) \mathrm{mho}
$$

The magnitude of equivalent admittance, $Y=\sqrt{(0.267)^{2}+(0.2)^{2}}=0.333 \mathrm{mho}$

Power factor of the circuit,

$$
\cos \phi=\frac{0.267}{0.333}=0.8 \text { (lagging) }
$$



Fig. 7.70 Circuit as per data

To improve the pf to 0.9 lagging, let the resistance of the resistor placed in parallel branch be changed to $R \Omega$.
Now, total conductance of the circuit, $G^{\prime}=\left(\frac{1}{4}+0.1\right)$ mho
Total susceptance of the circuit, $B^{\prime}=B=-0.2$ mho
Phase angle of the circuit, $\phi^{\prime}=\cos ^{-1} 0.9=-25.84^{\circ}$ (negative sign shows lagging)
Phase angle of the circuit is given as
or

$$
\begin{aligned}
\phi^{\prime} & =\tan ^{-1} \frac{B^{\prime}}{G^{\prime}}=\tan ^{-1} \frac{-0.2}{\frac{1}{R}+0.1}=-25.84^{\circ} \\
\frac{0.2}{\frac{1}{R}+0.1} & =-0.4843 \quad \text { or } \quad R=3.2 \Omega
\end{aligned}
$$

## Example 7.49

Give the series equivalent of the parallel circuit as shown in Figure 7.71(a).


Fig. 7.71 (a) Given circuit (b) Equivalent circuit

## Solution:

Conductance of Branch I, $G_{1}=\frac{R_{1}}{R_{1}^{2}+X_{\mathrm{L}}^{2}}$
Conductance of Branch II, $G_{2}=\frac{R_{2}}{R_{2}^{2}+X_{\mathrm{G}}^{2}}$
Susceptance of Branch I, $B_{1}=\frac{-X_{\mathrm{L}}}{R_{2}^{2}+X_{\mathrm{L}}^{2}}$ (inductive)
Susceptance of Branch II, $B_{2}=\frac{-X_{\mathrm{C}}}{R_{2}^{2}+X_{\mathrm{C}}^{2}}$ (capacitive)

Total conductance of the circuit, $G=G_{1}+G_{2}=\frac{R_{1}}{R_{1}^{2}+X_{\mathrm{L}}^{2}}+\frac{R_{2}}{R_{2}^{2}+X_{\mathrm{C}}^{2}}$
Total susceptance of the circuit, $B=B_{1}+B_{2}=\frac{-X_{\mathrm{L}}}{R_{2}^{2}+X_{\mathrm{L}}^{2}}+\frac{R_{2}}{R_{2}^{2}+X_{\mathrm{C}}^{2}}$
Resultant admittance of the given parallel circuit, $\bar{Y}=\bar{Y}_{1}+\bar{Y}_{2}=\left(G_{1}+G_{2}\right)+j\left(B_{1}+B_{2}\right)$

$$
=G+j B
$$

or

$$
\begin{aligned}
Y & =\sqrt{\left(G_{1}+G_{2}\right)^{2}+\left(B_{1}+B_{2}\right)^{2}} \angle \tan ^{-1} \frac{B_{1}+B_{2}}{G_{1}+G_{2}} \\
& =\sqrt{G^{2}+B^{2}} \angle \tan ^{-1} \frac{B}{G}
\end{aligned}
$$

Resistance of the equivalent series circuit, $R_{\mathrm{S}}=Z \cos \phi=\frac{1}{Y} \times \frac{G_{1}+G_{2}}{Y}=\frac{G}{Y^{2}}$
Reactance of the equivalent series circuit, $X_{\mathrm{S}}=Z \sin \phi=\frac{1}{Y} \times \frac{B_{1}+B_{2}}{Y}=\frac{B}{Y^{2}}$
Hence, the equivalent series circuit is shown in Figure 7.71(b) whether it is inductive or capacitive depends upon the value of the net susceptance $B$. If $B$ is negative, the circuit shown in Figure 7.71 (b) will be inductive and in case $B$ is positive, circuit will be capacitive.

## Example 7.50

Consider the circuit shown in Figure 7.72(a). Find the steady state current flowing through each branch. Estimate the impedance, resistance, and reactance of a series circuit that will draw the same current at the same power factor from the line.


Fig. 7.72 (a) Given circuit (b) Equivalent circuit

## Solution:

Here, supply voltage, $\bar{V}=230 \pm j 0=230 \angle 0^{\circ}$ (reference vector)

$$
\begin{gathered}
R_{1}=5 \Omega, L=0.03 \mathrm{H} \\
R_{2}=8 \Omega, C=500 \mu \mathrm{~F} \\
X_{\mathrm{L}_{1}}=2 \pi f L=2 \pi \times 50 \times 0.03=9.425 \Omega
\end{gathered}
$$

$$
\begin{aligned}
X_{C_{2}} & =\frac{1}{2 \pi f C} \\
& =\frac{1}{2 \pi \times 50 \times 500 \times 10^{-6}}=6.366 \Omega
\end{aligned}
$$

## Branch I

Impedance, $\quad \bar{Z}_{1}=R_{1}+j X_{\mathrm{L}_{1}}$

$$
=5+j 9.425=10.67 \angle 62.05^{\circ} \Omega
$$

Admittance, $\quad \bar{Y}_{1}=\frac{1}{\bar{Z}_{1}}=\frac{1}{10.67 \angle 62.05^{\circ}}$

$$
=0.09372 \angle-62.05^{\circ} \mathrm{mho}=(0.04393-j 0.0828) \mathrm{mho}
$$

Current, $\quad \bar{I}_{1}=\bar{V} \bar{Y}_{1}=230 \times 0.09372 \angle-62.05^{\circ}$

$$
=21.56 \angle-62.05^{\circ} \mathrm{A}
$$

## Branch II

Impedance, $\bar{Z}_{2}=R_{2}-j X_{\mathrm{C}_{2}}=8-j 6.366=10.22 \angle-38.51^{\circ} \Omega$
Admittance, $\bar{Y}_{2}=\frac{1}{\bar{Z}_{2}}=\frac{1}{10.22 \angle-38.51^{\circ}}$

$$
=0.0987 \angle 38.51^{\circ} \mathrm{mho}=(0.07653+j 0.0609) \mathrm{mho}
$$

Current, $\bar{I}_{2}=\bar{V} \bar{Y}_{2}=230 \times 0.0978 \angle 38.51^{\circ}=22.494 \angle 38.51^{\circ} \mathrm{A}$
Total admittance of the parallel circuit,

$$
\begin{aligned}
\bar{Y}=\bar{Y}_{1}+\bar{Y}_{2} & =0.04393-j 0.0828+0.07653+j 0.0609 \\
& =(0.12046-j 0.0219) \mathrm{mho}=0.1224 \angle-10.3^{\circ}
\end{aligned}
$$

Impedance of the equivalent series circuit,

$$
\begin{aligned}
\bar{Z}=\frac{1}{\bar{Y}} & =\frac{1}{0.1224 \angle-10.3^{\circ}}=8.17 \angle 10.3^{\circ} \Omega \\
& =(8.038+j 1.46) \Omega
\end{aligned}
$$

Value of equivalent resistance, $R=8.038 \Omega$
Value of equivalent reactance, $X=1.46 \Omega$ (inductive)

## Example 7.51

For Figure 7.73, find $\bar{I}_{1}, \bar{I}_{2}$, and $\bar{I}_{3}$ and draw the complete phasor diagram to scale indicating these currents and voltages across elements.


Fig. 7.73 Given circuit

## Solution:

Taking supply voltage as the reference vector,

$$
\bar{V}=100 \angle 0^{\circ}
$$

From Figure 7.73, we get

$$
\begin{aligned}
& \bar{Z}_{1}=10 \pm j 0=10 \angle 0^{\circ} \\
& \bar{Z}_{2}=0+j 10=10 \angle 90^{\circ} \\
& \bar{Z}_{3}=5 \pm j 0=5 \angle 0^{\circ}
\end{aligned}
$$

When impedance $Z_{2}$ and $Z_{3}$ are in parallel,

$$
\begin{aligned}
\therefore \quad \frac{1}{\bar{Z}_{23}}=\frac{1}{\bar{Z}_{2}}+\frac{1}{\bar{Z}_{3}} & =\frac{1}{10 \angle 90^{\circ}}+\frac{1}{5 \angle 0^{\circ}} \\
& =0.1 \angle-90^{\circ}+0.2 \angle 0^{\circ}=-j 0.1+0.2=0.2236 \angle-26.56^{\circ} \\
\bar{Z}_{23} & =\frac{1}{0.2236 \angle-26.56^{\circ}}=4.472 \angle 26.56^{\circ}=4+j 2 \Omega
\end{aligned}
$$

Impedance $Z_{1}$ is in series with $Z_{23}$, and therefore, circuit impedance can be given as

$$
\begin{aligned}
& \bar{Z}=\bar{Z}_{1}+\bar{Z}_{23}=10+4+j 2=14+j 2=14.142 \angle 8.13^{\circ} \Omega \\
\therefore \quad & \text { Current } \bar{I}_{1}=\frac{\bar{V}}{\bar{Z}}=\frac{100 \angle 0^{\circ}}{14.142 \angle 8.13^{\circ}}=7.07 \angle-8.13^{\circ} \mathrm{A}
\end{aligned}
$$

Voltage across $10 \Omega$ resistor, $\bar{V}_{1}=\bar{I}_{1} \bar{Z}_{1}=7.07 \angle-8.13^{\circ} \times 10 \angle 0^{\circ}=70.7 \angle-8.13^{\circ} \mathrm{V}$
Voltage across parallel combination;

$$
\bar{V}_{23}=\bar{I}_{1} \bar{Z}_{23}=7.07 \angle-8.13 \times 4.472 \angle 26.56^{\circ}=31.62 \angle 18.43^{\circ}
$$

$\therefore \quad$ Current, $\bar{I}_{2}=\frac{\bar{V}_{23}}{\bar{Z}_{2}}=\frac{31.62 \angle 18.43^{\circ}}{10 \angle 90^{\circ}}=3.162 \angle-71.57^{\circ} \mathrm{A}$
Current,

$$
\bar{I}_{3}=\frac{\bar{V}_{23}}{\bar{Z}_{3}}=\frac{31.62 \angle 18.43^{\circ}}{5 \angle 0^{\circ}}=6.324 \angle 18.43^{\circ} \mathrm{A}
$$



Fig. 7.74 Phasor Diagram

Taking scale, $25 \mathrm{~V}=1 \mathrm{~cm}$ and $2 \mathrm{~A}=1 \mathrm{~cm}$; the phasor diagram representing all the quantities to the scale are shown in Figure 7.74.
$\mathrm{OA}=100 \mathrm{~V}=4 \mathrm{~cm}$
$\mathrm{OB}=70.7 \mathrm{~V}=2.828 \mathrm{~cm}$
$\mathrm{OC}=31.62 \mathrm{~V}=1.2648 \mathrm{~cm}$
$\mathrm{OF}=6.324 \mathrm{~A}=3.162 \mathrm{~cm}$
$\mathrm{OE}=3.162 \mathrm{~A}=1.581 \mathrm{~cm}$
$\mathrm{OD}=7.07 \mathrm{~A}=3.535 \mathrm{~cm}$

## Example 7.52

Calculate (i) total current and (ii) equivalent impedance for four-branched circuit of Figure 7.75.
(U.P.T.U.)


Fig. 7.75 Given circuit

## Solution:

Admittance of Branch I, $\quad \bar{Y}_{1}=\frac{1}{20+j 0}$

$$
=(0.05+j 0) \mathrm{mho}
$$

Admittance of Branch II, $\quad \bar{Y}_{2}=\frac{1}{0+j 10}$

$$
=(0-j 0.1) \mathrm{mho}
$$

Admittance of Branch III, $\bar{Y}_{3}=\frac{1}{0-j 20}=(0+j 0.05) \mathrm{mho}$
Admittance of Branch IV, $\bar{Y}_{4}=\frac{1}{5+j 8.66}=(0.05-j 0.0866) \mathrm{mho}$

Equivalent admittance of the circuit, $\bar{Y}=\bar{Y}_{1}+\bar{Y}_{2}+\bar{Y}_{3}+\bar{Y}_{4}$

$$
\begin{aligned}
& =(0.05+j 0)+(0-j 0.1)+(0+j 0.05)+(0.05-j 0.0866) \\
& =(0.1-j 0.1366)=0.1693 \angle-53.79^{\circ} \mathrm{mho}
\end{aligned}
$$

(i) Total current, $\bar{I}=\bar{V} \bar{Y}=200 \angle 30^{\circ} \times 0.1693 \angle-53.79^{\circ} \mathrm{mho}$

$$
=33.86 \angle-23.79^{\circ} \mathrm{A}
$$

(ii) Equivalent impedance, $\bar{Z}=\frac{1}{\bar{Y}}=\frac{1}{0.1693 \angle-53.79^{\circ}}=5.91 \angle 53.79^{\circ} \Omega$

## Example 7.53

Figure 7.76 shows a series-parallel circuit. Find (i) Admittance of each parallel branch (ii) Total circuit impedance (iii) Supply current and power factor (iv) Total power supplied by the source.
 N

Fig. 7.76 Given circuit


Fig. 7.77 Circuit as per data

## Solution:

The given circuit is redrawn with additions as shown in Figure 7.77.
(i) Admittance of parallel Branch I

$$
\begin{aligned}
\bar{Y}_{2} & =\frac{1}{\bar{Z}_{2}}=\frac{1}{4+j 3}=\frac{1}{4+j 3} \times \frac{4-j 3}{4-j 3} \\
& =\frac{4-j 3}{16+9}=\frac{4}{25}-\frac{j 3}{25}=(0.16-j 0.12) \mathrm{mho}
\end{aligned}
$$

Admittance of parallel Branch III

$$
\begin{aligned}
\bar{Y}_{3} & =\frac{1}{\bar{Z}_{3}}=\frac{1}{6-j 8}=\frac{1}{6-j 8} \times \frac{6+j 8}{6+j 8} \\
& =\frac{6+j 8}{36+64}=\frac{6+j 8}{100}=(0.06+j 0.08) \Omega
\end{aligned}
$$

(ii) Total admittance of parallel combination $(2,3)$

$$
\begin{aligned}
\bar{Y}_{23} & =\bar{Y}_{2}+\bar{Y}_{3} \\
& =(0.16-j 0.12)+(0.06+j 0.08) \\
& =(0.22-j 0.04) \mathrm{mho}
\end{aligned}
$$

Total circuit impedance

$$
\begin{aligned}
\bar{Z}=\bar{Z}_{1}+\frac{1}{\bar{Y}_{23}} & =(1.6+j 7.2)+\frac{1}{(0.22-j 0.04)} \\
Z_{1}+\frac{1}{Y_{23}} & =(1.6+j 7.2)+\frac{1}{(0.22-j 0.04)} \\
& =(1.6+j 7.2)+\frac{1}{(0.22-j 0.04)} \times \frac{0.22+j 0.04}{0.22+j 0.04} \\
& =(1.6+j 7.2)+\frac{0.22+j 0.04}{(0.0484+0.0016)} \\
& =(1.6+j 7.2)+\frac{0.22+j 0.04}{0.05}=1.6+j 7.2+4.4+j 0.8 \\
\bar{Z} & =(6.0+j 8.0) \Omega
\end{aligned}
$$

(iii) Supply current,

$$
I=\frac{V}{Z}=\frac{100}{\sqrt{(6)^{2}+(8)^{2}}}=10 \mathrm{~A}
$$

Supply power factor,

$$
\cos \phi=\frac{R}{|Z|}=\frac{6.0}{\sqrt{6^{2}+8^{2}}}=0.6 \text { (lagging) }
$$

(iv) Total power supplied by the source,

$$
P=V I \cos \phi=100 \times 10 \times 0.6=600 \mathrm{~W}
$$

## Example 7.54

Three impedances $(2+j 4),(3-j 5)$ and $(1-j 3)$ are connected in parallel. The combination is in series with a coil of resistance $3 \Omega$ and inductance 0.02 H to $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Find (i) the complex expression for the total impedance of the circuit, (ii) current taken from the supply, and (iii) power factor.
(U.P.T.U. Tut.)

## Solution:

The circuit is shown in Figure 7.78
Let us consider that various branches are connected in parallel.
Admittance of Branch I,

$$
\begin{aligned}
\bar{Y}_{1} & =\frac{1}{2+j 4}=\frac{2-j 4}{20} \\
& =(0.1-j 0.2) \\
& =0.224 \angle-63.4^{\circ} \mathrm{mho}
\end{aligned}
$$



Fig. 7.78 Circuit as per data

Admittance of Branch II, $\bar{Y}_{2}=\frac{1}{3-j 5}=\frac{3+j 5}{34}$

$$
=(0.088+j 0.147)=0.171 \angle 59^{\circ} \mathrm{mho}
$$

Admittance of Branch III, $\bar{Y}_{3}=\frac{1}{1-j 3}=\frac{1+j 3}{10}=(0.1+j 0.3)=0.316 \angle 71.56^{\circ} \mathrm{mho}$
Admittance of parallel Branch BC, $\bar{Y}_{\mathrm{BC}}=\bar{Y}_{1}+\bar{Y}_{2}+\bar{Y}_{3}$

$$
\begin{aligned}
& =(0.1-j 0.2)+(0.088+j 0.147)+(0.1+j 0.3) \\
& =(0.288+j 0.247) \mathrm{mho}=0.38 \angle 40.62^{\circ} \mathrm{mho}
\end{aligned}
$$

Impedance of parallel Branch BC, $\bar{Z}_{\mathrm{BC}}=\frac{1}{\bar{Y}_{\mathrm{BC}}}=\frac{1}{0.38 \angle 40.62^{\circ}}$

$$
=2.63 \angle-40.62^{\circ}=(2-j 1.71) \Omega
$$

Impedance of series branch, $\bar{Z}_{\mathrm{AB}}=(3+j 2 \pi \times 50 \times 0.02)$

$$
=(3+j 6.28)=6.96 \angle 64.47^{\circ} \Omega
$$

(i) Total impedance of the circuit, $\bar{Z}=\bar{Z}_{\mathrm{AB}}+\bar{Z}_{\mathrm{BC}}=(3+j 6.28)+(2-j 1.71)$

$$
=(5+j 4.57) \mathrm{W}
$$

(ii) Current taken from the supply, $\bar{I}=\frac{\bar{V}}{\bar{Z}}=\frac{230 \angle 0^{\circ}}{5+j 4.57}$

$$
=\frac{230 \angle 0^{\circ}}{6.77 \angle 42.43^{\circ}}=33.97 \angle-42.43^{\circ} \mathrm{A}
$$

(iii) Power factor $=\cos \phi=\cos \left(-42.43^{\circ}\right)=0.738$ (lagging)

## Example 7.55

In the circuit shown in Figure 7.79, determine the voltage at 50 Hz to be applied across AB in order that a current of 10 A flows in the capacitor.
(U.P.T.U. June 2001)


Fig. 7.79 Given circuit

## Solution:

## Branch I

$$
\begin{gathered}
R_{1}=5 \Omega ; X_{\mathrm{L}}=2 \pi \phi L_{1}=2 \pi \times 50 \times 0.0191=6 \Omega \\
\bar{Z}_{1}=(5+j 6) \Omega=7.81 \angle 50.2^{\circ} \Omega
\end{gathered}
$$

Branch II

$$
\begin{aligned}
R_{2}=7 \Omega ; X_{\mathrm{C}_{2}} & =\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 398 \times 10^{-6}}=8 \Omega \\
\bar{Z}_{2} & =(7-j 8 \Omega)=10 \angle-48.8^{\circ} \Omega
\end{aligned}
$$

## Branch III

$$
\begin{array}{r}
R_{3}=8 \Omega ;=2 \pi f L_{3}=2 \pi \times 50 \times 0.0318=10 \Omega \\
\bar{Z}_{3}=(8+j 10) \Omega=12.806 \angle 51.34^{\circ} \Omega
\end{array}
$$

Let the current flowing through Branch II be taken as reference phasor,

$$
\bar{I}_{2}=(10+j 0)=10 \angle 0^{\circ} \mathrm{A}
$$

Voltage drop across parallel combination,

$$
\begin{aligned}
\bar{V}_{\mathrm{AC}}=\bar{I}_{2} \bar{Z}_{2} & =10 \angle 0^{\circ} \times 10.63 \angle-48.8^{\circ} \\
& =106.3 \angle-48.8^{\circ} \mathrm{V}=(70.02-j 79.98) \mathrm{V}
\end{aligned}
$$

Current through Branch I, $=\bar{I}_{1}=\frac{\bar{V}_{\mathrm{AC}}}{\bar{Z}_{1}}=\frac{106.3 \angle-48.8^{\circ}}{7.81 \angle 50.2^{\circ}}$

$$
=13.61 \angle-99^{\circ} \mathrm{A}=(-2.13-j 13.44) \mathrm{A}
$$

Circuit current, $\bar{I}=\bar{I}_{1}+\bar{I}_{2}=(-2.13+j 13.44)+(10+j 0)$

$$
=(7.87-j 13.44)=15.57 \angle-59.65^{\circ} \mathrm{A}
$$

Voltage drop across Branch III, $\bar{V}_{\mathrm{CB}}=\bar{I} \bar{Z}_{3}=15.57 \angle-59.65^{\circ} \times 12.806 \angle 51.34^{\circ}$

$$
=199.4 \angle-8.31^{\circ} \mathrm{V}=(197.31-j 28.82) \mathrm{V}
$$

Voltage applied across $\mathrm{AB}, \bar{V}_{\mathrm{AB}}=\bar{V}_{\mathrm{AC}}+\bar{V}_{\mathrm{CB}}=(70.02-j 79.98)+(197.31-j 28.82)$

$$
=(267.33-j 108.8) \mathrm{V}=28.62 \angle 22.15^{\circ} \mathrm{V}
$$

## Example 7.56

An inductive coil of $(6+j 8) \Omega$ impedance is connected to $100 \mathrm{~V}, 50 \mathrm{~Hz}$ AC supply. It is desired to improve power factor of supply current to 0.8 lagging by connecting a capacitor (i) in series with the coil and (ii) in parallel with the coil. Determine the value of the capacitance in each case.
(U.P.T.U., Feb. 2002)

## Solution:

Let a capacitor of capacitance $C_{\mathrm{S}}$ be connected in series with the coil, as shown in Figure 7.80(a), so as to increase the pf of supply current to 0.8 lagging.
Resistance of the circuit, $R=6 \Omega$
Inductive reactance of the coil, $X_{\mathrm{L}}=8 \Omega$
Capacitive reactance of the capacitor, $X_{\mathrm{C}}=\frac{1}{2 \pi f C_{\mathrm{S}}}$
Effective reactance of the circuit, $X=X_{\mathrm{L}}-X_{\mathrm{C}}$
Improved pf of the circuit, $\cos \phi=0.8$ lagging

(a)

(b)

Fig. 7.80 (a) Capacitor connected in series (b) Capacitor connected in parallel

$$
\tan \phi=\tan \cos ^{-1} 0.8=0.75
$$

Since, $\quad \frac{X}{R}=\tan \phi \Rightarrow X=R \tan \phi=6 \times 0.75=4.5 \Omega$
or

$$
X_{\mathrm{L}}-X_{\mathrm{C}}=4.5 \quad \text { or } \quad X_{\mathrm{C}}=8-4.5=3.5 \Omega
$$

or

$$
\frac{1}{2 \pi f C_{\mathrm{s}}}=3.5 \Omega
$$

or

$$
C_{S}=\frac{1}{2 \pi f \times 3.5}=\frac{1}{2 \pi \times 50 \times 3.5}=909.46 \mu \mathrm{~F}
$$

(ii) Let a capacitor of capacitance $C_{\mathrm{p}}$ be connected across the coil, as shown in Figure 7.80(b), so as to increase the pf of supply current to 0.8 lagging.

Conductance of coil, $G=\frac{R}{R^{2}+X^{2}}=\frac{6}{6^{2}+8^{2}}=0.06 \mathrm{mho}$
Susceptance of coil, $B_{\mathrm{L}}=\frac{X}{R^{2}+X^{2}}=\frac{8}{6^{2}+8^{2}}=0.08 \mathrm{mho}$ (inductive)
Capacitive susceptance of capacitor $C_{\mathrm{p}}, B_{\mathrm{C}}=2 \pi f C_{\mathrm{p}}$

Since,

$$
\frac{B}{G}=\tan \phi \quad \text { or } \quad \frac{B_{\mathrm{L}}-B_{\mathrm{C}}}{G}=0.75
$$

or

$$
B_{\mathrm{L}}-B_{\mathrm{C}}=0.75 \times G=0.75 \times 0.06=0.045 \mathrm{mho}
$$

$$
B_{\mathrm{C}}=B_{\mathrm{L}}-0.045=0.08-0.045=0.035 \mathrm{mho}
$$

or

$$
\begin{aligned}
2 \pi f C_{\mathrm{p}} & =0.035 \\
C_{\mathrm{p}} & =\frac{0.035}{2 \pi \times 50}=111.4 \mu \mathrm{~F}
\end{aligned}
$$

## Example 7.57

In the circuit shown in Figure 7.81, determine the magnitude of line current $I$ and its phase difference with respect to the applied voltage in terms of circuit parameters. Draw the phasor diagram of the circuit.

## Solution:

Current in the three components:


Fig. 7.81 Given circuit

$$
\overline{I_{\mathrm{R}}}=\frac{V}{R} ; \overline{I_{\mathrm{L}}}=\frac{V}{j \omega L}=\frac{j V}{j^{2} \omega L}=-j \frac{V}{\omega L} ; \overline{I_{\mathrm{C}}}=j \omega C V
$$

Line current,

$$
\begin{aligned}
\bar{I}=\overline{I_{\mathrm{R}}}+\overline{I_{\mathrm{L}}}+\overline{I_{\mathrm{C}}} & =\frac{V}{R}-j \frac{V}{\omega L}+j \omega C V \\
& =V\left(\frac{1}{R}-j\left(\frac{1}{\omega L}-\omega C\right)\right)
\end{aligned}
$$



Fig. 7.82 Phasor diagram

Current magnitude, $I=V \sqrt{\left(\frac{1}{R}\right)^{2}+\left(\frac{1}{\omega L}-\omega C\right)^{2}}$
Phase difference between voltage and current,

$$
\phi=\tan ^{-1} \frac{(1 / \omega L)-\omega C}{R} \text { negative }
$$

$\therefore i=I_{\mathrm{m}} \sin \left(\omega t+\phi_{\mathrm{L}}-\phi\right)$. The phasor diagram is shown in Figure 7.82.

## Example 7.58

In the circuit shown in Figure 7.83(a), the reactance of the capacitor $C_{1}$ is $4 \Omega$, the reactance of capacitor $C_{2}$ is $8 \Omega$, and the reactance of $L$ is $8 \Omega$. A sinusoidal voltage of 120 V is applied to the circuit. Find (ii) current in each branch and (iii) power loss in the circuit. (U.P.T.U., Feb 2002)


Fig. 7.83 (a) Given circuit (b) Circuit as per data

## Solution:

Here, $X_{\mathrm{C}_{1}}=4 \Omega ; \bar{Z}_{\mathrm{ab}}=-j 4 \Omega ; X_{\mathrm{C}_{2}}=8 \Omega$; i.e., $\bar{Z}_{2}=-j 8 \Omega$;

$$
X_{L_{3}}=8 \Omega ; \bar{Z}_{3}=R_{3}+j X_{L_{3}}=(4+j 8) \Omega=8.94 \angle 63.44^{\circ}
$$

The circuit is redrawn as shown in Figure 7.83(b).
Impedance of parallel branch, $\bar{Z}_{\mathrm{bc}}=\frac{\bar{Z}_{2} \bar{Z}_{3}}{\bar{Z}_{2}+\bar{Z}_{3}}=\frac{(-j 8)(4+j 8)}{-j 8+(4+j 8)}$

$$
=\frac{64-j 32}{4}=(16-j 8)=17.89 \angle-26.565^{\circ} \Omega
$$

Total impedance of the circuit, $\bar{Z}=\bar{Z}_{\mathrm{ab}}+\bar{Z}_{\mathrm{bc}}=(-j 4)+(16-j 8)=(16-j 12)=20 \angle-36.87^{\circ} \Omega$
Circuit current, $\bar{I}=\frac{\bar{V}}{\bar{Z}}=\frac{120 \angle 0^{\circ}}{20 \angle-36.87^{\circ}}$

$$
=6 \angle 36.87^{\circ} \mathrm{A}=(4.8+j 3.6) \mathrm{A}
$$

Current in branch $\bar{I}_{\mathrm{ab}}=\bar{I}=(4.8+j 3.6) \mathrm{A}$
Voltage across $\bar{V}_{\mathrm{ab}}=\bar{I}_{\mathrm{ab}} \bar{Z}_{\mathrm{ab}}=6 \angle 36.87^{\circ} \times 4 \angle-90^{\circ} \mathrm{V}$

$$
=24 \angle-53.13^{\circ}=(14.4-j 19.2) \mathrm{V}
$$

Voltage across $\bar{V}_{\mathrm{bc}}=\bar{I} \times \bar{Z}_{\mathrm{bc}}=6 \angle 36.87^{\circ} \times 17.89 \angle-26.56^{\circ} \mathrm{V}$

$$
=107.33 \angle 10.31^{\circ}=(105.6+j 19.2) \mathrm{V}
$$

Current in the capacitor $C_{2}=\frac{V_{\mathrm{bc}}}{-j 8}=\frac{107.33 \angle 10.31}{8 \angle-90^{\circ}}$

$$
=13.416 \angle 100.31^{\circ} \mathrm{A}=(-2.4+j 3.2) \mathrm{A}
$$

Current in $\bar{Z}_{2}=\frac{\bar{V}_{\mathrm{BC}}}{\bar{Z}_{3}}=\frac{107.33 \angle 10.31^{\circ}}{8.94 \angle 63.44^{\circ}} \mathrm{A}$

$$
=12 \angle-53.13^{\circ} \mathrm{A}=(7.2-j 9.6) \mathrm{A}
$$

(iii) Power loss in the circuit, $P=V \times I \times \cos \phi$

$$
=120 \times 6 \times \cos 36.87^{\circ}=576 \mathrm{~W}
$$

## Example 7.59

A potential difference of $230 \angle 30^{\circ} \mathrm{V}$ is applied to two circuits connected in parallel. The currents in the respective branches are $23 \angle 60^{\circ}$ and $46 \angle 30^{\circ}$. Find (i) circuit current in rectangular form, (ii) active power in each branch and in the whole circuit, and (iii) reactive power in each branch and in the whole circuit.

## Solution:

Applied voltage, $\bar{V}=230 \angle 30^{\circ}$

$$
\text { Current in Branch I, } \bar{I}_{1}=23 \angle 60^{\circ} \text {; current in Branch II, } \bar{I}_{2}=46 \angle 30^{\circ}
$$

## In rectangular form

Branch current, $\bar{I}_{1}=23 \cos 60^{\circ}+j 23 \sin 60^{\circ}=23 \times 0.5+j 23 \times 0.866=(11.5+j 19.92) \mathrm{A}$
Branch current, $\bar{I}_{2}=46 \cos 30^{\circ}+j 46 \sin 30^{\circ}=46 \times 0.866+j 46 \times 0.5=(39.84+j 23) \mathrm{A}$
Circuit current, $\bar{I}=\bar{I}_{1}+\bar{I}_{2}=(11.5+j 19.92)+(39.84+j 23)=(51.34+j 42.92) \mathrm{A}$
Apparent power consumed in Branch I,

$$
\begin{aligned}
S_{1} & =P_{1}+j P_{\mathrm{r} 1}=\bar{V}^{*} \bar{I}_{1}=230 \angle 30^{\circ} \times 23 \angle-60^{\circ}=230 \times 23 \angle\left(30^{\circ}-60^{\circ}\right) \\
& =5,290 \cos 30^{\circ}-5,290 \sin 30^{\circ}=5,290 \times 0.866-j 5,290 \times 0.5=(4,581.14-j 2,645) \mathrm{VA}
\end{aligned}
$$

Active power, $P_{1}=4,581.14 \mathrm{~W}$; reactive power, $P_{\mathrm{r} 1}=2,645 \mathrm{VAR}$
Apparent power consumed in second branch

$$
\begin{aligned}
S_{2}=P_{2}+j P_{\mathrm{r} 2}=\bar{V} * \bar{I}_{2} & =230 \angle 30^{\circ} \times 46 \angle-30^{\circ}=230 \times 46 \angle 0^{\circ} 10,580 \angle 0^{\circ} \\
& =(10,580 \pm j 0) \mathrm{VA}
\end{aligned}
$$

Active power, $P_{2}=10,580 \mathrm{~W}$; reactive power, $P_{\mathrm{r} 2}=0$
Total active power, $P=P_{1}+P_{2}=4,581.14+10,580=15,161.14 \mathrm{~W}$
Total reactive power, $P_{\mathrm{r}}=P_{\mathrm{r} 1}+P_{\mathrm{r} 2}=2,645+0=2,645$ VAR

## Example 7.60

A series R-L-C circuit has $R=10 \Omega, L=0.1 \mathrm{H}$, and $C=8 \mu \mathrm{~F}$. Determine (i) resonant frequency, (ii) $Q$-factor of the circuit at resonance, and (iii) the half-power frequencies.
(U.P.T.U. Sep. 2001)

## Solution:

Here, $R=10 \Omega ; L=0.1 \mathrm{H} ; C=8 \mu \mathrm{~F}=8 \times 10^{-6} \mathrm{~F}$
(i) Resonant frequency, $f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}=\frac{1}{2 \pi \sqrt{0.1 \times 8 \times 10^{-6}}}=177.95 \mathrm{~Hz}$
(ii) $Q$-factor of the circuit at resonance $=\frac{1}{R} \sqrt{\frac{L}{C}}=\frac{1}{10} \sqrt{\frac{0.1}{8 \times 10^{-6}}}=11.18$
(iii) Lower half-power frequency, $f_{1}=f_{\mathrm{r}}-\frac{R}{4 \pi L}=177.95-\frac{10}{4 \pi \times 0.1}=169 \mathrm{~Hz}$

Upper half-power frequency, $f_{2}=f_{\mathrm{r}}+\frac{R}{4 \pi L}=177.95-\frac{10}{4 \pi \times 0.1}=185.8 \mathrm{~Hz}$

## Example 7.61

Determine the frequency at which the voltage $V_{0}$ in Figure 7.84 is zero.
(U.P.T.U. Tut.)

## Solution:

Voltage $V_{0}$ will come out to be zero in the given circuit only when voltage drop across 100 mH inductor becomes equal to voltage drop across $0.05 \mu \mathrm{~F}$ capacitor. This condition can be obtained by making the supply frequency equal to resonant frequency.

$$
\text { i.e., } \begin{aligned}
f_{\mathrm{r}} & =\frac{1}{2 \pi \sqrt{L C}} \\
& =\frac{1}{2 \pi \sqrt{100 \times 10^{-3} \times 0.05 \times 10^{-6}}}=2,250 \mathrm{~Hz}
\end{aligned}
$$



Fig. 7.84 Given circuit

## Example 7.62

If the bandwidth of a resonant circuit is 10 kHz and the lower half-power frequency is 120 kHz , find out the value of the upper half-power frequency and the quality factor of the circuit.
(U.P.T.U. June 2004)

## Solution:

Bandwidth of the resonant circuit, $\Delta f=10 \mathrm{kHz}$
Lower half-power frequency, $f_{1}=120 \mathrm{kHz}$
We know that lower half-power frequency, $f_{1}=f_{\mathrm{r}}-\frac{\Delta f}{2}$
Resonant frequency, $f_{\mathrm{r}}=f_{1}+\frac{\Delta f}{2}=120+\frac{10}{2}=125 \mathrm{kHz}$
Upper half-power frequency, $f_{2}=f_{\mathrm{r}}+\frac{\Delta f}{2}=120+\frac{10}{2}=130 \mathrm{kHz}$
Quality factor of the resonant circuit, $Q_{\mathrm{r}}=\frac{f_{\mathrm{r}}}{\Delta f} \frac{125}{10}=12.5$

## Example 7.63

A coil of resistance $40 \Omega$ and inductance 0.75 H forms a part of a series circuit for which resonant frequency in 55 Hz . If the supply is $250 \mathrm{~V}, 50 \mathrm{~Hz}$, find (i) line current, (ii) power factor, (iii) power consumed, and (iv) voltage across the coil.
(U.P.T.U. June 2001)

## Solution:

The circuit is shown in Figure 7.85.
Resonant frequency, $f_{\mathrm{r}}=55 \mathrm{~Hz}$
At resonant frequency, $X_{\mathrm{L}}=X_{\mathrm{C}}$

$$
\begin{aligned}
\therefore \quad 2 \pi f_{\mathrm{r}} L & =\frac{1}{2 \pi f_{\mathrm{r}} C} \quad \text { or } \quad C=\frac{1}{\left(2 \pi f_{\mathrm{r}}\right)^{2} L} \\
& =\frac{1}{(2 \pi \times 55)^{2} \times(0.75)}=11.16 \mu \mathrm{~F}
\end{aligned}
$$



Fig. 7.85 Circuit as per data

## At 250 V, 50 Hz

Inductive reactance of the coil, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.75=235.62 \Omega$
Capacitive reactance of the capacitor, $X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 11.16 \times 10^{-6}}=285.1 \Omega$
Reactance of circuit, $X=X_{\mathrm{L}}-X_{\mathrm{C}}=235.62-285.1=-49.48 \Omega$
Impedance of the circuit, $Z=\sqrt{R^{2}+X^{2}}=\sqrt{(40)^{2}+(-49.48)^{2}}=63.63 \Omega$
(i) Line current, $I=\frac{V}{Z}=\frac{250}{63.63}=3.93 \mathrm{~A}$
(ii) Power factor, $\cos \phi=\frac{R}{Z}=\frac{40}{63.63}=0.629$ (leading) (since $X_{\mathrm{C}}>X_{\mathrm{L}}$ )
(iii) Power consumed, $P=I^{2} R=(3.93)^{2} \times 40=618 \mathrm{~W}$

Impedance of the coil, $Z_{\text {coil }}=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(40)^{2}+(235.62)^{2}}=239 \Omega$
(iv) Voltage across the coil, $V_{1}=I Z_{\text {coil }}=3.93 \times 239=939.3 \mathrm{~V}$

## Example 7.64

A series circuit consists of a resistance of $4 \Omega$, an inductance of 0.5 H , and a variable capacitance in series across a $100 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate (i) the value of capacitance to produce resonance, (ii) voltage across the capacitance, and (iii) $Q$-factor of the circuit.
(U.P.T.U. July 2002)

## Solution:

Inductive reactance, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.5=157.08 \Omega$
At resonance $X_{\mathrm{C}}=X_{\mathrm{L}}=157.08 \Omega$
Value of capacitance required, $C=\frac{1}{2 \pi f X_{\mathrm{C}}}=\frac{1}{2 \pi \times 50 \times 157.08}=20.26 \mu \mathrm{~F}$
Current flowing through the circuit at resonance,

$$
I=\frac{V}{Z}=\frac{V}{R}=\frac{100}{4}=25 \mathrm{~A}(Z=R \text { at resonance })
$$

(ii) Voltage across capacitor, $V_{\mathrm{C}}=I X_{\mathrm{C}}=25 \times 157.08=3,927 \mathrm{~V}$
(iii) $Q$-factor of the circuit, $Q=\frac{2 \pi f_{\mathrm{r}} L}{R}=\frac{157.08}{4}=39.27$

## Example 7.65

A $20 \Omega$ resistor is connected is series with an inductor and a capacitor across a variable frequency 25 V supply. When the frequency is 400 Hz , the current is at its maximum value of 0.5 A and the potential difference across the capacitor is 150 V . Calculate the resistance and inductance of the inductor.
(U.P.T.U., Jan. 2003)

## Solution:

Here, $R=20 \Omega ; V=25 \mathrm{~V}$

$$
\text { At } f=400 \mathrm{~Hz}, I_{\max }=0.5 \mathrm{~A}
$$

In a series $\mathrm{R}-\mathrm{L}-\mathrm{C}$ circuit, the current is maximum at resonance, and therefore,
or

$$
I_{\max }=\frac{V}{R}
$$

$$
\text { circuit resistance, } R=\frac{V}{I_{\max }}=\frac{25}{0.5}=50 \Omega
$$

Voltage across the capacitor, $V_{\mathrm{C}}=\frac{I_{\max }}{2 \pi f C}$
$\therefore$ Circuit capacitance, $C=\frac{I_{\max }}{2 \pi f V_{\mathrm{C}}}=\frac{0.5}{2 \pi \times 400 \times 150}=1.33 \mu \mathrm{~F}$
Under resonant condition, $X_{\mathrm{L}}=X_{\mathrm{C}}$
or

$$
2 \pi f L=\frac{1}{2 \pi f C}
$$

$\therefore \quad$ Inductance of inductor, $L=\frac{1}{(2 \pi f)^{2} \times C}=\frac{1}{(2 \pi \times 400)^{2} \times 1.3263 \times 10^{-6}}$ $=0.119 \mathrm{H}$

## PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by AC parallel circuits?
2. Define admittance, conductance, and susceptance.
3. Convert the complex number $(\bar{Z}=3+j 4)$ into polar form.
4. How two complex numbers are added?

## Test Questions

1. Explain the vector (or phasor) method of solving AC parallel circuit (considering two branches only.)
2. What is admittance triangle? How it is related to impedance triangle?
3. How a phasor is represented mathematically in rectangular form, trigonometric form, and polar form?

## Numericals

1. Two coils are connected in parallel across a $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. At the supply frequency, their impedances are 6 and $10 \Omega$, respectively, and their resistances are 2 and $3 \Omega$, respectively. Calculate the (i) current in each coil, (ii) total current, and (iii) total power.
(Ans. $33.33 \mathrm{~A} ; 20 \mathrm{~A} ; 53.3 \mathrm{~A}$;
3,421.6 W)
2. Estimate the current that will flow through a coil having an inductance of 0.02 H and a resistance of $5 \Omega$ when connected to $200 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Find the capacitance of a capacitor when connected in series with a $5 \Omega$ resistor will take the same current as the coil. Find the current taken by the two circuits when connected in parallel.
(Ans. $506.6 \mu \mathrm{~F}, 31.02 \mathrm{~A}$ )
3. A resistance of $10 \Omega$ is in series with an inductive reactance of $6.28 \Omega$ and the combination is in parallel with a circuit consisting of a resistance of $20 \Omega$ in series with a capacitance of $100 \mu \mathrm{~F}$. The abovementioned branches are in series with a resistance of $15 \Omega$ and an inductive reactance of $15.7 \Omega$. Calculate the equivalent resistance, reactance, and impedance of the whole circuit. Frequency of the system may be taken as 50 Hz .
(Ans. $25.9 \Omega, 18.56 \Omega, 31.86 \Omega$ )
4. Two impedances given by $Z_{1}=(10+j 5) \Omega$ and $Z_{2}(8+j 6) \Omega$ are joined in parallel and connected across a voltage $(200+j 0) \mathrm{V}$. Calculate the branch currents and the total current. Further, calculate the total power consumed.
(Ans. $17.89 \mathrm{~A}, 20 \mathrm{~A}, 37.3 \mathrm{~A}, 6400 \mathrm{~W}$ )


Fig. 7.86 Parallel circuit


Fig. 7.87 Phasor diagram

### 7.27 PARALLEL RESONANCE

An AC circuit containing an inductor and capacitor in parallel is said to be in parallel resonance when the circuit current is in phase with the applied voltage. Consider an inductor of $L$ Henry having some resistance $R \Omega$ connected in parallel with a capacitor of capacitance $C$ Farad across a supply voltage of $V \mathrm{~V}$ as shown in Figure 7.86. The phasor diagram of the circuit is shown in Figure 7.87.

The circuit current $I_{\mathrm{r}}$ will only be in phase with the supply voltage when

$$
I_{\mathrm{C}}=I_{\mathrm{L}} \sin \phi_{\mathrm{L}}
$$

Since at resonance, the reactive component of current is suppressed, the circuit draws minimum current under this condition.

### 7.27.1 Resonant Frequency

The value of $X_{\mathrm{L}}(=2 \pi f L)$ and $X_{\mathrm{C}}(=1 / 2 \pi f C)$ can be changed by changing the supply frequency. When frequency increases, the value of $X_{\mathrm{L}}$ and consequently the value of $Z_{\mathrm{L}}$ increases. This decreases the magnitude of current $I_{\mathrm{L}}$ that also lags behind the voltage $V$ by a progressively greater angle. On the other hand, the value of $X_{\mathrm{C}}$ decreases and consequently the value of $I_{\mathrm{C}}$ increases. At some frequency $f_{\mathrm{r}}$ (called resonance frequency), $I_{\mathrm{C}}=I_{\mathrm{L}} \sin \phi_{\mathrm{L}}$ and resonance occurs.
$\therefore$ At parallel resonance, $I_{\mathrm{C}}=I_{\mathrm{L}} \sin \phi_{\mathrm{L}}$
where $I_{\mathrm{L}}=\frac{V}{Z_{\mathrm{L}}} ; \sin \phi_{\mathrm{L}}=\frac{X_{\mathrm{L}}}{Z_{\mathrm{L}}} \quad$ and $\quad I_{\mathrm{C}}=\frac{V}{X_{\mathrm{C}}} \quad \therefore \frac{V}{X_{\mathrm{C}}}=\frac{V}{Z_{\mathrm{L}}} \times \frac{X_{\mathrm{L}}}{Z_{\mathrm{L}}} \quad$ or $\quad X_{\mathrm{L}} X_{\mathrm{C}}=Z_{\mathrm{L}}^{2}$
or

$$
\begin{equation*}
\frac{\omega L}{\omega C}=Z_{\mathrm{L}}^{2}=\left(R^{2}+X_{\mathrm{L}}^{2}\right) \tag{7.17}
\end{equation*}
$$

or

$$
\frac{L}{C}=R^{2}+\left(2 \pi f_{\mathrm{r}} L\right)^{2} \quad \text { or } \quad 2 \pi f_{\mathrm{r}} L=\sqrt{\frac{L}{C}-R^{2}}
$$

$$
\begin{equation*}
f_{\mathrm{r}}=\frac{1}{2 \pi L} \sqrt{\frac{L}{C}-R^{2}}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}} \tag{7.18}
\end{equation*}
$$

If $R$ is very small as compared to $L$, then
resonance frequency, $f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}$

### 7.27.2 Effect of Parallel Resonance

At parallel resonance, line current, $I_{\mathrm{r}}=I_{\mathrm{L}} \cos \phi$
or

$$
\frac{V}{Z_{\mathrm{r}}}=\frac{V}{Z_{\mathrm{L}}} \times \frac{R}{Z_{\mathrm{L}}} \quad \text { or } \quad \frac{1}{Z_{\mathrm{r}}}=\frac{R}{Z_{\mathrm{L}}^{2}}
$$

or

$$
\frac{1}{Z_{\mathrm{r}}}=\frac{R}{L / C}=\frac{C R}{L} \quad\left(\text { since } Z_{\mathrm{L}}^{2}=L / C\right. \text { from Equation (7.17)) }
$$

$\therefore$ Circuit impedance, $Z_{\mathrm{r}}=L / C R$
This expression shows that

1. Circuit impedance $Z_{\mathrm{r}}(=L / C R)$ is a pure resistive because there is no frequency terms present. If the value of $L, R$, and $C$ is in henry, ohm, and farad, then the value of $Z_{\mathrm{r}}$ is in ohms.
2. The value of $Z_{\mathrm{r}}$ is very high because the ratio $L / C$ is very large at parallel resonance.
3. The value of circuit current $I_{\mathrm{r}}\left(=V / Z_{\mathrm{r}}\right)$ is very small because the value of $Z_{\mathrm{r}}$ is very high.
4. The current flowing through the capacitor and coil is much greater than the line current because the impedance of each branch is quite low than circuit impedance $Z_{r}$.

Since a parallel resonant circuit can draw a very small current and power from the mains, it is often regarded as rejected circuit.

### 7.27.3 Resonance Curve

A current-frequency curve for a typical parallel resonant circuit is shown in Figure 7.88. The value of line current $I_{\mathrm{r}}\left(=V_{\mathrm{r}} / Z_{\mathrm{r}}\right)$ is minimum at resonance.

### 7.28 Q-FACTOR OF A PARALLEL RESONANT CIRCUIT

We have seen that at parallel resonance, the current circulating between the two branches is many times greater than the line current drawn from the mains. This current simplification produced by the resonance is called the $Q$-factor of the parallel resonant circuit.

$$
Q \text {-factor }=\frac{\text { Current circulating between } L \text { and } C}{\text { Line current }}=\frac{I_{\mathrm{C}}}{I_{\mathrm{r}}}
$$



Fig. 7.88 Graph between supply frequency and circuit current

Now, $\quad I_{\mathrm{C}}=V / X_{\mathrm{C}}=2 \pi f_{\mathrm{r}} C V \quad$ and $\quad I_{\mathrm{r}}=\frac{V}{L / C R}$
$\therefore \quad Q$-factor $=\frac{2 \pi f_{\mathrm{r}} C V}{V} \times \frac{L}{C R}=\frac{2 \pi f_{\mathrm{r}} L}{R} \quad \ldots$ same as for series circuit
or

$$
Q \text {-factor }=\frac{2 \pi L}{R} \cdot \frac{1}{2 \pi \sqrt{L C}}
$$

$$
\left(\Theta f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}} \text { neglecting } R\right)
$$

or $\quad Q$-factor $=\frac{1}{R} \sqrt{\frac{L}{C}} \quad$ or $\quad Q$-factor $=\sqrt{\frac{L}{C}}($ neglecting $R)$
The value of $Q$-factor is same as for series resonance.

### 7.29 COMPARISON OF SERIES AND PARALLEL RESONANT CIRCUITS

The comparison of series and parallel resonant circuits is given in Table 7.1.
Table 7.1 Comparison of Series and Parallel Resonant Circuits

| S.No. | Particulars | Series Circuit | Parallel Circuit |
| :--- | :--- | :--- | :--- |
| 1. | Impedance | Minimum, i.e., $Z_{\mathrm{r}}=R$ | Maximum, i.e., $Z_{\mathrm{r}}=V / C R$ |
| 2. | Current | Maximum, i.e., $I_{\mathrm{r}}=V / R$ | Minimum, i.e., $I_{\mathrm{r}}=V / Z_{\mathrm{r}}$ |
| 3. | Resonant frequency | $f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}$ |  |
| 4. | Power factor | Unity | $f_{\mathrm{r}}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}}$ |
| 5. | $Q$-factor | $X_{L} / R$ | Unity |
| 6. | Amplification | It amplifies voltage | $X_{L} / R$ |

## Example 7.66

A parallel circuit consists of a coil having $15 \Omega$ resistance and 300 mH inductance in parallel with a capacitor of capacitance $4 \mu \mathrm{~F}$. Determine (i) the resonant frequency, (ii) dynamic impedance of the circuit, and (iii) $Q$-factor of the circuit at resonance.

Solution:
(i) Resonant frequency, $f_{\mathrm{r}}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}}$

Or

$$
f_{\mathrm{r}}=\frac{1}{2 \pi} \sqrt{\frac{1}{0.3 \times 4 \times 10^{-6}}-\frac{(15)^{2}}{(0.3)^{2}}}=145.27 \mathrm{~Hz}
$$

(ii) Dynamic impedance, $Z_{\mathrm{r}}=\frac{L}{C R}=\frac{0.3}{4 \times 10^{-6} \times 15}=5,000 \Omega$
(iii) $Q$-factor $=\frac{2 \pi f_{\mathrm{r}} L}{R}=\frac{2 \pi \times 145.27 \times 0.3}{15}=18.255$

## Example 7.67

A coil of $10 \Omega$ resistance and 0.1 H inductance is connected in parallel with a capacitor of $10 \mu \mathrm{~F}$. Calculate the frequency at which the circuit will act as a non-inductive resistance of $R$ ohm. Find also the value of $R$, input current at resonant frequency, and the ratio of the circulating current to the supply current at resonant frequency. The applied voltage is $100 \mathrm{~V}(\mathrm{rms})$.

## Solution:

Capacitance, $C=10 \mu \mathrm{~F}=10 \times 10^{-6} \mathrm{~F} ; R=10 \Omega ; L=0.1 \mathrm{H} ; V=100 \mathrm{~V}$

The circuit will act as a resistive circuit at resonant frequency

$$
f_{\mathrm{r}}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}}=\frac{1}{2 \pi} \sqrt{\frac{1}{0.1 \times 10 \times 10^{-6}}-\frac{(10)^{2}}{(0.1)^{2}}}=158.36 \mathrm{~Hz}
$$

Dynamic resistance $($ or impedance $)=\frac{L}{C R}=\frac{0.1}{10 \times 10^{-6} \times 10}=1,000 \Omega$
Input current at resonant frequency, $I=\frac{V}{L / C R}=\frac{100}{1,000}=0.1 \mathrm{~A}$
The ratio of circulating current to line current at resonant frequency,

$$
\frac{I_{\mathrm{C}}}{I}=\frac{2 \pi f_{\mathrm{r}} L}{R}=\frac{2 \pi \times 158.36 \times 0.1}{10}=9.95
$$

## Example 7.68

An inductive circuit of resistance $2 \Omega$ and inductance 0.014 H is connected to a $250 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Which capacitance if placed in parallel will produce resonance?
(U.P.T.U. Tut.)

## Solution:

Here, $R=2 \Omega ; L=0.014 \mathrm{H} ; V=250 \mathrm{~V} ; f=50 \mathrm{~Hz}$
Inductive reactance, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.014=4.398 \Omega$
Impedance,

$$
Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(2)^{2}+(4.398)^{2}}=4.83 \Omega
$$

At resonance,

$$
\begin{aligned}
& Z=\sqrt{\frac{L}{C}} \\
& C=\frac{L}{Z^{2}}=\frac{0.014}{(4.83)^{2}}=600 \mu \mathrm{~F}
\end{aligned}
$$

## Example 7.69

A coil of $15 \Omega$ resistance and 0.3 H inductance is connected in parallel with a variable capacitor across a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate the (i) capacitance of the capacitor for resonance, (ii) effective impedance of the circuit, and (iii) current supplied from the mains.

## Solution:

The circuit is shown in Figure 7.89.
At resonance,
or

$$
\begin{aligned}
& f_{\mathrm{r}}= \frac{1}{2 \pi}\left(\frac{1}{L C}\right. \\
&\left.-\frac{R^{2}}{L^{2}}\right)^{1 / 2} \\
& \frac{1}{L C}=4 \pi^{2} f_{\mathrm{r}}^{2}+\frac{R^{2}}{L^{2}} \\
& \frac{1}{0.3 \times C}=4 \pi^{2}(50)^{2}+\frac{(15)^{2}}{(0.3)^{2}} \quad \text { or } \quad C=32.94 \mu \mathrm{~F}
\end{aligned}
$$

Effective impedance of the circuit,

$$
Z_{\mathrm{r}}=\frac{L}{C R}=\frac{0.3}{32.94 \times 10^{-6} \times 15}=607.18 \Omega
$$

Line current, $I_{\mathrm{r}}=\frac{V}{Z_{\mathrm{r}}}=\frac{230}{607.18}=0.3788 \mathrm{~A}$

## Example 7.70

In the circuit shown in Figure 7.90, find the value of $R$ such that the impedance of the whole circuit should be independent of the frequency of the supply. If voltage $=200 \mathrm{~V}, L=0.16 \mathrm{H}$, and $C=100 \mu \mathrm{~F}$, calculate the power loss in the circuit.
(U.P.T.U. Feb. 2002)


Fig. 7.90 Given circuit

## Solution:

Impedance of Branch I, $\bar{Z}_{1}=(R+j \omega L)$
Impedance of Branch II, $\bar{Z}_{2}=\left(R-\frac{j}{\omega C}\right)$
Impedance of the whole circuit,

$$
\begin{aligned}
\bar{Z}=\frac{\bar{Z}_{1} \bar{Z}_{2}}{\bar{Z}_{1}+\bar{Z}_{2}} & =\frac{(R+j \omega L)\left(R-\frac{j}{\omega C}\right)}{(R+j \omega L)+\left(R-\frac{j}{\omega C}\right)} \\
& =\frac{R^{2}+\frac{L}{C}+j R\left(\omega L-\frac{j}{\omega C}\right)}{2 R+j\left(\omega L-\frac{j}{\omega C}\right)}
\end{aligned}
$$

It is observed that $j$ term in the numerator of the abovementioned expression is $R$ times the denominator. Similarly, if the real term, that is, $\left(R^{2}+\frac{L}{C}\right)$ is also $R$ times of the real term in denominator, that is, $R \times 2 R$, then the term consisting of $\omega$ will disappear. Therefore, the impedance of the circuit will become independent of the supply frequency.

If

$$
\begin{aligned}
R^{2}+\frac{L}{C} & =R \times 2 R \quad \text { or } \quad R^{2}=\frac{L}{C} \\
R & =\sqrt{\frac{L}{C}}
\end{aligned}
$$

Now, $L=0.16 \mathrm{H}$ and $C=100 \times 10^{-6} \mathrm{~F}$

$$
\therefore \quad R=\sqrt{\frac{0.16}{100 \times 10^{-6}}}=40 \Omega
$$

Power loss in the circuit, $P=\frac{V^{2}}{R}=\frac{(200)^{2}}{40}=1,000 \mathrm{~W}$ or 1 kW

## Example 7.71

What are the features of resonance in parallel circuits? Calculate the value of $C$ that results in resonance for the circuit shown in Figure 7.91 when frequency is $1,000 \mathrm{~Hz}$ and find $Q$-factor for each branch.
(U.P.T.U. June 2001)

## Solution:

Admittance of Branch I,

$$
\begin{aligned}
\bar{Y}_{1} & =\frac{1}{4+j 8}=\frac{1}{4+j 8} \times \frac{4-j 8}{4-j 8}=\frac{4-j 8}{80} \\
& =(0.05-j 0.1) \mathrm{mho}
\end{aligned}
$$



Fig. 7.91 Given circuit

Admittance of Branch II,

$$
\begin{aligned}
\bar{Y}_{2} & =\frac{1}{5-j X_{\mathrm{C}}}=\frac{1}{5-j X_{\mathrm{C}}} \times \frac{5+j X_{\mathrm{C}}}{5+j X_{\mathrm{C}}}=\frac{5+j X_{\mathrm{C}}}{25+j X_{\mathrm{C}}^{2}} \\
& =\left(\frac{5}{25+X_{\mathrm{C}}^{2}}+j \frac{X_{\mathrm{C}}}{25+X_{\mathrm{C}}^{2}}\right) \text { (mho) }
\end{aligned}
$$

At resonance, the quadrature components of the admittance of the two branches must be equal,
i.e.,

$$
\frac{X_{\mathrm{C}}}{25+X_{\mathrm{C}}^{2}}=0.1 \text { or } 0.1 X_{\mathrm{C}}^{2}-X_{\mathrm{C}}+2.5=0
$$

or

$$
X_{\mathrm{C}}^{2}-10 X_{\mathrm{C}}+25=0 \quad \text { or } \quad\left(X_{\mathrm{C}}-5\right)^{2}=0
$$

or

$$
X_{\mathrm{C}}=5 \Omega
$$

and capacitance,

$$
C=\frac{1}{2 \pi f_{\mathrm{r}} X_{\mathrm{C}}}=\frac{1}{2 \pi \times 1,000 \times 5}=31.83 \mu \mathrm{~F}
$$

$Q$-factor for Branch $\mathrm{I}=\frac{X_{\mathrm{L}}}{R}=\frac{\omega L}{R}=\frac{8}{4}=2$
$Q$-factor for Branch II $=\frac{X_{\mathrm{C}}}{R}=\frac{5}{5}=1$

## SUMMARY

1. AC Circuit: The path for the flow of AC is called AC circuit.
2. Pure resistive circuit: Circuit contains pure resistance.
$v=V_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin \omega t ; I_{\mathrm{m}}=V_{\mathrm{m}} / R ; P=V I ; \mathrm{pf}=1 ;$
Current is in phase with voltage vector.
3. Pure inductive circuit: Circuit contains pure inductance.
$v=V_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin (\omega t-\pi / 2) ; I_{\mathrm{m}}=V_{\mathrm{m}} / X_{\mathrm{L}} ; X_{\mathrm{L}}=2 \pi f L ; P=0 ; \mathrm{pf}=0$ lag
Current lags behind voltage vector of $90^{\circ}$ (i.e., $\pi / 2$ )
4. Pure capacitive circuit: Circuit contains pure capacitance.
$v=V_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin (\omega t+\pi / 2) ; I_{\mathrm{m}}=V_{\mathrm{m}} / X_{\mathrm{C}} ; X_{\mathrm{C}}=1 / 2 \pi f C ; P=0 ; \mathrm{pf}=0$ lead;
Current leads the voltage vector by $90^{\circ}$ (i.e., $\pi / 2$ ).
5. $R-L$ series circuit: Circuit contains resistance and inductance in series. It is generally called inductive circuit.
$v=V_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin (\omega t-\phi) ; \phi=\tan ^{-1} X_{\mathrm{L}} / R ; I_{\mathrm{m}}=V_{\mathrm{m}} / Z ; Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}} ;$
$P=V I \cos \phi=I^{2} R ; \mathrm{pf}=\cos \phi=R / Z$ lag;
6. Power triangle: The three components of a right-angled power triangle are as follows:
(i) True power: It is the base or horizontal component of the power triangle that represents the actual or true power consumed in an AC circuit.
(ii) $P=V I \cos \phi$; unit of true power is watt (W), kilowatt ( $\mathrm{kW)}$ ) or megawatts (MW)
(iii) Reactive power: It is the perpendicular or vertical component of the power triangle that represents reactive power of the circuit. This is the power that reacts in circuit, in fact, it is that power that is supplied by the source in a quarter cycle and the same is fed back in the next quarter cycle.
(iv) $P_{\mathrm{r}}=V I \sin \phi$; units of reactive power are volt-ampere-reactive (VAR), kVAR or MVAR.
(v) Apparent power: It is the hypotenuse of the power triangle. It is the power that looks to be consumed in the circuit but actually it is not so.
$P_{\mathrm{a}}=V I$; units of apparent power are volt-ampere (VA), kVA or MVA.
7. Power factor and its importance: $\mathrm{pf}=\cos \phi=R / Z=$ true power/apparent power Ill-effects of poor pf: (i) greater conductor size, (ii) poor efficiency, (iii) poor voltage regulation, and (iv) larger kVA rating of equipment.
8. $R-C$ series circuit: Circuit contains resistance and capacitance in series. It is generally called capacitive circuit.
$v=V_{\mathrm{m}} \sin \omega t ; i=I_{\mathrm{m}} \sin (\omega t+\phi) ; \phi=\tan ^{-1} X_{\mathrm{C}} / R ; I_{\mathrm{m}}=V_{\mathrm{m}} / Z ; Z=\sqrt{R^{2}+X_{\mathrm{C}}^{2}} ;$
$P=V I \cos \phi=I^{2} R ; \mathrm{Pf}=\cos \phi=R / Z$ leading;
9. $R-L-C$ series circuit: Circuit contains resistance, inductance, and capacitance in series. This circuit behaves as follows:
(i) When $X_{\mathrm{L}}>X_{\mathrm{C}} \ldots \ldots . . .$. inductive circuit............pf lagging.... 0 to 1
(ii) When $X_{\mathrm{L}}<X_{\mathrm{C}}$..........capacitive circuit........pf leading.... 0 to 1
(iii) When $X_{\mathrm{L}}=X_{\mathrm{C}} \ldots \ldots . . . .$. resistive circuit.........pf unity.... 1 only
10. Series resonance: An R-L-C circuit is said to be in series resonance, when
$X_{\mathrm{L}}=X_{\mathrm{C}} ; Z_{\mathrm{r}}=\sqrt{R^{2}+\left(X_{\mathrm{L}}-X_{\mathrm{C}}\right)^{2}}=R$
Resonance frequency, $f_{\mathrm{r}}=1 / 2 \pi \sqrt{L C}$
Current is maximum, that is, $I_{\mathrm{r}}=V / Z_{\mathrm{r}}=V / R$
$Q$-factor $=2 \pi f_{\mathrm{r}} L / R=\frac{1}{R} \sqrt{\frac{L}{C}}$
11. AC Parallel circuit: An AC circuit in which number of branches (each branch contains number of component in series) are connected in parallel is called an AC parallel circuit.
12. Method of solving parallel AC circuits: The following methods are used:
(i) Phasor or vector method: The magnitude and direction of the currents flowing through various branches are determined and are represented as phasors. The phasors are resolved to determine the resultant.
(ii) Admittance method: In this method, the component values are changed as per admittance triangle that contains conductance and susceptance. Then, solution is obtained as per the need.
Conductance, $g=Y \cos \phi=R / Z^{2}$ and
Susceptance, $b=Y \sin \phi=X / Z^{2}$ positive for capacitance and negative for inductance
Admittance, $Y=\sqrt{G^{2}+B^{2}}$ where $G=g_{1}+g_{2}+\ldots+g_{\mathrm{n}}$;
$B=b_{1}+b_{2}+\ldots+b_{\mathrm{n}}$
Current, $I=V Y$
(iii) Method of phasor algebra or symbolic method or J-method: In this method, the quantities are represented in complex form, triangular form, or polar form, and then, the solution is achieved.
Let $V$ is a reference quantity, then it is represented as

$$
\bar{V}=V \pm j 0=V \angle 0^{\circ}
$$

Let $I$ is current lagging behind the voltage vector by angle $\phi$, then it is represented as

$$
\bar{I}=a-j b \text {, where } a=\text { horizontal vector }=I \cos \phi \text { and } b=\text { vertical vector }=I \sin \phi
$$

$$
\bar{I}=I \cos \phi-j \sin \phi \text { or } \bar{I}=I \angle-\phi^{\circ}(\text { polar form })
$$

For addition and subtraction, complex form is used; however, for multiplication and division, polar form is used.
13. Parallel resonance: A parallel circuit, containing inductor and capacitor in parallel, is said to be in parallel resonance when circuit current is in phase with the applied voltage.
$I_{\mathrm{C}}=I_{\mathrm{L}} \sin \phi$; resonance frequency, $f_{\mathrm{r}}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}}$
If $R$ is very small and neglected, $f_{\mathrm{r}}=\frac{1}{2 \pi \sqrt{L C}}$
Current is minimum, i.e., $I_{\mathrm{r}}=V / Z_{\mathrm{r}} ; Q$-factor $=2 \pi f_{\mathrm{r}} L / R=\frac{1}{R} \sqrt{\frac{L}{C}}$

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. All the domestic appliances are connected in $\qquad$ to the supply.
2. The maximum value of pf in an AC circuit can be $\qquad$ .
3. The product of rms value of voltage and current is called $\qquad$ .
4. The voltage vector in a pure capacitive circuit will $\qquad$ the current vector by $90^{\circ}$.
5. The power drawn by a pure inductor is $\qquad$ -.
6. In an AC circuit, the ratio of resistance to reactance is called $\qquad$ .
7. In an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit, the current will be maximum when $X_{\mathrm{L}}=$ $\qquad$ .
8. In L-C parallel circuit, at resonance, the current drawn by the circuit will be $\qquad$ .
9. In an AC circuit, the reciprocal of impedance is called $\qquad$ _.
10. The consumer having high pf will draw $\qquad$ current than the consumer having low pf for the same load.
11. In a pure inductive circuit, the voltage will $\qquad$ the current vector by $90^{\circ}$.
12. To improve pf of a circuit, a capacitor is connected in $\qquad$ to the load.
13. At resonance, the pf of the circuit is $\qquad$ .
14. For a given load, if reactive power is low, the pf will be $\qquad$ .
15. The power consumed in a pure capacitive circuit is $\qquad$ .
16. The power factor of a pure resistive circuit is $\qquad$ .
17. The power factor of a pure inductive circuit will be $\qquad$ .
18. In an $\mathrm{L}-\mathrm{C}$ parallel circuit, the current drawn will be minimum at $\qquad$ frequency.
19. If voltage applied across a circuit is $200 \angle 0^{\circ}$ and impedance of the circuit is $20 \angle 60^{\circ}$, the current flowing through it will be $\qquad$ A.
20. The currents flowing through the two branches of a parallel circuit are $(10-j 6) \mathrm{A}$ and $(6+j 8) \mathrm{A}$, the total current will be $\qquad$ A.

## OBJECTIVE TYPE QUESTIONS

1. The power factor at resonance in $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit is
(a) zero
(b) unity
(c) 0.5 lagging
(d) 0.5 leading
2. The current in resonance parallel $\mathrm{L}-\mathrm{C}$ circuit will be
(a) very small
(b) very large
(c) zero
(d) infinity
3. An R-L-C series circuit contains resistance $2.5 \Omega$, inductance 0.08 H , and capacitance of $0.8 \mu \mathrm{~F}$, the pf of the circuit will be
(a) unity
(b) zero
(c) lagging
(d) leading
4. If $a=4 \angle 30^{\circ}$ and $b=2 \angle 60^{\circ}$, then the value of $a / b$ will be
(a) $2 \angle 30^{\circ}$
(b) $2 \angle 90^{\circ}$
(c) $2 \angle-30^{\circ}$
(d) $-2 \angle 30^{\circ}$
5. The conjugate of the complex quantity $(4-j 5)$ will be
(a) $-4-j 5$
(b) $4+j 5$
(c) $-4+j 5$
(d) $4-j 5$
6. The inductive reactance is measured in ohm because it
(a) reduces the magnitude of AC
(b) increases the magnitude of AC
(c) is the product of frequency and inductance
(d) has a back emf
7. When a sinusoidal voltage is applied across an $\mathrm{R}-\mathrm{L}$ series circuit having $R=X_{\mathrm{L}}$, the phase angle will be
(a) $90^{\circ}$
(b) $0^{\circ}$
(c) $30^{\circ}$
(d) $45^{\circ}$
8. When a sinusoidal voltage is applied across a circuit having resistance and inductance in parallel so that $R=X_{\mathrm{L}}$, the phase angle will be
(a) $45^{\circ}$
(b) $30^{\circ}$
(c) $0^{\circ}$
(d) $90^{\circ}$
9. If an AC voltage is applied across a capacitor, the AC flows through the circuit because
(a) of high peak value of voltage
(b) capacitor offers low opposition
(c) electrons can pass through capacitor
(d) capacitor charges and discharges according to the supply voltage
10. At higher frequencies, the value of capacitive reactive
(a) increases
(b) decreases
(c) remains the same
(d) increases and also depends upon applied voltage
11. In an $\mathrm{L}-\mathrm{C}$ series circuit, at resonance the
(a) impedance is maximum
(b) voltage across $C$ is minimum
(c) current is maximum
(d) current is minimum
12. In an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit, at resonance frequency, the voltage across the resistance is
(a) much lower than applied voltage
(b) equal to applied voltage
(c) much higher than applied voltage
(d) function of value of inductance
13. In a pure inductive AC circuit,
(a) voltage leads the current vector by $90^{\circ}$
(b) current leads the voltage vector by $90^{\circ}$
(c) voltage lags the current vector by $90^{\circ}$
(d) current is in phase with voltage vector
14. In an AC circuit, which of the following expression is true for apparent power?
(a) $V_{\text {av }} I_{\text {av }}$
(b) $V I \cos \phi$
(c) $V_{\text {rms }} I_{\text {rms }}$
(d) $V I \sin \phi$
15. The impedance $Z_{1}=6+j 9$ and $Z_{2}=6-j 9$ are added. The resultant impedance will be
(a) $15 \angle 45^{\circ}$
(b) $12 \angle 0^{\circ}$
(c) $15 \angle 0^{\circ}$
(d) None of the above
16. If impedance $Z_{1}=10 \angle 20^{\circ}$ and $Z_{2}=20 \angle 10^{\circ}$, the value of $Z_{1} Z_{2}$ will be
(a) $200 \angle 10^{\circ}$
(b) $5 \angle 10^{\circ}$
(c) $\frac{1}{5} \angle 30^{\circ}$
(d) $200 \angle 30^{\circ}$
17. If impedance $Z_{1}=100 \angle 20^{\circ}$ and $Z_{2}=20 \angle 10^{\circ}$, the value of $Z_{1} / Z_{2}$ will be
(a) $200 \angle 10^{\circ}$
(b) $5 \angle 10^{\circ}$
(c) $200 \angle 30^{\circ}$
(d) $5 \angle 30^{\circ}$
18. What will happen if the frequency of power supply in a pure capacitive circuit is doubled?
(a) the current will also be doubled
(b) the current will reduce to half
(c) the current will remain the same
(d) the current will increase to four-fold
19. A pure inductance when connected across $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply, it consumes 40 W . This consumption is
(a) due to reactance of inductor
(b) due to current flowing through the inductor
(c) due to big size of the inductor
(d) None of the above; the statement is false
20. The power dissipated in the pure capacitance of an $\mathrm{R}-\mathrm{C}$ series circuit will be
(a) zero
(b) small
(c) higher than dissipated in resistance
(d) equal to dissipated in resistance

## NUMERICALS

1. A 230 V water heater operating for 3 h daily on AC mains is found to give 250 kcal per hour, the efficiency of heater system being $90 \%$. Determine (i) the rms and the maximum value of current, (ii) resistance of the heater, and (iii) also write down the equations for voltage and current when the supply frequency is 50 Hz . (Ans. $1.41 \mathrm{~A} ; 1.993 \mathrm{~A} ; 163.1 \Omega ; v=325.27 \sin 314 t ; i=1.993 \sin 314 t$ )
2. An inductive coil has negligible resistance and inductance of 0.1 H . It is connected across a $220 \mathrm{~V}, 50$ Hz supply. Find the current and power. Further, write down the expression for instantaneous value of applied voltage and current.
(Ans. 7 A ; zero; $v=311 \sin 314 t, i=9.9 \sin (314 t-\pi / 2)$ )
3. A capacitor has a capacitance of $30 \mu \mathrm{~F}$. Find its capacitive reactance for frequency of 25 and 50 Hz . Find, in each case, the current if the supply voltage is 440 V . (Ans. $212.2 \Omega ; 106.1 \Omega ; 2.073 \mathrm{~A} ; 4.146 \mathrm{~A}$ )
4. The instantaneous current in a pure inductance of 5 H is expressed as $i=10 \sin (314 t-\pi / 2)$. A capacitor is connected in parallel with the inductor. What should be the capacitance of the capacitor to receive the same amount of energy as inductance at the same terminal voltage?
(Punjab Univ.) (Hints: energy $\frac{1}{2} L I_{\mathrm{m}}^{2}=\frac{1}{2} C V_{\mathrm{m}}^{2}$ ) (Ans. $2.028 \mu \mathrm{~F}$ )
5. A coil having a resistance of $7 \Omega$ and an inductance of 31.8 mH is connected to $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate (i) the circuit current, (ii) phase angle, (iii) power factor, and (iv) power consumed.
(Ans. $18.85 \mathrm{~A} ; 55^{\circ} \mathrm{lag} ; 0.573 \mathrm{lag} ; 2,484.24 \mathrm{~W}$ )
6. A voltage $e=200 \sin 100 \pi t$ is applied to a coil having $R=200 \Omega$ and $L=638 \mathrm{mH}$. Find the expression for the current and also determine the power taken by the coil.
(Ans. $\left.0.706 \sin \left(314 t-45.06^{\circ}\right) ; 50 \mathrm{~W}\right)$
7. A circuit consists of a pure resistance and a coil connected in series. Power dissipated in the resistance and in the coil are $1,000 \mathrm{~W}$ and 200 W , respectively. Voltage drops across the resistance and the coil are 200 V and 300 V , respectively. Determine the reactance of the coil and the supply voltage.
(P.T.U.) (Ans. $59.46 \Omega ; 382 \mathrm{~V}$ )
8. A $100 \mathrm{~V}, 60 \mathrm{~W}$ lamp is to be operated at $220 \mathrm{~V}, 50 \mathrm{~Hz}$ mains. What (i) pure resistance and (ii) pure inductance should be placed in series with the lamp, which enable it to run without being damaged? Which method would be more economical?
(Ans. $200 \Omega ; 1.038 \mathrm{H}$ )
9. A coil is connected in series with a non-inductive resistance of $30 \Omega$ across, $240 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. The reading of a voltmeter across the coil is 180 V and across the resistance is 130 V . Determine (i) power absorbed by the coil, (ii) inductance of the coil, (iii) resistance of the coil, and (iv) power factor of the whole circuit.
(P.T.U.) (Ans. $137.95 \mathrm{~W} ; 130 \mathrm{mH} ; 7.36 \Omega ; 0.674 \mathrm{lag})$
10. Two coils are connected in series having resistance and inductive reactance 5 and $6 \Omega, 3$ and $7 \Omega$, respectively. A sinusoidal voltage of $200 \mathrm{~V}, 50 \mathrm{~Hz}$ is applied across the combination. Calculate (i) current, power factor, and power absorbed in the whole circuit, (ii) voltage drop across each coil, and (iii) power factor and power absorbed in each coil.
(Ans. 13.1 A, 0.5241 lag, 1,373 W; $102.3 \mathrm{~V}, 99.77 \mathrm{~V} ; 0.64 \mathrm{lag}, 858 \mathrm{~W}, 0.394$ lag, 515 W )
11. A coil is placed in series with a non-inductive resistance that consumes $5,000 \mathrm{~W}$ at 100 V . When a voltage of 104 V is applied to the circuit, the voltage across the coil is 66 V and across the resistor is 50 V . Calculate power absorbed by the coil and its power factor.
(U.P.T.U. Type) (Ans. 990 W, 0.6 lagging)
12. In a series circuit, a voltage of 10 V at 25 Hz produces 100 mA , while the same voltage at 75 Hz produces 60 mA . Draw the inductive circuit diagram and insert the values of the constants.
(U.P.T.U. Type) (Ans. $0.3001 \mathrm{H}, 88.19 \Omega$ )
13. A resistor $R$ in series with a capacitor $C$ is connected to a $50 \mathrm{~Hz}, 230 \mathrm{~V}$ supply. Find the value of C so that resistance absorbs 500 W at 150 V . Find also the maximum charge and maximum stored energy in C .
(Ans. $60.85 \mu \mathrm{~F} ; 15 \times 10^{-3} \mathrm{C} ; 1.85 \mathrm{~J}$ )
14. A coil of resistance $100 \Omega$ and inductance $100 \mu \mathrm{H}$ is connected in series with a 100 pF capacitor. The circuit is connected to a 50 V variable frequency supply. Calculate (i) the resonant frequency, (ii) current at resonance, (iii) voltage across $L$ and $C$ at resonance, and (iv) $Q$-factor of the circuit.
(Ans. $\left.1.59 \times 10^{6} \mathrm{~Hz} ; 0.5 \mathrm{~A} ; 500 \mathrm{~V} ; 500 \mathrm{~V} ; 10\right)$
15. A voltage source of frequency 5 kHz is applied to an inductor in series with variable capacitor. When the capacitor is set to $1.2 \mu \mathrm{~F}$, the current has its maximum value, whereas it reduces to one-third when the capacitor is changed to $6 \mu \mathrm{~F}$. Find the resistance of the coil and $Q$-factor.
(U.P.T.U. Type) (Ans. $7.5 \Omega ; 3.536$ )
16. A capacitor of capacitance $C$ farad is connected in series with a coil having $75 \Omega$ resistance and 12 H inductance. Calculate the value of $C$ at resonance when the circuit is connected across $220 \mathrm{~V}, 60 \mathrm{~Hz}$ supply.
(Ans. $0.587 \mu \mathrm{~F}$ )
17. Determine the parameter of an $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit that will resonate at $10,000 \mathrm{~Hz}$, has a bandwidth of $1,000 \mathrm{~Hz}$, and draws 15.3 W from a 200 V generator operating at the resonant frequency of the circuit.
(Ans. $2614 \Omega, 416 \mathrm{mH}, 609 \mathrm{pF}$ )
18. A coil of resistance $15 \Omega$ and inductance 0.05 H is connected in parallel with a non-inductive resistor of $20 \Omega$. Find (i) the current in each branch of the circuit, (ii) the total current supplied, and (iii) the phase angle of the combination when a voltage of 230 V at 50 Hz is applied across the circuit. Draw the relevant phasor diagram. (U.P.T.U. Type) (Ans. $10.59 \mathrm{~A}, 11.5 \mathrm{~A} ; 20.31 \mathrm{~A} ; 22.14^{\circ}$ lagging)
19. A single-phase motor draws 10 A at 230 V at a power factor of 0.8 lagging. Calculate the value of the capacitor that when connected across the terminals of the motor will bring the line current in phase with the voltage. Frequency of supply may be assumed as 50 Hz .
(U.P.T.U. Type) (Ans. $83 \mu \mathrm{~F}$ )
20. An inductive coil is connected in parallel with a condenser and the combination is connected across a supply of sinusoidal emf. Derive the expression for current flowing through the two branches, and hence, find the conditions for obtaining the minimum input current and the value of the maximum impedance of the circuit.
(Ans. $Z_{\mathrm{r}}=\frac{L}{C R}$ )
21. Calculate the impedance and admittance of the branches $A B$ and $C D$ of the circuit shown in Figure 7.92. Further, find the resultant impedance and admittance of the circuit.
(Ans. $10.48 \Omega ; 0.0954 \mathrm{mho} ; 25.43 \Omega ; 0.03938 \mathrm{mho} ; 7.52 \Omega ; 0.133 \mathrm{mho})$


Fig. 7.92 Given circuit
22. Two circuits A and B are connected in parallel to a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Circuit A consists of a resistance $10 \Omega$ in series with an inductive reactance of $10 \Omega$ and circuit B consists of a resistance $20 \Omega$ in series with a capacitive reactance of $10 \Omega$. Determine total current drawn from the supply by admittance method.
(Ans. 21.81 A )
23. A coil that has $6 \Omega$ and 25.5 mH inductance is energised from a $220 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate the current. A capacitor is then connected in parallel with the coil so that the overall pf is increased to
unity. Calculate the capacitance of the capacitor. What current will be flowing in the main supply cable when the capacitor is connected in the circuit and what is the power supplied to the circuit?
(Ans. $22 \mathrm{~A}, 254.65 \mu \mathrm{~F} ; 36.67 \mathrm{~A}, 8067 \mathrm{~W}$ )
24. For the parallel circuit shown in Figure 7.93, the values of various parameters are
$R_{1}=70 \Omega$ (non-inductive); coil $-R_{\mathrm{C}}=30 \Omega ; L_{\mathrm{C}}=0.54 \mathrm{H} ; R_{2}=100 \Omega ; X_{\mathrm{C}}=157 \Omega($ at 50 Hz$)$
(i) Determine (in magnitude and phase) the branch currents and the total current when the supply voltage is $240 \mathrm{~V}, 50 \mathrm{~Hz}$. (ii) Draw a neat phasor diagram indicating the current and the voltage across coil and condenser. (iii) What current the circuit would draw if the AC source is replaced by an equivalent DC source? (Ans. $1.2187 \mathrm{~A}, 59.48^{\circ} \mathrm{lag} ; 1.289 \mathrm{~A} ; 57.5^{\circ}$ lead; $1.312 \mathrm{~A} ; 1.63^{\circ}$ lead; 2.4 A (DC))
25. For the circuit shown in Figure 7.94, the instantaneous value of the supply voltage is $340 \sin 314 t$. Find the total rms value of current flowing from the source, currents in $L_{2}$, and $C_{2}$. Draw the phasor


Fig. 7.93 Given circuit
diagram of the circuit showing applied voltage, total current, and voltage across choke $L_{1}$.
(Ans. 0.9045 A, $0.687 \mathrm{~A}, 1.5916 \mathrm{~A})$
26. A circuit consisting of branches $A$ and $B$ are connected in parallel across $220 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Branch A consists of a resistance of $7 \Omega$ in series with 0.0125 H inductor and Branch B consists of a resistance


Fig. 7.94 Given circuit
of $8 \Omega$ in series with $1,000 \mu \mathrm{~F}$ capacitor. Find the current in each branch and total current. Draw the phasor diagram.
(Ans. $27.41 \angle-29.29^{\circ} \mathrm{A} ; 25.55 \angle 21.69^{\circ} \mathrm{A} ; 47.8 \angle-4.76^{\circ} \mathrm{A}$ )
27. A parallel circuit consists of a $2.5 \mu \mathrm{~F}$ capacitor and a coil whose resistance and inductance are $15 \Omega$ and 260 mH , respectively. Determine the resonant frequency, $Q$ of the coil, and dynamic impedance of the circuit.
(Ans. $197 \mathrm{~Hz}, 21.45,6,933.33 \Omega$ )
28. A coil has a resistance of $500 \Omega$ and inductance of $350 \mu \mathrm{H}$. Find the capacitance of a capacitor that when connected in parallel with the coil will produce resonance with a supply frequency of $10^{6} \mathrm{~Hz}$.

If a second capacitor of capacitance 30 pF is connected in parallel with the first capacitor, find the frequency at which resonance will occur.
(Ans. $68.8 \mathrm{pF}, 0.825 \mathrm{MHz}$ )

## VIVA VOCE/REASONING QUESTIONS

1. The power consumed in a pure inductive circuit is zero, why?
2. The power consumed in a pure capacitive circuit is zero, why?
3. Actual power consumed in an AC circuit is given by relation $V I \cos \phi$ and not $V I$, why?
4. Is pf more than one, if not why?
5. It is said that the pf should be high, why?
6. In AC series circuit at resonant frequency, the value of circuit impedance reduces to resistance, why?
7. To improve pf of a motor, a capacitor is connected in parallel with it not in series, why?
8. We prefer the use of phasor algebra in solving parallel AC circuit problems, why?

## SHORT ANSWER TYPE QUESTIONS

1. What is an AC circuit?
2. What are the differences between $D C$ circuit and $A C$ circuit?
3. What is the phase difference between the voltage and the current in a resistor?
4. What is the instantaneous power in a resistor?
5. What is the average power in a resistor?
6. Why does the induced emf in the inductor oppose the applied voltage?
7. What does a pure inductance alone do in a circuit?
8. What is the phase difference between the current and the voltage in an inductor (pure inductance)?
9. Define inductive reactance.
10. What is the instantaneous power in an inductor?
11. What is the value of average power in an inductor?
12. What does a pure capacitor alone do in a circuit?
13. What is the phase difference between voltage and current in a capacitor?
14. Define capacitive reactance.
15. Give the instantaneous power of a capacitor.
16. What is the value of average power in a capacitor?
17. What is a series $\mathrm{R}-\mathrm{L}$ circuit?
18. What is the impedance of $\mathrm{R}-\mathrm{L}$ series circuit?
19. What is apparent power of an AC circuit?
20. What is real power of an AC circuit?
21. What is power factor?
22. What is reactive power of an AC circuit?
23. What is active component of an $A C$ current?
24. What is a reactive component of an AC current?
25. What do you understand by $\mathrm{R}-\mathrm{C}$ series circuit?
26. What is the impedance of $\mathrm{R}-\mathrm{C}$ series circuit?
27. What is meant by $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit?
28. What is the impedance of $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit?
29. What is parallel AC circuit?
30. What is polar form of representing an AC quantity?
31. What is rectangular form of representing an AC quantity?
32. What is mean by resonance circuit?
33. What happens when $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit is at resonance?
34. How does series resonance result in voltage amplification?
35. $Q$ of a coil is 20 . What does it mean?
36. What are important points to be noted for a parallel circuit when it is at resonance?
37. At parallel resonance, why impedance is a pure resistance?

## TEST QUESTIONS

1. Explain why the phasor of voltage across an inductor leads its current by $90^{\circ}$ and the phasor of voltage across a capacitor lags its current by $90^{\circ}$.
2. Determine phase angle relationship between alternating voltage and current in a purely inductive and a purely capacitance circuit under steady condition.
3. Explain with mathematical expression that power consumed in a pure inductance in zero.
(P.T.U.)
4. Explain with mathematical expression that power consumed in a pure capacitance is zero.
5. Show that power consumed in a pure resistive circuit is not constant but it is fluctuating.
6. Develop the expression for the mean power consumed over a cycle of a single-phase sinusoidal supply delivering power to load comprising of a resistance $R$ in series with an inductance $L$.
7. Define the term 'watt-less current'.
8. What do you understand by real power, reactive power, and apparent power?
(U.P.T.U.)
9. What is power factor? Discuss the practical importance of pf
(U.P.T.U.)
10. Explain the use of capacitance for pf improvement.
11. What is power factor in $\mathrm{R}-\mathrm{L}-\mathrm{C}$ series circuit and its leading, lagging, and resonance conditions?
12. The series resonant circuit is often regarded as the acceptor circuit and the parallel circuit as the rejector circuit. Explain.
13. Define $Q$-factor for the series resonant circuit and express it in terms of the circuit parameters.
14. What do you understand by the operator $j$ ?
15. Define admittance, conductance, and susceptance.
(U.P.T.U.)

## ANSWERS

Fill in the Blanks

1. parallel
2. one
3. apparent power
4. lag behind
5. zero
6. pf
7. $X_{\mathrm{C}}$
8. minimum
9. admittance
10. less
11. lead
12. parallel
13. unity
14. high
15. zero
16. unity
17. zero lagging
18. resonance
19. $10 \angle-60^{\circ}$
20. $(16+j 2)$

Objective Type Questions

1. (b)
2. (a)
3. (d)
4. (c)
5. (b)
6. (a)
7. (d)
8. (a)
9. (d)
10. (b)
11. (c)
12. (b)
13. (a)
14. (c)
15. (b)
16. (d)
17. (b)
18. (a)
19. (d)
20. (a)


## LPARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is polyphase system?
* Why polyphase system is preferred over single-phase system?
* What are star and delta connections?
* What is the relation between phase and line voltages and phase and line currents in star and delta connections?
* How power is measured in three-phase circuits?
* How power factor of the load affects the readings of two wattmeters used to measure power in three-phase balanced load circuits?


### 8.1 INTRODUCTION

Although single-phase system is employed for the operation of almost all the domestic and commercial appliances, for example, lamps, fans, electric irons, refrigerators, TV sets, washing machines, exhaust fans, computers, etc. However, it has its own limitations in the field of generation, transmission, distribution, and industrial applications. Due to this, it has been replaced by polyphase system. Polyphase (three-phase) system is universally adopted for generation, transmission, and distribution of electric power because of its unchangeable superiority. In this chapter, we shall confine our attention to three-phase system and its practical utility in the field of engineering.

### 8.2 POLYPHASE SYSTEM

Poly means many (more than one) and phase means windings or circuits, each of them having a single alternating voltage of the same magnitude and frequency. Hence, a polyphase system
is essentially a combination of two or more voltages having same magnitude and frequency, but displaced from one another by equal electrical angle. This angular displacement between the adjacent voltages is called phase difference and depends upon the number of phases.

$$
\text { Phase difference }=\frac{360 \text { electrical degrees }}{\text { Number of phases }}
$$

However, the abovementioned relation does not hold good for two-phase system, where the voltages are displaced by $90^{\circ}$ electrical. Thus, an AC system having a group of (two or more than two) equal voltages of same frequency arranged to have equal phase difference between them is called a polyphase system.

The polyphase system may be two-phase system, three-phase system, or six-phase system. However, for all practical purposes, three-phase system is invariably employed. Therefore, whenever a polyphase system is mentioned, we mean by that a three-phase system unless stated otherwise.

### 8.3 ADVANTAGES OF THREE-PHASE SYSTEM OVER SINGLE-PHASE SYSTEM

The following are the main advantages of three-phase system over single-phase system:

1. Constant power: In single-phase circuits, the power delivered is pulsating. Even when the voltage and current are in phase, the power is zero twice in each cycle. While in polyphase system, power delivered is almost constant when the loads are balanced.
2. High rating: The rating (output) of a three-phase machine is nearly 1.5 times the rating (output) of a single-phase machine of the same size.
3. Power transmission economics: To transmit the same amount of power over a fixed distance at a given voltage, three-phase system requires only $75 \%$ of the weight of conducting material as required by single-phase system.
4. Superiority of three-phase induction motors: The three-phase induction motors have widespread field of applications in the industries because of the following advantages:
(a) Three-phase induction motors are self-starting, whereas single-phase induction motors have no starting torque without using auxiliary means.
(b) Three-phase induction motors have higher power factor and efficiency than that of single-phase induction motors.

### 8.4 GENERATION OF THREE-PHASE EMFS

In a three-phase system, there are three equal voltages (or emfs) of the same frequency having a phase difference of $120^{\circ}$. These voltages can be produced by a three-phase, AC generator having three identical windings (or phases) displaced $120^{\circ}$ electrical apart. When these windings are rotated in a stationary magnitude field (see Fig. 8.1(a)) or when these windings are kept stationary and the magnetic field is rotated (see Fig. 8.1(b)), an emfs is induced in each winding or phase. These emfs are of same magnitude and frequency, but are displaced from one another by $120^{\circ}$ electrical.


Fig. 8.1 (a) Coils rotating in stationary magnetic field (b) Magnetic field rotating in stationary coils (c) Wave diagram of induced emfs in three coils (d) Phasor diagram of induced emfs in three coils

Consider three identical coils $\mathrm{a}_{1} \mathrm{a}_{2}, \mathrm{~b}_{1} \mathrm{~b}_{2}$, and $\mathrm{c}_{1} \mathrm{c}_{2}$ mounted, as shown in Fig 8.1(a) and (b). Here, $\mathrm{a}_{1}, \mathrm{~b}_{1}$, and $\mathrm{c}_{1}$ are the start terminals, while $\mathrm{a}_{2}, \mathrm{~b}_{2}$, and $\mathrm{c}_{2}$ are the finish terminals of the three coils. It may be noted that a phase difference of $120^{\circ}$ electrical is maintained between the corresponding start terminals $\mathrm{a}_{1}, \mathrm{~b}_{1}$, and $\mathrm{c}_{1}$. Let the three coils mounted on the same axis be rotated (or the magnetic field system be rotated keeping coils stationary) in anticlockwise direction at $\omega$ radians/s, as shown in Figure 8.1(a) and (b), respectively.

Three emfs are induced in the three coils, respectively. Their magnitudes and directions, at this instant, are as follows:

1. The emf induced in coil $a_{1} a_{2}$ is zero (consider start terminal $a_{1}$ ) and is increasing in the positive direction, as shown by wave $e_{\mathrm{ala} 2}$ in Figure 8.1(c).
2. Coil $\mathrm{b}_{1} \mathrm{~b}_{2}$ is $120^{\circ}$ (electrical) behind coil $\mathrm{a}_{1} \mathrm{a}_{2}$. The emf induced in this coil is negative and is becoming maximum negative (consider start terminal $\mathrm{b}_{1}$ ) as shown by $e_{\mathrm{b} 1 \mathrm{~b} 2}$ in Figure 8.1(c).
3. Coil $c_{1} c_{2}$ is $120^{\circ}$ (electrical) behind $\mathrm{b}_{1} \mathrm{~b}_{2}$ or $240^{\circ}$ (electrical) behind $\mathrm{a}_{1} \mathrm{a}_{2}$. The emf induced in this coil is positive and is decreasing (consider start terminal $\mathrm{c}_{1}$ ) as shown by wave $e_{\text {clc2 }}$ in Figure 8.1(c).

### 8.4.1 Phasor Diagram

The emfs induced in three coils are of the same magnitude and frequency, but are displaced by $120^{\circ}$ (electrical) from each other as shown by phasor diagram in Figure 8.1(d). These can be represented by the equations:

$$
e_{\mathrm{ala} 2}=E_{\mathrm{m}} \sin \omega t ; e_{\mathrm{blb} 2}=E_{\mathrm{m}} \sin (\omega t-2 \pi / 3) ; e_{\mathrm{clc} 2}=E_{\mathrm{m}} \sin (\omega t-4 \pi / 3)=E_{\mathrm{m}} \sin \left(\omega t-240^{\circ}\right)
$$

### 8.5 NAMING THE PHASES

The three phases may be represented by numbers (1,2, and 3 ) or by letters ( $a, b$, and $c$ ) or by colours (red, yellow, and blue, i.e., R, Y, and B). In India, they are named by R, Y, and B, that is, red, yellow, and blue.

### 8.6 PHASE SEQUENCE

In a three-phase system, there are three voltages having same magnitude and frequency displaced by an angle of $120^{\circ}$ electrical. They are attaining their positive maximum value in a particular order. The order in which the voltages (or emfs) in the three phases attain their maximum positive value is called the phase sequence.

The emfs induced in the three coils or phases attain their positive maximum value in the order of $\mathrm{a}_{1} \mathrm{a}_{2}, \mathrm{~b}_{1} \mathrm{~b}_{2}$, and $\mathrm{c}_{1} \mathrm{c}_{2}$; therefore, the phase sequence is $\mathrm{a}, \mathrm{b}$, and c . However, if the coils or phases are being named out as $R, Y$, and $B$ in place of $a, b$, and $c$, respectively, then the phase sequence will be RYB.

The sequence RYB (or YBR or BRY) is considered as positive phase sequence, while the RBY (or BYR or YRB) is considered as negative phase sequence. The sequence knowledge of phase sequence is essential in the following important applications:

1. The direction of rotation of three-phase induction motors depends upon the phase sequence of three-phase supply. To reverse the direction of rotation, the phase sequence of the supply given to the motor has to be changed.
2. The parallel operation of three-phase alternators and transformers is only possible if phase sequence is known.

### 8.7 DOUBLE-SUBSCRIPT NOTATION

An alternating quantity is generally represented by a dou-ble-subscript notation. In this notation, two letters are placed at the foot of the symbol for voltage or current, as shown in Figure 8.2. This conveys the following two scenarios:

1. The subscript of the symbol for voltage or current indicates the portion of the circuit where the quantity is located.
2. The order of the subscript indicates the positive direction of the quantity in which it acts.


Fig. 8.2 Circuit to represent double subscript notation

For instant, the current is represented as $I_{\mathrm{ab}}$. It means that

1. The portion $a b$ of the circuit is considered.
2. The current flows from a to $b$.

The double-subscript notation is very useful in solving AC circuits having a number of voltages and currents.


Fig. 8.3 Three-phases supplying power independently

### 8.8 INTERCONNECTION OF THREE PHASES

In a three-phase AC generator, three are three windings. Each winding has two terminals (start and finish). If a separate load is connected across each phase winding as shown in Figure 8.3, then each phase supplies an independent load through a pair of leads (wires). Thus, six wires will be required in this case to connect the load to generator. This will make the whole system complicated and expensive.

In order to reduce the number of the conductors, the three-phase windings of the AC generator are suitably interconnected. The following are the two universally adopted methods of interconnecting the three phases:

1. Star or wye $(\mathrm{Y})$ connection
2. Mesh or delta ( $\Delta$ ) connection.

### 8.9 STAR OR WYE (Y) CONNECTION

In star or wye $(\mathrm{Y})$ connections, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point. The three line conductors are run from the remaining three free terminals called line conductors. Ordinarily, only three wires are carried to the external circuit giving three-phase, three-wire star-connected system. However, sometimes a fourth wire is carried from the star point to the external circuit, called neutral wire, giving threephase, four-wire star-connected system.

As shown in Figure 8.4, the finish terminals $\mathrm{a}_{2}, \mathrm{~b}_{2}$, and $\mathrm{c}_{2}$ of the three windings are connected to form a star or neutral point. From the remaining three free terminals, three conductors are run, named R, Y, and B. The current flowing through each phase is called phase current $I_{\mathrm{ph}}$ and current


Fig. 8.4 (a) 3-phases connected in star (b) Star connected system
flowing through each line conductor is called line current $I_{\mathrm{L}}$. Similarly, voltage across each phase is called phase voltage ( $E_{\mathrm{ph}}$ ) and voltage across two line conductors is called line voltage ( $E_{\mathrm{L}}$ ).

### 8.9.1 Relation between Phase Voltage and Line Voltage

The connections are shown in Figure 8.5(a). Since the system is balanced, three voltages $E_{\mathrm{NR}}$, $E_{\mathrm{NY}}$, and $E_{\mathrm{NB}}$ are equal in magnitude, but displaced from one another by $120^{\circ}$ electrical. Their phasor diagrams are shown in Figure 8.5(b). The arrow heads on emfs and currents indicate the positive direction and not their actual direction at any instant.
Now, $E_{\mathrm{NR}}=E_{\mathrm{NY}}=E_{\mathrm{NB}}=E_{\mathrm{ph}}($ in magnitude $)$


Fig. 8.5 (a) Circuit representing phase and line voltages and currents in star connections (b) Phasor diagram of phase and line voltages in star connections

It may be seen that between any two lines, there are two phase voltages.
Tracing the loop NRYN, we get $\overline{E_{\mathrm{NR}}}+\overline{E_{\mathrm{RY}}}-\overline{E_{\mathrm{NY}}}=0$
or

$$
\overline{E_{\mathrm{RY}}}=\overline{E_{\mathrm{NY}}}-\overline{E_{\mathrm{NR}}} \text { (vector difference) }
$$

To find the vector sum of $E_{\mathrm{NY}}$ and $-E_{\mathrm{NR}}$, reverse the vector $E_{\mathrm{NR}}$ and add it vectorially with $E_{\mathrm{NY}}$ as shown in Figure 8.5(b).
Therefore,

$$
E_{\mathrm{RY}}=\sqrt{E_{\mathrm{NY}}^{2}+E_{\mathrm{NR}}^{2}+2 E_{\mathrm{NY}} E_{\mathrm{NR}} \cos 60^{\circ}}
$$

or

$$
E_{\mathrm{L}}=\sqrt{E_{\mathrm{ph}}^{2}+E_{\mathrm{ph}}^{2}+2 E_{\mathrm{ph}} E_{\mathrm{ph}} \times 0.5}=\sqrt{3 E_{\mathrm{ph}}^{2}}=\sqrt{3} E_{\mathrm{ph}} \quad \text { (in magnitude) }
$$

Similarly,

$$
\overline{E_{\mathrm{YB}}}=\overline{E_{\mathrm{NB}}}-\overline{E_{\mathrm{NY}}} \quad \text { or } \quad E_{\mathrm{L}}=\sqrt{3} E_{\mathrm{ph}}=\overline{E_{\mathrm{NR}}}-\overline{E_{\mathrm{NB}}} \quad \text { or } \quad E_{\mathrm{L}}=\sqrt{3} E_{\mathrm{ph}}
$$

Hence, in star connections, line voltage $=\sqrt{3} \times$ phase voltage.

### 8.9.2 Relation between Phase Current and Line Current

From Figure 8.5(a), it is clear that same current flows through phase winding as well as the line conductor since line conductor is just connected in series with the phase winding.

$$
I_{\mathrm{R}}=I_{\mathrm{NR}} ; I_{\mathrm{Y}}=I_{\mathrm{NY}} \text { and } I_{\mathrm{B}}=I_{\mathrm{NB}}
$$

where $I_{\mathrm{NR}}=I_{\mathrm{NY}}=I_{\mathrm{NB}}=I_{\mathrm{ph}}$ (phase current) and $I_{\mathrm{R}}=I_{\mathrm{Y}}=I_{\mathrm{B}}=I_{\mathrm{L}}$ (line current)
Hence, in star connections, line current = phase current.

### 8.10 MESH OR DELTA ( $\Delta$ ) CONNECTION

In delta $(\Delta)$ or mesh connections, the finish terminal of one winding is connected to start terminal of the other winding and so on, which forms a closed circuit. The three line conductors are run from three junctions of the mesh called line conductors, as shown in Figure 8.6.


Fig. 8.6 (a) Three phases connected in delta (b) Delta connected system

To obtain delta connection, $a_{2}$ is connected with $b_{1}, b_{2}$ is connected with $c_{1}$, and $c_{2}$ is connected with $a_{1}$, as shown in Figure 8.6(a). Three conductors R, Y, and B are run from the three junctions called line conductors. The current flowing through each phase is called phase current $\left(I_{\mathrm{ph}}\right)$ and the current flowing through each line conductor is called line current $\left(I_{\mathrm{L}}\right)$, as shown in Figure $8.6(\mathrm{~b})$. Similarly, voltage across each phase is called phase voltage $\left(E_{\mathrm{ph}}\right)$ and voltage across two line conductors is called line voltage $\left(E_{\mathrm{L}}\right)$.

### 8.10.1 Relation between Phase Voltage and Line Voltage

From Figure 8.7(a), it is clear that voltage across terminals 1 and 2 is the same as across terminals R and Y . Therefore, $E_{12}=E_{\mathrm{RY}}$; similarly, $E_{23}=E_{\mathrm{YB}}$ and $E_{31}=E_{\mathrm{BR}}$, where $E_{12}=E_{23}=E_{31}=E_{\mathrm{ph}}$ (phase voltage) and $E_{\mathrm{RY}}=E_{\mathrm{YB}}=E_{\mathrm{BR}}=\mathrm{E}_{\mathrm{L}}$ (line voltage) Hence, in delta connection, line voltage $=$ phase voltage .

(a)

(b)

Fig. 8.7 (a) Circuit representing phase and line voltages and currents in delta connections (b) Phasor diagram of phase and line currents in delta connections

### 8.10.2 Relation between Phase Current and Line Current

Since the system is balanced, and therefore, three phase currents $I_{12}, I_{23}$, and $I_{31}$ are equal in magnitude, but displaced from one another by $120^{\circ}$ electrical. Their phasors are shown in Figure 8.7(b).
Thus,

$$
I_{12}=I_{23}=I_{31}=I_{\mathrm{ph}}(\text { in magnitude })
$$

In Figure 8.7(a), it may be seen that current is divided at every junction 1, 2, and 3.
By applying Kirchhoff's first law at junction 1,
incoming currents $=$ outgoing currents

$$
\overline{I_{31}}=\overline{I_{\mathrm{R}}}+\overline{I_{12}} \text { or } \overline{I_{\mathrm{R}}}=\overline{I_{31}}-\overline{I_{12}}(\text { vector difference })
$$

To find the vector sum of $I_{31}$ and $-I_{12}$, reverse the vector $I_{12}$ and add it vectorially with $I_{31}$, as shown in Figure 8.7(b).

$$
I_{\mathrm{R}}=\sqrt{I_{12}^{2}+I_{12}^{2}+2 I_{31} I_{12} \cos 60^{\circ}}=\sqrt{I_{\mathrm{ph}}^{2}+I_{\mathrm{ph}}^{2}+2 I_{\mathrm{ph}} I_{\mathrm{ph}} \times 0.5}\left(-I_{\mathrm{R}}=I_{\mathrm{I}}\right)
$$

or

$$
I_{L}=\sqrt{3 I_{\mathrm{ph}}^{2}}=\sqrt{3} I_{\mathrm{ph}} \text { (in magnitude) }
$$

Similarly,

$$
\begin{aligned}
& \overline{I_{\mathrm{Y}}}=\overline{I_{12}}-\overline{I_{23}} \quad \text { or } \quad I_{\mathrm{L}}=\sqrt{3} I_{\mathrm{ph}} \quad \text { and } \\
& \overline{I_{\mathrm{B}}}=\overline{I_{23}}-\overline{I_{31}} \quad \text { or } \quad I_{\mathrm{L}}=\sqrt{3} I_{\mathrm{ph}}
\end{aligned}
$$

### 8.11 CONNECTIONS OF THREE-PHASE LOADS

Similar to three-phase supply, the three-phase loads may also be connected in star or delta.

The three-phase loads connected in star and delta are shown in Figure 8.8(a) and (b), respectively. The three-phase loads may be balanced or unbalanced. If the three loads (impedances) $Z_{1}, Z_{2}$, and $Z_{3}$ are having same magnitude and phase angle, then the three-phase load is said to

(a)

(b)

Fig. 8.8 (a) Three phase load connected in star (b) Three phase load connected in delta
be a balanced load. Under such connections, all the phase or line currents and all the phase or line voltages are equal in magnitude. Throughout this book, balanced three-phase system will be considered unless stated otherwise.

### 8.12 POWER IN THREE-PHASE CIRCUITS

Power in single-phase system or circuit is given by the relation:

$$
P=V I \cos \phi
$$

where $V=$ voltage of single phase (i.e., $V_{\mathrm{ph}}$ );
$I=$ current of single phase (i.e., $\left.I_{\mathrm{ph}}^{\mathrm{ph}}\right)$; and
$\cos \phi=$ power factor of the circuit.
In three-phase circuits (balanced load), the power is just the sum of powers in three phases

In star connections,

$$
P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi
$$

$$
P=3 \frac{V_{\mathrm{L}}}{\sqrt{3}} I_{\mathrm{L}} \cos \phi \quad\left(\text { since } V_{\mathrm{ph}}=V_{\mathrm{L}} / \sqrt{3} \text { and } I_{\mathrm{ph}}=I_{\mathrm{L}}\right.
$$

or

$$
P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi
$$

In delta connections,

$$
\begin{array}{ll}
P=3 V_{\mathrm{L}} \frac{I_{\mathrm{L}}}{\sqrt{3}} \cos \phi & \left(\text { since } V_{\mathrm{ph}}=V_{\mathrm{L}} \text { and } I_{\mathrm{ph}}=I_{\mathrm{L}} / \sqrt{3}\right) \\
P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi &
\end{array}
$$

Thus, the total power in a three-phase balanced load, irrespective of connections (star or delta), is given by the relation $\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$. Its units are kW or W .

Apparent power,

$$
\begin{aligned}
& P_{\mathrm{a}}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \\
& P_{\mathrm{r}}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi
\end{aligned}
$$

Reactive power,
(kVAR or VAR)
The comparison between star and delta connected systems is shown in Table 8.1.
Table 8.1 Comparison between Star and Delta Systems

| S. No | Particulars | Star | Delta |
| :--- | :--- | :--- | :--- |
| 1. | Connections | Similar ends of the three phases are <br> joined together to get neutral or star <br> point. Line conductors run from the <br> remaining three terminals. | Dissimilar ends of the three <br> phases are joined together to <br> form a closed delta. Three line <br> conductors run from each <br> joint. |
| 2. | Relation between line <br> and phase voltages and <br> currents. | $V_{\mathrm{L}}=\sqrt{3} \mathrm{~V}_{\mathrm{ph}}$ <br> $\mathrm{I}_{\mathrm{L}}=I_{\text {ph }}$ | $V_{\mathrm{L}}=V_{\text {ph }}$ <br> $\mathrm{L}_{\mathrm{L}}=3 I_{\mathrm{ph}}$ |
| 3. | Neutral terminal | A neutral wire can be run from the <br> star point. | Neutral wire cannot be run. |

## Table 8.1 (Continued)

| S. No | Particulars | Star | Delta |
| :--- | :--- | :--- | :--- |
| 4. | Voltages to be obtained | Two voltages can be obtained in <br> star-connected four-wire system: <br> 1. Phase voltage between any one <br> line conductor and neutral | Only one voltage between the <br> lines can be obtained. |
| 5. | Lighting and power load | Lighting load can be connected <br> conductor between any two line |  |
| 6. | Applications |  | Ligetween any phase (line conductor) <br> and neutral, while power load is <br> connected across lines. | | Only power loads can be |
| :--- |
| connected between the lines. |
| These connections are used for |
| domestic (single phase) as well as |
| power (three phase) loads. Protective |
| devices (relays, etc.) are connected |
| between line and neutral where |
| neutral is properly earthed. |$\quad$| These connections are used |
| :--- |
| in transformers for power |
| transmission. Induction motors |
| are usually connected in delta. |

## Example 8.1

Three $100-\Omega$ resistors are connected first in star and then in delta across 415 V , three-phase supply. Calculate the line and phase currents in each case and also the power taken from the source.

## Solution:

The resistors are connected in star as shown in Figure 8.9.
Phase voltage, $V_{\text {ph }}=V_{\mathrm{L}} / \sqrt{3}=415 / \sqrt{3}=239.6 \mathrm{~V}$
Phase current, $I_{\mathrm{ph}}=V_{\mathrm{ph}} / Z_{\mathrm{ph}}=239.6 / 100=2.396 \mathrm{~A}$


Fig. 8.9 Resistors connected in star

Line current, $I_{\mathrm{L}}=I_{\mathrm{ph}}=2.396 \mathrm{~A}$
Power drawn, $P=3 I_{\mathrm{ph}}^{2} R_{\mathrm{ph}}=3 \times(2.396)^{2} \times 100=1,722 \mathrm{~W}$
The resistors are connected in delta as shown in Figure 8.10.

$$
\begin{aligned}
& V_{\mathrm{ph}}=V_{\mathrm{L}}=415 \mathrm{~V} \\
& I_{\mathrm{ph}}=V_{\mathrm{ph}} / Z_{\mathrm{ph}}=415 / 100=4.15 \mathrm{~A} \\
& I_{L}=\sqrt{3} \times 4.15=7.188 \mathrm{~A}
\end{aligned}
$$

Power drawn, $P=3 I_{\mathrm{ph}}^{2} R_{\mathrm{ph}}=3 \times(4.15)^{2} \times 100=5,166 \mathrm{~W}$


Fig. 8.10 Resistors connected in delta

## Example 8.2

Three similar coils, each having a resistance of $8 \Omega$ and an inductance of 0.0191 H in series in each phase, is connected across a 400 V , three-phase 50 Hz supply. Calculate the line current, power input, kVA, and kVAR taken by the load.
(U.P.T.U. 2006-07)

## Solution:



Fig. 8.11 Circuit as per data

The circuit is shown in Figure 8.11.
Here, $R=8 \Omega, L=0.0191 \mathrm{H}, f=50 \mathrm{~Hz}$

$$
X_{\mathrm{L}}=2 \pi f_{\mathrm{L}}=2 \pi \times 50 \times 0.0191=6 \Omega
$$

Line voltage, $V_{\mathrm{L}}=400 \mathrm{~V}$
Phase voltage, $V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$

$$
Z_{\mathrm{ph}}=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{8^{2}+6^{2}}=10 \Omega
$$

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{231}{10}=23.1 \mathrm{~A}$
Line current, $I_{\mathrm{L}}=I_{\mathrm{ph}}=23.1 \mathrm{~A}$.
Power factor, $\cos \phi=\frac{R_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{8}{10}=0.8$ lagging
Power input, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi=\sqrt{3} \times 400 \times 23.1 \times 0.8=12,800 \mathrm{~W}=12.8 \mathrm{~kW}$
kVA taken by the load, $\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \times 10^{-3}=\sqrt{3} \times 400 \times 23.1 \times 10^{-3}=16 \mathrm{kVA}$
kVAR taken by the load $=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi=\sqrt{3} \times 400 \times 23.1 \times 0.6 \times 10^{-3}=9.6 \mathrm{VAR}$

## Example 8.3



Fig. 8.12 Circuit as per data

A three-phase, 400 V supply is connected to a three-phase star-connected balanced load. The line current is 20 A and the power consumed by the load is 12 kW . Calculate the impedance of the load, phase current, and power factor.
(U.P.T.U. 2006-07)

## Solution:

The circuit is shown in Figure 8.12.
Here, $P=12 \mathrm{~kW}=12,000 \mathrm{~W} ; V_{\mathrm{L}}=400 \mathrm{~V} ; I_{\mathrm{L}}=20 \mathrm{~A}$
Phase voltage, $V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$
Phage current, $I_{\mathrm{ph}}=I_{\mathrm{L}}=20 \mathrm{~A}$
Phase impedance, $Z_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{I_{\mathrm{ph}}}=\frac{231}{20}=11.55 \Omega$

In three-phase balanced load, power

$$
P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi
$$

Therefore, power factor,

$$
\begin{aligned}
\cos \phi & =\frac{P}{\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}}} \\
& =\frac{12,000}{\sqrt{3} \times 400 \times 20} \\
& =0.866
\end{aligned}
$$

## Example 8.4

A three-phase balanced delta-connected load is connected to a three-phase, $400 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. It draws a line current of 34.64 A at 0.8 power factor lagging. Determine resistance and inductance of each branch. Further, determine the power drawn by each phase. (U.P.T.U. 2005-06)

## Solution:

The circuit is shown in Figure 8.13. Line voltage, $V_{\mathrm{L}}=400 \mathrm{~V}, f=50 \mathrm{~Hz}$
Line current, $I_{\mathrm{L}}=34.64 \mathrm{~A}$, power factor, $\cos \phi=0.8$ (lag).
Phase current, $I_{\mathrm{ph}}=\frac{I_{\mathrm{L}}}{\sqrt{3}}=\frac{34.64}{\sqrt{3}}=20 \mathrm{~A}$

$$
\begin{aligned}
\cos \phi & =0.8 \\
\Rightarrow \phi & =\cos ^{-1}(0.8) \\
& =36.87^{\circ}
\end{aligned}
$$

Phase voltage, $V_{\text {ph }}=V_{\mathrm{L}}=400 \mathrm{~V}$
Impedance of each branch, $Z_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{I_{\mathrm{ph}}}=\frac{400}{20}=20 \Omega$


Fig. 8.13 Circuit as per data

Power factor, $\cos \phi=\frac{R_{\mathrm{ph}}}{Z_{\mathrm{ph}}}$
Resistance of each branch, $R_{\mathrm{ph}}=Z_{\mathrm{ph}} \cos \phi=20 \times 0.8=16 \Omega$
Inductive reactance, $X_{\mathrm{ph}}=\sqrt{Z_{\mathrm{ph}}^{2}-R_{\mathrm{ph}}^{2}}=\sqrt{(20)^{2}-(16)^{2}}=12 \Omega$
Inductance, $L=\frac{X_{\mathrm{ph}}}{2 \pi f}=\frac{12}{2 \pi \times 50}=38.2 \mathrm{mH}$
Power drawn by each phase, $P_{\mathrm{ph}}=V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi=400 \times 20 \times 0.8=6,400 \mathrm{~W}$

## Example 8.5

A three-phase, three-wire, Y-connected system has 150 V from phase to phase. Each phase has $Z=5 \angle-30^{\circ}$. Find current in each phase and total power drawn from the mains. Draw the phasor diagram.
(U.P.T.U. 2007-08)

## Solution:

Line voltage, $V_{\mathrm{L}}=150 \mathrm{~V}$
Impedance per phase, $\bar{Z}_{\mathrm{ph}}=5 \angle-30^{\circ}$
Phase voltage, $V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{150}{\sqrt{3}}=50 \sqrt{3}$
Let us consider $V_{\text {ph }}(\mathrm{R})$ as a reference vector,

$$
\begin{aligned}
& \overline{V_{\mathrm{ph}(\mathrm{R})}}=50 \sqrt{3} \angle 0^{\circ} \\
& \bar{V}_{\mathrm{ph}(\mathrm{Y})}=50 \sqrt{3} \angle-120^{\circ} \\
& \bar{V}_{\mathrm{ph}(\mathrm{~B})}=50 \sqrt{3} \angle-240^{\circ}
\end{aligned}
$$

Current in each phase, $\overline{I_{\mathrm{ph}(\mathrm{R})}}=\frac{\overline{V_{\mathrm{ph(R)}}}}{Z_{\mathrm{ph}}}=\frac{50 \sqrt{3} \angle 0^{\circ}}{5 \angle-30^{\circ}}$

$$
\begin{aligned}
& =10 \sqrt{3} \angle 30^{\circ} \\
\overline{I_{\mathrm{ph}(\mathrm{Y})}} & =\frac{\overline{V_{\mathrm{ph}(\mathrm{Y})}}}{Z_{(\mathrm{ph})}}=\frac{50 \sqrt{3} \angle-120^{\circ}}{5 \angle-30^{\circ}}=10 \sqrt{3} \angle-90^{\circ} \\
\overline{I_{\mathrm{ph}(\mathrm{~B})}} & =\frac{\overline{V_{\mathrm{ph}(\mathrm{~B})}}}{Z_{(\mathrm{ph})}}=\frac{50 \sqrt{3} \angle-240^{\circ}}{5 \angle 30^{\circ}}=10 \sqrt{3} \angle-210^{\circ}
\end{aligned}
$$

Since it is a balanced system,
power, $P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi$,
where

$$
\cos \phi=\cos 30=\frac{\sqrt{3}}{2}
$$

Therefore, $P=3 \times 50 \sqrt{3} \times 10 \sqrt{3} \times \frac{\sqrt{3}}{2}=3,897 \mathrm{~W}=3.897 \mathrm{~kW}$
Phasor diagram is shown in Figure 8.14.


Fig. 8.14 Phasor diagram

## Example 8.6

A 400 V , three-phase voltage is applied to a balanced three-phase load of phase impedance $\Omega$ ( 15 $+j 20$ ). Find (i) the phasor current in each line. (ii) What is the power consumed per phase? (iii) What is the phasor sum of three line currents?
(U.P.T.U. Tut.)

## Solution:

The circuit is shown in Figure 8.15
Phase voltage, $V_{\text {ph }}=V_{\mathrm{L}}=400 \mathrm{~V}$
Phase impedance, $Z_{\mathrm{ph}}=\sqrt{R_{\mathrm{ph}}^{2}+X_{\mathrm{ph}}^{2}}=\sqrt{15^{2}+20^{2}}$

$$
=25 \Omega
$$

(i) Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{400}{25}=16 \mathrm{~A}$

Line current, $I_{\mathrm{L}}=\sqrt{3} I_{\text {ph }}=\sqrt{3} \times 16$

$$
=27.7 \mathrm{~A}
$$

(ii) Power consumed in each phase


Fig. 8.15 Circuit as per data

$$
=I_{\mathrm{ph}}^{2} R_{\mathrm{ph}}=16^{2} \times 15=3,840 \mathrm{~W}
$$

(iii) Phasor sum would be zero because the three currents are equal in magnitude and have a mutual phase difference of $120^{\circ}$.

## Example 8.7

Three similar coils, each having a resistance of $15 \Omega$ and an inductance of 0.04 H , are connected in star to a three-phase 50 Hz supply, 200 V between lines. Calculate the line current. If they are now connected in delta, calculate the phase current, line current, and the total power absorbed in each phase.

## Solution:

Here, $R=15 \Omega ; L=0.04 \mathrm{H}$

$$
X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.04=12.57 \Omega
$$

$$
Z=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(15)^{2}+(12.57)^{2}}=19.58 \Omega
$$

When the coils are connected in star as shown in Figure 8.16, voltage across each coil, that is, phase voltage.


Fig. 8.16 Circuit as per data


Fig. 8.17 Circuit as per data

$$
V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{200}{\sqrt{3}}=115.47 \mathrm{~V}
$$

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z}=\frac{115.47}{19.58}=5.9 \mathrm{~A}$
Since, current $I_{\mathrm{L}}=I_{\mathrm{ph}}=5.9 \mathrm{~A}$
The coils are connected in delta as shown in Figure 8.17.
Phase voltage, $V_{\text {ph }}=V_{\mathrm{L}}=200 \mathrm{~V}$
Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z}=\frac{200}{19.58}=10.215 \mathrm{~A}$
Line current, $\quad I_{\mathrm{L}}=\sqrt{3} I_{\mathrm{ph}}=\sqrt{3} \times 10.215$

$$
=17.69 \mathrm{~A}
$$

Power factor of the load,
Power absorbed in each phase, $\phi=\frac{R}{Z}=\frac{15}{19.58}=0.7661$
lagging
Power absorbed in each phase, $P_{\mathrm{ph}}=V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi$

$$
\begin{aligned}
& =200 \times 10.215 \times 7,661 \\
& =1,565 \mathrm{~W}
\end{aligned}
$$

Total power absorbed, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi=\sqrt{3} \times 200 \times 17.69 \times 0.7661=4,695 \mathrm{~W}$

## Example 8.8

A balanced star-connected load of $(8+j 6) \Omega$ per phase is connected to a balanced three-phase 400 V AC supply. Find the line current, power factor, power, and volt-ampere drawn by the load.
(U.P.T.U. 2003-04)

## Solution:

The circuit is shown in Figure 8.18.


Fig. 8.18 Circuit as per data

Here, $V_{\mathrm{L}}=400 \mathrm{~V} ; R=8 \Omega ; X_{\mathrm{L}}=6 \Omega$
Phase voltage, $V_{\mathrm{ph}}=\frac{V_{L}}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$
Phase impedance, $Z_{\mathrm{ph}}=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}$

$$
=\sqrt{(8)^{2}+(6)^{2}}=10 \Omega
$$

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{231}{10}=23.1 \mathrm{~A}$
Line current, $I_{\mathrm{L}}=I_{\mathrm{ph}}=23.1$

Power factor, $\cos \phi=\frac{R}{Z}=\frac{8}{10}=0.8 \mathrm{lag}$
Power drawn, $P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi=3 \times 231 \times 23.1 \times 0.8=12,800 \mathrm{~W}$
Volt-ampere drawn, $P_{\mathrm{a}}=3 V_{\mathrm{ph}} I_{\mathrm{ph}}=3 \times 231 \times 23.1=16,000$

## Example 8.9

Three identical coils connected in delta across $400 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase AC supply, take a line current of 17.32 A at power factor of 0.8 lagging. Calculate (i) the phase current, (ii) the resistance and inductance of each coil, and (iii) the power drawn by each coil. (U.P.T.U. Feb. 2002)

## Solution:

The circuit is shown in Figure 8.19.
Line voltage, $V_{\mathrm{L}}=400 \mathrm{~V}$
Phase voltage, $V_{\text {ph }}=V_{\mathrm{L}}=400 \mathrm{~V}$
Line current, $I_{\mathrm{L}}=17.32 \mathrm{~A}$ (given)
Power factor, $\cos \phi=0.8 \mathrm{lag}$
(i) Phase current, $I_{\mathrm{ph}}=\frac{I_{\mathrm{L}}}{\sqrt{3}}=\frac{17.32}{\sqrt{3}}=10 \mathrm{~A}$ Impedance of each coil, $Z_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{I_{\mathrm{ph}}}=\frac{400}{10}=40 \Omega$


Fig. 8.19 Circuit as per data
(ii) Resistance of each coil, $R_{\mathrm{ph}}=Z_{\mathrm{ph}} \cos \phi$

$$
\begin{aligned}
& =40 \times 0.8 \\
& =32 \Omega
\end{aligned}
$$

Inductive reactance of each coil, $X_{\mathrm{ph}}=\sqrt{Z_{\mathrm{ph}}^{2}-R_{\mathrm{ph}}^{2}}$

$$
=\sqrt{40^{2}-32^{2}}=24 \Omega
$$

Inductance of each coil, $L=\frac{X_{\mathrm{ph}}}{2 \pi f}=\frac{24}{2 \pi \times 50}=0.0764 \mathrm{H}=76.4 \mathrm{mH}$
(iii) Power drawn by each coil $=I_{\mathrm{ph}}^{2} R_{\mathrm{ph}}=10^{2} \times 32=3,200 \mathrm{~W}$

## Example 8.10

Each phase of a delta-connected load has a resistance of $25 \Omega$, an inductance of 0.15 H , and a capacitance of $120 \mu \mathrm{~F}$. The load is connected across a $400 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase supply. Determine the line current, active power, and volt-ampere reactive power.

## Solution:

The circuit is shown in Figure 8.20.
Resistance per phase, $R=25 \Omega$
Inductive reactance per phase, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 50 \times 0.15=47.124 \Omega$
Capacitive reactance per phase, $X_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 50 \times 120 \times 10^{-6}}=26.524 \Omega$
Equivalent reactance per phase, $X=X_{\mathrm{L}}-X_{\mathrm{C}}=47.124-26.524=20.6 \Omega$
Impedance per phase, $Z_{\mathrm{ph}}=\sqrt{R^{2}+X^{2}}=\sqrt{(25)^{2}+(20.6)^{2}}=32.4 \Omega$
Power factor, $\cos \phi=\frac{R_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{25}{32.4}=0.7716$ lag


Fig. 8.20 Circuit as per data

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{400}{32.4}$

$$
=12.35 \mathrm{~A} \quad\left(\_V_{\mathrm{L}}=V_{\mathrm{ph}}\right)
$$

Line current, $I_{\mathrm{L}}=\sqrt{3} I_{\text {ph }}=\sqrt{3} \times 12.35$

$$
=21.38 \mathrm{~A}
$$

Active power, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$

$$
\begin{aligned}
& =\sqrt{3} \times 400 \times 21.38 \times 0.7716 \\
& =11,430 \mathrm{~W} \\
& =11.43 \mathrm{~kW}
\end{aligned}
$$

Reactive power $=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi$

$$
\begin{aligned}
& =\sqrt{3 \times 400 \times 21.38 \times 0.6361} \quad\left(\_\sin \phi=\sin \cos ^{-1} 0.7716=0.6361\right) \\
& =9,420 \mathrm{VAR}=9.42 \mathrm{kVAR}
\end{aligned}
$$

## Example 8.11

A balanced star-connected load is supplied from a symmetrical three-phase, 400 V system. The current in each phase is 30 A and lags $30^{\circ}$ behind the phase voltage. Find (i) the phase voltage and (ii) the total power. Draw the phasor diagram showing the currents and voltages.

## Solution:

Line voltage, $V_{\mathrm{L}}=400 \mathrm{~V}$
(i) Phase voltage, $V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{400}{\sqrt{4}}=231 \mathrm{~V}$

Phase current, $I_{\mathrm{ph}}=30 \mathrm{~A}$
Power factor, $\cos \phi=\cos 30^{\circ}=0.866$ (lag)
(ii) Total power, $P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi=3 \times 231 \times 30 \times 0.866$

$$
=18,000 \mathrm{~W} \text { or } 18 \mathrm{~kW}
$$

Phasor diagram for currents and voltages is shown in Figure 8.21(b).


Fig. 8.21 (a) Circuit Diagram (b) Phasor Diagram for Currents and Voltages

## Example 8.12

A three-phase, three-wire RYB system, with effective line voltage of 150 V , has a balanced Y-connected load of $Z=5 \angle-30^{\circ} \Omega$ in each phase. Obtain the currents, power supplied to load, and draw voltage-current phasor diagram.
(U.P.T.U. June 2001)

## Solution:

Phase voltage, $V_{\text {ph }}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{150}{\sqrt{3}}=86.6 \mathrm{~V}$ Taking phase voltage $V_{\mathrm{ph}}$ as reference phasor

$$
V_{\mathrm{ph}}=86.6 \angle 0^{\circ} \mathrm{V}
$$

Impedance per phase, $Z_{\mathrm{ph}}=5 \angle-30^{\circ} \Omega$

$$
\begin{aligned}
I_{\mathrm{ph}} & =\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}} \\
& =\frac{86.6 \angle 0^{\circ}}{5 \angle-30^{\circ}} \\
& =17.32 \angle 30^{\circ} \mathrm{A}
\end{aligned}
$$



Phasor diagram
Fig. 8.22 Phasor diagram as per data

Power factor, $\cos \phi=\cos 30^{\circ}=0.866$ lag
Power supplied to load, $P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \phi$

$$
\begin{aligned}
& =3 \times 86.6 \times 17.32 \times 0.866 \\
& =3,897 \mathrm{~W}
\end{aligned}
$$

Phasor diagram is shown in Figure 8.22.

## Example 8.13

A three-phase motor operating of a $400-\mathrm{V}$ balanced system develops 18.65 kW at an efficiency of 0.87 per unit and a power factor of 0.82 . Calculate the line current and the phase current if the windings are delta connected.

## Solution:

In a three-phase system, power input to motor, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$
where

$$
\begin{aligned}
& P=\frac{\text { Power developed }}{\text { Efficiency }}=\frac{18.65 \times 1,000}{0.87}=2,1436.8 \mathrm{~W} \\
& V_{\mathrm{L}}=400 \mathrm{~V}
\end{aligned}
$$

and power factor

$$
(\cos \phi)=0.82
$$

Therefore,

$$
2,1436.8=\sqrt{3} \times 400 \times I_{\mathrm{L}} \times 0.82
$$

or line current, $I_{\mathrm{L}}=37.73 \mathrm{~A}$
As the motor is connected in delta,
phase current, $I_{\text {ph }}=I_{\mathrm{L}} / \sqrt{3}=37.73 / \sqrt{3}=27.78 \mathrm{~A}$

## Example 8.14

A balanced three-phase star-connected load of 120 kW takes a leading current of 85 A , when connected across a three-phase $1,100 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Obtain the values of the resistance, reactance, and capacitance of the load per phase and also calculate the power factor of the load.
(U.P.T.U. June 2004)


Fig. 8.23 Circuit as per data

## Solution:

The circuit diagram is shown in Figure 8.23.
Supply voltage, $V_{\mathrm{L}}=1,100 \mathrm{~V}$
Supply current, $I_{\mathrm{L}}=85 \mathrm{~A}$ (leading)
Power supplied, $P=120 \mathrm{~kW}$

$$
=120 \times 10^{3} \mathrm{~W}
$$

Power factor of the load,

$$
\begin{aligned}
\cos \phi & =\frac{P}{\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}}} \\
& =\frac{120 \times 10^{3}}{\sqrt{3} \times 1,100 \times 85} \\
& =0.741 \text { (leading) }
\end{aligned}
$$

Impedance of the load per phase, $Z_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{I_{\mathrm{ph}}}=\frac{V_{\mathrm{L}} / \sqrt{3}}{I_{\mathrm{L}}}=\frac{1,100 / \sqrt{3}}{85}=7.472 \Omega$

Resistance of the load per phase, $R_{\mathrm{ph}}=Z_{\mathrm{ph}} \cos \phi=7.472 \times 0.741=5.536 \Omega$
Capacitive reactance of the load per phase, $X_{\mathrm{C}}=\sqrt{Z_{\mathrm{ph}}^{2}-R_{\mathrm{ph}}^{2}}=\sqrt{(7.472)^{2}-(5.536)^{2}}$

$$
=5.02 \Omega
$$

Capacitance of the load per phase, $C=\frac{1}{2 \pi f X_{\mathrm{C}}}=\frac{1}{2 \pi \times 50 \times 5.02}$

$$
=634.3 \mu \mathrm{~F}
$$

## Example 8.15

Each phase of a star-connected load consists of a non-reactive resistance of $100 \Omega$ in parallel with capacitance of $31.8 \mu \mathrm{~F}$. Calculate the line current, the power absorbed, the total kVA , and the power factor when it is connected to a 400 V , three-phase, 50 Hz supply.
(U.P.T.U. Tut.)

## Solution:

The circuit diagram is shown in Figure 8.24.
Phase voltage, $V_{\mathrm{ph}}=\frac{V}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$
Taking phase voltage as reference phasor,

$$
\bar{V}_{\mathrm{ph}}=231 \angle 0^{\circ} \mathrm{V}
$$

Admittance per phase,

$$
\begin{aligned}
\bar{Y}_{\mathrm{ph}} & =\frac{1}{R}+j \omega C \\
& =\frac{1}{100}+j 2 \pi \times 50 \times 31.8 \times 10^{-6} \\
& =(0.01+j 0.01)=0.01414 \angle 45^{\circ} \mathrm{mho}
\end{aligned}
$$



Fig. 8.24 Circuit as per data

Line current, $\bar{I}_{\mathrm{L}}=\bar{I}_{\mathrm{ph}}=\bar{V}_{\mathrm{ph}}{\overline{Y_{\mathrm{ph}}}}$

$$
=231 \angle 0^{\circ} \times 0.01414 \angle 45^{\circ}=3.266 \angle 45^{\circ} \mathrm{A}
$$

Therefore, line current, $I_{\mathrm{L}}=3.266 \mathrm{~A}$
Power factor, $\cos \phi=\cos 45^{\circ}=0.707$ (leading)
Power absorbed, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$

$$
=\sqrt{3} \times 400 \times 3.266 \times 0.707=1,600 \Omega
$$

Total $\mathrm{kVA}=\frac{\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}}}{1,000}=\frac{\sqrt{3} \times 400 \times 3.266}{1,000}=2.263$

## Example 8.16

Three identical resistors of $20 \Omega$ each are connected in star to a $415 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase supply. Calculate (i) the total power consumed. If they are connected in delta, then calculate (ii) the total power consumed. If one of the resistors is opened, then calculate (iii) the power consumed.
(UP.T.U. Jan. 2003)

## Solution:

(i) When the resistors are star connected, as shown in Figure 8.25(a), total power consumed, $P=3 \times \frac{V_{\mathrm{ph}}^{2}}{R}=3 \times \frac{\left(V_{\mathrm{L}} / \sqrt{3}\right)^{2}}{R}=3 \times \frac{415 \times 415}{3 \times 20}$

$$
=8,612 \mathrm{~W}=8.612 \mathrm{~kW}
$$

(ii) When the resistors are delta connected, as shown in Figure 8.25(b),
total power consumed, $P=3 \times \frac{V_{\mathrm{ph}}^{2}}{R}=3 \times \frac{V_{\mathrm{L}}^{2}}{R}=3 \times \frac{(415)^{2}}{20}$

$$
=25,836 \mathrm{~W}=25.836 \mathrm{~kW}
$$

(iii) When one of the resistor is open circuited, then let us consider the following two cases: In the case of star connections, as shown in Figure 8.25(c),


Fig. 8.25 (a) Resistors connected in star (b) Resistors connected in delta
(c) One of the resistors is open circuited in star connections (d) One of the resistor is open circuited in delta connections

Total power consumed, $P=2 \times \frac{V_{\mathrm{ph}}^{2}}{R}=2 \times \frac{\left(V_{\mathrm{L}} / 2\right)^{2}}{R}=2 \times \frac{(415)^{2}}{2 \times 2 \times 20}$

$$
=4,306 \mathrm{~W}=4.306 \mathrm{~kW}
$$

In the case of delta connections, as shown in Figure 8.25(d),
Total power consumed, $P=2 \times \frac{V_{\mathrm{L}}^{2}}{R}=2 \times \frac{(415)^{2}}{20}=17,224 \mathrm{~W}=17.224 \mathrm{~kW}$

## Example 8.17

The line current is $I$ if balanced three-phase impedances are star connected. Show that the line current will be $3 I$ if the same impedances are delta connected.

## Solution:

The three impedances each of $Z \Omega$ are connected in star, as shown in Figure 8.26(a).
Line current $=$ phase current $=I$, and therefore, phase voltage $=I Z$
Line voltage $=\sqrt{3}$ phase voltage $=\sqrt{3} \mathrm{IZ}$
When the same impedances, each of $Z \Omega$, are connected in delta, as shown in Figure 8.26(b),
Phase voltage $=$ line voltage $=\sqrt{3} \mathrm{IZ}$
Phase current $=\sqrt{3} I Z / Z=\sqrt{3} I$
Line current $=\sqrt{3}$ phase current $=\sqrt{3} \times \sqrt{3} I=3 I$.

(a)

(b)

Fig. 8.26 (a) Impedances connected in star (b) Impedances connected in delta

## Example 8.18

Non-inductive loads of $8 \mathrm{~kW}, 6 \mathrm{~kW}$, and 4 kW are connected between neutral and red, yellow and blue phase, respectively, of a three-phase, four-wire system. The line voltage is 400 V . Find the current in each line conductor and neutral.

## Solution:

The three loads are connected as shown in Figure 8.27.
Load between red phase and neutral,

$$
P_{1}=8 \mathrm{~kW}=8 \times 10^{3} \mathrm{~W}
$$



Fig. 8.27 Circuit as per data

Load between yellow phase and neutral,

$$
P_{2}=6 \mathrm{~kW}=6 \times 10^{3} \mathrm{~W}
$$

Voltage across each load, that is, phase voltage,

$$
V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}
$$

Phase current in red phase, $I_{\mathrm{ph}}=\frac{P_{1}}{V_{\mathrm{ph}}}=\frac{8 \times 10^{3}}{231}$

$$
=34.64 \mathrm{~A}
$$

Phase current in yellow phase, $I_{\mathrm{ph} 2}=\frac{P_{2}}{V_{\mathrm{ph}}}=\frac{6 \times 10^{3}}{231}$

$$
=25.98 \mathrm{~A}
$$

Phase current in blue phase, $I_{\mathrm{ph} 3}=\frac{P_{3}}{V_{\mathrm{ph}}}=\frac{4 \times 10^{3}}{231}=17.32 \mathrm{~A}$
Therefore, line current, $I_{\mathrm{R}}=I_{\mathrm{ph} 1}=34.64 \mathrm{~A}$, line current, $I_{\mathrm{Y}}=I_{\mathrm{ph} 2}=25.98 \mathrm{~A}$, and line current, $I_{\mathrm{B}}=I_{\mathrm{ph} 3}=$


Fig. 8.28 Phasor diagram 17.32 A.

The three phase currents $I_{\mathrm{ph} 1}, I_{\mathrm{ph} 2}$, and $I_{\mathrm{ph} 3}$ are displaced from each other by $120^{\circ}$, as shown in Figure 8.28. The current in the neutral is the vector sum of the three phase currents.

By resolving them horizontally and vertically, we get

$$
\begin{aligned}
I_{\mathrm{xx}} & =I_{\mathrm{ph} 1} \cos 0^{\circ}-I_{\mathrm{ph} 2} \cos 60^{\circ}-I_{\mathrm{ph} 3} \cos 60^{\circ} \\
& =34.64 \times 1-25.98 \times 0.5-17.32 \times 0.5=13 \mathrm{~A} \\
I_{\mathrm{yy}} & =0-I_{\mathrm{ph} 2} \sin 60^{\circ}+I_{\mathrm{ph} 3} \sin 60^{\circ} \\
& =-25.98 \times 0.866+17.32 \times 0.5=-7.5 \mathrm{~A}
\end{aligned}
$$

Current in the neutral, $I_{\mathrm{N}}=\sqrt{I_{\mathrm{xx}}^{2}+I_{\mathrm{yy}}^{2}}=\sqrt{(13)^{2}+(7.5)^{2}}=15 \mathrm{~A}$

## Example 8.19

Three $20-\Omega$ non-inductive resistors are connected in star across a three-phase 400 V supply. Three other non-inductive resistors are connected in delta across the same supply, so as to take the same line current. What are the resistance values of these other resistors and what is the current flowing through each of them?
(U.P.T.U. Tut.)

## Solution:

When the resistors are connected in star, as shown in Figure 8.29(a),

$$
I_{\mathrm{L}_{1}}=I_{\mathrm{ph}_{1}}=\frac{V_{\mathrm{ph}_{1}}}{R_{\mathrm{ph}_{1}}}=\frac{V_{\mathrm{L}} / \sqrt{3}}{R_{\mathrm{ph}_{1}}}=\frac{400}{\sqrt{3} \times 20}=\frac{20}{\sqrt{3}} \mathrm{~A}
$$

When the resistors are connected in delta, as shown in Figure 8.29(b),

$$
\begin{aligned}
& I_{\mathrm{L}_{2}}=I_{\mathrm{L}_{1}}=\frac{20}{\sqrt{3}} \mathrm{~A} \\
& I_{\mathrm{ph}_{2}}=\frac{I_{\mathrm{L}}}{\sqrt{3}}=\frac{20}{\sqrt{3} \times \sqrt{3}}=\frac{20}{3}=6.667 \mathrm{~A} \\
& V_{\mathrm{ph}_{2}}=V_{\mathrm{L}}=400 \mathrm{~V} \\
& R_{\mathrm{ph}_{2}}=\frac{V_{\mathrm{ph}_{2}}}{I_{\mathrm{ph}_{2}}}=\frac{400 \times 3}{20}=60 \Omega
\end{aligned}
$$


(a)

(b)

Fig. 8.29 (a) Resistors connected in star (b) Resistors connected in delta

## Example 8.20

Three $20 \mu \mathrm{~F}$ capacitors are star connected across a $400 \mathrm{~V}, 50 \mathrm{~Hz}$ three-phase, three-wire supply.
(i) Calculate the current in each line. (ii) If one of the capacitors is open circuited, find line currents and potential difference across each of the two capacitors. (iii) If one of the capacitors is short circuited, calculate the line currents.

## Solution:

When the three capacitors are connected in star as shown in Figure 8.30.
(i) Line voltage, $V_{\mathrm{L}}=400 \mathrm{~V}$

Phase voltage, $V_{\text {ph }}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$
Impedance of each phase, $X_{\mathrm{C}}=\frac{1}{2 \pi f c}$

$$
\begin{aligned}
& =\frac{10^{6}}{2 \times \pi \times 50 \times 20} \\
& =159.15 \Omega
\end{aligned}
$$

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{X_{\mathrm{C}}}=\frac{231}{159.15}=1.45 \mathrm{~A}$


Fig. 8.30 Three capacitors connected in star
$\qquad$

Current in each line, $I_{\mathrm{L}}=I_{\mathrm{ph}}=1.45 \mathrm{~A}$
(ii) When one of the capacitors is open circuited (see Fig. 8.31)

Here, the remaining two capacitors are in series.


Fig. 8.31 One of the capacitor is open circuited

Total capacitance $=\frac{20 \times 20}{20+20}=10 \mu \mathrm{~F}$
Capacitive reactance, $X_{\mathrm{C}}=\frac{10^{6}}{2 \pi \times 50 \times 10}$

$$
=318.3 \Omega
$$

Line current (in line 1 and 2),

$$
I_{\mathrm{L}}=\frac{400}{318.3}=1.256 \mathrm{~A}
$$

Current in line $3=0 \mathrm{~A}$
Potential difference across each capacitor $=\frac{400}{2}=200 \mathrm{~V}$
(iii) When one of the capacitors is short circuited (see Fig. 8.32)

Line voltage

$$
V_{\mathrm{L}}=400 \mathrm{~V}
$$

Voltage between line 1 and $3=400 \mathrm{~V}$
Reactance between line 1 and $3=\frac{1}{2 \pi f_{C}}$

$$
\begin{aligned}
& =\frac{10^{6}}{2 \pi \times 50 \times 20} \\
& =159.15 \Omega
\end{aligned}
$$



Fig. 8.32 One of the capacitor is short circuited

Current in line $1=\frac{400}{159.15}=2.513 \mathrm{~A}$

$$
\begin{aligned}
\text { Similarly, current in line } 2 & =\frac{400}{159.15} \\
& =2.513 \mathrm{~A}
\end{aligned}
$$

The currents in line 1 and 2 are equal in magnitude, but differ in phase by $60^{\circ}$.

Current in line $3=$ vector sum of currents in line 1 and 2

$$
=2 \times 2.513 \times \cos 30^{\circ}=2 \times 2.513 \times \frac{\sqrt{3}}{2}=4.353 \mathrm{~A}
$$

## Example 8.21

Three identical impedances $12 \angle 30^{\circ} \Omega$ in a delta connection and three identical impedances of $5 \angle 45^{\circ} \Omega$ in a star connection are on the same three-phase three-wire $173 / 100 \mathrm{~V}$ system. Find the line currents and the total power.
(P.T.U.)

## Solution:

Line voltage, $V_{\mathrm{L}}=173 \mathrm{~V}$; phase voltage, $V_{\mathrm{ph}}=100 \mathrm{~V}$
Impedances connected in delta,

$$
\overline{Z_{1}}=12 \angle 30^{\circ}=12\left(\cos 30^{\circ}+j \sin 30^{\circ}\right)=12(0.866+j 0.5)=(10.392+j 6) \Omega
$$

Resistance $R_{1}=10.392 \Omega$; inductive reactance, $X_{\mathrm{L} 1}=6 \Omega$
Power factor, $\cos \phi_{1}=\cos 30^{\circ}=0.866$ lagging and $\tan \phi_{1}=\tan \cos ^{-1} 0.866=0.773$
Phase current, $I_{\mathrm{ph} 1}=\frac{V_{\mathrm{ph}}}{Z_{1}}=\frac{100}{12}=8.333 \mathrm{~A}$
Line current, $I_{\mathrm{L} 1}=\sqrt{3} I_{\text {ph } 1}=\sqrt{3} \times 8.333=14.434 \mathrm{~A}$
Power, $P_{1}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi_{1}=\sqrt{3} \times 173 \times 14.434 \times 0.866=3,745.56 \mathrm{~W}$
Reactive power, $P_{\mathrm{r} 1}=P_{1} \times \tan \phi_{1}=3,745.56 \times 0.5772=2,162.5 \mathrm{VAR}$
Impedances connected in star, $\overline{Z_{2}}=5 \angle 45^{\circ}=5\left(\cos 45^{\circ}+j \sin 45^{\circ}\right)$

$$
=5(0.707+j 0.707)=(3.535+j 3.535) \Omega
$$

Resistance, $R_{2}=5.535 \Omega$; inductive reactance $X_{\mathrm{L} 2}=5.535 \Omega$
Power factor,

$$
\cos \phi_{2}=\cos 45^{\circ}=0.707
$$

and

$$
\tan \phi_{2}=\tan 45^{\circ}=1
$$

Phase current,

$$
I_{\mathrm{ph} 2}=\frac{V_{\mathrm{ph}}}{Z_{2}}=\frac{100}{5}=20 \mathrm{~A}
$$

Line current,

$$
I_{\mathrm{L} 2}=I_{\mathrm{ph} 2}=20 \mathrm{~A}
$$

Power input,

$$
P_{2}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi_{2}=\sqrt{3} \times 173 \times 20 \times 0.707=4,237 \mathrm{~W}
$$

Reactive power,

$$
P_{\mathrm{r} 2}=P 2 \tan \phi_{2}=4,237 \times 1=4,237 \mathrm{VAR}
$$

Total active power,

$$
P=P_{1}+P_{2}=3,745.56+4,237=7,982.56 \mathrm{~W}
$$

Total reactive power,

$$
P_{\mathrm{r}}=P_{\mathrm{r} 1}+P_{\mathrm{r} 2}=2,162.5+4,237=6,399.5 \mathrm{VAR}
$$

Total apparent power, $\quad P_{\mathrm{a}}=\sqrt{P^{2}+P_{\mathrm{r}}^{2}}=\sqrt{(7,982.56)^{2}+(6,339.5)^{2}}$

$$
=10,231 \mathrm{VA}
$$

Resultant power factor, $\quad \cos \phi=\frac{P}{P_{\mathrm{a}}}=\frac{7,982.56}{1,0231}=0.7802$ lagging

Total line current,

$$
\begin{aligned}
I_{\mathrm{L}} & =\frac{P_{\mathrm{a}}}{\sqrt{3} \times V_{\mathrm{L}}}=\frac{10,231}{\sqrt{3} \times 173} \\
& =34.14 \mathrm{~A}
\end{aligned}
$$

## Example 8.22

A 400/230 V three-phase, four-wire system supplies an unbalanced load represented by the following impedances in ohm connected between the neutral and $\mathrm{R}, \mathrm{Y}$, and B phases, respectively, $(16+j 12),(6-j 8)$, and 23 . The phase sequence is RYB. Calculate the current in each line conductor of the cable and the neutral. Further, find the total power absorbed.
(Pune Univ.)

