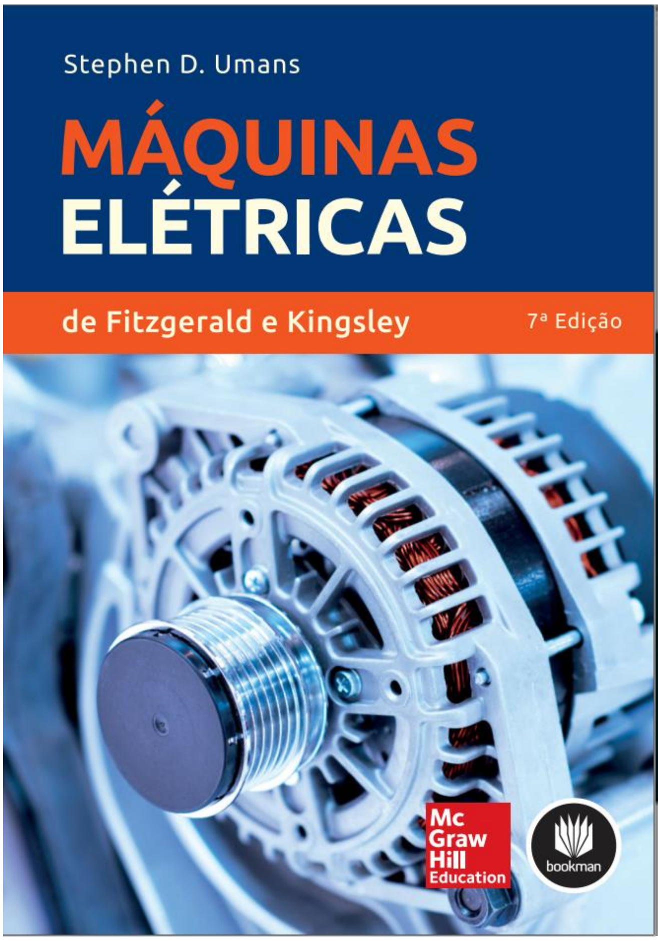
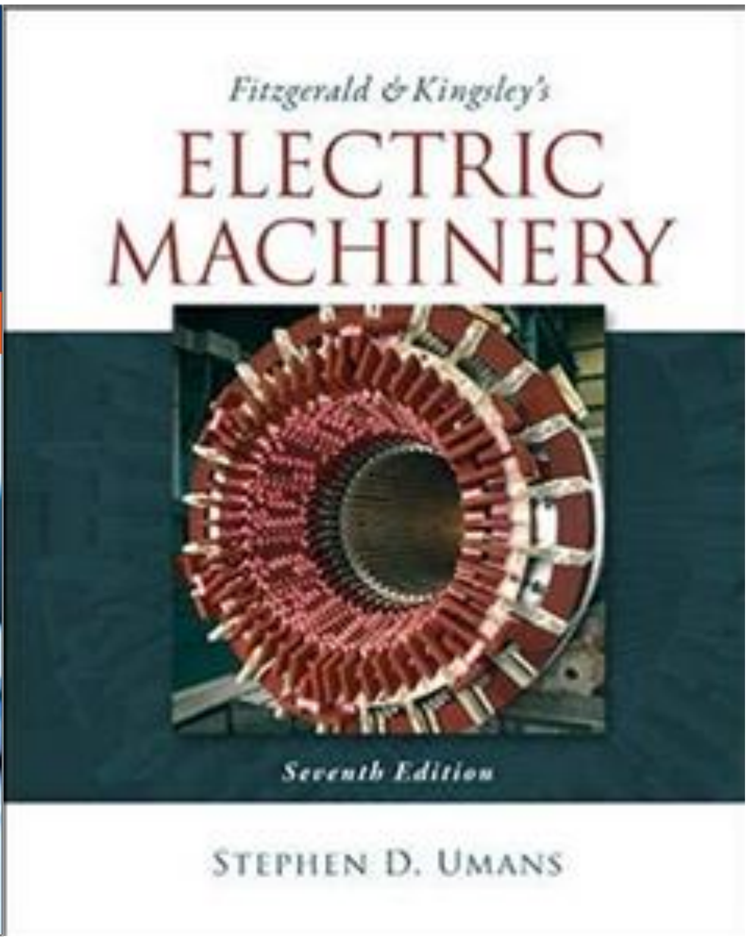
SOLUTION MANUAL

Fitzgerald & Kingsley's Electric Machinery [7th Edition]

Máquinas elétricas de Fitzgeral e Kingsley [7th edição]





PROBLEM SOLUTIONS: Chapter 1

Problem 1-1

Part (a):

$$\mathcal{R}_\mathrm{c} = \frac{l_\mathrm{c}}{\mu A_\mathrm{c}} = \frac{l_\mathrm{c}}{\mu_\mathrm{r}\mu_0 A_\mathrm{c}} = 0 \quad \mathrm{A/Wb}$$

$$\mathcal{R}_\mathrm{g} = \frac{g}{\mu_0 A_\mathrm{c}} = 5.457 \times 10^6 \quad \mathrm{A/Wb}$$

Part (b):

$$\Phi = \frac{NI}{\mathcal{R}_\mathrm{c} + \mathcal{R}_\mathrm{g}} = 2.437 \times 10^{-5} \quad \mathrm{Wb}$$

Part (c):

$$\lambda = N\Phi = 2.315 \times 10^{-3} \quad \mathrm{Wb}$$

Part (d):

$$L = \frac{\lambda}{I} = 1.654 \quad \mathrm{mH}$$

Problem 1-2

Part (a):

$$\mathcal{R}_\mathrm{c} = \frac{l_\mathrm{c}}{\mu A_\mathrm{c}} = \frac{l_\mathrm{c}}{\mu_\mathrm{r}\mu_0 A_\mathrm{c}} = 2.419 \times 10^5 \quad \mathrm{A/Wb}$$

$$\mathcal{R}_\mathrm{g} = \frac{g}{\mu_0 A_\mathrm{c}} = 5.457 \times 10^6 \quad \mathrm{A/Wb}$$

Part (b):

$$\Phi = \frac{NI}{\mathcal{R}_{\mathrm{c}} + \mathcal{R}_{\mathrm{g}}} = 2.334 \times 10^{-5} \quad \mathrm{Wb}$$

Part (c):

$$\lambda = N\Phi = 2.217 \times 10^{-3} \quad \mathrm{Wb}$$

Part (d):

$$L = \frac{\lambda}{I} = 1.584 \quad \mathrm{mH}$$

Problem 1-3

Part (a):

$$N = \sqrt{\frac{Lg}{\mu_0 A_{\mathrm{c}}}} = 287 \text{ turns}$$

Part (b):

$$I = \frac{B_{\mathrm{core}}}{\mu_0 N/g} = 7.68 \quad \mathrm{A}$$

Problem 1-4

Part (a):

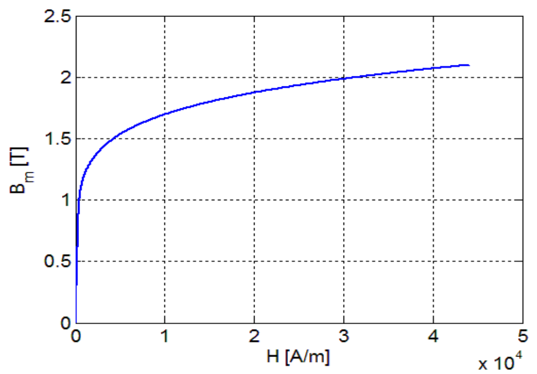
$$N = \sqrt{\frac{L(g + l_{\mathrm{c}}\mu_0/\mu)}{\mu_0 A_{\mathrm{c}}}} = \sqrt{\frac{L(g + l_{\mathrm{c}}\mu_0/(\mu_{\mathrm{r}}\mu_0))}{\mu_0 A_{\mathrm{c}}}} = 129 \text{ turns}$$

Part (b):

$$I = \frac{B_{\mathrm{core}}}{\mu_0 N/(g + l_{\mathrm{c}}\mu_0/\mu)} = 20.78 \quad \mathrm{A}$$

Problem 1-5

Part (a):



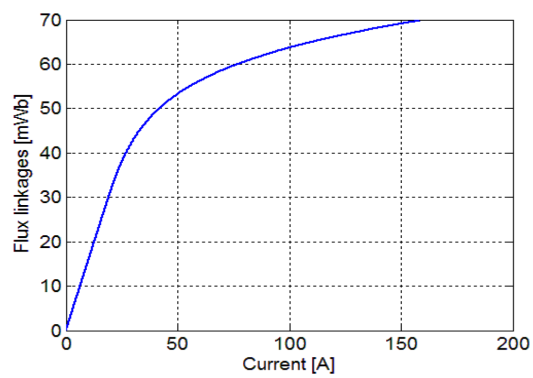
Part (b):

$$B_g = B_m = 2.1 \text{ T}$$

For $B_m = 2.1$ T, $\mu_r = 37.88$ and thus

$$I = \left(\frac{B_m}{\mu_0 N}\right)\left(g + \frac{l_c}{\mu_r}\right) = 158\ A$$

Part (c):



Problem 1-6

Part (a):

$$B_g = \frac{\mu_0 N I}{2g}$$

$$B_{\rm c} = B_{\rm g}\left(\frac{A_{\rm g}}{A_{\rm c}}\right) = \left(\frac{\mu_0 NI}{2g}\right)\left(1 - \frac{x}{X_0}\right)$$

Part (b): Will assume $l_{\rm c}$ is "large" and $l_{\rm p}$ is relatively "small". Thus,

$$B_{\rm g}A_{\rm g} = B_{\rm p}A_{\rm g} = B_{\rm c}A_{\rm c}$$

We can also write

$$2gH_{\rm g} + H_{\rm p}l_{\rm p} + H_{\rm c}l_{\rm c} = NI;$$

and

$$B_{\rm g} = \mu_0 H_{\rm g}; \qquad B_{\rm p} = \mu H_{\rm p} \qquad B_{\rm c} = \mu H_{\rm c}$$

These equations can be combined to give

$$B_{\rm g} = \left(\frac{\mu_0 NI}{2g + \left(\frac{\mu_0}{\mu}\right) l_{\rm p} + \left(\frac{\mu_0}{\mu}\right)\left(\frac{A_{\rm g}}{A_{\rm c}}\right) l_{\rm c}}\right) = \left(\frac{\mu_0 NI}{2g + \left(\frac{\mu_0}{\mu}\right) l_{\rm p} + \left(\frac{\mu_0}{\mu}\right)\left(1 - \frac{x}{X_0}\right) l_{\rm c}}\right)$$

and

$$B_{\rm c} = \left(1 - \frac{x}{X_0}\right) B_{\rm g}$$

Problem 1-7

From Problem 1-6, the inductance can be found as

$$L = \frac{NA_{\rm c}B_{\rm c}}{I} = \frac{\mu_0 N^2 A_{\rm c}}{2g + \frac{\mu_0}{\mu}\left(l_{\rm p} + (1 - x/X_0)\, l_{\rm c}\right)}$$

from which we can solve for $\mu_{\rm r}$

$$\mu_\mathrm{r} = \frac{\mu}{\mu_0} = \frac{L\Big(l_\mathrm{p} + (1 - x/X_0)\,l_\mathrm{c}\Big)}{\mu_0 N^2 A_\mathrm{c} - 2gL} = 88.5$$

Problem 1-8

Part (a):

$$L = \frac{\mu_0 (2N)^2 A_\mathrm{c}}{2g}$$

and thus

$$N = 0.5\sqrt{\frac{2gL}{A_\mathrm{c}}} = 38.8$$

which rounds to $N = 39$ turns for which $L = 12.33$ mH.

Part (b): $g = 0.121 \;\; \text{cm}$

Part(c):

$$B_\mathrm{c} = B_\mathrm{g} = \frac{2\mu_0 NI}{2g}$$

and thus

$$I = \frac{B_\mathrm{c} g}{\mu_0 N} = 37.1 \text{ A}$$

Problem 1-9

Part (a):

$$L = \frac{\mu_0 N^2 A_\mathrm{c}}{2g}$$

and thus

$$N = \sqrt{\frac{2gL}{A_{\rm c}}} = 77.6$$

which rounds to $N = 78$ turns for which $L = 12.33$ mH.

Part (b): $g = 0.121$ cm

Part(c):

$$B_{\rm c} = B_{\rm g} = \frac{\mu_0 (2N)(I/2)}{2g}$$

and thus

$$I = \frac{2B_{\rm c} g}{\mu_0 N} = 37.1 \text{ A}$$

Problem 1-10

Part (a):

$$L = \frac{\mu_0 (2N)^2 A_{\rm c}}{2(g + (\frac{\mu_0}{\mu}) l_{\rm c})}$$

and thus

$$N = 0.5 \sqrt{\frac{2(g + (\frac{\mu_0}{\mu}) l_{\rm c}) L}{A_{\rm c}}} = 38.8$$

which rounds to $N = 39$ turns for which $L = 12.33$ mH.

Part (b): $g = 0.121$ cm

Part(c):

$$B_{\mathrm{c}} = B_{\mathrm{g}} = \frac{2\mu_0 NI}{2(g + \frac{\mu_0}{\mu} l_{\mathrm{c}}0)}$$

and thus

$$I = \frac{B_{\mathrm{c}}(g + \frac{\mu_0}{\mu} l_{\mathrm{c}})}{\mu_0 N} = 40.9 \text{ A}$$

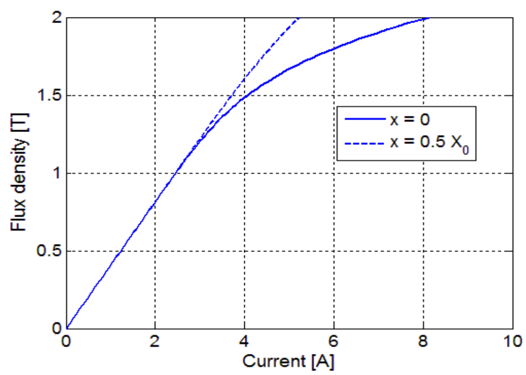
Problem 1-11

Part (a): From the solution to Problem 1-6 with $x = 0$

$$I = \frac{B_{\mathrm{g}}\left(2g + 2\left(\frac{\mu_0}{\mu}\right)(l_{\mathrm{p}} + l_{\mathrm{c}})\right)}{\mu_0 N} = 1.44 \text{ A}$$

Part (b): For $B_{\mathrm{m}} = 1.25$ T, $\mu_{\mathrm{r}} = 941$ and thus $I = 2.43$ A

Part (c):



Problem 1-12

$$g = \frac{\mu_0 N^2 A_{\mathrm{c}}}{L} - \left(\frac{\mu_0}{\mu}\right) l_{\mathrm{c}} = 7.8 \times 10^{-4} \text{ m}$$

Problem 1-13

Part (a):

$$l_\mathrm{c} = 2\pi\left(\frac{R_\mathrm{i} + R_\mathrm{o}}{2}\right) - g = 22.8 \text{ cm}$$

$$A_\mathrm{c} = h(R_\mathrm{o} - R_\mathrm{i}) = 1.62 \text{ cm}^2$$

Part (b):

$$\mathcal{R}_\mathrm{c} = \frac{l_\mathrm{c}}{\mu A_\mathrm{c}} = 0$$

$$\mathcal{R}_\mathrm{g} = \frac{g}{\mu_0 A_\mathrm{c}} = 7.37 \times 10^6 \text{ H}^{-1}$$

Part (c):

$$L = \frac{N^2}{\mathcal{R}_\mathrm{c} + \mathcal{R}_\mathrm{g}} = 7.04 \times 10^{-4} \text{ H}$$

Part (d):

$$I = \frac{B_\mathrm{g} A_\mathrm{c}(\mathcal{R}_\mathrm{c} + \mathcal{R}_\mathrm{g})}{N} = 20.7 \text{ A}$$

Part (e):

$$\lambda = LI = 1.46 \times 10^{-2} \text{ Wb}$$

Problem 1-14

See solution to Problem 1-13

Part (a):

$$l_c = 22.8 \text{ cm}$$

$$A_c = 1.62 \text{ cm}^2$$

Part (b):

$$\mathcal{R}_c = 1.37 \times 10^6 \text{ H}^{-1}$$

$$\mathcal{R}_g = 7.37 \times 10^6 \text{ H}^{-1}$$

Part (c):

$$L = 5.94 \times 10^{-4} \text{ H}$$

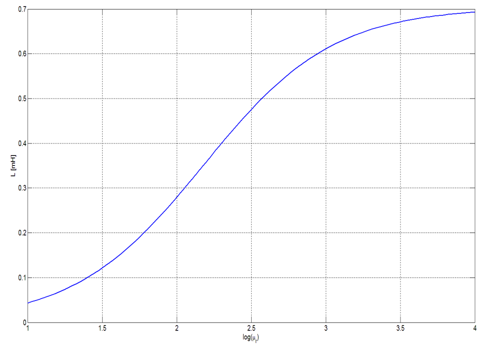
Part (d):

$$I = 24.6 \text{ A}$$

Part (e):

$$\lambda = 1.46 \times 10^{-2} \text{ Wb}$$

Problem 1-15



μ_{r} must be greater than 2886.

Problem 1-16

$$L = \frac{\mu_0 N^2 A_{\mathrm{c}}}{g + l_{\mathrm{c}}/\mu_{\mathrm{r}}}$$

Problem 1-17

Part (a):

$$L = \frac{\mu_0 N^2 A_{\mathrm{c}}}{g + l_{\mathrm{c}}/\mu_{\mathrm{r}}} = 36.6 \text{ mH}$$

Part (b):

$$B = \frac{\mu_0 N^2}{g + l_{\mathrm{c}}/\mu_{\mathrm{r}}} I = 0.77 \text{ T}$$

$$\lambda = LI = 4.40 \times 10^{-2} \text{ Wb}$$

Problem 1-18

Part (a): With $\omega = 120\pi$

$$V_{\rm rms} = \frac{\omega N A_{\rm c} B_{\rm peak}}{\sqrt{2}} = 20.8 \text{ V}$$

Part (b): Using L from the solution to Problem 1-17

$$I_{\rm peak} = \frac{\sqrt{2} V_{\rm rms}}{\omega L} = 1.66 \text{ A}$$

$$W_{\rm peak} = \frac{L I_{\rm peak}^2}{2} = 9.13 \times 10^{-2} \text{ J}$$

Problem 1-19

$B = 0.81$ T and $\lambda = 46.5$ mWb

Problem 1-20

Part (a):

$$R_3 = \sqrt{(R_1^2 + R_2^2)} = 4.49 \text{ cm}$$

Part (b): For

$$l_{\rm c} = 4l + R_2 + R_3 - 2h;$$

and

$$A_{\rm g} = \pi R_1^2$$

$$L = \frac{\mu_0 A_{\rm g} N^2}{g + (\mu_0/\mu) l_{\rm c}} = 61.8 \text{ mH}$$

Part (c): For $B_{\rm peak} = 0.6$ T and $\omega = 2\pi 60$

$$\lambda_{\text{peak}} = A_{\text{g}} N B_{\text{peak}}$$

$$V_{\text{rms}} = \frac{\omega \lambda_{\text{peak}}}{\sqrt{2}} = 23.2 \text{ V}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = 0.99 \text{ A}$$

$$W_{\text{peak}} = \frac{1}{2} L I_{\text{peak}}^2 = \frac{1}{2} L (\sqrt{2} I_{\text{rms}})^2 = 61.0 \text{ mJ}$$

Part (d): For $\omega = 2\pi 50$

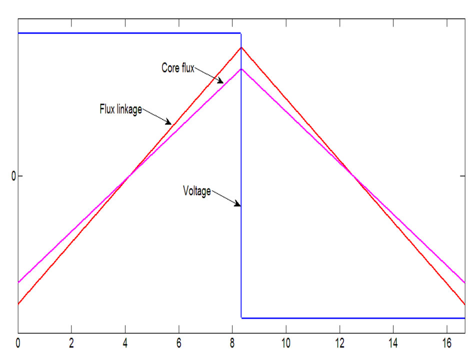
$$V_{\text{rms}} = 19.3 \text{ V}$$

$$I_{\text{rms}} = 0.99 \text{ A}$$

$$W_{\text{peak}} = 61.0 \text{ mJ}$$

Problem 1-21

Part (a);



Part (b):

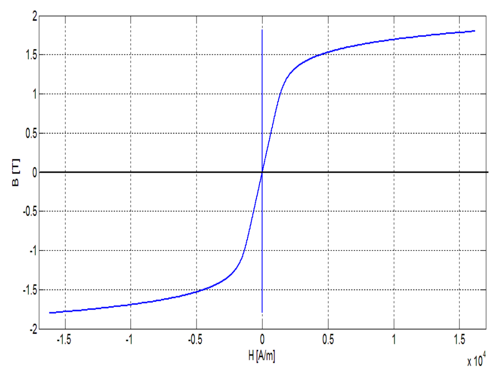
$$E_{\text{max}} = 4fNA_{\text{c}}B_{\text{peak}} = 118\ V$$

part (c): For $\mu = 1000\mu_0$

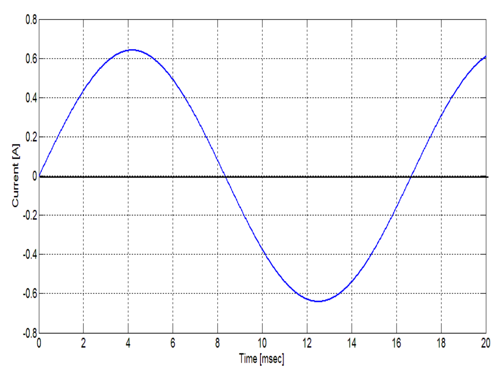
$$I_{\text{peak}} = \frac{l_{\text{c}}B_{\text{peak}}}{\mu N} = 0.46\ \text{A}$$

Problem 1-22

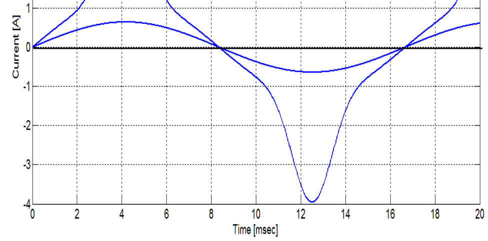
Part (a);



Part (b): $I_{\text{peak}} = 0.6$ A

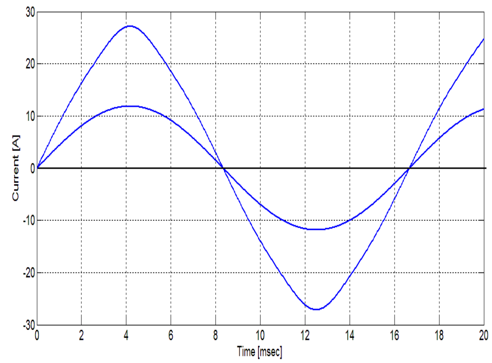


Part (c): $I_{\text{peak}} = 4.0$ A



Problem 1-23

For part (b), $I_{\text{peak}} = 11.9$ A. For part (c), $I_{\text{peak}} = 27.2$ A.



Problem 1-24

$$L = \frac{\mu_0 A_c N^2}{g + (\mu_0/\mu) l_c}$$

$$B_c = \frac{\mu_0 N I}{g + (\mu_0/\mu) l_c}$$

Part (a): For $I = 10$ A, $L = 23$ mH and $B_c = 1.7$ T

$$N = \frac{LI}{A_c B_c} = 225 \text{ turns}$$

$$g = \frac{\mu_0 N I}{B_c} - \frac{\mu_0 l_c}{\mu} = 1.56 \text{ mm}$$

Part (b): For $I = 10$ A and $B_c = B_g = 1.7$ T, from Eq. 3.21

$$W_g = \left(\frac{B_g^2}{2\mu_0}\right) V_g = 1.08 \text{ J}$$

$$W_{\rm core} = \left(\frac{B_{\rm c}^2}{2\mu}\right) V_{\rm g} = 0.072 \text{ J}$$

based upon

$$V_{\rm core} = A_{\rm c} l_{\rm c} \quad V_{\rm g} = A_{\rm c} g$$

Part (c):

$$W_{\rm tot} = W_{\rm g} + W_{\rm core} = 1.15 \text{ J} = \frac{1}{2} L I^2$$

Problem 1-25

$$L_{\rm min} = 3.6 \text{ mH} \quad L_{\rm max} = 86.0 \text{ mH}$$

Problem 1-26

Part (a):

$$N = \frac{LI}{BA_{\rm c}} = 167$$

$$g = \frac{\mu_0 NI}{2BA_{\rm c}} = 0.52 \text{ mm}$$

Part (b):

$$N = \frac{LI}{2BA_{\rm c}} = 84$$

$$g = \frac{\mu_0 NI}{BA_{\rm c}} = 0.52 \text{ mm}$$

Problem 1-27

Part (a):

$$N = \frac{LI}{BA_{\mathrm{c}}} = 167$$

$$g = \frac{\mu_0 NI}{2BA_{\mathrm{c}}} - (\mu_0/\mu)l_{\mathrm{c}} = 0.39 \text{ mm}$$

Part (b):

$$N = \frac{LI}{2BA_{\mathrm{c}}} = 84$$

$$g = \frac{\mu_0 NI}{BA_{\mathrm{c}}} - (\mu_0/\mu)l_{\mathrm{c}} = 0.39 \text{ mm}$$

Problem 1-28

Part (a): $N = 450$ and $g = 2.2$ mm

Part (b): $N = 225$ and $g = 2.2$ mm

Problem 1-29

Part (a):

$$L = \frac{\mu_0 N^2 A}{l} = 11.3 \text{ H}$$

where

$$A = \pi a^2 \quad l = 2\pi r$$

Part (b):

$$W = \frac{B^2}{2\mu_0} \times \text{Volume} = 6.93 \times 10^7 \text{ J}$$

where

$$\text{Volume} = (\pi a^2)(2\pi r)$$

Part (c): For a flux density of 1.80 T,

$$I = \frac{lB}{\mu_0 N} = \frac{2\pi r B}{\mu_0 N} = 6.75 \text{ kA}$$

and

$$V = L\left(\frac{\Delta I}{\Delta t}\right) = 113 \times 10^{-3}\left(\frac{6.75 \times 10^3}{40}\right) = 1.90 \text{ kV}$$

Problem 1-30

Part (a):

$$\text{Copper cross} - \text{sectional area} \equiv A_{\text{cu}} = f_{\text{w}} ab$$

$$\text{Copper volume} = \text{Vol}_{\text{cu}} = f_{\text{w}} b\Big((a + \frac{w}{2})(h + \frac{w}{2}) - wh\Big)$$

Part (b):

$$B = \frac{\mu_0 J_{\text{cu}} A_{\text{cu}}}{g}$$

Part (c):

$$J_{\text{cu}} = \frac{NI}{A_{\text{cu}}}$$

Part (d):

$$P_{\rm diss} = \rho J_{\rm cu}^2 {\rm Vol}_{\rm cu}$$

Part (e):

$$W_{\rm stored} = \frac{B^2}{2\mu_0} \times \text{gap volume} = \frac{\mu_0 J_{\rm cu}^2 A_{\rm cu}^2 wh}{2g}$$

Part (f):

$$\frac{W_{\rm store}}{P_{\rm diss}} = \frac{\frac{1}{2}I^2 L}{I^2 R}$$

and thus

$$\frac{L}{R} = 2\left(\frac{W_{\rm stored}}{P_{\rm diss}}\right) = \frac{\mu_0 A_{\rm cu}^2 wh}{g\rho {\rm Vol}_{\rm cu}}$$

Problem 1-31

$$P_{\rm diss} = 6.20 \text{ W} \quad I = 155 \text{ mA} \quad N = 12,019 \text{ turns}$$

$$R = 258\ \Omega \quad L = 32 \text{ H} \quad \tau = 126 \text{ msec} \quad \text{Wire size} \ = \ 34 \text{ AWG}$$

Problem 1-32

Part (a) (i):

$$B_{\rm g1} = \frac{\mu_0 N_1}{g_1}\, I_1 \qquad B_{\rm g2} = \frac{\mu_0 N_1}{g_2}\, I_1$$

(ii)

$$\lambda_1 = N_1(A_1 B_{\rm g1} + A_2 B_{\rm g2}) = \mu_0 N_1^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right) I_1$$

$$\lambda_2 = N_2 A_2 B_{\mathrm{g}2} = \mu_0 N_1 N_2 \frac{A_2}{g_2} I_1$$

Part (b) (i):

$$B_{\mathrm{g}1} = 0 \qquad B_{\mathrm{g}2} = \frac{\mu_0 N_2}{g_2}\, I_2$$

(ii)

$$\lambda_1 = N_1(A_1 B_{\mathrm{g}1} + A_2 B_{\mathrm{g}2}) = \mu_0 N_1 N_2 \frac{A_2}{g_2} I_2$$

$$\lambda_2 = N_2 A_2 B_{\mathrm{g}2} = \mu_0 N_2^2 \frac{A_2}{g_2} I_2$$

Part (c) (i):

$$B_{\mathrm{g}1} = \frac{\mu_0 N_1}{g_1}\, I_1 \qquad B_{\mathrm{g}2} = \frac{\mu_0 N_1}{g_2}\, I_1 + \frac{\mu_0 N_2}{g_2}\, I_2$$

(ii)

$$\lambda_1 = N_1(A_1 B_{\mathrm{g}1} + A_2 B_{\mathrm{g}2}) = \mu_0 N_1^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right) I_1 + \mu_0 N_1 N_2 \frac{A_2}{g_2} I_2$$

$$\lambda_2 = N_2 A_2 B_{\mathrm{g}2} = \mu_0 N_1 N_2 \frac{A_2}{g_2} I_1 + \mu_0 N_2^2 \frac{A_2}{g_2} I_2$$

Part (d):

$$L_1 = \mu_0 N_1^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right) \qquad L_2 \mu_0 N_2^2 \frac{A_2}{g_2}$$

$$L_{12} = \mu_0 N_1 N_2 \frac{A_2}{g_2}$$

Problem 1-33

$$\mathcal{R}_{\rm g} = \frac{g}{\mu_0 A_{\rm c}} \qquad \mathcal{R}_1 = \frac{l_1}{\mu A_{\rm c}}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu A_{\rm c}} \qquad \mathcal{R}_{\rm A} = \frac{l_A}{\mu A_{\rm c}}$$

Part (a):

$$L_1 = \frac{N_1^2}{\mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_{\rm A}/2}$$

$$L_{\rm A} = L_{\rm B} = \frac{N^2}{\mathcal{R}}$$

where

$$\mathcal{R} = \mathcal{R}_{\rm A} + \frac{\mathcal{R}_{\rm A}(\mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_{\rm A} + \mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2}$$

Part (b):

$$L_{\rm 1B} = -L_{\rm 1A} = \frac{N_1 N}{2(\mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_{\rm A}/2)}$$

$$L_{12} = \frac{N^2(\mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2)}{2\mathcal{R}_{\rm A}(\mathcal{R}_{\rm g} + \mathcal{R}_1 + \mathcal{R}_2) + \mathcal{R}_{\rm A}^2}$$

Part (c):

$$v_1(t) = L_{1\text{A}}\frac{di_{\text{A}}}{dt} + L_{1\text{B}}\frac{di_{\text{B}}}{dt} = L_{1\text{A}}\frac{d(i_{\text{A}} - i_{\text{B}})}{dt}$$

Problem 1-34

Part (a):

$$L_{12} = \frac{\mu_0 N_1 N_2 D(w - x)}{2g}$$

Part (b):

$$v_2(t) = -\omega I_o \left(\frac{\mu_0 N_1 N_2 D w \epsilon}{4g}\right) \cos \omega t$$

Problem 1-35

Part (a):

$$H = \frac{2N_1 i_1}{(R_o + R_i)}$$

Part (b):

$$v_2(t) = N_2 w(n\Delta)\frac{dB(t)}{dt}$$

Part (c):

$$v_0(t) = GN_2 w(n\Delta)B(t)$$

Problem 1-36

Must have

$$\left(\frac{\mu_0}{\mu}\right) l_\mathrm{c} < 0.05\left(g + \left(\frac{\mu_0}{\mu}\right) l_\mathrm{c}\right) \;\Rightarrow\; \mu = \frac{B}{H} > 19\mu_0\left(\frac{l_\mathrm{c}}{g}\right)$$

For $g = 0.05$ cm and $l_\mathrm{c} = 30$ cm, must have $\mu > 0.014$. This is satisfied over the approximate range $0.65\ \mathrm{T} \le B \le 1.65\ \mathrm{T}$.

Problem 1-37

Part (a): See Problem 1-35. For the given dimensions, $V_\mathrm{peak} = 20$ V, $B_\mathrm{peak} = 1$ T and $\omega = 100\pi$ rad/sec

$$N_1 = \frac{V_\mathrm{peak}}{\omega(R_\mathrm{o} - R_\mathrm{i})(n\Delta)} = 79 \text{ turns}$$

Part (b): (i)

$$B_\mathrm{peak} = \frac{V_{0,peak}}{GN_2(R_0 - R_\mathrm{i})(n\Delta)} = 0.83 \text{ T}$$

(ii)

$$V_\mathrm{peak} = \omega N_1(R_\mathrm{o} - R_\mathrm{i})(n\Delta)B_\mathrm{peak} = 9.26 \text{ V}$$

Problem 1-38

Part (a): From the M-5 dc-magnetization characteristic, $H_\mathrm{c} = 19$ A-turns/m at $B_\mathrm{c} = B_\mathrm{g} = 1.3$ T. For $H_\mathrm{g} = 1.3\ \mathrm{T}/\mu_0 = 1.03 \times 10^6$ A-turns/m

$$I = \frac{H_\mathrm{c}(l_\mathrm{A} + l_\mathrm{C} - g) + H_\mathrm{g} g}{N_1} = 30.2\ A$$

Part(b):

$$W_{\text{gap}} = gA_{\text{C}}\left(\frac{B_{\text{g}}^2}{2\mu_0}\right) = 3.77 \text{ J}$$

For $\mu = B_{\text{c}}/H_{\text{c}} = 0.0684$ H/m

$$W_{\text{c}} = (l_{\text{A}}A_{\text{A}} + l_{\text{B}}A_{\text{B}} + (l_{\text{C}} - g)A_{\text{C}})\left(\frac{B_{\text{c}}^2}{2\mu}\right) = 4.37 \times 10^{-3} \text{ J}$$

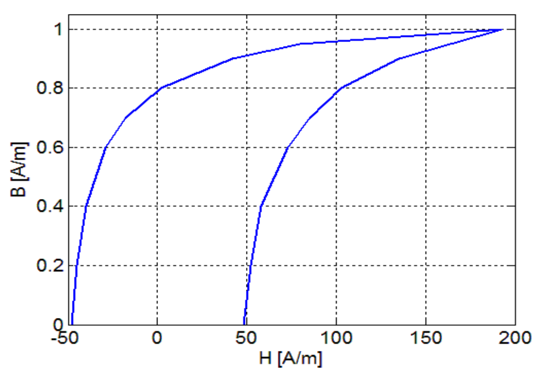
$$L = \frac{2(W_{\text{gap}} + W_{\text{c}})}{I^2} = 8.26 \text{ mH}$$

Part (c):

$$L = \frac{2W_{\text{gap}}}{I^2} = 8.25 \text{ mH}$$

Problem 1-39

Part (a):



Part (b): Loop area = 191 J/m^3

Part (c):

$$\text{Core loss} = \frac{f \times \text{Loop area}}{\rho}$$

For $f = 60$ Hz, $\rho = 7.65 \times 10^3$ kg/m^3, Core loss = 1.50 W/kg

Problem 1-40

$B_{\text{rms}} = 1.1$ T and $f = 60$ Hz,

$$V_{\text{rms}} = \omega N A_{\text{c}} B_{\text{rms}} = 86.7 \quad \text{V}$$

Core volume $= A_{\text{c}} l_{\text{c}} = 1.54 \times 10^{-3}$ m^3. Mass density $= 7.65 \times 10^3$ kg/m^3. Thus, the core mass $= (1.54 \times 10^{-3})(7.65 \times 10^3) = 11.8$ kg.

At $B = 1.1$ T rms $= 1.56$ T peak, core loss density = 1.3 W/kg and rms VA density is 2.0 VA/kg. Thus, the core loss $= 1.3 \times 11.8 = 15.3$ W. The total exciting VA for the core is $2.0 \times 11.8 = 23.6$ VA. Thus, its reactive component is given by $\sqrt{23.6^2 - 15.3^2} = 17.9$ VAR.

The rms energy storage in the air gap is

$$W_{\text{gap}} = \frac{g A_{\text{c}} B_{\text{rms}}^2}{\mu_0} = 5.08 \quad \text{J}$$

corresponding to an rms reactive power of

$$\text{VAR}_{\text{gap}} = \omega W_{\text{gap}} = 1917 \quad \text{VAR}$$

Thus, the total rms exciting VA for the magnetic circuit is

$$\text{VA}_{\text{rms}} = \sqrt{15.3^2 + (1917 + 17.9)^2} = 1935 \quad \text{VA}$$

and the rms current is $I_{\text{rms}} = \text{VA}_{\text{rms}}/V_{\text{rms}} = 22.3$ A.

Problem 1-41

Part(a): Area increases by a factor of 4. Thus the voltage increases by a factor of 4 to $e = 1096\ cos(377t)$.

Part (b): l_c doubles therefore so does the current. Thus $I = 0.26$ A.

Part (c): Volume increases by a factor of 8 and voltage increases by a factor of 4. There $I_{\phi,\mathrm{rms}}$ doubles to 0.20 A.

Part (d): Volume increases by a factor of 8 as does the core loss. Thus $P_c = 128$ W.

Problem 1-42

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B = 0.47$ T and $H =$ -360 kA/m. Thus the maximum energy product is 1.69×10^5 J/m^3.

Thus,

$$A_\mathrm{m} = \left(\frac{0.8}{0.47}\right) 2\ \mathrm{cm}^2 = 3.40\ \mathrm{cm}^2$$

and

$$l_\mathrm{m} = -0.2\ \mathrm{cm}\ \left(\frac{0.8}{\mu_0(-3.60 \times 10^5)}\right) = 0.35\ \mathrm{cm}$$

Thus the volume is $3.40 \times 0.35 = 1.20$ cm^3, which is a reduction by a factor of 5.09/1.21 = 4.9.

Problem 1-43

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately) $B = 0.63$ T and $H =$ -470 kA/m. Thus the maximum energy product is 2.90×10^5 J/m^3.

Thus,

$$A_\mathrm{m} = \left(\frac{0.8}{0.63}\right) 2\ \mathrm{cm}^2 = 2.54\ \mathrm{cm}^2$$

and

$$l_{\rm m} = -0.2\,{\rm cm}\,\left(\frac{0.8}{\mu_0(-4.70\times10^5)}\right) = 0.27\,{\rm cm}$$

Thus the volume is $2.54\times0.25 = 0.688$ cm^3, which is a reduction by a factor of $5.09/0.688 = 7.4$.

Problem 1-44

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B = 0.47$ T and $H =$ -360 kA/m. Thus the maximum energy product is 1.69×10^5 J/m^3. Thus, we want $B_{\rm g} = 1.3$ T, $B_{\rm m} = 0.47$ T and $H_{\rm m} = -360$ kA/m.

$$h_{\rm m} = -g\left(\frac{H_{\rm g}}{H_{\rm m}}\right) = -g\left(\frac{B_{\rm g}}{\mu_0 H_{\rm m}}\right) = 2.87 \quad {\rm mm}$$

$$A_{\rm m} = A_{\rm g}\left(\frac{B_{\rm g}}{B_{\rm m}}\right) = 2\pi Rh\left(\frac{B_{\rm g}}{B_{\rm m}}\right) = 45.1 \quad {\rm cm}^2$$

$$R_{\rm m} = \sqrt{\frac{A_{\rm m}}{\pi}} = 3.66 \quad {\rm cm}$$

Problem 1-45

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately) $B = 0.63$ T and $H =$ -482 kA/m. Thus the maximum energy product is 3.03×10^5 J/m^3. Thus, we want $B_{\rm g} = 1.3$ T, $B_{\rm m} = 0.47$ T and $H_{\rm m} = -360$ kA/m.

$$h_{\rm m} = -g\left(\frac{H_{\rm g}}{H_{\rm m}}\right) = -g\left(\frac{B_{\rm g}}{\mu_0 H_{\rm m}}\right) = 2.15 \quad {\rm mm}$$

$$A_{\rm m} = A_{\rm g}\left(\frac{B_{\rm g}}{B_{\rm m}}\right) = 2\pi Rh\left(\frac{B_{\rm g}}{B_{\rm m}}\right) = 31.3 \quad {\rm cm}^2$$

$$R_{\mathrm{m}} = \sqrt{\frac{A_{\mathrm{m}}}{\pi}} = 3.16 \quad \mathrm{cm}$$

Problem 1-46

For $B_{\mathrm{m}} = \mu_R(H_{\mathrm{m}} - H_{\mathrm{c}})$, the maximum value of the product $-B_{\mathrm{m}}H_{\mathrm{m}}$ occurs at $H_{\mathrm{m}} = H_{\mathrm{c}}/2$ and the value is $B_{\mathrm{r}}^2/4\mu_{\mathrm{R}}$.

T [C]	$-(B_{\mathrm{m}}H_{\mathrm{m}})_{\max}$ [kJ/m^3]	Corresponding H_{m} [kA/m]	$B_{\mathrm{m}}[T]$
20	253.0	-440.0	0.57
60	235.7	-424.7	0.56
80	223.1	-413.2	0.54
150	187.5	-378.8	0.50
180	169.0	-359.6	0.47
210	151.5	-340.5	0.45

Problem 1-47

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately) $B_{\mathrm{m}} = 0.63$ T and $H_{\mathrm{m}} =$ -470 kA/m. The magnetization curve for neodymium-iron-boron can be represented as

$$B_{\mathrm{m}} = \mu_{\mathrm{R}}H_{\mathrm{m}} + B_{\mathrm{r}}$$

where $B_{\mathrm{r}} = 1.26$ T and $\mu_R = 1.067\mu_0$. The magnetic circuit must satisfy

$$H_{\mathrm{m}}d + H_{\mathrm{g}}g = Ni; \quad B_{\mathrm{m}}A_{\mathrm{m}} = B_{\mathrm{g}}A_{\mathrm{g}}$$

part (a): For $i = 0$ and $B_{\mathrm{g}} = 0.6$ T, the minimum magnet volume will occur when the magnet is operating at the maximum energy point.

$$A_{\mathrm{m}} = \left(\frac{B_{\mathrm{g}}}{B_{\mathrm{m}}}\right) A_{\mathrm{g}} = 6.67 \quad \mathrm{cm}^2$$

$$d = -\left(\frac{H_{\mathrm{g}}}{H_{\mathrm{m}}}\right) g = 3.47 \quad \mathrm{mm}$$

part (b): Want $B_{\rm g} = 0.8$ T when $i = I_{\rm peak}$

$$I_{\rm peak} = \frac{\left[B_{\rm g}\left(\frac{dA_{\rm g}}{\mu_{\rm R}A_{\rm m}} + \frac{g}{\mu_0}\right) - \frac{B_{\rm r}d}{\mu_{\rm R}}\right]}{N} = 6.37 \text{ A}$$

Because the neodymium-iron-boron magnet is essentially linear over the operating range of this problem, the system is linear and hence a sinusoidal flux variation will correspond to a sinusoidal current variation.

Problem 1-48

Part (a): From the solution to Problem 1-46, the maximum energy product for neodymium-iron-boron at 180 C occurs at (approximately) $B_{\rm m} = 0.47$ T and $H_{\rm m} =$ -360 kA/m. The magnetization curve for neodymium-iron-boron can be represented as

$$B_{\rm m} = \mu_{\rm R}H_{\rm m} + B_{\rm r}$$

where $B_{\rm r} = 0.94$ T and $\mu_R = 1.04\mu_0$. The magnetic circuit must satisfy

$$H_{\rm m}d + H_{\rm g}g = 0; \quad B_{\rm m}A_{\rm m} = B_{\rm g}A_{\rm g}$$

For $B_{\rm g} = 0.8$ T, the minimum magnet volume will occur when the magnet is operating at the maximum energy point.

$$A_{\rm m} = \left(\frac{B_{\rm g}}{B_{\rm m}}\right)A_{\rm g} = 15.3 \quad \text{cm}^2$$

$$d = -\left(\frac{H_{\rm g}}{H_{\rm m}}\right)g = 5.66 \quad \text{mm}$$

Part(b): At 60 C, $B_{\rm r} = 1.12$ T. Combining

$$B_{\rm m} = \mu_{\rm R}H_{\rm m} + B_{\rm r}$$

$$H_{\rm m}d + H_{\rm g}g = 0$$

$$A_{\mathrm{m}}B_{\mathrm{m}} = A_{\mathrm{g}}A_{\mathrm{g}} = \mu_0 H_{\mathrm{g}} A_{\mathrm{g}}$$

gives

$$B_{\mathrm{g}} = \left(\frac{\mu_0 d A_{\mathrm{m}}}{\mu_0 d A_{\mathrm{g}} + \mu_{\mathrm{R}} g A_{\mathrm{m}}}\right) B_{\mathrm{r}} = 0.95 \text{ T}$$

PROBLEM SOLUTIONS: Chapter 2

Problem 2-1

At 60 Hz, $\omega = 120\pi$.

$$\text{primary:} \quad (V_{\text{rms}})_{\text{max}} = N_1 \omega A_{\text{c}} (B_{\text{rms}})_{\text{max}} = 3520 \quad \text{V, rms}$$

$$\text{secondary:} \quad (V_{\text{rms}})_{\text{max}} = N_2 \omega A_{\text{c}} (B_{\text{rms}})_{\text{max}} = 245 \quad \text{V, rms}$$

At 50 Hz, $\omega = 100\pi$. Primary voltage is 2934 V, rms and secondary voltage is 204 V, rms.

Problem 2-2

$$N = \frac{\sqrt{2} V_{\text{rms}}}{\omega A_c B_{\text{peak}}} = 147 \quad \text{turns}$$

Problem 2-3

$$N = \sqrt{\frac{300}{75}} = 2 \quad \text{turns}$$

Problem 2-4

Part (a):

$$R1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = 9.38\ \Omega \quad I_1 = \frac{V_1}{I_1} = 1.28\ \text{A}$$

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1 = 48\ \text{V} \quad P_2 = \frac{V_2^2}{R_2} = 14.6\ \text{W}$$

Part (b): For $\omega = 2\pi f = 6.28 \times 10^3\ \Omega$

$$X_1 = \omega L = 2.14\ \Omega \quad I_1 = \left|\frac{V_1}{R_1 + jX_1}\right| = 1.25\ \text{A}$$

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1 = 0.312\ \text{A} \quad V_2 = I_2 R_2 = 46.8\ \text{V}$$

$$P_2 = V_2 I_2 = 14.6\ \text{W}$$

Problem 2-5

For $\omega = 100]\pi$, $Z_{\text{L}} = R_{\text{L}} + j\omega L = 5.0 + j0.79\ \Omega$

$$V_{\text{L}} = 110\left(\frac{20}{120}\right) = 18.3\ \text{V} \quad I_{\text{L}} = \left|\frac{V_{\text{L}}}{I_{\text{L}}}\right| = 3.6\ \text{A}$$

and

$$I_{\text{H}} = I_{\text{L}}\left(\frac{120}{20}\right) = 604\ \text{mA}$$

Problem 2-6

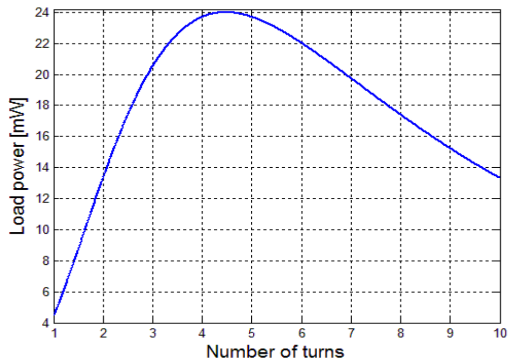
The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source (1.5 kΩ). Thus the transformer turns ratio N to give maximum power must be

$$N = \sqrt{\frac{R_{\text{s}}}{R_{\text{load}}}} = 4.47$$

Under these conditions, the source voltage will see a total resistance of $R_{\text{tot}} = 3\ \text{k}\Omega$ and the source current will thus equal $I = V_{\text{s}}/R_{\text{tot}} = 4$ mA. Thus, the power delivered to the load will equal

$$P_{\text{load}} = I^2(N^2 R_{\text{load}}) = 24\ \text{mW}$$

Here is the desired MATLAB plot:



Problem 2-7

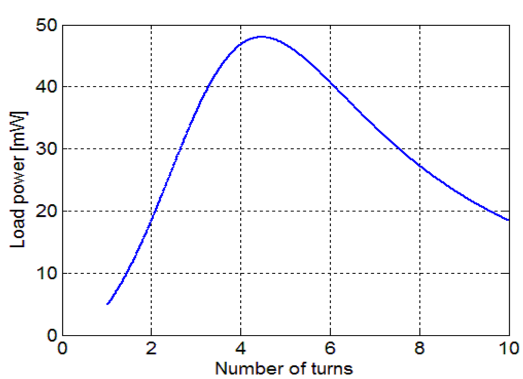
The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source (1.5 kΩ). Thus the transformer turns ratio N to give maximum power must be

$$N = \sqrt{\frac{X_s}{R_{\text{load}}}} = 4.47$$

Under these conditions, the source voltage will see a total impedance of $Z_{\text{tot}} = 1.5 + j1.5$ kΩ whose magnitude is $1.5\sqrt{2}$ kΩ. The current will thus equal $I = V_s/|Z_{\text{tot}}| = 5.7\sqrt{2}$ mA. Thus, the power delivered to the load will equal

$$P_{\text{load}} = I^2(N^2 R_{\text{load}}) = 48 \quad \text{mW}$$

Here is the desired MATLAB plot:



Problem 2-8

$$V_2 = V_1 \left(\frac{X_{\rm m}}{X_{\rm l_1} + X_{\rm m}} \right) = V_1 \left(\frac{L_{\rm m}}{L_{\rm l_1} + L_{\rm m}} \right) = 119.86 \quad \rm V$$

Problem 2-9

Part (a): Referred to the secondary

$$L_{\rm m,2} = \frac{L_{\rm m,1}}{N^2} = 37.9 \quad \rm mH$$

Part(b): Referred to the secondary, $X_{\rm m} = \omega L_{\rm m,2} = 14.3\,\Omega$, $X_2 = 16.6$ mΩ and $X_{\rm l} = 16.5$ mΩ. Thus,

$$\text{(i)} \quad V_1 = N \left(\frac{X_m}{X_m + X_2} \right) V_2 = 7961 \quad \rm V$$

and

$$\text{(ii)} \quad I_{\rm sc} = \frac{V_2}{X_{\rm sc}} = \frac{V_2}{X_2 + X_m || X_1} = 3629 \quad \rm A$$

Problem 2-10

Part (a):

$$I_1 = \frac{V_1}{X_{\rm l_1} + X_{\rm m}} = 4.98 \quad \text{A}; \qquad V_2 = N V_1 \left(\frac{X_{\rm m}}{X_{\rm l_1} + X_{\rm m}} \right) = 6596 \quad \rm V$$

Part (b): Let $X'_{\rm l_2} = X_{\rm l_2}/N^2$ and $X_{\rm sc} = X_{\rm l_1} + X_{\rm m} || (X_m + X'_{\rm l_2})$. For $I_{\rm rated} = 45\,\text{kVA}/230\text{ V} = 196$ A

$$V_1 = I_{\rm rated} X_{\rm sc} = 11.4 \quad \rm V$$

$$I_2 = \frac{1}{N} \left(\frac{X_{\rm m}}{X_{\rm m} + X_{\rm l_2}} \right) I_{\rm rated} = 6.81 \quad \rm A$$

Problem 2-11

Part (a): At 60 Hz, all reactances increase by a factor of 1.2 over their 50-Hz values. Thus

$$X_{\mathrm{m}} = 55.4\ \Omega \qquad X_{\mathrm{l,1}} = 33.4\ \mathrm{m}\Omega \qquad X_{\mathrm{l,2}} = 30.4\ \Omega$$

Part (b): For $V_1 = 240$ V

$$I_1 = \frac{V1}{X_1 + X_{\mathrm{m}}} = 4.33\ \mathrm{A} \quad V_2 = NV_1\left(\frac{X_{\mathrm{m}}}{X_{\mathrm{m}} + X_{\mathrm{l,1}}}\right) = 6883\ \mathrm{V}$$

Problem 2-12

The load voltage as referred to the high-voltage side is $V_{\mathrm{L}}' = 447 \times 2400/460 = 2332$ V. Thus the load current as referred to the high voltage side is

$$I_{\mathrm{L}}' = \frac{P_{\mathrm{L}}}{V_{\mathrm{L}}'} = 18.0\ \mathrm{A}$$

and the voltage at the high voltage terminals is

$$V_{\mathrm{H}} = |V_{\mathrm{L}}' + jX_{\mathrm{l,1}}I_{\mathrm{L}}'| = 2347\ \mathrm{V}$$

and the power factor is

$$\mathrm{pf} = \frac{P_{\mathrm{L}}}{V_{\mathrm{H}}I_{\mathrm{L}}'} = 0.957\ \mathrm{lagging}$$

here we know that it is lagging because the transformer is inductive.

Problem 2-13

At 50 Hz, $X_l = 39.3 \times (5/6) = 32.8\ \Omega$. The load voltage as referred to the high-voltage side is $V_\mathrm{L}' = 362 \times 2400/460 = 1889$ V. Thus the load current as referred to the high voltage side is

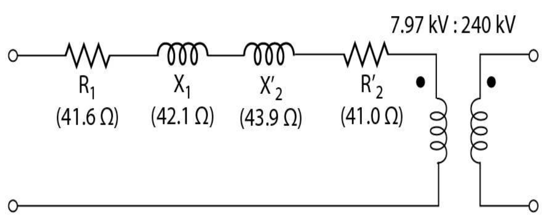
$$I_\mathrm{L}' = \frac{P_\mathrm{L}}{V_\mathrm{L}'} = 18.3 \text{ A}$$

and the voltage at the high voltage terminals is

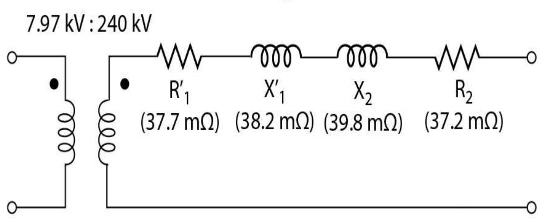
$$V_\mathrm{H} = |V_\mathrm{L}' + jX_{l,1} I_\mathrm{L}'| = 1981 \text{ V}$$

Problem 2-14

Part (a):



(i)



(ii)

Part (b):

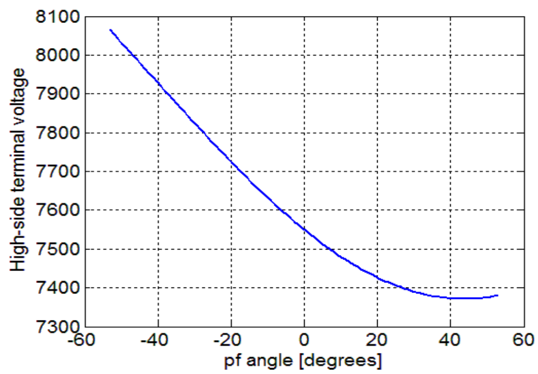
$$\hat{I}_\mathrm{load} = \frac{40 \text{ kW}}{240 \text{ V}}\, e^{j\phi} = 166.7\, e^{j\phi} \text{ A}$$

where ϕ is the power-factor angle. Referred to the high voltage side, $\hat{I}_\mathrm{H} = 5.02\, e^{j\phi}$ A.

$$\hat{V}_{\mathrm{H}} = Z_{\mathrm{H}} \hat{I}_{\mathrm{H}}$$

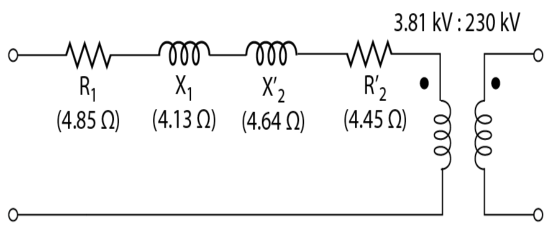
Thus, (i) for a power factor of 0.87 lagging, $V_{\mathrm{H}} = 7820$ V and (ii) for a power factor of 0.87 leading, $V_{\mathrm{H}} = 7392$ V.

part (c):

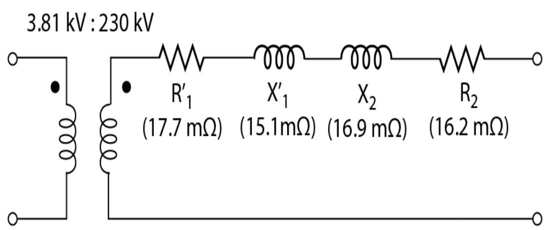


Problem 2-15

Part (a):



(i)



(ii)

Part (b):

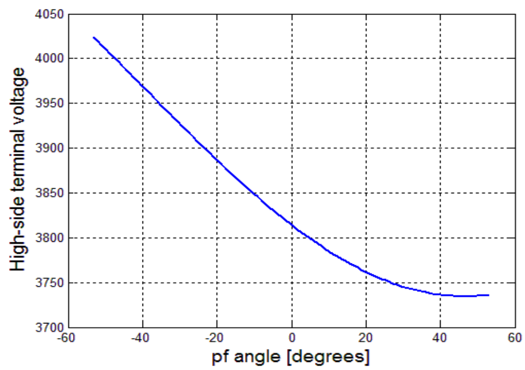
$$\hat{I}_{\rm load} = \frac{40\ \text{kW}}{240\ \text{V}}\, e^{j\phi} = 326.1\, e^{j\phi}\ \text{A}$$

where ϕ is the power-factor angle. Referred to the high voltage side, $\hat{I}_{\rm H} = 19.7\, e^{j\phi}$ A.

$$\hat{V}_{\rm H} = Z_{\rm H}\hat{I}_{\rm H}$$

Thus, (i) for a power factor of 0.87 lagging, $V_{\rm H} = 3758$ V and (ii) for a power factor of 0.87 leading, $V_{\rm H} = 3570$ V.

part (c):



Problem 2-16

Part (a): $\hat{I}_{\rm load} = (178/.78)$ kVA/2385 V = 72.6 A at $\angle = \cos^{-1}(0.78) = 38.7°$

$$\hat{V}_{\rm t,H} = N(\hat{V}_{\rm L} + Z_{\rm t}\,\hat{I}_{\rm load}) = 34.1\ \text{kV}$$

Part (b):

$$\hat{V}_{\rm send} = N(\hat{V}_{\rm L} + (Z_{\rm t} + Z_{\rm f})\,\hat{I}_{\rm load}) = 33.5\ \text{kV}$$

Part (c): $\hat{I}_{\rm send} = \hat{I}_{\rm load}/N$ and

$$S_{\rm send} = P_{\rm send} + jQ_{\rm send} = \hat{V}_{\rm send}\hat{I}^*_{\rm send} = 138\ \text{kW} - j93.4\ \text{kVAR}$$

Thus $P_{\text{send}} = 138$ kW and $Q_{\text{send}} = -93.4$ kVAR.