## Solution:

Impedances connected between neutral and red, yellow, blue phases, respectively, are

$$
\begin{aligned}
& \overline{Z_{\mathrm{R}}}=(16+j 12)=20 \angle 36.87^{\circ} \\
& \overline{Z_{\mathrm{Y}}}=(6-j 8)=10 \angle-52^{\circ} 13^{\circ} \\
& \overline{Z_{\mathrm{B}}}=23=23 \angle 0^{\circ}
\end{aligned}
$$

The three impedances connected to the line are shown in Figure 8.33.
The potential difference between the three lines and neutral are


Fig. 8.33 Circuit as per data

$$
\begin{aligned}
& \overline{V_{\mathrm{RN}}}=230 \angle 0^{\circ}(\text { reference vector }) \\
& \overline{V_{\mathrm{YN}}}=230 \angle-120^{\circ} \\
& \overline{V_{\mathrm{BN}}}=230 \angle-240^{\circ}
\end{aligned}
$$

Current in the three line conductors are

$$
\begin{aligned}
\overline{I_{\mathrm{R}}} & =\frac{\overline{V_{\mathrm{RN}}}}{\overline{Z_{\mathrm{R}}}}=\frac{230 \angle 0}{20 \angle 36.87^{\circ}}=11.5 \angle-36.87^{\circ} \\
& =11.5\left(\cos 36.87^{\circ}-j \sin 36.87^{\circ}\right)=(9.2-j 6.9) \mathrm{A} \\
\overline{I_{\mathrm{Y}}} & =\frac{\overline{V_{\mathrm{YN}}}}{\overline{Z_{\mathrm{Y}}}}=\frac{230 \angle-120^{\circ}}{10 \angle-53.13^{\circ}}=23 \angle-66.87^{\circ} \\
& =23\left(\cos 66.87-j 66.87^{\circ}\right)=(9.035-j 21.15) \mathrm{A} \\
\overline{I_{\mathrm{B}}} & =\frac{\bar{V}_{\mathrm{BN}}}{\bar{Z}_{\mathrm{B}}}=\frac{230 \angle-240^{\circ}}{23 \angle 0^{\circ}}=10 \angle-240^{\circ} \\
& =10\left(\cos 240^{\circ}-j \sin 240^{\circ}\right)=(-5+j 8.66) \mathrm{A}
\end{aligned}
$$

The three voltages and currents are shown vectorially in the phasor diagram (see Fig. 8.34). Current in the neutral,
or
Fig. 8.34 Phasor diagram

$$
\begin{aligned}
\bar{I}_{\mathrm{N}} & =\bar{I}_{\mathrm{R}}+\bar{I}_{\mathrm{Y}}+\bar{I}_{\mathrm{B}} \\
& =9.2-j 6.9+9.305-j 21.15-5+j 8.66 \\
& =(13.235-j 19.39) \mathrm{A} \\
I_{\mathrm{N}} & =\sqrt{(13.235)^{2}+(19.39)^{2}} \\
I_{\mathrm{N}} & =23.476 \mathrm{~A} \quad I_{\mathrm{R}}=11.5 \mathrm{~A} \\
I_{\mathrm{Y}} & =23 \mathrm{~A} \quad \quad I_{\mathrm{B}}=10 \mathrm{~A}
\end{aligned}
$$

## 国完 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What is polyphase system?
2. What are the advantages of three-phase system over single-phase system?
3. What are star and delta connections?
4. What is the relation between line voltage and phase voltage in star connections?
5. What is the relation between line current and phase current in delta connections?
6. What is the relation between line current and phase current in star connections?
7. What is three-phase balanced and unbalanced load?
8. What quantity of electric power is consumed by a three-phase balanced load?

## Test Questions

1. In three-phase star connections, derive a relation between phase voltage and line voltage.
2. In three-phase delta-connected system, derive a relation between (i) phase voltage and line voltage and (ii) phase current and line current.
3. What is the significance of phase sequence in a three-phase system?
4. Compare star- and delta-connected system.
5. A three-phase delta-connected load draws a power of 3 kW . What power will be drawn by it, if one of the load is disconnected?

## Numericals

1. Three identical impedances are connected in delta to a three-phase, $400 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. The line current is 34.65 A and total power taken from supply is 14.14 kW . Calculate the value of resistance and inductance of each phase
(Ans. $11.78 \Omega, 51.45 \mathrm{mH}$ )
2. Three $100 \Omega$ non-inductive resistances are connected in (i) star and (ii) delta across a $400 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate the power taken from the supply in each case. In the event of one of the resistance getting opened, what would be the value of the total power taken from the supply in each case.
(Ans. 1,600 W, 4,800 W, $800 \mathrm{~W}, 3,200 \mathrm{~W}$ )
3. A 400 V , three-phase system is connected to a balanced star load with the load impedance in each phase as $40 \angle 60^{\circ} \Omega$. Find (i) the line and phase currents and (ii) draw phasor diagram showing line voltages, line currents, and phase currents.
(Ans. $5.775 \mathrm{~A}, 5.775 \mathrm{~A})$

### 8.13 POWER MEASUREMENT IN THREE-PHASE CIRCUITS

In $A C$ circuits, power is measured with the help of a wattmeter. A wattmeter is an instrument that consists of two coils called current coil and potential coil. The current coil having low resistance is connected in series with the load so that it carries the load current. The potential coil have high resistance is connected across the load and carries the current proportional to the potential difference across the load.

For measuring power in a polyphase system, more than one wattmeter is required or more than one reading is made by one wattmeter. The first method is more convenient and the number of wattmeters required to measure power in a given polyphase system is determined by Blondel's theorem.

According to Blondel, a French engineer, when power is supplied by $K$-wire AC system, the number of wattmeters required to measure power is one less than the number of wires, that is, ( $K-1$ ), regardless the load is balanced or unbalanced.

Hence, three wattmeters are required to measure power in three-phase, four-wire system, while only two wattmeters are required to measure power in three-phase, three-wire system.

### 8.14 THREE-WATTMETER METHOD

This method is employed to measure power in three-phase, four-wire system. However, this method can also be employed in three-phase, three-wire delta-connected load, where power consumed by each load is required to be determined separately. The connections for both star- and delta-connected loads are shown in Figure 8.35(a) and (b), respectively.

The total power is given by the algebraic sum of readings of three wattmeters.

$$
\text { Total power, } P=W_{1}+W_{2}+W_{3}
$$

Except for three-phase, four-wire unbalanced load, three-phase power can be measured by using only two wattmeters.


Fig. 8.35 (a) Connections of three wattmeter for power measurement in star connected loads (b) Connections of three wattmeters for power measurement in delta connected load

### 8.15 TWO-WATTMETER METHOD

This method can be employed to measure power in a three-phase, three-wire star-or delta-connected balanced or unbalanced load. In this method, the current coil of the wattmeters are connected in any two lines, say R and Y , and the potential coil of each wattmeter is connected across the same line and the third line (i.e., B), as shown in Figure 8.36.
It can be proved that the sum of the power measured by the two wattmeters $W_{1}$ and $W_{2}$ is equal to the total instantaneous power absorbed by the three loads $Z_{1}, Z_{2}$, and $Z_{3}$.

Let us consider the star connections (see Fig. 8.36 (a)).


Fig. 8.36 (a) Connections of two wattmeters for power measurement in star connected load (b) Connections of two wattmeters for power measurement in delta connected load

Instantaneous current through current coil of $W_{1}=i_{\mathrm{R}}$
Instantaneous potential difference across potential coil of $W_{1}=e_{\mathrm{RN}}-e_{\mathrm{BN}}$
Instantaneous power measured by $W_{1}=i_{\mathrm{R}}\left(e_{\mathrm{RN}}-e_{\mathrm{BN}}\right)$
Instantaneous current through current coil of $W_{2}=i_{\mathrm{Y}}$
Instantaneous potential difference across potential coil of $W_{2}=\left(e_{\mathrm{YN}}-e_{\mathrm{BN}}\right)$
Instantaneous power measured by $W_{2}=i_{\mathrm{Y}}\left(e_{\mathrm{YN}}-e_{\mathrm{BN}}\right)$

$$
\begin{aligned}
W_{1}+W_{2} & =i_{\mathrm{R}}\left(e_{\mathrm{RN}}-e_{\mathrm{BN}}\right)+i_{\mathrm{Y}}\left(e_{\mathrm{YN}}-e_{\mathrm{BN}}\right) \\
& =i_{\mathrm{R}} e_{\mathrm{RN}}+i_{\mathrm{Y}} e_{\mathrm{YN}}-e_{\mathrm{BN}}\left(i_{\mathrm{R}}+i_{\mathrm{Y}}\right) \\
& =i_{\mathrm{R}} e_{\mathrm{RN}}+i_{\mathrm{Y}} e_{\mathrm{YN}}+i_{\mathrm{B}} e_{\mathrm{BN}}\left(-i_{\mathrm{R}}+i_{\mathrm{Y}}+i_{\mathrm{B}}=0\right)
\end{aligned}
$$

$=$ Total power absorbed in the three loads at any instant (i.e., $P$ )
or

$$
P=W_{1}+W_{2}
$$

Let us consider the delta connections (see Fig. 8.36 (b)).
Instantaneous current through current coil of $W_{1}=i_{\mathrm{R}}=i_{1}-i_{3}$
Instantaneous potential difference across potential coil of $W_{1}=e_{\mathrm{RB}}$
Instantaneous power measured by $W_{1}=e_{\mathrm{RB}}\left(i_{1}-i_{3}\right)$
Instantaneous current through current coil of $W_{2}=i_{\mathrm{Y}}=i_{2}-i_{1}$
Instantaneous potential difference across potential coil of $W_{2}=e_{\mathrm{YB}}$
Instantaneous power measured by $W_{2}=e_{\mathrm{YB}}\left(i_{2}-i_{1}\right)$

$$
\begin{aligned}
W_{1}+W_{2} & =e_{\mathrm{RB}}\left(i_{1}-i_{3}\right)+e_{\mathrm{YB}}\left(i_{2}-i_{1}\right)=i_{1} e_{\mathrm{RB}}+i_{2} e_{\mathrm{YB}}-i_{3} e_{\mathrm{RB}}-i_{1} e_{\mathrm{YB}} \\
& =i_{2} e_{\mathrm{YB}}+i_{3} e_{\mathrm{BR}}-i_{1}\left(e_{\mathrm{YB}}+e_{\mathrm{BR}}\right)\left(-e_{\mathrm{RB}}=e_{\mathrm{BR}}\right) \\
& =i_{1} e_{\mathrm{RY}}+i_{2} e_{\mathrm{YB}}+i_{3} e_{\mathrm{BR}}\left(-e_{\mathrm{RY}}+e_{\mathrm{YB}}+e_{\mathrm{BR}}=0\right)
\end{aligned}
$$

$=$ Total power absorbed in the three loads at any instant (i.e., $P$ )
or

$$
P=W_{1}+W_{2}
$$

It may be noted that power measured by the two wattmeters, at any instant, is the instantaneous power absorbed by the three loads connected in three phases. In fact, this power is the average power drawn by the load since the wattmeters read the average power because of the inertia of their moving system.

### 8.16 TWO-WATTMETER METHOD (BALANCED LOAD)

The two-wattmeter method can be explained somewhat more clearly by considering a balanced load. In this case, we shall prove that power measured by the two wattmeters (i.e., sum of two wattmeter readings) is equal to $\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$, which is the actual power consumed in a threephase balanced load.

The connection diagram for a three-phase balanced load (connected in star) is shown in Figure 8.37(a). Considering load to be an inductive one, the phasor diagram is shown in Figure 8.37(b). The three-phase voltages $V_{\mathrm{RN}}, V_{\mathrm{YN}}$, and $V_{\mathrm{BN}}$ displaced by an angle of $120^{\circ}$ electrical are shown in phasor diagram. The phase currents lag behind their respective phase voltages by an angle $\phi$.


Fig. 8.37 (a) Connection of two wattmeters for power measurement in balanced 3-phase load (b) Phase diagram

Current through current coil (c.c.) of $W_{1}=I_{\mathrm{R}}$
Potential difference across potential coil (p.c.) of $W_{1}=\overline{V_{\mathrm{RB}}}=\overline{V_{\mathrm{RN}}}=\overline{V_{\mathrm{BN}}}$
To obtain $V_{\mathrm{RB}}$, reverse the phasor $V_{\mathrm{BN}}$ and add it vectorially to phasor $V_{\mathrm{RN}}$, as shown in Figure 8.37(b).

The phase difference between $V_{\mathrm{RB}}$ and $I_{\mathrm{R}}$ is $\left(30^{\circ}-\phi\right)$.
Power measured by wattmeter

$$
W_{1}=V_{\mathrm{RB}} I_{\mathrm{R}} \cos (30-\phi)
$$

Current through c.c. of $W_{2}=I_{\mathrm{Y}}$
Potential difference across p.c. of $W_{2}=\overline{V_{\mathrm{YB}}}=\overline{V_{\mathrm{YN}}}-\overline{V_{\mathrm{BN}}}$
To obtain $V_{\mathrm{YB}}$, reverse the phasor $V_{\mathrm{BN}}$ and add it vectorially to phasor $V_{\mathrm{YN}}$ as shown in Figure 8.37(b).

The phase difference between $V_{\mathrm{YB}}$ and $I_{\mathrm{Y}}$ is $\left(30^{\circ}+\phi\right)$.
Power measured by wattmeter, $W_{2}=V_{\mathrm{YB}}=V_{\mathrm{BR}}=V_{\mathrm{L}}$
Wattmeter reading, $W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)$
Wattmeter reading, $W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)$
Sum of the two wattmeter readings,

$$
W_{1}+W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)+V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)
$$

$$
\begin{aligned}
& =V_{\mathrm{L}} I_{\mathrm{L}}\left[\cos \left(30^{\circ}-\phi\right)+\cos \left(30^{\circ}+\phi\right)\right] \\
& =V_{\mathrm{L}} I_{\mathrm{L}}\left[\cos 30^{\circ} \cos \phi+\sin 30^{\circ} \sin \phi+\cos 30^{\circ} \cos \phi-\sin 30^{\circ} \sin \phi\right] \\
& =V_{\mathrm{L}} I_{\mathrm{L}}\left(2 \cos 30^{\circ} \cos \phi\right)=V_{\mathrm{L}} I_{\mathrm{L}}\left(2 \times \frac{\sqrt{3}}{2} \times \cos \phi\right)=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi \\
& =\text { Total power absorbed by a three-phase balanced load }(P) \\
P & =W_{1}+W_{2}
\end{aligned}
$$

Thus, the sum of the readings of the two wattmeters is equal to the power absorbed in a threephase balanced load.

### 8.16.1 Determination of Power Factor from Wattmeter Readings

We have seen that

$$
\begin{gather*}
W_{1}+W_{2}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi  \tag{8.1}\\
W_{1}-W_{2}=V_{\mathrm{L}} I_{\mathrm{L}}\left(\cos \left(30^{\circ}-\phi\right)-\cos \left(30^{\circ}+\phi\right)\right)  \tag{8.2}\\
=2 V_{\mathrm{L}} I_{\mathrm{L}} \sin 30 \sin \phi=V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi
\end{gather*}
$$

Dividing Equation (8.2) by (8.1), we get
or

$$
\begin{aligned}
\frac{W_{1}-W_{2}}{W_{1}+W_{2}} & =\frac{V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi}{\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi} \\
\tan \phi & =\sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}}
\end{aligned}
$$

Power factor

$$
\cos \phi=\cos \tan ^{-1} \phi=\cos \tan ^{-1} \sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}}
$$

### 8.16.2 Determination of Reactive Power from Two Wattmeter Readings

Multiplying Equation (8.2) by $\sqrt{3}$, we get

$$
\sqrt{3}\left(W_{1}-W_{2}\right)=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi=P_{r}
$$

Reactive power,

$$
P_{\mathrm{r}}=\sqrt{3}\left(W_{1}-W_{2}\right)
$$

### 8.17 EFFECT OF POWER FACTOR ON THE TWO WATTMETER READINGS

For lagging power factor, the wattmeter readings are

$$
W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right) \text { and } W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)
$$

It is clear that the readings of the two wattmeters do not only depend upon load, but they also depend upon the phase angle $\phi$ or the power factor of the load.

### 8.17.1 Power Factor Is Unity ( $\cos \phi=1$ ) or $\phi=\mathbf{0}^{\circ}$

For pure resistance load, the power factor is unity, that is, $\cos \phi=1$ or $\phi=0^{\circ}$.
Then, two wattmeter readings will be

$$
\begin{aligned}
& W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-0\right)=V_{\mathrm{L}} I_{\mathrm{L}} \cos 30^{\circ} \\
& W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+0\right)=V_{\mathrm{L}} I_{\mathrm{L}} \cos 30^{\circ}, \text { i.e., } W_{1}=W_{2}
\end{aligned}
$$

Hence, when power factor is unity, both the wattmeters give equal reading.

### 8.17.2 Power Factor Is $\mathbf{0 . 5}(\cos \phi=0.5)$ or $\phi=60^{\circ}$

Under such conditions, the two wattmeter readings will be

$$
\begin{aligned}
& W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-60^{\circ}\right)=V_{\mathrm{L}} I_{\mathrm{L}} \cos 30^{\circ} \\
& W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+60^{\circ}\right)=0
\end{aligned}
$$

Hence, when the power factor is 0.5 , one of the wattmeter $W_{2}$ gives zero reading (no deflection) and whole of the power is measured by the other wattmeter $W_{1}$.

### 8.17.3 Power Factor Is More Than 0.5 But Less Than One (i.e., $1>\cos \phi>$ 0.5 ) or $\mathbf{6 0}>\boldsymbol{>}>\mathbf{0}^{\circ}$

Under such conditions, the readings of the wattmeters will be

$$
\begin{array}{ll}
W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right) & \text { (positive, larger) } \\
W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right) & \text { (negative, smaller) }
\end{array}
$$

Hence, when the power factor is more than 0.5 but less than one, both the wattmeters give positive (upscale) readings. However, wattmeter $W_{1}$ gives larger reading than wattmeter $W_{2}$.

Now, total power, $P=W_{1}+W_{2} ; P=$ larger reading + smaller readings

### 8.17.4 Power Factor Is Less Than O.5 But More Than 0 (i.e., $0.5>\cos \phi>$ 0) or $\mathbf{9 0}{ }^{\circ}>\boldsymbol{\phi}>\mathbf{6 0}{ }^{\circ}$

Under such conditions, the readings of the wattmeters will be

$$
\begin{array}{ll}
W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right) & \text { (positive, larger) } \\
W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right) & \text { (negative, smaller) }
\end{array}
$$

Hence, when the power factor is less than 0.5 but more than zero, one of the wattmeters $W_{2}$ gives negative downscale reading, while the other wattmeter $W_{1}$ gives positive upscale reading. In order to obtain upscale reading on $W_{2}$, either the connections of potential coil or current coil of this wattmeter are reversed. However, the reading is considered as negative.

Total power, $P=W_{1}+W_{2}$, where the value of $W_{2}$ is negative.

### 8.17.5 Power factor Is $\mathbf{O}(\boldsymbol{\operatorname { c o s }} \boldsymbol{\phi}=\mathbf{0})$ or $\boldsymbol{\phi}=\mathbf{9 0}{ }^{\circ}$

For pure inductive or capacitive loads, the power factor is zero lagging or zero leading, that is, $\cos \phi=0$ or $\phi=90^{\circ}$. The two wattmeter readings will be

$$
\begin{aligned}
W_{1} & =V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)=V_{\mathrm{L}} I_{\mathrm{L}}\left(30^{\circ}-90^{\circ}\right) \\
& =V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(-60^{\circ}\right)=V_{\mathrm{L}} I_{\mathrm{L}} \cos 60^{\circ}(\text { positive }) \\
W_{2} & =V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+90^{\circ}\right) \\
& =V_{\mathrm{L}} I_{\mathrm{L}} \cos 120^{\circ}=-V_{\mathrm{L}} I_{\mathrm{L}} \cos 60^{\circ} \text { (negative) }
\end{aligned}
$$

Hence, when the power factor is zero, the reading of the two wattmeters will be equal in magnitude but opposite in sign.

Now, total power,

$$
P=W_{1}+W_{2}
$$

Since the reading of wattmeter $W_{2}$ is considered as negative,

$$
P=W_{1}+\left(-W_{2}\right)=W_{1}-W_{2}=0 .
$$

## Example 8.23

In the two-wattmeter method of power measurement in a three-phase circuit, the readings of the two wattmeters are $1,000 \mathrm{~W}$ and 550 W . What is the power factor of the load?
(U.P.T.U. Sept. 2001)

## Solution:

Reading of wattmeter $W_{1}=1,000 \mathrm{~W}$ and
Reading of wattmeter $W_{2}=550 \mathrm{~W}$
Power factor of load, $\cos \phi=\cos \tan ^{-1} \sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}}$

$$
=\cos \tan ^{-1} \sqrt{3} \times \frac{1,000-550}{1,000+550}=0.893(\mathrm{lag})
$$

## Example 8.24

In a power measurement, the wattmeter readings were 5,000 and $1,000 \mathrm{~W}$. Calculate power and power factor if both the meters read positive values
(U.P.T.U. Tut.)

## Solution:

Reading of wattmeter $W_{1}=5,000 \mathrm{~W}$
Reading of wattmeter $W_{2}=1,000 \mathrm{~W}$
Total power, $P=W_{1}+W_{2}=5,000+1,000=6,000 \mathrm{~W}$
Power factor, $\cos \phi=\cos \tan ^{-1} \sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}}=\cos \tan ^{-1} \sqrt{3} \times \frac{5,000-1,000}{5,000+1,000}=0.655$ (lag)

## Example 8.25

A balanced three-phase, star-connected load draws power from a 440 V supply. The two wattmeters connected in the circuit indicate $W_{1}=4.2 \mathrm{~kW}$ and $W_{2}=0.8 \mathrm{~kW}$. Calculate the power, power factor, and current in the circuit.

## Solution:

$$
W_{1}=4.2 \mathrm{~kW} ; W_{2}=0.8 \mathrm{~kW}
$$

Total power, $P=W_{1}+W_{2}=4.2+0.8=5.0 \mathrm{~kW}$
Now, $\tan \phi=\sqrt{3} \frac{\left(W_{1}-W_{2}\right)}{\left(W_{1}+W_{2}\right)}=\sqrt{3} \times \frac{(4.2-0.8)}{(4.2)+(0.8)}=1.1778$
Power factor, $\cos \phi=\cos \tan ^{-1} 1.778=0.6472$
Now, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$

$$
5 \times 1,000=\sqrt{3} \times 440 \times I_{\mathrm{L}} \times 0.6472 \quad \text { or } \quad I_{\mathrm{L}}=10.137 \mathrm{~A}
$$

## Example 8.26

For a certain load, one of the wattmeter reads 20 kW and the other 5 kW after the voltage of this wattmeter has been reversed. Calculate the power and the power factor of the load.
(U.P.T.U. July 2002)

## Solution:

Wattmeter reading, $W_{1}=20 \mathrm{~kW}$
Wattmeter reading, $W_{2}=-5 \mathrm{~kW}$
Power, $P=W_{1}+W_{2}=20-5=15 \mathrm{~kW}$
Power factor of the circuit

$$
\begin{aligned}
& =\cos \tan ^{-1} \sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \\
& =\cos \tan ^{-1} \sqrt{3} \times \frac{20-(-5)}{20+(-5)}=0.3273(\mathrm{lag})
\end{aligned}
$$

## Example 8.27

The power input to a $2,000 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase motor running on full load at an efficiency of $90 \%$ is measured by two wattmeters that indicate 300 kW and 100 kW , respectively. Calculate (i) the input power and (ii) power factor.

## Solution:

Here, $W_{1}=300 \mathrm{~kW}$ and $W_{2}=100 \mathrm{~kW}$
(i) Input power, $P=W_{1}+W_{2}=300+100=400 \mathrm{~kW}$

$$
\text { Phase angle, } \begin{aligned}
\phi & =\tan ^{-1} \sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \\
& =\tan ^{-1} \sqrt{3} \times \frac{300-100}{300+100}=40.89^{\circ}(\mathrm{lag})
\end{aligned}
$$

(ii) Power factor $=\cos \phi=\cos 40.89^{\circ}=0.756$ (lagging)

## Example 8.28

The power input to a $2,000 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase motor running on full load at an efficiency of $90 \%$ is measured by two wattmeters that indicate 300 kW and 100 kW , respectively. Calculate (i) input, (ii) power factor, (iii) line current, and (iv) H.P. output.
(P.T.U.)

## Solution:

$$
W_{1}=300 \mathrm{~kW} ; W_{2}=1,000 \mathrm{~kW}
$$

Input power,

$$
P=W_{1}+W_{2}=300+100=400 \mathrm{~kW}
$$

Now,

$$
\tan \phi=\sqrt{3} \frac{\left(W_{1}-W_{2}\right)}{\left(W_{1}+W_{2}\right)}=\frac{\sqrt{3}(300-100)}{(300+100)}=0.866
$$

Power factor,

$$
\cos \phi=\cos \tan ^{-1} 0.866=0.756
$$

Now,

$$
P=\sqrt{3} V_{L} I_{L} \cos \phi
$$

$$
\begin{aligned}
400 \times 1,000 & =\sqrt{3} \times 2,000 \times I_{\mathrm{L}} \times 0.756 \quad \text { or } \quad I_{\mathrm{L}}=152.74 \mathrm{~A} \\
\text { Output } & =\text { input } \times \text { efficiency }=400 \times 0.9=360 \mathrm{~kW} \\
& =\frac{360}{0.7355}=489.46 \text { H.P. }
\end{aligned}
$$

## Example 8.29

In a two-wattmeter method, power measured was 30 kW at 0.7 power factor lagging. Find the reading of each wattmeter.
(U.P.T.U. 2006-07)

## Solution:

Here, $P=30 \mathrm{~kW} ; \cos \phi=0.7$ lagging
Let $W_{1}$ and $W_{2}$ be the readings of two wattmeters.
Then, $P=W_{1}+W_{2}=30 \mathrm{~kW}$
Now, $\cos \phi=0.7$ lagging; $\tan \phi=\tan \cos ^{-1} 0.7=1.0202$
We know that in two-wattmeter method of power measurements.
or

$$
\begin{align*}
\tan \phi & =\sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \text { or } 1.0202=\sqrt{3} \times \frac{W_{1}-W_{2}}{30}  \tag{8.4}\\
W_{1}-W_{2} & =\frac{30 \times 1.0202}{\sqrt{3}}=17.67 \mathrm{~kW}
\end{align*}
$$

Solving Equations (8.3) and (8.4), we get

$$
W_{1}=23.835 \mathrm{~kW} \quad \text { and } \quad W_{2}=6.165 \mathrm{~kW}
$$

## Example 8.30

Draw the connection diagram for measurement of power in a three-phase Y-connected load using two-wattmeter method. In one such experiment, the load supplied was 30 kW at 0.7 power factor lagging. Find the reading of each wattmeter.
(U.P.T.U. June 2001)

## Solution:

Total power supplied, $P=W_{1}+W_{2}=30 \mathrm{~kW}$
Power factor, $\cos \phi=0.7$ (lagging)
Phase angle, $\phi=\cos ^{-1} 0.7=45.57^{\circ}$ (lagging)

$$
\begin{align*}
\tan \phi & =\tan 45.57^{\circ}=1.0202 \\
\tan \phi & =\sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \text { or } 1.0202=\sqrt{3} \times \frac{W_{1}-W_{2}}{30}  \tag{8.6}\\
W_{1}-W_{2} & =\frac{1.0202 \times 30}{\sqrt{3}}=17.67 \mathrm{~kW}
\end{align*}
$$

or

Solving Equations (8.5) and (8.6), we get

$$
W_{1}=23.835 \mathrm{~kW} \quad \text { and } \quad W_{2}=6.165 \mathrm{~kW}
$$

## Example 8.31

A three-phase 500 V motor has 0.4 power factor lagging. Two wattmeters are connected to measures the input, they show the total input to be 30 kW . Find the reading of each wattmeter.
(U.P.T.U.)

## Solution:

Total power, $P=W_{1}+W_{2}=30 \mathrm{~kW}$
Power factor, $\cos \phi=0.4$ lag

$$
\tan \phi=\tan \cos ^{-1} 0.4=2.29
$$

$$
\begin{gather*}
\tan \phi=\sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \text { or } W_{1}-W_{2}=\frac{\tan \phi\left(W_{1}+W_{2}\right)}{\sqrt{3}} \\
W_{1}-W_{2}=\frac{2.291 \times 30}{\sqrt{3}}=39.683 \mathrm{~kW} \tag{8.8}
\end{gather*}
$$

Solving Equations (8.7) and (8.8), we get

$$
W_{1}=34.8415 \mathrm{~kW} \text { and } W_{2}=-4.8415 \mathrm{~kW}
$$

## Example 8.32

Two wattmeters are used to measure power in a three-phase balanced system. What is the power factor when (i) both the meters read equal, (ii) both the meters read equal but one is negative, and (iii) one reads twice the other.
(U.P.T.U.)

## Solution:

(i) When both the meters read equal, that is, $W_{1}=W_{2}$

Now, $\tan \phi=\sqrt{3} \frac{\left(W_{1}-W_{2}\right)}{\left(W_{1}+W_{2}\right)}=\sqrt{3} \frac{\left(W_{2}-W_{2}\right)}{\left(W_{2}+W_{2}\right)}=0$
Power factor, $\cos \phi=\cos \tan ^{-1} 0=1$
(ii) When both the meters read equal but one is negative, that is, $W_{1}=-W_{2}$

$$
\tan \phi=\sqrt{3} \frac{\left(-W_{2}-W_{2}\right)}{\left(-W_{2}+W_{2}\right)}=\sqrt{3} \frac{\left(-2 W_{2}\right)}{0}=\infty
$$

Power factor, $\cos \phi=\cos \tan ^{-1}=\infty=0$
(iii) When one meter reads twice the other, that is, $W_{1}=2 W_{2}$

$$
\tan \phi=\sqrt{3} \frac{\left(2 W_{2}-W_{2}\right)}{\left(2 W_{2}+W_{2}\right)}=\sqrt{3} \times \frac{1}{3}=\frac{1}{\sqrt{3}}
$$

Power factor, $\cos \phi=\cos \tan ^{-1} \frac{1}{\sqrt{3}}=0.866$

## Example 8.33

A three-phase balanced load connected across a three- $\phi, 400 \mathrm{~V}$ AC supply draws a line current of 10 A . Two wattmeters are used to measure input power. The ratio of two wattmeter readings is $2: 1$. Find the readings of the two wattmeters.
(U.P.T.U. Feb 2002)

## Solution:

The ratio of the two wattmeter readings, $\frac{W_{1}}{W_{2}}=\frac{2}{1}=2$

We know,

$$
\tan \phi=\sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}}=\sqrt{3} \times \frac{\frac{W_{1}}{W_{2}}-1}{\frac{W_{1}}{W_{2}}+1}=\sqrt{3} \times \frac{2-1}{2+1}
$$

or

$$
\begin{equation*}
\tan \phi=\sqrt{3} \times \frac{1}{3}=\frac{1}{\sqrt{3}} \tag{8.9}
\end{equation*}
$$

Power factor,

$$
\cos \phi=\cos \tan ^{-1} \frac{1}{\sqrt{3}}=0.866
$$

Input power,

$$
\begin{align*}
P=W_{1}+W_{2} & =\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi=\sqrt{3} \times 400 \times 10 \times 0.866  \tag{8.10}\\
& =6,000 \mathrm{~W}
\end{align*}
$$

From Equation (8.9), we get

$$
\begin{equation*}
W_{1}-W_{2}=\frac{\tan \phi}{\sqrt{3}} \times W_{1}+W_{2}=\frac{1}{\sqrt{3} \times \sqrt{3}} \times 6,000=2,000 \mathrm{~W} \tag{8.11}
\end{equation*}
$$

Solving Equations (8.10) and (8.11), we get

$$
W_{1}=4,000 \mathrm{~W} \text { and } W_{2}=2,000 \mathrm{~W}
$$

## Example 8.34

Two wattmeters are connected to measure the input to a balanced three-phase circuit read 5 kW and 1 kW , respectively. Find the power factor of the circuit when (i) both the readings are positive and (ii) the latter reading is obtained after reversing the connections of the current coil of the wattmeter.

## Solution:

(i) When both the readings are positive

$$
\tan \phi=\sqrt{3} \frac{\left(W_{1}-W_{2}\right)}{\left(W_{1}+W_{2}\right)}=\sqrt{3} \times \frac{5-1}{5+1}=1.1547
$$

Power factor, $\cos \phi=\cos \tan ^{-1} 1.1547=0.6546$
(ii) When the latter reading is obtained after reversing the connections of the current coil of the wattmeter (i.e., reading $W_{2}$ is negative)

$$
\tan \phi=\sqrt{3} \frac{\left(W_{1}-W_{2}\right)}{\left(W_{1}+W_{2}\right)}=\sqrt{3} \times \frac{5+1}{5-1}=2.598
$$

Power factor, $\cos \phi=\cos \tan ^{-1} 2.598=0.3592$

## Example 8.35

Three equal impedances, each consisting of $R$ and $L$ in series are connected in star and are supplied from a $400 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase, three-wire balanced supply system. The power input to the load is measured by two-wattmeter method and the two wattmeters read 3 kW and 1 kW . Determine the values of R and L connected in each phase.
(U.P.T.U. June 2003)

## Solution:

Wattmeter reading, $W_{1}=3 \mathrm{~kW}$
Wattmeter reading, $W_{2}=1 \mathrm{~kW}$
Total power, $P=W_{1}+W_{2}=3+1=4 \mathrm{~kW}$
Power factor of the circuit, $\cos \phi=\cos \tan ^{-1} \sqrt{3} \times \frac{W_{1}-W_{2}}{W_{1}+W_{2}}=\cos \tan ^{-1} \sqrt{3} \times \frac{3-1}{3+1}$

$$
=0.7559 \text { (lagging) }
$$

Line current,

$$
I_{\mathrm{L}}=\frac{P}{\sqrt{3} V_{\mathrm{L}} \cos \phi}=\frac{4 \times 1,000}{\sqrt{3} \times 400 \times 0.7559}=7.6376 \mathrm{~A}
$$

Impedance of the circuit per phase, $Z_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{I_{\mathrm{ph}}}=\frac{400 / \sqrt{3}}{7.6376}=30.237 \Omega$
(since in $\gamma$ connection $I_{\text {ph }}=I_{\mathrm{L}}$ )
Resistance per phase, $R_{\mathrm{ph}}=Z_{\mathrm{ph}} \cos \phi=30.237 \times 0.7559=22.856 \Omega$
Reactance per phase, $X_{\mathrm{L}}=\sqrt{Z_{\mathrm{ph}}^{2}-R_{\mathrm{ph}}^{2}}=\sqrt{(30.237)^{2}-(22.856)^{2}}$

$$
=19.796 \Omega
$$

Inductance per phase, $L=\frac{X_{\mathrm{L}}}{2 \pi f}=\frac{19.796}{2 \pi \times 50}=0.063 \mathrm{H}=63 \mathrm{mH}$

## Example 8.36

A 400 V , three-phase, induction motor has an output of $20 \mathrm{H} . \mathrm{P}$. and operates at a power factor of 0.85 lagging with an efficiency of $90 \%$. Calculate the readings on each of the two wattmeters connected to measure the input.

## Solution:

$$
\text { Motor output }=20 \text { H.P. }=20 \times 0.7355=14.71 \mathrm{~kW}
$$

Motor input, $P=\frac{\text { Output }}{\eta}=\frac{14.71}{0.9}=16.344 \mathrm{~kW}$
Power factor, $\cos \phi=0.85$; therefore, $\tan \phi=\tan \cos ^{-1} 0.85=0.6197$
Let the readings of two wattmeters be $W_{1}$ and $W_{2}$.
Now, $W_{1}+W_{2}=P=16.344 \mathrm{~kW}$

$$
\begin{equation*}
\tan \phi=\sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \quad \text { or } \quad 0.6197=\sqrt{3} \frac{W_{1}-W_{2}}{16.344} \quad \text { or } \quad W_{1}-W_{2}=5.848 \mathrm{~kW} \tag{8.12}
\end{equation*}
$$

Solving Equations (8.12) and (8.13), we get $W_{1}=11.096 \mathrm{~kW}$ and $W_{2}=5.248 \mathrm{~kW}$

## Example 8.37

A star-connected three-phase load has a resistance of $8 \Omega$ and an inductive reactance of $6 \Omega$ in each phase. It is fed from a 400 V , three-phase balanced supply. Determine line current, power factor, active, and reactive powers. Draw phasor diagram showing phase and line voltages and currents. If power measurement is made using two-wattmeter method, what will be readings of both wattmeters?

## Solution:

The circuit is shown in Figure 8.38(a) and its phasor diagram is shown in Figure 8.38(b).
Phase voltage, $V_{\mathrm{ph}}=\frac{V_{\mathrm{L}}}{\sqrt{3}}=\frac{400}{\sqrt{3}}=231 \mathrm{~V}$
Load impedance per phase, $Z_{\mathrm{ph}}=\sqrt{R^{2}+X_{\mathrm{L}}^{2}}=\sqrt{(8)^{2}+(6)^{2}}=10 \Omega$

## 424

Phase current, $I_{\mathrm{ph}}=\frac{V_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{231}{10}=23.1 \mathrm{~A}$
Load current, $I_{\mathrm{L}}=I_{\mathrm{ph}}=23.1 \mathrm{~A}$
Power factor, $\cos \phi=\frac{R_{\mathrm{ph}}}{Z_{\mathrm{ph}}}=\frac{8}{10}=0.8$ lagging
Active power, $P=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi=\sqrt{3} \times 400 \times \frac{400}{\sqrt{3} \times 10} \times 0.8$

$$
=12,800 \mathrm{~W}=12.8 \mathrm{~kW}
$$

Reactive power $P_{\mathrm{r}}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \sin \phi$
where

$$
\begin{aligned}
\sin \phi & =\sin \cos ^{-1} 0.8=0.6 \\
P_{\mathrm{r}} & =\sqrt{3} \times 400 \times \frac{400}{\sqrt{3} \times 10} \times 0.6=9,600 \mathrm{VAR}=9.6 \mathrm{kVAR}
\end{aligned}
$$

When power in measured using two wattmeters,

$$
\begin{gathered}
W_{1}+W_{2}=P \\
W_{1}+W_{2}=12,800 \mathrm{~W} . \\
\cos \phi=0.8 ; \phi=\cos ^{-1} 0.8=36.87^{\circ} \\
\sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}}=\tan 36.87^{\circ}=0.75 \\
W_{1}-W_{2}=\frac{0.75}{\sqrt{3}} \times\left(W_{1}+W_{2}\right) \\
W_{1}-W_{2}=\frac{0.75}{\sqrt{3}} \times 12,800
\end{gathered}
$$



Fig. 8.38 (a) Circuit as per data (b) Phasor diagram

$$
\begin{equation*}
W_{1}-W_{2}=5,542.56 \mathrm{~W} \tag{8.15}
\end{equation*}
$$

Solving Equations (8.14) and (8.15), we get

$$
\begin{aligned}
& W_{1}=9,171.28 \mathrm{~W}=9.17 \mathrm{~kW} \\
& W_{2}=3,628.72 \mathrm{~W}=3.628 \mathrm{~kW}
\end{aligned}
$$

## Example 8.38

Calculate the reading of each wattmeter in the circuit shown in Figure 8.39. The load impedance $Z=40 \angle-30^{\circ}$.
(U.P.T.U. 2004-05)


Fig. 8.39 Given circuit

## Solution:

Phase voltages,

$$
\begin{aligned}
& \bar{V}_{\mathrm{an}}=240 \angle 0^{\circ} \\
& \bar{V}_{\mathrm{bn}}=240 \angle 120^{\circ} \\
& \bar{V}_{\mathrm{cn}}=240 \angle-120^{\circ}
\end{aligned}
$$

The supply is connected in star, while load is connected in delta. Line voltage, that is, voltage across AB

$$
\left|V_{\mathrm{AB}}\right|=\left|V_{\mathrm{L}}\right|=\sqrt{3} \times\left|V_{\mathrm{an}}\right|=\sqrt{3} \times 240=415.69 \mathrm{~V}
$$

Phase voltage for delta-connected load, $\left|V_{\text {ph }}\right|=\left|V_{\text {ph }}\right|=415.69 \mathrm{~V}$
Impedance of each branch, $\bar{Z}=40 \angle-30^{\circ}$
Phase angle, $\phi=30^{\circ}$ and power factor, $\cos \phi=\cos 30^{\circ}=0.866$
Power current, $\left|I_{\mathrm{ph}}\right|=\frac{\left|V_{\mathrm{ph}}\right|}{\left|Z_{\mathrm{ph}}\right|}=\frac{415.69}{40}=10.39 \mathrm{~A}$

Power consumed by the load, $P=3 V_{\text {ph }} I_{\mathrm{ph}} \cos \phi$

$$
\begin{equation*}
=3 \times 415.69 \times 10.39 \times 0.866=11,223 \mathrm{~W} \tag{8.16}
\end{equation*}
$$

Sum of the wattmeter readings, $W_{1}+W_{2}=P=11,223 \mathrm{~W}$

$$
\begin{align*}
\tan \phi & =\sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}} \\
W_{1}-W_{2} & =\frac{\tan 30 \times\left(W_{1}+W_{2}\right)}{\sqrt{3}}=\frac{\frac{1}{\sqrt{3}} \times 11,223}{\sqrt{3}} \\
W_{1} & -W_{2}=3,741 \mathrm{~W} \tag{8.17}
\end{align*}
$$

Adding Equations (8.16) and (8.17), we get $2 W_{1}=14,964$
or

$$
W_{1}=7,482 \mathrm{~W}
$$

From Equation (8.17),

$$
W_{2}=11,223-7,482=3,741 \mathrm{~W}
$$

This problem can also be solved alternatively as follows:
Phase voltages are

$$
\begin{aligned}
& \bar{V}_{\mathrm{an}}=240 \angle 0^{\circ} ; \bar{V}_{\mathrm{bn}}=240 \angle 0^{\circ} ; \\
& \bar{V}_{\mathrm{cn}}=240 \angle-120^{\circ}
\end{aligned}
$$

Voltage across $\mathrm{AB}=V_{\mathrm{AB}}=V_{\mathrm{ab}}=\sqrt{3} \times 240 \mathrm{~V}$ as supply is star connected.
Since load is delta connected, $V_{\mathrm{L}}=$ line voltage

$$
=V_{\mathrm{AB}}=V_{\mathrm{ab}}=3 \times 240 \mathrm{~V}=\text { phase voltage }=V_{\mathrm{ph}}
$$

Current in phase $A B$

$$
\bar{I}_{\mathrm{AB}}=\bar{I}_{\mathrm{ph}}=\frac{\bar{V}_{\mathrm{ph}}}{\bar{Z}_{\mathrm{ph}}}=\frac{\sqrt{3} \times 240}{40 \angle-30^{\circ}}=\sqrt{3} \times 6 \angle 30^{\circ}
$$

Line current for delta load, $\quad \bar{I}_{\mathrm{L}}=\sqrt{3} \bar{I}_{\mathrm{ph}}=\sqrt{3} \times \sqrt{3} \times 6 \angle 30^{\circ}=18 \angle 30^{\circ}$

$$
\phi=30^{\circ}
$$

Reading of wattmeter $W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)$

$$
=\sqrt{3} \times 240 \times 18 \cos \left(30^{\circ}+30^{\circ}\right)=\sqrt{3} \times 240 \times 18 \times 0.5=3,741 \mathrm{~W}
$$

Reading of wattmeter $W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)$

$$
=\sqrt{3} \times 240 \times 18 \cos \left(30^{\circ}-30^{\circ}\right)=\sqrt{3} \times 240 \times 18 \times 1=7,482 \mathrm{~W}
$$

## SUMMARY

1. Poly-phase system: A polyphase system is essentially a combination of two or more voltages having same magnitude and frequency but displaced from one another by equal electrical angle.
2. Advantages of 3-phase system: It provides constant power, higher rating, economical transmission system and superior operation of 3-phase induction motors.
3. Interconnections of three-phases: The three phases are interconnected either in star or delta.
4. Relation between phase and line voltage and currents in star and delta connections:
(i) In star connections: Line voltage $=\sqrt{3} \times$ Phase voltage; line current $=$ Phase current
(ii) In delta connections: Line voltage $=$ Phase voltage; line current $=\sqrt{3}$ Phase current
5. Power in 3-phase circuits: $\mathrm{P}=\sqrt{3} V_{L} I_{L} \cos \phi=3 V_{p h} I_{p h} \cos \phi$ in both star and delta connections
6. Power measurement in 3-phase circuits: Power in 3-phase circuits can be measured by (i) threewattmeter method (ii) two-wattmeter method
$P=W_{1}+W_{2}+W_{3}$ (in 3-wattmeter method); $P=W_{1}+W_{2}$ (in 2-wattmeter method)
7. Measurement of pf by two wattmeter method (for balanced load)
$\cos \phi=\tan ^{-1} \sqrt{3} \frac{W_{1}-W_{2}}{W_{1}+W_{2}}$
8. Effect of pf on wattmeter reading (in case of balanced load-two wattmeter method)
(i) When $\mathrm{pf}=1 ; W_{1}=W_{2}$
(ii) When $\mathrm{pf}=0.5$; either $W_{1}=0$ or $W_{2}=0$
(iii) When $\mathrm{pf}<0.5$; one of the wattmeter will give downscale reading and to get up-scale reading the connections of its current coil are reversed but the power measured by this wattmeter is taken as negative.

## TEST YOUR PREPARATION

## 7 FILLIN THE BLANKS

1. In a three-phase system, the phase difference between the two adjacent emfs is $\qquad$ .
2. For transmission of electric power, $\qquad$ system in used.
3. In a star-connected system, the phase difference between line and phase voltage is $\qquad$ -.
4. In a three-phase delta-connected system, the apparent power is given by the relation $\qquad$ .
5. In a three-phase delta-connected system, $I_{\text {ph }}=$ $\qquad$ $I_{\mathrm{L}}$.
6. For the same size, the rating of three-phase motor will be $\qquad$ than the single-phase motor.
7. In a two-wattmeter method of power measurement in a three-phase balanced system, the two wattmeters will read equally, when the power factor is $\qquad$ .
8. By two-wattmeter method, to determine power factor, the relation used is $\tan \phi=$ $\qquad$ $/\left(W_{1}+W_{2}\right)$.
9. In two-wattmeter method of power measurement, one wattmeter gives downscale reading, when the power factor lies between $\qquad$ .
10. In two-wattmeter method, when the power factor of the load is zero leading or lagging, the two wattmeters will give $\qquad$ reading.

## OBJECTIVE TYPE QUESTIONS

1. For the same rating, the size of three-phase machine to that of single-phase machine will be
(a) more
(b) less
(c) same
(d) None of the above.
2. To transmit the same amount of power over a particular distance at a given voltage, the amount of conducting material required in a three-phase system to that of single-phase system will be
(a) 1.5 times
(b) 0.5 times
(c) 3 times
(d) 0.75 times
3. The phase sequence of a three-phase system is RYB. The same phase sequence can be represented as
(a) RBY
(b) BYR
(c) YBR
(d) YRB
4. In a balanced three-phase, star-connected system, the phase difference between phase voltages and their respective line voltages are
(a) $30^{\circ}$
(b) $60^{\circ}$
(c) $120^{\circ}$
(d) $45^{\circ}$
5. In a balanced three-phase, star-connected system, the relation between phase voltage ( $V_{\mathrm{ph}}$ ) and line voltage $\left(V_{\mathrm{L}}\right)$ is
(a) $V_{\text {ph }}=3 V_{\mathrm{L}}$
(b) $V_{\text {ph }}=0.577 V_{\mathrm{L}}$
(c) $V_{\mathrm{ph}}=V_{\mathrm{L}} \sqrt{2}$
(d) None of above
6. A three-phase load is said to be a balanced load, if all the three phases have the same
(a) impedance
(b) power factor
(c) both a and b
(d) None of above
7. Three $100-\Omega$ resistors are connected in star across a 400 V , three-phase supply, if one of the resistors is disconnected, then the line current will be
(a) 2 A
(b) 4 A
(c) $4 / \sqrt{3} \mathrm{~A}$
(d) 43 A
8. Three $50-\Omega$ resistors are connected in delta across a 400 V , three-phase supply, if one of the resistor is disconnected, then the phase current will be
(a) $8 / \sqrt{3} \mathrm{~A}$
(b) $8 \times \sqrt{3} \mathrm{~A}$
(c) 8 A
(d) $4 \sqrt{3} \mathrm{~A}$
9. In a three-phase, balanced load, the power consumed is given by the relation
(a) $3 V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi$
(b) $3 V_{\text {ph }} I_{\text {ph }} \cos \phi$
(c) both a and b
(d) None of the above
10. When three $10-\Omega$ resistors are connected in star across a 400 V , three-phase supply, each resistor must have a power rating.
(a) $5,290 \mathrm{~W}$
(b) $2,300 \mathrm{~W}$
(c) $4,000 \mathrm{~W}$
(d) $4,600 \mathrm{~W}$
11. Three delta-connected resistors absorb 30 kW when connected to a 400 V , three-phase supply. When they are connected in star across the same supply the power absorbed will be
(a) 60 kW
(b) 90 kW
(c) 20 kW
(d) 10 kW
12. Three identical resistances, each of $15 \Omega$, are connected in star across a 400 V , three-phase supply. The value of resistance the each phase of the equivalent delta-connected load would be
(a) $5 \Omega$
(b) $45 \Omega$
(c) $30 \Omega$
(d) $7.5 \Omega$
13. Three capacitors, each of $150 \mu \mathrm{~F}$, are connected in delta. The value of capacitance in each leg of the equivalent star-connected load would be
(a) $50 \mu \mathrm{~F}$
(b) $150 \mu \mathrm{~F}$
(c) $900 \mu \mathrm{~F}$
(d) $450 \mu \mathrm{~F}$
14. Wattmeter is an instrument that measures
(a) instantaneous power
(b) average real power
(c) apparent power
(d) reactive power
15. In two-wattmeter method of three-phase power measurement, one of the wattmeter gives downscale reading, when the power factor is
(a) unity
(b) 0.5
(c) between 1 and 0.5
(d) between 0 and 0.5
16. When the power factor of three-phase load is zero leading in a two-wattmeter method,
(a) one wattmeter gives zero reading
(b) both the wattmeters give zero reading
(c) both the wattmeters give equal but opposite reading
(d) None of above
17. For power measurement in a three-phase balanced load, the two wattmeters will read
(a) $W_{1}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)$ and $W_{2}=V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)$
(b) $W_{1}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}-\phi\right)$ and $W_{2}=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \left(30^{\circ}+\phi\right)$
(c) $W_{1}=V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \left(30^{\circ}-\phi\right)$ and $W_{2}=V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \left(30^{\circ}+\phi\right)$
(d) $W_{1}=\sqrt{3} V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \left(30^{\circ}-\phi\right)$ and $W_{2}=\sqrt{3} V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \left(30^{\circ}+\phi\right)$
18. In two-wattmeter method of three-phase power measurement in balanced load having 0.5 power factor lagging,
(a) one wattmeter reads zero
(b) one wattmeter reads downscale
(c) both the wattmeters read equally
(d) both the wattmeters give equal but opposite readings.
19. In two-wattmeter method of power measurement in three-phase balanced load, both the wattmeters give equal reading when the load power factor is
(a) 0.5
(b) 0
(c) between 0.5 and 1
(d) 1
20. In a three-phase, three-wire unbalanced load, power cannot be measured by two wattmeters.
(a) true
(b) false

## NUMERICALS

1. Three similar coils each having a resistance of $15 \Omega$ and an inductance of 0.04 H are connected in star to a three-phase, 50 Hz supply, 200 V between lines. Calculate the line current. If they are now connected in delta, calculate the phase current, line current, and the total power absorbed in each case.
(Ans. $5.9 \mathrm{~A}, 10.215 \mathrm{~A}, 17.69 \mathrm{~A}, 1,565 \mathrm{~W}, 4,695 \mathrm{~W}$ )
2. Three inductive coils each of inductances 50 mH are connected in star to a three-phase, $200 \mathrm{~V}, 50 \mathrm{~Hz}$ system. Calculate the inductance of each coil, which when connected in delta to the same supply will take the same line current.
(Ans. 150 mH )
3. Show that the power consumption is three times, when three identical impedances are connected in delta across a balanced three-phase supply in comparison to that when the same impedances are connected in star across the same supply.
4. Three capacitors each of $150 \mu \mathrm{~F}$ are connected in delta to a 400 V , three-phase, 50 Hz supply. Determine the line current. What will be the capacitance of each of the three capacitors such that when connected in star across the same supply, the line current remains the same?
(Ans. $32.62 \mathrm{~A}, 450 \mu \mathrm{~F}$ )
5. Three impedances $Z_{\mathrm{A}}=12 \angle 0^{\circ} \Omega, Z_{\mathrm{B}}=12 \angle 30^{\circ} \Omega$, and $Z_{\mathrm{C}}=10 \angle 45^{\circ} \Omega$ are connected in star to a three-phase four-wire, 416 V supply system, the voltages being $V_{\mathrm{AB}}=V_{\mathrm{L}} \angle-120^{\circ}, V_{\mathrm{BC}}=V_{\mathrm{L}} \angle 0^{\circ}, V_{\mathrm{CA}}$ $=V_{\mathrm{L}} \angle 120^{\circ} \mathrm{V}$. Determine the three line currents and the current in the neutral.
(Ans. $20 \mathrm{~A}, 20 \mathrm{~A}, 24 \mathrm{~A}, 15.9 \mathrm{~A})$
6. An induction motor on no-load gave the two wattmeter readings of 400 W and 100 W , respectively. The readings of the latter wattmeter are obtained after reversing the connection of its current coil. Calculate (i) power factor of the motor at no-load and (ii) phase difference of the current and voltage in two wattmeters.
(Ans. $0.3273 \mathrm{lag}, 100.89^{\circ} ;-49.89^{\circ}$ )
7. A three-phase 500 V motor load has an output of 80 H .P. with an efficiency of $90 \%$ and power factor 0.866 . Calculate (i) the current in each phase of motor if the motor is delta connected and (ii) the readings of two wattmeters connected in the lines to measure the input power.
(B.U. Jan. 1993) (Ans. $50.327 \mathrm{~A}, 43.586 \mathrm{~kW}, 21.793 \mathrm{~kW}$ )
8. Two wattmeters are connected to measure the input to a balanced three-phase circuit $2,000 \mathrm{~W}$ and 500 W , respectively. Find the power factor of the circuit when (i) both readings are positive, and (ii) the latter reading is obtained after reversing the connections of the current coil of the wattmeter.
(Ans. 0.6934, 0.3273)

## VIVA VOICE OR REASONING QUESTIONS

1. Three-phase system is preferred over single-phase system. Why?
2. The knowledge of phase sequence is very important in industries and power system. Why?
3. In star connections, the voltage between the two line terminals is not double to that of phase voltage but it is $\sqrt{3}$ times. Why?
4. When measuring power by two wattmeters in three-phase system, the connection of the current coil of one of the wattmeter is reserved. Why?
5. When measuring power by two wattmeters in three-phase system, one of the wattmeter does not read. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What do you understand by the following terms:
(i) three-phase balanced supply, (ii) three-phase balanced load, (iii) three-phase unbalanced supply, (iv) three-phase unbalanced load, and (v) single phasing.
2. Write down the advantages of three-phase system over single phase.
(U.P.T.U. Tut.)
3. What are the two ways of connecting a three-phase system? Draw their phasor diagrams and write down the relationship between phase and line voltages and currents for these systems.
4. Show that the total power in a three-phase balanced load is $P=\sqrt{3} V I \cos \phi$ where $V$ is the rms line voltage, $I$ is the rms line current, and $\phi$ is the phase angle between phase voltage and phase current.
5. Compare three-phase star- and delta-connected systems.
6. While measuring power in a three-phase balanced network using two wattmeters, one of the meters gives zero reading. Explain this condition with the help of phasor diagram and suitablederivation.

## TEST QUESTIONS

1. If the voltage of R -phase is represented by ${V_{\mathrm{R}}}=V_{\mathrm{m}} \sin \omega t$, write down similar expression for the voltage of Y-phase and B-phase. The phase sequence being RBY.
2. What are the advantages of three-phase system over single-phase system?
(U.P.T.U. Tut.)
3. What do you understand by three-phase power supply? Describe star and delta connections by showing line voltages and phase voltages, line currents, and phase currents.
(U.P.T.U. Tut.)
4. Derive the relation between phase and line voltages and currents for a balanced three-phase star-connected system.
(U.P.T.U. 2004-05)

Or
For star-connected system in a three-phase circuit, prove that $V_{\mathrm{L}}=\sqrt{3} V_{\mathrm{ph}}$ and $I_{\mathrm{L}}=I_{\mathrm{ph}}$.
(U.P.T.U. 2006-07)

Or
Derive the relationship between the line and the phase voltages of an alternator. (U.P.T.U. 2006-07)
5. Write down the relationship between line voltage and line current with phase voltage and phase current in star-connected and delta-connected circuits.
(P.T.U.)
6. Derive expression for power in three-phase star and delta connections.
(U.P.T.U. 2005-06)
7. Obtain the line and phase relationships for voltages and currents for a delta-connected three-phase power system. Further, obtain the expression of total power and per phase power.
(U.P.T.U. Tut.)
8. Develop an expression for the total power in a balanced three-phase load.
(U.P.T.U. 2004-05)
9. Describe the basic features of a balanced three-phase system.
10. Show that the power consumption is one third when three identical impedances are connected in star across a balanced three-phase supply in comparison to that when the same impedances are connected in delta across the same supply.
11. Explain three-wattmeter method for a three-phase power measurement. What is the necessity of using three wattmeters?
(U.P.T.U. Tut.)
12. Explain the methods to measure power in three-phase circuits.
(U.P.T.U. 2006-07)
13. Explain two-wattmeter method to measure power in a three-phase unbalanced load.
14. How power is measured by two wattmeters in a three-phase balanced load? Explain with a neat circuit and phasor diagram.
(U.P.T.U. 2007-08)
15. How the power factor of a three-phase balanced load can be determined using two wattmeters?
(U.P.T.U. 2007-08)
16. Under what conditions, the two wattmeters read equal and opposite when connected to measure power in three-phase balanced load.
17. While measuring power by two wattmeters in a three-phase balanced load, one of wattmeters gives downscale reading. Describe the circuit conditions. How upscale reading can be obtained?
18. Prove that the power in a three-phase balanced circuit can be deduced from the readings of two wattmeters with the help of suitable vector diagrams. Discuss the nature of power factor when (i) the two wattmeters read equal and positive values, (ii) the two wattmeters read equal and opposite values, and (iii) the one wattmeter reads zero value.
(U.P.T.U. Feb. 2001)

## ANSWERS

## Fill in the Blanks

1. $120^{\circ}$ electrical
2. three-phase, three-wire
3. $30^{\circ}$
4. $\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}}$ or $3 V_{\mathrm{ph}} I_{\mathrm{ph}}$
5. $1 / \sqrt{3}$
6. more ( 1.5 times)
7. unity
8. $\sqrt{3}\left(W_{1}-W_{2}\right)$
9. 0 and 0.5
10. equal and opposite

## Objective Type Questions

1. (b)
2. (d)
3. (c)
4. (a)
5. (b)
6. (c)
7. (a)
8. (c)
9. (c)
10. (a)
11. (d)
12. (b)
13. (d)
14. (b)
15. (d)
16. (c)
17. (a)
18. (a)
19. (d)
20. (b)


## LEARNING OBJECTIVES

After the completion of this unit, students or readers will be able to understand the following:
*What is the concept and importance of measurement?

* What are electrical instruments?
* How are the electrical instruments classified?
*What are the different types of electrical instruments?
* What are essentials of indicating instruments?
*What is deflecting torque and how it is provided in an indicating instrument?
* What is controlling torque and how it is provided in an indicating instrument?

What is the importance of damping torque and how it is provided in the indicating instruments?

* How can we differentiate between indicating and integrating instruments?
* What are moving iron instruments?
* What is the basic principle of operation and constructional details of an attraction and repulsion-type moving iron instrument?
* What can be the errors that may occur in moving iron instruments and how these can be minimised or taken care?
* How can a moving iron instrument used as an ammeter or a voltmeter.
* What is a PMMC instrument?
*What is the basic working principle and construction of a PMMC instrument?
* What can be the errors in PMMC instruments and how these can be minimised?
* How can we differentiate between an ammeter and a voltmeter?
* How can the range of an ammeter or voltmeter extended?
* What are the dynamometer-type instruments?
* Why dynamometer-type instruments are used only as a wattmeter to measure power and not as a voltmeter or ammeter?
* How a wattmeter measures actual power in AC circuits?
* What different errors may crop in wattmeters and how these can be removed or compensated?
* What are induction-type wattmeters and how these are different to dynamometer-type wattmeters?
* What is induction-type energy meter and how it is different to induction-type wattmeter?
* What type of errors may occur in induction-type energy meters and how these can be minimised or compensated?


### 9.1 INTRODUCTION

In the field of engineering, various types of quantities such as physical, chemical, mechanical, thermal, and electrical are involved. For records, these quantities are required to be measured by different methods using different types of instruments. Electrical measuring instruments are widely used in the modern engineering world because of their accuracy, convenience, and reliability. These instruments are not only used to measure the electrical quantities but are also used to measure other non-electrical quantities such as temperature, strain, deformation, velocity, and pressure with the help of transducers. Thus, all the engineering students must be in acquaintance with these instruments. In this chapter, we shall focus our attention on construction and working principles of some of the important electrical instruments such as ammeter, voltmeter, wattmeter, and energy meter.

### 9.2 CONCEPT OF MEASUREMENTS

To measure the magnitude of a quantity, a fixed quantity of that kind is taken as basis, and then, whole quantity is compared with it. The fixed quantity that is taken as basis is called unit and the process of comparing the quantity with this unit is termed as measurement. In other words, measurement means recording of the magnitude of given quantity in terms of its base units, for example, length of a given object (say a rod) is measured as $x$ metre, where $x$ is the magnitude and metre is the unit that represents a standard length as accepted by Bureau of International Standards. The process of comparing the given quantity with fixed unit (or standard) is called measurements.

### 9.3 INSTRUMENTS AND THEIR CLASSIFICATION

A device used to measure some quantity is known as instrument. An instrument is used to determine the magnitude of given quantity. Broadly, instruments may be classified as mechanical, electrical, and electronic instruments.

### 9.3.1 Electrical Instruments

In electrical system, there are various electrical quantities, such as current, voltage, power, energy, frequency, power factor, resistance, inductance, and capacitance that are required to be measured. To measure these quantities, various instruments such as ammeter, voltmeter, wattmeter, energy meter, frequency meter, power factor meter, and ohm meter (or RLC meter) are used, respectively. These instruments are known as electrical instruments.

The instruments, such as ammeter, voltmeter, energy meter, and megger, are used to measure electrical quantities are called electrical instruments.

## Classification of electrical instruments

The electrical instruments may be broadly classified as follows:

1. Absolute instruments: The instruments that give the value of the quantity to be measured in terms of constants of the instrument are called absolute instruments. Such instruments do not require any previous calibration. The common example of this type of instrument is tangent galvanometer. The tangent galvanometer gives the value of current in terms of the tangent of deflection produced by the current, the radius, and number of turns of wire and the horizontal component of earth's field. These instruments are seldom used except in standard laboratories for standardising the instruments.
2. Secondary instruments: The instruments that determine the electrical quantity to be measured directly in terms of deflection are called secondary instruments. Such instruments are calibrated with standard instruments that have already been calibrated before using them. These instruments are generally used in practical life. The secondary instruments are further classified as follows:
(a) Indicating instruments: The instruments that indicate the magnitude of electrical quantity being measured instantaneously are called indicating instruments.


Fig. 9.1 Indicating instruments (a) Ammeter (b) Voltmeter (c) Wattmeter

In such instruments, a pointer moves over the calibrated scale to indicate the magnitude of electrical quantity being measured. Ordinary ammeters, voltmeters, and wattmeters fall into this category (Fig. 9.1).
(b) Integrating instruments: The instruments that add up the electrical quantity, that is, electrical energy and measure the total energy (in kWh ) in a given period are called integrating instruments.


Fig. 9.2 Integrating instruments (a) Induction type energy meter (b) Digital energy meter

In such instruments, there are sets of dials or gears that register the total quantity of electricity or the total amount of electrical energy supplied to a circuit in a given period. The energy meters fall into this category (Fig. 9.2).
(c) Recording instruments: The instruments that give a continuous record of the variations of the electrical quantity being measured are called recording instruments. These instruments more or less are the indicating instruments; however, in this case, the moving system carries an inked pen that rests slightly on a chart or graph wrapped over a drum moving at a uniform slow speed (Fig. 9.3). The motion of the drum is in a direction perpendicular to that of the deflection of the pen. The path traced out indicates the variations in the electrical quantity being measured during the given period.

(a)

(b)

(c)

Fig. 9.3 Recording instruments (a) Paper wrapped over a drum (b) Inked pen tracing a graph (c) Graph traced on a paper

These instruments are used when we want to preserve the information that could be obtained later on. A simple example of these instruments is load curve plotter placed at the generating stations to plot variation of load at the generating station with respect to time. The other example is Electrocardiograph (ECG) that records the heart beats and shows the circulation of blood or effectiveness of heart (i.e., pumping system) to diagnose heart disease.

## Effects utilised in electrical instruments to produce deflecting torque

The principle of operation of electrical instruments depends upon various effects produced by electric current and voltage. The various effects and the electrical instruments in which these effects are utilised for their operation are mentioned in Table 9.1.

## Table 9.1 Effects Utilized in Various Electrical Instruments

| Effect produced | Instruments in which this effect is generally utilised |
| :--- | :--- |
| (1) Magnetic effect | Ammeters and voltmeters |
| (2) Electrodynamic effect | Ammeters, voltmeters, and wattmeters, usually wattmeters. |
| (3) Electromagnetic induction effect | Ammeters, voltmeters, wattmeters and energy meters. |
|  | Usually wattmeters and energy meters. |
| (4) Electrostatic effect | Voltmeters only |
| (5) Heating or thermal effect | Ammeters and voltmeters |
| (6) Chemical effect | DC ampere-hour meters |

## Essentials of indicating instruments

In indicating instruments, a pointer moves over the calibrated scale to indicate the magnitude of electrical quantity being measured. The pointer is attached to a moving system pivoted in jewelled bearings. The forces or torques required for satisfactory operation of indicating instruments are as follows:

1. Deflecting torque: The deflecting or operating torque $\left(T_{\mathrm{d}}\right)$ is produced by making use of any one of the effects such as magnetic, electrodynamic, and electromagnetic induction. The actual method of torque production depends upon the type of instrument. The deflecting torque is required to move the moving system (and hence, the pointer attached to it) from zero position when the instrument is connected in the circuit to measure the electrical quantity.

The torque produced by any of the abovementioned effects due to which moving system moves (or pointer deflects) from its zero position in an indicating instrument is called deflecting torque.
2. Controlling torque: The controlling or restoring torque ( $T_{\mathrm{c}}$ ) opposes the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques (i.e., deflecting torque and controlling torque) are equal. Thus, it ensures that the deflection of the pointer is according to the magnitude of electrical quantity being measured. If this torque was not provided, the pointer would continue to move indefinitely and the deflection shall be independent of the value of electrical quantity being measured.

The other function of the controlling torque is to bring the pointer back to zero when the instrument is removed from the circuit or when the deflecting torque acting in the instrument is removed. If this torque was not provided, the pointer once deflected would not return to zero position on removing the deflecting torque. This torque is either obtained by a spring or by gravity in the indicating instruments.

Thus, in indicating instruments, the torque that brings the pointer to zero position when the instrument is disconnected from the circuit or the torque that allows the pointer to deflect in accordance to the magnitude of electrical quantity is called controlling torque. It always opposes the deflecting torque.

In spring control, one or two phosphor bronze spiral hair-springs are attached to the moving spindle. The other ends of the springs are attached to the fixed body or frame. In this case, the controlling torque $\left(T_{c}\right)$ is directly proportional to the angle of deflection $(\theta)$, that is, $T_{\mathrm{c}} \infty \theta$.

In gravity control, a small adjustable weight is attached to the moving system in such a way that due to gravity, it tries to bring the pointer to zero position when it is deflected. In this case, the controlling torque $\left(T_{\mathrm{c}}\right)$ is proportional to the sine of angle of deflection $(\theta)$, i.e., $T_{\mathrm{c}} \infty \sin \theta$.
3. Damping torque: When deflecting torque is applied to the moving system, it deflects the pointer. While the controlling torque controls the deflection and tries to stop the pointer at its final position, say at $F$, as shown in Figure 9.4, where deflection torque is equal to controlling torque (i.e., $T_{\mathrm{d}}=T_{\mathrm{c}}$ ). However, due to inertia, the pointer oscillates around its final position $F$ before coming to rest. These oscillations are undesirable because it causes delay in taking the reading. Then what to do? In order to avoid these oscillations and to bring the pointer quickly to its final deflected position, damping torque is provided. The damping torque opposes the movement (forward or backward) of the pointer and operates only when the system is moving. In fact, damping torque acts like a brake on the moving system.

Thus, in indicating instruments, the torque that suppresses the undue oscillations of the pointer and brings the pointer to its final position quickly is called damping torque. It always acts opposite to motion.
(a) Underdamping: When the damping torque provided in an indicating instrument is such that the pointer oscillates around its final position for long period, the instrument is called underdamped and the condition is referred to underdamping (Fig. 9.5). It delays in taking the reading.


Fig. 9.4 Oscillating pointer


Fig. 9.5 Graphical representation of damping torque
(b) Overdamping: When the damping torque provided in an indicating instrument is such that the pointer rises slowly and takes sufficient time to reach its final position, the instrument is called overdamped and the condition is referred to overdamping (Fig. 9.5). It makes the instrument sluggish.
(c) Critical damping: When the damping torque provided in an indicating instrument is such that the pointer rises quickly and obtains its final position immediately, the instrument is called critically damped and the condition is referred to critical damping (Fig. 9.5). This is what we require in indicating instruments.

### 9.4 METHODS OF PROVIDING CONTROLLING TORQUE

The controlling or restoring or balancing torque is provided in the indicating instruments by two methods, namely spring control and gravity control.

### 9.4.1 Spring Control

In this method, two phosphor bronze spiral hair-springs $A$ and $B$ are attached to the moving system (spindle) of the indicating instrument as shown in Figure 9.6(a). However, only one spring is used in moving iron instruments. In fact, one end of each spring is attached with the spindle and the other end is attached to some fixed point (may be outer frame or body of the instrument). The two springs are wound in opposite direction to compensate for change in temperature. The moving system is statically balanced in all positions by providing balancing weights as shown in Figure 9.6(a). To adjust the zero of the pointer, an arrangement, called zero adjuster is provided on the pointer as shown in Figure 9.6(b).


Fig. 9.6 Spring control to provide controlling torque (a) Two springs attached to the spindle in opposite direction (b) Pointer with zero adjuster

Under ordinary conditions, when instrument is not in use, the two springs are in their natural position, the controlling torque is zero and the pointer is at zero position. When the instrument is connected in the circuit, deflecting torque ( $T_{\mathrm{d}}$ ) deflects the pointer. With the deflection of the pointer, one of the springs is unwound, while the other gets twisted. The resultant twist in the springs provides controlling torque $\left(T_{\mathrm{c}}\right)$ that is directly proportional to the angle of deflection $\theta$ of the moving system, that is, $T \propto \theta$. The pointer comes to the position of rest when controlling torque is equal to deflecting torque, that is, $T_{\mathrm{c}}=T_{\mathrm{d}}$.

If

$$
\begin{aligned}
& T_{\mathrm{d}} \propto I \text { (in the case of permanent magnet moving coil instrument), then } \\
& I \propto \theta\left(\text { since } T_{\mathrm{c}} \propto \theta \text { and } T_{\mathrm{d}}=T_{\mathrm{c}}\right)
\end{aligned}
$$

Hence, in spring-controlled instruments (only in which $T_{\mathrm{d}} \propto I$ ), scales are uniform.

## Important points for consideration

The spring should have a fairly large number of turns so that the deformation per unit length is kept small on full-scale deflection. Moreover, it also ensures that controlling torque is proportional to the angle of deflection. However, the stress in spring should be limited to such a value that there is no permanent set in it.

## Advantages

1. When controlling torque is provided by spring, the instrument can be placed in any position.
2. The hair spring has almost negligible weight, and therefore, practically there is no increase in weight of the moving system.
3. Spring-controlled instruments (only in which $T_{\mathrm{d}} \propto I$ ) have uniform scales.

## Disadvantages

1. The change in temperature affects the length of spring, and hence, the controlling torque.
2. Since the springs are very delicate, the accidental stresses may damage them.
3. Due to fatigue, springs deteriorate with time.

### 9.4.2 Gravity Control

In this method, a small adjustable weight $W$ is attached to the moving system in such a way that it produces a controlling torque when the moving system is in deflected position. The controlling torque can be varied by adjusting the position of the control weight upon the arm as shown in Figure 9.7(a).

At zero position of the pointer, the control weight $W$ is in the vertical position, and therefore, no controlling torque is produced. However, under the action of deflecting torque, the pointer is deflected through an angle $\theta$ from its zero position, as shown in Figure 9.4(a) (dotted position). Due to gravity, the control weight would try to come back to its original (i.e., vertical) position and produces controlling torque. Under the stationary position, controlling torque must be equal to deflecting torque (i.e., $T_{\mathrm{c}}=T_{\mathrm{d}}$ ).


Fig. 9.7 Gravity control to provide controlling torque (a) Position of pointer with and without deflection (b) Forces acting on weight at deflected position
$l=$ distance of control weight from axis of rotation and
$\theta=$ angle through which pointer or weight has deflected.
In the deflected position, weight $W$ can be resolved into two components, that is, $W \cos \theta$ and $W \sin \theta$. Only the component $W \sin \theta$ provides the controlling torque.
or

$$
T_{\mathrm{c}}=W l \sin \theta
$$

$$
T_{\mathrm{c}} \propto \sin \theta(\text { since } W \text { and } l \text { have fixed values })
$$

If

$$
\begin{aligned}
& T_{\mathrm{d}} \propto I \text {, then } \\
& I \propto \sin \theta\left(T_{\mathrm{d}}=T_{\mathrm{c}} \text { at the position of rest }\right)
\end{aligned}
$$

Hence, in gravity-controlled instruments, scales are not uniform, but are crowded in the beginning.

## Advantages

1. It is very cheap.
2. It is a very simple method.
3. It is not affected by change in temperature.
4. It is free from fatigue and does not deteriorate with time.
5. The controlling torque can be varied easily.

## Disadvantages

1. The instruments in which controlling torque is provided, with gravity control method, have non-uniform scales.
2. The instrument has to be kept in vertical position.
3. The control weight adds to the weight of the moving system that decreases torque or weight ratio and reduces the sensitivity of the instrument.

### 9.5 METHODS OF PROVIDING DAMPING TORQUE

To obtain critical damping (i.e., the value of damping that is sufficient to enable the point to rise quickly to its deflected position without overshooting), adjustments are made while designing the instruments. Damping torque or force can be provided by the following methods.

### 9.5.1 Air Friction Damping

Two methods of air friction damping are illustrated in Figure 9.8(a) and (b). In the first case, a light aluminium piston, and in the second case, a vane is attached to the moving system. The piston (or vane) attached to the spindle moves in an air chamber, having cross-section either circular or rectangular, closed at one end.


Fig. 9.8 Air friction damping (a) Air friction damping due to movement of piston in a cylinder (b) Air friction damping due to movement of vane in a chamber

The clearance between the piston and the sides of the chamber should be very small and uniform. When the pointer is deflected upscale to read the quantity to be measured, the piston moves out of the chamber, so that the pressure in the closed space $S$ falls. The pressure on the open side of the piston is greater than the closed space that opposes the motion. Thus, the arrangement restricts the quick movement of the pointer and does not allow it to overshoot from its final position. However, if the pointer overshoots slightly, it has to come back. In this case, the piston is pushed into the air chamber compressing the air in the closed space $S$. This increases the pressure in the closed space that restricts the movement of the piston, and thus, opposes the rapid downward movement of the pointer. Hence, the necessary damping is produced and pointer comes to rest at its final position quickly.

### 9.5.2 Fluid Friction Damping

In this method of damping, light disc or vanes are attached to the spindle of the moving system and dipped into a pot of damping oil as shown in Figure 9.9(a) and (b). The motion of the moving system is always opposed by the friction of the damping oil on the vanes. The damping force acting on the vanes increases with the increase in the speed of the moving system. The damping force always acts in opposite direction to that of rotation and is zero when the vanes are stationary.


Fig. 9.9 Fluid friction damping (a) Side view (b) Top view

In this method of damping, no care is required as in the air friction damping. However, this method is not suitable for portable instruments because of oil contained in the instrument may leak through it. The other disadvantages of this method are creeping of oil and the necessity of using the instrument always in the vertical position.

### 9.5.3 Eddy Current Damping

In this method of damping, when the sheet of conducting but non-magnetic material such as copper or aluminium moves in a magnetic field so as to cut through magnetic lines of force, eddy currents are induced in the sheet and a force exists between these currents and the magnetic field. This force always acts in opposite direction to that of the cause producing it, that is, motion, according to Lenz's law. This provides necessary damping torque. The magnitude of damping torque is directly proportional to the speed of the moving system.

Two methods of eddy current damping are shown in Figure 9.10. In Figure 9.10, a thin aluminium disc, mounted on the spindle carrying the pointer of the instrument, is allowed to rotate in the air gap of a damping magnet (permanent); when the spindle rotates, disc cuts through the magnetic lines of force and eddy currents are induced in it. Thus, a force acts on the disc that opposes the motion and provides the necessary damping torque.


Fig. 9.10 Eddy-current damping (a) Eddy-current damping for disc (b) Eddy-current damping in an aluminium former

Figure 9.10(b) shows the ${ }^{1}$ eddy current damping employed in permanent-magnet moving coil instruments. The operating coil (coil that produces deflecting torque) is wound on a thin light aluminium former to which pointer is attached. When the coil wound on the former moves in the field of the permanent magnet, eddy currents are induced in the aluminium former. Thus, a force acts on the former that opposes the motion and provides the necessary damping.

Differentiation of indicating and integrating instruments is given in Table 9.2.
Table 9.2 Difference Between Indicating and Integrating Instruments
S.No. Indicating instruments Integrating instruments

1. These instruments indicate the magnitude of electrical quantity being measured instantaneously.
2. Controlling torque is provided in these instruments.
3. These instruments are equipped with damping torque.
4. In these instruments, registering mechanism is not required.

These instruments add up the electrical quantity and measure the total quantity in a given period.
No controlling torque is provided in these instruments.

In these instruments, damping torque is not required.

In these instruments, registering mechanism is provided.

## Example 9.1

It is found that 10 A current passes through a resistor of $10 \Omega$ value. The resistor has $5 \%$ tolerance. The error in measurement of current can be as high as $5 \%$. What is the maximum error in the measurement of power calculated from the measured value of current and the nominal value of resistor?
(U.P.T.U. Tut.)

## Solution:

Power dissipated in the resistor $P=I^{2} R$
The limiting error in power dissipation $=2 \times 5+5=15 \%$
Hence, power dissipation, $P=I^{2} R=10^{2} \times 10=1000 \mathrm{~W} \pm 15 \%$, that is, $1000 \pm 150 \mathrm{~W}$

[^9]Thus, the maximum error that crop-up during measuring power while calculated from the measured value of current and nominal value of resistor is $\pm 150 \mathrm{~W}$.

## 国政 PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by measurements?
2. What do you mean by an instrument? How will you classify them?
3. What do you mean by electrical instruments?
4. How will you classify the instruments on the basis of measurements?
5. What are various types of secondary instruments?
6. What effects are used to obtain deflecting torque in electrical instrument?
7. What are essentials of indicating instruments?
8. What do you mean by deflecting torque?
9. What is controlling torque in indicating instruments?
10. What do you understand by damping torque?
11. How controlling torque is provided in indicating instruments?
12. How damping torque is provided in indicating instruments?

## Test Questions

1. Name the electrical quantities need to be measured.
2. Give the various types of electrical measuring instruments with examples.
3. Give one example of an integrating instrument.
4. Give one example of recording-type instrument.
5. What is the maximum value of power factor?
6. Differentiate between indicating and recording-type instruments.
7. Differentiate between indicating and integrating-type instruments.
8. How will you differentiate between indicating and integrating instruments? Give examples for both.
9. Explain briefly how reading is shown by indicating-, integrating-, and recording-type instruments. Name each type of instrument also.
10. Explain the role of deflecting and controlling torque in an indicating instrument.
11. Explain the various methods of providing controlling torque in indicating instruments.
12. What is the role of damping torque? Explain.
13. What are the various methods for providing damping torque in an instrument?

### 9.6 MEASURING ERRORS

Measurement is essentially an act by which a comparison is made between a given quantity and a quantity of the same kind chosen as a unit of measurement, that is, standard quantity. To make this comparison, a device is used called measuring instrument. When an unknown quantity is measured, the value thus obtained is considered to be a true value but it is seldom true. There is always some difference between the measured value and the true or exact value of the unknown quantity.

The difference between the measured value $\left(A_{\mathrm{m}}\right)$ and the actual or true value $A$ of the unknown quantity is called absolute error of measurement ( $\delta A$ ).

$$
\varepsilon_{0}=\delta A=A_{\mathrm{m}}-A
$$

The absolute value of error $\delta A$ does not indicate precisely the accuracy of measurement. For example, while measuring the circumference of earth if the absolute error is 1 cm , it is
considered to be negligible; however, if this error occurs while measuring the circumference of a cricket ball, then it is quite significant. Therefore, it is preferred to represent the error in terms of relative error than to represent in absolute form.

### 9.6.1 Relative Error

The ratio of absolute error to the true value of quantity to be measured is termed as relative error.
i.e.,

$$
\text { relative error, } \varepsilon_{\mathrm{r}}=\frac{\delta A}{A}=\frac{\varepsilon_{0}}{A}=\frac{\text { Absolute error }}{\text { True value }}
$$

When the absolute error $\varepsilon_{0}$ or $\delta A$ is negligible, that is, when the difference between measured value $A_{\mathrm{m}}$ and true value $A$ is negligible, then relative error may be expressed as

$$
\varepsilon_{\mathrm{r}}=\frac{\delta A}{A_{\mathrm{m}}}
$$

The relative error may be represented as a fraction or as a percentage.
i.e.,

$$
\% \varepsilon_{\mathrm{r}}=\frac{\varepsilon_{0}}{A} \times 100
$$

### 9.7 ERRORS COMMON TO ALL TYPES OF INSTRUMENTS

The errors that are common to all types of instruments are given as follows:

1. Temperature error: Apart from room temperature, the rise in temperature occurs due to the heating effect of current flowing through the operating coil. This causes the change in resistance of the working coil and resistance of the other resistors connected in the instrument. Consequently, an erroneous reading is obtained while measuring an electrical quantity. Such an error is not serious in the case of ammeters, but it causes serious effect in the case of voltmeters. This error can be reduced by providing sufficient ventilation and cooling. This error can be further minimised by providing a series resistance coil of material having very small temperature co-efficient of resistance and working coil is made of copper wire.
2. Friction error: When deflecting torque occurs, the spindle of the moving system pivoted in the jewel bearing rotates and this causes friction; in this manner, it affects the instrument reading and hence it is called friction error. This error is more prominent when the deflecting torque is very small. To minimise this error, a moving system of light weight is designed.
3. Observational error: This error is usually due to misreading of the scale. It can be minimised by placing parallax mirror with the scale and using it effectively.

### 9.8 MOVING IRON INSTRUMENTS

These instruments are quite cheap in cost, simple in construction, and reasonably accurate at fixed power supply frequency. These instruments can be used both on AC and DC. Therefore,
these are widely used in laboratories and on switching panels. These instruments are used either as voltmeter or ammeter only. The pictorial view of these instruments is shown in Figures 9.11 and 9.12 , respectively.


Fig. 9.11 Voltmeter


Fig. 9.12 Ammeter

Moving iron instruments are of two types, namely attraction type and repulsion type.

### 9.8.1 Attraction-type Moving Iron Instruments

## Principle

The basic principle of an attraction-type moving iron instrument is illustrated in Figure 9.13. When a soft iron piece (or vane) is placed in the magnetic field of a current-carrying coil, it is


Fig. 9.13 Attraction type moving-iron instruments attracted towards the centre of the coil. This is because the piece tries to occupy a position of minimum reluctance. Thus, a force of attraction is exerted on the soft iron piece and deflection in the needle takes place. Hence, the name attraction-type moving iron instrument.

## Construction

The sectional view of an attraction-type moving iron instrument is shown in Figures 9.14 and 9.15. It consists of a stationary hollow cylindrical coil. An oval-shaped soft iron piece is mounted eccentrically to the spindle to which a pointer (needle) is attached. The controlling torque is provided by spring control method while damping torque is provided by air friction, as shown in Figures 9.14 and 9.15 .

## Working

When the instrument is connected in the circuit, an operating current (i.e., current to be measured in ammeter and current proportional to voltage to be measured in
voltmeter) flows through the stationary coil. A magnetic field is set up and the soft iron piece is magnetised that is attracted towards the centre of the coil, as shown in Figure 9.13. Thus, the pointer attached to the spindle is deflected over the calibrated scale.

If current in the coil is reversed, the direction of magnetic field produced by the coil will reverse. In turn, this will also reverse the magnetism produced in the soft iron piece. Hence, the direction of deflecting torque remains unchanged. Thus, these instruments can be used on DC as well as on AC system.

## Deflecting torque

The deflecting torque $T_{\mathrm{d}}$ depends upon the force acting on the soft iron piece. Let
$H=$ field strength produced by the coil;
$m=$ pole strength of the soft iron piece, and $(m \propto H)$

Pulling force acting on the movable iron piece,

$$
F \propto m \times H \text { or } F \propto H^{2}
$$

Now, $H \propto I$

$$
\therefore \quad F \propto I^{2}
$$

As deflecting torque, $T_{\mathrm{d}} \propto F$

$$
\therefore \quad T_{\mathrm{d}} \propto I^{2}
$$

The controlling torque $T_{\mathrm{c}}$ is provided by the spiral spring
$\therefore \quad T_{\mathrm{c}} \propto \theta$ (where $\theta$ is angle of deflection)
In steady position of deflection, $T_{\mathrm{c}}=T_{\mathrm{d}}$
$\therefore \quad \theta \infty I^{2}$ ( $I$ is the rms value of current in AC)
Since deflection $\theta \propto I^{2}$, the scale of such an instrument is non-uniform, being crowded in the beginning. However, by choosing proper dimensions, shape and position of soft iron piece (vane), it is possible to design and construct an instrument with a scale that is very nearly uniform over a considerable part of its length.


Fig. 9.14 Sectional view of attraction type moving iron instrument


Fig. 9.15 Attraction type moving iron instrument with air friction damping

### 9.8.2 Repulsion-type Moving Iron Instruments

## Principle

The basic principle of a repulsion-type moving iron instrument is that the repulsive forces will act between two similarly magnetised iron pieces when placed near to each other.

## Construction

The sectional view of such an instrument is shown in Figure 9.16. It consists of a fixed cylindrical hollow coil that carries the operating current. Inside the coil, there are two soft iron pieces (rods or vanes) placed parallel to each other and along the axis of the coil. One of the rod or vane is fixed and the other is movable connected to the spindle. A pointer is attached to the spindle that gives deflection on the scale. The controlling torque is provided by spring control method, while damping torque is provided by air friction, as shown in Figure 9.16.


Fig. 9.16 (a) Sectional view of repulsion type moving iron instrument (b) Top view of repulsion type moving iron instrument

## Working

When the instrument is connected in the circuit, the operating current flows through the coil. A magnetic field is set up along the axis of the coil. This field magnetises both the iron pieces, and therefore, both the pieces attain similar polarities. A force of repulsion acts between the two; therefore, movable piece moves away from the fixed piece. Thus, the pointer attached to the spindle deflects over the calibrated scale.

If current in the coil is reversed, the direction of magnetic field produced by the coil is reversed. Although the polarity of the magnetised soft iron pieces is reversed but still they are magnetised similarly and repel each other. Hence, the direction of deflecting torque remains unchanged. Thus, these instruments can be used on DC as well as on AC system.

## Deflecting torque

The deflecting torque depends upon the repulsive force acting between the similarly magnetised iron pieces. Let,

$$
\begin{aligned}
H & =\text { field strength produced by the coil; } \\
m_{1} & =\text { pole strength of the fixed iron piece; }\left(m_{1} \propto H\right) \\
m_{2} & =\text { pole strength of the movable iron piece } ;\left(m_{2} \propto H\right)
\end{aligned}
$$

$\therefore$ Repulsive force acting on the movable iron piece,

$$
F \propto m_{1} m_{2} \quad \text { or } \quad F \propto H^{2}
$$

Now,

$$
H \propto I
$$

$\therefore$

$$
F \propto I^{2}
$$

As deflecting torque, $T_{\mathrm{d}} \propto F$

$$
\therefore \quad T_{\mathrm{d}} \propto I^{2}
$$

The controlling torque $T_{\mathrm{c}}$ is provided by the spring

$$
\therefore \quad T_{\mathrm{c}} \propto \theta(\text { where } \theta \text { is angle of deflection })
$$

At steady position of deflection, $T_{\mathrm{c}} \propto T_{\mathrm{d}}$
$\therefore \quad \theta \propto I^{2}$ ( $I$ is the rms value of current in AC)
Since deflection $\theta \propto I^{2}$, and therefore, the scale of such an instrument is non-uniform, being crowded in the beginning. However, by using tongue-shaped iron pieces, scale of such instruments can be made almost uniform.

### 9.8.3 Advantages and Disadvantages of Moving Iron Instruments

## Advantages

1. They are cheap in cost, mechanically robust, and simple in construction.
2. They can be used on both AC and DC.
3. They are reasonably accurate.
4. They possess high operating torque.
5. They can withstand overloads momentarily.

## Disadvantages

1. They cannot be calibrated with a high degree of precision with DC on account of the effect of hysteresis in the iron rods or vanes.
2. They have non-uniform scale crowded at the beginning, and therefore, it is difficult to get accurate readings at this end.
3. They are not very sensitive.
4. Power consumption is quite high.
5. Errors are introduced due to change in frequency in the case of AC measurements.

### 9.8.4 Errors in Moving Iron Instruments

There are two types of errors that occur in moving iron instruments:

## Errors with both DC and AC

The following errors may occur in moving iron instruments when these are used either on DC or AC.

1. Error due to hysteresis: Because of hysteresis in the iron parts of operating system, the readings are higher for descending value but lower for ascending values. The errors due to hysteresis are considerably reduced by using Mumetal or Permalloy that have negligible hysteresis loss.
2. Error due to stray magnetic fields: Since the operating magnetic field of these instruments is comparatively weak, and therefore, stray fields (fields other than the operating magnetic field) affect these instruments considerably. Thus, the stray fields cause serious errors. These errors can be minimised by using an iron case or a thin iron shield over the working parts.
3. Error due to temperature: In moving iron instruments, the change in temperature affects mainly the temperature coefficient of spring. With the change in temperature, stiffness of the spring varies that causes errors. However, for voltmeters both the temperature co-efficient of spring and temperature co-efficient of resistance of voltmeter circuit may balance each other.

However, in the case of shunt connected instruments, it is observed that the uncompensated instruments tend to read low by approximately $0.2 \% /{ }^{\circ} \mathrm{C}$ rise in temperature. Temperature compensation may be affected by connecting a resistor called the swamping resistor in series with the moving coil. The swamp resistor is made of Manganin combined with copper in the ratio of $20: 1$ to $30: 1$, which is hardly affected by temperature variations. The total resistance of the moving coil and swamping resistor increases slightly with a rise in temperature, but only just enough to overcome the effect of springs and magnets, so that the overall temperature effect is zero.

## Errors with AC only

## Error due to change in frequency

The change in frequency produces change in impedance of the coil and change in magnitude of eddy currents. The increase in impedance of the coil with the increase in frequency causes serious errors in the case of voltmeters only. However, this error can be eliminated by connecting a condenser of suitable value in parallel with the swamp resistance ' $r$ ' of the instrument. The impedance of the whole circuit of the instrument becomes independent of frequency if $C=L / r^{2}$, where $C$ is the capacitance of the condenser.

## Ranges

1. Ammeters: From about $0-20 \mathrm{~mA}$ to $0-800$, maximum without current transformer.
2. Voltmeters: From about $0-1 \mathrm{~V}$ to $0-800$ V, maximum without potential transformer.

### 9.8.5 Applications of Moving Iron Instruments

The moving iron instruments are used as ammeters and voltmeters only. These instruments can work on both AC and DC system.

## Ammeter

An instrument that is used to measure electric current in an electric circuit is called an ammeter. An ammeter is connected in series with the circuit or load whose current is to be measured
(Fig. 9.17). The operating coil of the instrument is to carry the whole of the current to be measured or fraction of it. When current flows through the operating coil, the desired deflecting torque is produced. Since an ammeter is connected in series, it should have low resistance to keep the circuit conditions to be the same. Hence, the operating coil of an ammeter should have a few turns of thick wire.

## Voltmeter



Fig. 9.17 Ammeter connected in series


Fig. 9.18 Voltmeter connected in parallel

## Example 9.2

A moving iron instrument gives full-scale deflection with 100 V . It has a coil of 20,000 turns and a resistance of $2,000 \Omega$. If the instrument is to be used as an ammeter to give full-scale deflection at 2 A , calculate the necessary number of turns in the coil.

## Solution:

Full-scale deflection current, $I=\frac{V}{R}=\frac{100}{2000}=0.05 \mathrm{~A}$
In moving iron instruments, the strength of magnetic field (and hence, deflecting torque) depends upon AT of the operating coil.
$\therefore$ Full-scale deflection AT $=N I=20,000 \times 0.05=1,000 \mathrm{AT}$
$\therefore$ Turns required to measure $=2 \mathrm{~A}=\frac{\text { Full-scale deflection AT }}{\text { Current }}=\frac{1000}{2}=500$
Learning outcome: The magnitude of deflecting torque depends upon the magnetic strength provided by the operating coil that further depends upon AT $(N I)$ of the coil (the larger the amount, the smaller is the number of turns for required flux).

## Example 9.3

The coil of a 250 V moving iron voltmeter has a resistance of $500 \Omega$ and an inductance of 1 H . The current taken by the instrument when placed on $250 \mathrm{~V}, \mathrm{DC}$ supply is 0.05 A . Determine the percentage error when the instrument is placed on $250 \mathrm{~V}, 100 \mathrm{~Hz}$ AC supply.

[^10]
## Solution:

Resistance of voltmeter coil, $R=500 \Omega$
Inductance of voltmeter coil, $L=1 \mathrm{H}$
Current taken by the instrument when placed on 250 V DC

$$
I_{\mathrm{dc}}=0.05 \mathrm{~A}
$$

Total ohmic resistance $=\frac{V}{I_{\mathrm{dc}}}=\frac{250}{0.05} \Omega$
$\therefore$ Series swamp resistance, $r=5,000-500=4,500 \Omega$
When the instrument is placed on $250 \mathrm{~V}, 100 \mathrm{~Hz} \mathrm{AC}$
Inductive reactance of the coil, $X_{\mathrm{L}}=2 \pi f L=2 \pi \times 100 \times 1=628.32 \Omega$
Impedance of the coil, $Z=\sqrt{(R+r)^{2}+X_{\mathrm{L}}^{2}}$

$$
=\sqrt{(5,000)^{2}+(628.22)^{2}}=5,039.32 \Omega
$$

Current flowing through the coil, $I_{\mathrm{ac}}=\frac{V}{Z}=\frac{250}{5,039.32}=0.04961 \mathrm{~A}$
Deflection or voltmeter reading with this current $=\frac{250 \times 0.04961}{0.05}=248.05 \mathrm{~V}$
Percentage error $=\frac{248.05-250}{250} \times 100=-0.78 \%$
Learning outcome: If moving iron instruments are calibrated on DC, they will not read accurately on AC if some compensation is not provided. Hence, an error crops in.

## Example 9.4

An AC voltmeter with a maximum scale reading of 50 V has an inductance of 0.09 H and a total resistance of $500 \Omega$. The coil is wound with copper wire having a resistance of $50 \Omega$ and the remainder of the voltmeter circuit consists of nine non-inductive resistances in series with the coil. Find the capacitance that should be placed across the non-inductive resistor to make the instrument read correctly both on DC as well as on AC.

## Solution:

Total resistance, $R_{\mathrm{E}}=500 \Omega$; coil resistance, $R_{\mathrm{C}}=50 \Omega$
swamp resistance, $r=R_{\mathrm{E}}-R_{\mathrm{C}}=500-50=450 \Omega$; inductance, $L=0.09 \mathrm{H}$
Capacitance required to be connected in parallel with the swamp resistance to make the instrument independent of frequency.

$$
C=\frac{L}{r^{2}}=\frac{0.09}{(450)^{2}}=0.444 \mu \mathrm{~F}
$$

Learning outcome: The frequency error in moving iron instruments can be eliminated by providing a suitable capacitor in parallel with the swamp resistance.

### 9.9 PERMANENT MAGNET MOVING COIL INSTRUMENTS

These instruments are very sensitive and accurate. These can be used only on DC as voltmeter and ammeter. The pictorial view of a voltmeter and ammeter is shown in Figures 9.19 and 9.20, respectively. The scale of such instruments is uniform, as shown in the Figure 9.20.


Fig. 9.19 Voltmeter


Fig. 9.20 Ammeter

### 9.9.1 Principle

The basic principle of permanent magnet-type moving coil instrument is that when a cur-rent-carrying conductor is placed in a magnetic field, a mechanical force is exerted on the conductor, as shown in Figure 9.21. The basic principle can also be stated that when a field $F_{\mathrm{r}}$ produced by the movable current-carrying coil tries to come in line with the main field $F_{\mathrm{m}}$, a deflecting torque is developed as shown in Figure 9.22. Due to the production of deflecting torque, the pointer deflects over the scale.


Fig. 9.21 Force exerted on a current carrying conductor placed in magnetic field


Fig. 9.22 Torque development by the alignment of two fields

### 9.9.2 Construction

The simple view of permanent magnet-type moving coil instrument is shown in Figure 9.23. It consists of a powerful permanent shoe magnet, a light rectangular coil of many turns of fine wire wound on a light aluminium former and a cylindrical iron core (a stationary part) inserted in
between the coil sides to reduce reluctance for the magnetic lines of force. The coil is mounted on the spindle and acts as the moving element. Two phosphor bronze spiral hair springs are attached to the spindle. The springs provide the controlling torque as well as they act as incoming and outgoing leads for the current. Eddy current damping is provided by the aluminium former over which the operating coil is wound.


Fig. 9.23 (a) Parts of PMMC Instrument (b) Eddy current damping in PMMC instrument

### 9.9.3 Working

When the instrument is connected in the circuit, a current flows through the operating coil mounted on the spindle. Since the coil is placed in the strong magnetic field of permanent magnets, a force is exerted on the current-carrying conductors of the coil that produces deflecting torque. Thus, the pointer attached to the spindle deflects over the calibrated scale.

If current in the coil is reversed, the direction of deflecting torque will also be reversed because field produced by the permanent magnets does not change. This will give an opposite direction of rotation. Thus, the instrument cannot be used on AC, and it can only be used on DC.

### 9.9.4 Deflecting Torque

The deflecting torque $T_{\mathrm{d}}$ depends upon the force acting on the coil sides.
Let
$B=$ flux density in tesla in the air gap;
$l=$ effective length of each coil side in metre;
$N=$ number of turns of the coil;
$r=$ distance in metre between centre of the coil and force;
$I=$ current flowing through the coil in ampere;
Force acting on each coil side,

$$
F=B I l N \mathrm{~N}
$$

Deflecting torque,

$$
T_{\mathrm{d}}=2 F r=2 B I l N r \mathrm{Nm}
$$

Since all other quantities are constant, except $I$

```
\therefore }\mp@subsup{T}{\textrm{d}}{}\propto
```

The controlling torque is provided by the springs

```
\therefore\quad}\quad\mp@subsup{T}{\textrm{c}}{}\propto0\mathrm{ (where }0\mathrm{ is angle of deflection)
```

At steady position of deflection,

$$
\begin{array}{cc} 
& T_{\mathrm{c}}=T_{\mathrm{d}} ; \\
\therefore & \theta \propto I
\end{array}
$$

Since deflection $\theta$ is proportional to the operating current flowing through the coil, and therefore, the scale of such instruments is uniform.

### 9.9.5 Advantages and Disadvantages of Permanent Magnet Moving Coil Instruments

## Advantages

1. These instruments have uniform scale.
2. Very effective; the reliable eddy current damping is provided.
3. No hysteresis loss, as the former is of aluminium.
4. Low power consumption because driving power is small.
5. No effect of stray magnetic field, as working field provided by the permanent magnets is very strong.
6. High torque or weight ratio, and therefore, such instruments require small operating current and are very sensitive.
7. These instruments are very accurate and reliable.

## Disadvantages

1. These instruments cannot be used for AC measurements.
2. These are costlier in comparison to moving iron instruments.
3. Friction and temperature might introduce some error.
4. Some errors are also caused due to ageing of control springs and permanent magnets.

### 9.9.6 Errors in Permanent Magnet Moving Coil Instruments

The main sources of errors in these instruments are due to the following:

1. Weakening of stiffness of springs due to ageing and rise in temperature.
2. Weakening of field produced by permanent magnet due to ageing and temperature effects.
3. Change in resistance of the moving coil with temperature.

However, these errors are negligibly small and as such these instruments are considered to be most accurate for measuring currents and voltages in DC circuits.

### 9.9.7 Range

## DC Ammeters

1. Without shunt (i.e., instrument alone) $0-5 \mu \mathrm{~A}$ to $050 \mu \mathrm{~A}$
2. Without internal shunts up to $0-200 \mathrm{~A}$
3. With external shunts, up to $0-5000 \mathrm{~A}$

## DC Voltmeters

1. Without series resistance or multiplier (i.e., instrument alone) $0-50 \mathrm{mV}$
2. With series resistance, up to $0-3,000 \mathrm{~V}$

## Example 9.5

In a moving coil instrument, the moving coil consists of 300 turns wound on a former of dimension $3 \mathrm{~cm} \times 2 \mathrm{~cm}$. The flux density in the air gap is 0.05 T . Determine the turning moment on the coil when carrying a current of 10 mA .

## Solution:

Deflecting force, $F=B I l N$
where

$$
N=300 ; l=0.03 \mathrm{~m} ; B=0.05 \mathrm{~T}
$$

$$
I=10 \times 10^{-3} \mathrm{~A} ; r=2 / 2=1 \mathrm{~cm}=0.01 \mathrm{~m}
$$

$$
\therefore \quad F=0.05 \times 10 \times 10^{-3} \times 0.03 \times 300=4.5 \times 10^{-3} \mathrm{~N}
$$

Turning moment on the coil, $T_{\mathrm{d}}=2 F \times r=2 \times 4.5 \times 10^{-3} \times 0.01$

$$
=9 \times 10^{-5} \mathrm{Nm}
$$

## Example 9.6

In a moving coil instrument, the moving coil consists of 100 turns wound on a square former of length 3 cm . The flux density in the air gap is 0.06 T . Calculate the turning moment acting on the coil when carrying a current of 12 mA .
(U.P.T.U.)

## Solution:

Number of turns on the coil, $N=100$
Length of former, $l=3 \mathrm{~cm}=0.03 \mathrm{~m}$
Width of former, $W=3 \mathrm{~cm}=0.03 \mathrm{~m}$
Radius for the turning, $r=\frac{W}{2}=\frac{0.03}{2}$
Field strength, $B=0.06 \mathrm{~T}$
Current flowing through the coil, $i=12 \mathrm{~mA}=0.012 \mathrm{~A}$
Turning moment or deflecting torque, $T_{\mathrm{d}}=2 \mathrm{Bil} N \times r$

$$
\begin{aligned}
& =2 \times 0.06 \times 0.012 \times 0.03 \times 100 \times 0.015 \\
& =6.48 \times 10^{-5} \mathrm{Nm}
\end{aligned}
$$

## Example 9.7

A moving coil comprises 100 turns of insulate copper wire wound on a former of length 3 cm and breadth 4 cm . The resistance of the coil is $2,000 \Omega$. The field strength of the magnets is 0.06 T. The torque exerted by the control spring is $0.02 \times 10^{-4} \mathrm{kgm} /$ degree. Estimate the deflection of the instrument when a voltage of 120 V is applied across it.

## Solution:

Number of turns of the coil, $N=100$
Mean length of the coil, $l=3 \mathrm{~cm}=0.03 \mathrm{~m}$
Radius of the coil, $r=\frac{4}{2} 2 \mathrm{~cm}=0.02 \mathrm{~m}$
Flux density, $B=0.06 \mathrm{~T}$
Current through the coil, $l=\frac{V}{R}=\frac{120}{2000} 0.06 \mathrm{~A}$
Deflecting force, $F=B I l N=0.06 \times 0.06 \times 0.03 \times 100=0.0108 \mathrm{~N}$
Deflecting torque, $T_{\mathrm{d}}=2 \mathrm{Fr}=2 \times 0.0108 \times 0.02=4.32 \times 10^{-4} \mathrm{Nm}$
Controlling torque $/$ degree $=0.02 \times 10^{-4} \mathrm{kgm}=0.02 \times 10^{-4} \times 9.81 \mathrm{Nm}$

$$
T_{\mathrm{c}} / \theta=0.1962 \times 10^{-4} \mathrm{Nm} / \text { degree }
$$

Now,

$$
=\frac{T_{\mathrm{c}}}{\theta} \times \theta=T d \quad \text { or } \quad \theta=\frac{T_{\mathrm{d}}}{T_{\mathrm{c}} / \theta}
$$

$\therefore$ Deflection,

$$
\theta=\frac{4.32 \times 10^{-4}}{0.1962 \times 10^{-4}} 22 \text { degree }
$$

## Example 9.8

A moving coil instrument has the following data:
number of turns $=100$; width of coil $=20 \mathrm{~mm}$; depth of coil $=30 \mathrm{~mm}$;
flux density in the gap $=0.1 \mathrm{~Wb} / \mathrm{m}^{2}$
Calculate the deflecting torque when carrying a current of 10 mA . Further, calculate the deflection if the control spring constant is $2 \times 10^{-6} \mathrm{Nm} /$ degree.

## Solution:

Here, $N=100 ; l=30 \mathrm{~mm}=0.03 \mathrm{~m} ; r=\frac{20}{2}=10 \mathrm{~mm}=0.01 \mathrm{~m}$

$$
B=0.1 \mathrm{~Wb} / \mathrm{m}^{2} ; I=10 \mathrm{~mA}=10 \times 10^{-3} \mathrm{~A}
$$

$$
T_{\mathrm{c}} / \theta=2 \times 10^{-6} \mathrm{Nm} / \text { degree }
$$

Now,

$$
\begin{aligned}
F & =B I l N=0.1 \times 10 \times 10^{-3} \times 0.03 \times 100=3 \times 10^{-3} \mathrm{~N} \\
T_{\mathrm{d}} & =2 F \times r=2 \times 3 \times 10^{-3} \times 0.01=0.06 \times 10^{-3} \mathrm{Nm}
\end{aligned}
$$

$$
\frac{T_{\mathrm{c}}}{\theta} \times \theta=T_{\mathrm{d}}
$$

$\therefore \quad$ Deflection, $\theta=\frac{T_{\mathrm{d}}}{T_{\mathrm{c}} / \theta}=\frac{0.06 \times 10^{-3}}{2 \times 10^{-6}}=30^{\circ}$.

## Example 9.9

A moving coil millivoltmeter has a resistance of $200 \Omega$ and the full-scale deflection is reached when a potential difference of 100 mV is applied across the terminals. The moving coil has effective dimensions of $30 \mathrm{~mm} \times 25 \mathrm{~mm}$ and is wound with 100 turns. The flux density of the gap is $0.2 \mathrm{~Wb} / \mathrm{m}^{2}$. Determine the control constant of the spring, if the final deflection is $100^{\circ}$.
(U.P.T.U.)

## Solution:

Here, $R_{\mathrm{m}}=200 \Omega ; V=100 \mathrm{mV} ; l=30 \mathrm{~mm}=30 \times 10^{-3} \mathrm{~m}$;

$$
r=\frac{W}{2}=\frac{25}{2} \times 10^{-3}=12.5 \times 10^{-3} \mathrm{~m}
$$

$$
N=100 ; B=0.2 \mathrm{~Wb} / \mathrm{m}^{2} ; \theta=100^{\circ}
$$

$$
\text { Full-scale deflection current, } i=\frac{\text { Full-scale potential difference }}{\text { Voltmeter resistance }}=\frac{100 \mathrm{mV}}{200 \Omega}
$$

$$
=0.5 \mathrm{~mA}=0.5 \times 10^{-3} \mathrm{~A}
$$

Deflecting torque, $T_{\mathrm{d}}=2 \operatorname{Bilr} N=2 \times 0.2 \times 0.5 \times 10^{-3} \times 30 \times 10^{-3} \times 12.5 \times 10^{-3} \times 100$

$$
=75 \times 10^{-7} \mathrm{Nm}
$$

Full-scale deflection, $\theta=100^{\circ}$
Control constant of spring $=\frac{T_{d}}{\theta}=\frac{75 \times 10^{-7}}{100}=0.75 \times 10^{-7} \mathrm{Nm} /$ degree.

### 9.10 DIFFERENCE BETWEEN AMMETER AND VOLTMETER

There is no fundamental difference in the operating principles of ammeters and voltmeters. Both are current operated devices (except electrostatic-type voltmeters), that is, deflecting torque is produced when current flows through their operating coils. In an ammeter, the deflecting torque is produced by current to be measured or by a definite fraction of it, whereas in a voltmeter, torque is produced by the current proportional to the voltage to be measured. Thus, the real difference between the two instruments is in the magnitude of the current producing the deflecting torque. The essential requirements of a measuring instrument are that its introduction into the circuit, where measurements are to be made, does not alter the circuit conditions and the power consumed by them for their operation should be very small.

An ammeter is connected in series with the circuit whose current is to be measured. Therefore, it should have a low resistance. On the other hand, a voltmeter is connected in parallel with the circuit whose voltage is to be measured, and therefore, it must have high resistance. Thus, we conclude that the difference is only in the resistance of the instrument; in fact, an ammeter can be converted into voltmeter by connecting a high resistance in series with it. Similarly, a voltmeter can be converted into an ammeter by connecting a shunt across the voltmeter.

## Example 9.10

A meter has a full-scale angle of $90^{\circ}$ at a current of 1 A . This meter has perfect square-law response. What is the current when the deflection angle is $45^{\circ}$ ? Draw the conclusion. (U.P.T.U.)

## Solution:

Here, $I_{1}=1 \mathrm{~A} ; \theta_{1}=90^{\circ}, \theta_{2}=45^{\circ}$
Since,

$$
\begin{gathered}
\theta \alpha I^{2} \\
=\frac{\theta_{2}}{\theta_{1}}=\frac{I_{2}^{2}}{I_{1}^{2}} \\
I_{2}=I_{1} \sqrt{\frac{\theta_{2}}{\theta_{1}}}=1 \times \sqrt{\frac{45^{\circ}}{90^{\circ}}}=0.707 \mathrm{~A} .
\end{gathered}
$$

$$
\therefore \quad=\frac{\theta_{2}}{\theta_{1}}=\frac{I_{2}^{2}}{I_{1}^{2}}
$$

or

Learning outcome: The result shows that when deflection is reduced to half, the current flowing through the meter is not reduced to half, it is $70.7 \%$ of the previous value. In meters that follows the square law, the scale is not uniform.

## Example 9.11

Two voltmeters one with a full-scale reading of 100 V and another with a full-scale reading of 200 V are connected in series across a 100 V supply. The internal resistance of both meters is the same. What are the readings?
(U.P.T.U.)

## Solution:

Two voltmeters connected in series are shown in Figure 9.24.
Let the resistance of each voltmeter be $R \Omega$
Total resistance,

$$
R_{\mathrm{t}}=R+R=2 R
$$

When 100 V is applied across the series combination,
Current,

$$
I=\frac{100}{2 R}
$$

Potential difference across voltmeter $V_{1}$ or the reading of voltmeter $V_{1}$,


Fig. 9.24 Circuit diagram

Potential difference across voltmeter $V_{2}$ or the reading of voltmeter $V_{2}$,

$$
=I \times R=\frac{100}{2 R}=50 \mathrm{~V} .
$$

## Example 9.12

Two voltmeters have the range $0-400 \mathrm{~V}$. The internal impedances are $30,000 \Omega$ and $20,000 \Omega$. If they are connected in series and 600 V be applied across them, what will be their readings?
(U.P.T.U.)

## Solution:

Total resistance when the two voltmeters are connected in series, as shown in Figure 9.25.

$$
R_{\mathrm{t}}=30,000+20,000=50,000 \Omega
$$

When 600 V is applied across the series combination,


Fig. 9.25 Two voltmeters connected in series

$$
\text { Current, } I=\frac{600}{50,000}=1.2 \times 10^{-2} \mathrm{~A}
$$

Potential difference across voltmeter $V_{1}$ or the reading of voltmeter $V_{1}$,

$$
=I R_{1}=1.2 \times 10^{-2} \times 30,000=360 \mathrm{~V} .
$$

Potential difference across voltmeter $V_{2}$ or the reading of voltmeter $V_{2}$,

$$
=I R_{2}=1.2 \times 10^{-2} \times 20,000=240 \mathrm{~V} .
$$

## Example 9.13

Two ammeters, one with a current scale of 10 A and resistance of $0.01 \Omega$ and the other with a current scale of 15 A and resistance of $0.005 \Omega$ are connected in parallel. What can be the maximum current carried by this parallel combination so that no meter reading goes out of the scale?
(U.P.T.U. Dec. 2003)


Fig. 9.26 Two ammeters connected in parallel

## Solution:

Two ammeters connected in parallel are shown in Figure 9.26.
Let the current carried by the parallel combination be $I$ ampere.
Maximum current carried by ammeter $A_{1}$, that is, $I_{1}=10 \mathrm{~A}$.
Voltage across the combination, say $V_{1}=I_{1} R_{\mathrm{m}_{1}}=10 \times 0.01=0.1 \mathrm{~V}$.
Then, current carried by the ammeter $A_{2}$, that is, $I_{2}=\frac{V_{1}}{R_{\mathrm{m}_{2}}}=\frac{0.1}{0.005}=20 \mathrm{~A}$
This current is more than the current that can be carried by the second ammeter, and hence, the proposal is wrong. Now, if the current carried by the second ammeter $A_{2}$, that is, $I_{2}{ }^{\prime}=15 \mathrm{~A}$ Then, voltage across the combination, say $V_{2}=I^{\prime}{ }_{2} R_{\mathrm{m}_{2}}=15 \times 0.05=0.075 \mathrm{~V}$

Current carried by the ammeter $A_{1}, I_{1}^{\prime}=\frac{V_{2}}{R_{\mathrm{m} 1}}=\frac{0.075}{0.01}=7.5 \mathrm{~A}$.
This current is less than the current that can be carried by the first ammeter $A_{1}$, and hence, the proposal is correct. Total maximum value of current that can be carried by the combination,

$$
I=I_{1}^{1}+I_{2}^{1}=15+7.5=22.5 \mathrm{~A} .
$$

### 9.11 EXTENSION OF RANGE OF AMMETERS AND VOLTMETERS

The moving coil instruments can carry maximum current of about 50 mA safely and the potential drop across the moving coil is about 50 mV . However, in practice, heavy currents and voltages are required to be measured. Therefore, it becomes necessary that the current and voltage being measured be reduced and brought within the range of instrument.

There are four common devices used for extending the range of the instruments, namely shunts, multipliers, current transformers, and potential transformers. The extension of range of current and potential transformers is employed for measurement of very high AC currents and voltages in power system.

1. Shunts: These are used for the extension of range of ammeters. Shunt is a resistance of small value, just like a strip, as shown in Figure 9.27, having minimum temperature co-efficient. It is always connected in parallel with the ammeter whose range is to be extended. The combination is connected in series with the circuit whose current is to be measured.
2. Multipliers: These are used for the extension of range of voltmeters. Multiplier is a non-inductive high resistance usually carbon resistors, as shown in Figure 9.28, connected in series with the instrument whose range is to be extended. The combination is connected across the circuit


Fig. 9.27 Shunts


Fig. 9.28 Carbon resistor whose voltage is to be measured.

### 9.11.1 Extension of Ammeter Range

The current range of a DC moving coil ammeter is extended by connecting a shunt $R_{\mathrm{S}}$ (low resistance) across the coil, circuit is shown in Figure 9.29.

Let $\quad I=$ current to be measured.
$I_{\mathrm{m}}=$ full-scale deflection current of ammeter;
$I_{\mathrm{s}}=$ shunt current;
$R_{\mathrm{m}}=$ resistance of ammeter;
$R_{\mathrm{s}}=$ shunt resistance,
From Figure 9.29, we have, $I=I_{\mathrm{m}}+I_{\mathrm{s}}$ or

$$
\begin{align*}
& \text { or } \\
& \text { or } \\
& I R_{\mathrm{s}}=I-I_{\mathrm{m}} \text { and } I_{\mathrm{s}} R_{\mathrm{s}}=I_{\mathrm{m}} R_{\mathrm{m}} \text { or }\left(I-I_{\mathrm{s}}\right)  \tag{i}\\
& \text { or }
\end{align*}
$$

or


Fig. 9.29 Circuit for extension of ammeter range

The ratio of the total current $I$ to be measured to the full-scale deflection current $I_{\mathrm{m}}$ is known as the multiplying power of the shunt or instrument constant. It may be denoted by $N$.

From Equation (i), we get $\quad N=1+\frac{R_{\mathrm{m}}}{R_{\mathrm{s}}} \quad$ or $\quad R_{\mathrm{s}}=\frac{R_{\mathrm{m}}}{N-1}$
Hence, for measurement of current $N$ times, the current range of instrument, the shunt resistance should be $\frac{1}{N-1}$ times the meter resistance.

## Example 9.14

A moving coil ammeter has a full-scale deflection of $50 \mu \mathrm{~A}$ and a coil resistance of $1,000 \Omega$.
What will be the value of the shunt resistance required for the instrument to be converted to read a full-scale reading of 1 A ?
(U.P.T.U.)

## Solution:

Full-scale deflection current, $I_{\mathrm{m}}=50 \times 10^{-6} \mathrm{~A}$
Instrument resistance, $R_{\mathrm{m}}=1000 \Omega$
Total current to be measured, $I=1 \mathrm{~A}$
Value of shunt resistance required, $R_{\mathrm{s}}=\frac{R_{\mathrm{m}}}{\frac{I}{I_{\mathrm{m}}}-1}=\frac{1,000}{\frac{1}{50 \times 10^{-6}}-1}=0.0500025 \Omega$.

## Example 9.15

The full-scale deflection current of an ammeter is 1 mA and its internal resistance is $100 \Omega$. If this meter is to have full-scale deflection at 5 A , what is the value of the shunt resistance to be used?
(U.P.T.U.)

## Solution:

Full-scale deflection current, $I_{\mathrm{m}}=1 \mathrm{~mA}=0.001 \mathrm{~A}$
Instrument resistance, $R_{\mathrm{m}}=100 \Omega$
Total current to be measured, $I=5 \mathrm{~A}$
Shunt current, $I_{\mathrm{s}}=I-I_{\mathrm{m}}=5-0.001=4.999 \mathrm{~A}$
Value of shunt resistance, $R_{\mathrm{s}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{I_{\mathrm{s}}}=\frac{0.001 \times 100}{4.999}=0.020004 \Omega$

## Example 9.16

Design a multi-range ammeter with ranges of $1 \mathrm{~A}, 5 \mathrm{~A}, 25 \mathrm{~A}$, and 125 A employing individual shunts in each case. A D'Arsonal movement with coil internal resistance of $730 \Omega$ and a fullscale current of 5 mA is available.

## Solution:

Meter resistance, $R_{\mathrm{m}}=730 \Omega$
Meter current, $I_{\mathrm{m}}=5 \mathrm{~mA}=0.005 \mathrm{~A}$
Shunt current, $I_{\mathrm{s}}=I-I_{\mathrm{m}}$

Shunt resistance, $R_{\mathrm{s}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{I_{\mathrm{s}}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{\left(I-I_{\mathrm{m}}\right)}$
For 1 A range, $R_{\mathrm{s} 1}=\frac{5 \times 10^{-3} \times 730}{(1-0.005)}=3.668 \Omega$
For 5 A range, $R_{\mathrm{s} 2}=\frac{5 \times 10^{-3} \times 730}{(5-0.005)}=0.7307 \Omega$
For 25 A range, $R_{\mathrm{s} 3}=\frac{5 \times 10^{-3} \times 730}{(25-0.005)}=0.146 \Omega$
For 125 A range, $R_{\mathrm{s} 4}=\frac{5 \times 10^{-3} \times 730}{(125-0.005)}=0.0292 \Omega$

### 9.11.2 Extension of Voltmeter Range

The voltage range of a DC moving coil voltmeter (instrument) is extended by connecting a multiplier $R$ (high resistance) in series with the coil. The circuit is shown in Figure 9.30.
Let $V=$ voltage to be measured;
$v=$ voltage across the meter;
$I_{\mathrm{m}}=$ full-scale deflection current of voltmeter;
$R=$ resistance in series with the coil to extend the range;
$R_{\mathrm{m}}=$ voltmeter resistance.
From the circuit shown in Figure 9.30, we have $V=I_{\mathrm{m}}\left(R+R_{\mathrm{m}}\right)$
or

$$
R+R_{\mathrm{m}}=\frac{V}{I_{\mathrm{m}}} \quad \text { or } \quad R=\frac{V}{I_{\mathrm{m}}}-R_{\mathrm{m}}
$$

Required resistance


Fig. 9.30 Circuit for extension of voltmeter range

$$
R=\frac{\text { Maximum voltage to be measured }}{\text { Full-scale deflection current of voltmeter }}-\text { Voltmeter resistance }
$$

From the circuit shown in Figure 9.30, we also get

$$
R=\frac{V-v}{I_{\mathrm{m}}} \quad \text { or } \quad R=\frac{v\left(\frac{V}{v}-1\right)}{I_{\mathrm{m}}}
$$

The ratio of voltage to be measured to the voltage across the voltmeter for which it is actually designed (i.e., $V / v$ ) is known as multiplying factor $(m)$.

$$
\therefore \quad R=\frac{v(m-1)}{I_{\mathrm{m}}} \text { or } R=(m-1) R_{\mathrm{m}}
$$

Hence, for the measurement of voltage, $m$ times the voltage range of the instrument the series multiplying resistance $R$ should be $(m-1)$ times the meter resistance $R_{\mathrm{m}}$.

Note: The ammeters or voltmeters used in the laboratories, etc., have shunt (low resistance) or multipliers (high resistance) incorporated in them, respectively. The scales are graduated and marked after taking into consideration the multiplying factor or instrument constant so that the circuit current or voltage be read directly from the scale.

The following are the disadvantages of using shunts and multipliers for measuring high currents and voltages:

1. In the case of AC measurements, inductance of the coil is also involved due to which accurate calibrations cannot be done.
2. It is dangerous to measure high voltages because the operator may come in contact with live leads accidently.
3. Shunts and multipliers do not give a straight line relation between the instrument current and the current in the shunts and multipliers in AC measurements.

## Example 9.17

The full-scale deflection current of a meter is 1 mA and its internal resistance is $100 \Omega$. If this meter is to have full-scale deflection when 100 V is measured. What should be the value of series resistor?
(U.P.T.U.)

## Solution:

Instrument resistance, $R_{\mathrm{m}}=100 \Omega$
Full-scale deflection current, $I_{\mathrm{m}}=1 \mathrm{~mA}=1 \times 10^{-3} \mathrm{~A}$
Voltage to be measured, $V=100 \mathrm{~V}$
Value of series resistor, $R=\frac{V}{I_{\mathrm{m}}}-R_{\mathrm{m}}=\frac{100}{1 \times 10^{-3}}-100=99,900 \Omega$

## Example 9.18

The electrical lab of a polytechnic has a $0-200 \mathrm{~V}$ range voltmeter with $20,000 \Omega / \mathrm{V}$ sensitivity. What modifications needed to extend the range of meter to $0-2,000 \mathrm{~V}$ ? What should be the power rating of resistor used?

## Solution:

Full-scale deflection voltage, $v=200 \mathrm{~V}$
Resistance of meter, $R_{\mathrm{m}}=20,000 \times 200=4 \times 10^{6} \Omega$
Meter current, $I_{\mathrm{m}}=\frac{v}{R_{\mathrm{m}}}=\frac{200}{4 \times 10^{6}}=50 \times 10^{-6} \mathrm{~A}$
Voltage to be measured, $V=2,000 \mathrm{~V}$
Voltage across series resistor $=2,000-200=1,800 \mathrm{~V}$
Resistance of the multiplier $=\frac{1800}{50 \times 10^{-6}}=36 \times 10^{6} \Omega$
Power rating of the resistor $=I_{\mathrm{m}}{ }^{2} R=\left(50 \times 10^{-6}\right)^{2} \times 36 \times 10^{6}=0.09 \mathrm{~W}$.

## Example 9.19

A moving coil instrument has a resistance of $5 \Omega$ and gives a full-scale deflection of 100 mV . Show how the instrument may be used to measure:
(i) voltage up to 50 V (ii) currents up to 10 A .
(U.P.T.U. 2006-07)

## Solution:

Instrument's resistance, $R_{\mathrm{m}}=5 \Omega$
Full-scale deflection voltage, $v=100 \mathrm{mV}$
(i) Let $R$ be the resistance to be connected in series for increasing the range of instrument as a voltmeter to 50 V .
Full-scale deflecting current, $I_{\mathrm{m}}=\frac{100 \times 10^{-3}}{5}=20 \mathrm{~mA}$
Voltage to be measured, $\quad V=I_{\mathrm{m}}\left(R_{\mathrm{m}}+R\right)$
or

$$
R=\frac{V}{I_{\mathrm{m}}}-R_{\mathrm{m}}=\frac{50}{20 \times 10^{-3}}-5=2,500-5=2,495 \Omega
$$

(ii) Let $R_{\mathrm{S}}$ be the resistance of the shunt to be connected in parallel to increase the range of instrument as ammeter to measure 10 A .
Full-scale deflection current, $I_{\mathrm{m}}=\frac{100 \times 10^{-3}}{5}=20 \times 10^{-3} \mathrm{~A}=0.02 \mathrm{~A}$
Shunt current, $I_{\mathrm{S}}=I-I_{\mathrm{m}}=10-0.02=9.98 \mathrm{~A}$
Now, $I_{\mathrm{s}} R_{\mathrm{s}}=I_{\mathrm{m}} R_{\mathrm{m}} \quad$ or $\quad R_{\mathrm{s}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{I_{\mathrm{s}}}=\frac{0.02 \times 5}{9.98}=0.01002 \Omega$.

## Example 9.20

A moving coil milliammeter gives full-scale deflection with 15 mA and has a resistance of $5 \Omega$. Calculate the resistance to be connected in (i) parallel to enable the instrument to read up to 1A and (ii) series to enable it to read up to 10 V .
(U.P.T.U. July 2002)

## Solution:

Here, $R_{\mathrm{m}}=5 \Omega$
Full-scale deflection current, $I_{\mathrm{m}}=15 \mathrm{~mA}=15 \times 10^{-3} \mathrm{~A}$
(i) Current to be measured, $I=1 \mathrm{~A}$

Multiplying power of shunt, $N=\frac{I}{I_{\mathrm{m}}}=\frac{I}{15 \times 10^{-3}}=66.667$
Let $R_{\mathrm{s}}$ be the resistance connected in parallel to enable the instrument to read up to 1 A

$$
R_{\mathrm{s}}=\frac{R_{\mathrm{m}}}{N-1}=\frac{5}{66.667-1}=0.07614 \Omega
$$

(ii) Voltage to be measured, $V=10 \mathrm{~V}$

Let $R$ be the resistance required to be connected in series for full-scale deflection with 10 V ,

$$
R=\frac{V}{I_{\mathrm{m}}}-R_{\mathrm{m}}=\frac{10}{15 \times 10^{-3}}-5=661.67 \Omega
$$

## Example 9.21

A moving coil instrument having internal resistance of $50 \Omega$ indicates full-scale deflection with a current of 10 mA . How can it be made to work as (i) a voltmeter to read 100 V on full-scale and (ii) an ammeter of 1 A on full-scale?
(U.P.T.U. June 2001)

## Solution:

Instrument resistance, $R_{\mathrm{m}}=50 \Omega$
Full-scale deflection current, $I_{\mathrm{m}}=10 \mathrm{~m} \mathrm{~A}=0.01 \mathrm{~A}$
(i) Let $R$ be the resistance required to be connected in series with the instrument to enable it to read 100 V on full-scale. Then,

$$
R=\frac{V}{I_{\mathrm{m}}}-R_{\mathrm{m}}=\frac{100}{0.01}-50=9,950 \Omega
$$

(ii) Let $R_{\mathrm{s}}$ be the resistance required to be connected across the instrument to enable it to read 1 A on full-scale,

$$
R_{\mathrm{s}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{I_{\mathrm{s}}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{I-I_{\mathrm{m}}}=\frac{0.01 \times 50}{1-0.01}=\frac{0.5}{0.99}=0.50505 \Omega
$$

## Example 9.22

A moving coil instrument has a resistance of $2 \Omega$ and it reads up to 250 V when a resistance of $5,000 \Omega$ is connected in series with it. Find the current range of the instrument when it is used as an ammeter with the coil connected across a shunt resistance of $2 \mathrm{~m} \Omega$.
(U.P.T.U. Feb. 2002)

## Solution:

Here, $R_{\mathrm{m}}=2 \Omega$
Current flowing through the instrument for full-scale deflection,

$$
\begin{aligned}
I_{\mathrm{m}} & =\frac{\text { Full-scale reading in volt }}{R_{\mathrm{m}}+\text { Series resistance }} \\
& =\frac{250}{2+5,000}=0.04998 \mathrm{~A} \text { or } 49.98 \mathrm{~m} \mathrm{~A}
\end{aligned}
$$

Shunt resistance, $R_{\mathrm{s}}=2 \times 10^{-3} \Omega$
Current through shunt resistance, $I_{\mathrm{s}}=\frac{I_{\mathrm{m}} R_{\mathrm{m}}}{R_{\mathrm{s}}}=\frac{49.98 \times 10^{-3} \times 2}{2 \times 10^{-3}}=49.98 \mathrm{~A}$
Current range of instrument $=$ Full-scale deflection current

$$
\begin{aligned}
& =I_{\mathrm{m}}+I_{\mathrm{s}}=0.04998+49.98 \\
& =50.02998 \mathrm{~A}=\text { say } 50 \mathrm{~A} .
\end{aligned}
$$

##  <br> PRACTICE EXERCISES

## Short Answer Questions

1. Why electrical instruments are widely used to measure electrical and non-electrical quantities?
2. What do you mean by moving iron instruments? How are they classified?
3. How deflecting, controlling, and damping torque is produced in attraction-type moving iron instruments?
4. How deflecting, controlling, and damping torque is produced in repulsion-type moving iron instruments?
5. Mention some important advantages of moving iron instruments.
6. Mention some important disadvantages of moving iron instruments.
7. What are the causes due to which errors occur in moving iron instruments?
8. What are moving coil instruments? How are these classified?
9. How deflecting, controlling, and damping torques are produced in PMMC instruments?
10. Mention some major advantages of PMMC instruments.
11. Mention some major disadvantages of PMMC instruments.
12. What are the major causes of errors in PMMC instruments?
13. Is there any difference between an ammeter and a voltmeter?
14. What should be done to extend the range of an ammeter? Mention some important mathematical relations?
15. What should be done to extend the range of a voltmeter? Mention some important mathematical relations.

## Test Questions

1. Give construction, principle, and working of an instrument that can measure both AC and DC .

Or
Explain the principle, construction, and working of moving iron instruments.
2. Describe with neat sketches the principle, construction, and working of moving iron repulsion-type instruments.
3. Why the scale of moving iron instruments is not uniform?
4. Describe the working of a moving iron instrument with a neat sketch. Describe the errors to which such an instrument is subjected to. Explain how these errors can be minimised.
5. Explain the concept of ammeter and voltmeter and difference between them.
6. Explain the construction, working principle, and uses of moving coil instrument.

Or
Explain construction and principle of operation of a moving coil-type ammeter.
7. Give merits and demerits of moving coil and moving iron instruments.
8. Give merits, demerits, sources of error, and applications of moving coil instruments.
9. Are there any instruments that can be used only for DC? Explain the working for measuring current and voltage.
10. Describe the construction and working of PMMC instrument. Describe the method of damping used in these instruments.
11. Differentiate between moving iron ammeter and permanent magnet moving coil ammeter.
12. How does current range of a PMMC instrument extended with the help of shunts? Describe the method of reducing errors due to temperature changes in shunt connected instruments.
13. A millimetre with a resistance of $10 \Omega$ is connected with a shunt of $0.005 \Omega$. What will be current flowing through the instrument if it is connected to a circuit in which a current of 10 A is flowing?
14. Describe methods of extension of range of an ammeter and a voltmeter.
15. Write short note on shunt and multipliers.
16. Explain the disadvantages of shunts and multipliers when used for extension of range of $A C$ instruments.

## Numericals

1. A moving iron instrument gives full-scale deflection with 250 V . It has a coil of 25,000 turns and a resistance of $2,500 \Omega$. If the instrument is to be used as an ammeter to give full-scale deflection of 5 A , calculate the necessary number of turns in the coil.
(Ans. 500)
2. A 10 V moving iron ammeter has a full-scale deflection of 40 mA on DC circuit. It reads $0.8 \%$ low on 50 Hz AC. Hence, calculate the inductance of the ammeter.
(Ans. 101.27 mH )
3. A 15 V moving iron voltmeter has a resistance of $500 \Omega$ and inductance of 0.12 H . Assuming that this instrument reads correctly on DC , what will be its reading on AC at 15 V when the frequency is
(i) 25 Hz and
(ii) 100 Hz ?
(A.M.I.E. Nov. 1994) (Ans. $14.99 \mathrm{~V}, 14.83 \mathrm{~V}$ )
4. The coil of 150 V moving iron voltmeter has a resistance of $500 \Omega$ and an inductance of 0.75 H . The current taken by the instrument when placed on a 150 V DC supply is 0.05 A . Estimate (i) the alteration of the reading between DC and AC at 100 Hz and (ii) the capacitance of the capacitor necessary to eliminate this frequency error.
(Ans. $-1.2 \%, 0.12 \mu \mathrm{~F}$ )
5. In a moving coil instrument, the moving coil consists of 300 turns wound on a former of dimensions $3 \mathrm{~cm} \times 2 \mathrm{~cm}$. The flux density in the air gap is 0.08 T . Determine the turning moment on the coil when carrying a current of 15 mA .
(July, 1992) (Ans. $2.16 \times 10^{-4} \mathrm{Nm}$ )
6. In a moving coil instrument, the moving coil consists of 400 turns wound on a square former of length 2 cm . The flux density in the air gap is 400 lines $/ \mathrm{cm}^{2}$. Calculate moment acting on the coil when carrying a current at 8 mA .
(Ans. $512 \times 10^{-7} \mathrm{Nm}$ )
7. In a moving coil instrument, the moving coil consists of 100 turns wound on a square former of length 3 cm . The flux density in the air gap is 600 lines $/ \mathrm{cm}^{2}$. Calculate the turning moment acting on the coil when carrying a current of 12 mA .
(Ans. $6.48 \times 10^{-5} \mathrm{Nm}$ )
8. A moving iron instrument has a full-scale angle of $120^{\circ}$ at a current of 5 A . This meter has perfect square-law response. What will be the deflection angle when the current passing through the meter is 2.5 A ? Draw the conclusion.
(Ans. 30 ${ }^{\circ}$ )
9. Two ammeters one with full-scale current of 1 mA and internal resistance of $100 \Omega$, and the other with a full-scale current of 10 mA and internal resistance of $25 \Omega$ are in parallel. What is the total current these two meters can carry without any meter reading getting out of scale?
(U.P.T.U.)
(Ans. 5 mA )
10. A moving coil ammeter can read up to 1 A has a resistance of $0.02 \Omega$. How could this instrument be adopted to read current up to 100 A ?
(Ans. $0.000202 \Omega$ )
11. A milliammeter with a resistance of $5 \Omega$ is connected with a shunt of $0.005 \Omega$. What will be the current flowing through the instrument, if it is connected to a circuit in which a current of 10 A is flowing?
(Ans. 9.99 mA )
12. A moving coil meter has resistance of $10 \Omega$ and gives full-scale deflection when carrying a current of 50 mA . Show how it can be adopted to measure a current of 100 A .
(Feb. 1994) (Ans. A shunt of $5 \mathrm{~m} \Omega$ has to be connected in parallel with the meter)
13. A milliammeter with a resistance of $5 \Omega$ is connected with a shunt of $0.01 \Omega$. What will be the current flowing through the instrument, if it is connected to a circuit in which a current of 5 A is flowing?
(Ans. 9.98 mA )
14. A moving coil instrument gives a full-scale deflection of 10 mA and potential difference across its terminals is 100 mV . Calculate series resistance for full-scale deflection corresponding to 10 V .
(Ans. 990 )
15. A moving coil voltmeter reading up to 20 mV has a resistance of $2 \Omega$. How this instrument be adopted to read voltage up to 300 V .
(Ans. 29,998 $\Omega$ )
16. A moving coil instrument gives a full-scale defection of 20 mA when a potential difference of 50 mV is applied. Calculate the series resistance to measure 500 V on full-scale. (Ans. 24,997.5 $\Omega$ )
17. A PMMC instrument gives full-scale reading of 25 mA when a potential difference across its terminals is 75 mV . Show how it can be used (a) as an ammeter for the range of $0-100 \mathrm{~A}$ and (b) as a voltmeter for the range of $0-750 \mathrm{~V}$. Further, find the multiplying factor of shunt and voltage amplification.
(Ans. $7.502 \times 10^{-4} \Omega ; 29997 \Omega ; 4,000 ; 10,000$ )
18. A moving coil ammeter reading up to 1 A has a resistance of $0.02 \Omega$. How could this instrument be adopted to read (i) voltage up to 250 V and (ii) current up to 10 A .
(Jan. 1993) (Ans. (i) multiplier: $249.98 \Omega$; (ii) shunt: $2.22 \times 10^{-3} \Omega$ )
19. A moving coil instrument gives full-scale deflection with 15 mA and has a resistance of $5 \Omega$. Calculate the resistance to be connected (i) in parallel to enable the instrument to read up to 1 A and (ii) in series to enable it to read up to 100 V .
(A.M.I.E. May 1974) (Ans. $0.0761 \Omega, 6,661.7 \Omega$ )
20. A moving coil instrument gives full-scale reading at 25 mA when a potential difference across its terminals is 75 mV . Show how it can be used to measure a current of 100 A and voltage up to 750 V . [Hints : $R_{\mathrm{m}}=\frac{v}{I_{\mathrm{m}}}=\frac{75}{25}=3 \Omega$ ]
(A.M.I.E. Nov. 1969) (Ans. $0.00075 \Omega, 29,997 \Omega$ )
21. A dynamometer-type instrument that gives a maximum reading of 20 mA has a resistance of $5 \Omega$. Show how it may be used to measure 400 V and 20 A .
(AMIE, Winter 1995) (Ans. 19,995 $\Omega, 0.005005 \Omega$ )

### 9.12 DYNAMOMETER-TYPE INSTRUMENTS

These are known as electrodynamics instruments. These are basically moving coil instruments. In these instruments, the operating field is produced by the fixed coil instead of permanent magnet. These instruments can be used either as ammeter or as voltmeter, but owing to the higher cost and lower sensitivity of dynamometer ammeters and voltmeters as compared with moving iron instruments, these are seldom used commercially as ammeter or voltmeter. However, these are generally used as wattmeter. Electrodynamic or dynamometer-type wattmeters are very important because they are commonly employed for measuring power in AC circuits. ${ }^{3}$

### 9.12.1 Dynamometer-type Wattmeters

An indicating instrument used to measure power in an electric circuit is called wattmeter. It measures power in watt or in kW . The pictorial view of a wattmeter is shown in Figure 9.31.

## Principle

The basic principle of dynamometer-type instruments is that when a current-carrying moving coil is placed in the magnetic field produced by the current-carrying fixed coil, a force is exerted on the coil sides of the moving coil and deflection takes place (Fig. 9.32). In other words, when the field produced by the current-carrying moving coil $\left(F_{\mathrm{r}}\right)$ tries to come in line with the field produced by the current-carrying fixed coil $\left(F_{\mathrm{m}}\right)$, a deflecting torque is exerted on the moving system and deflection takes place as shown in Figure 9.33.


Fig. 9.31 Wattmeter
${ }^{3}$ In DC circuits, usually power is not measured by wattmeters because in DC circuits power is just a product of voltage and current $(P=V I)$. However, in AC circuits, power factor of the circuit is also involved and dynamometer-type wattmeters give average power of the circuit directly on its scale.


Fig. 9.32 Working principle of wattmeter


Fig. 9.33 Working principle of watt-
$\qquad$

## Construction

The dynamometer-type wattmeter essentially consists of two coils called fixed coil and moving coil. The fixed coil is split into two equal parts that are placed parallel to each other, as shown in Figure 9.32. The two fixed coils are air-cored to avoid hysteresis effects when used on AC.


Fig. 9.34 (a) Constructional features of a wattmeter (b) Circuit diagram of a wattmeter

The fixed coil is connected in series with the load and carries the circuit current. It is, therefore, called current coil.

The moving coil is pivoted between the two parts of the fixed coil and is mounted on the spindle. The moving coil is connected in parallel with the load and carries the current proportional to the voltage applied across the load. It is, therefore, called potential coil. Generally, a high resistance is connected in series with the moving coil to limit the current through it. By limiting the current, the moving coil is made thin and light in weight which in turn increases the sensitivity of the instrument.

The controlling torque is provided by springs that also serve the additional purpose of leading current into and out of the moving coil. Air friction damping (not shown in Fig. 9.34(b)) is employed in such instruments.

## Working

When power is required to be measured in a circuit, the instrument is connected in the circuit as shown in Figure 9.35. The current coil connected in series with the load carries the load current and the potential coil connected in parallel with the load carries the current proportional to the voltage across the load. The fixed coil produces a field $F_{\mathrm{m}}$ (Fig. 9.33) and moving coil produces a field $F_{\mathrm{r}}$. This field $F_{\mathrm{r}}$ tries to come in line with the main field $F_{\mathrm{m}}$, which produces a deflecting torque on the moving coil. Thus, the pointer attached to the spindle of the moving coil deflects. The deflection is controlled by the controlling torque produced by the springs.

In the case of AC system, the direction of flow of current is reversed in both the coils simultaneously in negative half cycle, but the direction of deflecting torque produced in the moving system remains the same. Hence, the dynamometer-type wattmeter can be used on DC as well as on AC supply system.


Fig. 9.35 Circuit diagram of a wattmeter

## Deflecting torque

1. In the case of DC system,

Let $V=$ voltage across the load and $I=$ load current
Current through the fixed coil, $I_{1} \propto I$
Current through the moving coil, $I_{2} \propto V$

Since coils are air-cored, the flux density produced by the fixed coil is directly proportional to the current $I_{1}$, i.e., $B \propto I_{1}$
Current-carrying moving coil is placed in the flux density produced by the fixed coil.
$\therefore$ Deflecting torque,

$$
T_{\mathrm{d}} \propto B I_{2} \propto I_{1} I_{2} \propto I V \propto \text { Power }
$$

Hence, deflecting torque is proportional to power.
2. In the case of AC system,

Let $\quad e=$ instantaneous voltage across load;
$i=$ instantaneous current through load;
$V=\mathrm{rms}$ value of voltage across load;
$I=\mathrm{rms}$ value of current through load;
$\cos \phi=$ power factor (lagging) of the load
Now, $\quad e=V_{\mathrm{m}} \sin \omega t \times i=I_{\mathrm{m}} \sin (\omega t-\phi)$
Instantaneous value of current through fixed coil, $i_{1} \propto i$
Instantaneous value of current through moving coil, $i_{2} \propto e$
Due to inertia of the moving system, the deflection will be proportional to the average torque.
Average deflecting torque, $T_{\mathrm{d}} \propto$ average of $\left(i_{2} \times i_{1}\right) \propto$ average of $(e \times i)$
Now, $e \times i=V_{\mathrm{m}} \sin \omega t \times I_{\mathrm{m}} \sin (\omega t-\phi)$

$$
=V_{\mathrm{m}} I_{\mathrm{m}} \times \frac{1}{2}[\cos \phi-\cos (2 \omega t-\phi)]=\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}}(2 \omega t-\phi)
$$

Average of $(e \times i)=$ Average of $\left[\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos \phi-\frac{V_{\mathrm{m}}}{\sqrt{2}} \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t-\phi)\right]$
As the average of $\frac{V_{\mathrm{m}}}{\sqrt{2}} \cdot \frac{I_{\mathrm{m}}}{\sqrt{2}} \cos (2 \omega t-\phi)$ over a complete cycle being sinusoidal quantity
is zero.
$\therefore$ Average of $(e \times i)=V I \cos \phi$
Substituting this value in Equation (9.1), we get,
Average deflecting torque, $T_{\mathrm{d}} \propto V I \cos \phi \propto$ True power
Hence, deflecting torque is proportional to true power in AC circuits.
The controlling torque is provided by springs,

$$
\therefore \quad T_{\mathrm{c}} \propto \theta(\text { where } \theta \text { is angle of deflection })
$$

In steady position of deflection, $T_{\mathrm{c}}=T_{\mathrm{d}}$ or $\theta \propto$ Power
Since deflection $\theta$ is proportional to the power to be measured (consumed by the load) that further depends upon product of $I$ and $V$, and therefore, dynamometer-type wattmeters have uniform scale being crowded in the beginning.

## Advantages

1. It can be used on DC as well as AC circuits.
2. It has almost uniform scale.
3. High degree of accuracy can be obtained by careful designing.

## Disadvantages

1. At low power factors, the inductances of the potential coil cause serious errors.
2. The reading of the instrument may be affected by stray fields acting on the moving coil. In order to prevent it, magnetic shielding is provided by enclosing the instrumental in an iron case.

## Errors in Dynamometer-type Wattmeters

The following are the serious errors that may occur in dynamometer-type wattmeters:

1. Error due to potential coil inductance: The inductance of potential coil is liable to cause error in the reading of the wattmeter. Because of this error, the wattmeter gives high reading on lagging power factors and low reading on the leading power factors. This error can be reduced or compensated by connecting a capacitor in parallel with a portion of multiplier.
2. Effect due to power loss in potential coil or current coil: Another possible error in the indicated power may be due to some voltage drop in the current coil or the current taken by the potential coil. This defect can be overcome by having an additional compensating winding that is connected in series with the potential coil but is so placed that it produces a field in opposite direction to that of the current coils, as shown in Figure 9.36.


Fig. 9.36 Compensating coil of a wattmeter
3. Error due to eddy currents: The alternating field of current coil induces eddy currents in the solid metal parts that set up their own magnetic field. This alters the magnitude and phase of the magnetic field causing deflection. Thus, the error is introduced in the instrument reading. To reduce this error, the solid metal parts are removed as far away from the current coil as possible.
4. Error due to stray magnetic field: The dynamometer-type wattmeter has relatively weak operating field. Therefore, stray fields affect the reading of this instrument considerably and cause serious errors. Hence, this type of instruments must be shielded against stray magnetic fields by using iron cases or providing thin iron shields over the working parts.

## Range

1. Current circuit $0-0.25$ A to $0-100 \mathrm{~A}$ without employing current transformers.
2. Potential circuit $0-5 \mathrm{~V}$ to $0-750 \mathrm{~V}$ without employing potential transformers.

### 9.13 INDUCTION-TYPE INSTRUMENTS

Induction-type instruments are used only for AC measurements. These instruments can be used either as ammeter, voltmeter, or wattmeter. However, the induction principle finds its widest application as a wattmeter and energy meter. Induction-type wattmeter is quite suitable to measure power in an AC circuit where supply voltage and frequency are constant.

### 9.13.1 Induction-type Wattmeter

## Principle

The basic principle of induction-type wattmeter is electromagnetic induction. When AC flows through two suitably located coils (current coil and potential coil), they produce rotating magnetic field that is cut by the metallic disc suspended between the coils; therefore, an emf is induced in the disc that circulates eddy currents in it. By the interaction of rotating magnetic field and eddy currents, electromagnetic deflecting torque is developed that causes the disc to rotate.

## Construction

An induction-type wattmeter, as shown in Figure 9.37, has the following main parts of the operating mechanism:

1. Deflecting mechanism: The deflecting mechanism of the meter consists of two electromagnets:
(a) Series magnet: It consists of a number of U-shaped laminations of silicon steel staggered together to form a core. A coil of thick wire having a few turns is wound on both the legs of $U$-shaped magnet, as shown in Figure 9.37. This coil is connected in series with the load. Thus, it is excited by the circuit current $I$ and is known as current coil. This magnet is placed below the aluminium disc and produces the magnetic field $\phi_{\text {se }}$ proportional to and in phase with line current $I$.
(b) Shunt magnet: It consists of a number of M-shaped laminations of silicon steel assembled together to form a core. A coil of thin wire having large number of turns in wound on the central limb of the magnet, as shown in Figure 9.37. This coil is connected across the load. Thus, it is excited by the current proportional to the supply voltage and is known as potential or pressure coil. This magnet is placed above the aluminium disc.

In order to obtain deflecting torque, current in the pressure coil must lag behind the supply voltage by $90^{\circ}$. For this, copper shading band (short circuiting copper ring) is provided on the central limb of the shunt magnet. The phase difference of $90^{\circ}$ is obtained by adjusting the position of this shading band. The shading band acts as short-circuited transformer secondary. Since its resistance is negligibly small as compared to its inductance, and therefore, current circulating in the shading band lags behind the supply voltage nearly by $90^{\circ}$. Thus, shunt magnet produces a field $\phi_{\text {sh }}$ proportional to applied voltage. This field is in phase with the current flowing through the pressure coil $I_{\mathrm{p}}$, but is in quadrature with the applied voltage.
2. Deflecting system: It consists of a light aluminium disc mounted on a vertical spindle. The aluminium disc is positioned in the air gap between the series and the shunt magnet. The spindle is supported by a cup-shaped jewelled bearing at the bottom end and has a spring journal bearing at the top end.


Fig. 9.37 Constructional features of an induction type energymeter
3. Controlling torque arrangement: Controlling torque is provided with the help of two spiral springs placed at the upper and lower side of the spindle wound in opposite direction, as shown in Figure 9.37.

$$
T_{\mathrm{c}} \propto \theta
$$

4. Damping torque arrangement: A permanent magnet positioned near the edge of the aluminium disc, as shown in Figure 9.37, is used to provide necessary damping. When aluminium disc moves in the field of the permanent magnet, flux is cut and eddy currents are induced in the disc. The direction of induced currents is such that it opposes the rotation (Lenz's Law), and thus, damping torque is produced. Since the induced currents are proportional to the speed of the disc $(N)$, the damping torque is proportional to the disc speed. Quick deflection means more damping torque, while slow deflection means smaller damping torque. The position of braking magnet is adjustable, and therefore, critical damping can be obtained by adjusting its position.

## Working

When wattmeter is connected in the circuit, the current coil carries the load current and pressure coil carries the current proportional to the supply voltage. The magnetic field produced by the series magnet (series coil) is in phase with the line current and the magnetic field produced by the shunt magnet (pressure coil) is in quadrature with the applied voltage (since the coil is highly


Fig. 9.38 Phasor diagram
inductive). Thus, a phase difference exists between the fluxes produced by the two coils. A simplified phasor diagram for an induction-type wattmeter is shown in Figure 9.38.

Let the load current $I$ lags behind the circuit voltage $V$ by an angle $\phi$, as shown in Figure 9.38. Due to shunt magnetic flux, an emf $E_{\text {sh }}$ is induced in the disc; while series magnetic flux produces an emf of $E_{\mathrm{se}}$ in the disc. Two opposite torques are acting in the disc due to $\phi_{\mathrm{sh}} I_{\mathrm{se}}$ and $f_{\mathrm{se}} I_{\mathrm{sh}}$. The phase angle between $\phi_{\mathrm{sh}}$ and $I_{\text {se }}$ is $\phi$ and between $\phi_{\text {se }}$ and $I_{\text {sh }}$ is $(180-\phi)$.
$\therefore$ Resultant average deflecting torque,
$T_{\mathrm{d}} \propto\left[\phi_{\mathrm{sh}} I_{\mathrm{se}} \cos \phi-\phi_{\mathrm{se}} I_{\mathrm{sh}} \cos (180-\phi)\right]$
$\propto\left[\phi_{\mathrm{sh}} I_{\mathrm{se}} \cos \phi+\phi_{\mathrm{se}} I_{\mathrm{sh}} \cos \phi\right] \propto\left[\phi_{\mathrm{sh}} I_{\mathrm{se}}+\phi_{\mathrm{se}} I_{\mathrm{sh}}\right] \cos \phi_{\mathrm{sh}}$
where $\phi_{\mathrm{sh}} \propto V ; \phi_{\mathrm{se}} \propto I ; I_{\mathrm{se}} \propto I$ and $I_{\mathrm{sh}} \propto V$
$\therefore \quad T_{\mathrm{d}} \propto[V I+I V] \cos \phi \propto V I \cos \phi$
$\propto$ True power of the circuit $(P)$
As $\quad T_{\mathrm{d}} \propto P$ and $T_{\mathrm{c}} \propto \theta$, the wattmeter carries a uniform scale.

## Advantages

Induction-type wattmeters have long scale, and they are free from stray field and have good effective damping.

## Disadvantages

1. These are more costly.
2. They can be used only for AC measurements.
3. Change in frequency may cause errors.
4. Sensitive to temperature.

## Range

1. Current circuit $0-0.25$ A to $0-100$ A without employing CTs.
2. Potential circuit $0-5 \mathrm{~V}$ to $0-750 \mathrm{~V}$ without employing PTs.

### 9.13.2 Comparison between Dynamometer and Induction-type Wattmeters

The comparison between dynamometer and induction type wattmeters is given in Table 9.3
Table 9.3 Comparison between Dynamometer and Induction-Type Wattmeters

| S.No. | Dynamometer-type Wattmeters | Induction-type Wattmeters |
| :--- | :--- | :--- |
| 1. | The instrument can be used on both DC and AC <br> systems. | The instrument can be used only on AC system. |
| 2. | The instrument can have high degree of accuracy, <br> if carefully designed. | The instrument is less accurate. It is accurate <br> only at stated frequency and temperature. |

## Table 9.3 (Continued)

| S.No. | Dynamometer-type Wattmeters | Induction-type Wattmeters |
| :--- | :--- | :--- |
| 3. | Power consumption is comparatively low. | Power consumption is comparatively high. |
| 4. | Weight of moving system is comparatively low. | Weight of moving system is comparatively high. |
| 5. | The instrument has uniform scale. | The instrument has uniform and long scale. |
| 6. | The instrument has comparatively weaker working <br> torque. | The instrument has comparatively stronger <br> working torque. |

### 9.13.3 Induction-type Single-phase Energy Meter

Although induction-type instruments can be used as ammeter, voltmeter, or wattmeter, the induction-type energy meters are more popular. Induction-type single-phase energy meter is used invariably to measure the energy consumed in any AC circuit in a prescribed period where supply voltage and frequency are constant. Energy meter is an integrating instrument that measures the total quantity of electrical energy supplied to the circuit in a given period.

## Principle

The basic principle of induction-type energy meter is electromagnetic induction. When AC flows through two suitably located coils (current coil and potential coil), they produce rotating magnetic field that is cut by the metallic disc suspended between the coils, and thus, an emf is induced in the disc that circulates eddy currents in it. By the interaction of rotating magnetic field and eddy currents, electromagnetic torque is developed that causes the disc to rotate. This is the same principle that is applied in single-phase induction motors.

## Construction

An induction-type single-phase energy meter, as shown in Figure 9.39, has the following main parts of the operating mechanism:

1. Driving system: The driving system of the meter consists of two electromagnets:
(a) Series magnet: It consists of a number of U-shaped laminations of silicon steel staggered together to form a core. A coil of thick wire having a few turns is wound on both the legs of $U$-shaped magnet, as shown in Figure 9.39. This coil is connected in series with the load. Thus, it is excited by the circuit current $I$ and is known as current coil. This magnet is placed below the aluminium disc and produces the magnetic field $f_{\text {se }}$ proportional to and in phase with line current $I$.
(b) Shunt magnet: It consists of a number of M-shaped laminations of silicon steel assembled together to form a core. A coil of thin wire having large number of turns is wound on the central limb of the magnet, as shown in Figure 9.39. This coil is connected across the load. Thus, it is excited by the current proportional to the supply voltage and is known as potential or pressure coil. This magnet is placed above the aluminium disc.

In order to obtain deflecting torque, current in the pressure coil must lag behind the supply voltage by $90^{\circ}$. For this, copper shading band (short circuiting copper ring) is provided on the central limb of the shunt magnet. The phase difference of $90^{\circ}$ is obtained by adjusting the position of this shading band. The shading band acts as
short-circuited transformer secondary. Since its resistance is negligibly small as compared to its inductance, current circulating in the shading band lags behind the supply voltage nearly by $90^{\circ}$. Thus, shunt magnet produces a field $\phi_{\mathrm{sh}}$ proportional to applied voltage. This field is in phase with the current flowing through the pressure coil $I_{\mathrm{p}}$ but is in quadrature with the applied voltage.
2. Moving system: It consists of a light aluminium disc mounted on a vertical spindle. The aluminium disc is positioned in the air gap between the series and the shunt magnet. The spindle is supported by a cup-shaped jewelled bearing at the bottom end and has a spring journal bearing at the top end. Since there is no control spring, the disc makes continuous rotation under the action of deflecting torque.
3. Braking system: A permanent magnet positioned near the edge of the aluminium disc, as shown in Figure 9.39, forms the braking system. When aluminium disc moves in the field of the braking magnet, flux is cut and eddy currents are induced in the disc. The direction of induced currents is such that it opposes the rotation (Lenz's Law), and thus braking torque is produced. Since the induced currents are proportional to the speed of the disc $(N)$, braking torque $\left(T_{\mathrm{B}}\right)$ is proportional to the disc speed, that is, $T_{\mathrm{B}} \propto N$

The position of braking magnet is adjustable, and therefore, braking torque can be adjusted by shifting the magnet to different radial positions. If the braking magnet is moved towards the centre of the disc, flux cut by the disc is less. Further, this reduces the induced current, and thus, the braking torque is reduced. Hence, by the inward movement of the magnet, braking torque decreases but the speed of the disc increases and vice versa.


Fig. 9.39 (a) Connection of 1-phase energymeter (b) Connections of cc and pc in an energymeter
4. Recording mechanism: The function of recording or registering mechanism is to record continuously a number on the dial that is proportional to the revolutions made by the moving system. The number of revolutions of the disc is a measure of the electrical energy passing through the meter.

## Working

When the energy meter is connected in the circuit, the current coil carries the load current and pressure coil carries the current proportional to the supply voltage. The magnetic field produced by the series magnet (series coil) is in phase with the line current and the magnetic field produced by the shunt magnet (pressure coil) is in quadrature with the applied voltage (since the coil is highly inductive). Thus, a phase difference exists between the fluxes produced by the two coils. This sets up a rotating field that interacts with the disc and produces a driving torque, and thus, the disc starts rotating. The number of revolutions made by the disc depends upon the energy passing through the meter. The spindle is geared to the recording mechanism so that electrical energy consumed in the circuit is directly registered in kWh .

The speed of the disc is adjusted by adjusting the position of the braking magnet. For example, if the energy meter registers less energy than the energy actually consumed in the circuit, then the speed of the disc has to be increased that is obtained by shifting the braking magnet near to the centre of the disc and vice versa.

## Theory

The supply voltage $V$ is applied across the pressure coil. The pressure coil is highly inductive as it has large number of turns and the reluctance of its magnetic circuit is very small owing to the presence of short air-gap length. Thus, the current $I_{\mathrm{p}}$ flowing through the pressure coil is proportional to the supply voltage and lags behind it by nearly $90^{\circ}$. This current produces a flux $\phi_{\mathrm{sh}}$ that is in phase with $I_{\mathrm{p}}$ and is proportional to applied voltage.

The circuit or load current $I$ flows through the current coil that lags behind the applied voltage $V$ by an angle $\phi$ depending upon the power factor $(\cos \phi)$ of the load. This current produces a flux $f_{\text {se }}$ that is in phase with $I$ and is proportional to circuit current, as shown in the phasor diagram (Fig. 9.40).
$E_{\text {sh }}$ and $E_{\text {se }}$ are the emfs induced in the disc by the shunt magnetic flux $\phi_{\mathrm{sh}}$ and series magnetic flux $\phi_{\mathrm{se}}$, respectively, and lag behind their respective fluxes by $90^{\circ}$. The eddy currents $I_{\text {sh }}$ and $I_{\text {se }}$ are set up by the induced emfs and are assumed to be in phase with their respective emfs. All the quantities are shown vectorially in the phasor diagram shown in Figure 9.40.

Thus, two opposite directed torques are produced due to $\phi_{\mathrm{sh}} I_{\text {se }}$ and $\phi_{\mathrm{se}} I_{\mathrm{sh}}$. The instantaneous value of the net torque is the difference of the two, that is, $\left(\phi_{\mathrm{sh}} I_{\text {se }}-\phi_{\mathrm{se}} I_{\mathrm{sh}}\right)$. From the phasor diagram, the phase angle between $\phi_{\mathrm{sh}}$ and $I_{\text {se }}$ is $\phi$ and between $\phi_{\mathrm{se}}$ and $I_{\text {sh }}$ is $(180-\phi)$.


Fig. 9.40 Phasor diagram
or
or
or or

Mean driving torque, $T_{\mathrm{d}} \propto \phi_{\mathrm{sh}} I_{\mathrm{se}} \cos \phi-\phi_{\mathrm{se}} I_{\mathrm{sh}} \cos (180-\phi)$

$$
T_{\mathrm{d}} \propto \phi_{\mathrm{sh}} I_{\mathrm{se}} \cos \phi+\phi_{\mathrm{se}} I_{\mathrm{sh}} \cos \phi
$$

$$
T_{\mathrm{d}} \propto\left(K_{1} V I \cos \phi\right)+\left(K_{2} I V \cos \phi\right)
$$

$$
T_{\mathrm{d}} \propto V I \cos \phi\left(K_{1}+K_{2}\right) \text { or } T_{\mathrm{d}} \propto V I \cos \phi
$$

$$
\begin{equation*}
T_{\mathrm{d}} \propto \text { power } \tag{i}
\end{equation*}
$$

We have seen that braking torque, $T_{\mathrm{B}} \propto N$, where $N$ is the speed of the disc.
For steady speed, the driving torque is equal to the braking torque

$$
\begin{equation*}
T_{\mathrm{d}}=T_{\mathrm{B}} \tag{ii}
\end{equation*}
$$

From Equations (i) and (ii), we get $N \propto P$, where $P$ is power.
Multiplying both sides by time ' $t$ ', we get

$$
N t \propto P t
$$

Since the product $N t$ represents the total number of revolutions of the disc in time $t$, and the product $P t$ represents the energy passing through the meter in time $t$. Thus, number of revolutions of the disc are directly proportional to electrical energy passing through the meter.

## Errors in an induction-type energy meter and their adjustments

The main requirement of an induction-type energy meter is that it should record the actual energy supplied to the load. However, following are the errors that may crop up in an energy meter, and to make the correction, some adjustments are suggested along with the errors:

1. Phase and speed errors: The phase error is introduced because the shunt magnetic flux does not lag behind the supply voltage by exactly $90^{\circ}$ due to some resistance of the coil and iron losses. The angle of lag is slightly less than $90^{\circ}$. Because of this error, the torque is not zero at zero power factor of the load, and therefore, energy meter registers some energy even though the actual energy passing through the meter is zero at zero power factor.

In order to remove this error, the flux produced by the shunt magnet should be made to lag behind the supply voltage exactly by $90^{\circ}$. This is accomplished by adjusting the position of copper shading band provided on the central limb of the shunt magnet. An error on the fast side under these conditions can be eliminated by bringing the shading band near to the disc and vice versa.

Sometimes, the speed of the disc of an energy meter is either faster or slower, when tested on a load having moderate power factor. Therefore, energy meter registers either more or less energy than the actual energy passing through it and an error is introduced. In order to remove this error, the radial position of the braking magnet is adjusted. Movement of the braking magnet, away from the centre of the disc, increases the braking torque that decreases the speed of the disc and vice versa.
2. Frictional error: This error is introduced due to friction at the rotor bearing and in the register mechanism. Because of this error, an unwanted braking torque acts on the moving system and meter registers less energy than the actual energy passing through it. This error is compensated by placing two short circuited bands on the outer limbs of the shunt magnet. These bands embrace the flux contained in the two outer limbs of the shunt magnet. An emf is induced and current circulates through them. This causes phase displacement between the enclosed flux and the main gap flux. As a result of this, a small driving torque is exerted on the disc solely by the pressure coil that compensates the frictional torque. The amount of this corrective torque is adjusted by the variation of
the position of the two bands and it should be just sufficient to overcome the frictional torque, without actually rotating the disc at no load.
3. Creeping error: The slow but continuous rotation of the energy meter, when only pressure coil is excited but no current is flowing through the current coil is called creeping. This error may be due to excessive friction compensation, excessive voltage supply, stray magnetic field, etc. In order to prevent creeping on no-load, two holes are drilled in the disc on the opposite side of the spindle at the same radius. This causes sufficient distortion of the field to prevent continuous rotation. The disc remains stationary when one of the holes comes under one of the poles of the shunt magnet.
4. Temperature error: By the change in temperature, the parameters of the coils change slightly that introduce a small error in the meter. However, this error is negligibly small and there is no need to provide any means to eliminate this error.
5. Frequency error: Since the energy meters are used normally at fixed frequency, and therefore, they are designed and adjusted to have a minimum error at declared supply frequency that is normally 50 Hz in India.

### 9.14 NAME PLATE OF ENERGY METER

The following information is provided by the name plate of a single-phase energy meter:

1. Voltage
2. Frequency
3. Current
4. Revolutions/kWh
5. Type and its number

Energy meter constant: The number of revolutions made by the disc per unit ( kWh ) of energy passing through the energy meter is called energy meter constant.

### 9.15 CONNECTIONS OF SINGLE-PHASE ENERGY METER TO SUPPLY POWER TO A DOMESTIC CONSUMER

The electricity or electrical energy consumed by a consumer is measured with the help of an energy meter. A supplier gives energy to a consumer through an energy meter and the load of the consumer is connected to the meter, as shown in Figure 9.41.


Fig. 9.41 Connections of energymeter to measure electrical energy of a domestic consumer

The reading of the energy meter is recorded every month (or after two months as the case may be), the difference between the present reading and the previous reading shows the energy consumption during the said period.

## Example 9.23

A 230 V single-phase energy meter has a constant load current of 20 A at unity power factor. If the meter disc makes 2,300 revolutions during 2 h , calculate the meter constant. If the power factor were 0.8 lagging, what would be the number of revolutions made by the disc in 3 h ?

## Solution:

Supply voltage, $V=230 \mathrm{~V}$; load current, $I=20 \mathrm{~A}$
Power factor, $\cos \phi=1$; time, $t=2 \mathrm{~h}$
Energy supplied $=V I \cos \phi \times t=230 \times 20 \times 1 \times 2=9,200 \mathrm{~Wh}=9.2 \mathrm{kWh}$
Number of revolutions made by the disc during this period, $N=2,300$
$\therefore \quad$ Meter constant, $K=\frac{\text { Number of revolution }}{\text { Energy supplied in } \mathrm{kWh}}=\frac{2,300}{9.2}=250$ revolutions $/ \mathrm{kWh}$.
If the power factor, $\cos \phi=0.8$
Energy supplied $=230 \times 20 \times 0.8 \times 3=11,040 \mathrm{~Wh}=11.04 \mathrm{kWh}$
$\therefore \quad$ Number of revolutions made by the disc, $N=$ meter constant $\times$ energy supplied

$$
=250 \times 11.04=2,760 .
$$

## Example 9.24

For a $5 \mathrm{~A}, 230 \mathrm{~V}$ energy meter, the number of revolutions per kWh is 480 . If upon test at full load u.p.f., the disc makes 5 revolutions in 32 s , calculate the percentage error.

## Solution:

Supply voltage, $V=230 \mathrm{~V}$; full load current, $I=5 \mathrm{~A}$
Energy constant, $K=480$ revolutions $/ \mathrm{kWh}$
Energy consumed at full load unity p.f. in $32 \mathrm{~s}=\frac{5 \times 230 \times 1 \times 32}{1000 \times 60 \times 60}=0.01022 \mathrm{kWh}$
Number of revolutions required to be made by the disc

$$
N=K \times \text { energy supplied }=480 \times 0.01022=4.9067 \text { revolutions }
$$

Actual number of revolutions made by the disc, $N^{\prime}=5$

$$
\therefore \quad \% \text { error }=\frac{N-N^{\prime}}{N}=100=\frac{4.9067-5}{4.9067} \times 100=-1.902 \%
$$

Energy meter actually registers more energy than the actual energy passing through it. Therefore, it causes loss to the consumer and gain to the supplier.

## Example 9.25

The name plate of a single-phase energy meter installed in a house reads 1,200 revolutions $/ \mathrm{kWh}$. If 5 lamps of 100 W each and 5 lamps of 60 W each are operated for 1 h , the disc makes 1,000 revolution. State whether the meter reads correctly or not.
(Jan. 1995)

## Solution:

Energy meter constant, $K=1,200$ revolutions $/ \mathrm{kWh}$
Energy consumed in $1 \mathrm{~h}=\frac{100 \times 5 \times 1+60 \times 5 \times 1}{1,000}=0.8 \mathrm{kWh}$
Number of revolutions required to be made by the disc

$$
N=K \times \text { energy consumed }=1,200 \times 0.8=960
$$

Actual number of revolutions made by the disc, $N^{\prime}=1,000$
Hence, the meter reads incorrectly. In this case, the meter is faster and it records more than the actual energy passing through it.

## Example 9.26

A $50 \mathrm{~A}, 230 \mathrm{~V}$ meter on full load test makes 61 revolutions in 37 s . If the normal disc speed is 500 revolutions $/ \mathrm{kWh}$, find the percentage error.
(U.P.T.U. 2005-06)

## Solution:

Energy meter constant, $K=500$ revolutions $/ \mathrm{kWh}$

$$
V=230 \mathrm{~V}, I=50 \mathrm{~A} ; t=37 \mathrm{~s}
$$

Actual energy consumed by load $=\frac{230 \times 50 \times 37}{1,000 \times 60 \times 60}$
Number of revolutions that should be made by energy meter disc during the consumption of actual energy $=0.118194 \times 500=59.1$
Actual revolution's made by the disc $=61$. The instrument is reading higher than the actual.

$$
\begin{aligned}
\% \text { Error } & =\frac{\text { Actual revolutions }- \text { Correction revolution }}{\text { Correct revolution }} \times 100 \\
& =\frac{61-59.1}{59.1} \times 100=3.22 \%
\end{aligned}
$$

This error should be subtracted from the measured values.

## Example 9.27

An energy meter revolves 10 revolutions of disc for one unit of energy. Find the number of revolutions made by it during an hour when connected across load, which takes 20 A at 210 V and 0.8 power factor leading. If energy meter revolves 35 revolutions, find the percentage error.
(U.P.T.U. 2004-05)

## Solution:

Meter constant, $K=10$ revolutions $/ \mathrm{kWh}$
Energy consumed in $1 \mathrm{~h}=\frac{V I \cos \phi}{1,000} \times t=\frac{210 \times 20 \times 0.8}{1000} \times 1=3.36 \mathrm{kWh}$
Number of revolutions required to be made by the disc

$$
=K \times \text { energy consumed }=10 \times 3.36=33.6
$$

Number of revolutions made by the disc of energy meter $=35$
Energy recorded by the energy meter $=\frac{35}{10}=3.5 \mathrm{kWh}$

$$
\begin{aligned}
\% \text { Error } & =\frac{\text { Energy measured }- \text { Energy actually consumed }}{\text { Energy actually consumed }} \times 100 \\
& =\frac{3.5 \times 3.36}{3.36} \times 100=4.167 \%
\end{aligned}
$$

## Example 9.28

A 230 V single-phase energy meter has a constant load current of 10 A at unity power factor. If the meter disc makes 1,150 revolution during 2 h , calculate the meter constant. If the power factor were 0.8 , what would be the number of revolutions made by the disc in the same time?

## Solution:

Supply voltage, $V=230 \mathrm{~V}$; load current, $I=10 \mathrm{~A}$; power factor, $\cos \phi=1$; and time, $t=2 \mathrm{~h}$.

$$
\therefore \quad \text { Energy supplied }=V I \cos \phi \times t=230 \times 10 \times 1 \times 2=4,600 \mathrm{~Wh}=4.6 \mathrm{kWh}
$$

Number of revolutions made by the disc during this period; $N=1,150$
$\therefore \quad$ Meter constant, $K=\frac{\text { Number of revolutions }}{\text { Energy supplied in } \mathrm{kWh}}=\frac{1,150}{4.6}=250$ revolutions $/ \mathrm{kWh}$
If the power factor, $\cos \phi=0.8$
Energy supplied $=230 \times 10 \times 0.8 \times 2=3,680 \mathrm{~Wh}=3.68 \mathrm{kWh}$
$\therefore \quad$ Number of revolutions made by the disc,
$N^{\prime}=$ meter constant $\times$ energy supplied $=250 \times 3.68=920$.

## Example 9.29

A single-phase energy meter has a registration constant of 100 revolutions $/ \mathrm{kWh}$. If the meter is connected to a load carrying 20 A at 230 V and 0.8 power factor for an hour, find the number of revolutions made by it. If it actually makes 360 revolutions, find the percentage error.
(U.P.T.U. Feb. 2002)

## Solution:

Energy consumed in $1 \mathrm{~h}=\frac{V \times I \cos \phi}{1,000}=\frac{230 \times 20 \times 0.8}{1,000}$
Number of revolutions required to be made if it is correct

$$
\begin{aligned}
& =3.68 \times \text { registration constant in revolution } / \mathrm{kWh} \\
& =3.68 \times 100=368
\end{aligned}
$$

Number of revolutions actually made $=360$

$$
\text { Percentage error }=\frac{360-368}{368} \times 100=-2.174 \% \text {. }
$$

The negative sign shows that the disc of the meter is rotating slow.

### 9.16 DIFFERENCE BETWEEN WATTMETER AND ENERGY METER

The difference between wattmeter and energy meter are given in Table 9.4.
Table 9.4 Difference between Wattmeter and Energy Meter

| S.No. | Wattmeter | Energy meter |
| :--- | :--- | :--- |
| 1. | It is an indicating instrument. | It is an integrating instrument. |
| 2. | It measures electric power. | It measures electrical energy. |
| 3. | It contains graduated scale to measure | It contains recording mechanism to record <br> electrical energy in a given period. |
| 4. | It has a device that provides controlling torque. | It has no controlling torque. |
| 5. | It indicates the power consumed by the circuit |  |
| at a particular instant. | It adds- up the energy consumed by the circuit in |  |
| a given period. |  |  |

### 9.17 DIGITAL MULTIMETER

A digital multimeter (DMM) displays the AC or DC voltages being measured directly as discrete numerals in the decimal number system. Numerical readout of DMM is very convenient and it eliminates observational error. The use of digital multimeters increases the speed with which the readings can be taken.

The DMM is a versatile and accurate instrument used in laboratories. On account of developments in the integrated circuits (IC) technology, it has become possible to reduce the size, power requirements, and cost of digital multimeters.

The basic function performed by a digital multimeter is an analogue to digital (A/D) conversion. For example, the voltage value may be changed to a proportional time interval, which starts and stops


Fig. 9.42 Block diagram of a digital multimeter
a clock oscillator. In turn, the oscillator output is applied to an electronic counter that is provided with a readout in terms of voltage. There are many ways of converting the analogue reading into digital form, but the most common way is to use ramp voltage. The operating principle of a ramp-type DMM is simple. A ramp voltage increases linearly from zero to a predetermined level in a predetermined time interval. The ramp voltage value is continuously compared with the voltage being measured. At the instant, the value of ramp voltage becomes equal to that of unknown voltage, a coincidence circuit called input comparator, generates a pulse that opens the gate as shown in Figure 9.42. The ramp voltage continues to decrease till it reaches the ground level. At this instance, another comparator generates a pulse and closes the gate. The time interval between the opening and the closing of gate is measured with an electronic time-interval counter. This count is displayed as a number of digits.


Fig. 9.43 Outer view of a digital multimeter

Figure 9.43 shows the pictorial view of a DMM. The main parts on the panel are as follows:

1. Digital display unit: It displays the reading in digits.
2. ON-OFF switch: It is used to switch ON and OFF the power of battery.
3. Input terminals: There are four sockets; one of them is common to which black lead is inserted. The other three sockets to which red lead is connected are: (a) $V$ - for measurement of DC and AC voltage and resistance; (b) A - for measurement of DC and AC current up to 2 A ; and (c) 10A - for measurement of DC and AC current up to 10 A .
4. Mode switch: For measurement of either DC or AC voltage or current, this switch is used to select the mode.
5. Range switch: The central switch is used to select the range of quantity (voltage, current, or resistance) to be measured.

## 国隕 PRACTICE EXERCISES

## Short Answer Questions

1. What are dynamometer-type instruments? Why these are used only as wattmeter?
2. What are dynamometer-type wattmeters? How deflecting, controlling, and damping torque is provided in such instruments?
3. Mention the main causes of error in dynamometer-type moving coil wattmeters.
4. What is the basic working principle of induction-type wattmeter? What is its field of application? Which type of controlling and damping torque is provided in such instruments?
5. What for single-phase energy meter is used? What is its basic working principle and field of application? Give some important mathematical relations.
6. What are the prominent errors that may occur in induction-type energy meters? How these errors can be adjusted or compensated?
7. What do you mean by energy meter constant?

## Test Questions

1. In an industry, a voltmeter and ammeter are connected along with a wattmeter. Will the reading of wattmeter be equal to the reading of voltmeter $\times$ reading of ammeter, support your arguments.
2. Describe the construction and expression for torque for an electrodynamometer-type wattmeter.

Or
Describe the principle of operation and construction of a dynamometer-type wattmeter. Why is the scale of such an instrument uniform?
3. Name and explain the instrument you will use for measuring $V I \cos \theta$. Give the circuit diagram for measuring $V I \cos \theta$ in three-phase circuit.
4. How can you measure power and power factor in a single-phase circuit? Draw the circuit diagram and explain.

Or
How can you find the power and power factor of a cooler? Explain with the help of a circuit diagram.
5. Discuss merits and demerits of dynamometer-type wattmeter.
6. What are sources of error in a wattmeter? How can they be minimised?
7. What are the merits and demerits of induction-type wattmeter?
8. How you can measure the electricity consumption in houses? Explain the process and working of the instrument.

Or
What type of meter is installed in housing electricity board? What and how it measures? What it cannot measure? Explain.
9. What is induction-type watt-hour meter? Explain its construction and working.
10. How wattmeter and energy meter differ?
11. What are the errors in induction-type energy meter and how these can be adjusted?

Or
Explain sources of error and various adjustments involved in single-phase induction-type energy meter.

Or
Write a note on friction compensation and creeping error in induction-type watt-hour meter.
12. Calculate the energy consumed by a heater of 2 kW rating if it works for 10 h .
13. The disc of an energy meter makes 600 revolutions per unit of energy. When a $1,000 \mathrm{~W}$ load is connected, the disc rotates at 10.2 rpm . If the load is on for 12 h , how many units are recorded as error?

## Numericals

1. A $40 \mathrm{~A}, 230 \mathrm{~V}$ energy meter on full load test makes 60 revolutions in 46 s . If the normal disc speed is 500 revolutions $/ \mathrm{kWh}$, find the percentage error with proper sign by assuming the load to be purely resistive.
(Ans. $+2.08 \%$ (fast)) (U.P.T.U. June 2004)
2. A single-phase energy meter has a constant of 1,200 revolutions $/ \mathrm{kWh}$. When a load of 200 W is connected, the disc rotates at 4.2 revolutions $/ \mathrm{min}$. If the load is on for 10 h , how many units are recorded as an error? Further, find percentage error.
(Ans. 0.1 kWh (more), 5\%)
3. An energy meter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 25 A at 230 V and 0.8 p.f. for an hour. If it actually makes 450 revolutions, find the percentage error and explain it from the consumer point of view.
(Ans. - $2.174 \%$ (less))
4. A single-phase energy meter has a constant speed of 1,300 revolutions $/ \mathrm{kWh}$. The disc revolves at 3.5 rpm when a load of 150 W is connected to it. If the load is on for 11 h , how many units are recorded as error? What is the percentage error? (Ans. $0.127 \mathrm{kWh}, 7.69 \%$ ) (U.P.T.U Sep. 2001)
5. A $50 \mathrm{~A}, 230 \mathrm{~V}$ meter on full load test makes 61 revolutions in 37 s . If the normal disc speed is 500 revolutions $/ \mathrm{kWh}$, find the percentage error.
(Ans. 3.223\%)
6. For a $5 \mathrm{~A}, 230 \mathrm{~V}$ meter, the number of revolutions $/ \mathrm{kWh}$ is 480 . If at unity power factor, the disc, upon test, makes 10 revolutions in 64 s , calculate the percentage error.
(Feb. 1994) (Ans. Required revolutions $=9.813 ; \%$ error $=-1.9$ )
7. A 230 V , single-phase energy meter has a constant load current of 4 A passing through the meter for 5 H at unity p.f. If the meter disc makes 1,104 revolutions during this period, what is the constant in revolutions $/ \mathrm{kWh}$. If the p.f. is 0.8 , what number of revolutions will the disc make in the same time?
(Ans. 240 revolutions $/ \mathrm{kWh}$, 883.2)
8. The name plate of single-phase energy meter installed in a house reads as 4,800 revolutions $/ \mathrm{kWh}$. If three lamps of 100 W each are operated for 4 min , the disc makes 100 revolutions. State whether the meter reads correctly or not.
(Ans. $N=96$ revolutions, Incorrect)
9. An energy meter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 20 A at 230 V and 0.8 p.f. for an hour. If it actually makes 360 revolutions, find the percentage error and explain it from the consumer point of view.
(Ans. 2.175\%)

## SUMMARY

1. Measurement: The fixed quantity which is taken as basis is called unit and the process of comparing the quantity with this unit is termed as measurement.
2. Electrical instruments: The instruments, such as ammeter, voltmeter, energy meter, and megger are used to measure electrical quantities are called electrical instruments.
3. Classification of electrical instruments: 1. Absolute instruments 2 . Secondary instruments.
4. Absolute instruments: The instruments that give the value of the quantity to be measured in terms of constants of the instrument are called absolute instruments.
5. Secondary instruments: The instruments that determine the electrical quantity to be measured directly in terms of deflection are called secondary instruments.
6. Indicating instruments: The instruments that indicate the magnitude of electrical quantity being measured instantaneously are called indicating instruments.
7. Integrating instruments: The instruments that add up the electrical quantity, such as electrical energy and measure the total energy (in kWh ) in a given period are called integrating instruments.
8. Recording instruments: The instruments that give a continuous record of the variations of the electrical quantity being measured are called recording instruments.
9. Essentials of indicating instruments: The forces or torques required for satisfactory operation of indicating instruments are called essentials of indicating instruments, such as: 1 . Deflecting torque 2. Controlling torque 3 . Damping torque.
10. Deflecting torque: The deflecting or operating torque $\left(\mathrm{T}_{\mathrm{d}}\right)$ is produced by making use of any one of the effects such as magnetic, electrodynamic, electromagnetic induction etc.
11. Controlling torque: The controlling or restoring torque ( $\mathrm{T}_{\mathrm{c}}$ ) opposes the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques (i.e. deflecting torque and controlling torque) are equal.
12. Damping torque: When deflecting torque is applied to the moving system, it deflects the pointer. While the controlling torque controls the deflection and tries to stop the pointer at its final position. But due to inertia the pointer oscillates around its final position. To bring the pointer at its final position quickly damping torque is provided. It always acts in opposite direction to motion.
13. Methods of providing controlling torque: Controlling torque is provided by
(i) Spring control method $\left(\mathrm{T}_{\mathrm{c}} \propto \theta\right)$ (ii) Gravity control method $\left(\mathrm{T}_{\mathrm{c}} \propto \sin \theta\right)$.
14. Methods of providing damping torque: (i) Air friction (ii) Fluid friction (iii) Eddy currents
15. Errors common to all types of instruments: These error are (i) Temperature error (ii) Friction error (iii) Observational error.
16. Types of moving iron instruments: (i) Attraction type (ii) Repulsion type.
17. Error in moving iron instruments: These are (i) Error due to hysteresis (ii) Error due to stray magnetic field (iii) Error due to temperature (iv) Error due to change in frequency.
18. Application of moving iron instruments: These can be used on AC as well as on DC.
19. Permanent magnet moving iron instruments (PMMI): It carries a permanent magnet, a coil wound on a light aluminium former and a pointer.
20. Application of PMMI: Used on DC only.

## TEST YOUR PREPARATION

## 1 FILL IN THE BLANKS

1. Observational errors are called $\qquad$ .
2. The deflection in moving iron instrument is proportional to $\qquad$ —.
3. Indicating instruments are $\qquad$ slightly less than critically damped.
4. In indicating instruments, controlling torque always opposes the $\qquad$ torque.
5. A meter, which measures the discrete value of a quantity, is called $\qquad$ instrument.
6. One unit of electrical energy means $\qquad$ _.
7. Wattmeter is an instrument that measures $\qquad$ power.
8. The instruments, which indicate electrical quantities at particular instant, are called $\qquad$ instruments.
9. Example of integrating instrument is $\qquad$ .
10. Example of indicating instrument is $\qquad$ and example of integrating-type instrument is $\qquad$ .
11. A wattmeter measures $\qquad$ power.
12. In energy meters, controlling torque is provided by $\qquad$ .
13. The example of an absolute instrument is $\qquad$ .
14. A tangent galvanometer is $\qquad$ instrument.
15. Damping torque $\qquad$ the deflecting torque.
16. Damping torque in energy meter is provided by $\qquad$ -
17. $\qquad$ is the part, commonly available in all meters.
18. Cost of gravity control system is $\qquad$ than spring control system.
19. Moving iron instruments have $\qquad$ scale.
20. The type of instrument is used for both $A C$ and $D C$ is $\qquad$ _.
21. The range of an ammeter can be extended by using $\qquad$ .
22. PMMC instruments can be used for $\qquad$ measurements only.
23. Moving coil instruments have $\qquad$ scale.
24. Voltmeters should have $\qquad$ resistance.
25. $\qquad$ type of instruments can be
26. The MI instrument has $\qquad$ scale.
27. The multiplier is made of $\qquad$ material.
28. Ammeters should have $\qquad$ internal electrical resistance.
29. The internal resistance of an ammeter is $\qquad$ -.
30. The scale of PMMC instruments is $\qquad$ —.
31. Moving coil (PMMC) instruments can be used for $\qquad$ .
32. The deflection in moving iron instrument is proportional to $\qquad$ -
33. An ammeter is always connected in $\qquad$ with load.
34. The power consumption in moving coil instruments is typically about $\qquad$
35. The scale of moving iron-type instruments is $\qquad$ —.
36. A high torque to weight ratio in an analogue indicating instrument indicates $\qquad$ friction loss.
37. Swamping resistance is connected in series with the meter in order to reduce $\qquad$ errors in shunted ammeters.
38. Moving coil instruments can be used on $\qquad$ . $\qquad$
39. Voltmeter having high input resistance will be more accurate (yes or no).
40. In moving iron instruments, the deflecting torque is directly proportional to the $\qquad$ .
41. In moving iron-type instruments $\qquad$ damping cannot be provided.
42. Draw the symbolic representation of a wattmeter.
43. Wattmeter reading $P=$ $\qquad$ .
44. Wattmeter measures $\qquad$ $(V I / V I \cos \theta$ or $V I \sin \theta)$
45. Electrodynamometer-type instruments have uniform or non-uniform scale.
46. A wattmeter measures $\qquad$ power.
47. Electrodynamic-type instruments have $\qquad$ scale.
48. Energy meters have $\qquad$ torque or weight ratio.
49. Creeping is the phenomenon, which occurs in $\qquad$ .
50. Energy meter is $\qquad$ type of instrument.
51. Energy meter measures $\qquad$ (VI/VI $\cos \theta$ or $V I \sin \theta$ or none)
52. Energy meter in hours measure $\qquad$ -
53. Energy is given by a product of $\qquad$ - $\qquad$ .
54. In relation $P=V I \cos \mathrm{f}$. $V$ and $I$ are the values of $\qquad$ .

## OBJECTIVE TYPE QUESTIONS

1. The instruments that give the value of the quantity to be measured in terms of constants of the instrument are called
(a) indicating instruments.
(b) absolute instruments.
(c) secondary instruments.
(d) integrating instruments.
2. The instruments that determine the electrical quantity to be measured directly in terms of deflection are called
(a) absolute instruments.
(b) integrating instruments.
(c) secondary instruments.
(d) recording instruments.
3. The instruments that indicate the magnitude of electrical quantity being measured instantaneously are called
(a) integrating instruments.
(b) recording instruments.
(c) indicating instruments.
(d) All of these
4. The deflecting torque in an indicating instrument
(a) brings the needle to zero position when instrument is disconnected.
(b) deflects the needle.
(c) brings the pointer quickly to its final deflected position.
(d) none of these.
5. The controlling torque in an indicating instrument
(a) brings the pointer back to zero position when instrument is disconnected.
(b) controls the deflection and tries to stop the pointer at its final position where deflecting torque is equal to controlling torque.
(c) reduces the oscillations of the pointer and brings the pointer quickly to its final position.
(d) Both (a) and (b).
6. Damping torque is provided in indicating instruments by
(a) air friction.
(b) fluid friction.
(c) eddy current.
(d) All of these
7. Controlling torque is provided in an indicating instrument by
(a) air friction.
(b) spring or gravity control.
(c) fluid friction.
(d) eddy currents.
8. Deflecting torque is provided in an indicating instrument by
(a) air friction.
(b) fluid friction.
(c) gravity control.
(d) None of these
9. Overdamping will
(a) make the pointer to rise quickly to its deflected position without overshooting.
(b) make the pointer slow and lethargic.
(c) make the pointer to oscillate about its final position and take some time to come to rest in its steady position.
(d) None of these
10. For better performance, the indicating instruments should be
(a) critically damped.
(b) overdamped.
(c) underdamped.
(d) lethargic.
11. Energy meter is
(a) an indicating instrument.
(b) an integrating instrument.
(c) a recording instrument.
(d) an absolute instrument.
12. In the instrument provided with spring control
(a) $T_{\mathrm{d}} \propto I$.
(b) $T_{c} \propto I$.
(c) $T_{\mathrm{c}} \propto \theta$.
(d) $T_{\mathrm{d}} \propto \theta$.
13. In the instruments provided with gravity control
(a) $T_{\mathrm{c}} \propto \theta$.
(b) $T_{\mathrm{d}} \propto \sin \theta$.
(c) $T_{\mathrm{d}} \propto \theta$.
(d) $T_{\mathrm{c}} \propto \sin \theta$
14. Damping torque is provided in
(a) indicating instruments.
(b) integrating instruments.
(c) recording instrument.
(d) absolute instruments.
15. Eddy current damping cannot be provided in the indicating instruments
(a) permanent magnet moving coil instruments.
(b) moving iron instruments.
(c) indication-type instruments.
(d) None of these.
16. The internal resistance of a voltmeter is
(a) zero.
(b) very small.
(c) very high.
(d) infinite.
17. The internal resistance of a voltmeter must be very high in order to
(a) have maximum loading effect.
(b) have more current supplied by the voltage source.
(c) unalter the circuit conditions.
(d) reduce the current through the meter.
18. The shunt used in the milliammeter
(a) will extend the range and increases the resistance.
(b) will extend the range and decreases the meter resistance.
(c) will decrease the range and meter resistance.
(d) will decrease the range and increases the meter resistance.
19. A moving coil (permanent magnet) instrument can be used to measure
(a) low frequency AC .
(b) high frequency AC .
(c) both DC and AC both.
(d) DC only.
20. In permanent magnet moving coil instruments, damping torque is provided by
(a) air friction.
(b) eddy currents.
(c) fluid friction.
(d) Either (a), (b), and (c)
21. In permanent magnet moving coil instruments, the scale is
(a) uniform.
(b) non-uniform.
(c) crowded at the end.
(d) uniform at the beginning.
22. To extend the range of an ammeter
(a) a high resistance is connected in series with it.
(b) a low resistance is connected in series with it.
(c) a low resistance is connected in parallel with it.
(d) a high resistance is connected in parallel with it.
23. To extend the range of a voltmeter,
(a) a high resistance is connected in series with it.
(b) a low resistance is connected in series with it.
(c) a low resistance is connected in parallel with it.
(d) a high resistance is connected in parallel with it.
24. The value of the resistance required to extend the range of voltmeter is given by the relation
(a) $R=\frac{R_{\mathrm{m}}}{m-1}$
(b) $R=\frac{R_{\mathrm{m}}}{m+1}$
(c) $R=(m-1) R_{\mathrm{m}}$
(d) $R=(m+1) R_{\mathrm{m}}$
25. The value of the resistance required to extend the range of an ammeter is given by the relation
(a) $R_{\mathrm{s}}=\frac{R_{\mathrm{m}}}{N+1}$
(b) $R_{\mathrm{s}}=\frac{R_{\mathrm{m}}}{N-1}$
(c) $R_{\mathrm{s}}=(N+1) R_{\mathrm{m}}$
(d) $R_{\mathrm{s}}=(N-1) R_{\mathrm{m}}$
where $N=$ multiplying power of the shunt.
26. The dynamometer-type wattmeter can be used to measure
(a) DC power only.
(b) AC power only.
(c) neither AC nor DC power.
(d) both DC and AC power.
27. The wattmeter
(a) has three connections two of which are used at a time.
(b) can measure DC power but not $60 \mathrm{c} / \mathrm{s}$ AC power.
(c) has voltage and current coils to measure the real power.
(d) only measures apparent power.
28. The one 'unit' of energy measured in AC circuit is equivalent to
(a) 1 W
(b) 1 kWh
(c) 1 Wh
(d) 1 kW
29. The steady speed of the disc in an energy meter is achieved when
(a) braking torque is zero.
(b) braking torque is more than operating torque.
(c) braking torque is half of the operating torque.
(d) operating torque is equal to braking torque.
30. Creeping is the phenomenon that occurs in
(a) ammeter.
(b) voltmeter.
(c) energy meter.
(d) wattmeter.
31. The induction-type single-phase energy meter is
(a) an ampere-hour meter.
(b) true watt-hour meter.
(c) wattmeter.
(d) volt-ampere-reactive meter.
32. Two holes are drilled in the disc of energy meter on the opposite side of the spindle
(a) to eliminate creeping on no load.
(b) for proper ventilation.
(c) to reduce weight of the disc for easy rotation.
(d) to increase the deflecting torque.
33. In an induction-type energy meter, the frictional error is compensated by
(a) adjusting the opposition of brake magnet.
(b) placing short circuiting band on the two side limbs of the shunt magnet.
(c) drilling two holes in the aluminium disc.
(d) adjusting the position of short circuiting band on the central limb of the shunt magnet.

## NUMERICALS

1. A 15 V moving iron voltmeter has a resistance of $500 \Omega$ and inductance of 0.12 H . Assuming that this instrument reads correctly on DC , what will be its reading on AC at 15 V when the frequency is 25 Hz and 100 Hz .
(Ans. $14.99 \mathrm{~V}, 14.83 \mathrm{~V}$ )
2. In a moving coil instrument, the moving coil consists of 100 turns wound on a square former of length 3 cm . The flux density in the air gap is 600 lines $/ \mathrm{cm}^{2}$. Calculate the turning moment acting on the coil when carrying a current of 12 mA .
(Ans. $6.48 \times 10^{-5} \mathrm{Nm}$ )
3. A moving coil instrument gives full-scale deflection with 15 mA and has a resistance of $5 \Omega$. Calculate the resistance to be connected (i) in parallel to enable the instrument to read up to 1 A and (ii) in series to enable it to read up to 100 V .
(Ans. $0.0761 \Omega, 6,661.7 \Omega$ )
4. A moving coil instrument gives full-scale reading at 25 mA when a potential difference across its terminals is 75 mV . Show how it can be used to measure a current of 100 A and voltage up to 750 V .

$$
\left[\text { Hints : } R_{\mathrm{m}}=\frac{v}{I_{\mathrm{m}}}=\frac{75}{25}=3 \Omega\right]
$$

(Ans. $0.00075 \Omega, 29,997 \Omega$ )
5. A dynamometer-type instrument that gives a maximum reading of 20 mA has a resistance of $5 \Omega$. Show how it may be used to measure 400 V and 20 A .
(Ans. $19,995 \Omega, 0.005005 \Omega$ )
6. A 230 V , single-phase energy meter has a constant load current of 4 A passing through the meter for 5 H at unity p.f. If the meter disc makes 1,104 revolutions during this period, what is the constant in revolutions $/ \mathrm{kWh}$ ? If the p.f. is 0.8 , what number of revolutions will the disc make in the same time?
(Ans. 240 revolutions/kWh, 883.2)
7. The name plate of single-phase energy meter installed in a house reads as 4,800 revolutions $/ \mathrm{kWh}$. If three lamps of 100 W each are operated for 4 min , the disc makes 100 revolutions. State whether the meter reads correctly or not.
(Ans. $N=96$ revolutions, incorrect)

## VIVA VOCE OR REASONING QUESTIONS

1. Damping torque is different to controlling torque. How?
2. There is no need of controlling torque in the case of integrating instruments. Why?
3. Damping torque is essential for indicating instruments. Why?
4. In gravity control method, controlling torque is proportional to sine of deflection angle $\theta$. Why?
5. Damping torque can be provided by eddy currents. How?
6. Eddy current damping is not possible is moving iron instruments. Why?
7. Moving iron instruments can be used both on DC and AC. Why?
8. For general purpose, moving iron voltmeters and ammeters are used. Why?
9. The scale of moving iron instruments is not uniform. Why?
10. The permanent magnet moving coil instruments cannot be used on AC. Why?
11. The voltmeters have very high resistance. Why?

Voltmeters are always connected in parallel to the load and have very high resistance. Why?
12. The ammeters have very low resistance. Why?

Or
Ammeters are always connected in series with the load and have very small resistance. Why?
13. To extend the range of an ammeter, shunts (very small resistances) are connected in parallel to the instrument, why?
14. In dynamometer-type wattmeter, current coil is the fixed coil and potential coil is moving coil, why not vice versa?
15. The scale of dynamometer-type wattmeter is non uniform. Why?
16. The disc of induction-type energy meter is made of aluminium and not of wood or iron. Why?
17. No controlling torque is provided in an induction-type energy meter. Why?
18. A copper short circuiting band is provided on the central limb of the shunt magnet of an induction-type single-phase energy meter. Why?
19. Two short circuiting bands of copper are provided on the side limbs of the shunt magnet of an induc-tion-type single-phase energy meter. Why?
20. Two holes are provided in the disc of an electromagnetic induction-type energy meter. Why?
21. If an energy meter indicates more energy than the actual energy consumed, then the braking magnet is brought near to the axis or away from the axis to remove the error. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What are absolute Instruments?
2. What do you understand by a secondary instrument?
3. How are secondary instruments classified?
4. What is an indicating Instrument?
5. What are recording instruments?
6. What are integrating instruments?
7. What is a deflecting torque?
8. What is the function of controlling torque in indicating instruments?

Or
Why is controlling mechanism provided in indicating instruments?
9. List out the advantages and disadvantages of gravity control when compared to spring control:
(ii) Cramped scale.
(iii) The instrument should be properly levelled to avoid zero error.
10. What is the necessity of damping torque in indicating instruments?

Or
Why do we require damping torque in an indicating instrument?
11. Why is fluid friction damping not much used nowadays?
12. What are the types of moving iron instrument?
13. Explain the principle of operation of attraction-type moving iron instruments.
14. What is the basic principle of repulsion-type moving iron instruments?
15. What are the advantages of moving iron instruments?
16. List the disadvantages and moving iron instruments.
17. Why is the graduation of moving iron instrument not uniform throughout?
18. Why are iron made of Mumetal employed in moving iron instruments?
19. Why are repulsion-type moving iron instruments more commonly used than attraction-type one?
20. Why eddy current damping is not used in moving iron?
21. What do you mean by a linear scale and squared scale?
22. What is the difference between an ammeter and a voltmeter?
23. How will you make an ammeter from moving iron instrument?
24. Why should an ammeter have low resistance?
25. How to develop a voltmeter from a moving iron instruments.
26. Why should voltmeter have high resistance?
27. Why do we use a multiplier with a voltmeter?
28. Ammeters and voltmeters are connected in series and parallel, respectively. Why?
29. What are the two types of moving coil instruments?
30. What is principle of permanent magnet-type moving coil instruments?
31. What are the advantage of PMMC instruments?
32. What are the disadvantages of PMMC instruments?
33. Why permanent magnet moving coil instruments cannot be used for AC measurements?
34. Compare moving coil instruments with moving iron instruments.
35. What is dynamometer-type wattmeter?
36. Why are dynamometer instruments insensitive?
37. Why is eddy current damping not used in dynamometer-type instruments?
38. Why is dynamometer-type instrument mainly used as a wattmeter?
39. Why dynamometer instruments are not usually used for DC measurement?
40. What are the advantages of dynamometer-type wattmeter?
41. What are the disadvantages of dynamometer-type wattmeter?
42. Why are dynamometer wattmeter always preferred to induction wattmeters for AC power measurement?
43. What is an energy meter?
44. What is induction-type energy meter?
45. What is the principle of induction-type energy meter?
46. Why the rotating system of the energy meter is made as small as possible?
47. Why is it necessary that the strength of brake magnet should remain constant during the use of the meter?
48. List the different parts of single-phase induction-type energy meter.
48. Why are of aluminium disc is preferred over copper disc?
49. Why does the rotating disc of an induction-type energy meter carry a small hole?
50. What is meant by compensating loop with reference to induction-type energy meter?

## TEST QUESTIONS

1. What is the difference between absolute and secondary instruments?

Or
Differentiate between primary and secondary instruments.
2. Classify instruments and clearly differentiate between absolute and secondary instruments. (P.T.U.)
3. Differentiate between indicating, recording, and integrating instruments.
4. Differentiate between recording and integrating type of instruments. Give two examples of each of these.
5. Explain the essentials of indicating instruments.
(P.T.U.)
6. Name different types of torques provided in indicating instruments. Explain why these torques are required.
7. Describe the following in the case of measuring instruments:
(i) deflecting torque; (ii) controlling torque; and (iii) damping torque.
(U.P.T.U. Sept. 2003)
8. Discuss common features of indicating instruments.
(U.P.T.U. Feb. 2002)
9. Explain the methods of providing controlling torque in indicating instruments.
(P.T.U.)
10. Describe how controlling torque is obtained by using a spring.
11. Describe how controlling torque is obtained by using gravity control method. Further, show that in this method, $T_{\mathrm{c}} \mu \sin \theta$, where $\theta$ is the deflection angle.
12. What is the purpose of damping torque is an indicating instrument? How is it usually provided?
(P.T.U.)
13. Describe the operating principle of measuring instruments. What are the various torques acting on the moving mechanism of the instruments? How are these obtained and what are their roles in the operation of the instruments?
(U.P.T.U. Sept. 2001)
14. Explain the methods of providing controlling torque in indicating instruments.
(P.T.U.)
15. Explain different methods of obtaining damping torque in indicating instruments.
16. Explain why eddy current damping is not possible in moving iron instruments.
17. Compare the spring control and gravity control method of providing controlling torque in indicating instruments.
18. Describe with neat sketch the principle, construction, and working of moving iron attraction-type instruments.
19. Describe with neat sketch the principle, construction, and working of moving iron repulsion-type instrument.

## Or

Explain with neat sketch the principle, construction and, working of a repulsion-type moving iron instrument. Show that deflection $\theta$ is directly proportional to square of the current flowing through the coil. State the advantages and disadvantages of moving iron instruments.
20. Explain the principle and construction of attraction-type moving iron instruments. Discuss their merits and demerits.
(U.P.T.U. Jan. 2003)
21. Explain the principle of operation of one type of moving iron instrument, showing how it is suitable for use on both AC and DC systems.
(U.P.T.U. July 2002)
22. Explain the principle of operation of a repulsion-type moving iron instrument. Discuss its merits and demerits.
(U.P.T.U. Feb. 2001)
23. How is deflecting torque produced in a repulsion-type moving iron instruments? Why is the graduation of the scale not uniform in these instruments?
(M.Univ. Aug. 1990)
24. Drive the general equation for deflection for a spring-controlled repulsion-type moving iron instrument.
(P.T.U.)
25. Give advantages and disadvantages of moving iron-type instruments.
(P.T.U.)
26. Discuss different types of errors present with moving iron type of instrument.
(B. Univ. Oct. 87)
27. Compare merits and demerits of moving coil and moving iron instruments.
(U.P.T.U. Feb. 02)
28. Mention the errors in moving iron instruments.
(P.T.U.)
29. Describe with neat sketch a permanent magnet moving coil ammeter and discuss its errors. (P.T.U.)
30. Discuss the construction and working of permanent magnet moving coil-type instrument with the help of a neat sketch. Further, write its advantages of disadvantages.
(P.T.U.)
31. Explain the difference between moving coil and moving iron instruments with examples.
(M. Univ. April. 1988)(U.P.T.U. Tut.)
32. Discuss different sources of errors in PMMC instruments
(P.T.U.)
33. Explain the construction of a permanent magnet moving coil instruments with a neat sketch. (P.T.U.)
34. Explain why PMMC instruments are used most widely. Enlist their advantages and disadvantages.
35. With the help of neat labelled diagrams, explain briefly the construction and principle of operation of permanent magnet moving coil type of indicating instruments.
(U.P.T.U. June 2001)
36. Derive the expression for torque produced in a moving coil type of instrument and explain briefly its working.
(U.P.T.U. June 2004)
37. Describe a PMMC instrument in detail. Further, discuss its advantages and disadvantages.
(U.P.T.U. Sept. 2001)
38. Enlist the advantages and disadvantages of permanent magnet moving coil (PMMC) instruments.
(U.P.T.U. Sept. 2001)
39. Describe with the aid of a labelled diagram, the constructional details of a moving coil instrument, and also explain how 'control' and 'damping' forces are obtained.
(U.P.T.U. July 2002)
40. What is difference between an ammeter and voltmeter?
(P.T.U.)
41. Explain how will you extend the range of an ammeter.
(P.T.U.)
42. Explain how will you extend the range of a voltmeter.
(P.T.U.)
43. Derive expressions for multiplying powers when the range of a moving coil instrument is increased both as an ammeter and as a voltmeter.
(P.T.U.)
44. How is the current range of a permanent magnet moving coil instrument extended with the help of a shunt? Illustrate with an example.
(B. Univ. June 1988) (U.P.T.U. Tut.)
45. Describe with neat sketch the principle, construction, and working of dynamometer-type wattmeter.
(P.T.U.)
46. Name the two coils of a wattmeter. What is their function? How are they connected? In a dynamometer wattmeter, which coil is excited by current and which coil is excited by voltage? Why is it so?
(P.T.U.)
47. Explain why electrodynamometer-type instrument can be used both for AC and DC measurements. Discuss the main sources of error in such instruments.
48. Explain the working of electrodynamometer type of instruments.
49. Explain the operating principle of an electrodynamic-type wattmeter.
50. Describe the construction and working principle of a dynamometer-type wattmeter and show how its deflecting force is proportional to the average value of power.
51. Mention advantages and disadvantages of dynamometer-type wattmeter.
(P.T.U.)
52. With appropriate mathematical derivations, explain the principle of electrodynamic wattmeter.
53. Discuss the sources of errors in an electrodynamometer-type wattmeter.
(P.T.U.)
54. Explain the working principle of an induction-type wattmeter with the help of a diagram.
55. Describe the working principle and construction of a single-phase induction-type wattmeter. Show that deflecting torque is proportional to average power of the circuit.
56. Describe the constructional details of a single-phase induction-type energy meter, with the help of a circuit diagram.
(P.T.U.)
57. Describe the principle of induction-type energy meter.
(P.T.U.)
58. Describe with neat sketch the construction and working of single-phase energy meter.
(P.T.U.)
59. Discuss the construction and working of a single-phase energy meter. What is friction compensation? How is it provided?
(P.T.U.)
60. Describe the construction of a single-phase induction-type energy meter. Show that the number of revolutions made by the disc is proportional to the energy supplied.
61. What do you mean by creeping in an induction-type energy meter? How it can be avoided?
62. What are the errors in induction-type energy meter?
(P.T.U.)
63. Explain different errors and their compensation in a single-phase energy meter.
64. Describe the functions of the following in a single-phase induction-type energy meter.
(i) shunt and series magnets; (ii) moving disc; and (iii) permanent brake magnet.
65. Explain the sources of errors in single-phase induction-type energy meters. How are they eliminated?
66. Explain how the following adjustments are made in a single-phase induction-type energy meter: (i) adjustment for friction compensation and (ii) overload.
(P.T.U.)
67. Describe a two-element, three-phase energy meter.
(P.T.U.)
68. Describe with neat sketch the principle, construction, and working of a wright maximum demand indicator and mention its uses.
(P.T.U.)
69. Why maximum demand indicators are used? Give its construction and working principle. (P.T.U.)

## ANSWERS

## Fill in the Blanks

1. random error
2. $r^{2}$
3. having usually
4. deflecting
5. indicating
6. 1 kWh
7. indicating
8. indicating
9. energy meter
10. ammeter, energy meter
11. true
12. no controlling torque is provided
13. tangent galvanometer
14. absolute
15. does not affect
16. no damping torque is required in energy meter
17. operating mechanism
18. less
19. non-uniform
20. moving iron type
21. DC
22. uniform
23. a shunt in parallel with operating coil
24. non-uniform
25. high resistance
26. high
27. moving iron
28. uniform
29. DC measurements
30. square of the current passing through it
31. 25 to $200 \mu \mathrm{~W}$
32. non-uniform
33. low
34. series
35. DC and AC
36. yes
37. $r^{2}$
38. temperature

39. $V I \cos \phi$
40. $V I \cos \theta$
41. non-uniform
42. active
43. non-uniform
44. low
45. Induction-type energy meter
46. integrating
47. none
48. $V I \cos \phi$
49. time and power
50. voltage and current

## Objective Type Questions

| 1. (b) | 2. (c) | 3. (c) | 4. (b) | 5. (d) |
| ---: | ---: | ---: | ---: | ---: |
| 6. (d) | 7. (b) | 8. (d) | 9. (b) | 10. (a) |
| 11. (b) | 12. (c) | 13. (d) | 14. (a) | 15. (b) |
| 16. (c) | 17. (c) | 18. (b) | 19. (d) | 20. (c) |
| 21. (a) | 22. (c) | 23. (a) | 24. (c) | 25. (b) |
| 26. (d) | 27. (c) | 28. (b) | 29. (d) | 30. (c) |
| 31. (b) | 32. (a) | 33. (d) |  |  |



## LEARNING OBJECTIVES

After the completion of this chapter, students or readers will be able to understand the following:
*What is a transformer and its necessity in power system?

* How a transformer transfers electric power from one circuit to the other, that is, what is basic principle of a transformer?
* What is core of a transformer? What is its material?
- What are transformer windings?
* How the single-phase small rating transformers are constructed?
* How a transformer behaves and when it is considered to be an ideal one?
* What are the various factors on which emf induced in a transformer winding depends?
* How a transformer behaves when it is at no-load?
* How a transformer behaves when it is loaded?
* How to draw phasor diagrams of a transformer to represent various alternating quantities (neglecting resistance and reactance ampere-turns balance)
* How resistance of transformer windings affect its performance?
* What are mutual and leakage fluxes in a transformer?
* How inductive reactance appears in a transformer due to leakage fluxes?
* How to determine equivalent resistance and reactance of transformer windings on its either side?
* How to draw an equivalent circuit of a transformer and how to simplify it?
* What is voltage regulation of a transformer?
* What are the major losses in a transformer?
* How efficiency of a transformer is calculated at various loads?
* How to determine the condition at which a transformer works at its maximum efficiency?
* What is an autotransformer?
* How can autotransformer differ from a potential divider?
* What are the advantage and disadvantages of an autotransformer in comparison to an ordinary two-winding transformer?
* Why ordinary two-winding transformers cannot be replaced by autotransformers?
* What are the major applications of autotransformers?


### 10.1 INTRODUCTION

Transformer is considered to be a backbone of a power system. One of the main reasons of adopting alternate current (AC) system instead of direct current (DC) system for generation, transmission, and distribution of electric power is that the alternating voltage can be increased or decreased conveniently by means of a transformer. In fact, for economical reasons, electric power is required to be transmitted at high voltages, whereas it has to be utilized at low voltages from safety point of view. This increase in voltage for transmission and decrease in voltage for utilization can only be achieved by using a transformer. Hence, it is described that transformer is backbone of a power system. In this chapter, we shall discuss the general features and principle of operation of single-phase transformers.

### 10.2 TRANSFORMER

A transformer is a static device that transfers AC electrical power from one circuit to the other at the same frequency, but the voltage level is usually changed.

The block diagram of a transformer is shown in Figure 10.1. When the voltage is raised on the output side $\left(V_{2}>V_{1}\right)$, the transformer is known as step-up transformer, whereas the transformer in which the voltage is lowered on the output side $\left(V_{2}<V_{1}\right)$ is called a step-down transformer.


Fig. 10.1 Block diagram of a transformer

### 10.2.1 Necessity

In our country, usually electrical power is generated at 11 kV . For economical reasons, AC power is transmitted at very high voltages ( 220 kV or 400 kV ) over long distances; therefore, a step-up transformer is applied at the generating station. Then, to feed different areas, voltages are stepped down to different levels (for economical reasons) by transformer at various substations. Ultimately, for the utilization of electrical power, the voltage is stepped down to $400 / 230 \mathrm{~V}$ for safety reasons.

Thus, transformer plays an important role in the power system. The pictorial view of a power transformer is shown in Figure 10.2. The important accessories are labelled on it.


Fig. 10.2 (a) Oil-immersed air natural cooled transformer (b) Single-phase transformer

### 10.2.2 Applications

Main applications of the transformers are given below:

1. Used to change the level of voltage and current in electric power systems.
2. As impedance-matching device for maximum power transfer in low-power electronic and control circuits.
3. As a coupling device in electronic circuits.
4. To isolate one circuit from another, since primary and secondary are not electrically connected.
5. To measure voltage and currents, these are known as instrument transformers.

Transformers are extensively used in AC power systems because of the following reasons:

1. Electric energy can be generated at the most economic level ( $11 \mathrm{kV}-33 \mathrm{kV}$ )
2. Stepping up the generated voltage to high voltage, extra high voltage (EHV) (voltage above 230 kV ), or to even ultrahigh voltage (UHV) ( 750 kV and above) to suit the power transmission requirement in order to minimize losses and increase transmission capacity of lines.
3. The transmission voltage is stepped down in many stages for distribution and utilization for domestic, commercial, and industrial consumers.

### 10.3 WORKING PRINCIPLE OF A TRANSFORMER

The basic principle of a transformer is electromagnetic induction.
A simple form of a transformer is shown in Figure 10.3(a). It essentially consists of two separate windings placed over the laminated silicon steel core. The winding to which AC supply connected is called primary winding and the winding to which load connected is called a secondary winding.

When AC supply of voltage $V_{1}$ is connected to primary winding, an alternating flux is set up in the core. This alternating flux when links with the secondary winding, an emf is induced in it called mutually induced emf. The direction of this induced emf is opposite to the applied voltage $V_{1}$, according to Lenz's law as shown in Figure 10.3(b).


Fig. 10.3 (a) Simple diagram of a transformer (b) Mutual flux linking with primary and secondary winding

The same alternating flux also links with the primary winding and produces self-induced emf $E_{1}$. This induced emf $E_{1}$ also acts in opposite direction to the applied voltage $V_{1}$ according to Lenz's law and hence called 'back emf'.

Although there is no electrical connection between primary and secondary winding, electrical power is transferred from primary circuit to secondary circuit through mutual flux.

The induced emf in the primary and secondary winding depends upon the rate of change of flux linkages, that is, $\left(N \frac{d \phi}{d t}\right)$. The rate of change of flux $(d \phi / d t)$ is the same for both primary and secondary windings. Therefore, an induced emf in primary winding is proportional to number of turns of the primary winding $\left(E_{1} \propto N_{1}\right)$, and in secondary winding, it is proportional to number of turns of the secondary winding $\left(E_{2} \propto N_{2}\right)$.
$\therefore$ In case $N_{2}>N_{1}$, the transformer is step-up transformer, and when $N_{2}<N_{1}$, the transformer is step-down transformer.
Turn ratio: The ratio of primary to secondary turns is called turn ratio, that is, turn ratio $=N_{2} / N_{1}$. Transformation ratio: The ratio of secondary voltage to primary voltage is called voltage transformation ratio of the transformer. It is represented by $K$.

$$
K=\frac{E_{2}}{E_{1}}=\frac{N_{2}}{N_{1}}\left(\text { since } E_{2} \propto N_{2} \text { and } E_{1} \propto N_{1}\right)
$$

### 10.4 CONSTRUCTION OF A SINGLE-PHASE SMALL RATING TRANSFORMER

Single-phase small rating transformers are shown in Figure 10.4. According to the core construction and the manner in which the primary and secondary are placed around it, the transformers are named as follows:

1. Core-type transformers
2. Shell-type transformers
3. Berry-type transformers


Fig. 10.4 (a), (b) and (c) Small rating l-phase transformers

### 10.4.1 Core-type Transformers

In a simple core-type transformer, the magnetic core is built up of laminations to form a rectangular frame. The laminations are cut in the form of L-shape strips as shown in Figure 10.5. In order to avoid high reluctance at the joints where laminations are butted against each other, the alternate layers are stacked differently to eliminate continuous joint as shown Figure 10.6(a).


Fig. 10.5 L-shaped laminations


Fig. 10.6 (a) Alternate layer of joints (b) E $\&$ I laminations

The upper horizontal portion of the core is known as a yoke and the vertical portion as a limb, as shown in Figure 10.7(b). This carries the windings. Usually, the cross-sectional area of yoke is kept 15 to 20 per cent more than the limbs because it reduces the flux density and consequently reduces the iron losses.

In actual transformer construction, the primary and secondary windings are interleaved to reduce the leakage flux. Half of each winding is placed side by side or concentrically on either limb or leg of the core as shown in Figure 10.7. However, for simplicity, the two windings are shown in Figure 10.3(a) located on separate limbs of the core.


Fig. 10.7 (a), (b) and (c) Method to place LV and HV winding in core-type transformer

While placing these windings, an insulation layer (Bakelite former) is provided between core and lower winding and between the two windings. To reduce the insulation, low-voltage winding is always placed nearer the core as shown in Figure 10.7(a). The windings used are form wound (usually cylindrical in shape) and the laminations are inserted later on.

### 10.4.2 Shell-type Transformers

In case of shell-type transformer, each lamination is cut in the form of long strips of $E$ 's and $I$ 's as shown in Figure 10.8. In order to avoid high reluctance at the joints where the laminations are butted against each other, the alternate layers are stacked differently to eliminate continuous joints.


Fig. 10.8 E and I laminations for shall type core construction

In a shell-type transformer, the core has three limbs. The central limb carries whole of the flux, whereas the side limbs carry half of the flux. Therefore, the width of the central limb is about double to that of the outer limbs.

Both the primary and secondary windings are placed on the central limb side by side or concentrically (Fig 10.9). The low-voltage winding is placed nearer the core and high-voltage winding is placed outside the low-voltage winding to reduce the cost of insulation placed between core and low-voltage winding. In this case, the windings are form wound is cylindrical shape and the core laminations are inserted later on.


Fig. 10.9 (a), (b) and (c) Method to place LV and HV winding in shall-type transformer

The whole assembly, that is, core and winding is then usually placed in tank filled with transformer oil. The transformer oil provides better cooling to the transformer and acts as a dielectric medium between winding and outer tank which further reduces the size of outer tank of the transformer.

The comparison between core type and shall type transformers is given in Table 10.1
Table 10.1 Comparison between Core-type and Shell Type Transformers

| S.No. | Core-type Transformer | Shell-type Transformer |
| :---: | :---: | :---: |
| 1. | The windings surround a considerable portion of the core. | The core surrounds considerable portion of the windings. |
| 2. | Windings are of form-wound and are of cylindrical-type. | Windings are of sandwich type. The coils are first in the form of pancakes, and complete winding consists of stacked discs. |
| 3. | More suitable for high-voltage transformers. | More suitable and economical for low-voltage transformers. |
| 4. | Mean length of coil turns is shorter. | Mean length of coil turn is longer. |
| 5. | Core has two limbs to carry the windings. | Core has three or more limbs but the central limb carries the windings. |

### 10.4.3 Berry-type Transformers

A commonly used shell-type transformer is the one known as Berry-transformer so-called after the name of its designer and is cylindrical in shape. The transformer core consists of laminations arranged in groups which radiate from the centre as shown (as top view) in Figure 10.10.

It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise-a humming sound-during operation.

### 10.5 AN IDEAL TRANSFORMER

To understand the theory, operation, and applications of a transformer, it is better to view a transformer first as an ideal device. For this, the following assumptions are made:

1. Its coefficient of coupling $(k)$ is unity.
2. Its primary and secondary windings are pure inductors having infinitely large value.
3. Its leakage flux and leakage inductances are zero.
4. Its self- and mutual inductances are


Fig. 10.10 Top view of berry-type transformer zero having no reactance or resistance.
5. Its efficiency is 100 per cent having no loss due to resistance, hysteresis, or eddy current.
6. Its transformation ratio (or turn ratio) is equal to the ratio of its secondary to primary terminal voltage and also as the ratio of its primary to secondary current.
7. Its core has permeability $(\mu)$ of infinite value.

Thus, an ideal transformer is one which has no ohmic resistance and no magnetic leakage flux, that is, all the flux produced in the core links with both primary and secondary. Hence, transformer has no copper losses and core losses. It means an ideal transformer consists of two purely inductive coils wound on a loss-free core. Although in actual practice, it is impossible to realize such a transformer, yet for convenience, it is better to start with an ideal transformer and then extend it to an actual transformer.

In an ideal transformer, there is no power loss, and therefore, output must be equal to input.

That is,

$$
E_{2} l_{2} \cos \phi=E_{1} I_{1} \cos \phi \quad \text { or } \quad E_{2} I_{2}=E_{1} I_{2} \quad \text { or } \quad \frac{E_{2}}{E_{1}}=\frac{l_{1}}{l_{2}}
$$

Since,

$$
E_{2} \propto N_{2} ; E_{1} \propto N_{1} \quad \text { and } \quad E_{1} \cong V_{1} ; E_{2} \cong V_{2}
$$

$$
\frac{V_{2}}{V_{1}}=\frac{E_{2}}{E_{1}}=\frac{N_{2}}{N_{1}}=\frac{l_{1}}{l_{2}}=K(\text { transformation ratio })
$$

Hence, primary and secondary currents are inversely proportional to their respective turns.
The ratio of secondary turns to primary turns is called transformation ratio of the transformer and is represented by $K$.

### 10.5.1 Behaviour and Phasor Diagram

Consider an ideal transformer whose secondary is open as shown in Figure 10.11(a). When its primary winding is connected to sinusoidal alternating voltage $V_{1}$, a current $I_{\text {mag }}$ flows through it. Since the primary coil is purely inductive, the current $I_{\text {mag }}$ lags behind the applied voltage $V_{1}$ by $90^{\circ}$. This current sets up alternating flux


Fig. 10.11 Ideal transformer (a) General view (b) Phasor diagram (c) Wave diagram
(or mutual flux $\phi_{\mathrm{m}}$ ) in the core and magnetizes it. Hence, it is called magnetizing current. Flux is in phase with $I_{\text {mag }}$ as shown in the phasor diagram and wave diagram in Figure 10.11(b) and (c), respectively. The alternating flux links with both primary and secondary windings. When it links with primary, it produces self-induced emf $E_{1}$ in opposite direction to that of applied voltage $V_{1}$. When it links with secondary winding, it produces mutually induced emf $E_{2}$ in opposite direction to that of applied voltage. Both the emfs $E_{1}$ and $E_{2}$ are shown in phasor diagram (Fig. 10.11(b)).

A transformer is analogous to mechanical gear drive because of the facts given in Table 10.2.

## Table 10.2 Comparison between Mechanical Gear Drive and Transformer

S. No. Mechanical Gear Drive

1. It transfers mechanical power from one shaft to the other shaft.
2. There is perfect ratio between the number of teeth and the speeds of the two gears, i.e., $\frac{T_{1}}{T_{2}}=\frac{N_{2}}{N_{1}}$ where
$T_{1}=$ No. of teeth of gear 1
$T_{2}=$ No. of teeth of gear
$N_{2}=$ Speed of gear 2
$N_{1}=$ Speed of gear 1

## Transformer

It transfers electrical power from one circuit to the other.
There is perfect ratio between the number of turns and the induced emf or current of the two windings, i.e.,
$\frac{N_{2}}{N_{1}}=\frac{E_{2}}{E_{1}}=\frac{I_{1}}{I_{2}}$
where $N_{2}=$ No. of secondary turns
$N_{1}=$ No. of primary turns
$E_{2}=E M F$ in secondary
$E_{1}=\mathrm{EMF}$ is primary
$I_{1}=$ Current in primary
$I_{2}=$ Current in secondary
3. Power is transferred through mechanical mesh. Power is transferred through magnetic flux.

### 10.6 TRANSFORMER ON DC

A transformer cannot work on DC supply. If a rated DC voltage is applied across the primary, a flux of constant magnitude will be set up in the core. Hence, there will not be any self-induced emf (which is only possible with the rate of change of flux linkages) in the primary winding to
oppose the applied voltage. As the resistance of the primary winding is very low, the primary current will be quite high as given by the Ohm's law.

$$
\text { Primary current }=\frac{\text { DC applied voltage }}{\text { Resistance of primary winding }}
$$

This current is much more than the rated full-load current of primary winding. Thus, it will produce lot of heat $\left(I^{2} R\right)$ loss and burns the insulation of the primary winding, and consequently, the transformer will be damaged. Hence, DC is never applied to a transformer.

### 10.7 EMF EQUATION

When sinusoidal voltage is applied to the primary winding of a transformer, a sinusoidal flux, as shown in Figure 10.12 is set up in the iron core which links with primary and secondary winding.
Let $\phi_{\mathrm{m}}=$ Maximum value of flux in Wb
$f=$ supply frequency in Hz (or c/s)
$N_{1}=$ No. of turns in primary
$N_{2}=$ No. of turns in secondary


Fig. 10.12 Wave diagram of mutual flux set-up in magnetic core

As shown in Figure 10.19, flux changes from $+\phi_{\mathrm{m}}$ to $-\phi_{\mathrm{m}}$ in half cycle, that is, $\frac{1}{2 f}$ second,

Average rate of change of flux

$$
=\frac{\phi_{\mathrm{m}}-\left(-\phi_{\mathrm{m}}\right)}{1 / 2 f}=4 f \phi_{\mathrm{m}} \mathrm{~Wb} / \mathrm{s}
$$

Now, the rate of change of flux per turn is the average induced emf per turn in volt.
$\therefore$ Average emf induced per turn $=4 f \phi_{\mathrm{m}} \backslash$ volt
For a sinusoidal wave,

$$
\frac{\text { RMS value }}{\text { Average value }}=\text { Form factor }=1.11
$$

$\therefore$ RMS value of emf induced/turn, $E=1.11 \times 4 f \phi_{\mathrm{m}}=4.44 f \phi_{\mathrm{m}}$ volt
Since primary and secondary have $N_{1}$ and $N_{2}$ turns, respectively.
$\therefore$ RMS value of emf induced in primary,

$$
\begin{align*}
E_{1} & =(\text { emf induced } / \text { turn }) \times \text { No. of primary turns } \\
& =4.44 N_{1} f \phi_{\mathrm{m}} \text { volt } \tag{10.1}
\end{align*}
$$

Similarly, rms value of emf induced in secondary,

From eq. (10.1), we get,

$$
\begin{equation*}
E_{2}=4.44 N_{2} f \phi_{\mathrm{m}} \text { volt } \tag{10.2}
\end{equation*}
$$

$$
\begin{equation*}
\frac{E_{1}}{N_{1}}=4.44 f \phi_{\mathrm{m}} \mathrm{volt} / \mathrm{turn} \tag{10.3}
\end{equation*}
$$

From eq. (10.1), we get,

$$
\begin{equation*}
\frac{E_{2}}{N_{2}}=4.44 f \phi_{\mathrm{m}} \mathrm{volt} / \mathrm{turn} \tag{10.4}
\end{equation*}
$$

Equations (10.3) and (10.4) clearly show that emf induced per turn on both the sides, that is, primary and secondary is the same.

Again, we can find the voltage ratio,

$$
\frac{E_{2}}{E_{1}}=\frac{4.44 N_{2}}{4.44 N_{1}} \frac{f \phi_{\mathrm{m}}}{f \phi_{\mathrm{m}}} \quad \text { or } \quad \frac{E_{2}}{E_{1}}=\frac{N_{2}}{N_{1}}=K \text { (transformation ratio) }
$$

Equations (10.1) and (10.2) can be written in the form of maximum flux density $B_{\mathrm{m}}$ using relation,

$$
\begin{array}{ll} 
& \phi_{\mathrm{m}}=B_{\mathrm{m}} \times A_{\mathrm{i}} \text { (where } A_{\mathrm{i}} \text { is iron area) } \\
\therefore \quad & E_{1}=4.44 N_{1} f B_{\mathrm{m}} A_{\mathrm{i}} \text { volts and } \quad E_{2}=4.44 N_{2} f B_{\mathrm{m}} A_{\mathrm{i}} \text { volt }
\end{array}
$$

## Example 10.1

The emf per turn for a single-phase $2310 / 220 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer is approximately 13 volt. Calculate the number of primary and secondary turns.
(P.T.U. Dec. 2009)

## Solution:

Here,

$$
\frac{E_{1}}{N_{1}}=\frac{E_{2}}{N_{2}}=13 \mathrm{~V}(\text { given }) ; \quad E_{1}=2310 \mathrm{~V} ; E_{2}=220 \mathrm{~V}
$$

$\therefore$ Primary turns,

$$
N_{1}=\frac{E_{1}}{13}=\frac{2310}{13}=177.69 \cong 178
$$

Secondary turns,

$$
N_{2}=\frac{E_{2}}{13}=\frac{220}{13}=16.92 \cong 17
$$

## Example 10.2

A power transformer has 1000 primary turns and 100 secondary turns. The cross-sectional area of the core is 6 sq cm and the maximum flux density while in operation is 10,000 Gauss. Calculate turns per volt for the primary and secondary windings.
(P.T.U. Dec. 2008)

## Solution:

Here,

$$
\begin{aligned}
& N_{1}=1000 ; N_{2}=100 ; A_{\mathrm{i}}=6 \mathrm{~cm}^{2}=6 \times 10^{-4} \mathrm{~m}^{2} \\
& B_{\mathrm{m}}=10000 \text { gauss }=10000 \times 10^{-8} \times 10^{4}=1 \text { tesla }
\end{aligned}
$$

We know, $\quad E_{1}=4.44 \times N_{1} \times f \times B_{\mathrm{m}} \times A_{\mathrm{i}}=4.44 \times 1000 \times 50 \times 1 \times 6 \times 10^{-4}=133.2 \mathrm{~V}$

$$
E_{2}=4.44 \times N_{2} \times f \times B_{\mathrm{m}} \times A_{\mathrm{i}}=4.44 \times 100 \times 50 \times 1 \times 6 \times 10^{-4}=13.32 \mathrm{~V}
$$

On primary side, number of turns $/$ volt $=\frac{N_{1}}{E_{1}}=\frac{1000}{133.2}=7.5$
On secondary side, number of turns/volt $=\frac{N_{2}}{E_{2}}=\frac{100}{13.32}=7.5$
The number of turns per volt or voltage per turn on primary and secondary remains the same.

## Example 10.3

A 25 kVA transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary is connected to $3000 \mathrm{~V}, 50 \mathrm{~Hz}$ mains, calculate (i) primary and secondary currents at full
load, (ii) the secondary emf, and (iii) the maximum flux in the core. Neglect magnetic leakage, resistance of the winding and the primary no-load current in relation to the full-load current.

## Solution:

(i) At full load,

$$
I_{1}=\frac{25 \times 10^{3}}{3000}=8.33
$$

Now,

$$
\frac{I_{1}}{I_{2}}=\frac{E_{2}}{E_{1}}=\frac{N_{2}}{N_{1}}
$$

Secondary current,

$$
I_{2}=\frac{N_{1}}{N_{2}} \times l_{1}=\frac{500}{40} \times 8.33=104.15 \mathrm{~A}
$$

(ii) Secondary emf.

$$
E_{2}=\frac{N_{2}}{N_{1}} \times E_{1}=\frac{40}{500} \times 3000=240 \mathrm{~V}
$$

(iii) Using relation,

$$
E_{1}=4.44 \times N_{1} \times f \times \phi_{\mathrm{m}}
$$

$$
3300=4.44 \times 500 \times 50 \times \phi_{\mathrm{m}}
$$

$$
\phi_{\mathrm{m}}=\frac{3000}{4.44 \times 500 \times 50}=27 \mathrm{~m} \mathrm{~Wb}
$$

## Example 10.4

The design requirements of an $11000 / 415 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase core type transformer are approximate emf/turn 15 V , maximum flux density 1.5 T . Find suitable number of primary and secondary turns and net cross-sectional area of core.

## Solution:

Here,

$$
\begin{aligned}
& E_{1}=11000 \mathrm{~V} ; E_{2}=415 \mathrm{~V} ; f=50 \mathrm{~Hz} ; B_{\mathrm{m}}=1.5 \mathrm{~T} \\
& \text { EMF/turn }=\frac{E_{1}}{N_{2}}=\frac{E_{2}}{N_{2}}=15
\end{aligned}
$$

No. of primary turns,

$$
N_{1}=\frac{E_{1}}{15}=\frac{11000}{15}=733.33
$$

No. of secondary turns,

$$
N_{2}=\frac{E_{2}}{15}=\frac{415}{15}=27.67
$$

Now,

$$
E_{1}=4.44 N_{1} \times f \times A_{\mathrm{i}} \times B_{\mathrm{m}}
$$

$$
\frac{E_{1}}{N_{1}}=4.44 \times f \times A_{\mathrm{i}} \times B_{\mathrm{m}} \quad \text { or } \quad 15=4.44 \times 50 \times A_{\mathrm{i}} \times 1.5
$$

$\therefore$ Net area,

$$
A_{\mathrm{i}}=\frac{15}{4.44 \times 50 \times 1.5}=0.045045 \mathrm{~m}^{2}=450.45 \mathrm{~cm}^{2}
$$

## Example 10.5

A single phase 50 Hz core-type transformer has rectangular cores $30 \times 20 \mathrm{~cm}$ and the maximum allowable density is $1.05 \mathrm{~Wb} / \mathrm{sq}$. m. Find the number of turns per limb on the high- and lowvoltage sides for a voltage ratio of $3300 / 200 \mathrm{~V}$. Take iron factor as 0.93 .

## Solution:

Gross cross-sectional area $=30 \times 20=600 \mathrm{~cm}^{2}$

$$
A_{\mathrm{gc}}=600 \times 10^{-4} \mathrm{~m}^{2}
$$

The iron factor is to be taken into consideration as the laminations are insulated from each other and

$$
\begin{aligned}
K_{\mathrm{i}} & =\frac{\text { Net Area of Cross-Section }}{\text { Gross Area of Cross-Section }}=\frac{A_{\mathrm{i}}}{A_{\mathrm{gc}}} \\
A_{\mathrm{i}} & =K_{\mathrm{i}} \times A_{\mathrm{gc}} \\
& =0.93 \times 600 \times 10^{-4}=558 \times 10^{-4} \mathrm{~m}^{2}
\end{aligned}
$$

EMF induced/turn

$$
\begin{aligned}
& =4.44 \times f \times B_{\max } \times A_{\mathrm{i}} \\
& =4.44 \times 50 \times 1.05 \times 558 \times 10^{-4}=13 \mathrm{~V}
\end{aligned}
$$

Primary turns

$$
=\frac{3300}{13} ; 254 \text { turns }
$$

Secondary turns

$$
=\frac{200}{03} ; 16 \text { turns }
$$

## Example 10.6

The secondary of a $500 \mathrm{kVA}, 4400 / 500 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer has 500 turns. Determine (i) emf per turn, (ii) primary turns, (iii) secondary full-load current, (iv) maximum flux, (v) gross cross-sectional area of the core for flux density of 1.2 tesla and iron factor is 0.92 , and (vi) if the core is of square cross-section finds the width of the limb.

## Solution:

Here, Rating $=500 \mathrm{kVA} ; E_{1}=V_{1}=4400 \mathrm{~V} ; E_{2}=V_{2}=500 \mathrm{~V}$

$$
f=50 \mathrm{~Hz} ; N_{2}=500 ; B_{\mathrm{m}}=1.2 \mathrm{~T} ; k_{\mathrm{i}}=0.92
$$

(i) emf per turn $=\frac{E_{2}}{N_{2}}=\frac{500}{500}=1.0 \mathrm{~V} /$ turn
(ii) emf per turn $=\frac{E_{2}}{N_{2}}=\frac{E_{1}}{N_{1}}=\frac{4400}{N_{1}}=1.0$
$\therefore$ Primary turns,

$$
N_{1}=\frac{4400}{1.0}=4440
$$

(iii) Secondary full-load current, $I_{2}=\frac{\mathrm{KVA} \times 1000}{V_{2}}=\frac{500 \times 1000}{500}=1000 \mathrm{~A}$
(iv) Maximum flux, $\phi_{\mathrm{m}}=\frac{E_{2}}{4.44 \times N_{2} \times f}=\frac{500}{4.44 \times 500 \times 50}=4.5 \mathrm{mWb}$
(v) Iron area of the core, $A_{\mathrm{i}}=\frac{\phi_{\mathrm{m}}}{B_{\mathrm{m}}}=\frac{4.5 \times 10^{-3}}{1.2}=37.54 \times 10^{-4} \mathrm{~m}^{2}=37.54 \mathrm{~cm}^{2}$

Gross area of the core, $A_{\mathrm{g}}=\frac{A_{\mathrm{i}}}{k_{\mathrm{i}}}=\frac{37.54}{0.92}=4.8 \mathrm{~cm}^{2}$
(vi) Width of squared limb $=\sqrt{A_{\mathrm{g}}}=\sqrt{40.8}=6.39 \mathrm{~cm}$

## Example 10.7

A $100 \mathrm{kVA}, 3300 / 200 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase transformer has 40 turns on the secondary, calculate (i) the values of primary and secondary currents, (ii) the number of primary turns, and (iii) the maximum value of the flux. If the transformer is to be used on a 25 Hz system, calculate (iv) the primary voltage, assuming that the flux is increased by 10 per cent and (v) the kVA rating of the transformer assuming the current density in the windings to be unaltered.

## Solution:

(i) Full-load primary current, $I_{1}=\frac{100 \times 1000}{3300}=30.3 \mathrm{~A}$

Full-load secondary current, $I_{2}=\frac{100 \times 1000}{200}=500 \mathrm{~A}$
(ii) No. of primary turns, $N_{1}=N_{2} \times \frac{E_{1}}{E_{2}}=40 \times \frac{3300}{200}=660$
(iii) We know, $E_{2}=4.44 \times f \times \phi_{\max } \times N_{2} \mathrm{~V}$

$$
\begin{aligned}
& 200 & =4.44 \times 50 \times \phi_{\max } \times 40 \\
\therefore \quad & \phi_{\max } & =\frac{200}{4.44 \times 50 \times 40}=0.0225 \mathrm{~Wb}
\end{aligned}
$$

(iv) As the flux is increased by $10 \%$ at 25 Hz
$\therefore$ Flux at $25 \mathrm{~Hz}, \phi_{\mathrm{m}}^{\prime}=0.0225 \times 1.1=0.02475 \mathrm{~Wb}$
$\therefore$ Primary voltage $=4.44 \times N_{1} \times f^{\prime} \times \phi_{\mathrm{m}}^{\prime}$ volt

$$
=4.44 \times 660 \times 25 \times 0.02475=1815 \mathrm{~V}
$$

(v) For the same current density, the full-load primary and secondary currents remain unaltered.
$\therefore \mathrm{kVA}$ rating of the transformer $=\frac{30.3 \times 1815}{1000}=55 \mathrm{kVA}$

## 目冒 PRACTICE EXERCISES

## Short Answer Questions

1. What essentially is a transformer? What are the broad areas of applications of transformer?
(P.T.U. Dec. 2008)
2. What do you mean by step-up and step-down transformer?
3. Why a transformer has iron core?
(P.T.U. May 2009)
4. What type of material is used for the construction of core in a transformer and why?
(P.T.U. Dec. 2010)
5. Why is the transformer core laminated?
(P.T.U. May 2009)
6. When a transformer is not connected to supply, how its windings are named?
7. When a transformer is connected to the supply, how its windings are named?
8. Is it possible for any voltage winding to serve as primary?
9. Is there a definite relation between the number of turns and voltages in transformers?
10. While arranging the lamination of transformer core, continuous joint is avoided, why?
11. Why low-voltage winding is placed near the core than high-voltage winding in case of core-type transformers.
12. What is the significance of turn ratio in a transformer?
(P.T.U. May 2009)
13. Define back emf in a transformer.
(P.T.U. Dec. 2008)
14. What is an ideal transformer?
(P.T.U. May 2009)
15. Under what conditions a transformer is considered to be an ideal one?
16. Can a transformer work on DC? Justify.
17. In place of rated $A C$ if rated $D C$ voltage is applied to a transformer, what will you expect?
18. A transformer is said to be analogous to mechanical gear. Why?
19. While drawing phasor diagram of an ideal transformer, the flux vector is drawn $90^{\circ}$ out of phase (lagging) to the supply voltage. Why?

## Test Questions

1. What is the basic principle underlying the operation of practically all transformers? How are current, voltage, and number of turns related to one another in the primary and secondary of the transformer?
(P.T.U. Dec. 2008)
2. Differentiate between core-type and shell-type transformers. What are Berry-type transformers?
3. Explain the construction and working principle of a transformer and derive its emf equation.
(P.T.U. May 2011)
4. Derive from the first principle, the basic transformer equation that relates the induced voltages in the primary and secondary windings to the frequency $(f)$, flux density $(B)$, number of turns ( $N_{\mathrm{p}}$ and $N_{\mathrm{S}}$ ) and area of cross-section.
(P.T.U. Dec. 2008)
5. Derive an expression for the emf induced in a transformer winding. Show that the emf induced per turn in primary is equal to the emf per turn in secondary winding.
(P.T.U. Dec. 2007)
6. Explain the behaviour of an ideal transformer with the help of wave and phasor diagrams.
7. A transformer is analogous to mechanical gear drive, justify.
8. Give the constructional details of core-type transformer (for small rating say 2 kVA )
(P.T.U. Dec. 2008)

## Numericals

1. A sinusoidal flux 0.02 Wb (max.) links with 55 turns of a transformer secondary coil. Calculate the rms value of the induced emf in the secondary. The supply frequency is 50 Hz . (Ans. 244.2 V)
2. The design requirements of a $6600 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase, core-type transformer are as follows: emf per turn 15 V ; maximum flux density 1.5 Tesla (i.e., $\mathrm{Wb} / \mathrm{m}^{2}$ ). Find a suitable number of primary and secondary turns and the net cross-sectional area of core. (Ans. 440; 26.67; $450 \mathrm{~cm}^{2}$ )
3. A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is $60 \mathrm{~cm}^{2}$. If the primary winding is connected to a 50 Hz supply at 500 V , calculate the value of maximum flux density in the core and the emf induced in secondary winding. Draw the vector diagram representing the condition.
(Ans. 0.938 Tesla, 1250 V)
4. A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of core is $60 \mathrm{~cm}^{2}$. If the primary winding be connected to 50 Hz supply at 520 V , calculate (i) the peak value of flux density in the core; (ii) the voltage induced in the secondary winding.
(Ans. 0.976 Tesla, 1300 V)
5. A $200 \mathrm{kVA}, 3300 / 240 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer has 80 turns on the secondary winding. Assuming an ideal transformer, calculate (i) primary and secondary current on full load; (ii) the maximum value of flux; (iii) the number of primary turns.
(Ans. 60.6 A; 833.3 A; $13.5 \mathrm{mWb} ; 1100$ turns)
6. A single-phase transformer has a core whose cross-sectional area is $150 \mathrm{~cm}^{2}$ and operates at a maximum flux density of $1.1 \mathrm{~Wb} / \mathrm{m}^{2}$ from a 50 Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of $4 \Omega$ impedance. Neglect any voltage drop in the transformer.
(Ans. 14.6 kVA$)$
7. A $3300 / 250 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer is built on a core having an effective cross-sectional area of $125 \mathrm{~cm}^{2}$ and 70 turns on the low-voltage winding. Calculate (a) the value of the maximum flux density (b) the number of turns on the high-voltage winding.
(Ans. $1.287 \mathrm{~Wb} / \mathrm{m}^{2}$; 924)
8. A 10 kVA , single-phase transformer has a turn ratio of $300 / 23$. The primary is connected to a $1500 \mathrm{~V}, 60 \mathrm{~Hz}$ supply. Find the secondary voltage on open circuit and the approximate values of the currents in the two windings on full load. Find also the maximum value of the flux.
(Ans. $115 \mathrm{~V} ; 6.67 \mathrm{~A} ; 87 \mathrm{~A} ; 18.77 \mathrm{mWb})$
9. A $100 \mathrm{kVA}, 3300 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns. How does the core flux vary with load?
(Ans. $30.3 \mathrm{~A} ; 250 \mathrm{~A} ; 16.4 \mathrm{mWb}$; 907)
10. A circular single turn coil, 25 cm diameter is threaded by a flux whose maximum value is $1 \mathrm{~Wb} / \mathrm{m}^{2}$ and which alternates 50 cycles $/ \mathrm{sec}$. Calculate from the first principles the emf induced in it.
(Ans. 11 V(approximately.))
11. A 125 kVA transformer having a primary voltage of 2000 V at 50 Hz has 182 primary turns and 40 secondary turns. Neglecting losses, calculate (a) the full-load primary and secondary current (b) the no-load secondary induced emf.
(Ans. (a) $62.5 \mathrm{~A}, 284.4 \mathrm{~A}$ (b) 439.5 V )
12. A single-phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is $80 \mathrm{~cm}^{2}$. If the primary winding is connected to 50 Hz supply at 500 V , calculate
(a) the peak value of flux density in the core.
(b) the voltage induced in the secondary.
(Ans. (a) $0.563 \mathrm{~Wb} / \mathrm{m}^{2}$ (b) 1200 V )

### 10.8 TRANSFORMER ON NO-LOAD

A transformer is said to be on no-load when secondary winding is open circuited and the secondary current $I_{2}$ is zero. In this case, neither the secondary winding has any effect on the magnetic flux in the core nor it has any effect on the primary current.

In actual transformer, the losses cannot be neglected. Therefore, if transformer is on no-load, a small current $I_{0}$ (usually $2 \%$ to $10 \%$ of the rated value) called exciting current is drawn by the primary. This current has to supply the iron losses (hysteresis and eddy current losses) in the core
and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as $I_{2}$ is zero).

Therefore, current $I_{0}$ lags behind the voltage vector $V_{1}$ by an angle $\phi_{0}$ (called hysteresis angle of advance) which is less than $90^{\circ}$, as shown in Figure 10.13(b). The angle of lag depends upon the losses in the transformer. The no-load current $I_{0}$ has two components:

1. One, $I_{\mathrm{w}}$ in phase with the applied voltage $V_{1}$, called active or working component. It supplies the iron losses and a small primary copper losses.
2. The other, $I_{\text {mag }}$ in quadrature with the applied voltage $V_{1}$, called reactive of magnetizing component. It produces flux in the core and does not consume any power.

From phasor (vector) diagram shown in Figure 10.13(b).


Fig. 10.13 (a) Transformer on no-load (b) Phasor diagram of transformer at no-load

Working component,

$$
I_{\mathrm{w}}=I_{0} \cos \phi_{0}
$$

Magnetizing component,

$$
I_{\mathrm{mag}}=I_{0} \sin \phi_{0}
$$

No-load current,

$$
I_{0}=\sqrt{I_{\mathrm{w}}^{2}+I_{\mathrm{mag}}^{2}}
$$

Primary p.f. at no-load,

$$
\cos \phi_{0}=\frac{I_{\mathrm{w}}}{I_{0}}
$$

No-load power input,

$$
P_{0}=V_{1} I_{0} \cos \phi_{0}
$$

Exciting resistance,

$$
R_{0}=\frac{V_{1}}{I_{\mathrm{w}}}
$$

Exciting reactance,

$$
X_{0}=\frac{V_{1}}{I_{\mathrm{mag}}}
$$



Fig. 10.14 Equivalent circuit of a transformer at no-load

The equivalent circuit of a transformer at no-load is shown in Figure 10.14. Here, $R_{0}$ represents the exciting resistance of the transformer that carries power loss component of no-load current, that is, $I_{\mathrm{w}}$ used to meet with the no-load losses in the transformer, whereas $X_{0}$ represents the exciting reactance of the transformer that carries watt-less component of no-load current, that is, $I_{\text {mag }}$ used to set up magnetic field in the core.

## Example 10.8

A $230 / 110 \mathrm{~V}$ single-phase transformer has a core loss of 100 W . If the input under no-load condition is 400 VA , find core loss current, magnetizing current, and no-load power factor angle.

## Solution:

Here,

$$
V_{1}=230 \mathrm{~V} ; V_{2}=110 \mathrm{~V} ; V_{2}=110 \mathrm{~V} ; P_{\mathrm{i}}=100 \mathrm{~W}
$$

Input at no-load $=400 \mathrm{VA}$
That is,

$$
V_{1} I_{0}=400
$$

or no-load current

$$
I_{0}=\frac{400}{230}=1.739 \mathrm{~A}
$$

Core loss current,

$$
I_{\mathrm{w}}=\frac{P_{\mathrm{i}}}{V_{\mathrm{l}}}=\frac{100}{230}=0.4348 \mathrm{~A}
$$

Magnetizing current, $I_{\text {mag }}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}=\sqrt{(1.739)^{2}-(0.4348)^{2}}=1.684 \mathrm{~A}$

No-load power factor,

$$
\cos \phi_{0}=\frac{I_{\mathrm{w}}}{I_{0}}=\frac{0.4348}{1.739}=0.25 \mathrm{lag}
$$

No-load power factor angle,

$$
\phi_{0}=\cos ^{-1} 0.25=75.52^{\circ}
$$

## Example 10.9

A single-phase, $50 \mathrm{kVA}, 2300 / 230 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer is connected to 230 V supply on the secondary side, the primary being open. The meter indicates the following readings:-

Power $=230 \mathrm{~W}$
Voltage $=230 \mathrm{~V}$
Current $=6.5 \mathrm{~A}$
Find (i) core loss, (ii) loss component of the current, and (iii) magnetizing current. Draw the phasor diagram for this condition.

## Solution:

Power input at no-load, $P_{0}=230 \mathrm{~W}$

Supply voltage,

$$
V_{1}=230 \mathrm{~V}
$$

Current at no-load,

$$
I_{0}=6.5 \mathrm{~A}
$$

(i) Since low-voltage winding resistance is not given, the copper losses cannot be separated, and therefore, whole of the power input will represent the iron or core losses.
$\therefore$ Core loss $=230 \mathrm{~W}$
(ii) Using relation $P_{0}=V_{1} J_{0} \cos \phi_{0}=V_{1} I_{\mathrm{w}}$

Loss component of current, $\quad I_{\mathrm{w}}=\frac{P_{0}}{V_{1}}=\frac{230}{230}=1.0 \mathrm{~A}$
(iii) Magnetizing current, $I_{\text {mag }}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}=\sqrt{(6.5)^{2}-(1.0)^{2}}=6.423 \mathrm{~A}$

Under the given condition, transformer is operated at no-load.
Where

$$
V_{1}=230 \mathrm{~V} ; I_{0}=6.5 \mathrm{~A} ; I_{\mathrm{w}}=1.0 \mathrm{~A}
$$

$$
I_{\text {mag }}=6.423 \mathrm{~A} ; E_{1}=230 \mathrm{~V} ; E_{2}=2300 \mathrm{~V}
$$

## Example 10.10

The no-load current of a transformer is 5 A at 0.25 p.f. when supplied at $230 \mathrm{~V}, 50 \mathrm{~Hz}$. The number of turns on primary winding are 200. Calculate (i) maximum value of flux in the core, (ii) core loss, (iii) magnetizing current, and (iv) exciting resistance and reactance of the transformer. Also draw its equivalent circuit.

## Solution:

(i) Using the relation, $E_{1}=4.44 N_{1} f \phi_{\mathrm{m}}$
or

$$
230=4.44 \times 220 \times 50 \times \phi_{\mathrm{m}}
$$

$\therefore$ Maximum value of flux $\phi_{\mathrm{m}}=518 \mathrm{~m} \mathrm{~Wb}$
(ii) Core loss, $P_{0}=V_{1} l_{0} \cos \phi_{0}=230 \times 5 \times 0.25=287.5 \mathrm{~W}$
(iii) No-load p.f., $\cos \phi_{0}=0.25 ; \sin \phi_{0}=\sin \cos ^{-1} 0.25=0.9682$

Magnetizing current component, $I_{\mathrm{m}}=I_{0} \sin \phi_{0}=5 \times 0.9682=4.84 \mathrm{~A}$
Exciting resistance,

$$
R_{0}=\frac{V_{1}}{I_{\mathrm{w}}}=\frac{230}{I_{0} \cos \phi_{0}}=\frac{230}{5 \times 0.25}=184 \Omega
$$

Exciting reactance,

$$
X_{0}=\frac{V_{1}}{\mathrm{I}_{\mathrm{mag}}}=\frac{230}{4.84}=47.52 \Omega
$$

The equivalent circuit is shown in Figure 10.15. The values of different quantities are mentioned in the solution itself.


Fig. 10.15 Equivalent circuit of a transformer at no-load

## Example 10.11

At open circuit, transformer of $10 \mathrm{kVA}, 500 / 250 \mathrm{~V}, 50 \mathrm{~Hz}$ draws a power of 167 W at 0.745 A , 500 V . Determine the magnetizing current, watt-full current, no-load power factor, hysteresis angle of advance, equivalent resistance and reactance of exciting circuit referred to primary side.

## Solution:

Here,

$$
V_{1}=500 \mathrm{~V} ; I_{0}=0.745 \mathrm{~A} ; P_{0}=167 \mathrm{~W}
$$

Watt-full component of current, $\quad I_{\mathrm{w}}=\frac{P_{0}}{V_{1}}=\frac{167}{500}=0.334 \mathrm{~A}$
Magnetising component of current, $\quad I_{\text {mag }}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}$

$$
=\sqrt{(0.745)^{2}-(0.334)^{2}}=0.666 \mathrm{~A}
$$

No-load power factor,

$$
\cos \phi_{0}=\frac{I_{\mathrm{w}}}{I_{0}}=\frac{0.334}{0.745}=0.448 \mathrm{lag}
$$

Hysteresis angle of advance,

$$
\phi_{0}=\cos ^{-1} 0.448=63.36^{\circ} \mathrm{lag}
$$

Exciting resistance,

$$
R_{0}=\frac{V_{1}}{I_{\mathrm{w}}}=\frac{500}{0.334}=1497 \Omega
$$

Exciting reactance,

$$
X_{0}=\frac{V_{1}}{I_{\operatorname{mag}}}=\frac{500}{0.666}=750 \Omega
$$

### 10.9 TRANSFORMER ON LOAD

When a certain load is connected across the secondary, a current $I_{2}$ flows through it as shown in Figure 10.16. The magnitude of current $I_{2}$ depends upon terminal voltage $V_{2}$ and impedance of the load. The phase angle of secondary current $I_{2}$ with respect to $V_{2}$ depends upon the nature of load, that is, whether the load is resistive, inductive, or capacitive.
(Neglecting winding resistance and leakage flux)
When a certain load is connected across the secondary, a current $I_{2}$ flows through it as shown in Figure 10.16. The magnitude of current $I_{2}$ depends upon terminal voltage $V_{2}$ and impedance of


Fig. 10.16 Transformer on load
the load. The phase angle of secondary current $I_{2}$ with respect to $V_{2}$ depends upon the nature of load, that is, whether the load is resistive, inductive, or capacitive.

The operation of the transformer on load is explained below with the help of number of diagrams:

1. When the transformer is on no-load as shown in Figure 10.17(a), it draws no-load current $I_{0}$ from the supply mains. The no-load current $I_{0}$ produces an mmf. $N_{1} I_{0}$ which sets up flux in the core.
2. When the transformer is loaded, current $I_{2}$ flows in the secondary winding. This secondary current $I_{2}$ produces an $\operatorname{mmf} N_{2} I_{2}$ which sets up flux $\phi_{2}$ in the core. This flux opposes the flux which is set up by the current $I_{0}$ as shown in Figure 10.17(b), according to Lenz's law.
3. Since $\phi_{2}$ opposes the flux, and therefore, the resultant flux tends to decrease and causes the reduction of self-induced emf $E_{1}$ momentarily. Thus, $V_{1}$ predominates over $E_{1}$ causing additional primary current $I_{1}^{\prime}$ drawn from the supply mains. The amount of this additional current is such that the original conditions, that is, flux in the core must be restored to original value $\phi$, so that $V_{1}=E_{1}$. The current $I_{1}$ is in phase opposition with $I_{2}$ and is called primary counterbalancing current. This additional current $I_{1}^{\prime}$ produces an $\mathrm{mmf} N_{1}$ $I_{1}^{\prime}$ which sets up flux, $\phi$, in the same direction as that of $\phi$ as shown in Figure 10.17(c) and cancels the flux $\phi_{2}$ set up by mmf $N_{2} I_{2}$.

$$
\begin{array}{ll}
\text { Now, } & N_{1} I_{1}^{\prime}=N_{2} I_{2}(\text { ampere-turns balance }) \\
\therefore & I_{1}^{\prime}=\frac{N_{2}}{N_{1}} I_{2}=K I_{2}
\end{array}
$$

4. Thus, the flux is restored to its original value as shown in Figure 10.17(d). The total primary current $I_{1}$ is the vector sum of current $I_{0}$ and $I^{\prime}$, that is, $I_{1}=I_{0}+I_{1}^{\prime}$.

This shows that flux in the core of a transformer remains the same from no-load to full load; this is the reason why iron losses in a transformer remain the same from no-load to full load


Fig. 10.17 (a), (b), (c) and (d) : Effect on the magnetic flux set-up in the core when transformer is loaded

### 10.10 PHASOR DIAGRAM OF A LOADED TRANSFORMER

(Neglecting voltage drops in the winding; ampere-turns balance)
Since the voltage drops in both the windings of the transformer are neglected,

$$
V_{1}=E_{1} \quad \text { and } \quad E_{2}=V_{2}
$$

While drawing the phasor diagram, the following important points are to be considered.

1. For simplicity, let the transformation ratio $K=1$ be considered, and therefore, $E_{2}=E_{1}$.
2. The secondary current $I_{2}$ is in phase, lags behind, and leads the secondary terminal voltage $V_{2}$ by an angle $\phi_{2}$ for resistive, inductive, and capacitive load, respectively.
3. The counterbalancing current $I_{1}^{\prime}=\frac{N_{2}}{N_{1}} I_{2}$ (i.e., $l_{1}^{\prime}=K I_{2}$ here $K=1 \therefore I_{1}^{\prime}=I_{2}$ ) and is $180^{\circ}$
out of phase with $I_{2}$.
4. The total primary current $I_{1}$ is the vector sum of no-load primary current $I_{0}$ and counter balancing current $I_{1}^{\prime}$

That is,

$$
\overline{I_{1}}=\overline{I_{0}}+\overline{I_{1}^{\prime}} \text { or } \overline{I_{1}}=\sqrt{\left(I_{0}\right)^{2}+\left(I_{1}^{\prime}+2 I_{0} I_{1}^{\prime} \cos \theta\right.}
$$

Where $\theta$ is the phase angle between $I_{0}$ and $I_{1}^{\prime}$
5. The p.f. on the primary side is $\cos \phi_{1}$ which is less than the load p.f. $\cos \phi_{2}$ on the secondary side. Its value is determined by the relation.

$$
\cos \phi_{1}=\frac{I_{0} \cos \phi_{0}+I_{1}^{\prime} \cos \phi_{2}}{I_{1}}
$$



Fig. 10.18 Phasor diagram (a) for resistive load (b) for inductive load (c) for capacitive load

The phasor diagrams of the transformer for resistance, inductive, and capacitive loads are shown in Figure 10.18(a), (b), and (c), respectively.

Alternately,
The primary current $I_{1}$ can also be determined by resolving the vectors, that is,

$$
\begin{gathered}
I_{\mathrm{v}}=I_{0} \cos \phi_{0}+I_{1}^{\prime} \cos \phi_{2} \quad\left[\text { where } \sin \phi_{0}=\sin \cos ^{-1}\left(\cos \phi_{0}\right)\right. \\
\left.I_{\mathrm{H}}=I_{0} \sin \phi_{0}+I_{1}^{\prime} \sin \phi_{2} \quad \text { and } \quad \sin \phi_{2}=\sin \cos ^{-1}\left(\cos \phi_{2}\right)\right] \\
I_{1}=\sqrt{\left(I_{\mathrm{v}}\right)^{2}+\left(I_{\mathrm{H}}\right)^{2}}
\end{gathered}
$$

## Example 10.12

A single-phase transformer with a ratio of $440 / 110 \mathrm{~V}$ takes a no-load current of 5 A at 0.2 p.f. lagging. If the secondary supplies a current of 120 A at p.f. of 0.8 lagging, calculate the primary current and p.f.

## Solution:

Transformation ratio, $K=\frac{E_{2}}{E_{1}}=\frac{110}{440}=0.25$
Let the primary counterbalancing current be $I_{1}^{\prime}$
Then,

$$
I_{1}^{\prime}=K I_{2}=0.25 \times 120=30 \mathrm{~A}
$$

Now,

$$
\begin{aligned}
\cos \phi_{0} & =0.2 ; \phi_{0}=\cos ^{-1} 0.2=78.46^{\circ} \\
\cos \phi_{2} & =0.8 ; \phi_{2}=\cos ^{-1} 0.8=36.87^{\circ} \\
\theta & =\phi_{0}-\phi_{2}=78.46^{\circ}-36.87^{\circ}=41.59^{\circ}
\end{aligned}
$$

$$
I_{1}=\sqrt{\left(I_{0}\right)^{2}+\left(I_{1}^{\prime}\right)^{2}+2 I_{0} I_{1}^{\prime} \cos \theta}
$$

$$
=\sqrt{(5)^{2}+(30)^{2}+2 \times 5 \times 30 \times \cos 41.59^{\circ}}=33.9 \mathrm{~A}
$$

Primary p.f., $\quad \cos \phi_{1}=\frac{I_{1}^{\prime} \cos \phi_{2}+I_{0} \cos \phi_{0}}{I_{1}}=\frac{30 \times 0.8+5 \times 0.2}{33.9}$

$$
=0.7375 \mathrm{lag}
$$

## Example 10.13

A single-phase transformer with a ratio of $6600 / 400 \mathrm{~V}$ (primary to secondary voltage) takes to no-load current of 0.7 A at 0.24 power factor lagging. If the secondary winding supplies a current of 120 A at a power factor of 0.8 lagging. Estimate the current drawn by the primary winding.

## Solution:

Here,

$$
I_{0}=0.7 \mathrm{~A} ; \cos \phi_{0}=0.24 \mathrm{lag} ; I_{2}=120 \mathrm{~A} ; \cos \phi_{2}=0.8 \mathrm{lag}
$$

Transformation ratio,

$$
K=\frac{V_{2}}{V_{1}}=\frac{400}{6600}=\frac{2}{33}
$$

Let the primary counterbalance current be $I_{1}$.

$$
\begin{array}{ll}
\therefore & N_{1} I_{1}^{\prime}=N_{2} I_{2} \\
\text { or } & I_{1}^{\prime}=\frac{N_{2}}{N_{1}} \times I_{2}=K I_{2}=\frac{2}{33} \times 120=7.273 \mathrm{~A} \\
\text { Now, } & \cos \phi_{0}=0.24 ; \phi_{0}=\cos ^{-1} 0.24=76.11^{\circ} \\
& \cos \phi_{2}=0.8 ; \phi_{2}=\cos ^{-1} 0.8=36.87^{\circ}
\end{array}
$$

Angle between vector $I_{0}$ and $I_{1}$ (Figure 10.14(b))

$$
\theta=76.11^{\circ}-36.87^{\circ}=39.24^{\circ}
$$

Current drawn by the primary,

$$
\begin{aligned}
I_{1} & =\sqrt{\left(I_{0}\right)^{2}+\left(I_{1}^{\prime}\right)^{2}+2 I_{0} I_{1}^{\prime} \cos \theta} \\
& =\sqrt{(0.7)^{2}+(7.273)^{2}+2 \times 0.7 \times 7.273 \times \cos 39.24^{\circ}}=7.827 \mathrm{~A}
\end{aligned}
$$

### 10.11 TRANSFORMER WITH WINDING RESISTANCE

In an actual transformer, the primary and secondary windings have some resistance represented by $R_{1}$ and $R_{2}$, respectively. These resistances are shown external to the windings in Figure 10.19. The resistance of the two windings can be transferred to either side in order to simplify the cal-


Fig. 10.19 Transformer with its windings resistances culations. The resistance is transferred from one side to the other in such a manner that percentage voltage drop remains the same when represented on either side.

Let the primary resistance $R_{1}$ be transferred to the secondary side and the new value of this resistance be $R_{1}^{\prime}$ called equivalent resistance of primary referred to secondary side as shown in Figure 10.20(a). $I_{1}$ and $I_{2}$ be the full-load primary and secondary currents, respectively.

Then,

$$
\frac{I_{2} R_{1}^{\prime}}{V_{2}} \times 100=\frac{I_{1} R_{1}}{V_{1}} \times 100(\% \text { voltage drops })
$$

or

$$
R_{1}^{\prime}=\frac{I_{1}}{I_{2}} \times \frac{V_{2}}{V_{1}} \times K^{2} R_{1}
$$

$\therefore$ Total equivalent resistance referred to secondary.

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=R_{2}+K^{2} R_{1}
$$

Now consider resistance $R_{2}$, when it is transferred to primary, let its new value be $R_{2}^{\prime}$ called equivalent resistance of secondary referred to primary as shown in Figure 10.20(c).

Then,

$$
I_{1} R_{2}^{\prime} \times 100=\frac{I_{2} R_{2}}{V_{2}} \times 100 \quad \text { or } \quad R_{2}^{\prime}=\frac{I_{2}}{I_{1}} \times \frac{V_{1}}{V_{2}} \times R_{2}=\frac{R_{2}}{K^{2}}
$$

$\therefore$ Total equivalent resistance referred to primary,

$$
R_{\mathrm{ep}}=R_{1}+R_{2}^{\prime}=R_{1}+\frac{R_{2}}{K^{2}}
$$



Fig. 10.20 (a) and (b) Equivalent resistance when referred to secondary side (c) and (d) Equivalent resistance when referred to primary side

### 10.12 MUTUAL AND LEAKAGE FLUXES

So far, it is assumed that when AC supply is given to the primary winding of a transformer, an alternating flux is set up in the core and whole of this flux links with both primary and secondary windings. However, in an actual transformer, both the windings produce some flux that links only with the winding that produces it.

The flux that links with both windings of the transformer is called mutual flux and the flux that links only with one winding of the transformer and not to the other is called leakage flux.

The primary ampere turns produce some flux $\phi_{1}$ which is set up in air and links only with primary winding as shown in Figure 10.21(a), is called primary leakage flux.


Fig. 10.21 (a) Representation of leakage flux on primary and secondary side (b) Representation of primary and secondary reactances

Similarly, secondary ampere turns produce some flux $\phi_{l_{2}}$ which is set up in air and links only with secondary winding is called secondary leakage flux.

The primary leakage flux $\phi_{l_{1}}$ is proportional to the primary current $I_{1}$ and secondary leakage flux $\phi_{l_{2}}$ is proportional to secondary current $I_{2}$. The primary leakage flux $\phi_{l_{1}}$ produces self-inductance $L_{1}\left(=N_{1} \phi_{1} / I_{1}\right)$ which in turn produces leakage reactance $X_{1}\left(=2 \pi f L_{1}\right)$. Similarly, secondary leakage flux $\phi_{l_{2}}$ produces leakage reactance $X_{2}\left(=2 \pi f L_{2}\right)$. The leakage reactance (inductive) has been shown external to the windings in Figure 10.21(b).

### 10.13 EQUIVALENT REACTANCE

To make the calculations easy, the reactance of the two windings can be transferred to any one side. The reactance from one side to the other is transferred in such a manner that percentage voltage drop remains the same when represented on either side.

Let the primary reactance $X_{1}$ be transferred to the secondary, and the new value of this reactance is $X_{1}^{\prime}$ called equivalent reactance of primary referred to secondary, as shown in Figure 10.22(a).


Fig. 10.22 (a) and (b) Equivalent reactance when referred to secondary side (c) and (d) Equivalent reactance when referred to primary side

Then,

$$
\begin{gathered}
\frac{l_{2} X_{1}^{\prime}}{V_{2}} \times 100=\frac{l_{1} X_{1}}{V_{1}} \times 100(\% \text { voltage drops }) \\
X_{1}^{\prime}=\frac{l_{1}}{l_{2}} \times \frac{V_{2}}{V_{1}} \times X_{1}=K^{2} X_{1}
\end{gathered}
$$

or
$\therefore$ Total equivalent reactance referred to secondary.

$$
X_{\mathrm{es}}=X_{2}+X_{1}^{\prime}=X_{2}+K^{2} X_{1}
$$

Now, let us consider secondary reactance $X_{2}$ when it is transferred to primary side its new value is $X_{2}^{\prime}$ called equivalent reactance of secondary referred to primary, as shown in Figure 10.22(c).

Then,

$$
\frac{l_{1} X_{2}^{\prime}}{V_{1}} \times 100=\frac{I_{2} X_{2}}{V_{2}} \times 100 \quad \text { or } \quad X_{2}^{\prime}=\frac{l_{2}}{l_{1}} \times \frac{V_{1}}{V_{2}} \times X_{2}=\frac{X_{2}}{K^{2}}
$$

$\therefore$ Total equivalent reactance referred to primary.

$$
X_{\mathrm{ep}}=X_{1}+X_{2}^{\prime}=X_{1}+\frac{X_{2}}{K^{2}}
$$

## Example 10.14

A $63 \mathrm{kVA}, 1100 / 220 \mathrm{~V}$ single-phase transformer has $R_{1}=0.16 \mathrm{ohm}, X_{1}=0.5 \mathrm{ohm}, R_{2}=0.0064 \mathrm{ohm}$ and $X_{2}=0.02$ ohm. Find equivalent resistance and reactance as referred to primary winding.
(P.T.U. May 2009)

## Solution:

Here, transformer rating $=63 \mathrm{kVA} ; V_{1}=1100 \mathrm{~V} ; V_{2}=220 \mathrm{~V}$

$$
R_{1}=0.16 \mathrm{ohm} ; X_{1}=0.5 \mathrm{ohm} ; R_{2}=0.0064 \mathrm{ohm} ; X_{2}=0.02 \mathrm{ohm}
$$

Transformation ratio,

$$
K=\frac{V_{2}}{V_{1}}=\frac{220}{1100}=0.2
$$

Equivalent resistance referred to secondary side,

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=R_{2}+R_{1} \times K^{2}=0.0064+0.16 \times(0.2)^{2}=0.0128 \mathrm{ohm}
$$

Equivalent reactance referred to secondary side,

$$
X_{\mathrm{es}}=X_{2}+X_{1}^{\prime}=X_{2}+X_{1} \times K^{2}=0.02+0.5 \times(0.2)^{2}=0.04 \mathrm{ohm}
$$

## Example 10.15

A $33 \mathrm{kVA}, 2200 / 220 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase transformer has the following parameters. Primary winding resistance $r_{1}=2.4 \Omega$, Leakage reactance $x_{1}=6 \Omega$ Secondary winding resistance $r_{2}=0.03 \Omega$ Leakage reactance $x_{2}=0.07 \Omega$. Then, find primary, secondary, and equivalent resistance and reactance.
(P.T.U. Dec. 2009)

## Solution:

Here, rating of transformer $=33 \mathrm{kVA} ; V_{1}=2200 \mathrm{~V} ; V_{2}=220 \mathrm{~V}$

$$
f=50 \mathrm{~Hz} ; R_{1}=2.4 \Omega ; X_{1}=6 \Omega ; R_{2}=0.03 \Omega ; X_{2}=0.07 \Omega
$$

Transformation ratio,

$$
K=\frac{V_{2}}{V_{1}}=\frac{220}{2200}=0.1
$$

Transformer resistance referred to primary side

$$
R_{\mathrm{ep}}=R_{1}+R_{2}^{\prime}=R_{1}+\frac{R_{2}}{K^{2}}=2.4+\frac{0.03}{(0.1)^{2}}=2.4+3=5.4 \Omega
$$

Transformer reactance referred to primary side

$$
X_{\mathrm{ep}}=X_{1}+X_{2}^{\prime}=X_{1}+\frac{X_{2}}{K^{2}}=6+\frac{0.07}{(0.1)^{2}}=6+7=13 \Omega
$$

Transformer resistance referred to secondary side

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=R_{2}+R_{1} \times K^{2}=0.03+2.4 \times(0.1)^{2}=0.054 \Omega
$$

Transformer reactance referred to secondary side

$$
X_{\mathrm{es}}=X_{2}+X_{1}^{\prime}=X_{2}+X_{1} \times K^{2}=0.07+6 \times(0.1)^{2}=0.13 \Omega
$$

## Example 10.16

A single-phase transformer having voltage ratio $2500 / 250 \mathrm{~V}$ (primary to secondary) has a primary resistance and reactance 1.8 ohm and 4.2 ohm , respectively. The corresponding secondary values are 0.02 and 0.045 ohm . Determine the total resistance and reactance referred to secondary side. Also, calculate the impedance of transformer referred to secondary side.

## Solution:

Here,

$$
R_{1}=1.8 \Omega ; X_{1}=4.2 \Omega ; R_{2}=0.02 \Omega ; X_{2}=0.045 \Omega
$$

Transformation ratio,

$$
K=\frac{V_{2}}{V_{1}}=\frac{250}{2500}=0.1
$$

Total resistance referred to secondary side,

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=R_{2}+R_{1} \times K^{2}=0.02+1.8 \times(0.1)^{2}=0.038 \Omega
$$

Total reactance referred to secondary side,

$$
X_{\mathrm{es}}=X_{2}+X_{1}^{\prime}=X_{2}+X_{1} \times K^{2}=0.045+4.2 \times(0.1)^{2}=0.087 \Omega
$$

Impedance of transformer referred to secondary side,

$$
Z_{\mathrm{es}}=\sqrt{\left(R_{\mathrm{es}}\right)^{2}+\left(X_{\mathrm{es}}\right)^{2}}=\sqrt{(0.038)^{2}+(0.087)^{2}}=0.095 \Omega
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. Even at no-load, a transformer draws current from the mains. Why?
2. When a transformer is at no-load, it is said that it acts as a load on the system. How?
3. What do you mean by exciting resistance and exciting reactance?
4. The power factor of a transformer at no-load is very low. Why?
5. What is the magnetizing current of transformer required for?
(P.T.U. May 2007)
6. What do you know about reactance in a transformer?
(P.T.U. Dec. 2009)
7. When load current of a transformer increases, how does the input current adjust to meet the new conditions?
(U.P.T.U. May 2010)
8. How does leakage flux occur in a transformer?
(U.P.T.U. Dec. 2011)
9. 'The main flux in a transformer remains practically invariable under all conditions of load'. Explain.
(U.P.T.U. Dec. 2009)

## Test Questions

1. Explain the principle of working of a transformer. Draw its phasor diagram on no-load.
(P.T.U. Dec. 2008)
2. What do you know about no-load current of a transformer and upon what factors does it depend?
3. Explain how flux in the transformer core remains fairly constant from no-load to full load (ampereturns balance).
(May 1994, 1997)
4. Draw and explain the phasor diagram of a transformer on load, neglecting winding resistance and reactance, at unity and lagging power factor.
5. Explain the working of a transformer at no-load and on load conditions.
(U.P.T.U. May 2006)
6. What should be the value of primary resistance when referred to secondary side?
7. What is the effect of leakage flux in a transformer?
8. Derive the value of total impedance of a transformer referred to primary side.

## Numericals

1. A single-phase, $50 \mathrm{kVA}, 2300 / 230 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer is connected to 230 V supply on the secondary side, the primary being open. The metre indicates the following readings: Power $=$ 187 W ; voltage $=230 \mathrm{~V}$; current $=6.5 \mathrm{~A}$. Find (i) core loss; (ii) loss component of the current; (iii) magnetising current.
(Ans. $187 \mathrm{~W} ; 0.813 \mathrm{~A} ; 6.45 \mathrm{~A})$
2. The no-load current of a transformer is 5 A at 0.25 p.f when supplied at $335 \mathrm{~V}, 50 \mathrm{~Hz}$. The number of turns on the primary winding is 200 . Calculate (a) the maximum value of flux in the core, (b) the core loss (c) the magnetising component of current, and (d) exciting resistance and reactance.
(N.U.) (Ans. $7.545 \mathrm{mWb} ; 418.75 \mathrm{~W} ; 4.84 \mathrm{~A} ; 268 \Omega, 69.2 \Omega$ )
3. The number of turns on the primary and secondary winding of a single phase transformer are 350 and 38 , respectively. If the primary winding is connected to $2.2 \mathrm{kV}, 50 \mathrm{c} / \mathrm{s}$ supply, determine (i) the secondary voltage on no-load. (ii) The primary current when secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging. (iii) the p.f. of the primary current.
(Ans. $238.8 \mathrm{~V}, 25.67 \mathrm{~A}, 0.7156 \mathrm{lag})$
4. The number of turns on the primary and secondary windings of a single-phase transformer are 350 and 38 , respectively. If the primary winding is connected to $2.2 \mathrm{kV}, 50 \mathrm{c} / \mathrm{s}$ supply, determine
(i) the secondary voltage on no-load.
(ii) the primary current when the secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 power factor lagging
(iii) the power factor of the primary current.
(Ans. 239 V; $25.65 \mathrm{~A} ; 0.715 \mathrm{lag}$ )
5. A single-phase transformer takes a no-load current of 4 A at a p.f. of 0.24 lagging. The ratio of turns in the primary to secondary is 4 . Find the current taken by the transformer primary when the secondary supplies a load current of 240 A at a power factor 0.9 lagging.
(B.U.) (Ans. 62.58 A$)$
6. A $2000 / 200 \mathrm{~V}$ transformer has primary resistance and reactance of 2 ohm and 4 ohm , respectively. The corresponding secondary values are 0.025 ohm and 0.04 ohm. Determine:
(i) Equivalent resistance and reactance of primary referred to secondary; (ii) Total resistance and reactance referred to secondary; (iii) Equivalent resistance and reactance of secondary referred to primary; (iv) Total resistance and reactance referred to primary.
(Ans. $0.02 \Omega ; 0.04 \Omega ; 0.045 \Omega ; 0.08 \Omega ; 2.5 \Omega ; 4 \Omega ; 4.5 \Omega ; 8 \Omega$ )
7. A $2000 / 200 \mathrm{~V}$ transformer has a primary resistance 2.3 ohm and reactance 4.2 ohm , the secondary resistance 0.025 ohm and reactance 0.04 ohm . Determine total resistance and reactance referred to primary side.
(Ans. $4.8 \Omega ; 8.2 \Omega$ )
8. A transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 ohm and 0.1 ohm , respectively. The leakage reactance of the primary and secondary are 1.1 ohm and 0.035 ohm, respectively. Calculate the equivalent impedance referred to the primary circuit.
(Ans. $3.426 \Omega$ )

### 10.14 ACTUAL TRANSFORMER

An actual transformer has (i) primary and secondary resistances $R_{1}$ and $R_{2}$, (ii) primary and secondary leakage reactance $X_{1}$ and $X_{2}$ (iii) iron and copper losses and (iv) exciting resistance $R_{0}$ and exciting reactance $X_{0}$. The equivalent circuit of an actual transformer is shown in Figure 10.23.

Primary impedance, $\bar{Z}_{1}=R_{1}+j X_{1}$


Fig. 10.23 Equivalent circuit of an actual transformer on load

Supply voltage is $V_{1}$. The resistance and leakage reactance of primary winding are responsible for some voltage drop in primary winding.

$$
\therefore \quad \bar{V}_{1}=\bar{E}_{1}+\bar{I}_{1}\left(R_{1}+j X_{1}\right)=\bar{E}_{1}+\bar{I}_{1} \bar{Z}_{1}
$$

Where

$$
\bar{I}_{1}=\bar{I}_{1}^{\prime}+\bar{I}_{0}
$$

Secondary impedance,

$$
\bar{Z}_{2}=R_{2}+j X_{2}
$$

Similarly, the resistance and leakage reactance of secondary winding are responsible for some voltage drop in secondary winding. Hence,

$$
\bar{V}_{2}=\bar{E}_{2}-\bar{I}_{2}\left(R_{2}+j X_{2}\right)=\bar{E}_{2}-\bar{I}_{2} \bar{Z}_{2}
$$

The phasor (vector) diagrams of an actual transformer for resistive, inductive, and capacitive loads are shown in Figure 10.24(a), 10.24(b), and 10.24(c), respectively. The drops in resistances are drawn in phase with current vectors and drops in reactance are drawn perpendicular to the current vectors.


Fig. 10.24 Phasor diagram of an actual transformer at load (a) for resistive load (b) for inductive load (c) for capacitive load

### 10.15 SIMPLIFIED EQUIVALENT CIRCUIT

While drawing simplified circuit of a transformer, the exciting circuit (i.e., exciting resistance and exciting reactance) can be omitted.

The simplified equivalent circuit of a transformer is drawn by representing all the parameters of the transformer either on the secondary or on the primary side. The no-load current $I_{0}$ is neglected as its value is very small as compared to full-load current, therefore, $I_{1}^{\prime}=I_{1}$

### 10.15.1 Equivalent Circuit When All the Quantities Are Referred to Secondary

The primary resistance when referred to secondary side, its value is $R_{1}^{\prime}=K^{2} R_{1}$ and the total or equivalent resistance of transformer referred to secondary, $R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}$. Similarly, the primary reactance when referred to secondary side, its value is $X_{1}^{\prime}=K^{2} X_{1}$ and the total or equivalent reactance of transformer referred to secondary, $X_{\text {es }}=X_{2}+X_{1}^{\prime}$. All the quantities when referred to the secondary side are shown in Figure 10.25.


Fig. 10.25 (a) and (b) Simplified equivalent circuit of a loaded transformer when all quantities are referred to secondary side

Total or equivalent impedance referred to secondary side,

$$
Z_{\mathrm{es}}=R_{\mathrm{es}}+j X_{\mathrm{es}}
$$

There is some voltage drop in resistance and reactance of transformer referred to secondary. Hence,

$$
\bar{V}_{2}=\bar{E}_{2}-\bar{I}_{2}\left(R_{\mathrm{es}}+j X_{\mathrm{es}}\right)=\bar{E}_{2}-\bar{I}_{2} \bar{Z}_{\mathrm{es}}
$$

## Phasor diagrams

The phasor (vector) diagrams of a loaded transformer when all the quantities are referred to secondary side for resistive, inductive, and capacitive loads are shown in Figure 10.26(a), 10.26 (b), and 10.26 (c), respectively. The voltage drops in resistances (vectors) are taken parallel to the current vector and the voltage drops in reactance (vectors) are taken quadrature to the current vector.


Fig. 10.26 Phasor diagram of a loaded transformer when all quantities are referred to secondary side (a) for resistive load (b) for inductive load (c) for capacitive load

### 10.15.2 Equivalent Circuit When All the Quantities Are Referred to Primary

In this case, to draw the equivalent circuit, all the quantities are to be referred to primary, as shown in Figure 10.27.


Fig. 10.27 (a) and (b) Simplified equivalent circuit of a loaded transformer when all quantities are referred to primary side

Secondary resistance referred to primary, $R_{2}^{\prime}=R_{2} / K^{2}$
Equivalent resistance referred to primary, $R_{\text {ep }}=R_{1}+R_{2}^{\prime}$
Secondary reactance referred to primary, $X_{2}^{\prime}=X_{1} / K^{2}$
Equivalent reactance referred to primary, $X_{\mathrm{ep}}=X_{1}+X_{2}^{\prime}$
Total or equivalent impedance referred to primary side,

$$
Z_{\mathrm{ep}}=R_{\mathrm{ep}}+j X_{\mathrm{ep}}
$$

There is some voltage drop in resistance and reactance of the transformer referred to primary side. Therefore,

$$
\bar{V}_{1}=\bar{E}_{1}+\bar{I}_{1}\left(R_{\mathrm{ep}}+j X_{\mathrm{ep}}\right)=\bar{E}_{1}+\bar{E}_{1} \bar{Z}_{\mathrm{ep}}
$$

## Phasor diagrams

The phasor diagram to transformer when all the quantities are referred to primary side for different types of loads are shown in Figure 10.28.


Fig. 10.28 Phasor diagram of a loaded transformer when all quantities are referred to primary side (a) for resistive load (b) for inductive load (c) for capacitive load

### 10.16 EXPRESSION FOR NO-LOAD SECONDARY VOLTAGE

For a loaded transformer, when all the quantities are referred to secondary side, its phasor diagram can be drawn as shown in Figure 10.29.

Complete the phasor diagram as shown in Figure 10.29. From the phasor diagram, we can derive the approximate and exact expressions for no-load secondary voltage.

### 10.16.1 Approximate Expression

1. For lagging p.f. (inductive load), consider right-angled triangle OEC (Figure 8.29(a)).

$$
\begin{gathered}
\mathrm{OC}=\mathrm{OE}=\mathrm{OA}+\mathrm{AD}+\mathrm{DE}=\mathrm{OA}+\mathrm{AD}+\mathrm{BF} \\
E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}+I_{2} X_{\mathrm{es}} \sin \phi_{2}
\end{gathered}
$$

or
2. For unity p.f. (resistive load), consider right-angled triangle OBC (Figure. 10.29(b)).

$$
\mathrm{OC} \cong \mathrm{OB}=\mathrm{OA}+\mathrm{AB} ; E_{2}=V_{2}+I_{2} R_{\mathrm{es}}
$$

3. For leading p.f. (capacitive load), consider right-angled triangle OEC (Figure 10.29(c)).

$$
\mathrm{OC} \cong \mathrm{OE}=\mathrm{OA}+\mathrm{AD}-\mathrm{DE}=\mathrm{OA}+\mathrm{AD}-\mathrm{BF}
$$

or

$$
E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}-I_{2} X_{\mathrm{es}} \sin \phi_{2}
$$



Fig. 10.29 Phasor diagram of a loaded transformer where $\mathbf{V}_{2}$ is taken as reference vector (a) for inductive load (b) for resistive load (c) for capacitive load

### 10.16.2 Exact Expression

1. For a lagging p.f. (inductive load), consider right-angled triangle OHC (Figure 10.29(a)).

$$
\begin{aligned}
\mathrm{OC} & =\sqrt{(\mathrm{OH})^{2}+(\mathrm{HC})^{2}}=\sqrt{(\mathrm{OG}+\mathrm{GH})^{2}+(\mathrm{HB}+\mathrm{BC})^{2}} \\
& =\sqrt{(\mathrm{OG}+\mathrm{AB})^{2}+(\mathrm{GA}+\mathrm{BC})^{2}} \\
\text { or } \quad E_{2} & =\sqrt{\left(V_{2} \cos \phi_{2}+I_{2} R_{\mathrm{es}}\right)^{2}+\left(V_{2} \sin \phi_{2}+I_{2} X_{\mathrm{es}}\right)^{2}}
\end{aligned}
$$

Primary p.f., $\cos \phi_{1}=\frac{\mathrm{OH}}{\mathrm{OC}}=\frac{\mathrm{OG}+\mathrm{GH}}{\mathrm{OC}}=\frac{\mathrm{OG}+\mathrm{AB}}{\mathrm{OC}}=\frac{V_{2} \cos \phi_{2}+I_{2} R_{\mathrm{es}}}{E_{2}}$
2. For unity p.f., (resistive load), consider right-angled triangle OBC (Figure 10.29(b)).

$$
\mathrm{OC}=\sqrt{(\mathrm{OB})^{2}+(\mathrm{BC})^{2}}
$$

or

$$
\mathrm{OC}=\sqrt{(\mathrm{OA}+\mathrm{AB})^{2}+(\mathrm{BC})^{2}} \quad \text { or } \quad E_{2}=\sqrt{\left(V_{2}+I_{2} R_{\mathrm{es}}\right)^{2}+\left(I_{2} X_{\mathrm{es}}\right)^{2}}
$$

Primary p.f., $\cos \phi=\frac{\mathrm{OB}}{\mathrm{OC}}=\frac{\mathrm{OA}+\mathrm{AB}}{\mathrm{OC}}=\frac{V_{2}+I_{2} R_{\mathrm{es}}}{E_{2}}$
3. For leading p.f. (capacitive load), consider right-angled triangle OHC (Figure 10.29(c)).

$$
\begin{aligned}
& \begin{aligned}
& \mathrm{OC}=\sqrt{(\mathrm{OH})^{2}+(\mathrm{HC})^{2}}=\sqrt{(\mathrm{OG}+\mathrm{GH})^{2}+(\mathrm{HB}-\mathrm{BC})^{2}} \\
&=\sqrt{(\mathrm{OG}+\mathrm{AB})^{2}+(\mathrm{GA}-\mathrm{BC})^{2}} \\
& \text { or } \\
& E_{2}=\sqrt{\left(V_{2} \cos \phi_{2}+I_{2} R_{\mathrm{es}}\right)^{2}+\left(V_{2} \sin \phi_{2}-I_{2} X_{\mathrm{es}}\right)^{2}} \\
& \text { Primary p.f., } \cos \phi_{1}=\frac{\mathrm{HC}}{\mathrm{OC}}=\frac{\mathrm{OG}+\mathrm{GH}}{\mathrm{OC}}=\frac{\mathrm{OG}+\mathrm{AB}}{\mathrm{OC}}=\frac{V_{2} \cos \phi_{2}+I_{2} R_{\mathrm{es}}}{E_{2}}
\end{aligned}
\end{aligned}
$$

### 10.17 VOLTAGE REGULATION

When a transformer is loaded, with a constant supply voltage, the terminal voltage changes due to voltage drop in the internal parameters of the transformer, that is, primary and secondary resistances and inductive reactances. The internal voltage drop also depends upon the load and its power factor. The algebraic difference between the no-load and full-load terminal voltage is measured in terms of voltage regulation.

At a constant supply voltage, the change in secondary terminal voltage from no-load to full load with respect to no-load voltage is called voltage regulation of the transformer.

Let, $E_{2}=$ Secondary terminal voltage at no-load.
$V_{2}=$ Secondary terminal voltage at full load.
Then, voltage regulation $\quad=\frac{E_{2}-V_{2}}{E_{2}}$ (per unit)
In the form of percentage,

$$
\% \operatorname{Reg}=\frac{E_{2}-V_{2}}{E_{2}} \times 100
$$

When all the quantities are referred to the primary side of the transformer

$$
\% \operatorname{Reg}=\frac{V_{1}-E_{1}}{V_{1}} \times 100
$$

### 10.18 APPROXIMATE EXPRESSION FOR VOLTAGE REGULATION

The approximate expression for the no-load secondary voltage is derived in Section 16.1.
For inductive load

$$
E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}+I_{2} X_{\mathrm{es}} \sin \phi_{2}
$$

or

$$
E_{2}-V_{2}=I_{2} R_{\mathrm{es}} \cos \phi_{2}+I_{2} X_{\mathrm{es}} \sin \phi_{2}
$$

or

$$
\frac{E_{2}-V_{2}}{E_{2}} \times 100=\frac{I_{2} R_{\mathrm{es}}}{E_{2}} \times 100 \cos \phi_{2}+\frac{I_{2}-X_{\mathrm{es}}}{E_{2}} \times 100 \sin \phi_{2}
$$

where,

$$
\frac{I_{2} X_{\mathrm{es}}}{E_{2}} \times 100=\text { percentage resistance drop and }
$$

$$
\frac{I_{2} X_{\mathrm{es}}}{E_{2}} \times 100=\text { percentage reactance drop }
$$

$\therefore \quad \%$ Reg $=\%$ resistance drop $\times \cos \phi_{2}+\%$ reactance drop $\times \sin \phi_{2}$

## Similarly

(ii) For resistive load: \% Reg $=\%$ resistance drop
(iii) For capacitive load

$$
\therefore \quad \% \text { Reg }=\% \text { resistance drop } \times \cos \phi_{2}-\% \text { reactance drop } \times \sin \phi_{2}
$$

## Example 10.17

A $10 \mathrm{kVA}, 2000 / 400 \mathrm{~V}$, single-phase transformer has resistance and leakage reactance as follows:
Primary winding: Resistance $=5.5 \Omega$ Reactance $=12 \Omega$
Secondary winding: Resistance $=0.2 \Omega$, Reactance $=0.45 \Omega$
Determine the value of the secondary voltage at full load, 0.8 p.f. lagging, when the primary supply voltage is 2000 V .

## Solution:

Transformer rating $=10 \mathrm{kVA}=10 \times 10^{3} \mathrm{VA}$
Primary induced voltage, $E_{1}=2000 \mathrm{~V}$
Secondary induced voltage, $E_{2}=400 \mathrm{~V}$
Primary resistance, $R_{1}=5.5 \Omega$; Primary reactance, $X_{1}=12 \Omega$
Secondary resistance, $R_{2}=0.2 \Omega$; Secondary reactance, $X_{2}=0.45 \Omega$
Load p.f.,

$$
\cos \phi_{2}=0.8 \text { lagging }
$$

Transformation ratio,

$$
K=\frac{E_{2}}{E_{1}}=\frac{400}{2000}=0.2
$$

Primary resistance referred to secondary side,

$$
R_{1}^{\prime}=K^{2} R_{1}=(0.2)^{2} \times 5.5=0.22 \Omega
$$

Equivalent resistance referred to secondary side,

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=0.2+0.22=0.42 \Omega
$$

Primary reactance referred to secondary side,

$$
X_{1}^{\prime}=K^{2} X_{1}=(0.2)^{2} \times 12=0.48 \Omega
$$

Equivalent reactance referred to secondary side,

$$
X_{\mathrm{es}}=X_{2}+X_{1}^{\prime}=0.45+0.48=0.93 \Omega
$$

Load p.f.,

$$
\cos \phi_{2}=0.8 \therefore \sin \phi_{2}=\sin \cos ^{-1} 0.8=0.6
$$

Full-load secondary current, $\quad I_{2}=\frac{10 \times 10^{3}}{400}=25 \mathrm{~A}$
As the primary supply voltage,

$$
V_{1}=E_{1}=2000 \mathrm{~V}
$$

Secondary induced voltage, $\quad E_{2}=K E_{1}=0.2 \times 2000=400 \mathrm{~V}$
Using the expression; $\quad E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}-I_{2} X_{\mathrm{es}} \sin \phi_{2}$
Secondary terminal voltage,

$$
\begin{aligned}
V_{2} & =E_{2}-I_{2} R_{\mathrm{es}} \cos \phi_{2}-I_{2} X_{\mathrm{es}} \sin \phi_{2} \\
& =400-25 \times 0.42 \times 0.8-25 \times 0.93 \times 0.6=400-8.4-13.95=377.65 \mathrm{~A}
\end{aligned}
$$

## Example 10.18

The ratio of turns of a single-phase transformer is 8 , the resistance of the primary and the secondary windings are $0.85 \Omega$ and $0.012 \Omega$, respectively, and the leakage reactance of these windings are $4.8 \Omega$ and $0.07 \Omega$, respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary when the secondary terminal are short circuited. Ignore the magnetizing current.

## Solution:

Ratio of turns,

$$
\frac{N_{1}}{N_{2}}=8
$$

Primary resistance,

$$
R_{1}=0.85
$$

Primary reactance,

$$
X_{1}=4.8 \Omega
$$

Transformation ratio,

$$
K=\frac{N_{2}}{N_{1}}=\frac{1}{8}
$$

Secondary resistance

$$
R_{2}=0.012 \Omega
$$

Secondary reactance,

$$
X_{2}=0.07 \Omega
$$

Secondary resistance referred to primary, $\quad R_{2}^{\prime}=\frac{R_{2}}{K^{2}}=0.012 \times 8 \times 8=0.768 \Omega$
Equivalent resistance referred to primary, $R_{\mathrm{ep}}=R_{1}+R_{2}^{\prime}=0.85+0.768=1.618 \Omega$
Secondary reactance referred to primary, $\quad X_{2}^{\prime}=\frac{X_{2}}{K^{2}}=0.07 \times 8 \times 8=4.48 \Omega$
Equivalent reactance referred to primary, $X_{\mathrm{ep}}=X_{1}+X_{2}^{\prime}=4.8+4.48=9.28 \Omega$
Equivalent impedance referred to primary, $Z_{\mathrm{ep}}=\sqrt{R_{\mathrm{ep}}^{2}+X_{\mathrm{ep}}^{2}}=\sqrt{(1.618)^{2}+(9.28)^{2}}$

$$
=9.42 \Omega
$$

Short circuit current referred to primary, $I_{1(\mathrm{sc})}=K I_{2(\mathrm{sc})}=\frac{1}{8} \times 150=18.75 \mathrm{~A}$
Voltage applied to the primary under short circuit condition,

$$
V_{1(\mathrm{sc})}=I_{1(\mathrm{sc})} \times Z_{\mathrm{ep}}=18.75 \times 9.42=176.625 \mathrm{~V}
$$

## Example 10.19

The primary and secondary windings of a $40 \mathrm{kVA}, 6600 / 250 \mathrm{~V}$ single-phase transformer have resistance of $10 \Omega$ and $0.02 \Omega$, respectively. The total leakage reactance is $35 \Omega$ as referred to primary winding. Find full-load regulation of at a p.f. 0.8 lagging

## Solution:

Rating of transformer, $=40 \mathrm{kVA}=40 \times 10^{3} \mathrm{VA}$
Transformation ratio, $K=\frac{250}{6600}=0.03788$
Primary resistance, $\quad R_{1}=10 \Omega$
Secondary resistance, $\quad R_{2}=0.02 \Omega$
Total resistance, referred to primary side,

$$
R_{\mathrm{ep}}=R_{1}+R_{2}^{\prime}=R_{1}+\frac{R_{2}}{K^{2}}=10+\frac{0.02}{(0.03788)^{2}}=23.94 \Omega
$$

Total reactance referred to primary side,

$$
\begin{gathered}
X_{\mathrm{ep}}=35 \Omega \\
V_{1}=\sqrt{\left(E_{1} \cos \phi+I_{1} R_{\mathrm{ep}}\right)^{2}+\left(E_{1} \sin \phi+I_{1} X_{\mathrm{ep}}\right)^{2}}
\end{gathered}
$$

Where, $\quad I_{1}=\frac{40 \times 10^{3}}{6600}=6.06 \mathrm{~A}$

$$
\begin{aligned}
\cos \phi & =0.8 ; \sin \phi=\sin \cos ^{-1} 0.8=0.6 \\
\therefore \quad V_{1} & =\sqrt{(6600 \times 0.8+6.06 \times 23.94)^{2}+(6600 \times 0.6+6.06 \times 35)^{2}}=6843.7 \mathrm{~V} . \\
\% \operatorname{Reg} & =\frac{V_{1}-E_{1}}{V_{1}} \times 100=\frac{6843.7-6600}{6843.7} \times 100=3.56 \%
\end{aligned}
$$

## Example 10.20

A 75 kVA single-phase transformer, $6600 / 230 \mathrm{~V}$, requires 310 V across the primary to the primary to circulate full-load current on short circuit, the power absorbed being 1.6 kW . Determine the voltage regulation and the secondary terminal voltage for half full load, 0.8 p.f. lagging.