Problem 2-17

Part (a):

pf = 0.78 leading:

part (a): $V_{\text{t,H}} = 34.1$ kV
part (b): $V_{\text{send}} = 33.5$ kV
part (c): $P_{\text{send}} = 138.3$ kW, $Q_{\text{send}} =$ -93.4 kVA

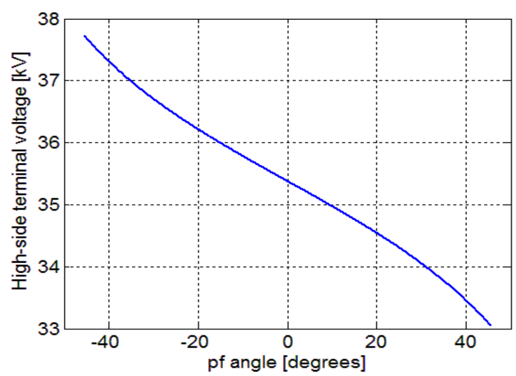
pf = unity:

part (a): $V_{\text{t,H}} = 35.0$ kV
part (b): $V_{\text{send}} = 35.4$ kV
part (c): $P_{\text{send}} = 137.0$ kW, $Q_{\text{send}} = 9.1$ kVA

pf = 0.78 lagging:

part (a): $V_{\text{t,H}} = 35.8$ kV
part (b): $V_{\text{send}} = 37.2$ kV
part (c): $P_{\text{send}} = 138.335$ kW, $Q_{\text{send}} = 123.236$ kVA

Part (b):



Problem 2-18

Following the methodology of Example 2.6, efficiency = 98.4 percent and regulation = 1.25 percent.

Problem 2-19

Part (a): The core cross-sectional area increases by a factor of two thus the primary voltage must double to 22 kV to produce the same core flux density.

Part (b): The core volume increases by a factor of $2\sqrt{2}$ and thus the excitation kVA must increase by the same factor which means that the current must increase by a factor of $\sqrt{2}$ to 0.47 A and the power must increase by a factor of $2\sqrt{2}$ to 7.64 kW.

Problem 2-20

Part (a):

$$|Z_{\mathrm{eq,H}}| = \frac{V_{\mathrm{sc,H}}}{I_{\mathrm{sc,H}}} = 14.1\ \Omega$$

$$R_{\mathrm{eq,H}} = \frac{P_{\mathrm{sc,H}}}{I_{\mathrm{sc,H}}^2} = 752\ \mathrm{m}\Omega$$

$$X_{\mathrm{eq,H}} = \sqrt{|Z_{\mathrm{eq,H}}|^2 - R_{\mathrm{eq,H}}^2} = 14.1\ \Omega$$

and thus

$$Z_{\mathrm{eq,H}} = 0.75 + j14.1\ \Omega$$

Part (b): With $N = 78/8 = 9.75$

$$R_{\mathrm{eq,L}} = \frac{R_{\mathrm{eq,H}}}{N^2} = 7.91\ \mathrm{m}\Omega$$

$$X_{\mathrm{eq,L}} = \frac{X_{\mathrm{eq,H}}}{N^2} = 148\ \mathrm{m}\Omega$$

and thus

$$Z_{\mathrm{eq,L}} = 7.9 + j148\ \mathrm{m}\Omega$$

Part (c): From the open-circuit test, the core-loss resistance and the magnetizing reactance as referred to the low-voltage side can be found:

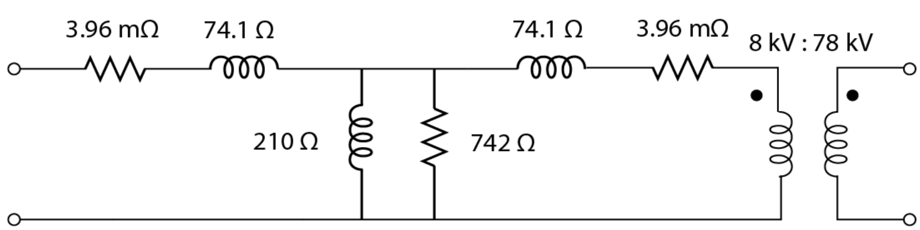
$$R_{\mathrm{c,L}} = \frac{V_{\mathrm{oc,L}}^2}{P_{\mathrm{oc,L}}} = 742\ \Omega$$

$$S_{\mathrm{oc,L}} = V_{\mathrm{oc,L}} I_{\mathrm{oc,L}} = 317\ \mathrm{kVA}; \quad Q_{\mathrm{oc,L}} = \sqrt{S_{\mathrm{oc,L}}^2 - P_{\mathrm{oc,L}}^2} = 305\ \mathrm{kVAR}$$

and thus

$$X_{\mathrm{m,L}} = \frac{V_{\mathrm{oc,L}}^2}{Q_{\mathrm{oc,L}}} = 210\ \Omega$$

The equivalent-T circuit for the transformer from the low-voltage side is thus:



Problem 2-21

Part (a):

$$|Z_{\rm eq,H}| = \frac{V_{\rm sc,H}}{I_{\rm sc,H}} = 14.1\ \Omega$$

$$R_{\rm eq,H} = \frac{P_{\rm sc,H}}{I_{\rm sc,H}^2} = 752\ {\rm m}\Omega$$

$$X_{\rm eq,H} = \sqrt{|Z_{\rm eq,H}|^2 - R_{\rm eq,H}^2} = 14.1\ \Omega$$

and thus

$$Z_{\rm eq,H} = 0.75 + j14.1\ \Omega$$

Part (b): With $N = 78/8 = 9.75$

$$R_{\rm eq,L} = \frac{R_{\rm eq,H}}{N^2} = 7.91\ {\rm m}\Omega$$

$$X_{\rm eq,L} = \frac{X_{\rm eq,H}}{N^2} = 148\ {\rm m}\Omega$$

and thus

$$Z_{\rm eq,L} = 7.9 + j148\ {\rm m}\Omega$$

Part (c): From the open-circuit test, the core-loss resistance and the magnetizing reactance as referred to the low-voltage side can be found:

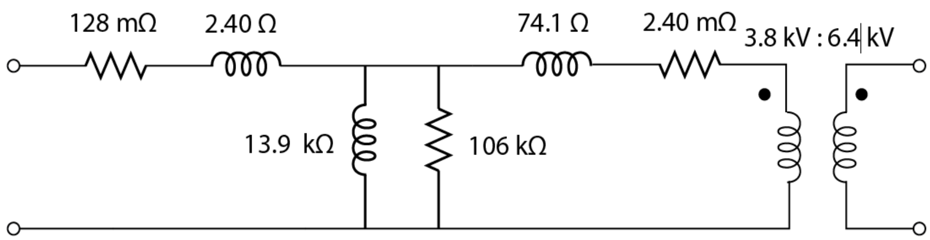
$$R_{\rm c,L} = \frac{V_{\rm oc,L}^2}{P_{\rm oc,L}} = 742\ \Omega$$

$$S_{\text{oc,L}} = V_{\text{oc,L}} I_{\text{oc,L}} = 317 \text{ kVA}; \quad Q_{\text{oc,L}} = \sqrt{S_{\text{oc,L}}^2 - P_{\text{oc,L}}^2} = 305 \text{ kVAR}$$

and thus

$$X_{\text{m,L}} = \frac{V_{\text{oc,L}}^2}{Q_{\text{oc,L}}} = 210 \ \Omega$$

The equivalent-T circuit for the transformer from the low-voltage side is thus:



Problem 2-22

Parts (a) & (b): For $V_{\text{oc,L}} = 7.96$ kV, $I_{\text{oc,L}} = 17.3$ A and $P_{\text{oc,L}} = 48$ kW

$$R_{\text{c,L}} = \frac{V_{\text{oc,L}}^2}{P_{\text{oc,L}}} = 1.32 \text{ k}\Omega \qquad R_{\text{c,H}} = N^2 R_{\text{c,L}} = 33.0 \text{ k}\Omega$$

$$Q_{\text{oc,L}} = \sqrt{S_{\text{oc,L}}^2 - P_{\text{oc,L}}^2} = \sqrt{(V_{\text{oc,L}} I_{\text{oc,L}} - P_{\text{oc,L}}^2)^2} = 129 \text{ kVAR}$$

$$X_{\text{m,L}} = \frac{V_{\text{oc,L}}^2}{Q_{\text{oc,L}}} = 491 \ \Omega \qquad X_{\text{m,H}} = N^2 X_{\text{m,L}} = 12.3 \text{ k}\Omega$$

For $V_{\text{sc,H}} = 1.92$ kV, $I_{\text{oc,L}} = 252$ A and $P_{\text{oc,L}} = 60.3$ kW

$$R_{\text{H}} = \frac{P_{\text{sc,H}}}{I_{\text{sc,H}}^2} = 950 \text{ m}\Omega$$

$$X_{\rm H} = \sqrt{Z_{\rm H}^2 - R_{\rm H}^2} = \sqrt{(V_{\rm sc,H}/I_{\rm sc,H})^2 - R_{\rm H}^2} = 7.56\ \Omega$$

$$R_{\rm L} = \frac{R_{\rm H}}{N^2} = 38.0\ {\rm m}\Omega \qquad X_{\rm L} = \frac{X_{\rm H}}{N^2} = 302\ {\rm m}\Omega$$

Part (c):

$$P_{\rm diss} = P_{\rm oc,L} + P_{\rm sc,H} = 108\ {\rm kW}$$

Problem 2-23

Parts (a) & (b): For $V_{\rm oc,L} = 3.81$ kV, $I_{\rm oc,L} = 9.86$ A and $P_{\rm oc,L} = 8.14$ kW

$$R_{\rm c,L} = \frac{V_{\rm oc,L}^2}{P_{\rm oc,L}} = 1.78\ {\rm k}\Omega \qquad R_{\rm c,H} = N^2 R_{\rm c,L} = 44.8\ {\rm k}\Omega$$

$$Q_{\rm oc,L} = \sqrt{S_{\rm oc,L}^2 - P_{\rm oc,L}^2} = \sqrt{(V_{\rm oc,L} I_{\rm oc,L} - P_{\rm oc,L}^2)^2} = 36.7\ {\rm kVAR}$$

$$X_{\rm m,L} = \frac{V_{\rm oc,L}^2}{Q_{\rm oc,L}} = 395\ \Omega \qquad X_{\rm m,H} = N^2 X_{\rm m,L} = 9.95\ {\rm k}\Omega$$

For $V_{\rm sc,H} = 920$ V, $I_{\rm oc,L} = 141$ A and $P_{\rm oc,L} = 10.3$ kW

$$R_{\rm H} = \frac{P_{\rm sc,H}}{I_{\rm sc,H}^2} = 518\ {\rm m}\Omega$$

$$X_{\rm H} = \sqrt{Z_{\rm H}^2 - R_{\rm H}^2} = \sqrt{(V_{\rm sc,H}/I_{\rm sc,H})^2 - R_{\rm H}^2} = 6.50\ \Omega$$

$$R_{\rm L} = \frac{R_{\rm H}}{N^2} = 20.6\ {\rm m}\Omega \qquad X_{\rm L} = \frac{X_{\rm H}}{N^2} = 259\ {\rm m}\Omega$$

Part (c):

$$P_{\text{diss}} = P_{\text{oc,L}} + P_{\text{sc,H}} = 18.4 \text{ kW}$$

Problem 2-24

Solution the same as Problem 2-22

Problem 2-25

Part (a): 7.69 kV:79.6 kV, 10 MVA

Part (b): 17.3 A, 48.0 kW

Part (c): Since the number of turns on the high-voltage side have doubled, this will occur at a voltage equal to twice that of the original transformer, i.e. 3.84 kV.

Part (d): The equivalent-circuit parameters referred to the low-voltage side will be unchanged from those of Problem 2-22. Those referred to the high-voltage side will have 4 times the values of Problem 2-22.

$$R_{\text{c,L}} = 1.32 \text{ k}\Omega \qquad R_{\text{c,H}} = 132 \text{ k}\Omega$$

$$X_{\text{m,L}} = 491\ \Omega \qquad X_{\text{m,H}} = 49.1 \text{ k}\Omega$$

$$R_{\text{L}} = 38.0 \text{ m}\Omega \qquad R_{\text{H}} = 3.80\ \Omega$$

$$X_{\text{L}} = 302 \text{ m}\Omega \qquad X_{\text{H}} = 30.2\ \Omega$$

Problem 2-26

Part (a): Under this condition, the total transformer power dissipation is 163.7 kW. Thus the efficiency is

$$\eta = 100 \times \frac{25\ \text{MW}}{25\ \text{MW} + 163.7\ \text{kW}} = 99.4\%$$

From Problem 2-20, the transformer equivalent series impedance from the low voltage side is $Z_{\text{eq,L}} = 7.91 + j148\ \text{m}\Omega$. The transformer rated current is $I_{\text{rated}} = 3125$ A and thus under load the transformer high-side voltage (neglecting the effects of magnetizing current) referred to the primary is

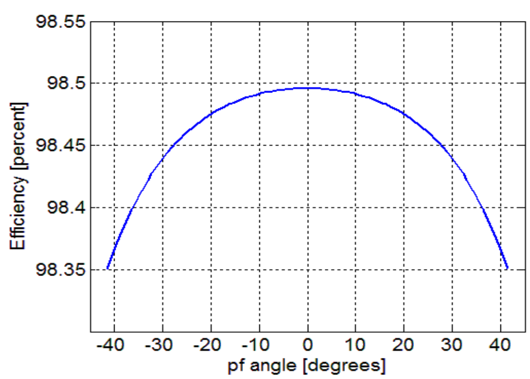
$$|V'_{\text{H}}| = |V_{\text{L}} - I_{\text{rated}} Z_{\text{eq,L}}| = 7.989\ \text{kV}$$

and thus the voltage regulation is $100 \times (7.989 - 8.00)/7.989 = 0.14\%$.

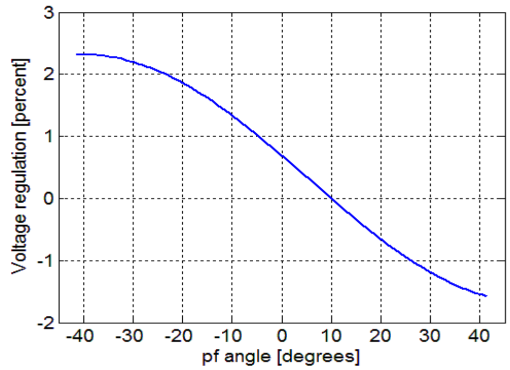
Part (b): Same methodology as part (a) except that the load is 22.5 MW and the current is $\hat{I} = I_{\text{rated}}\ \angle\phi$ where $\phi = \cos^{-1}(0.9) = 25.8°$. In this case, the efficiency is 99.3% and the regulation is 1.94%.

Problem 2-27

Part (a):



Part (b);



Problem 2-28

Part (a): The transformer loss will be equal to the sum of the open-circuit and short-circuit losses, i.e. 313 W. With a load of $0.85 \times 25 = 21.25$ kW, the efficiency is equal to

$$\eta = \frac{21.25}{21.25 + 0.313} = 0.9855 = 98.55\%$$

Part (b): The transformer equivalent-circuit parameters are found as is shown in the solution to Problem 2-23.

$$R_{\mathrm{c,L}} = 414\ \Omega \qquad R_{\mathrm{c,H}} = 41.4\ \mathrm{k}\Omega$$

$$X_{\mathrm{m,L}} = 193\ \Omega \qquad X_{\mathrm{m,H}} = 19.3\ \mathrm{k}\Omega$$

$$R_{\mathrm{L}} = 17.1\ \mathrm{m}\Omega \qquad R_{\mathrm{H}} = 1.71\ \Omega$$

$$X_{\mathrm{L}} = 64.9\ \mathrm{m}\Omega \qquad X_{\mathrm{H}} = 6.49\ \Omega$$

The desired solution is 0.963 leading power factor, based upon a MATLAB search for the load power factor that corresponds to rated voltage at both the low- and high-voltage terminals.

Problem 2-29

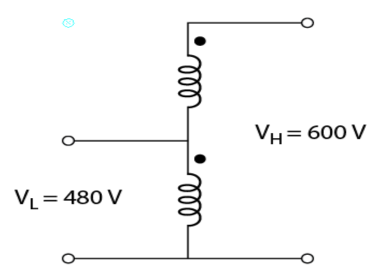
Efficiency = 98.4% and regulation = 2.38%.

Problem 2-30

The voltage rating is 280 V:400 V. The rated current of the high voltage terminal is equal to that of the 120-V winding, $I_{\text{rated}} = 45 \times 10^3/120 = 375$ A. Hence the kVA rating of the transformer is $400 \times 375 = 150$ kVA.

Problem 2-31

Part (a):



Part (b): The rated current of the high voltage terminal is equal to that of the 120-V winding, $I_{\text{rated}} = 10^4/120 = 83.3$ A. Hence the kVA rating of the transformer is $600 \times 83.3 = 50$ kVA.

Part (c): The full load loss is equal to that of the transformer in the conventional connection, $P_{\text{loss}} = (1 - 0.979)$ 10 kW = 180 W. Hence as an autotransformer operating with a load at 0.93 power factor ($P_{\text{load}} = 0.93 \times 50$ kW = 46.5 kW), the efficiency will be

$$\eta = \frac{46.5 \text{ kW}}{46.78 \text{ kW}} = 0.996 = 99.6 \text{ percent}$$

Problem 2-32

Part (a): The voltage rating is 78 kV:86 kV. The rated current of the high voltage terminal is equal to that of the 8-kV winding, $I_{\text{rated}} = 50 \times 10^6/8000 = 3.125$ kA. Hence the kVA rating of the transformer is 86 kV $\times$ 3.125 kA = 268.8 MVA.

Part (b): The loss at rated voltage and current is equal to 164 kW and hence the efficiency will be

$$\eta = \frac{268.8 \text{ MW}}{268.96 \text{ MW}} = 0.9994 = 99.94 \text{ percent}$$

Problem 2-33

MATLAB script should reproduce the answers to Problem 2-32.

Problem 2-34

Part (a): 7.97 kV:2.3 kV; 188 A:652 A; 1500 kVA

Part (b): 13.8 kV:1.33 kV; 109 A:1130 A; 1500 kVA

Part (c): 7.97 kV:1.33 kV; 188 A:1130 A; 1500 kVA

part (d): 13.8 kV:2.3 kV; 109 A:652 A; 1500 kVA

Problem 2-35

Part (a):

(i) 68.9 kV:230 kV, 225 MVA
(ii) $Z_{\text{eq}} = 0.087 + j1.01\,\Omega$
(iii) $Z_{\text{eq}} = 0.97 + j11.3\,\Omega$

Part (b):

(i) 68.9 kV:133 kV, 225 MVA
(ii) $Z_{\text{eq}} = 0.087 + j1.01\,\Omega$
(iii) $Z_{\text{eq}} = 0.32 + j3.77\,\Omega$

Problem 2-36

Part (a):

(i) 480 V:13.8 kV, 675 kVA
(ii) $Z_{\text{eq}} = 0.0031 + j0.0215\,\Omega$
(iii) $Z_{\text{eq}} = 2.57 + j17.8\,\Omega$

Part (b):

(i) 480 V:7.97 kV, 675 MVA
(ii) $Z_{\text{eq}} = 0.0031 + j0.0215\,\Omega$
(iii) $Z_{\text{eq}} = 0.86 + j5.93\,\Omega$

Problem 2-37

Following the methodology of Example 2.8, $V_{\text{load}} = 236$ V, line-to-line.

Problem 2-38

Part (a): The rated current on the high-voltage side of the transformer is

$$I_{\text{rated,H}} = \frac{25 \text{ MVA}}{\sqrt{3} \times 68 \text{ kV}} = 209 \text{ A}$$

The equivalent series impedance reflected to the high-voltage side is

$$Z_{\text{eq,H}} = N^2 Z_{\text{eq,L}} = 1.55 + j9.70\ \Omega$$

and the corresponding line-neutral voltage magnitude is

$$V_{\text{H}} = I_{\text{rated,H}}\,|Z_{\text{eq,H}}| = 2.05 \text{ kV}$$

corresponding to a line-line voltage of 3.56 kV.

Part (b): The apparent power at the high-voltage winding is $S = 18/.75 = 24$ MVA and the corresponding current is

$$I_{\rm load} = \frac{24\ {\rm MVA}}{\sqrt{3} \times 68\ {\rm kV}} = 209\ {\rm A}$$

The power factor angle $\theta = -\cos^{-1}(0.75) = -41.4°$ and thus

$$\hat{I}_{\rm load} = 209\ \angle{-41.4°}$$

With a high-side line-neutral voltage $V_{\rm H} = 69\ {\rm kV}/\sqrt{3} = 39.8$ kV, referred to the high-voltage side, the line-neutral load voltage referred to the high-voltage side is thus

$$V'_{\rm load} = |V_{\rm H} - \hat{I}_{\rm load} Z_{\rm eq,H}| = 38.7\ {\rm kV}$$

Referred to the low-voltage winding, the line-neutral load voltage is

$$V_{\rm load} = \left(\frac{13.8}{69}\right) V'_{\rm load} = 7.68\ {\rm kV}$$

corresponding to a line-line voltage of 13.3 kV.

Problem 2-39

Part (a): The line-neutral load voltage $V_{\rm load} = 24\ {\rm kV}/\sqrt{3} = 13.85$ kV and the load current is

$$\hat{I}_{\rm load} = \left(\frac{375\ {\rm MVA}}{\sqrt{3}\, 24\ {\rm kV}}\right) e^{j\phi} = 9.02 e^{j\phi}\ {\rm kA}$$

where $\phi = \cos^{-1} 0.89 = 27.1°$.

The transformer turns ratio $N = 9.37$ and thus referred to the high voltage side, $V'_{\rm load} = N V_{\rm load} = 129.9$ kV and $\hat{I}'_{\rm load} = \hat{I}_{\rm load}/N = 962 e^{j\phi}$ A. Thus, the transformer high-side line-neutral terminal voltage is

$$V_{\rm H} = |V'_{\rm L} + jX_{\rm t}\hat{I}'_{\rm load}| = 127.3\ {\rm kV}$$

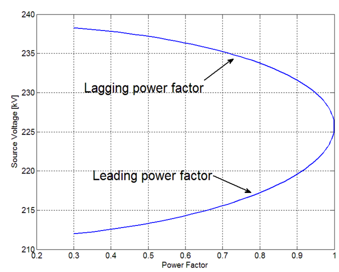
corresponding to a line-line voltage of 220.6 kV.

Part (b): In a similar fashion, the line-neutral voltage at the source end of the feeder is given by

$$V_{\mathrm{s}} = |V'_{\mathrm{L}} + (Z_{\mathrm{f}} + jX_{\mathrm{t}})\hat{I}'_{\mathrm{load}}| = 126.6 \text{ kV}$$

corresponding to a line-line voltage of 219.3 kV.

Problem 2-40



Problem 2-41

Part (a): For a single transformer

$$R_{\mathrm{eq,H}} = \frac{P_{\mathrm{sc}}}{I_{\mathrm{sc}}^2} = 342 \text{ m}\Omega$$

$$S_{\mathrm{sc}} = V_{\mathrm{sc}} I_{\mathrm{sc}} = 8.188 \text{ kVA}$$

$$Q_{\mathrm{sc}} = \sqrt{S_{\mathrm{sc}}^2 - P_{\mathrm{sc}}^2} = 8.079 \text{ kVAR}$$

and thus

$$X_{\mathrm{eq,H}} = \frac{Q_{\mathrm{sc}}}{I_{\mathrm{sc}}^2} = 2.07 \ \Omega$$

For the there-phase bank with the high-voltage side connected in Δ, the transformer series impedance reflected to the high-voltage side will be 1/3 of this value. Thus

$$Z_{\rm t,H} = \frac{R_{\rm eq,H} + jX_{\rm eq,H}}{3} = 114 + j689 \text{ m}\Omega$$

Part (b): Referred to the high voltage side, the line-neutral load voltage is $V_{\rm load} = 2400/sqrt3 = 1386$ V and the 450-kW load current will be

$$I_{\rm load} = \frac{P_{\rm load}}{3V_{\rm load}} = 108 \text{ A}$$

Thus the line-neutral source voltage is

$$V_{\rm s} = |V_{\rm load} + (Z_{\rm t,H} + Z_{\rm f})I_{\rm load}| = 1.40 \text{ kV}$$

corresponding to a line-line voltage of 2.43 kV.

Problem 2-42

Part (a): The transformer turns ratio is $N = 13800/120 = 115$. The secondary voltage will thus be

$$\hat{V}_2 = \frac{V_1}{N}\left(\frac{jX_{\rm m}}{R_1 + jX_1 + jX_m}\right) = 119.87 \ \angle 0.051^\circ$$

Part (b): Defining $R'_{\rm L} = N^2 R_{\rm L} = 9.92$ MΩ and

$$Z_{\rm eq} = jX_{\rm m}||(R'_2 + R'_{\rm L} + jX'_2)$$

$$\hat{V}_2 = \frac{V_1}{N}\left(\frac{Z_{\rm eq}}{R_1 + jX_1 + Z_{\rm eq}}\right) = 119.80 \ \angle 0.012^\circ$$

Part (c): Defining $X'_{\rm L} = N^2 X_{\rm L} = 9.92$ MΩ and

$$Z_{\rm eq} = jX_{\rm m}||(R'_2 + jX'_{\rm L} + jX'_2)$$

$$\hat{V}_2 = \frac{V_1}{N}\left(\frac{Z_{\mathrm{eq}}}{R_1 + jX_1 + Z_{\mathrm{eq}}}\right) = 119.78\ \angle 0.083°$$

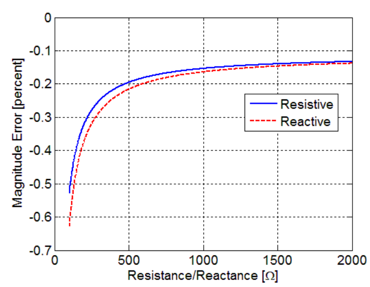
Problem 2-43

Following the methodology of Part (c) of Problem 2-42 and varying X_{L} one finds that the minimum reactance is 80.9Ω.

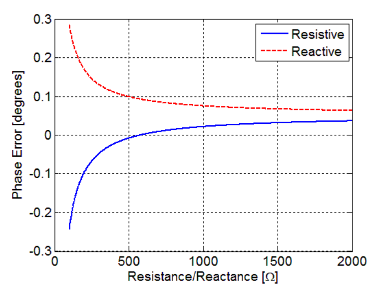
Problem 2-44

This solution uses the methodology of Problem 2-42.

Part (a):



Part (b):



Problem 2-45

Part (a): For $I_1 = 150$ A and turns ratio $N = 150/5 = 30$

$$\hat{I}_2 = \frac{I_1}{N}\left(\frac{jX_\mathrm{m}}{R_2' + j(X_\mathrm{m} + X_2')}\right) = 4.995\ \angle 0.01°\ \mathrm{A}$$

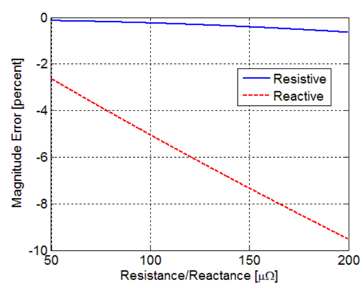
Part (b): With $R_\mathrm{b} = 0.1\ \mathrm{m}\Omega$ and $R_\mathrm{b}' = N^2 R_\mathrm{b} = 90\ \mathrm{m}\Omega$

$$\hat{I}_2 = \frac{I_1}{N}\left(\frac{jX_\mathrm{m}}{R_2' + R_\mathrm{b}' + j(X_\mathrm{m} + X_2')}\right) = 4.988\ \angle 2.99°\ \mathrm{A}$$

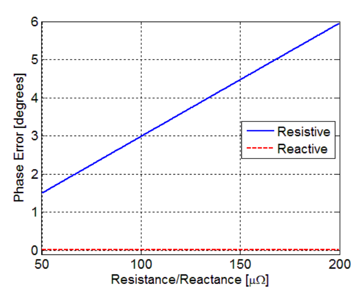
Problem 2-46

This solution uses the methodology of Problem 2-45.

Part (a):



Part (b):



Problem 2-47

The base impedance on the high-voltage side of the transformer is

$$Z_\mathrm{base,H} = \frac{V^2_\mathrm{rated,H}}{P_\mathrm{rated}} = 136.1\ \Omega$$

Thus, in Ohms referred to the high-voltage side, the primary and secondary impedances are

$$Z = (0.0029 + j0.023)Z_{\rm base,H} = 0.29 + j23.0\ {\rm m}\Omega$$

and the magnetizing reactance is similarly found to be $X_{\rm m} = 172\ \Omega$.

Problem 2-48

From the solution to Problem 2-20, as referred to the low voltage side, the total series impedance of the transformer is $7.92 + j148.2\ {\rm m}\Omega$, the magnetizing reactance is 210 Ω and the core-loss resistance is 742 Ω. The low-voltage base impedance of this transformer is

$$Z_{\rm base,L} = \frac{(8 \times 10^3)^2}{25 \times 10^6} = 2.56\ \Omega$$

and thus the per-unit series impedance is $0.0031 + j0.0579$, the per-unit magnetizing reactance is 82.0 and the per-unit core-loss resistance is 289.8.

Problem 2-49

From the solution to Problem 2-23, as referred to the low voltage side, the total series impedance of the transformer is $20.6 + j259\ {\rm m}\Omega$, the magnetizing reactance is 395 Ω and the core-loss resistance is 1780 Ω. The low-voltage base impedance of this transformer is

$$Z_{\rm base,L} = \frac{(3.81 \times 10^3)^2}{2.5 \times 10^6} = 5.81\ \Omega$$

and thus the per-unit series impedance is $0.0035 + j0.0446$, the per-unit magnetizing reactance is 68.0 and the per-unit core-loss resistance is 306.6.

Problem 2-50

Part (a): (i) The high-voltage base impedance of the transformer is

$$Z_{\rm base,H} = \frac{(7.97 \times 10^3)^2}{2.5 \times 10^3} = 2.54\ {\rm k}\Omega$$

and thus the series reactance referred high-voltage terminal is

$$X_{\rm H} = 0.075 Z_{\rm base,H} = 191\ \Omega$$

(ii) The low-voltage base impedance is 2.83 Ω and thus the series reactance referred to the low-voltage terminal is 212 mΩ.

Part (b):

(i) Power rating: 3×25 kVA = 75 kVA
Voltage rating: $\sqrt{3} \times 7.97$ kV : $\sqrt{3} \times 266$ V = 13.8 kV : 460 V
(ii) The per-unit impedance remains 0.075 per-unit
(iii) Referred to the high-voltage terminal, $X_{\rm H} = 191\ \Omega$
(iv) Referred to the low-voltage terminal, $X_{\rm L} = 212$ mΩ

Part (c):

(i) Power rating: 3×25 kVA = 75 kVA
Voltage rating: $\sqrt{3} \times 7.97$ kV : 266 V = 13.8 kV : 266 V
(ii) The per-unit impedance remains 0.075 per-unit
(iii) Referred to the high-voltage terminal, $X_{\rm H} = 191\ \Omega$
(iv) Referred to the low-voltage terminal, the base impedance is now $Z_{\rm base,L} = 266^2/(75 \times 10^3) = 0.943\ \Omega$ and thus $X_{\rm L} = 0.943 \times 0.075 =\ 70.8{\rm m}\Omega$

Problem 2-51

Part (a): 500 V at the high-voltage terminals is equal to $500/13.8 \times 10^3 = 0.0362$ per unit. Thus the per-unit short-circuit current will be

$$I_{\rm sc} = \frac{0.0363}{0.075} = 0.48 \text{ perunit}$$

(i) The base current on the high-voltage side is

$$I_{\rm base,L} = \frac{75 \times 10^3}{\sqrt{3} \times 13.8 \times 10^3} = 3.14\ A$$

and thus the short-circuit current at the high-voltage terminals will equal

$$I_{\mathrm{sc,H}} = 0.48 \times 3.14 = 1.51 \text{ A}$$

(ii) The base current on the low-voltage side is

$$I_{\mathrm{base,L}} = \frac{75 \times 10^3}{\sqrt{3} \times 460} = 94.1 \ A$$

and thus the short-circuit current at the low-voltage terminals will equal

$$I_{\mathrm{sc,L}} = 0.48 \times 94.1 = 45.4 \text{ A}$$

Part (b): The per-unit short-circuit current as well as the short-circuit current at the high-voltage terminals remains the same as for Part (a). The base current on the low-voltage side is now

$$I_{\mathrm{base,L}} = \frac{75 \times 10^3}{\sqrt{3} \times 266} = 163 \ A$$

and thus the short-circuit current at the low-voltage terminals will equal

$$I_{\mathrm{sc,L}} = 0.48 \times 163 = 78.6 \text{ A}$$

Problem 2-52

Part (a): On the transformer 26-kV base, the transformer base impedance is

$$Z_{\mathrm{base,t}} = \frac{26^2}{850} = 0.795 \ \Omega$$

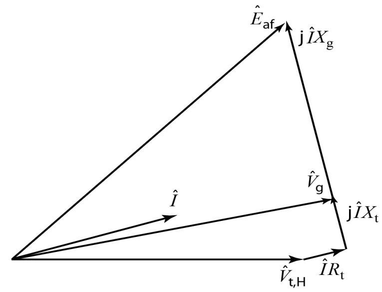
and on the same voltage base, the generator base impedance is

$$Z_{\mathrm{base,g}} = \frac{26^2}{800} = 0.845 \ \Omega$$

Thus, on the transformer base, the per-unit generator reactance is

$$X_{\mathrm{g}} = 1.28\left(\frac{Z_{\mathrm{base,g}}}{Z_{\mathrm{base,t}}}\right) = 1.36 \text{ perunit}$$

Part (b):



Part (c): In per-unit on the transformer base,

$$V_{\mathrm{t,H}} = 1.0 \text{ per unit} \quad P = \frac{750}{850} = 0.882 \text{ per unit} \quad S = \frac{P}{0.9} = 0.980 \text{ per unit}$$

and thus

$$\hat{I} = 0.98e^{j\phi}$$

where $\phi = \cos^{-1}(0.9) = 25.8°$

Thus, the per-unit generator terminal voltage on the transformer voltage base is

$$\hat{V}_{\mathrm{g}} = V_{\mathrm{t,H}} + (R_{\mathrm{t}} + jX_{\mathrm{t}})\hat{I} = 0.979 \quad \angle 3.01° \text{ per} - \text{unit}$$

which corresponds to a terminal voltage of 0.979×26 kV $= 25.4$ kV.

The per-unit generator internal terminal voltage on the transformer voltage base is

$$\hat{E}_{\mathrm{af}} = V_{\mathrm{t,H}} + (R_{\mathrm{t}} + jX_{\mathrm{t}} + jX_{\mathrm{g}})\hat{I} = 1.31 \quad \angle 72.4° \text{ per} - \text{unit}$$

which corresponds to a terminal voltage of 1.31×26 kV $= 34.1$ kV.

In per unit, the generator complex output power is

$$S = \hat{V}_{\mathrm{g}}\hat{I}^{*} = 0.884 - j0.373 \text{ per unit}$$

and thus the generator output power is $P_{\mathrm{gen}} = 0.884 \times 850 = 751.4$ MW. The generator power factor is

$$\text{pf} = \frac{P}{|S|} = 0.92$$

and it is leading.

PROBLEM SOLUTIONS: Chapter 3

Problem 3-1

By analogy to Example 3.1,

$$T = -2B_0Rl\left[I_1 \sin\alpha + I_2 \cos\alpha\right] = -7.24 \times 10^{-2}\left[I_1 \sin\alpha + I_2 \cos\alpha\right] \quad \text{N·m}$$

Thus

Part (a): $T = -0.362 \cos\alpha \quad \text{N·m}$

Part (b): $T = -0.362 \sin\alpha \quad \text{N·m}$

Part (c): $T = -0.579\,(I_1 \sin\alpha + I_2 \cos\alpha) \quad \text{N·m}$

Problem 3-2

$$T = -0.579 \quad \text{N} \cdot \text{m}$$

Problem 3-3

Part (a): From the example, the magnetic flux density in each air gap is $B_\text{g} = 0.65$ T. Since the iron permeability is assumed to be infinite, all of the stored energy is in the air gap. The total air gap volume is $2gA_\text{g}$ and thus the stored energy is found from Eq. 3.21 as

$$W_\text{fld} = 2gA_\text{g}\left(\frac{B_\text{g}^2}{2\mu_0}\right) = 67.2 \text{ J}$$

Part (b): (i) The winding inductance is given by

$$L = \frac{\mu_0 N^2 A_\text{g}}{2g} = 1.26 \text{ H}$$

(ii)

$$\lambda = LI = 12.6 \text{ Wb}$$

(iii) From Eq. 3.19

$$W_{\text{fld}} = \frac{\lambda^2}{2L} = 63 \text{ J}$$

Problem 3-4

$$W_{\text{fld}} = \frac{L(x)i^2}{2}$$

Part (a): For $i = 7.0$ A and $x = 1.30$ mm, $W_{\text{fld}} = 1.646$ J.

Part (b): For $i = 7.0$ A and $x = 2.5$ mm, $W_{\text{fld}} = 1.112$ J. Thus $\Delta W_{\text{fld}} = -0.534$ J.

Problem 3-5

Part (a): For $x = x_0$, $L = L_0$.

$$W_{\text{fld}} = \left(\frac{L_0}{2}\right) I_0^2 \sin^2(\omega t)$$

and

$$< W_{\text{fld}} >= \frac{L_0 I_0^2}{4} = 0.858 \text{ J}$$

Part (b):

$$< P_{\text{diss}} >= \frac{R_{\text{w}} I_0^2}{2} = 3.31 \text{ W}$$

Problem 3-6

Part (a):

$$L = \frac{\mu_0 A_{\rm g} N^2}{2g}$$

Part (b):

Problem 3-7

Problem 3-8

Part (a):

$$v_{\rm c}(t) = V_0 e^{-t/RC}$$

Part (b):

$$W(0) = \frac{1}{2}CV_0^2 \qquad W(\infty) = 0$$

$$W(t) = \frac{1}{2}CV_0^2 e^{-2t/RC}$$

Part (c):

$$i_{\mathrm{R}} = \frac{v_{\mathrm{c}}(t)}{R} = \frac{V_0 e^{-t/RC}}{R}$$

$$P_{\mathrm{diss}} = i_{\mathrm{R}}^2 R = \frac{V_0^2 e^{-2t/RC}}{R}$$

$$W_{\mathrm{diss}} = \int_0^\infty P_{\mathrm{diss}} dt = \frac{1}{2}CV_0^2$$

Problem 3-9

Part (a):

$$i_{\mathrm{L}}(t) = I_0 e^{-(R/L)t}$$

where $I_0 = V_0/R$.

Part (b):

$$W(0) = \frac{1}{2}LI_0^2 \qquad W(\infty) = 0$$

$$W(t) = \frac{1}{2}LI_0^2\, e^{-2(R/L)t}$$

Part (c):

$$P_{\text{diss}} = i_{\text{L}}^2(t)R = I_0^2 R\, e^{-2(R/L)t}$$

$$W_{\text{diss}} = \int_0^{\infty} P_{\text{diss}} dt = \frac{1}{2} L I_0^2$$

Problem 3-10

Part (a): The power dissipated is equal to

$$P_{\text{diss}} = I^2 R = 1.3 \times 10^6$$

and the stored energy is

$$W = \frac{1}{2} L I^2$$

Given that $\tau = L/R = 4.8$, we can find the stored energy as

$$W = \frac{1}{2}\, \tau\, P_{\text{diss}} = 0.5 \times (1.3 \times 10^6) \times 4.8 = 3.12 \text{ MJ}$$

Part (b):

$$i(t) = I_0(0.7 + 0.3e^{-t/\tau})$$

where I_0 is the field current prior to reducing the terminal voltage. Thus, the stored energy is

$$W(t) = \frac{1}{2} L i^2(t) = W_0(0.49 + 0.21e^{-t/\tau} + 0.09e^{-2t/\tau})$$

where $W_0 = 3.12$ MJ.

Problem 3-11

Part (a):

$$W_{\text{fld}}(\lambda, x) = \frac{\lambda^2}{2L(x)} = \left(\frac{\lambda^2}{2L_0X_0^2}\right)x^2$$

Part (b):

$$f_{\text{fld}}(\lambda, x) = -\left.\frac{\partial W_{\text{fld}}}{\partial x}\right|_{\lambda} = -\left(\frac{\lambda^2}{L_0X_0^2}\right)x$$

Part (c): For $i = I_0$,

$$\lambda = L(x)I_0 = \frac{L_0I_0}{(x/X_0)^2}$$

and thus from Part (b)

$$f_{\text{fld}}(x) = -\frac{L_0I_0^2X_0^2}{x^3}$$

The force acts to decrease x.

Problem 3-12

Part (a): Four poles

Part (b):

$$T_{\text{fld}} = \frac{I_0^2}{2}\left(\frac{dL(\theta_{\text{m}})}{d\theta_{\text{m}}}\right) = -L_2I_0^2\,\sin 2\theta_{\text{m}}$$

Problem 3-13

Part (a):

$$T_{\text{fld}} = \frac{I_0^2}{2}\left(\frac{dL(\theta_{\text{m}})}{d\theta_{\text{m}}}\right) = -3L_6 I_0^2\ \sin 6\theta_{\text{m}}$$

Part (b):

$$\lambda(t) = I_0(L_0 + L_6 \sin(6\theta_{\text{m}}))$$

and thus

$$v(t) = \frac{d\lambda(t)}{dt} = 6I_0 L_6 \cos 6\theta_{\text{m}}$$

The power which must be supplied to the coil is

$$p(t) = I_0 v(t) = 6I_0^2 L_6 \cos 6\theta_{\text{m}}$$

Problem 3-14

The coil inductance is equal to $L = \mu_0 N^2 A_{\text{c}}/(2g)$ and hence the lifting force is equal to

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL}{dg} = -\left(\frac{\mu_0 N^2 A_{\text{c}}}{4g^2}\right) i^2$$

where the minus sign simply indicates that the force acts in the direction to reduce the gap (and hence lift the mass). The required force is equal to 118 N (the mass of the slab times the acceleration due to gravity, 9.8 m/sec^2). Hence, setting $g = g_{\text{mim}}$ and solving for i gives

$$i_{\text{min}} = \left(\frac{2g_{\text{min}}}{N}\right)\sqrt{\frac{f_{\text{fld}}}{\mu_0 A_{\text{c}}}} = 223 \quad \text{mA}$$

and $v_{\text{min}} = i_{\text{min}} R = 0.51$ V.

Problem 3-15

Part (a): Combining

$$Ni = H_\mathrm{g} g + H_{\mathrm{g}_1} g_1$$

and

$$\pi R^2 B_\mathrm{g} = 2\pi R h_1 B_{\mathrm{g}_1}$$

where $B_\mathrm{g} = \mu_0 H_\mathrm{g}$ and $B_{\mathrm{g}_1} = \mu_0 H_{\mathrm{g}_1}$ gives

$$B_g = \frac{\mu_0 N i}{(g + R g_1/(2h_1))}$$

Part (b):

$$\lambda = N\pi R^2 B_\mathrm{g} = \frac{\mu_0 \pi R^2 N^2 i}{(g + R g_1/(2h_1))}$$

$$L(g) = \frac{\lambda}{i} = \frac{\mu_0 \pi R^2 N^2}{(g + R g_1/(2h_1))}$$

Part (c):

$$W'_\mathrm{fld} = \frac{1}{2} L(g) i^2 = \frac{\mu_0 \pi R^2 N^2 i^2}{2(g + R g_1/(2h_1))}$$

Part (d):

$$f_\mathrm{fld} = \left.\frac{\partial W'_\mathrm{fld}}{\partial g}\right|_i = -\frac{\mu_0 \pi R^2 N^2 i^2}{2(g + R g_1/(2h_1))^2}$$

The force acts to decrease g.

Problem 3-16

Part (a): From Part (a) of Problem 3-15

$$I_0 = \frac{B_{\rm g}(g + Rg_1/(2h_1))}{\mu_0 N} = 0.535 \text{ A}$$

Part (b):

Part (c): Using the expressions derived in Problem 3-15

(i)

$$\Delta W_{\rm fld} = \frac{1}{2}(L(g_{\rm min}) - L(g_{\rm max}))I_0^2 = 5.49 \text{ J}$$

(ii)

$$E_{\rm ext} = \int_{g_{\rm min}}^{g_{\rm max}} f_{\rm fld} dx = 5.49 \text{ J}$$

(iii)

$$E_{\rm gen} = \Delta W_{\rm fld} + E_{\rm ext} = 10.98 \text{ J}$$

Problem 3-17

Part (a):

$$\pi R^2 B_\delta = 2\pi R h B_{\rm g} \qquad \pi R^2 H_\delta = 2\pi R h H_{\rm g}$$

$$\delta H_\delta + g H_{\rm g} = Ni$$

$$B_\delta = \mu_0 H_\delta = \frac{\mu_0 N}{(\delta + (R/(2h))g)}$$

$$\lambda = \pi R^2 N B_\delta = \left(\frac{\mu_0 \pi R^2 N^2}{(\delta + (R/(2h))g)}\right) i$$

$$L = \frac{\lambda}{i} = \frac{\mu_0 \pi R^2 N^2}{(\delta + (R/(2h))g)}$$

Part (b):

$$f_{\rm fld} = -\left.\frac{\partial W_{\rm fld}}{\partial \delta}\right|_\lambda = \left(\frac{\lambda^2}{2L^2}\right)\frac{dL}{d\delta} = \left(\frac{i^2}{2}\right)\frac{dL}{d\delta}$$

Thus

$$\text{(i)} \quad f_{\rm fld} = -\left(\frac{1}{2\mu_0 \pi R^2 N^2}\right)\lambda^2$$

and

$$\text{(ii)} \quad f_{\rm fld} = -\left(\frac{\mu_0 \pi R^2 N^2}{2(\delta + (R/(2h))g))^2}\right) i^2$$

Part (c): The net force includes both $f_{\rm fld}$ and the spring force.

$$f_{\text{net}} = f_{\text{fld}}(\delta, I_0) + K(\delta_0 - \delta)$$

Part (c):

Stable equilibrium will occur at the position where $f_{\text{net}} = 0$ and $\frac{df_{\text{net}}}{d\delta} < 0$; in this case at $x = 4.25$ mm.

Problem 3-18

Part (a):

$$L = \frac{\mu_0 A_{\text{c}}(2N)^2}{2g} = \frac{2\mu_0 A_{\text{g}} N^2}{g}$$

Part (b):

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL}{dg} = -2.22 \times 10^3 \text{ N}$$

$$P = \frac{|f_{\text{fld}}|}{2A_{\text{c}}} = 1.15 \times 10^6 \text{ N/m}^2$$

Problem 3-19

Part (a):

Part (b): From

$$2Ni = 2H_\mathrm{c}l_\mathrm{c} + 2H_\mathrm{g}g$$

$$B_\mathrm{c} = B_\mathrm{g} = B = \mu_0 H_\mathrm{g}$$

we can solve for i as a function of B_c

$$i = \frac{H_\mathrm{c}(B_\mathrm{c})l_\mathrm{c} + B_\mathrm{c}g/\mu_0}{N}$$

where we have recognized that H_c can be expressed as a function of B_c via a spline fit with MATLAB.

For an infinitely-permeable core

$$B = \frac{\mu_0 Ni}{g}$$

Part (c): From the solution to part (b) we see that, at a current of 10 A, the core is indistinguishable from a core with infinite permeability. Thus, as in the solution to Problem 3-18, we can find that

$$f = 987 \text{ N} \quad \text{and} \quad P = 5.09 \times 10^5 \text{ N/m}^2$$

Part (d): From the plot of part (b), we see that for $i = 20$ A the non-linearity of the core material must be taken into account. As suggested by the hint in the problem statement, we will calculate the force as

$$f_{\text{fld}} = \left.\frac{W'_{\text{fld}}}{dg}\right|_i \approx \frac{W'(g = 5.01 \text{ mm}, i) - W'(g = 5.00 \text{ mm}, i)}{0.01 \text{ mm}}$$

From Eq. 3.41

$$W'(g,i) = \int_0^i \lambda(g,i')di' = 2NA_c \int_0^i B(g,i')di'$$

For each value of g, $B(g,i)$ can be represented by a MATLAB spline fit as was done in part (b) and the integration can be performed numerically in MATLAB.

$$f = 2417 \text{ N} \quad \text{and} \quad P = 1.26 \times 10^6 \text{ N/m}^2$$

Problem 3-20

$$L = \frac{\mu_0 A_c N^2}{g}$$

$$f(t) = \left(\frac{\lambda(t)}{2L^2}\right)\frac{dL}{dg} = -\left(\frac{1}{2\mu_0 N^2 A_c}\right)\lambda^2(t)$$

$$\lambda(t) = \int v(t)dt = \frac{\sqrt{2}\,V_{\text{rms}}}{\omega}\cos(\omega t)$$

where $V_{\rm rms} = 120$ and $\omega = 120\pi$. Thus

$$f(t) = -\left(\frac{V_{\rm rms}^2}{\mu_0\,\omega^2 N^2 A_{\rm c}}\right)\cos^2(\omega t) = -233\cos^2(\omega t)\ \mathrm{N}$$

The time-averaged value is one half of the peak force

$$< f(t) > \;\; = -116.5\cos^2(\omega t)\ \mathrm{N}$$

Because the $\lambda(t)$ does not vary with the air-gap length, the force will not vary.

Problem 3-21

Part (a):

$$B_{\rm s} = \frac{\mu_0 i}{s}$$

Part (b):

$$\phi_{\rm s} = B_{\rm s} x l = \frac{\mu_0 x l}{s}$$

Part (c): Note that as the coil moves upward in the slot, the energy associated with the leakage flux associated withing the coil itself remains constant while the energy in the leakage flux above the coil changes. Hence to use the energy method to calculate the force on the coil it is necessary only to consider the energy in the leakage flux above the slot.

$$W_{\rm fld} = \int \frac{B_{\rm s}^2}{2\mu_0} dV = \frac{\mu_0 x l i^2}{2s}$$

Because this expression is explicity in terms of the coil current i and becasue the magnetic energy is stored in air which is magnetically linear, we know that $W'_{\rm fld} = W_{\rm fld}$. We can therefore find the force from

$$f_{\rm fld} = \frac{dW'_{\rm fld}}{dx} = \frac{\mu_0 l i^2}{2s}$$

This force is positive, acting to increase x and hence force the coil further into the slot.

Part (d): $f_{\text{fld}} = 10.2$ N/m.

Problem 3-22

$$W'_{\text{fld}} = \left(\frac{\mu_0 H^2}{2}\right) \times \text{ coil volume} = \left(\frac{\mu_0 \pi r_0^2 N^2}{2h}\right) i^2$$

Thus

$$f = \frac{dW'_{\text{fld}}}{dr_0} = \left(\frac{\mu_0 \pi r_0 N^2}{h}\right) I_0^2$$

and hence the pressure is

$$P = \frac{f}{2\pi r_0 h} = \left(\frac{\mu_0 N^2}{2h^2}\right) I_0^2$$

The pressure is positive and hence acts in such a direction as to increase the coil radius r_0.

Problem 3-23

Part (a):

$$W_{\text{fld}}(q, x) = \int_0^q v(q', x) dq'$$

Part (b):

$$f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}}{\partial x}\right|_q$$

Part (c):

$$W'_{\text{fld}} = vq - dW_{\text{fld}} \Rightarrow dW'_{\text{fld}} = q\,dv + f_{\text{fld}}\,dx$$

Thus

$$W'_{\text{fld}} = \int_0^v q(v', x)\,dv'; \quad f_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}}{\partial x}\right|_v$$

Problem 3-24

Part (a):

$$W_{\text{fld}} = \int_0^q v(q', x)\,dq' = \frac{q^2}{2C} = \frac{xq^2}{2\epsilon_0 A}$$

$$W'_{\text{fld}} = \int_0^v q(v', x)\,dv' = \frac{Cv^2}{2} = \frac{\epsilon_0 A v^2}{2x}$$

Part (b):

$$f_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}}{\partial x}\right|_v = \frac{Cv^2}{2} = \frac{\epsilon_0 A v^2}{2x^2}$$

and thus

$$f_{\text{fld}}(V_0, \delta) = \frac{\epsilon_0 A V_0^2}{2\delta^2}$$

Problem 3-25

Part (a):

$$T_{\text{fld}} = \left(\frac{V_{\text{dc}}^2}{2}\right)\frac{dC}{d\theta} = \left(\frac{Rd}{2g}\right)V_{\text{dc}}^2$$

Part (b): In equilibrium, $T_{\text{fld}} + T_{\text{spring}} = 0$ and thus

$$\theta = \theta_0 + \left(\frac{Rd}{2gK}\right) V_{\text{dc}}^2$$

Part (c):

Problem 3-26

Part (a):

$$L_{11} = \frac{\mu_0 N_1^2 A}{2g_0} \qquad L_{22} = \frac{\mu_0 N_2^2 A}{2g_0}$$

Part (b):

$$L_{12} = \frac{\mu_0 N_1 N_2 A}{2g_0}$$

Part (c):

$$W'_{\text{fld}} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2 = \frac{\mu_0 A}{4g_0} \left(N_1 i_1 + N_2 i_2\right)^2$$

Part (d):

$$f_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}}{\partial g_0}\right|_{i_1, i_2} = -\frac{\mu_0 A}{4g_0^2} \left(N_1 i_1 + N_2 i_2\right)^2$$

Problem 3-27

Part (a):

$$W'_{\text{fld}} = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2 = I^2\left(L_{11} + L_{22} + 2L_{12}\right)\sin^2\omega t$$

$$T_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}}{\partial \theta}\right|_{i_1,i_2} = -6.2 \times 10^{-3} I^2 \sin\theta \sin^2\omega t \quad \text{N·m}$$

Part (b):

$$T_{\text{avg}} = -3.1 \times 10^{-3} I^2 \sin\theta \quad \text{N·m}$$

Part (c): $T_{\text{avg}} = -0.31$ N·m.

Parts (d) and (e): The curve of spring force versus angle is plotted as a straight line on the plot. The intersection with each curve of magnetic force versus angle gives the equilibrium angle for that value of current. For greater accuracy, MATLAB can be used to search for the equilibrium points. The results of a MATLAB analysis give:

I	θ
5	52.5°
7.07	35.3°
10	21.3°

part (f):

Problem 3-28

Neglecting resistances, with winding 2 short-circuited, its flux linkage will be fixed at zero. Thus

$$\lambda_2 = L_{22}i_2 + L_{12}i_1 \Rightarrow i_2 = -\left(\frac{L_{12}}{L_{22}}\right)i_1$$

$$W'_{\text{fld}} = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2 = \left(\frac{1}{2}L_{11} - \frac{1}{2}\,\frac{L_{12}^2}{L_{22}}\right)i_1^2$$

$$\begin{aligned} T_{\text{fld}} &= \left.\frac{\partial W'_{\text{fld}}}{\partial \theta}\right|_{i_1,i_2} = -\,\frac{L_{12}}{L_{22}}\,\frac{dL_{12}}{d\theta}\,i_1^2 \\ &= -3.34\sin{(2\theta)}i^2 \end{aligned}$$

Problem 3-29

Part (a): Winding 1 produces a radial magnetic which, under the assumption that $g << r_0$,

$$B_{\mathrm{r},1} = \frac{\mu_0 N_1}{g}\, i_1$$

The z-directed Lorentz force acting on coil 2 will be equal to the current in coil 2 multiplied by the radial field $B_{\mathrm{r},1}$ and the length of coil 2.

$$f_{\mathrm{z}} = 2\pi r_0 N_2 B_{\mathrm{r},1} i_2 = \frac{2\pi r_0 \mu_0 N_1 N_2}{g}\, i_1 i_2$$

Part (b): The self inductance of winding 1 can be easily written based upon the winding-1 flux density found in part (a)

$$L_{11} = \frac{2\pi r_0 l \mu_0 N_1^2}{g}$$

The radial magnetic flux produced by winding 2 can be found using Ampere's law and is a function of z.

$$B_{\mathrm{z}} = \begin{cases} 0 & 0 \le z \le x \\ -\frac{\mu_0 N_2 i_2 (z-x)}{gh} & x \le z \le x+h \\ -\frac{\mu_0 N_2 i_2}{g} & x+h \le z \le l \end{cases}$$

Based upon this flux distribution, one can show that the self inductance of coil 2 is

$$L_{22} = \frac{2\pi r_0 \mu_0 N_2^2}{g}\left(l - x - \frac{2h}{3}\right)$$

Part (c): Based upon the flux distribution found in part (b), the mutual inductance can be shown to be

$$L_{12} = \frac{2\pi r_0 \mu_0 N_1 N_2}{g}\left(x + \frac{h}{2} - l\right)$$

Part (d):

$$f_{\text{fld}} = \frac{d}{dx}\left[\frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2\right] = -\frac{\pi r_0\mu_0 N_2^2}{g}i_2^2 + \frac{2\pi r_0\mu_0 N_1 N_2}{g}i_1 i_2$$

Note that this force expression includes the Lorentz force of part (a) as well as a reluctance force due to the fact that the self inductance of coil 2 varies with position x. Substituting the given expressions for the coil currents gives:

$$f_{\text{fld}} = -\frac{\pi r_0\mu_0 N_2^2}{g}I_2^2\cos^2\omega t + \frac{2\pi r_0\mu_0 N_1 N_2}{g}I_1 I_2\cos\omega t$$

Problem 3-30

The solution follows that of Example 3.8 with the exception of the magnet properties of samarium-cobalt replaced by those of neodymium-boron-iron for which $\mu_{\text{R}} = 1.06\mu_0$, $H_{\text{c}}' = -940$ kA/m and $B_{\text{r}} = 1.25$ T.

The result is

$$f_{\text{fld}} = \begin{cases} \text{-203 N} & \text{at x = 0 cm} \\ \text{-151 N} & \text{at x = 0.5 cm} \end{cases}$$

Problem 3-31

Part (a): Because there is a winding, we don't need to employ a "fictitious" winding. Solving

$$H_{\text{m}}d + H_{\text{g}}g_0 = Ni; \quad B_{\text{m}}wD = B_{\text{g}}(h-x)D$$

in combination with the constitutive laws

$$B_{\text{m}} = \mu_{\text{R}}(H_{\text{m}} - H_{\text{c}}); \quad B_{\text{g}} = \mu_0 H_{\text{g}}$$

gives

$$B_{\text{m}} = \frac{\mu_0(Ni + H_{\text{c}}d)}{\frac{d\mu_0}{\mu_{\text{R}}} + \frac{wg_0}{(h-x)}}$$

Note that the flux in the magnetic circuit will be zero when the winding current is equal to $I_0 = -H_{\rm c}d/N$. Hence the coenergy can be found from integrating the flux linkage of the winding from an initial state where it is zero (i.e. with $i = I_0$) to a final state where the current is equal to i. The flux linkages are given by $\lambda = NwDB_{\rm m}$ and hence

$$W'_{\rm fld}(i,x) = \int_{I_0}^{i} \lambda(i',x)di' = \frac{\mu_0 wDN}{\frac{d\mu_0}{\mu_{\rm R}} + \frac{wg_0}{(h-x)}}\left[\frac{Ni^2}{2} + H_{\rm c}\left(i + \frac{H_{\rm c}d}{2N}\right)\right]$$

The force is then

$$f_{\rm fld} = \frac{dW'_{\rm fld}}{dx} = \frac{-\mu_0 w^2 DNg_0}{(\frac{\mu_0 d(h-x)}{\mu_{\rm R}} + wg_0)^2}\left[\frac{Ni^2}{2} + H_{\rm c}\left(i + \frac{H_{\rm c}d}{2N}\right)\right]$$

(i) for $i = 0$,

$$f_{\rm fld} = \frac{dW'_{\rm fld}}{dx} = \frac{-\mu_0 w^2 Dg_0 (H_{\rm c}d)^2}{2(\frac{\mu_0 d(h-x)}{\mu_{\rm R}} + wg_0)^2}$$

where the minus sign indicates that the force is acting upwards to support the mass against gravity.