## Solution:

Transformer output $=75 \mathrm{kVA}=75 \times 10^{3} \mathrm{VA}$
Primary induced voltage, $E_{1}=6600 \mathrm{~V}$
Secondary induced voltage, $E_{2}=230 \mathrm{~V}$
At short circuit, primary voltage, $V_{1(\mathrm{sc})}=310 \mathrm{~V}$
At short circuit, power absorbed, $P_{(\mathrm{sc})}=1.6 \mathrm{~kW}=1.6 \times 10^{3} \mathrm{~W}$
Load p.f.

$$
\cos \phi_{2}=08 \text { lagging }
$$

Primary current at full load, $\quad I_{1}=\frac{75 \times 10^{3}}{6600}=11.36 \mathrm{~A}$
Primary current at short circuit $\quad I_{1(\mathrm{sc})}=I_{1}=11.36 \mathrm{~A}$
Equivalent resistance referred to primary, $R_{\mathrm{ep}}=\frac{P_{\mathrm{sc}}}{\left(I_{\mathrm{lsc}}\right)^{2}}=\frac{1.6 \times 10^{3}}{(11.36)^{2}}=12.39 \Omega$
Equivalent impedance referred to primary, $Z_{\mathrm{ep}}=\frac{V_{\mathrm{lsc}}}{I_{\mathrm{lsc}}}=\frac{310}{11.36}=27.29 \Omega$
Equivalent reactance referred to primary, $X_{\mathrm{ep}}=\sqrt{Z_{\mathrm{ep}}^{2}-R_{\mathrm{ep}}^{2}}=\sqrt{(27.29)^{2}-(12.39)^{2}}=24.32 \Omega$

Transformation ratio,

$$
K=\frac{230}{6600}
$$

Equivalent resistance referred to secondary.

$$
R_{\mathrm{es}}=K^{2} R_{\mathrm{ep}}=\frac{230 \times 230 \times 12.39}{6600 \times 6600}=0.015 \Omega
$$

Secondary current at full load, $\quad I_{2}=\frac{75 \times 10^{3}}{230}=326 \mathrm{~A}$
Secondary current at half load, $\quad I_{2 \mathrm{hl}}=\frac{I_{2}}{2}=\frac{326}{2}=163 \mathrm{~A}$
Load p.f.,

$$
\cos \phi_{2}=0.8 \text { lag } \therefore \sin \phi_{2}=\sin \cos ^{-1} 0.8=0.6
$$

Secondary terminal voltage at half full load,

$$
\begin{aligned}
V_{2} & =E_{2}-I_{2 \mathrm{hl}} R_{\mathrm{es}} \cos \phi_{2}-I_{2 \mathrm{hl}} X_{\mathrm{es}} \sin \phi_{2} \\
& =230-163 \times 0.015 \times 0.8-163 \times 0.0295 \times 0.6=225.16 \mathrm{~V}
\end{aligned}
$$

Voltage regulation $=\frac{E_{2}-V_{2}}{E_{2}} \times 100=\frac{230-225.16}{230} \times 100=2.1 \%$

## Example 10.21

A $20 \mathrm{kVA}, 2500 / 500 \mathrm{~V}$, single-phase transformer has the following parameters:

$$
\begin{array}{ll}
\text { H.V. winding } & \text { L.V. winding } \\
r_{\mathrm{i}}=8 \Omega & r_{2}=0.3 \Omega \\
x_{1}=17 \Omega & x_{2}=0.7 \Omega
\end{array}
$$

Find the voltage regulation and secondary terminal voltage at full load for a power factor of
(i) 0.8 lagging
(ii) 0.8 leading

The primary voltage is held constant at 2500 V .
(P.T.U. May 2006)

## Solution:

Here, rating of transformer $=20 \mathrm{kVA} ; E_{1}=2500 \mathrm{~V} ; E_{2}=500 \mathrm{~V}$

$$
R_{1}=8 \Omega ; X_{1}=17 \Omega ; R_{2}=0.3 \Omega ; X_{2}=0.7 \Omega
$$

Transformer resistance referred to L.V. (secondary) side

$$
\begin{aligned}
& R_{\mathrm{es}}=R_{2}+R_{1} \times K^{2}=0.3+8 \times\left(\frac{500}{2500}\right)^{2}=0.62 \Omega \\
& X_{\mathrm{es}}=X_{2}+X_{1} \times K^{2}=0.7+17 \times(0.2)^{2}=1.38 \Omega
\end{aligned}
$$

Secondary full-load current, $I_{2}=\frac{20 \times 1000}{500}=40 \mathrm{~A}$
For p.f. 0.8 lagging, $\cos \phi=0.8 ; \sin \phi=\sin \cos ^{-1} 0.8=0.6$

$$
\begin{aligned}
V_{2} & =E_{2}-I_{2} R_{\mathrm{es}} \cos \phi-I_{2} X_{\mathrm{es}} \sin \phi \\
& =500-40 \times 0.62 \times 0.8-40 \times 1.38 \times 0.6=447.04 \mathrm{~V} \\
\% \mathrm{Reg} & =\frac{E_{2}-V_{2}}{E_{2}} \times 100=\frac{500-447.04}{500} \times 100=10.59 \%
\end{aligned}
$$

For p.f. 0.8 leading, $\cos \phi=0.8 ; \sin \phi=\sin \cos ^{-1} 0.8=0.6$

$$
\begin{aligned}
V_{2} & =E_{2}-I_{2} R_{\mathrm{es}} \cos \phi+I_{2} X_{\mathrm{es}} \sin \phi \\
& =500-40 \times 0.62 \times 0.8+40 \times 1.38 \times 0.6=513.28 \mathrm{~V} \\
\% \mathrm{Reg} & =\frac{E_{2}-V_{2}}{E_{2}} \times 100=\frac{500-513.28}{500} \times 100=-2.6 \%
\end{aligned}
$$

## Example 10.22

A $10 \mathrm{kVA}, 500 / 100 \mathrm{~V}$ transformer has the following circuit parameters referred to primary: Equivalent resistance, $R_{\text {eq }}=0.3 \Omega$; Equivalent reactance, $X_{\text {eq }}=5.2 \Omega$. When supplying power to a lagging load, the current, power, and voltage measured on primary side were $20 \mathrm{~A}, 8 \mathrm{Kw}$, and 500 V , respectively. Calculate the voltage on the secondary terminals under these conditions. Draw the relevant phasor diagram.

## Solution:

Rating of transformer $=10 \mathrm{kVA}=10 \times 10^{3} \mathrm{VA}$
Power factor on primary side, $\cos \phi_{1}=\frac{P}{V_{1} I_{1}}=\frac{8 \times 10^{3}}{500 \times 20}=0.8 \mathrm{lag}$

$$
\sin \phi_{1}=\sin \cos ^{-1} 0.8=0.6
$$

Primary induced emf, $E_{1}=V_{1}-I_{1} R_{\mathrm{ep}} \cos \phi_{1}-I_{1} X_{\mathrm{ep}} \sin \phi_{1}$

$$
=\sin 500-20 \times 0.3 \times 0.8-20 \times 5.2 \times 0.6=432.8 \mathrm{~V}
$$

Transformation ratio, $K=\frac{100}{500}=0.2$
Secondary induced emf, $E_{2}=E_{1} \times K=432.8 \times 0.2=86.56 \mathrm{~V}$
Secondary terminal voltage on load,
$V_{2}=E_{2}=86.56 \mathrm{~V}$ (since all the parameters are referred to primary side)
The equivalent circuit and phasor diagram is shown in Figure 10.30(a) and (b), respectively.


Fig. 10.30 (a) Equivalent circuit of the given transformer (b) Phasor diagram

## Example 10.23

Calculate the regulation of transformer in which ohmic loss is $1 \%$ of the output and reactance drop $5 \%$ of the voltage when the power factor is (i) 0.8 lagging, (ii) 0.8 leading, and (iii) unity.

## Solution:

Ohmic loss or resistance drop $=1 \%$; reactance drop $=5 \%$
(i) When p.f., $\cos \phi_{2}=0.8$ lagging; $\sin \phi_{2}=\sin \cos ^{-1} 0.8=0.6$
$\%$ Reg $=\%$ resistance drop $\times \cos \phi_{2}+\%$ reactance drop $\times \sin \phi_{2}$

$$
=1 \times 0.8+5 \times 0.6=3.8 \%
$$

(ii) When p.f., $\cos \phi_{2}=0.8$ leading; $\sin \phi_{2}=\sin \cos ^{-1} 0.8=0.6$
$\%$ Reg $=\%$ resistance drop $\times \cos \phi_{2}-\%$ reactance drop $\times \sin \phi_{2}$
$=1 \times 0.8-5 \times 0.6=-2.2 \%$
(iii) When p.f. is unity $\% \operatorname{Reg}=\%$ resistance drop $=1 \%$

## 国改 PRACTICE EXERCISES

## Short Answer Questions

1. What is voltage regulation of a transformer?
(P.T.U. Dec. 2009, May 2010)
2. Define regulation of a transformer and give its importance.
(P.T.U. Dec. 2010)
3. Why does voltage drop in a transformer?
4. How does the approximate equivalent circuit of a transformer differ from its exact equivalent circuit?
5. Is the regulation at rated load of a transformer same at 0.8 p.f. lagging and 0.8 p.f. leading?
(U.P.T.U. May 2000)
6. For negative voltage regulation, which type of load should be connected to a transformer?
7. Is the percentage impedance of a transformer same on primary and on secondary?
(AMIE Summer 97)
8. What is meant by equivalent circuit of a transformer?

## Test Questions

1. Draw the phasor diagram of a single-phase transformer on-load and describe it.
(P.T.U. May 2007)
2. Draw and discuss the phasor diagram of a transformer on load, when it is taking a resistive load.
(P.T.U. Dec. 2010)
3. Draw and briefly explain about transformer equivalent circuit.
(P.T.U. Dec. 2009; U.P.T.U. Nov. 2006)
4. Draw and explain equivalent circuit of a transformer, and hence, draw the phasor diagram for resistive loading of transformer.
(P.T.U. May 2011)
5. Draw a complete phasor diagram for a transformer, when the load p.f. is lagging.
(U.P.T.U. Dec. 2008)
6. Draw and explain the phasor diagram of a transformer on load. How it affects the power factor of the loaded transformer?
(P.U. May 2009)
7. Draw approximate equivalent circuit of a transformer referred to primary side and indicate how it differs from the exact equivalent circuit.
(A.M.I.E., Summer 96)
8. Define voltage regulation of a transformer and deduce expression for voltage regulation.
(U.P.T.U. June 2006)
9. What is regulation? How it can be obtained from equivalent circuit parameters? (P.U. July 98)
10. Explain what is meant by regulation of a transformer.
(P.T.U. Dec. 2007)

## Numericals

1. A single-phase transformer with a ratio $1: 2$ has primary resistance of $0.25 \Omega$ and reactance of $0.5 \Omega$ and the corresponding values for the secondary are $0.8 \Omega$ and $1.8 \Omega$, respectively. Determine the no-load secondary terminal voltage of the transformer, if it is delivering 10 A and 400 V at 0.8 p.f. lagging.
(Ans. 424.6 V )
2. A $230 / 460 \mathrm{~V}$, single-phase transformer has a primary resistance of $0.2 \Omega$ and reactance $0.5 \Omega$. The corresponding values for the secondary are $0.75 \Omega$ and $1.8 \Omega$, respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging. (B.U. Feb. 1988) (Ans. 424.8 V )
3. A $10 \mathrm{kVA}, 2000 / 400 \mathrm{~V}$ single-phase transformer has the following resistances and reactances.

Primary winding: resistance $5.0 \Omega$, leakage reactance $12 \Omega$.
Secondary winding: resistance $0.2 \Omega$, leakage reactance $0.48 \Omega$. Determine the secondary terminal voltage at full load, 0.8 power factor lagging when the primary supply is 2000 V .
(Ans. 377.1 V)
4. A $5 \mathrm{kVA}, 200 / 400 \mathrm{~V}$, single-phase transformer gave the following test results.

Secondary short circuited: 8 V applied to primary circulating full-load current in the secondary, power 80 W .
Calculate the secondary terminal voltage at full load (i) at unity p.f. (ii) p.f. 0.8 lagging (iii) 0.8 leading.
(Ans. $393.5 \mathrm{~V} ; 386.1 \mathrm{~V} ; 402.9 \mathrm{~V}$ )
5. A $230 / 440 \mathrm{~V}$ transformer has a primary resistance of $0.25 \Omega$ and a reactance of $0.6 \Omega$ and the corresponding value for the secondary are $0.8 \Omega$ and $1.8 \Omega$, respectively. Find the approximate secondary terminal voltage when supplying (i) 10 A at 0.707 p.f. lagging (ii) 10 A at 0.707 p.f. leading.
(Ans. $359.6 \mathrm{~V} ; 416.15 \mathrm{~V}$ )
6. The primary and secondary windings of a $30 \mathrm{kVA}, 6000 / 230 \mathrm{~V}$ transformer have resistance of $10 \Omega$ and $0.016 \Omega$, respectively. The reactance of the transformer as referred to primary is $34 \Omega$. Calculate (i) primary voltage required to circulate full-load current when the secondary is short-circuited and
(ii) the percentage regulation of the transformer when delivering full load at a power factor of 0.8 lagging.
(Ans. 199.5 V; 3.1\%)
7. A 100 kVA transformer has 400 turns in the primary and 80 turns in the secondary. The primary and secondary resistances are $0.3 \Omega$ and $0.01 \Omega$, respectively, and the corresponding leakage reactances are $1.1 \Omega$ and $0.035 \Omega$, respectively. The supply voltage is 2200 V . Calculate
(i) total impedance of the transformer referred to primary circuit and (ii) the voltage regulation and the secondary terminal voltage for full-load having power factors
(i) 0.8 lagging and (ii) 0.8 leading.
(Ans. $2.05 \Omega ; 3.364 \% ; 425.2 \mathrm{~V} ;-1.54 \% ; 446.78 \mathrm{~V}$ )
8. A single-phase transformer with a ratio of $4: 1$ has primary resistance and reactance of $0.5 \Omega$ and $1.5 \Omega$ and the corresponding values for the secondary are $0.034 \Omega$ and $0.1 \Omega$, respectively. Determine the percentage regulation when delivering 120 A at 600 V at p.f. (i) 0.707 lagging (ii) 0.8 leading.
(Ans. 3.63\%; -1.3\%)
9. A $17.28 \mathrm{MVA}, 66 / 11 \mathrm{kV}$ transformer has $1 \%$ resistance and $1 \%$ leakage reactance drop.

Find resistance and reactance of the transformer in ohms as referred to
(i) the high-voltage winding.
(ii) the low-voltage winding.
(Ans. $2.52 \Omega ; 7.56 \Omega ; 0.068 \Omega ; 0.209 \Omega$ )
10. Calculate the value of voltage regulation at 0.8 p.f. lagging for a transformer with resistance drop $2 \%$ and reactance drop $4 \%$ of the voltage.
(Ans. 4.0\%)

### 10.19 LOSSES IN A TRANSFORMER

The losses that occur in an actual transformer are core or iron losses and copper losses.

1. Core or iron losses: When AC supply is given to the primary winding of a transformer, an alternating flux is set up in the core; therefore, hysteresis and eddy current losses occur in the magnetic core.
(a) Hysteresis loss: When the magnetic material is subjected to reversal of flux, power is required for the continuous reversal of molecular magnets. This power is dissipated in the form of heat and is known as hysteresis loss $\left(P_{\mathrm{h}}=K_{\mathrm{h}} V f B_{\mathrm{m}}^{1.6}\right)$. This loss can be minimized by using silicon steel material for the construction of core.
(b) Eddy current loss: Since flux in the core of a transformer is alternating, it links with the magnetic material of the core itself also. This induces an emf in the core and circulates eddy currents. Power is required to maintain these eddy currents. This power is dissipated in the form of heat and is known as eddy current loss $\left(P_{\mathrm{e}}=K_{\mathrm{e}} V f^{2} t^{2} B_{\mathrm{m}}^{2}\right)$. This loss can be minimized by making the core of thin laminations.
It is already seen in article-8 that the flux set up in the core of the transformer remains constant from no-load to full load. Hence, iron loss are independent of the load and are known as constant losses.
2. Copper losses: Copper losses occur in both primary and secondary windings due to their ohmic resistance. If $I_{1}, I_{2}$ are the primary and secondary currents and $R_{1}, R_{2}$ are the primary and secondary resistances, respectively.

Then, total copper losses $=I_{1}^{2} R_{1}+I_{2}^{2} R_{2}=I_{1}^{2} R_{\text {ep }}=I_{2}^{2} R_{\text {es }}$
The currents in the primary and secondary winding vary according to the load, and therefore, these losses vary according to the load and are known as variable losses.

### 10.20 EFFICIENCY OF A TRANSFORMER

The efficiency of a transformer is defined as the ratio of output to the input power, the two being measured in same units (either in watts or in kW ).
Transformer efficiency, $\eta=\frac{\text { Output power }}{\text { Input power }}=\frac{\text { Output power }}{\text { Output power }+ \text { Losses }}$
or

$$
\eta=\frac{\text { Output power }}{\text { Output power }+ \text { Iron losses }+ \text { Coper losses }}=\frac{V_{2} I_{2} \cos \phi_{2}}{V_{2} I_{2} \cos \phi_{2}+P_{\mathrm{i}}+P_{\mathrm{c}}}
$$

Where $V_{2}=$ Secondary terminal voltage
$I_{2}=$ Full-load secondary current
$\cos \phi_{2}=$ p.f. of the load
$P_{\mathrm{i}}=$ lron losses $=$ Hysteresis losses + eddy current losses (constant losses)
$P_{\mathrm{c}}=$ Full-load copper losses $=I_{2}^{2} R_{\text {es }}$
(variable losses)
If $x$ is the fraction of the full load, the efficiency of the transformer at this fraction is given by the relation

$$
\eta_{\mathrm{x}}=\frac{x \times \text { Output at full load }}{x \times \text { Output at full load } \times P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}}=\frac{x V_{2} I_{2} \cos \phi_{2}}{x V_{2} I_{2} \cos \phi_{2}+p_{\mathrm{i}}+x^{2} I_{2}^{2} R_{\mathrm{es}}}
$$

The copper losses vary as the square of the fraction of the load.

### 10.21 CONDITION FOR MAXIMUM EFFICIENCY

The efficiency of a transformer at a given load and p.f. is expressed by the relation

$$
\eta=\frac{V_{2} I_{2} \cos \phi_{2}}{V_{2} I_{2} \cos \phi_{2}+p_{\mathrm{i}}+I_{2}^{2} R_{\mathrm{es}}}=\frac{V_{2} \cos \phi_{2}}{V_{2} \cos \phi_{2}+P_{\mathrm{i}} / I_{2}+I_{2} R_{\mathrm{es}}}
$$

The terminal voltage $V_{2}$ is approximately constant. Thus, for a given p.f., efficiency depends upon the load current $I_{2}$. In expression (i), the numerator is constant and the efficiency will be maximum, if denominator is minimum. Thus, the maximum condition is obtained by differentiating the quantity in the denominator with respect to the variables $I_{2}$ and equating that to zero, that is,
or

$$
\begin{gathered}
\frac{d}{d I_{2}}\left(V_{2} \cos \phi_{2}+\frac{P_{\mathrm{i}}}{I_{2}}+I_{2} R_{\mathrm{es}}\right)=0 \\
0-\frac{P_{\mathrm{i}}}{I_{2}^{2}}+R_{\mathrm{es}}=0 \quad \text { or } \quad I_{2}^{2} R_{\mathrm{es}}=P_{\mathrm{i}} \\
\text { Copper losses }=\text { iron losses }
\end{gathered}
$$

Thus, the efficiency of a transformer will be maximum when copper (or variable) losses are equal to iron (or constant) losses.

$$
\therefore \quad \eta_{\max }=\frac{V_{2} I_{2} \cos \phi_{2}}{V_{2} I_{2} \cos \phi_{2}+2 P_{\mathrm{i}}}
$$

$$
\left(\text { since } P_{\mathrm{c}}=P_{\mathrm{i}}\right)
$$

From equation (ii), the value of output current $I_{2}$ at which the efficiency of the transformer will be maximum is given by

$$
I_{2}=\sqrt{\frac{P_{\mathrm{i}}}{R_{\mathrm{es}}}}
$$

If $x$ is the fraction of full-load kVA at which the efficiency of the transformer is maximum.
Then, copper losses $=x^{2} P_{\mathrm{c}}\left(\right.$ where $P_{\mathrm{c}}$ is the full-load Cu losses $)$
Iron losses $=P_{\mathrm{i}}$
For maximum efficiency,

$$
x^{2} P_{\mathrm{c}}=P_{\mathrm{i}} ; \quad x=\sqrt{\frac{P_{\mathrm{i}}}{P_{\mathrm{c}}}}
$$

$\therefore$ Output kVA corresponding to maximum efficiency

$$
\begin{aligned}
& =x \times \text { full-load } \mathrm{kVA}=\text { full-load } \mathrm{kVA} \times \sqrt{\frac{P_{\mathrm{i}}}{P_{\mathrm{c}}}} \\
& =\text { full-load kVA } \times \sqrt{\frac{\text { Iron losses }}{\text { Copper losses at full load }}}
\end{aligned}
$$

## Example 10.24

The primary and secondary windings of a 500 kVA transformer have resistance of $0.42 \Omega$ and $0.0011 \Omega$, respectively. The primary and secondary voltages are 600 V and 400 V , respectively. The iron loss is 2.9 kW . Calculate the efficiency at half full load at a power factor of 0.8 lagging.

## Solution:

Transformer rating, $=500 \mathrm{kVA}$
Primary resistance, $R_{1}=0.42 \Omega$
Secondary resistance, $R_{2}=0.0011 \Omega$
Primary voltage, $E_{1}=6600 \mathrm{~V}$
Secondary voltage, $E_{2}=400 \mathrm{~V}$
Iron losses, $P_{\mathrm{i}}=2.9 \mathrm{~kW}$
Fraction of the load, $x=\frac{1}{2}=0.5$
Load p.f., $\cos \phi=0.8$ lagging
Transformation ratio,

$$
K=\frac{E_{2}}{E_{1}}=\frac{400}{6600}=\frac{2}{33}
$$

Primary resistance referred to secondary, $R_{1}^{\prime}=K^{2} R_{1}=\frac{2}{33} \times \frac{2}{33} \times 0.42=0.00154 \Omega$
Total resistance referred to secondary, $R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=0.0011+0 \cdot 00154=0.00264 \Omega$
Full-load secondary current, $I_{2}=\frac{\mathrm{kVA} \times 10^{3}}{E_{2}}=\frac{500 \times 10^{3}}{400}=1250 \mathrm{~A}$
Copper losses at full load, $\quad P_{\mathrm{c}}=I_{2}^{2} R_{\mathrm{es}}=(1250)^{2} \times 0.00264$

$$
=4125 \mathrm{~W}=4.125 \mathrm{~kW}
$$

Efficiency of transformer at any fraction (x) of the load,

$$
\begin{aligned}
\eta_{\mathrm{x}} & =\frac{x \mathrm{kVA} \cos \phi}{x \mathrm{kVA} \cos \phi+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}} \times 100 \\
& =\frac{0.5 \times 500 \times 0.8}{0.5 \times 500 \times 0.8+2.9+(0.5)^{2} \times 4.125} \times 100=98.07 \%
\end{aligned}
$$

## Example 10.25

In a $25 \mathrm{kVA}, 2000 / 200 \mathrm{~V}$ power transformer, the iron and full-load copper losses are 350 W and 400 W , respectively. Calculate the efficiency at unity power factor at (i) full load and (ii) half load.

## Solution:

$$
\eta_{\mathrm{x}}=\frac{x \mathrm{kVA} \times 1000 \times \cos \phi}{x \mathrm{kVA} \times 1000 \times \cos \phi+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}}
$$

Where

$$
\cos \phi=1 ; P_{\mathrm{i}}=350 \mathrm{~W} ; P_{\mathrm{c}}=400 \mathrm{~W}
$$

(i) At full load $x=1$

$$
\therefore \quad \eta=\frac{1 \times 25 \times 1000 \times 1}{1 \times 25 \times 1000 \times 1+350+1 \times 1 \times 140} \times 100=97.087 \%
$$

(ii) At half-load; $x=0.5$

$$
\therefore \quad \eta=\frac{0.5 \times 25 \times 1000 \times 1}{0.5 \times 25 \times 1000 \times 1+350+(0.5)^{2} \times 400} \times 100=96.525 \%
$$

## Example 10.26

A $220 / 400 \mathrm{~V}, 10 \mathrm{kVA}, 50 \mathrm{~Hz}$, single-phase transformer has copper loss of 120 W at full load. If it has an efficiency of $98 \%$ at full load, unity power factor, determine the iron losses. What would be the efficiency of the transformer at half full load at 0.8 p.f. lagging?

## Solution:

$$
\begin{aligned}
& \eta_{\mathrm{x}}=\frac{x \mathrm{kVA} \times 1000 \times \cos \phi}{x \mathrm{kVA} \times 100 \times \cos \phi+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}} \times 100 \\
& 98=\frac{1 \times 10 \times 1000 \times 1}{1 \times 10 \times 1000 \times 1+P_{\mathrm{i}}+1 \times 1 \times 120} \times 100 \quad \text { or } \quad P_{\mathrm{i}}=84.08 \mathrm{~W}
\end{aligned}
$$

When

$$
\begin{aligned}
x & =1 / 2 \text { and } \cos \phi=0.8 \\
\eta_{\mathrm{x}} & =\frac{0.5 \times 10 \times 1000 \times 0.8}{0.5 \times 10 \times 1000 \times 0.8+84.08+(0.5)^{2} \times 120} \times 100=97.23 \%
\end{aligned}
$$

## Example 10.27

The efficiency of a 400 kVA , single-phase transformer is $98.77 \%$ when delivering full load at 0.8 power factor and $99.13 \%$ at half load and unity power factor. Calculate (i) the iron loss (ii) the full-load copper loss.

## Solution:

Efficiency of a transformer at any fraction $x$ of the load

$$
\eta_{\mathrm{x}}=\frac{x \mathrm{kVA} \cos \phi}{x \mathrm{kVA} \cos \phi+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}} \times 100
$$

Case I: $x=1 ; \cos \phi=0.8 ; \eta_{\mathrm{x}}=98.77 \%$

$$
\begin{array}{lc}
\therefore & 98.77=\frac{1 \times 400 \times 0.8}{1 \times 400 \times 0.8+P_{\mathrm{i}}+(1)^{2} P_{\mathrm{c}}} \times 100 \\
\therefore \text { or } & P_{\mathrm{i}}+P_{\mathrm{c}}=3.985 \mathrm{~kW}
\end{array}
$$

Case II: $x=0.5 ; \cos \phi=1 ; \eta_{\mathrm{x}}=99.13$

$$
\begin{align*}
\therefore & 99.13= \\
\text { or } & \frac{0.5 \times 400 \times 1}{0.5 \times 400 \times 1 \times P_{\mathrm{i}}+(0.5)^{2} P_{\mathrm{c}}} \times 100 \\
& P_{\mathrm{i}}+0.25 P_{\mathrm{c}}=1.755 \mathrm{~kW} \tag{10.6}
\end{align*}
$$

Subtracting eq. (10.6) from (10.5), we get

$$
0.75 P_{\mathrm{c}}=2.23 \mathrm{~kW} \text { or } P_{\mathrm{c}}=2.973 \mathrm{~kW}
$$

and

$$
P_{\mathrm{i}}=3.985-2.973=1.012 \mathrm{~kW}
$$

## Example 10.28

A 440/110 V transformer has an effective primary resistance of $0.3 \Omega$ and a secondary resistance of $0.02 \Omega$. If iron loss on normal input voltage is 150 W , calculate the secondary current at which maximum efficiency will occur. What is the value of this maximum efficiency for unity power factor load?

## Solution:

Primary resistance, $R_{1}=0.3 \Omega$
Secondary resistance, $R_{2}=0.02 \Omega$
Iron losses, $P_{\mathrm{i}}=150 \mathrm{~W}$
Load power factor, $\cos \phi=1$
Primary induced voltage, $E_{1}=440 \mathrm{~V}$
Secondary induced voltage, $E_{2}=110 \mathrm{~V}$
Transformation ratio, $\quad K=\frac{E_{2}}{E_{1}}=\frac{110}{440}=\frac{1}{4}$
Primary resistance referred to secondary,

$$
R_{1}^{\prime}=k^{2} R_{1}=\frac{1}{4} \times \frac{1}{4} \times 0.3=0.01875 \Omega
$$

Equivalent resistance referred to secondary,

$$
R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}=0.02+0.01875=0.03875 \Omega
$$

We know the condition for max, efficiency is
Copper losses $=$ Iron losses, that is, $I_{2}^{2} R_{\text {es }}=P_{\mathrm{i}}$
Secondary current at which the efficiency is maximum,

$$
I_{2}=\sqrt{\frac{P_{\mathrm{i}}}{R_{\mathrm{es}}}}=\sqrt{\frac{50}{0.03875}}=62.22 \mathrm{~A}
$$

The maximum efficiency, $\eta_{\max }=\frac{I_{2} V_{2} \cos \phi}{I_{2} V_{2} \cos \phi+2 P_{\mathrm{i}}} \times 100$

$$
=\frac{62.22 \times 110 \times 1}{62.22 \times 110 \times 1+2 \times 150} \times 10095.8 \%
$$

## Example 10.29

The efficiency of a $1000 \mathrm{kVA}, 110 / 220 \mathrm{~V}, 50 \mathrm{~Hz}$. Single-phase transformer is $98.5 \%$ at half load and 0.8 power factor leading and $98.9 \%$ at full-load unity power factor. Determine (i) iron loss and (ii) full-load copper loss.
(P.T.U. Dec. 2005)

## Solution:

Here, Rating of transformer $=1000 \mathrm{kVA}$
We know,

$$
\% \eta_{\mathrm{x}}=\frac{x \mathrm{kVA} \times 1000 \times p . f .}{x \mathrm{kVA} \times 1000 \times p . f .+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}} \times 100
$$

(i) Where $\% \eta_{0.5}=98.5 ; x=0.5 ;$ p.f. $=0.8$ leading

$$
\begin{array}{ll}
\therefore & 98.5=\frac{0.5 \times 1000 \times 1000 \times 0.8}{0.5 \times 1000 \times 1000 \times 0.8+P_{\mathrm{i}}+(0.5)^{2} P_{\mathrm{c}}} \times 100 \\
\text { or } & 98.5=\frac{400 \times 10^{5}}{4 \times 10^{5}+p_{\mathrm{i}}+0.25 P_{\mathrm{c}}} \quad \text { or } \quad P_{\mathrm{i}}+0.25 \mathrm{P}_{\mathrm{c}}=6100 \tag{10.7}
\end{array}
$$

(ii) When $\% \eta_{\mathrm{fi}}=98.8 ; x=1$; p.f. $=1$

$$
\begin{align*}
98.8 & =\frac{1 \times 1000 \times 1000 \times 1 \times 100}{1 \times 1000 \times 1000 \times 1+P_{\mathrm{i}}+P_{\mathrm{c}}} \\
\text { or } 10 \times 10^{5}+P_{\mathrm{i}}+P_{\mathrm{c}} & =\frac{10 \times 10^{5} \times 100}{98.8}=10.121 \times 10^{5} \quad \text { or } \quad P_{\mathrm{i}}+P_{\mathrm{c}}=12100 \tag{10.8}
\end{align*}
$$

Subtracting eq. (10.7) from (10.8), we get

$$
0.75 P_{\mathrm{c}}=6000 \text { or } P_{\mathrm{c}}=8000 \mathrm{~W}
$$

From eq. (10.8), we get $P_{\mathrm{i}}=12100-8000=4100 \mathrm{~W}$

## Example 10.30

A 50 kVA transformer on full load has a copper loss of 600 and iron loss of 500 W . Calculate the maximum efficiency and the load at which it occurs.

## Solution:

$$
\% \eta=\frac{\text { Output }}{\text { Output }+ \text { Iron loss }+ \text { Copper loss }} \times 100
$$

Efficiency will be maximum when Cu . loss $=$ lron loss $=500 \mathrm{~W}$
Fraction at which the efficiency is maximum, $x=\sqrt{\frac{P_{\mathrm{i}}}{P_{\mathrm{c}}}}=\sqrt{\frac{500}{600}}=0.9128$

Load at which the efficiency is maximum, that is,
Output

$$
\begin{aligned}
& =x \times \mathrm{kVA}=0.9128 \times 50=45.64 \mathrm{kVA} \\
& =45.64 \times 1=45.64 \mathrm{~kW}(\text { since } \cos \phi=1) \\
\eta_{\mathrm{m}} & =\frac{45.64 \times 1000}{45.64 \times 1000+500+500} \times 100=97.85 \%
\end{aligned}
$$

## Example 10.31

In a $25 \mathrm{kVA}, 1100 / 400 \mathrm{~V}$, single-phase transformer, the iron and copper losses at full load are 350 W and 400 W , respectively. Calculate the efficiency on unity power at half load.

Determine the load at which maximum efficiency occurs.
(May, 1984)

## Solution:

Transformer rating $=25 \mathrm{kVA}$
Iron losses, $P_{\mathrm{i}}=350 \mathrm{~W}$
Full-load copper losses, $P_{\mathrm{c}}=400 \mathrm{~W}$
Load power factor $\cos \phi=1$
Fraction of the load $x=\frac{1}{2}=0.5$
Efficiency of transformer at any fraction of the load,

$$
\begin{aligned}
\eta_{\mathrm{x}} & =\frac{x \times \mathrm{kVA} \times 12^{3} \times \cos \phi}{x \times \mathrm{kVA} \times 10^{3} \times \cos \phi+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}} \times 100 \\
& =\frac{0.5 \times 25 \times 10^{3} \times 1}{0.5 \times 25 \times 10^{3} \times 1+350+(0.5)^{2} \times 400} \times 100=96.52 \%
\end{aligned}
$$

Output kVA corresponding to maximum efficiency

$$
=\text { Rated kVA } \sqrt{\frac{P_{\mathrm{i}}}{P_{\mathrm{c}}}}=25 \times \sqrt{\frac{350}{400}}=23.385 \mathrm{kVA}
$$

Output power or load on maximum efficiency.

$$
\begin{aligned}
& =\text { output } \mathrm{kVA} \text { for max efficiency } \times \text { p.f. } \\
& =23.385 \times 1=23.385 \mathrm{~kW}
\end{aligned}
$$

## Example 10.32

A $100 \mathrm{kVA}, 2$ winding transformer has an iron loss of 1 kW and a cu loss on a normal output current of 1.5 kW . Calculate the kVA loading at which the efficiency is maximum and its efficiency at this loading: (i) at unit p.f. and (ii) at 8 p.f. lagging.

## Solution:

Here, rated capacity $=100 \mathrm{kVA}$; Iron loss, $P_{\mathrm{i}}=1 \mathrm{~kW}$
Full-load Cu loss, $P_{\mathrm{c}}=1.5 \mathrm{~kW}$
Output kVA corresponding to maximum efficiency

$$
=x \times \text { rated } \mathrm{kVA}=\sqrt{\frac{P_{\mathrm{i}}}{P_{\mathrm{c}}}} \times \operatorname{rated} \mathrm{kVA}=\sqrt{\frac{1}{1.5}} \times 100=81.65 \mathrm{kVA}
$$

(i) At unity p.f.

$$
\eta=\frac{81.65 \times 1}{81.65 \times 1+1+1} \times 100=\frac{81.65}{83.65} \times 100=97.6 \%
$$

(ii) At 0.8 p.f. lagging

$$
\eta=\frac{81.65 \times 0.8}{81.65 \times 0.8+1+1} \times 100=\frac{65.32}{67.32} \times 100=97.03 \%
$$

### 10.22 ALL-DAY EFFICIENCY

The efficiency discussed so far is the ordinary or commercial efficiency which is given by the ratio of output power to input power, that is,
Commercial efficiency, $\quad \eta=\frac{\text { Output power }}{\text { Input power }}$
The load on certain transformers fluctuate throughout the day. The distribution transformers are energized for 24 h , but they deliver very light loads for major portion of the day. Thus, iron losses occur for whole day, but copper losses occur only when the transformer are loaded. Hence, the performance of such transformers cannot be judged by the commercial efficiency, but it can be judged by all-day efficiency also known as operational efficiency or energy efficiency which is computed on the basis of energy consumed during a period of 24 h .

The all-day efficiency is defined as the ratio of output in kWh (or Wh) to the input in kWh (or Wh ) of a transformer over 24 h .

$$
\begin{equation*}
\therefore \quad \text { All day efficiency }=\eta_{\text {all day }}=\frac{\text { Output in } \mathrm{kWh}}{\text { Input in } \mathrm{kWh}} \tag{for24h}
\end{equation*}
$$

To find this all-day efficiency, we have to know the load cycle on the transformer.

## Example 10.33

A 20 kVA transformer on domestic load, which can be taken as of unity power factor, has a fullload efficiency of $94.3 \%$, the copper loss then being twice the iron loss. Calculate its all-day efficiency on the following daily cycle; no-load for 10 h , half load for 8 h , and full load for 6 h .

## Solution:

Full-load output $=20 \times 1=20 \mathrm{~kW}$
Full-load input $=\frac{\text { Output }}{\eta}=\frac{20}{95.3} \times 100=20.986 \mathrm{~kW}$
Total losses, $P_{\mathrm{i}}+P_{\mathrm{c}}=20.986-20=0.986 \mathrm{~kW}$
Now, $P_{\mathrm{c}}=2 P_{\mathrm{i}}$ (given) $P_{\mathrm{i}}+2 P_{\mathrm{i}}=0.986 \mathrm{~kW}$
Or lron losses, $P_{\mathrm{i}}=0.3287 \mathrm{~kW}$
Full-load copper losses $=2 \times 0.3287=0.6574 \mathrm{~kW}$
kWh output in $24 \mathrm{~h}=\frac{1}{2} \times 20 \times 8+1 \times 20 \times 6=200 \mathrm{kWh}$
Iron losses for $24 \mathrm{~h}=0.3287 \times 24=7.89 \mathrm{kWh}$
Copper losses for $24 \mathrm{~h}=\mathrm{cu}$. losses for 8 h at $\frac{1}{2}$ full load +cu . losses for 6 h at full load

$$
=\left(\frac{1}{2}\right)^{2} \times 0.6574 \times 8+0.6574 \times 6=5.259 \mathrm{kWh}
$$

input in $24 \mathrm{~h}=\mathrm{kWh}$ output in $24 \mathrm{~h}+$ iron and cu losses in kWh for 24 h

$$
=200+7.89+5.259=213.149 \mathrm{kWh}
$$

All day efficiency, $\eta_{\text {all day }}=\frac{\mathrm{kWh} \text { output in } 24 \mathrm{hrs}}{\mathrm{kWh} \text { input in } 24 \mathrm{hrs}} \times 100=\frac{200}{213.49} \times 100=93.83 \%$

## Example 10.34

A 5 kVA single-phase transformer has a core loss of 50 W and full-load ohmic loss of 100 W . The daily variation of load on the transformer is as follows:

| 7 am to 1 pm | 3 kW at power factor 0.6 lagging. |
| :--- | :--- |
| 1 pm to 6 pm | 2 kW at power factor 0.8 lagging. |
| 6 pm to 1 am | 5 kW at power factor 0.9 lagging. |
| 1 am to 7 am | No-load |

Determine the all-day efficiency.
(P.T.U., Dec. 2006)

## Solution:

Transformer rating $=5 \mathrm{kVA} ; P_{\mathrm{i}}=50 \mathrm{~W} ; P_{\mathrm{c}}=100 \mathrm{~W}$
Load variation in tabulated form is given below:

| Timings | Duration <br> in hr | Load in kW | p.f | Load in kVA <br> $\mathbf{k W}$ | fraction of Load <br> $-\frac{\text { Actual kVA }}{\mathbf{p f}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 7 AM to 1 PM | 6 | 3 | 0.6 | $3 / 0.6=5$ | $5 / 5=1$ |
| 1 PM to 6 PM | 5 | 2 | 0.8 | $2 / 0.8=2.5$ | $2.5 / 5=0.5$ |
| 6 PM to 1 AM | 7 | 5 | 0.9 | $5 / 0.9=5.55$ | $5.55 / 5=1.11$ |
| 1 AM to 7 AM | 6 | 0 | 0 | 0 | 0 |

kWh output in $24 \mathrm{~h}=3 \times 6+2 \times 5+5 \times 7+0 \times 6=63 \mathrm{kWh}$
Iron losses in $24 \mathrm{~h}=P_{\mathrm{i}}$ in $\mathrm{kW} \times 24=\frac{50}{1000} \times 24=1.2 \mathrm{kWh}$
Copper losses in $24 \mathrm{~h}=(1)^{2} \times \frac{100}{1000} \times 6+(0.5)^{2} \times \frac{100}{1000} \times 5+(1.11)^{2} \times \frac{100}{1000} \times 7+(0)^{2} \frac{100}{1000} \times 6$

$$
=0.6+0.125+0.8625+0=1.5875 \mathrm{kWh}
$$

All-day efficiency, $\eta_{\text {allday }}=\frac{\text { Output in } \mathrm{kWh} \text { in } 24 \mathrm{hr}}{\text { Output in } \mathrm{kWh} \text { in } 24 \mathrm{hr}+\mathrm{P}_{\mathrm{i}} \text { in } \mathrm{kWh} \text { in } 24 \mathrm{hr}+P_{\mathrm{c}} \text { in } \mathrm{kWh} \text { in } 24 \mathrm{hr}}$

$$
=\frac{63}{63+1.2+1.5875} \times 100=95.763 \%
$$

## 国隕 PRACTICE EXERCISES

## Short Answer Questions

1. What are no-load losses occurring in the transformer?
(P.T.U. May 2010)
2. Why efficiency of a transformer in high as compared to other electrical machines?
(P.T.U. Dec. 2010)
3. Why is the efficiency of a transformer is as high as $96 \%$ ?
4. What is normally the efficiency of a transformer?
5. Are transformers normally considered to be efficient devices?
6. How can eddy current loss is reduced?
7. How may the iron loss be reduced to a minimum?
8. Is the efficiency of a transformer same at rated load 0.8 p.f. lagging and 0.8 p.f. leading?
(U.P.T.U. 2001)
9. If supply frequency is doubled, which loss component in transformer will be doubled?
(A.M.I.E. Winter 2001)
10. Derive conditions for maximum efficiency of a transformer.
(P.T.U. May 2010)
11. Define all-day efficiency and give its significance.
(P.T.U. May 2011)
12. Define efficiency and all-day efficiency of a transformer.
(P.T.U. May 2011)
13. What is meant by all-day efficiency of a power transformers and why is it lower than commercial efficiency?
(P.T.U. June 2005)

## Test Questions

1. What are the various losses in a transformer? Where do they occur and how do they vary with load?
2. State various losses which take place in a transformer. On what factors do they depend? Explain the steps to minimize their losses.
(U.P.T.U. May 2008)
3. Define efficiency of a transformer and find the condition for obtaining maximum efficiency.
(P.T.U. Dec. 2007)
4. State and prove the condition for maximum efficiency of a transformer.
(P.U. 2005)
5. Write short note on 'All-day efficiency of a transformer'.
6. What is all-day efficiency of a transformer? How does it differ from ordinary efficiency?
(P.U. 1996)

## Numericals

1. The primary and secondary windings of a 500 kVA transformer have resistance of $0.42 \Omega$ and $0.0011 \Omega$, respectively. The primary and secondary voltages are 6600 V and 400 V , respectively. The iron loss is 2.9 kW . Calculate the efficiency at half full load at a power factor of 0.8 lagging.
(Ans. 98.07\%)
2. At full-load current, the iron and copper losses in a 100 kVA transformer are each equal to 2.5 kW . Find the efficiency of the transformer at a load of 75 kVA and 0.8 power factor lagging.
(Ans. 93.8\%)
3. A 50 kVA transformer has an efficiency of $98 \%$ at full load at 0.8 p.f. and an efficiency of $96.9 \%$ at $\frac{1}{4}$ full load, 0.8 p.f. Determine the iron loss and full-load copper loss. (Ans. $287 \mathrm{~W} ; 529 \mathrm{~W}$ )
4. A $440 / 110 \mathrm{~V}$ transformer has an effective primary resistance of $0.3 \Omega$ and a secondary resistance of $0.02 \Omega$. If iron loss on normal input voltage is 150 W , calculate the secondary current at which maximum efficiency will occur. What is the value of this maximum efficiency for unity power factor load?
(Ans. 62.22 A; 95.8\%)
5. In a $25 \mathrm{kVA}, 1100 / 400 \mathrm{~V}$, single-phase transformer, the iron and copper loss at full load are 350 and 400 W , respectively. Calculate the efficiency on unity power factor at half load. Determine the load on maximum efficiency.
(Ans. $9652 \% ; 23.85 \mathrm{~kW}$ )
6. A 40 kVA transformer has a core loss of 450 W and a full-load copper loss of 800 W . If the power factor of the load is 0.8 , Calculate: (i) the full-load efficiency; (ii) the maximum efficiency.
(Ans. 96.24\%; 96.385\%)
7. A 100 kVA transformer supplies a lighting and power load. The iron loss is 960 W and the copper loss is 960 W at full load. The transformer is operated continuously at the rated voltage as per the following schedule in a day 100 kVA at 0.8 p.f. for $4 \mathrm{~h} ; 50 \mathrm{kVA}$ at 0.6 p.f. for 8 h and 5 kVA at 0.95 p.f. for 12 h . What will be the all-day efficiency of the transformer?
(Ans. 94.5\%)
8. A transformer has its maximum efficiency of $98 \%$ at 15 kVA at unity p.f. compare its all-day efficiencies for the following load cycle: (i) full load of $20 \mathrm{kVA} 12 \mathrm{~h} /$ day and no-load rest of the day; (ii) Full load $4 \mathrm{~h} /$ day and 0.4 full-load rest of the day.
(Ans. $\eta_{2} / \eta_{1}=1.0053$ )
9. A transformer has a maximum efficiency of $98 \%$ at 15 kVA at unity p.f. It is loaded as follows: $12 \mathrm{~h}-2 \mathrm{~kW}$ at p.f. $0.5 ; 6 \mathrm{~h}-12 \mathrm{~kW}$ at p.f. $0.8 ; 6 \mathrm{~h}-18 \mathrm{~kW}$ at p.f. 0.9 , calculate all-day efficiency of the transformer.
(Ans. 96.98\%)

### 10.23 TRANSFORMER TESTS

All the transformers are tested before placing them in the field. By performing these tests, we can determine the parameters of a transformer to compute its performance characteristics (like voltage regulation and efficiency.).

To furnish the required information, open circuit and short circuit tests are conducted conveniently without actually loading the transformer.

### 10.23.1 Open-circuit or No-load Test

This test is carried out to determine the no-load loss or core loss or iron loss and no-load current $I_{0}$ which is helpful in finding the no-load parameters, that is, exciting resistance $R_{0}$ and $X_{0}$ exciting reactance of the transformer.

This test is usually carried out on the low-voltage side of the transformer, that is, a watt meter W , a voltmeter V, and an ammeter A are connected in low-voltage winding (say primary). The primary winding is then connected to the normal rated voltage $V_{1}$ and frequency as given on the name plate of the transformer. The secondary side is kept open or connected to a voltmeter $\mathrm{V}^{\prime}$ as shown in Figure 10.31(a).


Fig. 10.31 (a) Circuit for open circuit test (b) Phasor diagram at no-load

Since the secondary (high-voltage winding) is open circuited, the current drawn by the primary is called no-load current $I_{0}$ measured by the ammeter A. The value of no-load current $I_{0}$ is very small usually 2 to $10 \%$ of the rated full-load current. Thus, the copper loss in the primary is negligibly small and no copper loss occurs in the secondary as it is open. Therefore, wattmeter reading $W_{0}$ only represents the core or iron losses for all practical purposes. These core losses are constant at all loads. The voltmeter $\mathrm{V}^{\prime}$ if connected on the secondary side measures the secondary induced voltage $V_{2}$.

The ratio of voltmeter readings, $\frac{V_{2}}{V_{1}}$ gives the transformation ratio of the transformer. The phasor diagram of transformer at no-load is shown in Figure 10.31(b).

Let the wattmeter reading $=W_{0}$
voltmeter reading $=V_{1}$
and ammeter reading $=I_{0}$
Then, iron losses of the transformer $P_{\mathrm{i}}=W_{0}$
that is, $V_{1} I_{0} \cos \phi_{0}=W_{0}$
No-load power factor, $\cos \phi_{0}=\frac{W_{0}}{V_{1} I_{0}}$

Working component,

$$
I_{\mathrm{w}}=\frac{W_{0}}{V_{1}}
$$

$$
\left(\_I_{\mathrm{w}}=I_{0} \cos \phi_{0}\right)
$$

Magnetising component

$$
I_{\mathrm{mag}}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}
$$

No-load, parameters, that is,


Fig. 10.32 Transformer at no-load with exciting circuit Equivalent exciting resistance, $\quad R_{0}=\frac{V_{1}}{I_{\mathrm{w}}}$ Equivalent exciting reactance, $\quad X_{0}=\frac{V_{1}}{I_{\text {mag }}}$ The Iron losses measured by this test are used to determine transformer efficiency and parameters of exciting circuit of a transformer shown in Figure 10.32.

### 10.23.2 Short Circuit Test

This test is carried out to determine the following:

1. Copper losses at full load (or at any desired load). These losses are required for the calculations of efficiency of the transformer.
2. Equivalent impedance ( $Z_{\mathrm{es}}$ or $Z_{\mathrm{ep}}$ ), resistance ( $R_{\mathrm{es}}$ or $R_{\mathrm{ep}}$ ) and leakage reactance ( $X_{\mathrm{es}}$ or $X_{\mathrm{ep}}$ ) of the transformer referred to the winding in which the measuring instruments are connected. Knowing equivalent resistance and reactance, the voltage drop in the transformer can be calculated and hence regulation of transformer is determined.
This test is usually carried out on the high-voltage side of the transformer, that is, a wattmeter W, voltmeter V, and an ammeter A are connected in high-voltage* winding (say secondary). The other winding (primary) is then short circuited by a thick strip or by connecting an ammeter $\mathrm{A}^{\prime}$ across the terminals as shown in Figure 10.33. A low voltage at normal frequency is applied to the high-voltage winding with the help of on autotransformer so that full-load current flows in
both the windings, measured by ammeters A and $\mathrm{A}^{\prime}$. Low voltage is essential, failing which an excessive current will flow in both the windings that may damage them.

Since a low voltage (usually 5 to $10 \%$ of normal rated voltage) is applied to the transformer winding, therefore, the flux set-up in the core is very small about $\frac{1}{30}$ th to $\frac{1}{8}$ th of normal flux.

The iron losses are negligibly small due to low value of flux as these losses are approximately proportional to the square of the flux. Hence, wattmeter reading $W_{c}$ only represents the copper losses in the transformer windings for all practical purposes. The applied voltage $V_{2 s \mathrm{c}}$ is measured by the voltmeter V which circulates the current $I_{2 \mathrm{sc}}$ (usually full-load current) in the impedance $Z_{\text {es }}$ of the transformer to the side in which instruments are connected as shown in Figure 10.33. Let the wattmeter reading $=W_{c}$


Fig. 10.33 Circuit for short circuit test


Fig. 10.34 Phasor diagram at short circuit
voltmeter reading $=V_{2 \mathrm{sc}}$
and ammeter reading $=I_{2 \mathrm{sc}}$
Then, full-load copper losses of the transformer,

$$
P_{\mathrm{c}}=\left(\frac{I_{2 \mathrm{f}}}{I_{2 \mathrm{sc}}}\right)^{2} W_{\mathrm{c}} \quad \text { and } \quad I_{2 \mathrm{sc}}^{2} R_{\mathrm{es}}=W_{\mathrm{c}}
$$

Equivalent resistance referred to secondary, $R_{\mathrm{es}}=\frac{W_{\mathrm{c}}}{I_{2 \mathrm{sc}}^{2}}$
From phasor diagram as shown in Figure 10.34

$$
I_{2 \mathrm{sc}} Z_{\mathrm{es}}=V_{2 \mathrm{sc}}
$$

$\therefore$ Equivalent impedance referred to secondary, $Z_{\text {es }}=V_{2 \mathrm{sc}} / I_{2 \mathrm{sc}}$
Equivalent reactance referred to secondary, $X_{\text {es }}=\sqrt{\left(Z_{\text {es }}\right)^{2}-\left(R_{\text {es }}\right)^{2}}$
After calculating $R_{\mathrm{es}}$ and $X_{\mathrm{es}}$, the voltage regulation of the transformer can be determined at any load and power factor.

## Example 10.35

Open-circuit and short-circuit tests were conducted on a $50 \mathrm{kVA}, 6360 / 240 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer in order to find its efficiency. The observations during these tests are as follows:
O.C. test: Voltage across primary winding $=6360 \mathrm{~V}$; primary current $=1.0 \mathrm{~A}$, and power input $=2 \mathrm{~kW}$.
S.C. test: Voltage across primary $=180 \mathrm{~V}$; current in secondary winding $=175 \mathrm{~A}$, and power input $=2 \mathrm{~kW}$.

Calculate the efficiency of the transformer, when supplying full load at p.f. of 0.8 lagging.
(A.U.)

## Solution:

O.C. test: $\quad W_{\mathrm{i}}=2000 \mathrm{~W} ; I_{2(\mathrm{fl)}}=\frac{50 \times 1000}{240}=208.33 \mathrm{~A}$
S.C. test: $\quad I_{2 \mathrm{sc}}=175 \mathrm{~A} ; W_{\mathrm{c}}=2000 \mathrm{~W}$
$\therefore \mathrm{Cu}$ loss at full load, $P_{\mathrm{c}}=W_{\mathrm{c}}\left(\frac{I_{2(\mathrm{fl})}}{I_{2 \mathrm{sc}}}\right)^{2}=2000\left(\frac{208.33}{175}\right)^{2}=2833 \mathrm{~W}$
$\therefore \quad$ Efficiency $=\frac{50 \times 10^{3} \times 0.8 \times 100 \%}{50 \times 10^{3} \times 0.8+2000+2833}=95.33 \%$

## Example 10.36

A $15 \mathrm{kVA}, 440 / 230 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer gave the following test results:
Open Circuit (L.V. side) 250 V, 1.8 A, 95 W.
Short circuit test (H.V. side) $\quad 80 \mathrm{~V}, 12.0 \mathrm{~A}, 380 \mathrm{~W}$.
Compute the parameters of the equivalent circuit referred to L.V. side.
(P.T.U. May 2005)

## Solution:

Transformer rating $=15 \mathrm{kVA} ; E_{1}=440 \mathrm{~V} ; E_{2}=230 \mathrm{~V} ; f=50 \mathrm{~Hz}$
Open circuit test (L.V. side); $V_{2}=250 \mathrm{~V} ; I_{0}=1.8 \mathrm{~A} ; W_{0}=95 \mathrm{~W}$
Short-circuit test (H.V. side); $V_{1(\mathrm{sc})}=80 \mathrm{~V} ; I_{1(\mathrm{sc})}=12 \mathrm{~A} ; W_{\mathrm{c}}=380 \mathrm{~W}$
From open-circuit test performed on L.V. side

$$
\begin{aligned}
& I_{\mathrm{w}}=\frac{W_{0}}{V_{2}}=\frac{95}{250}=0.38 \mathrm{~A} \\
& I_{\mathrm{mag}}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}=\sqrt{(1.8)^{2}-(0.38)^{2}}=1.75943 \mathrm{~A}
\end{aligned}
$$

Exciting resistance,

$$
R_{0}=\frac{V_{2}}{I_{\mathrm{w}}}=\frac{250}{0.38}=658 \Omega
$$

Exciting reactance,

$$
X_{0}=\frac{V_{2}}{I_{\mathrm{mag}}}=\frac{250}{1.75943}=142 \Omega
$$

From short-circuit test performed on H.V. side

$$
\begin{aligned}
& Z_{\mathrm{ep}}=\frac{V_{1(\mathrm{sc})}}{I_{1(\mathrm{sc})}}=\frac{80}{12}=6.667 \Omega \\
& R_{\mathrm{ep}}=\frac{W_{\mathrm{c}}}{\left(I_{1(\mathrm{sc})}\right)^{2}}=\frac{380}{(12)^{2}}=2.639 \Omega \\
& X_{\mathrm{ep}}=\sqrt{Z_{\mathrm{ep}}^{2}-R_{\mathrm{ep}}^{2}}=\sqrt{(6.667)^{2}-(2.639)^{2}}=6.122 \Omega
\end{aligned}
$$

Transformation ratio,

$$
K=\frac{E_{2}}{E_{1}}=\frac{230}{440}=0.5227
$$

Transformer resistance and reactance referred to L.V. (secondary) side

$$
\begin{gathered}
R_{\mathrm{es}}=R_{\mathrm{ep}} \times K^{2}=2.639 \times(0.5227)^{2}=0.7211 \Omega \\
X_{\mathrm{es}}=X_{\mathrm{ep}} \times K^{2}=6.122 \times(0.5227)^{2}=1.673 \Omega
\end{gathered}
$$

## Example 10.37

The following test data are obtained on a $5 \mathrm{kVA}, 220 / 440 \mathrm{~V}$ single-phase transformer; O.C. test -220 V, 2 A, 100 W on L.V. side; S.C. test -40 V. 11.4 A, 200 W on H.V. side. Determine the percentage efficiency and regulation at full load 0.9 p.f. lag.

## Solution:

From O.C. test, Iron losses, $P_{\mathrm{i}}=100 \mathrm{~W}$
From S.C. test, Copper losses, $W_{\mathrm{c}}=200 \mathrm{~W}$ (at the load at which test is performed)
Full-load current on H.V. side, $\quad I_{2}=\frac{\mathrm{kVA} \times 1000}{V_{2}}=\frac{5 \times 1000}{440}=11.4 \mathrm{~A}$
that is, S.C test is performed at full load since $I_{2 \mathrm{sc}}=I_{2}$
Full-load copper loss, $P_{\mathrm{c}}=W_{\mathrm{c}}=200 \mathrm{~W}$
Efficiency,

$$
\begin{aligned}
\eta & =\frac{\mathrm{kVA} \times 1000 \times \cos \phi_{2}}{\mathrm{kVA} \times 1000 \times \cos \phi_{2}+p_{\mathrm{i}}+p_{\mathrm{c}}} \times 100 \\
& =\frac{5 \times 1000 \times 0.9}{5 \times 1000 \times 0.9+100+200} \times 100=93.75 \%
\end{aligned}
$$

From S.C. test: $\quad R_{\mathrm{es}}=\frac{\text { Wattmeter reading }}{(\text { Ammeter reading })^{2}}=\frac{200}{(11.4)^{2}}=1.539 \Omega$

$$
\begin{aligned}
& Z_{\mathrm{es}}=\frac{\text { Voltmeter reading }}{\text { Ammeter reading }}=\frac{40}{11.4}=3.509 \Omega \\
& X_{\mathrm{es}}=\sqrt{\left(Z_{\mathrm{es}}\right)^{2}-\left(R_{\mathrm{es}}\right)^{2}}=\sqrt{(3.509)^{2}-(1.539)^{2}}=3.153 \Omega
\end{aligned}
$$

Here $\cos \phi_{2}=0.9 ; \sin \phi_{2}=\sin \cos ^{-1} 0.9=0.4359$
$E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}+I_{2} X_{\mathrm{es}} \sin \phi_{2}$

$$
=440+11.4 \times 1.539 \times 0.9+11.4 \times 3.153 \times 0.4359=471.46 \mathrm{~V}
$$

$$
\% \operatorname{Reg}=\frac{E_{2}-V_{2}}{E_{2}} \times 100=\frac{471.46-440}{471.46} \times 100=6.67 \%
$$

## Example 10.38

The O.C. and S.C. tests on a $5 \mathrm{kVA}, 230 / 160 \mathrm{~V}, 50 \mathrm{~Hz}$, transformer gave the following data.
O.C. test (H.V. side) - $230 \mathrm{~V}, 0.6 \mathrm{~A}, 80$ watt
S.C. test (L.V. side) - $6 \mathrm{~V}, 15 \mathrm{~A}, 20$ watt

Calculate the efficiency of transformer on full load at 0.8 p.f. lagging.
(P.U. June, 1996)

## Solution:

From open circuit test, iron losses, $P_{\mathrm{i}}=80 \mathrm{~W}$
As short-circuit test is performed on L.V. side, $I_{2 \mathrm{sc}}=15 \mathrm{~A}$

Full-load secondary current,

$$
I_{2}=\frac{5 \times 1000}{160}=31.25 \mathrm{~A}
$$

Copper losses measured at S.C. test, $W_{\mathrm{c}}=20 \mathrm{~W}$

Full-load copper losses, $P_{\mathrm{c}}=\left(\frac{I_{2}}{I_{2 \mathrm{sc}}}\right)^{2} W_{c}=\left(\frac{31.25}{15}\right)^{2} \times 20=86.8 \mathrm{~W}$
Efficiency of transformer at full load 0.8 p.f. lagging

$$
\begin{aligned}
\eta & =\frac{\mathrm{kVA} \times 1000 \times \cos \phi}{\mathrm{kVA} \times 1000 \times \cos \phi+P_{\mathrm{i}}+P_{\mathrm{c}}} \times 100 \\
& =\frac{5 \times 1000 \times 0.8}{5 \times 1000 \times 0.8+80+86.8} \times 100=96 \%
\end{aligned}
$$

## Example 10.39

The following results were obtained on a 50 kVA transformer:
(a) Open-circuit tests: Primary voltage 3300 V , secondary voltage 415 V , power 430 W
(b) Short-circuit test: Primary voltage 124 V , primary current 15.3 A , primary power 525 W secondary current full-load value.
Calculate:
(i) The efficiency at full load and at half load for 0.7 power factor.
(ii) The voltage regulation for power factor 0.7: (i) lagging (ii) leading
(iii) The secondary terminal voltages corresponding to (a) and (b)
(P.T.U. Dec. 2007)

## Solution:

Rating of transformer $=50 \mathrm{kVA}$; Power factor $=0.7$
Open-circuit test (primary): $V_{1}=3300 \mathrm{~V} ; V_{2}=415 \mathrm{~V} ; W_{0}=430 \mathrm{~W}$
Short-circuit test (primary): $V_{1(\mathrm{sc})}=124 \mathrm{~V} ; I_{1(\mathrm{sc})}=15.3 \mathrm{~A} ; W_{\mathrm{c}}=525 \mathrm{~W}$
Short-circuit test is performed at full-load secondary current,
$\therefore$ Full-load copper losses,

$$
P_{\mathrm{c}}=W_{\mathrm{c}}=525 \mathrm{~W}
$$

Iron losses,

$$
P_{\mathrm{i}}=W_{0}=430 \mathrm{~W}
$$

When p.f., $\cos \phi=0.7$
(i) Full-load efficiency, $\eta_{\mathrm{fl}}=\frac{\mathrm{kVA} \times 1000 \times \cos \phi}{\mathrm{kVA} \times 1000 \times \cos \phi+P_{\mathrm{i}}+P_{\mathrm{c}}}$

$$
=\frac{50 \times 1000 \times 0.7}{50 \times 1000 \times 0.7+430+525} \times 100=97.34 \%
$$

Efficiency at half load, $\eta_{0.5}=\frac{0.5 \mathrm{kVA} \times 1000 \times \cos \phi}{0.5 \mathrm{kVA} \times 1000 \times \cos \phi+P_{\mathrm{i}}+(0.5)^{2} P_{\mathrm{c}}}$

$$
=\frac{0.5 \times 50 \times 1000 \times 0.7}{0.5 \times 50 \times 1000 \times 0.7+430+(0.5)^{2} \times 525} \times 100=96.89 \%
$$

(ii) Transformer impedance referred to primary, $Z_{\text {ep }}=\frac{V_{1(\mathrm{sc})}}{I_{1(\mathrm{sc})}}=\frac{124}{15.3}=8.1 \Omega$

Transformer resistance referred to primary, $R_{\mathrm{ep}}=\frac{W_{\mathrm{c}}}{\left(I_{1(\mathrm{sc})}\right)^{2}}=\frac{525}{(15.3)^{2}}=2.243 \Omega$
Transformer reactance referred to primary, $X_{\mathrm{ep}}=\sqrt{Z_{\mathrm{ep}}^{2}-R_{\mathrm{ep}}^{2}}$

$$
=\sqrt{(8.1)^{2}-(2.243)^{2}}=7.783 \Omega
$$

At 3300 V , primary full-load current, $I_{1}=\frac{50 \times 1000}{3300}=15.15 \mathrm{~A}$
For p.f., $\quad \cos \phi=0.7$ lag; $\sin \phi=\sin \cos ^{-1} 0.7=0.714$

$$
\begin{aligned}
E_{1} & =V_{1}-I_{1} R_{\mathrm{ep}} \cos \phi-I_{1} X_{\mathrm{cp}} \sin \phi \\
& =3300-15.15 \times 2.243 \times 0.7-15.15 \times 7.783 \times 0.714 \\
& =3300-23.787-84.189=3192 \mathrm{~V} \\
\% R_{\mathrm{eg}} & =\frac{V_{1}-E_{1}}{V_{1}} \times 100=\frac{3300-3192}{3300} \times 100=3.27 \%
\end{aligned}
$$

For p.f., $\quad \cos \phi=0.7$ leading; $\sin \phi=\sin \cos ^{-1} 0.7=0.714$

$$
\begin{aligned}
E_{1}^{\prime} & =V_{1}-I_{1} R_{\mathrm{ep}} \cos \phi+I_{1} X_{\mathrm{cp}} \sin \phi \\
& =3300-15.15 \times 2.243 \times 0.7+15.15 \times 7.783 \times 0.714 \\
& =3300-23.787+84.189=3360 \mathrm{~V} \\
\% R_{\mathrm{eg}} & =\frac{V_{1}-E_{1}}{V_{1}} \times 100=\frac{3300-3360}{3300} \times 100=-1.82 \%
\end{aligned}
$$

(iii) Secondary terminal voltage at 0.7 p.f. lagging:

$$
V_{2}=E_{2}=K \times E_{1} \text { where } K=\frac{415}{3300}=\frac{415}{3300} \times 3192=401.4 \mathrm{~V}
$$

Secondary terminal voltage at 0.7 p.f. leading

$$
V_{2}^{\prime}=E_{2}^{\prime}=K \times E_{1}^{\prime}=\frac{415}{3300} \times 3360=422.5 \mathrm{~V}
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. Why short-circuit test is performed on high-voltage side of transformer?
2. Why core losses are neglected in short-circuit test and copper loss is neglected in open-circuit test of a transformer?
(P.T.U. Dec. 2002)
3. Why are iron losses or core losses assumed to remain constant in a power transformer from no-load to full load?
(Allahabad Univ. 1989)
4. Why O.C. test in a transformer is normally carried out on a low-voltage side?
5. How can iron loss be measured?
6. How can copper loss be measured?

## Test Questions

1. Write about open-circuit test of a transformer.
(P.T.U. Dec. 2009)
2. Draw and explain the circuit diagram of open-circuit test in single-phase transformer. What is the use of this test?
(P.T.U. May 2011)
3. From iron losses, how eddy current losses and hysteresis losses can be separated?
4. Explain how can you determine the parameters of equivalent circuit of a transformer no-load and short-circuit test.
(P.T.U. May 2008)
5. How will you carry out short-circuit and open-circuit test of a transformer?
(P.T.U. Dec. 2008)
6. What are load losses occurring in a single-phase transformer? Explain the test used to determine variable losses.
(P.T.U. May 2010)

## Numericals

1. In a $440 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer the total iron loss is 2500 W . When the applied voltage is 220 V at 25 Hz , the corresponding loss is 850 W . Calculate the eddy current loss at normal frequency and voltage.
(Ans. 1600 W ).
2. When a transformer is connected to a $1000 \mathrm{~V}, 50 \mathrm{~Hz}$ supply, the core loss is 1000 W , of which 650 W is hysteresis and 350 W is eddy current loss. If the applied voltage is raised to 2000 V and the frequency to 100 Hz , find the new core losses.
(Ans. 2700 W )
3. The iron loss in a transformer core at normal flux density was measured at frequencies of 30 and 50 Hz , the results being 30 W and 54 W , respectively. Calculate (a) the hysteresis loss and (b) the eddy current loss at 50 Hz .
(Ans. $44 \mathrm{~W}, 10 \mathrm{~W}$ )
4. A $10 \mathrm{kVA}, 200 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer gave the following test results: Open-circuittest $-200 \mathrm{~V}, 1.25 \mathrm{~A}, 120 \mathrm{~W}$, on low-voltage side; short-circuit test-20 V, $25 \mathrm{~A}, 200 \mathrm{~W}$, on high-voltage side.
(i) Calculate the magnetizing current at normal voltage and frequency; (ii) obtain the efficiency when the transformer is supplying rated load at 0.8 p.f. lagging.
(Ans. 1.0966 A; 96.15\%)
5. A $250 / 500 \mathrm{~V}$ transformer gave the following test results: - short-circuit test with low-voltage winding short circuited; $20 \mathrm{~V}, 12 \mathrm{~A}, 100 \mathrm{~W}$; open-circuit test on low-voltage side; $250 \mathrm{~V}, 1 \mathrm{~A}, 30 \mathrm{~W}$. Determine the efficiency of the transformer when the output is $10 \mathrm{~A}, 500 \mathrm{~V}$ at 0.8 p.f. lagging.
(Ans. 96.4\%)
6. A transformer has its maximum efficiency of $98 \%$ at 15 kVA at unity p.f. compare its all-day efficiencies for the following load cycle: (i) Full load of $20 \mathrm{kVA} 12 \mathrm{~h} /$ day and no-load rest of the day; (ii) Full load $4 \mathrm{~h} /$ day and 0.4 full-load rest of the day.
(Ans. $\eta_{2} / \eta_{1}=1.0053$ )
7. The O.C. and S.C. tests on a $5 \mathrm{kVA}, 230 / 110 \mathrm{~V}, 50 \mathrm{c} / \mathrm{s}$ transformer gave the following data: O.C. test (H.V. side); $230 \mathrm{~V}, 0.6$ A, 80 W ; S.C. test (L.V. side); $6 \mathrm{~V}, 15 \mathrm{~A}, 20 \mathrm{~W}$. Calculate the efficiency of the transformer on full load at 0.8 p.f. lagging. Also calculate the voltage on the secondary side under full-load conditions at 0.8 p.f. leading.
(Ans. 93.82\%, 117.4 V )
8. A $5 \mathrm{kVA}, 200 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase transformer gave the following test data: Open-circuit test (L.V. side); $200 \mathrm{~V}, 0.7 \mathrm{~A}, 60 \mathrm{~W}$; short-circuit test (H.V. side); $22 \mathrm{~V}, 16 \mathrm{~A}, 120 \mathrm{~W}$. If the transformer operates on full load, determine (i) the percentage regulation at 0.9 p.f. lagging (ii) efficiency at 0.8 p.f. lagging.
(Ans. 3.08\%, 96.77\%)
9. A $5 \mathrm{kVA}, 400 / 200 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase transformer gave the following results:

No-load: $400 \mathrm{~V}, 1 \mathrm{~A}, 50 \mathrm{~W}$ (L.V side) short circuit: $12 \mathrm{~V}, 10 \mathrm{~A}, 40 \mathrm{~W}$ (H.V side)
Calculate (a) the components of no-load current (b) the efficiency and regulation at full load and power factor of 0.8 lagging.
(Ans. 0.125 A; $0.992 \mathrm{~A} ; 97.8 \%$; 3.13\%) (P.U. 1957)
10. A $250 / 500 \mathrm{~V}$, transformer gave the following test results:

Short-circuit test: with low-voltage winding short-circuited: $20 \mathrm{~V}, 12 \mathrm{~A} 100 \mathrm{~W}$
Open-circuit test: $250 \mathrm{~V}, 1 \mathrm{~A}, 80 \mathrm{~W}$ on low-voltage side.
Determine the circuit constants, insert these on the equivalent circuit diagram and calculate the applied voltage and efficiency when the output is 10 A at 500 V and 0.8 power factor lagging.
(Ans. $781.3 \Omega ; 263.8 \Omega ; 257.4 \mathrm{~V} ; 96.4 \%$ )

### 10.24 AUTOTRANSFORMERS

A transformer, in which a part of the winding is common to both primary and secondary circuits, is called an autotransformer. In a two-winding transformers, primary and secondary windings are electrically isolated, but in an autotransformer, the two windings are not electrically isolated rather a section of the same winding acts as secondary or primary of the transformer.

### 10.24.1 Construction

The core of an autotransformer may be rectangular (Figure 10.35(a)) or circular ring type (Figure 10.36(a)) in shape. A single winding is wound around one or two limbs of the rectangular core as shown in Figure 10.35(b) or it is wound over the ring as shown in Figure 10.36(b). Terminal ' B ' is taken as a common point from which one terminal for primary and one terminal of the secondary is taken out. The second terminal of the secondary is connected to point 'C' which may be fixed or movable as shown in Figure 10.35(b) and 8.36(b). The number of turns between AB are taken as $N_{1}$ and the number of turns between BC are taken as $N_{2}$ as shown in Figure 10.35(c) and 8.36(c). Thus, one section of the same winding acts as a primary
and the other section of the same winding acts as a secondary. When the number of secondary turns $N_{2}$ is less than the primary turns $N_{1}$ (i.e., $N_{2}<N_{1}$ ) as shown in Figure 10.35 (c) and 10.36(c), the autotransformer works as step-down transformer, whereas it works as a step-up transformer if number of secondary turns $N_{2}$ is more than primary turns $N_{1}$ as shown in Figure $10.35(\mathrm{~d})$ and $10.36(\mathrm{~d})$.


Fig. 10.35 (a) Rectangular core of an auto-transformer (b) Single winding placed on the core (c) Electric circuit for step-down auto-transformer (d) Electric circuit for step-up auto-transformer (e) Electric circuit of an auto-transformer


Fig. 10.36 (a) Circular core of an auto-transformer (b) Single winding placed on the core (c) electric circuit for step-down auto-transformer (d) Electric circuit for step-up auto-transformer (e) \& (f) Pictorial view of an auto-transformer (or variac)

The pictorial view of a single-phase autotransformer used in laboratories is shown in Figure 10.36(e \& f). Here, point C is attached to a movable arm which carries a carbon brush. The brush moves over number of turns wound over a circular laminated core and its position determines the output voltage.

### 10.24.2 Working

When AC voltage $V_{1}$ is applied to winding AB , an exciting current starts flowing through the full winding AB if the internal impedance drop is neglected, then the voltage per turn in winding AB is $V_{1} / N_{1}$ and, therefore, the voltage across BC is $\left(V_{1} / N_{1}\right) N_{2}$.

When switch S is closed, as shown in Fig. 10.37(a), a current $I_{2}$ starts flowing through the load and current $I_{1}$ is drawn from the source [see Fig. 10.37(b)]. Neglecting losses,
or
Input power $=$ Output power

$$
V_{1} I_{1} \cos \phi_{1}=V_{2} I_{2} \cos \phi_{2}
$$



Fig. 10.37 (a) Winding of an auto-transformer placed around a limit of the core (b) Electric circuit of an auto-transformer

If internal (or leakage) impedance drops and losses are neglected, then

Hence

$$
\begin{aligned}
\cos \phi_{1} & =\cos \phi_{2} \\
V_{1} I_{1} & =V_{2} I_{2}
\end{aligned}
$$

or

$$
\begin{equation*}
\frac{V_{2}}{V_{1}}=\frac{I_{2}}{I_{1}}=\frac{N_{2}}{N_{1}}=K \tag{10.9}
\end{equation*}
$$

Here, $K$ is less than unity. The expression is identical to a two-winding transformer.
Let at any instant, the exciting current flows from $A$ to $B$ and it establishes a working mmf directed vertically upward in the core. When switch S is closed, the current in winding BC must flow from B to C , in order to create an mmf opposing the exciting or working mmf , as per Lenz's law. Since the working mmf in a transformer remains constant at its no-load value, the primary must draw additional current $I_{1}$ from the source, in order to neutralise the effect of current $I_{\mathrm{BC}}$. In winding $\mathrm{AB}, I_{1}$ flows from A to B while in winding $\mathrm{BC}, I_{2}$ flows from B to C . Therefore, the current in winding BC is $I_{1}$ from C to B and $I_{2}$ from B to C . Here, the current $I_{2}$ is greater than $I_{1}$ (because $V_{2}<V_{1}$ ) and their mmfs. are opposing each other at every instant, therefore,
mmf of winding

$$
I_{\mathrm{BC}}=I_{2}-I_{1}
$$

$$
\begin{aligned}
\mathrm{AC} & =I_{1}\left(N_{1}-N_{2}\right)=I_{1} N_{1}-I_{1} N_{2} \\
& =I_{2} N_{2}-I_{1} N_{2}=\left(I_{2}-I_{1}\right) N_{2} \quad\left[\therefore I_{1} N_{1}=I_{2} N_{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& =I_{\mathrm{BC}} N_{2} \\
& =\text { mmf of winding } \mathrm{CB} .
\end{aligned}
$$

It is, therefore, seen that the transformer action takes place between winding, section AC and winding section BC . In other words, the volt-amperes across winding AC are transferred by transformer action to the load connected across winding BC .
$\therefore$ Power transformed in VA $=V_{\mathrm{AC}} I_{\mathrm{AC}}=\left(V_{1}-V_{2}\right) I_{1}$
Total power to be transferred or input power in $\mathrm{VA}=V_{1} I_{1}$

$$
\begin{equation*}
\therefore \quad \frac{\text { Transformed power in VA }}{\text { Input power in VA }}=\frac{\left(V_{1}-V_{2}\right) I_{1}}{V_{1} I_{1}}=1-\frac{V_{2}}{V_{1}}=(1-K) \tag{10.10}
\end{equation*}
$$

Power transformed $=(1-K) \times$ power input
Out of the input volt-amperes $V_{1} I_{1}$, only $V_{\mathrm{AC}} I_{\mathrm{AC}}=\left(V_{1}-V_{2}\right) I_{1}$ is transformed to the output by transformer action. The remaining power in volt-ampere required for the output, are conducted directly to the secondary from the primary (due to electrical connection).
$\therefore \quad$ Power conducted in VA $=$ Total power input in VA - transformed power in VA

$$
\begin{array}{cc} 
& =V_{1} I_{1}-\left(V_{1}-V_{2}\right) I_{1}=V_{2} I_{1} \\
\therefore & \frac{\text { Power conducted in VA }}{\text { Power input in VA }}=\frac{V_{2} I_{1}}{V_{1} I_{1}}=K \tag{10.11}
\end{array}
$$

Power conducted $=K \times$ power input

Hence,

$$
\begin{equation*}
\frac{\text { Transformed power }}{\text { Input power }}=1-K \tag{10.12}
\end{equation*}
$$

And

$$
\begin{equation*}
\frac{\text { Conducted power }}{\text { Innut nower }}=K \tag{10.13}
\end{equation*}
$$

Considering eqn. (10.12),

$$
\frac{\text { Transformed power }}{\text { Input power }}=1-K=1-\frac{V_{2}}{V_{1}}=\frac{V_{1}-V_{2}}{V_{1}}
$$

or

$$
\frac{\text { Inductively transformed power }}{\text { Total power }}=\frac{\text { High voltage }- \text { Low voltage }}{\text { High voltage }}
$$

### 10.25 AUTOTRANSFORMER V/S POTENTIAL DIVIDER

At first sight, an autotransformer appears to be similar to a resistance potential divider. But this is not so, as described below.

1. A resistive potential divider cannot step up the voltage, whereas it is possible in an autotransformer.
2. The potential divider has more losses and is, therefore, less efficient.
3. In a potential divider, almost entire power to load flows by conduction, whereas in autotransformer, a part of the power is conducted and the rest is transferred to load by transformer action.
4. In a potential divider, the input current, must always be more than the output current, this is not so in an autotransformer. If the output voltage in autotransformer is less than the input voltage, the load current is more than the input current.

### 10.26 SAVING OF COPPER IN AN AUTOTRANSFORMER

Volume, and hence weight of copper (or aluminium), is proportional to the length and area of $X$-section of the conductor. The length of conductor is proportional to number of turns, whereas area of $X$-section is proportional to the current flowing through it. Hence, the weight of copper is proportional to the product of current and number of turns.

Now, with reference to Figure 10.38(a), weight of copper required in an autotransformer.

$$
\begin{array}{ll} 
& W t_{\mathrm{a}}=\text { weight of } \mathrm{Cu} \text { in section AC }+ \text { weight of } \mathrm{Cu} \text { in section CB } \\
\therefore & W t_{a} \infty I_{1}\left(N_{1}-N_{2}\right)+\left(I_{2}-I_{1}\right) N_{2} \infty I_{1} N_{1}+I_{2} N_{2}-2 I_{1} N_{2}
\end{array}
$$



Fig. 10.38 (a) Electric circuit of an auto-transformer (b) Equivalent circuit

If an ordinary two-winding transformer is to perform the same duty, then with reference to Figure 10.38(b). Total weight of copper required in the ordinary transformer.

$$
W t_{0}=\text { weight of } \mathrm{Cu} \text { on its primary }+ \text { weight of } \mathrm{Cu} \text { on its secondary. }
$$

$$
\therefore \quad W t_{0} \infty I_{1} N_{1}+I_{2} N_{2}
$$

Now, the ratio of weight of copper in autotransformer to the weight of copper in an ordinary transformer,

$$
\frac{W t_{a}}{W t_{0}}=\frac{I_{1} N_{1}+I_{2} N_{2}-2 I_{1} N_{2}}{I_{1} N_{1}+I_{2} N_{2}}=\frac{I_{1} N_{1}+I_{2} N_{2}}{I_{1} N_{1}+I_{2} N_{2}}=\frac{2 I_{1} N_{2}}{I_{1} N_{1}+I_{2} N_{2}}
$$

$$
\begin{aligned}
& =1-\frac{2 I_{1} N_{2} / I_{1} N_{1}}{I_{1} N_{1} / I_{1} N_{1}+I_{2} N_{2} / I_{1} N_{1}}=1-K \\
W t_{\mathrm{a}} & =(1-K) W t_{0}
\end{aligned}
$$

or
Saving of copper affected by using an autotransformer
$=w t$. of cu required in an ordinary transformer -wt . of copper required in an autotransformer.
$=W t_{0}-W t_{\mathrm{a}}=W t_{0}-(1-K) W t_{0}=K \times W t_{0}$
$\therefore$ Saving $=K \times \mathrm{Wt}$. of copper required for two-winding transformer
Hence, saving in copper increases as the transformation ratio approaches to unity, therefore, autotransformers are used when $K$ in nearly equal to unity.

### 10.27 ADVANTAGES OF AUTOTRANSFORMER OVER TWO-WINDING TRANSFORMER

1. Quantity of conducting material required is less: The quantity of conducting material required for an autotransformer having same rating as that of an ordinary two-winding transformer is only $(1-K)$ times, that is, quantity of conducting material required for autotransformer $=(1-K)$ quantity of conducting material required for an ordinary two-winding transformer. Thus, the cost of autotransformer is less as compared to two-winding transformer of the same rating.
2. Quantity of magnetic material required is less: During designing, the window dimensions are decided from the consideration of insulation and conductor material. For an autotransformer, a reduction in conductor material means lower window area and, therefore, smaller core length is needed. It shows that for the same core area, the weight of autotransformer core is less. Hence, there is further saving in core material. Thus, autotransformer is more economical than a two-winding transformer when $K$ approaches to unity.
3. Operate at higher efficiency: Owing to the reduction in conductor and core materials, the ohmic losses in conductor and the core loss are lowered. Consequently, an autotransformer has higher efficiency than a two-winding transformer of the same rating.
4. Operate at better voltage regulation: Reduction in the conductor material means lower value of ohmic resistance. A part of the winding being common, leakage flux or the leakage reactance is less. In other words, an autotransformer has lower value of leakage impedance, and hence, autotransformer has lower value of leakage impedance and has better voltage regulation than a two-winding transformer of the same output.

### 10.28 DISADVANTAGES OF AUTOTRANSFORMERS

Although autotransformers have less cost, better regulation and low losses as compared ordinary two-winding transformer of same rating. But still they are not widely used due to one major disadvantage that the secondary winding is not insulated from the primary. If an autotransformer is used to supply low voltage from a high voltage and there is a break in the secondary winding, full primary voltage comes across the secondary terminals which may be dangerous
to the operator and equipment (load). Therefore, it is advisable not to use an autotransformer for interconnecting high-voltage and low-voltage system. Their use is only limited to the places where slight variation of output voltage from the input voltage is required. The other disadvantages are as follows:

1. The effective per unit impedance of an autotransformer is smaller compared to a two-winding transformer. The reduced internal impedance results in a larger short-circuit (fault) current.
2. In an autotransformer, there is a loss of isolation between input and output circuits. This is particularly important in three-phase transformers where one may wish to use a different winding and earthing arrangement on each side of the transformer.

### 10.29 APPLICATIONS OF AUTOTRANSFORMERS

1. Single-phase and three-phase autotransformers are employed for obtaining variable output voltages at the output. When used as variable ratio autotransformers, these are known by their trade names, such as variac, dimmerstat, and autostat.

A variable ratio autotransformer (or variac) has a toroidal core and toroidal winding. A sliding contact with the winding is made by carbon brush, as shown in Figure 10.36(b) and Figure 10.39. The position of the sliding contact can be varied by a hand wheel which changes output voltage. These


Fig. 10.39 Auto-transformer as a variac are mostly used in laboratories.
2. Autotransformers are also used as boosters for raising the voltage in an AC feeder.
3. As furnace transformers, for getting a convenient supply to suit the furnace winding from normal 230 V AC supply
4. Autotransformers with a number of tapings are used for starting induction motors and synchronous motors. When autotransformers are used for this purpose, these are known as autostarters.

## Example 10.40

Determine the core area, the number of turn, and the position of the tapping point for a 500 kVA , 50 Hz single phase, $6600 / 5000 \mathrm{~V}$ autotransformer, assuming the following approximate values: emf per turn 8 V , maximum flux density 1.3 tesla.

## Solution:

We know,

$$
\begin{aligned}
E & =4.44 f \mathrm{~B}_{\mathrm{m}} \mathrm{~A}_{\mathrm{i}} N \\
\frac{E}{N} & =4.44 f \mathrm{~B}_{\mathrm{m}} \mathrm{~A}_{\mathrm{i}} \\
8 & =4.44 \times 50 \times 1.3 \times \mathrm{A}_{\mathrm{i}}
\end{aligned}
$$

or
or
or

$$
\begin{aligned}
\mathrm{A}_{\mathrm{i}}=\frac{8}{4.44 \times 50 \times 1.3} & =0.02772 \mathrm{~m}^{2} \\
& =277.2 \mathrm{~cm}^{2}
\end{aligned}
$$

Turns on the primary side, $\quad N_{1}=\frac{6600}{8}=825$
Turns on the secondary side $\quad N_{2}=\frac{5000}{8}=625$
Hence, tapping should be 200 turns from high-voltage end or 625 turns from the common end as shown in


Fig. 10.40 Circuit as per data Figure 10.40 .

## Example 10.41

An autotransformer having 1500 turns is connected across a 500 V AC supply. What secondary voltage will be obtained if a tap is taken at 900th turn?

## Solution:

Supply voltage,

$$
V_{1}=500 \mathrm{~V}
$$

Total turns,

$$
N_{1}=1500
$$

Secondary turns,

$$
N_{2}=900
$$

$$
=\frac{V_{1}}{N_{1}}=\frac{500}{1500}=\frac{1}{3} \mathrm{~V}
$$

Secondary voltage,

$$
V_{2}=\text { Voltage per turn } \times N_{2}=\frac{1}{3} \times 900=300 \mathrm{~V}
$$

## Example 10.42

An autotransformer supplies a load of 10 kW at 250 V and at unity power factor. If the primary voltage is 500 V , then determine the following: (i) transformation ratio, (ii) secondary current, (iii) primary current, (iv) number of turns across secondary if total number of turns is 500 , (v) power transformed, and (vi) power conducted directly from the supply mains to load.

## Solution:

(i) Transformation ratio, $K=\frac{V_{2}}{V_{1}}=\frac{250}{500}=0.5$
(ii) Secondary current, $I_{2}=\frac{k W \times 1000}{V_{2} \cos \phi}=\frac{10 \times 1000}{250 \times 1}=40 \mathrm{~A}$
(iii) Primary current, $I_{1}=K I_{2}=0.5 \times 40=20 \mathrm{~A}$
(iv) Turns across secondary, $N_{2}=K N_{1}=0.5 \times 500=250$
(v) Power transformed $=\operatorname{Load} \times(I-K)=10(1-0.5)=5 \mathrm{~kW}$
(vi) Power conducted directly from supply mains $=10-5=5 \mathrm{~kW}$

## Example 10.43

A $400 / 100 \mathrm{~V}, 5 \mathrm{kVA}$, two-winding transformer is to be used as an autotransformer to supply power at 400 V from 500 V source. Draw the connection diagram and determine the kVA output of the autotransformer.


Fig. 10.41 Circuit as per data

## Solution:

For a two-winding transformer

$$
\begin{aligned}
& V_{1} I_{1}=\mathrm{kVA} \times 1000 \text { or } I_{1}=\frac{5 \times 1000}{400}=12.5 \mathrm{~A} \\
& V_{2} I_{2}=\mathrm{kVA} \times 1000 \text { or } I_{2}=\frac{5 \times 1000}{100}=50 \mathrm{~A}
\end{aligned}
$$

Figure 10.41 shows the use of two-winding transformer as an autotransformer to supply power at 400 V from a 500 V source.

Here, $V_{1}^{\prime}=500 \mathrm{~V}, V_{2}^{\prime}=400 \mathrm{~V}$

Transformation ratio,

$$
\begin{gathered}
K=\frac{V_{2}^{\prime}}{V_{1}^{\prime}}=\frac{400}{500}=0.8 \\
I_{1}^{\prime}=K \times I_{2}^{\prime}=0.8 I_{2}^{\prime}
\end{gathered}
$$

Current through 400 V winding,

$$
I_{B C}=I_{2}^{\prime}-I_{1}^{\prime}=I_{2}^{\prime}-0.8 I_{2}^{\prime}=0.2 I_{2}^{\prime}
$$

Since the current rating of 400 V winding is 12.5 A

$$
0.2 I_{2}^{\prime}=12.5 \text { or } I_{2}^{\prime}=\frac{12.5}{0.2}=62.5 \mathrm{~A}
$$

The kVA output of the autotransformer $=\frac{V_{2}^{\prime} I_{2}^{\prime}}{1000}=\frac{400 \times 62.5}{1000}=25$

## Example 10.44

The primary and secondary voltages of an autotransformer are 250 V and 200 V , respectively. Show with the aid of a diagram the current distribution in the windings when the secondary


Fig. 10.42 Circuit as per data current is 100 A and calculate the economy of copper in this particular case (in percentage).

## Solution:

Transformation voltage, $\quad K=\frac{V_{2}}{V_{1}}$

$$
=\frac{200}{250}=0.8
$$

Secondary load current, $I_{2}=100 \mathrm{~A}$
Primary current, $I_{1}=K I_{2}=0.8 \times 100=80 \mathrm{~A}$
The current distribution is shown in Figure 10.42
Economy in copper $=K=0.8$ or $80 \%$

### 10.30 CLASSIFICATION OF TRANSFORMERS

The transformers are often classified according to their applications. Following are the important types of transformers:

1. Power transformers: These transformers are used to step up the voltage at the generating station for transmission purposes and then to step down the voltage at the receiving stations. These transformers are of large capacity (generally above 500 kVA ). These transformer usually operate at high average load, which would cause continuous capacity copper loss, thus affecting their efficiency. To have minimum losses during 24 h , such transformers are designed with low copper losses.
2. Distribution transformers: These transformers are installed at the distribution substations to step down the voltage. These transformers are continuously energised causing the iron losses for all the 24 h , Generally, the load on these transformers fluctuate from no-load to full load during this period. To obtain high efficiency, such transformers are designed with low iron losses.
3. Instrument transformers: To measure high voltages and currents in power system potential transformer (P.T.) and current transformer (C.T.) are used, respectively. The potential transformers are used to decrease the voltage, and current transformers are used to decrease the current up to measurable value. These are also used with protective devices.
4. Testing transformers: These transformers are used to step up voltage to a very high value for carrying out the tests under high voltage, for example, for testing the dielectric strength of transformer oil.
5. Special purpose transformer: The transformers may be designed to serve special purposes, and these may be used with furnaces, rectifiers, welding sets, etc.
6. Autotransformers: These are single-winding transformers used to step down the voltages for starting of large three-phase squirrel cage induction motors.
7. Isolation transformer: These transformers are used only to isolate (electrically) the electronic circuits from the main electrical lines, and therefore, their transformation ratios are usually one.
8. Impedance matching transformer: These transformers are used at the output stage of the amplifier for impedance matching to obtain maximum output from the amplifiers.

### 10.31 POWER TRANSFORMER AND ITS AUXILIARIES

The transformers used in the power system for transfer of electric power or energy from one circuit to the other are called power transformers. The rating of a transformer includes voltage, frequency, and kVA . The kVA rating is the kVA output that a transformer can deliver at the rated voltage and frequency under general service conditions without exceeding the standard limit of temperature rise (usually $45^{\circ}$ to $60^{\circ} \mathrm{C}$ ). The power transformer has the following important parts:

1. Magnetic circuit: The magnetic circuit comprises of transformer core. The transformer core may be core type or shell type in construction. The power transformers used in the power system are mostly three-phase transformers. In a core-type three-phase transformer core has three limbs of equal area of cross-section.
2. Electrical circuit: In three-phase transformers, there are three primary (H.V.) windings and three secondary (L.V.) windings. Whole of the L.V. winding is wound over one limb
next to the core, then whole of the H.V. winding is wound over the L.V. winding. In between the L.V. winding and H.V. winding and between core and L.V. winding, insulation is provided.
3. Transformer oil: Transformer oil is a mineral oil obtained by fractional distillation of crude petroleum. The oil is used only in the oil-cooled transformers. The oil not only carries the heat produced due to losses in the transformer, by convection from the windings and core to the transformer tank, but also has even more important function of insulation.

When transformer delivers power, heat is produced due to the iron and copper losses in the transformer. This heat must be dissipated effectively; otherwise, the temperature of the winding will increase. The raise in temperature further increases with the losses. Thus, the efficiency of the transformer will decrease. As there is no rotating part in the transformer, it is difficult to cool down the transformer as compared to rotating machines. Various methods are adopted to cool down the transformers of different rating. The common methods are air natural cooling, oil immersed natural cooling, oil immersed forced oil circulation natural cooling, oil immersed forced oil circulation with air blast cooling, oil immersed forced oil circulation with water cooling, etc.

Generally, for cooling of distribution transformers, oil immersed natural cooling method is adopted. Cooling tubes or small cooling radiators are used with the main tanks, as shown in Figure 10.43, to increase the surface area for the dissipation of heat.


Fig. $\mathbf{1 0 . 4 3}$ Pictorial view of a $\mathbf{2 0 0} \mathbf{~ k V A}, \mathbf{l l ~ k V} / \mathbf{4 0 0} \mathbf{V}$ oil-immersed natural cooled distribution transformer
4. Tank cover: A number of parts are arranged on the tank cover of which most important are as follows:
(a) Bushing: The internal winding of the transformer is connected to the lines through copper rods or bars which are insulated from the tank cover, these are known as bushings. Up to 33 kV , ordinary porcelain bushing can be used. Above this voltage, oil-filled bushings or condenser bushing are employed.
(b) Oil conservator tank: Oil conservator is also known as an oil expansion chamber. It is a small cylindrical air-tight and oil-tight vessel. The oil conservator is connected with a tube to the main transformer tank at the tank cover. This tank is partially filled with oil. The expansion and contraction of oil changes the oil level in the conservator.
(c) Breather: The transformer oil should not be allowed to come in contact with atmospheric air, since a small amount of moisture causes a great decrease in the dielectric strength of transformer oil. All the tank fittings are made air-tight. When oil level in the oil conservator changes, air moves in and out of the conservator. This action is known as breathing. The breathed air is made to pass through an apparatus called breather to abstract moisture. Breather contains silica gel or some other drying agent such as calcium chloride. This ensures that only dry air enters the transformer tank.
(d) Buchholz relay: This is installed in between the main tank and the oil conservator. It is a gas relay which gives warning of any fault developing inside the transformer, and if the fault is dangerous, the relay disconnects the transformer circuit. This relay is installed in the transformer having capacity more than 750 kVA .
All the important parts of a $200 \mathrm{kVA}, 11 \mathrm{kV} / 400 \mathrm{~V}$ oil immersed natural cooled distribution transformer are shown in Figure 10.43.

## 国禺 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What is an autotransformer?
(P.T.U. May 2007, May 2008)
2. What is an autotransformer? What are its applications?
(P.T.U. Dec. 2008)
3. What is the principle of autotransformer?
(P.T.U. May 2011)
4. How autotransformer is different to a potential divider?
5. Write the advantages of autotransformer.
(P.T.U. Dec. 2009)
6. Why an autotransformer has higher efficiency than other transformers?
(P.T.U. May 2009)
7. Why conservator tank is used in power transformer?
8. What is the function of breather in a transformer?

## Test Questions

1. Write short notes on an autotransformer.
(P.T.U. Dec. 2008)
2. What is an autotransformer? How does it differ from conventional two-winding transformer? State its one application.
(P.T.U. Dec. 2010)
3. Make a comparison between an autotransformer and a potential divider.
4. Write down the working principle, construction, and operation of auto transformer.
(P.T.U. May 2009)
5. What are the advantages of using an autotransformer over two-winding transformer?
(P.T.U. May 2007)
6. What are the disadvantages of using a two-winding transformer over an autotransformer?
(P.T.U. May 2008)
7. What are the disadvantages of autotransformer in comparison with two-winding transformer?
8. Discuss the advantage, disadvantages, and applications of autotransformer as compared to twowinding transformer.
(P.T.U. May 2009)
9. Why is autotransformer not used as a distribution transformer?
10. Name the auxiliaries of a power transformer and describe the function of bushing, conservator, Breather, and Buchholz relay.

## Numericals

1. An autotransformer is used to step down from 240 V to 200 V . The complete winding consist of 438 turns and secondary delivers a current of 15 ampere. Determine (i) Secondary turns (ii) the primary current (iii) the current in the secondary winding. Neglect the effect of the magnetizing current.
(Ans. 365; 12.5 A; 2.8 A )
2. An autotransformer supplies a load of 3 kW at 115 V at a power factor of unity. If the primary voltage applied is 230 V . Calculate (i) the power transformed (ii) the power conducted directly from the supply lines to the load.
(Ans. $1.5 \mathrm{~kW}, 1.5 \mathrm{~kW}$ )
3. The primary and secondary voltages of an autotransformer are 440 and 352 V , respectively. Calculate the value of the currents in distribution with the help of the diagram. Calculate the economy of copper in this case.
(Ans. $\left.I_{2}=100 \mathrm{~A} ; I_{1}=80 \mathrm{~A} ; 80 \%\right)$

## SUMMARY

1. Transformer: A static device which transfers AC electrical power from one circuit to the other at same frequency but usually at different voltage level is called a transformer. It works on the basic principle of electromagnetic induction (mutually induced emf).
2. Ideal transformer: Losses are neglected; $E_{1} l_{1} \cos \phi=E_{2} l_{2} \cos \phi$
3. Transformer on $D C$ : It is never applied on DC; otherwise, it will burn since there is no counter of self-induced emf and it draws heavy current on DC.
4. EMF equation: $E_{2}=4.44 N_{2} f \phi_{\mathrm{m}}=4.44 N_{2} f B_{\mathrm{m}} A_{\mathrm{i}}$
5. Transformer on no-load: At no-load, it draws power to meet with iron losses, $P_{0}=V_{1} I_{0} \cos \phi_{0}$

$$
I_{\mathrm{w}}=I_{0} \cos \phi_{0} ; I_{\mathrm{m}}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}
$$

6. Transformation on load: When a transformer is loaded, the secondary ampere-turns set up a field in opposition direction to the main field. To neutralize this effect, transformer primary draws extra current from the mains. Flux remains the same from no-load to full load.

$$
\begin{aligned}
I_{1}^{\prime} N_{1}= & I_{2} N_{2} ; I_{1}=\sqrt{I_{0}^{2}+I_{1}^{\prime 2}+2 I_{0} I_{1}^{\prime} \cos \left(\phi_{0}-\phi_{2}\right)} ; \\
& \cos \phi_{1}=\left(I_{0} \cos \phi_{0}+I_{1}^{\prime} \cos \phi_{2}\right) / I_{1}
\end{aligned}
$$

7. Transformer winding resistance: $R_{1} ; R_{2} ; R_{1}^{\prime}=R_{1} \times K^{2} ; R_{\mathrm{es}}=R_{2}+R_{1}^{\prime}$;

$$
R_{2}^{\prime}=R_{2} / K^{2} ; R_{\mathrm{ep}}=R_{1}+R_{2}^{\prime}
$$

8. Leakage flux: A part of the flux produced by a winding which is not linking with the other is called leakage flux. It develops leakage reactance $X_{1}$ and $X_{2}, X_{1}^{\prime}=X_{1} / K^{2} ; X_{\text {es }}=X_{2}+X_{1}^{\prime} ; X_{2}^{\prime}=X_{2} / K^{2} ; X_{\mathrm{cp}}=X_{1}+X_{2}^{\prime}$.
9. Voltage regulation: At constant supply voltage, the change in secondary terminal voltage from no-load to full load is called voltage regulation. It is generally taken as percentage of no-load voltage.

$$
\% \operatorname{Reg}=\frac{E_{2}-V_{2}}{E_{2}} \times 100
$$

10. Relation between $E_{2}$ and $V_{2}$ : $E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}+I_{2} X_{\mathrm{cs}} \sin \phi_{2}$ (lagging p.f.); $E_{2}=V_{2}+I_{2} R_{\mathrm{cs}}$ (unity p.f.); $E_{2}=V_{2}+I_{2} R_{\mathrm{es}} \cos \phi_{2}-I_{2} X_{\mathrm{es}} \sin \phi_{2}$ (leading p.f.)

$$
\% \operatorname{Reg}=\% R \times \cos \phi_{2}+\% X \times \sin \phi_{2} \text { (lagging p.f.) }
$$

11. Losses in a transformer: lron loss $\left(P_{\mathrm{i}}\right)$ and Cu . loss ( $P_{\mathrm{c}}$ at full load)
12. Efficiency of a transformer: $\eta=\frac{V_{2} I_{2} \cos \phi_{2}}{V_{2} I_{2} \cos \phi_{2}+P_{\mathrm{i}}+P_{\mathrm{c}}} ; \quad \eta_{\mathrm{x}}=\frac{x V_{2} I_{2} \cos \phi_{2}}{x V_{2} I_{2} \cos \phi_{2}+P_{\mathrm{i}}+x^{2} P_{\mathrm{c}}}$
13. Condition for max. efficiency: $\mathrm{Cu} . \operatorname{loss}=$ lron loss; $I_{2}=\sqrt{P_{\mathrm{i}} / R_{\mathrm{es}}}$
14. All-day efficiency: $\eta_{\text {allday }}=\frac{\text { Output in } \mathrm{kWh}}{\text { Input in } \mathrm{kWh}}$ (for 24 h )
15. Open-circuit test: It is performed to determine (i) Iron losses $P_{\mathrm{i}}=$ Wattmeter reading, $P_{\mathrm{i}}=P_{0}=V_{1} I_{0} \cos \phi_{0}$, (ii) Working component $I_{\mathrm{w}}=P_{0} / V_{1}$, and (iii) Magnetising component $I_{\mathrm{m}}=\sqrt{I_{0}^{2}-I_{\mathrm{w}}^{2}}$.
16. Short-circuit test: It is performed to determined (i) copper losses $P_{\mathrm{c}}=$ Wattmeter reading (when test is performed at full load) and (ii) Parameters of the transformer on the side in which instruments are connected, that is, $Z_{\mathrm{es}}=V_{2(\mathrm{sc})} / I_{2} ; R_{\mathrm{es}}=W_{\mathrm{c}} / I_{2}^{2} ; X_{\mathrm{es}}=\sqrt{Z_{\mathrm{es}}^{2}-R_{\mathrm{es}}^{2}}$
17. Autotransformer: A transformer, having only one winding, a part of which acts as primary and the other as secondary is called an auto transformer. Saving in copper $=\mathrm{K} \times \mathrm{Wt}$. of Cu . required for two-winding transformer.
18. Power transformer and its auxiliaries: A transformer employed in the power system to step up or step down the voltage as per the need is called power transformer. The main auxiliaries of these transformers are conservator tank, cooling tubes, breather, Buchholz relay, temperature gauge, etc.

## TEST YOUR PREPARATION

## 7 FILLIN THE BLANKS

1. Transformer steps up or steps down $\qquad$ .
2. Iron losses in a transformer consist of $\qquad$ and $\qquad$ .
3. Transformer core is made of laminations to reduce $\qquad$ -
4. Transformer core is made of silicon steel to reduce $\qquad$ .
5. In transformer, $E_{1}=$ $\qquad$ $\phi_{\mathrm{m}} f N_{1}$ volt.
6. Transformer is a $\qquad$ electric machine.
7. To determine the iron losses in the transformer $\qquad$ test is performed.
8. To determine the copper losses in the transformer $\qquad$ test is performed.
9. The condition for maximum efficiency of a transformer is copper losses = $\qquad$
10. For step up transformer, the transformation ratio is $\qquad$ than unity.

## OBJECTIVE TYPE QUESTIONS

1. The phase relationship between the primary and secondary voltages of a transformer is
(a) 90 degree out of phase
(b) in the same phase
(c) 180 degree out of phase
(d) None of the above
2. The transformer is analogous of gear trains.
(a) true
(b) false
3. Transformer core is laminated
(a) because it is difficult to fabricate solid core
(b) because laminated core provides high flux density
(c) to reduce eddy current losses.
(d) to avoid hysteresis losses.
4. The use of higher flux density in the transformer design
(a) reduces the weight per kVA
(b) increases weight per kVA
(c) has no relation with the weight of transformer
(d) increases the weight per kW .
5. The induced emf in the transformer secondary will depend on
(a) frequency of the supply only.
(b) Number of turns in secondary only.
(c) Frequency and flux in core.
(d) Frequency, number of secondary turns and flux in the core
6. The mutual flux links both the windings of transformer and hence contributes to the transfer of energy from primary to the secondary.
(a) true
(b) false
7. A $4: 1$ voltage step-up transformer has 150 V across the primary and $600 \Omega$ resistance across the secondary. Assuming $100 \%$ efficiency, the primary current equals.
(a) $1 / 4 \mathrm{~A}$
(b) 400 mA
(c) 4 A
(d) 1 A
8. A transformer with output of 250 kVA at 3000 V , has 600 turns on its primary and 60 turns on secondary winding. What will be the transformation ratio of the transformer?
(a) 10
(b) 0.1
(c) 100
(d) 0.01
9. A transformer with output of 250 kVA at 3000 V , has 600 turns on the its primary and 60 turns on secondary winding. What will be the voltage on the primary side?
(a) 300 V
(b) 30000 V
(c) 30 V
(d) $3 \times 10^{5} \mathrm{~V}$
10. The primary applied voltage in an ideal transformer on no-load is balanced by
(a) primary induced emf
(b) secondary induced emf
(c) secondary voltage
(d) iron and copper losses
11. A transformer is never connected in the DC line because
(a) there is no need to step up or step down the DC voltage
(b) Faraday's law is not valid as the rate of changed of flux is zero.
(c) Losses in the DC circuit are high.
(d) It is not economical.
12. No-load primary input is practically equal to the iron loss in the transformer because primary current is very small
(a) true
(b) false
13. If $R_{1}$ is the resistance of primary winding of the transformer and K is transformer ratio then the equivalent primary resistance referred to secondary will be
(a) $K R_{1}^{2}$
(b) $K R_{1}$
(c) $K^{2} R_{1}$
(d) $R_{1} / K^{2}$
14. A transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary
(a) true
(b) false
15. The percentage resistance and reactance have the same value whether referred to primary or secondary of the transformer.
(a) true
(b) false
16. A good transformer must have regulation as high as possible,
(a) true
(b) false
17. If the supply frequency to the transformer is increased the iron loss
(a) will not change
(b) will be zero
(c) will icrease
(d) will decrease
18. The eddy current loss in the transformer occurs in the
(a) primary winding
(b) core
(c) secondary winding
(d) None of the above.
19. The eddy current losses in the transformer will be reduced if
(a) the number of turns in the primary winding is reduced
(b) the number to turns in the secondary winding is reduced
(c) the laminations are thick
(d) the laminations are thin
20. Which of the following electrical machine has the highest efficiency?
(a) DC generator
(b) AC generator
(c) transformer
(d) induction motor
21. The efficiency of a transformer is independent of p.f.
(a) true
(b) false
22. The condition for maximum efficiency of the transformer is that
(a) copper losses are half of the iron losses
(b) copper losses are square of the iron losses
(c) copper losses are equal to the iron losses
(d) copper losses are zero
23. If the iron losses and full-load copper losses are given, then the load at which the efficiency of a transformer is maximum and is given by
(a) full load $\times \frac{\text { iron loss }}{\text { f. 1. cu loss }}$
(b) full load $\times \sqrt{\frac{\text { iron loss }}{\text { f. 1. cu loss }}}$
(c) full load $\times\left(\frac{\text { iron loss }}{\text { f. i. cu. loss }}\right)$
(d) full load $\times \sqrt{\frac{\text { f. 1. cu. loss }}{\text { iron loss }}}$
24. The short circuit test in the transformer is performed to determine
(a) the iron loss at any load
(b) the hysteresis loss
(c) the copper loss at any load or at full load
(d) the eddy current loss

## NUMERICALS

1. A sinusoidal flux 0.2 Wb (max.) links with 55 turns of a transformer secondary coil. Calculate the rms value to the induced emf in the secondary. The supply frequency is 50 Hz .
(Ans. 244.2 V )
2. The design requirements of a $6600 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$, single-phase, core-type transformer are: emf per turn 15 V ; maximum flux density 1.5 Tesla (i.e., $\mathrm{Wb} / \mathrm{m}^{2}$ ). Find a suitable number of primary and secondary turns and the net cross-sectional area of core.
(Ans. 440; 26.67; $450 \mathrm{~cm}^{2}$ )
3. A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is $60 \mathrm{~cm}^{2}$. If the primary winding is connected to a 50 Hz supply at 500 V , calculate the value of maximum flux density in the core and the emf induced in secondary winding. Draw the vector diagram representing the condition.
(Ans. 0.938 Tesla, 1250 V )
4. A single-phase, $50 \mathrm{kVA}, 2300 / 230 \mathrm{~V}, 50 \mathrm{~Hz}$ transformer is connected to 230 V supply on the secondary side, the primary being open. The meter indicate the following readings: Power $=187$ watt;

Voltage $=230 \mathrm{~V}$; Current $=6.5$ A. Find (i) core loss; (ii) loss component of the current;(iii) magnetizing current.
(Ans. $187 \mathrm{~W} ; 0.813 \mathrm{~A} ; 6.45 \mathrm{~A})$
5. A transformer on no-load has a core loss of 50 W , draws a current of $2 \mathrm{~A}(\mathrm{rms})$, has an induced emf of 230 V (rms). Determine the no-load power factor, core-loss component of exciting current and magnetizing current. Neglect leakage flux and winding resistance.
(U.P.T.U. Tut.) (Ans. 0.1087 (lagging); 0.2174 A; 1.988 A)
6. The number of turns on the primary and secondary winding of a single and transformer are 350 and 38 , respectively. If the primary winding is connected to $2.2 \mathrm{kV}, 50 \mathrm{c} / \mathrm{s}$ supply, determine (i) the secondary voltage on no-load. (ii) The primary current when secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging. (iii) the p.f. of the primary current.
(Ans. 238.8 V, 25.67, 0.7156 lag)
7. A $2000 / 200 \mathrm{~V}$ transformer has primary resistance and reactance of $2 \Omega$ and $4 \Omega$, respectively. The corresponding secondary values are $0.025 \Omega$ and $0.04 \Omega$. Determine: (i) Equivalent resistance and reactance of primary referred to secondary; (ii) Total resistance and reactance referred to secondary; (iii) Equivalent resistance and reactance of secondary referred to primary; (iv) Total resistance and reactance referred to primary.
(Ans. $0.02 \Omega ; 0.04 \Omega ; 0.045 \Omega ; 0.08 \Omega ; 2.5 \Omega ; 4 \Omega ; 4.5 \Omega ; 8 \Omega$ )
8. A $33 \mathrm{kVA}, 2200 / 220 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase transformer has the following parameters: Primary winding (H.V. side); resistance $r_{1}=2.4 \Omega$, leakage reactance $x_{1}=6.0 \Omega$. Secondary winding (I.V. side): resistance $r_{2}=0.03 \Omega$, leakage reactance $x_{2}=0.07 \Omega$. Find the primary resistance and leakage reactance referred to secondary.
(U.P.T.U. Tut.) (Ans. $0.024 \Omega$ and $0.06 \Omega$ )
9. A single-phase transformer with a ratio of $4: 1$ has primary resistance and reactance of $0.5 \Omega$ and $1.5 \Omega$ and the corresponding values for the secondary are $0.034 \Omega$ and $0.1 \Omega$, respectively. Determine the percentage regulation when delivering 120 A at 600 V at p.f. (i) 0.707 lagging (ii) 0.8 leading.
(Allahabad Univ.) (Ans. 3.53\%; -3.26\%)
10. Calculate the value of voltage regulation at 0.8 p.f. lagging for a transformer with resistance drop $2 \%$ and reactance drop $4 \%$ of the voltage.
(Ans. 4.0\%)
11. A $230 / 460 \mathrm{~V}$, single-phase transformer has a primary resistance of $0.2 \Omega$ and reactance $0.5 \Omega$. The corresponding values for the secondary are $0.75 \Omega$ and $1.8 \Omega$, respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging.
(B.U. Feb. 1988) (Ans. 424.8 V)
12. The primary and secondary windings of a 500 kVA transformer have resistance of $0.42 \Omega$ and $0.0011 \Omega$, respectively. The primary and secondary voltages are 6600 V and 400 V , respectively. The iron loss is 2.9 kW . Calculate the efficiency at half full load at a power factor of 0.8 lagging.
(Ans. 98.07\%)
13. A 50 kVA transformer has an efficiency of $98 \%$ at full load at $0.8 \mathrm{p} . \mathrm{f}$ and an efficiency of $96.9 \%$ at $\frac{1}{4}$ full load, 0.8 p.f. Determine the iron loss and full-load copper loss.
(Ans. 287 W; 529 W)
14. A $440 / 110 \mathrm{~V}$ transformer has an effective primary resistance of $0.3 \Omega$ and a secondary resistance of $0.02 \Omega$. If iron loss on normal input voltage is 150 W , calculate the secondary current at which maximum efficiency will occur. What is the value of this maximum efficiency for unity power factor load?
(Ans. 62.22 A; 94.8\%)
15. In a $25 \mathrm{kVA}, 1100 / 400 \mathrm{~V}$, single-phase transformer, the iron and copper loss at full load are 350 and 400 W , respectively. Calculate the efficiency on unity power factor at half load. Determine the load on maximum efficiency.
(Ans. 96.52\%; 23.85 kW )
16. A 100 kVA transformer supplies a lighting and power load. The iron loss is 960 W and the copper loss is 960 W at full load. The transformer is operated continuously at the rated voltage as per the following schedule in a day 100 kVA at 0.8 p.f. for $4 \mathrm{~h} ; 50 \mathrm{kVA}$ at 0.6 p.f. for 8 h . and 5 kVA at 0.95 p.f. for 12 h . What will be the all-day efficiency of the transformer?
(Ans. 94.5\%)
17. A transformer has its maximum efficiency of $98 \%$ at 15 kVA at unity p.f. compare its all-day efficiencies for the following load cycle: (i) Full load of 20 kVA $12 \mathrm{~h} /$ day and no-load rest of the day; (ii) Full load $4 \mathrm{~h} /$ day and 0.4 full-load rest of the day.
(Ans. $\eta_{2} / \eta_{1}=1.0053$ )
18. The O.C. and S.C. tests on a $5 \mathrm{kVA}, 230 / 110 \mathrm{~V}, 50 \mathrm{c} / \mathrm{s}$ transformer gave the following data; O.C. test (H.V. side); $230 \mathrm{~V}, 0.6 \mathrm{~A}, 80 \mathrm{~W}$; S.C. test (L.V. side); $6 \mathrm{~V}, 15 \mathrm{~A}, 20 \mathrm{~W}$. Calculate the efficiency of the transformer on full load at 0.8 p.f. lagging. Also calculate the voltage on the secondary side under full-load conditions at 0.8 p.f. leading.
(Ans. 93.82\%, 117.4 V )
19. A $5 \mathrm{kVA}, 200 / 400 \mathrm{~V}, 50 \mathrm{~Hz}$ single-phase transformer gave the following test data: Open circuit test (L.V. side); $200 \mathrm{~V}, 0.7 \mathrm{~A}, 60 \mathrm{~W}$; short circuit test (H.V. side); $22 \mathrm{~V}, 16 \mathrm{~A}, 120 \mathrm{~W}$. If the transformer operates on full load, determine: (i) the percentage regulation at 0.9 p.f. lagging (ii) efficiency at 0.8 p.f. lagging.
(Ans. 3.08\%; 96.77\%)
20. The following results were obtained on a $50 \mathrm{kVA}, 2400 / 120 \mathrm{~V}$ single-phase transformer:

1. O.C. test on the L.V. side $396 \mathrm{~W}, 9.65 \mathrm{~A}, 120 \mathrm{~V}$
2. S.C. test on the H.V. side $810 \mathrm{~W}, 20.8 \mathrm{~A}, 92 \mathrm{~V}$

Find the parameters of the equivalent circuit as referred to L.V. side.
(U.P.T.U. 2002-03) (Ans. $R_{\mathrm{o}}=36.364 \Omega ; X_{\mathrm{o}}=13.233 \Omega ; R_{\mathrm{ep}}=0.00468 \Omega ; X_{\mathrm{cp}}=0.01 \Omega$ )

## VIVA VOCE/REASONING QUESTIONS

1. Transformer is analogous to mechanical gear drive. How?
2. Core and winding of a power transformer is usually placed in transformer oil. Why?
3. Calcium carbonate or silica gel is placed in the transformer breather. Why?
4. The core of the transformer is not made of wood. Why?
5. A transformer is called a static electrical machine. Why?
6. In a transformer by electromagnetic induction, emf is induced in the secondary winding. Is it also induced in the transformer tank which is a conducting material state, Why?
7. Transformer cannot be used on DC. Why?
8. While assembling transformer core, $E$ and $I$ sections are assembled alternately in upper and lower portion. Why?
9. While drawing phasor diagram of an ideal transformer, the flux vector is shown $90^{\circ}$ lagging from the applied voltage $V_{1}$. Why?
10. Is the mutual flux and leakage flux are the same. State Why?
11. There is no electrical connection between primary and secondary winding of a transformer still when we change the load on the secondary, the current drawn by the primary changes. Why?
12. While performing short circuit test, rated voltage is never applied to the transformer. Why?
13. Under open circuit test, it is said that the copper losses are almost zero. Why?
14. The copper losses of a transformer are square of the fraction $x^{2}$ of the load. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What is a transformer?
2. What are step-up and step-down transformers?
3. What are the applications of step-up and step-down transformers?
4. Which are the two winding present in a transformer?
5. Explain the working principle of a transformer.
6. What are the functions of a transformer?
7. Define voltage transformation ratio.
8. From construction point of view, name different types of transformers.
9. Why the core of a transformer is laminated?
10. What is a transformer oil? List out its functions.
11. What is the transformer tank?
12. What do you understand by breathing in transformers?
13. Why is a transformer also called the static device?
14. What is a Buchholz relay?
15. Why do we use iron core in a transformer?
(U.P.T.U. Tut.)
16. Explain with reasons what happens when a power transformer is connected to DC supply of the same voltage rating?
17. Why is the efficiency of a transformer maximum among electrical equipments? Explain.
(P.T.U. 1999)
18. Why are the iron-cored transformers not used at high frequencies?
19. Where is core-type construction suitable for a transformer?
20. Where is shell-type transformer used?
21. What are the advantages of oil over air as a cooling medium in transformers?
22. What is the difference between an ideal and practical transformers?
(P.T.U. Jan. 2000)
23. What is meant by magnetizing current?
24. What do you know about no-load current of a transformer?
25. When the load current of a transformer increases, how does the input current adjust to meet the new conditions.
(U.P.T.U. Tut.)
26. Why the main flux in a transformer remains practically constant from no-load to full load?
27. How does leakages flux occur in a transformer?
(U.P.T.U. Tut.)

## TEST QUESTIONS

1. What is a transformer? What is its necessity in the power system?
2. Explain the working principle of a transformer.
3. State why the core of a transformer is laminated?
(U.P.T.U.) (B.U. Dec. 1985)
4. State why silicon steel in selected for the core of a transformer?
(U.P.T.U.) (B.U. Dec. 1985)
5. Give the constructional details of a core-type transformer.
6. In a transformer, explain how power is transferred from one winding to the other.
7. Show that $\left(E_{1} / E_{2}\right)=\left(l_{2} / l_{1}\right)=\left(T_{1} / T_{2}\right)$ in a transformer.
8. What happens when DC voltages is applied to the primary of a transformer?
9. Derive an expression for the emf induced in a transformer winding. Show that the emf induced per turn in primary is equal to the emf per turn in secondary.
(U.P.T.U. 2001, 2003)
10. Explain the behaviour of a transformer on no-load.
(U.P.T.U.; A.M.I.E. Summer 1992; Summer 1988)
11. 'The main flux in a transformer remains practically invariable under all conditions of load'. Explain.
12. Draw and explain the phasor diagram of a loaded transformer (neglecting voltage drop due to resistance and leakage reactance).
13. Draw the phasor diagram and equivalent circuit of a single-phase transformer.
14. Draw a neat phasor diagram showing the performance of a transformer on-load.
15. Draw and explain the phasor diagram of single-phase transformer connected to a lagging p.f. load.
16. 'The overall reactance of transformer decreases with load'. Explain.
(Hints: $L=N \phi / I$; when $I$ increases, $L$ decreases)
17. Explain what is meant by regulation of a transformer.
18. What are the various losses in a transformer? Where do they occur and how do they vary with load?
19. Define efficiency of a transformer and find the condition for obtaining maximum efficiency.
20. Write short note on 'All-day efficiency of a transformer'.
21. Distinguish between 'power efficiency' and 'all-day efficiency' of a transformer.
22. What information can be obtained from the open circuit test of transformer? How can you get these information?
(AMIE; W, 1983)
23. Explain open circuit and short circuit tests of a single-phase transformer giving circuit diagram for each test. Also mention uses of these tests.
(U.P.T.U.)
24. Write the function of the following auxiliaries of the transformer:
(i) Breather
(ii) Cooling tubes
(iii) Conservator tank
(iv) Bushings
(v) Buckholz relay.

## answers

## Fill in the Blanks

1. voltage
2. hysteresis, eddy current losses
3. eddy current losses
4. hysteresis losses
5. 4.44
6. static
7. short circuit
8. iron losses
9. more

## Objective Type Questions

| 1. (c) | 2. (a) | 3. (c) | 4. (a) | 5. (d) |
| ---: | ---: | ---: | ---: | ---: |
| 6. (a) | 7. (c) | 8. (b) | 9. (b) | 10. (b) |
| 11. (b) | 12. (a) | 13. (c) | 14. (a) | 15. (a) |
| 16. (b) | 17. (c) | 18. (b) | 19. (d) | 20. (c) |
| 21. (b) | 22. (c) | 23. (b) | 24. (b) |  |



## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:
*What is electromechanical energy conversion device? When and how it works either as a generator or as a motor?

* What is a DC generator and how it converts mechanical energy into electrical energy?
* What is a DC motor and how it converts electrical energy into mechanical energy?
* What are the main constructional features of a DC machine, material used for different parts, and their functions?
* What is the working principle of a DC generator?
*What are the factors on which emf induced in the armature depends?
* What are different types of DC generators on the basis of field excitation?
* How voltage is built-up in DC shunt generator and what is critical resistance?
* What is a DC motor and how it converts electrical energy into mechanical energy?
* What is back emf and its significance?
* What are the factors on which torque developed in a DC motor depends?
* How DC motors are classified on the basis of their field excitation?
* What are the performance characteristics of DC motors and how they are selected for different applications?
* What is the need of a starter and how starters limit the in-rush flow of starting current and protects the motor?
* What are the various losses of a DC machine, and how these are categorised?


## Il. I INTRODUCTION

A DC machine is an electromechanical energy conversion device. It can convert mechanical power $(\omega T)$ into DC electrical power $(E I)$ and is known as a DC generator. On the other hand, when it converts DC electrical power into mechanical power, it is known as a DC motor.

Although battery is an important source of DC electric power, it can supply a limited power. There are some applications where large quantity of DC power is required (e.g., electroplating electrolysis) and, at such places, DC generators are used to deliver power.

It has been seen that AC motors are invariably applied in the industry for the conversion of electrical power into mechanical power; however, at the places where wide range of speeds and good speed regulation are required (such as electric traction), DC motors have to be applied.

In short, we can say that DC machines have their own role in the field of engineering. In this chapter, we shall study the common topics of DC machines.

### 11.2 ELECTROMECHANICAL ENERGY CONVERSION DEVICES (MOTORS AND GENERATORS)

A device (machine) that makes possible the conversion of energy from electrical to mechanical form or from mechanical to electrical form is called an electro-mechanical energy conversion device or electro-mechanical transducer, as shown in Figure 11.1.


Fig. Il.l Electro-mechanical energy conversion

Depending upon the conversion of energy from one to the other, the electromechanical device can be named as generator or motor.

### 11.3 ELECTRIC GENERATOR AND MOTOR

Depending upon the energy conversion, a DC machine may work as a generator or motor.

### 11.3.1 Generator

An electromechanical device (electrical machine) that converts mechanical energy or power $(\omega T)$ into electrical energy or power $(E I)$ is called generator.


Fig. 11.2 Generator

Generators are used in hydroelectric power plants, steam power plants, diesel power plants, nuclear power plants, and automobiles. In these power plants, various natural sources of energy are first converted into mechanical energy, and then, it is converted into electrical energy with the help of generators. The block diagram of energy conversion, when the electromechanical device works as a generator, is shown in Figure 11.2.

### 11.3.2 Motor

An electromechanical device (electrical machine) that converts electrical energy or power ( $E I$ ) into mechanical energy or power $(\omega T)$ is called a motor.

Electric motors are used for driving industrial machines, for example, hammer presses, drilling machines, lathes, shapers, blowers for furnaces, etc., and domestic appliances, for example, refrigerators, fans, water pumps, toys, mixers, etc. The block diagram of energy conversion, when the electromechanical device works as a motor, is shown in Figure 11.3.


Fig. 11.3 Motor

Note: The same electromechanical device is capable of operating either as a motor or generator depending upon whether the input power is electrical or mechanical. Thus, the motor and generator actions are reversible.
The conversion of energy either from electrical to mechanical or from mechanical to electrical takes place through magnetic field. During the conversion, whole of the energy in one form is not converted in the other useful form. In fact, the input power is divided into the following three parts:

1. Most of the input power is converted into useful output power.
2. Some of the input power is converted into heat losses $\left(I^{2} R\right)$ that are due to the flow of current.

### 11.4 MAIN CONSTRUCTIONAL FEATURES

The complete assembly of various parts in a scattered form of a DC machine is shown in Figure 11.4. The essential parts of a DC machine are described as follows:


Fig. 11.4 Main parts of DC machine

1. Magnetic frame or yoke: The outer cylindrical frame to which main poles and interpoles are fixed and by means of which the machine is fixed to the foundation is called the yoke. It serves the following two purposes:
(a) It provides mechanical protection to the inner parts of the machine.
(b) It provides a low reluctance path for the magnetic flux.

The yoke is made of cast iron for smaller machines, and for larger machines, it is made of cast steel or fabricated rolled steel since these materials have better magnetic properties as compared to cast iron.
2. Pole core and pole shoes: The pole core and pole shoes are fixed to the magnetic frame or yoke by bolts. They serve the following purposes:
(a) They support the field or exciting coils.
(b) They spread out the magnetic flux over the armature periphery more uniformly.


Fig. 11.5 Pole core and pole shoe
(c) Since pole shoes have large X -section, the reluctance of magnetic path is reduced. Usually, the pole core and pole shoes are made of thin cast steel or wrought iron laminations that are riveted together under hydraulic pressure as shown in Figure 11.5.
3. Field or exciting coils: Enamelled copper wire is used for the construction of field or exciting coils. The coils are wound on the former (see Fig. 11.6) and then placed around the pole core as shown in Figure 11.5. When DC is passed through the field winding, it magnetises the poles that produce the required flux. The field coils of all the poles are connected in series in


Fig. 11.6 Field winding such a way that when current flows through them, the adjacent poles attain opposite polarity as shown in Figure 11.7.
4. Armature core: It is cylindrical in shape and keyed to the rotating shaft. At the outer periphery, slots are cut, as shown in Figure 11.8, and they accommodate the armature winding. The armature core shown in Figure 11.8 serves the following purposes:
(a) It houses the conductors in the slots.
(b) It provides an easy path for magnetic flux.

Since armature is a rotating part of the machine, reversal of flux takes place in the core. Hence, hysteresis losses are produced. To minimise these losses, silicon steel material is used for its construction. The rotating armature


Fig. 11.7 Connections of field winding cuts across the magnetic field that induces an emf in it. This emf circulates eddy currents that result in eddy current loss in it. To reduce these losses, armature core is laminated. In other words, we can say that about 0.3 to 0.5 mm thick stampings are used for its construction. Each lamination or stamping is insulated from the other by varnish layer (see Fig. 11.8).
5. Armature winding: The insulated conductors housed in the armature slots are suitably connected. This is known as armature winding. The armature winding is the heart of a DC machine. It is a place where conversion of power takes place, that is, in the case of generator, mechanical power is converted into electrical power; while in the case of motor, electrical power is converted into mechanical power. On the basis
of connections, there are two types of armature windings, namely lap winding and wave winding.
(a) Lap winding: In lap winding, the conductors are connected in such a way that the number of parallel paths are equal to the number of poles. Thus, if machine has $P$ poles and $Z$ armature conductors, then there will be $P$ parallel paths and each path will have $Z / P$ conductors in series. In this case, the number of brushes is equal to the number of parallel paths. Out of which, half the brushes are positive and the remaining (half)


Fig. 11.8 Armature core are negative.
(b) Wave winding: In wave winding, the conductors are so connected that they are divided into two parallel paths, irrespective of the number of poles of the machine. Thus, if machine has $Z$ armature conductors, there will be only two parallel paths each having $Z / 2$ conductors in series. In this case, the number of brushes is equal to two, that is, number of parallel paths.
6. Commutator: It is the most important part of a DC machine and serves the following purposes:
(a) It connects the rotating armature conductors to the stationary external circuit through brushes.
(b) It converts the AC induced in the armature conductors into unidirectional current in the external load circuit during the generator action, whereas it converts the alternating torque into unidirectional (continuous) torque produced in the armature during the motor action.
The commutator is of cylindrical shape and is made up of wedge-shaped hard-drawn copper segments. The segments are insulated from each other by a thin sheet of mica. The segments are held together by means of two V-shaped rings that fit into the V-grooves cut into the segments. Each armature coil is connected to the commutator segment through riser. The sectional view of the commutator assembly is shown in Figure 11.9.


Fig. 11.9 Commutator
7. Brushes: The brushes are pressed upon the commutator and form the connecting link between the armature winding and the external circuit. They are usually made of high grade carbon because carbon is a conducting material, and at the same time, in powdered form, it provides imbricating effect on the commutator surface. The brushes are held in particular position around the commutator by brush holders and rocker.
8. Brush rocker: It holds the spindles of the brush holders. It is fitted on to the stationary frame of the machine with nut and bolts. By adjusting its position, the position of the brushes over the commutator can be adjusted to minimise the sparking at the brushes.
9. End housings: End housings are attached to the ends of the main frame and support bearings. The front housing supports the bearing and the brush assemblies, whereas the rear housing usually supports the bearing only.
10. Bearings: The ball or roller bearings are fitted in the end housings. The function of the bearings is to reduce friction between the rotating and the stationary parts of the machine. Mostly, high carbon steel is used for the construction of bearings as it is very hard material.
11. Shaft: The shaft is made of mild steel with a maximum breaking strength. The shaft is used to transfer mechanical power from or to the machine. The rotating parts such as armature core, commutator, and cooling fan are keyed to the shaft.

### 11.5 ARMATURE RESISTANCE

The resistance between the armature terminals is called armature resistance. It is generally represented by $R_{a}$. The value of armature resistance is usually quite small (less than $1 \Omega$ ). Armature resistance depends upon the following factors:

1. Length, area of cross-section, and material of armature winding.
2. Type of armature winding, that is, lap or wave winding. This will show the manner in which the conductors (i.e., their series-parallel combination) are connected.

### 11.6 SIMPLE LOOP GENERATOR AND FUNCTION OF COMMUTATOR

For simplicity, consider only one coil AB placed in the strong magnetic field. The two ends of the coil are joined to slip rings $\mathrm{A}^{\prime}$ and $\mathrm{B}^{\prime}$, respectively. Two brushes rest on these slip rings, as shown in Figure 11.10.


Fig. 11.10 Direction of induced emf/current in internal and external circuit of rotating coil at different instants using two slip-rings

When this coil is rotated in counter clockwise direction at an angular velocity of $\omega$ radians $/ \mathrm{s}$, the magnetic flux is cut by the coil and an emf is induced in it. The position of the coil at various instants is shown in Figure 11.10 and the corresponding value of the induced emf and its direction is shown in Figure 11.11. The induced emf is alternating and the current flowing through the external resistance is also alternating, that is, at the second instant, current flows in external resistance from M to L ; while at the fourth instant, it flows from L to M as shown in Figure 11.11.


Fig. 11.11 Graphical representation of current in external circuit at various instants

### 11.6.1 Commutator Action

Now, consider that the two ends of the coil are connected to only one slip ring split into two parts (segment), that is, $\mathrm{A}^{\prime \prime}$ and $\mathrm{B}^{\prime \prime}$. Each part is insulated from the other by a mica layer. Two brushes rest on these parts of the ring as shown in Figure 11.12.


Fig. 11.12 Direction of induced emf/current in internal and external circuit of a rotating coil at different instants using split-ring

In this case, when the coil is rotated in counter clockwise direction at an angular velocity of $\omega$ radians/s, the magnetic flux is cut by the coil and an emf is induced in it. The magnitude of emf induced in the coil at various instants will remain the same as shown in Figure 11.13.

However, the flow of current in the external resistor or circuit will become unidirectional, that is, at the second instant, the flow of current in the external resistor is from $M$ to $L$ as well as the flow of current in the external resistor is from $M$ to $L$ in the fourth instant, as shown in Figure 11.12. Its wave shape is shown in Figure 11.13.


Fig. 11.13 Graphical representation of current in external circuit at various instants

Hence, an AC is converted into unidirectional current in the external circuit with the help of a split ring (i.e., commutator).

In an actual machine, there are number of coils connected to the number of segments of the ring called commutator. The emf or current delivered by these coils to the external load is shown in Figure 11.14(a). The actual flow of current flowing in the external load is shown by the firm line that fluctuates slightly. The number of coils placed on the armature are even much more than this and a pure DC is obtained at the output as shown in Figure 11.14(b).

Thus, in actual machine working as a generator, the function of commutator is to convert the AC produced in the armature into DC in the external circuit.


Fig. 11.14 (a) Pulsating current in external circuit using number of coils (b) Graphical representation of actual current in external circuit of a DC generator

### 11.7 EMF EQUATION

Let $P=$ number of poles of the machine
$\phi=$ flux per pole in Wb
$Z=$ total number of armature conductors
$N=$ speed of armature in rpm
$A=$ number of parallel paths in the armature winding
In one revolution of the armature, flux cut by one conductor $=P \phi \mathrm{~Wb}$

Time taken to complete one revolution, $t=60 / \mathrm{N} \mathrm{s}$
Therefore, average induced emf in one conductor is

$$
e=\frac{P \phi}{t}=\frac{P \phi}{60 / N}=\frac{P \phi N}{60} \mathrm{~V}
$$

The number of conductors connected in series in each parallel path $=Z / A$

Therefore, average induced emf across each parallel path or across the armature terminals

$$
E=\frac{P \phi N}{60} \times \frac{Z}{A}=\frac{P Z \phi N}{60 A} \mathrm{~V}
$$



Fig. 11.15 No. of conductors under the influence of one pole
or

$$
E=\frac{P Z \phi n}{A} \text { where } n \text { is the speed in r.p.s, that is, } n=\frac{N}{60}
$$

For a given machine, the number of poles and number of conductors per parallel path $(Z / A)$ are constant.

$$
E=K \phi n, \text { where } K=\frac{P Z}{A} \text { is a constant or } E \propto \phi n
$$

or $E=K_{1} \phi N$, where $K_{1}=\frac{P Z}{60 A}$ is another constant or $E \propto \phi N$
or $E \propto \phi \omega$, where $\omega=\frac{2 \pi N}{60}$ is the angular velocity in radian/s

Thus, we conclude that the induced emf is directly proportional to flux per pole and speed. Moreover, the polarity of the induced emf depends upon the direction of magnetic field and the direction of rotation. If either of the two is reversed, the polarity of induced emf, that is, brushes is reversed; however, when both are reversed, the polarity does not change.

This induced emf is fundamental phenomenon to all DC machines whether they are working as a generator or motor. However, when the machine is working as a generator, this induced emf is called generated emf and is represented as $E_{\mathrm{g}}$,i.e., $E_{\mathrm{g}}=\frac{P Z \phi N}{60 A} \mathrm{~V}$.

## Example 11.1

An 8-pole lap-wound DC generator has 960 conductors, a flux of 40 mWb per pole and is driven at 400 rpm . Find OC emf.

## Solution:

Open-circuit emf, $E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}$
where $\phi=40 \mathrm{mWb}=40 \times 10^{-3} \mathrm{~Wb} ; Z=960 ; N=400 \mathrm{rpm} ; P=8$
$A=P=8$ (lap winding)

$$
E_{\mathrm{g}}=\frac{40 \times 10^{-3} \times 960 \times 400 \times 8}{60 \times 8}=256 \mathrm{~V}
$$

## Example 11.2

Calculate the voltage induced in the armature winding of a 4-pole, wave wound DC machine having 500 conductors and running at $1,000 \mathrm{rpm}$. The flux per pole is 30 mWb .
(P.T.U. May 2000)

## Solution:

Here,

$$
P=4 ; A=2 \text { (wave wound) } ; Z=728 ; N=1,800 \mathrm{rpm}
$$

$$
\phi=35 \mathrm{mWb}=35 \times 10^{-3} \mathrm{~Wb}
$$

Generated voltage, $E_{\mathrm{g}}=\frac{\phi Z N P}{60 \mathrm{~A}}=\frac{35 \times 10^{-3} \times 728 \times 1,800 \times 4}{60 \times 2}=1,528.8 \mathrm{~V}$

## Example 11.3

A 4-pole, lap-wound armature has 144 slots with two coil sides per slot, where each coil has two turns. If the flux per pole is 20 mWb and the armature rotates at 720 rpm , what is the induced emf (i) across the armature and (ii) across each parallel path?
(PTU May 2001)

## Solution:

Here, $P=4 ; A=P=4$ (lap wound); $\phi=20 \mathrm{mWb}=20 \times 10^{-13} \mathrm{~Wb} ; N=720 \mathrm{rpm}$
Number of slots $=144$ with two coil sides per slot and each coil has two turns
Therefore, $Z=144 \times 2 \times 2=576$
Induced emf across armature, $E_{\mathrm{g}}=\frac{\phi \mathrm{ZNP}}{60 \mathrm{~A}}=\frac{20 \times 10^{-3} \times 576 \times 720 \times 4}{60 \times 4}=138.24 \mathrm{~V}$
Voltage across each parallel path $=E_{\mathrm{g}}=138.24 \mathrm{~V}$

## Example 11.4

A 6-pole machine has an armature with 90 slots and 8 conductors per slot, the flux per pole is 0.05 Wb and rms at $1,000 \mathrm{rpm}$. Determine induced emf if winding is (i) lap connected and (ii) wave connected.
(U.P.T.U. May 2003)

## Solution:

Here, $P=6 ; \phi=0.05 \mathrm{~Wb} ; N=1,000 \mathrm{rpm}$
Number of slots $=90$ with each slot having 8 conductors
Therefore, $Z=90 \times 8=720$
(i) When lap connected: $A=P=6$

$$
\text { Induced emf, } E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}=\frac{0.05 \times 720 \times 1,000 \times 6}{60 \times 6}=600 \mathrm{~V}
$$

(ii) When wave connected: $A=2$

$$
\text { Induced emf, } E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}=\frac{0.05 \times 720 \times 1,000 \times 6}{60 \times 2}=1,800 \mathrm{~V}
$$

## Example 11.5

An 8-pole DC generator has 600 armature conductors and a useful flux of 0.06 Wb . What will be the emf generated if it is lap wound and runs at $1,000 \mathrm{rpm}$ ? What must be the speed at which it is to be driven to induce the same voltage if it is wave wound? (U.P.T.U. May 2001, Type)

## Solution:

Here, $P=8 ; Z=600 ; \phi=0.06 \mathrm{~Wb} ; N=1,000 \mathrm{rpm}$

$$
A=P=8(\text { when lap wound })
$$

Induced emf,

$$
E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}=\frac{0.06 \times 600 \times 1000 \times 8}{60 \times 8}=600 \mathrm{~V}
$$

when wave wound, let the speed be $N^{\prime}$ rpm but $E_{\mathrm{g}}=600 \mathrm{~V}$
Now,

$$
N^{\prime}=\frac{E_{\mathrm{g}} \times 60 A}{\phi Z P}=\frac{600 \times 60 \times 2}{0.06 \times 600 \times 8}=250 \mathrm{rpm}
$$

## Example 11.6

A wave-wound armature of an 8-pole generator has 51 slots. Each slot contains 16 conductors. The voltage required to be generated is 300 V . What would be the speed of coupled prime mover if flux per pole is 0.05 Wb ?

If the armature is rewound as lap-wound machine and run by same prime mover, what will be the generated voltage?
(Univ. Question)

## Solution:

Here, $P=8 ; \phi=0.05 \mathrm{~Wb}$; number of slots $=51$; conductors per slot $=16$
Therefore, $Z=51 \times 16=816$
When the machine is wave wound, $A=2$ and $E_{\mathrm{g}}=300 \mathrm{~V}$
Now,

$$
E_{\mathrm{g}}=\frac{\phi Z N P}{60 A} \quad \text { or } \quad 300=\frac{0.05 \times 816 \times N \times 8}{60 \times 2}
$$

Therefore, speed

$$
N=\frac{300 \times 60 \times 2}{0.05 \times 816 \times 8}=110.3 \mathrm{rpm}
$$

When the machine is rewound as lap winding, $A=P=8$ and $N=110.3 \mathrm{rpm}$

$$
E_{\mathrm{g}}=\frac{0.05 \times 816 \times 110.3 \times 8}{60 \times 8}=75 \mathrm{~V}
$$

## Example 11.7

A 6-pole, lap-wound armature rotating at 350 rpm is required to generate 260 V . The effective flux per pole is about 0.05 Wb . If the armature has 120 slots, determine the suitable number of conductors per slot, and hence, determine the actual value of flux required to generate the same voltage.
(A.M.I.E. Summer 2001)

## Solution:

Here, $P=6 ; A=P=6 ; N=350 \mathrm{rpm} ; E_{\mathrm{g}}=260 \mathrm{~V} ; \phi=0.05 \mathrm{~Wb}$

Now,

$$
\begin{gathered}
E_{\mathrm{g}}=\frac{\phi Z N P}{60 A} \text { or } 260=\frac{0.05 \times Z \times 350 \times 6}{60 \times 6} \\
Z=\frac{260 \times 60 \times 6}{0.05 \times 350 \times 6}=\frac{260 \times 24}{7}
\end{gathered}
$$

Number of conductors or slot $=\frac{Z}{\text { Number of slots }}=\frac{260 \times 24}{7 \times 120}=7.43 \cong 8($ an integer $)$
For 8 conductors or slot, $Z=120 \times 8=960$
Actual value of flux required, $\phi=\frac{E_{\mathrm{g}} \times 60 A}{Z N P}=\frac{260 \times 60 \times 6}{960 \times 350 \times 6}=0.0464 \mathrm{~Wb}$

## Example 11.8

The emf generated by a 4-pole DC generator is 400 V , when the armature is driven at $1,200 \mathrm{rpm}$. Calculate the flux per pole if the wave-wound generator has 39 slots having 16 conductors per slot.

Solution:
Induced emf,

$$
E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}
$$

where $P=4 ; E_{\mathrm{g}}=400 \mathrm{~V} ; N=1,200 \mathrm{rpm} ; Z=39 \times 16=624 ; A=2$ (wave winding)
Therefore, flux per pole, $\phi=\frac{E_{\mathrm{g}} \times 60 A}{Z N P}=\frac{400 \times 60 \times 2}{624 \times 1,200 \times 4}=0.016 \mathrm{~Wb}=16 \mathrm{mWb}$

## Example 11.9

The armature of a 4-pole 250 V , lap-wound generator has 500 conductors and rms of 400 rpm . Determine the useful flux per pole. If the number of turns in each field coil is 1,000 , what is the average induced emf in each field coil on breaking its connection if the magnetic flux set-up by it dies away completely in 0.1 s ?

## Solution:

Here, $P=4 ; E_{\mathrm{g}}=250 \mathrm{~V} ; Z=500 ; A=P=4 ; N=400 \mathrm{rpm}$

$$
\phi=\frac{E_{\mathrm{g}} \times 60 A}{Z N P}=\frac{260 \times 60 \times 4}{500 \times 400 \times 4}=0.075 \mathrm{~Wb}
$$

Number of turns of exciting winding, $N_{\mathrm{t}}=1,000$

$$
e=N_{\mathrm{t}} \frac{d \phi}{d t}=1,000 \times \frac{0.075-0}{0.1}=750 \mathrm{~V}
$$

## Example 11.10

A 4-pole generator with wave-wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 Wb . At what speed must the armature rotate to give an induced emf of 220 V ? What will be the emf developed if the winding is lap connected and the armature rotates at the same speed?

## Solution:

Induced emf, $E_{\mathrm{g}}=\frac{\phi Z N P}{60 A}$
where $\phi=0.01 \mathrm{~Wb} ; Z=51 \times 24=1,224 ; E=220 \mathrm{~V} ; P=4 ; A=2$ (wave winding).

$$
220=\frac{0.01 \times 1,224 \times N \times 4}{60 \times 2} \quad \text { or } \quad N=\frac{220 \times 60 \times 2}{0.01 \times 1,224 \times 4}=539.21 \mathrm{rpm}
$$

For lap winding, $A=P=4$

$$
E_{\mathrm{g}}=\frac{0.01 \times 1224 \times 539.21 \times 4}{60 \times 4}=110 \mathrm{~V}
$$

## PRACTICE EXERCISES

## Short Answer Questions

1. Why is the armature core of a DC machine laminated?
2. In small DC machine, cast iron yokes are preferred. Why?
3. What for field coils are provided in a DC machine?
4. What for brushes are employed in DC generator?
5. What is the function of armature in a DC generator?
6. For what type of DC machine, wave winding is employed?
7. What is the function of commutator in a DC machine in (i) generator action and (ii) motor action
8. Which are the different types of armature windings commonly used in DC machines?
9. Which is the best-suited material for commutator segments?
10. For what type of DC machine, lap winding is employed?
11. Why are the graphite or carbon brushes preferred over copper brushes for use in DC machines?
12. Write down the emf equation of a DC generator.
(A.M.I.E. Winter 2002)

## Test Questions

1. Name major parts of a DC machine. Draw the sketch and show the path of magnetic flux in a 4-pole DC machine.
(P.T.U. May 2002)
2. Compare lap and wave winding. Where each type of winding is used and why?
(P.T.U. Dec. 2010, May 2012)
3. What is the principle of operation of a DC generator? Why is a commutator and brush arrangement necessary for the operation of a DC generator?
4. Describe the principle of operation and brief constructional details of a DC machine.
(M.P. Univ. May 2002)
5. With the help of sketches, describe the main parts and working principle of a DC generator.
6. Enumerate all the parts of a DC machine. State the material and the function of each part.
(U.P.T.U. Dec. 2002)
7. Explain in brief the construction of a DC machine.
(M.D. Univ. May 2003)
8. How can induced emf in the armature conductors of a DC generator be made unidirectional for external load?
(M.P. Univ. 2001)
9. Derive the emf equation of a DC machine.
(P.T.U. May 2011)
10. Derive the expression for induced voltages and armature MMF in a DC machine. (P.T.U. May 2010)
11. Derive an equation for emf in a DC machine.
(U.P.T.U. Dec. 2003)

## Numericals

1. Calculate the voltage induced in the armature winding of a 4 -pole, lap-wound DC machine having 500 conductors and running at $1,000 \mathrm{rpm}$. The flux per pole is 30 mWb . What will be induced emf if the armature is rewound for wave winding?
(Ans. $250 \mathrm{~V} ; 500 \mathrm{~V}$ ) (U.P.T.U. Dec. 2004)
2. A 4-pole, wave-wound armature has 72 slots with two coil-sides per slot, each coil has four turns. If the flux per pole is 20 mWb and the armature rotates at 720 rpm , what is the induced emf (i) across the armature and (ii) across each parallel path?
(Ans. $276.48 \mathrm{~V} ; 276.48 \mathrm{~V}$ )
(P.T.U. Dec. 2003)
3. A 4 -pole machine has an armature with 90 slots and 8 conductors per slot, the flux per pole is 0.05 Wb and runs at $1,200 \mathrm{rpm}$. Determine induced emf if winding is (i) lap connected and (ii) wave connected.
(Ans. 720 V, 1440 V) (U.P.T.U. Dec. 2005)
4. An 8 -pole DC generator has 400 armature conductors and a useful flux of 0.05 Wb . What will be the emf generated if it is lap wound and runs at $1,500 \mathrm{rpm}$ ? What must be the speed at which it is to be driven to induce the same voltage if it is wave wound?
(Ans. 500 V; 375 rpm ) (U.P.T.U. Dec. 2005)
5. A wave-wound armature of a 6 -pole generator has 51 slots. Each slot contains 20 conductors. The voltage required to be generated is 250 V . What would be the speed of coupled prime mover if flux per pole is 0.07 Wb . If the armature is rewound as lap-wound machine and run by same prime mover, what will be the generated voltage? (Ans. 70 rpm ; 83.3 V ) (A.M.I.E. Winter 2003)
6. A 4-pole wave-wound armature rotating at 400 rpm is required to generate 300 V . The effective flux per pole is about 0.05 Wb . If the armature has 120 slots, determine the suitable number of conductors per slot and hence determine the actual value of flux required to generate the same voltage.
(Ans. 4; 46.875 mWb ) (U.P.T.U. May 2008)
7. A 4-pole DC generator has rated armature current of 180 A . If the armature is lap wound, determine the current flowing through each parallel path of the armature. What will be its value if the armature is wave wound?
(Ans. $45 \mathrm{~A} ; 90 \mathrm{~A}$ )
8. A 4-pole lap-connected DC machine has an armature resistance of $0.18 \Omega$. Find the armature resistance of the machine when rewound for wave connections. (Ans. $0.72 \Omega$ ) (U.P.T.U. Dec. 2005)

### 11.8 TYPES OF DC GENERATORS

DC generators are generally classified according to the methods of their field excitation. On the basis of this criteria, they can be classified as follows:

1. Separately excited DC generators
2. Self-excited DC generators-these are further classified as follows:
(a) Shunt-wound DC generators
(b) Series-wound DC generators
(c) Compound-wound DC generators.

Furthermore, compound-wound generators can be classifies as long shunt and short shunt. Except these generators, there are also permanent magnet type DC generators. In these generators, no field winding is placed around the poles. The field produced by the poles of these machines fairly remains constant. Although these machines are very compact but are used only in small sizes like dynamos in automobiles. The main disadvantage of these machines is that the flux produced by the magnets deteriorates with the passage of time that changes the characteristics of the machine.