(ii) The maximum force occurs when $x = h$

$$f_{\rm max} = -\frac{\mu_0 wD(H_{\rm c}d)^2}{2} = -M_{\rm max}a$$

where a is the acceleration due to gravity. Thus

$$M_{\rm max} = \frac{\mu_0 wD(H_{\rm c}d)^2}{2a}$$

Part (b): Want

$$f(I_{\rm min,x=h} = -a\,\frac{M_{\rm max}}{2} = -\frac{\mu_0 wD(H_{\rm c}d)^2}{4}$$

Substitution into the force expression of part (a) gives

$$I_{\min} = (2 - \sqrt{2})(-H_c d) = -0.59 H_c d$$

Problem 3-32

Part (a): Combining

$$H_m d + H_g g = 0; \quad \pi r_0^2 B_m = 2\pi r_0 l B_g$$

$$B_g = \mu_0 H_g; \quad B_m = \mu_R (H_m - H_{rmc})$$

gives

$$B_g = \frac{-H_c d \mu_0}{g + \left(\frac{\mu_0}{\mu_R}\right)\left(\frac{2ld}{r_0}\right)}$$

Part (b): The flux linkages of the voice coil can be calculate in two steps. First calculate the differential flux linkages of a differential section of the voice coil of dN_2 turns at height z' above the bottom of the voice coil (which is at $z = x$).

$$d\lambda_2 = dN_2 \int_{z'}^{l} B_g (2\pi r_0) dz = \left[\frac{(-H_c d \mu_0)(2\pi r_0)(l - z')}{g + \left(\frac{\mu_0}{\mu_R}\right)\left(\frac{2ld}{r_0}\right)}\right] dN_2$$

Recognizing that $dN_2 = (N_2/h)dz'$ we can now integrate over the coil to find the total flux linkages

$$\lambda_2 = \int_x^{x+h} d\lambda_2 = \frac{N_2(-H_c d \mu_0)(2\pi r_0)(l - x - h/2)}{g + \left(\frac{\mu_0}{\mu_R}\right)\left(\frac{2ld}{r_0}\right)}$$

Part (c): Note from part (a) that the magnet in this case can be replaced by a winding of $N_1 i_1 = -H_c d$ ampere-turns along with a region of length d and permeability μ_R. Making this replacement from part (a), the self inductance of the winding can be found

$$\lambda_{11} = N_1\Phi_{11} = 2\pi r_0 h N_1 B_{\rm g} = \frac{2\pi r_0 h N_1^2 d\mu_0}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)} i_1$$

and thus

$$L_{11} = \frac{2\pi r_0 h N_1^2 d\mu_0}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)}$$

Similar, the mutual inductance with the voice coil can be found from part (b) as

$$L_{12} = \frac{\lambda_2}{i_1} = \frac{N_1\lambda_2}{-H_{\rm c}d} = \frac{N_2 N_2 \mu_0 (2\pi r_0)(l - x - h/2)}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)}$$

We can now find the coenergy (ignoring the term $L_{22}i_2^2/2$)

$$\begin{aligned} W'_{\rm fld} &= \frac{1}{2}L_{11}i_1^2 + L_{12}i_1 i_2 \\ &= \frac{\mu_0 (H_{\rm c}d)^2 \pi r_0 h}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)} + \frac{\mu_0 N_2 (-H_{\rm c}d)(2\pi r_0 d)(l - x - \frac{h}{2})}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)} i_2 \end{aligned}$$

Part (d):

$$f_{\rm fld} = \frac{dW'_{\rm fld}}{dx} = -\frac{\mu_0 N_2 (-H_{\rm c}d)(2\pi r_0 d)}{g + \left(\frac{\mu_0}{\mu_{\rm R}}\right)\left(\frac{2ld}{r_0}\right)}$$

Problem 3-33

Part (a):

$$H_{\rm m}t_{\rm m} + H_{\rm x}x + H_{\rm g}g = 0; \quad \pi(R_3^2 - R_2^2)B_{\rm m} = \pi R_1^2 B_{\rm x} = 2\pi R_1 h B_{\rm g}$$

$$B_{\rm g} = \mu_0 H_{\rm g}; \quad B_{\rm x} = \mu_0 H_{\rm x}; \quad B_{\rm m} = \mu_{\rm R}(H_{\rm m} - H_{\rm c})$$

where $\mu_{\rm R} = 1.05\mu_0$ and $H_{\rm c}' = -712$ kA/m.

Solving gives

$$B_{\rm g} = \left(\frac{\mu_0 R_1(-H_{\rm c}t_{\rm m})}{2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_{\rm m}}{\mu_R(R_3^2 - R_2^2)}} \right) = 0.624 \text{ T}$$

and

$$B_{\rm x} = \left(\frac{2h}{R_1} \right) B_{\rm g} = 0.595 \text{ T}$$

Part (b): We can replace the magnet by an equivalent winding of $Ni = -H_{\rm c}t_{\rm m}$. The flux linkages of this equivalent winding can then be found to be

$$\lambda = N(2\pi R_1 h)B_{\rm g} = \left(\frac{2\pi\mu_0 h R_1^2 N^2}{2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_{\rm m}}{\mu_R(R_3^2 - R_2^2)}} \right) i = Li$$

The force can then be found as

$$\begin{aligned} f_{\rm fld} &= \frac{i^2}{2}\frac{dL}{dx} = \frac{-2\pi\mu_0(hR_1)^2(Ni)^2}{\left(2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_{\rm m}}{\mu_R(R_3^2 - R_2^2)}\right)^2} \\ &= \frac{-2\pi\mu_0(hR_1)^2(-H_{\rm c}t_{\rm m})^2}{\left(2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_{\rm m}}{\mu_R(R_3^2 - R_2^2)}\right)^2} = -195 \quad \text{N} \end{aligned}$$

Part (c):

Part (d): The spring force will be of the form

$$f_{\text{spring}} = -K(x - X_0) - f_{\text{fld}}(X_0)$$

to guarantee that there will be equilibrium at $x = X_0$. It also must be larger than the magnitude of the magnetic force at $x = 0$ ($f_{\text{fld}}(0) = -246$ N). Thus, we must have

$$K > \frac{f_{\text{fld}}(X_0) + |f_{\text{fld}}(0)|}{X_0} = \frac{-195 + 246}{5 \times 10^{-4}} = 103 \text{ N/mm}$$

Problem 3-34

Part (a): If the plunger is stationary at $x = 0.5X_0$, the inductance will be constant at $L(0.5X_0) = 0.5L_0$. Thus

$$i(t) = \frac{V_0}{R_c}\,(1 - e^{-t/\tau})$$

where $\tau = L(0.5X_0)/R_c$. The magnetic force will thus be

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL}{dx} = -\frac{L_0}{2X_0}\left(\frac{V_0}{R_c}\right)^2 (1 - e^{-t/\tau})^2$$

A force of this magnitude must therefore be applied to maintain the plunger at this position.

Part (b): The steady-state current will be equal to $I_0 = V_0/R_c$

and since the force is independent of x we can write that at equilibrium

$$f_{\text{net}} = 0 = K_0(X_0 - x_0) - \frac{L_0}{2X_0}\left(\frac{V_0}{R_c}\right)^2$$

and thus

$$x_0 = 0.5X_0 - \frac{L_0}{2K_0X_0}\left(\frac{V_0}{R_c}\right)^2$$

Problem 3-35

Part (a): Following the derivation of Example 3.1, for a rotor current of 8 A, the torque will be give by $T = T_0 \sin\alpha$ where $T_0 = -0.0048$ N·m. The stable equilibrium position will be at $\alpha = 0$.

Part (b):

$$J\frac{d^2\alpha}{dt^2} = T_0 \sin\alpha$$

Part (c): The incremental equation of motion is

$$J\frac{d^2\alpha}{dt^2} = T_0\alpha$$

and the natural frequency is

$$\omega = \sqrt{\frac{T_0}{J}} = 0.62 \text{ rad/sec}$$

corresponding to a frequency of 0.099 Hz.

Problem 3-36

As long as the plunger remains within the core, the inductance is equal to

$$L = \frac{\mu_0 d\pi N^2}{ag}\left(\left(\frac{a}{2}\right)^2 - x^2\right)$$

where x is the distance from the center of the solenoid to the center of the core. Hence the force is equal to

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL}{dx} = -\frac{\mu_0 d\pi N^2 i^2 x}{ag}$$

Analogous to Example 3.10, the equations of motor are

$$f_t = -M\frac{d^2x}{dt^2} - B\frac{dx}{dt} - K(x - l_0) - \frac{\mu_0 d\pi N^2 i^2 x}{ag}$$

The voltage equation for the electric system is

$$v_t = iR + \frac{\mu_0 d\pi N^2}{ag}\left(\left(\frac{a}{2}\right)^2 - x^2\right)\frac{di}{dt} - \frac{2\mu_0 d\pi N^2 x}{ag}\frac{dx}{dt}$$

These equations are valid only as long as the motion of the plunger is limited so that the plunger does not extend out of the core, i.e. ring, i.e. between the limits $-a/2 < x < a/2$.

Problem 3-37

Part (a): From the solution to Problem 3-34, $x_0 = 1.0$ cm

Part (b): This is a 3'rd order system which can be expressed in the following form:

$$\frac{dx}{dt} = x_1$$

$$\frac{dx_1}{dt} = \frac{f_{\text{fld}} + f_{\text{spring}}}{M}$$

$$\frac{di}{dt} = \frac{v - R_c i - i\frac{dL(x)}{dx}x_1}{L(x)}$$

where

$$f_{\text{fld}} = -\left(\frac{L_0}{2X_0}\right)i^2$$

and

$$f_{\text{spring}} = K_0(X_0 - x)$$

Part (c):

$$f = \frac{1}{2\pi}\sqrt{\frac{K_0}{M}} = 7.2 \text{ Hz}$$

Part (d):

(i)

(ii)

Problem 3-38

PROBLEM SOLUTIONS: Chapter 4

Problem 4-1

Part (a):

$$\omega_m = \text{rpm} \times \frac{\pi}{30} = 125.7 \text{ rad/sec}$$

Part (b):

$$f = \text{rpm} \times \frac{\text{poles}}{120} = 60 \text{ Hz} \quad (= 120\pi \text{ rad/sec})$$

Part (c): The frequency will be 50 Hz if the speed is $(5/6) \times 1200 = 1000$ rpm.

Problem 4-2

(i)

$$v_{\text{b}}(t) = \sqrt{2}V_{\text{a}} \cos{(\omega t - 2\pi/3)} \qquad v_{\text{c}}(t) = \sqrt{2}V_{\text{a}} \cos{(\omega t + 2\pi/3)}$$

(ii)

$$v_{\text{ab}}(t) = v_{\text{a}}(t) - v_{\text{b}}(t) = \sqrt{6}V_{\text{a}} \cos{(\omega t + \pi/6)}$$

Problem 4-3

(i) With the wind turbine rotating at 0.5 r/sec = 30 rpm, the generator speed will be 300 rpm which will produce a line-line voltage of $480 \times 300/900 = 160$ V at a frequency of $f = 300 \times 8/120 = 20$ Hz.

(ii) At 1.75 r/sec = 105 rpm, the generator voltage will be 560 V line-line at a frequency of 70 Hz.

Problem 4-4

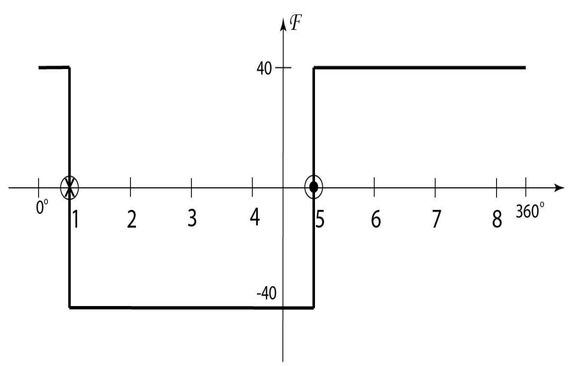
Part (a): Induction motor Part (b): 6 poles

Problem 4-5

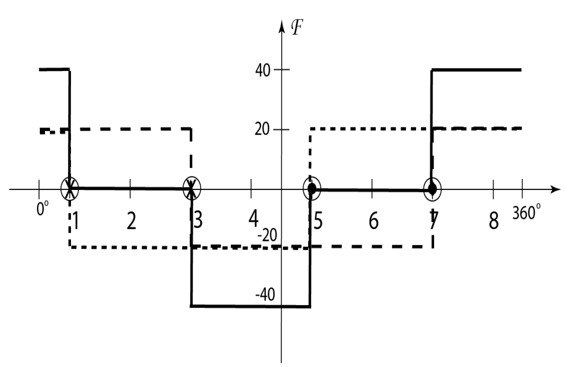
$$\text{rpm} = f \times \frac{120}{\text{poles}} = 6000$$

Problem 4-6

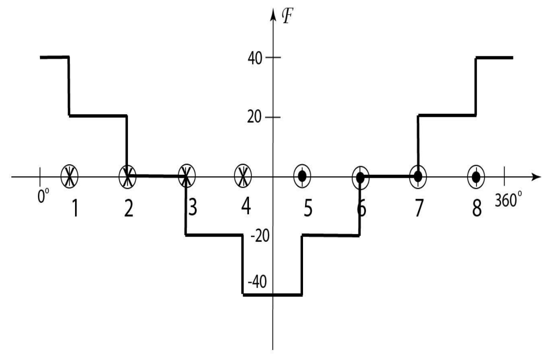
Part (a):



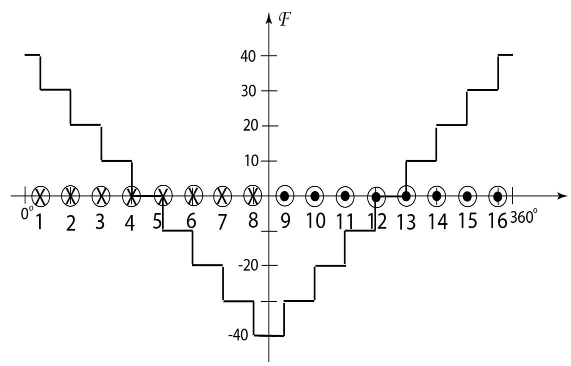
Part (b):



Part (c):



Part (d):



Problem 4-7

With one phase open-circuited, the motor becomes equivalent to a single-phase motor, with equal positive- and negative-traveling mmf waves.

Problem 4-8

The mmf and flux waves will reverse direction. Reversing two phases is the procedure for reversing the direction of a three-phase induction motor.

Problem 4-9

$$\mathcal{F}_1 = F_{\text{max}} \cos\theta_{\text{ae}} \cos\omega_{\text{e}} t = \frac{F_{\text{max}}}{2} \left(\cos\left(\theta_{\text{ae}} - \omega_{\text{t}}\right) + \cos\left(\theta_{\text{ae}} + \omega_{\text{t}}\right)\right)$$

$$\mathcal{F}_2 = F_{\text{max}} \sin\theta_{\text{ae}} \sin\omega_{\text{e}} t = \frac{F_{\text{max}}}{2} \left(\cos\left(\theta_{\text{ae}} - \omega_{\text{t}}\right) - \cos\left(\theta_{\text{ae}} + \omega_{\text{t}}\right)\right)$$

and thus

$$\mathcal{F}_{\text{total}} = \mathcal{F}_1 + \mathcal{F}_2 = F_{\text{max}} \cos\left(\theta_{\text{ae}} - \omega_{\text{t}}\right)$$

Problem 4-10

For n odd

$$\left| \frac{\int_{-\beta/2}^{\beta/2} \cos(n\theta) d\theta}{\int_{-\pi/2}^{\pi/2} \cos(n\theta) d\theta} \right| = \sin\left(\frac{n\theta}{2}\right)$$

For $\beta = 5\pi/6$,

$$\sin\left(\frac{n\theta}{2}\right) = \begin{cases} 0.97 & \text{n} = 1 \\ 0 & \text{n} = 3 \\ 0.26 & \text{n} = 5 \end{cases}$$

Problem 4-11

Part (a): Rated speed = 900 r/min

Part (b): From Eq. 4.45

$$I_{\text{r}} = \frac{\pi g B_{\text{ag1,peak}}(\text{poles})}{4\mu_0 k_{\text{r}} N_{\text{r}}} = 36.4 \quad \text{A}$$

Part (c): From Eq. 4.47

$$\Phi_\mathrm{P} = \left(\frac{2}{3}\right) lRB_\mathrm{ag1,peak} = 0.713 \quad \mathrm{Wb}$$

Problem 4-12

From the solution to Problem 4-11, $\Phi_\mathrm{P} = 0.713$ Wb.

$$V_\mathrm{rms} = \frac{\omega N \Phi_\mathrm{P}}{\sqrt{2}} = 7.60 \quad \mathrm{kV}$$

Problem 4-13

From the solution to Problem 4.9, $\Phi_\mathrm{P} = 0.713$ Wb.

$$V_\mathrm{rms} = \frac{\omega k_\mathrm{w} N_\mathrm{a} \Phi}{\sqrt{2}} = 7.99 \quad \mathrm{kV}$$

Problem 4-14

Part (a): Because this generator is Δ-connected, the rms phase voltage V_rms and line-line voltages are both equal to 575 V. Thus from Eq. 4.52

$$\Phi_\mathrm{P} = \frac{\sqrt{2}\, V_\mathrm{rms}}{\omega_\mathrm{e} k_\mathrm{w} N_\mathrm{ph}} = 0.191 \quad \mathrm{Wb}$$

and thus from Eq. 4-45

$$B_\mathrm{peak} = \left(\frac{\mathrm{poles}}{2}\right) \frac{\Phi_\mathrm{P}}{2lr} = 1.39 \quad \mathrm{T}$$

Part (b): From Eq. 4.45

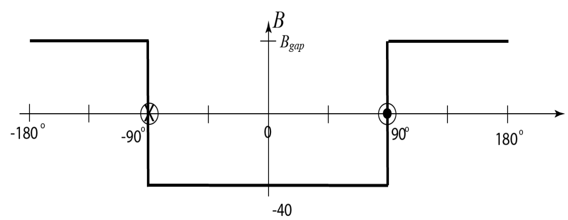
$$I_\mathrm{f} = \frac{\pi g \times \mathrm{poles}}{4\mu_0 k_\mathrm{f} N_\mathrm{f}} = 20.1 \quad \mathrm{A}$$

Part (c): Operating at 50 Hz with the same air-gap flux (corresponding to a field current of 20.1 A), the open-circuit voltage will be $(5/6) \times 575 = 479$ V. The armature turns must be increase from 12 to 18 turns/phase, which would produce a terminal voltage of 718 V. If the field current is then reduced to $20.1 \times (690/718) = 19.3$ A, the desired open-circuit voltage of 690 V will be achieved.

Problem 4-15

Part (a): The air-gap flux density is plotted below and is of amplitude

$$B_{\text{gap}} = \frac{\mu_0 N_{\text{f}} I_{\text{f}}}{2g} = 1.16 \quad \text{T}$$



The peak fundamental amplitude is

$$B_{\text{peak},1} = \left(\frac{4}{\pi}\right) B_{\text{gap}} = 1.47 \quad \text{T}$$

Part (b): In terms of the stator inner radius r, axial-length l and turns N_{ph}, the peak flux linkage of the stator winding is equal to

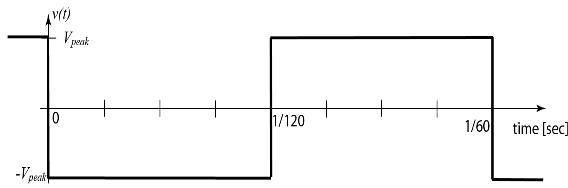
$$\lambda_{\text{peak}} = \pi r l B_{\text{gap}} N_{\text{ph}}$$

and it varies as a saw-tooth function of time, giving rise to a square-wave of voltage, shown in the plot below, of magnitude

$$V_{\text{peak}} = \frac{2\lambda_{\text{peak}}}{T/2} = 476.7 \quad \text{V}$$

where $T = 1/60$ sec. The rms-fundamental component of this voltage is

$$V_{1,\mathrm{rms}} = \frac{4}{\pi}\frac{V_{\mathrm{peak}}}{\sqrt{2}} = 477 \quad \mathrm{V}$$



Problem 4-16

$$\begin{aligned}\mathcal{F}_{\mathrm{a}} &= i_{\mathrm{a}}[A_1 \cos\theta_{\mathrm{a}} + A_3 \cos 3\theta_{\mathrm{a}} + A_5 \cos 5\theta_{\mathrm{a}}] \\ &= I_{\mathrm{a}} \cos\omega t[A_1 \cos\theta_{\mathrm{a}} + A_3 \cos 3\theta_{\mathrm{a}} + A_5 \cos 5\theta_{\mathrm{a}}]\end{aligned}$$

Similarly, we can write

$$\begin{aligned}\mathcal{F}_{\mathrm{b}} &= i_{\mathrm{b}}[A_1 \cos(\theta_{\mathrm{a}} - 120^\circ) + A_3 \cos 3(\theta_{\mathrm{a}} - 120^\circ) + A_5 \cos 5(\theta_{\mathrm{a}} - 120^\circ)] \\ &= I_{\mathrm{a}} \cos(\omega t - 120^\circ)[A_1 \cos(\theta_{\mathrm{a}} - 120^\circ) + A_3 \cos 3\theta_{\mathrm{a}} + A_5 \cos(5\theta_{\mathrm{a}} + 120^\circ)]\end{aligned}$$

and

$$\begin{aligned}\mathcal{F}_{\mathrm{c}} &= i_{\mathrm{c}}[A_1 \cos(\theta_{\mathrm{a}} + 120^\circ) + A_3 \cos 3(\theta_{\mathrm{a}} + 120^\circ) + A_5 \cos 5(\theta_{\mathrm{a}} + 120^\circ)] \\ &= I_{\mathrm{a}} \cos(\omega t + 120^\circ)[A_1 \cos(\theta_{\mathrm{a}} + 120^\circ) + A_3 \cos 3\theta_{\mathrm{a}} + A_5 \cos(5\theta_{\mathrm{a}} - 120^\circ)]\end{aligned}$$

The total mmf will be

$$\begin{aligned}\mathcal{F}_{\mathrm{tot}} &= \mathcal{F}_{\mathrm{a}} + \mathcal{F}_{\mathrm{b}} + \mathcal{F}_{\mathrm{c}} \\ &= \frac{3}{2} I_{\mathrm{a}}[A_1 \cos(\theta_{\mathrm{a}} - \omega t) A_5 \cos(5\theta_{\mathrm{a}} + \omega t)] \\ &= \frac{3}{2} I_{\mathrm{a}}[A_1 \cos(\theta_{\mathrm{a}} - \omega t) A_5 \cos 5\left(\theta_{\mathrm{a}} + (\frac{\omega t}{5})\right)]\end{aligned}$$

We see that the combined mmf contains only a fundamental space-harmonic component that rotates in the forward direction at angular velocity ω and a 5'th space-harmonic that rotates in the negative direction at angular velocity $\omega/5$.

Problem 4-17

The turns must be modified by a factor of

$$\left(\frac{48}{24}\right)\left(\frac{1800}{1400}\right) = \frac{9}{14} = 2.57$$

Problem 4-18

From Eq. 4.54

$$\Phi_P = \frac{30E_a}{N(\text{poles})n} = 5.55 \text{ mWb}$$

Problem 4-19

Part (a): For a peak air-gap flux density of $B_{\text{peak}} = 1.45$ T, the air-gap flux per pole can be found from Eq. 4.47 as

$$\Phi_P = \frac{2}{\text{poles}} 2lrB_{\text{peak}} = 25.9 \text{ mWb}$$

For this Y-connected machine, the rms line-neutral voltage will be $V_{\text{rms}} = 415/\sqrt{3} = 239.6$ V. The number of series turns per phase can be found from Eq. 4.52

$$N_{\text{ph}} = \frac{V_{\text{rms}}}{\sqrt{2}\pi f_e k_w \Phi_P} = 44.5$$

which in practice will be either 44 or 45 turns.

Part (b): If the motor is to be Δ connected, the number of turns must be increased by a factor of $\sqrt{3}$ to 77.1 (i.e. 77) turns.

Problem 4-20

From Eq. B.27

$$L = \frac{4\mu_0 lr}{\pi g}\left(\frac{2k_{\mathrm{w}}N_{\mathrm{ph}}}{\mathrm{poles}}\right)^2 = 35.4 \ \ \mathrm{mH}$$

Problem 4-21

Part (a): From Eq. 4.52

$$\Phi_{\mathrm{P}} = \frac{\sqrt{2}\,V_{\mathrm{rms}}}{\omega_{\mathrm{e}}k_{\mathrm{w}}N_{\mathrm{ph}}} = 10.8 \ \ \mathrm{mWb}$$

and from Eq. 4.47

$$B_{\mathrm{peak}} = \left(\frac{\mathrm{poles}}{2}\right)\frac{\Phi_{\mathrm{P}}}{2lr} = 0.52 \ \ \mathrm{T}$$

Part (b): From Eq. 4.45

$$I_{\mathrm{f}} = \frac{\pi B_{\mathrm{peak}} g}{2\mu_0 k_{\mathrm{r}} N_{\mathrm{r}}} = 0.65 \quad \mathrm{A}$$

Part (c):

$$L_{\mathrm{af}} = \frac{\lambda_{\mathrm{a,peak}}}{I_{\mathrm{f}}} = \frac{\sqrt{2}\,V_{\mathrm{rms}}/\omega}{I_{\mathrm{f}}} = 0.69 \quad \mathrm{H}$$

Problem 4-22

Part (a): From Eq. 4.47

$$\Phi_{\mathrm{P}} = \left(\frac{2}{\mathrm{poles}}\right)2lrB_{\mathrm{peak}} = 2.31 \ \ \mathrm{Wb}$$

and from Eq. 4.52 the rms phase voltage, in this case equal to the line-line voltage is

$$V_{\mathrm{rms,line-line}} = \sqrt{2}\pi f_{\mathrm{e}} k_{\mathrm{w}} N_{\mathrm{ph}} \Phi_{\mathrm{P}} = 13.8 \;\; \mathrm{kV}$$

Part (b): From Eq. 4.45

$$I_{\mathrm{f}} = \frac{\pi B_{\mathrm{peak}} g}{2\mu_0 k_{\mathrm{r}} N_{\mathrm{r}}} = 1415 \;\; \mathrm{A}$$

Problem 4-23

Part (a): Based upon the solution to Problem 4-22, to achieve an open-circuit voltage of 22 kV (corresponding to a line-neutral phase voltage of 12.7 kV) at 50 Hz, the turns should be increased by a factor of

$$\left(\frac{60}{50}\right)\left(\frac{12.7}{13.8}\right) = 1.10$$

which would correspond to a winding of 26.5 turns/phase. As a practical matter, the choice would thus be 26 or 27 turns per phase, corresponding to an open circuit voltage of 21.6 kV or 22.4 kV.

Part (b): For a 26-turn winding, based upon the value of field current calculated in Problem 4-22, the desired open-circuit voltage will be achieved at a field current of $1415 \times (26.5/26) = 1442$ A. For a 27-turn winding, the field current will be equal to $1415 \times (26.5/27) = 1389$ A.

Problem 4-24

For the given machine, the design requires am armature winding with 26 series turns/phase and a field winding with 200 series turns.

Problem 4-25

From Eq. 4.47

$$\Phi_{\text{peak}} = \left(\frac{2}{\text{poles}}\right) 2lrB_{\text{peak}} = 3.12 \quad \text{Wb}$$

From Eq. 4.9

$$F_{\text{r,peak}} = \frac{4k_{\text{r}}N_{\text{r}}I_{\text{r,max}}}{\pi \times \text{poles}} = 2.01 \times 10^5 \quad \text{A}$$

From Eq. 4.83

$$T_{\text{peak}} = \frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{\text{peak}} F_{\text{r,peak}} = 3.94 \times 10^6 \text{ N·m}$$

and thus

$$P_{\text{peak}} = T_{\text{peak}}\omega_{\text{m}} = 743 \quad \text{MW}$$

Problem 4-26

From Eq. 4.47

$$\Phi_{\text{peak}} = \left(\frac{2}{\text{poles}}\right) 2lrB_{\text{peak}} = 27.8 \quad \text{mWb}$$

From Eq. 4.9

$$F_{\text{r,peak}} = \frac{4k_{\text{r}}N_{\text{r}}I_{\text{r,max}}}{\pi \times \text{poles}} = 415 \quad \text{A}$$

From Eq. 4.83

$$T_{\text{peak}} = \frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{\text{peak}} F_{\text{r,peak}} = 18.1 \text{ N·m}$$

and thus

$$P_{\text{peak}} = T_{\text{peak}}\omega_{\text{m}} = 6.82 \ \ \text{kW}$$

Problem 4-27

The result will be that of Problem 4-26.

Problem 4-28

Part (a):

$$\begin{aligned} T &= i_{\text{a}}i_{\text{f}}\frac{dM_{\text{af}}}{d\theta_0} + i_{\text{b}}i_{\text{f}}\frac{dM_{\text{bf}}}{d\theta_0} \\ &= Mi_{\text{f}}\left(i_{\text{b}}\cos\theta_0 - i_{\text{a}}\sin\theta_0\right) \end{aligned}$$

This expression applies under all operating conditions.

Part (b):

$$T = 2MI_0^2(\cos\theta_0 - \sin\theta_0) = 2\sqrt{2}\,MI_0^2\sin\left(\theta_0 - \pi/4\right)$$

Provided there are any losses at all, the rotor will come to rest at $\theta_0 = \pi/4$ for which $T = 0$ and $dt/d\theta_0 < 0$.

Part (c):

$$\begin{aligned} T &= \sqrt{2}\,MI_{\text{a}}I_{\text{f}}(\sin\omega t\cos\theta_0 - \cos\omega t\sin\theta_0) \\ &= \sqrt{2}\,MI_{\text{a}}I_{\text{f}}\sin\left(\omega t - \theta_0\right) = \sqrt{2}\,MI_{\text{a}}I_{\text{f}}\sin\delta \end{aligned}$$

Part (d):

$$\begin{aligned} v_{\text{a}} &= R_{\text{a}}i_{\text{a}} + \frac{d}{dt}\left(L_{\text{aa}}i_{\text{a}} + M_{\text{af}}i_{\text{f}}\right) \\ &= \sqrt{2}\,I_{\text{a}}(R_{\text{a}}\cos\omega t - \omega L_{\text{aa}}\sin\omega t) - \omega MI_{\text{f}}\sin\left(\omega t - \delta\right) \end{aligned}$$

$$
\begin{aligned}
v_\mathrm{b} &= R_\mathrm{a} i_\mathrm{b} + \frac{d}{dt}\left(L_\mathrm{aa} i_\mathrm{b} + M_\mathrm{bf} i_\mathrm{f}\right) \\
&= \sqrt{2}\, I_\mathrm{a}(R_\mathrm{a}\sin\omega t + \omega L_\mathrm{aa}\cos\omega t) + \omega M I_\mathrm{f}\cos(\omega t - \delta)
\end{aligned}
$$

Problem 4-29

$$
\begin{aligned}
T &= M I_\mathrm{f}(i_\mathrm{b}\cos\theta_0 - i_\mathrm{a}\sin\theta_0) \\
&= \sqrt{2}\, M I_\mathrm{f}\left[(I_\mathrm{a} + I'/2)\sin\delta + (I'/2)\sin(2\omega t + \delta)\right]
\end{aligned}
$$

The time-averaged torque is thus

$$
< T >= \sqrt{2}\, M I_\mathrm{f}(I_\mathrm{a} + I'/2)\sin\delta
$$

Problem 4-30

Part (a):

$$
\begin{aligned}
T &= \frac{i_\mathrm{a}^2}{2}\frac{dL_\mathrm{aa}}{d\theta_0} + \frac{i_\mathrm{b}^2}{2}\frac{dL_\mathrm{bb}}{d\theta_0} + i_\mathrm{a} i_\mathrm{b}\frac{dL_\mathrm{ab}}{d\theta_0} + i_\mathrm{a} i_\mathrm{f}\frac{dM_\mathrm{af}}{d\theta_0} + i_\mathrm{b} i_\mathrm{f}\frac{dM_\mathrm{bf}}{d\theta_0} \\
&= \sqrt{2}\, I_\mathrm{a} I_\mathrm{f} M\sin\delta + 2 I_\mathrm{a}^2 L_2\sin 2\delta
\end{aligned}
$$

Part (b): Motor if $T > 0$, $\delta > 0$. Generator if $T < 0$, $\delta < 0$.

Part (c): For $I_\mathrm{f} = 0$, there will still be a reluctance torque $T = 2I_\mathrm{a}^2 L_2\sin 2\delta$ and the machine can still operate.

Problem 4-31

Part (a):

$$
v = \frac{f}{\lambda} = 45.5 \quad \mathrm{m/sec}
$$

Part (b): The synchronous rotor velocity is 45.5 m/sec.

Part (c): For a slip of 0.055, the rotor velocity will be $(1 - 0.055) \times 45.5 = 43.0$ m/sec.

Problem 4-32

From Eq. 4.93, recognizing that a balanced three-phase current will result in 1.5 times the peak flux as a single current and that $I_{\mathrm{rms}} = I_{\mathrm{peak}}/\sqrt{2}$

$$I_{\mathrm{rms}} = \frac{B_{\mathrm{peak}}}{\sqrt{2}} \left(\frac{g}{\mu_0}\right) \left(\frac{2}{3}\right) \left(\frac{\pi}{4}\right) \left(\frac{2 \times \mathrm{poles}}{k_{\mathrm{w}} N_{\mathrm{ph}}}\right) = 199 \text{ A}$$

Problem 4-33

Part (a): Defining $\beta = 2\pi/\mathrm{wavelength}$

$$\Phi_{\mathrm{p}} = w \int_0^{\pi/\beta} B_{\mathrm{peak}} \cos \beta x dx = \frac{2wB_{\mathrm{peak}}}{\beta} = 2.12 \text{ mWb}$$

Part (b): Since the rotor is 5 wavelengths long, the armature winding will link 12 poles of flux with 12 turns per pole. Thus, $\lambda_{\mathrm{peak}} = 144\Phi_{\mathrm{p}} = 0.305$ Wb.

Part (c): $\omega = \beta v$ and thus

$$V_{\mathrm{rms}} = \frac{\omega \lambda_{\mathrm{peak}}}{\sqrt{2}} = 50.2 \quad \mathrm{V, rms}$$

PROBLEM SOLUTIONS: Chapter 5

Problem 5-1

Basic equations are $T \propto \Phi_{\rm R} F_{\rm f} \sin\delta_{\rm RF}$. Since the field current is constant, $F_{\rm f}$ is constant, Note also that the resultant flux is proportional to the terminal voltage and inversely to the frequency $\Phi_{\rm R} \propto V_{\rm t}/f$. Thus we can write

$$T \propto \frac{V_{\rm t}\sin\delta_{\rm RF}}{f} \quad \text{or} \quad \delta_{\rm RF} \propto \sin^{-1}\left(\frac{fT}{V_{\rm t}}\right)$$

Similarly, because the power is proportional to the product of the torque and the mechanical speed which is in turn proportional to the frequency ($P \propto fT$), we can write

$$\delta_{\rm RF} \propto \sin^{-1}\left(\frac{P}{V_{\rm t}}\right)$$

Part (a): $\delta_{\rm RF}$ reduced to 30.1°

Part (b): $\delta_{\rm RF}$ unchanged

Part (b): $\delta_{\rm RF}$ unchanged

Part (d): $\delta_{\rm RF}$ increased to 36.3°

Problem 5-2

Part (a): The windings are orthogonal and hence the mutual inductance is zero.

Part (b): Since the two windings are orthogonal, the phases are uncoupled and hence the flux linkage under balanced two-phase operation is unchanged by currents in the other phase. Thus, the equivalent inductance is simply equal to the phase self-inductance.

Problem 5-3

$$L_{\mathrm{ab}} = -\frac{1}{2}\left(L_{\mathrm{aa0}} - L_{\mathrm{al}}\right) = -2.47 \ \ \mathrm{mH}$$

$$L_{\mathrm{s}} = \frac{3}{2}\left(L_{\mathrm{aa0}} - L_{\mathrm{al}}\right) + L_{\mathrm{al}} = 7.79 \ \ \mathrm{mH}$$

Problem 5-4

From

$$L_{\mathrm{ab}} = -\frac{1}{2}\left(L_{\mathrm{aa0}} - L_{\mathrm{al}}\right) \quad \text{and} \quad L_{\mathrm{s}} = \frac{3}{2}\left(L_{\mathrm{aa0}} - L_{\mathrm{al}}\right) + L_{\mathrm{al}}$$

we can write

$$L_{\mathrm{al}} = L_{\mathrm{s}} + 3L_{\mathrm{ab}} = 4.1 \ \ \mathrm{mH}$$

and

$$L_{\mathrm{aa0}} = \frac{2}{3}(L_{\mathrm{s}} - L_{\mathrm{al}}) = 20.2 \ \ \mathrm{mHx2}$$

Problem 5-5

Part (a):

$$L_{\mathrm{af}} = \frac{\sqrt{2}\,V_{\mathrm{l-l,rms}}}{\sqrt{3}\,\omega I_{\mathrm{f}}} = 58.0 \ \mathrm{mH}$$

Part (b): Voltage = (50/60) × (345/515) × 13.8 kV = 7.70 kV.

Problem 5-6

Part (a): The line-line voltage will be $13.8/sqrt3 = 7.97$ kV and the line-neutral voltage will be $13.8/3 = 4.6$ kV.

Part (b): $L_{\rm af}$ is reduced by $\sqrt{3}$ from that of the solution to Problem 5-5, Part (a). Thus $L_{\rm af} = 33.6$ mH.

Problem 5-7

Part (a): The magnitude of the phase current is equal to

$$I_{\rm a} = \frac{40 \times 10^3}{0.9 \times \sqrt{3}\,575} = 44.6 \text{ A}$$

and its phase angle is $-\cos^{-1} 0.9 = -25.8°$. Thus

$$\hat{I}_{\rm a} = 44.6 e^{-j25.8°}$$

Then

$$\hat{E}_{\rm af} = V_{\rm a} - jX_{\rm s}\hat{I}_{\rm a} = \frac{575}{\sqrt{3}} - j4.65 \times 44.6 e^{-j25.8°} = 305.6 \angle -37.7° \text{ V}$$

The field current can be calculated from the magnitude of the generator voltage

$$I_{\rm f} = \frac{\sqrt{2} E_{\rm af}}{\omega L_{\rm af}} = 10.9 \text{ A}$$

Part (b):

$$\hat{E}_{\rm af} = 380.9 \angle -29.4° \text{ V}; \quad I_{\rm f} = 13.6 \text{ A}$$

Part (c):

$$\hat{E}_{\rm af} = 461.9 \angle -23.9° \text{ V}; \quad I_{\rm f} = 16.5 \text{ A}$$

Problem 5-8

Same basic solution as that of Problem 5-7 except replace $\hat{I}_{\rm a}$ by $-\hat{I}_{\rm a}$.

Part (a):

$$\hat{E}_{\rm af} = 461.9 \angle 23.9^\circ \text{ V}; \quad I_{\rm f} = 16.5 \text{ A}$$

Part (b):

$$\hat{E}_{\rm af} = 380.9 \angle 29.4^\circ \text{ V}; \quad I_{\rm f} = 13.6 \text{ A}$$

Part (c):

$$\hat{E}_{\rm af} = 305.6 \angle 37.7^\circ \text{ V}; \quad I_{\rm f} = 10.9 \text{ A}$$

Problem 5-9

Part (a): The magnitude of the phase current is equal to

$$I_{\rm a} = \frac{40 \times 10^3}{0.9 \times \sqrt{3}\, 575} = 44.6 \text{ A}$$

and its phase angle is $-\cos^{-1} 0.9 = -25.8^\circ$. Thus

$$\hat{I}_{\rm a} = 44.6 e^{-j25.8^\circ}$$

Then

$$\hat{E}_{\rm af} = V_{\rm s} - j(X_{\rm s} + X_{\rm f}\hat{I}_{\rm a} = \frac{575}{\sqrt{3}} - j5.60 \times 44.6 e^{-j25.8^\circ} = 316.8 \angle -45.2^\circ \text{ V}$$

The field current can be calculated from the magnitude of the generator voltage

$$I_{\rm f} = \frac{\sqrt{2} E_{\rm af}}{\omega L_{\rm af}} = 11.3 \text{ A}$$

Finally, the motor line-neutral terminal voltage is calculated as

$$V_a = V_s - jX_f\hat{I}_a = 315.8 \ \text{V, line-neutral} = 547.0 \text{ V, line-line}$$

Part (b):

$$\hat{E}_{af} = 415.5 \angle -37.0^\circ \text{ V}; \quad I_f = 14.8 \text{ A}$$

$$V_a = 334.7 \text{ V, line-neutral} = 579.7 \text{ V, line-line}$$

Part (c):

$$\hat{E}_{af} = 495.0 \angle -27.0^\circ \text{ V}; \quad I_f = 17.7 \text{ A}$$

$$V_a = 352.5 \text{ V, line-neutral} = 610.6 \text{ V, line-line}$$

Problem 5-10

Same basic solution as that of Problem 5-9 except replace $\hat{I}_a$ by $-\hat{I}_a$.

Part (a):

$$\hat{E}_{af} = 495.0 \angle 27.0^\circ \text{ V}; \quad I_f = 17.7 \text{ A}$$

$$V_a = 352.5 \text{ V, line-neutral} = 610.6 \text{ V, line-line}$$

Part (b):

$$\hat{E}_{af} = 415.5 \angle 37.0^\circ \text{ V}; \quad I_f = 14.8 \text{ A}$$

$$V_a = 334.7 \text{ V, line-neutral} = 579.7 \text{ V, line-line}$$

Part (c):

$$\hat{E}_{\rm af} = 316.8\ \angle 45.2^\circ\ {\rm V}; \quad I_{\rm f} = 11.3\ {\rm A}$$

$$V_{\rm a} = 315.8\ {\rm V,\ line\text{-}neutral} = 547.0\ {\rm V,\ line\text{-}line}$$

Problem 5-11

Part (a):

$$L_{\rm af} = \frac{\sqrt{2}\,V_{\rm l-l,rms}}{\sqrt{3}\omega I_{\rm f}} = 40.7\ {\rm mH}$$

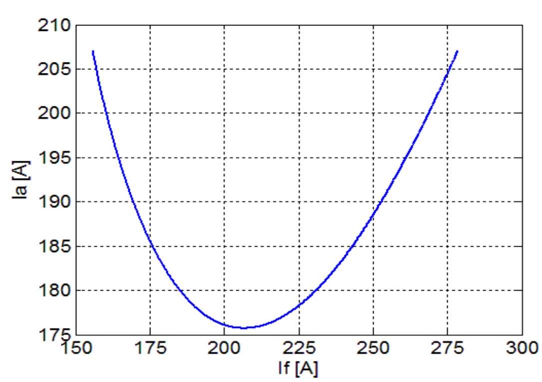
Part (b):

$$\hat{I}_{\rm a} = \frac{700 \times 10^3}{\sqrt{3}\,2300} = 175.7\ {\rm A}$$

$$\hat{E}_{\rm af} = V_{\rm a} - jX_{\rm s}\hat{I}_{\rm a} = 1.87\ \angle - 44.7^\circ\ {\rm kV}$$

$$I_{\rm f} = \frac{\sqrt{2}E_{\rm af}}{\omega L_{\rm af}} = 206.7\ {\rm A}$$

part (c): See plot below. Minimum current will when the motor is operating at unity power factor. From the plot, this occurs at a field current of 207 A.



Problem 5-12

Part (a):

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = \frac{(26 \times 10^3)^2}{910 \times 10^6} = 0.743\ \Omega$$

$$L_{\text{s}} = \frac{X_{\text{s,pu}} Z_{\text{base}}}{\omega} = 3.84\ \text{mH}$$

Part (b):

$$L_{\text{al}} = \frac{X_{\text{al,pu}} Z_{\text{base}}}{\omega} = 0.34\ \text{mH}$$

Part (c):

$$L_{\text{aa}} = \frac{2}{3}(L_{\text{s}} - L_{\text{al}}) + L_{\text{al}} = 2.34\ \text{mH}$$

Part (d):

$$L_{\text{af}} = \frac{\sqrt{2}\, V_{\text{l-l,rms}}}{\sqrt{3}\omega I_{\text{f}}} = 55.0\ \text{mH}$$

Problem 5-13

Part (a):

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = \frac{(11 \times 10^3)^2}{350 \times 10^6} = 0.346\ \Omega$$

$$X_{\text{s}} = X_{\text{s,pu}} Z_{\text{base}} = 0.408\ \Omega$$

$$X_{\text{s,u}} = X_{\text{s,u,pu}} Z_{\text{base}} = 0.408\ \Omega$$

Part (b):

$$\text{AFCS} = X_{\text{s,pu}} \times \text{AFNL} = 1.18 \times 427 = 504 \text{ A}$$

Problem 5-14

From the given data, we see that AFNL = 1690 A. The rated current of this generator is

$$I_{\text{rated}} = \frac{850 \times 10^6}{\text{sqrt3} \times 26 \times 10^3} = 18.9 \ \text{ kA}$$

and thus we see that AFSC = 3260 A. We can also calculate Z_{base}

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = \frac{(26 \times 10^3)^2}{850 \times 10^6} = 0.795 \ \Omega$$

Part (a):

$$\text{SCR} = \frac{\text{AFNL}}{\text{AFSC}} = \frac{1690}{3260} = 0.52$$

Part (b):

$$X_{\text{s,pu}} = \frac{1}{\text{SCR}} = 1.93$$

$$X_{\text{s}} = X_{\text{s,pu}} Z_{\text{base}} = 1.53 \ \Omega$$

Part (c): From the given data, the field current required to produce rated open-circuit voltage on the air-gap line is

$$\text{AFNL}_{\text{ag}} = 1690 \left(\frac{26.0}{29.6} \right) = 1485 \text{ A}$$

$$X_{\text{s,u,pu}} = \frac{\text{AFSC}}{\text{AFNL}_{\text{ag}}} = 2.20$$

$$X_{\mathrm{s,u}} = X_{\mathrm{s,u,pu}} Z_{\mathrm{base}} = 1.75\ \Omega$$

Problem 5-15

From the given data, we see that AFNL = 218 A. The rated current of this generator is

$$I_{\mathrm{rated}} = \frac{4.5 \times 10^6}{\mathrm{sqrt3} \times 4160} = 625 \quad \mathrm{A}$$

and thus we see that AFSC = 203 A. We can also calculate Z_{base}

$$Z_{\mathrm{base}} = \frac{V_{\mathrm{base}}^2}{P_{\mathrm{base}}} = \frac{4160^2}{4.5 \times 10^6} = 3.85\ \Omega$$

Part (a):

$$\mathrm{SCR} = \frac{\mathrm{AFNL}}{\mathrm{AFSC}} = \frac{218}{203} = 1.07$$

Part (b): From the given data, the field current required to produce rated open-circuit voltage on the air-gap line is

$$\mathrm{AFNL}_{\mathrm{ag}} = 218\left(\frac{4160}{4601}\right) = 197\ \mathrm{A}$$

$$X_{\mathrm{s,u,pu}} = \frac{\mathrm{AFSC}}{\mathrm{AFNL}_{\mathrm{ag}}} = 1.03$$

$$X_{\mathrm{s,u}} = X_{\mathrm{s,u,pu}} Z_{\mathrm{base}} = 3.96\ \Omega$$

Part (c):

$$X_{\mathrm{s,pu}} = \frac{1}{\mathrm{SCR}} = 0.93$$

$$X_s = X_{s,pu} Z_{base} = 3.58\ \Omega$$

Part (d):

$$L_s = \frac{X_s}{\omega_e} = 9.50\ \ \mathrm{mH}$$

where $\omega_e = 120\pi$.

$$L_{al} = \frac{X_{al,pu} Z_{base}}{\omega_e} = 1.43\ \ \mathrm{mH}$$

$$L_{a,ag} = \frac{2}{3}(L_s - L_{al}) = 5.38\ \mathrm{mH}$$

Problem 5-16

Check answers with those of Problems 5-14 and 5-15.

Problem 5-17

Part (a):

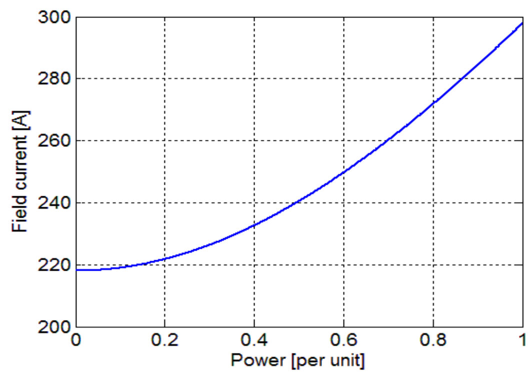
$$\mathrm{AFNL} = 913\ \ \mathrm{A}; \quad \mathrm{AFNL_{ag}} = 772\ \ \mathrm{A}; \quad \mathrm{AFSC} = 925\ \ \mathrm{A}$$

Part (b):

$$\mathrm{SCR} = 0.97; \quad X_{s,pu} = 1.01; \quad X_{s,u} = 1.10\ \Omega$$

$$X_{s,u,pu} = 1.20; \quad X_{s,u} = 1.30\ \Omega$$

Problem 5-18



Problem 5-19

At rated power, unity power factor, the armature current will be $I_a = 4.5$ MW$/(\sqrt{3} \times 4160$ V$) = 624$ A. From Eq.5.39 we can find the phase resistance at 140 C to be

$$R_a(140 \text{ C}) = R_a(25 \text{ C})\left(\frac{234.5 + 140}{234.5 + 25}\right) = 60.6 \text{ m}\Omega$$

The power dissipated in the armature winding will then equal $P_{\text{arm}} = 3 \times 624^2 \times 0.0606 = 70.9$ kW.

Working in per unit, the per-unit field current can be found from

$$E_{\text{af,pu}} = |V_a - (R_{\text{a,pu}} + jX_{\text{s,pu}})\hat{I}_{\text{a,pu}}|$$

where

$$R_{\text{a,pu}} = \frac{R_a(140\ )}{Z_{\text{base}}} = 0.0158$$

and $\hat{I}_{\text{a,pu}} = 1.0$. Thus $E_{\text{af,pu}} = 1.35$

$$I_f = \text{AFNL} \times E_{\text{af,pu}} = 295 \text{ A}$$

At 125°C, the field-winding resistance will be

$$R_{\mathrm{f}}(125\ \mathrm{C}) = R_{\mathrm{f}}(25\ \mathrm{C})\left(\frac{234.5+125}{234.5+25}\right) = 0.302\ \Omega$$

and hence the field-winding power dissipation is $P_{\mathrm{field}} = 295^2 \times 0.302 = 26.3$ kW.

The total loss will then be

$$P_{\mathrm{tot}} = P_{\mathrm{core}} + P_{\mathrm{arm}} + P_{\mathrm{friction/windage}} + P_{\mathrm{field}} = 167.3\ \mathrm{kW}$$

Hence the input power will equal 4.667 MW and the efficiency will equal $4.5/4.667 = 0.974 = 97.4\%$.

Part (b): Same method as Part (a) except that the per-unit terminal current will be

$$\hat{I}_{\mathrm{a,pu}} = \frac{3.5}{0.8 \times 4.5} e^{j\theta} = 0.972 e^{j\theta}$$

where $\theta = \cos^{-1}(0.8) = 36.9°$. For this condition, the input power is 3.649 MW and the efficiency is 95.9%.

Problem 5-20

Part (a): Working in per unit

$$\hat{E}_{\mathrm{af}} = V_{\mathrm{eq}} + j(X_{\mathrm{eq}} + X_{\mathrm{s}})\hat{I}_{\mathrm{a}}$$

where $V_{\mathrm{eq}} = 1.0$. The per-unit power is $P = 110/125 = 0.88$ per-unit and since the system is operating at unity power factor at the equivalent source the per-unit armature current

$$\hat{I}_{\mathrm{a}} = \frac{P}{V_{eq}} = 0.88$$

Thus $\hat{E}_{\mathrm{a}} = 1.66\angle 52.9°$ and thus $E_{\mathrm{af}} = 1.66$ per-unit $= 12.2$ kV, line-line. Under this condition, $I_{\mathrm{f}} = 1.66 \times AFNL = 538$ A.

Similarly

$$\hat{V}_{\rm a} = V_{\rm eq} + jX_{\rm eq}\hat{I}_{\rm a} = 1.01\angle 8.51^\circ$$

and thus $V_{\rm a} = 1.01$ per-unit $= 11.12$ kV. We see that the terminal current leads the terminal voltage by 8.51° and the power factor is thus pf $= \cos(8.51^\circ) = 0.989$ leading.

Part (b): In per-unit, the generator terminal voltage is equal to $\hat{V}_{\rm a} = 1.0\angle\delta_{\rm t}$ where, from analogy with Eq. 5.43,

$$\delta_{\rm t} = \sin^{-1}\left(\frac{PX_{\rm eq}}{V_{\rm eq}V_{\rm a}}\right) = 8.60^\circ$$

The current can be found as

$$\hat{I}_{\rm a} = \frac{(\hat{V}_{\rm t} - V_{\rm eq})}{jX_{\rm eq}} = 0.882\angle 4.30^\circ$$

The base current is equal to $I_{\rm base} = S_{\rm base}/(\sqrt{3} \times V_{\rm base}) = 6.56$ kA and thus $I_{\rm a} = 5.79$ kA.

The solution proceeds as in Part (a).

$$E_{\rm af} = 1.60 \text{ per-unit} = 17.6\ \text{ kV}$$

$$\text{pf} = 0.997 \text{ lagging}$$

Problem 5-21

Part (a): Work in per unit.

$$Z_{\rm base} = \frac{V_{\rm rated}^2}{P_{\rm rated}} = 17.3\ \Omega$$

and thus in per unit, $X_{\rm s} = 19.4/17.3 = 1.12$. With unity per-unit terminal voltage ($V_{\rm a} = 1.0$ and 0.5 per unit real power ($P = 0.5$) at unity power factor, $I_{\rm a} = P/V_{\rm a} = 0.5$ and

$$\hat{E}_{\rm af} = V_{\rm a} - jX_{\rm s}I_{\rm a} = 1.15\angle - 0.51^\circ$$

and thus $I_{\rm f} = AFNL \times Eaf = 142$ A.

Part (b): With $P = 0.8$, $V_{\rm a}$ and $E_{\rm af} = 1.15$, we can find the power angle δ from Eq. 5.43 as

$$\delta = \sin^{-1}\left(\frac{PX_{\rm s}}{V_{\rm a}E_{\rm af}}\right) = -51.47^\circ$$

and thus

$$\hat{I}_{\rm a} = \frac{(V_{\rm a} - \hat{E}_{\rm af})}{jX_{\rm s}} = 0.84\angle - 17.7^\circ$$

Thus the power factor is pf $= \cos(-17.7^\circ) = 0.95$ lagging.

Part (c): The same methodology as Part (a) but with P = 0.8 per unit. The result is $I_{\rm f} = 191$ A.

Problem 5-22

Will work in per unit. From the solution of Problem 5-15, $X_{\rm s} = 0.93$, $X_{\rm s,u} = 1.03$ and AFNL = 218 A.

Part (a): The motor is operating at unity per-unit terminal voltage ($V_{\rm a} = 1.0$) and per-unit real power $P = 3.6/4.5 = 0.80$. At a power factor of 0.87, the per-unit apparent power is $S = P/0.87 = 0.920$. Thus the terminal current is

$$\hat{I}_{\rm a} = \left(\frac{S}{V_{\rm a}}\right)e^{{\rm j}\theta} = 0.92e^{{\rm j}\theta}$$

where $\theta = \cos^{-1}(0.87) = 29.5^\circ$. Thus, using the saturated synchronous reactance

$$\hat{E}_{\rm af} = V_{\rm a} - jX_{\rm s}\hat{I}_{\rm a} = 1.51\angle - 25.4^\circ$$

and the field current is $I_{\mathrm{f}} = E_{\mathrm{af}} \times \mathrm{AFNL} = 330$ A.

Part (b): If the machine speed remains constant and the field current is not reduced, the terminal voltage will increase to the value corresponding to 330 A of field current on the open-circuit saturation characteristic. Based upon a MATLAB spline fit of the given data, this corresponds to a terminal voltage of 5.37 kV line-to-line.

Problem 5-23

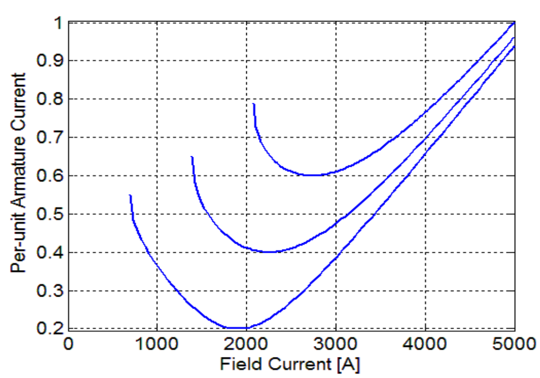
Part (a): The minimum field current will correspond to a power angle of $\delta = 90°$. For $V_{\mathrm{a}} = 1.0$ per unit and for a given per-unit power P, the corresponding per-unit generated voltage will be

$$E_{\mathrm{af}} = \frac{PX_{\mathrm{s}}}{V_{\mathrm{a}}}$$

and the corresponding field current will be given by $I_{\mathrm{f}} = \mathrm{AFNL} \times E_{\mathrm{af}}$. Thus

P	Minimum I_{f} [A]
0.2	692
0.4	1385
0.6	2077

Part (b):



Problem 5-24

Part (a): The per-unit terminal voltage and terminal current are both equal to unity. For

$$\hat{I}_{\rm a} = 1.0e^{j\theta}$$

For any given value of θ the per-unit real power will be equal to $P = \text{Re}[Va\hat{I}_{\rm a}^*] = \cos\theta$ and the corresponding generated voltage will be

$$\hat{E}_{\rm af} = V_{\rm a} + jX_{\rm s}\hat{I}_{\rm a}$$

A MATLAB search over all values of θ in the range $-\pi/2 \leq \theta \leq \pi/2$ gives the corresponding values of P and $E_{\rm af}$. The maximum value of P such that $E_{\rm af} \leq 1.75$ is $P = 0.874$ per unit. The correspond reactive power is 0.486 per unit and the power factor is 0.874 leading, hence the generator is absorbing reactive power.