### 11.9 SEPARATELY EXCITED DC GENERATORS

A DC generator in which current is supplied to the field winding from an external DC source is called a separately excited DC generator. The flux produced by the poles depends upon the field current with in the unsaturated region of magnetic material of the poles (i.e., $\phi \propto I_{\mathrm{f}}$ ), but in the saturated region, the flux remains constant. Its conventional diagram is shown in Figure 11.16.

## Important relations

Here, $I_{\mathrm{a}}=I_{\mathrm{L}}$, where $I_{\mathrm{a}}$ is armature
Current and $I_{\mathrm{L}}$ is the line current
Terminal voltage, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}$
If brush contact drop per brush $\left(v_{\mathrm{b}}\right)$ is known,

$$
V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}
$$

Power developed $=E_{\mathrm{g}} I_{\mathrm{a}}$
Power output $=V I_{\mathrm{L}}=V I_{\mathrm{a}}$


Fig. 11.16 Conventional diagram of a separately excited DC generator

### 11.10 SELF-EXCITED DC GENERATORS

A DC generator whose field winding is excited by the current supplied by the generator itself is called a self-excited DC generator. In a self-excited DC generator, the field coils may be connected in parallel with the armature, in series with the armature or partly in series, and partly in parallel with the armature winding. Accordingly, the self-excited generators may be classified as follows:

1. Shunt-wound generators: In this generator, the field winding is connected across the armature winding forming a parallel or shunt circuit. Therefore, full terminal voltage is applied across the field winding. A very small current $I_{\text {sh }}$ flows through it, because this winding has many turns of fine wire having very high resistance $R_{\mathrm{sh}}$ (of the order of $100 \Omega$ ). Its conventional diagram is shown in Figure 11.17.

## Important relations

Shunt field current, $I_{\text {sh }}=V / R_{\text {sh }}$
where $R_{\mathrm{sh}}$ is the shunt field winding resistance. The field current $I_{\text {sh }}$ is practically constant at all loads, and therefore, the DC shunt machine is considered to be constant flux machine.
Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}$; terminal voltage, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}$ Including brush contact drop, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$ Power developed $=E_{\mathrm{g}} I_{\mathrm{a}} ;$ power output $=V I_{\mathrm{L}}$
2. Series-wound generators: In this generator, the field winding is connected in series with the armature winding forming a series circuit. Therefore, full line current $I_{\mathrm{L}}$ or armature current $I_{\mathrm{a}}$ flows through it. Since the series field winding carries fullload current, it has a few turns of thick wire having low resistance (usually of the order of less than $1 \Omega$ ). Its conventional diagram is shown in Figure 11.18.


Fig. 11.17 Conventional diagram of a shunt wound DC generator


Fig. 11.18 Conventional diagram of a series wound DC generator

## Important relations

Series field current, $I_{\text {se }}=I_{\mathrm{L}}=I_{\mathrm{a}}$
Series field winding resistance $=R_{\text {se }}$
Terminal voltage, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-I_{\mathrm{se}} R_{\mathrm{se}}=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)$
Including brush contact drop, $V=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 v_{\mathrm{b}}$
Power developed $=E_{g} I_{\mathrm{a}}$; power output $=V I_{\mathrm{L}}=V I_{\mathrm{a}}$
Note: The flux developed by the series field winding is directly proportional to the current flowing through it (i.e., $\phi \propto I_{\text {se }}$ ). However, it is only true before magnetic saturation. After saturation, flux becomes constant even if the current flowing through it is increased.
3. Compound-wound generators: In a compound-wound generator, there are two sets of field windings on each pole. One of them is connected in series (having few turns of thick wire) and the other is connected in parallel (having many turns of fine wire) with armature. A compound-wound generator may be classified as fol-


Fig. 11.19 Conventional diagram of a long shunt compound wound DC generator lows:
(a) Long shunt: In this generator, the shunt field winding is connected in parallel with the combination of both armature and series field winding. The conventional diagram of lone shunt compound generator is shown in Figure 11.19.
Important relations
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}$
Series field current, $I_{\text {se }}=I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}$
Terminal voltage, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-I_{\mathrm{se}} R_{\mathrm{se}}$

$$
=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)
$$

Including brush contact drop, $V=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 v_{\mathrm{b}}$
Power developed $=E_{\mathrm{g}} I_{\mathrm{a}}$; power output $=V I_{\mathrm{L}}$
(b) Short shunt: In this generator, the shunt field winding is connected in parallel with only armature winding. The conventional diagram of short-shunt compound generator is shown in Figure 11.20.


Fig. 11.20 Conventional diagram of a short shunt compound wound DC generator

Important relations
Series field current, $\quad I_{\text {se }}=I_{\mathrm{L}}$
Shunt field current, $I_{\mathrm{sh}}=\frac{V+I_{\mathrm{L}} R_{\mathrm{se}}}{R_{\mathrm{sh}}}=\frac{E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}}{R_{\mathrm{sh}}}$

$$
I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}}
$$

Terminal voltage, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-I_{\mathrm{L}} R_{\mathrm{se}}$
Including brush contact drop, $V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-I L R_{\mathrm{se}}-2 v_{\mathrm{b}}$
Power developed $=E_{\mathrm{g}} I_{\mathrm{a}} ;$ power output $=V I_{\mathrm{L}}$

### 11.10.1 Cumulative and Differential Compound-wound Generators

In compound-wound DC generators, the field is produced by the shunt as well as series winding. Generally, the shunt field is stronger than the series field. When the series field assist the shunt field, the generator is called as cumulative compound-wound generator (see Fig. 11.21(a)). However, when the series field opposes the shunt field, the generator is known as differential compound-wound generator (see Fig. 11.21(b)).


Fig. 11.21 (a) Cumulatively compound
(b) Differentially compound

## Example ll. 11

A 200 V , 8-pole, lap-connected DC shunt generator supplied $60,40 \mathrm{~W}, 200 \mathrm{~V}$ lamps. It has armature and field circuit resistances of $0.2 \Omega$ and $200 \Omega$, respectively. Calculate the generated emf, armature current, and current in each armature conductor.
(U.P.T.U. 2004-05)

## Solution:

The conventional circuit is shown in Figure 11.22. Here, $V=200 \mathrm{~V} ; P=8 ; A=P=8$ (lap winding); $R_{\mathrm{a}}=0.2 \Omega ; R_{\mathrm{sh}}=200 \Omega$
Load $=60 \times 40=2,400 \mathrm{~W}$
Load current, $I_{\mathrm{L}}=\frac{\text { Load }}{\mathrm{V}}=\frac{2,400}{200}=12 \mathrm{~A}$
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{200}{200}=1 \mathrm{~A}$


Fig. 11.22 Conventional diagram as per data

Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}$

$$
=12+1=13 \mathrm{~A}
$$

Generated emf, $E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}=200+13 \times 0.2$

$$
=202.6 \mathrm{~V}
$$

Current in each armature conductor, $I_{\mathrm{C}}=\frac{I_{\mathrm{a}}}{A}=\frac{13}{8}=1.625 \mathrm{~A}$

## Example 11.12

A $20 \mathrm{~kW}, 200 \mathrm{~V}$ shunt generator has an armature resistance of $0.05 \Omega$ and a shunt field resistance of $200 \Omega$. Calculate the power developed in the armature when it delivers rated output.
(U.P.T.U. 2006-07)


Fig. 11.23 Conventional diagram as per data

## Solution:

The conventional circuit is shown in Figure 11.23.
For shunt generator: $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}$
Power delivered to load, $P_{\mathrm{L}}=V I_{\mathrm{L}}$
Power developed in armature, $P_{\mathrm{g}}=E_{\mathrm{g}} I_{\mathrm{a}}$
Now,

$$
\begin{aligned}
& I_{\mathrm{L}}=\frac{20 \times 10^{3}}{200}=100 \mathrm{~A} \\
& I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{200}{200}=1 \mathrm{~A} \\
& I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{200}{200}=1 \mathrm{~A}
\end{aligned}
$$

$$
\begin{aligned}
I_{\mathrm{a}} & =100+1=101 \mathrm{~A} \\
E_{\mathrm{g}} & =V+I_{\mathrm{a}} R_{\mathrm{a}}=200+101 \times 0.05=205.05 \mathrm{~V} \\
P_{\mathrm{g}} & =E_{\mathrm{g}} I_{\mathrm{a}}=205.05 \times 101=20,710.05 \mathrm{~W}=20.71 \mathrm{~kW}
\end{aligned}
$$

## Example 11.13

A 4-pole DC generator with wave-connected armature has 41 slots, and 12 conductors/slot. Armature resistance $R_{\mathrm{a}}=0.5 \Omega$; shunt resistance is $R_{\mathrm{sh}}=200 \Omega$; flux per pole $=125 \mathrm{mWb}$; and speed $N=1,000 \mathrm{rpm}$. Calculate voltage across $10 \Omega$ load resistance across the armature terminal.
(U.P.T.U. 2007-08)

## Solution:

The conventional circuit is shown in Figure 11.24.
Here, $P=4 ; R_{\mathrm{a}}=0.5 \Omega ; R_{\mathrm{sh}}=200 \Omega ; \phi=125 \mathrm{mWb}=125 \times 10^{-3} \mathrm{~Wb} ; N=1,000 \mathrm{rpm} ; R_{\mathrm{L}}=10 \Omega$
Number of slots $=41$
Number of conductors/slot $=12$
Armature conductors, $Z=41 \times 12=492$
Number of parallel paths, $A=2$ (wave winding)
Generated emf, $E_{\mathrm{g}}=\frac{P \phi \mathrm{ZN}}{60 A}$

$$
\begin{aligned}
& =\frac{4 \times 125 \times 10^{-3} \times 492 \times 1,000}{60 \times 2} \\
& =2,050 \mathrm{~V}
\end{aligned}
$$



Load current, $I_{\mathrm{L}}=\frac{V}{R_{\mathrm{L}}}=\frac{V}{10} \mathrm{~A}$
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{V}{200} \mathrm{~A}$
Fig. 11.24 Conventional diagram as per data Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}=\frac{\mathrm{V}}{10}+\frac{\mathrm{V}}{200}=\frac{21 \mathrm{~V}}{200} \mathrm{~A}$

Now,

$$
E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}
$$

or
or

$$
\begin{aligned}
2,050 & =V+\frac{21 V}{200} \times 0.05 \text { or } 2,050=\frac{421 \mathrm{~V}}{400} \\
V & =\frac{2,050 \times 400}{421}=1,947.74 \mathrm{~V}
\end{aligned}
$$

## Example ll. 14

A 4-pole DC shunt generator with wave-wound armature has 40 slots each having 12 conductors. Armature resistance is $1 \Omega$ and shunt field resistance is $200 \Omega$. The flux per pole is 25 mWb . If a load of $50 \Omega$ is connected across the armature terminals, calculate the voltage across the load when the generator is driven at $1,000 \mathrm{rpm}$. What will be the load voltage if the generator is lap wound?
(U.P.T.U. 2006-07)

## Solution:

The conventional circuit is shown in Figure 11.25.
Here, $P=4, R_{\mathrm{a}}=1 \Omega ; R_{\text {sh }}=200 \Omega ; \phi=25 \mathrm{mWb}=25 \times 10^{-3} \mathrm{~Wb} ; N=1,000 \mathrm{rpm}$
Number of slots $=40$; number of conductors/slot $=12$
Total armature conductors,

$$
\begin{gathered}
Z=40 \times 12=480 \\
R_{\mathrm{L}}=50 \Omega
\end{gathered}
$$

Load resistance,
Number of parallel path, $A=2$ (for wave winding)
Generated emf,

$$
\begin{aligned}
& E_{\mathrm{g}}=\frac{\phi Z N P}{60 \mathrm{~A}} \\
& E_{\mathrm{g}}=\frac{25 \times 10^{-3} \times 480 \times 1,000 \times 4}{60 \times 2}=400 \mathrm{~V}
\end{aligned}
$$

Load current, $I_{\mathrm{L}}=\frac{V}{R_{\mathrm{L}}}=\frac{V}{50} \mathrm{~A}$
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{V}{200} \mathrm{~A}$
Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}=\frac{V}{50}+\frac{V}{200}=\frac{5 \mathrm{~V}}{200}=\frac{V}{40} \mathrm{~A}$
Now, $\quad E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}$

$$
400=V+\frac{V}{40} \times 1 \quad \text { or } \quad 400=\frac{41}{40} \mathrm{~V}
$$

or $\quad V=\frac{400 \times 40}{41}=390.24 \mathrm{~V}$
If the generator is lap wound, then $A=P=4$

$$
E_{\mathrm{g}}=\frac{25 \times 10^{-3} \times 480 \times 1,000 \times 4}{60 \times 4}=200 \mathrm{~V}
$$



Fig. 11.25 Conventional diagram as per data
and

$$
\begin{gathered}
E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}} \\
200=V+\frac{V}{40} \times 1 \quad \text { or } \quad 200=\frac{41}{40} \mathrm{~V} \\
V=\frac{200 \times 40}{41}=195.122 \mathrm{~V} .
\end{gathered}
$$

## Example 11.15

A 4-pole DC shunt generator with a wave-wound armature having 390 conductors has to supply a load of 500 lamps each of 100 W at 250 V . Allowing 10 V for the voltage drop in the connecting leads between the generator and the load and brush drop of 2 V . Calculate the speed at which the generator should be driven. The flux per pole is 30 mWb and the value of $R_{\mathrm{a}}=0.05 \Omega$ and $R_{\mathrm{sh}}=65 \Omega$.

## Solution:

The conventional circuit diagram of the DC shunt generator is shown in Figure 11.26.
Total load $=500 \times 100 \mathrm{~W}$


Fig. 11.26 Conventional diagram as per data

$$
I_{\mathrm{L}}=\frac{500 \times 100}{250}=200 \mathrm{~A}
$$

Voltage drop in leads, $V_{\mathrm{L}}=10 \mathrm{~V}$
Voltage across shunt field winding,

$$
\begin{aligned}
V_{\text {sh }} & =V+V_{\mathrm{L}}=250+10=260 \mathrm{~V} \\
I_{\mathrm{sh}} & =V_{\text {sh }} / R_{\mathrm{sh}}=260 / 65=4 \mathrm{~A} \\
I_{\mathrm{a}} & =I_{\mathrm{L}}+I_{\mathrm{sh}}=200+4=204 \mathrm{~A}
\end{aligned}
$$

Armature drop $=I_{\mathrm{a}} R_{\mathrm{a}}=204 \times 0.05=10.2 \mathrm{~V}$
Total brush drop, $2 v_{\mathrm{b}}=2 \mathrm{~V}$
Generated emf, $E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}+V_{\mathrm{L}}+2 \nu_{\mathrm{b}}$

$$
=250+10.2+10+2=272.2 \mathrm{~V}
$$

Now,

$$
E_{\mathrm{g}}=\frac{P \phi N Z}{60 A} \text { or } 272.2=\frac{4 \times 30 \times 10^{-3} \times N \times 390}{60 \times 2}
$$

or

$$
N=\frac{272.2 \times 60 \times 2}{4 \times 30 \times 10^{-3} \times 390}=698 \mathrm{rpm}
$$

## Example 11.16

A 4-pole DC shunt generator with a shunt field resistance of $100 \Omega$ and an armature resistance of $1 \Omega$ has 378 wave-connected conductors in its armature. The flux per pole is 0.02 Wb . If a load resistance of $10 \Omega$ is connected across the armature terminals and the generator is driven at 1,000 rpm, calculate the power absorbed by the load.

## Solution:

The conventional circuit is shown in Figure 11.27.
Generated emf, $E_{\mathrm{g}}=\frac{P Z \phi N}{60 A}=\frac{4 \times 378 \times 0.02 \times 1,000}{60 \times 2}=252 \mathrm{~V}$

Line current, $I_{\mathrm{L}}=\frac{V}{R_{\mathrm{L}}}=\frac{V}{10}$ (where $V$ is terminal voltage)
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{V}{100}$
Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}=\frac{V}{10}+\frac{V}{100}=(0.11 \mathrm{~V})$
Using the relation, $E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}} ; 252=V+0.11 V \times 1.0$
Terminal voltage, $V=227 \mathrm{~V}$


Fig. 11.27 Conventional diagram as per data

Load current, $\quad I_{\mathrm{L}}=\frac{V}{R_{\mathrm{L}}}=\frac{227}{10}=22.7 \mathrm{~A}$
Power absorbed by the load, $P=V I_{\mathrm{L}}=227 \times 22.7=5.153 \mathrm{~kW}$

## Example 11.17

A short-shunt cumulative compound DC generator supplies 7.5 kW at 230 V . The shunt field, series field, and armature resistance are $100,0.3$ and $0.4 \Omega$, respectively. Calculate the induced emf and the load resistance.

## Solution:

The conventional circuit is shown in Figure 11.28.
From Figure 11.29,

$$
\begin{gathered}
I_{\mathrm{L}}=\frac{7.5 \times 1,000}{230}=32.61 \mathrm{~A} \\
I_{\mathrm{sh}}=\frac{V+I_{\mathrm{L}} R_{\mathrm{se}}}{R_{\mathrm{sh}}}=\frac{230+32.61 \times 0.3}{100}=2.39 \mathrm{~A} \\
I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}}=32.61+2.39=35 \mathrm{~A}
\end{gathered}
$$

Induced emf, $E_{\mathrm{g}}=V+I_{\mathrm{L}} R_{\mathrm{se}}+I_{\mathrm{a}} R_{\mathrm{a}}$

$$
=230+32.61 \times 0.3+35 \times 0.4=253.78 \mathrm{~V}
$$



Fig. 11.28 Conventional diagram as per data

Load resistance,

$$
R_{\mathrm{L}}=\frac{V^{2}}{P}=\frac{(230)^{2}}{7.5 \times 1,000}=7.053 \Omega
$$

## Example 11.18

A 20 kW compound generator works on full-load with a terminal voltage of 230 V . The armature, series, and shunt field resistance are $0.1,0.05$, and $115 \Omega$, respectively. Calculate the generated emf when the generator is connected as shunt.
(Pb. Univ. Dec. 1994)

## Solution:

The conventional circuit is shown in Figure 11.29.
Load $=20 \mathrm{~kW}=20 \times 10^{3} \mathrm{~W}$

$$
V=230 \mathrm{~V} ; R_{\mathrm{a}}=0.1 \Omega ; R_{\mathrm{se}}=0.05 \Omega
$$



Fig. 11.29 Conventional diagram as per data

$$
R_{\mathrm{sh}}=115 \Omega
$$

$$
I_{\mathrm{L}}=\frac{20 \times 10^{3}}{230}=86.96 \mathrm{~A}
$$

Shunt field current, $\quad I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{230}{115}=2 \mathrm{~A}$
Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\text {sh }}=86.96+2=88.96 \mathrm{~A}$
Generated emf, $E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}+I_{\mathrm{a}} R_{\mathrm{se}}$

$$
=230+88.96 \times 0.1+88.96 \times 0.05
$$

$$
=243.3 \mathrm{~V}
$$

### 11.11 VOLTAGE BUILD-UP IN SHUNT GENERATORS

The shunt generator is a self-excited DC generator whose field winding is supplied with the current from the output of the generator itself. However, question arises how it can supply current to the field winding before the voltage being generated? And if the field current is not supplied, how can the voltage be generated? Let us find out its answer from the following explanation.

The open-circuit characteristics of a DC shunt generator are shown in Figure 11.30(b). The shunt field resistance is represented by a straight line OX. When armature is rotated at a constant speed of $\omega$ radians/s, the small residual flux of the poles is cut by the armature conductors, and very small emf ( $o a$ ) is induced in the armature. Now, if key $(K)$ connected in the shunt field winding, as shown in the Figure 11.30(a) is closed, current $o b$ flows in the field winding. This current increases the flux produced by the poles and voltage generated in the armature is increased to $o c$ that further increases the field current to od; further, this builds up the voltage. This building up action comes to an end at point $f$ where the O.C.C. intersects the shunt field resistance line OX. It is because beyond this point, the induced voltage is less than that required to maintain the corresponding field current. Thus, the final current in the field winding is ef and the final voltage build-up by the generator for a given O.C.C. is oe as shown in Figure 11.30(b).


Fig. 11.30 (a) Conventional diagram of a shunt generator where field winding is open circuited (b) Open circuit characteristics of a DC shunt generator

### 11.12 CRITICAL FIELD RESISTANCE OF A DC SHUNT GENERATOR

The open-circuit characteristic of a DC shunt generator are shown in Figure 11.31. The line $O X$ is drawn in such a way that its slope gives the field winding resistance,
i.e.,

$$
R_{\mathrm{sh}}=\frac{\mathrm{OB}(\text { in volt })}{\mathrm{OC}(\text { in ampere })}
$$

In this case, the generator can build up a maximum voltage OB with a shunt field resistance $R_{\text {sh }}$. A line OY represents a small resistance. With this resistance, the generator can build up a maximum voltage OF that is slightly more than OB. If the field resistance is increased, the slope of the resistance line increases. Consequently, the maximum voltage that the generator can build up, at a specified speed, decreases. If the value of $R_{\mathrm{sh}}$ is increased to such an extent that the resistance line does not cut the no-load characteristics at all (OZ), then it is apparent that the voltage will not be built-up (i.e., the generator fails to excite).

If the resistance line (OP) just coincide with the slope of the curve, at this value of field resistance, the generator will

(Field current)
Fig. 11.31 Representation of critical resistance just excite. This resistance given by the tangent to O.C.C. is called the critical resistance at a specified speed. Thus, the slope of the tangent drawn on O.C.C. is called critical resistance.

1. Critical resistance of a field winding: It is that maximum value resistance of a field winding that is required to build-up voltage in a generator. If the value of field resistance is more than this value, the generator would not build-up the voltage.
2. Critical load resistance: The minimum value of load resistance on a DC shunt generator with which it can be in position to build up is called its critical load resistance.
3. Critical speed of a DC shunt generator: It is the speed of a DC shunt generator at which shunt field resistance will represent the critical field resistance.

### 11.13 CAUSES OF FAILURE TO BUILD-UP VOLTAGE IN A GENERATOR

There may be one or more of the following reasons due to which a generator fails to build-up voltage:

1. When the residual magnetism in the field system is destroyed.
2. When the connections of the field winding are reversed. This, in fact, destroys the residual magnetism due to which generator fails to build-up voltage.
3. In the case of shunt-wound generators, the other causes may be
(a) the resistance of shunt field circuit may be more than the critical resistance.
(b) the resistance of load circuit may be less than critical resistance.
(c) the speed of rotation may be below the rated speed.
4. In the case of series-wound generators, the other causes may be
(a) the load circuit may be open - it may be due to faulty contact between brushes and commutator or commutator surface may be greasy or dirty and making no contact with the brushes.
(b) the load circuit may have high resistance.

### 11.13.1 Rectification

If the generator is not building up because of the absence of residual magnetism due to any reason, the field coils should be connected to a DC source for a small period in order to magnetise the poles.

## 国最 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What are the requirements of self-excited DC machine?
(P.T.U. May 2010)

Or
What are the conditions to be fulfilled for a DC shunt generator to build-up emf?
(P.T.U. May 2009)
2. Under what circumstances, does a DC shunt generator fail to build up?
(P.T.U. Dec. 2009)
3. Define critical resistance of DC shunt generator.
(P.T.U. Dec. 2012)
4. The permanent magnet generators are rarely used in industry. Why?
5. Why field coils of a DC shunt generator are wound with a large number of turns of fine wire?
6. Why is the series field winding wound with few turns of thick wire?
7. What is the critical resistance of field circuit?
(U.P.T.U. Dec. 2003)

## Test Questions

1. Describe with suitable diagram various types of DC machines.
(P.T.U. May 2010)
2. Name and explain various methods of excitation of DC machines.
(P.U. Dec. 1992)
3. Explain the process of voltage build-up in a DC shunt generator. What is the field circuit critical resistance?
(U.P.T.U. 2002-03)
4. What could be the causes for the failure of voltage build-up of DC self-excited generator? How can the problem be remedied?
(A.M.I.E. Summer 2002)
5. Explain the process of 'building up' of voltage in a shunt generator. Under what conditions may it fail to build-up the voltage?
(U.P.T.U. 2002-03)
6. Define critical field resistance relating to DC shunt generator. Further, explain the voltage build-up process when the generator is driven at constant speed.
(A.M.I.E. Summer 1998)

## Numericals

1. A 4 -pole machine running at $1,000 \mathrm{rpm}$ has an armature with 90 slots having 6 conductors per slot. The flux per pole is $6 \times 10^{-2} \mathrm{~Wb}$. Determine the induced emf as a DC generator, if the coils are lap connected. If the current per conductor is 100 A , determine the electrical power output of the machine.
(Ans. $540 \mathrm{~V} ; 216 \mathrm{~kW}$ )
2. A 110 V DC shunt generator delivers a load current of 50 A . The armature resistance is $0.2 \Omega$ and the field circuit resistance is $55 \Omega$. The generator, rotating at a speed of $1,800 \mathrm{rpm}$., has 6 -pole lapwound armature having 360 conductors. Calculate (i) the no-load voltage in the armature and (ii) the flux per pole.
(B.U. Feb. 1983) (Ans. $120.4 \mathrm{~V} ; 0.011 \mathrm{~Wb})$
3. A 10 kW , 6-pole DC generator develops an emf of 200 V at $1,500 \mathrm{rpm}$. The armature has a lapconnected winding. The average flux density over a pole pitch is 0.9 T . The length and diameter of the armature are 0.25 m and 0.2 m , respectively. Calculate (i) the flux per pole, (ii) the total number
of active conductors in the armature, and (iii) the torque developed by the machine when the armature supplies a current of 60 A
(GATE 1997) (Ans. $23.6 \mathrm{mWb}, 340,76.4 \mathrm{Nm}$ )
4. A $30 \mathrm{~kW}, 300 \mathrm{~V}, \mathrm{DC}$ shunt generator has armature and field resistance of $0.05 \Omega$ and $100 \Omega$, respectively. Calculate the total power developed in the armature when it delivers full-load.
(A.M.I.E. Summer 1975) (Ans. 31.43 kW )
5. A 4-pole shunt generator with lap-connected armature having field and armature resistances of 50 $\Omega$ and $0.1 \Omega$, respectively, supplies a $2,400 \mathrm{~W}$ load at a terminal voltage of 100 V . Calculate the total armature current, the current per armature path, and the generated emf.
(P.T.U. Dec. 2001) (Ans. 26 A; 6.5 A; 102.6 V)
6. A shunt generator has an induced emf of 127 V on open circuit. When the machine is on load, the terminal voltage is 120 V . Find the load current, if the field resistance is $15 \Omega$ and the armature resistance $0.02 \Omega$.
(B.U. May 1986) (Ans. 342 A)
7. A 4-pole DC shunt generator with lap-connected armature supplies a load of 100 A at 200 V . The armature resistance is $0.1 \Omega$ and the shunt field resistance is $80 \Omega$. Find (i) total armature current, (ii) current per conductor, and (iii) emf generated.
(Ans. 102.5 A, 25.6 A, 210.25 V)
8. A 4-pole shunt generator with lap-connected armature has armature and field resistances of 0.2 and $50 \Omega$, respectively, and supplies to 100 lamps of $60 \mathrm{~W}, 200 \mathrm{~V}$ each. Calculate the total armature current, current per path, and the generated emf. Allow a brush drop of 1 V at each brush.
(Ans. $34 \mathrm{~A}, 8.5 \mathrm{~A}, 208.8 \mathrm{~V}$ )

### 11.14 DC MOTOR

An electromechanical energy conversion device (electrical machine) that converts DC electrical energy or power $(E I)$ into mechanical energy or power $(\omega T)$ is called a DC motor.

Electric motors are used for driving industrial machines such as hammers, presses, drilling machines, lathes, rollers in paper and steel industry, and blowers for furnaces and domestic appliances such as refrigerators, fans, water pumps, toys, and mixers. The block diagram of energy conversion, when the electromechanical device works as a motor, is shown in Figure 11.32.


Fig. 11.32 Motor

### 11.15 WORKING PRINCIPLE OF DC MOTORS

The operation of a DC motor is based on the principle that when a current carrying conductor is placed in a magnetic field, a mechanical force is experienced by it. The direction of this force is determined by Fleming's left-hand rule and its magnitude is given by the relation:

$$
F=\text { Bil Newton }
$$

For simplicity, consider only one coil of the armature placed in the magnetic field produced by a bipolar machine (see Fig. 11.33(a)). When DC supply is connected to the coil, current flows through it that sets up its own field, as shown in Figure 11.33(b). By the interaction of the two
fields (i.e., field produced by the main poles and the coil), a resultant field is set up, as shown in Figure 11.33(c). The tendency of this is to come to its original position, that is, in straight line due to which force is exerted on the two coil sides and torque is produced; this torque rotates the coil.


Fig. 11.33 (a) Field produced by main poles (b) Field produced by current carrying coil (c) Resultant field and direction of force exerted on conductors


Fig. 11.34 Position of main field and rotor field, torque development by the alignment of two fields

Alternately, it can be said that the main poles produce a field $F_{\mathrm{m}}$. Its direction is marked in Figure 11.34. When current is supplied to the coil (armature conductors), it produces its own field marked as $F_{\mathrm{r}}$. This field tries to come in line with the main field and an electromagnetic torque is developed in the clockwise direction, as shown in Figure 11.35.

In actual machine, a large number of conductors are placed on the armature. All the conductors placed under the influence of one pole (say North pole) carry the current in one direction (outward). While the other conductors placed under the influence of other pole, that is South pole, carry the current in opposite direction, as shown in Figure 11.35. A resultant rotor field is produced. Its direction is marked by the arrow arrowhead $F_{\mathrm{r}}$. This rotor field $F_{\mathrm{r}}$ tries to come in line with the main field $F_{\mathrm{m}}$ and torque $\left(T_{\mathrm{e}}\right)$ is developed. Thus, rotor rotates. It can be seen that to obtain a continuous torque, the direction of flow of current in each conductor or coil side must be reversed when it passes through the magnetic neutral axis (MNA). This is achieved with the help of a commutator.

### 11.15.1 Function of a Commutator

The function of a commutator in DC motors is to reverse the direction of flow of current in each armature conductor when it passes through the MNA to obtain continuous torque.

### 11.16 BACK EMF

It has been seen that when current is supplied to the armature conductors, as shown in Figure 11.36(a), placed in the main magnetic field, torque is developed; thus, the armature rotates.

Simultaneously, the armature conductors cut across the magnetic field and an emf is induced in these conductors. The direction of this induced emf in the armature conductors is determined by Fleming's right-hand rule and is marked in Figure 11.36(b).


Fig. 11.36 (a) Flow of rotor current due to applied voltage (b) Direction of induced emf in rotor conductors

It can be seen that the direction of this induced emf is opposite to the applied voltage. That is why this induced emf is called back $\operatorname{emf}\left(E_{\mathrm{b}}\right)$. The magnitude of this induced emf is given by the relation:

$$
E_{\mathrm{b}}=\frac{P Z \phi N}{60 A} \text { or } E_{\mathrm{b}}=\frac{Z P}{60 A} \phi N \text { or } E_{\mathrm{b}} \propto \phi N\left(\text { since } \frac{Z P}{60 A} \text { are constant }\right)
$$

Further, $N \propto \frac{E_{\mathrm{b}}}{\phi} \quad$ shows that speed of motor is inversely proportional to magnetic field or flux.

A simple conventional circuit diagram of the machine working as motor is shown in Figure 11.37. In this case, the supply voltage is always greater than the induced or back emf (i.e., $V>E_{\mathrm{b}}$ ). Therefore, current is always supplied to the motor from the mains and the relation among the various quantities will be $E_{\mathrm{b}}=V-I_{\mathrm{a}} R_{\mathrm{a}}$.

### 11.16.1 Significance of Back EMF

The current flowing through the armature is given by the relation:


Fig. 11.37 Conventional circuit diagram of a DC motor $I_{\mathrm{a}}=\frac{V-E_{\mathrm{b}}}{R_{\mathrm{a}}}$. When mechanical load applied on the motor increases, its speed decreases that reduces the value of $E_{\mathrm{b}}$. As a result, the value $\left(V-E_{\mathrm{b}}\right)$ increases that consequently increases $I_{\mathrm{a}}$. Hence, motor draws extra current from the mains. Thus, the back emf regulates the input power as per the extra load.

### 11.17 TORQUE EQUATION

We know that when a current carrying conductor is placed in the magnetic field, a force is exerted on it that exerts turning moment or torque $(F \times r$ ) (see Fig. 11.38). This torque is produced due to electromagnetic effect, and hence it is called electromagnetic torque.


Fig. 11.38 Force exerted on a single conductor

Let $P=$ number of poles
$\phi=$ flux per pole in Wb
$r=$ average radius of armature in metre
$l=$ effective length of each conductor in metre
$Z=$ total armature conductors
$I_{\mathrm{a}}=$ total armature current
$A=$ number of parallel paths
Average force on each conductor, $F=$ Bil Newton
Torque due to one conductor $=F \times r$ Newton metre
Total torque developed in the armature, $T=Z F r$ Newton metre or

$$
\begin{equation*}
T=Z B i l r \tag{11.1}
\end{equation*}
$$

Now, current in each conductor, $i=I_{\mathrm{a}} / A$
Average flux density, $B=\phi / a$
where ' $a$ ' is the X -sectional area of flux path at radius $r$.
Obviously,

$$
a=\frac{2 \Pi r l}{P} \mathrm{~m}^{2} ; \text { therefore, } B=\frac{\phi P}{2 \Pi r l} \mathrm{~T}
$$

Substituting these values in Equation (11.1), we get

$$
T=Z \times \frac{\phi p}{2 \pi r l} \times \frac{I_{\mathrm{a}}}{A} \times l \times r \quad \text { or } \quad T=\frac{P Z \phi I_{\mathrm{a}}}{2 \pi A} \mathrm{Nm}
$$

Alternately, the power developed in the armature is given as
or

$$
\begin{aligned}
E I_{\mathrm{a}} & =\omega T \\
E I_{\mathrm{a}} & =\frac{2 \pi N}{60} \times T \\
\frac{\phi Z N P}{60 A} \times I_{\mathrm{a}} & =\frac{2 \pi N}{60} \times T \\
T & =\frac{P Z \phi N}{2 \pi A} \mathrm{Nm} \text { (similar to the earlier equation) }
\end{aligned}
$$

For a particular machine, the number of poles $(P)$ and number of conductors per parallel path $(Z / A)$ are constant.

Therefore,

$$
T=\mathrm{K} \frac{\phi I_{\alpha}}{2 \pi}, \text { where } \mathrm{K}=\frac{P Z}{A} \text { is a constant }
$$

The constant $K$ for a given machine is the same for the emf equation as well as for the torque equation.

Further, $T=\mathrm{K}_{2} \phi I_{\mathrm{a}}$, where $\mathrm{K}_{2}=\frac{P Z}{2 \pi A}$ is another constant. Thus, $T \propto \phi I_{\mathrm{a}}$
Thus, we conclude that torque produced in the armature is directly proportional to flux per pole and armature current. Moreover, the direction of electromagnetic torque developed in the armature depends upon the direction of flux or magnetic field and the direction of flow of current in armature conductors. If either of the two is reversed, the direction of torque produced is reversed, and hence, the direction of rotation. However, when both are reversed, the direction of torque does not change.

### 11.18 SHAFT TORQUE

In DC motors, whole of the electromagnetic torque ( $T_{\mathrm{e}}$ ) developed in the armature is not available at the shaft. A part of it is lost to overcome the iron and mechanical (friction and windage) losses. Therefore, shaft torque $\left(T_{\text {sh }}\right)$ is somewhat less than the torque developed in the armature. Thus, in the case of DC motors, the actual torque available at the shaft for doing useful mechanical work is known as shaft torque.

### 11.18.1 Brake Horse Power

In the case of motors, the mechanical power (H.P.) available at the shaft is known as brake horse power (BHP). If $T_{\text {sh }}$ is the shaft torque in Nm and $N$ is speed in rpm, then

$$
\begin{aligned}
& \text { useful output power }=\omega T_{\text {sh }}=2 \pi N T_{\text {sh }} / 60 \mathrm{~W} \\
& \qquad \text { Output in } \mathrm{BHP}=\frac{2 \pi N T_{\text {sh }}}{60 \times 735.5}
\end{aligned}
$$

### 11.19 COMPARISON OF GENERATOR AND MOTOR ACTION

It has been seen that the same machine can be used as a DC generator or as a DC motor. When it converts mechanical energy (or power) into electrical energy (or power), it is called a DC generator and when it is used for reversed operation, it is called a DC motor. Table 11.1 gives the comparison between the generator and the motor action.

Table 11.1 Comparison between the Generator and the Motor Action


(a)

Fig. 11.39 (a) Generator

1. In generator action, the rotation is due to mechanical torque, and therefore, $T_{\mathrm{m}}$ and $\omega$ are in the same direction.
2. The frictional torque $T_{\mathrm{f}}$ acts in opposite direction to rotation $\omega$.
3. Electromagnetic torque $T_{\mathrm{e}}$ acts in opposite direction to mechanical torque $T_{\mathrm{m}}$ so that $\omega T_{\mathrm{m}}=\omega T_{\mathrm{e}}+\omega T_{\mathrm{f}}$

Motor Action

Fig. 11.39 (b) Motor

1. In motor action, the rotation is due to electromagnetic torque, and therefore, $T_{\mathrm{e}}$ and $\omega$ are in the same direction.
2. The frictional torque $T_{\mathrm{f}}$ acts in opposite direction to rotation $\omega$.
3. Mechanical torque $T_{\mathrm{m}}$ acts in opposite direction to electromagnetic torque $T_{\mathrm{e}}$ so that $\omega T_{\mathrm{e}}=\omega T_{\mathrm{m}}+\omega T_{\mathrm{f}}$.

## Table 11.1 (Continued)

| Generator Action | Motor Action |
| :--- | :--- |
| 4. In generator action, an emf is induced in the | 4. In motor action, current is impressed to the |
| armature conductors that circulate current in the | armature against the induced emf $(e)$, and |
| armature when load is connected to it. Hence, | therefore, current flows in the opposite direction to |
| both $e$ and $i$ are in the same direction. | that of induced emf. |
| 5. In generator action, $E>V$ | 5. In motor action, $E<V$ |
| 6. In generator action, the torque angle $\theta$ is leading. | 6. In motor action, the torque angle $\theta$ is lagging. |
| 7. In generator action, mechanical energy is | 7. In motor action, electrical energy is converted into |
| converted into electrical energy | mechanical energy. |

## Example 11.19

A 50 H.P., 400 V , 4-pole, $1,000 \mathrm{rpm}$, DC motor has flux per pole equal to 0.027 Wb . The armature having 1,600 conductors is wave connected. Calculate the gross torque when the motor takes 70 A .

## Solution:

Torque developed, $T=\frac{P \phi \mathrm{ZI}_{\mathrm{a}}}{2 \pi A}$
where $P=4 ; \phi=0.027 \mathrm{~Wb} ; Z=1,600 ; I_{\mathrm{a}}=70 \mathrm{~A} ; A=2$ (wave connected)

$$
T=\frac{4 \times 0.027 \times 1,600 \times 70}{2 \times \pi \times 2}=963 \mathrm{Nm}
$$

## Example 11.20

The induced emf in a DC machine is 200 V at a speed of $1,200 \mathrm{rpm}$. Calculate the electromagnetic torque developed at an armature current of 15 A .

## Solution:

Here, $E_{\mathrm{b}}=200 \mathrm{~V} ; N=1,200 \mathrm{rpm} ; I_{\mathrm{a}}=15 \mathrm{~A}$
Now, power developed in the armature,

$$
\begin{aligned}
\omega T_{\mathrm{e}}=E_{\mathrm{b}} I_{\mathrm{a}} \text { or } T_{\mathrm{e}}=\frac{E_{b} I_{a}}{\omega} & =\frac{E_{\mathrm{b}} I_{\mathrm{a}}}{2 \pi N} \times 60 \quad\left(\Theta \omega=\frac{2 \pi N}{60}\right) \\
& =\frac{200 \times 15}{2 \pi \times 1,200} \times 60=23.87 \mathrm{Nm}
\end{aligned}
$$

## Example ll. 21

A 4-pole DC motor has a wave-wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 mWb . Calculate H.P. of the motor when running at $1,440 \mathrm{rpm}$.

## Solution:

Torque developed,

$$
T=\frac{P Z \varphi I_{\mathrm{a}}}{2 \pi A}=\frac{4 \times 594 \times 7.5 \times 10^{-3} \times 40}{2 \pi \times 2}=56.72 \mathrm{Nm}
$$

Power developed $=\omega T$ watt, where $\omega=\frac{2 \pi N}{60}$

$$
\begin{aligned}
& \text { H.P. }=\frac{\text { Power developed }}{735.5} ; \text { H.P. }=\frac{\omega T}{735.5}=\frac{2 \pi N T}{60 \times 735.5} \\
& \text { H.P. }=\frac{2 \times 1,440 \times 56.72}{60 \times 735.5}=11.63
\end{aligned}
$$

## Example 11.22

A 6-pole, lap-wound DC motor takes 340 A when the speed is 400 rpm . The flux per pole is 0.05 Wb and the armature has 864 turns. Neglecting mechanical losses, calculate the BHP of the motor.
(P.T.U. May 2010)

## Solution:

Here, $P=6 ; A=P=6$ (lap wound); $I_{\mathrm{L}}=340 \mathrm{~A} ; N=400 \mathrm{rpm}$,
$\phi=0.05 \mathrm{~Wb}$; number of turns $=864$
$Z=864 \times 2=1,728$
Back emf, $E_{\mathrm{b}}=\frac{\phi Z N P}{60 A}=\frac{0.05 \times 1,728 \times 400 \times 6}{60 \times 6}=576 \mathrm{~V}$
Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}=340 \mathrm{~A}$
Power developed $=E_{\mathrm{b}} \times I_{\mathrm{a}}=576 \times 340=195,840 \mathrm{~W}$
Neglecting losses, $\mathrm{BHP}=\frac{E_{\mathrm{b}} I_{\mathrm{b}}}{735.5}=\frac{195,840}{735.5}=266.27$

### 11.20 TYPES OF DC MOTORS

On the basis of the connections of armature and their field winding, DC motors can be classified as follows.

### 11.20.1 Separately Excited DC Motors

The conventional diagram of a separately excited DC motor is shown Figure 11.40. Its voltage equation will be
$E_{\mathrm{b}}=V-I_{\mathrm{a}} R_{\mathrm{a}}-2 \nu_{\mathrm{b}}\left(\right.$ where $v_{\mathrm{b}}$ is voltage drop per brush)

### 11.20.2 Self-excited DC Motors

These motors can be further classified as follows:

1. Shunt motors: Their conventional diagram is shown in Figure 11.41.
Important relations: $I_{\text {sh }}=V / R_{\text {sh }} ; I_{\mathrm{a}}=I_{\mathrm{L}}-I_{\text {sh }}$


Fig. 11.40 Conventional diagram of a separately excited DC motor
$E_{\mathrm{b}}=V-I_{\mathrm{a}} R_{\mathrm{a}}-2 \nu_{\mathrm{b}}$ (where $v_{\mathrm{b}}$ is voltage drop per brush)
2. Series motor: Its conventional diagram is shown in Figure 11.42.

Important relations: $I_{\mathrm{L}}=I_{\mathrm{a}}=I_{\mathrm{se}} ; E_{\mathrm{b}}=V-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 v_{\mathrm{b}}$


Fig. 11.41 Conventional diagram of a shunt wound DC motor


Fig. 11.42 Conventional diagram of a series wound DC motor


Fig. 11.43 Conventional diagram of a compound wound DC motor
3. Compound motor: Its conventional diagram (for long shunt) is shown in Figure 11.43.

$$
I_{\mathrm{sh}}=\frac{V}{R_{\mathrm{sh}}} ; I_{\mathrm{a}}=I_{\mathrm{L}}-I_{\mathrm{sh}} ; E_{\mathrm{b}}=V-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 \nu_{\mathrm{b}}
$$

In all the above mentioned voltage equations, the brush voltage drop $v_{\mathrm{b}}$ is sometimes neglected since its value is very small.

The compound motor can be further subdivided as follows:
(a) Cumulative compound motors: In these motors, the flux produced by both the windings is in the same direction, that is,

$$
\phi_{\mathrm{r}}=\phi_{\mathrm{sh}}+\phi_{\mathrm{se}}
$$

(b) Differential compound motors: In these motors, the flux produced by the series field winding is opposite to the flux produced by the shunt field winding, that is,

$$
\phi_{\mathrm{r}}=\phi_{\mathrm{sh}}-\phi_{\mathrm{se}}
$$

## Example 11.23

A 4-pole DC motor has a wave-wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 mWb . Calculate H.P. of the motor when running at $1,440 \mathrm{rpm}$

## Solution:

Torque developed,

$$
T=\frac{P Z \varphi I_{\mathrm{a}}}{2 \pi A}=\frac{4 \times 594 \times 7.5 \times 10^{-3} \times 40}{2 \pi \times 2}=56.72 \mathrm{Nm}
$$

Power developed $=\omega T$ watts, where $\omega=\frac{2 \pi N}{60}$
H.P. $=\frac{\text { Power developed }}{735.5}$
H.P. $=\frac{\omega T}{735.5}=\frac{2 \pi N T}{60 \times 735.5}$ or H.P. $=\frac{2 \times 1,440 \times 56.72}{60 \times 735.5}=11.63$

## Example 11.24

A 6-pole lap-wound shunt motor has 500 conductors in the armature. The resistance of armature path is $0.05 \Omega$. The resistance of shunt field is $25 \Omega$. Find the speed of the


Fig. 11.44 Conventional diagram as per data motor when it takes 120 A from a DC mains of 100 V supply. Flux per pole is $2 \times 10^{-2} \mathrm{~Wb}$.

## Solution:

The conventional diagram of the motor is shown in Figure 11.44

$$
\begin{aligned}
I_{\text {sh }} & =\frac{V}{R_{\text {sh }}}=\frac{100}{25}=4-\mathrm{A} \\
I_{\mathrm{a}} & =I_{\mathrm{L}}-I_{\text {sh }}=120-4=116-\mathrm{A} \\
E_{\mathrm{b}} & =V-I_{\mathrm{a}} R_{\mathrm{a}}
\end{aligned}
$$

$$
=100-116 \times 0.05=94.2 \mathrm{~V}
$$

Now,

$$
E_{\mathrm{b}}=\frac{P \phi Z N}{60 A} \quad \text { or } \quad 94.2=\frac{6 \times 2 \times 10^{-2} \times 500 \times \mathrm{N}}{60 \times 6}
$$

or

$$
N=565.2 \mathrm{rpm}
$$

## Example 11.25

A 6-pole, 440 V DC motor has 936 wave-wound armature conductors. The useful flux per pole is 25 mWb . The torque developed is 45.5 kgm . If armature resistance is $0.5 \Omega$, then calculate (i) armature current and (ii) speed.
(P.T.U.)

## Solution:

Number of poles, $P=6$
Number of armature conductors, $Z=936$
Flux per pole, $\phi=25 \mathrm{mWb}=25 \times 10^{-3} \mathrm{~Wb}$
Number of parallel path, $A=2$ (wave-wound armature)
Terminal voltage, $V=440 \mathrm{~V}$
Armature resistance, $R_{\mathrm{a}}=0.5 \Omega$
Torque developed, $T=45.5 \mathrm{kgm}=45.5 \times 9.81=446.35 \mathrm{Nm}$
(i) Using the relation, $T=\frac{P Z \phi I_{a}}{2 \pi A}$

Armature current, $I_{\mathrm{a}}=\frac{2 \pi A \times T}{P Z \phi}=\frac{2 \pi \times 2 \times 446.35}{6 \times 936 \times 25 \times 10^{-3}}=39.95 \mathrm{~A}$
(ii) Induced emf, $E=V-I_{\mathrm{a}} R_{\mathrm{a}}$ (motor action)

$$
=440-39.95 \times 0.5=420 \mathrm{~V}
$$

Using the relation, $\omega T=E I_{\mathrm{a}} \quad$ or $\quad \frac{2 \pi N}{60} \times T=E I_{\mathrm{a}}$
Speed

$$
N=\frac{60 \times E I_{\mathrm{a}}}{\pi T}=\frac{60 \times 420 \times 39.95}{2 \pi \times 446.35}=359 \mathrm{rpm}
$$

## Example 11.26

A 400 V DC motor takes an armature current of 100 A when its speed is $1,000 \mathrm{rpm}$. If the armature resistance is $0.25 \Omega$, then calculate the torque produced in Nm .
(P.T.U.)

## Solution:

Terminal voltage, $V=400 \mathrm{~V}$
Armature current, $I_{\mathrm{a}}=100 \mathrm{~A}$
Armature resistance, $R_{\mathrm{a}}=0.25 \Omega$
Speed, $N=1,000 \mathrm{rpm}$
Induced emf, $E=V-I_{\mathrm{a}} R_{\mathrm{a}}$ (motor action)

$$
=400-100 \times 0.25=375 \mathrm{~V}
$$

Using the relation, $\omega T=E I_{\mathrm{a}} \quad$ or $\quad \frac{2 N T}{60}=E I_{\mathrm{a}}\left[\right.$ Because $\left.\omega=\frac{2 N T}{60}\right]$
Therefore, torque produced, $T=\frac{60 E I_{\mathrm{a}}}{2 N}=\frac{60 \times 375 \times 1,000}{2 \times 1,000}=358.1 \mathrm{Nm}$

## Example 11.27

The electromagnetic torque developed in a DC machine is 80 Nm for an armature current of 30 A. What will be the torque for a current of 15 A? Assume constant flux. What is the induced emf at a speed of 900 rpm and an armature current of 15 A ?

## Solution:

Torque developed, $T_{1}=80 \mathrm{Nm}$
Armature current, $I_{\mathrm{a} 1}=30 \mathrm{~A}$
Armature current, $I_{\mathrm{a} 2}=15 \mathrm{~A}$
Let the torque developed is $T_{2} \mathrm{Nm}$ when the armature current is 15 A .
Now,

$$
T \propto f I_{\mathrm{a}}
$$

When flux $f$ is constant, $T \propto I_{\text {a }}$
Let the torque developed is $T_{2} \mathrm{Nm}$ when the armature current is 15 A .
Now,

$$
T \alpha \phi I_{a}
$$

When flux $\phi$ is constant, $T \propto I_{\mathrm{a}}$
Therefore,

$$
\begin{gathered}
\frac{T_{2}}{T_{1}}=\frac{I_{\mathrm{a} 2}}{I_{\mathrm{a} 1}} \\
T_{2}=\frac{I_{a 2}}{I_{a 1}} \times T_{1}=\frac{15}{30} \times 80=40 \mathrm{Nm}
\end{gathered}
$$

Power developed in the armature $=E_{2} I_{\mathrm{a} 2}=\omega_{2} T_{2}$
where

$$
\omega_{2}=\frac{2 \pi N_{2}}{60}=\frac{2 \pi \times 900}{60}=30 \pi
$$

Induced emf $=E_{2}=\frac{\omega_{2} T_{2}}{I_{\mathrm{a} 2}}=\frac{30 \pi \times 40}{15}=251.33$

### 11.21 CHARACTERISTICS OF DC MOTORS

The performance of a DC motor can be easily judged from its characteristic curves known as motor characteristics. The characteristics of a motor are those curves that show relation between the two quantities. On the basis of these quantities, the following characteristics can be obtained:

1. Speed and armature current, that is, $N-I_{\mathrm{a}}$ characteristics: It is the curve drawn between speed $N$ and armature current $I_{a}$. It is also known as speed characteristics.
2. Torque and armature current, that is, $\boldsymbol{T}-\boldsymbol{I}_{\mathrm{a}}$ characteristics: It is the curve drawn between torque developed in the armature $T$ and armature current $I_{\mathrm{a}}$. It is also known as electrical characteristic.
3. Speed and torque, that is, $\boldsymbol{N} \boldsymbol{-} \boldsymbol{T}$ characteristics: It is the curve drawn between speed $N$ and torque developed in the armature $T$. It is also known as mechanical characteristics.

The following important relations must be kept in mind while discussing the motor characteristics:

$$
E_{\mathrm{b}} \propto N \phi \quad \text { or } \quad N \propto \frac{E_{\mathrm{b}}}{\phi} \quad \text { and } \quad T \propto \phi I_{\mathrm{a}}
$$

### 11.22 CHARACTERISTICS OF SHUNT MOTORS

The conventional diagram of this motor is shown in Figure 11.45. In these motors, the shunt field current $I_{\text {sh }}=V / R_{\text {sh }}$ remains constant since the supply voltage $V$ is constant. Hence, the flux in DC shunt motors is practically constant (although at heavy loads, somewhat flux decreases due to armature reaction).

1. $\boldsymbol{N}-\boldsymbol{I}_{\mathrm{a}}$ characteristics: We know that, $N \propto \frac{E_{\mathrm{b}}}{\phi}$. Since flux is constant, $N \propto E_{\mathrm{b}}$ or $N \propto V-I_{\mathrm{a}} R_{\mathrm{a}}$. If the armature drop $\left(I_{\mathrm{a}} R_{\mathrm{a}}\right)$ is negligible, the speed of the motor will remain constant for all values of load as shown by the dotted line AB in Figure 11.46. However, strictly speaking, as the armature current increases due to the increase of load, armature drop $I_{\mathrm{a}} R_{\mathrm{a}}$ increases and speed of the motor decreases slightly as shown by the straight line AC in Figure 11.46 (neglecting armature reaction). Moreover, the characteristic curve does not start from a point of zero armature current because a small current, no-load armature current $I_{\mathrm{a} 0}$, is necessary to maintain rotation of the motor at no-load.

Since there is no appreciable change in the speed of a DC shunt motor from no-load to full-load, it is considered to be a constant speed motor. This motor is best suited where almost constant speed is required and the load may be thrown off totally and suddenly.
2. $\boldsymbol{T}-\boldsymbol{l}_{\mathrm{a}}$ characteristics: We know that, $T \propto \phi l_{\mathrm{a}}$. Since flux is constant, $T \propto l_{\mathrm{a}}$. Hence, the electrical characteristic (i.e., $T-I_{\mathrm{a}}$ ) is a straight line passing through the origin as shown in Figure 11.47. It is clear from the characteristic curve that a large armature current is required at the start, if machine is on heavy load. Thus, shunt motor should never be started on load.
3. $\boldsymbol{N}-\boldsymbol{T}$ characteristics: The $N-T$ characteristic is derived from the first two characteristics. When load torque increases, armature current $I_{a}$ increases, but speed decreases slightly. Thus, with the increase in load or torque, the speed decreases slightly as shown in Figure 11.48.


Fig. 11.45 Conventional diagram of a DC shunt motor


Fig. $11.46 \quad \mathrm{~N}-\mathrm{I}_{\mathrm{a}}$ characteristics of DC shunt motor


Fig. 11.47 T-I characteristics of DC shunt motor


Fig. $11.48 \mathrm{~N}-\mathrm{T}$ characteristics of DC shunt motor


Fig. 11.49 Conventional diagram of a DC series motor


Fig. $11.50 \quad \mathrm{~N}-\mathrm{I}_{\mathrm{a}}$ characteristics of a DC series motor


Fig. 11.51 T-I ${ }_{\mathrm{I}}$ characteristics of a DC series motor

### 11.23 CHARACTERISTICS OF SERIES MOTORS

The conventional diagram a series motor is shown in Figure 11.49. In these motors, the series field winding carries the armature current. Therefore, the flux produced by the series field winding is proportional to the armature current before magnetic saturation, but after magnetic saturation flux becomes constant.

1. $\boldsymbol{N}-\boldsymbol{I}_{\mathrm{a}}$ Characteristics: We know that $N \propto \frac{E_{\mathrm{b}}}{\phi}$ where, $E_{\mathrm{b}}$ $=V-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)$. When armature current increases, the induced emf (back emf) $E_{\mathrm{b}}$ decreases, due to $I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{es}}\right)$ drop. While flux $\phi$ increases as $\phi \propto I_{\mathrm{a}}$ before magnetic saturation. However, under normal conditions, $I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)$ drop is quite small and may be neglected.
Considering $E_{\mathrm{b}}$ to be constant,

$$
N \propto \frac{1}{\phi} \propto \frac{1}{I_{\mathrm{a}}}
$$

Thus, before magnetic saturation, the $N-I_{\mathrm{a}}$ curve follows the hyperbolic path as shown in Figure 11.50. In this region, the speed decreases abruptly with the increase in load or armature current.

After magnetic saturation, flux becomes constant, then,

$$
N \propto E_{\mathrm{b}} \propto V-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)
$$

Thus, after magnetic saturation, the $N-I_{a}$ curve follows a straight line path and speed decreases slightly, as shown in Figure 11.50. From this characteristic, it is concluded that the series motor is a variable speed motor, that is, its speed changes when the armature current (or load) changes. As the load on this motor decreases, speed increases. If this motor is connected to the supply without load, armature current will be very small and hence speed will be dangerously high that may damage the motor due to heavy centrifugal forces.

Therefore, a series motor is never started on no-load. However, to start a series motor, mechanical load (not belt driven load because belt slips over the pulley) is put on it first and then started.
2. $\boldsymbol{T}-\boldsymbol{I}_{\mathrm{a}}$ characteristics: We know that, $T \propto \phi I_{\mathrm{a}}$. In series motors, before magnetic saturation $\phi \propto I_{a}$. Hence, before magnetic saturation, the electromagnetic torque produced in the armature is proportional to the square of the armature current. Therefore, this portion of the curve ( OA ) is a parabola passing through the origin, as shown in Figure 11.51. However, after magnetic saturation, the flux $\phi$ becomes constant.
Therefore,

$$
T \propto I_{\mathrm{a}}
$$

Hence, after magnetic saturation, the curve (AB) becomes a straight line. It is seen that before magnetic saturation $T \propto I_{\mathrm{a}}{ }^{2}$. When load is applied to this motor at start, it takes large current and heavy torque is produced that is proportional to square of this current. Thus, this motor is capable to pick up heavy loads at the start and best suited for electric traction.
3. $\boldsymbol{N}-\boldsymbol{T}$ characteristics: This characteristic is derived from the first two characteristics. At low value of load, $I_{\mathrm{a}}$ and torque are small, but the speed is very high. As load increases, $I_{\mathrm{a}}$ and torque increases, but the speed decreases rapidly. Thus, for increasing torque, speed decreases rapidly as shown in Figure 11.52.


Fig. $11.52 \mathrm{~N}-\mathrm{T}$ characteristics of a DC series motor

### 11.24 CHARACTERISTICS OF COMPOUND MOTORS

There are two types of compound-wound DC motors, namely cumulative compound motors and differential compound motors. Cumulative compound motors are most common. The characteristics of these motors lie between the shunt and the series motors. The $N-I_{\mathrm{a}}$ characteristics, $T-I_{\mathrm{a}}$ characteristics, and $N-T$ characteristics are shown in Figure 11.53(a), (b), and (c), respectively.


Fig. 11.53 (a) $N-I_{a}$, (b) $T-I_{a}$ and (c) $N-T$ characteristics of shunt series and cumulatively compound wound motors

However, the $N-I_{\mathrm{a}}, T-I_{\mathrm{a}}$, and $N-T$ characteristics of a differential compound motor are shown in Figure 11.54(a), (b), and (c), respectively.


Fig. 11.54 (a) $N-I_{a}$, (b) $T-I_{a}$ and (c) N-T characteristics of shunt, series and differentially compound wound motors

### 11.25 APPLICATIONS AND SELECTION OF DC MOTORS

As per the characteristics of DC motors, different types of DC motors are applied for different jobs as follows:

1. Separately excited motors: Very accurate speeds can be obtained by these motors. Moreover, these motors are best suited where speed variation is required from very low value to high value. These motors are used in steel rolling mills, paper mills, diesel-electric propulsion of ships, etc.
2. Shunt motors: From the characteristics of a shunt motor, we have seen that it is almost constant speed motor. Therefore, it is used under the following conditions:
(a) Where the speed between no-load to full-load has to be maintained almost constant.
(b) Where it is required to drive the load at various speeds (various speeds are obtained by speed control methods) and any one of the speed is required to be maintained almost constant for a relatively long period.
As such, the shunt motors are most suitable for industrial drives such as lathes, drills, grinders, shapers, spinning and weaving machines, and line shafts in the group drive.
3. Series motors: The characteristics of a series motor reveal that it is variable speed motor, that is, the speed is low at high torques and vice versa. Moreover, at light loads or at no-load, the motor attains dangerously high speed. Therefore, it is employed under the following conditions:
(a) Where high torque is required at the time of starting to accelerate heavy loads quickly.
(b) Where the load is subjected to heavy fluctuations and speed is required to be adjusted automatically.
As such the series motors are most suitable for electric traction, cranes, elevators, vacuum cleaners, hair driers, sewing machines, fans and air compressors, etc.
Note: The series motors are always directly coupled with loads or coupled through gears. Belt loads are never applied to series motor because the belt may slip over the pulley or it may break. Then, the motor will operate at light loads or at no-load and will attain dangerously high speed that may damage the motor.
4. Compound motors: The important characteristic of this motor is that the speed falls appreciably on heavy loads as in a series motor; however, at light loads, the maximum speed is limited to safe value. Therefore, it is used under the following conditions:
(a) Where high torque is required at the time of starting and where the load may be thrown off suddenly.
(b) Where the load is subjected to heavy fluctuations.

As such the cumulative compound motors are best suited for punching and shearing machines, rolling mills, lifts and mine hoists, etc.

### 11.26 NECESSITY OF STARTER FOR A DC MOTOR

Under normal operating conditions, the voltage equation for a motor is given as

$$
E_{\mathrm{b}}=V-I_{\mathrm{a}} R_{\mathrm{a}} \quad \text { or } \quad I_{\mathrm{a}} R_{\mathrm{a}}=V-E_{\mathrm{b}}
$$

The armature current is given by the relation; $I_{\mathrm{a}}=\frac{V-E_{\mathrm{b}}}{R_{\mathrm{b}}}$
When the motor is at rest, the induced emf $E_{\mathrm{b}}$ in the armature is zero $\left(E_{\mathrm{b}} \propto N\right)$. Consequently, if full voltage is applied across the motor terminals, the armature will draw heavy current $\left(I_{\mathrm{a}}=V / R_{\mathrm{a}}\right)$ because armature resistance is relatively small. This heavy starting current has the following effects:

1. It will blow out the fuses and prior to that it may damage the insulation of armature winding due to excessive heating effect if starting period is more.
2. Excessive voltage drop will occur in the lines to which the motor is connected. Thus, the operation of the appliances connected to the same line may be impaired, and in some cases, they may refuse to work.

To avoid this heavy current at start, a variable resistance is connected in series with the armature, as shown in Figure 11.55, called a starting resistance or starter. Thus, the armature current is limited to safe value $\left(I_{\mathrm{a}}=\frac{V}{R_{\mathrm{a}}+R}\right)$. Once the motor picks up speed, emf is built up and current is reduced $\left(I_{\mathrm{a}}=\frac{V-E_{\mathrm{b} 1}}{R_{\mathrm{a}}+R}\right)$. After that, the starting resistance is gradually reduced. Ultimately, whole of the resistance is taken out of circuit when the motor attains normal speed.

Another important feature of a starter is that it contains protective devices such as overload protection coil (or relay) that provides necessary protection to the motor against overloading and no-volt release coil.


Fig. 11.55 A resistor connected in series with the armature that limits the starting current

### 11.27 STARTERS FOR DC SHUNT AND COMPOUND-WOUND MOTORS

The basic function of a starter is to limit the current in the armature circuit during starting or accelerating period. Starters are always rated on the basis of output power and voltage of the motor with which they are to be employed (e.g., 10 H.P., 250 V shunt motor starter). A simplest


Fig. 11.56 (a) Starter for DC shunt motor and (b) Starter for
DC compound motor
type of starter is just a variable resistance (a rheostat) connected in series with the armature alone (not with the motor as a whole), as shown in Figure 11.56(a) and (b).

It may be noted that shunt field is kept independent of starting resistance. It is because when supply is connected, it receives normal rated voltage and sets up maximum (rated) flux. A high value of flux results in a low operating speed and a high motor torque for a particular value of starting current since speed is inversely proportional to flux per pole $\left(N \propto \frac{1}{\phi}\right)$, whereas motor torque is proportional to product of flux per pole and armature current $\left(T \propto \phi I_{\mathrm{a}}\right)$. Hence, for a given load torque, the motor will accelerate quickly and reduces the starting period. Thus, the heating effect to armature winding is reduced.

For all practical application, this starter is further modified that includes protective devices such as overload release and no-volt release. The overload release protects the motor against overloading, that is, when the motor is overloaded (or short circuited) this relay brings the plunger to its OFF position. On the other hand, the no-volt release brings the plunger to its OFF position so that the motor may not start again without starter. For shunt and compound motors, there are two standard types of starters, namely three-point starter and four-point starter.

### 11.28 THREE-POINT SHUNT MOTOR STARTER

The schematic connection diagram of a shunt motor starter is shown in Figure 11.57. It consists of starting resistance $R$ divided into several sections. The tapping points of starting resistance are connected to number of studs. The last stud of the starting resistance is connected to terminal A to which one terminal of the armature is connected. The positive supply line is connected to the line terminal L through main switch. From line terminal, supply is connected to the starting lever $S L$ through overload release coil OLRC. A spring $S$ is placed over the lever to bring it to the OFF position, when the supply goes OFF. A soft iron piece $S I$ is attached with the starting lever that is pulled by the no-volt release coil under normal running condition. The far end of the brass strip $B S$ is connected to the terminal Z through a no-volt release coil NVRC. One end of the shunt field winding is connected to Z terminal of the starter. An iron piece is lifted by OLRC under abnormal condition to short circuit the no-volt release coil. The negative supply line is connected directly to the other ends of shunt field winding and armature of the DC shunt motor.

### 11.28.1 Operation

First, the main switch is closed with starting lever resting in OFF position. The handle is then turned clockwise to the first stud and brass strip. As soon as it comes in contact with the first stud, whole of the starting resistance $R$ is inserted in series with the armature and the field winding is directly connected across the supply through brass strip. As the handle is turned further, the starting resistance is cut out of the armature circuit in steps, and finally, entire starting resistance is cut out of armature |circuit.

### 11.28.2 No-volt Release Coil and Its Function

A no-volt release coil is a small electromagnet having many turns of fine wire. It is connected in series with shunt field winding, and therefore, it carries a small field current. When the handle is turned to on


Fig. 11.57 Three-point starter for DC shunt motors position, the no-volt release coil is magnetised by the field current and holds the starting lever at on position. In the case of failure or disconnection of the supply, this coil is demagnetised and the lever comes to the OFF position due to spring tension. Consequently, the motor is disconnected from the supply. If the spring with the no-volt release coil is not used, the lever would remain in ON position in the case of supply failure. And again, when the supply comes, the motor would be connected directly to the lines without starter.

The other important advantage of connecting the no-volt release coil in series with the shunt field winding is that due to an accident if the circuit of field winding becomes open, the NVRC will be demagnetised and the starting lever is immediately pulled back to the OFF position by the spring. Otherwise, the motor would have attained dangerously high speed.

### 11.28.3 Overload Release Coil and Its Function

An overload release coil is an electromagnet having small number of turns of thick wire. It is connected in series with the motor and carries the line current. When the motor is overloaded (or short circuited), a heavy current more than predetermined value will flow through it. Then, the iron piece (armature or plunger) is lifted and short circuits the no-volt release coil. Hence, the starting lever is released and pulled back to the OFF position due to spring tension. Thus, the motor is disconnected from the supply and is protected against overloading.

### 11.29 LOSSES IN A DC MACHINE

A DC machine is used to convert mechanical energy into electrical energy or vice versa. While doing so, the whole of input energy does not appear at the output but a part of it is lost in the form of heat in the surroundings. This wasted energy is called losses in the machine.

These losses affect the efficiency of the machine. A reduction in these losses leads to high efficiency. Thus, the major objective in the design of a DC machine is to reduce these losses. The various losses occurring in a DC machine can be sub-divided as follows:

### 11.29.1 Copper Losses

The various windings of the DC machine, made of copper, have some resistance. When current flows through them, there will be power loss proportional to the square of their respective currents. These power losses are called copper losses. In general, the various copper losses in a DC machine are as follows:

1. Armature copper loss $=I_{\mathrm{a}}^{2} R_{\mathrm{a}}$
2. Shunt field copper loss $=I_{\mathrm{sh}}^{2} R_{\text {sh }}$
3. Series field copper loss $=I_{\mathrm{se}}^{2} R_{\mathrm{se}}$
4. Interpole winding copper loss $=I_{\mathrm{i}}^{2} R_{\mathrm{i}}=I_{\mathrm{a}}^{2} R_{\mathrm{i}}$
5. Brush contact loss $=I_{\mathrm{a}}^{2} R_{\mathrm{b}}=2 I_{\mathrm{a}} v_{\mathrm{b}}$
6. Compensating winding copper loss $=I_{\mathrm{a}}^{2} R_{\mathrm{c}}$

The loss due to brush contact is generally included in armature copper loss.

### 11.29.2 Iron Losses

The losses that occur in the iron parts of a DC machine are called iron losses or core losses or magnetic losses. These losses can be classified as follows:

1. Hysteresis loss: Whenever a magnetic material is subjected to reversal of magnetic flux, this loss occurs. It is due to retentivity (a property) of the magnetic material. It is expressed with reasonable accuracy by the following expression:

$$
P_{\mathrm{h}}=\mathrm{K}_{\mathrm{h}} V f B_{\mathrm{m}}^{1.6}
$$

where $\mathrm{K}_{\mathrm{h}}=$ hysteresis constant in $\mathrm{J} / \mathrm{m}^{3}$
In other words, energy loss per unit volume of magnetic material during one magnetic reversal and its value depends upon the nature of material.
$V=$ volume of magnetic material in $\mathrm{m}^{3}$
$f=$ frequency of magnetic reversal in cycle per second
$B_{\mathrm{m}}=$ maximum flux density in the magnetic material in Tesla
It occurs in the rotating armature. To minimise this loss, the armature core is made of silicon steel that has low hysteresis constant.
2. Eddy current loss: When flux linking with the magnetic material changes (or flux is cut by the magnetic material), an emf is induced in it, which circulates eddy currents through it. These eddy currents produce eddy current loss in the form of heat. It is expressed with reasonable accuracy by the expression:

$$
P_{\mathrm{e}}=\mathrm{K}_{\mathrm{e}} V f^{2} t^{2} B_{\mathrm{m}}^{2}
$$

where $\mathrm{K}_{\mathrm{e}}=$ constant called co-efficient of eddy current and its value depends upon the nature of magnetic material.
$t=$ thickness of lamination in m
$V, f$, and $B_{\mathrm{m}}$ are the same as earlier

The major part of this loss occurs in the armature core. To minimise this loss, the armature core is laminated into thin sheets ( 0.3 to 0.5 mm ), since this loss is directly proportional to the square of thickness of the laminations.

### 11.29.3 Mechanical Losses

As the armature of a DC machine is a rotating part, some power is required to overcome:
(a) Air friction of rotating armature (windage loss)
(b) Friction at the bearing and friction between brushes and commutator (friction loss). These losses are known as mechanical losses. To reduce these losses, proper lubrication is done at the bearings.

### 11.30 CONSTANT AND VARIABLE LOSSES

The losses in a DC machine may also be sub-divided into two categories:

1. Constant losses: The losses in a DC machine that remain the same at all loads are called constant losses. The constant losses in a DC machine are iron losses, mechanical losses, and shunt field copper losses.
2. Variable losses: The losses in a DC machine that vary with load are called variable losses. The variable losses in a DC machine are armature copper loss and series field copper loss, and interpole winding copper loss.

### 11.31 STRAY LOSSES

The sum of the iron loss and mechanical loss in a DC machine are known as stray losses, that is, stray losses $=$ iron loss + mechanical loss.

### 11.32 POWER FLOW DIAGRAM

When machine is working as a generator, the mechanical power $\left(\omega T_{\mathrm{m}}\right)$ is supplied to the generator that is converted into electrical power $\left(V I_{\mathrm{L}}\right)$. During the conversion, various losses occur in the machine. The power flow diagram for a DC generator is shown in Figure 11.58.


Fig. 11.58 Power flow diagram of a DC genertator

Although losses in a DC machine are the same whether it works as a generator or as a motor, but the flow of power is opposite. The power flow diagram for a DC motor is shown in Figure 11.59.


Fig. 11.59 Power flow diagram of a DC motor

### 11.33 EFFICIENCY OF A DC MACHINE

The ratio of output power to the input power of a DC machine is called its efficiency.
Efficiency,

$$
\eta=\frac{\text { Output }}{\text { Input }}
$$

### 11.33.1 Machine Working as a Generator

$$
\begin{aligned}
\text { Power output } & =V I_{\mathrm{L}} \text { watt } \\
\text { Power input } & =\text { power output }+ \text { variable losses }+ \text { constant losses }
\end{aligned}
$$

Since the shunt field current $I_{\text {sh }}$ is very small as compared to line current, we may consider, $I_{\mathrm{L}} \cong I_{\mathrm{a}}$ (neglecting $I_{\text {sh }}$ )

Therefore, variable losses $=I_{\mathrm{L}}^{2} R_{\mathrm{a}} ;$ constant losses $=P_{\mathrm{c}}$ (say)
Then, power input $V I_{\mathrm{L}}+I_{\mathrm{L}}^{2} R_{\mathrm{a}}+P_{\mathrm{c}} ; \eta=\frac{V I_{\mathrm{L}}}{V I_{\mathrm{L}}+I_{\mathrm{L}}^{2} R_{\mathrm{a}}+P_{\mathrm{c}}}$

### 11.33.2 Machine Working as a Motor

Power input $=V I L$
Power output $=$ power input - variable losses - constant losses $=V I_{\mathrm{L}}-I_{\mathrm{L}}^{2} R_{\mathrm{a}}-P_{\mathrm{c}}$

$$
\eta=\frac{V I_{\mathrm{L}}-I_{\mathrm{L}}^{2} R_{\mathrm{a}}-P_{\mathrm{c}}}{V I_{\mathrm{L}}}
$$

## Example 11.28

A DC generator is connected to a 220 V DC mains. The current delivered by the generator to the mains is 100 A . The armature resistance is $0.1 \Omega$. The generator is driven at a speed of 500 rpm . Calculate (i) the induced emf, (ii) the electromagnetic torque, (iii) the mechanical power input to the armature neglecting iron, windage, and friction losses, (iv) electrical power output from the armature, and (v) armature copper loss.

## Solution:

(i) The induced emf, $E_{\mathrm{g}}=V+I_{\mathrm{a}} R_{\mathrm{a}}$

$$
=220+0.1 \times 100=230 \mathrm{~V}
$$

Using the relation, $\omega T=E_{\mathrm{g}} I_{\mathrm{a}}$
(ii) Electromagnetic torque, $T=\frac{E_{\mathrm{g}} I_{\mathrm{a}}}{\omega}=\frac{E_{\mathrm{g}} I_{\mathrm{a}}}{2 \pi N} \times 60 \quad\left[\Theta \omega=\frac{2 \pi N}{60}\right]$

$$
=\frac{230 \times 100 \times 60}{2 \pi \times 500}=439.27 \mathrm{Nm} .
$$

(iii) Neglecting iron, windage, and friction losses

$$
\begin{aligned}
\text { Input to armature } & =\omega T \text { or } E_{\mathrm{g}} I_{\mathrm{a}} \\
& =\frac{2 \pi N T}{60}=\frac{2 \pi \times 500 \times 439.27}{60}=23,000 \mathrm{~W}
\end{aligned}
$$

(iv) Electrical power output $=V I_{\mathrm{a}}=220 \times 100=22,000 \mathrm{~W}$
(v) Armature copper losses $=I_{a}^{2} R_{\mathrm{a}}=(100)^{2} \times 0.1=1,000 \mathrm{~W}$

## Example 11.29

A shunt generator supplies 195 A at 220 V . Armature resistance is $0.02 \Omega$ and shunt field resistance is $44 \Omega$. If the iron and friction losses amount to $1,600 \mathrm{~W}$, find (i) emf generated, (ii) copper losses, and (iii) BHP of the engine driving the generator.

## Solution:

Shunt field current,

$$
I_{\mathrm{sh}}=\frac{V}{R_{\mathrm{sh}}}=\frac{220}{44}=5 \mathrm{~A}
$$

Armature current, $I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}}=195+5=200 \mathrm{~A}$
Generated or induced emf, $E=V+I_{\mathrm{a}} R_{\mathrm{a}}$

$$
=220+200 \times 0.02=224 \mathrm{~V}
$$

Armature copper loss $=I_{\mathrm{a}}^{2} R_{\mathrm{a}}=(200)^{2} \times 0.02=800 \mathrm{~W}$
Shunt field copper loss $=I_{\mathrm{sh}}^{2} R_{\mathrm{sh}}=(5)^{2} \times 44=1,100 \mathrm{~W}$
Total copper losses $=800+1,100=1,900 \mathrm{~W}$
Output power $=V I_{\mathrm{L}}=220 \times 195=42,900 \mathrm{~W}$
Input power $=42,900+1,600+1,900=46,400 \mathrm{~W}$
BHP of the engine driving the generator $=\frac{46,400}{735.5}=63.08$ H.P.

## Example 11.30

A $10 \mathrm{~kW}, 200 \mathrm{~V}, 1,200 \mathrm{rpm}$ series DC generator has armature resistance of $0.1 \Omega$ and field winding resistance of $0.3 \Omega$. The frictional and windage loss of the machine is 200 W and brush contact drop is 1 V per brush. Find the efficiency of the machine and the load current at which this machine has maximum efficiency.
(U.P.T.U. 2004-05)

## Solution:

The conventional circuit is shown in Figure 11.60.
Load $=10 \mathrm{~kW}=10 \times 10^{3} \mathrm{~W} ; V=200 \mathrm{~V} ; N=1,200 \mathrm{rpm} ; R_{\mathrm{a}}=0.1 \Omega$
$R_{\text {se }}=0.3 \Omega ; V_{\mathrm{b}}=1 \mathrm{~V} / \mathrm{brush}$; friction and windage loss $=$


Fig. 11.60 Conventional diagram as per data 200 W

$$
\text { Load current, } \begin{aligned}
I_{\mathrm{L}} & =\frac{\text { Load }}{V} \\
& =\frac{10 \times 10^{3}}{200}=50 \mathrm{~A} \\
I_{\mathrm{a}} & =I_{\mathrm{L}}=50 \mathrm{~A}
\end{aligned}
$$

Armature copper loss $=I_{\mathrm{a}}^{2} R_{\mathrm{a}}$

$$
=(50)^{2} \times 0.1=250 \mathrm{~W}
$$

Series field copper loss $=I_{\mathrm{a}}^{2} R_{\mathrm{sc}}=(50)^{2} \times 0.3=750 \mathrm{~W}$
Total copper loss $=250+750=1,000 \mathrm{~W}$
Brush contact loss $=2 V_{\mathrm{b}} \times I_{\mathrm{a}}=2 \times 1 \times 50=100 \mathrm{~W}$
Friction and windage loss $=200 \mathrm{~W}$
Efficiency will be maximum when,
iron loss $=$ copper loss $=1,000 \mathrm{~W}$
total losses $=$ iron loss + copper loss + brush contact loss + friction and windage loss

$$
=1,000+1,000+100+200=2,300 \mathrm{~W}
$$

Output $=10 \mathrm{~kW}=10 \times 1,000=10,000 \mathrm{~W}$
Input $=$ output + losses $=10,000+2,300=12,300 \mathrm{~W}$

$$
\begin{aligned}
\eta_{\max } & =\frac{\text { Output }}{\text { Input }} \times 100 \\
& =\frac{10,000}{12,300} \times 100=81.3 \%
\end{aligned}
$$

## Example 11.31

A 17 H.P., $230 \mathrm{~V}, 1,200 \mathrm{rpm}$ shunt motor has four parallel armature path and 868 armature conductors. The armature circuit resistance is $0.18 \Omega$. At rated speed and rated output, the armature current is 70 A and the field current is 1.5 A . Calculate (i) the electromagnetic torque, (ii) the flux per pole, (iii) the rotational losses, and (iv) the efficiency.

## Solution:

The conventional circuit is shown in Figure 11.61.
Here, $V=230 \mathrm{~V}$

$$
\begin{aligned}
N & =1,200 \mathrm{rpm} ; A=4 ; P=4 ; Z=868 \\
R_{\mathrm{a}} & =0.18 \Omega \\
I_{\mathrm{a}} & =70 \mathrm{~A} ; I_{\mathrm{sh}}=1.5 \mathrm{~A}
\end{aligned}
$$

Output, $P_{0}=17 \times 735.5=12,503.5 \mathrm{~W}$

$$
\begin{aligned}
I_{\mathrm{L}} & =I_{\mathrm{a}}+I_{\mathrm{sh}}=70+1.5=71.5 \mathrm{~A} \\
E_{\mathrm{b}} & =V-I_{\mathrm{a}} R_{\mathrm{a}} \\
& =230-70 \times 0.18=217.4 \mathrm{~V}
\end{aligned}
$$



Fig. 11.61 Conventional diagram as per data

Electromagnetic torque, $T_{\mathrm{e}}=\frac{E_{\mathrm{b}} \times I_{\mathrm{a}}}{\omega}=\frac{E_{\mathrm{b}} \times I_{\mathrm{a}}}{2 \pi N / 60}=\frac{217.4 \times 70 \times 60}{2 \pi \times 1,200}=121.1 \mathrm{Nm}$

Now,

$$
E_{\mathrm{b}}=\frac{\phi Z N P}{60 \mathrm{~A}}
$$

Flux per pole, $\phi=\frac{60 A \times E_{\mathrm{b}}}{Z N \times P}=\frac{60 \times 4 \times 217.4}{868 \times 1,200 \times 4}=0.0125 \mathrm{~Wb}$

$$
=12.5 \mathrm{mWb}
$$

Power input, $P_{\text {in }}=V I_{\mathrm{L}}=230 \times 71.5=16,445 \mathrm{~W}$
Total losses $=P_{\text {in }}-P_{0}=16,445-12,503.5=3,941.5 \mathrm{~W}$
Total copper loss $=$ armature copper loss + shunt field copper loss

$$
\begin{aligned}
I_{\mathrm{a}}^{2} R_{\mathrm{a}}+V I_{\text {sh }} & =(70)^{2} \times 0.18+230 \times 1.5 \\
& =882+345=1,227 \mathrm{~W}
\end{aligned}
$$

Rotational loss $=$ total loss - copper loss

$$
=3,941.5-1,227=2,714.5 \mathrm{~W} \text { (neglecting iron loss) }
$$

Overall efficiency, $\eta=\frac{\text { Output }}{\text { Input }} \times 100=\frac{12,503.5}{16,445} \times 100=76.03 \%$.

## Example 11.32

A 250 V shunt motor draws a current of 10 A at no-load. The resistance of the armature including brushes is $0.2 \Omega$. The resistance of the field winding is $125 \Omega$. The current drawn by the motor on full-load is 100 A . Calculate the full-load motor output and its efficiency.

## Solution:

The conventional circuit is shown in Figure 11.62.
Terminal voltage, $V=250 \mathrm{~V}$
Armature resistance, $R_{\mathrm{a}}=0.2 \Omega$
Shunt field resistance, $R_{\mathrm{sh}}=125 \Omega$
No-load line current, $I_{\text {L } 0}=10 \mathrm{~A}$


Fig. 11.62 Conventional diagram as per data

Full-load line current, $I_{\mathrm{Lf}}=100 \mathrm{~A}$
Input at no-load $=V I_{\mathrm{L} 0}=250 \times 10=2,500 \mathrm{~W}$
Shunt field current, $I_{\text {sh }}=\frac{V}{R_{\text {sh }}}=\frac{250}{125}=2 \mathrm{~A}$
No-load armature current, $I_{\mathrm{a} 0}=I_{\mathrm{L} 0}-I_{\mathrm{sh}}=10-2=8 \mathrm{~A}$ Variable armature copper losses at no-load

$$
I_{\mathrm{a} 0}^{2} R_{\mathrm{a}}=(8)^{2} \times 0.2=12.8 \mathrm{~W}
$$

Constant losses $=2,500-12.8+2,487.2 \mathrm{~W}$
Full-load armature current,

$$
I_{\mathrm{af}}=I_{\mathrm{Lf}}-I_{\mathrm{sh}}=100-2=98 \mathrm{~A}
$$

Variable, armature copper losses at full-load

$$
=I_{\mathrm{af}}^{2} R_{\mathrm{a}}=(98)^{2} \times 0.2=1,920.8 \mathrm{~W}
$$

Total losses at full-load $=$ constant + variable losses

$$
=2,487 \cdot 2+1,920.8=4,408 \mathrm{~W}
$$

Input power at full-load $=V_{\mathrm{Lf}} \times I_{\mathrm{af}}=250 \times 100=25,000 \mathrm{~W}$
Output power at full-load $=$ input power - total losses $=25,000-4,408=20,592 \mathrm{~W}$
Efficiency, $\eta=\frac{\text { Output power }}{\text { Input power }} \times 100=\frac{20,592}{25,000} \times 100=82.37 \%$

## PRACTICE EXERCISES

## Short Answer Questions

1. How does a DC motor differ from a DC generator in construction?
2. Why is a commutator needed in DC motors?
(M.P. Univ. Dec. 2002)
3. Why the emf generated in the armature of a DC motor is called the back emf? (U.P.T.U. May 2004)
4. How can the direction of rotation of a DC shunt motor be reversed?
5. How can the direction of rotation of a DC series motor be changed?
6. Why the shaft torque is always less than electromagnetic torque developed by the armature of a motor?
7. Why is load not applied through belt to a series motor?
8. Why is the starting torque of a DC series motor more than that of a DC shunt motor of the same rating?
9. Why series motors are preferred for electric traction?
10. What would you expect if the field winding of a $D C$ shunt motor gets disconnected while in normal operation?
(A.M.I.E. Winter 1992)
11. Why should a DC series motor be not switched $O N$ without load?
(P.T.U. Dec. 2012)
12. What is difference between cumulative compound and differential compound-wound motors?
13. What is the field of application of DC shunt motor and DC series motor?
(P.T.U. Jan. 2000)
14. Why the DC motors are fitted with gear drives?
15. Give two applications of series motor and shunt motor each.
(P.T.U. Dec. 2002)
16. What would happen if a DC motor is directly switched ON to the supply, without any starter?
17. Why is the starting current high in a DC motor?
(UPSC 2002)
18. Why a starter is necessary for a DC motor?
(P.T.U. May 2011)

Or
Why starters are used for DC motors?
(P.T.U. May 2010)
19. What is the function of a no-volt release coil provided in a DC motor starter? (P.T.U. Dec. 2009)

## Test Questions

1. Explain the principle of operation of DC motors. What is back emf in DC motors? What is its significance?
(U.P.T.U. Dec. 2003)
2. Give the concept of counter emf in a DC motor.
(Pb. Univ. Dec. 1992)
3. What is significance of back emf in DC machines and derive the expression for it?
4. How is back emf produced in a DC motor? Further, derive an expression for this emf.
(R.T.U. 2004)
5. What is back emf? Is the back emf greater or lesser than the applied voltage? Why? By what amount do the two voltages differ? Write voltage equation of a motor.
(Pb. Univ. May 1988)
6. How can the direction of rotation of a DC shunt motor be reversed? What is the effect of reversing the line terminals?
(Pb. Univ. 1998)
7. Derive the torque equation of a DC machine.
(UPSC 1996)
8. Define an expression for the torque developed by a DC motor in terms of $\phi, Z, P, A$, and $L_{\mathrm{a}}$, where the symbols have usual meaning.
(Pb. Univ. 2003)
9. Explain the motor and generator actions of a DC machine.
(U.P.T.U. Sept. 2001)
10. Mention different types of DC motors.
(R.T.U. 2005)
11. Explain the speed-current, torque-current, and speed-torque characteristics of a DC shunt motor.
(UPSC 1994)
12. Describe the speed-torque characteristics of DC shunt motors under regenerative braking conditions. Give advantages and disadvantages of regenerative braking.
(P.T.U. Dec. 2009)
13. What are the important characteristics of DC motors? Sketch all these characteristics for DC series motors.
(P.T.U. June 2003)
14. Explain the speed-current, torque-current, and speed-torque characteristics of DC series motor.
(UPSC 1992)
15. Explain the speed-armature current characteristics of series and shunt motors.
(M.D.U. July 2001)
16. List applications of DC shunt, DC series, and DC compound motors. (M.P. Univ. June 2003)
17. Write a short note on DC motor starters.
(P.T.U. May 2010)
18. Explain why a starter is required for starting a DC motor. Describe a 3 -point starter having no-volt and overload protections for starting a DC shunt motor. What modification is made in a 4 -point starter?
(U.P.T.U. May 2002)

## Numericals

1. A DC series motor draws 50 A at 230 V . Resistance of armature and series field winding is $0.2 \Omega$ and $0.1 \Omega$, respectively. Calculate (i) brush voltage (ii) back emf, (iii) power wasted in armature, and (iv) mechanical power developed. (U.P.T.U. 2003-04) (Ans. $225 \mathrm{~V}, 215 \mathrm{~V}, 500 \mathrm{~W}, 10.75 \mathrm{~kW}$ )
2. A 230 V DC shunt motor draws a current of 36.86 A , its field resistance is $230 \Omega$, and armature resistance is $0.28 \Omega$. Find the input power, armature current, and back emf.
(P.T.U. Dec. 2000) (Ans. 8,477.8 W, 35.86 A, 220 V)
3. A 120 V DC shunt motor has an armature resistance of $0.2 \Omega$ and a field resistance of $60 \Omega$. At full-load, it draws a current is 60 A at a speed of $1,800 \mathrm{rpm}$. If the brush contact drop is 3 V , find the speed of the motor at half load.
(P.T.U. June 2000) (Ans. 1,902)
4. A $250 \mathrm{~V}, 25 \mathrm{~kW}$ DC shunt machine has armature and field resistances of $0.06 \Omega$ and $100 \Omega$, respectively. Determine the total armature power developed when the machine is working (i) as generator delivering 25 kW output and (ii) as a motor taking 25 kW input.
(U.P.T.U. 2003-04) (Ans. $26.26 \mathrm{~kW} ; 23.8 \mathrm{~kW}$ )

## SUMMARY

1. DC Generator: A machine that converts mechanical power into DC electrical power is called a DC generator. Its basic principle is electromagnetic induction.
2. Commutator action: It converts AC produced in the armature into DC in the external (load) circuit.
3. $E M F$ equation: $E_{\mathrm{g}}=\frac{P Z N \phi}{60 A} \mathrm{~V} ; A=P$ (for lap winding); $A=2$ (for wave winding)
4. Types of $D C$ generators and important relations:

Separately excited: $I_{\mathrm{a}}=I_{\mathrm{L}} ; V=E_{\mathrm{g}}+I_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$
Shunt-wound: $I_{\mathrm{sh}}=V / R_{\mathrm{sh}} ; I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}} ; V=E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$
Series-wound: $I_{\mathrm{a}}=I_{\mathrm{se}}=I_{\mathrm{L}} ; V=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 v_{\mathrm{b}}$
Compound (long shunt): $I_{\text {sh }}=V / R_{\text {shh }} ; I_{\mathrm{se}}=I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}} ; V=E_{\mathrm{g}}-I_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 u_{\mathrm{b}}$
Compound (short shunt): $I_{\text {sh }}=\left(V+I_{\mathrm{L}} R_{\mathrm{se}}\right) / R_{\mathrm{sh}}=\left(E_{\mathrm{g}}-I_{\mathrm{a}} R_{\mathrm{a}}\right) / R_{\mathrm{sh}} ; I_{\mathrm{a}}=I_{\mathrm{L}}+I_{\mathrm{sh}}$
$I_{\mathrm{se}}=I_{\mathrm{L}} ; V=E_{\mathrm{g}}-I_{\mathrm{se}} R_{\mathrm{se}}-I_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$
5. Voltage build-up in shunt generators: For building up voltage in shunt generators, they must have the residual magnetism and the value of shunt field resistance must be less than critical resistance.
6. DC Motor: A machine that converts DC electrical power into mechanical power is known as DC motor. Its working depends upon the basic principle that when a current carrying conductor is placed in the magnetic field, a force is exerted on it and torque develops.
7. Functions of commutator: Commutator reverses the direction of flow of current in the armature conductors when they cross the magnetic nutral axis (MNA) to obtain continuous torque.
8. Back emf: When DC supply is given to motor, armature rotates. The armature conductors cut across the main magnetic field and an emf is induced in them in opposite direction to that of supply voltage called back emf $\left(E_{\mathrm{b}}\right)$.

$$
E_{\mathrm{b}}=P Z N \phi / 60 A ; E_{\mathrm{b}} \propto N \phi \quad \text { or } \quad N \propto E_{\mathrm{b}} / \phi ; E_{\mathrm{b}}<V \quad \text { and } \quad E_{\mathrm{b}}=V-l_{\mathrm{a}} R_{\mathrm{a}}
$$

9. Torque equation: $T=P Z l_{\mathrm{a}} \phi / 2 \pi A$ or $T \propto \phi l_{\mathrm{a}}$
10. Types of $D C$ motors and important relations:

Separately excited: $l_{\mathrm{a}}=l_{\mathrm{L}} ; E_{\mathrm{b}}=V-l_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$
Shunt motors: $l_{\mathrm{sh}}=V / R_{\mathrm{sh}} ; l_{\mathrm{a}}=l_{\mathrm{L}}-l_{\mathrm{sh}} ; E_{\mathrm{b}}=V-l_{\mathrm{a}} R_{\mathrm{a}}-2 v_{\mathrm{b}}$
Series motors: $l_{\mathrm{se}}=l_{\mathrm{a}}=l_{\mathrm{L}} ; E_{\mathrm{b}}=V-l_{\mathrm{a}}\left(R_{\mathrm{a}}+R_{\mathrm{se}}\right)-2 v_{\mathrm{b}}$
Compound motors: Cumulative $\phi_{\mathrm{r}}=\phi_{\mathrm{sh}}+\phi_{\mathrm{sc}}$; differential $\phi_{\mathrm{r}}=\phi_{\mathrm{sh}}-\phi_{\mathrm{se}}$
11. Important characteristics: shunt motor - constant speed motor.

Series motor - it builds heavy torque at load and has the ability to pick it up. Therefore, it is best suited for electric traction. It is never operated at no-load since it picks up dangerously high speed at no-load and is damaged.
12. Speed control of DC motors: The speeds of DC motors can be controlled very accurately by employing field control and armature control methods.
13. Speed regulation $=\frac{N L \text { speed }-F L \text { speed }}{F L \text { speed }}$
14. Starter: A device used to limit the inrush flow of current at start is called a starter. It also contains the protective devices such as no-volt release coil and overload release coil.
15. Losses in a DC machine: The various losses in a DC machine are
(a) Copper loss: armature copper loss $\left(I_{\mathrm{a}}^{2} R_{\mathrm{a}}\right)$, shunt field copper loss $\left(I_{\mathrm{sh}}^{2} R_{\mathrm{sh}}\right)$, series field copper loss $\left(I_{\mathrm{se}}^{2} R_{\mathrm{se}}\right)$, interpole winding copper loss ( $I_{\mathrm{i}}^{2} R_{\mathrm{i}}$ ), etc.
(b) Iron loss: hysteresis loss and eddy current loss.
(c) Mechanical loss: friction and windage loss.

Constant losses $=$ iron loss + mechanical loss + shunt field copper loss
Variable losses = armature copper loss and series field copper loss.
Stray losses $=$ iron loss + mechanical loss
16. Efficiency:

$$
\begin{aligned}
& \eta=\frac{\text { Output }}{\text { Input }} \times 100 ; \text { for generator, } \eta=\frac{V I_{\mathrm{L}}}{V I_{\mathrm{L}}+I_{\mathrm{L}}^{2} R_{\mathrm{a}}+P_{\mathrm{c}}} \times 100 \\
& \text { for motor, } \eta=\frac{V I_{\mathrm{L}}+I_{\mathrm{L}}^{2} R_{\mathrm{a}}-P_{\mathrm{c}}}{V I_{\mathrm{L}}} \times 100
\end{aligned}
$$

17. Condition for maximum efficiency: variable losses $=$ constant losses
18. Swinberne's test: It is performed to determine constant losses of a DC machine, it is performed at no-load.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. Interpoles are used $\qquad$ .
2. DC Series motor is used for $\qquad$ .
3. DC Series generator is used for $\qquad$ -
4. The two types of windings used in DC machines are $\qquad$ and $\qquad$ .
5. DC Motor starter is used to $\qquad$ the starting current.
6. A 4-pole, DC lap winding will have $\qquad$ parallel paths.
7. If a DC motor is driven by an external prime mover, it will run as a $\qquad$ .
8. Wave winding of a 4 -pole DC motor will have $\qquad$ parallel paths.
9. The resistance of field winding of a shunt motor is $\qquad$ than the resistance of the armature winding.
10. There is $\qquad$ difference in the construction of a DC generator and motor.

## OBJECTIVE TYPE QUESTIONS

1. The induced emf in a DC machine is proportional to
(a) field flux only.
(b) speed of armature only.
(c) armature current only.
(d) field flux and speed both.
2. The electromagnetic torque developed in a DC machine depends upon
(a) armature current.
(b) magnetic field.
(c) magnetic field and armature current both.
(d) speed.
3. The induced emf (back emf) has no relation with electromagnetic torque (armature torque) in DC machine.
(a) True
(b) False
4. The yoke of a DC machine is made of
(a) copper.
(b) carbon.
(c) cast iron.
(d) silicon steel.
5. The armature core of a DC machine is made of
(a) copper.
(b) carbon.
(c) cast iron.
(d) silicon steel.
6. The segments of the commutator of a DC machine are made of
(a) brass.
(b) copper.
(c) carbon.
(d) silicon steel.
7. The segments of the commutator of a DC machine are insulated from each other by
(a) rubber.
(b) porcelain.
(c) mica.
(d) varnish.
8. The brushes of a DC machine are made of
(a) iron.
(b) brass.
(c) mica.
(d) carbon.
9. The number of parallel paths in wave-wound armature are
(a) equal to the number of poles of the machines.
(b) equal to two, irrespective of the number of poles.
(c) equal to the number of commutator segments.
(d) equal to the number of armature conductors.
10. The number of parallel paths in lap-wound armature are
(a) equal to the number of poles of the machine.
(b) equal to two, irrespective of the number of poles.
(c) equal to the number of commutator segments.
(d) equal to the number of armature conductors.
11. The polarity of induced emf in a generator can be reversed by reversing the direction of
(a) rotation.
(b) field flux.
(b) rotation and field flux both.
(d) either rotation or field flux.
12. The direction of electromagnetic torque developed in the armature of a DC machine can be reversed by reversing the direction of
(a) rotation.
(b) field flux.
(c) armature current.
(d) either field flux or armature current.
13. When the machine operates as a generator at load, the relation between induced emf and terminal voltage is
(a) $E_{\mathrm{g}}>V$
(b) $E_{\mathrm{g}}<V$
(c) $E_{\mathrm{b}}=V$
(d) $E_{\mathrm{b}} \times V=1$
14. A thicker wire is used in the DC series field winding than $D C$ shunt field winding in $D C$ machines
(a) to prevent mechanical vibrations.
(b) to produce large flux.
(c) because it carries the load current, which is much higher than shunt field current for the same rating of DC machines.
(d) to provide strength.
15. Which one of the following motors has high starting torque?
(a) DC shunt motor
(b) DC series motor
(c) Both shunt and series motor
(d) None of the above
16. As the load is increased, the speed of DC shunt motor will
(a) increase proportionately.
(b) remain almost constant.
(c) increase slightly.
(d) reduce slightly.
17. The speed of the DC shunt motor increases as the armature torque increases.
(a) True
(b) False
18. It is preferable to start a DC series motor with some mechanical load because
(a) it may develop excessive speed otherwise and get damaged.
(b) it will not run at no-load.
(c) a little load will act as a starter to the motor.
(d) None of the above
19. If the excitation to the field of DC motor is constant then the torque developed in the motor is proportional to
(a) field flux.
(b) speed.
(c) armature current.
(d) magnetic flux.
20. The DC series motors are preferred for traction applications because
(a) the torque is proportional to armature current.
(b) the torque is proportional to square root of armature current.
(c) the torque is proportional to square of armature current and the speed is inversely proportional to the torque.
(d) torque and speed are inversely proportional to armature current.
21. Which of the motor is used to drive the constant speed line shafting, lathes, blowers and fan?
(a) DC shunt motor
(b) DC series motor
(c) Cumulative compound motor
(d) None of the above
22. For which of the following machine, residual magnetism is a requirement to build-up voltage output?
(a) Separately excited generator
(b) Self-excited generator
(c) All types of generators
(d) None of the above
23. For DC shunt motors, the speed is dependent on induced emf only because
(a) flux is proportional to armature current.
(b) flux is practically constant in DC shunt motors.
(c) armature drop is negligible.
(d) induced emf is equal to armature drop.
24. What would happen if the field of DC shunt motor is opened?
(a) The speed of motor will be reduced.
(b) It will continue to run at its normal speed.
(c) The speed of the motor will be enormously high and may damage it.
(d) The current in the armature will decrease.
25. The speed of the DC motor can be varied
(a) by varying field current only.
(b) by varying armature resistance only.
(c) by varying supply voltage only.
(d) All the above
26. A diverter across the armature of DC motor cannot be used for getting speeds lower than the rated speed.
(a) True
(b) False
27. In series-parallel control method, when two DC series motors are connected in series, the speed of the set is
(a) same as in parallel.
(b) half of the speed of the motors when connected in parallel.
(c) one-fourth of the speed of motors when connected in parallel.
(d) four times of the speed of motors when connected in parallel.
28. The diverter in DC machine is basically potential divider.
(a) True
(b) False
29. In the DC machines, iron losses occur in
(a) the yoke.
(b) the pole shoe.
(c) the armature.
(d) the field.
30. The horse power obtained from the shaft torque is called
(a) Brake horse power (BHP).
(b) Indicated horse power (IHP).
(c) Fractional horse power (FHP).
(d) None of the above
31. In a DC shunt machine, the shunt field copper losses are practically constant.
(a) True
(b) False
32. The efficiency of DC machine is maximum when
(a) iron losses are equal to mechanical losses.
(b) variable losses are equal to constant losses.
(c) field copper losses are equal to constant losses.
(d) stray losses are equal to copper losses.

## NUMERICALS

1. An 8 -pole lap-connected armature of a DC machine has 960 conductors, a flux of 40 mWb per pole and a speed of 400 rpm Calculate the emf generated on open circuit. If the abovementioned armature were wave connected, at what speed must it be driven to generate 400 V ?
(A.M.I.E. D, W, 1989) (Ans. 256 V; 156.25 rpm )
2. In a given DC machine, if $P=8, Z=400, N=300 \mathrm{rpm}$, and $\phi=100 \mathrm{mWb}$, calculate $E_{\mathrm{g}}$ with winding (i) lap connected and (ii) wave connected. (B.U. Dec. 1981) (Ans. $200 \mathrm{~V}, 800 \mathrm{~V}$ )
3. A 110 V DC shunt generator delivers a load current of 50 A . The armature resistance is $0.2 \Omega$ and the field circuit resistance is $55 \Omega$. The generator rotating at a speed of $1,800 \mathrm{rpm}$ has 6 -pole lap-wound armature having 360 conductors. Calculate (i) the no-load voltage in the armature and (ii) the flux per pole.
(B.U. Feb. 1983) (Ans. $120.4 \mathrm{~V} ; 0.011 \mathrm{~Wb}$ )
4. A $30 \mathrm{~kW}, 300 \mathrm{~V}, \mathrm{DC}$ shunt generator has armature and field resistance of $0.05 \Omega$ and $100 \Omega$, respectively. Calculate the total power developed in the armature when it delivers full-load.
(A.M.I.E. D, S, 1975) (Ans. 31.43 kW )
5. A shunt generator has an induced emf of 127 V on open circuit. When the machine is on load, the terminal voltage is 120 V . Find the load current, if the field resistance is $15 \Omega$ and the armature resistance $0.02 \Omega$.
(B.U. May 1986) (Ans. 342 A)
6. A 4-pole shunt generator with lap-connected armature has armature and field resistances of 0.2 and $50 \Omega$, respectively, and supplies to 100 lamps of 60 W 200 V each. Calculate the total armature current, current per path, and the generated emf Allow a brush drop of 1 V at each brush.
(P.T.U.) (Ans. 34 A; $8.5 \mathrm{~A} ; 208.8 \mathrm{~V}$ )
7. A long shunt compound generator supplies a load current of 50 A at 220 V . Shunt field resistance is $110 \Omega$, series field resistance is $0.01 \Omega$, and armature resistance $0.02 \Omega$. Determine the emf generated and power developed in the armature. Take contact drop per brush as 1.5 V .(Ans. $224.56 \mathrm{~V} ; 11.677 \mathrm{~kW}$ )
8. A 4-pole lap-wound DC generator has a useful flux per pole of 0.07 Wb . The armature winding consists of 220 turns each of $0.004 \Omega$ resistance. Calculate the terminal voltage when running at 900 rpm if the armature current is 50 A .
$\left(\right.$ Ans. $\left.R_{\mathrm{a}}=0.055 \Omega ; 459.25 \mathrm{~V}\right)$
9. A 500 V shunt generator has a full-load current of 100 A and stray losses being 1.5 kW . Armature and field resistances are 0.3 and $250 \Omega$, respectively. Calculate the input power and efficiency.
(U.P.T.U.) (Ans. $55.621 \mathrm{~kW} ; 89.89 \%$ )
10. A shunt generator delivers full-load current of 200 A at 240 V . The shunt field resistance is $60 \Omega$ and full-load efficiency is $90 \%$. The stray losses are 800 W . Find (i) armature resistance and (ii) current at which maximum efficiency occurs.
(Ans. $0.858 \Omega ; 139.22 \mathrm{~A}$ )
11. A 50 H.P., $400 \mathrm{~V}, 4$-pole, $1,000 \mathrm{rpm}$, DC motor has flux per pole equal to 0.027 Wb . The armature having 1,600 conductors is wave connected. Calculate the gross torque when the motor takes 75 A .
(U.P.T.U.) (Ans. 1,031.32 Nm)
12. A 6 -pole, 440 V DC motor has 936 wave-wound armature conductors. The useful flux per pole is 25 mWb . The torque developed is 45.5 kgm . If armature resistance is $0.5 \Omega$, calculate (i) armature current and (ii) speed.
(Ans. $39.95 \mathrm{~A}, 359 \mathrm{rpm}$ )
13. A $25 \mathrm{~kW}, 250 \mathrm{~V}, \mathrm{DC}$ shunt generator has armature and field resistance of $0.06 \Omega$ and $100 \Omega$, respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.
(Ans. $26.255 \mathrm{~kW} ; 23.8 \mathrm{~kW}$ )
14. A 250 V DC shunt motor runs at $1,000 \mathrm{rpm}$ and takes 6 A at no-load. The armature resistance of the motor is $0.04 \Omega$ and shunt field resistance is $250 \Omega$. Calculate the change in speed when the motor is loaded and takes 51 A from the lines. Neglect armature reaction.
(Ans. 7.2 rpm )
15. A 250 V DC shunt motor runs at $1,000 \mathrm{rpm}$ and on no-load takes 5 A . Armature and field resistances are $0.2 \Omega$ and $250 \Omega$, respectively. Calculate the speed when loaded and taking current of 50 A . Assume that the flux gets weakened by $3 \%$ due to armature reaction.
(Ans. 993.7 rpm )
16. A DC shunt machine connected to 250 V mains, has an armature resistance of $0.12 \Omega$ and field resistance of $100 \Omega$. Calculate the ratio of speed as a generator to motor when the line current in each case is 180 A .
(P.T.U.) (Ans. 1.0797)
17. A series motor with series field and armature resistance of 0.06 and $0.04 \Omega$, respectively, is connected across 200 V . The armature takes 40 A and speed is $1,000 \mathrm{rpm}$. Determine its speed when the armature takes 75 A and excitation is increased by $10 \%$.
(Ans. 892.86 rpm$)$
18. A series motor with series field and armature resistance of 0.04 and $0.06 \Omega$, respectively, is connected across 250 V . The armature takes 50 A and speed is $1,000 \mathrm{rpm}$. Determine the speed when the armaure takes 70 A . Assume that the magnetic circuit is unsaturated. (U.P.T.U.) (Ans. 708.45 rpm )
19. A series motor having a resistance of $1 \Omega$ between terminals drives a fan for which the torque varies as the square of the speed. When connected to 200 V DC supply, it runs at 250 rpm and takes 20 A . The speed is to be raised to 400 rpm by increasing the voltage. Calculate the voltage and current assuming flux directly proportional to current.
(Ans. $32 \mathrm{~A} ; 492.8 \mathrm{~V}$ )
20. A 230 V , DC motor takes a no-load current of 2 A and runs at a speed of $1,200 \mathrm{rpm}$ If the full-load current is 40 A , find (i) speed on full-load (ii) percentage speed regulation. Assume that the flux remains constant and armature resistance is $0.25 \Omega$.
(Ans. 1,150.3 rpm; 4.32\%)
21. A DC shunt motor runs at $1,000 \mathrm{rpm}$ on 200 V supply. Its armature resistance is $0.8 \Omega$ and the current taken is 40 A , in addition to the field current. What resistance do you connect in series with the armature to reduce the speed to 600 rpm the current in the armature remaining the same? Neglect armature reaction.
(Ans. $1.68 \Omega$ )
22. A 4-pole DC shunt motor takes 22.5 A from a 250 V supply, $R_{\mathrm{a}}=0.5 \Omega$ and $R_{\mathrm{f}}=125 \Omega$. The armature is wave wound with 300 conductors. If the flux per pole is 0.02 Wb . Calculate (i) the speed, (ii) torque developed, and (iii) power developed.
(Ans. 1,198.75 rpm; $39.15 \mathrm{Nm} ; 4.915 \mathrm{~kW}$ )
23. A 200 V DC series motor takes 60 A . Armature resistance $0.08 \Omega$ and series field resistance $0.05 \Omega$. If iron and friction losses are twice to copper losses at this load, find the BHP and efficiency.
(Ans. 14.4 H.P. 88.3\%)
24. An 8-pole DC generator has 500 armature conductors and a useful flux of 0.05 Wb . What will be the emf generated, if it is lap connected and runs at $1,200 \mathrm{rpm}$ ? What must be the speed at which it is to be driven to produce the same emf, if it is wave wound?
(U.P.T.U. Dec. 2001) (Ans. 500 V, 300 rpm )
25. A 4-pole DC shunt generator with lap-connected armature has field and armature resistances of $80 \Omega$ and $0.1 \Omega$, respectively. It supplies power to 50 lamps rated for $100 \mathrm{~V}, 60 \mathrm{~W}$ each. Calculate the total armature current and the generated emf by allowing a contact drop of 1 V per brush.
(U.P.T.U. June-2004)
26. An 8 -pole DC shunt generator has 778 wave-connected armature conductors running at 500 rpm , supplies a load of $12.5 \Omega$ resistance at a terminal voltage of 250 V . The armature resistance is $0.24 \Omega$ and the field resistance is $250 \Omega$. Find out the armature current, the induced emf, and the flux per pole.
(Agra Univ. 1982; Bombay Univ. 1988)

## VIVA VOCE QUESTIONS

1. Armature core of a DC machine is always laminated. Why?
2. Can we use wood for the construction of a yoke in a DC machine? Why?
3. No residual flux and no building up of voltage in DC shunt generators. Why?
4. Shunt field resistance should not be more than critical resistance. Why?
5. In the case of DC motors, the induced emf in armature is also called back emf. Why?
6. DC Series motors are never started at no-load. Why?
7. Belt loads are never recommended for DC series motors. Why?
8. For punching, DC compound motors are best suited. Why?
9. For electric traction, DC series motors are best suited. Why?
10. For speed control of DC motors, flux control method is preferred over armature control method. Why?
11. In electric traction, voltage control method of speed control of DC series motors is employed. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What is a generator?
2. What are the basic requirements of a DC generator to generate electricity?
3. What is the working principle of a DC generator?
4. What is armature of a DC machine?
5. What are the functions of armature core?
6. What is the function of field system?
7. What are the main functions of yoke?
8. How will you make yoke for small and large machines?
9. What are the main functions of pole shoes?
10. What are field coils?
11. What is the function of collecting brushes?
12. Give the emf equation of $D C$ generator.
13. What do you understand by separately excited generators?
14. What are self-excited generators?
15. What are the advantages of carbon brushes?
16. What are the disadvantages of carbon brushes?
17. Why armature of DC machines is made up of silicon steel?
18. Why is the armature winding placed on the rotor of DC machine?
19. What are the advantages of a separately excited generator over a self-excited Generator?
20. What are the disadvantages of separately excited generator?
21. What are the factors that may affect the voltage build-up of a DC shunt generator?
22. Why is series generator not generally used?
23. How can you improve the design of a DC machine?
24. Name the losses that occur in a DC generator.
25. What is a DC motor?
26. Can we use the same DC machine as a generator and as a motor?
27. What is the working principle of a DC motor?
28. What is back emf?
29. What are the difference types of DC motor?
30. What is the condition for developing maximum mechanical power in a DC motor?
31. What is the important advantage of a DC series motor?
32. What are the causes of sparking at the brushes?
33. What are the causes of overheating of a DC motor?

## TEST QUESTIONS

1. Explain generator and motor action of a DC machine.
2. What is an electromechanical energy conversion device? How mechanical power is converted into electrical power in these devices? Explain.
3. Name the various parts of a DC machine and give the function of each part.
4. Explain the principle of action of a DC generator. Describe briefly its important parts.
5. What is the principle commercially adopted to transform mechanical energy to electrical energy? Illustrate in the case of a simple DC generator. How can the generator be made to supply DC?
6. Explain how commutator works in a DC machine to generate DC voltage.
7. Derive emf equation of a DC generator (or DC machine).
(P.T.U., U.P.T.U.)
8. Explain how can you distinguish a lap and wave winding. How can you recognise the winding of a DC machine by counting its brushes?
9. What are the different types of excitation employed for DC generators?
10. Explain the principle of action of a DC shunt generator.
11. Define O.C.C. (no-load characteristics) of a DC generator. Explain how it is obtained for a given generator.
12. Explain how a shunt generator builds up. What limits the voltage to which a generator can build up?
13. What is critical field resistance in a DC generator?
14. Mention the different reasons for the drop in the terminal voltage of a shunt generator when it is loaded.
15. Under what circumstances, does a DC shunt generator fails to build up?
16. What are the various energy losses in a DC machine and how do they vary with load?
17. Derive a condition for maximum efficiency of a DC generator.
18. Explain the principle of operation of a DC motor.
19. Explain with suitable diagram the working of a DC motor.
20. Explain the function of commutator in a DC motor.
21. What is back emf? Explain.
(U.P.T.U., P.T.U.)
22. On what factors, does the torque developed by a DC motor depends? Or
Derive an expression for torque development in a DC motor.
(U.P.T.U. 2002)
23. Mention the various types of DC motors and their uses.
(U.P.T.U.)
24. With the help of speed-armature current characteristics, show that a shunt motor runs at almost constant speed irrespective of the load.
25. Sketch the speed-torque curve of a DC series motor and discuss its nature. What are the applications for DC series motors?
(U.P.T.U.)
26. A DC series motor should not be started without load. Why?
27. Show that a series motor develops high starting torque.
28. Using characteristics, explain why a DC series motor is suitable for electric traction and should never be started without a load on it.
(P.T.U.)
29. Sketch the speed-load and torque-load characteristics of DC cumulative compound motor and comment on the shape of the characteristics. Indicate where such a motor can be ideally used.
30. Following are some of the applications that need suitable motors. Motors available are series, shunt, cumulative compound, and differential compound DC motors. Mention the motor used for the following applications. Give reasons for your answer. Blower, shears, diesel electric locomotives, cranes, hoists centrifugal pumps, elevators, and rolling mills.
31. Write a general expression for the speed of a DC motor in terms of supply voltage and flux per pole.
32. Describe the speed control methods of DC shunt motors.
33. Describe briefly the methods of speed control used for DC series motors.
34. Why is a starter necessary for a DC motor?
35. Describe with neat sketch the working of a starter for a DC shunt motor.
36. Explain briefly the functions of (i) no-volt release and (ii) overload release in a 3-point starter used to start a DC shunt motor.

## ANSWERS

## Fill in the Blanks

1. reduce sparking at the brushes
2. electric traction, where high starting torque is required
3. boosting up the supply voltage 4. lap and wave
4. limit or restrict
5. four
6. generator
7. two
8. more
9. No

## Objective Type Questions

| 1. (d) | 2. (c) | 3. (b) | 4. (c) | 5. (d) |
| ---: | ---: | ---: | ---: | ---: |
| 6. (b) | 7. (c) | 8. (d) | 9. (b) | 10. (a) |
| 11. (d) | 12. (d) | 13. (a) | 14. (c) | 15. (b) |
| 16. (d) | 17. (b) | 18. (a) | 19. (c) | 20. (c) |
| 21. (a) | 22. (b) | 23. (b) | 24. (c) | 25. (d) |
| 26. (b) | 27. (c) | 28. (b) | 29. (c) | 30. (a) |
| 31. (a) | 32. (b) |  |  |  |



## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:
*What are the various parts of an induction motor?
. How a revolving field is developed in the stator of a three-phase induction motor?
What is the working principle of an induction motor?

* Why three-phase induction motor is called an asynchronous motor?
*What are the various losses in an induction motor?
*Why starter is needed to start a three-phase induction motor?
* What is a push button (DOL) and start-delta starter?
* What are the major applications of three-phase induction motors?


### 12.1 INTRODUCTION

Induction machines are also called asynchronous machines, that is, the machines that never run at a synchronous speed. Whenever we say induction machine we mean to say induction motor. Induction motors may be either single phase or three phase. The single-phase induction motors are usually built in small sizes (up to 3 H.P.). Three-phase induction motors are the most commonly used AC motors in the industry, because they have simple and rugged construction, low cost, high efficiency, reasonably good power factor, self-starting torque, and low maintenance. Nearly, more than 90 per cent of the mechanical power used in industry is provided by threephase induction motors.

In this chapter, all the important aspects of a three-phase induction motor are discussed.

### 12.2 CONSTRUCTIONAL FEATURES OF A THREE-PHASE INDUCTION MOTOR

A three-phase induction motor consists of two main parts, namely stator and rotor.

1. Stator: It is the stationary part of the motor. It has three main parts, namely outer frame, stator core, and stator winding.
(a) Outer frame: It is the outer body of the motor. Its function is to support the stator core and to protect the inner parts of the machine. For small machines, the fame is casted, but for large machines, it is fabricated.

To place the motor on the foundation, feet are provided in the outer frame as shown in Figure 12.1.
(b) Stator core: The stator core is to carry the alternating magnetic field which produces hysteresis and eddy current losses; therefore, core is built up of high grade silicon steel stamping. The stampings are assembled under hydraulic pressure and are keyed to the frame. Each stamping is insulated from the other with a thin varnish layer. The thickness to the stamping usually varies from 0.3 to 0.5 mm . Slots are punched on the inner periphery of the stampings, as shown in Figure 12.2, to accommodate stator winding.


Fig. 12.1 Stator of 3-phase induction motor


Fig. 12.2 Stator stamping
(c) Stator winding: The stator core carries a three-phase winding which is usually supplied from a three-phase supply system. The six terminals of the winding (two of each phase) are connected in the terminal box of the machine. The stator of the motor is wound for definite number of poles, the exact number being determined by the requirement of speed. It will be observed that greater the number of poles, the lower is the speed and vice-versa, since $N_{\mathrm{s}} \propto \frac{1}{P}\left(Q N_{\mathrm{s}}=\frac{120 f}{P}\right)$. The three-phase winding may be connected in star or delta externally through a starter.
2. Rotor: It is the rotating part of the motor. There are two types of rotors, which are employed in threephase induction motors, namely squirrel-cage rotor and phase-wound rotor.
(a) Squirrel-cage rotor: The motors employing this type of rotor are known as 'squirrel-cage induction motors'. Most of the induction motors are of this type because of simple and rugged construction of rotor. A squirrel-cage rotor consists of a laminated cylindrical core


Fig. 12.3 Squirrel cage rotor having semi-closed circular slots at the outer periphery. Copper or aluminium bar conductors are placed in these slots and short circuited at each end by copper or aluminium rings, called short-circuiting rings, as shown in Figure 12.3. Thus, the rotor winding is permanently short circuited, and it is not possible to add any external resistance in the rotor circuit.

The rotor slots are usually not parallel to the shaft but are skewed. Skewing of rotor has the following advantages:
(i) It reduces humming, thus ensuring quiet running of a motor.
(ii) It results in a smoother torque curves for different positions of the rotor.
(iii) It reduces the magnetic locking of the stator and rotor.
(iv) It increases the rotor resistance due to the increased length of the rotor bar conductors.
(b) Phase-wound rotor: Phase-wound rotor is also called slip-ring rotor and the motors employing this type of rotor are known as 'phase-wound or slip-ring induction motors'. Slip-ring rotor consists of a laminated cylindrical core having semi-closed slots at the outer periphery and carries a three-phase insulated winding. The rotor is wound for the same number of poles as that of stator. The three finish terminals are connected together forming star point, and the three start terminals are connected to three copper slip-rings fixed on the shaft (Figure


Fig. 12.4 Phase-wound rotor 12.4).

In this case, depending upon the requirement, any external resistance can be added in the rotor circuit. In this case also, the rotor is skewed.

A mild steel shaft is passed through the centre of the rotor and is fixed to it with key. The purpose of shaft is to transfer mechanical power.

### 12.3 PRODUCTION OF REVOLVING FIELD

Consider a stator on which three different windings represented by three concentric coils $a_{1} a_{2}$, $b_{1} b_{2}$, and $c_{1} c_{2}$, respectively, are placed $120^{\circ}$ electrically apart.

Let a three-phase supply, as shown in Figure 12.5, is applied to the stator. Three-phase currents will flow through the three coils and produce their own magnetic fields. The positive half


Fig. 12.5 Wave diagram of 3-phase AC supply with instants $\mathrm{t}_{1} \mathrm{t}_{2}$ and $\mathrm{t}_{3}$
cycle of the alternating current (AC) is considered as inward flow of current in the start terminals and negative half cycle is considered as outward flow of current in the start terminals. The direction of flow of current is opposite in the finish terminals of the same coil.

Let at any instant $t_{1}$, current in coil side $a_{1}$ be inward and in $b_{1}$ and $c_{1}$ outward, whereas the current in the other sides of the same coils is opposite, that is, in coil side $a_{2}$ is outward and $b_{2}$ and $c_{2}$ is inward. The resultant field and its direction $\left(F_{\mathrm{m}}\right)$ are marked in Figure 12.6.

At instant $t_{2}$, when $\theta$ is $60^{\circ}$, current in coil sides $a_{1}$ and $b_{1}$ is inward and in $c_{1}$ is outward, whereas the current in the opposite sides is opposite. The resultant field and its direction is shown in Figure 12.7, which is rotated through an angle $\theta=60^{\circ}$ from its previous position.
At instant $t_{3}$ when $\theta$ is $120^{\circ}$, current in coil side $b_{1}$ is inward and in $c_{1}$ and $a_{1}$ is outward. The resultant field and its direction is shown in Figure 12.8, which is rotated through an angle $\theta=120^{\circ}$ electrical from its first position.


Fig. 12.6 Position of resultant field at instant $t_{1}$

Fig. 12.7 Position of resultant field at instant $t_{2}$

Fig. 12.8 Position of resultant field at instant $t_{3}$

Thus, in one cycle, the resultant field completes one revolution. Hence, we conclude that when three-phase supply is given to a three-phase wound stator, a resultant field is produced which revolves at a constant speed, called synchronous speed ( $\left.N_{\mathrm{s}}=120^{\circ} f / P\right)$.

In this case, we have observed that when supply from phase 1,2 , and 3 is given to coil $a_{1} a_{2}$, $b_{1} b_{2}$, and $c_{1} c_{2}$, respectively, an anticlockwise rotating field is produced. If the supply to coil $a_{1} a_{2}, b_{1} b_{2}$, and $c_{1} c_{2}$ is given from phase 1,3 , and 2 , respectively, the direction of rotating field is reversed. Therefore, to reverse the direction of rotation of rotating field, the connections of any two supply terminals are inter changed.

### 12.4 PRINCIPLE OF OPERATION

When three-phase supply is given to the stator of a three-phase wound induction motor, a revolving field is set up in the stator. At any instant, the magnetic field set-up by the stator is shown in Figure 12.9.

The direction of resultant field is marked by an arrow head $F_{\mathrm{m}}$. Let this field is rotating in an anticlockwise direction at an angular speed of $\omega_{\mathrm{s}}$ radians per second, that is, synchronous speed.


Fig. 12.9 (a) Induced emf/current in rotor conductors at an instant (b) Phasor representation of stator and rotor field at an instant

The stationary rotor conductors cut the revolving field and due to electromagnetic induction an emf is induced in the rotor conductors. As the rotor conductors are short circuited, current flows through them in the direction as marked in the figure. Rotor current carrying conductors set up a resultant field $F_{\mathrm{r}}$. This field tries to come in line with the stator main field $F_{\mathrm{m}}$. Due to this, an electromagnetic torque $T_{\mathrm{e}}$ is developed in the anticlockwise direction. Therefore, rotor starts rotating in same direction in which stator field is revolving.

### 12.4.1 Alternate Explanation

Reproducing section X of Figure 12.9(a) as shown in Figure 12.10, when the revolving stator field (refer Figure 12.10(a)) cuts the stationary rotor conductors, an emf is induced in the conductors by induction. As rotor conductors are short circuited, current flows through them, as marked in Figure 12.10(b) which sets up field around them. A resultant field is set up, as shown in Figure 12.10(c) which exerts force on the rotor conductors. Therefore, the rotor starts rotating in the same direction in which stator field is revolving.


Fig. 12.10 (a) Field produced by stator winding at an instant (b) Field produced around rotor conductors at that instant (c) Resultant field around rotor conductors

The rotor picks up speed and tries to attain the synchronous speed but fails to do so. It is because if the rotor attains the synchronous speed, then the relative speed between revolving stator field and rotor will be zero, no emf will be induced in rotor conductors. No emf means no current, no rotor field $F_{r}$, and hence, no torque is produced. Therefore, an induction motors never runs at synchronous speed. It always seems at a speed less than synchronous speed.

Since the principle of operation of this motor depends upon electromagnetic induction, hence the name induction motor.

### 12.5 REVERSAL OF DIRECTION OF ROTATION OF THREE-PHASE INDUCTION MOTORS

In Figure 12.2, it has been observed that a revolving field is set up in the stator of a three-phase induction motor, when three-phase supply is given to its winding and the direction of rotation depends upon the supply sequence.

In Figure 12.3, it has been observed that rotor of a three-phase induction motor rotates in the same direction as that of the revolving field.

The direction of rotation of the revolving field or that of the rotor can be reversed if the sequence of supply is reversed. The supply sequence can be reversed by interchanging the connections of any two supply leads at the stator terminals.

Hence, the direction of rotation of a three-phase induction motor can be reversed by interchanging the connections of any two supply leads at the stator terminals.

### 12.6 SLIP

The rotor of an induction motor always rotates at a speed less than synchronous speed. The difference between the flux speed $(N s)$ and the rotor speed $(N)$ is called slip. It is usually expressed as a percentage of synchronous speed $(N s)$ and is represented by symbol $S$.

Mathematically,
\% slip,

$$
\% S=\frac{N_{\mathrm{s}}-N}{N_{\mathrm{s}}} \times 100
$$

or fractional slip,

$$
S=\frac{N_{\mathrm{s}}-N}{N_{\mathrm{s}}}
$$

Rotor speed,

$$
N=N_{\mathrm{s}}(l-S)
$$

The difference between synchronous speed and rotor speed is called slip speed, that is,

$$
\text { Slip speed }=N_{\mathrm{s}}-N
$$

The value of slip at full-load varies from about 6 per cent small motors to about 2 per cent for large motors.

### 12.6.1 Importance of Slip

Slip plays an important role in the operation of an induction motor. We have already seen that the difference between the rotor speed and synchronous speed of flux determine the rate at which the flux is cut by rotor conductors and hence the magnitude of induced emf, that is, $e_{2} \propto N_{\mathrm{s}}-N$

Rotor current,

$$
\begin{gathered}
i_{2} \propto e_{2} \text { and torque, } T \propto i_{2} \\
T=K\left(N_{\mathrm{s}}-N\right) \\
T=K N_{\mathrm{s}}\left(\frac{N_{\mathrm{s}}-N}{N_{\mathrm{s}}}\right)
\end{gathered}
$$

or

$$
T=K_{1} S
$$

Hence

$$
T \propto S
$$

Thus, greater the slip greater will be the induced emf or rotor current, and hence, larger will be the torque developed.

At no-load, induction motor requires small torque to meet with the mechanical, iron, and other losses, and therefore, slip is small. When the motor is loaded, greater torque is required to drive the load, and therefore, the slip increases and rotor speed decreases slightly.

Therefore, it is observed that slip in an induction motor adjusts itself to such a value to meet the required driving torque under normal operation.

### 12.7 FREQUENCY OF ROTOR CURRENTS

The frequency of rotor currents depends upon the relative speed between rotor and stator field. When the rotor is stationary, the frequency of rotor currents is the same as that of the supply frequency. But once the rotor starts to rotate, the frequency of rotor currents depends upon slip speed $\left(N_{\mathrm{s}}-N\right)$. Let at any speed $N$, the frequency of rotor currents be $f_{\mathrm{r}}$.

Then,

$$
f_{\mathrm{r}}=\frac{\left(N_{\mathrm{S}}-N\right) \times P}{120}=\frac{\left(N_{\mathrm{S}}-N\right)}{N_{\mathrm{S}}} \times \frac{N_{\mathrm{S}} \times P}{120}=S \times f
$$

## Example 12.1

A three-phase, four-pole, 50 Hz , induction motor runs at 1460 rpm . Determine its percentage slip.
(U.P.T.U. June 2004)

## Solution:

Synchronous speed,

$$
N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rp}
$$

Speed of motor,

$$
N=1460 \mathrm{rpm}
$$

Slip,

$$
S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}} \times 100=\frac{1500-1460}{1500} \times 100
$$

$$
=2.667 \%
$$

## Example 12.2

In a three-phase slip-ring, four-pole induction motor, the rotor frequency is found to be 2.0 Hz , while connected to a 400 V , three-phase, 50 Hz supply. Determine motor speed in rpm.

## Solution:

Synchronous speed, $\quad N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$

Slip,

$$
S=\frac{\text { Frequency of rotor emf }}{\text { Supply frequency }}=\frac{f_{\mathrm{r}}}{f}=\frac{2}{50}=0.04
$$

Speed of motor on load, $N=N_{\mathrm{S}}(1-S)=1500(1-0.04)=1440 \mathrm{rpm}$

## Example 12.3

A three-phase, four-pole induction motor is supplied from three-phase, 50 Hz AC supply. Calculate (i) synchronous speed, (ii) rotor speed when slip is $4 \%$, and (iii) rotor frequency when rotor runs at 600 rpm .
(U.P.T.U. 2005-06)

## Solution:

Here, $P=4 ; f=50 \mathrm{~Hz} ; S=4 \%=0.04$
(i) Synchronous speed, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$
(ii) Rotor speed, $N=N_{\mathrm{S}} \times(1-S)=1500 \times(1-0.04)=1440 \mathrm{rpm}$
(iii) When rotor speed is 600 rpm

Slip,

$$
S=\frac{N_{s}-N}{N_{s}}=\frac{1500-600}{1500}=0.6
$$

Rotor frequency, $\quad f_{\mathrm{r}}=S \times f=0.6 \times 50=30 \mathrm{~Hz}$

## Example 12.4

A three-phase slip-ring, four-pole induction motor has rotor frequency 2.0 Hz while connected to 400 V , three-phase, 50 Hz supply. Determine slip and rotor speed.
(U.P.T.U. 2006-07)

## Solution:

No. of poles, $P=4$
Supply frequency, $f=50 \mathrm{~Hz}$
Rotor frequency, $f_{\mathrm{r}}=2 \mathrm{~Hz}$
Now,

$$
f_{\mathrm{r}}=S \times f \quad \therefore \quad \text { Slip }=\frac{f_{\mathrm{r}}}{f}=\frac{2}{50}=0.04
$$

Synchronous speed,

$$
N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}
$$

Rotor speed,

$$
N=N_{\mathrm{S}} \times(1-S)=1500 \times(1-0.04)=1440 \mathrm{rpm}
$$

## Example 12.5

A three-phase, four-pole induction motor operates from a supply whose frequency is 50 Hz . Calculate its synchronous speed, speed of rotor when slip is 0.04 and frequency of rotor currents at standstill.

## Solution:

Synchronous speed, $\quad N_{\mathrm{S}}=\frac{120 \times f}{p}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$
Speed of rotor when the slip is 0.04

$$
N=(1-S) \times N_{\mathrm{s}}=(1-0.04) \times 1500=1440 \mathrm{rpm}
$$

Frequency of rotor currents at standstill
At standstill $=S=1$ and $N=0$
$\therefore \quad$ Frequency of rotor current $f_{\mathrm{r}}=S \times f=1 \times 50=50 \mathrm{~Hz}$

## Example 12.6

A 12-pole, three-phase alternator driven at a speed of 500 rpm supplies power to an eightpole, three-phase induction motor. If the slip of the motor is 0.03 pu , then calculate the speed.
(U.P.T.U. July 2002)

## Solution:

No. of poles of the alternator, $P_{\mathrm{a}}=12$
Speed of alternator, $N_{\mathrm{a}}=500 \mathrm{rpm}$
No. of poles of the induction motor, $P_{\mathrm{m}}=8$; slip $S=0.03 \mathrm{pu}$
Supply frequency delivered by the alternator, $f=\frac{P_{\mathrm{a}} \times N_{\mathrm{a}}}{120}=\frac{12 \times 500}{120}=50 \mathrm{~Hz}$
Synchronous speed of three-phase induction motor, $N_{\mathrm{S}}=\frac{120 \times f}{P_{\mathrm{m}}}=\frac{120 \times 50}{8}=750 \mathrm{rpm}$
Speed of three-phase induction motor, $N=N_{\mathrm{S}} \times(1-S)=750 \times(1-0.03)=727.5 \mathrm{rpm}$

## Example 12.7

A 12-pole, three-phase alternator is coupled to an engine running at 500 rpm . It supplied a threephase induction motor having a full-load speed of 1440 rpm . Find the percentage slip, frequency of rotor current and number of poles of the motor.
(U.P.T.U. 2005-06)

## Solution:

No. of poles of three-phase alternator, $P_{\mathrm{a}}=12$
Speed of engine, $N_{\text {S(a) }}=500 \mathrm{rpm}$
Frequency of generated voltage,

$$
f=\frac{N_{\mathrm{S}(\mathrm{a})} \times P_{\mathrm{a}}}{120}=\frac{500 \times 12}{120}=50 \mathrm{~Hz}
$$

For three-phase induction motor,
Synchronous speed,

$$
N_{\mathrm{S}}=\frac{120 \times f}{P_{\mathrm{m}}}
$$

$N_{\mathrm{S}}$ in nearly equal to $N$

$$
\therefore \quad P_{\mathrm{m}}=\frac{120 \times f}{N}=\frac{120 \times 50}{1440} \cong 4.16
$$

But poles are always even in number

$$
\therefore \quad P_{\mathrm{m}}=4
$$

$\therefore$ Synchronous speed,

$$
\begin{gathered}
N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm} \\
S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}}=\frac{1500-1440}{1500}=0.04
\end{gathered}
$$

Frequency of rotor current, $f_{\mathrm{r}}=S \times f=0.04 \times 50=2 \mathrm{~Hz}$

## Example 12.8

A motor-generator set used for providing variable frequency AC supply consists of a threephase, 10 -pole synchronous motor and a 24-pole, three-phase synchronous generator. The motorgenerator set is fed from a 25 Hz , three-phase AC supply. A six-pole, three-phase induction motor is electrically connected to the terminals of the synchronous generator and runs at a slip of $5 \%$. Determine (i) the frequency of the generated voltage of the synchronous generator, (ii) the speed at which the induction motor is running.
(U.P.T.U. Feb. 2001)

## Solution:

Given, No. of poles of synchronous motor, $P_{\mathrm{sm}}=10$
No. of poles of synchronous generator, $P_{\text {sg }}=24$
No. of poles of induction motor, $P_{\mathrm{im}}=6$
Supply frequency, $f=25 \mathrm{~Hz}$
Slip of induction motor, $S=5 \%=0.05$
Speed of synchronous motor, $N_{\mathrm{Sm}}=\frac{120 \times f}{P_{\mathrm{Sm}}}=\frac{120 \times 25}{10}=300 \mathrm{rpm}$
Frequency of emf generated by synchronous generator,

$$
f_{\mathrm{g}}=\frac{P_{\mathrm{sg}} \times N_{\mathrm{sm}}}{120}=\frac{24 \times 300}{120}=60 \mathrm{~Hz}
$$

Synchronous speed of the induction motor (revolution field),

$$
N_{\mathrm{s}}=\frac{120 \times f_{\mathrm{g}}}{P_{\mathrm{im}}}=\frac{120 \times 60}{6}=1200 \mathrm{rpm}
$$

Running speed of the induction motor,

$$
N=N_{\mathrm{s}} \times(1-S)=1200 \times(1-0.05)=1140 \mathrm{rpm}
$$

## 国聂 <br> PRACTICE EXERCISES

## Short Answer Questions

1. Define three-phase induction motor.
2. How will you classify three-phase induction motors?
3. What do you understand by revolving field?
4. What is the working principle of three-phase induction motor?
5. How can you reverse the direction of rotation of a three-phase induction motor?
6. What do you mean by slip in an induction motor?
7. What do you mean by rotor frequency?

## Test Questions

1. Name the various parts of a squirrel-cage induction motor and explain the construction of stator with neat diagram.
2. Why stator core of a three-phase induction motor is laminated?
3. Explain with neat sketch, how revolving field is developed in the stator of three-phase induction motor?
4. Explain the working principle of a three-phase induction motor with the help of a neat sketch.
5. What is slip? Show that torque is proportional to slip under running condition.

## Numericals

1. If a three-phase, four-pole induction motor is supplied from a three-phase, 50 Hz AC supply, determine the following:
(i) its synchronous speed,
(ii) rotor speed when slip is $3 \%$
(iii) rotor frequency when rotor runs at 1000 rpm .
(Ans. $1500 \mathrm{rpm}, 1455 \mathrm{rpm}, 1.5 \mathrm{~Hz}$ )
2. A six-pole, three-phase induction motor operates from a supply of frequency 50 Hz . Calculate its synchronous speed, speed of rotor at a slip of $4 \%$ and frequency of rotor currents at standstill and running condition.
(Ans. $1000 \mathrm{rpm}, 960 \mathrm{rpm}, 50 \mathrm{~Hz}, 2 \mathrm{~Hz}$ )
3. An eight-pole, three-phase alternator is coupled to an engine running at 900 rpm supplies power to a four-pole, three-phase induction motor. If the slip of the motor is 0.04 pu , determine its rotor speed and rotor current frequency at running condition.
(Ans. $1728 \mathrm{rpm}, 2.4 \mathrm{~Hz}$ )

### 12.8 SPEED OF ROTOR FIELD OR MMF

When three-phase currents are supplied to the stator winding of a polyphase induction motor, a resultant field is set up, which rotates at a constant speed called synchronous speed $\left(N_{\mathrm{s}}=120 \times f / P\right)$.

This rotating field induces polyphase emfs in the rotor winding, and if rotor winding is closed, polyphase currents circulate in it. These currents set up a revolving field in the rotor that rotates at a speed $N_{\mathrm{r}}=120 \times f_{\mathrm{r}} / P$ with respect to rotor.

Now,

$$
N_{\mathrm{r}}=120 \times S \times f / P=S N_{\mathrm{s}}
$$

When rotor itself is rotating at a speed $N \mathrm{rpm}$ in the space,
$\therefore \quad$ Speed of rotor field in space $=N+N_{\mathrm{r}}=(1-S) \times N_{\mathrm{s}}+S \times N_{\mathrm{s}}=N_{\mathrm{s}}-S \times N_{\mathrm{s}}+S \times N_{\mathrm{s}}=N_{\mathrm{s}}$

Therefore, rotor magnetic field also rotates in space at the same speed and in the same direction as that of stator field. Hence, the two fields are magnetically locked with each other and are stationary with respect to each other.

### 12.9 ROTOR EMF

The revolving magnetic field set up in the stator by polyphase currents is common to both stator and rotor winding. This field induces emfs in both windings. The stator-induced emf per phase is given by the relation

$$
\begin{equation*}
E_{1}=4.44 \times k w_{1} \times T_{1} \times f \times \phi_{\mathrm{m}} \tag{12.1}
\end{equation*}
$$

Where $k w_{1}=$ winding factor, that is, product or coil span factor $k_{\mathrm{c}}$ and distribution factor $k_{\mathrm{d}}$.
$T_{1}=$ No. of turns/phase of stator winding
$f=$ stator or supply frequency and
$\phi_{\mathrm{m}}=$ maximum value of flux.
The rotor-induced emf/phase, $E_{2}=4.44 \times k w_{2} \times T_{2} \times f_{\mathrm{r}} \times \phi_{\mathrm{m}}$
Where $f_{\mathrm{r}}$ is the rotor current frequency, and under stationary condition, that is, at the start $f_{\mathrm{r}}=f$.
Therefore, rotor-induced emf/phase at standstill or start, $E_{2 \mathrm{~s}}=4.44 \times k w_{2} \times T_{2} \times f \times \phi_{\mathrm{m}}$
Dividing equation (12.2) by (12.1), we get,

$$
\frac{E_{2 \mathrm{~s}}}{E_{1}}=\frac{4.44 \times k w_{2} \times T_{2} \times f \times \phi_{\mathrm{m}}}{4.44 \times k w_{1} \times T_{1} \times f \times \phi_{\mathrm{m}}} \propto \frac{T_{2}}{T_{1}}=K \quad \text { (i.e.,transformation ratio) }
$$

From equation (12.2), induced emf in the rotor under running condition,

$$
E_{2}=4.44 \times k w_{2} \times T_{2} \times(S \times f) \times \phi_{\mathrm{m}}=S \times E_{2 \mathrm{~s}}
$$

The induced emf in the rotor circuit is maximum at the start and varies according to the value of slip under running condition. Since the value of normal slip under loaded condition is nearly 5 per cent, the rotor-induced emf is, therefore, nearly 5 per cent of the maximum value.

### 12.10 ROTOR RESISTANCE

Since the rotor winding is made of some conducting material (copper or aluminium), it has a definite resistance ( $R=\rho \times l / a$ ). Its value remains constant and is denoted by $R_{2}$.

## 12.Il ROTOR REACTANCE



Fig. 12.11 Leakage flux

Total flux produced by the rotor currents does not link with the stator winding. The part of rotor flux that links the rotor conductors but not with the stator winding is called leakages flux and hence develops leakage inductance $\left(L_{2}\right)$. The leakage flux and hence the inductance is very small, if the rotor conductors are placed at the outermost periphery of the rotor as shown in Figure 12.11. Depending upon the rotor current frequency, rotor reactance will be developed.

Rotor reactance,

$$
X_{2}=2 \times \pi \times f_{\mathrm{r}} \times L_{2}=2 \times \pi \times S \times f \times L_{2}=S \times\left(2 \times \pi \times f \times L_{2}\right)
$$

When the rotor is standstill, that is, at the start, when slip, $S=1$
The value of rotor reactance $=X_{2 \mathrm{~s}}=2 \times \pi \times f \times L_{2}$
Therefore, under normal running, rotor reactance, $X_{2}=S \times X_{2 \mathrm{~s}}$

### 12.12 ROTOR IMPEDANCE

The total opposition offered to the flow of rotor current by the rotor circuit is called the rotor impedance.

Rotor impedance,

$$
\bar{Z}_{2}=R_{2}+j X_{2}=R_{2}+j S X_{2 \mathrm{~s}}
$$

Magnitude of rotor impedance,

$$
Z_{2}=\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}
$$

### 12.13 ROTOR CURRENT AND POWER FACTOR

The rotor circuit diagram of an induction motor is shown in Figure 12.12.
Under running condition
Rotor-induced emf $=E_{2}=S \times E_{2 \mathrm{~s}}$
Rotor impedance,

$$
\begin{aligned}
Z_{2} & =\sqrt{R_{2}^{2}+R_{2}^{2}} \\
& =\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}
\end{aligned}
$$

Rotor current,

$$
\begin{aligned}
I_{2} & =\frac{E_{2}}{Z_{2}}=\frac{E_{2}}{\sqrt{\left(R_{2}\right)^{2}+\left(X_{2}\right)^{2}}} \\
& =\frac{S E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}
\end{aligned}
$$

Rotor power factor,

$$
\cos \phi_{2}=\frac{R_{2}}{Z_{2}}=\frac{R_{2}}{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}
$$

### 12.14 SIMPLIFIED EQUIVALENT CIRCUIT OF ROTOR

The various parameters and electrical quantities are represented on the circuit diagram, as shown in Figure 12.13. The rotor current is given by the following expression:

$$
I_{2}=\frac{S E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}
$$



Fig. 12.13 Rotor circuit


Fig. 12.14 Equivalent rotor circuit

The other expression for the rotor current is

$$
I_{2}=\frac{E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}} \quad \begin{gathered}
\text { (dividing the numerator and } \\
\text { denominator by } S \text { ) }
\end{gathered}
$$

This expression gives a convenient form of equivalent circuit as shown in Figure 12.14.

The resistance is a function of slip and can be split into two parts; $\frac{R_{2}}{S}=R_{2}+R_{2}\left(\frac{1-S}{S}\right)$
where $R_{2}\left(\frac{1-S}{S}\right)$ represents electrical load on the rotor.
Therefore, the final simplified equivalent rotor circuit is shown in Figure 12.15(a). Where $R_{2}$ is rotor resistance and $X_{2 \mathrm{~s}}$ is standstill leakage reactance. The resistance $R_{2}\left(\frac{1-S}{S}\right)$ is fictitious
resistance representing load. resistance representing load.

The power consumed by this fictitious resistance, that is, $I_{2}{ }^{2} R_{2}\left(\frac{1-S}{S}\right)$ is the electrical power, which is converted into mechanical power to pick the load. After subtracting the mechanical losses, we get the output power available at the shaft.

Therefore, electrical power converted into mechanical power $=I_{2}^{2} R_{2}\left(\frac{1-S}{S}\right) \mathrm{W}$
From the simplified equivalent circuit, the phasor diagram of rotor circuit is drawn as shown in Figure 12.15(b).

Rotor current $I_{2}$ lags behind the rotor standstill induced emf $E_{2 \mathrm{~s}}$ by an angle $\phi$.
The voltage drop across $R_{2}$, that is, $I_{2} R_{2}$ and across $R_{2}\left(\frac{1-S}{S}\right)$, that is, $I_{2} R_{2}\left(\frac{1-S}{S}\right)$ are in phase with current $I_{2}$, whereas the voltage drop in $X_{25}$, that is, $I_{2} X_{2 \mathrm{~s}}$ leads the current $I_{2}$ by $90^{\circ}$.

The vector sum of all the three drops is equal to $E_{2 s}$, that is,

$$
E_{2 \mathrm{~s}}=I_{2} \sqrt{\left(R_{2} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}
$$

Power factor of rotor circuit,

$$
\cos \phi=\frac{R_{2} / S}{\sqrt{\left(R_{2} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}
$$



Fig. 12.15 (a) Simplified rotor circuit (b) Phasor diagram for rotor circuit

## Example 12.9

A three-phase, $440 \mathrm{~V}, 50 \mathrm{H} . \mathrm{P} ., 50 \mathrm{~Hz}$ induction motor runs at 1450 rpm , when it delivers rated output power. Determine: (i) number of poles in the machine, (ii) speed of rotating air gap field, (iii) rotor-induced voltage if stator to rotor turns ratio is 1:0.80. Assume the winding factors are the same, and (iv) frequency of rotor current.
(U.P.T.U. 2004-05)

## Solution:

(i) No. of poles, $P=\frac{120 \times f}{N}=\frac{120 \times 50}{1450}=4$ pole
(ii) Speed of rotating air gap field, $N_{\mathrm{s}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$
(iii) $\frac{T_{\mathrm{s}(\mathrm{ph})}}{T_{\mathrm{r}(\mathrm{ph})}}=\frac{E_{\mathrm{s}(\mathrm{ph})}}{E_{\mathrm{r}(\mathrm{ph})}}=\frac{1}{0.8}$

$$
E_{\mathrm{r}(\mathrm{ph})}=0.8 \times E_{\mathrm{s}(\mathrm{ph})}=0.8 \times 440=342 \mathrm{~V}
$$

(iv) Frequency of rotor currents, $f_{\mathrm{r}}=S \times f$

Where

$$
S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}}=\frac{1500-1450}{1500}=0.0333
$$

$$
\therefore \quad f_{\mathrm{r}}=0.0333 \times 50=1.665 \mathrm{~Hz}
$$

## Example 12.10

A three-phase, 50 Hz induction motor has six poles and operates with a slip of $5 \%$ at a certain load. Determine (i) the speed of the rotor with respect to the stator, (ii) the frequency of rotor current, (iii) the speed of the rotor magnetic field with respect to rotor, (iv) the speed of the rotor magnetic field with respect to stator, and (v) the speed of the rotor magnetic field with respect to the stator magnetic field.
(U.P.T.U. Feb. 2002)

## Solution:

Supply frequency, $f=50 \mathrm{~Hz}$
Number of poles, $P=6$
Slip,

$$
S=5 \%=0.05
$$

Synchronous speed, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{P}=1000 \mathrm{rpm}$
(i) Speed of rotor with respect to the stator, $N=N_{\mathrm{S}} \times(1-S)=1000 \times(1-0.05)=950 \mathrm{rpm}$
(ii) Frequency of rotor current, $f_{\mathrm{r}}=S \times f=0.05 \times 50=2.5 \mathrm{~Hz}$
(iii) Speed of rotor magnetic field with respect to the rotor

$$
\begin{aligned}
N_{\mathrm{r}} & =\frac{120 \times f_{\mathrm{r}}}{P}=\frac{120 \times 2.5}{6} \\
& =50 \mathrm{rpm}
\end{aligned}
$$

(iv) Speed of rotor magnetic field with respect to the stator $=N+N_{\mathrm{r}}=950+50=1000 \mathrm{rpm}$
(v) Rotor field and stator field are revolving at the same speed of 1000 rpm , and therefore, speed of rotor field with respect to stator field is zero.

## Example 12.11

A three-phase, 50 Hz induction motor has a full-load speed of 960 rpm . Calculate (i) slip, (ii) number of poles, (iii) frequency of the rotor-induced emf, (iv) speed of the rotor field with respect to rotor structure, (v) speed of rotor field with respect to stator structure, and (vi) speed of rotor field with respect to the stator field.
(U.P.T.U. Tut)

## Solution:

Supply frequency, $f=50 \mathrm{~Hz}$
Full-load running speed, $N=960 \mathrm{rpm}$
All the machines have even number of poles such as $2,4,6,8, \ldots$
When, $P=2, N_{\mathrm{S}}=3000 \mathrm{rpm} ; P=4, N_{\mathrm{S}}=1500 \mathrm{rpm} ; P=6, N_{\mathrm{S}}=1000 \mathrm{rpm}$

$$
P=8, N_{\mathrm{S}}=750 \mathrm{rpm} .
$$

The nearest synchronous speed more than 960 rpm is 1000 rpm , and therefore, $P=6$

$$
N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{6}=1000 \mathrm{rpm}
$$

(i) Slip, $\quad S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}} \times 100=\frac{1000-960}{1000} \times 100$

$$
=4 \%=0.04
$$

(iii) Frequency of rotor-induced emf, $f_{\mathrm{r}}=S \times f=0.04 \times 50=2 \mathrm{~Hz}$
(iv) Speed of the rotor field with respect to rotor structure

$$
N_{\mathrm{r}}=\frac{120 \times f_{\mathrm{r}}}{P}=\frac{120 \times 2}{6}=40 \mathrm{rpm}
$$

(v) Speed of the rotor field with respect to stator structure

$$
\begin{aligned}
& =\text { Speed of rotor }+ \text { speed of rotor field with respect to rotor structure } \\
& =N+N_{\mathrm{r}}=960+40=1000 \mathrm{rpm}
\end{aligned}
$$

(vi) Rotor field and stator field are revolving at the same of 1000 rpm , and therefore, speed of rotor field with respect to stator field is zero.

## Example 12.12

A balanced, three-phase, $50 \mathrm{c} / \mathrm{s}$ voltage is applied to a three-phase, four-pole induction motor. When the motor delivers rated output, the slip is found to be 0.05 . Determine (i) the speed of the revolving field relative to the stator structure, (ii) the frequency of the rotor currents, (iii) the speed of the rotor mmf relative to the rotor structure, (iv) the speed of the rotor mmf relative to the stator structure, (v) the speed of the rotor mmf relative to the stator field distribution, and (vi) are the conditions right for the development of the net unidirectional torque?
(P.T.U.)

## Solution:

Here, $P=4, f=50 \mathrm{~Hz}, S=0.05$
(i) The speed of the revolving field relative to the stator structure

$$
\text { that is, } N_{\mathrm{s}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}
$$

(ii) $f_{\mathrm{r}}=S \times f=0.05 \times 50=2.5 \mathrm{~Hz}$
(iii) The speed of rotor mmf relative to the rotor structure,

$$
N_{\mathrm{r}}=\frac{120 \times f_{\mathrm{r}}}{P}=\frac{120 \times 2.5}{4}=75 \mathrm{rpm}
$$

(iv) Rotor speed, $N=N_{\mathrm{s}} \times(1-S)=1500 \times(1-0.05)=1425 \mathrm{rpm}$

The speed of the rotor mmf relative to the stator structure

$$
=N+N_{\mathrm{r}}=1425+75=1500 \mathrm{rpm}
$$

(v) The speed of the rotor mmf relative to the stator field distribution

$$
=N_{\mathrm{s}}-\left(N+N_{\mathrm{r}}\right)=1500-1500=0
$$

(vi) Yes. The given conditions completely satisfy the development of net unidirectional torque.

## Example 12.13

A three-phase induction motor has a rotor for which the resistance per phase is $0.1 \Omega$ and reactance per phase when stationary is $0.4 \Omega$. The rotor-induced emf per phase is 100 V when stationary. Calculate rotor current and rotor p.f. (i) when rotor is stationary and (ii) when running with a slip of $5 \%$.
(P.T.U.)

## Solution:

Here, $R_{2}=0.1 \Omega ; X_{2 \mathrm{~s}}=0.4 \Omega ; E_{2 \mathrm{~s}}=100 \mathrm{~V}$
(i) When the rotor is stationary

Rotor current, $I_{2 \mathrm{~s}}=\frac{E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}=\frac{100}{\sqrt{(0.1)^{2}+(0.4)^{2}}}=242.5 \mathrm{~A}$
Rotor power factors, $\cos \phi_{2 \mathrm{~s}}=\frac{R_{2}}{\sqrt{\left(R_{2}\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}=\frac{0.1}{\sqrt{(0.1)^{2}+(0.4)^{2}}}=0.2425 \mathrm{lag}$
(ii) When rotor is running with a slip of $5 \%$, that is, $S=0.05$

Rotor current, $I_{2}=\frac{S E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}=\frac{0.04 \times 100}{\sqrt{(0.1)^{2}+(0.05 \times 0.4)^{2}}}=49 \mathrm{~A}$
Rotor power factor, $\cos \phi_{2}=\frac{R_{2}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}=\frac{0.1}{\sqrt{(0.1)^{2}+(0.05 \times 0.4)^{2}}}=0.98 \mathrm{lag}$

### 12.15 STATOR PARAMETERS

Similar to rotor, the stator winding of the motor also has resistance $R_{1}$. The flux produced by stator winding linking with its own turns only (leakage flux) produces leakage reactance $X_{1}$.

Of the total voltage $V$ applied to the stator, a part of it is consumed by stator resistance $\left(I_{1} R_{1}\right)$ and leakage reactance $\left(I_{1} X_{1}\right)$ and the remaining is utilized in establishing mutual flux that links with both stator and rotor winding. When it links with the stator winding, it produces selfinduced emf $E_{1}$.

$$
\bar{E}_{1}=\bar{V}_{1}-\bar{I}_{1} \bar{R}_{1}-\bar{I}_{1} \bar{X}_{1}
$$

### 12.16 INDUCTION MOTOR ON NO-LOAD (ROTOR CIRCUIT OPEN)

In slip-ring, induction motor rotor circuit can be opened. Under this condition, when stator is connected to three-phase supply, it draws a very small current called no-load current $I_{0}$. This current has two components, that is, working component $I_{\mathrm{w}}$ and magnetizing component $I_{\text {mag }}$. Working component is in phase with the supply voltage, and it supplies the stator iron losses. Whereas magnetizing component lags behind the supply voltage $V$ by $90^{\circ}$ and produces the mutual flux that links with stator and rotor winding and induces $E_{1}$ and $E_{25}$.

The equivalent circuit and phasor diagram of the motor under this condition is shown in Figure 12.16.

(a)

(b)

Fig. 12.16 (a) Equivalent circuit of an induction motor with open circuited rotor (b) Phasor diagram of an induction motor with open circuited rotor

### 12.17 INDUCTION MOTOR ON LOAD

When load is applied on the induction motor, its speed decreases slightly and slip increases. Therefore, rotor current $I_{2}$ increases. Simultaneously, to meet with this load, motor draws extra current from the supply mains similar to that of a transformer. In fact, power is transferred through magnetic field or flux.

The complete circuit diagram and phasor diagram of a loaded induction motor is shown in Figure 12.17(a) and (b), respectively.

Here, $X_{0}-$ exciting reactance $=\frac{V}{I_{\text {mag }}}$


Fig. 12.17 (a) Equivalent circuit of an induction motor (b) Phasor diagram of an induction motor
$R_{0}-$ exciting resistance $=\frac{V}{I_{\mathrm{w}}}$
All other abbreviations have their usual meaning.

### 12.17.1 Causes of Low-power Factor

The basic principle of operation of an induction motor is mutual induction. When three-phase supply is given to a three-phase wound stator of an induction motor, a revolving field is set up in the stator. This field (flux) is also set up in the air between stator and rotor, which links with rotor conductors and emf is induced in them by mutual induction.

To set up the mutual flux, induction motor draws magnetizing current ( $I_{\text {mag }}$ ) from the mains which lags behind the voltage by $90^{\circ}$ as shown in Figure 12.16 (b) and $12.17(\mathrm{~b})$. The magnitude of this current is quite large because of high reluctance of the air gap between stator and rotor.

The power factor of the induction motor is minimum at no-load as shown in Figure 12.16(b) since the magnetizing current has its dominating effect. However, the power factor increases with increase in load on the induction motor and is maximum at full-load as shown in Figure 12.17(b). Therefore, it is advised to operate the induction at full-load.

Therefore, because of air gap, induction motor draws large magnetizing current and operates at low lagging power factor.

### 12.18 LOSSES IN AN INDUCTION MOTOR

The major losses in an induction motor are as follows:

1. Stator losses: The losses that occur in the stator of an induction motor are called stator losses.
(a) Stator copper losses: $I_{1}^{2} R_{2}$ (per phase)
(b) Stator iron losses: These are the hysteresis and eddy current losses.
2. Rotor losses: The losses that occur in the rotor of an induction motor are called rotor losses.
(a) Rotor copper losses: $I_{2}^{2} R_{2}$ (per phase)
(b) Rotor iron losses: Since under normal running condition, rotor frequency is very small. These losses are so small that they are neglected.
3. Mechanical losses: The sum of windage and friction losses is called mechanical losses.

### 12.19 POWER FLOW DIAGRAM

Electrical power input is given to the stator. There are stator copper and iron losses and the remaining power, that is, stator output is transferred to the rotor through magnetic flux called rotor input. In the rotor, there are rotor copper losses, and the remaining power is converted into mechanical power called mechanical power developed in the rotor.

Then, there are mechanical losses, and the remaining power is available at the shaft called mechanical power output.

The power flow diagram is shown in Figure 12.18.


Fig. 12.18 Power flow diagram

### 12.20 RELATION BETWEEN ROTOR COPPER LOSS, SLIP, AND ROTOR INPUT

We have seen that the electrical power developed in the rotor is converted into mechanical power which is given by the following relation:

Mechanical power developed in the rotor $=I_{2}^{2} R_{2}\left(\frac{I-S}{S}\right)$
The rotor copper losses $=I_{2}^{2} R_{2}$

From power flow diagram,
Rotor input $=$ Mechanical power developed + rotor copper losses

$$
\begin{equation*}
=I_{2}^{2}\left(\frac{I-S}{S}\right)+I_{2}^{2} R_{2}=\left(\frac{I_{2}^{2}}{S}\right) \tag{12.5}
\end{equation*}
$$

From eq. (12.3) and (12.4), we get, $\frac{\text { Rotor copper loss }}{\text { Mechanical power developed }}=\frac{I_{2}^{2} R_{2}}{I_{2}{ }^{2} R_{2}\left(\frac{1-S}{S}\right)}$
$\therefore$ Rotor copper loss $=\left(\frac{S}{1-S}\right)$ Mechanical power developed
From eq. (12.4) and (12.5), we get, $\frac{\text { Rotor copper loss }}{\text { Rotor input }}=\frac{I_{2}^{2} R_{2}}{I_{2}^{2} R_{2} / S}$
$\therefore$ Rotor copper loss $=S \times$ Rotor input
Note: All the values are the phase values.

### 12.21 ROTOR EFFICIENCY

The ratio of rotor output (i.e., mechanical power developed in rotor neglecting mechanical losses) to the rotor input is called the rotor efficiency.

$$
\text { Rotor efficiency }=\frac{\text { Mechanical Power developed }}{\text { Rotor input }}=\frac{I_{2}^{2} R_{2}\left(\frac{1-S}{S}\right)}{I_{2}^{2} R / S}=(1-S)
$$

## Example 12.14

The power input to a three-phase induction motor is 80 kW . The stator losses in total 1.5 kW . Find the total mechanical power developed if the motor is running with a slip of $4 \%$.

## Solution:

Stator output or rotor input $=$ Stator input - stator losses $=80-1.5=78.5 \mathrm{~kW}$
Rotor copper losses $=\mathrm{S} \times$ Rotor input $=0.04 \times 78.5=3.14 \mathrm{~kW}$
Mechanical power developed $=$ Rotor input - Rotor copper losses

$$
=78.5-3.14=75.36 \mathrm{~kW}
$$

## Example 12.15

A 10 H.P., four-pole, 25 Hz , three-phase, wound rotor induction motor is taking 9100 W from the line. Core loss is 290 W , stator copper loss is 568 W , rotor copper loss in 445 W , friction and windage losses are 100 W . Determine (i) power transferred across air gap, (ii) mechanical power in watt developed by rotor, (iii) mechanical power output in watt, (iv) efficiency, and (v) slip.
(P.T.U.)

## Solution:

Power input to motor or stator $=9100 \mathrm{~W}$
Power transferred across air gap $=$ Stator input - Stator core loss - Stator copper loss

$$
=9100-290-568=8242 \mathrm{~W}
$$

Mechanical power developed in rotor $=$ rotor input - Rotor copper loss $=8242-445=7797$
Rotor output $=$ Mechanical power developed - Mechanical loss

$$
=7797-100=7697 \mathrm{~W}
$$

$$
\text { Motor efficiency }=\frac{\text { Output }}{\text { Input }} \times 100=\frac{7697}{9100} \times 100=84.58 \%
$$

Slip,

$$
S=\frac{\text { Rotor copper loss }}{\text { Rotor input }}=\frac{445}{8242}=0.05399
$$

## Example 12.16

The power input to the rotor of a $440 \mathrm{~V}, 50 \mathrm{~Hz}$, three-phase, six-pole induction motor is 50 kW . The rotor emf makes 120 cycles per minutes. Friction and windage losses are 2 kW . Calculate (i) slip, (ii) rotor speed, (iii) rotor copper losses, (iv) mechanical power developed, (v) output power, and (vi) output torque.

## Solution:

Here, $V_{\mathrm{L}}=440 \mathrm{~V} ; P=6 ; f=50 \mathrm{~Hz}$
Power input to rotor or rotor input $=50 \mathrm{~kW}$
Friction and windage loss $=2 \mathrm{~kW}$
(i) Synchronous speed, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{6}=1000 \mathrm{rpm}$

Frequency of rotor emf, $f_{\mathrm{r}}=\frac{120}{60}=2 \mathrm{~Hz}$
Now,

$$
f_{\mathrm{r}}=S \times f
$$

Slip,

$$
S=\frac{f_{\mathrm{r}}}{f}=\frac{2}{50}=0.04
$$

(ii) Rotor speed, $N=N_{\mathrm{S}} \times(1-S)$

$$
=1000 \times(1-0.04)=960 \mathrm{rpm}
$$

(iii) Rotor copper loss $=S \times$ Rotor input

$$
=0.04 \times 50=2 \mathrm{~kW}
$$

(iv) Mechanical power developed $=$ Rotor input - rotor copper loss

$$
=50-2=48 \mathrm{~kW}
$$

(v) Output power $=$ Mechanical power developed - friction and windage loss

$$
=48-2=46 \mathrm{~kW}
$$

(vi) Output torque $=\frac{\text { Power output }}{\omega}=\frac{46000}{2 \pi N / 60}=\frac{46000 \times 60}{2 \pi \times 960}$

$$
=457.57 \mathrm{Nm}
$$

## Example 12.17

A four-pole, three- $\phi$ induction motor runs at 1440 rpm . Supply voltage is 500 V at 50 Hz . Mechanical power output is 20.3 H.P. and mechanical loss is 2.23 H.P. Calculate (i) mechanical power developed, (ii) rotor copper loss, and (iii) efficiency.
(U.P.T.U. 2007-08)

## Solution:

Here, $P=4 ; N=1440 \mathrm{rpm} ; V_{\mathrm{L}}=500 \mathrm{~V} ; f=50 \mathrm{~Hz}$
Mechanical power output $=20.3$ H.P. $=20.3 \times 735.5=14931 \mathrm{~W}$
Mechanical loss $=2.23$ H.P. $=2.23 \times 735.5=1640 \mathrm{~W}$
(i) Mechanical power developed $=$ Mechanical power output + mechanical loss

$$
=14931+1640=16571 \mathrm{~W}
$$

(ii) Synchronous speed, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$

Slip,

$$
S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}}=\frac{1500-1440}{1500}=0.04
$$

Rotor copper loss $\quad=\frac{S}{1-S} \times$ mechanical power developed

$$
=\frac{0.04}{1-0.04} \times 16571=690 \mathrm{~W}
$$

(iii) Rotor input $=$ mechanical power developed + Rotor copper loss

$$
=16571+690=17261 \mathrm{~W}
$$

Since stator losses are not given, considering them to be equal to rotor copper loss, that is,

$$
\text { Stator losses }=\text { rotor copper loss }=690 \mathrm{~W}
$$

Power input to the motor $=$ Rotor input + stator loss

$$
=17261+690=17951 \mathrm{~W}
$$

Efficiency,

$$
\begin{aligned}
\eta & =\frac{\text { Power output }}{\text { Power input }} \times 100 \\
& =\frac{14931}{17951} \times 100=83.17 \%
\end{aligned}
$$

## Example 12.18

A six-pole, three-phase induction motor develops 30 H.P. including 2 H.P. mechanical losses at a speed of 950 rpm on 550 V 50 Hz mains. The power factor is 0.88 lagging. Calculate (i) slip, (ii) rotor copper loss, (iii) total input if stator losses are 2 kW , (iv) efficiency, and (v) line current.
(U.P.T.U. June 2001)

## Solution:

Data given, $P=6 ; N=950 \mathrm{rpm} ; V_{\mathrm{L}}=550 \mathrm{~V} ; f=50 \mathrm{~Hz}$
Mechanical power developed $=30$ H.P.; mechanical loss $=2$ H.P.
Stator loss $=2 \mathrm{~kW}$
Synchronous speed, $\quad N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1000 \mathrm{rpm}$
(i) Slip, $S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}}=\frac{1000-950}{1000}=0.05$
(ii) Rotor copper loss $=\frac{S}{1-S} \times$ mechanical power developed

$$
=\frac{0.05}{1-0.05} \times 30 \times 735.5=\frac{0.05}{0.95} \times 22065=1161 \mathrm{~W}
$$

(iii) Total input $\quad$ Mechanical power developed + Rotor copper loss + stator loss

$$
=22065+1161+2000=25226 \mathrm{~W}
$$

Output $=$ Mechanical power developed - Mechanical loss
$=30-2=28$ H.P. $=28 \times 735.5=20594 \mathrm{~W}$
(iv) Efficiency, $\quad \eta=\frac{\text { Output }}{\text { Input }}=\frac{20594}{25226}=0.8164=81.64 \%$
(v) Line current, $I_{\mathrm{L}}=\frac{\text { Input }}{\sqrt{3} \times V_{\mathrm{L}} \times \cos \phi}=\frac{25226}{\sqrt{3} \times 550 \times 0.88}=30.1 \mathrm{~A}$

## Example 12.19

A 400 V , six-pole, 50 Hz , three-phase induction motor develops 20 kW inclusive of mechanical losses when running at 980 rpm and the power factor being 0.85 . Calculate (i) slip, (ii) rotor current frequency, (iii) total input if the stator loss is 1500 W , and (iv) line current.
(U.P.T.U. Tut)

## Solution:

Solution:
Synchronous speed of motor, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{6}=1000 \mathrm{rpm}$
(i) Slip, $S=\frac{N_{\mathrm{S}}-N}{N_{\mathrm{S}}}=\frac{1000-980}{1000}=0.02=2 \%$
(ii) Rotor current frequency, $f_{\mathrm{r}}=S \times f=0.02 \times 50=1 \mathrm{~Hz}$

Rotor output $=20 \mathrm{~kW}$
Power input to rotor $=\frac{\text { Rotor output }}{1-S}=\frac{20}{1-0.02}=20.408 \mathrm{~kW}$
(iii) Power input to stator $=$ Rotor input + stator losses

$$
=20.408+1.5=21.908 \mathrm{~kW}
$$

(iv) Line current supplied to motor $I_{\mathrm{L}}=\frac{\text { Stator input in } \mathrm{kW} \times 1000}{\sqrt{3} V_{\mathrm{L}} \cos \phi}$

$$
=\frac{21.908 \times 1000}{\sqrt{3} \times 400 \times 0.85}=37.2 \mathrm{~A}
$$

## Example 12.20

A 400 V , six-pole, 50 Hz , three-phase induction motor develops $20 \mathrm{H} . P$. inclusive of mechanical losses when running at 965 rpm , the power factor being 0.87 lagging. Calculate (i) the slip,
(ii) rotor copper losses, (iii) the total input if the stator losses are 1500 W , (iv) line current, and (v) the number of cycles made per minute by the rotor emf.

## Solution:

Here,

$$
V_{\mathrm{L}}=400 \mathrm{~V} ; P=6 ; f=50 \mathrm{~Hz} ; N=965 \mathrm{rpm}
$$

$$
\cos \phi=0 \times 4 \text { lag; stator copper loss }=1500 \mathrm{~W}
$$

Synchronous speed, $\quad N=\frac{120 \times f}{P}=\frac{1220 \times 50}{6}=1000 \mathrm{rpm}$.
(i) Slip, $S=\frac{N_{\mathrm{s}}-N}{N_{\mathrm{s}}} \times 100=\frac{1000-965}{1000} \times 100=3.5 \%$

Mechanical power developed $=20$ H.P. $=20 \times 735.5=14710 \mathrm{~W}$
(ii) Rotor copper losses $=\left(\frac{S}{1-S}\right)$ mechanical power developed

$$
=\frac{0.035}{1-0.035} \times 14710=533 \times 5 \mathrm{~W}
$$

(iii) Input to stator $=$ mechanical power developed + rotor copper loss + stator copper loss

$$
=14710+533.5+1500=16743.5 \mathrm{~W}
$$

(iv) Line current, $I_{\mathrm{L}}=\frac{\text { Input }}{\sqrt{3} V_{\mathrm{L}} \cos \phi}=\frac{167443.5}{\sqrt{3} \times 400 \times 0.87}=27.8 \mathrm{~A}$

Rotor frequency, $f_{\mathrm{r}}=S \times f=0.035 \times 50=1.75 \mathrm{~Hz}$ or c/s
(v) No. of cycles made per minute by rotor emf $=1.75 \times 60=105$ cycle $/ \mathrm{min}$.

## Example 12.21

A four-pole, three-phase, 50 Hz induction motor supplies a useful torque of 159 Newton metre. Calculate at $4 \%$ slip: (i) the rotor input, (ii) motor input, (iii) motor efficiency, if the friction and windage losses are totally 500 W and stator losses are 1000 W .
(P.T.U.)

## Solution:

No. of poles, $P=4$; frequency $f=50 \mathrm{~Hz}$
Torque at shaft, $T_{\mathrm{m}}=159 \mathrm{Nm}$; slip, $S=4 \%=0.04$
Mechanical losses $=500 \mathrm{~W}$; stator losses $=1000 \mathrm{~W}$
Synchronous speed, $N_{\mathrm{S}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$
Rotor speed, $\quad N=N_{\mathrm{s}} \times(1-S)=1500 \times(1-0.04)=1440 \mathrm{rpm}$.
Angular speed, $\quad \omega=\frac{2 \times \pi \times N}{60}$
Rotor output $\quad=\omega T_{\mathrm{m}}=\frac{2 \times \pi \times N}{60} T_{\mathrm{m}}=\frac{2 \times \pi \times 1440}{60} \times 159=23977 \mathrm{~W}$
Mechanical power developed in rotor $=$ Rotor output + Mechanical losses $=23977+500=24477 \mathrm{~W}$ Rotor Cu loss $=\frac{S}{1-S}$ Mechanical power developed $=\frac{0.04}{1-0.04} \times 24477=1020 \mathrm{~W}$
(i) $\therefore$ Rotor input $=$ Mechanical power developed + Rotor Cu loss

$$
=24477+1020=25497 \mathrm{~W}
$$

(ii) Motor input $=$ Rotor input + Stator losses $=25497+1000=26497 \mathrm{~W}$
(iii) Motor efficiency, $\eta=\frac{\text { Output }}{\text { Input }} \times 100=\frac{23977}{26497} \times 100=90.49 \%$

## Example 12.22

A three-phase induction motor has an efficiency of $90 \%$ and runs at a speed of 480 rpm . The motor is supplied from 400 V mains and it takes a current of 75 A at 0.77 p.f. Calculate the bhp (metric) of the motor and pull on the belt when driving the line shaft through pulley of 0.75 m diameter.

## Solution:

Supply voltage, $V_{\mathrm{L}}=400 \mathrm{~V}$; rotor speed, $N=480 \mathrm{rpm}$
Motor efficiency, $\eta=90 \%=0.9$; Current drawn from mains, $I_{\mathrm{L}}=75 \mathrm{~A}$
Motor p.f., $\quad \cos \phi=0.77$ lag. Diameter of pulley, $d=0.75 \mathrm{~m}$
Radius of pulley

$$
r=\frac{0.75}{2}=0.375 \mathrm{~m}
$$

Input power $=\sqrt{3} V_{\mathrm{L}} I_{\mathrm{L}} \cos \phi=\sqrt{3} \times 400 \times 75 \times 0.77=40010 \mathrm{~W}$
Output power $=$ Input power $\times \eta=40010 \times 0.9=36009 \mathrm{~W}$
Bhp of the motor $=\frac{\text { Output power }}{735.5}=\frac{36009}{735.5}=48.958$
Angular speed, $\omega=\frac{2 \times \pi \times N}{60}=\frac{2 \times \pi \times 480}{60}=16 \pi$
Torque at the shaft, $T_{\mathrm{m}}=\frac{\text { Output power }}{\omega}=\frac{36009}{16 \times \pi}=716.376 \mathrm{Nm}$
Now, torque, $T_{\mathrm{m}}=$ Pull on the belt $\times$ radius of pulley
$\therefore$ Pull on the belt $=\frac{T_{\mathrm{m}}}{r}=\frac{716.376}{0.375}=1910.34 \mathrm{~N}=\frac{1910.34}{9.81}=194.73 \mathrm{~kg}$

### 12.22 TORQUE DEVELOPED BY AN INDUCTION MOTOR

We have already seen that the electrical power of three-phase induction motor converted into mechanical power is given by the following relation:

$$
\begin{equation*}
P_{0}=3 I_{2}^{2} R_{2}\left(\frac{1-S}{S}\right) \tag{12.6}
\end{equation*}
$$

also,

$$
\begin{equation*}
P_{0}=\omega T \tag{12.7}
\end{equation*}
$$

Where, $\omega=$ angular speed of the rotor in $\mathrm{rad} / \mathrm{sec}$. and $T=$ torque developed by an induction motor in Nm. Equating eq. (12.6) and (12.7), we get
or

$$
\begin{array}{cc}
\omega T=3 I_{2}^{2} R_{2}\left(\frac{1-S}{S}\right)=3 \frac{I_{2}^{2} R_{2}}{S} \times \frac{1-S}{\omega} & \\
T=\frac{3}{\omega_{2}} \times \frac{I_{2}^{2} R_{2}}{S} & {\left[\text { since } \omega=\omega_{\mathrm{s}} \times(1-S)\right]}
\end{array}
$$

Where $\omega_{\mathrm{s}}=$ angular synchronous speed in rad $/ \mathrm{sec}$.
As

$$
I_{2}=\frac{S E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}
$$

$$
\therefore \quad T=\frac{3}{\omega_{2}}\left(\frac{S E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}\right)^{2} \times \frac{R_{2}}{S}
$$

or

$$
T=\frac{3}{\omega_{\mathrm{s}}} \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}=\frac{3}{\omega_{\mathrm{s}}} \times \frac{E_{2 \mathrm{~s}}^{2} R_{2} / S}{\left[\left(R_{2} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}\right]}
$$

This is the expression for full-load torque.

### 12.23 CONDITION FOR MAXIMUM TORQUE AND EQUATION FOR MAXIMUM TORQUE

The full-load torque developed in an induction motor is given by the relation:

$$
T=\frac{3}{\omega_{2}} \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}
$$

$$
T \propto \frac{S R_{2}}{R_{2}^{2}+S^{2} X_{2 \mathrm{x}}^{2}} \quad\left(\text { since } \frac{3}{\omega_{\mathrm{s}}} E_{2 \mathrm{~s}}^{2} \text { is constant }\right)
$$

The torque developed will be maximum at a particular value of slip. As slip $(S)$ is a variable quantity, therefore, to obtain the condition for maximum torque, the above expression for torque is differentiated with respect to $S$ and equated to zero.

$$
\frac{d T}{d S}=\frac{\left(R_{2}^{2}+S^{2} X_{2 \mathrm{~s}}^{2}\right) R_{2}-S R_{2}\left(0+2 S X_{2 \mathrm{~s}}^{2}\right)}{\left(R_{2}^{2}+S^{2} X_{2 \mathrm{~s}}^{2}\right)^{2}}=0
$$

or

$$
\left(R_{2}^{2}+S^{2} X_{2 \mathrm{~s}}^{2}\right) R_{2}=2 R_{2} S^{2} X_{2 \mathrm{~s}}^{2} \quad \text { or } \quad R_{2}^{2}=\left(S X_{2 \mathrm{~s}}\right)^{2} \quad \text { or } \quad R_{2}=S X_{2 \mathrm{~s}}
$$

or
$S=R_{2} / X_{2 \mathrm{~s}}$ is the slip at which torque is maximum.
To obtain the expression for maximum torque substitute, the value of $R_{2}=S X_{2 \mathrm{~s}}$ in the expression for full-load torque, we get,

$$
\text { Maximum torque, } T_{\mathrm{m}}=\frac{3}{\omega_{\mathrm{s}}} \times \frac{S E_{2 \mathrm{~s}}^{2}\left(S X_{2 \mathrm{~s}}\right)}{\left[\left(S X_{2 \mathrm{~s}}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}=\frac{3 E_{2 \mathrm{~s}}^{2}}{2 \omega X_{2 \mathrm{~s}}}
$$

Therefore, the maximum torque is independent of rotor resistance, but it is inversely proportional to rotor reactance at standstill (i.e., $X_{2 \mathrm{~s}}$ ). Therefore, to achieve higher value of maximum torque, the leakage reactance of the rotor should be kept minimum. This is achieved (i) by placing the
rotor conductors very near to the outer periphery of the rotor and (ii) by reducing the air gap between stator and rotor to smallest possible value.

### 12.24 STARTING TORQUE

Initially, rotor is stationary and the value of slip is one, that is, $S=1$.
Therefore, to obtain the expression for starting torque, substitute the value of slip, $S=1$ in the expression of full-load torque.

$$
\therefore \quad \text { Starting torque, } T_{s}=\frac{3}{\omega_{\mathrm{s}}}=\frac{E_{2 \mathrm{~s}}^{2} R_{2}}{\left[\left(R_{2}\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}\right]}
$$

Sometimes maximum torque is required at start. In that case, in the condition for maximum torque, substitute the value of $S=1$.

$$
R_{2}=S X_{2 \mathrm{~s}}=X_{2 \mathrm{~s}}(\text { since, } S=1 \text { at start })
$$

Therefore, to obtain maximum torque at start, the value of rotor resistance must be equal to rotor leakage reactance at standstill. Therefore, at start, some external resistance is added in the rotor circuit. This is only possible in case of slip-ring induction motors. This is the reason, why slip-ring induction motors are applied where heavy loads are required to be picked up at start such as in lifts, cranes, elevators, etc. Once the motor picks up the load, the external resistance is gradually reduced to zero.

In case of squirrel-cage induction motors, the rotor resistance is fixed and is kept quite low in comparison to rotor reactance; otherwise, the rotor copper losses would be high and the efficiency of the motor would fall to low value. However, to obtain higher starting torque in case of squirrel-cage induction motors another cage is embedded in the rotor and the motor is called double-cage induction motor.

### 12.25 RATIO OF STARTING TO MAXIMUM TORQUE

Starting torque is given by the expression:

$$
\begin{equation*}
T_{\mathrm{s}}=\frac{3}{\omega_{\mathrm{s}}} \frac{E_{2 \mathrm{~s}}^{2} R_{2}}{\left[R_{2}^{2}+X_{2 \mathrm{~s}}^{2}\right]} \tag{12.8}
\end{equation*}
$$

Maximum torque is given by the expression.

$$
\begin{equation*}
T_{\mathrm{m}}=\frac{3 E_{2 \mathrm{~s}}^{2}}{2 \omega_{\mathrm{s}} X_{2 \mathrm{~s}}} \tag{12.9}
\end{equation*}
$$

The ratio of starting to maximum torque is obtained by dividing equation (12.8) by (12.9):

$$
\frac{T_{\mathrm{s}}}{T_{\mathrm{m}}}=\frac{2 R_{2} X_{2 \mathrm{~s}}}{R_{2}^{2}+X_{2 \mathrm{~s}}^{2}}=\frac{2 R_{2} / X_{2 \mathrm{~s}}}{\left(R_{2} / X_{2 \mathrm{~s}}\right)+1}
$$

Putting

$$
\frac{R_{2}}{X_{2 \mathrm{~s}}}=a ; \frac{T_{\mathrm{s}}}{T_{\mathrm{m}}}=\frac{2 a}{a^{2}+1}
$$

### 12.26 RATIO OF FULL-LOAD TORQUE TO MAXIMUM TORQUE

Full-load torque is given by the expression:

$$
\begin{equation*}
T=\frac{3}{\omega_{\mathrm{s}}} \cdot \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[\left(R_{2}\right)^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]} \tag{12.10}
\end{equation*}
$$

Maximum torque is given by the expression:

$$
\begin{equation*}
T_{\mathrm{m}}=\frac{3 E_{2 \mathrm{~s}}^{2}}{2 \omega X_{2 \mathrm{~s}}} \tag{12.11}
\end{equation*}
$$

To obtain the ratio of full-load torque to maximum torque, divide equation (12.10) by the (12.11).

Putting

$$
\begin{gathered}
\frac{T}{T_{\mathrm{m}}}=\frac{2 R_{2} X_{2 \mathrm{~s}} S}{\left[\left(R_{2}\right)^{2} \times\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}=\frac{2 S R_{2} / X_{2 \mathrm{~s}}}{\left(R_{2} / X_{2 \mathrm{~s}}\right)^{2}+(S)^{2}} \\
\frac{R_{2}}{X_{2 \mathrm{~s}}}=a ; \frac{T}{T_{m}}=\frac{2 S a}{a^{2}+S^{2}}
\end{gathered}
$$

### 12.27 EFFECT OF CHANGE IN SUPPLY VOLTAGE ON TORQUE

The torque developed by the induction motor, when it is running with slip $S$, is given by the following expression:
or

$$
\begin{aligned}
& T=\frac{3}{\omega_{\mathrm{s}}} \times \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[R_{2}^{2}+\left(S X_{2 \mathrm{~s}}^{2}\right)\right]}=\frac{\mathrm{K} S E_{2 \mathrm{~s}}^{2} R_{2}}{R_{2}^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}} \quad\left(\begin{array}{l}
\text { constant } \left.\mathrm{K}=\frac{3}{\omega_{\mathrm{s}}}\right) \\
T=\frac{\mathrm{K}^{\prime} S R_{2} V^{2}}{R_{2}^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}} \quad\left[\begin{array}{l}
\text { since } E_{2 \mathrm{~s}} \propto \phi \propto V(\text { applied voltage }) \\
\text { and } \mathrm{K}^{\prime} \text { is another constant }
\end{array}\right]
\end{array}\right.
\end{aligned}
$$

At full-load, the slip $S$ is very low, and therefore, the value of $\left(S^{2} X_{2 \mathrm{~s}}^{2}\right)$ is so small that it can be neglected in comparison to $R_{2}$.

$$
\begin{aligned}
T=\frac{\mathrm{K}^{\prime} R_{2} S V^{2}}{R_{2}^{2}} & =\frac{\mathrm{K}^{\prime}}{R_{2}} S V^{2}=\mathrm{K}^{\prime \prime} S V^{2} \quad\left(\text { where constant } \mathrm{K}^{\prime \prime}=\frac{\mathrm{K}^{\prime}}{R_{2}}\right) \\
T & \propto S V^{2}
\end{aligned}
$$

or
Therefore, when the supply voltage $V$ is changed, it changes the torque $T$ developed by the motor under running condition. With the decrease in supply voltage, torque decreases abruptly and in order to maintain the same torque to pick up the load slip increases or speed decreases. Hence, the motor draws extra current from the supply mains that may overheat the motor. If the motor is operated continuously under this condition, it may burn.

### 12.28 TORQUE-SLIP CURVE

The full-load torque developed by an induction motor is given by the expression:

$$
T=\frac{3}{\omega_{\mathrm{s}}} \times \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[R_{2}^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}
$$

To draw the torque-slip or torque-speed curve, the following points are considered:


Fig. 12.19 Torque-slip curve of an induction motor

1. At synchronous speed $\left(N_{\mathrm{s}}\right)$, slip, $S=0$ and torque $T=0$.
2. When rotor speed is very near to synchronous speed, that is, when the slip is very low, the value of the term $\left(S X_{2 s}\right)^{2}$ is very small in comparison to $\left.R_{2}^{2}\left[i . e .,\left(S X_{2 \mathrm{~s}}\right)^{2} \ll R_{2}^{2}\right)\right]$ and is neglected.
Therefore, torque is given by the expression:

$$
T=\frac{3}{\omega_{\mathrm{s}}} \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{R_{2}^{2}}=\mathrm{K} S \quad \text { or } \quad T \propto S
$$

Therefore, at low values of slip, torque is approximately proportional to slip $S$ and the torque-slip curve is a straight line, as shown in Figure 12.19.
3. As the slip increases torque increases and attains its maximum value when $S=R_{2} / X_{2 s}$. This maximum value of torque is also known as break down or pull out torque.
4. With further increase in slip due to increase in load beyond the point of maximum torque, that is, when slip is high, the value of term $\left(S X_{25}\right)^{2}$ is very large in comparison to $R_{2}^{2}\left[\right.$ i.e., $\left.\left.\left(S X_{2 \mathrm{~s}}\right)^{2} \gg R_{2}^{2}\right)\right]$. Therefore, $R_{2}^{2}$ is neglected as compared to $\left(S X_{2 s}\right)^{2}$ and the torque is given by the expression.

$$
T=\frac{3}{\omega_{\mathrm{s}}} \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{X^{2} X_{2 \mathrm{~s}}^{2}}=K^{\prime} \frac{1}{S} \quad \text { or } \quad T \propto \frac{1}{S}
$$

Thus, at higher value of slip (i.e., the slip beyond that corresponding to maximum torque), torque is approximately inversely proportional to slip $S$ and the torque-slip curve is a rectangular hyperbola, as shown in Figure 12.19.
Therefore, with the increase in slip beyond the point of maximum torque, due to increase in load, torque decreases. The result is that the motor could not pick-up the load and slows down and eventually stops. This results in blocked rotor or short-circuited motor.

### 12.29 TORQUE-SPEED CURVE AND OPERATING REGION



Fig. 12.20 Torque-speed curve of an induction motor

The torque-speed curve of an induction motor is shown in Figure 12.20. It is the same curve which is already drawn, the only difference is that speed is taken on the abscissa instead of slip.

From the curve, it is clear that induction motor develops the same torque at point X and Y . However, at point X , the motor is unstable because with the increase in load, speed decreases and the torque developed by the motor also decreases. Therefore, the motor could not pick up the load and the result is that the motor slows down and eventually stops. The miniature circuit breakers will be tripped open if the circuit has been so protected.

At point $Y$, the motor is stable because in this region with the increase in load speed decreases but the torque developed by the motor increases. Thus the motor will be in position to pick up the extra load effectively.

Therefore, on the torque-speed curve, region BC is the unstable region and region AB is the stable or operating region of the induction motor as shown in Figure 12.20.

### 12.30 EFFECT OF ROTOR RESISTANCE ON TORQUE-SLIP CURVE

To observe the effect of rotor resistance on torqueslip curve, consider a slip-ring induction motor in which additional resistance in the rotor circuit can be introduced through slip-rings. The rotor reactance $X_{2 \mathrm{~s}}$ remains constant. It has already been observed that maximum value of the torque developed by an induction motor is independent of rotor resistance $R_{2}$, but is inversely proportional to rotor standstill reactance $X_{2 s}$. Therefore, the effect of change in rotor resistance will change the value of slip at which this maximum torque occurs. Greater the rotor resistance, greater the value of slip at which the maximum torque occurs since


Fig. 12.21 Effect of rotor resistance on the torque-slip curve of an induction motor

$$
S=\frac{R_{2}}{X_{2 \mathrm{~s}}} \text { at which torque is maximum }
$$

The torque-slip curves are shown in Figure 12.21 for various values of rotor resistance $R_{2}$ keeping rotor reactance $X_{2 \mathrm{~s}}$ constant. When $R_{2}$ is 0.1 times of $X_{2 \mathrm{~s}}$, the maximum torque will occur at $\operatorname{slip} S=\frac{R_{2}}{X_{2 \mathrm{~s}}}=\frac{0.1 \times X_{2 \mathrm{~s}}}{X_{2 \mathrm{~s}}}=0.1$. Now, if the rotor resistance $R_{2}$ is increased, by adding some resistance externally, to the value so that it becomes 0.2 times of $X_{2 \mathrm{~s}}$, then maximum torque would occur at a slip $S=\frac{R_{2}}{X_{2 \mathrm{~s}}}=\frac{0.2 \times X_{2 \mathrm{~s}}}{X_{2 \mathrm{~s}}}=0.2$ and so on.

The maximum value of the torque can be obtained even at the start by adding that much resistance in the rotor circuit so that $R_{2}$ becomes equal to $X_{2 s}$.

The following important points may be noted from the above discussions:

1. The maximum torque developed by an induction motor remains constant since it is independent of the rotor resistance.
2. The slip at which maximum torque occurs varies with the variation of the rotor resistance.
3. The starting torque increased with the increase in the value of rotor resistance.
4. The maximum torque is obtained at the start when rotor resistance is made equal to rotor reactance at standstill, that is, $R_{2}=X_{2 \mathrm{~s}}$.

## Example 12.23

A four-pole, 50 Hz , three-phase induction motor has a rotor resistance of $0.21 \Omega$ per phase and standstill reactance of $0.7 \Omega$ per phase. Calculate the speed at which maximum torque is developed.
(P.T.U.)

## Solution:

Here,

$$
P=4 ; f=50 \mathrm{~Hz} ; R_{2}=0.21 \Omega ; X_{2 \mathrm{~s}}=0.7 \Omega
$$

Condition for maximum torque is $R_{2}=S \times X_{2 \mathrm{~s}}$
$\therefore$ Torque will be maximum at a slip, $S=\frac{R_{2}}{X_{2 \mathrm{~s}}}=\frac{0.21}{0.7}=0.03$
Synchronous speed, $N_{\mathrm{s}}=\frac{120 \times f}{P}=\frac{120 \times 50}{4}=1500 \mathrm{rpm}$
Speed at which the torque will be maximum, $N=N_{\mathrm{s}} \times(1-S)=1500 \times(1-0.03)=1455 \mathrm{rpm}$

### 12.31 COMPARISON OF SQUIRREL-CAGE AND PHASE-WOUND INDUCTION MOTORS

For the comparison of two types of induction motors, the same output is considered. The comparison given in Table 12.1 is made on the basis of construction, cost, losses, maintenance, starting, performance, etc.
Table 12.1 Comparison of Squirrel-cage and Phase-wound Induction Motors

| S.No. | Particulars | Squirrel-cage Induction Motor | Phase-wound Induction Motor |
| :---: | :---: | :---: | :---: |
| 1. | Construction | It is simple in construction and mechanically robust. | Its rotor is phase wound which needs care. |
| 2. | Cost | It is cheaper in construction cost | It is more costly |
| 3. | Maintenance | It requires less maintenance. | Because of slip-rings, brushes etc. it requires more maintenance. |
| 4. | Utilization of slot space | Space of rotor slots is better utilized since rotor conductors are in shape of bars | Whole of the slots space is not utilized since winding is placed in the slots |
| 5. | Copper losses | It has got small copper losses because of low rotor resistance | It has got more copper losses |
| 6. | Efficiency | Its efficiency is high. | It has lower efficiency |
| 7. | Cooling | The heat can be dissipated more efficiently because the end rings are bare thus there is more space for providing a cooling fan. | In this case, overhang of the winding occupies space and less space is left for the provision of a good cooling fan. Thus, cooling is not quite efficient. |
| 8. | Starting torque | It develops very poor starting torque due to low resistance which cannot be increased by ordinary means. | In this case an external resistance can be added in the rotor circuit at the start to improve its starting torque. |
| 9. | Starter | Starters are applied on stator side and starting methods are quite simple | Starter is applied on rotor side through slip-rings and brush gears |

### 12.32 NECESSITY OF A STARTER

The current drawn by a motor from the mains depends upon the rotor current. The rotor current under running condition is given by the expression:

$$
I_{2}=\frac{S E_{2 \mathrm{~s}}}{\sqrt{R_{2}^{2}+\left(S X_{2 \mathrm{~s}}\right)^{2}}}
$$

At start slip $S=1$, therefore, rotor current. $I_{2 \mathrm{~s}}=\frac{E_{2 \mathrm{~s}}}{\sqrt{R_{2}^{2}+X_{2 \mathrm{~s}}^{2}}}$

This current is very large as compared to its full-load current. Therefore, when a squirrel-cage induction motor is directly connected to the supply mains, it draws very large current (nearly 5 to 7 times of the full-load current) from the mains. This heavy current may not be dangerous for the motor because it occurs for a short duration of time, but it causes the following affects:

1. It produces large voltage drop in the distribution lines and therefore affects the voltage regulation of the supply system.
2. It adversely affects the other motors and loads connected to the same lines.

Hence, it is not advisable to start large capacity induction motors by direct switching. Rather, such motors should be started by means of some starting device known as starter.

The function of a starter is to limit the initial rush of current to a predetermined value.

A starter also has some protective devices to protect the induction motors against over loading.

### 12.33 STARTING METHODS OF SQUIRREL-CAGE INDUCTION MOTORS

The various starters that are employed to restrict the initial rush of current in squirrel-cage induction motors are Direct On Line (DOL) starter, Star-delta starter, and autotransformer starter.

### 12.33.1 Direct On Line (DOL) Starter

It is a starter by which the motor is switched ON direct to the supply mains by switching conductor. With normal industrial motors, this operation results in a heavy rush of current of the order of five to seven times of the normal full-load current. This high current rapidly decreases as the motor picks up speed but it is at a very low power factor and thus tends to disturb the voltage of the supply in the distribution lines. For this reason, the supply authorities limit the size of motor up to 5 H.P., which can be started by this starter. An automatic DOL starter is shown in Figure 12.22.


Fig. 12.22 Direct-on-line starter

A direct on line starter essentially consists of a contactor having four normally open (NO) contacts and a contactor coil also known as no-volt coil or no-volt release. There are two push buttons ON and OFF which are used to start and stop the motor. To protect motor against overload, thermal or magnetic over-load coils are connected in each phase.

To start the motor, the ON push button (green) is pressed which energies the no-volt coil by connecting it across two phases. The no-volt coil pulls its plunger in such a direction that all the normally open (NO) contacts are closed and motor is connected across supply through three contacts. The fourth contact serves as a hold on contact which keeps the no-volt coil circuit closed even after the ON push button is released. To stop the motor, OFF push button (red) is pressed momentarily, which de-energises the no volt coil opening the main contacts.

When the motor is over loaded, the thermal overload relay contact, connected in the control circuit opens thus disconnecting the No-volt relay from the supply. Overload protection is achieved by thermal element overload relay.

### 12.33.2 Star-Delta Starter

This method is based upon the principle that is star connections, voltage across each winding is phase voltage, that is, one-third times the line voltage, whereas the same winding when connected in delta will have full-line voltage across it. So at start, connections of the motor are made in star fashion so that reduced voltage is applied across each winding. After the motor attains speed, the same windings through a change-over switch, as shown in Figure 12.23 are connected in delta across the same supply. The starter is provided with overload and under voltage protection devices also.


Fig. 12.23 Star-delta starter

Since at start stator windings are connected in star connection, so voltage across each phase winding is reduced to $1 / \sqrt{3}$ of line voltage, and therefore, starting current/phase becomes equal to

$$
I_{\mathrm{sc}} / \sqrt{3}=\text { Starting line current }
$$

Starting line current by direct switching with stator winding connected in delta $=\sqrt{3} I_{\text {sc }}$

$$
\therefore \quad \frac{\text { Line current with star delta starter }}{\text { Line current with direct switching }}=\frac{I_{\mathrm{sc}} / \sqrt{3}}{\sqrt{3} I_{\mathrm{sc}}}=\frac{1}{3}
$$

Therefore, it concludes that when a three-phase motor is started by a star-delta starter, the current drawn by it is limited to one-third of the value that it would draw without starter.

### 12.33.3 Autotransformer Starter

In the previous method, the current can only be reduced to one-third times the short circuit current, whereas, in this method, the voltage applied across the motor and hence current can be reduced to a very low value at the time of start. At the time of start, the motor is connected to supply through autotransformer by a six-pole double-throw switch. When the motor is accelerated to about full speed, the operating handle is moved to run position. By this, motor is directly connected to the line as shown in Figure 12.24.

Overload protection and under voltage protection is provided as explained in the first method. Although this type of starter is expensive but is most suitable for both the star-connected and delta-connected induction motors. It is most suitable for starting of large motor.


Fig. 12.24 Auto-transformer starter

Large size motors draw huge amount of current from the mains if they are connected to mains without starter. However, if they are connected to the mains through star-delta starter, the current is limited to one-third value which is still, so large that it would disturb the other loads connected to the same lines. Hence, to limit the initial rush of current to low values autotransformer starters are preferred. With the help of autotransformer starters, we can limit the starting current to any predetermined value as explained below:

Let the motor be started by an autotransformer having transformation ratio $K$.
If $I_{\text {sc }}$ is the starting current when normal voltage is applied.
Applied voltage to stator at start $=K V$
Then motor input current $I_{\mathrm{s}}=K I_{\text {sc }}$
Supply current $=$ Primary current of Auto transformer

$$
=K \times \text { Secondary current of autotransformer }=K K I_{\mathrm{sc}}=K^{2} I_{\mathrm{sc}}
$$

If 20 per cent (i.e., $1 / 5$ th) voltage is applied to the motor through autotransformer starter, the current drawn from the mains is reduced to $\left(\frac{1}{5}\right)^{2}$, that is, $1 / 25$ th times.

### 12.34 STARTING METHOD OF SLIP-RING INDUCTION MOTORS

To start a slip-ring induction motor, a three-phase rheostat is connected in series with the rotor circuit through brushes as shown in Figure 12.25. This is called rotor rheostat starter. This is made of three separate variable resistors joined together by means of a three-phase armed handle which forms a star point. By moving the handle, equal resistance in each phase can be introduced.

At start, whole of the rheostat resistance is inserted in the rotor circuit and the rotor current is reduced to

$$
I_{2 \mathrm{~s}}=\frac{E_{2 \mathrm{~s}}}{\sqrt{\left(R_{2}+R\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}
$$



Fig. 12.25 Slip-ring induction motor starter

Correspondingly, it reduces the current drawn by the motor from the mains at start.
When the motor picks up speed, the external resistance is reduced gradually and ultimately whole of the resistance is taken out of circuit and slip-rings are short-circuited.

By inserting external resistance in the rotor circuit, not only the starting current is reduced but at the same time starting torque is increased due to improvement in power factor:

At starts:
Power factor without starter,

$$
\cos \phi_{\mathrm{s}}=\frac{R_{2}}{\sqrt{\left(R_{2}\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}
$$

Power factor with starter,

$$
\cos ^{\prime} \phi_{\mathrm{s}}=\frac{\left(R_{2}+R\right)}{\sqrt{\left(R_{2}+R\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}}}
$$

Hence, $\cos ^{\prime} \phi_{\mathrm{s}} \gg \cos \phi_{\mathrm{s}}$

## Example 12.24

A 10 H.P. three-phase induction motor with full-load efficiency and p.f. of 0.83 and 0.8 , respectively, has a short circuit current of 3.5 times full-load current. Estimate the line current at the instant of starting the motor from a 500 V supply by means of star-delta starter.
(P.T.U.)

## Solution:

Output of the motor $=10$ H.P. $=7355 \mathrm{~W}$
Power factor,

$$
\begin{gathered}
\cos \phi=0.8 \\
\eta=0.83
\end{gathered}
$$

Efficiency,
Supply voltage (line value), $V_{\mathrm{L}}=500 \mathrm{~V}$
Input to the motor $=\frac{\text { output }}{\eta}=\frac{7335}{0.83}=8861.45 \mathrm{~W}$
Full-load current, $\quad I_{f l}=\frac{\text { Input }}{\sqrt{3} V_{\mathrm{L}} \cos \phi}=\frac{8861.45}{\sqrt{3} \times 500 \times 0.8}=12.79 \mathrm{~A}$

Ratio of,

$$
\frac{I_{\mathrm{sc}}}{I_{\mathrm{f} 1}}=3.5
$$

$\therefore$ Short circuit current,

$$
I_{\mathrm{sc}}=3.5 \times 12.79=44.76 \mathrm{~A}
$$

Staring current,

$$
I_{\mathrm{s}}=\frac{I_{\mathrm{sc}}}{3}=\frac{44.76}{3}=14.92 \mathrm{~A}
$$

## Example 12.25

Determine the starting torque of a three-phase induction motor in terms of full-load torque when started by means of (i) star-delta starter and (ii) an autotransformer starter with 50 per cent tapings.

The motor draws a starting current of 5 times the full-load current when started direct on line. The full-load slip is 4 per cent.

Solution:
Ratio of short circuit current to full-load current, $\frac{I_{\mathrm{sc}}}{I_{\mathrm{fl}}}=5$
Full-load slip, $S=0.04$
(i) For star-delta starter,

Ratio of starting torque to full-load torque,

$$
\frac{T_{\mathrm{s}}}{T_{\mathrm{f}}}=\frac{1}{3}\left(\frac{I_{\mathrm{sc}}}{I_{\mathrm{fl}}}\right)^{2} \times S=\left(\frac{1}{3}\right) \times(5)^{2} \times 0.04=0.3333
$$

$\therefore$ Starting torque $=33.33 \%$ of full-load torque
(ii) For autotransformer starter,

For autotransformer starter,
Transformation ratio or tapings, $K=50 \%=\frac{1}{2}$
Ratio of starting torque to full-load torque

$$
\frac{T_{\mathrm{s}}}{T_{\mathrm{f}}}=K^{2}\left(\frac{I_{\mathrm{sc}}}{I_{\mathrm{fl}}}\right)^{2} \times S=\left(\frac{1}{2}\right)^{2} \times(5)^{2} \times 0.04=0.25
$$

$\therefore$ Starting torque $=25 \%$ of full-load torque

## Example 12.26

A 20 H.P., three-phase, six-pole, $50 \mathrm{~Hz}, 400 \mathrm{~V}$ induction motor runs at 960 rpm on full-load. If it takes 120 A on direct starting, find the ratio of starting torque to full-load torque with a star-delta starter. Full-load efficiency and p.f. are $90 \%$ and 0.85 .
(U.P.T.U. Tut)

## Solution:

Output power $=20 \mathrm{H} . \mathrm{P} .=20 \times 735.5=14710 \mathrm{~W}$
No. of poles, $P=6$
Supply voltage (line value), $V_{\mathrm{L}}=400 \mathrm{~V}$
Supply frequency, $f=50 \mathrm{~Hz}$
Short circuit current, $I_{\text {sc }}=120 \mathrm{~A}$
Efficiency, $\eta=90 \%=0.9$
Power factor, $\cos \phi=0.85$
Rotor speed, $N=960 \mathrm{rpm}$

Full-load current,

$$
I_{\mathrm{f}}=\frac{14170}{\sqrt{3} \times 400 \times 0.9 \times 0.85}=27.75 \mathrm{~A}
$$

Synchronous speed,

$$
N_{\mathrm{s}}=\frac{120 \times f}{p}=\frac{120 \times 50}{6}=1000 \mathrm{rpm}
$$

Slip,

$$
S=\frac{N_{\mathrm{s}}-N}{N_{\mathrm{s}}}=\frac{1000-960}{1000}=0.04
$$

Ratio of starting torque to full-load torque,

$$
\begin{aligned}
\frac{T_{\mathrm{s}}}{T_{\mathrm{f}}}=\frac{1}{3} \times\left(\frac{I_{\mathrm{sc}}}{I_{\mathrm{f}}}\right)^{2} & \times S=\frac{1}{3} \times\left(\frac{120}{27.75}\right)^{2} \times 0.04 \\
& =0.2493
\end{aligned}
$$

## Example 12.27

Find the suitable tapping on an autotransformer starter for an induction motor required to start the motor with $36 \%$ of full-load torque. The short circuit current of the motor is 5 times the fullload current and the full-load slip is $4 \%$. Determine also the starting current in the supply mains as a percentage of full-load current.
Solution:
Ratio of starting torque to full-load torque, $\frac{T_{\mathrm{s}}}{T_{\mathrm{f}}}=0.36$
Ratio of short-circuit current to full-load current, $\frac{I_{\mathrm{sc}}}{I_{\mathrm{f}}}=5$
Full-load slip, $S=0.04$

Now,

$$
\begin{aligned}
\frac{T_{\mathrm{s}}}{T_{\mathrm{f}}} & =K^{2}\left(\frac{I_{\mathrm{sc}}}{I_{\mathrm{f}}}\right)^{2} \times S \\
0.36 & =K^{2} \times(5)^{2} \times 0.04
\end{aligned}
$$

$\therefore$ Transformation ratio or tapping of autotransformer,

$$
K=\sqrt{\frac{0.036}{5 \times 5 \times 0.04}}=0.6 \text { or } 60 \%
$$

In an autotransformer starter, ratio of starting current to short-circuit current,

$$
\begin{array}{ll} 
& \frac{I_{\mathrm{s}}}{I_{\mathrm{sc}}}=K^{2} \quad \text { Where } I_{\mathrm{sc}}=5 I_{\mathrm{f}} \\
\therefore \quad & \frac{I_{\mathrm{sc}}}{I_{\mathrm{f}}}=5 K^{2}=5 \times(0.6)^{2}=1.8 \text { or } 180 \%
\end{array}
$$

## Example 12.28

Find the ratio of starting to full-load current for 10 kW input, 415 V , three-phase induction motor with star-delta starter at full-load efficiency 0.9 and the full-load p.f. 0.8 . The short circuit current is 40 A at 210 V and the magnetizing current is negligible.
(P.T.U.)

## Solution:

Input power,

$$
P=10 \mathrm{~kW}=1000 \mathrm{~W}
$$

Supply voltage (line value),

$$
V_{\mathrm{L}}=415 \mathrm{~V}
$$

Power factor,

$$
\cos \phi=0.8
$$

Short circuit current at

$$
210 \mathrm{~V}=40 \mathrm{~A}
$$

Short-circuit current at $415 \mathrm{~V}, \quad I_{\mathrm{sc}}=\frac{415 \times 40}{210}=79 \mathrm{~A}$
Full-load current,

$$
I_{\mathrm{f}}=\frac{p}{\sqrt{3} V_{\mathrm{L}} \cos \phi}=\frac{10000}{\sqrt{3} \times 415 \times 0.8}=17.39 \mathrm{~A}
$$

In case of star-delta starter
Starting current, $\quad I_{\mathrm{s}}=\frac{I_{\mathrm{sc}}}{3}=\frac{79}{3}=26.33 \mathrm{~A}$
Ratio of starting to full-load current, $\frac{I_{\mathrm{s}}}{I_{\mathrm{f}}}=\frac{26.33}{17.39}=1.514$

### 12.35 APPLICATIONS OF THREE-PHASE INDUCTION MOTORS

The applications of squirrel-cage induction motors and slip-ring (phase wound) induction motors are given below:

1. Squirrel-cage induction motors: These motors are mechanically robust and are operated almost at constant speed. These motors operate at high-power factor and have high over load capacity. However, these motors have low-starting torque. (i.e., these motors cannot pick up heavy loads) and draw heavy current at start. On the bases of these characteristics, these motors are best suited for the following:
(a) Printing machinery
(b) Flour mills
(c) Saw mills
(d) Shaft drives of small industries
(e) Pumps
(f) Prime movers with small generators
2. Slip-ring (or phase-wound) induction motors: These motors have all the important characteristics (advantage) of squirrel-cage induction motors and at the same time have the ability to pick up heavy loads at start drawing smaller current from the mains. Accordingly, these motors are best suited for the following:
(a) Rolling mills
(b) Lifts and hoists
(c) Big flour mills
(d) Large pumps
(e) Line shafts of heavy industries
(f) Prime moves with medium and large generators

### 12.36 COMPARISON BETWEEN INDUCTION MOTOR AND SYNCHRONOUS MOTOR

The comparison between induction motor and synchronous motor is given in Table 12.2.
Table 12.2 Comparison between Induction and Synchronous Motors

| S. No. | Particulars | Induction Motor | Synchronous Motor |
| :--- | :--- | :--- | :--- |
| 1. | Starting <br> DC excitation | It is a self-starting motor. <br> Its basic principle is mutual induction <br> and no excitation is required. | It is inherently not a self-starting motor. <br> It requires DC for field excitation. |
| 3. | Speed | Its speed is always less than <br> synchronous speed and speed <br> decreases with the increase in load. | It only runs at a synchronous speed and <br> its speed is not affected by load. With <br> the increase in load the torque angle $d$ <br> increases. |
| 4. | Power factor | It runs at lagging p.f. which may <br> become very low at light loads. | It can be operated under wide range of <br> power factors both lagging and leading <br> by changing its excitation. <br> It runs only at synchronous speed. The <br> only way to change its speed is to vary <br> the supply frequency. |
| 5. | Speed control | Its speed can be controlled. | It is also used to supply mechanical load <br> and in addition it improves the p.f. of the <br> system. <br> It is costlier and complicated to |
| 7. | Cosplications | Its application is limited to the supply <br> of mechanical load. | It is very cheap to manufacture and <br> mechanically robust. |
| 8. | Maintenance | It requires less maintenance. | It requires more maintenance. |

### 12.37 SPEED CONTROL OF INDUCTION MOTORS

The speed of an induction motor is given by the relation

$$
N=N_{\mathrm{s}} \times(1-S) \quad \text { or } \quad N=\frac{120 \times f}{P} \times(1-S)
$$

Hence, the speed of the motor depends upon three factors, that is, frequency, slip, and number of poles for which the motor is wound. Therefore, the speed of an induction motor can be controlled by changing or controlling any one of these three quantities.

### 12.37.1 Speed Control by Changing the Slip

The speed of an induction motor can be changed by changing its slip, and the slip can be changed by the following:

1. Changing the rotor circuit resistance
2. Changing the supply voltage
3. Injecting voltage in the rotor circuit

## Speed control by changing the rotor circuit resistance

In the wound-type motor, the slip may be changed by introducing resistance in the rotor circuit, and hence, speed is changed.


Fig. 12.26 Effect of rotor resistance on speed of induction motor

Torque developed in an induction motor is given by the expression:

$$
T=\frac{3}{\omega_{\mathrm{s}}} \frac{E_{2 \mathrm{~s}}^{2} R_{2} / S}{\left[\left(R_{2} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}\right.}
$$

The torque will remain constant if $\frac{R_{2}}{S}$ is constant. For a given torque, the slip at which a motor works is proportional to the rotor resistance.

The torque-speed curve (dotted) of a slip-ring induction rotor is shown in Figure 12.26. When an external resistance is added in the rotor circuit, speed decreases for the same torque $T$, so that ratio $\frac{R_{2}}{S}$ remains constant. trol are as follows:

1. Poor efficiency: By the introduction of external resistance in the rotor circuit, there is extra power loss $\left(I_{2}^{2} R\right)$ in the rotor circuit which reduces the overall efficiency of the motor.
2. Poor speed regulation: When speed of the induction motor is controlled by adding some external resistance in the rotor circuit, the change in speed is larger when load on the machine


Fig. 12.27 Torque-speed curves by adding resistance in rotor circuit changes from one value to the other. Hence, the machine operates at a poor regulation.

For illustration, refer to Figure 12.27. When the rotor resistance is $R_{1}$ and the load on the machine changes from half load to full-load, the speed of the motor decreases from $N_{1}$ to $N_{2}$. However, when some resistance is added in the rotor circuit so that its value becomes $R_{2}$ (i.e., $R_{2}>R_{1}$ ), then the speed changes from $N_{3}$ to $N_{4}$ when load on the motor changes from half load to full-load. It is very clear that $N_{3}-N_{4}$ is larger than $N_{1}-N_{2}$.

Hence, the mTachine operates at a poor regulation.

## Speed control by controlling the supply voltage

Slip or speed of a motor can also be changed by controlling the voltage fed to the motor. We have already seen that the torque developed by the motor is directly proportional to the
square of the supply voltage. If the supply voltage is decreased, the torque developed by the motor decreases rapidly ( $T \propto V_{2}$ ) and to pick up the load slip increases or speed decreases.

For illustration, look at Figure 12.28. At rated voltage and given load, the speed of the motor is $N_{1}$. If the supply voltage is reduced (say to $90 \%$ ), the speed of the motor decreases to $N_{2}$ to pick up the given load. This method is never used for the speed control of threephase large induction motors because the voltage control devices are very costly and bulky. However, this method is usually employed with single-phase induction motors, for example, ceiling fans, etc.


Fig. 12.28 Torque-speed curves at different voltages

## Speed control by injecting voltage in the rotor circuit

The speed of an induction motor can also be controlled by injecting a voltage at slip frequency directly into the rotor circuit. This method was first of all introduced by K.H. Schrage of Sweden and the motor in which this method is employed is called Schrage motor. If the injected emf has a component directly opposite to the rotor-induced emf, the motor speed decreases. On the other hand, if the injected emf has a component in phase with the rotor-induced emf, the motor speed increases and may rises beyond the synchronous speed.

Nowadays, Schrage motors are not preferred because of their heavy cost and bulky construction but these are still employed in large printing presses like newspaper printing.

### 12.37.2 Speed Control by Changing the Supply Frequency

The frequency of the power supply is constant, and therefore, to control the speed of an induction motor by this method, the induction motor is connected to the alternator operating independently. To control the speed, the frequency of the alternator is changed. This is a costly affair.

Recent improvements in the capabilities of controlled rectifiers (SCR) and continued decreases in the cost of their manufacturing, it has made it possible to control the speed of induction motor by controlling the supply frequency fed to the motor. By this method 5 to 10 per cent of rated speed of induction motors can be controlled. However, if the speed is to be controlled beyond this value, the motor has to be designed accordingly.

### 12.37.3 Speed Control by Changing the Poles

By means of suitable switch, the stator winding connections can be changed in such a manner that the number of stator poles is changed. This changes the actual speed of motor since actual speed of the motor is approximately inversely proportional to the number of poles.

By suitable connections, one winding can give two different speeds.
Suppose there are four coils per phase. If these are connected in such a way that they carry current in same direction then it will form eight poles altogether as shown in Figure 12.29(a).


Fig. 12.29 (a) Stator winding connections for 8 -poles (b) Stator winding connections for 4-poles

Now if the connections are such that the alternate coils carry current in opposite directions, we get four poles altogether as shown in Figure 12.29(b).

If more than two speeds are required, two separate winding are housed in same slots, and if each is arranged to give two speeds, then two windings can give four different speeds.

In squirrel-cage motors, the rotor poles are adjusted automatically. However, in wound-type motors, care has to be taken to change the rotor poles accordingly.

## PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by rotor resistance and rotor reactance?
2. Mention the particulars on the basis of which squirrel-cage and phase-wound induction motors are compared?
3. Why a three-phase induction motor is connected to the mains through a starter?
4. What is a DOL starter?
5. To what extent the starting current of a three-phase induction motor can be reduced by employing star-delta starter?
6. To start large squirrel-cage induction motors, an autotransformer starter is preferred. Why?

## Test Questions

1. Explain that induced emf in the rotor circuit of an induction motor is maximum at the start.
2. Explain that rotor magnetic field rotates at synchronous speed with respect to stator.
3. What do mean by rotor reactance? How slip affects its value?
4. Draw and explain torque-slip curve of a three-phase induction motor.
5. When a resistance is connected in series with the rotor circuit of a slip-ring induction motor, how it affects the torque?
6. Make a comparison between squirrel-cage and slip-ring induction motor.
7. What is the necessity of a starter in case of squirrel-cage induction motor? Explain the function of over-load relay used in the starter of an induction motor.
8. Explain how a star-delta starter limits the starting current of an induction motor.
9. Explain the working of an autotransformer starter.
10. How a starter placed in the rotor circuit of a slip-ring induction motor limits the starting current supplied to the motor?

## Numericals

1. A three-phase, 50 Hz induction motor has four poles and operates with a slip of 4 per cent at a certain load. Calculate (i) the speed of rotor with respect to the stator (ii) the frequency of rotor current (iii) the speed of rotor magnetic field with respect to rotor, (iv) the speed of rotor magnetic field with respect to the stator, and (v) the speed of the rotor magnetic field with respect to the stator magnetic field.
(Ans. $1440 \mathrm{rpm}, 2 \mathrm{~Hz}, 60 \mathrm{rpm}, 1500 \mathrm{rpm}$, zero)
2. A three-phase, $400 \mathrm{~V}, 20 \mathrm{H} . P ., 50 \mathrm{~Hz}$ induction motor runs at 1455 rpm when it delivers rated load. Determine (i) number of poles of the machine, (ii) speed of rotating air gap field, (iii) rotor-induced voltage if the stator to rotor turns ratio is $1: 80$, assuming that the winding factors are same for stator and rotor, and (iv) frequency of rotor current.
(Ans. 4, $1500 \mathrm{rpm}, 320 \mathrm{~V}, 1.5 \mathrm{~Hz}$ )

## SUMMARY

1. Three-phase induction motor: A machine that converts three-phase AC electric, power into mechanical power by using an electromagnetic induction phenomenon is called a three-phase induction motor.
2. Types of induction motors: According to the construction of rotor, there are two types of induction motors namely squirrel-cage induction motor and phase-wound induction motor.
3. Production of revolving field: When a three-phase supply is given to a three-phase wound stator of an induction motor, a resultant field of magnitude $1.5 \phi_{\mathrm{m}}$ is step-up which revolves in space at a constant speed called synchronous speed $N_{\mathrm{s}}\left(N_{\mathrm{s}}=120 \times f / P\right)$.
4. Working principle: At start, stationary rotor conductors cut across the revolving magnetic field and an emf is induced in them by the electromagnetic induction phenomenon. Current flows through the rotor conductors as they are short circuited and produce rotor field. By the interaction of rotor and stator magnetic field, torque develops and rotor starts rotating in the same direction as that of the revolving field.
5. Reversal of direction of rotation: The direction of rotation of three-phase induction motor can be reversed by interchanging the connections of any two supply leads at the stator terminals.
6. Slip: The rotor of a three-phase motor never obtains synchronous speed because at that speed there would be no relative speed between rotor conductors and stator revolving field and induction phenomenon is not possible. Its speed is always less than synchronous speed.
The difference between synchronous speed and rotor speed is called slip. It is generally denoted as a fraction of synchronous speed. $S=\left(N_{\mathrm{s}}-N\right) / N_{\mathrm{s}}$.
7. Frequency of rotor currents: $f_{\mathrm{r}}=S \times f$
8. Rotor emf: At stand still, $E_{2 \mathrm{~s}}=44 k w_{2} \mathrm{~T}_{2} f \phi$ or $E_{2 \mathrm{~s}}=K E_{1}$ Under running conditions $E_{2}=S E_{2 \mathrm{~s}}$
9. Rotor resistance: $R_{2}$
10. Rotor reactance: At standstill - $X_{25}$; at running conditions, $X_{2}=S X_{2 \mathrm{~s}}$
11. Rotor current: $I_{2}=E_{2} / Z_{2}$ where $E_{2}=S E_{2 s}$ and $Z_{2}=\sqrt{R_{2}^{2}+\left(S X_{2 s}\right)^{2}}$
12. Rotor p.f: $\cos \phi_{2}=R_{2} / Z_{2}=\sqrt{R_{2}^{2}+\left(S X_{2 s}\right)^{2}}$
13. Power flow: Input electric power to stator-stator cu. loss-stator iron loss $=$ stator output or rotor input. Rotor input-rotor cu. loss $=$ Mechanical power developed in rotor.
Mechanical power developed in rotor - Mechanical loss $=$ output mech mechanical power at the shaft.
14. Relation between rotor Cu loss and slip: Rotor cu. loss $=S \times$ Rotor input.
15. Torque developed: $T=\frac{3}{\omega_{\mathrm{s}}} \frac{S E_{2 \mathrm{~s}}^{2} R_{2}}{\left[\left(R_{2}^{2}\right)+\left(S X_{2 \mathrm{~s}}\right)^{2}\right]}=\frac{3}{\omega_{\mathrm{s}}} \frac{E_{2 s}^{2} R_{2} / S}{\left[\left(R_{\mathrm{s}} / S\right)^{2}+\left(X_{2 \mathrm{~s}}\right)^{2}\right]}$
16. Condition for max. torque: $R_{2}=S X_{2 \mathrm{~s}}$ or $S=R_{2} / X_{2 \mathrm{~s}}$
17. Max. Torque: $T_{\mathrm{m}}=\frac{3 E_{2 \mathrm{~s}}^{2}}{2 \omega_{\mathrm{s}} X_{2 \mathrm{~s}}}$ and $T_{\mathrm{m}} \propto \frac{1}{X_{2 \mathrm{~s}}}$
18. Effect of change in supply voltage: $T \propto S V^{2}$, therefore, when voltage decreases, torque developed by the motor decreases abruptly but load is same hence to pick this load slip increases or speed decreases.
19. Effect of change in rotor resistance: In case of slip-ring induction motors, some resistance is added in the rotor circuit; the value of slip at which maximum torque occurs changes. But the value of maximum torque remains the same because maximum torque is independent of rotor resistance

$$
\left(T_{\mathrm{m}}=\frac{3 E_{2 \mathrm{~s}}^{2}}{2 \omega_{\mathrm{s}} X_{2 \mathrm{~s}}}\right)
$$

20. Comparison of squirrel-cage and phase-wound induction motors: Squirrel case induction motors are more robust, cheap in construction, have higher efficiency and therefore, invariably employed in industries for the conversion of electrical power into mechanical power.
21. Starter: A device used to limit the inrush flow of current at start is known as starter. It contains no volt coil and over-load coil for motor protection.
22. Direct On Line (DOL) Starter: This starter does not limit the inrush flow of starting current but it contains No-volt and Over-load relays to protect the motors. It is used with the motors of smaller size (less than 3 H.P.).
23. Star-delta Starter: It connects the stator winding first in star and then in delta thus reduced the starting current to $\frac{1}{3}$ rd value. It is employed with most of the three-phase squirrel-cage induction motors.
24. Autotransformer Starter: It provides lower voltages to the stator winding at the start and has the ability to reduce the starting current to any predetermined value. It is used to start three-phase squirrel-cage induction motors of very large sizes.
25. Starting of slip-ring induction motors: In slip-ring induction motors, the starting current is limited by adding resistance in the rotor circuit at the start.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. The starting torque at a three-phase induction motor can be increased by increasing the rotor $\qquad$ .
2. In induction motors, the torque is directly proportional to $\qquad$ .
3. When load on an induction motor increases, its slip will $\qquad$ .
4. A squirrel-cage induction motor has $\qquad$ slip-rings.
5. The mechanical load across the induction motor is equivalent to electrical load of $\qquad$ .
6. To determine efficiency of an induction motor, we have to perform $\qquad$ test and $\qquad$ test.
7. When a three-phase, four-pole, 50 Hz induction motor runs at 1440 rpm its slip is $\qquad$ .
8. Induction motors having wound rotor are known as $\qquad$ type motors.
9. The resistance in the rotor circuit of a squirrel-cage induction motor $\qquad$ increased.
10. The core of a rotor of an induction motor is laminated in order to reduce $\qquad$ losses.
11. The condition for maximum torque in a three-phase induction motor is $\qquad$ .
12. The magnetising current of three-phase induction motor is high as compared to transformer because of $\qquad$ of magnetic path.
13. The slip of an induction motor at standstill is equal to $\qquad$ -.
14. Laminated stator core is used in three-phase induction motor to reduce $\qquad$ .
15. At normal load, the slip of an induction motor is usually $\qquad$ -.
16. The slip of an induction motor at synchronous speed will be $\qquad$ —.
17. A revolving field can be produced when $\qquad$ phase supply is fed to a $\qquad$ phase winding.
18. A three-phase, four-pole, 50 Hz induction motor rotates at 1440 rpm the frequency of the rotorinduced emf at running condition is $\qquad$ Hz.
19. A three-phase, six-pole, 50 Hz induction motor rotates at 960 rpm . The frequency of the rotor-induced emf at standstill (start) condition is $\qquad$ Hz.
20. The value of maximum torque developed by an induction motor is inversely proportional to $\qquad$ .

## OBJECTIVE TYPE QUESTIONS

1. Stator core of an induction motor is made of
(a) laminated cast iron.
(b) mild steel.
(c) silicon steel stampings.
(d) soft wood.
2. The stator winding of an induction motor can be designed for
(a) any number of pole.
(b) any even number of poles.
(c) any odd number of poles.
(d) only for four poles.
3. The rotor of squirrel-cage induction motor is skewed because
(a) it reduces humming thus ensures quite running of the motor.
(b) it results in a smoother torque curves for different positions of the rotor.
(c) it avoids the magnetic looking of the stator and rotor.
(d) All the above
4. Slip-rings of phase-wound induction motor are made of
(a) wood.
(b) cast iron.
(c) steel.
(d) cooper.
5. There is no electrical connection between stator and rotor, still power is transferred from stator to rotor through
(a) magnetic flux.
(b) air
(c) water
(d) magnet.
6. In a large induction motor usually the value of full-load slip is
(a) $0.4 \%$.
(b) $20 \%$.
(c) $3 \%$ to $5 \%$.
(d) $6 \%$ to $15 \%$.
7. At start, the slip of the induction motor is
(a) zero.
(b) 0.5 .
(c) one.
(d) infinite.
8. Under running conduction, the rotor reactance is directly proportional to
(a) induced emf.
(b) rotor current.
(c) slip.
(d) supply voltage.
9. At start the rotor power factor is
(a) very high.
(b) very low.
(c) unity.
(d) zero.
10. The rotor copper losses of an induction motor are directly proportional to
(a) input to the motor.
(b) output of the motor.
(c) rotor resistance.
(d) slip.
11. The condition for maximum torque is
(a) $R_{2}=S Z_{2 s}$.
(b) $X_{2 \mathrm{~s}}=S R_{2}$.
(c) $R_{2}=X_{2 s}$.
(d) $R_{2}=S X_{2 s}$.
12. The function of a starter is
(a) to start the motor.
(b) to start and stop the motor.
(c) to limit the starting current.
(d) to limit the applied voltage.
13. Which of the following statement is most appropriate if $T$ is the starting torque developed in the rotor
and $V$ is the supply voltage to the stator?
(a) $T \propto V$
(b) $T \propto V^{2}$
(c) $T \propto \frac{1}{V}$
(d) $T \propto \frac{1}{\sqrt{V}}$
14. The torque developed in the cage induction motor with autotransformer starter having tapping at transformation ratio K is
(a) $K \times$ torque with direct switching.
(b) $K /$ torque with direct switching.
(c) $K^{2} \times$ torque with direct switching.
(d) $K^{2} /$ torque with direct switching.
15. In a three-phase slip-ring induction motor, if some resistance is added in the rotor circuit, then
(a) its starting torque will decrease and maximum torque will increase.
(b) its both starting torque and maximum torque will increase.
(c) its starting torque will increase but the maximum torque will remain the same.
(d) its starting torque will remain the same but maximum torque will increase.
16. The power factor of an induction motor will be high when
(a) rotor is locked.
(b) motors are running at full-load.
(c) motor is running at no-load.
(d) All the above
17. Nowadays die cast aluminium rotors are used in three-phase, squirrel-cage induction motors because
(a) aluminium in cheaper than copper.
(b) aluminium is lighter than copper.
(c) aluminium has high resistivity than copper.
(d) aluminium is easy to cast because of its low melting point and is easily available.
18. The maximum torque developed by an induction motor is
(a) inversely proportional to rotor reactance.
(b) inversely proportional to rotor resistance.
(c) inversely proportional to supply voltage.
(d) inversely proportional to slip.
19. A 400 V , three-phase, 50 Hz induction motor rotates at 960 rpm on full-load. The motor is wound for
(a) 2 poles.
(b) 4 poles.
(c) 6 poles.
(d) 8 poles.
20. A three-phase, $400 \mathrm{~V}, 50 \mathrm{~Hz}$, four-pole induction motor cannot run at 1500 rpm because
(a) at this speed motor will draw such a heavy current which may damage the motor.
(b) at this speed motor bearings may be damaged.
(c) at this speed, emf will not be induced in the rotor circuit and hence no torque will be developed.
(d) All the above
21. Torque developed by a three-phase, 400 V , induction motor is 200 Nm . If the supply voltage is reduced to 200 V , the developed torque will be
(a) 100 Nm
(b) 50 Nm
(c) 75 Nm
(d) 200 Nm
22. The stator and rotor core of an induction motor are made up of laminations
(a) to reduce reluctance.
(b) to reduce cooper losses.
(c) to reduce hysteresis loss.
(d) to reduce eddy current loss.

## NUMERICALS

1. If the emf in the stator of an eight-pole induction motor has a frequency of 50 Hz and that in rotor 1.5 Hz find the speed at which motor is running and its slip.
(U.P.T.U. Type) (Ans. $727.5 \mathrm{rpm}, 0.03$ )
2. An eight-pole alternator runs at 750 rpm and supplies power to a six-pole induction motor which has full-load slip of $3 \%$. Find the full-load speed of the induction motor and the frequency of its rotor emf.
(Ans. $970 \mathrm{rpm}, 1.5 \mathrm{~Hz}$ )
3. It is desired to obtain a speed of approximately 700 rpm with a three-phase induction motor. Determine number of poles for (i) 60 Hz motor and (ii) 25 Hz motor. If the rated load slip of each motor is $5 \%$, determine rated speed for each motor.
(U.P.T.U. Type) (Ans. 10, $684 \mathrm{rpm}, 4,712.5 \mathrm{rpm})$
4. Determine the number of poles, the slip and the frequency of rotor currents at rated load for a threephase, 3.7 kW induction motor rated at:
(i) $220 \mathrm{~V}, 50 \mathrm{~Hz}, 1440 \mathrm{rpm}$
(ii) $120 \mathrm{~V}, 400 \mathrm{~Hz}, 3800 \mathrm{rpm}$
(Ans. $4.4 \%, 2 \mathrm{~Hz}, 12.5 \%, 20 \mathrm{~Hz})$
5. A balanced three-phase, 50 Hz voltage is applied to a three-phase six-pole induction motor. When the motor delivers rated output the slip is found to be 0.04 . Determine the following:
(i) The speed of the revolving field relative to the stator structure.
(ii) The frequency of the rotor currents.
(iii) The speed of the rotor mmf relative to the rotor structure.
(iv) The speed of the rotor mmf relative to the stator structure.
(v) The speed of the rotor mmf relative to the stator field distribution.
(vi) Are the conditions right for the development of the net unidirectional torque?
(U.P.T.U. Type) (Ans. $1000 \mathrm{rpm}, 2 \mathrm{~Hz}, 40 \mathrm{rpm}, 1000 \mathrm{rpm}$, zero, yes)
6. A six-pole, three-phase, 50 Hz induction motor has a star connected rotor. The rotor has resistance and standstill reactance of 0.25 ohm and 2.5 ohm per phase, respectively. The induced emf between sliprings at start is 100 V . If the full-load speed is 960 rpm . Calculate (i) the slip, (ii) rotor-induced emf per phase, (iii) the rotor current and power factor at standstill, and (iv) the rotor current and power factor at rated load.
(Ans. $4 \%, 57.735 \mathrm{~V}, 22.98 \mathrm{~A}, 0.0995$ lagging, $8.577 \mathrm{~A}, 0.9285$ lagging)
7. The power supplied to a three-phase induction motor is 40 kW and the corresponding stator losses are 1.5 kW . Calculate the following:
(i) The total mechanical power developed and the rotor $I R$ losses when the slip is 0.04 .
(ii) The output power of the motor if the friction and windage losses are 0.8 kW .
(iii) The efficiency of the motor. Neglect the rotor iron losses.
(U.P.T.U. Type) (Ans. $36.96 \mathrm{~kW}, 1.54 \mathrm{~kW}, 36.16 \mathrm{~kW}, 90.4 \%)$
8. The shaft output of a three-phase induction motor is 75 kW . The friction and windage losses are 1000 W . The stator core losses are 4000 W and the stator copper losses are 2500 W . If the slip is $3.5 \%$ what is the efficiency of the motor?
(Ans. 87.97\%)
9. A 400 V , six-pole, 50 Hz , three-phase induction motor develops $20 \mathrm{H} . P$. inclusive of mechanical losses when running at 995 rpm , the power factor being 0.87 . Calculate the following:
(i) Slip
(ii) The rotor copper losses
(iii) The line current. The stator copper loss is 1500 W .
(U.P.T.U. Type) (Ans. $0.005,73.92 \mathrm{~W}, 27 \mathrm{~A})$
10. A 50 H.P., six-pole, three-phase induction motor delivers full-load output at 955 rpm and with 0.86 p.f. when connected to $500 \mathrm{~V}, 50 \mathrm{~Hz}$ mains. Friction and windage losses total $2 \mathrm{H} . \mathrm{P}$. and stator losses are 1.5 kW . Determine for this load (i) total rotor Cu losses, (ii) the efficiency, and (iii) the line current.
(U.P.T.U. Type) (Ans. $1.802 \mathrm{~kW}, 88.51 \%, 55.78 \mathrm{~A})$
11. A six-pole, three-phase induction motor runs at a speed of 960 rpm and the shaft torque is 135.7 Nm . Calculate the rotor copper loss if the friction and windage losses amount of 150 W . The frequency of supply is 50 Hz .
(P.T.U.) (Ans. 574.67 W)
12. An induction motor has an efficiency of $85 \%$ when loaded at $50 \mathrm{H} . P$. At this load, stator copper loss and rotor copper loss each equal iron loss. Mechanical losses are one third of the no-load loss. Calculate rotor copper loss, rotor input, and slip. (Hints: Input $=\frac{\text { Output }}{\eta}$; losses = Input - Output. If rotor copper loss $=K$, then total losses $\left.=K+K+K+\frac{1}{3} K\right) \quad$ (P.T.U.) (Ans. 1946.9 W, 39370.88 W, 0.04945)
13. If the motor has a rotor resistance of 0.02 ohm and standstill reactance of 0.1 ohm . What must be the value of total resistance of a starter for the rotor circuit for maximum torque to be exerted at starting.
(P.T.U.) (Ans. 0.08 ohm)
14. A three-phase induction motor, at standstill, has a rotor voltage of 100 V between slip-rings when they are open-circuited. The rotor winding is star-connected and has a leakage reactance of 1 ohm per phase at standstill and a resistance of 0.2 ohm per phase. Calculate the following:
(i) The rotor current when the slip is $4 \%$ and the slip-rings are short-circuited
(ii) The slip and the rotor current when the rotor is developing maximum torque. Assume the flux to remain constant.
(P.T.U.) (Ans. $11.323 \mathrm{~A}, 40.82 \mathrm{~A})$
15. Determine the starting torque of a three-phase induction motor in terms of full-load torque when started by means of:
(i) Star-delta starter
(ii) An autotransformer starter with $60 \%$ tapings.

The motor draws a starting current of 4.5 times the full-load current when started direct on line. The full-load slip is $5 \%$.
(Ans. 33.75\%, 36.45\%)
16. A 15 bhp (metric), three-phase, six-pole, $50 \mathrm{~Hz}, 400 \mathrm{~V}$ induction motor runs at 950 rpm on full-load. If it takes 90 A in direct switching, find the ratio of the starting torque to full-load torque with a star-delta starter. Full-load efficiency and p.f. are $90 \%$ and 0.8 , respectively.
(Ans. 0.276)
17. Find the suitable tapping on an autotransformer starter for an induction motor required to start the motor with $31.5 \%$ of full-load torque. The short circuit current of the motor is 6 times the full-load current and the full-load slip is $35 \%$. Determine also the starting current in the supply mains as a percentage of full-load current.
(P.S.B./H.S.B. May 1987) (Ans. 50\%, 150\%)
18. Find the ratio of starting to full-load current is 15 H.P. 400 V , three-phase induction motor with stardelta starter. The full-load p.f. is 0.85 , full-load efficiency is $88 \%$ short circuit current is 40 A at 200 V . Ignore the magnetizing current.
(P.S.B./H.S.B. Dec. 1986) (Ans. 1.2526)

## VIVA-VOCE/REASONING QUESTIONS

1. What are the different types of three-phase induction motors? Name them.
2. Can the outer frame an induction motor be made of plastic? Why?
3. The rotor of an inductor motor is skewed. Why?
(P.T.U.)
4. Can we reverse the direction of rotation of a three-phase induction motor by just interchanging the connections of any two terminals at the input, if so state why?
5. In a squirrel-cage rotor, no insulation is provided between the rotor conductors and slots. Why?
6. An induction motor is also called asynchronous motor. Why?
7. Can an induction motor run at synchronous speed, state why?
8. When three-phase supply is given to the stator of a three-phase wound induction motor, a revolving field is set-up. When the same supply is given to a three-phase transformer no revolving field is set up, state why?
9. The rotor conductors are placed at the outermost periphery of the rotor. Why?
10. The air gap between stator and rotor of an inductor motor is kept as small as possible, why?
(U.P.T.U.)
11. The direct on line starter does not limit the starting current of an induction motor still it is called a starter not a switch. Why?
(P.T.U.)
12. For very large three-phase induction motors, autotransformer starters are preferred over star-delta starters. Why?
13. Large induction motors are not started direct on line. Why?
14. The iron losses in the rotor of an induction motor are generally neglected. Why?
15. In lifts which type of three-phase induction motor is employed, state why?
16. An induction motor always draws power at lagging power factor. Why?
17. When load on an induction motor increases, its speed decreases. Why?
18. The rotor of a slip-ring induction motor is always connected in star. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What is significant of an induction motor?
(M.K. Univ. Nov. 1995)
2. What is a three-phase Induction Motor?
3. How does three-phase induction motor differ from the DC machine?
4. What are the two types of three-phase induction motors? Which type is generally preferred?
(M.S. Univ. Nov. 1996)
5. What are the different parts of a three-phase Induction Motor?
6. What are the two types of rotors? Which type of rotor is employed more commonly and why?
7. What are end rings?
8. What do you understand by synchronous speed?
9. Induction motors are also called 'asynchronous' motors, why?
10. The air gap between the rotor and stator of a three-phase Induction Motor is kept as small as possible, why?
11. The rotor of a slip-ring induction motor is always wound for the same number of poles as its stator, why?
12. Usually closed type slots are used for small induction motors, why? Which type of induction motor develops higher starting torque?
13. The rotor conductors in a squirrel-cage induction motor are always short circuited, why?
14. What happens when the ends of rotor conductors of a squirrel-cage induction motor are kept open?
15. Explain what is rotating magnetic field and how it is used in electrical machinery?
(P.T.U. Dec. 2001)
16. When in a rotating magnetic fields produced? What is its importance in electrical machinery? (P.T.U. Jan. 2000)
17. On what factor does the direction of rotation of a three-phase induction motor depend?
18. How can the direction of rotation of the three-phase induction motor be reversed?
(Madras Univ. April-1996)
19. What happens when in a three-phase induction motor, the connections of any two phase are interchanged?
20. The speed of an induction motor cannot be equal to synchronous speed. Explain why?
21. What is slip of an induction motor?
(P.T.U. Nov. 1995)
22. What is the speed of rotor mmf of a three-phase induction motor with respect to its stator mmf?
23. In case of a three-phase induction motor, the slip is always positive, why?
(P.T.U. 1996)
24. Show that in induction motor, the rotor always run in the direction of stator field.
(P.T.U. Dec. 2002)
25. How does the slip vary with load?
(P.T.U. May 2001)
26. At what slip, the torque developed in an induction motor will be maximum?
(M.S. Univ. Nov. 1996)
27. Is the maximum torque of a three-phase induction motor depends on the rotor resistance?
28. When the applied rated voltage per phase is reduced to one-half, what will be the starting torque of a squirrel-cage induction motor m terms of its starting torque with full voltage?
(M.K. Univ. Nov. 1996)
29. What is the condition for maximum torque at starting in a three-phase induction motor?
(Pune Univ. Nov. 1996)
30. How is the mechanical power output represented in the circuit model of an induction motor?
(Pune Univ. Nov. 1996)
31. An induction motor can be called a generalized transformer. Explain.
(P.T.U., May 2001)
32. The reactance of the rotor of an induction motor varies greatly between starting and running conditions. Explain why?
(P.T.U. June 2000)
33. List the advantages of three-phase induction motor.
34. What are the disadvantages of three-phase induction motors as compared to DC shunt motors?
35. Starters are used for starting of three-phase induction motors. Why?
(P.T.U. Dec. 2003)
36. How many terminals do you expect on the terminal box of a squirrel-cage induction motor to be started by star-delta starter?
37. In what ratio the line current and starting torque are reduced when started by star-delta starter?
38. What is the serious objection to the practice of employing reduced voltage for the starting of a squirrel-cage induction motor?
39. What protections are provided in the starters used to start three-phase induction motors?
40. Rotor resistance starting is preferred to reduced voltage staring of a wound rotor induction motor. Why?
41. Is it possible to add an external resistance in the rotor circuit of a three-phase cage induction motor? Give reason for your answer.
(Madras Univ. April 1997)
42. What are the advantages of wound rotor motors over squirrel-cage motor?
43. What are the disadvantages of wound rotor motors compared to squirrel-cage motor?
44. What is meant by single phasing?
(Madras Univ. Elect. Machines-II, Nov. 1996)
45. What is the usual proportion of three-phase induction motors, three-phase synchronous motors, DC motors and fractional horse power motors in a typical distribution system?
(P.T.U. Jan. 2000)
46. What is the place of application of cage and wound rotor induction motors.
(P.T.U., Jan. 2000)

## TEST QUESTIONS

1. Explain the construction of three-phase squirrel-cage induction motor.
2. Discuss how a rotating field is produced in a three-phase induction motor. How does the rotating field help in the production of torque?
3. Derive the relationship between the frequency of rotor currents and supply frequency in case of a three-phase induction motor.
(U.P.T.U. 2003-04)
4. If one of the phase of a three-phase induction motor is blown off while running without load, what will happen to its rotation.
5. Can induction motor (three phase) run at synchronous speed? Explain your answer.
(U.P.T.U.)
6. Explain the terms slip, slip frequency, wound rotor, cage rotor.
7. How can the direction of rotation of three-phase induction motor be reversed?
8. Explain the principle of working of a three-phase induction motor and give the expression of percentage slip.
(U.P.T.U. 2001, 2003-04)
9. How much torque does an induction motor develop at synchronous speed? Explain your answer.
10. Derive an expression for the rotor copper loss in terms of slip and input to the rotor.
(P.T.U.)
11. Obtain an expression for torque under running condition, for a three-phase induction motor and then deduce the condition for maximum torque.
12. Derive the simplified equation for torque of an induction motor.
13. Draw and explain the slip-torque characteristics of a three-phase slip-ring induction motor. Mark on it starting and maximum torque.
(U.P.T.U. 2002-03)
14. Draw the torque speed characteristics of polyphase induction motor and clearly indicate the effect of change in rotor resistance.
15. Explain briefly the effect of increasing the rotor resistance of an induction motor on (i) starting torque and (ii) running torque.
16. State the difference between squirrel-cage rotor and wound rotor type of induction motors.
(P.T.U.)
17. Compare relative advantages of cage rotor-type induction motor and wound rotor type induction motor.
18. Give a list of advantages and disadvantages of three-phase induction motors.
19. Compare cage- and wound-type induction motors.
(U.P.T.U. 2003-04)
20. Why starter is necessary for starting three-phase induction motors? Name various method of starting three-phase squirrel-cage induction motors and explain any one method in detail.
(U.P.T.U. 2002-03)
21. Name the various starters employed for the starting of three-phase squirrel-cage induction motor. Explain star-delta starter in detail.
(U.P.T.U. 2003-04)
22. Give various methods of starting large three-phase induction motors. Explain auto transformer starter in detail.
23. Calculate the ratio between the line currents drawn by an induction motor when started directly on line and through a star-delta starter.
24. How many terminals do you expect to find on the terminal box of three-phase squirrel-cage induction motor to be used for starting by star-delta starter.
25. Explain how three-phase wound-type induction motor is started.
26. State the causes of low-power factor of an induction motors.
27. Explain why the no-load current in an induction motor is much higher than that of an equivalent transformer.

## ANSWERS

## Fill in the Blanks

1. resistance
2. slip
3. increase
4. no
5. $R_{2}\left(\frac{1}{S}-1\right)$
6. no-load test; locked rotor test
7. 0.04
8. slip-ring
9. cannot be
10. iron (eddy current)
11. $R_{2}=S \times X_{2 \text { s }}$
12. high reluctance (air gap)
13. one
14. iron (eddy current)
15. three, three
16. 2
17. 50
18. 0.03 to 0.05
19. zero
20. standstill rotor reactance

## Objective Type Questions

1. (c)
2. (b)
3. (c)
4. (d)
5. (a)
6. (c)
7. (c)
8. (c)
9. (b)
10. (d)
11. (d)
12. (c)
13. (b)
14. (c)
15. (c)
16. (b)
17. (d)
18. (a)
19. (c)
20. (c)
21. (b)
22. (d)


## LEARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:
*Why single-phase induction motors are inherently not self-starting?

* What arrangements are made to make them self-starting?
*What are the different types of single-phase induction motors?
*What are the characteristics and applications of various types of single-phase induction motors?
* How the speed of a fan motor (single-phase, capacitor run motor) is controlled?


### 13.1 INTRODUCTION

Although three-phase induction motors are invariably employed in the industry which involves bulk power conversion from electrical to mechanical, single-phase induction motors are mostly used for small power conversion.

In practice, the motors that have output less than one horse power or one kilowatt are called 'fractional horse power or fractional kilowatt motors'. A single-phase alternating current induction motor with fractional kilowatt performs variety of services in homes, offices, business concerns, factories, etc. Single-phase induction motors are used only in the domestic appliances such as refrigerators, fans, washing machines, hair driers, mixers, etc. In this chapter, the general principles, operation, and performance of single-phase induction motors are discussed.

### 13.2 NATURE OF FIELD PRODUCED IN SINGLE-PHASE INDUCTION MOTORS

The field produced in a single-phase induction motor can be explained by double revolving field theory, which is discussed as follows:

This theory is based on the 'Ferraris Principle' that pulsating field produced in single-phase motor can be resolved into two components of half the magnitude and rotating in opposite direction at the same synchronous speed.

Therefore, the alternating flux that passes across the air gap of single-phase induction motor at standstill consists of combination of two fields of same strength which are revolving with same speed, one in clockwise direction and the other in anticlockwise direction. The strength of each one of these fields will be equal to one half of the maximum field strength of the actual alternating field as shown in Figure 13.1(a).

Let $\phi_{\mathrm{m}}$ be the pulsating field that has two components each of magnitude $\phi_{\mathrm{m}} / 2$. Both are rotating at the same angular speed $\omega_{\mathrm{s}} \mathrm{rad} / \mathrm{s}$ but in opposite direction as shown in Figure 13.1(a). The resultant of the two fields is $\phi_{\mathrm{m}} \cos \theta$. This shows that resultant field varies according to cosine of the angle $\theta$. The wave shape of the resultant field is shown in Figure 13.1(b).


Fig. 13.1 (a) Two vectors each of magnitude $\phi_{\mathrm{m}} / 2$ rotating in opposite direction at same angular velocity (b) Wave shape of resultant field

Therefore, an alternating field can be represented by the two fields each of half the magnitude rotating at same angular speed of $\omega_{\mathrm{s}} \mathrm{rad} / \mathrm{s}$ but in opposite direction.

### 13.3 TORQUE PRODUCED BY SINGLE-PHASE INDUCTION MOTOR

The two revolving fields will produce torques in opposite directions. Let the two revolving fields be field No. 1 and field No. 2 revolving in clockwise and anticlockwise direction. Field No. 1 produces torque in clockwise direction, whereas the field No. 2 produces torque in anticlockwise direction. The clockwise torque is plotted as positive and anticlockwise as negative. At standstill, slip for both fields is one. Synchronous speed in clockwise direction will give condition of zero slip for field 1 but it will be 2 for field No. 2. Similarly, synchronous speed in a counter clockwise


Fig. 13.2 Torque developed in a single-phase motor
direction will give condition of zero slip for field 2 but it will be 2 for field No. 1. The resultant torque developed in the rotor is shown by the curve passing through zero position as shown in Figure 13.2. For the resultant torque, the starting torque (i.e., torque at slip $=1$ ) is zero. And except at starting, there is always some magnitude of resultant torque, which shows if this type of motor once started in any direction, it will develop torque in that direction and rotor will pick up the required speed.

From the analysis discussed earlier, sin-gle-phase induction motor with single winding develops no starting torque but if the rotor is rotated in any direction by some auxiliary means will develop torque in the same direction in which it has been rotated to start. Hence, it is difficult to find out the auxiliary means to give the starting torque to the motor.

### 13.4 TYPES OF MOTORS

Various methods (means) are employed to obtain the starting torque in single-phase induction motors. Accordingly, they are classified as follows:

1. Split-phase motors
2. Shaded pole motors
3. AC series motors or Commutator motors.

### 13.5 SPLIT-PHASE MOTORS

In split phase motors, the single-phase winding is split into two parts, one of them is called a starting winding and the other is called a running winding.

### 13.5.1 Construction

The outer frame and stator core of a split-phase motor is similar to the outer frame and stator core of a three-phase induction motor. It is provided with an auxiliary stator winding called starting winding in addition to main winding. These windings are placed in the stator slots. Both the windings are put in parallel as shown in Figure 13.3(a). The purpose is to obtain two different currents sufficiently displaced from each other so that a revolving field is produced. The main winding, which is highly inductive, is connected across the line in the usual manner. The auxiliary or starting winding has a greater resistances and lesser reactance as compared to main winding.

The current in the starting winding $I_{\mathrm{s}}$ lags the supply voltage by lesser angle $\phi_{\mathrm{s}}$, whereas the current in the main winding $I_{\mathrm{m}}$ being highly inductive lags the supply voltage by greater angle $\phi_{\mathrm{m}}$ as shown in Figure 13.3(b). These two currents have a phase difference of $\theta^{\circ}$ electrical. Therefore, a revolving field is set up in the stator, and a starting torque is developed in the rotor. Consequently, rotor starts rotating and picks up the speed. A centrifugal switch that normally closed is incorporated in series with the starting winding. When the motor attains a speed about 75 per cent of synchronous speed, the centrifugal switch is opened automatically with the help of


Fig. 13.3 (a) Circuit diagram of a split-phase motor (b) Phasor diagram
centrifugal force and puts the starting winding out of circuit. The auxiliary winding made of thin wire will be over heated and damaged, if the centrifugal switch failed to open.

### 13.5.2 Performance and Characteristics

A typical torque speed characteristics are shown in Figure 13.4, and the starting torque is about twice the full-load torque. Initially, the current is about six to eight times. The speed falls with increase in load only by 5 per cent to 7 per cent; otherwise, it is a constant speed motor. Speed is governed by the following relation:

$$
N_{\mathrm{s}}=\frac{120 f}{P} \mathrm{rpm}
$$

Actual speed is less than synchronous speed $N_{\mathrm{S}}$. For the same weight, its rating is about 60 per cent to that of the poly-phase induction motor. It has lower pf and lesser efficiency. Pf is about 0.6 and


Fig. 13.4 Graph between speed and torque efficiency is also about 60 per cent.

### 13.5.3 Applications

As starting torque is not so high, this machine is not used where large starting torque is required. It is used for smaller sizes about 0.25 H.P. It is widely used in washing machines, fans, blowers, woodworking tools, grinders, and various other low-starting torque applications.

### 13.5.4 Reversal of Direction of Rotation

The direction of rotation of a single-phase (split phase) induction motor can be reversed by reversing (interchanging) the connections of either starting winding or running winding.

### 13.6 CAPACITOR MOTORS

It is also a split-phase motor. In this motor, a capacitor is connected in series with the starting winding. This is an improved form of the above-mentioned split-phase motor. In these motors, the angular displacement between $I_{\mathrm{S}}$ and $I_{\mathrm{m}}$ can be made nearly $90^{\circ}$, and high starting torques can be obtained since starting torque is directly proportional to sine of angle $\theta$. The capacitor in the starting winding may be connected permanently or temporarily. Accordingly, capacitor motors may be classified as follows:

1. Capacitor start motors
2. Capacitor run motors
3. Capacitor start and capacitor run motors.

### 13.6.1 Capacitor Start Motors

In the capacitor start induction motor, capacitor $C$ is of large value such that the motor will give high-starting torque. Capacitor employed is of short time duty rating.


Fig. 13.5 (a) Circuit diagram of a capacitor start motor (b) Phasor diagram

Capacitor is electrolytic type. Electrolytic capacitor $C$ is connected in series with the starting winding along with centrifugal switch $S$ as shown in Figure 13.5(a). When the motor attains


Fig. 13.6 Graph between speed and torque the speed of about 75 per cent of synchronous speed, starting winding is cut off. The construction of the motor and winding is similar to usual split-phase motor. It is used where high-starting torque is required such as refrigerators.

## Performance and characteristics

Speed is almost constant with 5 per cent slip. This type of motor develops high-starting torque about 4 to 5 times the full-load torque. It draws low-starting current. A typical torque speed curve is shown in Figure 13.6. The direction of rotation can be changed by interchanging the connection of either starting or running winding.

### 13.6.2 Capacitor Run Motors (Fan Motors)

In these motors, a paper capacitor is permanently connected in the starting winding as shown in Figure 13.7(a). In this case, electrolytic capacitor cannot be used since this type of capacitor is designed only for short-time rating and hence cannot be permanently connected in the winding. Both main and starting winding are of equal rating.


Fig. 13.7 (a) Circuit diagram of a capacitor motor (b) Graph between speed and torque

## Performance and characteristics

Starting torque is lower about 50 to 100 per cent of full-load torque. Power factor is improved about unity. Efficiency is improved to about 75 per cent. The characteristics have been shown in Figure 13.7(b). It is often used in fans, room coolers, portable tools, and other domestic and commercial electrical appliances.

### 13.6.3 Capacitor Start and Capacitor Run Motors

In this case, two capacitors are used: one for starting purpose and other for running purpose as shown in Figure 13.8(a). The capacitor used for starting purpose $C_{\mathrm{S}}$ is of electrolytic type and is disconnected from the supply when the motor attains 75 per cent of synchronous speed with the help of centrifugal switch $S$, whereas the other capacitor $C_{\mathrm{R}}$, which remains in the circuit of starting winding during running condition, is a paper capacitor. This type of motor gives best running and starting operation. Starting capacitor $C_{\mathrm{S}}$ always has higher value than the value of running capacitor $C_{\mathrm{R}}$.

## Performance and characteristics

Such motors operate as two-phase motors giving best performance and noiseless operation. If starting torque is high, starting current is low and give better efficiency and higher pf . But it is highly expensive. A typical torque speed curve is shown in Figure 13.8(b).


Fig. 13.8 (a) Circuit diagram of a capacitor start capacitor run motor (b) Graph between speed and torque

### 13.7 SHADED POLE MOTOR

In shaded pole motors, starting torque is obtained by providing a shading band or a short circuiting copper ring (band) around almost one-third portion of the projected pole of the machine.

### 13.7.1 Construction

Shaded pole motor is constructed with salient poles in stator. Each pole has its own exciting winding as shown in Figure 13.9(a). A one-third portion of each pole core is surrounded by a copper strip forming a closed loop called the shading band as shown in Figure 13.9(a) and (b). Rotor is usually squirrel cage type.

(a)

(b)

Fig. 13.9 (a) Constructional details of a shaded pole motor (b) Pole with shading band and exciting winding

### 13.7.2 Principle

When a single-phase supply is given to the stator (exciting) winding, it produces alternating flux. When the flux is increasing in the pole, a portion of the flux attempts to pass through the shaded portion of the pole. This flux induces voltage and current in the copper ring, and according to Lenz's law, the direction of current is such that it opposes the causes, that is, an increase in flux in shaded portion. Hence, in the beginning, the greater portion of flux passes through unshaded side of each pole and resultant lies on unshaded side of the pole. When the flux reaches its maximum
value, its rate of change is zero, thereby the emf and hence current in the shading coil becomes zero. Flux is uniformly distributed over the pole phase and the resultant field lies at the centre of the pole. After that the main flux tends to decrease, the current induced in the shading coil now tends to increase the flux on the shaded portion of the pole and resultant lies on the shaded portion of the pole as shown in Figure 13.10.

Hence, a revolving field is set up which rotates from unshaded portion of the pole to the shaded portion of the pole as marked by the arrow head in Figure 13.10. Therefore, by electromagnetic induction, a starting torque develops in the rotor and the rotor starts rotating. Then, its rotor picks up the speed.


Fig. 13.10 Position of resultant field at various instants

### 13.7.3 Performance and Characteristics

A typical speed torque characteristics is shown in Figure 13.11. Starting torque is very small about 50 per cent of full-load torque. Efficiency is low because of continuous power loss in shading coil. These motors are used for small fans, electric clocks, gramophones, etc.

Its direction of rotation depends upon the position of the shading coil, that is, which portion of the pole is wrapped with shading coil. The direction of rotation is from unshaded portion of pole to shaded portion. Its direction of rotation cannot be reversed unless the position of the poles is reversed.


Fig.13.11 Graph between speed and torque

### 13.8 RELUCTANCE START MOTOR

The stator of a reluctance start motor is constructed with salient poles. The starting torque is achieved by creating non-uniform air gap of the salient poles as shown in Figure 13.12. Each pole is excited by its own winding carrying the same current as shown in Figure 13.13. The rotor is usually squirrel cage type.


Fig. 13.12 Constructional details of a reluctance start motor


Fig. 13.13 Vector position of fields developed in reluctance start motors

The non-uniform air gap between stator poles and rotor offers different reluctance to the magnetic lines of force. The flux $\left(\phi_{\mathrm{A}}\right)$ set up in the portion having greater air gap will be more in phase with the current than the flux $\left(\phi_{\mathrm{B}}\right)$ set up in the portion having smaller air gap. This can be illustrated more clearly by considering two coils one having air core and the other having iron core as shown in Figure 13.14. In air-cored coil, there is no core loss, $I$ lags behind the voltage vector by $90^{\circ}$. This current is the magnetizing current and sets up field $\phi_{\mathrm{A}}$ in phase with $I$, whereas, in iron-cored coil, there are core losses, $I$ lags behind the voltage vector by an angle $\alpha$ (less than $90^{\circ}$ ). This current has two components, $I_{\mathrm{e}}$ (energy component) in phase with voltage to meet with losses and $I_{\mathrm{m}}$ (magnetizing component) in quadrature to voltage vector to set up field $\phi_{\mathrm{i}}$ in the core. Hence, field $\phi_{\mathrm{i}}$ lags behind the current vector $I$ by an angle $\theta$.


Fig. 13.14 (a) and b) Vector position of various quantities with air core and iron core

Hence, the fluxes set up by the two portions of the poles will lag behind the current by different angles and are displaced in time from one another. Therefore, the resultant magnetic axis will shift across the poles from the longer air gap region to the shorter air gap region since $\phi_{\mathrm{A}}$ is in more phase with current $I$ than $\phi_{\mathrm{B}}$. Consequently, the rotor starts rotating in the same direction. Once the rotor starts rotating, it will continue to rotate like other types of single-phase inductor motors.

It is evident that the direction of rotation of these motors is fixed by the construction (i.e., stator poles) and cannot be reversed.

Reluctance start motors have very small starting torque, low efficiency, and poor power factor, and therefore, their applications are limited. For most of the small power applications, shaded pole motors are preferred.

### 13.9 AC SERIES MOTOR OR COMMUTATOR MOTOR

When a single-phase AC supply is given to a DC series motor, a unidirectional torque is developed in it. In fact, during positive half cycle, same current flows in the series field winding and armature winding which develops a torque in one direction (clockwise). During the negative half cycle, current flowing through series field winding is reversed, and at the same time, current
flowing through the armature also reverses, and therefore, torque is developed in the same direction (i.e., clockwise direction). Hence, a continuous rotation is obtained.
Mathematically,
Torque in DC series motors, $T \propto \phi_{\mathrm{se}} I_{\mathrm{a}}$ where $\phi_{\mathrm{se}}$ is the series field winding flux, and $I_{\mathrm{a}}$ is the armature current.

Now, when AC supply is given to series motor, $T \propto \phi_{\mathrm{se}} I_{\mathrm{a}}$ for positive half cycle. For negative half cycle, $T \propto\left(-\phi_{\mathrm{se}}\right)\left(-I_{\mathrm{a}}\right) \propto \phi_{\mathrm{se}} I_{\mathrm{a}}$. Therefore, same torque is produced during positive and negative half cycle. However, the following are some modifications necessary in a DC series motor for its satisfactory operation on AC:

1. The iron structure of field and yoke are laminated.
2. For AC series motors, the series field winding is so designed that it would produce smaller magneto-motive force (mmf) than that it would have been produced by series field winding of DC series motor. This is done by reducing the number of turns. The smaller field mmf would result in reduced air gap flux. Therefore, in order to develop the necessary torque, the number of armature conductors has to be increased proportionately.
3. An increase in armature conductors would result in increased inductive reactance of the armature so the net inductive reactance may not be reduced. In order to overcome this difficulty, compensating winding is connected in series with armature as shown in Figure 13.15. This completely neutralizes the inductive effect of armature winding.
4. The reluctance of the magnetic circuit is reduced to have high flux with reduced mmf So magnetic material used should be of high permeability and air gap should be small.

As shown in the vector diagram (Figure 13.16), large voltage drop occurs in resistance and reactance of the armature and field winding. Voltage left for operation is only $E$.

$$
\bar{E}=\bar{V}-\left(\bar{I} \overline{R_{\mathrm{s}}}+\bar{I} \overline{X_{\mathrm{s}}}+\bar{I} \overline{R_{\mathrm{a}}}+\bar{I} \overline{X_{\mathrm{a}}}\right)
$$

Where $I R_{\mathrm{S}} \rightarrow$ voltage drop in series winding resistance
$I X_{\mathrm{S}} \rightarrow$ voltage drop in series winding reactance
$I R_{\mathrm{a}} \rightarrow$ voltage drop in armature resistance
$I X_{\mathrm{a}} \rightarrow$ voltage drop in armature reactance.


Fig. 13.15 Circuit diagram of an AC series motor


Fig. 13.16 Phasor diagram for series motor


Fig. 13.17 Graph between Torque and Speed for DC and AC series motor

### 13.9.1 Performance and Characteristics

The speed-torque characteristic for DC and AC series motors are shown in Figure 13.17. The torque varies as square of the current and the speed varies inversely proportional to the current approximately. The efficiency will not be good as that of corresponding DC machine because of greater eddy current loss and effects of pf .

These motors have their wide applications where high speed (20000 rpm.) is required, for example, mixer grinders, blowers, hair dryers, etc.

### 13.10 UNIVERSAL MOTOR

A motor that can be operated on both AC and DC supply at the rated voltage is called universal motor.

Basically, universal motor is an AC series motor. It is just an improved form of a DC series motor. The core size of an universal motor is more than the core size of a DC series motor of the same rating.

### 13.10.1 Construction

The motor has two main parts, namely stator and rotor.

1. Stator: It is the stationary part of the motor. It consists of magnetic frame (or yoke), pole core, and pole shoe and field or exciting winding as shown in Figure 13.18.

The magnetic frame, pole core, and pole shoe are made of silicon steel stampings. These stampings are insulated from each other by varnish layer. The hysteresis losses are very small in silicon steel, and eddy current losses are reduced due to stampings. The field winding made of enamelled copper is wound around the poles to produce the required flux.
2. Rotor: It is the rotating part of the motor. It consists of shaft, armature, armature winding and commutator as shown in Figure 13.19.


Fig. 13.18 Constructional details of an universal motor


Fig. 13.19 Construction of rotor for universal motor

Shaft is a part of rotor which transfers mechanical power or energy to the load. It is made up of mild steel. Armature is made up of stampings of silicon steel material since it carries the magnetic field. It is keyed to the shaft. Slots are cut at its outer periphery to accommodate armature winding. The ends of armature winding are braced to the commutator segments. Commutator is made up of wedge-shaped segments forming a ring. The wedges are insulated from each other by an insulating layer of micanite. The commutator is also keyed to the shaft.

Carbon brushes are pressed over the commutator surface to deliver current to the machine.

### 13.10.2 Principle

When a current carrying conductor is placed in the magnetic field, a force is exerted on it and torque develops. In other words, when the rotor field produced by the rotor current carrying conductors, tries to come in line with the main field, torque develops, and rotor rotates.

### 13.10.3 Working

Both armature winding and stator field winding are connected in series as shown in Figure 13.20. When single-phase AC supply is given to the motor, current flows through the field winding and armature winding. The field winding sets up main stator field $F_{\mathrm{m}}$, and the armature winding sets up rotor field $F_{\mathrm{r}}$ as shown in Figure 13.21. Rotor field $F_{\mathrm{r}}$ tries to align itself with the main field $F_{\mathrm{m}}$ and an anticlockwise torque is produced.

During negative half cycle, the direction of flow of current in both the field winding and the armature winding is reversed as shown in Figure 13.22. The two windings set up their fields in the direction as shown in Figure 13.22, and anticlockwise torque is produced in the rotor. Therefore, unidirectional torque is produced in the motor.

To obtain continuous torque, commutator reverses the direction of flow of current in the coil or conductors which cross the magnetic neutral axis (MNA).


Fig. 13.20 Circuit diagram of AC series motor


Fig. 13.21 Direction of torque development and rotation at an instant


Fig. 13.22 Direction of torque development and rotation at an instant

### 13.10.4 Applications

In large sizes of $\frac{3}{4}$ H.P. these are used in vacuum cleaners and industrial sewing machines. In smaller sizes of $\frac{1}{4}$ H.P. or less, these are used in electric hand drills, mixers, can openers, blenders, electric shavers, hair dryers, etc.

### 13.11 SPEED CONTROL OF SINGLE-PHASE INDUCTION MOTORS (FAN REGULATOR)

The speed of a single-phase induction motor (capacitor run split-phase induction motor) can be controlled by changing voltage supplied to it. The voltage applied across the motor can be regulated (below rated value) by conventional fan regulator and electronic (using TRIAC) regulator.

1. Conventional fan regulation: A conventional fan regulation is shown in Figure 13.23. In this case, a tapped resistor or inductor is connected in series with the fan motor. The speed of fan can be reduced as and when desired by increasing the regulator resistance or inductance. This reduces the voltage applied across the fan motor due to voltage drop in the regulator resistance or inductance. These regulators have been replaced by the electronic regulators due to heavy energy losses in these regulators.


Fig. 13.23 Conventional speed regulator for fan
2. Electronic (using TRIAC) regulator: An electronic fan regulator using a diac-triac pair is shown in Figure 13.24.

When the switch S is closed, during each half (positive or negative) of the AC supply, the capacitors $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$ are charged at a rate determined by the variable resistor $\mathrm{R}_{2}$. When the voltage across $\mathrm{C}_{3}$ exceeds the breakover voltage of the diac, the diac conducts heavily. This allows the capacitor $\mathrm{C}_{3}$ to discharge through the diac into the gate of the triac. Consequently, the triac is triggered and load (fan) circuit is closed. The voltage (or power) applied to the load depends upon the firing angle of the triac which further


Fig. 13.24 Electronic speed regulator for fan
depends upon the rate of charging and discharging of capacitor $\mathrm{C}_{3}$. The rate of charging of capacitor $\mathrm{C}_{3}$ is controlled by a variable resistor $\mathrm{R}_{2}$. Therefore, by changing the value of resistor $\mathrm{R}_{2}$, the voltage applied across the load or the power fed to the load can be varied.

If reader likes to fabricate a fan regulator circuit, he/she may use the various components as per the following list:
$\mathrm{C}_{1}-0.1 \mu \mathrm{~F}, 400 \mathrm{~V}$; R1—470 W, $\frac{1}{2} \mathrm{~W}$; diac—D84
$\mathrm{C}_{2}-0.1 \mu \mathrm{~F}, 400 \mathrm{~V}$; R2-470 K (carbon volume control) triac—BT136 in parallel with 220 K fixed

$$
\mathrm{C}_{3}-0.047 \mu \mathrm{~F}, 100 \mathrm{~V} ; \mathrm{R} 3-10 \mathrm{~K}, \frac{1}{2} \mathrm{~W} ; \text { coil-123 turns of R4—15 K, } \frac{1}{2} \mathrm{~W} ; 24 \mathrm{SWG}
$$

## SUMMARY

1. Single-phase induction motor: A machine that converts single-phase AC electric power into mechanical power by an electromagnetic induction phenomenon is called single-phase induction motor.
2. Torque development in a single-phase induction motor: When single-phase supply is given to the stator of a single-phase induction motor, two fields of magnitude $\varphi_{\mathrm{m}} / 2$ are produced which rotate in opposite direction at synchronous speed. An equal and opposite torque is developed by the two fields, and the resultant torque is zero at start. Therefore, single-phase induction motor is not a self-starting motor. However, if the rotor is rotated in either direction by some external means, torque develops, and rotor picks up the speed in that direction.
3. Split-phase motor: To obtain starting torque in single-phase induction motors, the single winding splits into two parts having different resistance and inductance. They carry currents at different angles which produce a resultant field revolving in space at synchronous speed, and this develops starting torque in the motor.
4. Capacitor motors: These may be capacitor start motors, capacitor run motor, and capacitor start and run motors.
5. Shaded pole motors: These motors have projected poles with one-third pole part rapped with a copper strip. This portion of the pole is called shaded part of the pole. Because of this, a revolving field is set up in the stator and torque is developed.
6. Reluctance start motor: Its stator is constructed with salient poles having non-uniform air gap. The axis of the magnetic flux shifts across the poles from longer air gap region to the shorter air-gap region. The required cage rotor starts rotating by induction phenomenon in the same direction to that of the rotation of stator field.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. The stator core of a single-phase, split winding induction motor is made of $\qquad$ —.
2. The direction of rotation of a single-phase, split winding induction motor can be reversed by interchanging $\qquad$ .
3. For a single-phase, 4-pole 50 Hz induction motor, if the rotor speed in 1440 rpm , then its slip is
$\qquad$ .
4. To obtain speeds more than 3000 rpm at 50 Hz $\qquad$ AC motors are used.
5. To operate compressor of a refrigerator usually $\qquad$ motor is employed.

## OBJECTIVE TYPE QUESTIONS

1. Stator core of a single-phase, split winding induction motor is made of
(a) laminated cast iron
(b) mild steel
(c) silicon steel stampings
(d) soft wood
2. Single-phase induction motor is self-starting motor.
(a) True
(b) False
3. Two field revolving theory is based upon 'Ferrari's Principle'.
(a) True
(b) False
4. According to 'Ferrari's Principle', an alternating field ( $\phi_{\mathrm{m}}$ ) can be represented by two fields of half the magnitude ( $\phi_{\mathrm{m}} / 2$ ) rotating in opposite direction at the same speed called synchronous speed.
(a) True
(b) False
5. In a single-phase induction motor at start, the two revolving fields produce
(a) unequal torques in the rotor conductors
(b) no torque in the rotor conductors
(c) equal and opposite torque in the rotor conductors
(d) equal torques in same direction in the rotor conductors
6. In split-phase induction motors, basically a phase difference is created between the current flowing through main winding and starting winding.
(a) True
(b) False
7. During running condition, if the starting winding of a split phase induction motor is disconnected,
(a) the motor will stop
(b) the motor winding will burn
(c) the main winding will be damaged
(d) the motor will continue to rotate
8. The starting torque of a capacitor start motor is
(a) more than a capacitor run motor
(b) less than a capacitor run motor
(c) less than a shaded pole motor
(d) less than a split-phase induction motor.
9. The function of the centrifugal switch is to disconnect the main winding during over load condition.
(a) True
(b) False
10. The centrifugal switch only disconnects the starting winding when motor is over loaded.
(a) True
(b) False
11. The direction of rotation of a split-phase induction motor can be reversed by
(a) reversing the connections of the supply terminals
(b) reversing the connections of the main winding only
(c) reversing the connections of the starting winding only
(d) Either (b) or (c)
12. The direction of rotation of the field in a shaded pole induction motor is from shaded part of the pole to unshaded part of the pole.
(a) True
(b) False
13. The rotor of a shaded pole induction motor rotates in opposite direction to that of the rotation of the revolving stator field.
(a) True
(b) False
14. The direction of rotation of the shaded pole induction motor cannot be reversed.
(a) True
(b) False
15. The efficiency of the shaded pole induction motor is very poor because of losses in the shading band.
(a) True
(b) False
16. The DC series motor can never be operated on AC.
(a) True
(b) False
17. The speed of a universal motor can never be more than synchronous speed.
(a) True
(b) False
18. Which motor is best suited for domestic refrigerator?
(a) Three-phase induction motor
(b) Universal motor
(c) Capacitor start motor
(d) Shaded pole motor
19. Which motor is best suited for sewing machine?
(a) Capacitor start motor
(b) Shaded pole motor
(c) Capacitor run motor
(d) Universal motor

## VIVA-VOCE/REASONING QUESTIONS

1. In case of single-phase capacitor run motor, if capacitor is damaged, the motor does not start. However if it is rotated in either direction, it picks up speed in that direction. Why?
2. The direction of rotation of a shaded pole motor cannot be reversed. Why?
3. For high speed (mixer guiders) which type of single phase AC motor is used and why?
4. The direction of rotation of a split-phase, single-phase induction motor can be reversed. How?
5. Can a series motor run at DC and AC both. How?

## SHORT ANSWER TYPE QUESTIONS

1. State the principle of double revolving field theory.
2. Name the two theories regarding single-phase induction motor. Which one of them is used extensively for explanation?
3. Is a single-phase induction motor self-starting?
4. Why single-phase induction motors are usually set on rubber spring mounts?
5. What is the effect of increasing rotor resistance in a single-phase induction motor?
6. In a single-phase induction motor, out of the two windings, that is, main winding and auxiliary winding, which should be more resistive?
7. What a split-phase motor?
8. How is the direction of rotation of a single-phase induction motor reversed?
9. State a resistance start motor has a high- or low-starting torque.
10. Does the capacitor start induction motor develop a high- or low-starting torque?
11. How does a capacitor start motor differ from a resistance start motor?
12. What is the advantage of using a capacitor start motor over a resistance start split-phase motor?
13. State the advantages of capacitor start capacitor run over capacitor start motor.
(PTU)
14. Define capacitor start and capacitor-run motor.
15. Name the motor being used in a ceiling fan.
16. What happens when the auxiliary winding of a capacitor motor is disconnected during running condition?
17. Does a shaded pole motor have much starting torque?
18. In what direction shaded pole motors rotates?
19. How can a shaded pole motor be reversed in the direction of rotation?
20. State the applications of shaded pole single-phase induction motors.
21. Why is the normal full-load slip of a single-phase induction motor higher than that of a three-phase motor?

## TEST QUESTIONS

1. A single-phase induction motor is not a self-starting motor. Explain.
(UPTU)
2. How will you make a single-phase induction motor self-starting?
3. Show that a single-phase sinusoidal field can be replaced by two fields rotating around the air gap in opposite directions. Sketch a torque-slip curve due to each of these two fields. How can the fact that
the single-phase induction motor has no starting torque be explained by these curves? How do they explain the fact of the motor accelerating in the direction in which it is started? State the manner in which the starting torque of a single-phase motor may be obtained by splitting the phase.
4. Explain why single-phase induction motor is not self-starting and discuss briefly any two methods used to produce starting torque in such motors.
(UPTU 2002-2003)
5. Explain why a single-phase induction motor does not develop starting torque.
(UPTU 2003-2004)
6. Describe along with connection diagram, the common starting methods of a single-phase induction motor.
(UPTU 2002-2003)
7. Explain the principle of operation of a single-phase induction motor.
(UPTU 2002-2004)
8. Explain the construction and working of a single-phase capacitor start induction motor.
9. Why is a capacitor employed with a ceiling fan?
10. Explain the working principle of a domestic electric fan. How is its speed controlled?
(UPTU 2001-2002)
11. A ceiling fan when switched on to a single-phase AC supply does not start rotating. Why?
12. Why should the auxiliary winding disconnected in a capacitor start single-phase induction motor after the motor picks-up speed?
(UPTU 2003-2004)
13. Explain the following types of single-phase induction motors with diagram and also draw their torque speed characteristics.
(Madras Univ. 1997)
(a) Split-phase induction motor
(b) capacitor start induction run motor.
14. Explain the construction (with sketch) and working of a capacitor-start capacitor-run single-phase induction motor. What are its advantages and practical applications?
15. How can you change the direction of rotation of a single-phase induction motor?
16. Describe the working and construction of a single-phase shaded pole motor.
17. Explain the working of a single-phase series motor. Name two electrical gadgets where these motors are used.
18. (a) Suggest suitable single-phase induction motors for following applications and give reasons for your choice:
(i) Ceiling fan;
(ii) Portable drilling machine;
(iii) Domestic refrigerators;
(iv) Sewing machine.
(b) Name five applications where you have seen single-phase induction motors being used.

## ANSWERS

## Fill in the Blanks

1. silicon steel
2. 0.04 or 4 per cent
3. the connections of either starting or running winding
4. commutator motors
5. capacitor start

Objective Type Questions

1. (c)
2. (b)
3. (a)
4. (a)
5. (c)
6. (a)
7. (d)
8. (a)
9. (b)
10. (b)
11. (d)
12. (b)
13. (b)
14. (a)
15. (a)
16. (b)
17. (b)
18. (c)
19. (d)


## LFARNING OBJFCTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* Why the machines used at different generating stations are called synchronous generators?
* What are the basic working principles of a synchronous machine?
* What are the various parts of a synchronous machine and what are their functions and material?
* When this machine works as a motor, how load and excitation affects its working?
* Why this machine is also known as synchronous condenser?
* How synchronous motor is made self-starting?
* What is hunting in synchronous machines?
* What are the major applications of synchronous motors?


### 14.1 INTRODUCTION

In an alternating current (AC) system, voltage level can be increased or decreased (as per requirement) very easily with the help of transformer, and therefore, this system is exclusively used for generation, transmission, and distribution of electric power. The mechanical power or energy is converted into electrical power or energy with the help of an AC machine called 'alternator or synchronous generator'. However, the same machine can be used to convert electrical power or energy into mechanical power or energy, and then, it is known as a synchronous motor. In fact,
the same machine can be operated as a generator or as a motor, and in general, it is known as a synchronous machine.

In this chapter, the salient features of synchronous machine working as a motor were discussed in detail.

### 14.2 SYNCHRONOUS MACHINE

A synchronous machine is an AC machine whose satisfactory operation depends upon the maintenance of the following relationship:

$$
N_{\mathrm{S}}=\frac{120 \times f}{P} \quad \text { or } \quad f=\frac{P \times N_{\mathrm{S}}}{120}
$$

Where $N_{\mathrm{s}}$ is the synchronous speed in rpm; $f$ is the supply frequency; and $P$ is the number of poles of the machine.

When connected to an electric power system, a synchronous machine always maintains this relationship. If a synchronous machine working as a motor fails to maintain this average speed $\left(N_{\mathrm{s}}\right)$, the machine will not develop sufficient torque to maintain its rotation and will stop. Then, the motor is said to be pulled out of step.

In case, the synchronous machine is operating as a generator, it has to be run at a fixed speed called synchronous speed to generate power at a particular frequency since all the electrical equipment and machines are designed to operate at this frequency. In India, the value of power frequency is 50 Hz .

### 14.3 BASIC PRINCIPLES

A synchronous machine is just an electromechanical transducer that converts mechanical energy into electrical energy or vice versa. The fundamental phenomena that make these conversions possible are as follows:

1. Law of electromagnetic induction: This relates to the production of emf, that is, emf is induced in a conductor whenever it cuts across the magnetic field (Fig. 14.1). This is called Faraday's first law of electromagnetic induction.
2. Law of interaction: This law relates to the phenomenon of production of force or torque, that is, whenever a current carrying conductor is placed in the magnetic field, by the interaction of the magnetic fields produced by the current carrying conductor and the main field, force is exerted on the conductor and torque is developed (Fig. 14.2).


Fig. 14.1 A coil rotating in a stationary magnetic field (generator action)


Fig. 14.2 A current carrying coil placed in a uniform magnetic field (motor action)

### 14.4 GENERATOR AND MOTOR ACTION

In generator action, an emf is induced in the armature conductors when they cut across the magnetic field. On closing the circuit, current flows through the armature conductors that produce another field. By the interaction of this field and main field, a force is exerted on the conductor, which acts opposite direction to that of rotation. This force against which the relative motion of conductors has to be maintained by the mechanical power supplied by the prime mover, and therefore, the mechanical power is converted into electrical power.

In motor action, a current is supplied to the machine, which flows through the armature conductors.
The armature conductors produce a field, which interacts with the main field. Therefore, a force is exerted on the conductors and rotation takes place (i.e., torque is developed). Once rotation occurs, an emf is induced in the conductors due to relative motion. This emf acts in opposite direction to the flow of current. The flow of current has to be maintained against this emf by applying external voltage source, and therefore, electrical power is converted into mechanical power.

### 14.5 PRODUCTION OF SINUSOIDAL ALTERNATING EMF

When a conductor or coil cuts across the magnetic field, an emf is induced in it by the phenomenon called electromagnetic induction. This can be achieved either by rotating a coil in the stationary magnetic field or by keeping the coil stationary and rotating the magnetic field. (The magnetic field can be rotated by placing the field winding on the rotating part of the machine.)

For illustration, Figures 14.3(a) and (b), two positions of a coil rotating in a stationary magnetic field is shown. Whereas in Figures 14.3(c) and (d), two positions of a rotating electromagnet in a coil placed on stationary armature is shown. At first instant, the emf induced in the coil is zero, since flux cut by the coil is zero. However, at second instant, the emf induced in the coil is maximum (positive). The two instants $t_{1}$ and $t_{2}$ are marked on the wave diagram shown in Figure 14.3(e). In one revolution, the induced emf completes one cycle and its wave shape is shown in Figure 14.3(e).


Fig. 14.3 (a) and (b) A coil rotating in a stationary magnetic field (c) and (d) A constant magnetic field produced by electromagnet rotating in a stationary coil (e) Wave diagram of induced emf

### 14.6 RELATION BETWEEN FREQUENCY SPEED AND NUMBER OF POLES

In Figure 14.4, a machine is shown having $P$ number of poles on the rotor revolving at a speed at $N_{\mathrm{s}} \mathrm{rpm}$. When a conductor passes through a pair of poles, one cycle of emf is induced in it.


Fig. 14.4 Wave shape of induced emf in a stationary coil when it passes through a pair of poles
$\therefore$ No. of cycle made per revolution $=\frac{P}{2}$
No. of revolutions made per second $=\frac{N_{\mathrm{S}}}{60}$
No. of cycles made per second

$$
\begin{aligned}
& =\text { No. of cycles } / \text { revolution } \times \text { No. of revolutions } / \mathrm{s} \\
f & =\frac{P}{2} \times \frac{N_{\mathrm{S}}}{60}=\frac{P \times N_{\mathrm{S}}}{120} \text { cycles } / \mathrm{s} \text { or Hz }
\end{aligned}
$$

### 14.7 CONSTRUCTIONAL FEATURES OF SYNCHRONOUS MACHINES

Only in small synchronous machines, the field system is placed on stator and armature winding on rotor, but in larger machines, the field winding is placed on the rotor and armature winding is placed on the stator. The rotating field and stationary armature system is preferred over stationary field and rotating armature system.

The important parts of a synchronous machine are stator, rotor, and miscellaneous.

1. Stator: The outer stationary part of the machine is called stator, and it has the following important parts:
(a) Stator frame: It is the outer body of the machine made of cast iron, and it protects the inner parts of the machine. It can be also made of any other strong material, since it is not to carry the magnetic field. Cast iron is used only because of its high mechanical strength.
(b) Stator core: The stator core is made of silicon steel material. It is made from number of stamping which are insulated from each other. Its function is to
provide an easy path for the magnetic lines of force and accommodate the stator winding.
(c) Stator winding: Slots are cut on the inner periphery of the stator core in which three-phase or single-phase winding is placed. Enamelled copper is used as winding material.
2. Rotor: The rotating part of the machine is called rotor. From construction point of view, there are two types of rotors, namely salient pole-type rotor and non-salient pole-type rotor.


Fig. 14.5 Sectional view of a salient pole type alternator
(a) Salient pole-type rotor: In this case, projected poles are provided on the rotor. Salient pole-type construction is suited for medium and low speeds and are usually employed at hydroelectric and diesel power plants as synchronous generators. Since the speed of these machines (generators) is quite low, to obtain the required frequency, the machines have large number of poles as shown in Figures 14.5 and 14.6. To accommodate such a large number of poles, these machines have larger diameter and small length. For a speed of 200 rpm (alternators coupled with water turbines), the diameter of the machines is as large as 14 m and length is only 1 m . The salient pole-type rotor has the following important parts:
(i) Spider: Spider is made of cast iron to provide an easy path for the magnetic flux. It is keyed to the shaft and at the outer surface, pole core and pole shoe are keyed to it (Fig. 14.6).

(a)

(b)

(c)

Fig. 14.6 (a) Rotor of a salient pole alternator (b) Pole shoe and pole core (c) Field winding
(ii) Pole core and pole shoe: It is made of laminated sheet material [Fig. 14.6(b)]. Pole core provides least reluctance path for the magnetic field and pole shoe distributes the field over the whole periphery uniformly to produce sinusoidal wave form of the generated emf.
(iii) Field winding or exciting winding: Field winding is wound on the former [Fig. 14.6(c)] and then placed around the pole core. Direct current (DC) supply is given to it through slip rings. When direct current flows through the field winding, it produces the required magnetic field.
(iv) Damper winding: At the outer most periphery, holes are provided in which copper bars are inserted and short-circuited at both the sides by rings forming damper winding.


Fig. 14.7 Sectional view of a non-salient pole type alternator


Field winding
Fig. 14.8 Rotor of a non-salient pole alternator
(b) Non-salient pole-type rotor: In this case, there are no projected poles, but the poles are formed by the current flowing through the rotor (exciting) winding. Non-salient pole-type construction is suited for the high speeds. The steam turbines rotate at a high speed $(3,000$ $\mathrm{rpm})$. When these turbines are used as prime mover for this machine working as a generator, a small number of poles are required for given frequency. Hence, these machines have smaller diameter and larger length. A non-salient pole-type rotor (Fig. 14.7) has the following parts:
(i) Rotor core: Rotor core is made of silicon steel stampings. It is keyed to the shaft. At the outer periphery, slots are cut in which exciting coils are placed. It provides an easy path to the magnetic flux.
(ii) Rotor winding or exciting winding: It is placed in rotor slots, and current is passed through the winding in such a way that poles are formed according to the requirement (Fig. 14.8).
3. Miscellaneous part: The following are few important miscellaneous parts:
(a) Brushes: Brushes are made of carbon and these just slip over the slip rings. DC supply is given to the brushes. The current flows from the brushes to the slip rings and then to the exciting winding.
(b) Bearings: Bearings are provided between the shaft and outer stationary body to reduce the friction. The material used for their construction is high carbon steel.
(c) Shaft: Shaft is made of mild steel. Mechanical power is taken or given to the machine through shaft.

### 14.8 ADVANTAGES OF ROTATING FIELD SYSTEM OVER STATIONARY FIELD SYSTEM

Following are the important advantages of rotating field system over stationary field system:

1. The armature winding is more complex than the field winding. Therefore, it is easy to place armature winding on stationary structure.
2. In the modern alternators (synchronous generators), high voltage is generated, and therefore, heavy insulation is provided, and it is easy to insulate the high-voltage winding when it is placed on stationary structure.
3. The size of the armature conductors is much more to carry heavy current, and therefore, high centrifugal stresses are developed. Therefore, it is preferred to place them on stationary structure.
4. The size of slip rings depends upon the magnitude of flow of current, and therefore, it is easy to deliver small current for excitation through slip rings of smaller size when rotating field system is used.
5. It is easier to build and properly balance high-speed rotors, when they carry the lighter field system.
6. The weight of rotor is small when field system is provided on rotor and as such friction losses are produced.
7. Better cooling system can be provided, when the armature is kept stationary.

### 14.9 THREE-PHASE SYNCHRONOUS MACHINES

Only small AC machines employed in household applications are single-phase machines. The large AC machines are usually three-phase machines. The major application of synchronous machines is as a generator employed at the generating stations.

At all the generating stations, three-phase synchronous generators are invariably employed because of the following reasons:

1. For the same size of frame and material, three-phase machines have nearly 1.5 times and output to that of single-phase machines.
2. Power can be transmitted and distributed more economically, when it is in the form of three phase than when it is in the form of single phase. Therefore, three-phase synchronous generators are employed for the generation of electrical power.
3. In the industries, for power conversion, three-phase induction motors are employed invariably since they are robust, more efficient, self-starting, operate at high-power factor, and very cheap in cost. Three-phase power is required for their operation, and therefore, it is preferred to generate (three-phase synchronous generators), transmit, and distribute electrical power adopting three-phase system.

### 14.10 EMF EQUATION

Let $P=$ No. of poles
$\phi=$ Flux per pole in Weber
$N=$ Speed in rpm
$f=$ frequency in Hz
$Z_{\mathrm{ph}}=$ No. of conductors connected in series per phase
$T_{\mathrm{ph}}=$ No. of turns connected in series per phase
${ }^{1} K_{\mathrm{c}}=$ Coil span factor
${ }^{2} K_{\mathrm{d}}=$ Distribution factor
Flux cut by each conductor during one revolution $=P \phi$ Weber
Time taken to complete one revolution $=\frac{60}{N}$ second
Average emf induced per conductor $=\frac{P \phi}{60 / N}=\frac{P \phi N}{60}$
Average emf induced per phase,

$$
\begin{aligned}
& =\frac{P \phi N}{60} \times Z_{\mathrm{ph}}=\frac{P \phi N}{60} \times 2 T_{\mathrm{ph}} \quad\left(\Theta T_{\mathrm{ph}}=\frac{Z_{\mathrm{ph}}}{2}\right) \\
& =4 \times \phi \times T_{\mathrm{ph}} \times \frac{P N}{120}=4 \phi f T_{\mathrm{ph}}
\end{aligned}
$$

RMS values of emf induced per phase,

$$
\begin{aligned}
& E_{\mathrm{ph}}=\text { Average value } \times \text { form factor } \\
& E_{\mathrm{ph}}=4 \phi f T_{\mathrm{ph}} \times 1.11=4.44 \times \phi \times f \times T_{\mathrm{ph}} \mathrm{~V}
\end{aligned}
$$

Taking into consideration, the coil span factor $\left(K_{\mathrm{c}}\right)$ and distribution factor $\left(K_{\mathrm{d}}\right)$ of the winding. Actual emf induced per phase

$$
E_{\mathrm{ph}}=4.44 K_{\mathrm{c}} K_{\mathrm{d}} \phi f T_{\mathrm{ph}} \mathbf{V}
$$

## Example 14.1

A three-phase 50 Hz , synchronous generator runs at 187.5 rpm . Find the number of poles of the machine? What type of prime mover would you expect for this machine?
(U.P.T.U.)

## Solution:

Frequency, $f=50 \mathrm{~Hz}$
Speed, $N_{\mathrm{s}}=187.5 \mathrm{rpm}$
${ }^{1}$ Coil span factor $\left(\boldsymbol{K}_{\mathbf{c}}\right)$ : The ratio of emf induced in a short-pitched coil to emf induced in a full-pitched coil is called coil span factor or pitch factor or chorded factor. When the winding is short pitched, coil span is less than the pole pitch. When the winding is full pitched, coil span is equal to the pole pitch. It is generally denoted by $K_{\mathrm{c}}$ and its value is always less than unity.
$\therefore$ Coil span factor, $K_{\mathrm{c}}=\frac{2 e \cos \beta / 2}{2 e}=\cos \beta / 2$
Where $\beta$ is the angle through which the coil is short pitched.
${ }^{2}$ Distribution factor $\left(\boldsymbol{K}_{\mathrm{d}}\right)$ : The ratio of induced emf in the coil group when the winding is distributed in number of slots to the induced emf in the coil group when the winding is concentrated in one slot is called a distribution factor or breadth factor. It is generally denoted by $K_{\mathrm{d}}$ and its value is always less than unity.
$\therefore$ Distribution factor, $K_{\mathrm{d}}=\frac{\sin \frac{m a}{2}}{m \sin \frac{\alpha}{2}}$ where, $m=$ No. of slots per pole per phase and
$a=180^{\circ} / \mathrm{No}$. of slots per pole, that is, slot pitch.

$$
P=\frac{120 \times f}{N_{\mathrm{s}}}=\frac{120 \times 50}{187.5}=32
$$

Since the speed of the synchronous generator is very low, the prime mover would be a water turbine (hydraulic turbine). For a large number of poles, the machine would be a salient pole type.

## Example 14.2

Calculate the no-load terminal voltage of a three-phase, four-pole, star-connected alternator running at 1500 rpm having following data: sinusoidally distributed flux per pole $=66 \mathrm{mWb}$; total no. of armature slots $=72$; no. of conductors per slot $=10$; distribution factor; $K_{d}=0.96$. Assume full-pitch windings.

## Solution:

For full-pitch winding, coil span factor, $K_{\mathrm{c}}=1$
Distribution factor is given, and therefore, it is not to be calculated.
No. of turns/phase,

$$
T_{\mathrm{ph}}=\frac{72 \times 10}{2 \times 3}=120
$$

Supply frequency,

$$
f=\frac{P \times N_{\mathrm{s}}}{120}=\frac{4 \times 1,500}{120}=50 \mathrm{~Hz}
$$

EMF induced per phase,

$$
\begin{aligned}
E_{\mathrm{ph}} & =4.44 K_{\mathrm{c}} K_{\mathrm{d}} f \phi T_{\mathrm{ph}} \\
& =4.44 \times 1 \times 0.96 \times 50 \times 66 \times 10^{-3} \times 120=1,688 \mathrm{~V}
\end{aligned}
$$

Since the alternator is star connected
No-load terminal voltage $, \quad E_{\mathrm{L}}=\sqrt{3} E_{\mathrm{ph}}=\sqrt{3} \times 1,688=2,924 \mathrm{~V}$

## Example 14.3

Calculate the no-load terminal voltage of a three-phase, eight-pole, star-connected alternator running at 750 rpm having following data:
Sinusoidally distributed flux per pole $=55 \mathrm{mWb}$
Total No. of armature slots $=72$
Number of conductors/slot $=10$
Distribution factor $=0.96$
Assume full-pitch windings.

## Solution:

No. of poles, $P=8$; speed, $N_{\mathrm{s}}=750 \mathrm{rpm}$
Flux, $\phi=55 \times 10^{-3} \mathrm{~Wb}$; no. of slots $=72$
No. of conductors/slot $=10$; distribution factor, $K_{d}=0.96$
For full-pitch winding,
Coil span factor, $K_{\mathrm{c}}=1$
Distribution factor is given, and therefore, it is not to be calculated.
No. of turns/phase, $\quad T_{\mathrm{ph}}=\frac{72 \times 10}{2 \times 3}=120$
Supply frequency,

$$
f=\frac{P N_{\mathrm{s}}}{120}=\frac{8 \times 750}{120}=50 \mathrm{~Hz}
$$

$$
\text { EMF induced per phase, } \quad \begin{aligned}
E_{\mathrm{ph}} & =4.44 K_{\mathrm{c}} K_{\mathrm{d}} f \phi T_{\mathrm{ph}} \\
& =4.44 \times 1 \times 0.96 \times 50 \times 10^{-3} \times 120=1,406.6 \mathrm{~V}
\end{aligned}
$$

Since the alternator is star-connected
No-load terminal voltage, $\quad E_{\mathrm{L}}=\sqrt{3} E_{\mathrm{ph}}=\sqrt{3} \times 1,406.6=2,436.3 \mathrm{~V}$

## 国夏 <br> PRACTICE EXERCISES

## Short Answer Questions

1. What is the relation between speed and frequency in a synchronous machine?
2. What is the basic principle of operation of synchronous generator?
3. What is the basic principle of operation of a synchronous motor?
4. Name the major parts of a salient pole alternator.
5. Name the parts of a non-salient pole type alternator.
6. What is the function and material used for the construction of spider used in salient pole alternator?
7. Why 3-phase machine is considered to be economical than a 1-phase machine of same capacity?
8. Define coil span factor.
9. Define distribution factor.
10. How coil span factor and distribution factor affect the magnitude of induced emf in an alternator.

## Test Questions

1. How a sinusoidal alternating emf is induced in a single-phase alternator?
2. Give the constructional details of a 3-phase salient pole alternator.
3. Give the constructional details of a 3-phase non-salient pole alternator.
4. Why rotating field system is preferred over stationary field system in large 3-phase alternators.
5. Derive an emf equation of a synchronous generator.

## Numericals

1. A 3 -phase, 50 Hz synchronous generator runs at 500 rpm . Find the number of poles of the generator? What type of prime mover would you expect for this machine? (Ans. 12, salient pole type)
2. A 3-phase, 6-pole, 1000 rpm , star connected alternator has flux per pole 50 m Wb , slots on the stator 108 and 6 conductors per slot. If distribution factor is 0.966 and coil span factor is 0.9848 , what will be the induced emf per phase and no-load terminal voltage. (Ans. $1140.5 \mathrm{~V}, 3583 \mathrm{~V}$ )

## 14.Il WORKING PRINCIPLE OF A THREE-PHASE SYNCHRONOUS MOTOR

When a three-phase supply is given to the stator of a three-phase wound synchronous motor, a revolving field is set up (anticlockwise) which rotates at a synchronous speed $\left(N_{\mathrm{S}}=\frac{120 \times f}{P}\right)$. This field is represented by the imaginary stator poles. At an instant as shown in Figure 14.9(a), the opposite poles of stator and rotor are facing each other (for simplicity, two pole machine is considered). As there is a force of attraction between them, an anticlockwise torque is produced in the rotor as the rotor poles are dragged by the stator-revolving poles or field.

After half a cycle, polarity of the stator poles is reversed, whereas the rotor poles could not change their position due to inertia. Therefore, like poles are facing each other and due to force of repulsion a clockwise torque is produced in the rotor as shown in Figure 14.9(b).

Hence, the torque produced in a three-phase synchronous motor is not unidirectional and as such this motor is not self-starting. However, if rotor of synchronous motor is rotated by some external means at the start so that it also reverses its polarity and since the polarity of stator poles is reversed after half a cycle, as shown in Figure 14.9(c), then a continuous force of attraction between stator and rotor poles will exist. This is called magnetic locking. Once the magnetic locking is obtained, the rotor poles are dragged by the stator-revolving field (imaginary poles) and a continuous torque is obtained. As the rotor poles are dragged by the stator revolving field, the rotor rotates at the same speed as that of stator-revolving field, that is, synchronous speed.

Therefore, a synchronous motor only runs at a constant speed called synchronous speed.


Fig. 14.9 (a) Torque developed at initial instant (b) Torque developed after $T / 2$ second (c) Torque developed after T/2 second when rotor is rotated by external means through half revolution

### 14.12 SYNCHRONOUS MOTOR ON LOAD

When a synchronous motor is connected to the lines and started by some external means, it starts rotating at synchronous speed. If the motor is running at no-load and has no losses, then the induced emf $E$ is equal and opposite to applied voltage $V$ as shown in Figure 14.10(a) and the stator and rotor poles are in line with each other as shown in Figure 14.10(b). The resultant emf and hence the current drawn by the motor is zero. Therefore, the motor is said to be floating on the lines.