Part (b): The machine will supply maximum reactive power when $P = 0$ and $E_{\rm af}$ is at its maximum value of 1.75 per unit. Under this condition.

$$\hat{I}_{\rm a} = \frac{E - V_{\rm a}}{jX_{\rm s}} = -j0.375 \text{ per-unit}$$

and $Q_{\rm max} = V_{\rm a}\hat{I}_{\rm a}^* = 0.375$ per unit.

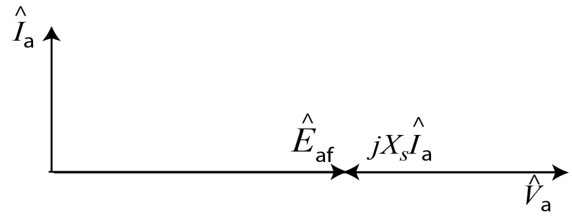
NOTE: The solution of this problem can be visualized in terms of a synchronous-machine capability curve as in Fig.5.19.

Problem 5-25

Part (a): $Z_{\rm base} = V_{\rm base}^2 P_{\rm base} = 4.23\ \Omega$.

$$X_{\rm s} = \frac{1}{\text{SCR}} = 0.60 \text{ per unit} = 2.52\ \Omega$$

Part (b): For $I_{\rm f} = 260$ A, $E_{\rm af} = 260/490 = 0.53$ per unit. With $V_{\rm a} = 1.0$, the corresponding phase diagram is



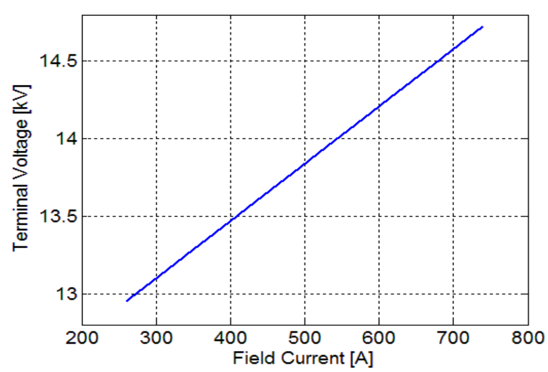
Part (c): $I_{base} = P_{base}/(\sqrt{3}V_{base})$ =1,883 A.

$$I_a = \frac{V_a - E_{af}}{X_s} = 0.79 \text{ per unit} = 1485 \quad \text{A}$$

Part (d): Under this condition, the generator appears inductive and it is absorbing reactive power from the system.

Part (e): $E_{af} = 1.51$ per unit, $I_a = 0.86 = 1614$ A (lagging) and the generator is supplying reactive power and looks capacitive.

Problem 5-26



Problem 5-27

Part (a): It was absorbing reactive power.

Part (b): The terminal voltage increased.

Part (c): Answers the same whether the machine is operating as a motor or a generator.

Problem 5-28

Part (a):

$$X_{\rm s} = \frac{413}{1520} = 0.272 \text{ per-unit}$$

Part (b): $P = 43/50 = 0.86$ and $S = P/0.9 = 0.956$, both in per unit. The power-factor angle is $-\cos^{-1} 0.9 = -25.8°$ and thus $\hat{I}_{\rm a} = 0.956\angle - 25.8°$.

$$\hat{E}_{\rm af} = V_{\rm a} + jX_s\hat{I}_{\rm a} = 1.137\angle 11.9° \quad \text{per-unit}$$

The field current is $I_{\rm f} = \text{AFNL} \times E_{\rm af} = 1729$ A. The rotor angle is 11.9° and the reactive power is

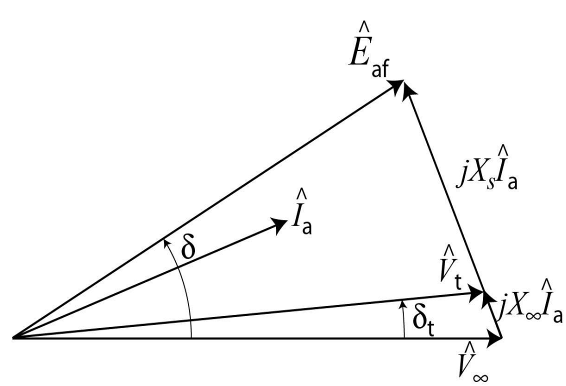
$$Q = \sqrt{S^2 - P^2} = 0.42 \text{ per unit} = 4.24 \text{ MVAR}$$

Part (c): Now $E_{\rm af} = 1.0$ per unit. Same method gives $Q = -0.10$ per unit = -5.09 MVAR. The rotor angle is increased to 13.5 degrees.

Problem 5-29

Part (a):

(i) Generic phasor diagram



(ii) Working in per-unit, $V_\infty = 1.0$, $V_\mathrm{t} = 26.3/26 = 1.012$ and $P = 375/550 = 0.682$. The base impedance is $Z_\mathrm{base} = 26^2/550 = 1.229\ \Omega$ and thus $X_\infty = 0.35$ per unit. From Eq. 5.43

$$\delta_\mathrm{t} = \sin^{-1}\left(\frac{PX_\infty}{V_\infty V_\mathrm{t}}\right) = 13.64°$$

(iii) With $\hat{V}_\mathrm{t} = V_\mathrm{t} e^{j\delta_\mathrm{t}}$, can find

$$\hat{I}_\mathrm{a} = \frac{\hat{I}_\mathrm{t} - V_\infty}{jX_\infty} = 0.68\angle 4.07°$$

For $I_\mathrm{base} = 550$ MVA/$(\sqrt{3} \times 26$ kV$) = 12.21$ kA and thus $I_\mathrm{a} = 0.68$ per unit $= 8.35$ kA.

(iv) At the generator terminal, $\hat{I}_\mathrm{a}$ lags $\hat{V}_\mathrm{t}$ by $13.64° - 4.07° = 9.57°$ and thus the terminal power factor is pf $= \cos 9.57° = 0.986$ lagging.

(v)

$$\hat{E}_\mathrm{af} = V_\infty + j(X_\infty = X_\mathrm{s})\hat{I}_\mathrm{a} = 1.65\angle 56.8°$$

Part (b):

(ii) $\delta_\mathrm{t} = 18.33°$

(iii) $\hat{I}_\mathrm{a} = 11.19$ kA

(iv) pf $= 0.981$ lagging

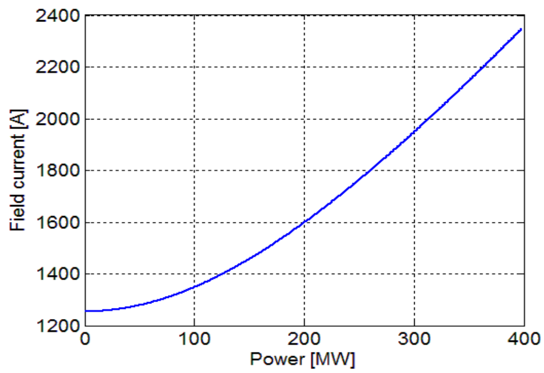
(v) $E_\mathrm{af} == 1.99\angle 67.3°$

Problem 5-30

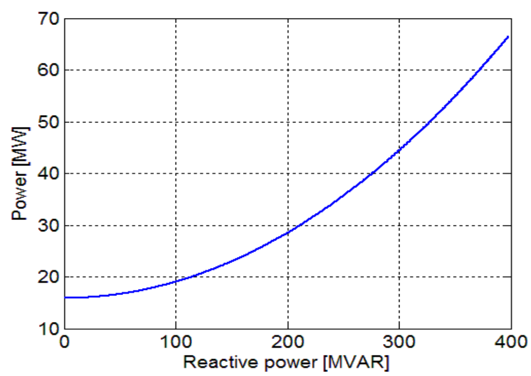
Part (a):

(i) $P_{\max} = 398$ MW

(ii)



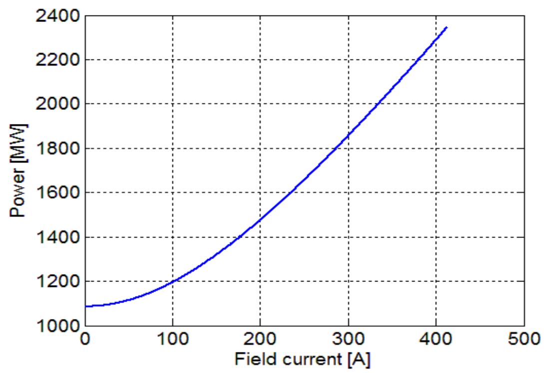
(iii)



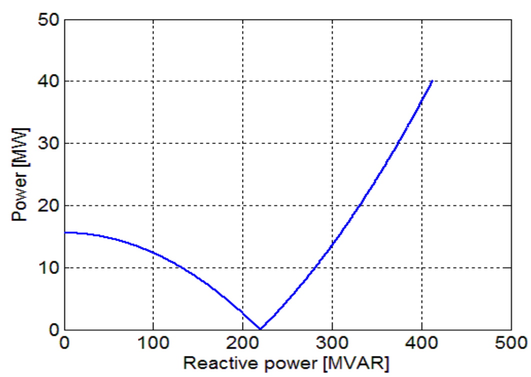
Part (b):

(i) $P_{\text{max}} = 398$ MW

(ii)



(iii)



Problem 5-31

Part (a): Will work in per unit on the generator base. With $Z_{\text{base}} = 26^2/450 = 1.50\ \Omega$, the per-unit transformer reactance is $X_{\text{t}} = .095/1.50 = 0.063$.

(i) With V_∞ and E_{af} both equal to unity, from Eq. 5.46 we see that

$$P_{\text{max}} = \frac{V_\infty E_{\text{af}}}{(X_{\text{t}} + X_{\text{s}}} = 0.58$$

and that the generated cannot be loaded to full load.

(ii) To achieve $P_{\mathrm{max}} = 1.0$, must have $E_{\mathrm{af}} = 1.793$, corresponding to a field current of $I_{\mathrm{f}} = AFNL \times E_{\mathrm{af}} = 3837$ A. Under this operating condition $\hat{E}_{\mathrm{af}} = j1.793$ and the current is equal to

$$\hat{I}_{\mathrm{a}} = \frac{\hat{E}_{\mathrm{af}} - V_{\infty}}{j(X_{\mathrm{s}} + X_{\mathrm{t}})} = 1.14\angle 29.15°$$

and the generator terminal voltage is

$$\hat{V}_{\mathrm{a}} = V_{\infty} + jX_{\mathrm{t}}\hat{I}_{\mathrm{a}} = 0.97\angle 3.75°$$

The generator reactive power is then

$$Q_{\mathrm{gen}} = \mathrm{Im}[\hat{V}_{\mathrm{a}}\hat{I}_{\mathrm{a}}^{*}] = -0.48 \text{ per unit} = -214 \text{ MVAR}$$

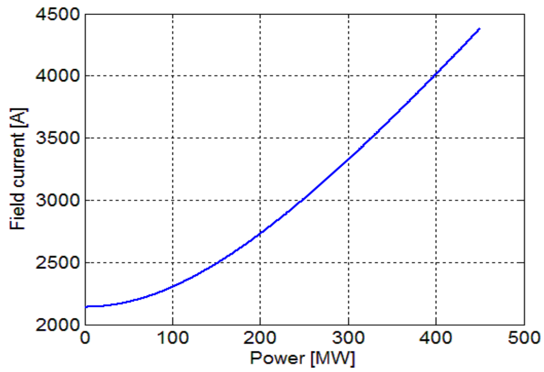
This is of course a totally unacceptable operating condition, both because the generator apparent-power output greatly exceeds its rating and because the power angle is at 90° which is at the verge of instability.

Part (b): On the 345 kV system, the base impedance is $Z_{\mathrm{base}} = 345^2/450 = 264.5\ \Omega$ and thus in per unit $X_{\infty} = 12.4/264.5 = 0.0469$. The solution to this part is identical to that of Part (a) with the exception that one must replace X_{t} by $X_{\mathrm{t}} + X_{\infty}$.

(i) $P_{\mathrm{max}} = 0.54$ therefore cannot load to full load.

(ii) $I_{\mathrm{f}} = 3938$ A and $Q_{\mathrm{gen}} = -0.40$ per unit = -180 MVAR.

Part (c):



Problem 5-32

Following the calculation steps of Example 5.15, $E_{\mathrm{af}} = 1.35$ per unit.

Problem 5-33

Part (a):

(i) Working in per unit and based upon the reactances calculated in the solution to Problem 5-31, let $X_{\mathrm{dT}} = X_{\mathrm{d}} + X_{\mathrm{t}} = 1.793$ per unit and $X_{\mathrm{qT}} = X_{\mathrm{q}} + X$. Setting E_{af} and X_{∞} both equal to unity, the power angle characteristic of Eq. 5.72 gives

$$P = 0.558 \sin \delta + 0.078 \sin (2\delta)$$

The maximum power P_{max} can be found by setting the derivative with respect to δ equal to 0. The result is $\delta_{\mathrm{max}} = 75.83°$ and $P_{\mathrm{max}} = 0.578 = 260$ MW and we see that under this condition the generator cannot supply its rated power.

(ii) The solution for I_{f} is most easily found by a MATLAB search over increasing values of E_{af} until $P_{\mathrm{max}} = 1.0$. The solution is $E_{\mathrm{af}} = 1.772$ and $I_{\mathrm{f}} = 3792$ A.

To find the generator reactive power, I will also use a MATLAB search taking advantage of the following two facts:

1. The per-unit power can be expressed in terms of the magnitude of the phase current I_{a} and its phase angle θ as

$$P = 1.0 = V_\infty I_a cos\theta$$

and thus for any given value of θ, $I_a = P/\cos\theta$.

2. We also know that the phasor $V_\infty + jX_{qT}\hat{I}_a$ must lie along the quadrature axis, i.e. that its angle must be 75.84°.

Combining these two facts, we can search over values of θ until we find the desired solution which is $\hat{I}_a = 1.147\angle 29.301°$.

We can now find the generator terminal voltage

$$\hat{V}_a = V_\infty + jX_t\hat{I}_a = 0.967\angle 3.74°$$

and the reactive power as

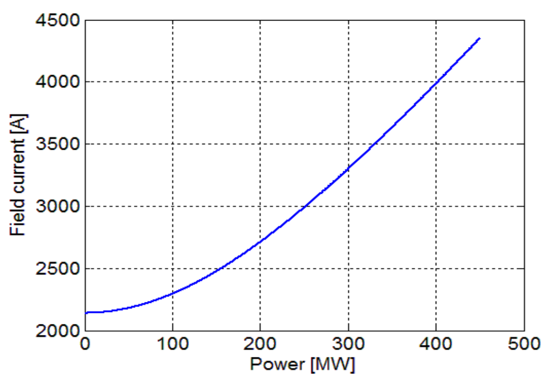
$$Q_{gen} = \mathrm{Im}[\hat{V}_a\hat{I}_a^*] = -0.48 \text{ per unit} = -215 \text{ MVAR}$$

Part (b): Same method but combine X_∞ with X_t

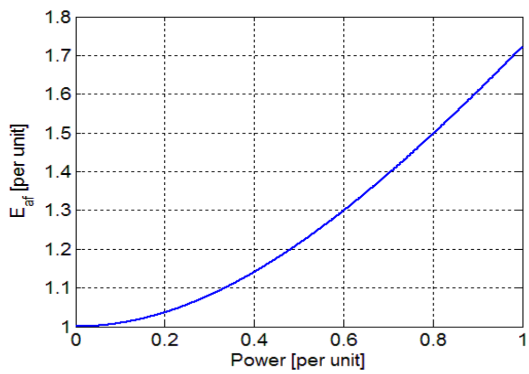
(i) $P_{max} = 0.562$ therefore cannot load to full load.

(ii) $I_f = 3897$ A and $Q_{gen} = -0.40$ per unit = -182 MVAR.

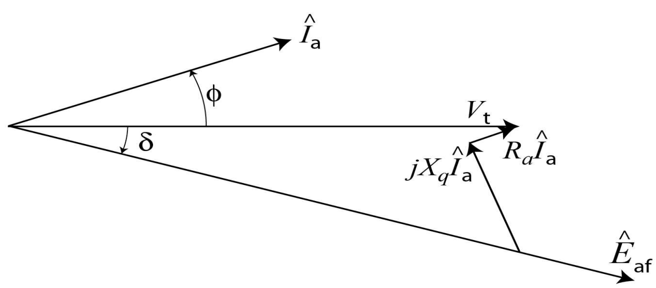
Part (c):



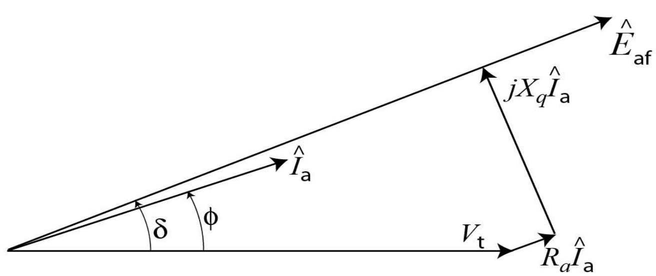
Problem 5-34



Problem 5-35



Problem 5-36



Problem 5-37

Will work in per unit. For $E_{af} = 0$, From Eq. 5.73

$$P_{\max} = \frac{V_{\rm t}^2}{2}\left(\frac{1}{X_{\rm q}} - \frac{1}{X_{\rm d}}\right) = 0.23 = 23\%$$

This maximum power occurs for $\delta = -45°$. Thus, using the motor reference direction for current

$$I_{\rm d} = \frac{V_{\rm q}}{X_{\rm d}} = -\frac{V_{\rm t}\cos\delta}{X_{\rm d}} = 0.615$$

$$I_{\rm q} = -\frac{V_{\rm t}\sin\delta}{X_{\rm q}} = 0.943$$

and thus $I_{\rm a} = \sqrt{I_{\rm d}^2 + I_{\rm q}^2} = 1.27$.

$$S = V_{\rm t} I_{\rm a} = 1.27$$

Hence

$$Q = \sqrt{S^2 - P^2} = 1.25$$

Problem 5-38

Working in per unit, for this operating condition $\hat{V}_{\rm a} = 1.0$ and $\hat{I}_{\rm a} = 1.0$.

Part (a): Can find δ from the angle of the phasor $\hat{V}'$ where

$$\hat{V}' = \hat{V}_{\rm a} - jX_{\rm q}\hat{I}_{\rm a} = 1.25\angle - 36.9°$$

Note the minus sign corresponding to motor notation. Thus $\delta = -36.9°$ and

$$V_{\rm q} = V_{\rm a}\cos\delta = 0.80 \qquad I_{\rm d} = I_{\rm d}\sin\delta = -0.60$$

and thus

$$E_{\mathrm{af}} = V_{\mathrm{q}} - X_{\mathrm{d}} I_{\mathrm{d}} = 1.49$$

and $I_{\mathrm{f}} = E_{\mathrm{af}} \times \mathrm{AFNL} = 1.49 \times \mathrm{AFNL}$

Part (b): Same method gives $I_{\mathrm{f}} = 1.52 \times \mathrm{AFNL}$

Problem 5-39

Part (a): Working in per unit, for this operating condition $\hat{V}_{\mathrm{a}} = 1.0$ and $\hat{I}_{\mathrm{a}} = 1.0$. Can find δ from the angle of the phasor $\hat{V}'$ where

$$\hat{V}' = \hat{V}_{\mathrm{a}} - jX_{\mathrm{q}}\hat{I}_{\mathrm{a}} = 1.26\angle - 37.6^{\circ}$$

Note the minus sign corresponding to motor notation. Thus $\delta = -37.6^{\circ}$ and

$$V_{\mathrm{q}} = V_{\mathrm{a}} \cos\delta = 0.79 \qquad I_{\mathrm{d}} = I_{\mathrm{d}} \sin\delta = -0.61$$

and thus

$$E_{\mathrm{af}} = V_{\mathrm{q}} - X_{\mathrm{d}} I_{\mathrm{d}} = 1.36$$

and $I_{\mathrm{f}} = E_{\mathrm{af}} \times \mathrm{AFNL} = 1.36 \times \mathrm{AFNL}$

Part (b): With $V_{\mathrm{a}} = 1.0$ $E_{\mathrm{af}} = 1.36$, use Eq. 5.73 to find the value of δ such that $P = 0.5$. A MATLAB search gives $\delta = -17.4^{\circ}$ and thus $\hat{E}_{\mathrm{af}} = 1.36\angle - 17.4^{\circ}$. We can then find

$$V_{\mathrm{d}} = V_{\mathrm{a}} \sin\delta = -0.298 \qquad V_{\mathrm{q}} = V_{\mathrm{a}} \cos\delta = 0.954$$

and

$$I_{\mathrm{d}} = \frac{V_{\mathrm{q}} - E_{\mathrm{af}}}{X_{\mathrm{d}}} = -0.436 \qquad I_{\mathrm{q}} = -\frac{V_{\mathrm{d}}}{X_{\mathrm{q}}} = -0.388$$

Recognizing that the quadrature axis lies along the phasor $e^{j\delta}$ and that the direct axis lags the quadrature axis by 90° and thus lies along the phase $-je^{j\delta}$ we can find the terminal current as

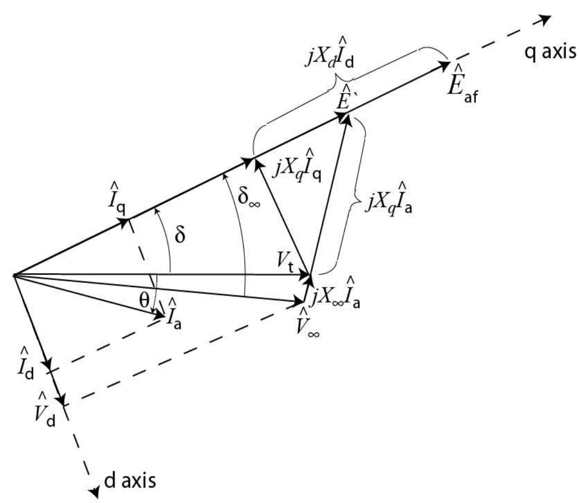
$$\hat{I}_{\rm a} = (-jI_{\rm d} + I_{\rm q})e^{j\delta} = 0.583\angle 30.99°$$

and the reactive power is equal to

$$Q = {\rm Im}[V_{\rm t}\hat{I}_{\rm a}^*] = -0.30$$

Problem 5-40

Part (a): Note that in the following figure the terminal voltage $V_{\rm t}$ has been chosen as the reference phasor and that all angles are defined as positive in the counter-clockwise direction and hence θ has a negative value as drawn.



Part (b): We know that $V_{\rm t} = 1.0$ and that $\hat{I}_{\rm a} = 1.0\angle\theta$ where $\theta = \cos^{-1}(0.95) = -18.19°$. We can thus locate the quadrature axis by the phasor $\hat{E}'$

$$\hat{E}' = V_{\rm t} + jX_{\rm q}\hat{I}_{\rm a} = 1.78\angle 55.8°$$

from which we see that $\delta = 55.8°$.

We can now calculate

$$V_{\rm q} = V_{\rm t}\cos\delta = 0.51 \qquad I_{\rm d} = I_{\rm a}\sin(\delta - \theta) = 0.79$$

and thus $E_{\rm af} = V_{\rm q} + X_{\rm d} I_{\rm d} = 1.88$.

We can also calculate

$$\hat{V}_\infty = V_{\rm t} - jX_\infty \hat{I}_{\rm a} = 0.91\angle - 1.8^\circ$$

and finally that $\delta_\infty = \delta - \angle\hat{V}_\infty = 57.5^\circ$.

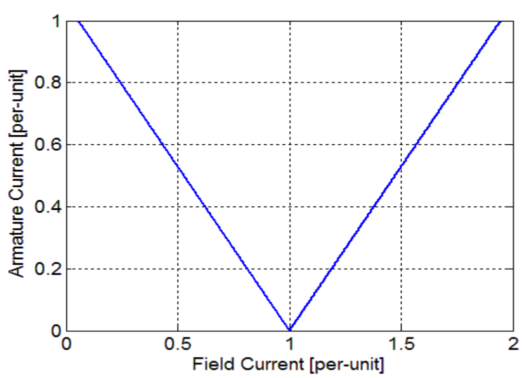
Problem 5-41

Part (a): (i) Define $X_{\rm dT} = X_{\rm d} + X_\infty$. With the generator supplying only reactive power, $\delta = 0$ and the infinite bus voltage $V_\infty = 1.0$ will lie along the quadrature axis and hence $V_{\rm q} = V_\infty$ and $\hat{I}_{\rm a}$ will lie along the direct axis and thus $I_{\rm d} = I_{\rm a}$. With $I_{\rm a}$ in the range $-1.0 \le I_{\rm a} \le 1.0$

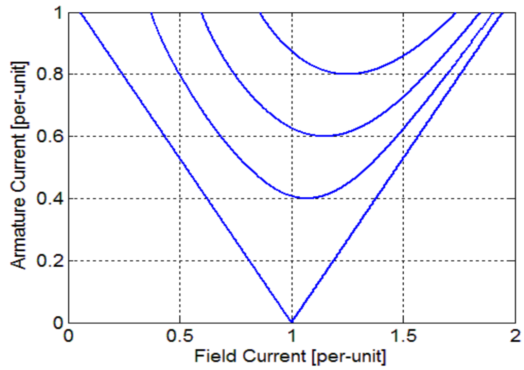
$$E_{\rm af} = V_{\rm q} - X_{\rm dT} I_{\rm a}$$

and we see that $E_{\rm af,min} = 0.13$ and $E_{\rm af,max} = 1.87$.

(ii)



Parts (b) and (c):



Problem 5-42

Will work in per unit. The base current of the synchronous condenser is Ibase $=$ 150 MVA/($\sqrt{3} \times 13.8$ kV $= 6.28$ kA

Part (a): For $V_\infty = 1.0$ and $V_{\rm t} = 13.95/13.8 = 1.011$

$$\hat{I}_{\rm a} = \frac{V_{\rm t} - V_\infty}{jX_{\rm t}} = -j0.169$$

and the terminal current is thus $I_{\rm a} = 0.169 \times 6.28$ kA $= 1.07$ kA.

$$Q = {\rm Im}[V_{\rm t}\hat{I}_{\rm a}^*] = 0.17 = 25.4 \text{ MVAR}$$

Note that $V_{\rm t}$ lies along the quadrature axis and thus $V_{\rm q} = 1.0$. Similarly $\hat{I}_{\rm a}$ lies along the direct axis and thus $\hat{I}_{\rm d} = 0.169$. Thus

$$E_{\rm af} = V_{\rm q} + jX_{\rm d}I_{\rm d} = 1.23$$

and $I_{\rm f} = \text{AFNL} \times E_{\rm af} = 3050$ A.

Part (b): In per-unit, $Q = -85/150 = -0.567$. We know that

$$Q = {\rm Im}[V_{\rm t}\hat{I}^*{\rm a}]$$

and that

$$\hat{I}_a = \frac{V_t - V_\infty}{jX_t}$$

Together these equations give

$$V_t^2 - V_\infty V_t - QX_t = 0$$

from which we find that $V_t = 0.96 = 13.3$ kV. We also can find that $I_a = -0.59 = -3.7$ kA and $I_f = 4300$ A.

Problem 5-43

Part (a):

$$f_e = \left(\frac{\text{poles}}{2}\right)\frac{\text{r/min}}{60} = 66.7 \text{ Hz}$$

Part (b): At 2000 r/min, $\omega_e = 2\pi \times 66.7 = 419$ rad/sec and the generated voltage is $E_{am} = 185$ V, line-line. With $V_t = 208$ V, line-line, $P = 10$ kW and $X_s = \omega_e L_s = 2.35\ \Omega$ we can find the power angle δ from Eq. 5.43

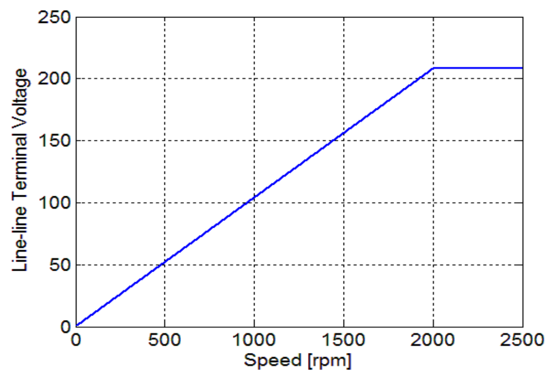
$$\delta = -\sin^{-1}\left(\frac{PX_s}{V_t E_{am}}\right) = -37.6^\circ$$

and the rated terminal current is

$$\hat{I}_a = \frac{V_t - E_{am}e^{j\delta}}{\sqrt{3}(jX_s)} = 31.6 \quad \text{A}\angle 28.6^\circ$$

The power factor is thus $\cos^{-1}(-28.6^\circ) = 0.88$ lagging.

Part (c):



Part (d): At 1500 r/min, $f_e = 50$ Hz, $X_s = 1.76\ \Omega$, $E_{am} = 138.8$ V, line-line and the terminal voltage must be limited to V_t =156 V, line-line. For any given value of power angle δ, the terminal current will be equal to

$$I_a = \left|\hat{I}_a\right| = \left|\frac{V_t - E_{am}e^{j\delta}}{\sqrt{3}(jX_s)}\right|$$

Using MATLAB to search for those values of δ such that $I_a \leq 31.6\ A$, we find that this occurs at $\delta = -37.6°$. From Eq. 5.73, the maximum power is thus 7.5 kW.

Part (e): At 2500 r/min, $f_e = 83.3$ Hz, $X_s = 2.93\ \Omega$, $E_{am} = 231.2$ V, line-line and the terminal voltage must be limited to $V_t = 208$ V, line-line. Again searching for the operating point at which rated current is reached, we find that the maximum power is 11.1 kW.

Problem 5-44

Part (a): For a 3-phase resistive load with $P = 6.8$ kW and $V_t = 189/\sqrt{3}$ V, line-neutral

$$I_a = \frac{P}{3\ \mathrm{V_t}} = 20.8\ A$$

Part (b): Under this operating condition the generated voltage is $E_{am} = 208/\sqrt{3}$ V generator is operating at unity power factor and the terminal current is thus $\hat{I}_a = 20.8\angle 0°$. We know that $E_{am} = 208$ V, line-line and that

$$\hat{E}_{am} = V_t + jX_s\hat{I}_a$$

A corresponding phasor diagram is a right triangle with V_{t} as its base, $\hat{E}_{\mathrm{am}}$ as its hypotenuse and the phasor $jX_{\mathrm{s}}\hat{I}_{\mathrm{a}}$ as its opposite side. Thus we can find

$$X_{\mathrm{s}}I_{\mathrm{a}} = \sqrt{E_{\mathrm{am}}^2 - V_{\mathrm{t}}^2} = 50.14 \quad \mathrm{V}$$

from which we find that $X_{\mathrm{s}} = 2.4\ \Omega$.

Part (c): Under this condition, we know that $P = 3V_{\mathrm{t}}I_{\mathrm{a}}7.5$ kW and that $V^2\mathrm{t} + (X_{\mathrm{s}}I_{\mathrm{a}})^2 = E_{\mathrm{am}}^2$. These equations can be solved simultaneously for V_{t} giving $V_{\mathrm{t}} = 105.6$ V corresponding to a line-line voltage of 183 V.

Problem 5-45

$$\hat{I}_{\mathrm{a}} = \frac{E_{\mathrm{a}}}{R_{\mathrm{a}} + R_{\mathrm{b}} + j\omega L_{\mathrm{a}}} = \frac{\omega K_{\mathrm{a}}}{R_{\mathrm{a}} + R_{\mathrm{b}} + j\omega L_{\mathrm{a}}}$$

Thus

$$|\hat{I}_{\mathrm{a}}| = \frac{\omega K_{\mathrm{a}}}{\sqrt{(R_{\mathrm{a}} + R_{\mathrm{b}})^2 + (\omega L_{\mathrm{a})^2}}} = \frac{K_{\mathrm{a}}}{L_{\mathrm{a}}\sqrt{1 + \left(\frac{R_{\mathrm{a}}+R_{\mathrm{b}}}{\omega L_{\mathrm{a}}}\right)}}$$

Clearly, I_{a} will remain constant with speed as long as the speed is sufficient to insure that $\omega >> (R_{\mathrm{a}} + R_{\mathrm{b}})/L_{\mathrm{a}}$

Problem 5-46

Will work in per-unit on a 25-kW, 460-V, 31.4 A, 8.46 Ω base for which $X_{\mathrm{d}} = 0.26$, $X_{\mathrm{q}} = 0.47$ and $E_{\mathrm{am}} = 0.924$ all with the motor operating at 3600 r/min.

Part (a): With $V_{\mathrm{t}} = 1.0$ and $P = 18/250.72 =$, we can find $\delta = -21.6°$ from Eq. 5.73 and thus

$$V_{\mathrm{d}} = V_{\mathrm{t}} \sin\delta = -0.73$$

$$V_{\mathrm{q}} = V_{\mathrm{t}} \cos\delta = 0.69$$

$$I_{\mathrm{d}} = \frac{V_{\mathrm{q}} - E_{\mathrm{am}}}{X_{\mathrm{d}}} = -0.92$$

$$I_{\mathrm{q}} = -\frac{V_{\mathrm{d}}}{X_{\mathrm{q}}} = 0.08$$

We can now find $\hat{I}_{\mathrm{a}}$ as

$$\hat{I}_{\mathrm{a}} = -jI_{\mathrm{d}}e^{j\delta} + I_{\mathrm{q}}e^{j\delta} = 0.92\angle 38.49^\circ$$

and thus $I_{\mathrm{a}} = 28.9$ A and the power factor is $\cos(38.49^\circ) = 0.78$ leading.

PROBLEM SOLUTIONS: Chapter 6

Problem 6-1

Part (a): Synchronous speed is 1500 r/min. Therefore,

$$s = \frac{1500 - 1458}{1800} = 0.028 = 2.8\%$$

Part (b): Rotor currents are at slip frequency, $f_{\mathrm{r}} = s \times 50 = 1.40$ Hz.

Part (c): The stator flux wave rotates at synchronous speed with respect to the stator: 1500 r/min = 314 rad/sec. It rotates at slip speed ahead of the rotor: $s \times 1500 = 40.8$ r/min = 8.8 rad/sec.

Part (d): The rotor flux wave is synchronous with that of the stator. Thus it rotates at synchronous speed with respect to the stator: 1500 r/min = 314 rad/sec. It rotates at slip speed ahead of the rotor: $s \times 1500 = 40.8$ r/min = 8.8 rad/sec.

Problem 6-2

The rotor-winding flux linkages are lower than those of the stator by the turns ratio 38.42 and, with the motor operating at a slip of 2.31%, the rotor frequency is only 2.31% of that of the stator. Thus the induced rotor winding voltage is equal to 193 V $\times$ (38/42) $\times$ 0.0231 = 4.0 V.

Problem 6-3

Part (a): The rotor slip can be found as the ratio of the frequency of the induced rotor voltage to the applied stator frequency

$$s = \frac{0.73}{50} = 0.0146$$

Based upon a 50-Hz, 6-pole synchronous speed of 1000 r/min, the rotor speed is equal to $(1-s)\times 1000 = 985.4$ r/min.

Part (b):

$$s = \frac{1800 - 1763}{1800} = 0.0206 = 2.06\%$$

and the frequency of the induced voltage is $f_{\mathrm{r}} = s \times 60 = 1.23$ Hz.

Problem 6-4

Part (a): A six-pole, 60-Hz motor has a synchronous speed of 1200 r/min. Therefore, with a no-load speed of 1198r/min, it is clear that this motor has 6 poles.

Part (b): At full load

$$s = \frac{(1200 - 1119)}{1200} = 0.0675 = 6.75\%$$

Part (c):

$$f_{\mathrm{r}} = s \times 60 \text{ Hz} = 4.05 \text{ Hz}$$

Part (d): The rotor field rotates at 1200 r/min (synchronous speed) with respect to the stator. As seen from the rotor, it rotates 81 r/min faster that then rotor.

Problem 6-5

Part (a): The wavelength of the fundamental flux wave is equal to the span of two poles or $\lambda = 6.7/10 = 0.67$ m. The period of the applied excitation is $T = 1/40 = 25$ msec. Thus the synchronous speed is

$$v_{\mathrm{s}} = \frac{\lambda}{T} = 26.8 \text{ m/sec} = 96.5 \text{ km/hr}$$

Part (b): Because this is an induction machine, the car in this case) will never reach synchronous speed.

Part (c):

$$s = \frac{96.5 - 89}{96.5} = 0.0775 = 7.75\%$$

The induced track currents will be a slip frequency, $f = s75 = 4.66$ Hz.

Part (d): For a slip of 7.75% and a car velocity of 75 km/hr, the synchronous velocity must be

$$v_{\mathrm{s}} = \frac{75}{1-s} = 81.3 \text{ km/hr}$$

Thus the electrical frequency must be

$$f_{\mathrm{e}} = 40\left(\frac{81.3}{96.5}\right) = 33.7 \text{ Hz}$$

and the track currents will be at a frequency of $s \times f_{\mathrm{e}} = 2.61$ Hz.

Problem 6-6

The voltage is proportional to the product of the number of turns N, the frequency f and the flux density. Thus for a constant flux density

$$\frac{N_1 f_1}{V_1} = \frac{N_2 f_2}{V_2}$$

and thus

$$N_{50} = 10 \times \frac{60 \times 400}{50 \times 208} = 23 \text{ turns}$$

Problem 6-7

Part (a): From Eq. 6.36 we see that because the torque is proportional to the square of the voltage, the torque-speed characteristic will simply be reduced by a factor of 4.

Part (b): Halving the applied frequency will have the synchronous speed. Neglecting the effects of stator resistance and leakage reactance, we see that the combination of halving the applied voltage and the applied frequency will reduce the factor $V_{1,\mathrm{eq}}^2/\omega_\mathrm{e}$ by a factor of two. Similarly, the factor

$$\frac{(R_2/s)}{(R_2/s)^2 + (X_{1,\mathrm{eq}} + X_2)^2}$$

will be twice its original value for each slip equal to twice the value original. Thus at those 2-times-slip points, the torque will be equal to that of the original full-voltage, full-frequency case. Thus the torque-speed characteristic will appear identical to the full-frequency but shifted such that its synchronous speed point is one-half of that of the original.

The torque-speed characteristics are plotted below.

Problem 6-8

Part (a): 1000 r/min

Part (b): The induction motor rotor is rotating at 1000 r/min in the clockwise direction. Its stator flux wave is rotating at 3000× (2/poles) = 1500 r/min in the counterclockwise direction. Thus, the rotor sees a flux wave rotating at 2500 r/min. Noting that a flux wave rotating at 1500 r/min with respect to the 4-pole rotor would produce 50-Hz voltages at the slip rings, we see that in this case the rotor frequency will be $f_\mathrm{r} = 50\times(2500/1500) = 83.3$ Hz.

Part (c): Now the stator flux wave will rotate at 1500 r/min in the clockwise direction and the rotor will see a flux wave rotating at 500 r/min. The induced voltage will therefore be at a frequency of 16.7 Hz.

Problem 6-9

Part (a): R_1 will decrease by a factor of 1/1.06 to 0.176 Ω.

Part (b): $X_{\rm m}$ will increase by a factor of 1/.85 to 45.7 Ω.

Part (c): R_2 will decrease by a factor of 3.5/5.8 to 0.106 Ω.

Part (d): All values will decrease by a factor of 3.

Problem 6-10

Part (a): The motor input impedance is given by

$$Z_{\rm in} = R_1 + jX_1 + (jX_{\rm m}||R_{\rm c})||(jX_2 + R_2/s)$$

where '$||$' represents "in parallel with". At a slip of 3.5%, $Z_{\rm in} = 6.74 + j2.42$ Ω. Thus,

$$\hat{I}_1 = \frac{V_1}{Z_{\rm in}} = 37.1\angle - 19.7^\circ \quad {\rm A}$$

where $V_1 = 460/\sqrt{3} = 265.6$ V.

The real input power is given by

$$P_{\rm in} = 3\,{\rm Re}[V_1\hat{I}_1^*] = 27.8 \ {\rm kW}$$

and the reactive power by

$$Q_{\rm in} = 3\,{\rm Im}[V_1\hat{I}_1^*] = 10.0 \ {\rm kVAR}$$

Part (b):

$$\hat{I}_2 = \hat{I}_1\left(\frac{jX_{\rm m}||R_{\rm c}}{(jX_{\rm m}||R_{\rm c}) + jX_2 + R_2/s}\right) = 36.0\angle - 15.2^\circ \quad {\rm A}$$

The output power is then given by

$$P_{\text{out}} = 3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\text{FW}} = 25.74 \text{ kW} - 270 \text{ W} = 25.47 \text{ kW}$$

and the power dissipated in the rotor is

$$P_{\text{rotor}} = 3I_2^2R_2 = 934 \text{ W}$$

Part(c): The voltage $\hat{V}_{\text{c}}$ across the motor core-loss resistance is given by

$$\hat{V}_{\text{c}} = V_1 - (R_1 + jX_1) = 248.9\angle - 7.97^\circ \quad \text{A}$$

and thus the core loss is given

$$P_{\text{core}} = 3\,\frac{V_{\text{c}}^2}{R_{\text{c}}} = 427 \text{ W}$$

and the efficiency is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 91.6\%$$

Problem 6-11

Same solution as Problem 6-10.

Problem 6-12

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\text{c}} = \frac{3V_1^2}{P_{\text{core}}} = 651 \ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1 - s) \times \text{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\text{in}} = jX_{\text{m}}||(R_1 + jX_1 + jX_{\text{m}}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\text{in}}}$$

The real input power is given by

$$P_{\text{in}} = 3\,\text{Re}[V_1 \hat{I}_1^*]$$

and the power factor is

$$\text{pf} = \frac{P_{\text{in}}}{3V_1 I_1}$$

The shaft output power is then given by

$$P_{\text{out}} = 3I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\text{FW}}$$

where

$$\hat{I}_2 = \hat{I}_1 \left(\frac{jX_{\text{m}}}{jX_{\text{m}} + jX_2 + R_2/s}\right)$$

and the shaft torque as

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_{\text{m}}}$$

where $\omega_{\text{m}} = (2/\text{poles}) \times \omega_{\text{e}}$

Finally, the efficiency is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

Thus

slip [%]	rpm	P_{in} [kW]	pf	P_{shaft} [kW]	T_{shaft} [N·m]	η [%]
1.0	1782	13.6	0.90	12.7	67.8	93.2
2.0	1764	26.0	0.93	24.5	132.7	94.3
3.0	1746	37.0	0.92	34.7	189.6	93.7

Problem 6-13

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\text{c}} = \frac{3V_1^2}{P_{\text{core}}} = 168\ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

The motor input impedance is given by

$$Z_{\text{in}} = R_{\text{c}}||(R_1 + jX_1 + jX_{\text{m}}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\text{in}}}$$

The shaft output power is given by

$$P_{\text{out}} = 3I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\text{FW}}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_{\text{c}}})\left(\frac{jX_{\text{m}}}{jX_{\text{m}} + jX_2 + R_2/s}\right)$$

Since the slip ($s = (1800 - 1780.7)/1800 = 0.0107$) and all other motor parameters are known, the value of R_2 which produces the desired output power can be found using a MATLAB search. The result is $R_2 = 21.1$ mΩ.

Problem 6-14

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_c = \frac{3V_1^2}{P_{\rm core}} = 271\ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1 - s) \times \mathrm{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\rm in} = R_c||(R_1 + jX_1 + jX_m||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\rm in}}$$

The real input power is given by

$$P_{\rm in} = 3\,\mathrm{Re}[V_1\hat{I}_1^*]$$

and the power factor is

$$\mathrm{pf} = \frac{P_{\rm in}}{3V_1I_1}$$

The shaft output power is then given by

$$P_{\rm out} = 3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\rm FW}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_c})\left(\frac{jX_m}{jX_m + jX_2 + R_2/s}\right)$$

and the shaft torque as

$$T_{\mathrm{out}} = \frac{P_{\mathrm{out}}}{\omega_{\mathrm{m}}}$$

where $\omega_{\mathrm{m}} = (2/\mathrm{poles}) \times \omega_{\mathrm{e}}$

Finally, the efficiency is given by

$$\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \times 100\%$$

Parts (a) and (b): The following table can be constructed from a MATLAB search for values of slip which match the desired levels of output power.

% of Full Load	P_{out} [kW]	slip [%]	rpm	I_1 [A]	pf	η [%]
100	75.0	2.23	1760	105.6	0.95	94.3
75	56.2	1.63	1771	78.9	0.95	94.3
50	37.5	1.07	1781	53.6	0.94	93.4
25	18.8	0.54	1790	30.0	0.88	89.5
0	0.0	0.03	1799	13.5	0.19	0.0

Problem 6-15

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\mathrm{c}} = \frac{3V_1^2}{P_{\mathrm{core}}} = 271\ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1 - s) \times \mathrm{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\mathrm{in}} = R_{\mathrm{c}}||(R_1 + jX_1 + jX_{\mathrm{m}}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\rm in}}$$

The real input power is given by

$$P_{\rm in} = 3\,{\rm Re}[V_1 \hat{I}_1^*]$$

and the power factor is

$${\rm pf} = \frac{P_{\rm in}}{3 V_1 I_1}$$

The shaft output power is then given by

$$P_{\rm out} = 3 I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\rm FW}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_{\rm c}})\left(\frac{jX_{\rm m}}{jX_{\rm m} + jX_2 + R_2/s}\right)$$

Finally, the efficiency is given by

$$\eta = \frac{P_{\rm out}}{P_{\rm in}} \times 100\%$$

Part (a): A MATLAB search for that value of slip which produces rated output power gives the following operating condition:

$P_{\rm out}$ [kW]	slip [%]	rpm	I_1 [A]	pf	η [%]
75.0	2.28	1759	107.6	0.95	92.6

Part (b): At 50 Hz, the terminal voltage will be equal to $460 \times (50/60) = 383.3$ V, line-line ($V_1 = 221.3$ V, line-neutral), the synchronous speed will be 1500 r/min and the reactance will be 5/6 of their 60-Hz values. The friction and windage loss will be $1250 \times (50/60)^3 = 723$ W. Because the voltage is proportional to frequency, the core loss, calculated by the loss in the core-loss resistance, will automatically vary with frequency. A MATLAB search over slip for the value which corresponds to the rated current of 94.1 A gives

slip [%]	rpm	I_1 [A]	P_{out} [kW]	pf	η [%]
2.37	1464	94.1	55.5	0.95	93.7

Problem 6-16

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\text{c}} = \frac{3V_1^2}{P_{\text{core}}} = 651\ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1 - s) \times \text{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\text{in}} = R_{\text{c}}||(R_1 + jX_1 + jX_{\text{m}}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\text{in}}}$$

The real input power is given by

$$P_{\text{in}} = 3\,\text{Re}[V_1 \hat{I}_1^*]$$

and the power factor is

$$\text{pf} = \frac{P_{\text{in}}}{3V_1 I_1}$$

The shaft output power is then given by

$$P_{\text{out}} = 3I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\text{FW}}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_c})\left(\frac{jX_m}{jX_m + jX_2 + R_2/s}\right)$$

Finally, the efficiency is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

Part (a): A MATLAB search for the slip which produces an output power of 37 kW gives a speed of 1741.3 rpm.

Part (b): Similarly, the motor will operate at a speed of 1799.5 rpm when the shaft power is zero (corresponding to an electromechanical power of 325 W to supply the friction/windage loss.

Part (c):

Problem 6-17

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_c = \frac{3V_1^2}{P_{core}} = 2.94 \ \ \text{k}\Omega$$

where $V_1 = 1905$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1-s) \times \text{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\text{in}} = R_{\text{c}}||(R_1 + jX_1 + jX_{\text{m}}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\text{in}}}$$

The real input power is given by

$$P_{\text{in}} = 3\,\text{Re}[V_1\hat{I}_1^*]$$

and the power factor is

$$\text{pf} = \frac{P_{\text{in}}}{3V_1I_1}$$

The shaft output power is then given by

$$P_{\text{out}} = 3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\text{FW}}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_{\text{c}}})\left(\frac{jX_{\text{m}}}{jX_{\text{m}} + jX_2 + R_2/s}\right)$$

Finally, the efficiency is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

Thus, at a speed of 1466 rpm, corresponding to a slip of 2.27%

slip [%]	rpm	P_{in} [kW]	P_{out} [kW]	pf	η [%]
2.27	1466	744	707	0.91	95.0

Problem 6-18

A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\rm c} = \frac{3V_1^2}{P_{\rm core}} = 192\ \Omega$$

where $V_1 = 265.6$ V is the line-neutral terminal voltage.

For a given slip s, the speed is equal to $(1-s) \times \text{rpm}_0$ where rpm_0 is the synchronous speed of 1800 rpm. The motor input impedance is given by

$$Z_{\rm in} = R_{\rm c}||(R_1 + jX_1 + jX_{\rm m}||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\rm in}}$$

The real input power is given by

$$P_{\rm in} = 3\,{\rm Re}[V_1\hat{I}_1^*]$$

and the power factor is

$$\text{pf} = \frac{P_{\rm in}}{3V_1I_1}$$

The shaft output power is then given by

$$P_{\rm out} = 3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\rm FW}$$

where

$$\hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_{\rm c}})\left(\frac{jX_{\rm m}}{jX_{\rm m} + jX_2 + R_2/s}\right)$$

Finally, the efficiency is given by

$$\eta = \frac{P_{\rm out}}{P_{\rm in}} \times 100\%$$

Parts (a) and (b): Using MATLAB to search over slip, the rated output power (120 kW) operating condition can be found. The solution with the copper squirrel cage is obtained by reducing R_2 by a factor of 1.5 to $R_2 = 23.0\ \ \mathrm{m\Omega}$.

Cage	slip [%]	rpm	$I_{\rm in}$	$P_{\rm in}$ [kW]	$P_{\rm out}$ [kW]	pf	η [%]
Aluminum	2.27	1173	171.2	127.6	120.0	0.94	94.0
Copper	1.50	1182	169.8	126.6	120.0	0.94	94.8

Part (c): Here is the complete table

Cage	slip [%]	rpm	$I_{\rm in}$	$P_{\rm in}$ [kW]	$P_{\rm out}$ [kW]	pf	η [%]
Aluminum	2.27	1173	171.2	127.6	120.0	0.94	94.0
Copper	1.50	1182	169.8	126.6	120.0	0.94	94.8
Aluminum	1.63	1180	127.2	95.8	90.0	0.95	94.0
Copper	1.08	1187	126.4	95.2	90.0	0.95	94.5
Aluminum	1.06	1187	86.1	64.5	60.0	0.94	93.0
Copper	0.71	1192	85.8	64.3	60.0	0.94	93.3
Aluminum	0.53	1194	47.8	33.8	30.0	0.89	88.8
Copper	0.35	1196	47.7	33.7	30.0	0.89	88.9

Problem 6-19

(i): The synchronous speed of a 60-Hz, 6-pole motor is 1200 rpm. The speed at a slip of 3.2% is $(1-0.032)\times 1200 = 1161.6$ rpm, corresponding to $\omega_{\rm m,rated} = 121.6$ rad/sec. Thus the rated torque is

$$T_{\rm rated} = \frac{P_{\rm rated}}{\omega_{\rm m,rated}} = 82.2\ \ \mathrm{N\cdot m}$$

(ii) From Eq. 6.32, $V_{\rm 1,eq} = 262.8$ V and from Eq. 6.33 = $Z_{\rm 1,eq} = 1.234 + j2.263\ \Omega$. Thus, from Eq. 6.39, $T_{\rm max} = 130.8$ N·m and from Eq. 6.38, $s_{\rm maxT} = 0.1000$ corresponding to a speed of 1080 rpm.

(iii) From Eq. 6.36 with $s = 0$, $T_{\text{start}} = 30.5$ N·m. The motor input impedance is given by

$$Z_{\text{in}} = R_1 + jX_1 + jX_{\text{m}}||(jX_2 + R_2)$$

where '$||$' represents "in parallel with" and the terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\text{in}}}$$

Thus, $I_{\text{start}} = 50.7$ A.

Problem 6-20

Assuming $R_1 = 0$ and defining

$$X = X_{1,\text{eq}} + X_2$$

$$k = \frac{0.5qV_{1,\text{eq}}^2}{\omega_{\text{s}}}$$

where $\omega_{\text{s}} = 2\omega_{\text{e}}/\text{poles}$.

We can write from Eq. 6.37

$$s_{\text{maxT}} = \frac{R_2}{X}$$

and from Eq. 6.39

$$T_{\text{max}} = \frac{0.5k}{X}$$

Similarly, we can write

$$T_{\text{start}} = \frac{kR_2}{R_2^2 + X^2}$$

and

$$T_{\text{rated}} = \frac{2kR_2/s_{\text{rated}}}{R_2^2 + X^2}$$

Part (a): From these equations, we can show that

$$s_{\text{maxT}}^2 - 2s_{\text{maxT}}\left(\frac{T_{\text{max}}}{T_{\text{start}}}\right) + 1 = 0$$

from which we find $s_{\text{maxT}} = 0.268$.

Part (b): We can also show that

$$\left(\frac{s_{\text{maxT}}}{s_{\text{rated}}}\right)^2 - 2\left(\frac{T_{\text{max}}}{T_{\text{rated}}}\right)\left(\frac{s_{\text{maxT}}}{s_{\text{rated}}}\right) + 1 = 0$$

from which we find that $s_{\text{rated}} = 0.061$.

Part (c): Finally, we can show that

$$\frac{I_{\text{start}}}{I_{\text{rated}}} = \sqrt{\frac{1 + (s_{\text{maxT}}/s_{\text{rated}})^2}{1 + s_{\text{maxT}}^2}} = 4.33 = 433\%$$

Problem 6-21

From Eq.6.36, we can find

$$R^* = \frac{R_{1,\text{eq}}}{R_2} = \frac{0.5/s_{\text{fl}} + 0.5s_{\text{fl}}/s_{\text{maxT}}^2 - T^*/s_{\text{maxT}}}{T^* - 1}$$

where $T^* = T_{\text{max}}/T_{\text{fl}}$ and that

$$\frac{T_{\rm start}}{T_{\rm fl}} = \frac{s_{\rm fl}(2s_{\rm maxT}^2 R^*/s_{\rm fl} + (s_{\rm maxT}/s_{\rm fl})^2)}{2(s_{\rm maxT}R^* + 1)}$$

from which we find that $T_{\rm start} = 1.58T_{\rm fl}$.

Problem 6-22

The motor input impedance is given by

$$Z_{\rm in} = R_1 + jX_1 + jX_{\rm m}||(jX_2 + R_2/s)$$

where '$||$' represents "in parallel with". The total impedance seen from the source is

$$Z_{\rm tot} = jX_{\rm s} + Z_{\rm in}$$

where $X_{\rm s}$ is the source impedance.

The terminal current is

$$\hat{I}_1 = \frac{V_{\rm s}}{Z_{\rm tot}}$$

where $V_{\rm s} = 332.0$ V is the line-neutral source voltage.

The line-neutral motor terminal voltage is equal to

$$\hat{V}_1 = V_{\rm s} - jX_{\rm s}\hat{I}_1$$

The real input power is given by

$$P_{\rm in} = 3\,{\rm Re}[\hat{V}_1\hat{I}_1^*]$$

Note that in this case, because we are considering generator action, the machine will be operating with negative slip and $P_{\rm in}$ will be negative, corresponding to positive generator output power.

A MATLAB search over negative slip shows that the machine will produce an electrical output power of 110 kW at a slip of -1.13%, corresponding to a speed of 1213.5 rpm. The machine terminal voltage will be 34.3 V, line-neutral (561.8 V, line-line).

Problem 6-23

The motor input impedance (not accounting for core loss) is given by

$$Z_{\mathrm{in}} = R_1 + jX_1 + jX_{\mathrm{m}}||(jX_2 + R_2/s)$$

The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\mathrm{in}}}$$

where $V_1 = 1386$ V is the line-neutral source voltage.

The real input power is given by

$$P_{\mathrm{in}} = -3\,\mathrm{Re}[V_1\hat{I}_1^*]$$

The electromechanical shaft output power is then given by

$$P_{\mathrm{out}} = 3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\mathrm{FW}}$$

where

$$\hat{I}_2 = \hat{I}_1\left(\frac{jX_{\mathrm{m}}}{jX_{\mathrm{m}} + jX_2 + R_2/s}\right)$$

Part (a): At a slip of 2.35%, the calculated motor input power is 1.560 MW. From the stated operating condition of rated output power and 95.2% efficiency, the actual motor input power is 1.5 MW/0.952 = 1.575 MW. Thus the motor core loss is equal to

$$P_{\mathrm{core}} = (1.575 - 1.560)\ \ \mathrm{MW} = 15\ \ \mathrm{kW}$$

The calculated output power is equal to 1.507 MW. Thus the friction and windage loss is equal to

$$P_{\mathrm{FW}} = (1.507 - 1.500) \ \ \mathrm{MW} = 7\ kW$$

Part (b): A core-loss resistance connected directly at the motor terminals will produce constant core loss. The core loss resistance can then be calculated as

$$R_{\mathrm{c}} = \frac{3V_1^2}{P_{\mathrm{core}}} = 373\ \Omega$$

Including the effects of core loss, the motor input impedance is

$$Z_{\mathrm{in}} = R_1 + jX_1 + (jX_{\mathrm{m}}||R_{\mathrm{c}})||(jX_2 + R_2/s)$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\mathrm{in}}}$$

For generator operating, the electrical output power is given by

$$P_{\mathrm{out}} = -3\,\mathrm{Re}[V_1\hat{I}_1^*]$$

The electromechanical shaft power is then given by

$$P_{\mathrm{em}} = -3I_2^2R_2\left(\frac{1-s}{s}\right) - P_{\mathrm{FW}}$$

where

$$\hat{I}_2 = \hat{I}_1\left(\frac{jX_{\mathrm{m}}||R_{\mathrm{c}}}{(jX_{\mathrm{m}}||R_{\mathrm{c}}) + jX_2 + R_2/s}\right)$$

and the shaft input power is equal to

$$P_{\mathrm{in}} = P_{\mathrm{FW}} + P_{\mathrm{em}}$$

and the terminal power factor is

$$\text{pf} = \frac{P_{\text{out}}}{3V_1 I_1}$$

Finally, the efficiency is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

At a slip of -2.35%

(i) $P_{\text{out}} = 1.58$ MW
(ii) $\eta = 97.10$ %
(iii) pf = 0.983

Part (c): From the infinite bus, the total impedance is equal to

$$Z_{\text{tot}} = Z_{\text{f}} + Z_{\text{in}}$$

where Z_{f} is the feeder impedance. The solution for the machine performance proceeds the same as that in part (b) with Z_{tot} replacing Z_{in}. The terminal voltage can be found as

$$\hat{V}_1 = V_{\text{inf}} - Z_{\text{f}} \hat{I}_1$$

At a slip of -2.35%, the power at the infinite bus is calculated as

$$P_{\text{inf}} = -3\,\text{Re}[V_{\text{inf}} \hat{I}_1^*] = 1.57 \ \ \text{MW}$$

and at the machine terminals

$$P_{\text{term}} = -3\,\text{Re}[\hat{V}_1 \hat{I}_1^*] = 1.59 \ \ \text{MW}$$

Problem 6-24

Problem 6-25

Part (a): Given $I_{2,\mathrm{maxT}}^2 R_2 = 8.5 I_{2,\mathrm{fl}}^2 R_2$. Thus $I_{2,\mathrm{maxT}} = \sqrt{8.5}\, I_{2,\mathrm{fl}}$. Ignoring R_1, $R_{1,\mathrm{eq}} = 0$ and we can write

$$\hat{I}_2 = \frac{\hat{V}_{\mathrm{eq}}}{R_2/s + j(X_{\mathrm{eq}} + X_2)}$$

and thus

$$\frac{\hat{I}_{2,\mathrm{fl}}}{\hat{I}_{2,\mathrm{maxT}}} = \frac{j(X_{\mathrm{eq}} + X_2) + R_2/s_{\mathrm{maxT}}}{j(X_{\mathrm{eq}} + X_2) + R_2/s_{\mathrm{fl}}}$$

Substitution from Eq. 6.37

$$(X_{1,\mathrm{eq}} + X_2) = \frac{R_2}{s_{\mathrm{maxT}}}$$

gives

$$\frac{\hat{I}_{2,\mathrm{fl}}}{\hat{I}_{2,\mathrm{maxT}}} = \frac{j + 1}{j + s_{\mathrm{maxT}}/s_{\mathrm{fl}}}$$

and thus

$$\frac{I_{2,\mathrm{fl}}}{I_{2,\mathrm{maxT}}} = \frac{|j+1|}{|j + s_{\mathrm{maxT}}/s_{\mathrm{fl}}|} = \frac{\sqrt{2}}{\sqrt{1 + (s_{\mathrm{maxT}}/s_{\mathrm{fl}})^2}}$$

Finally, we can solve for s_{maxT}

$$s_{\mathrm{maxT}} = 4.00 s_{\mathrm{fl}} = 0.104 = 10.4\%$$

Part (b): Taking the ratio of Eqs. 6.39 and 6.36 with $R_{1,\mathrm{eq}} = 0$ and substitution of Eq. 6.37 gives

$$\frac{T_{\mathrm{max}}}{T_{\mathrm{fl}}} = \frac{0.5[(R_2/s_{\mathrm{fl}})^2 + (X_{1,\mathrm{eq}} + X_2)^2]}{(X_{1,\mathrm{eq}} + X_2)(R_2/s_{\mathrm{fl}})} = \frac{0.5[1 + (s_{\mathrm{maxT}}/s_{\mathrm{fl}})^2]}{(s_{\mathrm{maxT}}/s_{\mathrm{fl}})} = 2.13$$

In other words, $T_{\mathrm{max}} = 2.13$ per unit.

Part (c): In a similar fashion, one can show that

$$\frac{T_{\mathrm{start}}}{T_{\mathrm{fl}}} = s_{\mathrm{fl}} \left(\frac{1 + (s_{\mathrm{maxT}}/s_{\mathrm{fl}})^2}{1 + s_{\mathrm{maxT}}^2} \right) = 0.44$$

In other words, $T_{\mathrm{start}} = 0.44$ per unit.

Problem 6-26

Part (a): $T \propto I_2^2 R_2/s$. Thus

$$\frac{T_{\mathrm{start}}}{T_{\mathrm{fl}}} = s_{\mathrm{fl}} \left(\frac{I_{2,\mathrm{start}}}{I_{2,\mathrm{fl}}} \right)^2 = 1.24$$

and thus $T_{\mathrm{start}} = 1.24$ per unit.

Part (b): Ignoring R_1, $R_{1,\mathrm{eq}} = 0$ and we can write

$$\hat{I}_2 = \frac{\hat{V}_{\rm eq}}{R_2/s + j(X_{\rm eq} + X_2)}$$

and thus

$$\frac{\hat{I}_{2,{\rm fl}}}{\hat{I}_{2,{\rm start}}} = \frac{j(X_{\rm eq} + X_2) + R_2}{j(X_{\rm eq} + X_2) + R_2/s_{\rm fl}}$$

Substitution from Eq. 6.37

$$(X_{1,{\rm eq}} + X_2) = \frac{R_2}{s_{\rm maxT}}$$

gives

$$\frac{\hat{I}_{2,{\rm fl}}}{\hat{I}_{2,{\rm start}}} = \frac{j + s_{\rm maxT}}{j + s_{\rm maxT}/s_{\rm fl}}$$

and thus

$$\frac{I_{2,{\rm fl}}}{I_{2,{\rm start}}} = \frac{\sqrt{1 + s_{\rm maxT}^2}}{\sqrt{1 + (s_{\rm maxT}/s_{\rm fl})^2}}$$

This can be solved for $s_{\rm maxT}$

$$s_{\rm maxT} = s_{\rm fl}\sqrt{\frac{1 - (I_{2,{\rm start}}/I_{\rm fl})^2}{(s_{\rm fl}I_{2,{\rm start}}/I_{\rm fl})^2 - 1}} = 0.210 = 21.0\%$$

Taking the ratio of Eqs. 6.39 and 6.36 with $R_{1,{\rm eq}} = 0$ and substitution of Eq. 6.37 gives

$$\frac{T_{\rm max}}{T_{\rm fl}} = \frac{0.5[(R_2/s_{\rm fl})^2 + (X_{1,{\rm eq}} + X_2)^2]}{(X_{1,{\rm eq}} + X_2)(R_2/s_{\rm fl})} = \frac{0.5[1 + (s_{\rm maxT}/s_{\rm fl})^2]}{(s_{\rm maxT}/s_{\rm fl})} = 3.08$$

In other words, $T_{\rm max} = 3.08$ per unit.

Problem 6-27

Neglecting R_1 and hence $R_{\mathrm{eq},1}$ gives from Eq. 6.38

$$s_{\mathrm{maxT}} = \frac{R_2}{X_{1,\mathrm{eq}} + X_2}$$

and from Eq. 6.39

$$T_{\mathrm{max}} = \frac{0.5 n_{\mathrm{ph}} V_{1,\mathrm{eq}}^2}{\omega_{\mathrm{s}}(X_{1,\mathrm{eq}} + X_2)} = \frac{0.5 n_{\mathrm{ph}} V_{1,\mathrm{eq}}^2 s_{\mathrm{maxT}}}{\omega_{\mathrm{s}} R_2}$$

If the frequency is reduced from 60 to 50 Hz, $X_{1,\mathrm{eq}} + X_2$ will drop by a factor of 5/6 and hence s_{maxT} will increase by a factor of 6/5 to $s_{\mathrm{maxT}} = 19.2\%$, corresponding to a speed of $1500(1 - 0.192) = 1212$ r/min.

T_{max} will increase as

$$\frac{(T_{\mathrm{max}})_{50}}{(T_{\mathrm{max}})_{60}} = \frac{(380/460)^2(6/5)}{5/6} = 0.983$$

or $(T_{\mathrm{max}})_{50} = 1140$ N·m.

Problem 6-28

Part (a): Solving the equations of chapter 6 with $s = 1$ for starting with MATLAB yields

I_{start} 456 A
$T_{\mathrm{start}} = 137$ N·m

part (b): (i) When the motor is connected in Y, the equivalent-circuit parameters will be three times those of the normal Δ connection. Thus

$R_1 = 0.099\ \Omega$
$R_2 = 0.135\ \Omega$
$X_1 = 0.84\ \Omega$
$X_2 = 0.93\ \Omega$
$X_m = 23.1\ \Omega$

(ii)

$$I_{\text{start}} = 152 \text{ A}$$
$$T_{\text{start}} = 45.8 \text{ N}\cdot\text{m}$$

Problem 6-29

The motor performance at any given slip can be found from Eqs. 6.32 though 6.36.

Part (a): Using a MATLAB search over slip, the speed at which the motor electromechanical power equals the load power is 973 rpm.

Part (b): The starting current is 141 A.

Part (c):

Problem 6-30

Part (a): At slip s, the motor input impedance is

$$Z_{\mathrm{in}} = R_{\mathrm{c}}||(R_1 + jX_1 + jX_{\mathrm{m}}||(jX_2 + R_2/s))$$

where '$||'$ represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\mathrm{in}}}$$

where $V_1 = 219.4$ V is the line-neutral source voltage. The real input power is given by

$$P_{\mathrm{in}} = -3\,\mathrm{Re}[V_1 \hat{I}_1^*]$$

The electromechanical shaft output power is then given by

$$P_{\mathrm{out}} = 3I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\mathrm{FW}}$$

where

$$\hat{I}_2 = \left(\hat{I}_1 - \frac{V_1}{R_{\mathrm{c}}}\right) \frac{jX_{\mathrm{m}}}{(j(X_{\mathrm{m}} + X_2) + R_2/s)}$$

A MATLAB search to find that slip which results in an electromechanical output power equal to the friction and windage loss of 150 W. As expected, this in an extremely small slip (0.01%). The input current and input power under this condition is $I_{\mathrm{nl}} = 14.61$ A and $P_{\mathrm{nl}} = 1467$ W.

Part (b): In this case, because the test is conducted at 12.5 Hz instead of the rating frequency of 50 Hz, the input impedance Z_{bl} must be evaluated with each reactance set equal to 25% of its 50-Hz value. The rated current of this motor is $I_{\mathrm{rated}} = 50$ kW$/(\sqrt{3}\,380$ V$) =$ 76.0 A. At this current value of blocked-rotor current I_{bl}, the voltage is

$$\hat{V}_{\mathrm{bl}} = I_{\mathrm{bl}} Z_{\mathrm{bl}}$$

and the blocked-rotor power is given by

$$P_{\rm bl} = 3\,{\rm Re}[\hat{V}_{\rm bl}\hat{I}_{\rm bl}^*]$$

In this case, we get $V_{\rm bl} = 17.9$ V, (30.9 V, line-line) and $P_{\rm bl}$ 2667 W.

Part (c): Starting with the no-load test results, the core loss can be calculated from Eq. 6.43 as

$$P_{\rm core} = P_{\rm nl} - 3I_{\rm nl}^2R_1 - P_{\rm FW} = 1277 \text{ W}$$

from which, assuming the core loss is modeled by a resistance $R_{\rm c}$ at the motor terminal

$$R_{\rm c} = \frac{3V_{\rm nl}^2}{P_{\rm core}} = 113\ \Omega$$

From Eq. 6.47,

$$S_{\rm nl} = 3V_{\rm nl}I_{\rm nl} = 9618 \text{ VAR}$$

and from Eq. 6.46

$$Q_{\rm nl} = \sqrt{S_{\rm nl}^2 - P_{\rm nl}^2} = 9505 \text{ VAR}$$

and from Eq. 6.48

$$X_{\rm nl} = \frac{Q_{\rm nl}}{3I_{\rm nl}^2} = 15.2\ \Omega$$

Next, consider the blocked-rotor test results at 12.5 Hz. From Eq. 6.51

$$S_{\rm bl} = 3V_{\rm bl}I_{\rm bl} = 4076 \text{ VAR}$$

and from Eq. 6.50

$$Q_{\mathrm{bl}} = \sqrt{S_{\mathrm{bl}}^2 - P_{\mathrm{bl}}^2} = 3083 \text{ VAR}$$

Thus, from Eq.6.52, X_{bl} as corrected to 60 Hz is

$$X_{\mathrm{bl}} = \left(\frac{f_{\mathrm{r}}}{f_{\mathrm{bl}}}\right)\left(\frac{Q_{\mathrm{bl}}}{3I_{\mathrm{bl}}^2}\right) = 0.712\ \Omega$$

Assuming $X_1 = X_2$, we can solve Eq. 6.60 to give

$$X_1 = X_2 = X_{\mathrm{nl}} - \sqrt{(X_{\mathrm{nl}}^2 - X_{\mathrm{bl}}X_{\mathrm{nl}})} = 0.360\ \Omega$$

and from Eq. 6.61 $X_{\mathrm{m}} = X_{\mathrm{nl}} - X_1 = 14.8\ \Omega$

Finally, from Eq. 6.53

$$R_{\mathrm{bl}} = \frac{P_{\mathrm{bl}}}{3I_{\mathrm{bl}}^2} = 0.154\ \Omega$$

and from Eq. 6.59

$$R_2 = (R_{\mathrm{bl}} - R_1)\left(\frac{X_2 + X_{\mathrm{m}}}{X_{\mathrm{m}}}\right)^2 = 0.095\ \Omega$$

Problem 6-31

The solution is the same as for Problem 6-30 except that all model parameters must be multiplied by 3, the rated voltage becomes 660 V and the rated current becomes 38.0 A.

Part (a): $I_{\mathrm{nl}} = 8.46$ A, $P_{\mathrm{nl}} = 1475$ W

Part (b): $V_{\mathrm{bl}} = 30.8$ V, (53.4 V, line-line), $P_{\mathrm{bl}} = 2652\ W$

Part (c):

$$R_1 = 0.189\ \Omega \quad R_2 = 0.285\ \Omega \quad R_{\mathrm{c}} = 339\ \Omega$$

$$X_1 = X_2 = 1.078\ \Omega \quad X_\mathrm{m} = 44.5\ \Omega$$

Problem 6-32

Part (a): $R_1 = 0.52/2 = 0.260\ \Omega$. The core loss can be calculated from Eq. 6.43 as

$$P_\mathrm{core} = P_\mathrm{nl} - 3I_\mathrm{nl}^2 R_1 - P_\mathrm{FW} = 1652\ \mathrm{W}$$

Part (b): Assuming the core loss is modeled by a resistance R_c at the motor terminal

$$R_\mathrm{c} = \frac{3V_\mathrm{nl}^2}{P_\mathrm{core}} = 3.20\ \mathrm{k}\Omega$$

From Eq. 6.47,

$$S_\mathrm{nl} = 3V_\mathrm{nl} I_\mathrm{nl} = 8366\ \mathrm{VAR}$$

and from Eq. 6.46

$$Q_\mathrm{nl} = \sqrt{S_\mathrm{nl}^2 - P_\mathrm{nl}^2} = 8013\ \mathrm{VAR}$$

and from Eq. 6.48

$$X_\mathrm{nl} = \frac{Q_\mathrm{nl}}{3I_\mathrm{nl}^2} = 605.6\ \Omega$$

Next, consider the blocked-rotor test results at 15 Hz. From Eq. 6.51

$$S_\mathrm{bl} = 3V_\mathrm{bl} I_\mathrm{bl} = 19.80\ \mathrm{kVAR}$$

and from Eq. 6.50

$$Q_\mathrm{bl} = \sqrt{S_\mathrm{bl}^2 - P_\mathrm{bl}^2} = 16.59\ \mathrm{kVAR}$$

Thus, from Eq.6.52, $X_{\rm bl}$ as corrected to 60 Hz is

$$X_{\rm bl} = \left(\frac{f_{\rm r}}{f_{\rm bl}}\right)\left(\frac{Q_{\rm bl}}{3I_{\rm bl}^2}\right) = 5.61\ \Omega$$

Assuming $X_1 = X_2$, we can solve Eq. 6.60 to give

$$X_1 = X_2 = X_{\rm nl} - \sqrt{(X_{\rm nl}^2 - X_{\rm bl}X_{\rm nl})} = 2.81\ \Omega$$

and from Eq. 6.61 $X_{\rm m} = X_{\rm nl} - X_1 = 603\ \Omega$.

Finally, from Eq. 6.53

$$R_{\rm bl} = \frac{P_{\rm bl}}{3I_{\rm bl}^2} = 0.913\ \Omega$$

and from Eq. 6.59

$$R_2 = (R_{\rm bl} - R_1)\left(\frac{X_2 + X_{\rm m}}{X_{\rm m}}\right)^2 = 0.659\ \Omega$$

Part (c): Including the effects of core loss, the motor input impedance is

$$\text{Part (c)}: \quad Z_{\rm in} = R_{\rm c}||(R_1 + jX_1 + jX_{\rm m}||(jX_2 + R_2/s))$$

$$\text{Part (d)}: \quad Z_{\rm in} = R_1 + jX_1 + (R_{\rm c}||jX_{\rm m})||(jX_2 + R_2/s))$$

where '$||$' represents "in parallel with". The terminal current is

$$\hat{I}_1 = \frac{V_1}{Z_{\rm in}}$$

For generator operating, the electrical output power is given by

$$P_{\rm out} = -3\,{\rm Re}[V_1\hat{I}_1^*]$$

The electromechanical shaft power is then given by

$$P_{\text{em}} = -3I_2^2 R_2 \left(\frac{1-s}{s}\right) - P_{\text{FW}}$$

where

$$\text{Part (c)} \quad \hat{I}_2 = (\hat{I}_1 - \frac{V_1}{R_{\text{c}}})\left(\frac{jX_{\text{m}}}{(jX_{\text{m}} + jX_2 + R_2/s)}\right)$$

$$\text{Part (d)} \quad \hat{I}_2 = \hat{I}_1\left(\frac{(R_{\text{c}}||jX_{\text{m}})}{((R_{\text{c}}||jX_{\text{m}}) + jX_2 + R_2/s)}\right)$$

and the shaft input power is equal to

$$P_{\text{in}} = P_{\text{FW}} + P_{\text{em}}$$

and the terminal power factor is

$$\text{pf} = \frac{P_{\text{out}}}{3V_1 I_1}$$

The stator I^2R loss is $P_{\text{stator}} = 3I_1^2 R_1$

Finally, the efficiency is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

At a slip of -2.35%

	Part (c)	Part (d)
Stator current [A]	59.8	60.1
Input power [kW]	229.8	229.6
Power factor	0.96	0.96
Stator I^2R [W]	2786	2822
Core loss I^2R [W]	1652	1474
Output power [kW]	217.6	217.6
Efficiency [%]	94.7	94.7

Problem 6-33

Because this is a blocked-rotor test, one can ignore the magnetizing reactance X_m. As a result, the motor input impedance can be approximated as

$$Z_\mathrm{in} \approx R_1 + R_2 + j(X_1 + X_2)$$

and R_2 can be calculated from the blocked-rotor power and current

$$R_2 = \frac{P_\mathrm{bl}}{3I_\mathrm{bl}^2} - R_1$$

where $R_1 = 33.9/2 = 16.95 \;\; \mathrm{m}\Omega$. This gives

Motor 1: $R_2 = 12.38 \;\; \mathrm{m}\Omega$
Motor 2: $R_2 = 47.07 \;\; \mathrm{m}\Omega$

The motor starting torque is proportional to $I_2^2 R_2 \approx I_\mathrm{bl}^2 R_2$ and thus the torque ratio is given by

$$\frac{T_\mathrm{motor2}}{T_\mathrm{motor1}} = \frac{(I_\mathrm{bl}^2)_\mathrm{motor2}(R_2)_\mathrm{motor2}}{(I_\mathrm{bl}^2)_\mathrm{motor1}(R_2)_\mathrm{motor1}} = \left(\frac{(R_2)_\mathrm{motor2}}{(R_2)_\mathrm{motor1}}\right)\left(\frac{(I_\mathrm{bl})_\mathrm{motor2}}{(I_\mathrm{bl})_\mathrm{motor1}}\right)^2$$

Part (a): For the same currents, the torque will be simply proportional to the resistance ratio and hence

$$\frac{T_\mathrm{motor2}}{T_\mathrm{motor1}} = 3.80$$

Part (b): From the given data, we see that for the same voltage, the current ratio will be $(I_2)_\mathrm{motor2}/(I_2)_\mathrm{motor1} = 70.5/60.7 = 1.16$ and hence

$$\frac{T_\mathrm{motor2}}{T_\mathrm{motor1}} = 5.13$$

Problem 6-34

	Part (a)	Part (b)
R_1 [Ω]	0.214	0.214
R_2 [Ω]	0.660	0.711
R_c [Ω]	2399	2260
X_1 [Ω]	3.041	3.072
X_2 [Ω]	3.717	3.754
X_m [Ω]	103.8	100.8

Problem 6-35

$s_{\mathrm{maxT}} \propto R_2$. Therefore

$$\frac{s_{\mathrm{maxT}}(R_2 + R_{\mathrm{ext}})}{s_{\mathrm{maxT}}(R_2)} = \frac{R_2 + R_{\mathrm{ext}}}{R_2}$$

$$R_2 = \frac{R_2 \times s_{\mathrm{maxT}}(R_2)}{s_{\mathrm{maxT}}(R_2 + R_{\mathrm{ext}}) - s_{\mathrm{maxT}}(R_2)} = 1.49\ \Omega$$

Problem 6-36

Part (a): Combining Eqs. 6.36, 6.38 and 6.39 we can show that

$$\frac{s_{\mathrm{maxT}}}{s_{\mathrm{fl}}} = \frac{T_{\mathrm{max}}}{T_{\mathrm{fl}}} + \sqrt{\left(\frac{T_{\mathrm{max}}}{T_{\mathrm{fl}}}\right)^2 - 1} = 3.50$$

and thus

$$s_{\mathrm{fl}} = \frac{s_{\mathrm{maxT}}}{4.27} = 0.0486 = 4.86\%$$

Part (b): The rotor rotor power dissipation at rated load is given by

$$P_{\mathrm{rotor}} = P_{\mathrm{rated}}\left(\frac{s_{\mathrm{fl}}}{1 - s_{\mathrm{fl}}}\right) = 4.86\ \mathrm{kW}$$

part (c): Again, combining Eqs. 6.36, 6.38 and 6.39 we can show that

$$T_{\mathrm{start,pu}} = 2\,T_{\mathrm{max,pu}}\left(\frac{s_{\mathrm{maxT}}}{1+s_{\mathrm{maxT}}^2}\right) = 0.744 \quad \mathrm{pu}$$

Rated-load speed is equal to

$$\omega_{\mathrm{m,fl}} = 2\pi f_{\mathrm{e}}\left(\frac{2}{\mathrm{poles}}\right)(1-s_{\mathrm{fl}}) = 99.63 \quad \mathrm{rad/sec}$$

and rated torque is equal to T_{rated} = 125 kW/99.63 rad/sec = 1.25×10^3 N·m. Thus $T_{\mathrm{start}} = T_{\mathrm{rated}} T_{\mathrm{start,pu}} = 933$ pu.

Part (d): If the rotor resistance is doubled, the motor impedance will be the same if the slip is also doubled. Thus, the slip will be equal to $s = 2s_{\mathrm{fl}} = 9.71\%$.

Part (e): Under these operating conditions, I_2 will be the same. Since the torque is proportional to $I_2^2 R_2/s$, it will be the same under both operating conditions and will thus be equal to $T = 1250$ N·m.

Problem 6-37

Parts (a) - (c):

Part (d):

$R_a[\Omega]$	0.08	0.4	0.8	2.4
rpm	1004	1725	1662	1479
P [kW]	3.8	19.4	17.4	12.2

Note from the plot that the motor cannot get the fan up to full speed with $R_2 = 0.08\ \Omega$ since there is a stable operating point at 1000 rpm.

Problem 6-38

With no external resistance, the line current is 210 percent of rated at a slip of $s_1 = 5.5\%$ which corresponds to an apparent rotor resistance of R_2/s. The starting current thus will be equal to 210 percent of rated if the rotor resistance at starting including the external resistance R_{ext} is equal to this value, i.e.

$$R_2 + R_{\text{ext}} = R_2/s_1$$

or

$$R_{\text{ext}} = R_2\left(\frac{1 - s_1}{s_1}\right)$$

For $R_2 = 95/2 = 47.5\ \text{m}\Omega/\text{phase}$, this gives $R_{\text{ext}} = 816\ \text{m}\Omega/\text{phase}$.

Torque is proportional to $I_2^2(R_2 + R_{\text{ext}})/s$. We gave chosen R_{ext} such that at starting with $s = 1$, both I_2 and $(R_2 + R_{\text{ext}})/s$ are equal to the values at $s = 5.5\%$ where $R_2 = 0$. Thus the starting torque under this condition will be equal to 210 percent of rated torque.

Problem 6-39

Part (a): The synchronous speed of this 8-pole 60-Hz motor motor is 900 rpm. Thus the full-load slip is $s_{\text{fl}} = 31/900 = 0.0344$ (3.44%). Combining Eqs.6.22 and 6.23 we find that

$$P_{\text{rotor}} = \left(\frac{s}{1 - s}\right)P_{\text{mech}}$$

Ignoring rotational loss, at full load $P_{\text{mech}} = 100$ kW and thus $P_{\text{rotor}} = 3.57$ kW.

Part (b): Assuming $R_1 = 0$, combining Eqs. 6.36, 6.38 and 6.39, we can show that

$$\frac{T_{\text{fl}}}{T_{\max}} = \frac{2(s_{\text{maxT}}/s_{\text{fl}})}{1 + (s_{\text{maxT}}/s_{\text{fl}})^2}$$

from which we find that $s_{\text{maxT}} = 19.7\%$, corresponding to a speed of 722.5 rpm.

Part (c): To achieve maximum torque at starting ($s = 1$), must have

$$R_2 + R_{\text{ext}} = \frac{R_2}{s_{\text{maxT}}}$$

For $R_2 = \;0.18\ \Omega$ we find that $R_{\text{ext}} = 0.73\ \Omega$.

Part (d): At the same flux level and 50 Hz, the applied voltage will be equal to $(50/60) \times 460 = 383$ V.

Part (e): Since the applied voltage and all the reactances will have values equal to 5/6 times their 60-Hz values, the torque, which is proportional to I^2R_2/s will be equal to the rated 60-Hz value if R_2/s is equal to 5/6 of its 60-Hz value. Thus, at 50-Hz

$$s_{\text{fl,50Hz}} = \left(\frac{6}{5}\right) s_{\text{fl,60Hz}} = 0.0413$$

The 50-Hz synchronous speed is 750 rpm and thus the full-load speed is 719 rpm.

Problem 6-40

Part (a):

- Speed = 1162.5 rpm
- $I_1 = 125.7$ A

- $P_{\text{load}} = 115.5$ kW
- $P_{\text{rotor}} = 3.7$ kW
- Efficiency = 95.1%

Part (b): The required external resistance is $R_{\text{ext}} = 0.34\ \Omega$.

- Speed = 1050 rpm
- $I_1 = 125.9$ A
- $P_{\text{load}} = 104.5$ kW
- $P_{\text{rotor}} = 3.7$ kW
- Efficiency = 85.9%
- $P_{\text{ext}} = 11.2$ kW

PROBLEM SOLUTIONS: Chapter 7

Problem 7-1

Part (a): The speed will be proportional to the armature terminal voltage.

Part (b): An increase in field current will cause the speed to decrease and vice versa.

Part (c): The speed will remain constant.

Problem 7-2

Part (a): For constant terminal voltage, the product nI_{f} (where n is the motor speed) is constant. Since $I_{\mathrm{f}} \propto 1/R_{\mathrm{f}}$

$$\frac{R_{\mathrm{f}}}{1420} = \frac{R_{\mathrm{f}} + 8}{1560}$$

and hence $R_{\mathrm{f}} = 81.1\ \Omega$.

Part (b):

$$n = 1420 \left(\frac{R_{\mathrm{f}} + 20}{R_{\mathrm{f}}} \right) = 1770 \text{ r/min}$$

Part (c):

$$n = 1420 \left(\frac{90}{125} \right) = 1022 \text{ r/min}$$

Problem 7-3

If we write $E_{\mathrm{a}} = \omega_{\mathrm{m}} K_{\mathrm{a}} I_{\mathrm{f}}$, we can find K_a. At no load given no-load condition with $E_{\mathrm{a}} = V_{\mathrm{a}} = 250$ V, $I_{\mathrm{f}} = 250/185 = 1.35$ A and $\omega_{\mathrm{m}} = 1850 \times \pi/30 = 193.7$ rad/sec. Thus,

$$K_{\mathrm{a}} = 0.956$$

Part (a): Here $I_{\mathrm{f}} = 1.35$ A, $I_{\mathrm{L}} = 290$ A and $I_{\mathrm{a}} = I_{\mathrm{L}} - I_{\mathrm{f}} = 288.6$ A. Thus

$$E_{\mathrm{a}} = V_{\mathrm{a}} - I\mathrm{a} R_{\mathrm{a}} = 237 \text{ V}$$

(i)

$$\omega_{\rm m} = \frac{E_{\rm a}}{K_{\rm a} I_{\rm f}} = 183.6 \text{ rad/sec}$$

corresponding to a speed of 1754 r/min.

(ii)

$$P_{\rm load} = E_{\rm a} I_{\rm a} = 68.40 \text{ kW}$$

(iii)

$$T_{\rm load} = \frac{P_{\rm load}}{\omega_{\rm m}} = 372.5 \text{ N} \cdot \text{m}$$

Part (b): If the terminal voltage is 200 V, $I_{\rm f} = 200/185 = 1.08$ A.

$$T_{\rm load} = \frac{E_{\rm a} I_{\rm a}}{\omega_{\rm m}} = (V_{\rm a} - E_{\rm a}) \left(\frac{K_{\rm a} I_{\rm f}}{R_{\rm a}} \right)$$

and thus

$$E_{\rm a} = V_{\rm a} - \frac{T_{\rm load} R_{\rm a}}{K_{\rm a} I_{\rm f}} = 183.5 \text{ V}$$

from which (i)

$$\omega_{\rm m} = \frac{E_{\rm a}}{K_{\rm a} I_{\rm f}} = 177.7 \text{ rad/sec}$$

corresponding to a speed of 1697 r/min.

(ii)

$$I_{\rm L} = \frac{V_{\rm a} - E_{\rm a}}{R_{\rm a} + I_{\rm f}} = 367.8 \text{ A}$$

Part (c): Let $T_0 = 372.5$ N·m and $\omega_{\rm m0} = 183.6$ rad/sec. We can write

$$T_0 \left(\frac{\omega_{\rm m}}{\omega_{\rm m0}} \right)^2 = k_{\rm a} I_{\rm f} I_{\rm a} = K_{\rm a} I_{\rm f} (V_{\rm a} - K_{\rm a} \omega_{\rm m} I_{\rm f})$$

from which we find that $\omega_{\rm m} = 178.8$ rad/sec corresponding to 1707 r/min.

$$I_{\rm L} = \frac{V_{\rm a} - \omega_{\rm m} K_{\rm a} I_{\rm f}}{R_{\rm a}} + I_{\rm f} = 344.6 \text{ A}$$

Problem 7-4

Part (a): $\omega_{\rm m}$ halved; $I_{\rm a}$ constant

Part (b): ω_m halved; I_a doubled

Part (c): ω_m halved; I_a halved

Part (d): ω_m constant; I_a doubled

Part (e): ω_m halved; I_a reduced by a factor of 4.

Problem 7-5

Part (a): Rated armature current = 35 kW/250-V = 140 A.
Part (b): At 1500 r/min, E_a can be determined directly from the magnetization curve of Fig. 7.32. The armature voltage can be calculated as

$$V_a = E_a + I_a R_a$$

and the power output as $P_{out} = V_a I_a$. With $I_a = 140$ A

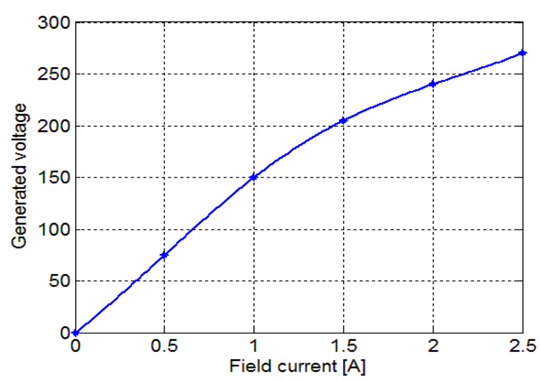
I_f [A]	E_a [V]	V_a [V]	P_{out} [kW]
1.0	150	137	19.1
2.0	240	227	31.7
2.5	270	257	35.9

Part (c): The solution proceeds as in part (b) but with the generated voltage equal to 1250/1500 = 0.833 times that of part (b)

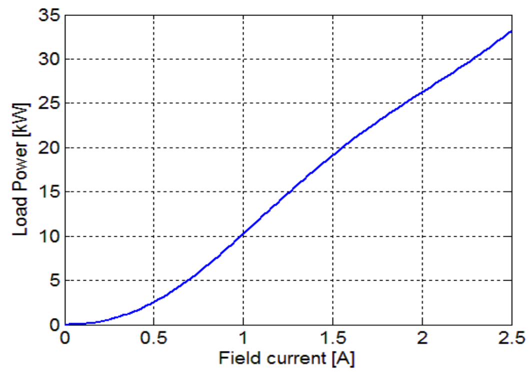
I_f [A]	E_a [V]	V_a [V]	P_{out} [kW]
1.0	125	112	15.6
2.0	200	187	26.1
2.5	225	212	29.6

Problem 7-6

Part (a):



Part (b):



Problem 7-7

Part (a): For a field current of 1.67 A, the generated voltage at a speed of 1500 r/min is $1.67 \times 150 = 250$ V.

Thus the generated voltage as a function of speed n is

$$E_\mathrm{a} = 250\left(\frac{n}{1500}\right)$$

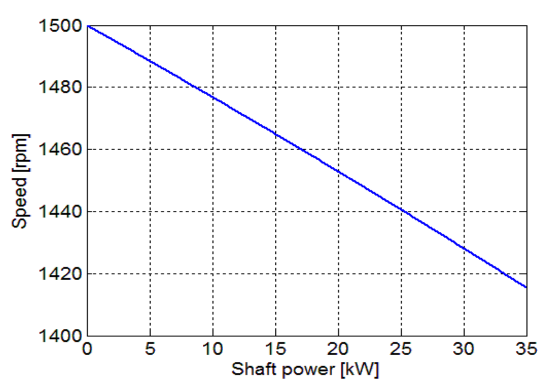
For a given motor speed

$$I_\mathrm{a} = \frac{V_\mathrm{a} - E_\mathrm{a}}{R_\mathrm{a}} = \frac{V_\mathrm{a} - (250/1500)n}{R_\mathrm{a}}$$

and the shaft power is

$$P = E_\mathrm{a} I_\mathrm{a} = 250\left(\frac{n}{1500}\right)\left(\frac{V_\mathrm{a} - (250/1500)n}{R_\mathrm{a}}\right)$$

We see that the shaft power is equal to zero when the speed is 1500 r/min and can find that the shaft power is equal to 35 kW at a speed of 1415 r/min. Here is the desired plot:



Part (b): We can solve for E_{a} as a function of power P by recognizing that

$$P = E_{\mathrm{a}} I_{\mathrm{a}} = E_{\mathrm{a}} \left(\frac{V_{\mathrm{a}} - E_{\mathrm{a}}}{R_{\mathrm{a}}} \right)$$

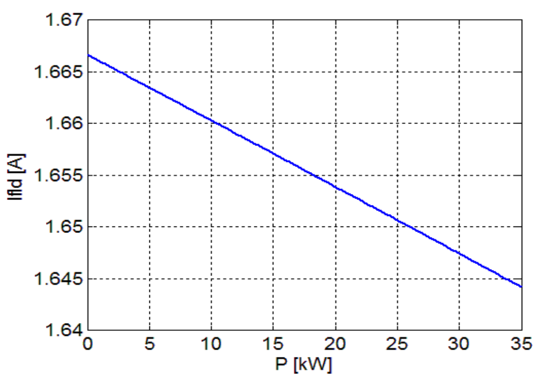
and thus

$$E_{\mathrm{a}} = \frac{V_{\mathrm{a}} + \sqrt{V_{\mathrm{a}}^2 - 4PR_{\mathrm{a}}}}{2}$$

where we have chosen the + sign in the quadratic solution since we know that $E_{\mathrm{a}} = V_{\mathrm{a}}$ when $P = 0$. Because the speed is constant, we can solve for the field current I_{f} as

$$I_{\mathrm{f}} = 1.67 \left(\frac{E_{\mathrm{a}}}{250} \right)$$

resulting in the following plot:



Problem 7-8

Part (a): Because the field current is constant at the value required to produce 250 V at 1500 r/min, saturation does not play a role. Thus the generated voltage as a function of speed n is

$$E_{\mathrm{a}} = 250 \left(\frac{n}{1500} \right)$$

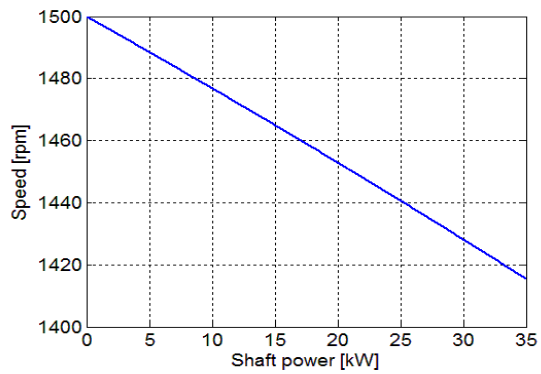
For a given motor speed

$$I_{\mathrm{a}} = \frac{V_{\mathrm{a}} - E_{\mathrm{a}}}{R_{\mathrm{a}}} = \frac{V_{\mathrm{a}} - (250/1500)n}{R_{\mathrm{a}}}$$

and the shaft power is

$$P = E_{\mathrm{a}} I_{\mathrm{a}} = 250 \left(\frac{n}{1500} \right) \left(\frac{V_{\mathrm{a}} - (250/1500)n}{R_{\mathrm{a}}} \right)$$

We see that the shaft power is equal to zero when the speed is 1500 r/min and can find that the shaft power is equal to 35 kW at a speed of 1415 r/min. Here is the desired plot:



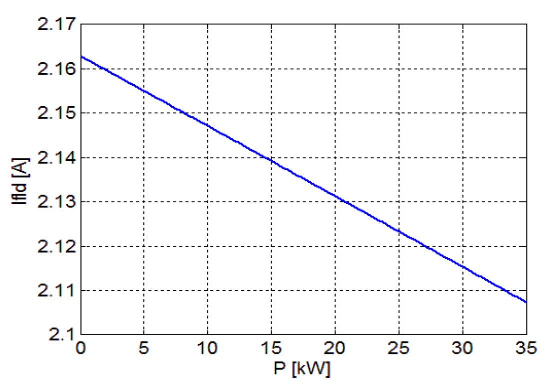
Part (b): We can solve for E_a as a function of power P by recognizing that

$$P = E_a I_a = E_a \left(\frac{V_a - E_a}{R_a} \right)$$

and thus

$$E_a = \frac{V_a + \sqrt{V_a^2 - 4PR_a}}{2}$$

where we have chosen the + sign in the quadratic solution since we know that $E_a = V_a$ when $P = 0$. Combining this solution with a spline fit of I_f as a function of E_a at 1500 r/min results in the following plot:



Problem 7-9

Part (a): At no load, the effect of armature resistance can be neglected and the 1700 r/min open-circuit voltage can be calculated by the intersection of the no-load magnetization characteristics (scaled by the speed ratio

1700/1750) $E_{\rm a}(I_{\rm f}$ and the curve describing the shunt field winding characteristic $E_{\rm a} = V_{\rm a} = I_{\rm f} R_{\rm f}$. The solution is $V_{\rm a} = E_{\rm a} = 561.6$ V.

Part (b): At a terminal voltage of 527 V, the field current is $I_{\rm f} = 527/163 = 3.23$ A. The generated voltage under this operating condition is equal to

$$E_{\rm a} = V_{\rm a} + R_{\rm a}(I_{\rm L} + I_{\rm f}) = 527 + 163 \times (180 + 3.23) = 537.4\ \Omega$$

corresponding to a field current (obtained from a spline fit of the magnetization curve scaled down to 1700 rpm) of 2.99 A. Thus the equivalent armature-reaction field current is $I_{\rm f,ar} = 0.24$ A and the armature reaction is $F_{\rm ar} = N_{\rm f} I_{\rm f,ar} = 284$ A.

Part (c): With the terminal current remaining at $I_{\rm L} =$ A, we will assume that the armature reaction remains the same as in part (b), corresponding to a equivalent field current of 0.24 A. Thus the 1750 r/min magnetization curve can be re-written as a function of a modified current $I'_{\rm f} = I_{\rm f} - 0.24$.

The terminal relations under this operating condition can be written as

$$E_{\rm a} = V_{\rm a} + R_{\rm a}(I_{\rm L} + I_{\rm f})$$

The field current is equal to $I_{\rm f} = V_{\rm a}/R_{\rm f}$, and with appropriate substitutions, we can write and expression for $E_{\rm a}$ as a function of $I'_{\rm f}$.

$$E_{\rm a} = I'_{\rm f}(R_{\rm a} + R_{\rm f}) + 0.24\,(R_{\rm a} + R_{\rm f}) + I_{\rm L} R_{\rm a}$$

The operating point is given by the intersection of this curve with the 1750 r/min magnetization characteristic. The solution is $I'_{\rm f} = 3.15$ A, $I_{\rm f} = 3.39\ A$ and $V_{\rm a} = R_{\rm f} I_{\rm f} = 552.4$ A.

Problem 7-10

Part(a): At a terminal voltage of 527 V, the field current is $I_{\rm f} = 527/163 = 3.23$ A. The generated voltage under this operating condition is equal to

$$E_{\rm a} = V_{\rm a} + R_{\rm a}(I_{\rm L} + I_{\rm f}) = 527 + 163 \times (180 + 3.23) = 537.4\ \Omega$$

corresponding to a field current (obtained from a spline fit of the magnetization curve scaled down to 1700 rpm) of 2.99 A. Thus the equivalent armature-reaction field current is $I_{\rm f,ar} = 0.24$ A and the armature reaction is $F_{\rm ar} = N_{\rm f} I_{\rm f,ar} = 284$ A-turns.

Part (b): Based upon the equivalent armature-reaction field current found in part (a), the effect of armature reaction on the 1750 r/min magnetization curve can be accounted for by representing it as a function of the current $I'_{\rm f}$ where

$$I'_{\rm f} = I_{\rm f} - 0.24\left(\frac{I_{\rm a}}{180}\right)$$

The effect of the motor speed n on the generated voltage can be accounted for by modifying the 1500 r/min as

$$E_{\mathrm{a}} = \left(\frac{n}{1750}\right) E_{\mathrm{a0}}$$

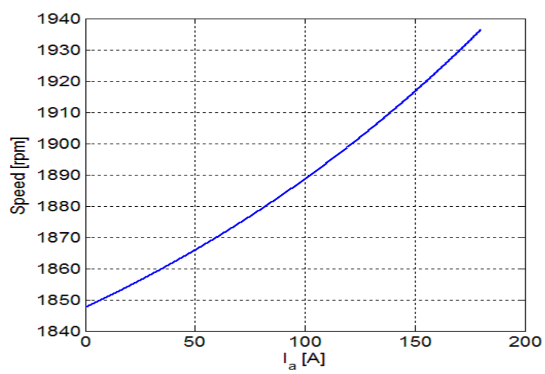
where E_{a0} is the voltage from the 1500 r/min magnetization characteristic.

At any given armature current I_{a}, the speed can be found by

1. Find the generated voltage as $E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{a}}$.
2. Find the magnetization curve generated voltage E_{a0} based upon the corresponding value of I'_{f}.
3. Find the speed as

$$n = \left(\frac{E_{\mathrm{a}}}{E_{\mathrm{a0}}}\right) 1500$$

Here is the resultant plot



Problem 7-11

At no load, $E_{\mathrm{a,nl}} = 250 - 7.4 \times 0.13 = 249.0$ V. At full load, $E_{\mathrm{a,fl}} = 250 - 152 * 0.13 = 230.2$ V. But, $E_{\mathrm{a}} \propto n\Phi$, thus

$$n_{\mathrm{fl}} = n_{\mathrm{nl}} \left(\frac{E_{\mathrm{a,fl}}}{E_{\mathrm{a,nl}}}\right) \left(\frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}}\right) = 1975 \left(\frac{230.2}{249.0}\right) \left(\frac{1}{0.92}\right) = 1984.7 \text{ r/min}$$

Problem 7-12

Part (a): The armature reaction at 80 A can be found from the separately-excited data. From the given data we see that the generated voltage is

$$E_{\mathrm{a}} = V_{\mathrm{a}} + I_{\mathrm{a}} R_{\mathrm{a}} = 262.2 \ \ \mathrm{V}$$

corresponding to a voltage of 273.6 V at 1195 r/min which in-turn corresponds to a field current of 1.735 A based upon a spline fit of the magnetization characteristic. We thus see that the equivalent armature-reaction field current is $I_{\rm f,ar} = 1.735 - 1.62 = 0.115$ A and the armature reaction is $AR = N_{\rm f} I_{\rm f,ar} =$ 74.8 A-turns.

Part (b): We first must calculate the resistance of the shunt field winding. At not load, the shunt-connected generator is operating at 1195 r/min and the terminal voltage is 230 A. Ignoring the small voltage drop produced by the field current across the armature resistance, the generated voltage is equal to 230 V and the corresponding field current from the magnetization curve is 1.05 A and thus $R_{\rm f} = 230/1.05 = 219\ \Omega$.

Under the desired full-load condition, with a terminal voltage $V_{\rm a} = 250$ V, a terminal current of $I_{\rm L} = 80\ A$ and at a speed of 1145 rpm, the shunt-field voltage will be equal to

$$V_{\rm f} = V_{\rm a} + I_{\rm L} R_{\rm s} = 250 + 80 \times 0.049 = 253.9 \ \ {\rm V}$$

and the shunt-field current will thus be equal to

$$I_{\rm f} = \frac{V_{\rm f}}{R_{\rm f}} = 1.16 \ \ {\rm A}$$

The generated voltage can thus be calculated as

$$E_{\rm a} = V_{\rm a} + I_{\rm L} R_{\rm s} + (I_{\rm L} + I_{\rm f}) R_{\rm a} = 266.3 \ \ {\rm A}$$

This corresponds to a magnetizing-curve voltage of $266.3 \times (1195/1145) =$ 277.9V and a magnetizing current of 1.848 A and Net mmf $= N_{\rm f} \times 1.848 =$ 1201 A-turns. We can solve for the required series-field turns/pole $N_{\rm s}$ from Eq. 7.28 as

$$N_{\rm s} = \frac{\text{Net mmf} - N_{\rm f} I_{\rm f} + F_{\rm ar}}{I_{\rm L}} = 6.5 \text{ turns/pole}$$

This can be rounded up to 7 turns/pole which will result in a voltage slight above 250 V or down to 6 turns/pole which will result in a slightly lower voltage.

Problem 7-13

From the given data, the generated voltage at $I_{\rm a} = 70$A and $n(70) =$ 1225 r/min is

$$E_{\rm a}(70) = V_{\rm a} - I_{\rm a}(R_{\rm a} + R_{\rm s}) = 300 - 70(0.13 + 0.09) = 284.6 \text{ V}$$

Similarly, the generated voltage at $I_{\rm a} = 25$ A is

$$E_{\rm a}(25) = 300 - 25(0.13 + 0.09) = 294.5 \text{ V}$$

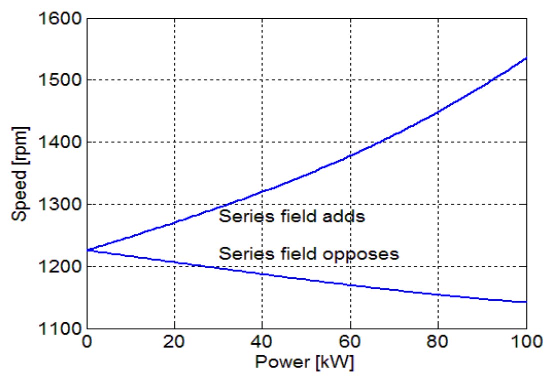
Since $E_a \propto n\Phi$

$$\frac{E_a(25)}{E_a(70)} = \left(\frac{n(25)}{n(70)}\right)\left(\frac{\Phi(25)}{\Phi(70)}\right)$$

Making use of the fact that $\Phi(25)/\Phi(70) = 0.54$, we can solve for $n(25)$

$$n(25) = n(70)\left(\frac{E_a(25)}{E_a(70)}\right)\left(\frac{\Phi(70)}{\Phi(25)}\right) = 2347 \text{ r/min}$$

Problem 7-14



Problem 7-15

Errata: Part (a) should read

> Calculate the field current, terminal current and torque corresponding to operation at a speed of 2400 r/min, an armature terminal voltage of 250 V and a load of 75 kW.

From the given data, we see that the generated voltage can be expressed as

$$E_a = nK'_a I_f$$

where

$$K'_a = \frac{250\text{V}}{2400 \text{ r/min} \times 4.5 \text{ A}} = 2.31 \times 10^{-2} \quad \frac{\text{V}}{(\text{r/min}) \cdot \text{A}}$$

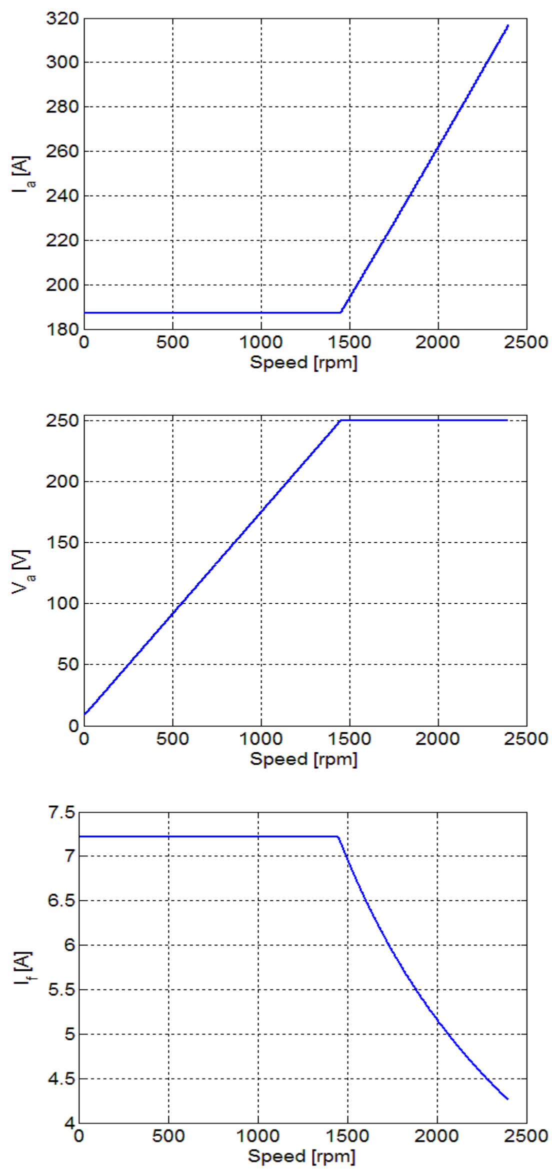
Part (a): We know that $V_a = 250$ V, $E_a = V_a - I_a R_a$ and $E_a I_a = 750$ kW. Solving gives $E_a = 236.7$ V and $I_a = 316.9$ A. Thus

$$I_f = \frac{E_a}{nK'_a} = 4.27 \quad \text{A}$$

At a speed of 2400 r/min, $\omega_{\mathrm{m}} = 80\pi$ and the torque is equal to $T = P/\omega_{\mathrm{m}} = 298.4$ N·m.

Part (b): The solution for E_{a} and I_{a} is similar to that of Part (a) but the power is reduced to $P = (1450/2400) \times 75$ kW $= 45.3$ kW with the result that $E_{\mathrm{a}} = 242.1$ V, $I_{\mathrm{a}} = 187.1$ A and $I_{\mathrm{f}} = 7.21$ A.

Part (c):



Problem 7-16

Errata: The second paragraph of the problem statement should read:

... whose torque varies linearly within this speed range.

From the given data, we see that the generated voltage can be expressed as

$$E_{\mathrm{a}} = nK'_{\mathrm{a}} I_{\mathrm{f}}$$

where

$$K'_{\mathrm{a}} = \frac{550\ \mathrm{V}}{3500\ \mathrm{r/min} \times 0.9\ \mathrm{A}} = 0.175 \quad \frac{\mathrm{V}}{(\mathrm{r/min}) \cdot \mathrm{A}}$$

The load torque at 3800 r/min ($\omega_{\mathrm{m}} = 126.7\pi$) is $T_{\mathrm{L}}(3800) = 180\ \mathrm{kW}/\omega_{\mathrm{m}} = 452.2$ N·m and the load torque at 1500 r/min ($\omega_{\mathrm{m}} = 50\pi$) is $T_{\mathrm{L}}(1500) = 125\ \mathrm{kW}/\omega_{\mathrm{m}} = 795.8$ N·m. Thus the load torque at speed n r/min is

$$\begin{aligned} T_{\mathrm{L}}(n) &= T_{\mathrm{L}}(1500) + \left(\frac{T_{\mathrm{L}}(3800) - T_{\mathrm{L}}(1500)}{2300}\right)(n - 1500) \\ &= 1.020 \times 10^3 - 0.149\, n \quad \mathrm{N \cdot m} \end{aligned}$$

Part (a): At each speed n, the load power is equal to $P_{\mathrm{L}}(n) = T_{\mathrm{L}}(n)(\pi/30)$. With $V_{\mathrm{a}} = 550$ V, $E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{a}}$ and $E_{\mathrm{a}} I_{\mathrm{a}} = P_{\mathrm{L}}$. We can solve these equations simultaneously for E_{a} and I_{a} and then the armature power dissipation as $P_{\mathrm{arm}} = I_{\mathrm{a}}^2 R_{\mathrm{a}}$ and the field current as $I_{\mathrm{f}} = E_{\mathrm{a}}/(nK'_{\mathrm{a}})$.

r/min	I_{f} [A]	I_{a} [A]	P_{arm} [kW]
1500	2.06	231.7	2.4
2500	1.23	315.9	4.5
3000	1.02	335.8	5.1
3800	0.81	336.5	5.1

Part (b): With the field current held constant that 0.81 A as found in Part (a), the generated voltage at speed n can be found from $E_{\mathrm{a}} = nK'_{\mathrm{a}}I_{\mathrm{f}}$, the armature current can be found as $I_{\mathrm{a}} = P_{\mathrm{L}}/E_{\mathrm{a}}$ and the terminal voltage will be $V_{\mathrm{a}} = E_{\mathrm{a}} + I_{\mathrm{a}}R_{\mathrm{a}}$.

r/min	V_{a} [V]	I_{a} [A]	P_{arm} [kW]
1500	237.8	592.1	15.8
2500	373.5	481.0	10.4
3000	441.4	425.4	8.1
3800	336.5	336.5	5.1

Problem 7-17

Given that the field winding has 1200 turns/pole, the 180 A-turns/pole of demagnetizing effect at a field current of 350 A is equivalent to an equivalent demagnetizing field current of 180/1200 = 0.15 A. From the given data, we see that the generated voltage can be expressed as

$$E_{\mathrm{a}} = nK'_{\mathrm{a}}I_{\mathrm{f}}$$

where

$$K'_{\mathrm{a}} = \frac{550\ \mathrm{V}}{3500\ \mathrm{r/min} \times 0.9\ \mathrm{A}} = 0.175 \quad \frac{\mathrm{V}}{(\mathrm{r/min})\cdot\mathrm{A}}$$

Including the effect of demagnetizing effect of the armature current we can thus write

$$E_{\mathrm{a}} = nK'_{\mathrm{a}}\left(I_{\mathrm{f}} - 0.15\left(\frac{I_{\mathrm{a}}}{350}\right)\right)$$

The load torque at 3800 r/min ($\omega_{\mathrm{m}} = 126.7\pi$) is $T_{\mathrm{L}}(3800) = 180\ \mathrm{kW}/\omega_{\mathrm{m}} = 452.2$ N·m and the load torque at 1500 r/min ($\omega_{\mathrm{m}} = 50\pi$) is $T_{\mathrm{L}}(1500) = 125\ \mathrm{kW}/\omega_{\mathrm{m}} = 795.8$ N·m. Thus the load torque at speed n r/min is

$$\begin{aligned} T_{\mathrm{L}}(n) &= T_{\mathrm{L}}(1500) + \left(\frac{T_{\mathrm{L}}(3800) - T_{\mathrm{L}}(1500)}{2300}\right)(n - 1500) \\ &= 1.020 \times 10^3 - 0.149\,n \quad \mathrm{N\cdot m} \end{aligned}$$

Part (a): At each speed n, the load power is equal to $P_{\mathrm{L}}(n) = T_{\mathrm{L}}(n)(\pi/30)$. With $V_{\mathrm{a}} = 550$ V, $E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}}R_{\mathrm{a}}$ and $E_{\mathrm{a}}I_{\mathrm{a}} = P_{\mathrm{L}}$. We can solve these equations simultaneously for E_{a} and I_{a} and then the armature power dissipation as $P_{\mathrm{arm}} = I_{\mathrm{a}}^2R_{\mathrm{a}}$. The field current can then be found as

$$I_{\mathrm{f}} = \frac{E_{\mathrm{a}}}{nK'_{\mathrm{a}}} + 0.15\left(\frac{I_{\mathrm{a}}}{350}\right)$$

r/min	I_f [A]	I_a [A]	P_{arm} [kW]
1500	2.16	231.7	2.4
2500	1.36	315.9	4.5
3000	1.17	335.8	5.1
3800	0.95	336.5	5.1

Part (b): With the field current held constant that 0.95 A as found in Part (a), the generated voltage at speed n can be found from

$$E_a = nK'_a\left(I_f - 0.15\left(\frac{I_a}{350}\right)\right)$$

The armature current is related to the power as as $I_a = P_L/E_a$ and thus we can find the armature current from the solution to

$$\left(\frac{0.15nK'_a}{350}\right)I_a^2 - nK'_a I_f I_a + P_L = 0$$

and the terminal voltage as $V_a = E_a + I_a R_a$.

r/min	V_a [V]	I_a [A]	P_{arm} [kW]
1500	197.2	768.6	26.6
2500	338.3	539.0	13.1
3000	416.4	453.7	9.3
3800	550.0	336.5	5.1

Problem 7-18

$$\omega_m = \frac{E_a}{K_a\Phi_d} = \frac{V_a - I_a R_a}{K_a\Phi_d}$$

$$I_a = \frac{T}{K_a\Phi_d}$$

Thus

$$\omega_m = \frac{1}{K_a\Phi_d}\left(V_a - \frac{TR_a}{K_a\Phi_d}\right)$$

The desired result can be obtained by taking the derivative of ω_m with Φ_d

$$\begin{aligned}
\frac{d\omega_m}{d\Phi_d} &= \frac{1}{K_a\Phi_d^2}\left(\frac{2TR_a}{K_a\Phi_d} - V_a\right) \\
&= \frac{1}{K_a\Phi_d^2}(2I_aR_a - V_a) \\
&= \frac{1}{K_a\Phi_d^2}(V_a - 2E_a)
\end{aligned}$$

From this we see that for $E_\mathrm{a} > 0.5V_\mathrm{a}$, $d\omega_\mathrm{a}/d\Phi_\mathrm{d} < 0$ and for $E_\mathrm{a} < 0.5V_\mathrm{a}$, $d\omega_\mathrm{a}/d\Phi_\mathrm{d} > 0$. Q.E.D.

Problem 7-19

Errata: In parts (a) and (c), the power should be 25 kW, not 30 kW.

... whose torque varies linearly within this speed range.

Part (a): The synchronous machine is operating at rated terminal voltage, rated power and unity power factor and hence at rated terminal voltage. Thus in per-unit,

$$\hat{E}_\mathrm{af} = \hat{V}_\mathrm{a} + jX_\mathrm{s}\hat{I}_\mathrm{a} = 1.0 + j\,0.781.0 = 1.27\angle 38.0^\circ$$

and thus $E_\mathrm{af} = 1.27$ per unit $= 1.27 \times 460/\sqrt{3} = 337$ V, line-neutral.