Fig. 14.10 (a) Phasor representation of applied voltage and induced emf under ideal condition

However, in actual machine, some losses are always present with the result induced emf $E$ falls back by an angle $\delta_{0}$ relative to the stator poles as shown in Figure 14.11(b). This causes a resultant voltage $E_{\mathrm{r}}$ across the armature circuit and motor draws no-load current $I_{0}\left(I_{0}=E_{\mathrm{r}} / Z_{\mathrm{s}}\right)$ from the mains. This no-load current lags behind the resultant voltage by an angle $\theta$ where $\theta=\tan ^{-1} \frac{X_{\mathrm{s}}}{R} ; X_{\mathrm{S}}$ is the synchronous reactance, and $R$ is the resistance of armature stator). Since resistance is very small as compared to synchronous reactance, and therefore, angle $\theta$ is nearly $90^{\circ}$. The power drawn by the motor at no-load is $V I_{0} \cos \phi_{0}$ which is sufficient to meet with the losses and make the motor running continuously at synchronous speed.


Fig. 14.11 (a) Phasor representation of $V$ and $E$ at no-load (b) Position of stator and rotor field at no-load

However, when load is applied to the shaft on the motor, the rotor poles fall back a little more (angle $\delta$ ) relative to stator poles as shown in Figure 14.12(b). Hence, the torque angle increases to $\delta$ with the increase in load. This increases the resultant voltage $E_{\mathrm{r}}$ which in turn increases the current $I\left(I=E_{\mathrm{r}} Z_{\mathrm{s}}\right)$ drawn by the motor from the mains.

(a)

(b)

Fig. 14.12 (a) Phasor representation of $E$ and $V$ when the motor is on-load
(b) Position of stator and rotor field when the motor is on-load

Therefore, a synchronous motor is able to supply the power to increasing mechanical load, not by decrease in speed, but by shifting the position of the rotor poles (or induced emf $E$ ) with respect to the stator poles or field.

When load applied on the shaft of the motor is further increased, the induced, emf $E$ falls back further. Hence, load angle (torque angle) $\delta$ increases with the increase in load. When $\delta$ increases, the resultant voltage $E_{\mathrm{r}}$ increases and so the armature current $I$. If too great mechanical load is applied to the synchronous motor, the rotor is pulled out of synchronism, after which it comes to stand still.

This maximum value of torque that a motor can develop without losing its synchronism is called pull-out torque.

### 14.13 EFFECT OF CHANGE IN EXCITATION

Consider a synchronous motor loaded with a constant mechanical load and normal (100\%) excitation, that is, having induced emf equal to applied voltage $V$ in magnitude.

At the given load, it takes a current of $I$ amperes lagging behind the applied voltage $V$ by an angle $\phi$. Since applied voltage $V$ is constant, for a constant power, $I \cos \phi$, that is, active component of current will remain constant.

When the excitation is decreased, the induced emf decreases to $E^{\prime}$ in magnitude, keeping torque angle $\delta$ to be the same for constant load as shown in Figure 14.13(b). This increases the resultant voltage to $E_{\mathrm{r}}^{\prime}$ which is also shifted in clockwise direction. With the increase in resultant voltage, current increases to $I^{\prime}\left(I^{\prime}=E_{\mathrm{r}}^{\prime} / Z_{\mathrm{s}}\right)$. Since the phase angle $\theta$ between the resultant voltage and current is constant, therefore, current is also shifted in clockwise direction. This increases the phase angle between voltage and current to $\phi^{\prime}$ which in turn decreases the power factor to cos $\phi^{\prime}$, but the active component of current $I^{\prime} \cos \phi^{\prime}$ remains the same.

Therefore, with the decreases in excitation, synchronous motor draws more current from the supply mains at lower (lagging) power factor.

When the excitation is increased, the induced emf increases in magnitude, keeping torque angle $\delta$ to be the same for constant load. This decreases the resultant voltage which is also shifted in anticlockwise direction.

Since the phase angle $\theta$ between the resultant voltage and current is constant, therefore, current is also shifted in anticlockwise direction and decreases in magnitude till it comes in phase with the voltage vector as shown in Figure 14.13(c). At this instant, the current drawn by the synchronous motor is minimum and that of the power factor is maximum (i.e., one).

(a)

(b)

(c)
(d)

Fig. 14.13 (a) (b) (c) and (d) Vector position of various quantities when excitation is changed

Now, if the excitation is further increased, the induced emf increases to $E^{\prime \prime}$ in magnitude. This increases the resultant voltage to $E_{\mathrm{r}}^{\prime \prime}$ which is also shifted to anticlockwise direction. Therefore, current is also shifted to anticlockwise direction and its magnitude increases to $I^{\prime \prime}$. This increases the phase angle between voltage and current to $\phi^{\prime \prime}$ in opposite direction as shown in Figure 14.13(d), which makes the power factor leading.

Therefore, when synchronous motor is over excited, it draws more current at a leading power factor from the supply mains. A synchronous motor operating under this condition is also called synchronous condenser.

### 14.14 V-CURVES

While changing the excitation of a three-phase synchronous motor, keeping the load to be the same, the curve plotted between field current (If) and armature or load current ( $I$ ) is called V-curve.

It is named as V-curve because its shape resembles with the shape of English alphabet ' $V$ '.
When the excitation of a three-phase synchronous motor taking constant power is varied, it changes the operating power factor of the motor. If

$$
P=3 V I \cos \phi
$$

Where
$P=$ power input
$V=$ terminal voltage (phase value)
$I=$ armature current (phase value)
$\cos \phi=$ power factor
then for constant power input $P$ and terminal voltage $V$, only increase in power factor causes decrease in armature current $I$ and vice versa. Armature current will be minimum at unity power factor and increases when the power factor decreases on either side (lagging or leading).

Hence, variation in excitation (field current) causes the variation in armature current. If we plot a curve taking field current If on X-axis and the armature current $I$ on Y-axis, the curve so obtained is called V-curve because of its shape. The V-curve at different power inputs are shown in Figure 14.14.


Fig. 14.14 V-curves of a synchronous motor

### 14.15 APPLICATION OF SYNCHRONOUS MOTOR AS A SYNCHRONOUS CONDENSER

The power factor of a synchronous motor can be controlled over a wide range by adjusting its excitation. At no-load, when the motor is over excited, it may draw the current from mains, which leads the voltage by large angle nearly $90^{\circ}$. Hence, the motor acts like a static capacitor and is known as a synchronous condenser.

Therefore, an over-excited synchronous motor operating at no-load is called a synchronous condenser or synchronous capacitor.

When an over-excited motor is operated on the same electrical system to which some industrial load (induction motors, induction furnaces, arc furnaces etc.) is operating at lagging power factor, the leading reactive power supplied by the synchronous motor compensates for the lagging reactive power of industrial load and improves the overall power factor of the system. In large industrial plants, which have a low lagging power factor load, it is often found economical to install an over-excited synchronous motor (synchronous condenser), even though the motor is not required to drive a load.

Consider an industrial load $P_{\mathrm{L}}$ operating at a power factor $\cos \phi_{1}$. When an over-excited motor drawing power $P_{\mathrm{m}}$ is connected in parallel with the existing load as shown in Figure 14.15(a), some of the lagging reactive power of the industrial load in compensated by the leading reactive power of the motor (i.e., $P_{\mathrm{rm}}$ ) which improves the over-all power factor to $\cos \phi_{2}$ as shown in Figure 14.15(b).


Fig. 14.15 (a) Connections of 3-phase synchronous condencer for power factor improvement (b) Phasor diagram for power factor improvement

## Example 14.4

A three-phase synchronous motor is connected in parallel with a load of 500 kW at 0.8 p.f. lagging and its excitation is adjusted until it raises the total p.f. to 0.9 lagging. If the mechanical load on the motor including losses is 125 kW , calculate the kVA input to the synchronous motor and its p.f.
(P.T.U.)

## Solution:

Industrial load,

$$
P_{\mathrm{L}}=500 \mathrm{~kW}
$$

Load p.f.,

$$
\cos \phi_{L}=\text { lagging } ; \tan \phi=\tan \cos ^{-1} 0.8=0.75
$$

Reactive power of the industrial load,

$$
P_{\mathrm{rL}}=P_{\mathrm{L}} \tan \phi=500 \times 0.75=375 \mathrm{kVAR}
$$

Motor load,

$$
P_{\mathrm{m}}=125 \mathrm{~kW}
$$

Total active power,

$$
P=P_{\mathrm{L}}+P_{\mathrm{m}}=500+125=625 \mathrm{~kW}
$$

Power factor of total load,

$$
\begin{aligned}
& \cos \phi=0.9 \mathrm{lag} \\
& \tan \phi=\tan \cos ^{-1} 0.9=0.4843
\end{aligned}
$$

Total reactive power,

$$
P_{\mathrm{r}}=P \tan \phi=625 \times 0.4843=302.7 \mathrm{kVAR}
$$

Reactive power supplied by synchronous motor,

$$
P_{\mathrm{rm}}=P_{\mathrm{r}}-P_{\mathrm{rL}}=302.7-375=-72.3 \mathrm{kVAR}
$$

Input of the motor in kVA ,

$$
\begin{aligned}
& P_{\mathrm{am}}=\sqrt{P_{\mathrm{m}}^{2}+P_{\mathrm{rm}}^{2}} \\
& P_{a m}=\sqrt{(125)^{2}+(72.3)^{2}}=144.4 \mathrm{kVA}
\end{aligned}
$$

Power factor of the motor,

$$
\cos \phi_{\mathrm{m}}=\frac{P_{\mathrm{m}}}{P_{\mathrm{am}}}=\frac{125}{144.4}=0.8656 \text { leading }
$$

## Example 14.5

A $50 \mathrm{~kW}, 400 \mathrm{~V}$, three-phase synchronous motor is operating at full-load with an efficiency of $92 \%$. If the field current is adjusted to make its power factor 0.8 leading, estimate the armature current.
(P.T.U.)

Solution:
Rated power,

$$
P=50 \mathrm{~kW}=50 \times 10^{3} \mathrm{~W}
$$

Line voltage,

$$
V_{\mathrm{L}}=400 \mathrm{~V}
$$

Efficiency,

$$
\eta=92 \%=0.92
$$

Power factor,

$$
\cos \phi=0.8 \text { leading }
$$

Armature current,

$$
I=\frac{P}{\sqrt{3} V_{L} \cos \phi \times \eta}=\frac{50 \times 10^{3}}{\sqrt{3} \times 400 \times 0.8 \times 0.92}=98 \mathrm{~A}
$$

### 14.16 CHARACTERISTICS OF SYNCHRONOUS MOTOR

A synchronous motor has the following important characteristics:

1. It is inherently not a self-starting motor.
2. For a given frequency, it operates only at one speed called synchronous speed given by the expression $N_{\mathrm{s}}=120 \times f / P$.
3. It can be operated under a wide range of power factors both lagging and leading.
4. In addition to the motor being used for mechanical load, it is also used as a power factor improvement equipment and is known as synchronous condenser.
5. At no-load, it draws a very small current from the mains to meet the internal losses of the motor. With the increase in load, the torque angle $\delta$ increases due to which motor draws more current from the mains. After the input current reaches maximum (torque angle $\delta$ in nearly $90^{\circ}$ ), no further increase in load is possible. If the motor is further loaded, it goes out of synchronism and stops.

### 14.17 METHODS OF STARTING OF SYNCHRONOUS MOTORS

Since a synchronous motor is inherently not self-starting, the following methods are generally adopted to start the synchronous motor:

1. By means of auxiliary motor: A small induction motor called the pony motor (auxiliary motor) is mounted on the same shaft or coupled to synchronous motor as shown in Figure 14.16. The auxiliary motor should have the same number of poles as that of synchronous motor or preferably one pole pair less so that it can rotate the motor nearly at synchronous speed. Initially, supply is given to the pony motor. When it rotates the rotor of the synchronous motor near to the synchronous speed, the main switch and DC switch of the main synchronous motor are closed. The rotor poles are pulled into synchronism with the rotating field (poles) of the armature (stator) of the main motor. Then, supply to the auxiliary motor is disconnected, and it acts as a load on the main motor.


Fig. 14.16 Starting of synchronous motor by auxiliary motor
2. Self-starting: This is a most common method of starting a synchronous motor. In this method, the motor is first started as a squirrel-cage induction motor by providing a special winding on the rotor poles known as damper or squirrel-cage winding. This damper winding consists of number of copper bars embedded into the slots or holes provided on the outer periphery of the pole shoes, where salient poles are employed, and then short circuiting these bars by brazing them to end rings as shown in Figure 14.17. In a non-salient pole machine, the damper winding conductors are placed in the rotor slots above the main field winding and short circuited by the end rings.


Fig. 14.17 Starting of synchronous motor by using damper winding

When the synchronous motor (armature) is connected to three-phase supply mains, a revolving field is set up which causes the rotor to rotate as a squirrel-cage induction motor. As soon as motor attains about $65 \%$ synchronous speed, the rotor winding is connected to DC mains (exciter), and the rotor field is magnetically locked with the stator rotating field and the motor starts running runs as a synchronous motor.

### 14.18 HUNTING

When a synchronous motor is loaded, the rotor poles slightly fall back in position with respect to the stator field (poles) by an angle $\delta$ known as power angle or torque angle or retarding angle. As the load is gradually increased, this angle $\delta$ also increases gradually so as to produce more torque for coping with the increased load. If the load is suddenly thrown off, angle $\delta$ decreases suddenly and the rotor poles are pulled into almost exact opposition to the stator poles, but due to inertia of rotor and rotor poles travel too far. They are then pulled back again, and therefore, oscillations are set up around the equilibrium position, corresponding to new load. If these oscillations are too large, they may throw the motor out of synchronism and stops.

The oscillation of the rotor about its equilibrium position is known as hunting.
The hunting (oscillations) can be prevented by providing damper winding or squirrel-cage winding on the rotor pole faces. This damper winding consists of number of copper bars embedded into the slots provided on the outer periphery of the poles shoes and then short circuited by end rings. When hunting takes place, there is relative motion of the rotor with respect to the stator field, which sets up eddy currents in this winding which flow in such a way that it suppresses with oscillations.

The hunting can also occur when the machine is operating as an alternator. In this case also because of sudden change in electrical output or mechanical input, oscillations are set up in the rotor called hunting, which can be prevented by providing damper winding on the rotor.

### 14.19 APPLICATIONS OF SYNCHRONOUS MOTORS

The important applications of synchronous motors are as follows:

1. These are used to improve the power factor of large industries.
2. These are used at the substations to improve the power factor.
3. These are used to control the voltage at the end of transmission lines by varying their excitation.
4. These are used in textile mills, cement factories, mining industries, and rubber mills for power applications.

These motors are mostly used to drive equipment, which are operated at constant speed continuously, such as centrifugal pumps, centrifugal fans, air compressors, motor generator sets, blowers, etc.

## PRACTICE EXERCISES

## Short Answer Questions

1. What are the factors on which the speed of a revolving field depends when a 3-phase AC supply is given to a 3-phase synchronous motor?
2. For ideal synchronous motor what will be the value of torque angle $\delta$ ?
3. What happens to the torque angle $\delta$, when load is applied on a synchronous motor?
4. What will you expect, if the excitation (field current) is increased supplied to synchronous motor?
5. Under what conditions, a synchronous motor is called a synchronous condenser?
6. In synchronous machines, hunting is caused due to inertia, justify.

## Test Questions

1. Explain that 3-phase synchronous motor is not self-starting.
2. Explain the working of a 3-phase synchronous motor under load.
3. Explain the effect of change in excitation in a 3-phase synchronous motor.
4. What do you mean by V-curves? What is their significance.
5. How a synchronous condenser is used to improve the power factor of a system?
6. Mention the characteristics and applications of a 3-phase synchronous motor.
7. What are the starting methods of a 3-phase synchronous motor? Explain any one with neat sketches.
8. What is hunting ? How this effect is reduced?

## Numericals

1. An $80 \mathrm{~kW}, 400 \mathrm{~V}, 3$-phase synchronous motor is operating at $90 \%$ of its full-load with an efficiency of $90 \%$. If the load current is adjusted to make its power factor 0.8 leading, calculate the armature current.
(Ans. 144.34 A)
2. A 3-phase synchronous motor is connected in parallel with a load of 650 kW at 0.707 p.f. lagging and its excitation is adjusted until it raises the total p.f. to 0.9 lagging. If the mechanical load on the motor including losses is 150 kW , calculate the kVA input to the synchronous motor and its p.f.
(Ans. $303 \mathrm{kVA}, 0.495$ leading)

## SUMMARY

1. Synchronous machines: A machines that rotates only at synchronous speed $N_{\mathrm{s}}$ is called a synchronous machine. Its satisfactory operation depends upon the relation.

$$
N_{\mathrm{S}}=\frac{120 \times f}{P}
$$

2. Alternator: An AC machine that converts mechanical power or energy into AC electrical power or energy at a desired frequency ( 50 Hz in India) is called an alternator. It is also called as synchronous generator or simply AC generator. Its basic principle of operation is electromagnetic induction.
3. Synchronous motor: An AC machine that converts electrical power or energy into mechanical power or energy and rotates only at synchronous speed is called synchronous motor.
The basic principle of operation of a synchronous motor is torque development by the alignment of two fields. In this machine, the two fields are magnetically locked and rotor is dragged by the stator revolving field.
4. Construction of synchronous machines: Usually, machines of large size have stationary armature and rotating field system because of economy and simple designing. As per rotor construction, there are two types of synchronous machines, namely salient pole-type and non-salient pole-type machines
5. Applications: (i) Salient pole-type machines (alternators) are operated at low speeds and are coupled with water turbines at hydroelectric power plants. These machines have large number of poles, larger diameter, and smaller length. (ii) Non-salient pole-type machines (alternators) are operated at high speeds and are coupled with steam turbines at thermal power plants.
These machines have small number of poles, smaller diameter and larger length.
6. Three-phase synchronous machines: Larger AC machines are always three-phase wound machines because of their high efficiency and economy.
7. EMF education: $E_{\mathrm{ph}}=4.44 K_{\mathrm{c}} K_{\mathrm{d}} \phi f T_{\mathrm{ph}}$
8. Principle of operation of a synchronous motor: When three-phase supply is given to the stator of a three-phase synchronous motor, a revolving field is set up. At the same time, DC supply is given to the rotor which sets up rotor field. Stator revolving field tries to drag the rotor field with it but due to inertia of rotor, the rotor could not rotate.
Hence, three-phase synchronous motor is not self-starting.
However, if the inertia of the rotor is removed by giving initial torque (with the help of a pony motor or by rotating it initially as an induction motor), the motor can be started. Once the rotor is started, the rotor field is magnetically locked with the stator revolving field and it starts rotating at synchronous speed.
9. Synchronous motor on-load: When synchronous motor is loaded, the torque angle $\delta$ between $V$ and $E$ increases but the speed remains the same.
10. Effect of change in excitation: When excitation is increased, the $p . f$. at which machine operates improves. An over-excited machine operates at leading $p$.f. and is called a synchronous condenser
11. V-curves: The curves plotted between field current (If) and armature or load current (I) of a synchronous machine working as a motor gives a shape of letter ' $V$ ' and hence are called V-curves.
12. Hunting: When load on a synchronous machine changes suddenly, the torque angle changes and the rotor tries to obtain the new position. Before obtaining the final position, the rotor oscillates around it. These oscillations of the rotor about its equilibrium position are known as hunting.
13. Damper winding: To reduce hunting, damper winding is provided.

## TEST YOUR PREPARATTON

## 7 FILL IN THE BLANKS

1. The satisfactory operation of synchronous machines depends upon the relation $N_{\mathrm{S}}=$ $\qquad$
2. Synchronous motor can be made to work on leading $p$.f. by increasing its $\qquad$ -.
3. The rating of an alternator is given in $\qquad$ .
4. The high-speed alternators are $\qquad$ type.
5. The speed regulation of a synchronous motor is $\qquad$ .
6. The synchronous motor working at a leading $p f$ is known as $\qquad$ .
7. The speed of four-pole, 25 Hz synchronous machine will be $\qquad$ rpm.
8. The decrease in excitation to synchronous motor causes $\qquad$ power factor.
9. When load on a synchronous machine is changed suddenly, the rotor oscillates around its final position, this phenomenon is called $\qquad$ _.
10. When load on a synchronous motor increases, its speed $\qquad$ .

## OBJECTIVE TYPE QUESTIONS

1. In an alternator, the eddy current and hysteresis losses occur in
(a) Iron of field structure only.
(b) Iron of armature structure only.
(b) Both (a) and (b)
(d) None of the above
2. The frequency of voltage generated in large alternators in India is
(a) in megacycles.
(b) in kilocycles.
(c) 60 Hz .
(d) 50 Hz .
3. The rotors preferred for alternators coupled to hydraulic turbines are
(a) salient pole type.
(b) cylindrical rotor type.
(c) solid rotor type.
(d) Any of the above
4. Salient pole-type alternators are generally employed with
(a) high-speed prime movers.
(b) low- and medium-speed prime movers.
(c) hydrogen-cooled prime movers.
(d) low-voltage alternators.
5. RMS value of voltage generated per phase in an alternator is given by
(a) $E_{\mathrm{ph}}=4.44 K_{\mathrm{c}} K_{\mathrm{d}} N \phi f$
(b) $E_{\mathrm{ph}}=4.44 K_{\mathrm{c}} K_{\mathrm{d}} N \phi$
(c) $E_{\mathrm{ph}}=4.44 K_{\mathrm{c}} K_{\mathrm{d}} N^{2} \phi f$
(d) $E_{\mathrm{ph}}=1.11 K_{\mathrm{c}} K_{\mathrm{d}} N \phi f$
6. The rating of alternators is usually expressed in
(a) full-load current.
(b) Horse power.
(c) kVA.
(d) kW .
7. Synchronous motors are not self-starting because
(a) starters cannot be used on these machines.
(b) starting winding is not providing on these machines.
(c) the direction of rotation is not reversed.
(d) the direction in instantaneous torque reverses after half cycle.
8. A pony motor is basically a
(a) DC series motor.
(b) DC shunt motor.
(c) double-winding $\mathrm{AC} / \mathrm{DC}$ motor.
(d) small induction motor.
9. A synchronous motor can be started by
(a) providing damper winding.
(b) pony motor.
(c) DC compound motor.
(d) Any of the above
10. A three-phase synchronous motor will have
(a) no slip rings.
(b) two slip rings.
(c) three slip rings.
(d) four slip rings.
11. Cage winding in a synchronous motor carries
(a) high starting and running current.
(b) no starting current.
(c) no running current.
(d) no starting and running current.
12. Slip rings in a synchronous motor carry
(a) DC.
(b) AC.
(c) Both (a) and (b)
(d) no current.
13. A Synchronous motor working on leading p.f. at no-load is known as
(a) condenser
(b) Synchronous condenser
(c) inverter
(d) convertor.
14. The maximum value of torque that a synchronous motor can develop without losing its synchronism is called
(a) slip torque.
(b) pull-out torque.
(c) breaking torque.
(d) synchronous torque.
15. The armature current in a synchronous motor will be least when p.f. is
(a) zero.
(b) unity.
(c) leading.
(d) lagging.
16. When the field of a synchronous motor is under excited, the p.f. will be
(a) zero.
(b) unity.
(c) lagging.
(d) leading.
17. Operating speed of a synchronous motor can be changed to new fixed value by
(a) changing the load.
(b) changing the supply voltage.
(c) changing frequency.
(d) using brakes.
18. A synchronous motor can be made self-starting by providing
(a) damper winding on rotor pole.
(b) damper winding on stator.
(c) damper winding on stator and rotor.
(c) None of the above
19. Synchronous speed for a synchronous motor is given by
(a) $200 \mathrm{f} / \mathrm{p}$.
(b) $120 \mathrm{f} / \mathrm{p}$.
(c) $120 \mathrm{p} / \mathrm{f}$.
(d) 120 f.p.
20. A synchronous motor can be used as a synchronous capacitor when it is
(a) under loaded.
(b) over loaded.
(c) under excited.
(d) over excited.

## NUMERICALS

1. A three-phase, 50 Hz , synchronous generator runs at 166.67 rpm . What is the number of poles? What prime over would you except for this machine?
(P.T.U.) (Ans. 36, Hydraulic turbine)
2. The stator core of a four-pole, three-phase alternator has 36 slots. It carries a short pitch three-phase winding with coil span equal to 8 slots. Determine the distribution and coil pitch factor.
(U.P.T.U. Type) (Ans. 0.9598, 0.9848)
3. A three-phase, 12-pole alternator has a star-connected winding with 108 slots and 10 conductors per slot. The coils are full pitched. The flux per pole is 0.05 Wb sinusoidally distributed and speed is 600 rpm . Calculate the line voltage on open circuit. Assume distribution factor equal to 0.96 .
(P.T.U.) (Ans. 3986.7 V)
4. A synchronous motor improves the p.f. of a load of 600 kW from 0.8 lagging to 0.9 lagging. Simultaneously, the motor carries a load of 150 kW . Find (i) reactive power supplied by the motor, (ii) $k V A$ rating of the motor, and (iii) the p.f. at which the motor operates.
(Ans. 60.19 kVAR leading, $161.62 \mathrm{kVA}, 0.928$ leading)

## VIVA-VOCE/REASONING QUESTIONS

1. A synchronous machine (generator or motor) is named as synchronous machine. Why?
2. The synchronous generators employed at hydroelectric power plant have larger diameter and smaller length. Why?
3. The synchronous generators employed at steam power plants have smaller diameter and larger length. Why?
4. In large synchronous machines, stationary armature and rotating field system is used. Why?
5. The outer frame of a synchronous machine may not be made of magnetic material (cast iron)., Why?
6. A synchronous motor cannot run at a speed less than synchronous speed. Why?
7. Synchronous machine is also called a synchronous condenser. When and why?
8. A three-phase synchronous motor is not self-starting. Why?
9. Damper winding is provided in synchronous machines. Why?

## SHORT ANSWER TYPE QUESTIONS

1. What factors determine the number of poles of a synchronous motor?
(U.P.T.U. Nov. 1995)
2. Why a three-phase synchronous motor will always run at synchronous speed?
(Madras Univ. April 1997)
3. A synchronous motor runs at $N_{\mathrm{s}} \mathrm{rpm}$ at full-load. What will be its speed if the load is reduced to half the full-load?
4. What is the effect of increase in excitation of a synchronous motor? (Madras Univ. April 1997)
5. How the power factor of a synchronous motor is changed keeping the shaft load undisturbed?
6. Does the change in excitation affect the synchronous motor speed and power factor?
(Madras Univ. April 1996)
7. What is meant by $V$-curves of a synchronous condenser?
(M.K. Univ. Nov. 1996)
8. What is the common starting method used for synchronous motor?
9. What is meant by synchronous condenser?
(M.K. Univ. Nov. 1995)
(M.K. Univ. Nov. 1996)
10. What are the advantages of synchronous motor?
11. What is the use of synchronous condenser?
(Madras Univ. April 1997)

## TEST QUESTIONS

1. Deduce the relation between number of poles frequency and speed of an alternator.
2. Name the various part of a synchronous machine. Give the function and material used for each of them.
3. Give the constructional details of cylindrical rotor alternator.
(P.T.U.)
4. Explain the difference between salient pole and cylindrical pole type of rotor used in alternators. Mention their applications.
5. Explain why the stator core of an alternator is laminated.
(U.P. Tech. Univ.)
6. List the advantages of making field system rotating and armature stationary in case of an alternator.
(P.T.U.; U.P.T.U.)
7. Derive an expression for induced emf for an alternator.
8. Why are the alternators rated in kVA ?
9. Explain the principle of operation of a synchronous motor.
(P.T.U.)
10. Explain how a synchronous motor is made self-staring.
(U.P. Tech. Univ.)
11. What is the effect on speed if load on a synchronous motor is increased?
(P.T.U.)
12. A synchronous motor is running at lagging p.f. at a given load. Explain with the help of phasor diagram of improving this p.f. to unity. Suggest industrial applications of this property of the synchronous motor.
13. Show by means of a phasor diagram how changing the excitation of a synchronous motor causes it to work (i) as a condenser and (ii) as an inductor.

Explain clearly the effects of excitation on performance of synchronous motor.
14. Draw and explain V-curves of synchronous machine.
(P.T.U.)
15. Explain any two methods of starting a synchronous motor.
16. Suggest the various application of a synchronous motor.
17. Explain the phenomenon of hunting in alternators.
18. What is hunting in synchronous machines? What are the constructional features incorporated to minimize hunting effect? In turbo-alternators, no additional features are incorporated to offset this effect. Why?

## ANSWERS

Fill in the Blanks

1. $\frac{120 \times f}{P} \mathrm{rpm}$
2. excitation
3. kVA
4. Non-salient
5. Zero
6. synchronous condenser
7. 750
8. lagging and poor
9. hunting
10. remains the same

Objective Type Questions

1. (b)
2. (d)
3. (a)
4. (b)
5. (a)
6. (c)
7. (d)
8. (d)
9. (d)
10. (b)
11. (c)
12. (a)
13. (c)
14. (d)
15. (d)
16. (c)
17. (c)
18. (a)
19. (b)
20. (d)

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## LPARNING OBJECTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What are natural sources of energy? What are renewable and non-renewable sources of energy?
* What is wind energy and its conversion into electrical energy?
*What is solar energy and its conversion into electrical energy?
* What is fuel cell and its application for the conversion of chemical energy into electrical energy?
*What is hydropower and hydroelectric power generation?
*What is tidal power or energy and its conversion into electrical energy?
* What is geothermal energy and its conversion into electrical energy?
*What is thermal (steam, diesel, and gas) energy and its conversion into electrical energy?
* What are nuclear power plants?
*What is cogeneration and distributed generation?


### 15.1 INTRODUCTION

Nature has blessed us with unlimited resources of energy. We are to see how we can exploit these resources to meet with our requirements. In fact, we require energy in one form or the other for our daily activities. For example, for cooking, we require heat energy obtained by burning
any fossil fuel such as wood, coal, kerosene, crude oil, or LPG. Practically, electrical energy is needed to perform various activities we do from the time we get up until we sleep. Petrol or diesel is required to produce energy for our vehicles, while industry uses energy in various forms. It shows that use of energy is very essential for the survival of the modern society.

It is not out of place to mention here that the various forms of energy that we are using at present were not available to the people in ancient times. The development of various sources of energy was slow but systematic. With the fear that the conventional sources of energy, such as petrol, LPG, CNG, and coal may not last long, efforts are being made to exploit alternative non-conventional sources of energy.

### 15.2 CLASSIFICATION OF SOURCES OF ENERGY

All sources of energy are classified as renewable sources of energy and non-renewable sources of energy.

### 15.2.1 Renewable Sources of Energy

The sources of energy that are present in an unlimited quantity in nature and get replenished through some natural process are called renewable sources of energy. Renewable sources of energy are also called inexhaustible sources of energy because these sources do not get exhausted by normal human activities. Wind, water, solar radiations, geothermal energy, biomass, ocean waves, etc., are renewable (or inexhaustible) sources of energy.

### 15.2.2 Non-renewable Sources of Energy

The sources of energy that are present in a limited quantity in nature and are not replenished by any natural process are called non-renewable sources of energy. Non-renewable sources of energy are also called exhaustible sources of energy, because these sources get exhausted by normal human activities over a certain period of time. Coal, petroleum, natural gas (fossil fuels), and fissionable materials like uranium are non-renewable (or exhaustible) sources of energy.

Electrical energy is very handy, efficient, flexible, and easy to control. Hence, first, every effort is made to convert all the renewable sources of energy to electrical energy, and then it can be used in any other form as per the requirement of consumer. In this chapter, we shall confine our attention on generation of electrical energy by exploiting renewable energy resources.

### 15.3 INTRODUCTION TO WIND ENERGY

The surface of earth is heated unevenly by the sun depending upon the angle of incidence of the sun rays and their duration. The heat energy absorbed by the earth's surface is transferred to air directly above it. The hot air develops pressure. As the earth's surface is headed unevenly, a pressure difference occurs in the atmosphere, which moves the air causing wind.

Air in motion (blowing air) is called wind. Wind contains kinetic energy. Total wind energy flowing through an imaginary area ' $A$ ' during time ' $t$ ' is given as,

$$
E=\frac{1}{2} m v^{2}=\frac{1}{2}(A v t \rho) v^{2}=\frac{1}{2} A t \rho v^{3}
$$

where $m=$ mass of moving air (wind)
$A=$ imaginary area perpendicular to direction of wind
$v=$ velocity of blowing air
$t=$ time of duration
$\rho=$ density of air
Avt $\rho$ is, therefore, mass ( $m$ ) of wind and Avt is the volume of air.
Therefore, kinetic energy of wind/unit volume $=\frac{1}{2} \rho v^{2}$

### 15.3.1 Utilization of Wind Energy

The kinetic energy of wind (wind energy) can be utilized for doing some useful work in the following ways:

1. Wind energy is used for transporting people and materials by using sailboats. Wind energy is used for pumping out underground water, grinding of grains such as wheat, grams, and maize by using windmills.
2. Wind energy is used for generating electricity by using windmills.

## Advantages of wind energy

Wind energy has the following advantages:

1. Wind energy is cheap and inexhaustible.
2. Wind energy does not cause any pollution.

## Disadvantages of wind energy

Wind flowing with a sufficient speed is not available everywhere and all the time. Thus, wind is not a dependable source of energy.

### 15.3.2 Factors Affecting Wind

Speed and direction of the wind are affected by the following factors:

1. Location on the earth
2. Rotation of the earth
3. Local conditions
4. Height from the ground

The wind blows fast at high altitudes because of the reduced influence of drag.

### 15.3.3 Wind Map of a Site

To use energy efficiently and on commercial scale, wind should have a speed of above $20 \mathrm{~km} / \mathrm{h}$. Therefore, before setting up a windmill, we should locate the areas where sufficient wind is available for most of the time. This is done by preparing wind maps of all the probable sites.

A graphical record of wind speed at a certain fixed height (say 10 m ) above ground at any given location for the whole year is termed as the wind map of that location.

## Information obtained from a wind map

A wind map provides the following information:

1. A wind map gives an idea about the annual average wind speed and the average wind speed for January and July at any location. January is the typical month for low speed wind, while July is for typical high speed wind.
2. A wind map gives information about the energy available (in kWh unit) per square metre area of wind stream at a height of 10 m from the ground.

### 15.3.4 Wind Power Capacity and Production



Fig. 15.1 Wind turbine

Wind turbines, as shown in Figure 15.1, are used to convert wind energy into electrical energy. In this case, a generator is mounted on the same shaft to which blades of the turbine are mounted. When wind pressure (by the kinetic energy of wind) is exerted on the blades, the turbine spins and electric power is generated by the generator.

At present, there are 2,00,000 wind turbines operating worldwide. Their total nameplate capacity was $2,82,482 \mathrm{MW}$ by the end of 2012 . The European Union alone passed some 1,00,000 MW nameplate capacity in September 2012, while the United States and China both surpassed 50,000 MW in August 2012.

### 15.3.5 Wind Farm

A wind farm or wind park is a group of wind turbines in the same location used to produce electrical energy. A large wind farm may consist of several hundred individual wind turbines and cover an extended area of hundreds of square kilometre, but the land between the turbines may be used for agricultural or other purposes. A wind farm may be located onshore or offshore.

## Onshore wind farms

Onshore wind farms operate on land, as shown in Figure 15.2, generally in places of high altitudes or in large open spaces where the wind tends to be the strongest.

## Advantages

Onshore wind farms offer several key advantages over offshore farms.

1. The turbines, for instance, are much easier and less expensive to set up, maintain, and operate than offshore turbines.
2. Onshore farms also have easier access to the utility grid with lesser cost.

## Disadvantages

1. Weaker and more turbulent winds are available than those of offshore wind farms.
2. These projects often face opposition from area residents due to aesthetic disruption of landscape as well as the noise pollution issues.

## Offshore wind farms

Offshore wind farms operate on the sea, as shown in Figure 15.3, some of them may be near the sea shore, where turbines can be installed comfortably.

## Advantages

1. Offshore wind farms take advantage of the strong, smooth ocean winds that create high quality wind capacities.
2. Noise pollution is also not a factor with offshore wind farms because their sound does not carry to the shore. Many offshore wind farms are also far enough out to sea to be out of visual sight from the shoreline.


Fig. 15.2 Onshore wind farm


Fig. 15.3 Offshore wind farm

## Disadvantages

1. The deeper waters far offshore make installing wind turbines much more difficult and costlier than onshore or near shore locations.
2. Offshore farms are much more difficult and expensive to connect to the utility grid than onshore or near shore farms.

However, despite the added expense of placing wind farms far offshore, the winds further out to sea offer greater potential wind capacity than those onshore or closer to the coast.

### 15.3.6 Largest Wind Farm in India

The largest wind farm in India is Jaisalmer Wind farm (see Fig. 15.4). Its present capacity is 1,064 MW. The development of the wind park was initiated by Suzlon in August 2011.

## 6

Basic Electrical Engineering


Fig. 15.4 Jaisalmer wind farm (India, Rajasthan)

### 15.3.7 Development of Wind Power in India

The development of wind power in India began in 1990s and has significantly increased in the last few years. Although relatively India is a newcomer to the wind power generation when compared with Denmark or the United States, but as on today, India has become the fifth largest installed wind power capacity in the world. In 2009-2010, India's growth rate was the highest among the other top four countries.

As of 31 January 2013, the installed capacity of wind power in India was $19,564.95 \mathrm{MW}$, mainly spread across different states as follows:

1. Tamil Nadu: $7,154 \mathrm{MW}$
2. Gujarat: 3,093 MW
3. Maharashtra: 2,976 MW
4. Karnataka: 2,113 MW
5. Rajasthan: 2,355 MW
6. Madhya Pradesh: 386 MW
7. Andhra Pradesh: 435 MW
8. Kerala: 35.1 MW
9. Orissa: 2 MW
10. West Bengal: 1.1 MW
11. Other states: 3.20 MW

It is estimated that $6,000 \mathrm{MW}$ of additional wind power capacity will be installed in India by 2015. The share of wind power is $8.5 \%$ of India's total installed power capacity. As on today (2013), India is the world's fifth largest wind power producer, with a generation capacity of 8,896 MW.

### 15.3.8 Variability

Electricity generated from wind power can be highly variable at several different timescales: hourly, daily, or seasonally. Annual variation also exists, but it is not significant. Because
instantaneous electrical generation and consumption must remain in balance to maintain grid stability, this variability can present substantial challenges to incorporate large amounts of wind power into a grid system. Intermittency and the non-dispatchable nature of wind energy production can raise costs for regulation, incremental operating reserve and (at high penetration levels) could require an increase in the already existing energy demand management, load shedding, storage solutions, or system interconnection with HVDC cables.

### 15.3.9 Reliability

Wind power hardly ever suffers major technical failures, since failures of individual wind turbines have hardly any effect on overall power, so that the distributed wind power is highly reliable and predictable.

### 15.3.10 Environmental Effects

In comparison to the environmental impact of traditional energy sources, the environmental impact of wind power is relatively minor in terms of pollution. Wind power consumes no fuel and emits no air pollution, unlike fossil fuel power sources. The energy consumed to manufacture and transport the materials used to build a wind power plant is equal to the new energy produced by the plant within a few months. While a wind farm may cover a large area of land, many land uses such as agriculture are compatible, with only small areas of turbine foundations and infrastructure made unavailable for use. However, there are some reports of negative effects from noise on people who live very close to wind turbines.

### 15.4 INTRODUCTION TO SOLAR ENERGY

Solar power is the conversion of sunlight into electricity. Sun is the primary source of energy. Sun radiates energy in the form of electromagnetic waves that include heat (infrared rays), light, and a small quantity of ultraviolet radiation. It is estimated that the sunlight falling on the earth per day delivers energy that is 50,000 times the total energy used all over the world in one year. One square metre of the earth's upper atmosphere receives 1.36 kJ of energy per second. Out of this, only $47 \%$, that is, 0.64 kJ of energy reaches on area of one square metre of the earth per second.

The solar energy received from sun can be converted directly into electricity using photovoltaic (PV) cells or indirectly with concentrated solar power (CSP), which normally focuses the sun's energy to boil water, which is then used to provide electric power. Mostly PV cells were initially and still are used to power small- and medium-sized applications from the calculator powered by a single solar cell to off-grid homes powered by a PV array (panel). They are an important and relatively inexpensive source of electrical energy where grid power is inconvenient, unreasonably expensive to connect, or simply unavailable. However, in the recent years, fast developments have taken place in this area due to which large solar power plants have come in picture. Let us see how CSP and PV technologies are used for the generation of electricity.

### 15.4.1 Concentrating Solar Power



Fig. 15.5 (a) Parabolic trough solar farm

In CSP systems, lenses or mirrors with tracking systems are used to focus a large area of sunlight into a small beam. The concentrated heat is then used as a heat source for a conventional power plant. A wide range of concentrating technologies are existing, but out of them, the most developed technologies are the parabolic trough, the concentrating linear Fresnel reflector, the Stirling dish, and the solar power tower. Various techniques are used to track the sun and focus light at a particular area. In all of these systems, a working fluid is heated by the concentrated sunlight and is then used for power generation or energy storage. Thermal storage efficiently allows us to generate electricity during all 24-h period. A diagram of a parabolic trough solar farm and an end view of how a parabolic collector focuses sunlight onto its focal point is shown in Figure 15.5(a).


Fig. 15.5 (b) End view of parabolic collection and how it focuses the sunlight

## Application of parabolic trough technology

A parabolic trough solar farm is shown in Figure 15.5(b). It consists of a linear parabolic reflector that concentrates light onto a receiver positioned along the reflector's focal line. The receiver is a tube positioned right above the middle of the parabolic mirror and is filled with a working fluid. The reflector is made to follow the sun during the daylight hours by tracking along a single axis. Parabolic trough systems provide the best land-use factor of any solar technology. The SEGS ${ }^{1}$ plants in California are using this technology.

## Application of concentrating linear Fresnel technology

Compact linear Fresnel reflectors are CSP-plants in which many thin mirror strips are used instead of parabolic mirrors to concentrate sunlight on two tubes with working fluid. This has the advantage that flat mirrors can be used, which are much cheaper than parabolic mirrors, and that more reflectors can be placed in the same amount of space, allowing more of the available sunlight to be used. Concentrating linear Fresnel reflectors can be used in either large or more compact plants.

## Application of Stirling dish technology

The Stirling solar dish that combines a parabolic concentrating dish and a Stirling engine, as shown in Figure 15.6, can also be used to drive an electric generator. The advantages of

[^11]Stirling solar over PV cells are higher efficiency of converting sunlight into electricity and longer lifetime. Parabolic dish systems give the highest efficiency among CSP technologies. The $50-\mathrm{kW}$ Big Dish in Canberra, Australia is an example of this technology.

## Application of solar power tower technology

A solar power tower uses an array of tracking reflectors (heliostats) to concentrate light on a central receiver atop a tower. Power towers are more cost effective, offer higher efficiency, and better energy storage capability among CSP technologies. The PS10 Solar Power Plant, shown in Figure


Fig. 15.6 Stirling dish type solar power plant 15.7, and PS20 solar power plant are examples of this technology.


Fig. 15.7 PS-10 Solar power plant

### 15.4.2 Photovoltaic Solar Power

A solar cell, or PV cell, is a device that converts light into electric current using the photoelectric effect. The first solar cell was constructed by Charles Fritts in 1880s. The German industrialist Ernst Werner von Siemens was among those who recognized the importance of this discovery. In 1931, the German engineer Bruno Lange developed a photocell using silver selenide $\left(\mathrm{Ag}_{2} \mathrm{Se}\right)$ in the place of copper oxide, although the prototype selenium cells converted less than $1 \%$ of incident light into electricity. Following the work of Russell Ohl in 1940s, researchers Gerald Pearson, Calvin Fuller, and Daryl Chapin created the silicon solar cell in 1954. These early solar cells cost 286 USD/W and their efficiency was 4.5-6\%.

## Photovoltaic power systems

A simplified schematics of a grid-connected residential PV power system is shown in Figure 15.8(a). Solar cells produce DC power that fluctuates with the sunlight's intensity. For practical


Fig. 15.8 (a) Grid connected residential PV power system
use, this usually requires conversion to certain desired voltages or AC through the use of inverters. Multiple solar cells are connected inside a module. Module are wired together to form arrays, then tied to an inverter, which produces AC power at the desired frequency.

Many residential systems are connected to the grid wherever available, especially in developed countries with large markets. In these grid-connected PV systems, use of energy storage is optional. In certain applications such as satellites, lighthouses, or in developing countries, batteries or additional power generators are often added as backups. Such stand-alone power systems permit operations at night and at other times of limited sunlight.

## Development and deployment

The early development of solar technologies starting in 1860s was driven by an expectation that coal would soon become scarce. However, development of solar technologies stagnated in


Fig. 15.8 (b) Nellis solar power plant, 14 MW power plat installed (2007) in Nevada, USA
the early 20th century in the face of the increasing availability, economy, and utility of coal and petroleum. In 1979, energy crisis caused a reorganization of energy policies around the world and brought renewed attention to developing solar technologies. Deployment strategies focused on incentive programs that were launched by number of countries.

Between 1970 and 1983, PV installations grew rapidly, but falling oil prices in the early 1980s moderated the growth of PV from 1984 to 1996. Since 1997, PV development has accelerated due to supply issues with oil and natural gas, global warming concerns, and the improving economic position of PV relative to other energy technologies. Photovoltaic production growth has averaged $40 \%$ per year since 2000. Many large PV power stations were developed between 2000 and 2010 and many of them are still under construction. A summary of electricity generation from solar energy is given in Table 15.1.

## Table 15.1 Summary of Electricity Generation from Solar

| Year | Energy (TWh) | $\%$ of Total |
| :--- | :---: | :--- |
| 2005 | 3.7 | 0.02 |
| 2006 | 5.0 | 0.03 |
| 2007 | 6.7 | 0.03 |
| 2008 | 11.2 | 0.06 |
| 2009 | 19.1 | 0.09 |
| 2010 | 30.4 | 0.14 |
| 2011 | 58.7 | 0.27 |
| 2012 | 93.0 | 0.41 |

The abovementioned data clearly shows that how fast this energy is being exploited. Rather, we have to make more efforts to exploit it further.

## World's largest PV power stations

Some of the important world's largest PV power stations are tabulated in Table 15.2.
Table 15.2 Some Important World's Largest PV Power Stations

| S.No. | PV Power Station | Country | Nominal <br> Power (MWh) | Production <br> (Annual GWh) | Notes |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1. | Agua Caliente | USA | 251.3 AC |  | 397 MW DC when completed |
| 2. | Solar Project <br> California Valley <br> Solar Ranch | USA | 250 |  | First 130 MW connected <br> February 2013 <br> Collection of 17 co-located <br> power plants; out of which <br> the largest is 25 MW |
| 3. | Charanka Solar <br> Park | India | 221 | 317 | Completed October 2011 <br> within a group of 570 MW of <br> co-located plants |
| 4 | Huanghe <br> Hydropower <br> Golmud Solar Park | China | 200 |  |  |

For the sake of comparison, the largest non-PV solar plant, the solar thermal SEGS in California has an installed capacity of 354 MW . The largest nuclear power station is rated more than 8,200 MW. The largest hydropower plant is three Gorges Dam with 22,500 MW installed capacity.

### 15.4.3 Economics

Despite the overwhelming availability of solar power, little was installed when compared to other power generation prior to 2012, due to the high installation cost. This cost has declined as more systems have been installed. Photovoltaic systems use no fuel and modules typically for last 25 to 40 years. The cost of installation is almost the only cost, as there is very little maintenance required. Installation cost is measured in \$/watt or ₹/watt. The cost of PV has fallen well below that of nuclear power and is set to fall further. It is estimated that for large-scale installations, prices below $\$ 1.00 /$ watt are now common. In some locations, PV has reached grid parity, the cost at which it is competitive with coal or gas-fired generation.

### 15.4.4 Solar Power Cost

Physicists have claimed that recent technological developments bring the cost of solar energy more in parity with that of fossil fuels.

### 15.4.5 Self-consumption

With net metering that is the total generated value.

### 15.4.6 Grid Parity

Grid parity, the point at which PV electricity is equal to or cheaper than grid power, is achieved first in areas with abundant sun and high costs for electricity such as in California and Japan. Grid parity has reached Hawaii and other islands that otherwise use fossil fuel (diesel fuel) to produce electricity, and most of US is expected to reach grid parity by 2015. General Electric's Chief Engineer predicted that grid parity without subsidies in sunny parts of the United States would reach by around 2015. Due to growing demand for PV electricity, more companies may enter into this market that may further lower the cost of PV cells in near future.

### 15.4.7 Environmental Impacts

Unlike fossil fuel-based technologies, solar power does not lead to any harmful emissions during operation, but the production of the panels leads to some amount of pollution.

## Greenhouse gases

The life-cycle greenhouse gas emissions of solar power are in the range of 22 to $46 \mathrm{~g} / \mathrm{kWh}$ depending on if solar thermal or solar PV is being analysed, respectively. This would be decreased to $15 \mathrm{~g} / \mathrm{kWh}$ in the future. Although this value is very low but still it is more than the life-cycle emission intensity of hydro, wind, and nuclear power.

## Cadmium

One issue that has often raised concerns is the use of cadmium in cadmium telluride solar cells (CdTe is only used in a few types of PV panels). Cadmium in its metallic form is a toxic substance that has the tendency to accumulate in ecological food chains. The amount of cadmium used in thin-film PV modules is relatively small $\left(5-10 \mathrm{~g} / \mathrm{m}^{2}\right)$, and with proper emission control
techniques in place, the cadmium emissions from module production can be almost zero. Current PV technologies lead to cadmium emissions of $0.3-0.9 \mu \mathrm{~g} / \mathrm{kWh}$ over the whole life cycle. Most of these emissions actually arise through the use of coal power for the manufacturing of the modules, and coal and lignite combustion lead to much higher emissions of cadmium. Life-cycle cadmium emissions from coal is $3.1 \mu \mathrm{~g} / \mathrm{kWh}$, lignite 6.2 , and natural gas $0.2 \mu \mathrm{~g} / \mathrm{kWh}$. Note that if electricity produced by PV panels were used to manufacture the modules instead of electricity from burning coal, cadmium emissions from coal power usage in the manufacturing process could be entirely eliminated.

## PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by renewable sources of energy?
2. What are non-renewable sources of energy?
3. What are the factors on which kinetic energy of wind depends?
4. Why the wind speed is higher at higher altitudes?
5. Which country is generating the largest amount of electricity by using wind energy?
6. What do you mean by wind farm?
7. What are the onshore wind farms?
8. What are the offshore wind farms?
9. In India, where the largest onshore wind farm is built?
10. Define solar power.
11. What do you mean by CSP?
12. What do you understand by PV solar power?

## Test Questions

1. What are non-renewable and renewable sources of energy? Why renewable sources of energy are preferred?
2. What is wind energy? What are the factors on which this energy depends? What do you mean by onshore and offshore wind farms?
3. Name the leading countries in which large quantity of electric power is generated by using wind energy. Which country is the leader in this aspect. In India, where the largest onshore wind farm is built?
4. What do you mean by solar power? How solar energy is converted into electrical energy?
5. Is it economical to generate electric power by using PV cells? Explain.

### 15.5 INTRODUCTION TO FUEL CELL

A device that converts the chemical energy from a fuel into electrical energy through a chemical reaction with oxygen or another oxidizing agent is called a fuel cell. Hydrogen is the most common fuel, but hydrocarbons such as natural gas and alcohols like methanol are used as fuel. Fuel cells are different from batteries. They require a constant source of fuel and oxygen or air to sustain the chemical reaction. This is the reason for the fuel cells to produce electricity continually for as long as these inputs are supplied.


Fig. 15.9 Scheme of a proton-conducting fuel cell

In 1838, a German Physicist Christian Friedrich Schönbein invented the first crude fuel cell. However, the first commercial use of fuel cells took place in NASA space programs to generate power for probes, satellites, and space capsules. Since then, fuel cells have been used in many other applications.

### 15.5.1 Applications

Fuel cells are used for primary and backup power for commercial, industrial, and residential buildings and in remote or inaccessible areas. These are used to power fuel cell vehicles, including automobiles, buses, forklifts, airplanes, boats, motorcycles, and submarines.

### 15.5.2 Main Constituents of Fuel Cell

There are many types of fuel cells, but they all consist of an anode (negative side), a cathode (positive side), and an electrolyte that allow charges to move between the two sides of the fuel cell as shown in Figure 15.9. Electrons are drawn from the anode to the cathode through an external circuit, producing DC electricity.
Fuel cells are classified on the basis of the type of electrolyte used in the cell. Fuel cells come in a variety of sizes. Individual fuel cells produce relatively small electrical potentials, about 0.7 V , so cells are 'stacked' or placed in series to increase the voltage and meet an application's requirements. In addition to electricity, fuel cells produce water, heat, and depending on the fuel source, very small amounts of nitrogen dioxide and other emissions. The energy efficiency of a fuel cell is generally between $40 \%$ and $60 \%$, or up to $85 \%$ efficiency, if waste heat is captured for use. The fuel cell market is also growing at a healthy pace and according to Pike Research, the stationary fuel cell market is predicted to reach 50 GW by 2020.

### 15.5.3 Development and Deployment

The principle of the fuel cell was discovered by German scientist Christian Friedrich Schönbein in 1838. The first fuel cell was demonstrated by Welsh scientist and barrister Sir William Robert Grove in February 1839.

In 1939, British engineer Francis Thomas Bacon successfully developed a $5-\mathrm{kW}$ stationary fuel cell.

In 1959, a team led by Harry Ihrig built a $15-\mathrm{kW}$ fuel cell tractor for Allis-Chalmers, which was demonstrated across the US at state fairs. This system used potassium hydroxide as the electrolyte and compressed hydrogen and oxygen as the reactants.

UTC ${ }^{2}$ Power was the first company to manufacture and commercialize a large, stationary fuel cell system for use as a co-generation power plant in hospitals, universities, and large office

[^12]buildings. UTC power continues to be the sole supplier of fuel cells to NASA for use in space vehicles, having supplied fuel cells for the Apollo missions, and the Space Shuttle program and is developing fuel cells for cell phone towers and other applications.

### 15.5.4 Types of Fuel Cells

Variety of fuel cells are available in the market; however, they all work in the same general manner. These are made up of three main segments, that is, the anode, the electrolyte, and the cathode. Two chemical reactions occur at the interfaces of the three different segments. The net result of the two reactions is that fuel is consumed, water or carbon dioxide is created, and an electric current is created, which can be used to power electrical devices, normally called as the load.

At the anode, a catalyst oxidizes the fuel, usually hydrogen, turning the fuel into a positively charged ion and a negatively charged electron. The electrolyte is a substance specifically designed, so ions can pass through it, but the electrons cannot. The freed electrons travel through a wire creating the electric current. The ions travel through the electrolyte to the cathode. Once reaching the cathode, the ions are reunited with the electrons and both of them react with a third chemical, usually oxygen, to create water or carbon dioxide, as shown in Figure 15.10 (the block diagram).


Fig. 15.10 Block diagram of a fuel cell

Thus, the most important design features in a fuel cell are as follows:

1. The electrolyte substance usually defines the type of fuel cell.
2. The most common fuel used is hydrogen.
3. The anode catalyst breaks down the fuel into electrons and ions. The anode catalyst is usually made up of very fine platinum powder.
4. The cathode catalyst turns the ions into the waste chemicals such as water or carbon dioxide. The cathode catalyst is often made up of nickel, but it can also be a nanomaterial -based catalyst.

A typical fuel cell produces a voltage from 0.6 V to 0.7 V at full rated load. Voltage decreases as current increases, due to the following factors:

1. Activation loss
2. Ohmic loss (voltage drop due to resistance of the cell components and interconnections)
3. Mass transport loss (depletion of reactants at catalyst sites under high loads, causing rapid loss of voltage).

## Fuel cell stack

To deliver the desired amount of energy, the fuel cells can be combined in series and parallel circuits to yield high voltage; further, parallel channel of configurations allow a high current to be supplied. Such a design is called a fuel cell stack. The cell surface area can be increased to allow strong current from each cell. In the stack, reactant gases must be distributed uniformly over all the cells to maximize the power output and to improve the efficiency.

### 15.5.5 Classification

On the basis of electrolyte and technology used, the fuel cells may be classified as follows:

1. Polymer electrolyte membrane (PEM): A fuel cell incorporating a solid polymer membrane as its electrolyte. Protons $\left(\mathrm{H}^{+}\right)$are transported from the anode to the cathode. The operating temperature range is generally from $60^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$.
2. Phosphoric acid fuel cell (PAFC): A type of fuel cell in which the electrolyte consists of concentrated phosphoric acid $\left(\mathrm{H}_{3} \mathrm{PO}_{4}\right)$. Protons $\left(\mathrm{H}^{+}\right)$are transported from the anode to the cathode. The operating temperature range is generally from $160^{\circ} \mathrm{C}$ to $220^{\circ} \mathrm{C}$.
3. Molten carbonate fuel cell (MCFC): A type of fuel cell that contains a molten carbonate electrolyte. Carbonate ions $\left(\mathrm{CO}_{3}^{2}\right)$ are transported from the cathode to the anode. Operating temperatures are typically near $650^{\circ} \mathrm{C}$.
4. Solid oxide fuel cell (SOFC): A type of fuel cell in which the electrolyte is a solid, nonporous metal oxide, typically zirconium oxide $\left(\mathrm{ZrO}_{2}\right)$ treated with $\mathrm{Y}_{2} \mathrm{O}_{3}$, and $\mathrm{O}_{2}$ is transported from the cathode to the anode. Any CO in the reformate gas is oxidized to $\mathrm{CO}_{2}$ at the anode. Temperatures of operation are typically from $800^{\circ} \mathrm{C}$ to $1,000^{\circ} \mathrm{C}$.

### 15.5.6 Important Terms Used in Fuel Cells

1. Anode: The electrode at which oxidation (a loss of electrons) takes place. For fuel cells and other galvanic cells, the anode is the negative terminal; for electrolytic cells (where electrolysis occurs), the anode is the positive terminal.
2. Aqueous solution: The solution made from, with, or by water.
3. Catalyst: A chemical substance that increases the rate of a reaction without being consumed. Therefore, after the reaction, it can potentially be recovered from the reaction mixture and is chemically unchanged. The catalyst lowers the activation energy required and allows the reaction to proceed more quickly or at a lower temperature. In a fuel cell, the catalyst facilitates the reaction of oxygen and hydrogen. It is usually made of platinum powder very thinly coated onto carbon paper or cloth. The catalyst is rough and porous so the maximum surface area of the platinum can be exposed to the hydrogen or oxygen. The platinum-coated side of the catalyst faces the membrane in the fuel cell.
4. Cathode: The electrode at which reduction (a gain of electrons) occurs. For fuel cells and other galvanic cells, the cathode is the positive terminal; for electrolytic cells (where electrolysis occurs), the cathode is the negative terminal.
5. Electrolyte: A substance that conducts charged ions from one electrode to the other in a fuel cell, battery, or electrolyser.
6. Fuel cell stack: Individual fuel cells connected in a series. Fuel cells are stacked to increase voltage.
7. Matrix: Something within or from which something else originates, develops, or takes form. Here, matrix denotes series-parallel combination of fuel cells.
8. Membrane: The separating layer in a fuel cell that acts as electrolyte (an ion-exchanger) as well as a barrier film separating the gases in the anode and cathode compartments of the fuel cell.
9. Solution:
(a) The process by which a solid, liquid, or gaseous substance is homogeneously mixed with a liquid or sometimes a gas or solid.
(b) A homogeneous mixture formed by this process.
(c) The condition of being dissolved.

### 15.5.7 Efficiency of Fuel Cell

The energy efficiency of a system or device that converts energy is measured by the ratio of the amount of useful energy to the total amount of input energy or it may be defined as the useful output energy as a percentage of the total input energy. In the case of fuel cells, useful output energy is measured as electrical energy produced by the system and the input energy is the energy stored in the fuel.

The maximum theoretical energy efficiency of a fuel cell is $83 \%$, operating at low power density and using pure hydrogen and oxygen as reactants (assuming no heat recapture). In actual practical, tank-to-wheel efficiency of a fuel cell vehicle is greater than $45 \%$ at low loads and average value is about $36 \%$.

### 15.5.8 Major Applications

## Power generation

Stationary fuel cells are used for backup power for commercial, industrial, and residential buildings. Fuel cells are very useful as power sources in remote locations, such as spacecraft, remote weather stations, large parks, communications centres, rural locations including research stations, and in certain military applications. A fuel cell system running on hydrogen can be compact and lightweight, and have no major moving parts. Because fuel cells have no moving parts and do not involve combustion, in ideal conditions, their reliability is as high as 99.9999\%.

There are many different types of stationary fuel cells, and hence, efficiencies vary; however, most of them are having efficiency between $40 \%$ and $60 \%$. However, when the fuel cell's waste heat is used to heat a building in a cogeneration system, this efficiency can increase to $85 \%$. This is significantly more efficient than traditional coal power plants, which are only about one-third energy efficient.

## Fuel cell electric vehicles (FCEVs)

Currently, there are no fuel cell vehicles available for commercial sale; however, more than 20 FCEVs prototypes and demonstration cars have already been released.

## Buses

In total, there are over 100 fuel cell buses deployed around the world today. Most buses are produced by UTC power, fuel cell buses have a $39-141 \%$ higher fuel economy than diesel buses. Fuel cell buses have been deployed around the world including Whistler (Canada), San Francisco (United States), Hamburg (Germany), Shanghai (China), London (England), and São Paulo (Brazil) as well as several others. The Fuel Cell Bus Club is a global cooperative effort in trial fuel cell buses. The following are the notable projects:

1. Twelve fuel cell buses are being deployed in the Oakland and San Francisco Bay area of California.
2. Daimler AG with 36 experimental buses powered by Ballard Power Systems fuel cells completed a successful three-year trial, in 11 cities, in January 2007.
3. A fleet of Thor buses with UTC power fuel cells were deployed in California operated by SunLine Transit Agency.

## Forklifts

Fuel cell-powered forklifts are one of the largest sectors of fuel cell applications in the industry. Most fuel cells used for material handling purposes are powered by PEM fuel cells, although some direct methanol fuel forklifts are coming onto the market. Fuel cell-powered forklifts provide significant benefits over both petroleum- and battery-powered forklifts; their benefits are that they produce no local emissions and can work for a full 8-h shift on a single tank of hydrogen, they can be refuelled in 3 min and have a lifetime of $8-10$ years. Fuel cell-powered forklifts are often used in refrigerated warehouses as their performance is not degraded by lower temperatures. The other vehicles that can be operated by using fuel cells may be motorcycles and bicycles, airplanes, boats, submarines, etc.

### 15.5.9 Some Other Common Applications

1. Providing power for base stations or cell sites
2. Distributed generation
3. Emergency power systems are a type of fuel cell system, which may include lighting, generators, and other apparatus, to provide backup resources in a crisis or when regular systems fail. They find uses in a wide variety of settings from residential homes to hospitals, scientific laboratories, data centres, etc.
4. Telecommunication equipment and modern naval ships
5. An uninterrupted power supply (UPS) provides emergency power, and depending on the topology, provide line regulation as well to connected equipment by supplying power from a separate source when utility power is not available. Unlike a standby generator, it can provide instant protection from a momentary power interruption.
6. Base load power plants
7. Solar hydrogen fuel cell water heating
8. Hybrid vehicles, pairing the fuel cell with either an $\mathrm{ICE}^{3}$ or a battery
(a) Notebook computers for applications where AC charging may not be readily available
(b) Portable charging docks for small electronics (e.g., a belt clip that charges your cell phone)
(c) Smart phones, laptops, and tablets
(d) Small heating appliances
(e) Food preservation achieved by exhausting the oxygen and automatically maintaining oxygen exhaustion in a shipping container, containing, for example, fresh fish.

### 15.5.10 World's Largest Fuel Cell Park

The world's largest, according to full cell energy, fuel cell production park is now operating in Daegu City, South Korea. Its output is 11.2 MW. Unlike other utility-scale renewable energy installations such as wind and solar farms, fuel cell parks have a small land footprint, which makes it ideally suited for countries with little land to spare.
'Development of this 58.8 MW fuel cell park illustrates how our stationary fuel cell power plants can be used to support the power grid as the combination of near-zero pollutants, modest land-use needs, and quiet operating nature of these stationary fuel cell power plants facilitates their siting in urban locations,' says Chip Bottone, fuel cell energy CEO.

Earlier this year, fuel cell energy won a contract for a 230 MW fuel cell plant for Seoul City. Last year, fuel cell energy opened an 11.2 MW fuel cell park in Daegu City, South Korea. The power goes to the grid and the waste heat is used by a municipal wastewater treatment facility under long-term power purchase agreements.
'Distributing a number of multi-megawatt fuel cell parks throughout an electrical service area enhances power reliability and energy security for electric utilities and their customer,' says Taehyoung Kim, Group Leader of POSCO's fuel cell division.

IBM is working with POSCO to develop South Korea's first renewable energy management system for a smart grid. In May, South Korea passed cap-and-trade legislation with a near unanimous vote. This country is the 8th biggest greenhouse gas emitter in the world, and it is the fastest growing source of emissions among industrialized nations, doubling since 1990. It sets a national target of cutting emissions $30 \%$ by 2020.

### 15.6 INTRODUCTION TO HYDROELECTRICITY

Hydroelectricity is the term that refers to electricity generated by hydropower, that is, the production of electrical power through the use of the gravitational force of falling water or flowing water. It is the most widely used form of renewable energy. Almost $25 \%$ of global electricity (i.e., $5,354 \mathrm{TW}-\mathrm{h})^{4}$ is produced by this source and is expected to increase about $3.1 \%$ each year for the next 25 years.

Hydropower is produced in more than 150 countries, with the Asia-Pacific region generating $32 \%$ of global hydropower. A pictorial view of Bhakhra hydroelectric power project is shown in Figure 15.11 , where Figure 15.12 shows a view of water turbine. China is the largest hydroelectricity producer representing around $17 \%$ of domestic electricity use. At present, there are three

[^13]

Fig. 15.11 Bhakhra hydroelectric power project


Fig. 15.12 Water turbine
hydroelectricity plants that are larger than 10 GW, namely Three Gorges Dam in China, Itaipu Dam across the Brazil/Paraguay border, and Guri Dam in Venezuela.

The cost of hydroelectricity is relatively low, making it a competitive source of renewable electricity. Hydropower is also a flexible source of electricity since plants can adjust the generation very quickly depending upon the changing energy demands. However, after erection of dam, the flow of rivers is usually interrupted that may harm local ecosystems. Once a hydroelectric complex is constructed, the project produces no direct waste and has a considerably lower output level of the greenhouse gas carbon dioxide $\left(\mathrm{CO}_{2}\right)$ than fossil fuel-powered energy plants.

### 15.6.1 Development in the Field of Hydropower

1. Hydropower has been used since ancient times to grind flour and perform other tasks.
2. In the mid-1770s, French engineer Bernard Forest de Bélidor published Architecture Hydraulique that described vertical- and horizontal-axis hydraulic machines. By the late 19th century, the electrical generator was developed and could now be coupled with hydraulics.
3. In 1878, the world's first hydroelectric power scheme was developed at Cragside in Northumberland, England by William George Armstrong. It was used to power a single arc lamp in his art gallery. The old Schoellkopf Power Station No. 1 near Niagara Falls in US began to produce electricity in 1881.
4. At the beginning of the 20th century, many small hydroelectric power plants were being constructed by commercial companies in mountains near metropolitan areas.
5. By 1920, $40 \%$ of the power produced in the United States was hydroelectric.
6. As the power plants became larger, their associated dams developed additional purposes to include flood control, irrigation, and navigation.
7. Hydroelectric power plants continued to become larger throughout the 20th century. Hydropower was referred to as white coal for its adequate power.
8. Three Gorges Dam in China at 22,500 MW is the largest hydroelectric power plant in the world.


Fig. 15.13 Cross-section of a conventional hydroelectric dam

### 15.6.2 Methods of Power Generation at Hydroelectric Power Plants

The following are the methods used at hydroelectric power stations.

## Conventional method (dams)

Most of the hydroelectric power is generated by using potential energy of dammed water in driving a water turbine coupled with a generator as shown in Figure 15.13. The power extracted from the water depends on the volume and on the difference in height between the source and the water's outflow. This height difference is called the head. The amount of potential energy (mgh) in water is proportional to the head. A large pipe (the 'penstock') delivers water to the turbine.

## Pumped storage method

This method produces electricity to supply high peak demands by moving water between reservoirs at different elevations. At times of low electrical demand, excess generation capacity is used to pump water into the higher reservoir. When there is higher demand, water is released back into the lower reservoir through a turbine. Pumped storage schemes currently provide the most commercially important means of large-scale grid energy storage and improve the daily capacity factor of the generating station.

## Run-of-the-river

Run-of-the-river hydroelectric power stations have either very small or no reservoir capacity. The water coming from upstream at high speed and allowed to bypass the dam, must be used for generation at that moment by converting kinetic energy of flowing water into electrical energy.

## Underground power station

An underground power station makes use of a large natural height difference between two waterways, such as a waterfall or mountain lake. In this case, an underground tunnel is constructed to take water from the high reservoir to the generating hall built in an underground natural or artificial cave near the lowest point of the water tunnel and a horizontal tailrace taking water away to the lower outlet waterway.

### 15.6.3 Classification of Hydroelectric Power Stations on the Basis of Size and Capacity

## Large power stations

There is no official demarcation for the capacity range of large hydroelectric power stations. However, the power plants from a few hundred megawatt to more than 10 GW are generally considered large hydroelectric power stations. Three Gorges Dam at 22.5 GW, Itaipu Dam at 14 GW , and Guri Dam at 10.2 GW are the large power stations. Large-scale hydroelectric power stations are more commonly seen as the largest power producing facilities in the world, with some hydroelectric facilities capable of generating more than double the installed capacities of the current largest nuclear power stations. Five largest hydroelectric power stations of the world are mentioned in Table 15.3.

Table 15.3. Five Largest Hydroelectric Power Stations in the World

| Rank | Power Station | Country | Capacity (MW) |
| :--- | :--- | :--- | :--- |
| 1. | Three Gorges Dam | China | 20,300 |
| 2. | Itaipu Dam | Brazil Paraguay | 14,000 |
| 3. | Guri Dam | Venezuela | 10,200 |
| 4. | Tucuruí Dam | Brazil | 8,370 |
| 5. | Grand Coulee Dam | United States | 6,809 |

## Small power stations

Although the definition of a small hydro project varies but a generating capacity of up to $10 \mathrm{MW})$ is generally accepted as the upper limit of small hydroelectric power stations. Small capacity is 85 GW and small hydro plants may be connected to conventional electrical distribution networks as a source of low-cost renewable energy. Alternatively, small hydro projects may be built in isolated areas that would be uneconomic to serve from a network, or in areas where there is no national electrical distribution network.

## Micro-hydroelectric power stations

Micro-hydro term is used for hydroelectric power stations that typically produce up to 100 kW of power. These power stations can provide power to an isolated home or small community or are sometimes connected to electric power networks. There are many of such power stations
around the world, particularly in developing nations as they can provide an economical source of energy without purchase of fuel. In many of the cases, micro-hydro stations complement PV solar energy systems because in those areas, water flow, and thus available hydro power, is the highest in the winter when solar energy is at a minimum.

## Pico-hydroelectric power stations

Pico-hydro is a term used for hydroelectric power generation having capacity below 5 kW . It is useful in small, remote communities that require only a small amount of electricity. For example, to power one or two fluorescent light bulbs and a TV for a few homes. Even smaller turbines of 200-300W may power a single home. Typically, pico-hydro setups are run-of-the-river type, that is, dams are not used, but pipes divert some of the water flow and pass it through the turbine before returning it to the stream.

### 15.6.4 Amount of Available Hydraulic Power

A hydropower resource can be evaluated by its available power. Power is a function of the hydraulic head and rate of water flow. The static head is proportional to the difference in height through which the water falls. Dynamic head is related to the velocity of moving water. Each unit of water has the ability to do some amount of work equal to its weight times the head.

The power available from falling water can be calculated from the flow rate and density of water, the height of fall, and the local acceleration due to gravity. In SI units, the power is

$$
P=\eta \rho Q g h
$$

where
$P=$ power in watt
$\eta=$ the dimensionless efficiency of the turbine
$\rho=$ the density of water in kilogram per cubic metre
$Q=$ the flow in cubic metre per second
$g=$ the acceleration due to gravity
$h=$ the height difference between inlet and outlet
To illustrate, calculate the power for a turbine having an efficiency of $85 \%$ with water at $1,000 \mathrm{~kg} / \mathrm{m}^{3}$ and a flow rate of $80 \mathrm{~m} / \mathrm{s}$, gravity of $9.80 \mathrm{~m} / \mathrm{s}^{2}$, and with a net head of 140 m .

In SI units: Power, $P=0.85 \times 1,000 \times 80 \times 9.8 \times 140 \mathrm{~W}=93.296 \times 10^{6} \mathrm{~W}$ or 93.296 MW .
Some hydropower systems such as water wheels can draw power from the flow of a body of water without necessarily changing its height. In this case, the available power is the kinetic energy of the flowing water. Over-shot water wheels can efficiently capture both types of energy.

### 15.6.5 Advantages of Hydroelectricity

1. Flexibility: Hydro is a flexible source of electricity, since plants can be ramped up and down very quickly to adapt to changing energy demands.
2. Low power costs: The major advantage of hydroelectricity is its low cost of generation due to elimination of the cost of fuel, low cost of operating labour, low maintenance cost, and long life.

When a dam serves multiple purposes, a hydroelectric plant may be added with relatively low construction cost, providing a useful revenue stream to offset the costs of dam operation. It has been calculated that the sale of electricity from a hydroelectric power plant usually cover the construction cost after 5 to 8 years of its full generation.

Usually, many hydroelectric projects supply public electricity networks, but some are created to serve specific industrial enterprises. Dedicated hydroelectric projects are often built to provide the substantial amounts of electricity needed for particular project.
3. Reduced $\mathrm{CO}_{2}$ emissions: They are claimed not to produce carbon dioxide directly. While some carbon dioxide is produced during manufacture and construction of the project, this is a tiny fraction of the operating emissions of equivalent fossil-fuel electricity generation.
4. Other uses of the reservoir-type hydro projects: In these projects, multiuse dams installed for irrigation support agriculture with a relatively constant water supply. Large hydro dams can control floods, which would otherwise affect people living downstream of the project. Reservoirs created by hydroelectric schemes often provide facilities for water sports and become tourist attractions themselves. In some countries, aquaculture in reservoirs is common.

### 15.6.6 Disadvantages of Hydroelectricity

1. Ecosystem damage and loss of land: Large reservoirs required for the operation of hydroelectric power stations result in submersion of extensive areas upstream of the dams, destroying biologically rich and productive lowland and valley forests, marshland, and grasslands. Hydroelectric projects can be disruptive to surrounding aquatic ecosystems both upstream and downstream of the plant site. Generation of hydroelectric power changes the downstream river environment.
2. Methane emissions (from reservoirs): It has been noted that the reservoirs of power plants in tropical regions produce substantial amounts of methane. This is due to plant material in flooded areas decaying in an anaerobic environment and forming methane, a greenhouse gas.
3. Relocation: When dams are erected, there is a need to relocate the people living in the area where reservoirs are planned.
4. Failure risks: In large hydroelectric power plants, a large quantity of water is blocked behind the dams, a failure due to poor construction, natural disasters or sabotage can destroy the surroundings and infrastructure. Dam failures have been some of the largest man-made disasters in history. The Banqiao Dam failure in Southern China has witnessed the deaths of 26,000 people, and another $1,45,000$ from epidemics. Millions were left homeless. Smaller dams and micro-hydro facilities create less risk.

### 15.6.7 Hydroelectric Power in India

India ranks 5th in terms of exploitable hydro potential on global scenario. In 2012, India is the 7th largest producer of hydroelectric power with $114 \mathrm{TW} / \mathrm{h}$. With installed capacity of 37 GW , it produces $3.3 \%$ of the world total.

### 5.6.8 Hydro Potential

India is naturally furnished with economically exploitable and viable hydro potential assessed to be about $84,000 \mathrm{MW}$ at $60 \%$ load factor. In addition, $6,780 \mathrm{MW}$ in terms of installed capacity from small, mini, and micro-hydel schemes have been assessed. Further, 56 sites for pumped storage schemes with an aggregate installed capacity of $94,000 \mathrm{MW}$ have been identified. It is the most widely used form of renewable energy. India is blessed with immense amount of hydroelectric potential and ranks 5th in terms of exploitable hydro potential on global scenario. The installed capacity as on 2011 is approximately $37,367.4$ MW, which is $21.53 \%$ of total electricity generation in India.

Bhakra Beas Management Board (BBMB), an illustrative state-owned enterprise in north India, has an installed capacity of 2.9 GW and generates $12,000-14,000$ million units per year. The cost of generation of energy after four decades of operation is about 20 paise $/ \mathrm{kWh}$. BBMB is a major source of peaking power and black start to the northern grid in India. Large reservoirs provide operational flexibility. BBMB reservoirs annually supply water for irrigation to 125 lakh ( 12.5 million) acres of agricultural land of partner states, enabling northern India in its green revolution. Five major hydroelectric power plants of India are given in Table 15.4.
Table 15.4 List of Five Major Hydropower Plants in India

| Name | Location | Operator | Configuration | Important Facts |
| :---: | :---: | :---: | :---: | :---: |
| Babail | Uttar Pradesh | Uttar Pradesh Jal Vidyut Nigam Ltd | $\begin{aligned} & 2 \times 1.5 \mathrm{MW} \\ & \text { tube } \end{aligned}$ | The Babail minihydel project was approved in Sep 1986 and was awarded to PGM in Sep 1988 as a ₹ 6.22 crore turnkey project. |
| Bhandardara-1 | Maharashtra | Dodson-Lindblom HydroPower Pvt. Ltd | $\begin{aligned} & 1 \times 14.4 \mathrm{MW} \\ & \text { Francis } \end{aligned}$ | This plant was acquired in 1996 from Maharashtra Water Resources Dept. and overhauled in 1997/98 with assistance from AHEC. |
| Belka | Uttar Pradesh | Uttar Pradesh Jal Vidyut Nigam Ltd | $\begin{aligned} & 2 \times 1.5 \mathrm{MW} \\ & \text { tube } \end{aligned}$ | The Belka and Babail minihydel projects were approved in September 1986 and Belka was awarded to FCC and PGM in July 1988 as a total ₹5.66 crore project. Construction on Belka did not start until December 1996 after delays in securing forest clearance and land acquisition. |
| Chenani I | Jammu and Kashmir | Jammu and <br> Kashmir Power <br> Development Corp | $\begin{aligned} & 5 \times 4.66 \text { MW } \\ & \text { Pelton } \end{aligned}$ | The Chenani I \& II projects in Udhampur district were inaugurated in 1971 by Prime Minister Indira Gandhi. They were closed on 25 February 2005, following a landslide that damaged a $700-\mathrm{m}$ diversion tunnel. Repairs were completed at a cost of ₹8 crore and the plants put back in service in June 2008. |

Table 15.4 (Continued)

| Name | Location | Operator | Configuration | Important Facts |
| :---: | :---: | :---: | :---: | :---: |
| Bhatghar | Maharashtra | Maharashtra State Power Generation Co. Ltd | $1 \times 16 \mathrm{MW}$ <br> Kaplan | The dam was part of the world's largest irrigation project known as Lloyd Barrage. This was a multipurpose scheme that was initiated in 1923 by Sir George Ambrose Lloyd, the Governor of Bombay. The project was opened in January 1932. |

### 15.7 INTRODUCTION TO TIDAL POWER

Tidal power or tidal energy is a form of hydropower that converts the energy of tides into useful forms of power, mainly electricity. Tidal forces are developed due to periodic variations in gravitational attraction exerted by celestial bodies. These forces create corresponding motions or currents in the world's oceans. Due to the strong attraction to the oceans, a bulge in the water level is created, causing a temporary increase in sea level. When the sea level is raised, water from the middle of the ocean is forced to move toward the shorelines, creating a tide. These forces are developing due to the consistent pattern of the moon's orbit around the earth. The magnitude and characteristic of this motion reflects the changing positions of the moon and sun relative to the earth, the effects of Earth's rotation, and local geography of the sea floor and coastlines.

A tidal generator converts the tidal energy into electrical energy as shown in Figure 15.14. Greater tidal variation and higher tidal current velocities can dramatically increase the potential of a site for tidal electricity generation. Because the earth's tides are ultimately due to gravitational interaction with the moon and sun and the earth's rotation, tidal power is practically inexhaustible and classified as a renewable energy resource.


Fig. 15.14 Tidal power generation

### 15.7.1 History and Development

Historically, tide mills have been used long back, both in Europe and on the Atlantic coast of North America. The incoming water was contained in large storage ponds, and as the tide went out, it turned waterwheels that were used to mill grains. It was only in the 19th century that the process of using falling water and spinning turbines to create electricity was introduced in US and Europe.

Although not yet widely used, tidal power has potential for future electricity generation. Tides are more predictable than wind energy and solar power.

### 15.7.2 Generating methods

Tidal power can be classified into three generating methods, namely tidal stream generator (TSG), tidal barrage, and dynamic tidal power. The following two are very common methods:

1. Tidal stream generator: It makes use of the kinetic energy of moving water to power turbines, in a similar way to wind turbines that use wind to power turbines. Some tidal generators can be built into the structures of existing bridges, involving virtually no aesthetic problems. The world's first commercial and grid-connected tidal stream gener-ator-SeaGen-is in Strangford Lough.
2. Tidal barrage: Tidal barrages make use of the potential energy created due to difference in height (or head) between high and low tides as shown in Figure 15.15. When using tidal barrages to generate power, the potential energy from a tide is seized through strategic placement of specialized dams. When the sea level rises and the tide begins to


Fig. 15.15 (a) High tide: water flows from sea to basin (b) Low tide: water flows from basin to sea
come in, the temporary increase in tidal power is channelled into a large basin behind the dam, holding a large amount of potential energy. With the receding tide, this energy is then converted into mechanical energy as the water is released through large turbines, a generator coupled to it converts mechanical energy into electrical energy.

Table 15.5 Some Important Current and Future Tidal Power Schemes

| Station | Capacity (MW) | Country | Comm. Ref. |
| :--- | :--- | :--- | :--- |
| Rance Tidal Power Station | 240 | France | 1966 |
| Sihwa Lake Tidal Power Station | 254 | South Korea | 2011 |
| Annapolis Royal Generating Station | 20 | Canada | 1984 |
| Jiangxia Tidal Power Station | 3.2 | China | 1980 |
| Kislaya Guba Tidal Power Station | 1.7 | Russia | 1968 |
| Incheon Tidal Power Station (under <br> construction) | 818 or 1,320 | South Korea | 2017 |

## Tidal power stations in India

There is no tidal power plant in India as such but Gujarat has given a proposal for the Kalpasar Project, which if started in this year, it will be completed in 2020.

### 15.8 INTRODUCTION TO GEOTHERMAL ENERGY

Below the earth's crust, there is a layer of hot and molten rock called magma. Heat is continually produced there, mostly from the decay of naturally radioactive materials such as uranium and potassium. The amount of heat within $10,000 \mathrm{~m}$ of earth's surface contains 50,000 times more energy than all the known oil and natural gas resources in the world.

The areas with the highest underground temperatures are in regions with active or geologically young volcanoes. These 'hot spots' occur at plate boundaries or at places where the crust is thin enough to let the heat through. The Pacific Rim, often called the Ring of Fire for its many volcanoes, has many hot spots, including some in Alaska, California, and Oregon. Nevada has hundreds of hot spots, covering much of the northern part of that state.

Seismically active hotspots are not the only places where geothermal energy can be found. There is a steady supply of mild heat, useful for direct heating purposes, at depths of anywhere from 4 to a few hundred metre below the surface virtually in any location on Earth. There is a vast amount of heat energy available from dry rock formations, which are very deep below the surface ( $4-10 \mathrm{~km}$ ). By using emerging technologies known as Enhanced Geothermal Systems (EGS), we may be able to capture this heat for electricity production on a much larger scale than conventional technologies allow.

If these resources can be tapped, they offer enormous potential for electricity production. The Geothermal Energy Association estimates that in US, all the 132 projects now under development could provide up to $6,400 \mathrm{MW}$ of new capacity. As EGS technologies improve and become competitive, even more of the largely untapped geothermal resource could be developed. In other words, this resource could one day supply nearly all of today's US electricity needs.

The geothermal resources in US have such a great potential that they can provide continuous base load electricity. As per the latest report, it is said that geothermal plants are comparable with those of coal and nuclear power. With the combination of both the size of the resource base and its consistency, geothermal can play an indispensable role in a cleaner, more sustainable power system.

### 15.8.1 Geothermal Electricity

The electricity generated from geothermal energy is called geothermal electricity. It is estimated that the electricity generating potential of geothermal energy vary from 35 to $2,000 \mathrm{GW}$. Current worldwide installed capacity is $10,715 \mathrm{MW}$, with the largest capacity in the United States (3,086 MW), Philippines, and Indonesia. India has announced a plan to develop the country's first geothermal power plant in Chhattisgarh.

### 15.8.2 Geothermal Electric Power Plant

The region between crust and core of the earth is called mantle. The mantle consists of molten rocks and hot gases. This heat of interior of the earth is called geothermal energy. To utilize this geothermal energy for the generation of electric power, a hole (well) is drilled in the earth up to metal, as shown in Figure 15.16. When water is poured in the well, steam at high pressure is produced. This steam and gases evolved at high pressure are brought back to the surface through a pipe by drilling another hole (or well), as shown in Figure 15.16. This high pressure steam is used to spin turbines. Electricity is generated by the generator coupled to the turbine shaft. The complete block diagram of a typical geothermal electric power plant is shown in Figure 15.17.


Fig. 15.16 Generator of electric power using geothermal energy


Fig. 15.17 Block diagram of a typical geothermal electric power plant

### 15.8.3 Types of Geothermal Power Stations

Geothermal power stations are similar to steam-turbine thermal power stations. Heat from a fuel source (in this case, it is the earth's core) is used to heat water or another working fluid. The working fluid is then used to turn a turbine of a generator, thereby producing electricity. The fluid is then cooled and returned to the heat source.

On the basis of technology used, geothermal power plants may be classified as follows:

1. Dry steam power plants: Dry steam plants are the simplest and oldest design. They directly use geothermal steam of $150^{\circ} \mathrm{C}$ or more to turn turbines. Figure 15.18 shows a general view of a dry steam power plant.
2. Flash steam power plants: Flash steam plants pull deep, high-pressure hot water into low pressure tanks and use the resulting flashed steam to drive turbines as shown in Figure 15.19 . They require fluid temperatures of at least $180^{\circ} \mathrm{C}$ or more.
3. Binary cycle power plants: Binary cycle power plants are the most recent development and can accept fluid temperatures as low as $57^{\circ} \mathrm{C}$. The moderately hot geothermal water is passed by a secondary fluid has much lower boiling point than water. This causes the secondary fluid to flash vaporize, which then drives the turbines. This is the most common type of geothermal electricity plant being constructed today. Both Organic Rankine and Kalina cycles are used. The thermal efficiency of this type plant is typically about $10-13 \%$.

### 15.8.4 Main Geothermal Power Plants

The largest group of geothermal power plants in the world is located at The Geysers, a geothermal field in California, United States. As of 2004, five countries (El Salvador, Kenya, the


Fig. 15.18 Dry steam plant


Fig. 15.19 Flash steam plant

Philippines, Iceand, and Costa Rica) generate more than $15 \%$ of their electricity from geothermal sources. At present, Geothermal electricity is generated in 24 countries, while there were also plants under construction in 11 other countries.

### 15.8.5 Environmental Impact

Fluids drawn from the deep earth carry a mixture of gases, notably carbon dioxide $\left(\mathrm{CO}_{2}\right)$, hydrogen sulphide $\left(\mathrm{H}_{2} \mathrm{~S}\right)$, methane $\left(\mathrm{CH}_{4}\right)$, and ammonia $\left(\mathrm{NH}_{3}\right)$. These pollutants contribute to global warming, acid rain, and noxious smells, if released. Existing geothermal electric plants emit an average of 400 kg of $\mathrm{CO}_{2}$ per megawatt-hour (MWh) of electricity, which is a small fraction of the emission intensity of conventional fossil fuel plants. Plants that experience high levels of acids and volatile chemicals are usually equipped with emission-control systems to reduce the exhaust. Geothermal plants could theoretically inject these gases back into the earth, as a form of carbon capture and storage. Thus, the environmental impact of these plants is nominal.

### 15.8.6 Economics

Geothermal power requires no fuel, and therefore, it is immune to fuel cost fluctuations. However, capital costs tend to be high. Drilling accounts for over half the costs, and exploration of deep resources entails significant risks. However, with modern technology, the drilling cost has been reduced. By and large, these power plants are quite economical where high temperatures are available near the earth crust, that is, at Pacific Rim (Ring of fire).

### 15.8.7 Future of Geothermal Energy

Geothermal energy has the potential to play a significant role in moving the United States (and other regions of the world) toward a cleaner, more sustainable energy system. It is one of the few
renewable energy technologies that, like fossil fuels, can supply continuous, base load power. The costs for electricity from geothermal facilities are also declining. Some geothermal facilities have realised at least $50 \%$ reductions in the price of electricity since 1980.

### 15.9 INTRODUCTION TO THERMAL- (STEAM, DIESEL, AND GAS ENERGY) ELECTRIC POWER STATIONS

All the electric power stations in which the electric power is generated by steam-driven turbines are called thermal- or steam-electric power stations. In these power stations, water is heated that turns into steam and spins a steam turbine. After it passes through the turbine, the steam is condensed in a condenser. The major variation in the design of steam-electric power plants is due


Fig. 15.20 Steam power plant to the different fuel sources. A simple pictorial view of a coal fed thermal power plant is shown in Figure 15.20.

Almost all coal, nuclear, geothermal, solar thermal electric as well as many natural gas power plants are steam-electric power plants. Natural gas is frequently combusted in gas turbines as well as in boilers. The waste heat from a gas turbine can be used to raise steam in a combined cycle plant that improves overall efficiency.

Throughout the world, most of the electric power is produced by steam-electric power plants, which produce about $86 \%$ of all electric generation. The only other types of plants that currently have a significant contribution are hydroelectric power plants. Photovoltaic panels, wind turbines, and binary cycle geothermal plants are also nonsteam electric power plants, but currently they do not produce much electricity.

### 15.9.1 History and Development

1. Reciprocating steam engines have been used for mechanical power sources since the 18th Century, with notable improvements being made by James Watt.
2. The very first commercial central electrical generating stations was erected in New York and London, in 1882, which used reciprocating steam engines.
3. As generator sizes increased, eventually turbines took over due to higher efficiency and lower cost of construction.
4. By 1920s, any central station larger than a few thousand kilowatt would use a turbine prime mover.
5. The first practical electricity generating system using a steam turbine was designed and made by Charles Parsons in 1885. The energy generated was used for lighting an exhibition in Newcastle. Since then, apart from getting bigger, turbine design has hardly changed and Parson's original design is still in use today. Despite the introduction of many alternative technologies in the intervening 120 years, over $80 \%$ of the world's electricity is still generated by steam turbines driving rotary generators.

### 15.9.2 Energy Conversion Processes

Electrical energy generation using steam turbines involves three energy conversions:

1. Extracting thermal energy from fuel (or other sources) and using it to raise steam.
2. Converting the thermal energy of the steam into kinetic energy in the turbine.
3. Using a rotary generator to convert the turbine's mechanical energy into electrical energy.

## Extracting thermal energy from fuel (or other sources) and using it to raise steam

Steam is mostly raised from fossil fuel sources, three of which are shown Figure 15.21 but any convenient source of heat can be used.


Fig. 15.21 Fossil fuel-powered steam turbine electricity generation

1. Thermal energy from fuel: In fossil-fuelled power plants, steam is raised by burning fuel, mostly coal but oil and gas are also used in a combustion chamber. Nowadays, in many cases, these fuels have been supplemented by limited amounts of renewable biofuels and agricultural waste. Combustion is the chemical process of burning the fuel releases heat by the chemical transformation (oxidation) of the fuel. Combustion can never be perfect. There will be losses due to impurities in the fuel, incomplete combustion and heat and pressure losses in the combustion chamber and boiler. Typically, these losses would amount to about $10 \%$ of the available energy in the fuel.
2. Thermal energy from nuclear power: Steam for driving the turbine can also be raised by capturing the heat generated by controlled nuclear fission in nuclear reactors.
3. Thermal energy from solar power: Similarly solar thermal energy can be used to raise steam, although this is less common.
4. Thermal energy from geothermal energy: Steam emissions by using geothermal energy are also used to power steam turbine power plants.

## Steam turbine (prime mover)

High pressure steam is fed to the turbine and passes along the machine axis through multiple rows of alternately fixed and moving blades. From the steam inlet port of the turbine towards the exhaust point, the blades and the turbine cavity are progressively larger to allow for the expansion of the steam.

The stationary blades act as nozzles in which the steam expands and emerges at an increased speed but at low pressure. (Bernoulli's conservation of energy principle is that the kinetic energy increases as pressure energy falls). As the steam impacts on the moving blades, it imparts some of its kinetic energy to the moving blades. There are two basic steam turbine types: impulse turbines and reaction turbines.

## Condenser

The exhausted steam from the low pressure turbine is condensed to water in the condenser that extracts the latent heat of vaporization from the steam. This causes the volume of the steam to go to zero, reducing the pressure abruptly to near vacuum conditions, thus increasing the pressure drop across the turbine enabling the maximum amount of energy to be extracted from the steam. The condensed water is then pumped back into the boiler as feed water to be used again.

For this process of condensation, it is true that condenser systems need a constant, ample supply of cooling water, and this is supplied in a separate circuit from the cooling tower that cools the condenser cooling water by direct contact with the air and evaporation of a portion of the cooling water in an open tower.

Water vapour seen as of smoke rising to the atmosphere from power plants is evaporating cooling water and not the working fluid (or steam).

## Electromechanical energy transfer (AC generator)

The steam turbine drives a generator to convert the mechanical energy into electrical energy. Typically, this will be a rotating field synchronous machine. The energy conversion efficiency of these high capacity generators can be as high as $95 \%$ to $99 \%$ for a very large machine.

## Ancillary systems

Apart from the basic steam raising and electricity generating plant, there are several essential automatic control and ancillary systems that are necessary to keep the plant operating safely at its optimum capacity. The most important of these systems are mentioned as follows:

1. Maintaining the system voltage and frequency
2. Keeping the plant components within their operating pressure, temperature, and speed limits
3. Current controls-matching the power output to the demand
4. Feeding the fuel to the combustion chamber and removing the ash
5. Pumps and fans for water and air flow
6. Lubrication systems
7. Pollution Control-separating harmful products from the combustion exhaust emissions
8. Cooling the generator
9. Electricity transmission equipment - transformers and high voltage switching
10. Overload protection, emergency shut down, and load shedding

### 15.9.3 Largest Power Plants in the World (Coal, Oil, and Gas Fired)

The largest thermal power plants in the world (coal, oil and Gas fired) are shown in the Table 15.6, 15.7 and 15.8 respectively.

Table 15.6 Coal Power Stations

| Rank | Station | Country | Capacity (MW) |
| :--- | :--- | :--- | :--- |
| 1 | Taichung Power Plant | Taiwan | 5,780 |
| 2 | Tuoketuo Power Station | China | 5,400 |
| 3 | Belchatów Power Station | Poland | 5,053 |
| 4 | Guodian Beilun Power Station | China | 5,000 |
| 5 | Waigaoqiao Power Station | China | 5,000 |
| 6 | Guohua Taishan Power Station | China | 5,000 |

Table 15.7 Fuel Oil Power Stations

| Rank | Station | Country | Capacity (MW) |
| :--- | :--- | :--- | :--- |
| 1 | Shoaiba power and | Saudi | 5,600 |
|  | desalination plant | Arabia |  |
| 2 | Kashima Power Station | Japan | 4,400 |
| 3 | Hirono Power Station | Japan | 3,800 |
| 4 | Surgut-1 Power Station | Russia | 3,268 |
| 5 | Peterhead Power Station | UK | 2,177 |

Table 15.8 Natural Gas Power Station

| Rank | Station | Country | Capacity (MW) |
| :--- | :--- | :--- | :--- |
| 1 | Surgut-2 Power Station | Russia | $5,597.1$ |
| 2 | Futtsu Power Station | Japan | 5,040 |
| 3 | Kawagoe Power Station | Japan | 4,802 |
| 4 | Higashi-Niigata Power Station | Japan | 4,600 |
| 5 | Tata Power Plant | Taiwan | 4,272 |

### 15.9.4 Diesel Generator Unit

A combination of a diesel engine and an electric generator (often an alternator) to generate electrical energy is called a diesel generator unit. Diesel generating sets are usually used in places without connection to the power grid as emergency power supply if the grid fails.

## Genset

The packaged combination of a diesel engine, a generator, and various ancillary devices (such as base, canopy, sound attenuation, control systems, circuit breakers, jacket water heaters, and starting system) is referred to as a 'generating set' or a 'genset' for short.

The size of the genset is available in the range from 8 to 30 kW (also 8 to 30 kVA single phase) for homes, small shops, and offices. For large industrial applications, these sets are available in the range from $8 \mathrm{~kW}(11 \mathrm{kVA})$ up to $2,000 \mathrm{~kW}(2,500 \mathrm{kVA}$ three phase) used for large office complexes and factories. A $2,000 \mathrm{~kW}$ set can be housed in a 12 m ISO container with fuel tank, controls, power distribution equipment, and all other equipment needed to operate as a standalone power station or as a standby backup to grid power. These units are referred to as power modules. Generally, these gensets are placed on large triple axle trailers for their easy handling. A combination of these modules are used for small power stations and these may use from 1 to 20 units per power section and these sections can be combined to involve hundreds of power modules. In these large sizes, the power module (engine and generator) are brought to site on trailers separately and are connected together with large cables and a control cable to form a complete synchronized power plant.

Diesel generators, sometimes as small as $200 \mathrm{~kW}(250 \mathrm{kVA})$ are widely used not only for emergency power, but also may have a secondary function of feeding power to utility grids either during peak periods, or periods when there is a shortage of large power generators.

Generally, on ships, diesel generators are employed not only to provide auxiliary power for lights, fans, etc., but also indirectly for main propulsion. With electric propulsion, the generators can be placed in a convenient position to allow more cargo to be carried.

### 15.9.5 Gas Power Station

A power station in which the chemical energy of natural gas (fuel) is converted into electrical energy is known as gas power station. The largest natural gas power plant is shown in Figure 15.22.

Referring to Figure 15.23, natural gas (1) is pumped into the gas turbine (2), where it is mixed with air (3) and ignited; during combustion, its chemical energy is converted into heat energy. The heat makes the combustion gas to expand. In the enclosed gas turbine, a pressure is built up.

The pressure drives the combustion gas over the blades of the gas turbine, causing it to spin, converting some of the heat energy into mechanical energy. An AC generator (4) is coupled to the shaft. The generator uses an electromagnetic field to convert this mechanical energy into electrical energy.

After passing through the gas turbine, the still-hot combustion gas is piped to the heat recovery steam generator (5). Here, it is used to heat pipes full of water, turning the water into steam, before escaping through the exhaust stack (6). Natural gas burns


Fig. 15.22 Surgut-2-the largest natural gas power at 5,600 MW very cleanly, but the stack is still built tall so that the exhaust gases (7) can disperse before they touch the ground. This ensures that it does not affect the quality of the air around the station.

The hot steam expands in the pipes, so when it emerges, it is under high pressure. These high-pressure steam jets spin the steam turbine (8), just like the combustion gas spins the gas turbine. The steam turbine coupled to a steam turbine generator (9), which converts the mechanical energy into electrical energy.

After passing through the turbine, the steam comes into contact with pipes surrounded by cold water, that is, condenses (10). The cold pipes cool the steam so that it condenses back into water. Then, it is piped back to the heat recovery steam generator to be reused.


Fig. 15.23 Gas power plant

### 15.10 INTRODUCTION TO NUCLEAR POWER PLANT

The power plants in which the energy released from the nucleus of an atom via nuclear fission that takes place in a nuclear reactor is used to generate electricity are called nuclear power plants. Unlike fossil-fuelled power plants, the only substance leaving the cooling towers of nuclear power plants is water vapour, and thus, it does not pollute the air or cause global warming.

Just as in many conventional thermal power stations, electricity in generated by harnessing the thermal energy released from burning fossil fuels. In nuclear power plants, the heat is removed from the reactor core by a cooling system and is used to generate steam, which drives a steam turbine connected to a generator producing electricity.


Fig. 15.24 Nuclear cycle used for power generation

The nuclear fuel cycle, shown in Figure 15.24, begins when uranium is mined, enriched, and manufactured into nuclear fuel, (1) which is delivered to a nuclear power plant. (2) After usage in the power plant, the spent fuel is delivered to a reprocessing plant (3) or to a final repository (4) for geological disposition. In reprocessing, $95 \%$ of spent fuel can potentially be recycled to be returned to usage in a power plant (5).

### 15.10.1 History and Development

1. The hope of nuclear energy for electricity generation began soon after the discovery in the early 20th century that radioactive elements, such as radium, released immense amounts of energy, according to the principle of mass-energy equivalence.
2. However, means of harnessing such energy was impractical, because intensely radioactive elements were, by their very nature, short lived.
3. This situation, however, changed in the late 1930s, with the discovery of nuclear fission.
4. In the United States, Fermi and Szilárd had both emigrated; this led to the creation of the first man-made reactor, known as Chicago Pile-1, which achieved criticality on December 2, 1942. This work became part of the Manhattan Project, which made enriched uranium and built large reactors to breed plutonium for use in the first nuclear weapons, which were used on the cities of Hiroshima and Nagasaki.
5. After World War II, the prospects of using 'atomic energy' for good, rather than simply for war, was advocated as a reason not to keep all nuclear research controlled by military organizations.
6. Electricity was generated for the first time by a nuclear reactor on December 20, 1951, at the EBR-I experimental station near Arco, Idaho, which initially produced about 100 kW .
7. On June 27, 1954, the USSR's Obninsk Nuclear Power Plant became the world's first nuclear power plant to generate electricity for a power grid and produced around 5 MW of electric power.


Fig. 15.25 Calder Hall nuclear power station in UK
8. The world's first commercial nuclear power station, shown in Figure 15.25, Calder Hall at Windscale, England, was opened in 1956 with an initial capacity of 50 MW (later 200 MW).
9. The first commercial nuclear generator to become operational in the United States was the Shippingport Reactor (Pennsylvania, December 1957).
10. The SM-1 Nuclear Power Plant, at Fort Belvoir, Virginia, was the first power reactor in US to supply electrical energy to a commercial grid, in April 1957, before Shippingport.
Health and safety concerns, the 1979 accident at Three Mile Island, and the 1986 Chernobyl disaster played a part in stopping new plant construction in many countries. Although the public policy organization, the Brookings Institution states that new nuclear units, at the time of publishing in 2006, had not been built in US because of soft demand for electricity, and cost overruns on nuclear plants due to regulatory issues and construction delays. By the end of 1970s, it became clear that nuclear power would not grow nearly as dramatically as once believed. Eventually, more than 120 reactor orders in US were ultimately cancelled and the construction of new reactors ground to a halt.

### 15.10.2 Nuclear Fission

The process in which an unstable nucleus of a heavy atom (like uranium-235) splits up into two medium weight nuclei with the liberation of enormous amount of energy is called nuclear fission. The nucleus of uranium- 235 atom is highly unstable. Therefore, when a neutron from outside is made to collide with a uranium- 235 nucleus, it breaks up or produces a fission to form smaller atoms with the liberation of a large amount of energy. The fission of uranium- 235 was first carried out by Fermi in 1941. In the fission process, when uranium- 235 atoms are bombarded with slow moving neutrons. The heavy uranium nucleus breaks up to produce two medium weight atoms of barium-139 and krypton-94 with the emission of neutrons. Enormous amount of energy is released during the fission of uranium. The equation for the nuclear fission is given as follows:

During fission process, some mass of uranium disappears (is lost) and a large amount of energy is produced.

Einstein theory of relativity shows that mass and energy are interchangeable as per the formula:

$$
E=m C^{2}
$$

where $E=$ energy in joules
$m=$ mass (which is lost during the process) in kg and
$C=$ velocity of light, that is, $3 \times 10^{8} \mathrm{~ms}^{-1}$
This energy is used for the generation of electrical energy. The abovementioned equation shows that during nuclear fission, one neutron is consumed and three neutrons are produced. These neutrons lead to further fission, causing nuclear chain relation. The nuclear fission process also emits radiations (rays of very short wavelength called gamma rays) that are harmful to living beings. Thus, we conclude that a nuclear fission produces the following elements:

1. Two medium weight nuclei (or two medium weight atoms $\mathrm{Ba}: 139$ and $\mathrm{Kr}: ~ 94$ )
2. Three neutrons (in some cases, two neutrons)
3. Radiations of very short wavelength called gamma rays.
4. Large amount of heat and light energy.

### 15.10.3 Fissioning of Uranium-235

In the spherical nucleus (see Fig. 15.26) of heavy atom U-235, the nuclear particles such as protons and neutrons are very close to each other because of which the nuclear force of attraction and the electrostatic force of repulsion are very delicately balanced. When a slow moving neutron attacks this nucleus and enters into it, the nucleus gets elongated, then the distance between nuclear particles (protons and neutrons) increases. The increased inter-particle distance weakens the nuclear force so that the electrostatic force of repulsion becomes more dominant. Due to the increased repulsion between the protons, a neck is developed in the nucleus as shown in Figure 15.26. The formation of neck decreases the nuclear attractive force further. Further, increased repulsion between protons disrupt the nucleus and breaks it into two smaller nuclei of barium-139 and krypton-94 along with the emission of three neutrons. A large amount of energy in the form of heat, light, and radiations is also released in the process.


Fig. 15.26 Nuclear change reaction

## Chain reaction

When a slow moving neutron strikes the nucleus of uranium-235 atom, the nucleus of U-235 undergoes fission producing nuclei of barium and krypton. It also produces three neutrons along with a tremendous amount of energy in the form of heat, light, and radiations. If we have a considerable quantity of uranium- 235 atoms, then the three neutrons produced by the fission of first U-235 nucleus would cause the fission of three more uranium atoms, producing $3 \times 3=9$ neutrons, and a lot more energy. These nine neutrons can cause the breaking of nine more uranium atoms, giving $9 \times 3=27$ neutrons and yet more energy. This nuclear reaction would propagate further. This process may continue like an unending chain and is called chain reaction.

A reaction in which particle that initiates (starts) the reaction is also produced during the reaction to carry on this reaction further and further is called a chain reaction.

The chain reaction taking place during the fission of uranium- 235 can be represented more clearly with the help of a diagram shown in Figure 15.27. Here, one neutron is being used and three neutrons are produced during the fission of each uranium nucleus. It shows that only one neutron is used in each fission process, but three neutrons are produced. This makes the fission process in U-235 a self-sustaining (or self-propagating) process called chain reaction.


Fig. 15.27 Nuclear chain reaction

## Explosive or uncontrolled fission reaction

That fission reaction that is deliberately allowed to go out of control by allowing all the neutrons produced during fission of uranium- 235 atoms to cause further fission, resulting in an explosion is called explosive or uncontrolled nuclear fission reaction.

It has been calculated that in an explosive fission reaction, about 1,020 uranium- 235 atoms undergo fission in just 1 min producing an unmanageable amount of energy. However, for all peaceful purposes, critical or controlled fission reaction is applied.

## Critical or controlled fission reaction

The fission reaction in which each fission of uranium-235 is allowed to retain just enough neutrons to ensure that the number of uranium atoms undergoing fission remains constant with time and does not go on multiplying endlessly is called a critical or controlled fission reaction.

In controlled or critical fission reaction, the energy is produced at a slow, steady, and manageable rate. To control the fission reaction, cadmium or boron material, which absorbs two neutrons quickly out of the three produced in each fission. Then, only one neutron is left for further fission. This releases manageable amount of energy. The first critical or controlled fission reaction was carried out on 2 December 1941 by Fermi.

### 15.10.4 Elements of a Nuclear Power Station

Some of the important parts of a nuclear power station are discussed in this sections

## Nuclear reactor

Nuclear reactor (or atomic reactor) is a kind of nuclear furnace for carrying out controlled fission of a radioactive material like uranium- 235 for producing energy. Nuclear reactor is the
major part of a nuclear power station where enormous amount of heat energy is produced by the controlled nuclear fission process. The following are the important functions of a nuclear reactor:

1. It provides neutrons with enough energy (slow speed) so that the same may be absorbed by the nuclei of certain very heavy atoms ( $\mathrm{U}-235$ ) causing fission. In the process, large amount of energy is liberated.
2. It controls the chain process according to the requirements.
3. It may also be used to form nuclear fuel.

The following are the brief description on the main parts of a nuclear reactor and they are shown in Figure 15.28.


Fig. 15.28 Nuclear reactor

1. Fuel elements: These are in shape of plates or rods. The fissionable material used in nuclear reactors for producing energy by the fission process is enriched uranium- 235 . Enriched uranium contains about $2 \%$ to $3 \%$ of easily fissionable uranium-235 isotope and the remaining is uranium-238 isotope, which does not undergo fission easily. It is important to note here that natural occurring uranium, which contains only $0.7 \%$ of ura-nium-235, cannot be used as a nuclear fuel directly in the reactor. It is because in natural uranium, the quantity of uranium-235, which is readily fissionable, is very small (only $0.7 \%$ ) and cannot sustain the nuclear chain reaction. Hence, the naturally occurring uranium is first processed to increase the percentage of fissionable uranium-235 in it. This process is called enrichment. The enriched uranium used in the reactors still contains both types of atoms, uranium-235 (2-3\%) and uranium-238 (97-98\%). However, U-235 only act as a real nuclear fuel to produce energy.
2. Coolant: The substance that is used to take away heat energy produced in the nuclear reactor to the heat exchanger is called coolant. In general, some pipes are embedded in the reactor through which some coolant is circulated to take out heat produced in the reactor to the heat exchanger. In most of the successful reactors, liquid sodium metal is used as a coolant. However, in some of the reactors, carbon dioxide gas or even water is used as a coolant.
3. Moderator: The substance that is used to slow down (or moderate) the speed of neutrons in the nuclear reactor to a level appropriate to cause fission of uranium-235 effectively is called moderator.

The fission of uranium-235 takes place only if the bombarding neutron is slow. It is because when the neutron is slow, it has more time to react with the nucleus of U-235 and causes fission. Once the fission starts, the neutrons emitted by the uranium-235 atoms have fairly high speed, so they cannot cause the fission of other U-235 atoms effectively. In such a case, the fission reaction will not become a self-propagating chain reaction and the fission may stop. Hence, a moderator (a substance that slows down the speed of emitted neutrons) is necessary to obtain self-propagating chain reaction (or self-sustained nuclear reaction).

The commonly used moderators in the nuclear reactors are graphite or heavy water. Graphite is a form of carbon element (C), while heavy water $\left(\mathrm{D}_{2} \mathrm{O}\right)$ is a compound of oxygen (O) with heavy hydrogen called deuterium (D). Graphite or heavy water is used around the uranium rods in the reactor to moderate (slow down) the speed of neutrons. This maintains the desired chain reaction of nuclei fission and prevents it from dying out.

In a nuclear reactor, during fission reaction, high speed neutrons are emitted that collide with the graphite nucleus or heavy water and lose some of their kinetic energy and get slowed down. These slow neutrons further split (fission) the nuclei of uranium-235 and liberate enormous amount of energy in the form of heat, light, and radiations.
4. Controlling rods: The rods or plates made of some neutron absorbing substance like cadmium or boron used in the nuclear reactors to control the nuclear chain reaction are called controlling rods.

An ordinary fission reaction produces a lot of neutrons at every step that cause further fission and ultimately a chain reaction of fission is produced that goes out of control. In such a case of uncontrolled fission, the nuclear reactor will become a nuclear bomb and may cause explosion. In order to keep the nuclear fission reaction of U-235 under control, controlling rods made of cadmium or boron are used. The controlling rods absorb the excessive neutrons being emitted by nuclear fission, so that the fission takes place at a steady rate and the required heat energy may be liberated. It is important to note here that the rate of nuclear fission is controlled by the insertion of controlling rods in the nuclear fuel. In other words, if the controlling rods are inserted more deeply into the fuel, they will absorb more neutrons and the fission will become slow and vice versa. Hence, the position of controlling rods into the fuel is so adjusted that a critical fission reaction is obtained. However, if the nuclear fission reaction in the nuclear reactor is required to be stopped, then cadmium or boron rods are fully inserted into the nuclear fuel.
5. Reflector: In order to keep the size of the reactor small, and hence the amount of fissionable material, it is necessary to conserve the neutrons. For this purpose, the reactor core is surrounded by a material that reflects the escaping neutrons back into the core.

This material is called the reflector. Very often, the reflector is of the same material as the moderator.
6. Pressure vessel: Its purpose is to house the reactor core.
7. Biological shield: It is made of concrete or steel and its purpose is to protect the operating personal from radiations produced in the core.

## Heat exchanger

It is an arrangement by which heat produced due to nuclear fission is extracted. Heat produced in the nuclear reactor is given to the working substance (coolant) that receives it and carries it to the heat exchanger. Here, the heat of the coolant is used for the conversion of water into steam. After giving up heat, the coolant is again fed to the reactor.

## Steam turbine

The steam produced by the heat exchanger is fed to the steam turbine through a valve that converts heat energy into mechanical energy.

## Alternator

An alternator is coupled to the steam turbine that converts mechanical energy into electrical energy. The function of the remaining elements, namely condenser, cooling tower, pumps, etc., have already been discussed under steam power stations.

## 15.II CONCEPT OF COGENERATION

Cogeneration is a thermodynamically efficient use of fuel. In thermal power stations while producing electricity, some energy must be discarded as waste heat, but in cogeneration, this thermal energy is put to use, as shown in Figure 15.29. All thermal power plants emit heat during electricity generation, which can be released into the natural environment through cooling towers, flue


Fig. 15.29 Block diagram for cogeneration
gas, or by other means. In contrast, combined heat and power (CHP) captures some or all of the by-product for heating water for district heating with temperatures ranging from approximately 80 to $130^{\circ} \mathrm{C}$. One of such arrangement is shown in Figure 15.30. This is also called combined heat and power district heating (CHPDH). Small CHP plants are an example of decentralized energy.

In fact, in the case of cogeneration, the supply of high-temperature heat first drives a gas- or steam turbine-powered generator and the resulting low-temperature waste heat is then used for water or space heating. Cogeneration was practiced in some of the earliest installations of electrical generation. Before central stations distributed power, industries were generating their own power and the exhausted steam was used for heating the buildings. Large offices, apartment buildings, hotels, and stores commonly generated their own power and used waste steam for building heating.

Thermal power plants (using fossil fuel such as coal, petroleum, or natural gas) and heat engines in general, do not convert all of their thermal energy into electricity. In most heat engines, a bit more than half is lost because of the excess heat (as per second law of thermodynamics and Carnot's theorem). By capturing the excess heat, CHP uses heat that would be


Fig. 15.30 Masnedo CHP Power Station in Denmark. This station burns straw as fuel. The adjacent greenhouses are heated by district heating from the plant wasted in a conventional power plant, potentially reaching an efficiency of up to $80 \%$ for the best conventional plants. A cogeneration plant is shown in Figure 15.31.


Fig. 15.31 A cogeneration plant in Metz, France. A 45MW boiler uses waste wood biomass as energy source and provides electricity and heat for $\mathbf{3 0 , 0 0 0}$ dwellings

This means that less fuel needs to be consumed to produce the same amount of useful energy. Steam turbines for cogeneration are designed for the extraction of steam at low pressures after it has passed through a number of turbine stages, or they may be designed for final exhaust at back pressure (non-condensing) or both. The extracted or exhaust steam is used for the process of heating, such as drying paper, evaporation, heat for chemical reactions, or distillation.

Some tricycle plants have used a combined cycle in which several thermodynamic cycles produce electricity. A heating system was used as a condenser of the power plant's bottoming cycle. For example, the RU-25 MHD generator in Moscow heated a boiler for a conventional steam power plant, whose condensate was then used for space heat.

A more modern system might use a gas turbine powered by natural gas, whose exhaust powers a steam plant and condensate provides heat. Tricycle plants can have thermal efficiencies above $80 \%$.

CHP is the most efficient when heat can be used on-site or very close to it. Overall, efficiency reduces when the heat is transported over long distances. This requires heavily insulated pipes, which are expensive and inefficient; while electricity can be transmitted along a comparatively simple wire, and over much longer distances for the same energy loss.

A car engine becomes a CHP plant in winter when the reject heat of engine is useful for warming the interior of the vehicle. The example illustrates the point that deployment of CHP depends on heat uses in the vicinity of the heat engine.

Cogeneration plants are commonly found in district heating systems of cities, hospitals, oil refineries, paper mills, wastewater treatment plants, thermal enhanced oil recovery wells, and industrial plants with large heating needs in cold countries.

### 15.12 CONCEPT OF DISTRIBUTED GENERATION

Distributed generation (DG) is an approach by which we produce electricity close to the end users of power by employing small-scale technologies of power generation, as shown in Figure 5.33. It reduces the burden on the main grid. DG technologies often consist of modular (and sometimes renewable energy) generators, and they offer a number of potential benefits. In many cases, distributed generators can provide lower cost electricity and higher power reliability and security with fewer environmental consequences than traditional power generators.

In contrast to the use of a few large-scale generating stations located far from load centres, DG systems employ numerous, but small plants and can provide power onsite. The system does not rely upon distribution-transmission and grid. DG technologies yield power in capacities that range from a fraction of 1 kW to about 100 MW .

### 15.12.1 Central Electricity Paradigm Versus Distributed Generation Versus Electricity Paradigm

In most of the countries, the current model for electricity generation and distribution is by centralized power plants. The power at these plants is typically produced by the combustion of fossil fuel or nuclear power. Centralized power models require distribution from the centre to outlying consumers, as shown in Figure 5.32. Power substations are erected in between anywhere from 10 to 100 km away from the actual users of the electric power. This requires transmission across the distance.

This system of centralized power plants has many disadvantages. In addition to the transmission distance issues, these systems contribute to greenhouse gas emission, the production of nuclear waste, inefficiencies and power loss over the lengthy transmission lines, environmental problems where the power lines are constructed, and security-related issues.

Many of these issues can be resolved by adopting DG. Distributed generation (DG) is often produced by small modular energy conversion units like solar panels or wind mill. These units can be stand alone or integrated into the existing energy grid. Sometimes, consumers who have installed solar panels may produce more energy than they require for daily use, and then they contribute more to the grid than they take out resulting in a win-win situation for both the power grid and the end user.


Fig. 15.32 Classic electricity paradigm


Fig. 15.33 Distributed generation

### 15.12.2 Some Examples of Distributed Generation Technologies

Distributed generation takes place on two-levels: the local level and the end-point level. Local level power generation plants often include renewable energy technologies that are site specific, such as wind turbines, geothermal energy production, and solar systems. These plants tend to be smaller and less centralized than the traditional model plants. Further, they have frequently more energy and are cost efficient and more reliable. Since these local-level DG producers often take into account the local context, they usually produce less environmentally damaging or disrupting energy than the larger central model plants.

Phosphorus fuel cells also provide an alternative route to a DG technology. These are not as environmentally reliant as the previously mentioned technologies. These fuel cells are able to provide electricity through a chemical process rather than a combustion process. This process produces little particulate waste.

At the end-point level, the individual energy consumer can apply many of the said technologies with similar effects. One DG technology frequently employed by end-point users is the modular internal combustion engine. Usually, all commercial buildings, hospitals, colleges, telephone exchanges, railway stations, etc., use these power generators as a backup to the normal power grid. It clearly shows that DG technologies can operate as isolated 'islands' of electric energy production or they can serve as small contributors to the power grid.

## 国贯 PRACTICE EXERCISES

## Short Answer Questions

1. What is a fuel cell?
2. What do you mean by fuel cell stack?
3. What do you understand by hydroelectricity?
4. Classify hydro-electric power plants on the basis of their capacity.
5. What is tidal power?
6. What is geothermal energy?
7. What do you mean by thermal power stations?
8. What is diesel generator unit?
9. What are gas power stations?
10. Define nuclear power plants.
11. What do you mean by nuclear fission?
12. Name the main parts of a nuclear reactor.
13. What do you mean by cogeneration?
14. What do you mean by distributed generation?

## Test Questions

1. What is fuel cell and fuel stack? How a fuel cell works?
2. How electric power is generated at hydroelectric power plant? Classify the hydroelectric power plants on the basis of their size?
3. Explain how electric power is generated by using geothermal energy.
4. How electric power is generated at nuclear power plant?
5. Give the concept of cogeneration and distributed generation.

## SUMMARY

1. Renewable sources of energy: The sources of energy that are present in an unlimited quantity in nature and get replenished through some natural process are called renewable sources of energy.
2. Non-renewable sources of energy: The sources of energy that are present in a limited quantity in nature and are not replenished by any natural process are called non-renewable sources of energy.
3. Kinetic energy of wind $=\frac{1}{2} m v^{2}$ joule

Kinetic energy of wind/unit volume $=\frac{1}{2} \rho v^{3}$ joule
4. Wind speed at higher altitudes: The wind blows faster at high altitudes because of the reduced influence of drag.
5. Largest generation using wind energy: China surpassed German and US in wind generation capacity in 2010.
6. Wind farm: A wind farm or wind park is a group of wind turbines in the same location used to produce electrical energy.
7. Onshore wind farm: Onshore wind farm is the turbine installations in hilly areas and should be on ridgelines; generally, it is 3 km or more inland from the nearest shoreline.
8. Offshore wind farm: When a large number of wind turbines are installed near the seashore or inside the sea are called offshore wind farms.
9. Largest wind farm in India: The largest wind farm in India is Jaisalmer wind farm. Its present capacity is $1,064 \mathrm{MW}$, The development of the wind park was initiated by Suzlon in August 2011.
10. Solar power: Solar power is the conversion of sunlight into electricity.
11. CPS system: In concentrating solar power (CSP) systems, lenses or mirrors with tracking systems are used to focus a large area of sunlight into a small beam.
12. Photovoltaic solar power: A solar cell or PV cell is a device that converts light into electric current using the photoelectric effect.
13. Fuel cell: A device that converts the chemical energy from a fuel into electrical energy through a chemical reaction with oxygen or another oxidizing agent is called a fuel cell.
14. Fuel cell stack: To deliver the desired amount of energy, the fuel cells can be combined in series and parallel circuits to yield higher voltage, and parallel channel of configurations allow a higher current to be supplied. Such a design is called a fuel cell stack.
15. Hydroelectricity: It is the term that refers to electricity generated by hydropower, that is, production of electrical power through the use of the gravitational force of falling water or flowing water.
16. Classification of hydroelectric power station: On the basis of size and capacity, these are classified as (i) large power stations, (ii) small power stations, (iii) micro-hydroelectric power stations, and (iv) pico-hydroelectric power stations.
17. Tidal power or tidal energy: It is a form of hydropower that converts the energy of tides into useful forms of power, mainly electricity.
18. Geothermal electricity: The electricity generated from geothermal energy is called geothermal electricity.
19. Thermal power stations: All the electric power stations in which the electric power is generated by steam driven turbines are called thermal- or steam-electric power stations.
20. Diesel generator unit: A combination of a diesel engine and an electric generator (often an alternator) to generate electrical energy is called a diesel generator unit.
21. Nuclear power plant: The power plants in which the energy released from the nucleus of an atom via nuclear fission that takes place in a nuclear reactor is used to generate electricity are called nuclear power plants.
22. Nuclear fission: The process in which an unstable nucleus of a heavy atom (like uranium-235) splits up into two medium weight nuclei with the liberation of enormous amount of energy is called nuclear fission.
23. Elements of a nuclear reactor: The important parts of a nuclear reactor are as follows: (i) fuel elements, (ii) coolant, (iii) moderator, (iv) controlling rods, (v) reflector, (vi) pressure vessel, and (vii) biological shield.
24. Cogeneration: Cogeneration is a thermodynamically efficient use of fuel. In fact, in the case of cogeneration, the supply of high temperature heat first drives a gas or steam turbine-powered generator and the resulting low-temperature waste steam is then used for water or space heating.
25. Distributed generation: Distributed generation is an approach by which we produce electricity close to the end users of power by employing small-scale technologies of power generation. It reduces the burden on the main grid.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. The sources of energy that are present in an unlimited quantity in nature and get replenished through some natural process are called $\qquad$ .
2. The sources of energy that are present in a limited quantity in nature and are not replenished by any natural process are called $\qquad$ .
3. Total wind energy flowing through an imaginary area ' $A$ ' during time ' $t$ ' is given as
4. The wind blows $\qquad$ at higher altitudes because of the reduced influence of drag.
5. A group of wind turbines in the same location used to produce electrical energy is called $\qquad$ -.
6. The power plants in which concentrated sunlight is used to heat up the working fluid to generate electricity are called $\qquad$ .
7. A device that converts the chemical energy from a fuel into electrical energy through a chemical reaction with oxygen or another oxidizing agent is called a $\qquad$ -.
8. A substance that conducts charged ions from one electrode to the other in a fuel cell, battery, or electrolyser is called $\qquad$ _.
9. The term used for hydroelectric power station that typically produces up to 100 kW of power is
$\qquad$ —.
10. The electricity generated from geothermal energy is called $\qquad$ .

## OBJECTIVE TYPE QUESTIONS

1. The substance used to take away heat energy produced in the nuclear reactor is called
(a) moderator.
(b) coolant.
(c) fuel.
(d) convector.
2. Cogeneration means
(a) generation of electricity at many places.
(b) generation of electricity by different sources.
(c) thermodynamically efficient use of fuel.
(d) clubbing of more than two generating stations.
3. The largest wind farm in India is located at
(a) Jaisalmer.
(b) Kota.
(c) Hyderabad.
(d) Goa.
4. One TW means
(a) $1 \times 10^{3} \mathrm{~W}$.
(b) $1 \times 10^{6} \mathrm{~W}$.
(c) $1 \times 10^{12} \mathrm{~W}$.
(d) $1 \times 10^{15} \mathrm{~W}$.
5. Coolant, moderator, controlling rods, reflector, and biological shield are the main parts of
(a) hydropower plant.
(b) coal-fed thermal power plant.
(c) geothermal power plant.
(d) nuclear power plant (nuclear reactor).

## SHORT ANSWER TYPE QUESTIONS

1. Classify the natural sources of energy.
2. What do you mean by renewable sources of energy?
3. What are the exhaustible sources of energy?
4. Why all the natural sources of energy are first converted into electrical energy and then utilized?
5. How wind energy is related to wind velocity?
6. Which are the strategic positions where wind energy is available in abundance?
7. What do you mean by TW?
8. What do you mean by wind farm or wind park?
9. How will you differentiate between onshore and offshore wind farms?
10. What do you mean by 'micro-sitting' related to wind farms?
11. Do you know where the largest wind farm is located in India?
12. Mention any adverse environmental effect of wind farm.
13. How solar energy or power is converted into electrical energy or power?
14. Name the various technologies used for conversion of solar power into electrical power in CSP plants.
15. How concentrating linear Fresnel technology is used in CSP plants?
16. What do you understand by stirling dish technology?
17. Draw the block diagram of $P V$ power generation for a residential building.
18. Why solar power generation is preferred over coal-fed thermal power generation?
19. What is a fuel cell?
20. What do you mean by efficiency of a fuel cell?
21. Name major applications of fuel cells.
22. How hydropower is used to generate electricity?
23. What are the different methods by which hydro-power is converted into electrical power?
24. How hydropower stations are classified on the basis of size and capacity?
25. What are the major advantages of hydroelectric power plants?
26. What are the disadvantages of hydroelectric power stations?
27. How tidal power is used for generation of electricity?
28. How geothermal energy is used to generate electrical energy?
29. What are different types of geothermal power stations?
30. What do you mean by thermal-electric power stations?
31. How thermal energy of fuel is converted into electrical energy in thermal power stations?
32. Draw the block diagram of a thermal power station.
33. What is the function of condenser in a thermal power station?
34. How does a gas power station work?
35. What is a nuclear power plant?
36. What do you mean by nuclear fission?
37. What is a chain reaction in nuclear fission?
38. What is nuclear reaction? Name its main parts.
39. How the chain reaction is controlled in a nuclear reactor?
40. What is the concept of a cogeneration?
41. Give the concept of distributed generation.

## TEST QUESTIONS

1. What are the natural sources of energy? How will you distinguish them as renewable and nonrenewable sources of energy? What are the various factors on which wind energy depends?
2. How altitude affects the wind velocity? Justify your answer. What do you mean by onshore and offshore wind farms?
3. How wind farms affect our environment? How will you rate the reliability of this source of energy?
4. Is solar power a renewable or non-renewable source of energy? Justify your answer. How this source of energy is used for the generation of electrical energy?
5. What are CSP generating plants? Which technologies can be employed in these plants to generate electrical energy?
6. How PV cells are employed for conversion of solar power into electrical power? What do you mean by PV solar power plant? Explain in detail.
7. What is a fuel cell? What are the main constituents of a fuel cell? Explain the working of a fuel cell.
8. Classify the fuel cells and define the following: (i) anode, (ii) catalyst, (iii) cathode, (iv) electrolyte, (v) fuel cell stack, (vi) membrane, and (vii) solution
9. What are the various applications of fuel cells?
10. Classify the fuel cells on the basis of electrolyte and technology used. Explain the working of any one of them in detail.
11. What do you mean by fuel cell stack? Define efficiency of a fuel cell. Name some of the major applications of fuel cell.
12. What are the different methods by which hydro energy is converted into electrical energy? What are the adverse effects of hydropower plants on the environment?
13. How hydropower plants are classified on the basis of size and capacity? With the help of a block diagram, explain the functioning of a small power station.
14. What are the advantages and disadvantages of hydroelectric power stations? How site is selected for their erection?
15. How tidal power is used to generate electricity? What do you mean by tidal stream generator and tidal barrage?
16. How geothermal energy is used to generate electric power? Explain the functioning of dry steam power plant, flash steam power plant, and binary cycle power plant.
17. With the help of a block diagram, explain the working of a thermal power station.
18. With the help of a block diagram, explain the working of a gas power station.
19. What is a nuclear power plant? What is the function of a nuclear reactor in a nuclear power station? How electric power is generated in a nuclear power station?
20. What do you mean by nuclear fission? What is nuclear chain reaction and how is it controlled in a nuclear reactor?
21. Draw a neat sketch of a nuclear reactor, label its components, and mention their material and function.
22. What do you understand by cogeneration?
23. What is the concept of distributed generation?

## q. ANSWERS

## Fill in the Blanks

1. renewable sources of energy
2. non-renewable sources of energy
3. $\frac{1}{2} \rho v^{2}$
4. wind farm
5. faster
6. Fuel cell
7. concentrated solar power station
8. micro-hydroelectric power station
9. electrolyte
10. geothermal electricity

Objective Type Questions

1. (b)
2. (c)
3. (a)
4. (c)
5. (d)


## LEARNING OBJECTIVES

After the completion of this unit, students or readers will be able to understand the following:

* What are the major generating stations and how electric power is generated in these power stations?
* How electric power is transmitted and distributed among various consumers?
* What are the major components used in transmission and distribution lines?
* What are substations and which activities are performed in these substations?
* What are the major equipments used in the substations?
* What are grid stations?


### 16.1 INTRODUCTION

The electrical energy is generated at the various power (or generating) stations such as hydroelectric power stations, steam power stations, and nuclear power stations, that are located at the favourable places where water energy or fuel (coal) is available in abundance. This electrical energy, generated at the power stations, has to be supplied to various consumers scattered at far off places. There is a large network of conductors between the power stations and consumers for this purpose. This network can be broadly divided into two parts, namely transmission and distribution system.

The system by which bulk power (or energy) is delivered from power stations to the load centres (big substations) is called 'transmission system'.

The system by which electrical power (or energy) is delivered from substations to the bulk and small power consumers is called 'distribution system'.

In this chapter, the various aspects of electric power supply system are discussed.

### 16.2 LAYOUT OF POWER SYSTEM

The generation, transmission, and distribution of electric power are called 'power system'. The layout of a typical power system is shown in Figure 16.1, whereas the single-line diagram of power system is shown in Figure 16.2. Generally, a power system consists of the following stages:

1. Power station: The electric power is generated at the power station by three-phase system employing number of alternators in parallel. Electric power is generated at the generating stations at voltages $3.3,11$, or 33 kV . However, in practice, electric power generated at 11 kV is mostly adopted. At the power stations, generated voltage ( 11 kV ) is ${ }^{1}$ stepped up to 132,220 , 400 or 750 kV (i.e., whichever is economical) depending upon the distance, the amount of power to be transmitted, and the system stability.
2. Transmission system: The system by which bulk power (or energy) is delivered from generating station to load centres (i.e., big substations) is called transmission system.
(a) Primary transmission: The electric power is transmitted in bulk from the generating station to the major load centres (i.e., main substations) by overhead lines at 132, 220 , or 400 kV . This forms the primary transmission system.

Therefore, the system by which huge electric power is transmitted through long distances at higher voltages (e.g., $750,400,220,132 \mathrm{kV}$, etc.) is called 'primary transmission system'.

For primary transmission, three-phase, three-wired system is always used, and the aluminium conductors with steel reinforcement (ACSR) are run over the steel towers.
(b) Secondary transmission: On the outskirt of the cities, there are substations where power is received at 132,220 , or 400 kV and is stepped down to 66 or 33 kV depending upon the amount of power to be fed to a particular area. Here, the power is transmitted at 66 or 33 kV by overhead lines. This constitutes secondary transmission system. Three-phase, three-wired system is always employed for secondary transmission.

Therefore, the system by which comparatively smaller quantity of electrical power is transmitted through smaller distances at lower voltages (e.g., $66 \mathrm{kV}, 33 \mathrm{kV}$, etc.) is called secondary transmission system.
3. Distribution system: The system by which power (or energy) is delivered from substations to the bulk and small power consumers is called distribution system. Broadly, distribution system may be subdivided as follows:
(a) Primary distribution: At suitable place near the cities or load centres, there are substations where secondary transmission voltage is stepped down to 11 kV , and the power is delivered by overhead lines at this voltage. This forms the ${ }^{2}$ primary

[^14]
Fig. 16.1 Layout of power system


Fig. 16.2 Single line diagram of power system
distribution system. The system by which electric power is distributed among bulk power consumers at 33 kV or 11 kV is called primary distribution system.

These 11 kV lines are run along all the important road sides of the city, and power is supplied to the bulk consumers (big industries) at this voltage. The bulk consumers install their own substation to stepdown the voltage to 400 V for the utilization of electric power. Three-phase, three-wired system is invariably employed for this purpose.
(b) Secondary distribution: The electric power from primary distribution lines is delivered to various substations called distribution substations. These substations are located near the consumers' localities. At these substations, voltage is stepped down to 400 V , and the power is distributed by three-phase, four-wire system. This forms secondary distribution system. The system by which electric power is distributed among small power consumers (for industrial, commercial, or domestic purposes) at $400 \mathrm{~V} / 230 \mathrm{~V}$ is called secondary distribution system.
The voltage between any two phases (i.e., line voltage) is 400 V , and the voltage between any one phase and neutral (i.e., phase voltage) is 230 V .
4. Utilization: The consumers utilize the electrical power (or energy) for different purposes. Accordingly, the single-phase domestic loads (e.g., lamps, fans, refrigerators, heaters, etc.) are connected between any one phase and neutral, whereas industrial loads (e.g., three-phase motors, furnaces, etc.) are connected across three-phase, 400 V lines.

A tree of power system is shown in Figure 16.3. The various parts of the tree represent different parts of power system. The major parts of the power system are discussed in detail in the following section.

### 16.3 GENERATION OF ELECTRICAL ENERGY

Since energy can be neither created nor destroyed, it can only be converted. Therefore, the generation of electrical energy is basically the conversion of some form of energy available in nature into electrical energy.

Therefore, the conversion of energy available in different forms in nature into electrical energy is known as generation of electrical energy.


Fig. 16.3 A tree representing power system

### 16.4 MAJOR GENERATING STATIONS

A large number of sources of energy (e.g., sum, wind, water, conventional fuel, nuclear fuel, tide in sea, etc.) are available in nature. However, some of them can be used commercially and economically for the generation of electrical energy in large quantity. According to the conversion of natural source, the generating stations may be classified as follows:

1. Hydroelectric power stations.
2. Coal-fed thermal power stations or steam power stations.
3. Diesel power stations.
4. Nuclear power stations.

The salient features of various power station are discussed as follows:

### 16.5 HYDROELECTRIC POWER STATIONS

The power stations where potential or kinetic energy (hydraulic energy) of water is converted into electrical energy are called hydroelectric power stations.


Fig. 16.4 Conventional diagram of a hydro-electric power station

To generate electrical energy by this method economically, ample quantity of water at sufficient head must be available. Therefore, water is collected from natural lakes or stored in the artificial reservoirs by constructing dams across flowing streams. As shown in Figure 16.4, when water falls through pen stock, on the blades of water turbines (prime mover), potential energy of water is converted into mechanical energy. Generators are coupled with the turbines, which convert mechanical energy into electrical energy.

Due to limited reserves of fuels and increasing demand of electric power, the hydroelectric power stations are becoming more and more popular nowadays. These stations are generally located in hilly areas where abundant water at suitable head is available. Various advantages and disadvantages of hydroelectric power stations are as follows:

### 16.5.1 Advantages of Hydroelectric Power Stations

1. These generating stations require no fuel, less maintenance, small number of operating staff, hence have smaller generating cost.
2. Their operating efficiency is high and has instant starting.
3. By erecting these power plants, floods are controlled and water is used for agriculture. Moreover, no pollutant is produced by these plants. Hence, our engineers always try to exploit this source of energy.

### 16.5.2 Disadvantages of Hydroelectric Power Stations

1. Capital investment is high and also transmission of power is highly expensive.
2. They require large area for reservoir and dam.
3. Erection of these power stations is time-consuming.
4. Long dry season may affect generation.

### 16.5.3 Elements of Hydroelectric Power Station

The important elements, as shown in Figure 16.4 and 16.5, of hydroelectric power station are as follows:

1. Catchment area: The surrounding area from where water collected in the reservoir is called catchment area.
2. Reservoir: The area behind the dam where water collected is called reservoir.
3. Dam: A cement concrete wall erected to block flowing water of stream is called a dam.
4. Spillways: These are the safety valves of dam. When level behind the dam rises beyond the danger mark, these valves are opened.
5. Valve house: It contains sluice valves to control supply of water to the penstock or turbine.
6. Penstock: The huge steel or reinforced steel concrete pipes that carry water from valve house to scroll case of turbine are called penstocks.
7. Surge tank: These are used to protect the penstocks against water hammer, which occurs due to opening or closing of sluice valves.
8. Racks: Racks (iron bars) are placed at the entrance of the tunnel to prevent the ingress of floating and other harmful material to the turbine.
9. Water turbines: Water turbines or water wheels are the prime movers, and they convert potential and kinetic energy of water into mechanical (rotatory motion) energy. These may be impulse turbines or reaction turbines depending upon the head of water fall.
10. Draft tube: It is an airtight pipe connected at the bottom of the scroll case of turbine. It allows the water, coming from the turbine, to discharge at the tail race.


Fig. 16.5 Hydro-electric power plant
11. Tail race: It is the outlet of the water. It leads the water to the outgoing stream.
12. Alternator: It is an electrical machine that converts mechanical energy to alternating electrical energy.
13. Control room: It is a place where controlling equipment, protective devices, indicating instruments, etc. are installed on the panels.

### 16.6 THERMAL POWER STATIONS

The power stations, where heat energy of coal combustion is converted into electrical energy, are called steam power stations.

In these power stations, the heat of combustion of coal is utilized for the conversion of water into steam which runs the steam engine or turbine. An alternator is coupled with the steam engine or turbine, which converts mechanical energy into electrical energy.

The flow (schematic) diagram of a modern coal-fed steam power station is shown in Figure 16.6. The different parts connected in the power station are as follows:

1. Coal storage
2. Coal handling
3. Boiler
4. Ash handling
5. Ash storage
6. Air draught fan
7. Air pre-heater
8. Supper heater
9. Economizer
10. Induced draught fan
11. Chimney
12. River
13. Raw water tank
14. Pump
15. Water treatment chamber
16. Feed water pump
17. Low-pressure water heater
18. Boiler feed water pump
19. High-pressure water heater
20. Valve
21. Steam turbine
22. Condenser
23. Water extraction pump
24. Cooling tower
25. Cold water circulating pump
26. Alternator

Figure 16.7 shows a simple block diagram of a coal-fed stream power station.

1. Boiler
2. Super heater
3. Steam turbine
4. Alternator
5. Condenser
6. Cooling tower
7. Extraction pump
8. Water heater
9. Feed water pump
10. Economizer

The whole arrangement can be subdivided into the following five stages for simple explanation:

1. Coal- and ash-handling arrangement: The coal transported to the storage site of the power plant by road, rail, or waterways whichever is the cheapest. From the storage site, coal is taken to the coal storage yard. From coal storage plant, it is delivered to the coal-handling plant where it is pulverized, that is, crushed into small pieces for completed combustion. The pulverized coal is fed to the boiler furnace by means of belt conveyors.

Fig. 16.6 Schematic diagram of a thermal power station


Fig. 16.7 Simplified block diagram of a thermal power station

In the boiler furnace, combustion of coal takes place, which can be controlled by controlling the great speed, quantity of coal entering the grate, and damper openings. After the combustion of coal, ash is produced which is sent to the ash precipitator. In the ash precipitator, unburnt coal is removed and again sent to the boiler furnace. Then, ash is removed to the ash-handling plant and delivered to ash storage. The removal of ash from the boiler is necessary for the proper burning of coal.
2. Air and flue gases arrangement: Air is drawn from the atmosphere by the air draught fan and given to the boiler furnace through air pre-heater. In the air pre-heater, air is heated up by utilizing the furnace temperature. Flue gases are produced after combustion of fuel which are circulated around boiler tubes and super heater. The flue gases are then drawn by the induced draught fan through economizer and air pre-heater and finally exhausted to the atmosphere through chimney.
3. Steam and feed water arrangement: In the boiler, heat of fuel gases is utilized, and water is converted into high-pressure steam, which is wet. Wet steam is passed through the super heater, where it is further superheated and attains higher temperature and becomes dry. The dry steam is supplied to the steam turbine through the regulating valve. After giving out its heat energy to the turbine, it is exhausted into the condenser where its latent heat is extracted and steam is condensed. The condensed steam (water) is extracted from the condenser by extraction pump. After passing through the low-pressure water heater, high-pressure water heater, and economizer, it is again fed to the boiler. In low-pressure water heater and high-pressure water heater, the feed water is heated up by utilizing the bled steam. In the economizer, the heat of flue gases is utilized to heat up the feed water.

The condensed steam (water) is not sufficient for the boiler, and therefore, it requires extra clean and soft water. The water taken from the river is stored in storage tank (raw water tank). Water is sucked from the storage tanks by pumps, purified and softened by chemicals and then delivered to the boiler by means of feed water pumps through low-pressure water heater, high-pressure water heater, and economizer.
4. Cooling arrangement: Steam is condensed in the condenser after giving heat energy to the turbine. The rejected heat must be dissipated to atmosphere. This is done by circulating cooling water in the condenser. This circulating water takes away heat from the condenser and becomes hot. It is cooled down with the help of cooling towers and again pumped onto the condenser.
5. Electrical plant: An alternator is coupled to the turbine, which converts mechanical energy of turbine into electrical energy. The electrical output from the alternator is delivered to the bus bars through transformer, circuit breakers, and isolators. An exciter (DC shunt generator) is coupled on the same shaft to give DC excitation to main poles of the alternator.

### 16.6.1 Advantages of Thermal Power Stations

1. The fuel used is not so expensive, and therefore, cost of generation is low.
2. Initial cost is less as compared to other power stations.
3. Such plants can be located near the load centre, and therefore, cost of transmission lines is less.

Since coal is available in India in abundance, and therefore, these plants are erected near all the major cities as and when need arises.

### 16.6.2 Disadvantages of Thermal Power Stations

1. Atmosphere gets polluted due to smoke, dust, and fumes.
2. Require more maintenance and a large number of operating staff which increases the cost of generation.

### 16.7 DIESEL POWER STATIONS

The power stations where diesel engines are used as prime movers and energy produced by the combustion of diesel oil is converted into electrical energy are called 'diesel power stations'.

In these power stations, the products of combustion of diesel oil are used to produce mechanical energy. Alternator is coupled with the diesel engine, which converts mechanical energy into electrical energy.

Because of heavy cost of diesel, these generating stations are not employed in India commercially to generate power. However, due to their easy installation and quick starting, these generating plants are used as standby generating stations at the places, such as hospitals, telephone exchanges, TV stations, railway stations, commercial and domestic buildings, and many other similar places.

### 16.8 NUCLEAR POWER STATIONS

The power stations where nuclear energy is converted into electrical energy are called nuclear power stations.

The reserves of natural resources such as coal and other fuels in the world are limited, and these will be exhausted soon. Therefore, scientists are looking towards the new sources of energy, and nuclear energy is one of such sources. It is interesting to note that 1 kg of atomic material produces the same heat which is produced by 2,700 tons of coal. Based on so far exploited world deposits of this fuel, it is estimated that total energy which could be generated from this source would be about 23 times that obtainable from all known sources of conventional energy put together.

The flow diagram of the nuclear power station is shown in Figure 16.8. It may be said that basically a nuclear power station is a steam power station in which steam boiler is replaced by the nuclear reactor and heat exchanger. A reactor is that part of a nuclear plant where the fission chain reaction (controlled) is made to take place and where the heat is generated to be picked up


Fig. 16.8 Flow diagram of a nuclear power station
by the cooling medium (coolant). The coolant delivers this heat to the heat exchanger to generate steam for the turbine. Steam after giving out its heat energy to the turbine is condensed in the condenser, which is again fed to the heat exchanges by feed water pumps. An alternator is coupled to the turbine, which converts mechanical energy into electrical energy.

The advantages of nuclear power station are as follows:

### 16.8.1 Advantages of Nuclear Power Stations

1. Transportation cost of fuel is low (almost negligible).
2. The running cost and transmission cost of these plants is low.

### 16.8.2 Disadvantages of Nuclear Power Stations

1. Initial cost and maintenance cost is high.
2. Dangerous radioactive product is produced and it is difficult to dispose-off such product.

Since the conventional fuels such as coal and diesel are going to be exhausted, and therefore, all the developed and developing countries are trying their best to exploit the nuclear energy for the generation of electrical energy.

### 16.9 TRANSMISSION OF ELECTRICAL POWER OR ENERGY

Usually, electrical power or energy is generated at the generating stations that are located far away from the consumers who are to utilize it.

The system by which bulk power or energy is delivered from generating station to load centres (i.e., big substations) is called transmission system.

The major components of a transmission system are as follows:

1. Transmission lines or conductors: They are used to transmit electrical power from one place to the other. The size of a conductor depends upon the current flowing through it, which further depends upon the system voltage and the power to be transmitted. Only stranded conductors, as shown in Figure 16.9, are used due to their high flexibility.

Usually, ACSR (aluminium conductor steel reinforced) conductors are employed to transmit electric power. In smaller diameter conductors, one central strand and in larger diameter conductors, seven central strands are made of galvanized steel, whereas the surrounding conductors are made of aluminium. The steel strands improve the tensile strength of the conductor to large extent. On the other hand, the surrounding aluminium strands carry almost all the current flowing through the conductor due to ${ }^{3}$ skin effect.
2. Supporting structure: The supporting structure for overhead lines is required to keep the bare conductors at a suitable height (as per Indian Electricity Rules) above the ground. The following line supports are generally used in overhead system:
(a) Steel towers: The broad base, lattice steel towers for single and double circuits are used as shown in Figure 16.10(a) and (b), respectively. These towers are invariably employed for transmission lines operating at $33 \mathrm{kV}, 66 \mathrm{kV}, 132 \mathrm{kV}, 220 \mathrm{kV}, 400 \mathrm{kV}$, and above.


Fig. 16.9 Stranded conductor (a) Isometric view (b) Sectional view
(b) RCC poles: The reinforced cement concrete poles have greater mechanical strength, longer life, and low maintenance. Moreover, they have good outlook. Therefore, these poles are used to carry the conductors of overhead lines operating at 11 kV . Figure 16.11(a) and (b) shows a single-circuit and double-circuit RCC poles, respectively.
(c) Steel poles: Steel poles are available in three sections, and accordingly, these are named as rail poles, tubular poles, and rolled steel joist poles. The sectional view of these poles is shown in Figure 16.12(a), (b), and (c), respectively. These poles are generally employed to carry the conductors of overhead lines operating at $400 \mathrm{~V}, 11 \mathrm{kV}$, and 33 kV .
(d) Wooden poles: Wooden poles are mechanically weak, require maintenance, have poor life but have high-insulating properties. These are rarely employed in urban areas. Generally, these were employed in rural areas for overhead lines operating

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Fig. 16.10 Steel towers (a) For single circuit (b) For double circuit


Fig. 16.11 RCC Poles (a) For double circuit (b) For single circuit
at 400 V . Double-circuit A-type and H-type wooden pole structures are shown in Figure 16.13(a) and (b), respectively. However, these poles are not employed in any area nowadays.
3. Insulators: The overhead line bar conductors are supported on the towers or poles. To insulate the conductor from the supports, insulators are provided. In fact, insulators prevent leakage of current from conductor to earth through supports. The various insulators employed in the overhead lines are as follows:
(a) Suspension-type insulators: The conductors of the lines operating at 66 kV and above are insulated from the supports with the help of suspension-type insulators shown in Figure 16.14.
(b) Pin-type insulators: The pin-type insulator is screwed on to a galvanized steel bolt which in turn is installed on the cross arm of the pole. These insulators are used to insu- late the conductors of 11 kV and 33 kV lines. A pin-type insulator with its bolts is shown in Figure 16.15.
(c) Shackle insulators: A shackle insulator with conductor is shown in Figure 16.16. These are use at 400 V distribution lines.
(d) Eggs or stay insulators: These insulators are used in the guy or stay wire, as shown in Figure 16.17, employed with the poles placed at the dead ends or at sharp
turns of low-voltage distribution lines. These insulators do not allow the leakage current, if any, to reach at the lower portion of the stay wire. Hence provides safety.
(e) Strain insulators: The insulators that take up the strain and are provided at the poles placed at the dead ends or at sharp


Fig. 16.12 (a) Rail-pole section (b) Tubu-lar-pole section (c) Rolled-steel joist poles


Fig. 16.14 Suspension type insulators


Fig. 16.15 (a) Pin type insulator (b) Stud for pin-type insulator
turns are called strain insulators. These insulators may be disc type or shackle type.
(f) Reel insulators: In LT lines, guard wire is placed between earth wire and neutral below the three-line conductors. To insulate earth wire from neutral, a small insulator is provided at the neutral as shown in Figure 16.18. This insulator is called reel insulator.