The dc machine is delivering 25 kW to the generator and thus we know that $E_\mathrm{a}I_\mathrm{a} = 25$ kW. Similarly, we know that $V_\mathrm{a} = E_\mathrm{a} + I_\mathrm{a}R_\mathrm{a} = 250$ V. Solving simultaneously gives $E_\mathrm{a} = 247.8$ V.

Part (b): Increase the dc-motor field excitation until $E_\mathrm{a} = V_\mathrm{a,dc} = 250$ V, in which case the dc motor input current will equal zero and it will produce no shaft power. The ac generator will thus operate at a power angle of zero and hence its terminal current will be

$$I_\mathrm{a} = \frac{E_\mathrm{a} - V\mathrm{a}}{jX_\mathrm{s}} = -j0.34 \text{ per-unit}$$

The rated current of this generator is

$$I_\mathrm{rated} = \frac{25 \text{ kW}}{\sqrt{3} \times 460} = 31.4 \text{ A}$$

and thus the armature current is 10.7 A.

Part (c): If one further increases the dc-machine field excitation the dc machine will act as a generator and the ac machine as a motor. For the purposes of this part of the problem, we will switch to generator notation for the dc machine (positive current out of the terminals) and motor notation for the ac machine (positive current into the terminals).

With 25 kW transferred to the dc source, $V_\mathrm{a}I_\mathrm{a} = 25$ kW and thus the dc-machine terminal current will be $I_\mathrm{a} = 100$ A and the generated voltage will be $E_\mathrm{a} = V_\mathrm{a} + I_\mathrm{a}R_\mathrm{a} = 252.2$ V.

The ac machine, operating in this case as a motor, must also supply the dc-machine $I_\mathrm{a}^2R_\mathrm{a} = 220$ W and thus a total power of $P = 25.22$ kW $= 1.009$ per unit.

With $E_{\mathrm{af}} = 1.27$ per-unit and $V = 1.0$ per-unit we can find

$$\delta = -\sin^{-1}\left(\frac{PX_{\mathrm{s}}}{V_{\mathrm{a}}E_{\mathrm{af}}}\right) = -38.3°$$

and thus $\hat{E}_{\mathrm{af}} = 1.27\angle 38.3°$. The per-unit terminal current is thus

$$\hat{I}_{\mathrm{a}} = \frac{\hat{E}_{\mathrm{af}} - V\mathrm{a}}{jX_{\mathrm{s}}} = 1.009\angle - 0.39° \text{ per-unit}$$

which equates to a terminal current of 31.7 A.

Problem 7-20

First find the demagnetizing mmf. At the rated condition,

$$E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}}R_{\mathrm{tot}} = 600 - 250 \times 0.125 = 568.8 \text{ V}$$

corresponding to a voltage of 379.2 V at 400 r/min. Using the MATLAB 'spline' function, the corresponding field current on the 400 r/min magnetizing curve is $I_{\mathrm{f}} = 237.3$ A. Thus, the demagnetizing mmf at a current of 250 A is equal to $250 - 237.3 = 12.7$ A and in general, the effective series-field current will be equal to

$$I_{\mathrm{s,eff}} = I_{\mathrm{a}} - 12.7\left(\frac{I_{\mathrm{a}}}{250}\right)^2$$

For a starting current of 470 A, the effective series field current will thus equal 425 A. Using the MATLAB 'spline()' function, this corresponds to a generated voltage of 486 V from the 400 r/min magnetization curve. The corresponding torque (which is the same as the starting torque at the same flux and armature current) can then be calculated as

$$T = \frac{E_{\mathrm{a}}I_{\mathrm{a}}}{\omega_{\mathrm{m}}} = \frac{486 \times 470}{400(\pi/30)} = 5453 \text{ N}\cdot\text{m}$$

Problem 7-21

The motor power is given by $P = E_{\mathrm{a}}I_{\mathrm{a}}$, where

$$E_{\mathrm{a}} = K_{\mathrm{a}}\Phi_{\mathrm{d}}\omega_{\mathrm{m}}$$

and where, from Eq. 7.6

$$K_{\mathrm{a}} = \frac{\text{poles}\, C_{\mathrm{a}}}{2\pi m} = \frac{4 \times 784}{2\pi \times 2} = 249.6$$

Thus, for $\Phi_{\mathrm{d}} = 5.9 \times 10^{-3}$, $E_{\mathrm{a}} = K_{\mathrm{a}}\Phi_{\mathrm{d}}\omega_{\mathrm{m}} = 1.47\omega_{\mathrm{m}} = 0.154n$ where n is the motor speed in r/min.

Part (a): From a spline fit of the fan characteristic, the fan power requirement at 1285 r/min is 20.5 kW. The total power requirement for the dc motor including rotational losses is thus $P = 21.63$ kW. At a speed of 1285 r/min, $E_\mathrm{a} = 198.1$ V, $I_\mathrm{a} = P/E_\mathrm{a} = 109.3$ A and $V_\mathrm{a} = E_\mathrm{a} + I_\mathrm{a}R_\mathrm{a} = 219.4$ V.

Part (b): Using MATLAB and its 'spline()' function to represent the fan characteristics, an iterative routine can be written to solve for the operating point (the intersection of the motor and fan characteristics). The result is that the motor will operate at a speed of 1064 r/min, with a terminal current of 82.1 A and the fan power will be 12.3 kW.

Problem 7-22

Part (a): Assuming negligible voltage drop across the armature resistance at no load, the field current can be found from the 1250 r/min magnetization curve by setting $E_\mathrm{a} = 230$ V. This can be most easily done using the MATLAB 'spline()' function. The result is $I_\mathrm{f} = 1.67$ A.

Part (b): At full load, $E_\mathrm{a} = V_\mathrm{a} - I_\mathrm{a}R_\mathrm{a} = 221.9$ V. From the no-load, 1250 r/min magnetization curve, the corresponding field current is 1.50 A (again obtained using the MATLAB 'spline()' function). Since the motor terminal voltage remains at 230 V, the shunt field current remains at 1.67 A, the effective armature reaction is

$$\begin{aligned}\text{Armature reaction} &= (1.67 - 1.5)\ \text{A} \times 1650\ \text{turns/pole} \\ &= 281\ \text{A} \cdot \text{turns/pole}\end{aligned}$$

Part (c): With the series field winding, $R_\mathrm{tot} = R_\mathrm{a} + R_\mathrm{s} = 0.187\ \Omega$. Thus, under this condition, $E_\mathrm{a} = V_\mathrm{a} - I_\mathrm{a}R_\mathrm{a} = 220.0$ V. This corresponds to a 1250 r/min generated voltage of 239.1 V and a corresponding field current (determined from the magnetization curve using the MATLAB 'spline()' function) of 1.90 A, corresponding to a total of 3140 A·turns. The shunt-field-winding current of 1.67 produces $1.67 \times 1650 = 2755$ A·turns and thus, the required series field turns will be

$$N_\mathrm{s} = \frac{3140 - (2755 - 281)}{53.7} = 12.4$$

or, rounding to the nearest integer, $N_\mathrm{s} = 12$ turns/pole.

Part (d): Now the effective field current will be

$$I_\mathrm{eff} = \frac{2755 - 281 + 21 \times 53.7}{1650} = 2.18\ \text{A}$$

From the 1250 r/min magnetization curve, $E_\mathrm{a} = 247.8$ V while the actual $E_\mathrm{a} = V_\mathrm{a} - R_\mathrm{tot}I_\mathrm{a} = 220.0$ V. Hence the new speed is

$$n = 1250\left(\frac{220.0}{247.8}\right) = 1110 \text{ r/min}$$

Problem 7-23

Part (a): At full load, 1770 r/min, with a field current of 0.468 A

$$E_\mathrm{a} = V_\mathrm{a} - I_\mathrm{a}R_\mathrm{tot} = 217.5\ V$$

where $R_\mathrm{tot} = 0.18 + 0.035 = 0.215\ \Omega$.

A 1770 r/min magnetization curve can be obtained by multiplying 230 V by the ratio of 1770 r/min divided by the given speed for each of the points in the data table. A MATLAB 'spline()' fit can then be used to determine that this generated voltage corresponds to a field-current of 0.429 A . Thus, the armature reaction is $(0.468 - 0.429)2400 = 93.7$ A·turns/pole.

Part (b): The electromagnetic power at full load of 12.5 kW is $E_\mathrm{a}I_\mathrm{a} =$ 12.66 kW and hence the rotational loss is $P_\mathrm{rot} = 160$ W. For $\omega_\mathrm{m} = 1770 \times (\pi/30) = 185.4$ rad/sec the electromagnetic torque is 12.66 kW$/\omega_\mathrm{m} = 68.3$ N·m and the rotational-loss torque is similarly 0.86 N·m.

Part (c): With a field current of 0.555 A and armature reaction of 175 A-turns, the effective field current will be

$$I_\mathrm{eff} = 0.555 \text{ A} - \frac{175 \text{ A} - \text{turns}}{2400 \text{ turns}} = 0.482 \text{ A}$$

From the 1770 r/min magnetization curve found in part (b), this corresponds to a generated voltage of $E_\mathrm{a} = 231.4$ V. Thus, the corresponding torque (which is the same as the starting torque at the same flux and armature current) can then be calculated as

$$T = \frac{E_\mathrm{a}I_\mathrm{a}}{\omega_\mathrm{m}} = \frac{231.4 \times 85}{1770(\pi/30)} = 106.1 \text{ N}\cdot\text{m}$$

Part (d): The shunt field current can be found from the 1770 r/min magnetization characteristic at a voltage of $E_\mathrm{a} = 230 \times (1770/1800) = 226.2$ V. The result is $I_\mathrm{f} = 0.458$ A and including the armature reaction found in Part (a) as well as effect of an N_s-turn series field winding, the effective field current will be

$$I_\mathrm{f,eff} = 0.458 - \left(\frac{93.7}{2400}\right) + 58.2\left(\frac{N_\mathrm{s}}{2400}\right)$$

With the addition of 0.045 Ω, the total resistance in the armature circuit will now be $R_\mathrm{tot} = 0.260\ \Omega$. The required generated voltage will thus be

$$E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{tot}} = 214.9\ V$$

This corresponds to 214.9(1770/1575) = 241.5 V on the 1770 r/min magnetization curve and a corresponding effective field current of $I_{\mathrm{a,eff}} = 0.532$ A.

$$N_{\mathrm{s}} = \frac{2400(I_{\mathrm{f,eff}} - 0.458) + 93.7}{58.2} = 6.1 \text{ turns} \tag{1}$$

Problem 7-24

Part (a): From the 1500 r/min magnetization curve (obtained using the MATLAB 'spline()' function), we see that the shunt field current is 0.550 A since the no-load generated voltage must equal 230 V. The full-load generated voltage, also at 1500 r/min, is

$$E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{a}} = 219.4\ V$$

and the corresponding field current from the magnetization curve is 0.463 A. Thus the armature reaction is equal to 1700 (0.550−0.463) = 147 A·turns/pole.

Part (b): The total effective armature resistance is now $R_{\mathrm{tot}} = 0.12+0.038 = 0.158\ \Omega$. Thus, the full-load generated voltage will be

$$E_{\mathrm{a}} = V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{tot}} = 210.3\ V$$

The net effective field current is now equal to 0.463+125(3/1700) = 0.684 A. The corresponding voltage at 1500 r/min (found from the magnetization curve using the MATLAB 'spline()' function) is 248.9 V and hence the full-load speed is

$$n = 1500 \left(\frac{210.3}{248.9} \right) = 1267 \text{ r/min}$$

Part (c): The effective field current under this condition will be

$$I_{\mathrm{eff}} = 0.55 + 190\ (3/1700) - 270/1700 = 0.727 \text{ A}$$

From the 1500 r/min magnetization curve (using the MATLAB 'spline()' function), this corresponds to a generated voltage of 249.9 V. Thus, the corresponding torque will be

$$T = \frac{E_{\mathrm{a}} I_{\mathrm{a}}}{\omega_{\mathrm{m}}} = \frac{249.9 \times 190}{1500\ (\pi/30)} = 302 \text{ N} \cdot \text{m}$$

which is the same as the starting current at this level of flux and armature current.

Problem 7-25

Part (a): For a constant torque load and with constant field current (and hence constant field flux), the armature current must remain unchanged and hence $I_a = 84$ A.

Part(b):

$$E_a = V_a - R_a I_a$$

Prior to adding the 1.2Ω resistor, $E_a = 350 - 0.21 \times 84 = 332.4$ V and when it is inserted $E_a = 350 - 1.41 \times 84 = 231.6$ V. Thus,

$$\text{Speed ratio} = \frac{332.4}{231.6} = 0.70$$

Problem 7-26

Part (a): At a speed of 1240 r/min and an armature current of 103.5 A, $E_a = 460 - 103.5 \times 0.082 = 451.5$ V. At rated load, $E_a = 460 - 171 \times 0.082 = 446.0$ V. Thus, rated-load speed is

$$n = 1240 \left(\frac{446.0}{451.5} \right) = 1133 \text{ r/min}$$

Part (b): The maximum value of the starting resistance will be required at starting.

$$\frac{460}{R_a + R_{max}} = 1.8 \times 171 = 308$$

and thus $R_{max} = 1.412\ \Omega$.

Part (c): For each value of $R_{tot} = R_a + R_{ext}$, the armature current will reach its rated value when the motor reaches a speed such that

$$E_a = 230 - 171 R_{tot,old}$$

At this point R_{tot} will be reduced such that the armature current again reaches 307.8 A. Based upon this algorithm, the external resistance can be controlled as shown in the following table:

n [r/min]	ΔR [Ω]
0	–
561.5	-0.664
873.4	-0.369
1046.7	-0.205
1143.0	-0.114
1196.5	-0.060

Problem 7-27

Part (a): At no load, $E_{\mathrm{a,nl}} = K_{\mathrm{m}}\omega_{\mathrm{m,nl}} = V_{\mathrm{a}}$. Thus

$$\omega_{\mathrm{m,nl}} = \frac{V_{\mathrm{a}}}{K_{\mathrm{m}}} = \frac{85}{0.21} = 357.1 \text{ rad/sec}$$

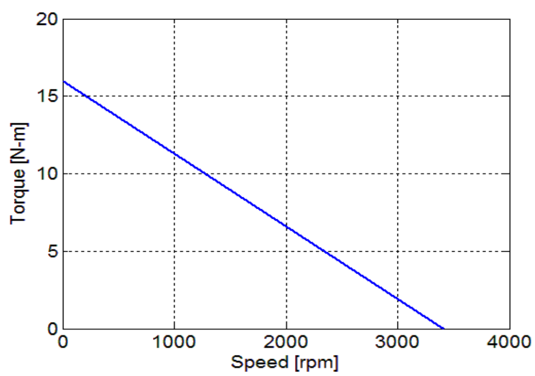
Hence, the no-load speed is $\omega_{\mathrm{m,nl}}(30/\pi) = 3410$ r/min.

Part (b): At zero speed, the current will be $I_{\mathrm{a}} = V_{\mathrm{a}}/R_{\mathrm{a}} = 57.1$ A and the corresponding torque will be $T = K_{\mathrm{m}}I_{\mathrm{a}} = 16.0$ N·m.

Part (c):

$$T = K_{\mathrm{m}}I_{\mathrm{a}} = \frac{K_{\mathrm{m}}(V_{\mathrm{a}} - E_{\mathrm{a}})}{R_{\mathrm{a}}} = \frac{K_{\mathrm{m}}(V_{\mathrm{a}} - K_{\mathrm{m}}\omega_{\mathrm{m}})}{R_{\mathrm{a}}}$$

Here is the desired plot, obtained using MATLAB:



Part (d): At speed ω_{m} the motor torque is equal to

$$T = K_{\mathrm{m}}\left(\frac{V_{\mathrm{a}} - \omega_{\mathrm{m}}K_{\mathrm{m}}}{R_{\mathrm{a}}}\right)$$

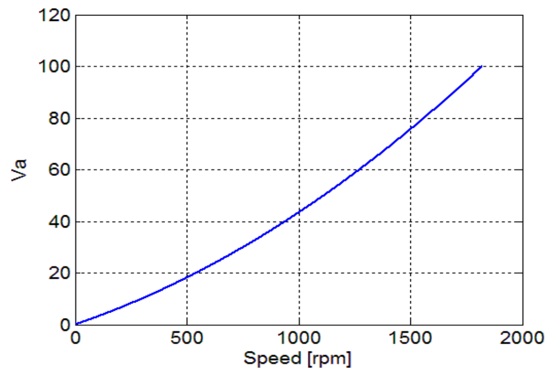
and the given pump torque is

$$T_{\mathrm{pump}} = 9.0\left(\frac{30\omega/\pi}{2000}\right)^2$$

A MATLAB search shows that the motor and pump torques will be equal at a speed of 1821 r/min.

Part (e): For a given speed ω_{m}, the pump torque is given as in Part (d) and the corresponding value of armature voltage can be found from

$$V_{\mathrm{a}} = \frac{R_{\mathrm{a}}T_{\mathrm{pump}}}{K_{\mathrm{m}}} + \omega_{\mathrm{m}}K_{\mathrm{m}}$$



Problem 7-28

Part (a): At no load, $\omega_{\mathrm{m,nl}} = 13,340\ (\pi/30) = 1397$ rad/sec and $E_{\mathrm{a,nl}} = V_{\mathrm{a}} - I_{\mathrm{a,nl}}R_{\mathrm{a}} = 8.60$ V. Thus

$$K_{\mathrm{m}} = \frac{E_{\mathrm{a,nl}}}{\omega_{\mathrm{m,nl}}} = 6.156 \times 10^{-3}\ \mathrm{V/(rad/sec)}$$

Part (b): The no load rotational losses are

$$P_{\mathrm{rot,nl}} = E_{\mathrm{a,nl}}I_{\mathrm{a,nl}} = 387\ \mathrm{mW}$$

Part (c): At zero speed, the current will be $I_{\mathrm{a}} = V_{\mathrm{a}}/R_{\mathrm{a}} = 1.01$ A and the corresponding torque will be $T = K_{\mathrm{m}}I_{\mathrm{a}} = 6.23$ mN·m.

Part (d): From Part (b), the rotational loss power is equal to

$$P_{\mathrm{rot}} = \left(\frac{n}{13340}\right)^3 \times 387\ \ \mathrm{mW} = \left(\frac{30\ \omega_{\mathrm{m}}/\pi}{13340}\right)^3 \times 387\ \ \mathrm{mW}$$

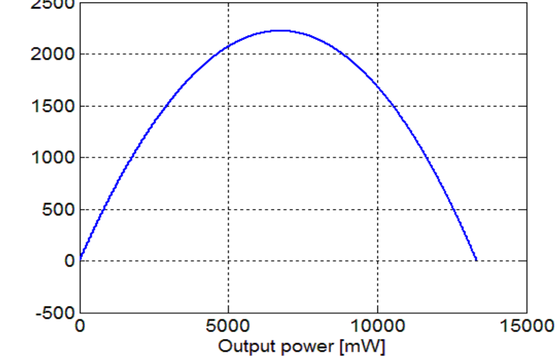
while the electromagnetic power is given by

$$P = E_{\mathrm{a}}I_{\mathrm{a}} = \frac{K_{\mathrm{m}}\omega_{\mathrm{m}}(V_{\mathrm{a}} - K_{\mathrm{m}}\omega_{\mathrm{m}})}{R_{\mathrm{a}}}$$

The output power is then given by

$$P_{\mathrm{out}} = P - P_{\mathrm{rot}}$$

A plot of P_{out} vs speed shows that the output power is equal to 2 W at two speeds.

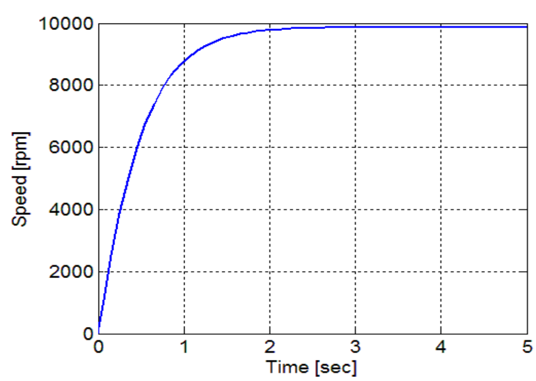


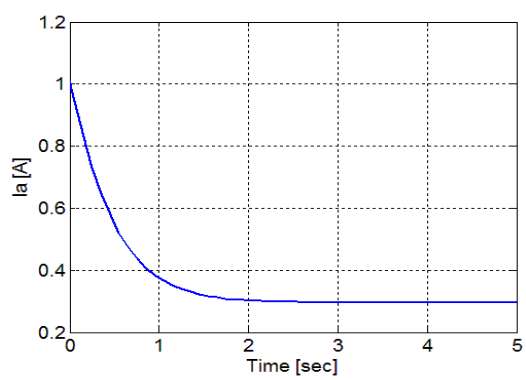
A MATLAB solution shows that the output power is equal to 2 W at 4624 r/min at which speed the efficiency is 32.8% and at 8845 r/min at which speed the efficiency is 56.0%.

Problem 7-29

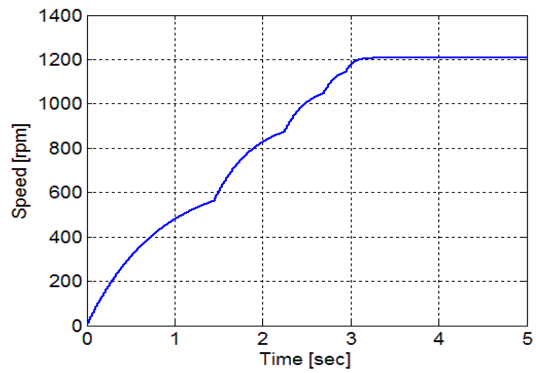
$K_{\rm m} = 8.28 \;\; \text{mV/(rad/sec)}$ and the rotational loss is 162 mW.

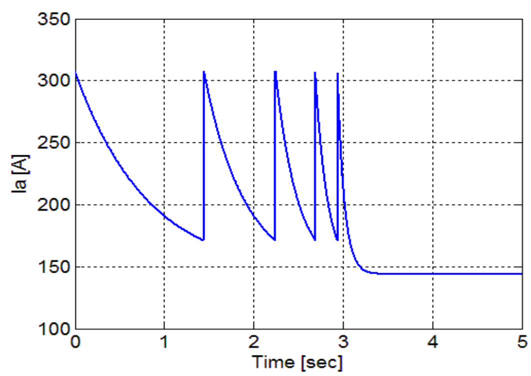
Problem 7-30





Problem 7-31


1400
1200
1000
800
600
400
200
0
Speed [rpm]
0
1
2
3
4
5
Time [sec]

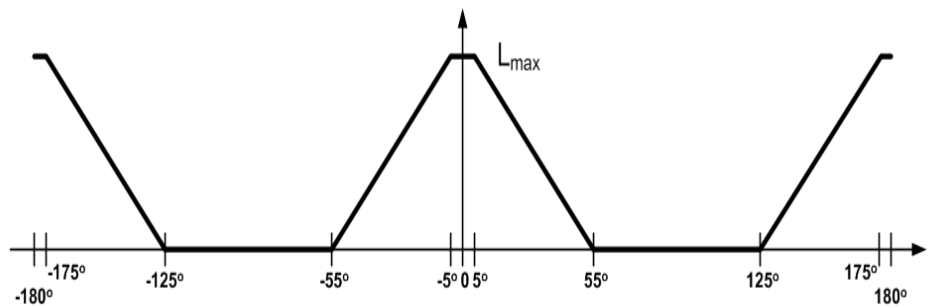

350
300
250
200
150
100
Ia [A]
0
1
2
3
4
5
Time [sec]

PROBLEM SOLUTIONS: Chapter 8

Problem 8-1

Part (a): In this case, $\beta = 50° = 0.278\pi$ rad, there is a 10° overlap region of constant inductance with

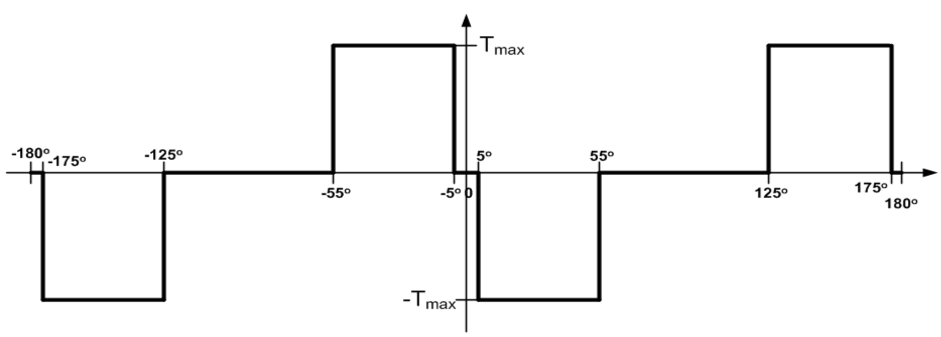
$$L_{\max} = \frac{N^2 \mu_0 \beta R D}{2g} = 154 \text{ mH}$$



Part (b): The inductance changes from 0 to $L_{\max}$ as the rotor rotates through angle β and thus

$$T_{\max,1} = \frac{L_{\max} I_1^2}{2\beta} = 8.8 \times 10^{-2} i_1^2 \text{ N} \cdot \text{m}$$

$$T_{\max,1} = \frac{L_{\max} I_2^2}{2\beta} = 8.8 \times 10^{-2} i_2^2 \text{ N} \cdot \text{m}$$



Part (c): $i_1 = i_2 = 5$ A;

(i) $\omega = 0 \quad T_{\text{net}} = 0$
(ii) $\omega = 45° \quad T_{\text{net}} = 0$ (iii) $\omega = 75° \quad T_{\text{net}} = 2.20$ N·m

Problem 8-2

When a single phase is excited, draw a closed contour through stator back iron, the poles of that phase, across the two air gaps and through the rotor. Applying Faraday's law, we see that 1/2 the mmf drop, i.e. the mmf of one phase coil, occurs across each of the two airgaps. If one then draws a similar contour through one pole of the excited phase and one pole of the un-excited phase, one sees that there is no additional mmf available to drive flux through the second phase.

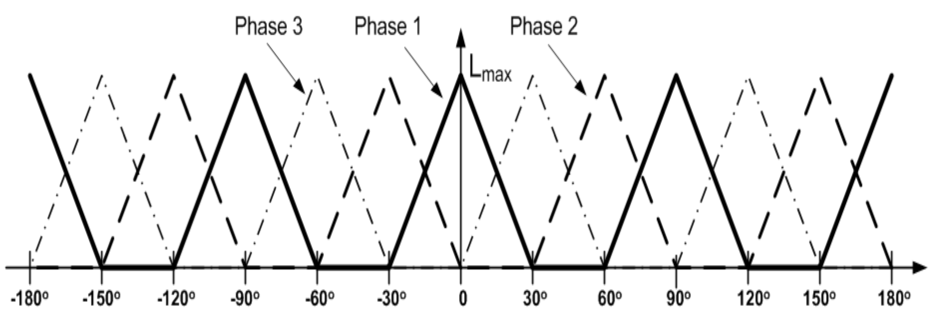
Problem 8-3

When a single phase is excited, draw a closed contour through stator back iron, the poles of that phase, across the two air gaps and through the rotor. Applying Faraday's law, we see that 1/2 the mmf drop, i.e. the mmf of one phase coil, occurs across each of the two airgaps. If one then draws a similar contour through one pole of the excited phase and one pole of the un-excited phase, one sees that there is no additional mmf available to drive flux through the remaining phases.

Problem 8-4

Parts (a) and (b):

$$L_{\max} = \frac{DR\alpha\mu_0 N^2}{2g} = 29.8 \text{ mH}$$

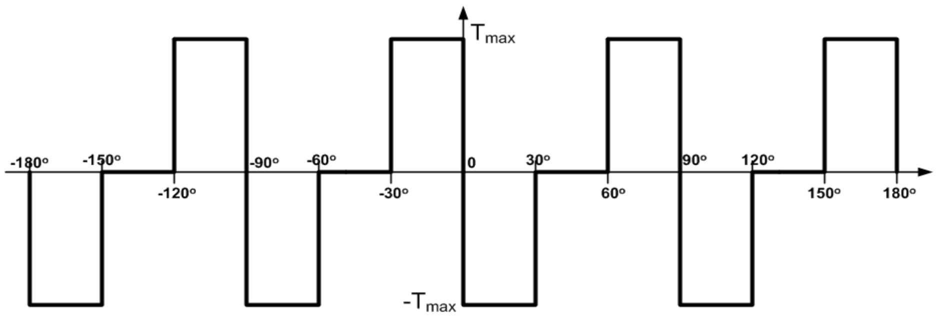


Part (c):

$$I_0 = \frac{2gB}{\mu_0 N} = 8.75 \text{ A}$$

Part (d):

$$T_{\max} = \frac{I0^2}{2}\left(\frac{L_{\max}}{\alpha/2}\right) = 4.37 \text{ N}\cdot\text{m}$$



Part (e):
phase 1 ON:
$-120° \leq \theta \leq -90°$, $-30° \leq \theta \leq 0°$, $60° \leq \theta \leq 90°$, $150° \leq \theta \leq 180°$

phase 2 ON:
$-150° \leq \theta \leq -120°$, $-60° \leq \theta \leq -30°$, $30° \leq \theta \leq 60°$, $120° \leq \theta \leq 150°$

phase 3 ON:
$-180° \leq \theta \leq -150°$, $-90° \leq \theta \leq -60°$, $0° \leq \theta \leq 30°$, $90° \leq \theta \leq 120°$

Part (f): The rotor will rotate 90° in 45 msec.

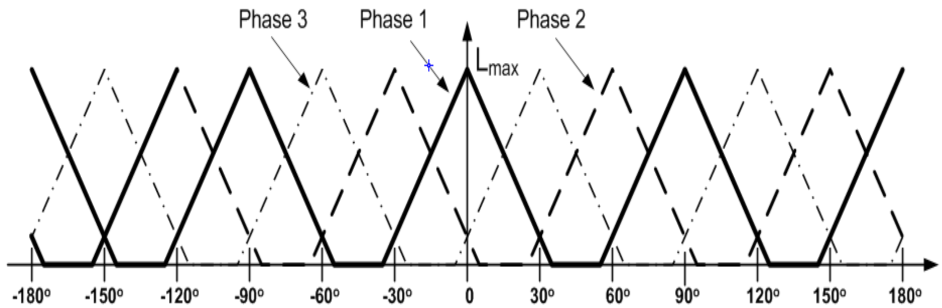
$$n = \frac{(1/4)\ \text{r}}{45\ \text{msec}} = 6.25\ \text{r/sec} = 375\ \text{r/min}$$

The rotor will rotate in the clockwise direction if the phase sequence is 1 - 2 - 3 - 1 ....

Problem 8-5

Parts (a) and (b):

$$L_{\max} = \frac{DR\alpha\mu_0 N^2}{2g} = 29.8\ \text{mH}$$

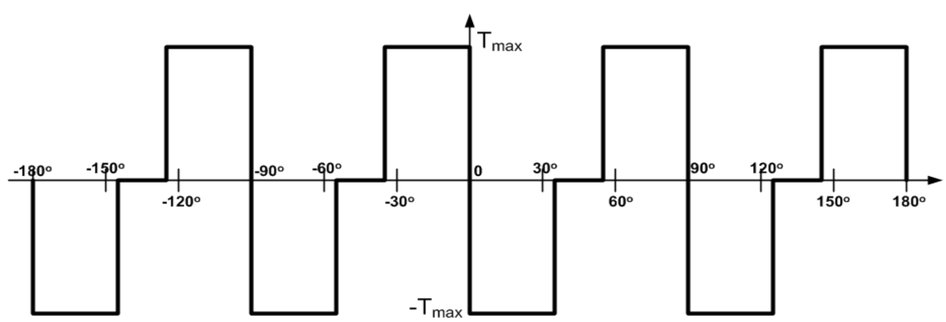


Part (c):

$$I_0 = \frac{2gB}{\mu_0 N} = 8.75 \text{ A}$$

Part (d):

$$T_{\max} = \frac{I0^2}{2}\left(\frac{L_{\max}}{\alpha/2}\right) = 4.37 \text{ N} \cdot \text{m}$$



Part (e):
phase 1 ON:
$-120° \leq \theta \leq -90°$, $-30° \leq \theta \leq 0°$, $60° \leq \theta \leq 90°$, $150° \leq \theta \leq 180°$

phase 2 ON:
$-150° \leq \theta \leq -120°$, $-60° \leq \theta \leq -30°$, $30° \leq \theta \leq 60°$, $120° \leq \theta \leq 150°$

phase 3 ON:
$-180° \leq \theta \leq -150°$, $-90° \leq \theta \leq -60°$, $0° \leq \theta \leq 30°$, $90° \leq \theta \leq 120°$

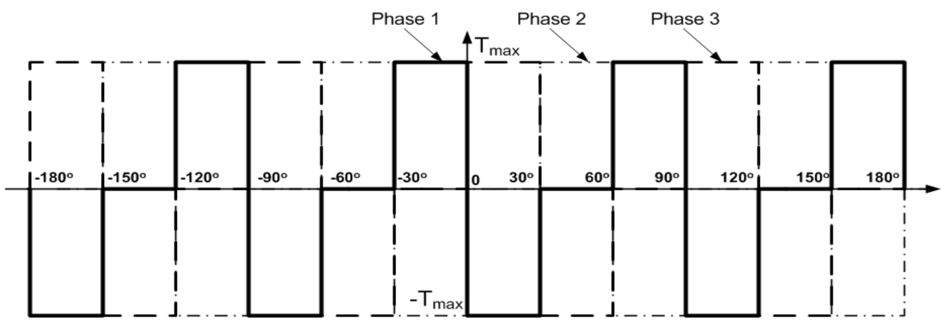
Part (f): The rotor will rotate 90° in 45 msec.

$$n = \frac{(1/4) \text{ r}}{45 \text{ msec}} = 6.25 \text{ r/sec} = 375 \text{ r/min}$$

The rotor will rotate in the clockwise direction if the phase sequence is 1 - 2 - 3 - 1 ....

Problem 8-6

A typical torque vs. position plot including the torque produced by all three phases such as that shown below shows that there is no rotor position at which it is possible to get uni-directional torque.



Problem 8-7

The rotor will rotate 15° as each consecutive phase is excited. Thus, the rotor will rotate 1 revolution in 24 sequences of phase excitation or 8 complete sets of phase excitation. Thus, the rotor will rotate 1 revolution in $8 \times 10 = 80!$ msec. Thus it will rotate at $1/0.08 = 12.5$ r/sec = 750 r/min.

Problem 8-8

Part (a): If phase 1 is shut off and phase 2 is turned on, the rotor will move to the left by $2\beta/3 \approx 4.29°$. Similarly, turning off phase 2 and turning on phase 3 will cause the rotor to move yet another 4.29°. Thus, starting with phase 1 on, to move 21.4° will require $21.4°/4.29° \approx 5$ steps, the sequence will be:

1 ON
1 OFF & 2 ON
2 OFF & 3 ON
3 OFF & 1 ON
1 OFF & 2 ON
2 OFF & 3 ON

Part (b): Clockwise is equivalent to rotor rotation to the right. The required phase sequence will be ... 1 - 3 - 2 - 1 - 3 - 2 .... The rotor will rotate $\approx 4.29°$/step and hence the rotor speed will be

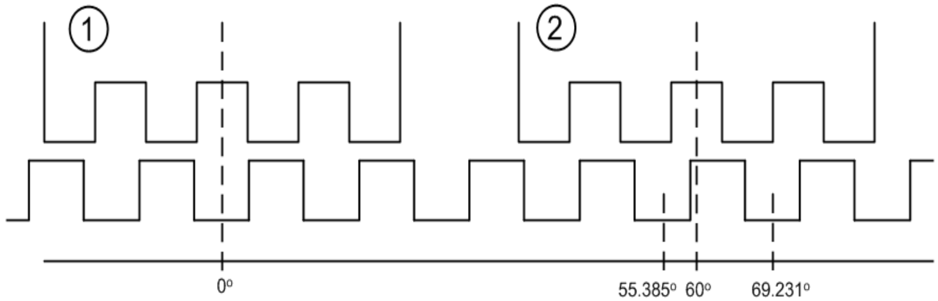
$$95 \text{ r/min} \times \frac{360°}{r} \times \frac{1 \text{ step}}{4.29°} = 7.97 \times 10^3 \text{ steps/min}$$

Thus the required step time is

$$\frac{\text{time}}{\text{step}} = \frac{1 \text{ min}}{7.97 \times 10^3 \text{ step}} \times \frac{60 \text{ sec}}{\text{min}} = 7.53 \text{ msec/step}$$

Problem 8-9

Part (a): When phase 1 is energized, the rotor will be aligned as shown in the following figure:



From the figure, we see that if phase 1 is turned off and phase 2 is energized, the rotor will rotate 4.615° degrees to the right (clockwise) to align with the phase-2 pole. Similarly, if phase 3 is excited after phase 1 is turned off, the rotor will rotate 4.615° degrees to the left (counterclockwise).

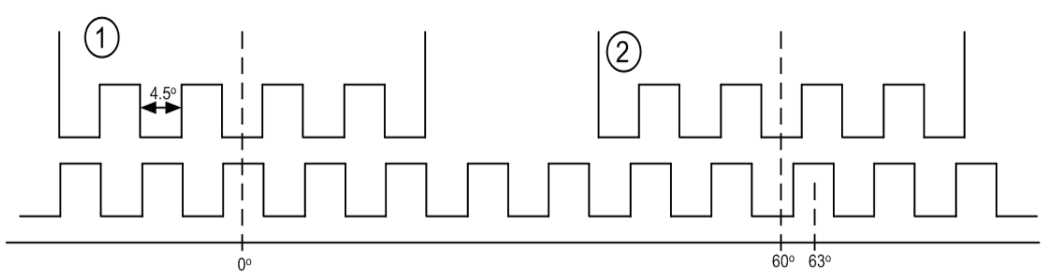
Part (b):

$$50 \text{ r/min} \times \frac{360°}{r} \times \frac{1 \text{ step}}{4.61°} = 3.905 \times 10^3 \text{ steps/min}$$

$$\frac{\text{time}}{\text{step}} = \frac{1 \text{ min}}{3.905 \times 10^3 \text{ step}} \times \frac{60 \text{ sec}}{\text{min}} = 15.4 \text{ msec/step}$$

The required phase sequence will thus be ... 1 - 3 - 2 - 1 - 3 - 2 - 1 ....

Problem 8-10



Part (a): As can be seen from the pole pattern, if phase 1 is shut off and phase 2 is turned on, the rotor will move to the left by 3°. Similarly, turning off phase 2 and turning on phase 3 will cause the rotor to move yet another 3. Thus, starting with phase 1 on, to move 21.4° will require $21.4°/3.0° \approx 7$ steps, the sequence will be:

1 ON
1 OFF & 2 ON
2 OFF & 3 ON
3 OFF & 1 ON
1 OFF & 2 ON
2 OFF & 3 ON

Part (b): Clockwise is equivalent to rotor rotation to the right. The required phase sequence will be ... 1 - 3 - 2 - 1 - 3 - 2 .... The rotor will rotate 3°/step and hence the rotor speed will be

$$95 \text{ r/min} \times \frac{360°}{r} \times \frac{1 \text{ step}}{3.0°} = 1.14 \times 10^4 \text{ steps/min}$$

Thus the required step time is

$$\frac{\text{time}}{\text{step}} = \frac{1 \text{ min}}{1.14 \times 10^4 \text{ step}} \times \frac{60 \text{ sec}}{\text{min}} = 5.26 \text{ msec/step}$$

Problem 8-11

Part (a): From Eq. 8.15, the differential equation governing the current buildup in phase 1 is given by

$$v_1 = \left[R + \frac{dL_{11}(\theta_\mathrm{m})}{d\theta_\mathrm{m}}\frac{d\theta_\mathrm{m}}{dt}\right] i_1 + L_{11}(\theta_\mathrm{m})\frac{di_1}{dt}$$

At 380 r/min,

$$\omega_\mathrm{m} = \frac{d\theta_\mathrm{m}}{dt} = 380 \text{ r/min} \times \frac{\pi}{30}\left[\frac{\text{rad/sec}}{\text{r/min}}\right] = 39.79 \text{ rad/sec}$$

From Fig. 8.4 (for $-60° \leq \theta_\mathrm{m} \leq 0°$))

$$L_{11}(\theta_\mathrm{m}) = L_l + \frac{L_\mathrm{max}}{\pi/3}\left(\theta_\mathrm{m} + \frac{\pi}{3}\right)$$

Thus

$$\frac{dL_{11}(\theta_\mathrm{m})}{d\theta_\mathrm{m}} = \frac{3L_\mathrm{max}}{\pi}$$

and

$$\frac{dL_{11}(\theta_\mathrm{m})}{d\theta_\mathrm{m}}\frac{d\theta_\mathrm{m}}{dt} = \left(\frac{3\,L_\mathrm{max}}{\pi}\right)\omega_\mathrm{m} = 4.86\ \Omega$$

which is much greater than the resistance $R = 0.2\,\Omega$

This will enable us to obtain an approximate solution for the current by neglecting the Ri term in Eq. 8.13. We must then solve

$$\frac{d(L_{11} i_1)}{dt} = v_1$$

for which the solution is

$$i_1(t) = \frac{\int_0^t v_1 dt}{L_{11}(t)} = \frac{V_1 t}{L_{11}(t)}$$

where $V_1 = 100$ V. Substituting

$$\theta_{\mathrm{m}} = -\frac{\pi}{3} + \omega_{\mathrm{m}} t$$

into the expression for $L_{11}(\theta_{\mathrm{m}})$ gives gives

$$L_{11}(t) = L_{\mathrm{l}} + \left(\frac{3\,L_{\mathrm{max}}}{\pi}\right)\omega_{\mathrm{m}}\,t$$

and thus

$$i_1(t) = \frac{100\,t}{0.005 + 4.86\,t}$$

which is valid until $\theta_{\mathrm{m}} = 0°$ at $t = t_1 = 26.31$ msec, at which point $i_1(t_1) = 19.8$ A.

Part (b): During the period of current decay, the solution proceeds as in Part (a). From Fig. 8.4, for $0° \leq \theta_{\mathrm{m}} \leq 60°$, $dL_{11}(\theta_{\mathrm{m}})/dt = -4.86\,\Omega$ and again the Ri term can again be ignored in Eq. 8.15. As a result, during this period, the phase-1 current can again be solved by integration

$$i_1(t) = i_1(t_1) + \frac{\int_{t_1}^t v_1 dt}{L_{11}(t)} = \frac{V_2\,(t - t_1)}{L_{11}(t)}$$

where $V_2 = -200$ V and

$$L_{11}(t) = L_l + \left(\frac{3\,L_{\mathrm{max}}}{\pi}\right)\omega_m(2t_1 - t)$$

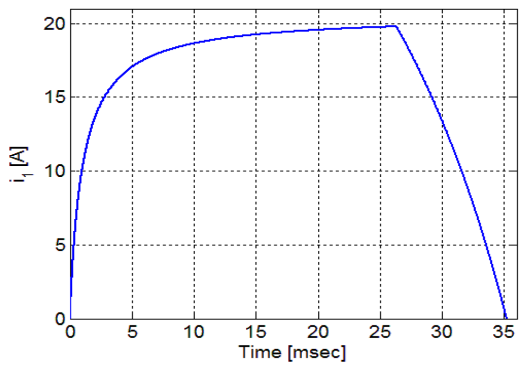
From this equation, we see that the current reaches zero at $t = 35.20$ msec.

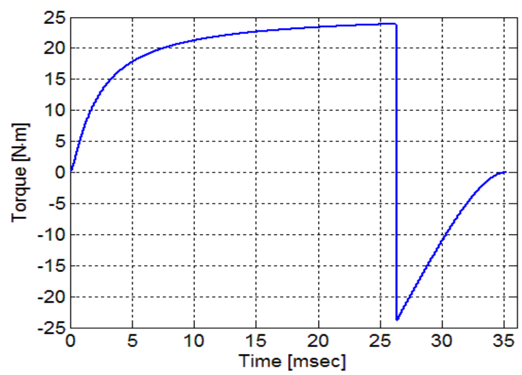
Part (c): The torque can be found from Eq. 8.9 by setting $i_2 = 0$. Thus

$$T_{\mathrm{mech}} = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta_{\mathrm{m}}}$$

Using MATLAB and the results of parts (a) and (b), the current waveform and torque are plotted below. The integral under the positive portion of the torque curve is 0.266 $\mathrm{N \cdot m \cdot sec}$ while that under the negative portion of the torque curve is 0.043 $\mathrm{N \cdot m \cdot sec}$. Thus we see that the negative torque produces a 16.2 percent reduction in average torque from that which would otherwise be available if the current could be reduced instantaneously to zero.

The current and torque are plotted below:





Problem 8-12

Part (a): From Eq. 8.15, the differential equation governing the current buildup in phase 1 is given by

$$v_1 = \left[R + \frac{dL_{11}(\theta_{\mathrm{m}})}{d\theta_{\mathrm{m}}}\frac{d\theta_{\mathrm{m}}}{dt}\right] i_1 + L_{11}(\theta_{\mathrm{m}})\frac{di_1}{dt}$$

At 450 r/min,

$$\omega_{\mathrm{m}} = \frac{d\theta_{\mathrm{m}}}{dt} = 450 \text{ r/min} \times \frac{\pi}{30}\left[\frac{\text{rad/sec}}{\text{r/min}}\right] = 47.12 \text{ rad/sec}$$

From Fig. 8.4 (for $-60^\circ \le \theta_{\mathrm{m}} \le 0^\circ$))

$$L_{11}(\theta_{\mathrm{m}}) = L_l + \frac{L_{\max}}{\pi/3}\left(\theta_{\mathrm{m}} + \frac{\pi}{3}\right)$$

Thus

$$\frac{dL_{11}(\theta_{\mathrm{m}})}{d\theta_{\mathrm{m}}} = \frac{3L_{\mathrm{max}}}{\pi}$$

and

$$\frac{dL_{11}(\theta_{\mathrm{m}})}{d\theta_{\mathrm{m}}}\frac{d\theta_{\mathrm{m}}}{dt} = \left(\frac{3\,L_{\mathrm{max}}}{\pi}\right)\omega_{\mathrm{m}} = 5.76\ \Omega$$

which is much greater than the resistance $R = 0.2\,\Omega$

This will enable us to obtain an approximate solution for the current by neglecting the Ri term in Eq. 8.13. We must then solve

$$\frac{d(L_{11}i_1)}{dt} = v_1$$

for which the solution is

$$i_1(t) = \frac{\int_0^t v_1 dt}{L_{11}(t)} = \frac{V_1 t}{L_{11}(t)}$$

where $V_1 = 100$ V. Substituting

$$\theta_{\mathrm{m}} = -\frac{\pi}{3} + \omega_{\mathrm{m}} t$$

into the expression for $L_{11}(\theta_{\mathrm{m}})$ gives gives

$$L_{11}(t) = L_{\mathrm{l}} + \left(\frac{3\,L_{\mathrm{max}}}{\pi}\right)\omega_{\mathrm{m}}\,t$$

and thus

$$i_1(t) = \frac{100\,t}{0.005 + 5.76\,t}$$

which is valid until $\theta_{\mathrm{m}} = 0°$ at $t = t_1 = 22.22$ msec, at which point $i_1(t_1) = 16.7$ A.

Part (b): During the period of current decay, the solution proceeds as in Part (a). From Fig. 8.4, for $0° \le \theta_{\mathrm{m}} \le 60°$, $dL_{11}(\theta_{\mathrm{m}})/dt = -5.76\,\Omega$ and again the Ri term can again be ignored in Eq. 8.15. As a result, during this period, the phase-1 current can again be solved by integration

$$i_1(t) = i_1(t_1) + \frac{\int_{t_1}^t v_1 dt}{L_{11}(t)} = \frac{V_2\,(t - t_1)}{L_{11}(t)}$$

where $V_2 = -250$ V and

$$L_{11}(t) = L_l + \left(\frac{3\,L_{\mathrm{max}}}{\pi}\right)\omega_m(2t_1 - t)$$

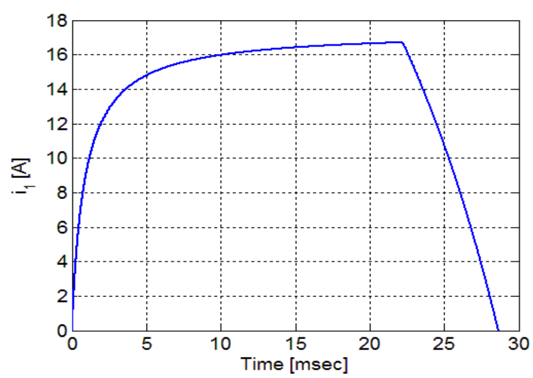
From this equation, we see that the current reaches zero at $t = 28.64$ msec.

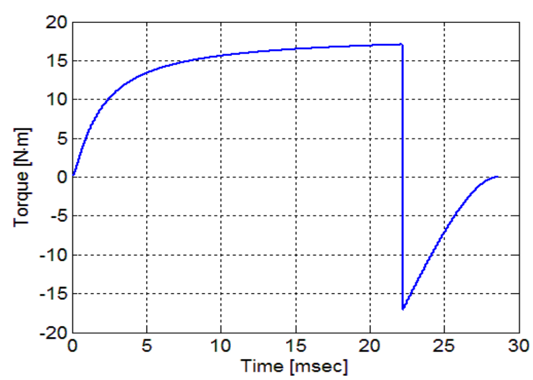
Part (c): The torque can be found from Eq. 8.9 by setting $i_2 = 0$. Thus

$$T_{\mathrm{mech}} = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta_{\mathrm{m}}}$$

Using MATLAB and the results of parts (a) and (b), the current waveform and torque are plotted below. The integral under the positive portion of the torque curve is 0.160 $\mathrm{N \cdot m \cdot sec}$ while that under the negative portion of the torque curve is 0.021 $\mathrm{N \cdot m \cdot sec}$. Thus we see that the negative torque produces a 13.3 percent reduction in average torque from that which would otherwise be available if the current could be reduced instantaneously to zero.

The current and torque are plotted below:



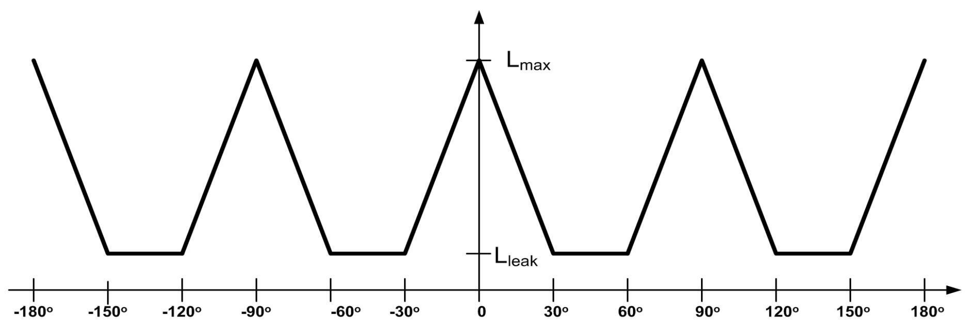


Problem 8-13

Advance angle = 2.5°: $T_{\mathrm{avg}} = 12.6$ N·m
Advance angle = 7.5°: $T_{\mathrm{avg}} = 22.4$ N·m

Problem 8-14

Part (a): The phase inductance looks like the plot of Problem 8.4, part (a), with the addition of the $L_{\mathrm{leak}} = 4.2$ mH leakage inductance. Now $L_{\max} = \frac{DR\alpha\mu_0 N^2}{2g} + L_{\mathrm{leak}} = 29.8 + 4.2 = 34.0$ mH.



Part (b): For $-30° \leq \theta \leq 0°$

$$\frac{dL}{d\theta} = \frac{29.8 \text{ mH}}{\pi/6 \text{ rad}} = 56.9 \text{ mH/rad}$$

$$\omega_{\mathrm{m}} = \frac{d\theta}{dt} = \frac{1675 \text{ r}}{\text{min}} \times \frac{2\pi\text{rad}}{\text{r}} \times \frac{1\text{min}}{60\text{sec}} = 175.4 \text{ rad/sec}$$

$$\frac{dL}{dt} = \omega_{\mathrm{m}} \frac{dL}{d\theta} = 9.98 \ \Omega$$

The governing equation is

$$v = iR + L\frac{di}{dt} + i\frac{dL}{dt}$$

Noting that $dL/dt >> R$, we can approximate this equation as

$$v \approx \frac{d(Li)}{dt}$$

and thus

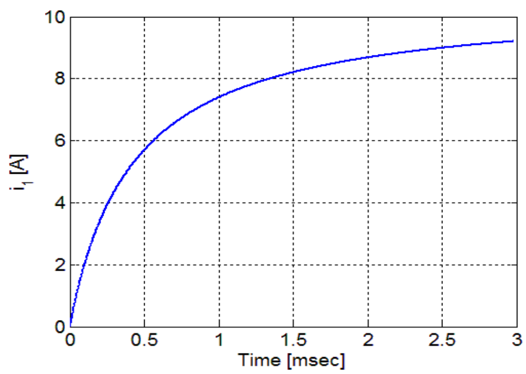
$$i(t) = \frac{\int v(t)\, dt}{L(t)}$$

Substituting $v(t) = 105$ V and $L(t) = 4.2 \times 10^{-3} + 7.21\, t$ then gives

$$i(t) = \frac{105\, t}{4.2 \times 10^{-3} + 9.98\, t}$$

which is valid over the range $0 \leq t \leq (\pi/6)/\omega_{\mathrm{m}}$ msec ($0 \leq t \leq 2.99$ msec), at which time $i(t) = 9.22$ A.

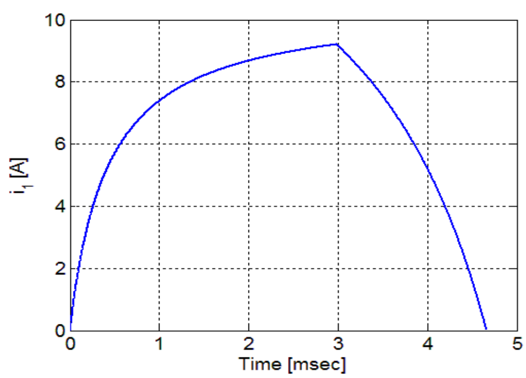
Here is the desired plot:



Part (c): During this time, starting at time $t = 2.99$ msec, $v(t) = -105$ V and $L(t) = 34.0 \times 10^{-3} - 9.98(t - 2.99 \times 10^{-3})$. Thus

$$i(t) = 16.5 + \frac{-105\,(t - 2.99 \times 10^{-3})}{34.0 \times 10^{-3} - 9.98(t - 2.99 \times 10^{-3})}$$

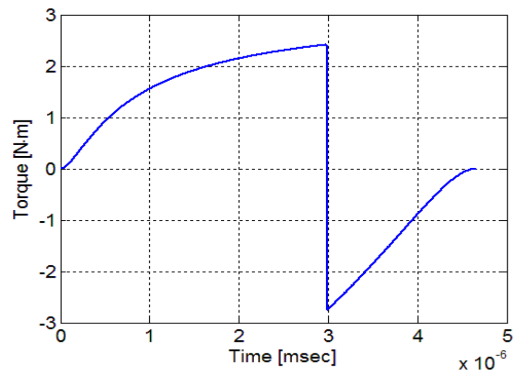
which reaches zero at $t = 4.66$ msec. Here is the plot of the total current transient.



Part (d): The torque is given by

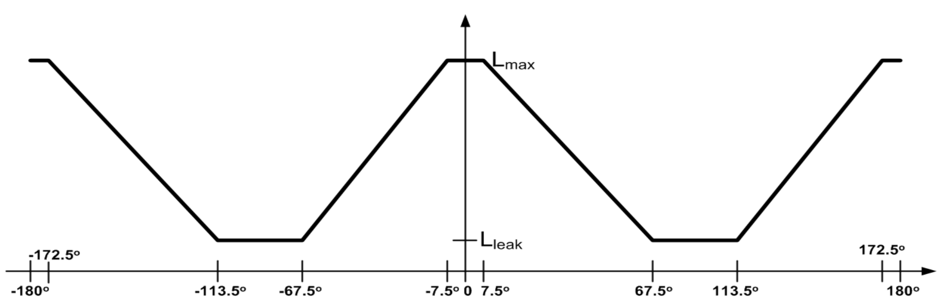
$$T = \frac{i^2}{2}\,\frac{dL}{d\theta}$$

Here is the plot:



Problem 8-15

Part (a): The plot of $L(\theta)$ is shown below



Here, from Examples 8.1 and 8.3, $L_{\text{leak}} = 5$ mH and $L_{\text{max}} = 133$ mH.

Part (b): The solution for $-67.5° \leq \theta \leq -7.5°$ ($0 \leq t \leq 2.5$ msec) is exactly the same as part (a) of Example 8.3

$$i(t) = \frac{100t}{0.005 + 51.2t} \text{ A}$$

For $-7.5° \leq \theta \leq 7.5°$ (2.5 msec $\leq t \leq 3.13$ msec), $dL/dt = 0$ and thus

$$v = iR + L\frac{di}{dt} \Rightarrow -100 = 1.5i + 0.133\,\frac{di}{dt}$$

This equation has an exponential solution with time constant $\tau = L/R =$ 88.7 msec.

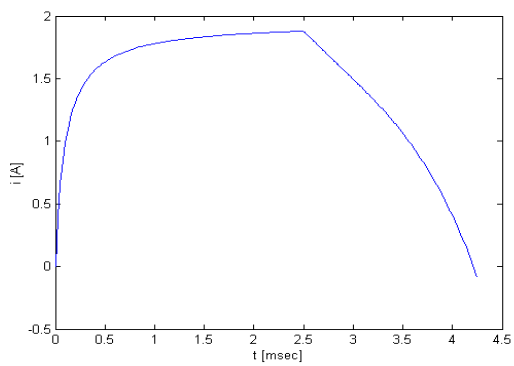
$$i = -66.7 + 68.6e^{-(t-0.0025)/0.0887}$$

At $t = 3.13$ msec, $i(t) = 1.39$ A.

Following time $t = 3.13$ msec, the solution proceeds as in Example 8.3. Thus

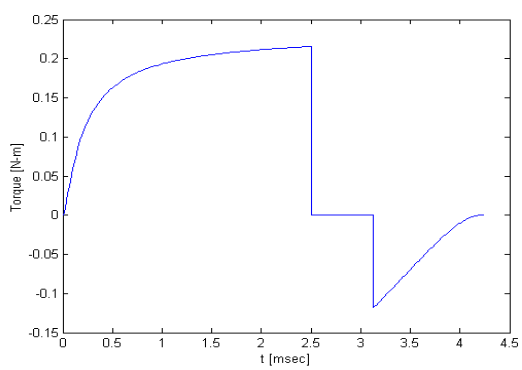
$$i(t) = 1.468 - \frac{100 - 3.13 \times 10^{-3}}{0.005 - 51.2(t - 5.63 \times 10^{-3})}$$

The current reaches zero at $t = 4.25$ msec. Here is the corresponding plot, produced by MATLAB



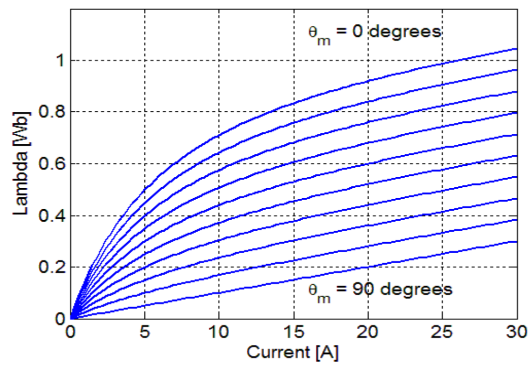
Part (c):

$$T = \frac{i^2}{2}\frac{dL}{d\theta}$$



Problem 8-16

Part (a):



Part (b): Using MATLAB as in Example 8.4, we find that area($W_{\rm net}$) = 17.81 joules and area($W_{\rm rec}$) = 9.87 joules and thus

$$\frac{\text{Inverter volt-ampere rating}}{\text{Net output power}} = \frac{\text{area}(W_{\rm rec} + W_{\rm net})}{\text{area}(W_{\rm net})} = 1.55$$

Part (c):

$$P_{\rm phase} = 2\left(\frac{\text{area}(W_{\rm net})}{T}\right) \text{ W} = 1484 \text{ W}$$

and thus $P_{\rm mech} = 2P_{\rm phase} = 2968$ W.