### 16.10 DISTRIBUTION SYSTEM

The part of power system by which electric power is distributed among various consumers for their local use is known as distribution system.

In general, the distribution system is part of power system that conveys electric power (or energy) from major substations (which are supplied by transmission lines) to the consumers as



Fig. 16.19 Line diagram of distribution system
per their requirement. A low-tension (LT) distribution system is shown in Figure 16.19 which comprises feeders, distributors, and service mains.

1. Feeder: A line or conductor that connects the major substation (or localized generating station) to the distributor is known as feeder. It is to feed the electric power (or energy) to the distributor. In Figure 16.19, SA, SB, and SD are the feeders. Since no tapping is taken from the feeder, generally, it carries same current throughout its length. The current carrying capacity is the main consideration taken into account while designing a feeder.
2. Distributor: A line or conductor to which various consumers are connected through service mains is known as distributor. It is to distribute electric power (or energy) among various consumers, and therefore, a number of tappings are taken from the distributor. Hence, it carries different currents along its length. In Figure 16.19 , AB, BCD, and DA are the distributors, while designing a distributor, voltage drop along the length of the distributor is the main consideration. It is because the fluctuations of voltage at the consumer terminals should not exceed beyond the permissible limits (i.e., $6 \%$ of the rated value).
3. Service mains: The cable (overhead or underground) that supplies electrical energy from the pole to consumer's terminals is called service main or service line.

In case of a single-phase supply (domestic loads), the service main has three wires, namely phase wire, neutral wire, and earth wire running from the supply pole to the consumer's terminals.

However, for industrial loads (e.g., workshop, cinema house, etc.), three-phase supply is given. For this supply, five wires are run from the pole to the consumer terminals (Figure 16.20), where three wires are for three phases, fourth wire is for neutral, and the fifth is for earth.


Fig. 16.20 Service mains

### 16.11 SUBSTATIONS

A substation is an assembly of apparatus that is installed to control transmission or distribution of electric power. Substation may be required to perform one or more of the following important functions:

1. To perform switching operation, that is, switch ON or OFF the power line.
2. To perform voltage transformation operation, that is, to step-up or step-down the voltage.
3. To perform power factor correction operation.

Of the above operation, probably voltage transformation is the most important operation of a substation. Generally, a substation receives power at one voltage and delivers it at higher or lower voltage.

### 16.11.1 Classification of Substations

According to design, substations are classified as follows:

1. Indoor substations: In these substations, the apparatuses are installed within the building of the substation and hence the name indoor sub-station. Such substations are usually designed for 11 kV but can be erected for 33 kV or 66 kV it the surrounding atmosphere is containing impurities that may damage the equipment.
2. Outdoor substations: In these substations, the apparatuses of the substations are installed in open and hence the name outdoor substation. Such substations can be designed to handle low, high, and extra high voltages. The outdoor substations may be further classified as follows:
(a) Pole-mounted substations (Fig. 16.21)
(b) Foundation-mounted substations

### 16.11.2 Key Diagram of a 66-kV Substation

The key diagram of a $220 / 66 \mathrm{kV}$ substation is shown in Figure 16.22. In Figure 16.22, all the equipments are shown on one side of the transformer, and similarly, all the equipments are connected on the other side of the transformer also.

The plan of the substation is shown in Figure. 16.23. Its elevations after sectioning at AA and BB are shown in Figure 16.24 and Figure 16.25, respectively.

### 16.11.3 Substation Equipment

Following are the important equipment/apparatus employed at the $66 \mathrm{kV} / 11 \mathrm{kV}$ substations:

1. Transformer: It is a static device that transforms AC electric power from one circuit to the other at the same frequency. It is used to step-up or step-down the voltage. At the substations except at the generating station, step-down transformers are employed.


Fig. 16.21 Pole mounted $11 \mathrm{kV} / 400 \mathrm{~V}$ sub-station
2. Circuit breaker: It is a device that makes and breaks the circuit under no-load, full-load, or fault conditions. It can be operated manually under normal conditions and automatically under abnormal conditions with the help of relays.
3. Isolators: These are knife switches that are operated only at no-load. Their main function is to isolate a portion of the circuit from the other. These are generally placed on both the sides of a circuit breaker in order to do repair and maintenance on the circuit


Fig. 16.22 Key diagram of a $220 / 66 \mathrm{kV}$ sub-station


Fig. 16.23 Plan of a sub-station


Fig. 16.24 View of a sub-station sectioned at A-A


Fig. 16.25 View of a sub-station sectioned at B-B
breaker without any danger. For maintenance, first, circuit breakers are opened then isolators are opened and properly earthed. Then, maintenance is done.
4. Current transformers: These are the instrument transformers. The secondary winding of CT is connected to the instruments placed on the panel boards. The secondary windings are also connected to the operating coils of various relays for their operation.
5. Lightening arrestors: These are connected between line and earth. Their purpose is to protect the transformer winding against over voltages.
6. Insulators: Generally, suspension and strain insulators are employed at the substations. They provide insulation between line conductors and earthed steel towers.
7. Wave traps: These are used in carrier communication circuits and are mounted on the lines.
8. CVTS: Capacitive voltage transformers (CVTS) are used in carrier line protection and carrier communication circuits.

### 16.12 INTERCONNECTED SYSTEM OF POWER STATIONS (GRID STATION)

The arrangement by which several generating stations are connected in parallel is called interconnected system of power stations or interconnected grid system.

To curtail the various problems of the power system and to improve the reliability of electric supply, various power stations are interconnected with each other. Although interconnections of power stations involve extra expenditure yet, it is preferred because of the following advantages:

1. Exchange of peak loads: At different power stations, the peak load may occur at different instants. Accordingly, the load of one generating station during peak load hours can be shifted to the other generating station only if they are interconnected with each other.
2. Ensures economical operation: The interconnection of power stations ensures economical operation. More efficient plants are operated continuously throughout the year at high-load factors, whereas less efficient plants are operated only during peak load hours.
3. Increases reliability of supply: In interconnected system, if a breakdown occurs at one station, the supply can be maintained from the other station. Therefore, interconnected system increases the reliability of supply.
4. Increases diversity factor: The load curves of different interconnected power stations are generally different. As a result of this, the overall maximum demand on the system is much reduced as compared to the sum of individual maximum demands on different power stations. This increases the diversity factor of the system.
5. Reduces plant reserve capacity: When the power stations work independently, each one must have a standby generating unit as a reserve unit, that is, reserve plant capacity. However, when a number of power stations are interconnected, there is no need to keep one unit as a reserve plant at each station. We can keep only one or two units at one or two generating stations as reserve capacity. This reduces the overall plant reserve capacity of the system.

## SUMMARY

1. Power supply system: The whole system by which electrical power is generated, transmitted, and distributed among the consumers for utilization is called the power supply system.
2. Power stations: The place where natural source of energy is converted into bulk electric power is called power station.
3. Transmission system: The system by which bulk electrical power or energy is delivered from power station to load centres (big substations) is called transmission system.
4. Distribution system: The system by which electrical power or energy is delivered from substations to the bulk and small power consumers is called distribution system.
5. Primary transmission system: The system by which huge electric power is transmitted through long distances at higher voltages (e.g., $400 \mathrm{kV}, 200 \mathrm{kV}, 132 \mathrm{kV}$, etc.) is called primary transmission system.
6. Secondary transmission system: The system by which comparatively smaller quantity of electric power is transmitted through smaller distances at lower voltages ( $66 \mathrm{kV}, 33 \mathrm{kV}$, etc.) is called secondary transmission system.
7. Primary distribution system: The system by which electric power is distributed among bulk power consumers at 33 kV or 11 kV is called primary distributed system.
8. Secondary distribution system: The system by which electric power is distributed among small power consumers (for industrial, commercial, or domestic purposes) at 400 V three-phase or 230 V sin-gle-phase is called secondary distribution system.
9. Hydroelectric power stations: The power stations where potential or kinetic energy (hydraulic energy) of water is converted into electrical energy are called hydroelectric power stations.
10. Thermal power stations: The power stations where heat energy of coal combustion is converted into electrical energy are called thermal power stations.
11. Diesel power stations: The power stations where diesel engines are used as prime movers and energy produced by the combustion of diesel oil is converted into electrical energy are called diesel power stations.
12. Nuclear power stations: The power stations where nuclear energy is converted into electrical energy are called nuclear power stations.
13. ACSR conductors are invariably employed for transmission and distribution of electric power.
14. The supports used in the overhead power system are as follows:
(i) Steel towers: For transmission lines operating at 66 kV and above.
(ii) RCC poles: For lines operating at $400 \mathrm{~V}, 11 \mathrm{kV}$, and 33 kV .
(iii) Steel poles: For lines operating at 400 V and 11 kV in urban areas.
(iv) Wooden poles: For lines operating at 400 V and 11 kV in rural areas but not used nowadays at all.
15. Insulators used in overhead lines: To provide insulation between line conductors and earthed supporting structure, various insulators are employed in the overhead lines.
(i) Suspension-type insulators: For lines operating at 66 kV and above.
(ii) Pin-type insulators: For lines operating at 11 kV and 33 kV .
(iii) Shackle insulators: For lines operating at $400 / 230 \mathrm{~V}$ and 33 kV .
(iv) Egg or stay insulators: Used in stay wires.
(v) Strain insulators: Used at dead end or sharp curves to take up strain disc type and shackle insulators. (vi) Reel insulators: Used with guard wires placed in LT distribution lines.
16. Feeder: A power line or conductor that joins the major substations to the distributors is known as feeder. It is the main criterion for its design in magnitude of current.
17. Distributor: A power line or conductor to which various consumers are connected is known as distributor. The main criterion for its design is voltage drop.
18. Service mains: The conductor by which consumers are connected to the distributor is called service mains. Its size is designed as per the connected load of the consumer.
19. Substations: A substation is an assembly of apparatus which is installed to control transmission or distribution of electric power. These may be classified as (i) indoor substation and (ii) outdoor substation.
20. Substation equipment: The major equipment of a substation are transformers, circuit breakers, isolates, current transformers, lighting arrestors, insulators, wave trap, CVTS, etc.
21. Grid station: The interconnection of various generating stations forms a grid station.

## TEST YOUR PREPARATION

FILL IN THE BLANKS

1. In India, electrical power is generally generated at voltage $\qquad$ kV .
2. Electrical power is transmitted by $\qquad$ phase, $\qquad$ wire system.
3. For transmission and distribution of electric power, the conductors used are called $\qquad$ .
4. The pipes that carry water from valve house to scroll case of turbine are called $\qquad$ .
5. The potential and kinetic energy of water is converted into mechanical energy (rotatory motion) with the help of $\qquad$ .
6. An alternator converts $\qquad$ energy into electrical energy.
7. In India, best suited and standby generating station is $\qquad$ power plant.
8. In nuclear power plants, the nuclear reaction is controlled in the part called $\qquad$ .
9. In ACSR conductors, the steel strands are placed at the $\qquad$ portion.
10. At 11 kV overhead line, the insulators used are called $\qquad$ insulators.

## OBJECTIVE TYPE QUESTIONS

1. The supporting structure used for 220 kV overhead transmission line is
(a) steel poles.
(b) lattice steel tower.
(c) RCC Poles.
(d) wooden poles.
2. At 132 kV transmission line, the insulators used are
(a) pin-type insulators.
(b) shackle insulators.
(c) suspension insulators.
(d) egg insulators.
3. Shackle insulators are used at
(a) 33 kV overhead line.
(b) 11 kV overhead line.
(c) 66 kV overhead line.
(d) 400 V , LT line.
4. Reel insulator is used with
(a) stay wire.
(b) guard wire.
(c) line wire.
(d) None of the above
5. The conductor or line that joints distributor with main substation is called
(a) service main.
(b) power main.
(c) feeder.
(d) distributor.
6. While designing distributor, the main criterion is
(a) voltage drop.
(b) power loss.
(c) heat dissipation.
(d) mechanical strength.
7. The system by which various generating stations are interconnected is known as
(a) power station.
(b) substation.
(c) grid station.
(d) power system.
8. The line or conductor used to connect consumers to the distributor is called
(a) main supply.
(b) service mains.
(c) delivery mains.
(d) supply mains.
9. The transformer installed at the generating station is called
(a) step-up transformer.
(b) step-down transformer.
(c) either or the two.
(d) distribution transformer.
10. In hydroelectric power stations, the penstocks are protected against water hammerage by providing
(a) surge tank.
(b) heavy load.
(c) racks.
(d) spill ways.

## VIVA-VOICE REASONING QUESTIONS

1. Electric power is not transmitted only at one voltage, but it is transmitted at different voltage levels. Why?
2. The transmission voltages are high, but the utilization voltage is very low. Why?
3. The generated voltage is 11 kV . Why?
4. We always prefer to generate electrical power at hydroelectric power plants. Why?
5. Diesel power stations are not employed to generate bulk power in India. Why?
6. Reactor is used in nuclear power plants. Why?
7. Conductors used in transmission and distribution lines is always standard. Why?
8. An egg insulator is employed in the stay wire. Why?
9. Reel insulator is placed at the neutral in a guard wire. Why?
10. Distributors are designed on the basis of voltage drop. Why ?

## SHORT ANSWER TYPE QUESTIONS

1. We step-up the voltage at the generating station and step-down it at various substations. Explain.
2. What is the function of following elements of a hydropower station: (i) dam (ii) penstock (iii) spillways (iv) surge tank (v) water turbine?
3. Explain the function of the following elements related to steam power station: (i) boiler (ii) super heater (iii) air pre-heater (iv) condenser (v) cooling tower (vi) economizer
4. Give the function of following elements of nuclear power station: (i) nuclear reaction (ii) heat exchanger.
5. Which type of generating station is preferred to work as standby generating station? State why?
6. Why stranded conductors are used for transmission and distribution lines?
7. Name the supporting structures used for overhead transmission and distribution lines. Also mention the voltage of the line for which these support are used.
8. Name the various types of insulators used for overhead transmission and distribution lines.
9. What do you mean by a feeder, distributor, and service mains?
10. What do you mean by a grid station?

## TEST QUESTIONS

1. Draw the layout of power system and explain its each part briefly.
2. Name the major generating stations and explain hydroelectric power station in detail.
3. With the help of a block diagram, show how power is generated at thermal power station.
4. Draw a simple line diagram of a nuclear power station and explain how electrical power is generated at these stations.
5. Name the supports used in transmission and distribution system. Draw a neat sketch a double-circuit lattice steel tower.
6. Name the insulators employed in transmission and distribution lines of electrical power. Draw a neat sketch of a string of suspension-type insulators.
7. How conductor is placed with the shackle-type insulators, show with the help of a neat sketch.
8. What do you understand by (i) feeder (ii) distributor (iii) service mains?
9. What do you mean by a substation, give their classification. Draw a neat sketch of a pole-mounted substation.
10. Name the various equipment installed at a substation and mention the function of each one of them.
11. Discuss the advantages and disadvantages of an interconnected grid system.

## ANSWERS

## Fill in the Blanks

1. 11
2. 3,3
3. mechanical
4. diesel
5. ACSR
6. penstocks
7. water turbine
8. reactor
9. inner
10. pin type

Objective Type Questions

1. (b)
2. (c)
3. (d)
4. (b)
5. (c)
6. (a)
7. (c)
8. (b)
9. (a)
10. (a)


## LFARNING OBJFCTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What is an electric shock?
* When does a person experience an electric shock?
* How a victim of electric is treated?
* What precautions should be made against electric shock?
* What are the electric safety measures?
* What is earthing and its purpose?
* What are different methods of earthing?
* What is double earthing and its purpose?
* What is a fuse and an MCB? What are their roles in electrical installations?
* What is an ELCB ot RCCB?


### 17.1 INTRODUCTION

The rapid industrial growth in twentieth century is mainly due to utilization of electrical energy in various fields. Now, electrical energy becomes a part of our day-to-day life.

Electricity is a good servant but a bad master if not handled properly. Therefore, while working with electrical installation and handling electrical equipment, one should always take care of his own and of other's safety. A little carelessness may result in an accident that may be fatal. Therefore, certain safety measures must be observed before dealing with electricity. In this chapter, various common safety measures and earthing of the equipments are discussed.

### 17.2 ELECTRIC SHOCK

When a person comes in contact with a live conductor, directly or indirectly, he gets a shock. The shock may be minor or severe. The severity of shock depends upon the following:

1. Nature of the current whether AC or DC: Since DC flows continuously and does not pass through zero current value as in AC , so DC is considered more dangerous than AC supply.
2. Duration of flow of current: Shock will be more severe if duration of contact with the live wire of a person is more. However, the severity of shock can be reduced by disengaging the person from live wire contact immediately.
3. Path of current through human body: Severity of shock also depends upon the path of the electric current through human body. A person has severe shock if current path involves his heart.

The effect of electric current when passes through human body is given in Table 17.1.
Table 17.1 Effect of Electric Current Through Human Body

| $1-10 \mathrm{~mA}$ | Prickling sensations |
| :--- | :--- |
| 10 mA | Muscle contraction: The person remains 'stuck' to the conductor |
| $20-30 \mathrm{~mA}$ | Muscles contraction can cause respiratory paralysis |
| $70-100 \mathrm{~mA}$ | Cardiac fibrillation: The heart begins to vibrate and no longer beats at steady rate. This <br> situation is dangerous since it is irreversible. |
| 500 mA | Immediate cardiac arrest resulting in death |

However, electrocution should not be viewed in terms of 'current' alone but in terms of 'current and voltage'. A person gets electrocuted by coming in contact with an object that has a different potential from his/her own. The difference in potential causes the current to flow through the body.

The human body has known limits:
Under normal dry conditions, voltage limit $=50 \mathrm{~V}$
In damp surroundings, voltage limit $=25 \mathrm{~V}$

### 17.3 ELECTRIC SHOCK TREATMENT

The victim of electric shock should be immediately treated as suggested below:

1. Victim should be removed immediately from the contact of live conductor.
2. Artificial respiration should be given immediately.
3. Treat the burns, if any, on recovery of the victim.
4. Finally, give call to a doctor.

For removing the victim from the contact of L.T. (Low Tension) live wire, any one of the following procedures can be followed:

1. Immediately 'Switch OFF' the supply. If switch is far away, then pull out the plug top.
2. Pull the victim by using wooden stick, dry clothes, dry rope, etc.
3. One can pull victim directly by standing on well-insulated footing such as rubber mat, dry board, dry wooden chair, electrician rubber gloves or even pull directly (to some extent) if wearing rubber sole shoes.
4. Cut the conductor by an axe, or axe like device having a large wooden handle.

For removing the victim from the contact of HT (high tension) live conductor, any one of the following procedures can be followed:

1. 'Switch OFF' the circuit breaker, if it is nearby.
2. Short the live conductors by throwing a bare wire or chain upon them. This will result in tripping of the circuit breaker, at the substation or power station, as the case may be.

### 17.4 METHODS OF ARTIFICIAL RESPIRATION

Once the victim has been removed from the contact of the live wire, next step is to give him artificial respiration. There are various methods of giving artificial respiration, which are detailed in this section.

### 17.4.1 Schafer's Method

The Schafer's method is the best method to give artificial respiration to the victim of electric shock. The various steps of this method are as follows:

1. Victim is laid on the stomach, with his face on one side and pull the arms forward as shown in Figure 17.1(a).
2. To allow proper breathing, victim's neck is cleared from clothing.
3. Clear the mouth of the victim from pan, tobacco, artificial teeth, etc.
4. Kneel over the victim as shown in Figure 17.1(b) and place both your hands flat on his back. Place your hand in such a manner that both of your thumbs nearly touch each other and fingers are spread on each side of the victim's lower ribs.


Fig. 17.1 Schafer's method of artificial respiration (a) Lay the victim on the stomach (b) Kneeling over the victim
5. Lean forward over the victim gradually and gently by putting your weight on the victim for a moment as shown in Figure 17.1(b).
6. Now slowly release the pressure and come to original position.

Repeat the process at least 12 to 15 times in one minute, till the victim starts natural breathing.

Schafer's method of artificial respiration is better method as compared with other two methods to be followed. Hence, this method should be adopted as and when required. However, this method cannot be used if it is not possible due to burns to lay the victim on his stomach.

### 17.4.2 Silvestre's Method

Schafer's method is the best method of artificial respiration; however, if the victim got some burns on the chest or anywhere on the front side of the body, only then this method is adopted. This method can be proceeded as follows:

1. Lay the victim on his back, as shown in Figure 17.2(a).
2. Remove the victim's clothes around his neck.
3. Clear the mouth of the victim from pan, tobacco, artificial teeth, etc.
4. Use pillow or rolled coat or any other cloth under the shoulders of the victim, so that his head falls backwards.
5. Tilt the head a little back. It will keep the tongue out of the throat allowing passage to air for breathing.
6. Now kneel in the position near the head of the victim as shown in Figure 17.2(b).
7. Stretch both arms of the victim backward by holding them below the elbows.
8. Keep the arms in this position for about 2 to 3 s .
9. Now bring the arms of the victim on each side of his chest (Fig. 17.2(b)), so as to compress his chest. Keep the victim's arms in same position also for 2 to 3 seconds.
10. Repeat the process for about 12 to 15 times in a minute, till victim starts natural breathing.


Fig. 17.2 Silvestre's method of artificial respiration (a) Lay the victim on the back (b) Kneeling near the victim's head

It should be assure that this method is to be adopted only when it is not convenient to make the victim lie on his stomach due to burns.

### 17.4.3 Third Method (Artificial Respirator Method)

It is the easiest and most hygienic method of artificial respiration, if the apparatus is available. When the victim has suffered an electric shock and is unconscious, an artificial respirator may be used. An artificial respirator consists of a rubber bulb mask and an air filter along with a transparent celluloid valve arrangement as shown in Figure 17.3(b). The air enters through the holes of rubber bulb and goes out through the outlet valve. The mask is placed on the mouth and nose of the patient as shown in Figure 17.3(a). The rubber bulb is pressed at the rate of 12 to 15 times per minute to bring his respiration back. This process should be continued regularly till the doctor advises to stop.


Fig. 17.3 (a) Artificial respiration (b) Respirator

The working principle of respirator is as follows: The rubber mask is fitted on the mouth and nose of the victim. When the rubber bulb is pressed, the air of the bulb passes through the air filter that lifts the inlet valve and closes the outlet valve. Now, this filtered air enters the lungs of the patient through the mask and nose. When the pressure on the bulb is released, the inlet valve closes and the outlet valve is opened, which gives path to the used air to go out.

Special Instructions for treatment are as follows:

1. Never give any drink to the unconscious man.
2. Violent operation of the process must be avoided because an internal injury in the affected organ may be harmed due to quick and excessive pressure.
3. If there is a burn on the body, it should be properly dressed after the recovery of the patient.
4. The patient should be kept warm.
5. No medicine should be given without the consent of the doctor.
6. An owner of the factory must provide and fix a chart explaining the methods of artificial respiration The chart should carry the name of the nearest doctor and his telephone number. Preferably, the address of the hospital and residential address of the doctor to be contacted should be given in the chart.

### 17.5 PRECAUTIONS AGAINST ELECTRIC SHOCK

The following precautions should be taken as preventive measures from electric shock while dealing with electrical equipment fittings or appliances:

1. Never work on live circuit.
2. Always stand on the insulating material, such as rubber mat, wooden board, etc., while switching on the main switch, motor switch, etc.
3. While switching ON the circuit, equipment, etc., ensure that your hands and feet are dry.
4. Avoid working at all those places where your head is liable to touch the live parts.
5. While working with electrical circuits/equipments, never come in contact with the metallic casing, earth conductor, cross arms, etc.
6. While working on the high-voltage circuit, avoid your direct contact with concrete flooring.
7. Never touch the person directly, while rescuing him from electric shock.
8. Consider all conductors as live, till you are not sure.

### 17.6 ELECTRIC SAFETY MEASURES

The following common safety measures should be followed while dealing with the electricity:

1. Always follow IER (Indian Electricity Rules) while dealing with electrical equipment and installations.
2. Consider all conductors (insulted or bare) as live conductors. So do not touch them till you are not sure.
3. Switch OFF the main switch and keep the fuse carriers with you while working on an electrical installation.
4. Single-way switches should be always connected in a live wire.
5. Fuse should be only provided in a live wire.
6. Before replacing the blown fuse or switching ON the tripped MCB (miniature circuit breaker), be sure that the defect has been rectified.
7. Do not work on live circuits. However, wherever the energized circuits are to be handled, then always use rubber mats, rubber sole shoes, rubber gloves, etc., as the case may be.
8. Never disconnect the appliance from the plug point by pulling the connecting cord.
9. Always use proper size of wire of proper voltage grade for all electrical appliances, equipments, wiring installation, etc.
10. Always use standard cable for connecting portable equipments, appliances, pendant holders, etc. Since standard conductor cable provides flexibility, so equipment can be handled conveniently.
11. All electrical connections should be made tight and these should be checked periodically, so as to avoid fire due to lose connections.
12. Always use water proof cables and fittings for all our door installations except distribution and transmission lines.
13. All non-current carrying metal parts of the equipment and of installation should be properly earthed.
14. All portable electrical equipments should be properly earthed. Therefore, for such equipments, always use three-core cable.
15. Always keep the earthing in good condition, that is, earth resistance should be kept very low all the time, since safety depends upon perfect earthing.
16. As far as possible, put off the main switch (or controlling switch), when a person is still in contact with a live conductor.
17. Do not disengage a person, who is still in contact of live circuit, by touching him directly. Push him only with a piece of dry wood or other such insulating material.
18. In case, a person is electrocuted, immediately apply artificial respiration and call the doctor.
19. Never use water for extinguishing fire due to electric current. Use only carbon tetrachloride, liquid carbon dioxide fire extinguishers. Sand can also be used for extinguishing the electric fire.

### 17.7 EARTHING

The process of connecting metallic bodies of all the electrical apparatus and equipment to the huge mass of earth by a wire of negligible resistance is called earthing.

When a body is earthed, it is basically connected to the huge mass of earth by a wire having negligible resistance. Therefore, the body attains zero potential, that is, potential of earth. This ensures that whenever a live conductor comes in contact with the outer body, the charge is released to the earth immediately.

### 17.7.1 Purpose of Earthing

The basic purpose of earthing is to protect the human body (operator) from electric shock. To illustrate the purpose of earthing, consider an electrical circuit shown in Figure 17.4 where an electrical appliance of resistance $R$ is connected to the supply through a fuse and a switch. When an operator touches the metallic body of the apparatus [Fig. 17.4(a)] having perfect insulation, the equivalent circuit is shown in Figure 17.4(b), where two parallel paths are formed. Since the insulation resistance $R_{\mathrm{i}}$ is very high as compared with appliance resistance $R$, whole current flows through appliance resistance and no current flows through human body (operator's body) resistance.

When earth fault occurs, the live (phase) wire directly comes in contact with the outer body and the insulation resistance reduces to zero as shown in Figure 17.4(c). Now, the body resistance is just in parallel with the appliance resistance. A heavy current flows through the human body and operator gets a severe shock.

However, if the metallic body or outer frame of the appliance is properly earthed, then under earth fault condition, the circuit will be as shown in Figure 17.4(d), where earth resistance $R_{\mathrm{e}}$ is just in parallel with the appliance resistance $R$ and body resistance $R_{\mathrm{b}}$. Since earth resistance is very small as compared with body resistance, almost whole of the fault current flows through the earth resistance and no current flows through the human body. Therefore, the operator is protected from electric shock. Moreover, the fault current is much more than the full-load current of the circuit that melts the fuse. Hence, the appliance is disconnected automatically from the supply mains.


Fig. 17.4 (a) Operator touching the metallic body of the apparatus (b) Under normal condition insulation and body resistance come in series (c) When frame comes in contact with live wire, the insulation resistance vanishes (d) When earthing is provided, low earth resistance come in parallel with body resistance

### 17.7.2 Equipment Earthing

According to Rule 61 of Indian Electricity Rules 1956, it is obligatory to earth by the following points and apparatus used in the power system, where the voltage is more than 125 V :

1. All the metal frames of motors, generators, transformers, and controlling equipment.
2. The steel tower and steel tubular or rail poles carrying overhead conductors.
3. The metal frames of portable electrical equipment such as heaters, table fans, electric iron, refrigerator, air conditioners, vacuum cleaners, etc.
4. Other metal parts such as conduits, switch gear casings, etc.
5. Earth terminal of all the three-pin outlet sockets.
6. In case of concentric cables, external conductor, that is, armouring of such cables.
7. Stay wires of overhead lines if stay insulator is not provided.

In case of insulation failure, the primary objective of connecting all the above points and apparatus to earth is to release the charge accumulated on them immediately to earth so that the person coming in contact may not experience electric stock. The other objective is that a heavy current when flows through the circuit operates the protective device (i.e., fuse or miniature circuit breaker), which opens the circuit.

Generally, the values of earth resistance given in Table 17.2 must be achieved while earthing.

Table 17.2 Value of Earth Resistance for Different Equipment

| Equipment to be earthed | Maximum Value of Resistance Under Worst Conditions |
| :--- | :--- |
| 1. Large power stations | $0.5 \Omega$ |
| 2. Major substations | $1.0 \Omega$ |
| 3. Small substations | $2.0 \Omega$ |
| 4. Factories substations | $1.0 \Omega$ |
| 5. Lattice steel towers | $3.0 \Omega$ |
| 6. Industrial machines and equipment | $0.5 \Omega$ |

The earth resistance depends upon the moisture contents in the soil and varies from month to month. Therefore, earth resistance must be checked periodically by earth tester and maximum permissible value be obtained by pouring water into the funnel.

### 17.7.3 System Earthing

A proper system has to be adopted while earthing. In fact, all the heavy power equipment should be earthed by two separate distinct earth wires following the different routes. The two earth connections are applied to improve the reliability. If one of the earth wires breaks or fails to carry the fault current, the other carries that current and provides the required protection. Moreover, in factories and substations, where more than one equipment is to be earthed, parallel connections should invariably be used. In no case series, connections are done, as even a single bad contact or break in the earthing lead will disconnect all the succeeding equipment from the earth.

Therefore, for proper earthing of heavy power equipment, double earthing system has to be adopted. Moreover, the number of apparatus must be connected in parallel to the earth.

### 17.7.4 Methods of Earthing

As discussed earlier, earthing means to connect metallic bodies of the apparatus with the general mass of earth by a wire of negligible resistance. There are various methods of achieving this connection, some of them are given below:

1. Strip earthing: This system of earthing employs the use of 5 SWG (standard wire gauge) copper wire or strip of cross section not less than $25 \mathrm{~mm} \times 1.6 \mathrm{~mm}$. The strips or wires are buried in horizontal trenches. This type of earthing is used where the earth bed has a rocky soil and excavation work is difficult.
2. Earthing through water mains: In this type of earthing, a stranded copper lead is used that is rounded on the pipe with the help of a steel binding wire and a properly designed earthing clip. The copper lead is soldered to make it solid. Before making connection to the water main, it should be ensured that G.I. pipe is used throughout.
3. Rod earthing: It is the cheapest method of earthing and is employed in sandy areas. In this method, a copper rod is hammered directly into the ground, and no excavation work is required. The earthing lead is joined to this rod with the help of nuts and bolts.
4. Pipe earthing: Taking into consideration, the factors such as initial cost, inspection, resistance measurement, etc., G.I. pipe earthing is best form of ground connection. Iron is the cheapest material and remains serviceable even if put in salty mass of earth. The
pipe used as earth electrode is galvanized and perforated. Its diameter is 38.1 mm and length is 2 m . The length may be increased to 2.75 m in case of dry soil. The diameter of pipe has very little effect on the resistance of the earth connection. To facilitate the driving in of the pipe into ground, it is provided with the tapered casting at the lower end. Another pipe of 19.05 mm diameter and of length 2.45 m is connected at the top of the above-mentioned pipe. The connection between these pipes is made through a reducing socket as shown in Figure 17.5.

The earthing lead should be soldered and connected to the pipe. Alternate layers of charcoal and salt are provided around the G.I. pipe to keep the surroundings moist enough. The salt is poured at the bottom and thereafter alternate layers of charcoal and salt are arranged.
5. Plate earthing: In this type of earthing, a copper or G.I. plate of dimensions not less than $60 \mathrm{~cm} \times 60 \mathrm{~cm} \times 3.18 \mathrm{~mm}$ or $60 \mathrm{~cm} \times 60 \mathrm{~cm} \times 6.35 \mathrm{~mm}$ is used as earth electrode instead of G.I. pipe. The plate is buried into ground in such a way that its face is vertical


Fig. 17.5 Pipe earthing


Fig. 17.6 Plate earthing
and the top is not less than 3 m below the ground level. The G.I. wire is used for G.I. plate and copper wire for copper plate earthing. The size of wire is selected according to the installation and fault current. The earthing lead is suitably protected placing it underground in a pipe as shown in Figure 17.6.

Alternate layers of charcoal and salt are used around the plate. The layers of charcoal shall be placed immediately over the plate, and thereafter, successive layers of salt and charcoal are laid to keep the surroundings sufficiently moist.

Note: Pipe earthing and plate earthing are considered to be the best as they have reasonably low value of earth resistance.

### 17.8 SIZE OF EARTH WIRE

In case of household wiring or installation, a 14 SWG hard drawn bare copper conductor is used as earth wire. For power installations, the size of earth wire depends upon the rating of the motors installed. For different ratings of motors, the size of earth wire can be selected from Table 17.3.

Table 17.3 The Size of Earth Wire for the Motors of Different Rating

| Capacity of Apparatus | Size of Earth Wire in SWG |  | Size of Earth Electrode |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Copper | G.I | Copper | G.I |
| Up to 10 H.P. | No. 8 | No. 8 | $\begin{aligned} & 60 \mathrm{~cm} \times 60 \mathrm{~cm} \\ & \times 3.18 \mathrm{~mm} \end{aligned}$ | $\begin{aligned} & 60 \mathrm{~cm} \times 60 \mathrm{~cm} \times \\ & 6.35 \mathrm{~mm} \end{aligned}$ |
| Above 10 H.P. and up to 15 H.P. | No. 8 | No. 6 | -do- | -do- |
| Above 15 H.P. and up to 30 H.P. | No. 6 | No. 2 | -do- | $\begin{aligned} & 90 \mathrm{~cm} \times 90 \mathrm{~cm} \times \\ & 6.35 \mathrm{~mm} \end{aligned}$ |
| Above 30 H.P. and up to 50 H.P. | No. 4 | - | $\begin{aligned} & 90 \mathrm{~cm} \times 90 \mathrm{~cm} \\ & \times 6.35 \mathrm{~mm} \end{aligned}$ | -do- |
| Above 50 H.P. and up to 100 H.P. | No. 2 or strip $12.7 \times$ $2.5 \times 4 \mathrm{~mm}$ | - | -do- | -do- |
| Above 100 H.P. | $\begin{aligned} & \text { Strip } 25.4 \times 2.5 \times \\ & 4 \mathrm{~mm} \end{aligned}$ | - | -do- | -do- |

The conductor used for earthing purpose should not be of cross section less than 14 SWG and greater than $64.52 \mathrm{sq} . \mathrm{mm}$ from mechanical considerations. From electrical considerations, the copper earth wire should not be of size less than half of the largest current carrying conductor.

### 17.9 DOUBLE EARTHING

For providing better safety, it is advisable to provide two separate earth wires, from two separate earth electrodes, connected to same metallic body of the equipment at two different points. This is known as double earthing. Double earthing is essential, as per Indian Electricity Rule, for metallic bodies of large rating equipment such as transformer, motors, etc. working at 400 V and above.

Advantages of double earthing are as follows:

1. Surety of safety, because if at any time, one earthing is ineffective, then another will provide earth path to fault current.
2. As the two earth wires are in parallel, the effective resistance from equipment to earth electrode is reduced.

## 园良 PRACTICE EXERCISES

## Short Answer Questions

1. What do you mean by electric shock?
2. How to proceed with the electric shock treatment?
3. What are the rules that suggest the safety measures?
4. What is earthing?
5. What is the purpose of earthing?
6. What do you understand by equipment earthing?
7. What do you understand by system earthing?
8. What are the different methods of earthing?
9. What do you mean by double earthing?

## Test Questions

1. Under what conditions, a person experiences an electric shock?
2. For electric shock treatment, what are the methods of artificial respiration? Explain any one in detail.
3. Explain Silvester's method of artificial respiration.
4. Mention 10 electric safety measures.
5. What is earthing? How does it protect a person from electric shock?
6. How will you differentiate between equipment earthing and system earthing?
7. What are the different methods of earthing? Explain plate earthing with a neat sketch?
8. Name different methods of earthing and explain pipe earthing with a neat sketch.

### 17.10 CAUSES OF ELECTRIC FIRE

The following are the causes of electric fire:

1. Use of inferior quality of materials.
2. Overloading of the circuits.
3. Proper protective devices are not employed.
4. Insulation of the wires is damaged.
5. When bare conductor come in contact with each other.
6. When bare conductors come in contact with the earthed points in the electrical installation.
7. Loose connections.

### 17.11 PREVENTION OF ELECTRIC FIRE

Fire due to electric current can be prevented by taking the following precautions:

1. Use superior quality of materials in the electrical installation and with each and every equipment.
2. By using protective devices of proper rating with the electrical circuits, so that when excessive current flows due to sustained overloading, short circuit fault or earth fault, the
protective devices should operate for disconnecting the supply to faulty circuit/equipment. This prevents damage to installation/equipment and danger of fire.
3. Overloading of every equipment and electrical installation should be avoided.
4. Well insulated and proper size of wires should be used.
5. The joints in the electrical system should mechanically and electrically be sound. For electrical installation, always use looping in system instead of jointing in system of wiring. There should not be any loose connection in the installation.
6. All connections in the electrical installation should be tight and these should be checked periodically, especially when aluminium wires have been used in the installation.
7. The electrical installation should be free from moisture effect. Wiring route should not be near to the water pipe installation.
8. In case of electric fire, never use water when circuit is live. To avoid the spreading of electric fire and for rescue operation, following fire fighting equipment should be kept ready for use.
(a) Fire extinguisher carbon tetra chloride (i.e., CTC) or foam type
(b) Fire extinguisher carbon dioxide

These fire extinguishers employ chemicals that are insulators and do not give shock when thrown on to the live parts. Hence, these are used for extinguishing fire due to electrical short circuits and sparks.

1. Buckets filled with sand
2. Ladder
3. Rope
4. Stretcher
5. First aid box, etc.

### 17.12 FUSE

A short piece of metal wire, inserted in series with the circuit, which melts when predetermined value of current flows through it and breaks the circuit is called a fuse.

A fuse is connected in series (Fig. 17.7) with the circuit to be protected and carries the load current without overheating itself under normal conditions. However, when abnormal condition occurs, an excessive current (more or equal to the predetermined value for which the fuse is designed) flows through it. This raises the temperature of the fuse wire to the extent that it melts and opens the circuit. This protects the machines or apparatus from damage that can be caused by the excessive current.
Time-current characteristics: The time required to blow out a fuse depends upon the magnitude of excessive current. Larger the current, smaller is the time taken by the fuse to blow out. Hence, a fuse has inverse time current characteristic as shown in Figure 17.8, which is desirable for a protective device.


Fig. 17.7 Electric circuit with fuse and switch


Fig. 17.8 Typical characteristics of 60A, 100A and 200A fuse

### 17.12.1 Advantages of Fuse

1. The cost of this protective device is very low.
2. It requires no maintenance.
3. It interrupts heavy current without noise or smoke.
4. The smaller size of fuse element imposes a current-limiting effect under short circuit.
5. The minimum time of operation can be predetermined by selecting proper material of the fuse wire.
6. The inverse time-current characteristic makes it suitable for over current protection.

### 17.12.2 Disadvantages of Fuse

1. Considerable time is lost in rewiring or replacing fuses after every operation.
2. On short circuit, determination between fuses in series can only be obtained if there is considerable difference in the relative sizes of the fuse concerned.

### 17.13 MINIATURE CIRCUIT BREAKER (MCB)

Miniature circuit breaker (MCB) is a device that ensures definite protection of wiring system and sophisticated equipment against over current and short circuit faults. The outer view and the internal details of a MCB are shown in Figures 17.9 and 17.10, respectively.


Fig. 17.9 Outer body of a miniature circuit breaker


Fig. 17.10 Internal parts of a miniature circuit breaker

### 17.13.1 Construction

Construction of an MCB can be explained by considering the following main parts:

1. Outer body or housing: The outer body or housing of an MCB is moulded from a special grade glass fibre-reinforced polyester with the help of an injection-moulding machine. The outer body and other polyester components of MCB are fire-retardant, anti-tracking and non-hygroscopic. These polyester parts and housing have the ability to withstand high-temperature and mechanical impacts.
2. Contacts: The contacts of an MCB are made of pure silver. This provides definite advantages such as long contact life, low contact resistance, ensures quick arc removal, and low-heat generation.
3. Operating mechanism: All the components of the operating mechanism are made of special plastic that they are self-lubricating that eliminates wear and tear, rust, and corrosion. These components are very light in weight and have low inertia, thereby ensure snap make the break ability. The reliability and ruggedness of the operating mechanism is, thus, maintained.
4. Arc extinguishing chamber: The arc produced during breaking of circuit is extinguished abruptly by providing a special arc chute chamber.
5. Fixing arrangement: The MCB-mounting clip gets easily snapped on to the Din-bar and can be removed easily by a simple operation with a screw driver. This saves the time that would have been required for fixing it with screws.
6. Mechanical interlocking of multiple MCBs: The levers of all the (3 or 4) multiple MCBs are connected internally. This ensures simultaneous tripping of all poles even if the fault develops in any one of the phases.

### 17.13.2 Working

MCB may operate under the following two different conditions:

1. Moderate overload condition: Detection of moderate overload conditions is achieved by the use of a thermometal that deflects in response to the current passing through it. The thermometal moves against the trip lever, releasing the trip mechanism.
2. Short circuit conditions: When the current flowing through the MCB reaches a predetermined level (as per its setting or rating), it pushes the solenoid plunger that releases the trip mechanism and simultaneously separates the contacts.

Under short circuit conditions, the current-limiting action is achieved by the use of a high speed, direct acting electromagnetic mechanism.

This mechanism forcibly separates the contacts and simultaneously releases the trip mechanism. A high arc voltage drop is rapidly introduced that limits the fault current to a duration of few milliseconds and achieves almost instantaneous interruption (the facts are shown graphically in Fig. 17.11).


Fig. 17.11 Operating time of an MCB

When the contacts are separated, the current still rises due to arc. This arc is extinguished quickly in the arc chute chamber and does not allow the current to reach theoretical maximum value. The total breaking time is reduced to less than 5 millisecond.

### 17.13.3 Applications

Since MCBs are available with different current ratings of $0.5,1.6,2,2.5,3,4,5,6,7.5,10$, $16,20,25,32,35,40$, and 63 A and voltage ratings of $240 / 415 \mathrm{~V} \mathrm{AC}$ and up to 220 V DC. Moreover, they have very small breaking time ( 5 millisecond), and therefore, these are generally employed to protect the important and sophisticated appliances used commercially and
for domestic purposes, such as computers, air conditioners, compressors, refrigerators, and many others.

### 17.14 EARTH LEAKAGE CIRCUIT BREAKER (ELCB)

In the industrial, commercial, and domestic buildings, sometimes (usually in rainy season) leakage to earth occurs. This leakage may cause electric shock or fire. Hence, the leakage to earth is very dangerous and needs protection.

ELCB is a device that provides protection against earth leakage faults.

### 17.14.1 Construction and Internal Circuit Details

The enclosures of the ELCB is moulded from high-quality insulating material. The materials are fire-retardant, anti-tracking, non-hygroscopic, impact resistant and can withstand high temperatures. The body contains spring-loaded mounting arrangement on din-channel that ensures snap fitting of ELCB into position. However, these also have the facility to screw-on directly to any surface with the help of two screws. A two-pole ELCB is used for single-phase supply and a four-pole ELCB is used for three-phase, four-wire supply. A four-pole ELCB is shown in Figure 17.12.

Internal wiring diagram of a two-pole ELCB is shown in Figure 17.13. As shown in Figure 17.13, an ELCB contains a core-balanced transformer (ferrite ring on which one or two turns of phase and neutral wire, and a few turns of operating coil of relay are wound) and a relay. A test button is placed between phase and neutral in series with a limiting resistor. The terminal designation and connection diagram for a two-pole and four-pole ELCB are shown in Figures 17.14 and 17.15 , respectively.


Fig. 17.12 Outer body of an earth leakage circuit breaker


Fig. 17.13 Electrical circuit of an earth leakage circuit breaker


Fig. 17.14 Schematic diagram of a 2-pole ELCB


Fig. 17.15 Schematic diagram of a 4-pole ELCB

### 17.14.2 Principle of Operation

Under normal conditions, the magnitude of currents flowing through the phase wire and neutral are the same and core of the core-balanced transformer does not carry any flux (i.e., two windings neutralize the flux). Therefore, no emf is induced in the operating coil of the relay wound on the same core. However, when the earth fault (earth leakage) occurs, the current in the phase wire becomes more than the neutral wire. This unbalancing sets up flux in the core of the core-balanced transformer, which in turn induces an emf in the operating coil of the relay. Hence, relay is energized and the plunger of the ELCB goes to the off position or disconnects the load from the supply. Therefore, ELCB protects the system against leakage.

## Use of test knob

A test knob is provided for the periodic checking of the mechanism and function of ELCB.

## PRACTICE EXERCISES

## Short Answer Questions

1. What are the causes of electric fire?
2. What is a fuse?
3. What are different types of fuses?
4. What is MCB? How does it work?
5. What is the full form of RCCB and its usage in wiring system?

## Test Questions

1. How electric fire can be prevented?
2. What is an electric fuse? Draw the characteristics of a fuse and mention its merits and demerits.
3. What are the different parts of an MCB and explain how it works.
4. Explain the working of an ELCB.

## SUMMARY

1. Electric shock: When more than 10 mA current passes through a human body, one experiences an electric shock.
2. Electric shock treatment: When a person experiences an electric shock, first of all he/she should be removed from the electric contact and then artificial respiration should be given to the patient till doctor reaches at the spot.
3. Safety measures: Indian electricity rules have suggested various safety measures while dealing with electricity.
4. Earthing: The process of connecting metallic bodies of all the electrical apparatus and equipment to the huge mass of earth by a wire of negligible resistance is called earthing.
5. Purpose of earthing: The basic purpose of earthing is to protect the human body (operator) from electric shock.
6. Equipment earthing: According to Indian electricity rules, it is obligatory to earth the metallic bodies of all the electrical equipment/apparatus/appliance that is operated at 125 V or more than this.
7. System earthing: All the electrical equipment/apparatus/appliance operated at 125 V or more must be properly earthed at least at two places called double earthing.
8. Methods of earthing: The following method may be employed for earthing (i) strip earthing (ii) earthing through water mains (iii) rod earthing (iv) pipe earthing (v) plate earthing. Pipe earthing is considered the best method.
9. Causes of electric fire: Electric fire may be caused by (i) using inferior quality material and (ii) due to overloading of circuit. (iii) Use of improper protective devices. (iv) Poor quality of insulation of wiring cables. (v) Loose connection, that is, sparking. (vi) Bare conductor coming in contact with earth.
10. Fuse: A short piece of metal wire, inserted in series with the circuit, which melts when more than predetermined value of current flows through it and breaks the circuit is called a fuse.
11. Types of fuses: (i) Low-voltage fuses, that is, (a) rewireable fuses (b) high breaking capacity fuses (HBC), or high-rupturing capacity (HRC) fuses (iii) high-voltage fuses, that is HBC fuses.
12. Miniature Circuit Breakers (MCBs): It is a protective device that is connected in series with load in phase wire. It trips or disconnects the load as and when current through it flows more than its predetermined value.
13. Earth Leakage Circuit Breaker (ELCB) or Residual Current Circuit Breaker (RCCB): It is a protective device that disconnects the circuit or load from the mains when earth fault develops in the circuit.

## TEST YOUR PREPARATION

## 7 FILL IN THE BLANKS

1. One experiences an electric shock when the current passing through one's body is more than
$\qquad$ _.
2. A short piece of wire connected in series with the circuit that melts when excessive current flows through it is called $\qquad$ -.
3. A fuse is always connected in $\qquad$ with the circuit to be protected.
4. A fuse element should have $\qquad$ melting point.
5. The fuse wire having low-melting point is an alloy of $\qquad$ .
6. The minimum value of current at which the fuse element melts is called $\qquad$ .
7. The basic purpose of earthing is to $\qquad$ _.
8. For a good earthing system, the value of earth resistance should be $\qquad$ .

## OBJECTIVE TYPE QUESTIONS

1. In motor wiring installations, double earthing is required.
(a) True
(b) False
2. The material used for fuse element is
(a) copper.
(b) aluminium.
(c) tin-lead alloy.
(d) Any of (a), (b), or (c)
3. The material used for fuse must have
(a) low-melting point and low-specific resistance.
(b) low-melting point and high-specific resistance.
(c) high-melting point and low-specific resistance.
(d) low-melting point with any specific resistance.
4. The rewirable fuses do not deteriorate with age.
(a) True
(b) False
5. In fluorescent tubes, the outer glass tube is internally coated with
(a) quarts.
(b) paint.
(c) telecom powder.
(d) phosphor.
6. Fuse is always connected
(a) in series with the circuit to be protected.
(b) in parallel with the circuit to be protected.
(c) Either (a) or (b)
(d) None of the above
7. Fuse is always connected in
(a) neutral.
(b) earth.
(c) phase.
(d) Any of (a), (b), or (c)
8. The basic purpose of earthing is that
(a) it avoids faults.
(b) it allows the current to flow in the circuit.
(c) it protects the operator from electric shock.
(d) it stops current to flow in the circuit.
9. When more than one equipment is to be earthed
(a) parallel connections should invariably be used.
(b) series connections should invariably be used.
(c) Either (a) or (b)
(d) None of the above
10. For proper earthing, according to Indian electricity rules, of heavy power equipment,
(a) single earthing is sufficient,
(b) double earthing system has to be adopted,
(c) half earthing is sufficient,
(d) Any of (a), (b), or (c)
11. Earth resistance should be very low.
(a) True
(b) False

## VIVA VOICE/REASONING QUESTIONS

1. Electric shock is dangerous for a human body. Why?
2. All the metallic bodies or frames of all the electrical equipment/apparatus are properly earthed. Why?
3. Fuse is always provided in phase wire. Why?
4. MCB is always provided in phase wire. Why?
5. Double earthing is provided for all the electrical equipment operated at 400 V and above. Why?
6. Pipe earthing is preferred over other types of earthing. Why ?

## SHORT ANSWER TYPE QUESTIONS

1. What do you mean by electric shock?
2. How one can prevent himself from electric shock?
3. If a person is electrocuted, what steps will you take?
4. What do you mean by earthing?
5. How earth resistance can be kept low in dry summer season?
6. What do you mean by double earthing?
7. Which type of earthing preferred in pipe earthing or plate earthing? Give reasons.
8. What is fuse wire?
9. How a fuse protects appliances?
10. A fuse wire should have low- or high-melting point. Explain the reason.
11. An MCB is just like a fuse or switch. Explain.
12. What is an ELCB? Mention its function.

## TEST QUESTIONS

1. Give brief description of any five safety measures you will take while handling the electrical equipment.
2. What do you understand by electric shock? Explain how it damages the human body.
3. How a person is disengaged from electrical line in case of an accident?
4. Explain in brief the action you will take in restoring a person who has suffered an electrical shock and is unconscious.
5. What are the causes of electric fire?
6. How can we prevent electric fire?
7. How can we prevent electric shock?
8. What is meant by a fuse? Enlist the desirable properties of a fuse wire.
9. Write short note on a rewirable fuse.
10. How does a fuse protect and maintain the life of an electrical equipment?
11. Write short note on HRC (or HBC) fuse.
12. Enumerate the disadvantages of a rewirable fuse.
13. What is earthing?
14. What is the purpose of earthing?
15. Explain system earthing and equipment earthing.
16. What do you understand by double earthing?
17. What are the various methods of earthing? Explain pipe earthing.

## ANSWERS

## Fill in the Blanks

1. 10 mA
2. fuse
3. series
4. low
5. tin and lead
6. fusing current
7. protect the operator
8. very small below $1 \Omega$

## Objective Type Questions

1. (a)
2. (d)
3. (a)
4. (b)
5. (d)
6. (a)
7. (c)
8. (c)
9. (a)
10. (b)
11. (a)


## LEARNING OBJEGTIVES

After the completion of this chapter, the students or readers will be able to understand the following:

* What are different types of cables used in wiring installations?
* Which are different types of wiring systems used in domestic, commercial, and industrial installations?
* How different wiring circuits are designed?
* What is illumination and what are different terms used their in?
* What are various laws of illumination?
* What are different types of electric lamps and what are their characteristics?
* What are different factors on which illumination depends?
* How lighting schemes are designed for different places as per required illumination


### 18.1 INTRODUCTION

The electric supply is given by the supplier up to energy meter. After energy meter, the electric supply is conveyed to the different places of the building with the help of a wiring system. Therefore, the wiring should be economical, safe, and good looking, which is the chief requirement of a building. With the modern developments, wiring techniques have undergone radical changes. In this chapter, various types of wiring systems and their applications are discussed.

### 18.2 TYPES OF CABLES

It should be necessary to know about various types of cables (insulated conductors) that are used for internal wiring systems before considering the various types of wiring systems suitable for any installation.

A solid or stranded conductor covered with insulation is known as a cable.
The cable may be a single core or multicore depending upon the number of conductors. Various types of insulating materials are employed for covering the conductors.

Accordingly, the cables (wiring conductors) may be classified as follows:

1. Vulcanized Indian Rubber (VIR) cables
2. Polyvinyl chloride (PVC) cables
3. Tough rubber sheathed (TRS) or Cab tire sheathed (CTS) cables
4. Lead sheathed cables
5. Weather proof cables

### 18.3 TYPES OF WIRING SYSTEMS

The main types of wirings usually employed in residential buildings, commercial buildings, and industries are as follows:

1. Cleat wiring
2. Casing and capping wiring
3. CTS or TRS wiring
4. Metal-sheathed wiring
5. Conduit wiring

### 18.3.1 Cleat Wiring

In this system of wiring, usually VIR or PVC conductors are employed. The conductors are supported in porcelain cleats that are placed at least 6 mm above the walls. The porcelain cleats are made of two parts, namely base and cap, that is, the lower one is known as base being having two or three grooves for the accommodation of conductors and the upper one is known as cap as shown in Figure 18.1. The conductors are run in the grooves, cap is placed over the base, and the whole assembly is fixed on to the wall with the help of wooden screws and gutties (wooden or PVC plugs) already cemented in the wall. The screw not only fixes the cleats on the wall but also tightens the grip of the wires between the two halves of the cleat.


Fig. 18.1 Porcelain cleats

## Advantages of cleat wiring

1. It is the cheapest system of wiring.
2. A little skill is required to lay the wiring.
3. This wiring can be installed very quickly.
4. It is the most suitable system for temporary wiring.
5. The wiring can be dismantled and recovered very quickly.
6. Inspection, alteration, and additions can be made easily.

## Disadvantages of cleat wiring

1. It gives a rubbish look.
2. It is rarely employed for permanent job.
3. The insulation of wire is damaged while whitewashing or distempering the lime falls over the wires.
4. Oil and smoke are also injurious to VIR.
5. Mechanical injuries may damage the conductor since there is no protecting cover.

### 18.3.2 Casing and Capping Wiring

In this system of wiring, generally, VIR wires are employed. The casing is a base that consists of rectangular PVC or wooden block of seasoned teak wood and has usually two grooves to accommodate wires. The casing is fixed on the wall with the help of wooden screws and gutties already cemented in the wall. The casing is usually placed 3 mm apart from the wall by means of porcelain discs in order to protect the casing from dampness. Then, the wires of opposite polarity are laid in different grooves. After placing the wires in the grooves of casing, the casing is covered by means of a rectangular strip of seasoned wood of the same width as the casing known as copping with the help of wooden screws. The assembled view of casing and capping with the VIR wires placed in the


Fig. 18.2 Casing-capping wiring grooves is shown in Figure 18.2.

## Advantages of casing and capping wiring

1. It gives better appearance than cleat wiring.
2. Its cost is quite low as compared to other systems of wiring except cleat wiring.
3. It is easy to install and repair.
4. Conductors are strongly insulated.
5. Capping provides protection against mechanical injury.

## Disadvantages of casing and capping wiring

1. It is not suitable in damp situations.
2. There is a risk of fire.
3. To make the job good looking highly skilled labour is required.

### 18.3.3 Cab Tire Sheathed or Tough Rubber Sheathed Wiring

In this system of wiring, generally CTS (Cab Tire Sheathed) or TRS (Tough Rubber Sheathed) conductors are employed These conductors are run on well-seasoned, perfectly straight, and wellvarnished teak wood batten of thickness 13 mm . The width of the batten is chosen depending upon the number of wires to be run on it. While doing this type of wiring, the batten is fixed on to the wall by means of wooden screws and gutties already cemented in the wall. The wires are held on the batten with the help of clips already fixed on the batten with the help of nails as shown in Figure 18.3.


Fig. 18.3 CTS or TRS wiring

## Advantages of tough rubber sheathed wiring

1. It is easy to install and repair.
2. It gives nice appearance.
3. The conductors have strong insulation, and therefore, it has longer life.
4. It is fireproof up to some extent.
5. Chemicals do not affect the conductor's insulation.

## Disadvantages of tough rubber sheathed wiring

1. The conductors are open and liable to mechanical injury, and therefore, this type of wiring cannot be used in workshops.
2. Its use in places open to sun and rain in restricted.

### 18.3.4 Metal-sheathed Wiring

This system of wiring is similar to CTS or TRS wiring. The main difference is that, in this case, VIR conductors covered with lead alloy sheath (metal-sheathed cable) are used. The metal-sheathed cables are run on the wooden batten. The batten is fixed on the wall by means of screws and gutties already cemented into the wall. The cables are held on the batten with the help of link clips.

## Advantages of metal-sheathed wiring

1. The conductors are protected against mechanical injury.
2. It can be suitably employed under damp situations.
3. It gives batter appearance.
4. It has longer life.
5. Conductors are protected against chemicals.
6. It can be installed in open space.

## Disadvantages of metal-sheathed wiring

1. The metal-sheathed cables are costlier than CTS or TRS wires.
2. In case of leakage, there is every rick of shock.
3. Skilled labour and proper supervision is required.

### 18.3.5 Conduit Wiring

In this system of wiring, VIR or TRS conductors are run in metallic or PVC tubes called conduits. This system of wiring provides best mechanical protection, safety against fire and shock. Therefore, it is considered to be the most suitable system of wiring for workshops and commercial buildings. The conduits can be either supported over the wall by means of saddles or buried under plaster.

Accordingly, there are two types of conduit wirings, namely surface conduit wiring and concealed conduit wiring. In surface conduit wiring, the conduit is run over the wall supported by means of saddles as shown in Figure 18.4, whereas in concealed conduit wiring, the conduit is embedded in the walls and ceiling by placing it in the cavity made previously in them. After placing the conduit, the insulated conductors (or cables) are drawn into them by means of G.I. (galvanized iron) wire known as pilot wire. A number of inspection boxes (conduit boxes) are provided along the run of conduit to facilitate the drawing of wires.


Fig. 18.4 Surface conduct wiring

## Advantages of conduit wiring

1. Conduit provides protection against mechanical injury and fire.
2. Conduit provides protection against chemicals.
3. Conductors are safely secured from moisture.
4. This wiring has far better look.
5. It has a longer life.

## Disadvantages of conduit wiring

1. It is costly system of wiring.
2. It requires more time for erection.
3. It requires highly skilled labour.

### 18.4 IMPORTANT LIGHTING ACCESSORIES

Some of the common lighting accessories are as follows:

1. Main switch: It is provided after the energy meter. It makes and breaks the phase and neutral connections simultaneously and controls the internal wiring circuits. In early days, switch-fuse units enclosed in iron boxes called iron clad switches were used. The following types of iron clad switches are employed in practice:
(a) Double-pole iron clad (DPLC) switches: These are used to control single-phase, two-wire circuits.
(b) Triple-pole iron clad (TPIC) switches: These are used to control three-phase, three-wire circuits.
(c) Triple-pole iron clad switches with neutral link (TPNIC): These are used to control three-phase, four-wire circuits.
However, nowadays, improved form of switches is employed. Fuse units with indicator light are used, which give better appearance.
2. Switch: Switch is a device that makes or breaks the circuit of an appliance such as lamp, fan, tube, socket, etc. There are two types of switches generally used in wiring system.
(a) Surface or tumbler switch: A tumbler switch is mounted on a round block or a teak wood board.
(b) Flush switch: A flush switch is generally mounted on teak wood board or Bakelite sheet.
Mostly, it is used in concealed conduit wiring system.
3. Ceiling rose: It is used to connect the pendent lamp holder, fan, or fluorescent tube to the wiring installation through flexible wire.
4. Three-pin outlet socket: It is used to give temporary connection of electrical appliances such as radios, table tans, television sets, electric iron, etc.
5. Lamp holder: It is used to hold the lamp. It has two spring-loaded pins to which phase and neutral is connected. The electric supply is conveyed through these pins to the lamp. There are various types of lamp holders (e.g., batten holder, pendent holder, angle holder, bracket holder, water-tight bracket holder, swivel lamp holder, etc.) employed in the wiring system depending upon the requirement.

### 18.5 IMPORTANT CIRCUITS

The supplier (P.S.E.B. Department) gives electric supply to energy meter. From the energy meter, the supply is connected to the consumer's main switch as shown in Figure 18.5, which controls the supply of internal wiring. From main switch, the supply is connected to main distribution board that carries a number of fuses and neutral link. Then, the supply is given to sub-main distribution board or to switch board.


Fig. 18.5 Energymeter and distribution board

The important wiring circuits are as follows:

1. Circuit to control one lamp with one switch: The circuit is shown in Figure 18.6. In this figure, the switch is in the off position. The phase wire is connected to the lamp through switch, whereas the neutral wire is connected directly.

A switch is always connected in phase, so that when switch is off and some maintenance is done at the holder, the mechanic may not get shock.
2. Circuit to control one lamp with two 2-way switches: The circuit is shown in Figure 18.7. This circuit is used in staircase wiring. The two positions of the switches are shown. When both the switches are in the same position (i.e., either ON or OFF) as shown in Figure 18.7(a), the lamp will glow.

When the two switches are in different positions (i.e., one is in ON position and the other is in OFF position) as shown in


Fig. 18.6 One lamp controlled by one single-way switch Figure 18.7(b), the lamp will remain dark.


Fig. 18.7 One lamp controlled by two 2 -way switches (a) Glowing position and (b) Dark position

## 8

3. Circuit to control one lamp, fan, and three-pin outlet socket independently by singleway switches placed on a switch board: The circuit is shown in Figure 18.8. A three-pin outlet socket, a fan regulator, and three switches are fixed on a switch board symmetrically as shown in Figure 18.8. The phase wire is given to all the appliances (i.e., lamp, fan, and three-pin outlet socket) through switches, whereas neutral wire is connected directly. An earth wire is connected to the earth terminal (large terminal) of three-pin outlet socket. In the fan circuit, a fan regulator is connected in series to regulate the fan speed.


Fig. 18.8 Switch board to control one lamp, one tube and on fan with regulator
4. Circuit to control three-phase induction motor: The circuit diagram to control a three-phase induction motor is shown in Figure 18.9. Only three wires are run from


Fig. 18.9 Electric wiring circuit to control 3-phase induction motor through a star-delta starter
meter board to main switchboard and main switchboard to motor control board. At the motor control board, a three-phase motor switch and a star/delta starter are fixed. From motor switch to starter, there are three wires, whereas six wires are run from starter to motor. Double earthing is provided in the motor installation as shown in Figure 18.9.

### 18.6 ILLUMINATION

Illumination is directly related to light. Light is a prime factor in the human life. All activities of human being basically depend upon light. Where natural light is not sufficient, artificial light is made available. Usually, artificial light is produced electrically because of its cleanliness, reliability, steady output, better control, efficiency, and low cost.

Generally, the terms light and illumination are used synonymously. But light is the cause, and illumination is the result. When light falls on a surface, the surface reflects a part of this light, which is received by our eyes. Then, the surface is said to be illuminated.

### 18.6.1 Terms Used in Illumination

Some of the important terms related to illumination are briefly described below. The knowledge of these terms helps in designing modern lighting schemes, selection of lamps, and their fittings.

1. Light: The part of radiant energy from a hot body, which produces the visual sensation on human eye, is called light.
2. Luminous flux: The total quantity of radiant energy per second responsible for visual sensation from a luminous body is called luminous flux. It is represented by a symbol $F$ or $\phi$ and is measured in 'lumens'.
3. Lumen: It is the unit of luminous flux. One lumen is defined as the luminous flux emitted per unit solid angle from a point source of one candle power.
4. Plane angle: The angle subtended at a point by two converging lines lying in the same plane is called plane angle. It is measured in radians and is equal to the ratio of the length of the arc to its radius as shown


Fig. 18.10 Plane angle

Fig. 18.11 Solid angle


The largest angle that can be subtended by the periphery of the circle on its centre is equal to $2 \pi$ radians.
5. Solid angle: The angle subtended by the partial surface area of a sphere at its centre, as shown in Figure 18.11, is called solid angle. It is measured in steradian and is equal to the ratio of the area of the surface to the square of radius of the sphere, that is,

$$
\omega=\frac{\text { Area of surface }}{(\text { radius })^{2}}=\frac{A}{r^{2}} \text { steradian }
$$

The largest solid angle that can be subtended by the surface area of the sphere is,

$$
\omega=\frac{4 \pi r^{2}}{r^{2}}=4 \pi \text { steradian }
$$

6. Steradian: The unit of solid angle. One steradian is defined as the solid angle that is subtended at the centre of a sphere by its surface having area equal to radius square, that is.,

$$
\omega=\frac{\text { Surface area }}{(\text { radius })^{2}}=\frac{r^{2}}{r^{2}}=1 \text { sterasdian }
$$

7. Candle power: The light-radiating capacity of a source is called its candle power. The number of lumens given out by a source per unit solid angle in the given direction is called its candle power. It is denoted by the symbol CP.

The total flux emitted by a source of one candle power
$=\mathrm{CP} \times$ solid angle subtended by a sphere at its centre
$=1 \times 4 \pi=4 \pi$ lumens
That is, $1 \mathrm{CP}=4 \pi$ lumens
8. Luminous intensity: Luminous intensity in any particular direction is the luminous


Fig. 18.12 Light source placed at the apex of a cone flux emitted by the source per unit solid angle in that direction. It is denoted by $I$ and its unit is candela or candle power (CP).

Consider a point source of light ' $S$ ' placed at the apex of a narrow cone of solid angle $\omega$ steradians as shown in Figure 18.12. If the luminous flux emitted in the direction of this solid angle is $\phi$ lumens, then luminous intensity of the source ' $S$ ' in this particular direction, $I=\frac{\phi}{\omega}$.
9. Mean horizontal candle power (MHCP): The mean horizontal candle power (MHCP) of a source of light is the mean or average of the candle powers in all directions on a horizontal plane that passes through the source.
10. Mean spherical candle power (MSCP): Generally, the luminous intensity of a source is different in different directions. The average candle power of a source is the average value of its candle power in all directions and is called mean spherical candle power (MSCP). It is given by total flux in lumens emitted by the source in all directions divided by $4 \pi$. Mathematically,

$$
\mathrm{MSCP}=\frac{\text { Total luminous fluc emitted by the source in all directions }}{4 \pi}
$$

11. Mean hemispherical candle power (MHSCP): It is the mean of the candle powers in all directions with in the hemisphere either above or below the horizontal plane. It is given by the total flux emitted in a hemisphere divided by the solid angle subtended at the point source by the hemisphere.

$$
\therefore \quad \text { MHSCP }=\frac{\text { Total luminous flux emitted in a hemisphere }}{2 \pi}
$$

12. Reduction factor: Reduction factor of a source of light is the ratio of its mean spherical candle power to its mean horizontal candle power.

That is, Reduction factor $=\frac{\text { MSCP }}{\text { MHCP }}$
13. Illumination: When light falls on a surface, it becomes visible, and the phenomenon is called illumination. It is defined as the luminous flux falling on a surface per unit area. It is denoted by the symbol $E$ and is measured in lumens per square metre or base or metre-candle.

Mathematically, $E=\frac{\phi}{A}$ lux
14. Lux or metre-candle: One metre candle or lux is defined as the illumination produced by a uniform source of one CP on the inner surface of a sphere of radius one metre.
15. Brightness or luminance: When our eye receives a great deal of light from a source or reflecting surface, it is looking very bright. Hence, brightness or luminance of a surface or source depends upon the luminous flux emitted by the surface or source.

Therefore, brightness or luminance is defined as the luminous flux emitted (or reflected) by the source or surface per unit projected area. It is denoted by symbol $B$.

To understand the significance of brightness or luminance, consider an ordinary filament lamp with clear glass bulb. In this case, the projected area will only be the filament that is very small, and hence, its brightness is very large. On the other hand, if milky white bulb is used with the same lamp, the projected area for the light to be emitted will be the area of the bulb. This reduces the brightness or luminance of the lamp.

The unit of brightness may be one of the following, depending upon the light emitted and the projected area:
(a) Nit-defined as candle per square metre
(b) Stilb-defined as candle per square cm
(c) Lambert-defined as lumens per square cm
16. Glare: In the human eye, the opening of pupil is controlled by its iris which depends upon the intensity of light received by the eye. If the eye is exposed to a very bright source of light, the pupil of the eye contracts automatically in order to reduce the amount of light admitted and prevent damage to the retina. This reduces the sensitivity of eye to see the surrounding objects. This effect is called glare. It is very common when one comes across strong motorcar headlight on the road.

Therefore, glare may be defined as the brightness within the field of vision of such a character that causes discomfort, interference in vision, and eye fatigue.
17. Lamp efficiency: The efficiency of a lamp is defined as the visible radiations emitted by it in lumens per watt.

Usually, the light sources do not radiate energy only in the visible spectrum. The radiant energy is also accompanied with infrared and ultraviolet radiations. The typical, spectra distribution curves for sunlight, gas-filled, and vacuum tungsten filament lamps are shown
in Figure 18.13. Sunlight produces majority of radiations in the visible spectrum. Energy radiations produced by tungsten lamp mostly fall in the infrared region. The amount of light radiations produced by these lamps is small. Therefore, these lamps have poor light-emitting efficiency.


Fig. 18.13 Light spectrum for sunlight, gas filled and vaccum filament lamp

If an electric lamp transforms whole of its electrical energy input into radiant energy at a wavelength of 5,500 $\AA$ (the most sensitive wave length to human eye), the lamp is said to be operating at its $100 \%$ efficiency. Under this condition, a lamp would produce 621 lumen per watt. But, in actual practice, the whole electrical power supplied to the lamp is not converted into luminous flux because most of the power is lost in the form of heat radiations by conduction, convection, and absorption. Only a small portion of radiant energy falls in the visual range of wave length $4,000 \AA$ to $7,500 \AA$. For ready reference, the efficiency of tungsten filament lamps and fluorescent tubes is given below:
Table 18.1 Comparison of Efficiency of Tungsten Filament Lamp and Fluorescent Lamps

|  | Tungsten Lamp |  | Fluorescent Lamp |
| :--- | :---: | :--- | :--- |
| Wattage | Efficiency in Lumens/Watt | Wattage | Efficiency in Lumens/Watt |
| 40 W | 11.5 | 20 W | 45 |
| 60 W | 14.0 | 40 W | 47.5 |
| 100 W | 16.3 | 80 W | 50 |
| 200 W | 18.0 | - | - |

18. Utilization factor or coefficient of utilization: Whole of the light emitted by the source does not reach the surface to be illuminated. Some of the light reaches directly while some portion goes towards walls ceiling and floor, etc.

Therefore, the coefficient of utilization or utilization factors may be defined as the ratio of the total lumens reaching the working plane to the total lumen emitted by the source.
That is, Coefficient of utilization $=\frac{\text { Total lumens reaching the working plane }}{\text { Total lumen emitted by the source }}$
19. Maintenance factor: The conditions in majority of the industries and buildings are dusty. Due to accumulation of dust, dirt, and smoke on the lamps, the light emitted by the sources goes on decreasing as compared to the initial conditions when the walls, ceilings, and lamp fittings were perfectly clean.

The ratio of the illumination under normal working conditions to the illumination when the things are perfectly clean is called maintenance factor.
That is, Maintenance factor $=\frac{\text { Illumination under normal working condition }}{\text { Illumination when the things are perfectly clean }}$
Its value varies between 0.5 and 0.8 .
20. Depreciation factor: In the beginning when the light sources are new and the conditions are perfectly neat and clean the illumination is more. But it deteriorates with the passage of time depending upon the conditions.

Depreciations factor may be defined as the ratio of illumination on the working plane at the initial state to the ultimate illumination at any moment. It is just a reciprocal to maintenance factor.

Deprecation factor $=\frac{\text { Illumination on the working plane at the initial stage }}{\text { Ultimate illumination on the working plane }}$
21. Waste light factor: In many lighting schemes, a surface is illuminated by a number of lamps or light sources; some of the light is always wasted due to overlapping and falling of light outside the edge of surface. Therefore, there is some wastage of light, and this effect is taken into account while computing total lumens required for illuminating the surface. The theoretical lumens are multiplied by waste light factor, and the value of which is about 1.2 for rectangular areas and 1.5 for irregular objects such as monuments, statues, etc.

Therefore, waste light factor is defined as the lumens required to illuminate an area to the theoretical lumen calculated to illuminate the same area at a particular level.
22. Space-height ratio: In order to achieve uniform illumination over a surface, the number of luminaries must be placed suitably. The space-height ratio plays a vital role in deciding the height of lamps from floor level and their spacing.

The ratio of horizontal distance between adjacent lamps and the height of their mounting above working plane is called space-height ratio.
That is, Space-height ratio $=\frac{\text { Horizontal distance between two lamps }}{\text { Mounting height of lamps above working plane }}$
For proper illumination, this ratio is generally kept between 1 and 1.5 .
23. Reflection factor: Whole of the light incident on a reflecting surface is not reflected. Some portion of it is absorbed by the surface. The ratio of the reflected light to the incident light is called reflection factor.
24. Specific consumption: It is the ratio of power input to the source of light to its candle power. Its unit is watts/CP.

### 18.7 LAWS OF ILLUMINATION

The illumination $(E)$ on a surface depends upon the luminous intensity, distance between the source and surface, and the direction of rays of light. It is governed by the following two laws of illumination.

### 18.7.1 Inverse Square Law

It states that the illumination of a surface is inversely proportional to the square of the distance of the surface from the source.
that is,

$$
E \propto \frac{1}{d^{2}}
$$

For illustration, consider portions $A_{1}$ and $A_{2}$ of two spheres of radii $r_{1}$ and $r_{2}$, respectively, as shown in Figure 18.14. A source of $S$ having intensity $I$ is placed at the centre of the spheres. Both areas are suspending the same solid angle of $\omega$ steradian.


Fig. 18.14 Projected area of two spheres having radius $r_{1}$ and $r_{2}$ respectively

Total luminous flux radiated in the direction of surface of the spheres,

$$
\phi=\omega \times I
$$

Illumination on the surface area $A_{1}$,

$$
E_{1}=\frac{\phi}{A_{1}}=\frac{\omega \times I}{\omega \times r_{1}^{2}}\left(\text { as } A_{1}=\omega \times r_{1}^{2}\right)=\frac{I}{r_{1}^{2}}
$$

Similarly, illumination on the surface are $A_{2}$,

$$
E_{2}=\frac{\phi}{A_{1}}=\frac{\omega \times I}{\omega \times r_{2}^{2}}=\frac{I}{r_{2}^{2}}
$$

Hence, the illumination on a surface is inversely proportional to the square of the distance between the surface and the light source provided that the distance between the surface and the source is sufficiently large so that the source is regarded as a point source and rays fall perpendicularly on the surface.

$$
E=\frac{I}{d_{2}}
$$

### 18.7.2 Lambert's Cosine Law

This law states that the illumination on any surface is proportional to the cosine of the angle between the direction of the incident flux and normal (perpendicular) to the area.

For illustration, consider luminous flux $\phi$ falling normal (perpendicular) to a plane having surface area ' $a$ '. Then, illumination at the surface, $E=\frac{\phi}{a}$. If the surface is inclined at an angle $\theta$, then the angle between the direction of flux and normal to the surface is also $\theta$ (Fig. 18.15).


Fig. 18.15 Plane perpendicular to the luminous flux and inclined at some angle

The projected area of this surface is $\frac{a}{\cos \theta}$ and hence its illumination

$$
E^{\prime}=\frac{\frac{\phi}{a}}{\cos \theta}=\frac{\phi}{a} \cos \theta
$$

It shows that illumination is proportional to $\cos \theta$, which proves the Lambert's consign law. Combining both the laws, the illumination on any surface is given by the relation,

$$
E=\frac{1}{d^{2}} \cos \theta
$$

### 18.8 ILLUMINATION AT A POINT ON THE PLANE SURFACE DUE TO LIGHT SOURCE SUSPENDED AT A HEIGHT (H)



Let us consider a point B on the plane surface PQ where illumination due to light source $S$ of luminous intensity $I$ mounted at a height $h$ from the plane PQ is to be determined (see Fig. 18.16).

Distance $\mathrm{SB}=\frac{h}{\cos \theta}$
Illumination at point B ,

$$
E=\frac{1}{(\mathrm{SB})^{2}} \cos \theta=\frac{1}{(h / \cos \theta)^{2}} \times \cos \theta=\frac{1}{h^{2}} \times \cos ^{3} \theta
$$

Fig. 18.16 Illumination at a point (B)

## Example 18.1

A lamp has a mean spherical candle power of 30 . Calculate the total flux of light from the lamp.

## Solution:

$$
\mathrm{MSCP}=30
$$

Total luminous flux emitted, $\phi=4 \pi \times M S C P$

$$
=4 \pi \times 30=377 \text { lumen }
$$

## Example 18.2

A 230-V lamp has a total flux of 2,000 lumens and takes a current of 0.4348 A. Calculate (i) lumens per watt (i.e., efficiency of the lamp) and (ii) the MSCP per watt.

## Solution:

Here, $V=230 \mathrm{~V} ; I=0.4348 \mathrm{~A} ; \phi=2,000$ lumen
Wattage of the lamp, $W=V \times I=230 \times 0.4348=100 \mathrm{~W}$

$$
\begin{aligned}
\text { MSCP } & =\frac{\phi}{4 \pi}=\frac{2,000}{4 \pi}=159.15 \\
\text { Lumen per watt } & =\frac{\phi}{W}=\frac{2,000}{100}=20 \\
\text { MSCP per watt } & =\frac{159.15}{100}=1.5915
\end{aligned}
$$

## Example 18.3

A lamp giving 500 CP in every direction below the lamp level is suspended 5 m above the ground. Calculate (i) the illumination at point ' $O$ ' on the ground vertically below it, (ii) total flux of light, and (iii) the total flux of light within a circular area of 1 metre diameter about the point ' O ' assuming illumination to be uniform within this area.

## Solution:

Intensity of lamp, $I=500 \mathrm{CP}$
Mounting height, $h=5 \mathrm{~m}$ (refer to Fig. 18.17)
(i) Illumination at point $\mathrm{O}, E=\frac{I}{h^{2}}=\frac{500}{25}=20$ lux
(ii) Total flux of light, $\phi=\mathrm{CP} \times 4 \pi$

$$
=500 \times 4 \pi=6,284 \text { lumen }
$$

(iii) Surface area within 1 m diameter,

$$
A=\frac{\pi d^{2}}{4}=\frac{\pi}{4}(1)^{2}=0.785 \mathrm{~m}^{2}
$$

Total lumens received in this area, $\phi=E \times A=20 \times 0.785$

$$
=15.70 \text { lumen }
$$



Fig. 18.17 Drawing as per data

## Example 18.4

A lamp emitting 900 lumens is placed inside a globe of frosted glass, the diameter of which is 0.3 m . The globe gives a uniform brightness of 240 milli lamberts in all directions. Calculate the CP of the globe and estimate the percentage of light absorbed by the globe.

## Solution:

Flux emitted by the lamp, $\phi=900$ lumen
1 lambert $=\frac{1}{\pi}$ candle per sq. cm of projected area
$\therefore \quad 240 \times 10^{-3}$ lambert $=\frac{240 \times 10^{-3}}{\pi}$ candle/sq.cm
$\therefore \quad$ Candle power $=$ projected area $\times$ candle $/$ sq. cm

$$
=\frac{\pi}{4}(30)^{2} \times \frac{240 \times 10^{-3}}{\pi}=54
$$

Flux emitted by the globe $=54 \times 4 \pi=680$ lumen
Flux absorbed by the globe $=900-680=220$ lumen

$$
\% \text { of light absorbed }=\frac{220}{900} \times 100=24.4 \%
$$

## Example 18.5

A lamp of 100 CP is placed 1 me below a plane mirror, which reflects $90 \%$ of light falling on it. The lamp in hung 4 m above ground. Find the illumination at a point on the ground 3 m away from the point vertically below the lamp.

## Solution:

Capacity of the source of light placed at $\mathrm{S}=100 \mathrm{CP}$
Height of source above ground, $\mathrm{SA}=4 \mathrm{~m}$ (refer Fig. 18.18)
Height of image of the source, $\mathrm{S}^{\prime} \mathrm{A}=6 \mathrm{~m}$


Fig. 18.18 Drawing as per data

$$
\mathrm{S}^{\prime} \mathrm{B}=\sqrt{(\mathrm{AB})^{2}+\left(\mathrm{AS}^{\prime}\right)^{2}}=\sqrt{(3)^{2}+(6)^{2}}=\sqrt{45}
$$

$$
\cos \theta^{\prime}=\frac{\mathrm{AS}^{\prime}}{\mathrm{S}^{\prime} \mathrm{B}}=\frac{6}{\sqrt{45}}
$$

Illumination at point $B=\frac{100}{5 \times 5} \times \frac{4}{5}+\frac{90}{45} \times \frac{6}{\sqrt{45}} \quad\left[\Theta S^{\prime} B=\sqrt{45}\right]=3.20+1.79=4.99$ lux

## Example 18.6

The candle power of a lamp in all directions below the horizontal is 200. If this lamp is suspended 2 m above the centre of a square table of 1 m side, determine the maximum and minimum illumination.

## Solution:

Intensity of the lamp, $I=20 \mathrm{CP}$
Height of suspension, $h=2 \mathrm{~m}$
The maximum illumination will be at the centre of the table and the minimum illumination will be at the corners (Fig. 18.19).


Fig. 18.19 Drawing as per data

The image of the source acts as a secondary source of light, so

Capacity of this secondary source $=0.9 \times 100=90 \mathrm{CP}$
Illumination at point $\mathrm{B}=$ Illumination due to S + Illumination due to $S^{\prime}$
$=\frac{100}{(\mathrm{SB})^{2}} \cos \theta+\frac{90}{\left(\mathrm{~S}^{\prime} \mathrm{B}\right)^{2}} \cos \theta^{\prime}$
where,

$$
\begin{aligned}
\mathrm{SB} & =\sqrt{\mathrm{AB}^{2}+\mathrm{AS}^{2}}=\sqrt{(3)^{2}+(4)^{2}}=5 \\
\cos \theta & =\frac{\mathrm{AS}}{\mathrm{SB}}=\frac{4}{5}
\end{aligned}
$$

Minimum illumination at point A, $E_{\min }=\frac{I}{h^{2}} \cos ^{3} \theta$

$$
=\frac{200}{2^{2}}\left[\frac{2 \sqrt{2}}{3}\right]^{3}=41.9 \text { lux }
$$

## Example 18.7

Two lamps are hung at a height of 6 m from the floor level. The distance between the lamps is 8 m . Lamp one is of 500 CP . If the illumination on the floor vertically below this lamp is 20 lux, find the candle power of the second lamp.
(A.M.I.E. Summer 1977)

## Solution:

Intensity of lamp $\mathrm{L}_{1}, I_{1}=500 \mathrm{CP}$
Let the intensity of lamp $\mathrm{L}_{2}$ be $I_{2}$
Horizontal distance between $\mathrm{L}_{1}$ and $\mathrm{L}_{2}=8 \mathrm{~m}$
Mounting height $=6 \mathrm{~m}$
From Figure 18.20, Distance $L_{2} A=\sqrt{8^{2}+6^{2}}=10 \mathrm{~m}$

$$
\cos \theta=\frac{6}{10}
$$

Illumination at point A, $E=$ Illumination due to $\mathrm{L}_{1}$


Fig. 18.20 Drawing as per data

$$
\begin{aligned}
E & =\frac{I_{1}}{\left(\mathrm{~L}_{1} \mathrm{~A}\right)^{2}}+\frac{I_{2}}{\left(\mathrm{~L}_{2} \mathrm{~A}\right)^{2}} \cos \theta \\
20 & =\frac{500}{6^{2}}+\frac{I_{2}}{10^{2}} \times \frac{6}{10}
\end{aligned}
$$

$$
\text { or } \quad \frac{500}{36}+\frac{6 I_{2}}{1,000}=20
$$

or

$$
I_{2}=1,018 \mathrm{CP}
$$

## Example 18.8

Two lamp posts are 16 m apart and are fitted with a 1,000 CP lamp each at a height of 6 m above ground. Calculate the illumination on the ground (i) under each lamp and (ii) midway between the two lamp posts.

## Solution:

Intensity of each lamp, $I=1,000 \mathrm{CP}$
Height of each lamp, $h=6 \mathrm{~m}$
Horizontal distance between two lamps $=16 \mathrm{~m}$
From Figure 18.21, distance $L_{1} Q=\sqrt{6^{2}+16^{2}}=\sqrt{292} \mathrm{~m}$
Distance $L_{1} M=L_{2} M=\sqrt{6^{2}+8^{2}}=10 \mathrm{~m}$


Fig. 18.21 Drawing as per data
(i) Illumination under each lamp, that is, at point P or Q will be same due to symmetry. $\therefore$ Illumination at point $\mathrm{Q}=$ Illumination due to $\mathrm{L}_{1}+$ Illumination due to $\mathrm{L}_{2}$

$$
\begin{aligned}
& =\frac{1}{\left(\mathrm{~L}_{1} \mathrm{Q}\right)^{2}} \cos \theta_{1}+\frac{1}{\left(\mathrm{~L}_{2} \mathrm{Q}\right)^{2}} \\
& =\frac{1,000}{292} \times \frac{6}{\sqrt{292}}+\frac{1,000}{36} \\
& =29 \text { lux }
\end{aligned}
$$

(ii) Illumination at point $\mathrm{M}=2$ (illumination due to $\mathrm{L}_{1}$ or $\mathrm{L}_{2}$ )

$$
\begin{aligned}
& =2\left[\frac{1}{\left(\mathrm{~L}_{1} \mathrm{M}\right)^{2}} \cos \theta_{2}\right]=2\left[\frac{1000}{10^{2}} \times \frac{6}{10}\right] \\
& =12 \text { lux }
\end{aligned}
$$

## 园㬝 PRACTICE EXERCISES

## Short Answer Questions

1. Name various types of cables used in wiring system.
2. What are the various types of wiring systems? What are the advantages of casing-capping wiring system?
3. Where do we prefer surface conduit wiring and why?
4. Where do we prefer cleat wiring?
5. Draw a circuit for staircase wiring.
6. Differentiate between light and illumination.
7. What is the difference between plane angle and solid angle?
8. Define luminance and glare.
9. What do you mean by utilization factor?
10. Define space-height ratio.

## Test Questions

1. Which type of wiring is used in first-class residential buildings? What are its advantages and disadvantages?
2. Draw the wiring diagram to control three-phase induction motor from energy meter to motor terminals.
3. Define the following terms used in illumination and mention their units: (i) luminous flux, (ii) plane angle, (iii) solid angle, (iv) luminous intensity, (v) brightness, and (vi) mean spherical candle power.
4. Define utilization factor, maintenance factor, depreciation factor, and waste light factor. What is their significance in illumination?
5. Define and explain laws of illumination.

## Numericals

1. In a street lighting scheme, lamps having luminous intensity of 1,000 lumens/steradian are hung at a height of 6 m . The distance between consecutive lamp posts is 8 m . Find the illumination under the lamp and at the centre in between the lamp posts.
(Ans. 33.7 lux, 32.1 lux)
2. The desired illumination on a working table at a point directly below lamp is to be 100 lumen $/ \mathrm{m}^{2}$. The lamp gives 400 candles (CP) uniformly below the horizontal plane. Find the height of the lamp. What will be the illumination at a point 2 m away from the vertical axis of the lamp?
(Ans. 2 m, 35.4 lux)
3. A lamp has CP of 1,000 measured in any direction below horizontal plane through the centre of the lamp and zero above the plane. What is the lumen output of the lamp? Calculate also illumination on horizontal surface 10 m below the lamp at a point 15 m away from the vertical through the lamp.
(Ans. 6,280 lumen, 1.69 lux)
4. A lamp having MSCP of 500 is hung at a height of 12 m above ground level. Determine (i) total flux emitted by the lamp. (ii) illumination directly below the lamp.
(Ans. (i) 6,280 lumen and (ii) 3.47 lux)
5. A lamp having a MSCP of 500 is hung at a height of 12 m above ground level. Determine: (i) total flux emitted by lamp, (ii) illumination directly below the lamp, and (iii) illumination at a point 5 m away on the ground from vertically below the lamp.
(Ans. (i) 6,280 lumen, (ii) 3.47 lux, and (iii) 2.73 lux)
6. A lamp of 500 W having a MSCP of 1,000 is suspended 2.5 m above a working plane. Calculate (i) illumination directly below the lamp, (ii) the lamp efficiency, (iii) total luminous flux in a radius of 25 cm just below the lamp, and (iv) number of units consumed in a year if lighting hours of the lamp are six per day.
(Ans. (i) 160, (ii) $25 \mathrm{~L} / \mathrm{W}$, (iii) 31.42, and (iv) 1,095 )
7. Two lamps of 36 and 16 CP , respectively, are 1 m apart. Where the screen should be placed on a straight line passing through the lamps so as to have equal illumination on it?
(HSB Nov. 1984) (Ans. 0.6 m from $\mathrm{L}_{1}$ )
8. A lamp of 200 CP is placed 1 m below a plane mirror, which reflects $90 \%$ of light falling on it. The lamp is hung 4 m above ground. Find the illumination at a point on the ground 4 m away from the point vertically below lamp.
(Ans. 9.98 lux)

### 18.9 ELECTRICAL METHODS OF PRODUCING LIGHT

The different methods of producing light electrically may be classified into three categories:

1. By establishing an arc between two electrodes such as in carbon arc lamps, flame arc lamps, and magnetite arc lamps.
2. By passing electric current through a filament, thus raising its temperature to incandescence such as in tungsten filament incandescent lamps.
3. By electric discharge through vapours or gases such as in vapour lamps, fluorescent tubes, and neon signs.

### 18.10 SOURCES OF LIGHT

Using the above phenomenon, velocity of electric lamps has been designed to meet with different requirements. Electric lamps may be classified as follows:

1. Arc lamps
2. Incandescent or filament lamps
3. Gaseous discharge lamps
(a) Sodium vapour lamp
(b) Mercury vapour lamp
(c) Fluorescent tube
(d) Compact fluorescent lamps

### 18.11 INCANDESCENT OR FILAMENT LAMPS

In the earlier stages, arc lamps were used for general lighting. But these lamps were complicated in construction, unsafe due to harmful radiations and very poor in efficiency, and therefore, these were replaced by filament lamps for general lighting.

### 18.11.I Working Principles

All of us know that when a room heater is switched ON the supply, it gives out faint red light along with heat. The working temperature of a room heater is about $750^{\circ} \mathrm{C}$, and at this temperature, the radiations are mostly in the red and infrared regions. The phenomenon where heat is accompanied by light is known as incandescence, and the construction of the filament lamp is based upon this phenomenon.

When an electric current is passed through a fine metallic wire, it raises the temperature of the wire. At low temperatures, heat is produced. As the temperature is increased, both heat and light radiations are radiated, and the amount of light radiations goes on increasing with the increase in wire temperature. An incandescent lamp essentially consists of a fine wire of highly resistive material placed in an evacuated glass bulb. Such lamps were first produced by Edison in 1879, in which he used carbonized paper filament. But a carbon filament cannot be operated at high temperatures, and therefore, the efficiency of those lamps was very poor about 4 lumens/watt. To obtain higher efficiency, the filament has to be operated at high temperatures about $2,500^{\circ} \mathrm{C}$. Therefore, the material employed for the construction of filament must have the following properties:

1. High melting point, so that it can be operated at high temperature.
2. High specific resistance, so that it can produce more heat and have smaller size.
3. Low temperature coefficient, so that filament resistance may not change at operating temperature.
4. Low vapour pressure, so that it may not vaporize.
5. Highly ductile and very strong mechanically to withstand vibrations.

After carbon, which could not be operated successfully beyond $1,850^{\circ} \mathrm{C}$, osmium, and tantalum having melting point $2,700^{\circ} \mathrm{C}$ and $2,900^{\circ} \mathrm{C}$, respectively, were tried. Later, when it became possible to draw tungsten into fine wires, it superseded all other filament materials. Tungsten filament can be safely operated at about $2,400^{\circ} \mathrm{C}-2,750^{\circ} \mathrm{C}$ in vacuum, with an efficiency of about 10 to 14 lumen/watt.

### 18.11.2 Construction

A tungsten filament lamp is shown in Figure 18.22 where a filament is enclosed in an evacuated glass bulb to prevent oxidation of the filament. But evacuation helps the evaporation resulting in blackening of the bulb from inside.


Fig. 18.22 Vacuum lamp


Fig. 18.23 Gas-filled coiled coil filament lamp

This not only reduces the efficiency of the lamp but also reduces life of the filament. Later on, it was observed that this difficulty can be overcome by filling the bulb with some chemically insert gas such as argon or nitrogen, which led to discovery of gas-filled lamps, shown in Figure 18.23.

## Gas-filled filament lamps

The basic working principle and constructing of this lamp is same as that of a vacuum filament lamp. In these lamps, to prevent evaporation of tungsten filament, the glass bulb is filled with some inner gas under pressure. The gases used for that purpose are generally argon and nitrogen. It is found that a mixture of $85 \%$ argon and about $15 \%$ nitrogen gave best results. The nitrogen is added to reduce the possibility of arcing within filament turns. By the use of inert gases, the tungsten filament can be safely operated at temperatures of about $2,500^{\circ} \mathrm{C}$ to $2,800^{\circ} \mathrm{C}$, this also increases the efficiency of lamp. The introduction of gas, however, gave rise to another problem. It increases loss of heat from filament due to convection, thus again lowering the efficiency of lamp. Since the loss of heat directly depends upon the surface area of the filament, therefore, it can be reduced by decreasing surface area of the filament. This is achieved by either using single spiral (coiled) or coiled-coil filaments as shown in Figure 18.24.


Fig. 18.24 Lamp's filament

In case of smaller wattage lamps, the loss of heat is comparatively more, so these are generally made vacuum type. Coiled coil filaments are used for wattages up to 100 W and for still higher wattages single coil filament are used. As the wattage of lamp increases, the filament diameter also increases to carry larger currents. But as the diameter increases, the ratio of surface area to cross sectional area decreases. Hence, lesser the cooling effect, higher the operating temperature and thus higher the efficiency.

The efficiency of a $40-\mathrm{W}$ lamp is about $10-12$ lumen/watt and that of a $100-\mathrm{W}$ lamp is about 14-to 18 lumen/watt, whereas the efficiency of a $200-\mathrm{W}$ and more lamp may be 20-25 lumen/ watt. The normal life of incandescent lamps is 1,000 working hours, but it largely depends upon the operating voltage. If the operating voltage is below the rated value, the life of the lamp increases; on the other hand, if the operating voltage is more than the rated value, the life of lamp decreases considerably.

### 18.12 GASEOUS DISCHARGE LAMPS

Before actually studying various discharge lamps, let us understand the gas discharge phenomenon. An atom has a nucleus and extra nucleus. The nucleus contains neutrons and protons, whereas electrons move around the nucleus in different orbits. Under normal conditions, an atom is electrically neutral. The electrons in the outer most orbit are called valance electrons and can be easily knocked out from the atom either by raising the temperature or by applying high potential across it. Consider an atom of a gas having one valence electron. When this electron is knocked out, the positively charged atom left behind is called positive ion. This process is called ionization. If potential is applied at the two ends of the space containing ionized gas, then the positive ions will move towards negative electrode, and electrons will move towards positive electrode. This is known as electric discharge. As the electrons move towards the positive electrode, they collide with other atoms, thus knocking out more electrons and creating more positive ions. This process of ionization is cumulative.

If this cumulative process of ionization is not checked, it will give rise to an electronic avalanche, which amounts to a virtual short circuit. The above condition is prevented by inserting a resistor or choke in series with the circuit which acts as a ballast. When the current in
the circuit tries to increase, the voltage drop across the chock or resistor increases, and therefore, the voltage across discharge tube decreases, which in turn reduces the acceleration of electrons. When electrons move with reduced velocity, the lesser number of electrons are knocked out and hence the current decreases. The reverse is also true when the current decreases, that is, increased voltage becomes available across the discharge tube which promotes ionization.

All discharge lamps essentially consist of a glass or quartz tube having two electrodes, a small quantity of gas or vapour at low pressure, a starting device, and a ballast. Due to the application of electric potential across the electrodes, the gas or vapours gets ionized and the tube is filled with a luminous discharge. The colour of light depends upon the nature of gas or vapour used. The discharge lamps may be broadly classified into two types:

1. Those which emit light of the same colour as is produced by the discharge through the gas or vapour as in sodium vapour, mercury vapour lamp, and neon signs.
2. Those which employ the phenomenon of fluorescence. In such lamps, the discharge produces ultraviolet radiations, and these radiations are made to strike upon a special material called phosphor coated on the inner surface of the discharge tube. The phosphors have the property of absorbing ultraviolet rays and then to reradiate them at longer wavelengths in the visible spectrum. Fluorescent tube is the most common example of this type.

### 18.13 SODIUM VAPOUR LAMPS

### 18.13.1 Construction

This lamp consists of a discharge tube made from a special heat-resistant glass, containing a small amount of metallic sodium, neon gas, and two electrodes. Neon gas is added to start the discharge and to develop enough heat to vapourize the sodium. Because of low pressure inside the tube, a sufficiently long tube is required to obtain more light. To reduce the overall dimensions of the lamp, this tube is generally bent into U-shape as shown in Figure 18.25.


Fig. 18.25 Low pressure sodium vapour lamp

### 18.13.2 Working Principle

All electric discharge lamps require a higher voltage at the time of starting and low voltage during operation. Generally, sodium vapour lamps are operated by a high leakage reactance transformer. At starting, a high voltage of about 450 volts is applied across the lamp, which is sufficient to start the discharge. When the lamp is fully operative after about $10-15 \mathrm{~min}$, the voltage across it falls to about 150 volts. Because of the high reactance of the circuit, its power factor is low. To improve the pf, a capacitor is connected across the supply as shown in Figure 18.25.

The characteristic light produced by this lamp is yellowish. This is produced at its optimum pressure of about 0.004 mm of mercury. This pressure is obtained at a temperature of about $280^{\circ} \mathrm{C}$ and so it becomes necessary to maintain this temperature. For this purpose, the U-tube is enclosed in a double-walled flask to prevent the loss of heat. The double-walled flask is interchangeable and can be fitted on to another U-tube. While replacing the inner U-tube, one must be very careful because if it is broken the sodium will come in contact with moisture which may result in fire.

### 18.13.3 Characteristics and Applications

The efficiency of a low-pressure sodium vapour lamp is very high (about 40-50 lumen/watt) and it produces a light of particular wave length having yellow colour. Sodium lamps are mainly employed for streets, highways, and airfield lighting where colour distinction is not so important.

### 18.14 HIGH-PRESSURE MERCURY VAPOUR LAMPS (M.A. TYPE)

In general, when we ask about the mercury vapour lamp that means we are asking for MA-type high-pressure mercury vapour lamp because it is the most commonly used mercury vapour lamp that can be operated at $200-250 \mathrm{~V} \mathrm{AC}$ having power range of $250-400 \mathrm{~W}$.

### 18.14.1 Construction

It consists of a discharge envelope, that is, a tube of hard glass enclosed in an outer bulb of ordinary glass. The space between the two bulbs is partially or completely evacuated to prevent heat loss by convection from the inner bulb. The outer bulb absorbs harmful ultra violet rays. The inner bulb contains argon and certain quantity of mercury. In addition to two main electrodes, an auxiliary or starting electrode having a high resistance in series is also provided. The main electrodes are made of tungsten wire in helical shapes as shown in Figure 18.26. The lamp has a screwed cap and is connected to the supply mains through a choke. A condenser is connected across the supply mains to improve the power factor.

### 18.14.2 Working Principle

When the supply is switched ON, full mains voltage is applied between the auxiliary electrode and neighbouring main electrode. This voltage breaks down the gap and a discharge through argon gas takes place. This enables the main discharge to commence. As the lamp warms up, mercury is vapourized, which increases the vapour pressure. This discharge, later on, takes up the shape of an intense arc. After 4-5 min, the lamp gives full brilliance. It gives greenish blue


Fig. 18.26 M.A. type M.V. lamp
colour light and its efficiency is about 40 lumen/ watt. The following points are to be noted in such lamps:

1. As the lamp is not operative when cold, so it takes about 5 min to give full brilliance.
2. Once the lamp is switched off, it will not restart again until and unless the pressure developed inside the tube lowers down. The lamp may be kept switched $O N$.
3. This lamp is always suspended vertically, and otherwise, there is a danger of convection currents to make the discharge touch the inner glass tube which may break it due to excessive heat.

### 18.15 FLUORESCENT TUBES

### 18.15.1 Construction

A fluorescent tube is a low-pressure mercury vapour lamp. It consists of a glass tube 25 mm in diameter and $0.6 \mathrm{~m}, 1.2 \mathrm{~m}$, and 1.5 m in length. The tube contains argon gas at low pressure about 2.5 mm of mercury and a few drops of mercury. At the two ends, two electrodes coated with some electron-emissive material are provided.

In this low-pressure mercury vapour lamp, considerable amount of radiation is in the ultraviolet range. By coating the inside of the tube by phosphor, these ultraviolet radiations are converted into visible light. Phosphors have the property of emitting visible light radiations when excited by ultraviolet radiations. Phosphors have definite characteristic colours. These are stable inorganic compounds and give a high output throughout the life of the lamp. The colours of fluorescence produced by various phosphors are given in Table 18.2.
Table 18.2 The Phosphor Used for a Particular Colour

|  | Phosphor | Colour |
| :--- | :--- | :--- |
| 1. | Zinc silicate | Green |
| 2. | Calcium tungstate | Blue |
| 3. | Cadium borate | Pink |
| 4. | Calcium halophosphate | Day light or white |
| 5. | Magnesium tungstate | Bluish white |

### 18.15.2 Working Principle

A choke is connected in series with the tube that acts as ballast and provides a high-voltage impulse or surge for starting the glow in the tube, whereas during running (operating) condition, the same choke absorbs some of the supply voltage and provides the remaining voltage of 110 V across the lamp, which is sufficient for its operation. A capacitor is connected across the circuit to improve the power factor. A starting switch or starter is usually provided in the circuit which puts the electrodes directly across the supply mains at the time of starting so that electrodes may get heated and emit sufficient electrons. The starting switches are of two types, namely ther-mal-type starter and glow-type starter. Usually, glow-type starter is used with fluorescent tubes.

## Working of fluorescent tube using glow starter

The glow-type starter is a voltage-operated device and consists of two bimetallic electrodes enclosed in a glass bulb filled with a mixture of hydrogen and helium. Normally, the contact between the bimetallic strips is open as shown in Figure 18.27.


Fig. 18.27 Glow type starter circuit

When supply is switched ON, the full mains voltage appears across the two bimetallic electrodes which causes a glow discharge in the glow switch. Due to this discharge, a small amount of heat is produced which in turn causes the bimetallic strips to bend in such a way so as to make contact with each other. At this instant, an appreciable amount of current flows through the choke, electrodes, and strips, which raises the temperature of tube electrodes to incandescence. After one or two seconds, the strips get cooled down and the contact is again opened. The opening of the contact in series with the choke provides a momentary high voltage across the electrodes which is sufficient to start the discharge. The starter cannot glow after the tube has started operating because the potential available across the strips does not remain too high to cause glow discharge. During operating condition, the starter does not consume any power, even if it is removed, it does not affected the working of the lamp.

### 18.15.3 Drawbacks and Remedial Measures

There are two main drawbacks in the fluorescent tubes, namely radio interference and flicker or stroboscopic effect. The radio interference can be minimized by connecting a capacitor of value 0.05 microfarad across the starter terminals.

Single lamp cannot be operated without flickering on 50 Hz supply. However, the flickering effect can be corrected by using a pair of lamps as shown in Figure 18.28. This circuit is called a split-phase circuit. Here, it is possible to minimize stroboscopic effect. This allows one tube to be operated at about maximum voltage, when the other is getting nearly zero voltage. There is an additional advantage in this circuit that the power factor is also improved.


Fig. 18.28 Split phase circuit

### 18.16 COMPARISON BETWEEN TUNGSTEN FILAMENT LAMPS AND FLUORESCENT TUBES

The comparison between the two lamps is given in Table 18.3.
Table 18.3 Comparison between Tungsten Filament Lamps and Fluorescent Tubes

| S.No. | Particulars | Tungsten Filament Lamp | Fluorescent Tubes |
| :--- | :--- | :--- | :--- |
| 1. | Effect of voltage <br> fluctuation | Voltage fluctuation has <br> comparatively more effect on the <br> light output. | Voltage fluctuation has comparatively <br> more effect on the light output. |
| 2. | Effect of voltage <br> on luminous <br> efficiency | Luminous efficiency per watt <br> increases with the increase in <br> applied voltage. | The luminous efficiency increases with <br> the increase in applied voltage and length <br> of the tube. |
| 3. | Effect of coloured <br> light on luminous <br> efficiency | Luminous efficiency of coloured <br> filament lamps is low because <br> coloured glass absorbs light. <br> It does not give light close to <br> natural light, so colour rendering <br> is defective. | Their efficiency is high because coloured <br> light is produced due to fluorescence. |
| Colour distortion gives light close to neutral light and |  |  |  |
| hence coloured objects can be properly |  |  |  |
| seen. |  |  |  |

Table 18.3 (Continued)

| S.No. | Particulars | Tungsten Filament Lamp | Fluorescent Tubes |
| :---: | :---: | :---: | :---: |
| 5. | Heat Radiant | Heat radiations are also present due to higher working temperature. | Heat radiations are negligible due to low operating temperature. |
| 6. | Brightness | It brightness is more. | Its brightness is less. |
| 7. | Working life | The average life of filament lamp is about 1,000 working hours, but it varies with the working voltage. A slight increase in voltage may damage the lamp. | Its average life is about 6,000 working hours. Life of fluorescent tube is not affected so much by variation in operating voltage but it depends upon the frequency of starting. |
| 8. | Initial cost | Initial cost per lamp is quite low. | Initial cost per tube is more. |
| 9. | Wiring cost | For the same light output, a large number of lamps are required which results in a higher wiring cost. | For the same light output, smaller number of tubes is required and hence wiring cost is low. |
| 10. | Effect of age on the output of the lamp | The output of the lamp reduces with the passage of time. | The output reduces more rapidly as compared to filament lamp. |
| 11. | Maintenance cost | Overall maintenance cost is low. | Overall maintenance cost is low. |

### 18.17 COMPACT FLUORESCENT LAMPS

The compact fluorescent lamps (CFL) are becoming more and more popular nowadays because of their low-power consumption, low running cost, longer life, attractive look, soothing light, and low maintenance. These lamps are available in different sizes and designs as shown in Figure 18.29. These may have single rod, double rod, triple rod, or spiral rod. These lamps are available in different power rating, such as $5,7,9,11,18,24$ watt and 220 V .


Fig. 18.29 Glow type starter circuit

### 18.17.1 Construction

Basically, a compact fluorescent lamp, shown in Figure 18.30, is a low-pressure mercury vapour lamp having two electrodes coated with electron-emissive material placed at the two ends of a glass tube. In the tube, one drop of mercury and argon gas is filled at low pressure. The tube is coated internally with some fluorescent material in the form of powder. By mixing different powders, the light of any desired colour (including day light) can be obtained. To improve its luminous efficiency, a small amount of halogen is also added to the inert gas. An electronic ballast is used with the lamp to start the glow. The luminous efficiency of these lamps varies from 50-90 lm/W. The technical data of these lamps are given in Table 18.4:


Fig. 18.30 Compact fluorescent lamp

Table 18.4 Technical Data of CFL

| Types of lamps | Wattage (W) | Lumen (LM) | Lamp voltage (V) | Current (mA) | Dimensions (mm) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | A | B | C |
| CFL 5 W | 5 | 250 | 34 | 180 | 65 | 89 | 108 |
| CFL 7 W | 7 | 400 | 47 | 175 | 91 | 115 | 138 |
| CFL 9 W | 9 | 600 | 60 | 170 | 129 | 145 | 168 |
| CFL 11 W | 11 | 900 | 91 | 160 | 199 | 215 | 238 |

### 18.17.2 Advantages of CFL

The following are the main advantages of CFL over the ordinary incandescent lamp:

1. Low-energy consumption.
2. Low maintenance costs.
3. It provides instant glow.
4. It provides light without heating the surroundings.
5. Excellent colour rendering properties. Hence, all colours look natural and desired ambience can be created in various interiors.
6. Low operational costs: The power consumption of CFL is 80 per cent less than incandescent lamps.
7. The compact size opened up possibilities for exciting and aesthetically appealing new luminaries.
8. Average life of compact fluorescent lamp is 8,000 working hour.

### 18.17.3 Applications of CFL

The compact size, lovely appearance, longer life, higher efficiency, low running and maintenance cost, instant glowing power make these lamps suitable for all places where uniform illumination (not flood light) is required. Some of such places are mentioned below:

Offices, shops, hotels, hospitals, conference halls, cinema halls, table lamp fittings, reception lobbies, residential buildings, emergency lighting systems, etc.

### 18.18 LIGHTING SCHEMES

Lighting schemes are classified according to the location, requirement, and purpose. In the broad sense, these may be classified as under the following:

1. Direct lighting: As it is clear from the heading, in this system, almost 90 to 95 per cent light falls directly on the object or the surface. The light is made to fall upon the surface with the help of deep reflectors. Such type of lighting scheme is mostly used in industries and commercial lighting. Although this scheme is most efficient, it is liable to cause glare and shadows.
2. Indirect lighting: In this system, light does not fall directly on the surface but more than 90 per cent light is directed upwards by using diffusing reflectors. Here, the ceiling acts as a source of light and light is uniformly distributed over the surface and glare is reduced to minimum. It also provides a shadowless illumination which is very useful for drawing offices and composing rooms. It is also used for decoration purposes in cinema halls, theatres, hotels, etc.
3. Semi-direct lighting: This is also an efficient system of lighting and chances of glare are also reduced. Here, transparent type shades are used through which about 60 per cent light is directed downward and 40 per cent is directed upward. This also provides a uniform distribution of light and is best suited for rooms with high ceilings.
4. Semi-indirect lighting: In this system, about 60 to 90 per cent of total light is thrown upward to the ceiling for diffused reflection and the rest reaches the working plane directly. A very small amount of light is absorbed by the bowl. It is mainly used for interior decoration.
5. General lighting: This system employs such type of luminaries, shades, and reflectors which give equal illumination in all directions.

### 18.19 DESIGN OF INDOOR LIGHTING SCHEMES

While designing a good lighting scheme, the following points must be kept in mind:

1. It should provide adequate illumination.
2. It should provide uniformly distributed light all over the working plane.
3. It should avoid glare and hard shadows as far as possible.
4. It should provide light of suitable colour.
5. It should provide luminaries suspended at suitable height so that these may not come in line with the vision.

The following factors are required to be considered while designing the lighting scheme:

1. Illumination level: This is the most vital factor in deciding the number and wattage of luminaries so that we are able to see and recognize the object properly. Colours of the body have the property of reflecting the light in different proportions, degree of illumination, to give necessary brightness to the objects depends upon size of the object, its distance from the viewer, contact between the object to be seen and its surroundings. It is also observed that moving objects require more illumination as compared to the stationary objects. Depending upon these points, the illumination level for different purposes is recommended (see Table 18.5).

Table 18.5 Work Place V/s Recommended Illumination

| Type of Work | Recommend Illumination Level in Lumens $/ \mathbf{m}^{2}$ |
| :--- | :---: |
| (1). Offices |  |
| Drawing office | 400 |
| Natural office work, book keeping, typing, etc. | 250 |
| Staircase, corridor, waiting room | 75 |
| 2. Schools |  |
| Class rooms | 250 |
| Art and drawing rooms | 400 |
| Laboratories |  |
| 3. Industry | 150 |
| $\quad$ Very fine work such as making watches | 1,000 |
| Fine bench and machine work | 400 |
| Ordinary bench work | 150 |
| Rough work forging etc. | 120 |
| 4. Shops |  |
| Sales premises and show rooms | 500 |
| Other premises | 250 |
| 5. Hotels |  |
| Bed rooms | 60 |
| Kitchen, lounge, and dining room | 80 |
| General | 80 |
| 6. Hospitals |  |
| Laboratory | 250 |
| Operating table | 3,500 |
| Wards and private rooms | 50 |

2. Quality of light: This means that the illumination should not be harmful to the viewers. It should be glare-free, shadowless, and contrast-free. Direct glare from the source of light is most common and is hindrance to vision. The presence of polished and glassy surfaces will cause indirect glare unless diffused light is used. Similarly, the formation of long and hard shadows causes fatigue to eyes and is the prime source of accidents. Hard and long shadows can be avoided by using a large number of lamps and adjusting the mounting height.
3. Co-efficient of Utilization: A surface to be illuminated receive light either directly from the lamps or reflected from the ceiling and walls or both. In all cases, the total flux reaching the surface will never be equal to the flux emitted by the lamp, due to absorption by reflectors, ceiling and walls. Hence, the ratio of lumens reaching at the working plane to the total lumens emitted by the light source is called coefficient of utilization or utilization factor.
Mathematically,

$$
\text { Utilization factor }=\frac{\text { Lumens reaching at the working plane }}{\text { Total lumens emitted by the source }}
$$

Therefore, the flux reaching at the working plane can be calculated by multiplying the total flux emitted by the source by a factor called utilization factor or coefficient of utilization.

The value of utilization factor depends upon the type of reflector, lighting schemes, condition of walls, and ceilings. Usually, its value varies from 0.5 to 0.8 .
4. Depreciation factor: The total flux emitted by a source and its fittings may be reduced due to deposition of dust and dirt upon their surfaces. Similarly, quantity of light reflected from the ceiling and walls also decreases with the passage of time. This reduction in light flux is taken into account by the depreciation factor, the value of which varies from 1.3 to 1.6 .

Hence, depreciation factor is defined as the ratio of total flux emitted by the source when all its fittings are neat and clean to the flux emitted by the same source under normal working conditions.
Mathematically,

$$
E=\frac{1}{d^{2}}
$$

5. Space-height ratio: The ratio of space (horizontal distance) between the two adjacent lamps to the vertical height of the lamps above the working plane is called space-height ratio.

In order to obtain almost uniform illumination over the working plane, the distance between the lamps should not be too much. An ideal scheme could be developed, when there is large number of small size lamps. But this would increase the installation cost. To have uniform illumination, the ratio of spacing between the lamps and their height above the working plane (i.e., space-height ratio) should normally lie between 1 and 1.5 .

### 18.19.1 Common Indoor Lighting Schemes

In case of factories, filament lamps and fluorescent tubes with direct reflectors are normally used. For offices, a slightly better quality of light is required, and therefore, filament lamps and fluorescent tubes with diffusing fittings are adopted. In domestic lighting, a small room can be well lighted by using a single-filament lamp or a fluorescent tube but for drawing rooms and other large rooms, a number of semi-direct fittings may be adopted.

### 18.20 METHODS OF LIGHTING CALCULATIONS

Lighting calculations means to find out a suitable number of lamps to be installed a particular place for proper illumination. There are three common methods employed for lighting calculations:

1. Watts per square metre method: This is an approximate method and generally employed for rough estimates. Here, an allowance of watts per square metre of an area to be illuminated is made on the assumption of overall efficiency of the system and degree of illumination.
2. Light flux method: This method is applicable to those sources of light which produce an approximately uniform illumination over the working plane. In this case, first, the size of lamps is selected and their lumen output is calculated. Then, lumens reaching at the working plane are determined by taking into account the depreciation and utilization factor, considering the following relations:

Lumens received on the working plane $=($ No. of lamps $\times$ wattage of each lamp $\times$ efficiency of each lamp in Lumens/watt $\times$ coefficient of utilization)/depreciation factor.

This method is usually employed for determining the lighting schemes of domestic and commercial buildings.
3. Point to point or inverse square law method: This is applied where illumination at a particular point is required and the candle power of the source is known. If the polar curve of a source or a lamp is known, the candle power in any particular direction can be calculated. Then, by applying inverse square law, the illumination at any point can be found out. If two or more lamps illuminate the same working plane, the illumination due to each can be added to get total illumination. This system is employed in street lighting and yard lighting.

## Example 18.9

A workshop measures $10 \mathrm{~m} \times 20 \mathrm{~m}$ and is lighted by 15 lamps which are each rated at 200 watts and have an efficiency of 15 lumen/watt. Assuming a depreciation factor of 1.5 and coefficient of utilization as 0.5 . Find the illumination on the working plane.

## Solution:

Area of the workshop ' A ' $=10 \times 20=200 \mathrm{~m}^{2}$.
No. of lamps $=15$
Efficiency = 15 lumens/watt
Depreciation factor $=1.5$
Utilization factor $=0.5$
Using the relation,
Lumens reaching the working plane $=\frac{\text { No. of lamps } \times \text { wattage } \times \text { eff. } \times \text { UF }}{\text { DF }}$

$$
\begin{array}{ll}
\therefore & \phi=\frac{15 \times 200 \times 15 \times 0.5}{1.5}=15,000 \text { lumens } \\
\therefore & \quad \text { Illumination, } E=\frac{\phi}{A}=\frac{15,000}{200}=75 \text { lux }
\end{array}
$$

## Example 18.10

Find the total saving in electrical load and percentage increases in illumination if instead of using twelve 150 W tungsten filament lamps, we use twelve 80 W fluorescent tubes. It may be assumed that (i) there is a choke loss of $25 \%$ of rated lamp wattage, (ii) average luminous efficiency throughout life for each lamp is 15 lumens/watt and for each tube $40 \mathrm{~lm} / \mathrm{W}$, and (iii) coefficient of utilization remains the same in both cases.
(May 2004)

## Solution:

Total power consumption of filament lamps $=12 \times 150=1,800 \mathrm{~W}$
Total power consumption of fluorescent tubes $=12 \times\left[80+\frac{25}{100} \times 80\right]=1,200 \mathrm{~W}$
Saving in electrical load $=1,800-1,200=600 \mathrm{~W}$
Lumen output of filament lamps $=12 \times 150 \times 15=27,000$
Lumen output of fluorescent tubes $=12 \times 80 \times 40=38,400$
Increase in lumen output $=38,400-27,000=11,400$
$\%$ increase in illumination $=\frac{11,400}{27,000} \times 100=42.22 \%$

## Example 18.11

A drawing hall in an engineering college is to be provided with a lighting installation. The hall is $30 \mathrm{~m} \times 20 \mathrm{~m} \times 8 \mathrm{~m}$ (height). The mounting height is 5 m and required level of illumination is 144 lux. Using metal filament lamps, estimate the size and number of single lamp luminaries and also draw their spacing layout. Assume the following:

Utilization coefficient $=0.6$
Maintenance factor $=0.75$
Space-height ratio $=1.0$
Lumens/watt for 300 W lamp $=13$
Lumens/watt for 500 W lamp $=16$
(Dec. 2004)

## Solution:

Area to be illuminated $=30 \times 20=600 \mathrm{~m}^{2}$
Total lumens required $=600 \times 144=86,400$
Gross lumens required $=\frac{86,400}{\text { Utilization factor } \times \text { Maintenance factor }}$

$$
=\frac{86,400}{0.6 \times 0.75}=1,92,000
$$

Lumen output of 500 W lamp $=500 \times 16=8,000$
No. of lamps required $=\frac{1,92,000}{8,000}=24$
Space-height ratio $=1$
Mounting height $=5 \mathrm{~m}$
Space between the lamps $=$ Space - height ratio $\times$ height

$$
=1 \times 5=5 \mathrm{~m}
$$

No. of lamps placed lengthwise $=\frac{30}{5}=6$
No. of lamps placed breadthwise $=\frac{20}{5}=4$
The spacing layout is shown in Figure 18.31.


Fig. 18.31 Layout of lamps

## Example 18.12

A drawing hall 30 metre by 15 metre with a ceiling height of 5 metre to be provided with a general illumination of 120 lux. Taking a coefficient of utilization of 0.5 and depreciation factor of 1.4 determine the number of fluorescent tubes required, their spacing, mounting height, and total wattage. Taking luminous efficiency of fluorescent tube as 40 lumen/watt for 80 watts tube.

## Solution:

Floor area to be illuminated $=15 \times 30=450 \mathrm{~m}^{2}$
Total lumens required at floor level $=450 \times 120$

$$
=54,000 \text { lumen }
$$

Lumens required to be emitted by the tubes or
Gross lumens $=\frac{54,000 \times \mathrm{DF}}{\mathrm{UF}}=\frac{54,000 \times 1.4}{0.5}=1,51,200$ lumen
Lumens output of each tube $=80 \times 40=3,200$ lumen
No. of tubes required $=\frac{1,51,200}{3200}=47.25$ (48 say)
Total wattage of tubes $=48 \times 80=3,840$
24 twin tube units, each containing two tubes of 80 watt each can be provided in the drawing hall in six rows having spacing of 5 m . Each row will have four units spaced 3.75 m as shown in Figure 18.32.


Fig. 18.32 Layout of lamps

Assuming mounting height as 3.5 m ,
Space-height ratio lengthwise $=\frac{5}{3.5}=1.43$
Space-height ratio breadthwise $=\frac{3.75}{3.5}=1.071$, which lies between 1 and 1.5 .

## Example 18.13

A hall measuring $15 \mathrm{~m} \times 45 \mathrm{~m}$ is to be illuminated by suitable lamps to give an average illumination of 45 lux. Assuming mounting height of lamps to be 3 m above the working plane,
utilization factor of 0.65 and depreciation factor of 1.3. Estimate the number and installation position of lamps. The lamps are to be selected from the following:

Wattage of lamp: 75, 100, 150, 200
Lumens output: 880; 1,280; 2,120; 2,880

## Solution:

The area to be illuminated $=15 \times 45=675 \mathrm{~m}^{2}$
Total lumens required $=675 \times 45=30,375$
Gross lumens required $=\frac{30,375 \times \mathrm{DF}}{\mathrm{UF}}=60,750$ lumens
Since the efficiency of higher wattage lamps is more and no. of lamps required will be less, so let us first try for 200 watt lamps,
Output of a 200 watt lamp $=2,880$ lumens
No. of lamps required $=\frac{60,750}{2,880}=21$
21 lamps can be arranged in 7 rows of 3 lamps each.
Lengthwise spacing $=\frac{45}{7}$
Breadthwise spacing $=\frac{15}{3}=5 \mathrm{~m}$
Space-height ratio in length and breadthwise will be $\frac{6.43}{3}=2.14$ and $\frac{5}{3}=1.66$, respectively, which is more than 1.5 and this arrangement cannot provide uniform illumination. Next, trying 150 watt size lamps.
Output of a 150 watt lamp $=2,120$ lumens
No. of lamps required $=\frac{60,750}{2,120}=29.1$
Arranging 30 lamps in 10 rows of 3 lamps each,
Lengthwise spacing $=\frac{45}{10}=4.5 \mathrm{~m}$
Breadthwise spacing $=\frac{15}{3}=5 \mathrm{~m}$
Space-height ratio lengthwise $=\frac{4.3}{3}=1.5$
Space-height ratio, breadthwise $=\frac{5}{3}=1.66$
Instead of keeping 2.5 m distance between the lamp and wall on breadth side, let it be 3 metre, then the distance between two consecutive lamps will be 4.5 m , thus giving space-height ratio as $\frac{4.5}{3}=1.5$
A suitable arrangement with 150 watt lamps can be had as shown in Figure 18.33.


Fig. 18.33 Layout of lamps

## Example 18.14

A drawing hall measuring $30 \mathrm{~m} \times 15 \mathrm{~m}$ with ceiling height of 5 m is to be provided with an average illumination of 150 lux. Assume suitable coefficients and find the number of lamps required, their spacing, mounting height, and total wattage.


Fig. 18.34 Layout of lamps

## Solution:

Area to be illuminated $=30 \times 15=450 \mathrm{~m}^{2}$
Average illumination $=150$ lux
Total lumens reqd. at floor level $=450 \times 150=67,500$ lumen
Assuming a utilization factor as 0.5 and maintenance factor as 0.9 ,
Gross lumens required $=\frac{67,500}{0.5 \times 0.9}=1,50,000$ lumen
If the efficiency of a $200-\mathrm{W}$ filament lamp is assumed to be 15 lumen/watt, then
Output of each lamp $=200 \times 15=3,000$ lumen
No. of lamps required $=\frac{1,50,000}{3,000}=50$

Total wattage $=50 \times 200=10,000$ watts
The lamps can be suitably arranged in 10 rows of 5 lamps each, that is, 10 lamps length wise and 5 lamps breadthwise as per layout shown in Figure 18.34, keeping a distance of $\frac{30}{10}$ or $\frac{15}{5}$, that is, 3 m in both ways between two consecutive lamps.
Assuming a mounting height of 3 m .
Space-height ratio (both ways) $=\frac{3}{3}=1$

## Example 18.15

The front of a building $50 \mathrm{~m} \times 16 \mathrm{~m}$ is illuminated by sixteen 1,000 watt lamps arranged so that uniform illumination on the surface is obtained. Assuming a luminous efficiency of 17.4 lumens/ watt, depreciation factor 1.3 , utilization factor 0.4 , and waste light factor 1.2 , determine the illumination on the surface.

## Solution:

Surface area to be illuminated by flood lights $=50 \times 16=800 \mathrm{~m}^{2}$
Total lumens output of 16 lamps $=1,000 \times 16 \times 17.4$
that is, Gross lumens $=2,78,400$ lumens
Total lumens reaching the surface $=\frac{\text { Gross lumens } \times \mathrm{UF}}{\mathrm{DF} \times \text { Waste light factor }}$

$$
=\frac{2,78,400 \times 0.4}{1.3 \times 1.2}=71,400 \text { lumens }
$$

Illumination on the surface, $E=\frac{71,400}{800}=89$ lux

## PRACTICE QUESTIONS

## Short Answer Questions

1. Which principles are used to produce light by electricity?
2. What should be the properties of the metal used to construct filament of an incandescent lamp?
3. Why coiled-coil filament is preferred over coiled filament used in incandescent lamps?
4. What is the basic principle of operation of a sodium vapour lamp?
5. Why fluorescent tube is preferred over incandescent lamp?
6. What is the necessity of choke in fluorescent tube?
7. What is the function of a glow starter used with a fluorescent tube?
8. Why CFLs are preferred over fluorescent tubes?
9. What are different types of lighting schemes?
10. What are the requirements of a good lighting scheme?

## Test Questions

1. What electrical methods are used to produce artificial light? Describe construction and working of a mercury vapour lamp.
2. What is the basic principle of operation of incandescent lamp? Why coiled-coil filament is preferred over coiled filament? Which metal is used to construct a filament of an incandescent lamp and why?
3. Explain the construction, working, advantages, disadvantages, and applications of a sodium vapour lamp.
4. Draw the circuit diagram of a fluorescent tube and explain the function of each component therein. In fluorescent tubes, how the stroboscopic effect is neutralized?
5. Explain the method of calculation for lighting scheme required to illuminate a hall at desired value of illumination.

## Numericals

1. A factory space of $30 \mathrm{~m} \times 10 \mathrm{~m}$ is to be illuminated with an average illumination of 75 lux by 200 watt lamps. The coefficient of utilization is 0.4 and depreciation factor is 1.4. Calculate the number of lamp required. Luminous efficiency of 200 watt lamp is 14.4 lumen/watt.
(May 1997) (Ans. 32)
2. It is required to provide an illumination of 100 lux in a workshop hall $40 \mathrm{~m} \times 10 \mathrm{~m}$ and efficiency of lamp is 14 lumens/watt. Calculate the number and rating of lamps and their positions when seven trusses are provided at mutual distance of 5 m . Take coefficient of utilization as 0.4 and depreciation factor 0.8 .
(Dec. 1997) (Ans. 21 lamps of 500 W each. 3 on each truss)
3. An illumination of 300 lux is to be provided in a class room $20 \mathrm{~m} \times 10 \mathrm{~m}$ with 40 W fluorescent lamps. Find out the number and layout of lamps in the lighting installation. Assume coefficient of utilization as 0.47 , depreciation factor as 0.8 and luminous output of 40 W fluorescent lamp as 2,400 lumen.
(Ans. 66 lamps)
4. The illumination in a drawing office $30 \mathrm{~m} \times 10 \mathrm{~m}$ is the have a value of 250 lux and is to be provided by a number of 300 watts filament lamps. If the coefficient of utilization is 0.4 and the depreciation factor 0.9 , determine the number of lamps required. The luminous efficiency of each lamp is 14 lumen per watt.
(Ans. 50)
5. The illumination in a drawing hall measuring $30 \mathrm{~m} \times 10 \mathrm{~m}$ is to have an average lighting of 300 lux and is to be provided by filament lamps. If the coefficient of utilization is 0.4 and depreciation factor 0.9 , calculate the number of lamps required. Design a suitable lighting scheme and draw a sketch showing the relative position of lamps. Given luminous efficiency for $100-\mathrm{W}$ lamp $=13.4$; $200-\mathrm{W}$ lamp $=14.4 ; 300-\mathrm{W}$ lamp $=16.0$.
(Ans. 52 lamps of 300 W each)
6. A foundry shop $80 \mathrm{~m} \times 20 \mathrm{~m}$ with a ceiling height of eight metre is to be illuminated by mercury vapour lamps, the desired illumination being 100 lux. Design the lighting installation giving the number, size, location, and mounting height. The lamp sizes available are $80 \mathrm{~W}, 125 \mathrm{~W}$, and 400 W , and the efficiency being 40 lumens/watt. Take coefficient of utilization as 0.5 and depreciation factor as 1.43. Show the arrangement on plan.
(Ans. 100 lamps of 125 watt in 5 rows of 20 lamps each; mounting height 4 m )

## SUMMARY

1. Types of wiring:
(i) Cleat wiring: Best method for temporary wiring.
(ii) Casing-capping wiring: Best suited for rural domestic wiring.
(iii) CTS or TRS wiring: Best suited for urban domestic wiring.
(iv) Metal sheathed wiring: Best suited for workshops.
(v) Conduit wiring:
(a) Surface conduit wiring: Best suited for workshops, industries, laboratories, etc.
(b) Concealed conduit wiring: Best suited for commercial and high first class buildings.
2. Light and Illumination: Light is the cause and illumination is the result due to which the object is visible.
3. Terms used in illumination: These are as follows:
(i) Light: That part of radiant energy from a hot body that produces the visual sensation on human eye is called light.
(ii) Luminous flux: The total quantity of radiant energy per second responsible for visual sensation from a luminous body is called Luminous flux.
(iii) Lumen: It is the unit of luminous flux.
(iv) Plane angle: The angle subtended at a point by two converging lines lying in the same plane is called plane angle.
(v) Solid angle: The angle subtended by the partial surface area of a sphere at its centre is called solid angle.
(vi) Steradian: The unit of solid angle.
(vii) Candle power: The light radiating capacity of a source is called its candle power.
(viii) Luminous intensity: Luminous intensity in any particular direction is the luminous flux emitted by the source per unit solid angle in that direction.
(ix) Illumination: When light falls on a surface, it becomes visible, the phenomenon is called illumination. It is defined as the luminous flux falling on a surface per unit area.
(x) Lux or metre-candle: One metre candle or lux is defined as the illumination produced by a uniform source of one CP on the inner surface of a sphere of radius one metre.
(xi) Brightness or luminance: Brightness or luminance is defined as the luminous flux emitted (or reflected) by the source or surface per unit projected area. It is denoted by symbol B. Things that are perfectly clean are called maintenance factors.
(xii) Unit of brightness: Depending upon the light emitted and the projected area, it may be
(a) Nit: Defined as candle per square metre.
(b) Stilb: Defined as candle per square cm .
(c) Lambert: Defined as lumens per square cm .
(xiii) Glare: It may be defined as the brightness within the field of vision of such a character so as to cause discomfort, interference in vision and eye fatigue.
(xiv) Lamp efficiency: The efficiency of a lamp is defined as the visible radiations emitted by it in lumens per watt.
(xv) Utilization factor or coefficient of utilization: The coefficient of utilization or utilization factors may be defined as the ratio of the total lumens reaching the working plane to the total lumen emitted by the source.
(xvi) Maintenance factor: The ratio of the illumination under normal working conditions to the illumination when the things are perfectly clean is called maintenance factor.
(xvii) Depreciation factor: Depreciation factor may be defined as the ration of illumination on the working plane at the initial state to the ultimate illumination at any moment. It is just reciprocal to maintenance factor.
(xviii) Waste light factor: Waste light factor is defined as the lumens required to illuminate an area to the theoretical lumen calculated to illuminate the same area at a particular level.
(xix) Space-height ratio: The ratio of horizontal distance between adjacent lamps and the height of their mounting above working plane is called space-height ratio.
(xx) Reflection factor: The ratio of the reflected light to the incident light is called reflection factor.
4. Law of illumination:
(i) Inverse Square Law: It states that the illumination of a surface is inversely proportional to the square of the distance of the surface from the source, that is, $E \propto 1 / d^{2}$.
(ii) Lambert's Cosine Law: This law states that the illumination on any surface is proportional to the cosine of the angle between the direction of the incident flux and normal (perpendicular) to the area.
5. Electrical methods of producing light:
(i) By establishing an arc between two electrodes such as in carbon arc lamps, flame arc Lamps, and magnetic arc lamps.
(ii) By passing electric current through a filament, thus raising its temperature to incandescence such as in tungsten filament incandescent lamps.
(iii) By electric discharge through vapours or gases such as in vapour lamps, fluorescent tubes, and neon signs.
6. Incandescent or filament lamps: In these lamps, a filament is heated to the extent that it becomes incandescent (i.e., it starts emitting light). The light emitted by the filament is used for vision.
7. Properties of the metal required for the construction of filament of an incandescent lamp:
(i) High melting point
(ii) High specific resistance
(iii) Low temperature coefficient
(iv) Low vapour pressure
(v) Highly ductile
8. Gas-filled incandescent lamps: An inert gas such as argon, nitrogen, is filled in an incandescent lamp to increase it luminous efficiency.
9. Gaseous discharge lamps: In these lamps, gaseous discharge took place between the electrodes. The discharged particles become luminous and start emitting light.
10. Sodium vapour lamps: These lamps have luminous efficiency of about 40 to 50 lumen/watt and emit yellowish light. These are usually employed for street lighting, highway lighting, airfield lighting, lighting in open yards, etc., where colour distinction is not so important.
11. Mercury vapour lamps: In these lamps. mercury vapours are used for discharge. There efficiency is about 35 to 45 lumen/watt. They emit bluish light. These are generally used for street lighting, highway lighting, airfield lighting, etc.
12. Fluorescent tubes: A fluorescent tube is basically a low-pressure mercury vapour lamp having an efficiency of about 40 lumen/watt. It provides almost day light. It has replaced the incandescent lamps. It needs high voltage (about 400 V ) at the start but low voltage (about 110 V ) during normal operation. To achieve this condition, usually a glow starter and a choke is used in the circuit.
13. Compact Fluorescent Lamps: These are available in different sizes and ratings. These are very efficient (about 80 lumen/watt).
14. Advantages of CFLs:
(i) Low operating cost
(ii) Low maintenance cost
(iii) Very long life (about 8,000 working hours)
(iv) High efficiency (about 80 lumen/watt)
(v) Instant glow
(vi) Compact size
(vii) Attractive look
15. Applications of CFLs: Because of various merits, these lamps have replaced incandescent lamps and large fluorescent tubes. These are invariably employed in shops, offices, hotels, hospitals, conference halls, cinema halls, residential building and many other places.
16. Bunsen grease spot photometer: It is used to measure the luminous intensity of a light source. It works on the basic principle of illumination $\frac{I_{s}}{d_{s}{ }^{2}}=\frac{I_{L}}{d_{L}{ }^{2}}$
17. Requirements of an indoor lighting scheme are as follows:
(i) It should provide adequate illumination.
(ii) It should provide uniformly distributed light all over the working plane.
(iii) It should avoid glare and hard shadows as far as possible.
(iv) It should provide light of suitable colour.
(v) It should provide luminaries suspended at suitable height so that these may not come in line with the vision.

## TEST YOUR PREPARATION

## FILL IN THE BLANKS

1. $\qquad$ connected lamps are used for decoration lighting at festivals and at marriage parties, to light a large number of lamps of low voltage rating controlled by a single switch. (series, parallel)
2. In a tube light, the electrodes are enclosed in a glass bulb filled with $\qquad$ gas. (argon, neon)
3. Fuses and switches should be inserted in the $\qquad$ . (phase, neutral)
4. Voltage between any one phase wire and neutral is $\qquad$ . (line voltages, phase voltage)
5. Solid angle subtended by a sphere at its centre is $\qquad$
6. The life of a filament lamp $\qquad$ with the increase in operating voltage.
7. The unit of luminous intensity is $\qquad$
8. The unit of luminous flux is $\qquad$
9. Candela is the unit of $\qquad$
10. The function of choke in a fluorescent lamp circuit is $\qquad$
11. Solid angle is measured in $\qquad$ which is equal to $\qquad$
12. Running cost of a fluorescent lamp is $\qquad$ than incandescent lamp.
13. With the increase in voltage, life of the lamp
14. The resistance of a 200 W lamp is $\qquad$ than 100 W lamp.
15. The unit of solid angle is $\qquad$
16. The efficiency of fluorescent tube is $\qquad$ lumen/watt.
17. The space-height ratio should be between $\qquad$ and $\qquad$ for proper illumination.
18. The unit to illumination is $\qquad$
19. The material of the filament in filament lamps is $\qquad$
20. The efficiency of filament lamp is about $\qquad$ to $\qquad$ lumen/watt.
21. The normal life of tungsten filament lamp is $\qquad$ working hours.
22. The sodium vapour lamp produces $\qquad$ light.
23. The efficiency of a low-pressure sodium vapour lamp is about $\qquad$ lumen/watt.
24. The efficiency of mercury vapour lamp is $\qquad$ lumen/watt.
25. $\qquad$ lamps are used for street lighting generally.
26. The normal working life of fluorescent tube is $\qquad$ hours.
27. To minimize the stroboscopic effect $\qquad$ of fluorescent tubes can be used.
28. The luminous efficiency of coloured filament lamps is $\qquad$ than ordinary filament lamps.
29. The ratio of lumens reaching the working plane to lumens emitted by lamps is called $\qquad$ factor.
30. Reduction factor of a source of light is the ratio of its mean spherical candle power to its $\qquad$
31. The solid angle subtended by sphere at its centre is $\qquad$
32. The efficiency of sodium vapour lamp is approximately $\qquad$
33. Steradian is the unit of $\qquad$
34. The fluorescent material used for coating the inside of fluorescent lamp is $\qquad$

## OBJECTIVE TYPE QUESTIONS

1. The cheapest internal wiring system is
(a) cleat wiring.
(b) casing-capping wiring.
(c) CTS wiring.
(d) conduit wiring.
2. The wiring system generally employed in public buildings (offices) is
(a) cleat wiring.
(b) casing-capping wiring.
(c) CTS wiring.
(d) conduit wiring.
3. From economy point of view, mostly the wiring employed in residential building is
(a) cleat wiring.
(b) casing-capping wiring.
(c) CTS wiring.
(d) conduit wiring.
4. From safety point of view, the most suitable wiring system is
(a) cleat wiring.
(b) casing-capping wiring.
(c) CTS wiring.
(d) conduit wiring.
5. For temporary fitting, the most suitable wiring system is
(a) cleat wiring.
(b) casing-capping wiring.
(c) CTS wiring.
(d) conduit wiring.
6. In concealed conduit wiring, the switches used are
(a) tumbler switches.
(b) flush switches.
(c) Pendent switches.
(d) None of these
7. In wiring installations, the appliances are always controlled by the switches connected in
(a) phase wire.
(b) neutral wire.
(c) earth wire.
(d) None of the above
8. In stair case wiring circuit, the lamp glows when two 2 -way switches are in the similar position.
(a) True
(b) False
9. In a three-pin outlet socket, all the three terminals (pins) are of the same size.
(a) True
(b) False
10. The efficiency of a light source is given in
(a) lumens per watt.
(b) lumens per unit area of the surface.
(c) lumens per solid angle.
(d) lumens per candle power.
11. Space-height ratio should ideally be equal to
(a) 2
(b) 1
(c) 0.5
(d) 3
12. Which of the following quantity has the unit of lux?
(a) Utilization factor
(b) Luminous flux
(c) Luminous intensity
(d) Illumination
13. Which of the following comparison between the filament lamp and the fluorescent lamp is correct?
(a) The fluorescent lamp has higher dazzle.
(b) The fluorescent lamp produces sharper shadows.
(c) The fluorescent lamp produces greater brightness.
(d) The average life of fluorescent lamp is five to seven time higher.
14. 1 Candela $=$
(Dec. 2004)
(a) $2 \pi$ lumens
(b) $4 \pi$ lumens
(c) $4 \pi r^{2}$
(d) None of the above
15. Tungsten filament lamp contains
(Dec. 2004)
(a) vacuum.
(b) nitrogen.
(c) oxygen.
(d) hydrogen.
16. Candela is the unit of
(May 2004)
(a) flux.
(b) luminous intensity.
(c) illumination.
(d) luminance.

## NUMERICALS

1. Two lamps each of $1,000 \mathrm{CP}$ are hung 20 m apart at a height of 6 m above the ground. Find out the illumination on the ground (i) under each lamp and (ii) midway between the lamps.
(Ans. 28.36 lux, 7.56 lux)
2. A lamp emits 400 lumens in all directions. What is its MSCP? This lamp is placed at a distance of 4 m from the plane surface. Calculate the illumination of the surface when it is (i) normal, (ii) inclined $60^{\circ}$, and (iii) parallel to the rays. (Ans. 31.85 CP (i) 1.99 , (ii) 0.995 , and (iii) 0 )
3. It is desired to illuminate a corridor with 100 CP lamps, three in number, 8 m apart are suspended 5 m above floor level. Calculate the intensity of illumination exactly vertically below the lamp positions on the surface of the floor
(Ans. 4.70, 5.19, 4.70)
4. Two lamps of 45 and 30 CP are placed 1.5 m apart. Locate the position of screen to be placed on a straight line passing through the lamps so as to have equal illumination. (Ans. 0.67 m from $\mathrm{L}_{2}$ )
5. Two lamps of 500 W , each with lamp efficiency of 25 lumens per watt are mounted on two lamps posts, 10 m apart. The posts have different heights of 3 m and 4 m , respectively. Calculate the illumination at a point midway between the lamp posts.
(Ans. 30.36 lux)
6. In a living room $5 \mathrm{~m} \times 4 \mathrm{~m} \times 3 \mathrm{~m}$ high, there is a metal filament lamp of 150 CP suspended 0.5 m below the ceiling and at the centre of the room. The lamp gives uniform illumination in all directions. Calculate the illumination due to the lamp at a bottom corner of the room on the horizontal phase of the floor.
(Ans. 5.54 lux)
7. Determine the average effective illumination of a room measuring $10 \mathrm{~m} \times 15 \mathrm{~m}$ illuminated by fifteen 150 watts lamps. The luminous efficiency of the lamp is to be taken as 14 lumens per watt and the coefficient of utilization as 0.35 .
(Ans. 73.5 lux)
8. A hall measuring $30 \mathrm{~m} \times 50 \mathrm{~m} \times 5 \mathrm{~m}$ is illuminated by indirect lighting employing several inverted light fittings. An average of 90 lumens per square metre is to be provided on a horizontal plane parallel to the floor and 1 m above it. The walls and ceiling are nicely white washed. Assume a suitable coefficient of utilization and design lighting scheme using filament lamps. Draw a neat sketch showing their relative positions in the hall. Given luminous efficiency, for 100 -watt filament lamp $=13.4$ and for 200 -watt filament lamp $=14.4$.
(Ans. 104 lamps of 200 W each)
9. factory space of $33 \mathrm{~m} \times 10 \mathrm{~m}$ is to be illuminated with an average illumination of 72 lux by 200 W lamps. The coefficient of utilization is 0.4 and depreciation factor is 1.4 . Calculate the number of lamps required. The lumens output of a 200 W lamp is 2,730 .
(Ans. 30 lamps)

## VIVA-VOCE/REASONING QUESTIONS

1. For domestic wiring, concealed conduct wiring system is preferred. Why?
2. One lamp is controlled by two switches in staircase wiring circuit. Why?
3. A starter is required to start the glow in a fluorescent tube. Why?
4. In three-pin outlet socket, why earth terminal is made of larger diameter?

## SHORT ANSWER QUESTIONS

1. What is CTS and VIR wire?
2. Name the various types of wiring systems commonly used in residential and commercial buildings. Explain any one in detail.
3. Explain different types of wiring systems with reference to their field of applications, advantages, and disadvantages.
4. What are the advantages of conduit wiring?
5. What types of wiring are used in workshops and why?

## TEST QUESTIONS

1. Name the most commonly used wiring system in (i) residential buildings and (ii) workshops. Compare the advantages and disadvantages of concealed conduit wiring and batten wiring.
2. What is meant by DPIC and TPIC switch? Where are they used?
3. Draw the circuit diagram showing one, three pin outlet socket controlled by a single-way switch.
4. Draw the circuit diagram showing one lamp controlled by two 2 -way switches and explain its working.
5. Why is the earth point of a three-pin plug made thicker and longer?
6. Draw a circuit to control one lamp, fan, and a three-pin outlet socket individually by a single-way switch.
7. Draw a fluorescent tube circuit. What is the function of each component used in the circuit? Why fluorescent tube is preferred over incandescent filament lamp?
8. Explain the working of a sodium vapour lamp. Why it is preferred for street lighting?

## ANSWERS

## Fill in the blanks

1. series
2. organ
3. phase
4. phase voltage
5. $4 \pi$ steradian
6. decreases
7. candle power
8. lumen
9. luminous intensity
10. steradian, $A / r^{2}$
11. less
12. to give a blast at start
13. steradian
14. 40
15. tungsten
16. 10,14
17. decreases
18. less
19. $50-60$
20. 80
21. a pair
22. lower
23. $1,1.5$
24. lux
25. 1,000
26. yellowish
27. M.V. lamps
28. 7,000
29. $4 \pi$ steradian
30. 50 lumen/watt
31. utilization
32. МНСР
33. solid angle
34. calcium halophosphate

## Objective Type Questions

1. (a)
2. (d)
3. (c)
4. (d)
5. (b)
6. (b)
7. (a)
8. (a)
9. (b)
10. (a)
11. (b)
12. (d)
13. (d)
14. (b)
15. (b)
16. (b)

[^0]:    ${ }^{1}$ Only electrons are supplied or removed from the body, since they have very small mass and are much mobile than protons. Moreover, the protons are powerfully attached in the nucleus and cannot be removed or detached.

[^1]:    ${ }^{2}$ Electric lines of force are the imaginary lines that do not actually exist. These are used only to represent an electric field. Usually, high field strength is represented by drawing lines of force close together, whereas the low field strength is represented by widely spaced lines.

[^2]:    ${ }^{3}$ Strictly speaking, sea level is considered to be the place of zero potential.
    ${ }^{4}$ We know that $F=9 \times 10^{9} \times \frac{Q \times 1}{d^{2}}$ as $d \rightarrow \infty ; F \rightarrow 0$
    ${ }^{5}$ However, in actual practice, earth is considered to be at zero potential since earth is such a huge conductor that its potential practically remains constant.

[^3]:    ${ }^{6}$ Although the derived unit of electric potential is joules/coulomb, it has been given a special name volt.

[^4]:    ${ }^{7}$ A negative sign is placed with the equation since work done in moving a unit positive charge $(+1 \mathrm{C})$ from B and A is against the electric field.
    ${ }^{8}$ The spark or arc may burn the dielectric such as paper, cloth, wood, and mica, whereas hard materials such as porcelain and glass may melt to form a hole or crack. This is called rupturing of material.

[^5]:    ${ }^{9}$ Since current cannot pass through an insulator but an electric field can; therefore, an insulator is also called as dielectric.

[^6]:    ${ }^{10}$ The electrons cannot jump from plate B to A as there is insulating material between the plates.
    ${ }^{11}$ Capacitance, $C=Q / V$; the unit of charge and voltage is coulomb and volt, respectively. Therefore, the unit of capacitance will be C/V. However, to honour Michel Faraday, the unit given to capacitance is farad (symbol F).

[^7]:    ${ }^{2}$ The number of negative plates is always one more than the positive plates.

[^8]:    ${ }^{1}$ In mathematics, $\sqrt{-1}$ is denoted by $i$ (iota); however, in electrical engineering, it is denoted by $j$ to avoid confusion. This is because letter $i$ is reserved for current.

[^9]:    ${ }^{1}$ With the introduction of an ammeter in the circuit, the circuit conditions should not alter, that is, the voltage drop in ammeter should be negligibly small. This is possible only if an ammeter has a low resistance.

[^10]:    ${ }^{2}$ With the introduction of a voltmeter in the circuit, the circuit conditions should not alter; this means that the current drawn by the voltmeter should be negligibly small. This is possible only if a voltmeter has a very high resistance.

[^11]:    ${ }^{1}$ SEGS—Solar Energy Generation System

[^12]:    ${ }^{2}$ UTC--United Technologies Corporation

[^13]:    ${ }^{3}$ ICE Internal Combustion Engine
    ${ }^{4}$ TW mean tera-watt, that is, $1 \times 10^{12}$ watt

[^14]:    ${ }^{1}$ For the transmission of electric power voltage level is increased due to economical reasons.
    ${ }^{2}$ Sometimes, power is supplied to the bulk customers (large factories) through 33 kV line. Then, 33 kV line is called the primary distributor.

[^15]:    ${ }^{3}$ The tendency of alternating current to concentrate near the surface of the conductor is known as skin effect.