Problem 8-17

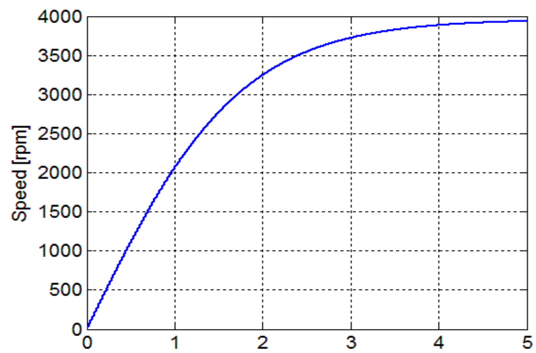
Part (a): Based upon the discussion in the text associated with Fig. 8.21, the following table can be produced:

θ_m	bit pattern	i_1	i_2
0°	1000	I_0	0
45°	1010	I_0	I_0
90°	0010	0	I_0
135°	0110	$-I_0$	I_0
180°	0100	$-I_0$	0
225°	0101	$-I_0$	$-I_0$
270°	0001	0	$-I_0$
315°	1001	I_0	$-I_0$

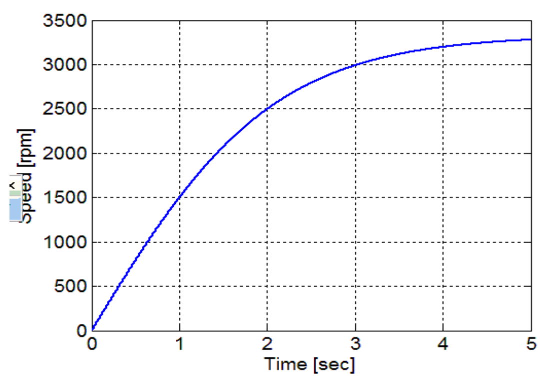
Part (b): There will be 8 pattern changes per revolution. At 1400 r/min = 23.33 r/sec, there must be 186.7 pattern changes per second, corresponding to a time of 5.36 msec between pattern changes.

Problem 8-18

Plot for $\gamma = 0$



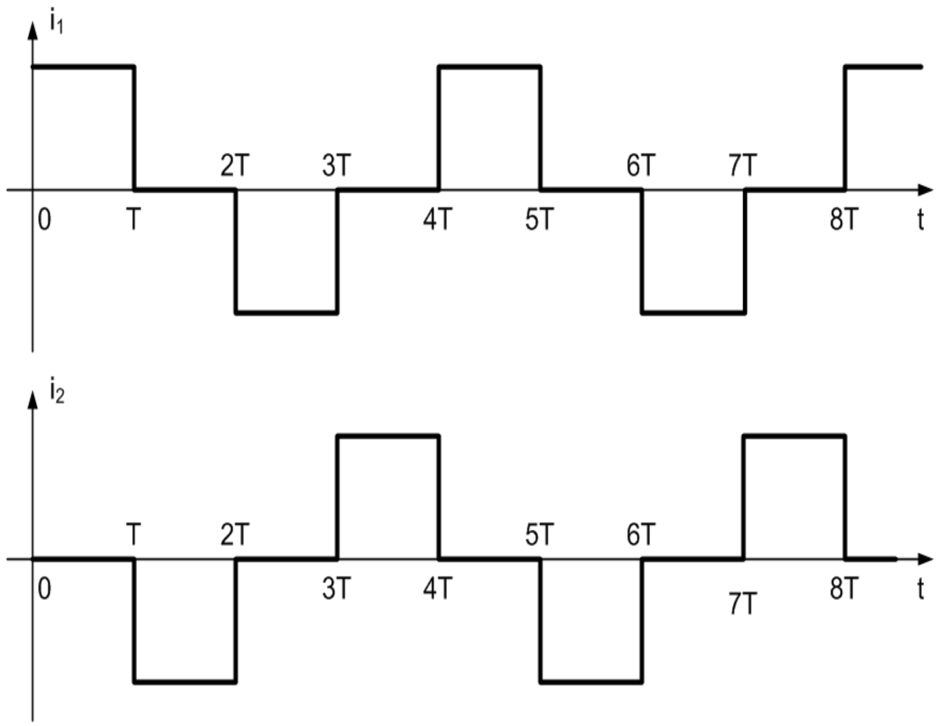
Plot for $\gamma = \pi/4$



Problem 8-19

Part (a): The rotor will rotate 2° counter clockwise.

Part (b): The phase excitation will look like (with T determined in Part (c)):



Part (c):

$$\frac{10 \text{ r}}{\text{min}} = \frac{3600°}{\text{min}} = \frac{60°}{\text{sec}} = \frac{2°}{33.33 \text{ msec}}$$

and thus $T = 33.33$ msec. The frequency will be

$$f = \frac{1}{4T} = 7.5 \text{ Hz}$$

Problem 8-20

Part (a): The displacement will be $360°/(3 \times 16) = 7.50°$.

Part (b): There will be one revolution of the motor for every 16 cycles of the phase currents. Hence

$$f = \left(\frac{750 \text{ r}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{16 \text{ cycles}}{\text{r}}\right) = 200 \text{ Hz}$$

PROBLEM SOLUTIONS: Chapter 9

Problem 9-1

Part (a):

$$\hat{I}_{\rm main} = \frac{\hat{V}}{Z_{\rm main}} = 10.3\angle - 56.4^\circ \text{ A}$$

$$\hat{I}_{\rm aux} = \frac{\hat{V}}{Z_{\rm aux}} = 7.4\angle - 49.2^\circ \text{ A}$$

Part (b): We want the angle of the auxiliary-winding current to lead that of the main winding by 90^c ($\pi/2$ rad). Thus, defining $Z'_{\rm aux} = Z_{\rm aux} + jX_{\rm C}$ ($X_{\rm C} = -1/\omega C$), we want

$$\angle Z'_{\rm aux} = \tan^{-1}\left(\frac{\text{Im}[Z_{\rm aux}] - X_{\rm C}}{\text{Re}[Z_{\rm aux}]}\right) = \angle Z_{\rm main} + \frac{\pi}{2}$$

Thus $X_{\rm C} = -19.2\ \Omega$ and $C = 138\ \mu$F.

Part (c):

$$\hat{I}_{\rm main} = \frac{\hat{V}}{Z_{\rm main}} = 10.3\angle - 56.4^\circ \text{ A}$$

$$\hat{I}_{\rm aux} = \frac{\hat{V}}{Z'_{\rm aux}} = 9.4\angle 33.6^\circ \text{ A}$$

$$\hat{V}_{\rm aux} = \hat{I}_{\rm aux} Z_{\rm aux} = 152.4\angle 82.6^\circ \text{ V}$$

Problem 9-2

For this problem

$$Z_{\rm main} = 6.43 + j8.09\ \Omega$$

$$Z_{\rm aux} = 10.6 + j10.17\ \Omega$$

Part (a):

$$\hat{I}_{\rm main} = \frac{\hat{V}}{Z_{\rm main}} = 11.6\angle - 51.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = \frac{\hat{V}}{Z_{\text{aux}}} = 8.2\angle - 43.8^\circ \text{ A}$$

Part (b): We want the angle of the auxiliary-winding current to lead that of the main winding by 90^c ($\pi/2$ rad). Thus, defining $Z'_{\text{aux}} = Z_{\text{aux}} + jX_{\text{C}}$ ($X_{\text{C}} = -1/\omega C$), we want

$$\angle Z'_{\text{aux}} = \tan^{-1}\left(\frac{\text{Im}[Z_{\text{aux}}] - X_{\text{C}}}{\text{Re}[Z_{\text{aux}}]}\right) = \angle Z_{\text{main}} + \frac{\pi}{2}$$

Thus $X_{\text{C}} = -18.6\ \Omega$ and $C = 171\ \mu$F.

Part (c):

$$\hat{I}_{\text{main}} = \frac{\hat{V}}{Z_{\text{main}}} = 11.6\angle - 51.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = \frac{\hat{V}}{Z'_{\text{aux}}} = 8.8\angle 38.6^\circ \text{ A}$$

$$\hat{V}_{\text{aux}} = \hat{I}_{\text{aux}} Z_{\text{aux}} = 130.0\angle 82.4^\circ \text{ V}$$

Problem 9-3

$C = 128\ \mu$F.

Problem 9-4

Part (a): For $V = 120$ V, $\hat{I}_{\text{main}} = 4.89\angle - 55.8^\circ$ A and $\hat{I}_{\text{aux}} = 3.32\angle\ 34.2^\circ$ A

$$P_{\text{in}} = V(\hat{I}_{\text{main}} + \hat{I}_{\text{aux}})^* = 659 \text{ W}$$

$$\text{pf} = \frac{P_{\text{in}}}{V|(\hat{I}_{\text{main}} + \hat{I}_{\text{aux}})|} = 0.93$$

$$\eta = 100 \times \frac{P_{\text{in}}}{P_{\text{out}}} = 75.8\%$$

Part (b):

$$Z_{\text{main}} = \frac{V}{\hat{I}_{\text{main}}} = 13.8 + j20.3\ \Omega$$

$$Z'_{\rm aux} = Z_{\rm aux} + jX_{\rm c} = \frac{V}{\hat{I}_{\rm aux}} = 29.9 - j20.3\ \Omega$$

where $X_{\rm c} = -1/(\omega C) = 64.2\ \Omega$. Thus

$$Z_{\rm aux} = 29.9 + j43.9\ \Omega$$

Part (c): For $N_{\rm main} = 180$, assuming the winding impedance are proportional to the square of the turns

$$N_{\rm aux} = N_{\rm main}\sqrt{\frac{|Z_{\rm aux}|}{|Z_{\rm main}|}} = 265$$

Part (d): The winding currents can be seen to be 90° out of phase. Thus there will be a single rotating flux wave if they produce the same ampere-turns.

$$N_{main} I_{\rm main} = 880 \qquad N_{aux} I_{\rm aux} = 880$$

Q.E.D.

Problem 9-5

The first step is to determine the values of the forward- and backward-field impedances at the assigned value of slip. The following relations, derived from Eq. 9.4, simplify the computations of the forward-field impedance $Z_{\rm f}$:

$$R_{\rm f} = \left(\frac{X^2_{\rm m,main}}{X_{22}}\right)\frac{1}{sk_{\rm 2,main} + 1/(sk_{\rm 2,main})} \qquad X_{\rm f} = \frac{X_{\rm 2,main}X_{\rm m,main}}{X_{22}} + \frac{R_{\rm f}}{sk_{\rm 2,main}}$$

where

$$X_{22} = X_{\rm 2,main} + X_{\rm m,main} \qquad \text{and} \qquad k_{\rm 2,main} = \frac{X_{22}}{R_{\rm 2,main}}$$

Substitution of numerical values gives, for $s = 0.035$,

$$Z_{\rm f} = R_{\rm f} + jX_{\rm f} = 28.2 + j50.3\,\Omega$$

Corresponding relations for the backward-field impedance $Z_{\rm b}$ are obtained by substituting $2 - s$ for s in these equations. When $(2 - s)k_{\rm 2,main}$ is greater than 10, as is usually the case, less than 1 percent error results from use of the following approximate forms:

$$R_{\rm b} = \frac{R_{\rm 2,main}}{2 - s}\left(\frac{X_{\rm m,main}}{X_{22}}\right)^2 \qquad X_{\rm b} = \frac{X_{\rm 2,main}X_{\rm m,main}}{X_{22}} + \frac{R_{\rm b}}{(2 - s)k_{\rm 2,main}}$$

Substitution of numerical values gives, for $s = 0.035$,

$$Z_{\mathrm{b}} = R_{\mathrm{b}} + jX_{\mathrm{b}} = 1.97 + j2.11\,\Omega$$

Addition of the series elements in the equivalent circuit of Fig. 9.10c gives

$$\begin{array}{rcl} R_{1,\mathrm{main}} + jX_{1,\mathrm{main}} & = & 2.02 + j2.79 \\ 0.5(R_{\mathrm{f}} + jX_{\mathrm{f}}) & = & 14.12 + j25.14 \\ \underline{0.5(R_{\mathrm{b}} + jX_{\mathrm{b}})} & = & \underline{0.98 + j1.06} \\ \text{Total Input } Z & = & 17.12 + j28.99 \;=\; 33.67\,\angle 59.4^\circ \end{array}$$

$$\text{Stator current } I = \frac{V}{Z} = \frac{110}{33.67} = 3.27\,\mathrm{A}$$

$$\text{Power factor} = \cos(51.7^\circ) = 0.509$$

$$\text{Power input} = P_{\mathrm{in}} = VI \times \text{power factor} = 110 \times 3.27 \times 0.509 = 183\,\mathrm{W}$$

The power absorbed by the forward field (Eq. 9.7) is

$$P_{\mathrm{gap,f}} = I^2(0.5R_{\mathrm{f}}) = 3.27^2 \times 14.1 = 151\,\mathrm{W}$$

The power absorbed by the backward field (Eq. 9.9) is

$$P_{\mathrm{gap,b}} = I^2(0.5R_{\mathrm{b}}) = 3.27^2 \times 0.98 = 10.5\,\mathrm{W}$$

The internal mechanical power (Eq. 9.14) is

$$P_{\mathrm{mech}} = (1-s)(P_{\mathrm{gap,f}} - P_{\mathrm{gap,b}}) = 0.965(151 - 11) = 140\,\mathrm{W}$$

Assuming that the core loss can be combined with the friction and windage loss, the rotational loss becomes $24 + 13 = 37$ W and the shaft output power is the difference. Thus

$$P_{\mathrm{shaft}} = 140 - 37 = 103\,\mathrm{W} = 0.138\,\mathrm{hp}$$

From Eq. 4.42, the synchronous speed in rad/sec is given by

$$\omega_{\mathrm{s}} = \left(\frac{2}{\mathrm{poles}}\right)\omega_{\mathrm{e}} = \left(\frac{2}{4}\right)120\pi = 188.5\,\mathrm{rad/sec}$$

or in terms of r/min from Eq. 4.44

$$n_s = \left(\frac{120}{\mathrm{poles}}\right) f_{\mathrm{e}} = \left(\frac{120}{4}\right) 60 = 1800\,\mathrm{r/min}$$

$$\begin{aligned}\text{Rotor speed} &= (1-s)(\text{synchronous speed})\\ &= 0.965 \times 1800 = 1737\,\text{r/min}\end{aligned}$$

and

$$\omega_{\rm m} = 0.965 \times 188.5 = 182\,\text{rad/sec}$$

The torque can be found from Eq. 9.14.

$$T_{\rm shaft} = \frac{P_{\rm shaft}}{\omega_{\rm m}} = \frac{103}{182} = 0.566\,\text{N}\cdot\text{m}$$

and the efficiency is

$$\eta = \frac{P_{\rm shaft}}{P_{\rm in}} = \frac{103}{183} = 0.563 = 56.3\%$$

As a check on the power bookkeeping, compute the losses:

$$\begin{aligned}I^2R_{1,\rm main} = (3.27)^2(2.02) &= 21.6\\ \text{Forward-field rotor } I^2R \text{ (Eq. 9.11)} = 0.035 \times 151 &= 3.5\\ \text{Backward-field rotor } I^2R \text{ (Eq. 9.1)2} = 1.965 \times 10.5 &= 20.6\\ \text{Rotational losses} &= \underline{37.0}\\ & \ 62.7\,\text{W}\end{aligned}$$

From $P_{\rm in} - P_{\rm shaft}$, the total losses = 60 W which checks within accuracy of computations.

Problem 9-6

The synchronous speed of a 6-pole, 60-Hz motor is 1200 r/min. Thus, operating at a slip of 0.065, its speed is $n = (1 - 0.065)1200 = 1122$ r/min.

$$Z_{\rm f} = jX_{\rm m,main} \;\text{ in parallel with }\; (jX_{2,\rm main} + R_{2,\rm main}/s) = 15.07 + j10.70\ \Omega$$

$$Z_{\rm b} = jX_{\rm m,main} \;\text{ in parallel with }\; (jX_{2,\rm main} + R_{2,\rm main}/(2-s)) = 0.61 + j1.10\ \Omega$$

$$Z = R_{1,\rm main} + jX_{1,\rm main} + 0.5(Z_{\rm f} + Z_{\rm b}) = 8.96 + j7.73\ \Omega$$

$$\hat{I}_{\rm main} = \frac{V_{\rm rated}}{Z} = 9.72\angle -40.8^\circ\ \text{A}$$

Thus $I_{\rm main} = 9.72$ A and the power factor is $\cos^{-1}(-40.8^\circ) = 0.76$ lagging.

$$P_{\rm in} = \text{Re}\,[V_{\rm rated}\hat{I}^*_{\rm main}] = 846\ \text{W}$$

$$\hat{I}_{\rm f} = \hat{I}_{\rm main}\left(\frac{jX_{\rm m,main}}{R_{\rm 2,main}/s + j(X_{\rm m,main} + X_{\rm 2,main})}\right) = 7.85 - j1.14 \ \ {\rm A}$$

$$\hat{I}_{\rm b} = \hat{I}_{\rm main}\left(\frac{jX_{\rm m,main}}{R_{\rm 2,main}/(2-s) + j(X_{\rm m,main} + X_{\rm 2,main})}\right) = 7.25 - j5.99 \ \ {\rm A}$$

$$P_{\rm gap,f} = 0.5 I_{\rm 2,f}^2 R_{\rm 2,main}/s = 711 \ \ {\rm W}$$

$$P_{\rm gap,f} = 0.5 I_{\rm 2,b}^2 R_{\rm 2,main}/(2-s) = 34 \ \ {\rm W}$$

For $\omega_{\rm s} = 40\pi$

$$T_{\rm mech} = \frac{P_{\rm gap,f} - P_{\rm gap,b}}{\omega_{\rm s}} = 5.93 \ {\rm N \cdot m}$$

and thus with $\omega_{\rm m} = (1-s)\omega_{\rm s}$, $P_{\rm out} = \omega_{\rm m} T_{\rm mech} - P_{\rm FW} - P_{\rm core} = 559$ W and the output torque is $T_{\rm out} = P_{\rm out}/\omega_{\rm m} = 4.8$ N·m. Finally, the efficiency is given by $\eta = 100(P_{\rm out}/P_{\rm in}) = 66.1\%$.

Problem 9-7

The synchronous speed of a 4-pole, 60-Hz motor is 1800 r/min. Thus, operating at a slip of 0.072, its speed is $n = (1 - 0.072)1800 = 1670$ r/min.

$$Z_{\rm f} = jX_{\rm m,main} \ \ \text{in parallel with} \ \ (jX_{\rm 2,main} + R_{\rm 2,main}/s) = 17.38 + j10.43 \ \Omega$$

$$Z_{\rm b} = jX_{\rm m,main} \ \ \text{in parallel with} \ \ (jX_{\rm 2,main} + R_{\rm 2,main}/(2-s)) = 0.85 + j0.73 \ \Omega$$

$$Z = R_{\rm 1,main} + jX_{\rm 1,main} + 0.5(Z_{\rm f} + Z_{\rm b}) = 9.66 + j6.41 \ \Omega$$

$$\hat{I}_{\rm main} = \frac{V_{\rm rated}}{Z} = 10.35\angle - 33.5^\circ \ {\rm A}$$

Thus $I_{\rm main} = 10.35$ A and the power factor is $\cos^{-1}(-33.5^\circ) = 0.83$ lagging.

$$P_{\rm in} = {\rm Re}\,[V_{\rm rated}\hat{I}_{\rm main}^*] = 1035 \ {\rm W}$$

$$P_{\rm gap,f} = I_{\rm main}^2 R_{\rm f} = 930.6 \ {\rm W}$$

$$P_{\rm gap,b} = I_{\rm main}^2 R_{\rm b} = 45.6 \ {\rm W}$$

$$P_{\text{mech}} = (1-s)(P_{\text{gap,f}} - P_{\text{gap,b}}) = 821.3$$

We will subtract rotational loss and core loss to get the shaft output power

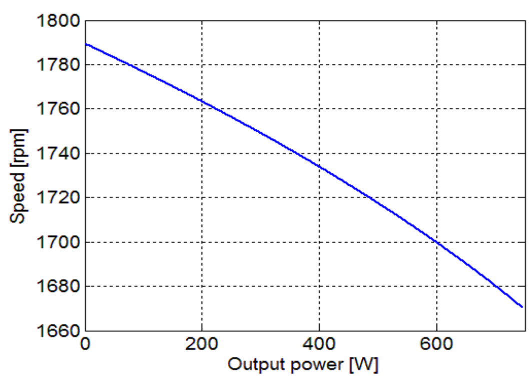
$$P_{\text{out}} = P_{\text{mech}} - P_{\text{rot}} - P_{\text{core}} = 746.6\ W$$

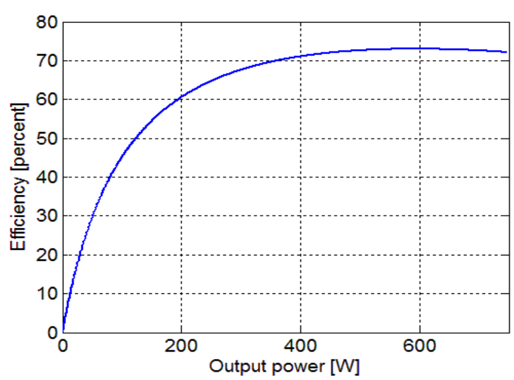
For $\omega_{\text{s}} = 60\pi$ and $\omega_{\text{m}} = (1-s)\omega_{\text{s}} = 174.9$ rad/sec

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_{\text{m}}} = 4.27\ \text{N}\cdot\text{m}$$

Finally, the efficiency is $\eta = 100 \times P_{\text{out}}/P_{\text{in}} = 72.1\%$.

Problem 9-8





Problem 9-9

Part (a): Doubling the number of turns will increase the main-winding parameters by a factor of 4:

$$R_{1,\mathrm{main}} = 2.20 \qquad R_{2,\mathrm{main}} = 6.80$$
$$X_{1,\mathrm{main}} = 3.32 \qquad X_{\mathrm{m,main}} = 166 \qquad X_{2,\mathrm{main}} = 2.88$$

Part (b): A MATLAB search can be used to find the desired result:

$$\mathrm{slip} = 0.06665 \qquad I_{\mathrm{main}} = 4.02 \text{ A} \qquad \eta = 72.6\%$$

Problem 9-10

Part (a): From Eq. 4.6, the peak amplitude, in space and time, of the mmf waves are given by

$$F_{\mathrm{peak}} = \frac{4}{\pi}\left(\frac{k_{\mathrm{w}}N_{\mathrm{ph}}}{\mathrm{poles}}\right) I_{\mathrm{peak}}$$

Thus

$$F_{\mathrm{main,peak}} = \frac{4}{\pi}\left(\frac{47}{4}\right)\left(18.9\sqrt{2}\right) = 400 \text{ A}\cdot\text{turns}$$

and

$$F_{\mathrm{aux,peak}} = \frac{4}{\pi}\left(\frac{73}{4}\right)\left(12.1\sqrt{2}\right) = 398 \text{ A}\cdot\text{turns}$$

part (b): The auxiliary winding current must be phase shifted by 90° from that of the main winding and the mmf amplitudes must be equal. Hence, I_{aux} should be increased slightly to

$$I_{\mathrm{aux}} = I\mathrm{main}\left(\frac{N_{\mathrm{main}}}{N_{\mathrm{aux}}}\right) = 12.2 \text{ A}$$

Problem 9-11

The internal torque is proportional to $R_{rmf} - R_{\mathrm{b}}$ and thus is equal to zero when $R_{\mathrm{f}} = R_{\mathrm{b}}$. From Example 9.2,

$$R_{\mathrm{f}} = \left(\frac{X_{\mathrm{m,main}}^2}{X_{22}}\right)\frac{1}{sQ_{2,\mathrm{main}} + 1/(sQ_{2,\mathrm{main}})}$$

and

$$R_{\mathrm{b}} = \left(\frac{X^2_{\mathrm{m,main}}}{X_{22}}\right) \frac{1}{(2-s)Q_{2,\mathrm{main}} + 1/((2-s)Q_{2,\mathrm{main}})}$$

We see that $R_{\mathrm{f}} = R_{\mathrm{b}}$ if $(2-s)Q_{2,\mathrm{main}} = 1/(sQ_{2,\mathrm{main}})$ or

$$s = 1 \pm \sqrt{1 + \frac{1}{Q_{2,\mathrm{main}}}}$$

and thus

$$n = n_{\mathrm{s}}(1-s) = \pm n_{\mathrm{s}} \sqrt{1 + \frac{1}{Q_{2,\mathrm{main}}}}$$

where n_{s} is the synchronous speed in r/min.

Problem 9-12

Part (a): The positive sequence voltage is

$$\hat{V}_{\mathrm{f}} = 0.5(\hat{V}_\alpha - j\hat{V}_\beta) = 213.2\angle - 6.9^\circ$$

and the negative sequence voltage is

$$\hat{V}_{\mathrm{f}} = 0.5(\hat{V}_\alpha + j\hat{V}_\beta) = 32.7\angle 51.5^\circ$$

Part (b): The total positive-sequence impedance of the motor is

$$Z_{\mathrm{tot,f}} = R_1 + jX_1 + Z_{\mathrm{f}} = R_1 + jX_1 + \frac{jX_{\mathrm{m}}(R_2/s + jX_2)}{R_2/s + j(X_{\mathrm{m}} + X_2)}$$

and the total negative-sequence impedance is

$$Z_{\mathrm{tot,b}} = R_1 + jX_1 + Z_{\mathrm{b}} = R_1 + jX_1 + \frac{jX_{\mathrm{m}}(R_2/(2-s) + jX_2)}{R_2/(2-s) + j(X_{\mathrm{m}} + X_2)}$$

We can find the sequence-component currents as

$$\hat{I}_{\mathrm{f}} = \frac{\hat{V}_{\mathrm{f}}}{Z_{\mathrm{tot,f}}} = 21.2\angle - 36.5^\circ \quad \mathrm{A}$$

$$\hat{I}_{\mathrm{b}} = \frac{\hat{V}_{\mathrm{b}}}{Z_{\mathrm{tot,b}}} = 21.3\angle - 27.9^\circ \quad \mathrm{A}$$

and the phase currents as

$$\hat{I}_\alpha = \hat{I}_\mathrm{f} + \hat{I}_\mathrm{b} = 33.5\angle - 33.4° \quad \mathrm{A}$$

$$\hat{I}_\beta = j(\hat{I}_\mathrm{f} - \hat{I}_\mathrm{b}) = 9.2\angle 41.9° \quad \mathrm{A}$$

Part (c):

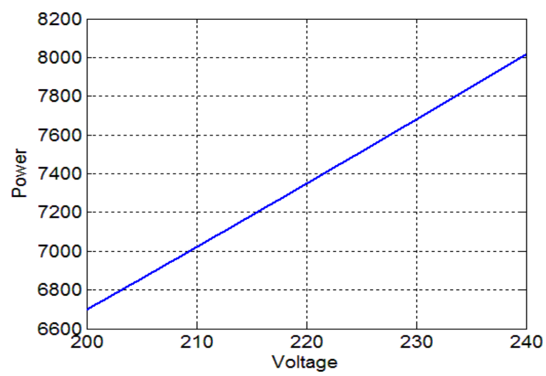
$$P_\mathrm{gap,f} = 2(\mathrm{Re}[\hat{V}_\mathrm{f}\hat{I}_\mathrm{f}^*] - I_\mathrm{f}^2 R_1) = 7620 \quad \mathrm{W}$$

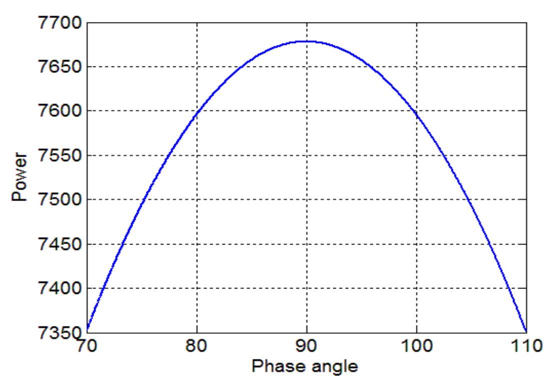
$$P_\mathrm{gap,b} = 2(\mathrm{Re}[\hat{V}_\mathrm{b}\hat{I}_\mathrm{b}^*] - I_\mathrm{b}^2 R_1) = 67 \quad \mathrm{W}$$

and thus

$$P_\mathrm{mech} = (1 - s)(P_\mathrm{gap,f} - P_\mathrm{gap,b}) = 7198 \quad \mathrm{W}$$

Problem 9-13





Problem 9-14

Part (a): Following the calculations of Example 9.3 with $s = 1$, $T_{\mathrm{mech}} =$ 14.8 N·m.

Part (b): Setting

$$\hat{V}_\alpha = 230\ \mathrm{V} \quad \hat{V}_\beta = 230e^{j90^\circ}\ \mathrm{V}$$

gives $T_{\mathrm{mech}} = 16.4$ N·m.

Part (c): Letting $\hat{V}_\alpha = V_\alpha$ and $\hat{V}_\beta = jV_\beta$ gives

$$V_{\mathrm{f}} = \frac{V_\alpha + jV_\beta}{2}; \quad V_{\mathrm{b}} = \frac{V_\alpha - jV_\beta}{2}$$

At starting, $Z_{\mathrm{f}} = Z_{\mathrm{b}}$. Let $Z = R_1 + jX_1 + Z_{\mathrm{f}}$. Thus

$$I_{\mathrm{f}} = \frac{V_{\mathrm{f}}}{Z} = \frac{V_\alpha + jV_\beta}{2Z}; \quad I_{\mathrm{b}} = \frac{V_{\mathrm{b}}}{Z} = \frac{V_\alpha - jV_\beta}{2Z}$$

$$T = \frac{P_{\mathrm{gap,f}} - P_{\mathrm{gap,b}}}{\omega_{\mathrm{s}}} = \frac{R_{\mathrm{f}}(I_{\mathrm{f}}^2 - I_{\mathrm{b}}^2)}{\omega_{\mathrm{s}}} = \left(\frac{R_{\mathrm{f}}}{|Z|^2}\right) V_\alpha V_\beta$$

Clearly, the same torque would be achived if the phase voltages were each equal in magnitude to $\sqrt{V_\alpha V_\beta}$.

Problem 9-15

The positive sequence source voltage is

$$\hat{V}_{\mathrm{f}} = 0.5(\hat{V}_\alpha - j\hat{V}_\beta) = 221.1\angle - 7.1^\circ$$

and the negative sequence source voltage is

$$\hat{V}_{\mathrm{f}} = 0.5(\hat{V}_\alpha + j\hat{V}_\beta) = 32.0\angle 58.7^\circ$$

The total forward impedance seen by the source including the feeder is

$$Z_{\mathrm{tot,f}} = Z_{\mathrm{feeder}} + R_1 + jX_1 + Z_{\mathrm{f}} = Z_{\mathrm{feeder}} + R_1 + jX_1 + \frac{jX_{\mathrm{m}}(R_2/s + jX_2)}{R_2/s + j(X_{\mathrm{m}} + X_2)}$$

Similarly, the total backward impedance is

$$Z_{\mathrm{tot,b}} = Z_{\mathrm{feeder}} + R_1 + jX_1 + Z_{\mathrm{b}} = Z_{\mathrm{feeder}} + R_1 + jX_1 + \frac{jX_{\mathrm{m}}(R_2/(2-s) + jX_2)}{R_2/(2-s) + j(X_{\mathrm{m}} + X_2)}$$

We can find the sequence-component currents as

$$\hat{I}_{\text{f}} = \frac{\hat{V}_{\text{f}}}{Z_{\text{tot,f}}} = 21.5\angle - 44.0^\circ \ \ \text{A}$$

$$\hat{I}_{\text{b}} = \frac{\hat{V}_{\text{b}}}{Z_{\text{tot,b}}} = 8.5\angle - 18.6^\circ \ \ \text{A}$$

The phase currents are thus

$$\hat{I}_\alpha = \hat{I}_{\text{f}} + \hat{I}_{\text{b}} = 29.4\angle = -36.9^\circ \ \ \text{A}$$

$$\hat{I}_\beta = j(\hat{I}_{\text{f}} - \hat{I}_{\text{b}}) = 14.3\angle 31.3^\circ \ \ \text{A}$$

and thus the terminal phase voltages are

$$\hat{V}_{\text{term},\alpha} = \hat{V}_\alpha - \hat{I}_\alpha Z_{\text{feeder}} = 205.2\angle - 8.3^\circ$$

$$\hat{V}_{\text{term},\beta} = \hat{V}_\beta - \hat{I}_\beta Z_{\text{feeder}} = 193.8\angle 69.2^\circ$$

Finally, the positive- and negative-sequence voltages at the terminals are

$$\hat{V}_{\text{term,f}} = 0.5(\hat{V}_{\text{term},\alpha} - j\hat{V}_{\text{term},\beta}) = 198.3\angle - 14.4^\circ$$

and the negative sequence voltage is

$$\hat{V}_{\text{term,b}} = 0.5(\hat{V}_{\text{term},\alpha} + j\hat{V}_{\text{term},\beta}) = 22.5\angle 60.8^\circ$$

From these calculations we see that at the source, the negative-sequence voltage is 14.5% of the positive-sequence voltage while at the terminals that ratio is reduced to 11.3%. Q.E.D.

Problem 9-16

The solution is based upon the following equations:

The positive sequence voltage is

$$\hat{V}_{\text{f}} = 0.5(\hat{V}_\alpha - j\hat{V}_\beta)$$

and the negative sequence voltage is

$$\hat{V}_{\text{f}} = 0.5(\hat{V}_\alpha + j\hat{V}_\beta) = 32.7\angle 51.5^\circ$$

For

$$Z_{\mathrm{tot,f}} = R_1 + jX_1 + Z_{\mathrm{f}}$$

where

$$Z_{\mathrm{f}} = R_{\mathrm{f}} + jX_{\mathrm{f}} = \frac{jX_{\mathrm{m}}(R_2/s + jX_2)}{R_2/s + j(X_{\mathrm{m}} + X_2)}$$

and

$$Z_{\mathrm{tot,b}} = R_1 + jX_1 + Z_{\mathrm{b}}$$

where

$$Z_{\mathrm{b}} = R_{\mathrm{b}} + jX_{\mathrm{b}} = \frac{jX_{\mathrm{m}}(R_2/(2-s) + jX_2)}{R_2/(2-s) + j(X_{\mathrm{m}} + X_2)}$$

we can find the sequence-component currents as

$$\hat{I}_{\mathrm{f}} = \frac{\hat{V}_{\mathrm{f}}}{Z_{\mathrm{tot,f}}}$$

$$\hat{I}_{\mathrm{b}} = \frac{\hat{V}_{\mathrm{b}}}{Z_{\mathrm{tot,b}}}$$

and the phase currents as

$$\hat{I}_{\alpha} = \hat{I}_{\mathrm{f}} + \hat{I}_{\mathrm{b}}$$

$$\hat{I}_{\beta} = j(\hat{I}_{\mathrm{f}} - \hat{I}_{\mathrm{b}})$$

The air-gap mechanical power P_{mech} and torque T_{mech} can be calculated as

$$P_{\mathrm{gap,f}} = 2(\mathrm{Re}[\hat{V}_{\mathrm{f}}\hat{I}_{\mathrm{f}}^*] - I_{\mathrm{f}}^2 R_1)$$

$$P_{\mathrm{gap,b}} = 2(\mathrm{Re}[\hat{V}_{\mathrm{b}}\hat{I}_{\mathrm{b}}^*] - I_{\mathrm{b}}^2 R_1)$$

$$P_{\mathrm{mech}} = (1-s)(P_{\mathrm{gap,f}} - P_{\mathrm{gap,b}})$$

$$T_{\mathrm{mech}} = \frac{P_{\mathrm{mech}}}{\omega_{\mathrm{m}}} = \frac{P_{\mathrm{gap,f}} - P_{\mathrm{gap,b}}}{\omega_{\mathrm{s}}}$$

and the shaft output power P_{out} and torque T_{out} are

$$P_{\mathrm{out}} = P_{\mathrm{mech}} - P_{\mathrm{rot}}$$

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_{\text{m}}}$$

Part (a): For $s = 0.042$, $T_{\text{mech}} = 6.61\ \text{N}\cdot\text{m}$ and the output torque is $T_{\text{out}} = 5.99$ N·m.

Part (b): At starting $s = 1.0$ and $T_{\text{mech}} = 17.87\ \text{N}\cdot\text{m}$.

Part (c): A MATLAB search gives $s = 0.053$.

Part (d): With phase β open, the analysis becomes that of an induction motor running on a single winding. With $\hat{V}_\alpha$ as in Part (c), the winding current will be

$$\hat{I}_\alpha = \frac{\hat{V}_\alpha}{R_1 + jX_1 + 0.5(Z_{\text{f}} + Z_{\text{b}})} = 8.22\angle - 51.7^\circ \quad \text{A}$$

and the output power P_{out} can be calculated as

$$P_{\text{gap,f}} = I_\alpha^2(0.5R_{\text{f}}) = 1048 \quad \text{W}$$

$$P_{\text{gap,b}} = I_\alpha^2(0.5R_{\text{b}}) = 40 \quad \text{W}$$

$$P_{\text{mech}} = (1-s)(P_{\text{gap,f}} - P_{\text{gap,b}}) = 965.7 \quad \text{W}$$

$$P_{\text{out}} = P_{\text{mech}} - P_{\text{rot}} = 859.7 \quad W$$

Part (e): From Eq.9.19 and 9.20, we see that for $\hat{I}_beta = 0$, $\hat{I}_{\text{f}} = \hat{I}_{\text{b}} = \hat{I}_\alpha/2$. Thus

$$\hat{V}_{\text{f}} = \hat{I}_{\text{f}} Z_{\text{tot,f}} = 192.9\angle - 3.9^\circ \quad \text{V}$$

$$\hat{V}_{\text{b}} = \hat{I}_{\text{b}} Z_{\text{tot,b}} = 30.5\angle 25.2^\circ \quad \text{V}$$

and

$$\hat{V}_\beta = j(\hat{V}_{\text{f}} - \hat{V}_{\text{b}}) = 166.9\angle 81.0^\circ \quad \text{V}$$

Problem 9-17

This problem can be solved using a MATLAB script similar to that written for Example 9.5.

Part (a): $T_{\text{start}} = 0.22$ N·m.

Part (b): $I_{\text{main}} = 24.2$ A; $I_{\text{aux}} = 3.8$ A

Part (c): $I = 26.8$ A and the power factor is 1.0

Part (d): $P_{\text{out}} = 2275$ W

Part (e): $P_{\text{in}} = 3242$ W and $\eta = 85.6\%$

Problem 9-18

This problem can be solved using a MATLAB script similar to that written for Example 9.5.

Part (a): 1426 rpm

Part (b): Current = 8.0 A and Efficiency = 80.4%

Problem 9-19

This problem can be solved using a MATLAB script similar to that written for Example 9.5. A search over capacitor values shows a maximum efficiency of 82.3% at an output power of 1.31 kW for a capacitance of 14.81 μF. The motor current is $I = 6.77$ A.

Problem 9-20

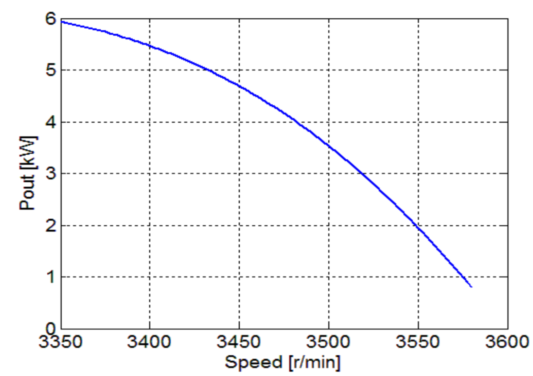
This problem can be solved using a MATLAB script similar to that written for Example 9.5. A search over capacitive values gives $C = 84.7$ μF and an efficiency of 86.3%.

Problem 9-21

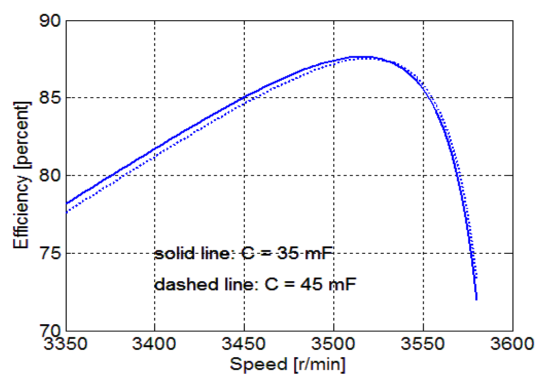
This problem can be solved using a MATLAB script similar to that written for Example 9.5. An iterative search shows that the minimum capacitance is 101 μF.

Problem 9-22

Part (a):



Parts (b) and (c):



PROBLEM SOLUTIONS: Chapter 10

Problem 10-1

Errata: Second sentence should read

The motor has an armature resistance of 163 mΩ ...

Part (a): From the no-load data

$$K_\mathrm{f} = \frac{E_\mathrm{a,nl}}{\omega_\mathrm{m,nl} I_\mathrm{f,nl}} = \frac{120}{(1708\pi/30) \times 0.85} = 0.789$$

Combining

$$T = \frac{E_\mathrm{a} I_\mathrm{a}}{\omega_\mathrm{m}}$$

and

$$V_\mathrm{a} = E_\mathrm{a} + I_\mathrm{a} R_\mathrm{a}$$

gives

$$\begin{aligned} E_\mathrm{a} &= 0.5\left(V_\mathrm{a} + \sqrt{V_\mathrm{a}^2 - 4\omega_\mathrm{m} T R_\mathrm{a}}\right) \\ &= 0.5\left(120 + \sqrt{120^2 - 4(1750\pi/30) \times 13.7 \times 0.243}\right) = 116.5 \text{ V} \end{aligned}$$

Thus

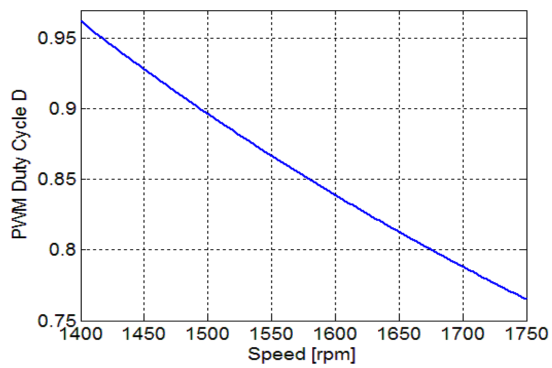
$$I_\mathrm{f} = \frac{E_\mathrm{a}}{\omega_\mathrm{m} K_\mathrm{f}} = 0.805 \text{ A}$$

and, defining $I_\mathrm{f,max} = V_\mathrm{rated}/R_\mathrm{f} = 1.14$,

$$D = \frac{I_\mathrm{f}}{I_\mathrm{f,max}} = 0.765$$

Part (b): Following the solution procedure of Part (a) $I_\mathrm{f} = 1.01$ A and $D = 0.962$.

Part (c):



Problem 10-2

Part (a): At 1750 r/min the torque is $T = 13.7(1750/1500)^{1.8} = 18.1$ N·m. From the no-load data

$$K_{\rm f} = \frac{E_{\rm a,nl}}{\omega_{\rm m,nl} I_{\rm f,nl}} = \frac{120}{(1708\pi/30) \times 0.85} = 0.789$$

Combining

$$T = \frac{E_{\rm a} I_{\rm a}}{\omega_{\rm m}}$$

and

$$V_{\rm a} = E_{\rm a} + I_{\rm a} R_{\rm a}$$

gives

$$\begin{aligned} E_{\rm a} &= 0.5\left(V_{\rm a} + \sqrt{V_{\rm a}^2 - 4\omega_{\rm m} T R_{\rm a}}\right) \\ &= 0.5\left(120 + \sqrt{120^2 - 4(1750\pi/30) \times 18.1 \times 0.243}\right) = 115.3 \text{ V} \end{aligned}$$

Thus

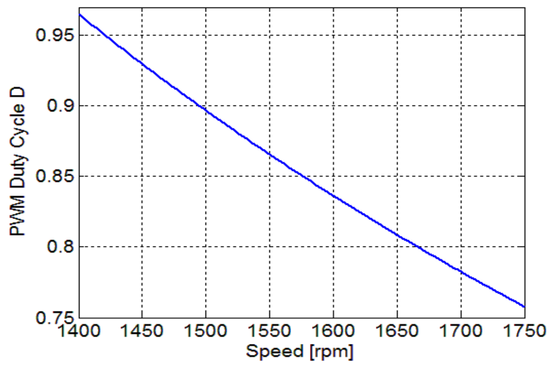
$$I_{\rm f} = \frac{E_{\rm a}}{\omega_{\rm m} K_{\rm f}} = 0.797 \text{ A}$$

and, defining $I_{\rm f,max} = V_{\rm rated}/R_{\rm f} = 1.14$,

$$D = \frac{I_{\rm f}}{I_{\rm f,max}} = 0.757$$

Part (b): At 1400 r/min the torque $T = 13.7(1400/1500)^{1.8} = 12.1$ N·m and following the solution procedure of Part (a) we get $I_{\rm f} = 1.02$ A and $D = 0.965$.

Part (c):



Problem 10-3

Part (a): From the no-load data

$$K_{\mathrm{f}} = \frac{E_{\mathrm{a,nl}}}{\omega_{\mathrm{m,nl}} I_{\mathrm{f,nl}}} = \frac{120}{(1708\pi/30) \times 0.85} = 0.789$$

Combining

$$T = \frac{E_{\mathrm{a}} I_{\mathrm{a}}}{\omega_{\mathrm{m}}}$$

and

$$V_{\mathrm{a}} = E_{\mathrm{a}} + I_{\mathrm{a}} R_{\mathrm{a}}$$

gives

$$\begin{aligned} E_{\mathrm{a}} &= 0.5\left(V_{\mathrm{a}} + \sqrt{V_{\mathrm{a}}^2 - 4\omega_{\mathrm{m}} T R_{\mathrm{a}}}\right) \\ &= 0.5\left(120 + \sqrt{120^2 - 4(1500\pi/30) \times 13.7 \times 0.}\right) = 117.0 \text{ V} \end{aligned}$$

Thus

$$I_{\mathrm{f},1} = \frac{E_{\mathrm{a}}}{\omega_{\mathrm{m}} K_{\mathrm{f}}} = 0.944 \text{ A}$$

and, defining $I_{\mathrm{f,max}} = V_{\mathrm{rated}}/R_{\mathrm{f}} = 1.14$,

$$D = \frac{I_{\mathrm{f},1}}{I_{\mathrm{f,max}}} = 0.897$$

Part (b): $I_{\mathrm{f},2} = D I_{\mathrm{f,max}} = 0.789$ A

$$T = \frac{E_\mathrm{a} I_\mathrm{a}}{\omega_\mathrm{m}} = \frac{K_\mathrm{f} I_{\mathrm{f},2}(V_\mathrm{a} - K_\mathrm{f} I_{\mathrm{f},2}\omega_\mathrm{m})}{R_\mathrm{a}}$$

and thus

$$\omega_\mathrm{m} = \frac{V_\mathrm{a} - T/(K_\mathrm{f} I_{\mathrm{f},2})}{K_\mathrm{f} I_{\mathrm{f},2}} = 186.8 \ \ \mathrm{rad/sec}$$

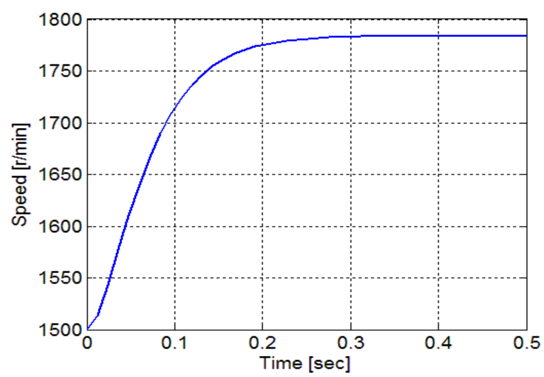
and thus $n = \omega_\mathrm{m}(30/\pi) = 1784$ r/min.

Part (c):

$$i_\mathrm{f}(t) = I_{\mathrm{f},2} + (I_{\mathrm{f},1} - I_{\mathrm{f},2})e^{-t/\tau} = 0.789 + 0.154e^{-t/\tau}$$

where $\tau = L_\mathrm{f}/R_\mathrm{f} = 29.8$ msec.

Part (d):



Problem 10-4

Part (a): From Example 10.1, $T_\mathrm{load} = T_0\ \omega_\mathrm{m}^2$ where $T_0 = 4.18 \times 10^{-4}$ N·m. The motor torque is given by

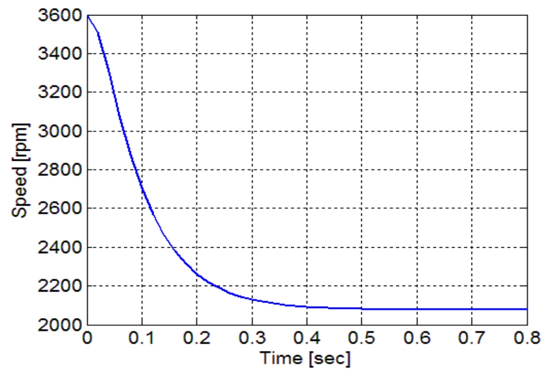
$$T = \frac{E_\mathrm{a} I_\mathrm{a}}{\omega_\mathrm{m}} = \frac{K_\mathrm{f} I_\mathrm{f}(V_\mathrm{a} - K_\mathrm{f} I_\mathrm{f}\omega_\mathrm{m})}{R_\mathrm{a}}$$

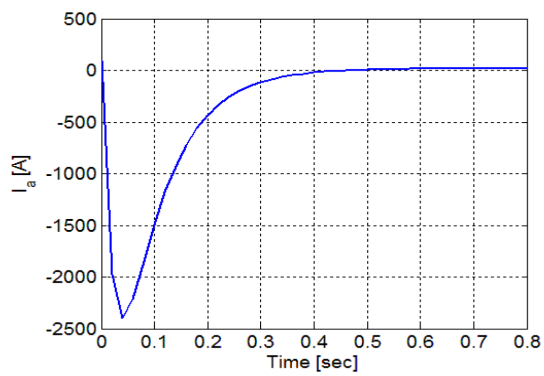
Equation $T = T_\mathrm{load}$ gives a quadratic in ω_m

$$\left(\frac{T_0 R_\mathrm{a}}{K_\mathrm{f} I_\mathrm{f}}\right)\omega_\mathrm{m}^2 + K_\mathrm{f} I_\mathrm{f}\omega_\mathrm{m} - V_\mathrm{a} = 0$$

Solving for ω_m with $I_\mathrm{f} = 110/187 = 0.588$ A gives $\omega_\mathrm{m} = 217.4$ rad/sec and $n = 2076$ r/min.

Part (b):





Note that this simulations indicates that such operation should not be attempted with this motor. The huge armature current which results would clearly overheat and probably damage the armature winding.

Problem 10-5

Part (a):

$$E_{\mathrm{a,nl}} = V_{\mathrm{a}} - R_{\mathrm{a}} I_{\mathrm{nl}} = 239.7 \text{ V}$$

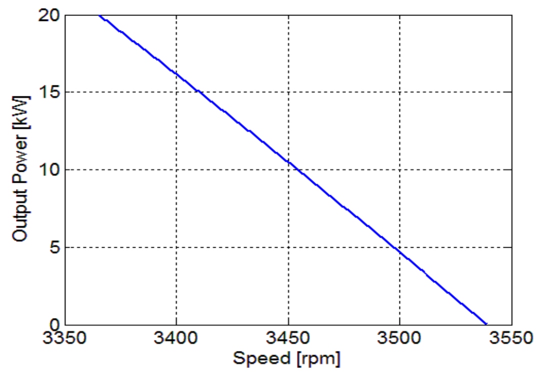
The rotational loss is given by $P_{\mathrm{rot}} = E_{\mathrm{a,nl}} I_{\mathrm{a,nl}} = 463$ W.

Based upon $I_{\mathrm{f}} = V_{\mathrm{a}}/R_{\mathrm{f}} = 1.22$ A,

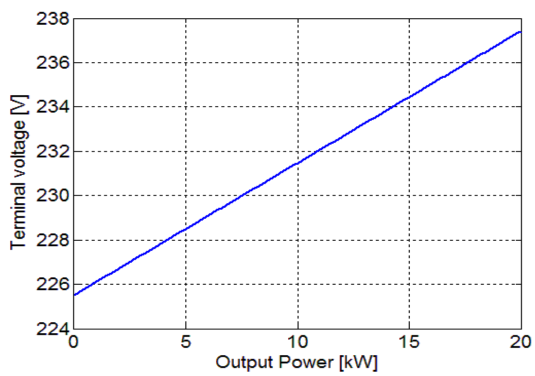
$$\omega_{\mathrm{m,nl}} = \frac{E_{\mathrm{a,nl}}}{I_{\mathrm{f}} K_{\mathrm{f}}} = 370.6 \text{ r/min}$$

and thus $n_{\mathrm{nl}} = 30\omega_{\mathrm{m,nl}}/\pi = 3539$ r/min.

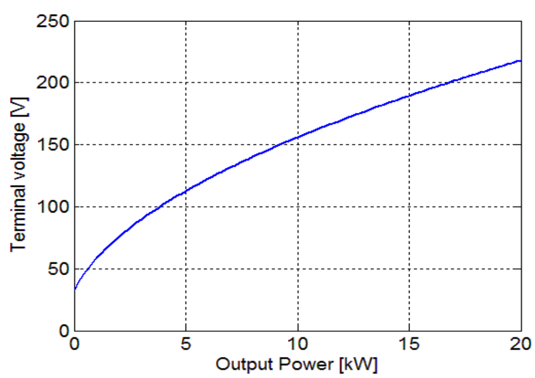
Part (b):



Part (c):



Part (d):



From a comparison of the results of Parts (c) and (d), one sees that this operation appears to be quite feasible but it requires a large variation in terminal voltage as compared to the case where the shunt field voltage is held constant.

Problem 10-6

Part (a):

$$R_a = \frac{V_{\text{rated}}}{I_{\text{stall}}} = 3.16\ \Omega$$

At no load, $E_{a,nl} = V_{\text{rated}} - I_{nl}R_a = 5.37$ V and thus

$$K_m = \frac{E_{a,nl}}{2\pi n_{nl}} = 3.41 \times 10^{-3}\ \text{V/rad/sec}$$

Part (b): $P_{\text{rot,nl}} = E_{a,nl}I_{nl} = 1.07$ W.

Part (c): At 12000 r/min

$$P_{\text{rot}} = P_{\text{rot,nl}}\left(\frac{12000}{15025}\right)^3 = 0.55\ \ \text{W}$$

and $E_a = 2\pi n K_m = 4.29$ V. Thus the armature current

$$I_a = \frac{V_a - E_a}{R_a} = 0.542\ \ \text{A}$$

and thus the propellor power P_{out} is

$$P_{\text{out}} = E_aI_a - P_{\text{rot}} = 1.8\ \ \text{W}$$

The motor input power is $P_{\text{in}} = V_aI_a = 3.25$ W and thus the efficiency is $\eta = 1.8/3.25 = 0.554 = 55.4\%$.

Problem 10-7

Part (a): The stall torque is $T_{\text{stall}} = 0.106$ oz·in $= 5.52 \times 10^{-4}$ N·m and $K_m = 2.14$ mV/(r/min)) $= 2.04 \times 10^{-3}$ V/(rad/sec). We thus can find I_{stall} as

$$I_{\text{stall}} = \frac{T_{\text{stall}}}{K_m} = 0.366\ \ \text{A}$$

and thus

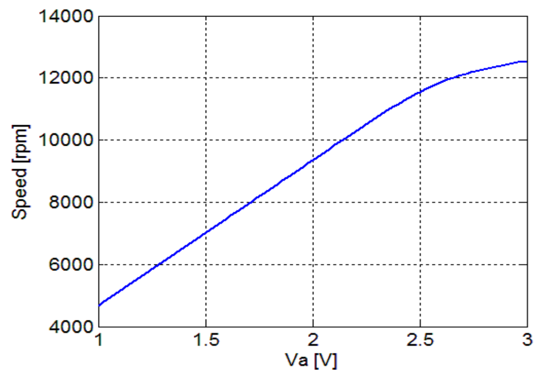
$$R_a = \frac{V_{\text{rated}}}{I_{\text{stall}}} = 8.19\ \Omega$$

Part (b): At no load, $E_{a,nl} = 13100 \times 2.14 \times 10^{-4} = 2.80$ V.

$$I_{nl} = \frac{V_a - E_{a,nl}}{R_a} = 24.0\ \text{mA}$$

and thus

$$P_{\rm rot} = E_{\rm a,nl} I_{\rm nl} = 67.3 \ \ {\rm mW}$$



Problem 10-8

Part (a): $n_{\rm nl} = 3580$ r/min $= 374/9$ rad/sec. $E_{\rm a,nl} = V_{\rm a} - I_{\rm nl} R_{\rm a} = 24.0$ V. Thus

$$K_{\rm m} = \frac{E_{\rm a,nl}}{\omega_{\rm m,nl}} = 63.9 \text{ mV/(rad/sec)}$$

Part (b): $P_{\rm rot,nl} = E_{\rm a,nl} I_{\rm nl} = 11.3$ W.

Part (c): For and given duty cycle, $V_{\rm a} = DV_{\rm rated}$. We can then calculate $E_{\rm a} = V_{\rm a} - I_{\rm a} R_{\rm a}$ and $\omega_{\rm m} = E_{\rm a}/K_{\rm m}$ ($n = \omega_{\rm m} \times (30/\pi)$) and the load power as

$$P_{\rm load} = E_{\rm a} I_{\rm a} - P_{\rm rot}$$

where

$$P_{\rm rot} = P_{\rm rot,nl} \left(\frac{n}{n_{\rm nl}} \right)^3$$

Here are the results:

D	I_a [A]	r/min	P_{load} [W]
0.80	14.70	3374	332.4
0.75	12.79	3177	264.0
0.70	11.55	2971	223.2
0.65	10.34	2764	186.1
0.60	9.20	2557	153.3
0.55	8.07	2349	123.7
0.50	7.02	2140	98.0

Problem 10-9

Part (a): $n_{\mathrm{nl}} = 3580$ r/min $= 374.9$ rad/sec. $E_{\mathrm{a,nl}} = V_{\mathrm{a}} - I_{\mathrm{nl}}R_{\mathrm{a}} = 24.0$ V. Thus

$$K_{\mathrm{m}} = \frac{E_{\mathrm{a,nl}}}{\omega_{\mathrm{m,nl}}} = 63.9 \text{ mV/(rad/sec)}$$

For a constant current, the torque $T = K_{\mathrm{I_a}}$ is constant. Thus, from

$$J\frac{d\omega_{\mathrm{m}}}{dt} = T$$

we get

$$\Delta t = \left(\frac{J}{T}\right)\Delta\omega_{\mathrm{m}}$$

For $\Delta\omega_{\mathrm{m}} = 374.9$ rad/sec, $\Delta t = 0.51$ sec.

Part (b): With constant $V_{\mathrm{a}} = 24$ V,

$$I_{\mathrm{a}} = \frac{V_{\mathrm{a}} - E_{\mathrm{a}}}{R_{\mathrm{a}}} = \frac{V_{\mathrm{a}} - K_{\mathrm{m}}\omega_{\mathrm{m}}}{R_{\mathrm{a}}}$$

and the torque is

$$T = \frac{E_{\mathrm{a}}I_{\mathrm{a}}}{\omega_{\mathrm{m}}} = \frac{K_{\mathrm{m}}(V_{\mathrm{a}} - K_{\mathrm{m}}\omega_{\mathrm{m}})}{R_{\mathrm{a}}}$$

The speed is governed by the differential equation

$$J\frac{d\omega_{\mathrm{m}}}{dt} = T = -\left(\frac{K_{\mathrm{m}}^2}{R_{\mathrm{a}}}\right)\omega_{\mathrm{m}} + \frac{K_{\mathrm{m}}V_{\mathrm{a}}}{R_{\mathrm{a}}}$$

whose solution is

$$\omega_{\mathrm{m}} = \omega_{\mathrm{m},\infty}(1 - e^{-t/\tau})$$

where $\omega_{\mathrm{m},\infty} = V_{\mathrm{a}}/K_{\mathrm{m}}$ and

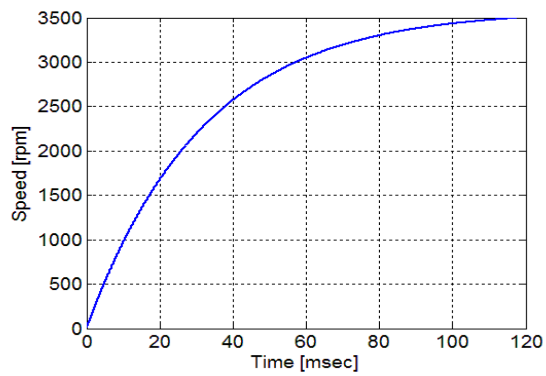
$$\tau = \frac{J R_{\mathrm{a}}}{K_{\mathrm{m}}^2}$$

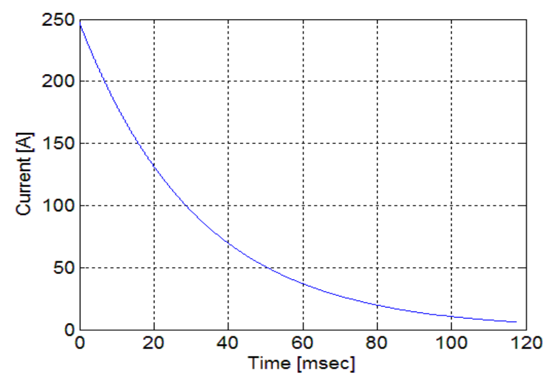
From this solution, the time to achieve a speed of ω_{m} is

$$t = -\tau \ln\left(1 - \frac{\omega_{\mathrm{m}}}{\omega_{\mathrm{m},\infty}}\right)$$

For a speed of 3500 r/min, $\omega_{\mathrm{m}} = 366.5$ and $t = 0.118$ sec.

Part (c):





Problem 10-10

Errata: Part (b): should read

The current source is supplying ....

Part (a):

$$I_{\mathrm{a,rated}} = \frac{P_{\mathrm{rated}}}{V_{\mathrm{rated}}} = 5.0 \text{ A}$$

$$T_{\mathrm{rated}} = \frac{P_{\mathrm{rated}}}{\omega_{\mathrm{m,rated}}} = 3.18 \text{ N} \cdot \text{m}$$

Part (b): For $n = 3120$ r/min, $\omega_{\mathrm{m}} = 326.7$ rad/sec and thus $E_{\mathrm{a}} = K_{\mathrm{m}}\omega_{\mathrm{m}} =$ 210.7 V. The rotational loss torque is

$$T_{\mathrm{rot}} = \frac{109}{3600 \times \pi/30} = 0.29 \text{ N·m}$$

The mechanical torque is

$$T_{\mathrm{mech}} = K_{\mathrm{a}} I_{\mathrm{a}} = 2.84 \text{ N·m}$$

and thus $T_{\mathrm{load}} = T_{\mathrm{mech}} - T_{\mathrm{rot}} = 2.55$ N·m and $P_{\mathrm{load}} = \omega_{\mathrm{m}} T_{\mathrm{load}} = 833$ W.

Part (c): The differential equation governing the motor speed is

$$J\frac{d\omega_{\mathrm{m}}}{dt} = T_{\mathrm{mech}} - T_{\mathrm{rot}} - T_{\mathrm{load}}$$

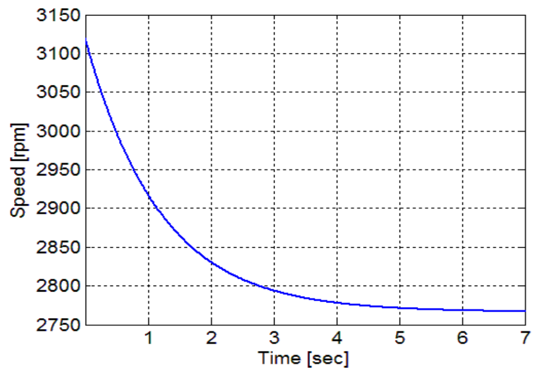
Here, $T_{\mathrm{mech}} = K_{\mathrm{m}} I_{\mathrm{a}} = 3.23$ N·m and, from part (b),

$$T_{\mathrm{load}} = 2.55 \left(\frac{\omega_{\mathrm{m}}}{326.7}\right) \text{ N} \cdot \text{m} = (7.83 \times 10^{-3})\omega_{\mathrm{m}} \text{ N·m}$$

The solution to the differential equation (expressed in terms of n the speed in r/min instead of ω_{m}) is

$$n = 2766 + 354 e^{-t/\tau}$$

where $\tau = 1.18$ s. Here is the plot



Problem 10-11

Part (a): With the motor operating at a speed of 3600 r/min ($\omega_{\rm m0} = 120\pi$) rad/sec, $E_{\rm a} = K_{\rm m}\omega_{\rm m} = 243.2$ V and thus

$$I_{\rm a} = \frac{P_{\rm rot}}{E_{\rm a}} = 0.448 \;\; \text{A}$$

and

$$V_{\rm a0} = E_{\rm a} + I_{\rm a}R_{\rm a} = 244.2 \;\; \text{V}$$

Part (b): With the motor operating at a speed of 3550 r/min rotational loss power drops to

$$P_{\rm rot} = \left(\frac{3550}{3600}\right) 109 = 107.5 \;\; \text{W}$$

and the total mechanical power supplied by the motor is

$$P_{\rm mech} = P_{\rm load} + P_{\rm rot} = 1307 \text{ W}$$

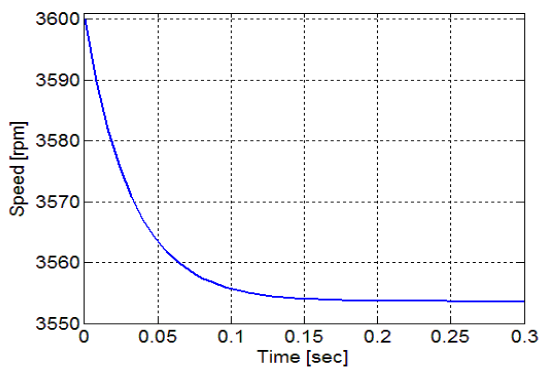
Thus

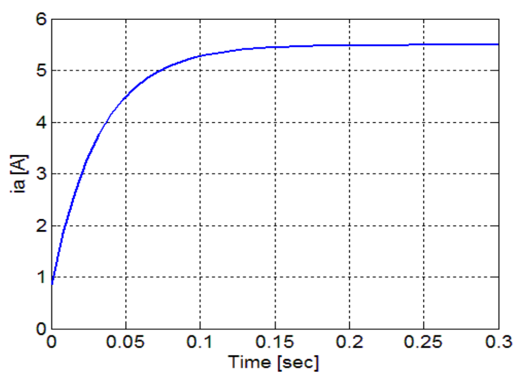
$$I_{\rm a} = \frac{P_{\rm mech}}{E_{\rm a}} = 5.45 \;\; \text{A}$$

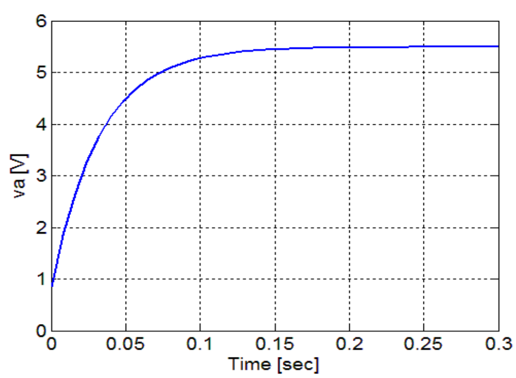
and $V_{\rm a} = E_{\rm a} + I_{\rm a}R_{\rm a} = 252.5$ V from which we find that

$$G = \frac{V_{\rm a} - V_{\rm a0}}{\omega_{\rm m0} - \omega_{\rm m}} = 1.582 \text{ V/(rad/sec)}$$

Problem 10-12







Problem 10-13

Part (a): At no load, $I_{\mathrm{a}} \approx 0$ and thus $V_{\mathrm{a}} \approx E_{\mathrm{a}}$. Rated field current for this motor is $I_{\mathrm{f}} = 300$ V/127 $\Omega = 2.36$ A and at 2150 r/min, $\omega_{\mathrm{m}} = 225.1$ rad/sec.

Thus from Eq. 10.1

$$V_{\mathrm{a0}} \approx E_{\mathrm{a}} = K_{\mathrm{f}} I_{\mathrm{f}}\, \omega_{\mathrm{m}} = 471.2 \text{ V}$$

Part (b): From Eq. 10.2

$$I_{\mathrm{a}} = \frac{T_{\mathrm{load}}}{K_{\mathrm{f}} I_{\mathrm{f}}} = 105 \text{ A}$$

and from Eq. 10.4

$$\omega_{\mathrm{m}} = \frac{(V_{\mathrm{a}} - I_{\mathrm{a}} R_{\mathrm{a}})}{K_{\mathrm{f}} I_{\mathrm{f}}} = 218.5 \text{ rad/sec}$$

corresponding to a speed of 2087 r/min.

(c) At a speed of 2125 r/min, $\omega_{\mathrm{m}} = 2125 \times (30/\pi) = 222.5$ rad/sec and thus

$$E_{\mathrm{a}} = K_{\mathrm{f}} I_{\mathrm{f}} \omega_{\mathrm{m}} = 466 \text{ V}$$

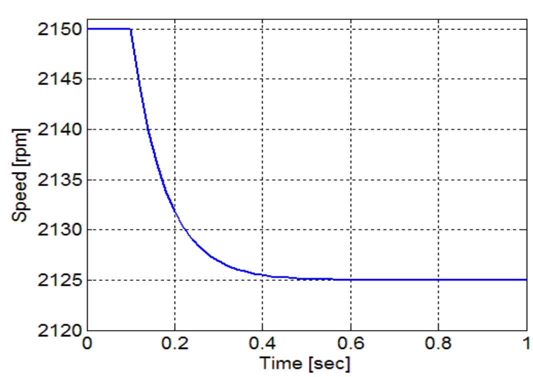
The armature current required to produce a steady-state load torque of 220 N·m is equal to 105 A independent of motor speed and thus we can find the required terminal voltage as

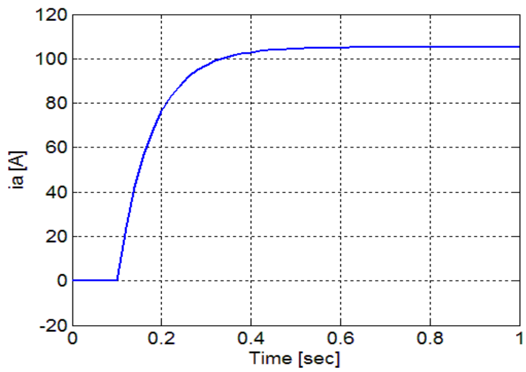
$$V_{\mathrm{a}} = E_{\mathrm{a}} + I_{\mathrm{a}} R_{\mathrm{a}} = 466 + 105 \times 0.132 = 479.6 \text{ V}$$

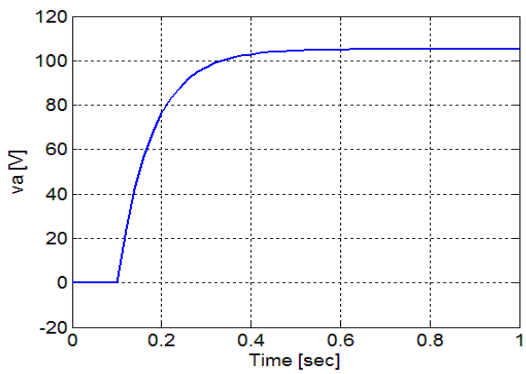
Solving for G from the block diagram of Fig. 10.7 gives

$$G = \frac{V_{\mathrm{a}} - V_{\mathrm{a0}}}{\omega_{\mathrm{ref}} - \omega_{\mathrm{m}}} = \frac{479.6 - 471.2}{225.1 - 222.5} = 3.23 \text{ V/(rad/sec)}$$

Part (d):







Problem 10-14

Because the frequency is varied, let us calculate the various inductances.

$$Z_{\rm base} = V_{\rm base}^2 P_{\rm base} = 3.07\ \Omega$$

and the base inductance is $L_{\rm base} = Z_{\rm base}/(\omega_{\rm base}) = 8.15$ mH. Thus $L_{\rm s} = X_{\rm s,pu} L_{\rm base} = 7.09$ mH and we can calculate $L_{\rm af}$ as

$$L_{\rm af} = \sqrt{\frac{2}{3}} \left(\frac{V_{\rm base}}{\omega_{\rm base} \times {\rm AFNL}} \right) = 38.5 \ \ {\rm mH}$$

Finally

$$I_{\rm rated} = \frac{P_{\rm rated}}{\sqrt{3}\, V_{\rm rated}} = 90.2 \ \ {\rm A}$$

Part (a): We'll work Part (a) in per unit. With $V_{\rm a} = 1.0$ pu, $I_{\rm a} = 1.0$ pu

$$\hat{E}_{\rm af} = V_{\rm a} - jX_{\rm s} I_{\rm a} = 1.33\angle - 41^\circ$$

and thus

$$I_{\mathrm{f}} = 1.33 \times \mathrm{AFNL} = 1.33 \times 27 = 35.8 \;\; \mathrm{A}$$

Part (b): Since the motor is operating at unity power factor, the power is calculated directly from the voltage and current as

$$P_{\mathrm{out}} = \sqrt{3} I_{\mathrm{rated}} \times 320 \mathrm{~V} = 50 \;\; \mathrm{kW}$$

and since it is a 2-pole motor, the speed at 40 Hz is 2400 r/min.

To calculate the field current, since the frequency is no longer the base frequency, it is easiest to work here in actual units with $X_{\mathrm{s}} = (80\pi)L_{\mathrm{s}} = 1.78\ \Omega$. The line-neutral terminal voltage is $V_{\mathrm{a}} = 320/sqrt3 = 184.8$ V and thus the line-neutral generated voltage is

$$E_{\mathrm{af}} = V_{\mathrm{a}} - jX_{\mathrm{s}} I_{\mathrm{rated}} = 244.9\angle - 41.0^{\circ} \;\; \mathrm{V}$$

corresponding to a line-line voltage magnitude of 424.2 V. If the motor were operating at 60 Hz, the same field current would produce a line-line generated voltage of $(60/40) \times 424.2 = 636.3$ V, corresponding to a per-unit value of $636.3/V_{\mathrm{rated}} = 1.33$ per unit. Thus, the field current is $I_{\mathrm{f}} = 1.33 \times \mathrm{AFNL} = 35.8$ A.

Part (c): The solution follows the identical methodology as the solution of Part (b) with an appropriate change in frequency. The result is: $P_{\mathrm{out}} = 75$ kW, Speed = 4500 r/min and $I_{\mathrm{f}} = 31.9$ A.

Problem 10-15

Part (a): Rated speed is $n_{\mathrm{rated}} = 120f/\mathrm{poles} = 1800$ r/min and the rated current is

$$I_{\mathrm{rated}} = \frac{P_{\mathrm{rated}}}{\sqrt{3}\, V_{\mathrm{rated}}} = 144.3 \mathrm{~A}$$

Part (b): With rated voltage, unity power factor, operation at an output power of $P_{\mathrm{out}} = 1000$ kW = 0.870 per-unit corresponds to a terminal voltage of $I_{\mathrm{a,pu}} = 0.87$ per-unit. Thus

$$E_{\mathrm{af,pu}} = V_{\mathrm{a,pu}} - jX_{\mathrm{s,pu}} I_{\mathrm{a,pu}} = 1.50\angle - 48.28^{\circ}$$

and hence $I_{\mathrm{f}} = 1.50 \times \mathrm{AFNL} = 147$ A.

Part (c): The inverter frequency is $(1325/1800) \times 60 = 44.2$ Hz and both the terminal voltage and the generated voltage will be reduced by the ratio $44.2/60 = 0.737$. The rated line-line terminal voltage is $4600/\sqrt{3} = 2656$ V and thus, under this operating condition $V_{\mathrm{a}} = 0.737 \times 2656 = 1958$ V and $E_{\mathrm{af}} = 0.737 \times 1.50 \times 2656 = 2936$ V. The load power is

$$P_{\rm load} = 1000 \times \left(\frac{1325}{1800}\right)^2 .7 = 437.3 \text{ kW}$$

At 60 Hz, the base impedance is $Z_{\rm base} = V_{\rm rated}^2/P_{\rm rated} = 18.4\ \Omega$ and thus $X_{\rm s} = X_{\rm s,pu}Z_{\rm base} = 23.7\ \Omega$ and at 44.2 Hz, $X_{\rm s}(44.2/60) \times 23.7 = 17.5\Omega$.

The power angle δ can be calculated as

$$\delta = -\sin^{-1}\left(\frac{X_{\rm s}P}{3V_{\rm a}E_{\rm af}}\right) = -46.1^\circ$$

and thus the terminal current is

$$\hat{I}_{\rm a} = \frac{V_{\rm a} - E_{\rm af}e^{j\delta}}{jX_{\rm s}} = 121.3\angle 27.5^\circ$$

and the power factor is thus pf = $\cos^{-1} 26.5^\circ = 0.89$ leading.

Part (d): For a power of 437.3 kW and a terminal voltage of 1958 V line-neutral, at unity power factor the terminal current will be

$$I_{\rm a} = \frac{437.3 \times 10^{-3}}{3 \times 1958} = 74.6 \text{ A}$$

and thus

$$E_{\rm af} = V_{\rm a} - jX_{\rm s}I_{\rm a} = 2349\angle - 33.7^\circ \ \ \text{V, line} - \text{neutral}$$

corresponding to a line-line voltage of 4069 V. With the motor operating at 60 Hz, the same field current would produce a generated voltage of $(60/44.2) \times 4069 = 5524$ V which is $(5524/4600) = 1.20$ per-unit. Thus,

$$I_{\rm f} = E_{\rm af,pu} \times \text{AFNL} = 117 \ \ \text{A}$$

Problem 10-16

Part (b):

- 1500 r/min:

  Terminal voltage: 3.83 kV, line-line
  Maximum power: 958 kW
  Field current: 160 A

- 2000 r/min:

  Terminal voltage: 34.60 kV, line-line
  Maximum power: 1150 kW
  Field current: 154 A

Problem 10-17

$L_{\mathrm{s}} = 5.23$ mH
$L_{\mathrm{af}} = 63.1$ mH
Rated torque = 530 N·m

Problem 10-18

Part (a):

$$L_{\mathrm{af}} = \frac{\sqrt{2}\, V_{\mathrm{base}}}{\sqrt{3}\, \omega_{\mathrm{base}}\, \mathrm{AFNL}} = 67.6 \text{ mH}$$

L_{s} can be calculated from the per-unit value of X_{s}.

$$Z_{\mathrm{base}} = \frac{V_{\mathrm{base}}^2}{P_{\mathrm{base}}} = 2.33\ \Omega$$

and $L_{\mathrm{base}} = Z_{\mathrm{base}}/\omega_{\mathrm{base}} = 12.4$ mH. Thus,

$$X_{\mathrm{s}} = \frac{X_{\mathrm{s}}}{Z_{\mathrm{base}}} = 2.06\ \Omega$$

$$L_{\mathrm{s}} = \frac{X_{\mathrm{s}}}{L_{\mathrm{base}}} = 5.46 \text{ mH}$$

Part (b): $\omega_{\mathrm{m,base}} = \omega_{\mathrm{base}}(2/\mathrm{poles}) = 60\pi$ and $T_{\mathrm{base}} = P_{\mathrm{base}}/\omega_{\mathrm{m,base}} = 663$ N·m. Thus, $T = 0.5\, T_{\mathrm{base}} = 332$ N·m.

$$i_{\mathrm{Q}} = \left(\frac{2}{3}\right)\left(\frac{2}{\mathrm{poles}}\right)\left(\frac{T}{L_{\mathrm{af}} I_{\mathrm{f}}}\right) = 112 \text{ A}$$

$$I_{\mathrm{a}} = \frac{i_{\mathrm{Q}}}{\sqrt{2}} = 79.2 \text{ A, rms}$$

Part (c):

$$E_{\mathrm{af}} = \frac{\omega_{\mathrm{base}} L_{\mathrm{af}} I_{\mathrm{f}}}{\sqrt{2}} = 263 \text{ V}$$

Because $i_{\mathrm{D}} = 0$, $\hat{I}_{\mathrm{a}}$ and $\hat{E}_{\mathrm{af}}$ both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_{\mathrm{a}} = |E_{\mathrm{af}} + jX_{\mathrm{s}} I_{\mathrm{a}}| = 309 \text{ V, l-n} = 536 \text{ V, l-l}$$

Problem 10-19

Part (a): At 60 Hz and at a speed of 1800 r/min, $\omega_{\mathrm{m,rated}} = 60\pi$ and thus the rated torque is

$$T_{\text{rated}} = \frac{P_{\text{rated}}}{\omega_{\text{m,rated}}} = 663\ \text{N·m}$$

For this operating condition, $\omega_{\text{m}} = 1515 \times (30/\pi) = 158.7$ r/min and $T = 0.8 * T_{\text{rated}} = 530.5$ N·m and thus $P = T\omega_{\text{m}} = 84.2$ kW.

Part (b):

$$i_{\text{Q}} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{T}{L_{\text{af}}I_{\text{f}}}\right) = 167.7\ \text{A}$$

$$I_{\text{a}} = \frac{i_{\text{Q}}}{\sqrt{2}} = 118.6\ \text{A, rms}$$

Part (c): $f_{\text{e}} = 60(1515/1800) = 50.5$ Hz.

Part (d): The line-neutral generated voltage is

$$E_{\text{af}} = \frac{\omega_{\text{base}}L_{\text{af}}I_{\text{f}}}{\sqrt{2}} = 237\ \text{V}$$

Because $i_{\text{D}} = 0$, $\hat{I}_{\text{a}}$ and $\hat{E}_{\text{af}}$ both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_{\text{a}} = |E_{\text{af}} + jX_{\text{s}}I_{\text{a}}| = 313\ \text{V, l-n} = 543\ \text{V, l-l}$$

Problem 10-20

Errata: Should read

> In order to achieve this operating condition with a reasonable armature terminal voltage, the field-oriented control algorithm is changed to one which results in unity terminal-power-factor operation at rated terminal voltage. Based upon that algorithm, calculate the field current, the armature current and the direct- and quadrature-axis currents i_{D} and i_{Q}.

Part (a): At 60 Hz and at a speed of 1800 r/min, $\omega_{\text{m,rated}} = 60\pi$ and thus the rated torque is

$$T_{\text{rated}} = \frac{P_{\text{rated}}}{\omega_{\text{m,rated}}} = 663\ \text{N·m}$$

With T_{ref} increased to $0.85T_{\text{rated}} = 564$ N·m

$$i_{\text{Q}} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{T_{\text{ref}}}{L_{\text{af}}I_{\text{f}}}\right) = 190.4\ \text{A}$$

$$I_a = \frac{i_Q}{\sqrt{2}} = 134.6 \text{ A, rms}$$

$$L_{af} = \frac{\sqrt{2}\,V_{base}}{\sqrt{3}\,\omega_{base}\,\text{AFNL}} = 67.6 \text{ mH}$$

$$E_{af} = \frac{\omega_{base} L_{af} I_f}{\sqrt{2}} = 263.1 \text{ V}$$

Because $i_D = 0$, $\hat{I}_a$ and $\hat{E}_{af}$ both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_a = |E_{af} + jX_s I_a| = 382.0 \text{ V, l-n} = 661.7 \text{ V, l-l} = 1.23 \text{ per unit}$$

Part (b): With the motor operating at 85% of rated torque at rated speed, unity power factor and rated terminal voltage, the armature current will be $I_{a,pu}$ =0.85 per unit. $I_{rated} = P_{rated}/(\sqrt{3}V_{rated}) = 133.6$ A and hence $I_a = 113.6$ A. Thus

$$\hat{E}_{af,pu} = V_{a,pu} - jX_{s,pu} I_{a,pu} = 1.25\angle - 36.9\ ^\circ$$

and $I_f = 1.25 \times \text{AFNL} = 21.6$ A.

We see that that $\delta = -36.9°$ and thus $i_{d,pu} = I_{a,pu} \sin\delta = -0.499$ per unit and $i_q = I_{a,pu} \sin\delta = 0.688$. Thus,

$$i_D = \sqrt{2}\, I_{rated}\; i_{d,pu} = -94.3 \ \text{ A}$$

$$i_Q = \sqrt{2}\, I_{rated}\; i_{q,pu} = 130.1 \ \text{ A}$$

Problem 10-21

Errata: Should read

> Consider a 450-kW, 2300-V, 50-Hz, 6-pole synchronous motor with a synchronous reactance of 1.32 per unit and AFNL = 11.7 A. It is to be operated under a field-oriented control such that the armature flux linkages remain at their rated value and with minimum armature current at each operating point. It will be used to drive a load whose torque varies quadratically with speed and whose torque at a speed of 1000 r/min is 4100 N·m. The complete drive system will include a speed-control loop such as that shown in Fig. 10.14b. Write a MATLAB script to plot the field current, direct- and quadrature axis currents, the armature current over the speed-range 0-1000 r/min.

For the purposes of this problem, we will need to calculate some basic machine characteristics. The rated line-neutral rms terminal voltage is $V_{\rm a,rated} = 2300/sqrt3$ and the rated electrical frequency is $\omega_{\rm e} = 120\pi$. From Eq.10.35, the rms armature flux linkages

$$(\lambda_{\rm a})_{\rm rms} = \frac{V_{\rm a,rated}}{\omega_{\rm e}} = 4.23 \text{ Wb}$$

$$Z_{\rm base} = \frac{V_{\rm rated}^2}{P_{\rm rated}} = 11.8\ \Omega$$

and thus

$$L_{\rm s} = \frac{X_{\rm s,pu} X_{\rm base}}{\omega_{\rm e}} = 49.4 \ \text{ mH}$$

From Eq.10.31, we can find

$$L_{\rm af} = \frac{\sqrt{2}\, V_{\rm a,rated}}{\omega_{\rm e} \times \text{AFNL}} = 51.1 \ \text{ mH}$$

The basic equations required to implement this controller are Eq.10.29

$$T_{\rm mech} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right) L_{\rm af} i_{\rm F} i_{\rm Q} = 4.5 L_{\rm af} i_{\rm F} i_{\rm Q}$$

from which we can solve for $i_{\rm F}$ in terms of the torque and $i_{\rm Q}$ as

$$i_{\rm F} = \frac{T_{\rm mech}}{4.5 L_{\rm af} i_{\rm Q}}$$

and Eq.10.35

$$(\lambda_{\rm a})_{\rm rms} = \sqrt{\frac{(L_{\rm s} i_{\rm D} + L_{\rm af} i_{\rm F})^2 + (L_{\rm s} i_{\rm Q})^2}{2}}$$

For low values of torque, operation with $i_{\rm D} = 0$ will result in minimum armature current and thus we can find an expression for $i_{\rm Q}$ in terms of the torque

$$(4.5 L_{\rm s})^2\, i_{\rm Q}^4 - 2(4.5 (\lambda_{\rm a})_{\rm rms})^2\, i_{\rm Q}^2 + T_{\rm mech}^2 = 0$$

Under this condition, we can solve for $i_{\rm Q}$ and $i_{\rm F}$ uniquely.

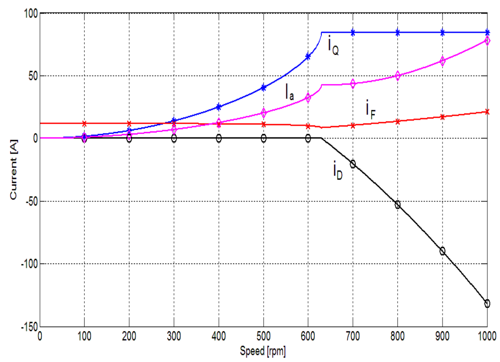
As the torque is raised, a point will be reached where the solution will result in an imaginary value for $i_{\rm Q}$. Above that level of torque, the algorithm will hold $i_{\rm Q}$ constant which will then uniquely determine $i_{\rm F}$ for a given level of torque.

$$i_{\rm F} = \frac{T_{\rm mech}}{4.5 L_{\rm af} i_{\rm Q}}$$

i_{D} can then be found as

$$i_{\mathrm{D}} = \frac{\sqrt{2(\lambda_{\mathrm{a}})^2_{\mathrm{rms}} - (L_{\mathrm{s}} i_{\mathrm{Q}})^2} - L_{\mathrm{af}} i_{\mathrm{F}}}{L_{\mathrm{s}}}$$

Here is the resultant plot



... saturated synchronous reactance of 1.15 per unit ...

Problem 10-22

Errata: First sentence should read
Some initial calculations:

$$Z_{\mathrm{base}} = \frac{V^2_{\mathrm{rated}}}{P_{\mathrm{rated}}} = 1.84\ \Omega$$

$$L_{\mathrm{base}} = \frac{Z_{\mathrm{base}}}{\omega_{\mathrm{e,rated}}} = 4.89\ \ \mathrm{mH}$$

$$L_{\mathrm{s}} = 1.15 L_{\mathrm{base}} = 5.62\ \ \mathrm{mH}$$

$$L_{\mathrm{af}} = \frac{\sqrt{2}\, V_{\mathrm{rated}}}{\sqrt{3}\, \omega_{\mathrm{e,rated}} \times \mathrm{AFNL}} = 56.2\ \ \mathrm{mH}$$

$$T_{\mathrm{rated}} = \frac{P_{\mathrm{rated}}}{\omega_{\mathrm{m,rated}}} = 663\ \mathrm{N{\cdot}m}$$

$$\lambda_{\mathrm{a,rated}} = \frac{V_{\mathrm{rated}}}{\sqrt{3}\, \omega_{\mathrm{e,rated}}} = 0.735\ \ \mathrm{Wb}$$

From Eq. 10.29 we know that

$$T_{\text{mech}} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right) L_{\text{af}} i_{\text{F}} i_{\text{Q}} = 3L_{\text{af}} i_{\text{F}} i_{\text{Q}} = T_{\text{rated}}$$

and from Eq. 10.35 we know that we must have

$$(\lambda_{\text{a}})_{\text{rms}} = \sqrt{\frac{(L_{\text{s}} i_{\text{D}} + L_{\text{af}} i_{\text{F}})^2 + (L_{\text{s}} i_{\text{Q}})^2}{2}} \le \lambda_{\text{a,rated}}$$

The easiest way (not necessarily the most elegant) to solve this problem is to write a MATLAB script which searches over a range of field current (e.g. $0.5 \times \text{AFNL} \le i_{\text{F}} \le 3 \times \text{AFNL}$) for the desired value based upon the following steps:

1. For each value of i_{f}, calculated i_{Q} as

$$i_{\text{Q}} = \frac{T_{\text{rated}}}{3L_{\text{af}} i_{\text{F}}}$$

2. Calculate the corresponding value of $(\lambda_{\text{a}})_{\text{rms}}$ assuming $i_{\text{D}} = 0$.

3. If $(\lambda_{\text{a}})_{\text{rms}} > \lambda_{\text{a,rated}}$, calculate the value of i_{D} such that $(\lambda_{\text{a}})_{\text{rms}} = \lambda_{\text{a,rated}}$

$$i_{\text{D}} = \frac{\sqrt{2(\lambda_{\text{a,rated}})^2 - (L_{\text{s}} i_{\text{Q}})^2} - L_{\text{af}} i_{\text{F}}}{L_{\text{s}}}$$

   Note that no solution will exist if $L_{\text{s}} i_{\text{Q}} > \lambda_{\text{a,rated}}$.

4. Calculate the rms armature current as

$$I_{\text{a}} = \frac{\sqrt{i_{\text{D}}^2 + i_{\text{Q}}^2}}{2}$$

5. Once this is done for each value of i_{F}, select the solution corresponding to the minimum value of I_{a}.

The minimum armature current occurs at $i_{\text{F}} = 29.2$ A. The result is:

$i_{\text{Q}} = 139.5$ A $\quad$ $i_{\text{D}} =$ -160.5 A
$I_{\text{a}} = 106.3$ A $= 0.71$ per unit
$V = 400$ V $\quad$ $P = 104$ kW

Problem 10-23

The motor parameters are calculated in the Examples 10.8 - 10.10. The relevant equations are Eq. 10.29 for the torque

$$T_{\rm mech} = \left(\frac{3}{2}\right)\left(\frac{\rm poles}{2}\right) L_{\rm af} i_{\rm F} i_{\rm Q} = 3 L_{\rm af} i_{\rm F} i_{\rm Q} = T_{\rm rated}$$

Eq. 10.35 for the rms line-neutral flux linkages

$$(\lambda_{\rm a})_{\rm rms} = \sqrt{\frac{(L_{\rm s} i_{\rm D} + L_{\rm af} i_{\rm F})^2 + (L_{\rm s} i_{\rm Q})^2}{2}}$$

an equation for the line-line voltage

$$V_{\rm a,l-l} = \sqrt{3}\,\omega_{\rm e} (\lambda_{\rm a})_{\rm rms}$$

where

$$\omega_{\rm e} = \left(\frac{\rm poles}{2}\right)\left(\frac{\pi}{30}\right) \times \rm rpm$$

and the equation for the rms line current

$$I_{\rm a} = \sqrt{\frac{i_{\rm D}^2 + i_{\rm Q}^2}{2}}$$

Note that because the rated machine operating speed is 1200 r/min, at the 1000 r/min operating speed, the relevant limit will be the rms armature flux linkages and at the 1400 r/min operating speed, the relevant limit will be terminal voltage.

The easiest way (not necessarily the most elegant) to solve this problem is to write a MATLAB script which performs a search based upon the following steps:

1. Search over a range of quadrature-axis currents, for example $(0.1 \times \sqrt{2}\, I_{\rm a,rated}) \le i_{\rm Q} \le (\sqrt{2}\, I_{\rm a,rated})$.

2. For each value of $i_{\rm Q}$, search over the range of de-magnetizing direct-axis currents: $-\sqrt{2 I_{\rm a,rated}^2 - i_{\rm Q}^2} \;\le\; i_{\rm D} \le 0$.

3. For each $i_{\rm Q}$, $i_{\rm D}$ pair, search over a range of field current, for example $0.5 \times \rm AFNL \le i_{\rm F} \le 3 \times AFNL$

4. For each $i_{\rm Q}$, $i_{\rm D}$, $i_{\rm F}$ triplet, calculate $T_{\rm mech}$, $(\lambda_{\rm a})_{\rm rms}$ and $V_{\rm a,l-l}$

5. Finally, search over all the resultant calculated operating points for the point which has the maximum value of $T_{\rm mech}$ subject to the constraints that $(\lambda_{\rm a})_{\rm rms} \le \lambda_{\rm a,rated}$ (for 1000 r/min) and $V_{\rm a} \le V_{\rm a,rated}$ (for 1400 r/min).

Part (a):

V_a = 183.2 V,l-l I_a = 118.1 A
i_Q = 129.2 A i_D = -105.9 A
i_F = 3.7 A P_{out} = 37.5 kW
T_{mech} = 99.9 % of rated

Part (b):

V_a = 219.7 V,l-l I_a = 118.1 A
i_Q = 120.0 A i_D = -116.2 A
i_F = 3.4 A P_{out} = 44.9 kW
T_{mech} = 85.6 % of rated

Problem 10-24

The relevant equations are Eq. 10.29 for the torque

$$T_{mech} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right) L_{af} i_F i_Q = T_{rated}$$

Eq. 10.35 for the rms line-neutral flux linkages

$$(\lambda_a)_{rms} = \sqrt{\frac{(L_s i_D + L_{af} i_F)^2 + (L_s i_Q)^2}{2}}$$

an equation for the line-line voltage

$$V_{a,l-l} = \sqrt{3}\,\omega_e (\lambda_a)_{rms}$$

where

$$\omega_e = \left(\frac{\text{poles}}{2}\right)\left(\frac{\pi}{30}\right) \times \text{rpm}$$

and the equation for the rms line current

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}}$$

Note that because the rated machine operating speed is 1200 r/min, at the 1000 r/min operating speed, the relevant limit will be the rms armature flux linkages and at the 1400 r/min operating speed, the relevant limit will be terminal voltage.

Part (a): Without a constraint on the level of terminal current, the easiest way (not necessarily the most elegant) to solve this problem is to write a MATLAB script which performs a search based upon the following steps:

1. Search over a range of field current, for example
   $0.5 \times \text{AFNL} \leq i_\text{F} \leq 3 \times \text{AFNL}$

2. For each value of i_F, find the value of i_Q corresponding to rated torque

$$i_\text{Q} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\frac{T\text{rated}}{L_\text{af} i_\text{F}}$$

3. Search over a range of de-magnetizing currents, for example
   $-\sqrt{2}\, I_\text{a,rated} \leq i_\text{D} \leq 0$

4. For each i_Q, i_D, i_F triplet, calculate the terminal voltage and the armature current.

5. Finally, search over all the resultant calculated operating points for the point which has the maximum value of I_a subject to the constraint that $V_\text{a} \leq V_\text{a,rated}$.

i_F = 29.0 A $\quad i_\text{Q}$ = 135.7 A $\quad i_\text{D}$ = -193.5 A
I_a = 167.1 A = 1.11 per unit
$V_\text{a,l-l}$ = 479.8 V $\quad P_\text{out}$ = 138.9 kW

Part (b): The easiest way (not necessarily the most elegant) to solve this problem is to write a MATLAB script which performs a search based upon the following steps:

1. Search over a range of quadrature-axis currents, for example
   $(0.1 \times \sqrt{2}\, I_\text{a,rated}) \leq i_\text{Q} \leq (\sqrt{2}\, I_\text{a,rated})$.

2. For each value of i_Q, search over the range of de-magnetizing direct-axis currents: $-\sqrt{2I^2_\text{a,rated} - i^2_\text{Q}} \leq i_\text{D} \leq 0$.

3. For each i_Q, i_D pair, search over a range of field current, for example $0.5 \times \text{AFNL} \leq i_\text{F} \leq 3 \times \text{AFNL}$

4. For each i_Q, i_D, i_F triplet, calculate T_mech, $(\lambda_\text{a})_\text{rms}$ and $V_\text{a,l-l}$

5. Finally, search over all the resultant calculated operating points for the point which has the maximum value of T_mech subject to the constraint $V_\text{a} \leq V_\text{a,rated}$.

i_F = 27.3 A $\quad i_\text{Q}$ = 129.7 A $\quad i_\text{D}$ = -168.5 A
I_a = 150.4 A = 1.00 per unit
$V_\text{a,l-l}$ = 480.0 V $\quad P_\text{out}$ = 125.0 kW

Problem 10-25

Part (a):

$$\Lambda_{\rm PM} = \frac{\sqrt{2}(E_{\rm af})_{\rm rated}}{\omega_{\rm e}} = \frac{\sqrt{2}\,(230/\sqrt{3})}{3500\,\pi/30} = 0.508 \;\; \text{Wb}$$

Part (b): For operation at 3600 r/min, the frequency will be 60 Hz and hence $X_{\rm s} = \omega_{\rm e} L_{\rm s} = 2.94\ \Omega$. $E_{\rm af} = (3600/3530)(230/\sqrt{3}) = 135.4$ V.

$$I_{\rm a,rated} = \frac{P_{\rm rated}}{\sqrt{3}\,V_{\rm rated}} = 6.28 \text{ A}$$

The armature current is given by

$$\hat{I}_{\rm a} = \frac{\left(V_a - \hat{E}_{\rm af}\right)}{jX_{\rm s}} = \frac{\left(V_a - E_{\rm af}e^{j\delta}\right)}{jX_{\rm s}}$$

where $V_{\rm a} = 230/\sqrt{3} = 132.8$ V.

Although the magnitude of $\hat{E}_{\rm af}$ is known, the angle δ required to give $|\hat{I}_{\rm a}| = I_{\rm a,rated}$ is not. A MATLAB script can be used to easily iterate to find that $\delta = -7.81°$. The motor power is then given by

$$P = -\left(\frac{3E_{\rm af}\,V_{\rm a}}{X_{\rm s}}\right)\sin\delta = 2.49 \text{ kW}$$

Then,

$$T = \frac{P}{\omega_{\rm m}} = 6.61 \text{ N}\cdot\text{m}$$

and

$$i_{\rm Q} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{T}{\Lambda_{\rm PM}}\right) = 8.67 \text{ A}$$

$$i_{\rm D} = -\sqrt{2I_{\rm a}^2 + i_Q^2} = -1.87 \text{ A}$$

Problem 10-26

Some preliminary calculations:

$$\Lambda_{\rm PM} = \frac{\sqrt{2}(E_{\rm af})_{\rm rated}}{\omega_{\rm e}} = \frac{\sqrt{2}\,(230/\sqrt{3})}{3500\,\pi/30} = 0.508 \;\; \text{Wb}$$

$$I_{\rm a,rated} = \frac{P_{\rm rated}}{\sqrt{3}\,V_{\rm rated}} = 6.28 \text{ A}$$

Part (a): At 4000 r/min, $f_e = (4000/3600)60 = 66.7$ Hz, $\omega_e = 2\pi f_e = 418.9$ rad/sec and thus $E_{af} = \omega_e \Lambda_{PM}/\sqrt{2} = 150.5$ V. The armature current is given by

$$\hat{I}_a = \frac{\left(V_a - \hat{E}_{af}\right)}{jX_s} = \frac{\left(V_a - E_{af}e^{j\delta}\right)}{jX_s}$$

where $V_a = 230/\sqrt{3} = 132.8$ V.

Although the magnitude of $\hat{E}_{af}$ is known, the angle δ required to give $|\hat{I}_a| = I_{a,rated}$ is not. A MATLAB script can be used to easily iterate to find that $\delta = -15.0°$. The motor power is then given by

$$P = -\left(\frac{3E_{af}\,V_a}{X_s}\right)\sin\delta = 2.38 \text{ kW}$$

and

$$T = \frac{P}{\omega_m} = 5.68 \text{ N}\cdot\text{m}$$

Part (b):

$$i_Q = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{T}{\Lambda_{PM}}\right) = 7.46 \text{ A}$$

$$i_D = -\sqrt{2I_a^2 + i_Q^2} = -4.81 \text{ A}$$

Problem 10-27

Errata: Part (c), first sentence should read

... in excess of that found in part (b), flux ...

The rated current of this motor is

$$I_{a,rated} = \frac{P_{rated}}{\sqrt{3}\,V_{rated}} = 30.1 \text{ A}$$

$$\Lambda_{PM} = \frac{\sqrt{2}\,V_a}{\omega_e} = \frac{\sqrt{2}\,(230/\sqrt{3})}{7675\,\pi/30} = 0.488 \text{ Wb}$$

Part (a): The torque will be maximized when $i_D = 0$ and $i_Q = \sqrt{2}\,I_{a,rated} = 42.5$ A and thus

$$T_{max} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right)\Lambda_{PM} i_Q = 31.1 \text{ N}\cdot\text{m}$$

Part (b):

$$(\lambda_a)_{rms} = \sqrt{\frac{\Lambda_{PM}^2 + \lambda_Q^2}{2}} = \sqrt{\frac{\Lambda_{PM}^2 + (L_s i_Q)^2}{2}} = 0.543 \text{ Wb}$$

Thus, to avoid exceeding rated terminal voltage, the electrical frequency of the motor must be limited to

$$\omega_{e,max} = \frac{V_{rated}}{\sqrt{3}\,(\lambda_a)_{rms}} = 722.4 \quad \text{rad/s}$$

and the corresponding motor speed will be

$$n = \omega_{e,max}\left(\frac{30}{\pi}\right) = 6899 \text{ r/min}$$

Part (c): At 9500 r/min, $\omega_e = 9500\pi/30 = 995$ rad/sec. In order to maintain rated terminal voltage, the rms line-to-neutral armature flux linkages must now be limited to

$$(\lambda_a)_{rms} = \frac{\left(V_{rated}/\sqrt{3}\right)}{\omega_e} = 0.279 \text{ Wb}$$

Thus solving

$$\begin{aligned}(\lambda_a)_{rms} &= \sqrt{\frac{(L_s i_D + \Lambda_{PM})^2 + (L_s i_Q)^2}{2}} \\ &= \sqrt{\frac{2(L_s I_a)^2 + 2L_s i_D \Lambda_{PM} + \Lambda_{PM}^2}{2}}\end{aligned}$$

for i_D gives

$$i_D = \frac{\sqrt{2(\lambda_a)_{rms}^2 - 2(L_s I_a)^2 - \Lambda_{PM}^2}}{2L_s \Lambda_{PM}}$$

Setting $I_a = I_{rated}$ gives $i_D = -25.5$ A and thus

$$i_Q = \sqrt{2I_{rated}^2 - i_D^2} = 34.0 \text{ A}$$

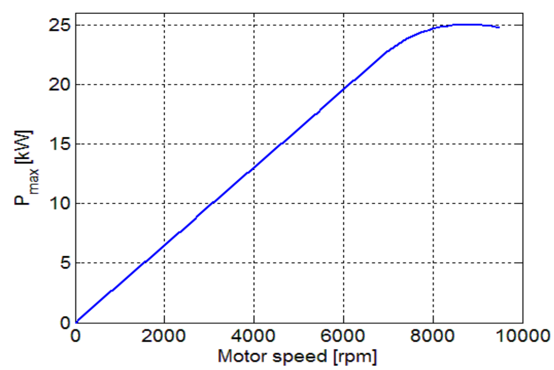
The motor torque is then given by

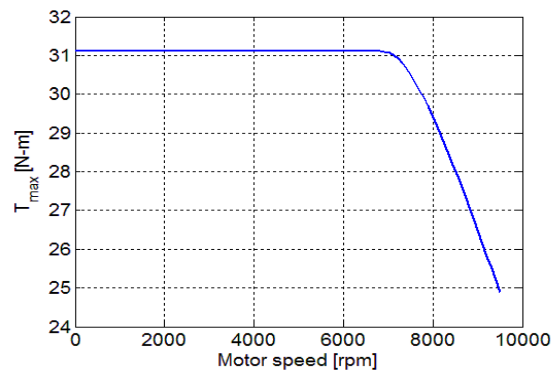
$$T = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right)\Lambda_{PM} i_Q = 24.9 \text{ N}\cdot\text{m}$$

Since this is a two-pole machine and $\omega_m = \omega_e$, the corresponding power will be $P = \omega_m T = 24.8$ kW and the motor power factor will be

$$\text{power factor} = \frac{P}{\sqrt{3}\,V_\text{rated} I_\text{rated}} = 0.99$$

Problem 10-28





Problem 10-29

The relevant equations are

$$T_\text{mech} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right)\Lambda_\text{PM} i_\text{Q}$$

$$(\lambda_\text{a})_\text{rms} = \sqrt{\frac{(L_\text{s} i_\text{D} + \Lambda_\text{PM})^2 + (L_\text{s} i_\text{Q})^2}{2}}$$

an equation for the line-line voltage

$$V_{\text{a,l}-\text{l}} = \sqrt{3}\,\omega_\text{e}(\lambda_\text{a})_\text{rms}$$

where

$$\omega_e = \left(\frac{\text{poles}}{2}\right)\left(\frac{\pi}{30}\right) \times \text{rpm}$$

and the equation for the rms line current

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}}$$

For this motor

$$\Lambda_{PM} = \frac{\sqrt{2}\,V_a}{\omega_e} = \frac{\sqrt{2}\,(475/\sqrt{3})}{19250\,\pi/30} = 0.192 \text{ Wb}$$

The easiest way (not necessarily the most elegant) to solve this problem is to write a MATLAB script which performs a search based upon the following steps:

1. Search over a range of i_Q, for example $0.3 \times \sqrt{2} I_{a,max} \leq i_Q \sqrt{2} I_{a,max}$
2. For each value of i_Q, search over the range of de-magnetizing direct-axis currents: $-\sqrt{2I_{a,max}^2 - i_Q^2} \leq i_D \leq 0$.
3. For each i_Q, i_D pair, calculate the output power, rms flux linkages and terminal voltage.
4. Finally, search over all the resultant calculated operating points for the point which has the maximum value of output power subject to the constraints that $(\lambda_a)_{rms} \leq \lambda_{a,rated}$ (for 16000 r/min) and $V_a \leq V_{a,rated}$ (for 25000 r/min).

16000 r/min

$P_{out} = 212$ kW	$I_a = 350.0$ A
$i_Q = 438.7$ A	$i_D =$ -229.2 A
$V_a = 395$ V, l-l	

25000 r/min

$P_{out} = 273$ kW	$I_a = 350.0$ A
$i_Q = 361.6$ A	$i_D =$ -338.0 A
$V_a = 475$ V, l-l	

Problem 10-30

Part (a): Following the analysis of Chapter 6

$$Z_{1,\mathrm{eq}} = R_{1,\mathrm{eq}} + X_{1,\mathrm{eq}} = \frac{jX_\mathrm{m}(R_1 + jX_1)}{R_1 + j(X_1 + X_\mathrm{m})} = 0.047 + j0.521\ \Omega$$

$$T_\mathrm{max} = \frac{0.5 n_\mathrm{ph} V_{1,\mathrm{eq}}^2}{\omega_\mathrm{s}(X_{1,\mathrm{eq}} + X_2)} = 484\ \mathrm{N \cdot m}$$

$$s_\mathrm{maxT} = \frac{R_2}{\sqrt{R_{1,\mathrm{eq}}^2 + (X_{1,\mathrm{eq}} + X_2)^2}} = 0.110 = 11.0\%$$

and the corresponding speed is 1602 r/min.

Part (b): At 60 Hz, $\omega_\mathrm{s} = 2\pi f(2/\mathrm{poles}) = 188.5$ rad/sec. At $s = 0.032$, $\omega_\mathrm{m} = (1 - s)\omega_\mathrm{s} = 182.5$ rad/sec. The torque is given by

$$T = \frac{1}{\omega_\mathrm{s}} \left[\frac{n_\mathrm{ph}\, V_{1,\mathrm{eq}}^2 (R_2/s)}{(R_{1,\mathrm{eq}} + (R_2/s))^2 + (X_{1,\mathrm{eq}} + X_2)^2} \right] = 265\ \mathrm{N \cdot m}$$

and the power is $P = \omega_\mathrm{m} T = 48.3$ kW.

Part (c): With the frequency reduced from 60 Hz to 45 Hz, ω_s, the terminal voltage and each reactance must each be scaled by the factor (45/60). With R_1 neglected, the torque expression reduces to

$$T = \frac{1}{\omega_\mathrm{s}} \left[\frac{n_\mathrm{ph} V_{1,\mathrm{eq}}^2 (R_2/s)}{(R_2/s)^2 + (X_{1,\mathrm{eq}} + X_2)^2} \right]$$

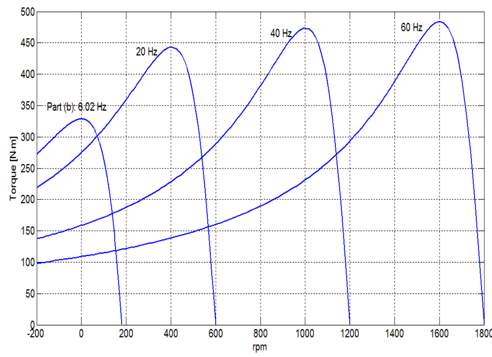
which leads to the following quadratic expression in R_2/s which in turn can be solved for s

$$\omega_\mathrm{s} T \left(\frac{R_2}{s} \right)^2 - n_\mathrm{ph} V_{1,\mathrm{eq}}^2 \left(\frac{R_2}{s} \right)^2 + (X_{1,\mathrm{eq}} + X_2)^2 \omega_\mathrm{s} T = 0$$

Solving gives a slip of 2.21%, corresponding to a speed of 1320 r/min. Substituting this value of slip into the torque expression including the effects of R_1 gives $T_\mathrm{mech} = 260$ N·m, quite close to the actual value of 265 N·m.

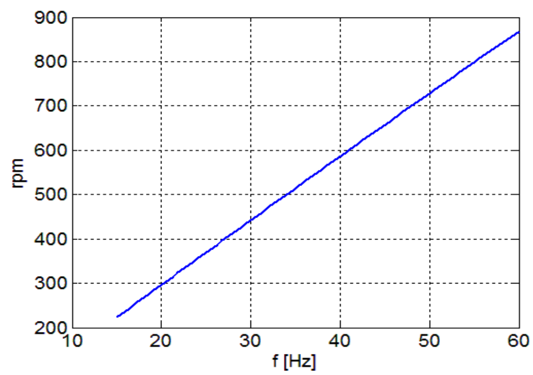
Problem 10-31

Part (a):



Part (b): $f = 6.02$ Hz which will produce a starting torque of 329 N·m.

Problem 10-32



Problem 10-33

The motor torque is a function of the ratio R_2/s. The slip with $R_{2,\text{ext}} = 0$ is

$$s_0 = \frac{1800 - 1732}{1800} = 0.0378$$

and that with $R_{2,\text{ext}} = 0.790\ \Omega$ is

$$s_1 = \frac{1800 - 1693}{1800} = 0.0594$$

Thus, solving

$$\frac{R_2}{s_0} = \frac{R_2 + 0.790}{s_1}$$

for R_2 gives $R_2 = 1.377\ \Omega$.

Problem 10-34

The motor torque is a function of the ratio R_2/s. The slip with $R_{2,\mathrm{ext}} = 0$ is

$$s_0 = \frac{1800 - 1732}{1800} = 0.0378$$

The desired operating speed corresponds to a slip of

$$s_1 = \frac{1800 - 1550}{1800} = 0.1389$$

Thus substituting the value of $R_2 = 1.377\ \Omega$ found in the solution to Problem 10.32 and solving

$$\frac{R_2}{s_0} = \frac{R_2 + R_{2,\mathrm{ext}}}{s_1}$$

for $R_{2,\mathrm{ext}}$ gives $R_{2,\mathrm{ext}} = 3.687\ \Omega$.

Problem 10-35

Part (a): If R_1 is assumed negligible, the torque expression becomes

$$T = \frac{1}{\omega_{\mathrm{s}}}\left[\frac{n_{\mathrm{ph}}\, V_{1,\mathrm{eq}}^2 (R_2/s)}{(R_2/s)^2 + (X_{1,\mathrm{eq}} + X_2)^2}\right]$$

Substituting the corresponding expressions for $T_{\max}$

$$T_{\max} = \frac{1}{\omega_{\mathrm{s}}}\left[\frac{0.5 n_{\mathrm{ph}} V_{1,\mathrm{eq}}^2}{X_{1,\mathrm{eq}} + X_2}\right]$$

$$s_{\mathrm{maxT}} = \frac{R_2}{X_{1,\mathrm{eq}} + X_2}$$

gives

$$T = T_{\max}\left(\frac{2}{s/s_{\mathrm{maxT}} + s_{\mathrm{maxT}}/s}\right)$$

Defining the ratio of full-load torque to maximum torque as

$$k \equiv \frac{T_{\text{fl}}}{T_{\text{max}}} = \frac{1}{2.37} = 0.422$$

the full-load slip can then be found as

$$s_{\text{fl}} = s_{\text{maxT}} \left(\frac{k}{1 + \sqrt{1 - k^2}} \right) = 0.0347 = 3.47\%$$

Part (b): The full load rotor power dissipation is given by

$$P_{\text{rotor}} = P_{\text{fl}} \left(\frac{s_{\text{fl}}}{1 - s_{\text{fl}}} \right) = 1800 \text{ W}$$

Part (c): At rated load, $\omega_{\text{m,rated}} = (1 - s_{\text{fl}})\omega_{\text{s}} == 180.7$ rad/sec. The rated torque is $T_{\text{rated}} = P_{\text{rated}}/\omega_{\text{m,rated}} = 415$ N·m. Setting $s = 1$ gives

$$T_{\text{start}} = T_{\text{max}} \left(\frac{2}{1/s_{\text{maxT}} + s_{\text{maxT}}} \right) = 72.6\% = 240 \text{ N} \cdot \text{m}$$

Part (d): If the stator current is at its full load value, this means that R_2/s is equal to its full load value and hence the torque will be equal to the full-load torque, 330 N·m.

Part (e): The slip will be twice the original full load slip or 6.95%.

Problem 10-36

Part (a):

$$s_{\text{fl}} = \frac{1200 - 1164}{1200} = 0.0300$$

and thus the full-load rotor power dissipation is equal to

$$P_{\text{rotor}} = P_{\text{fl}} \left(\frac{s_{\text{fl}}}{1 - s_{\text{fl}}} \right) = 1392 \text{ W}$$

Part (b): If R_1 is assumed negligible, the torque expression becomes

$$T = \frac{1}{\omega_{\text{s}}} \left[\frac{n_{\text{ph}} V_{1,\text{eq}}^2 (R_2/s)}{(R_2/s)^2 + (X_{1,\text{eq}} + X_2)^2} \right]$$

Substituting the corresponding expressions for T_{max}

$$T_{\text{max}} = \frac{1}{\omega_{\text{s}}} \left[\frac{0.5 n_{\text{ph}} V_{1,\text{eq}}^2}{X_{1,\text{eq}} + X_2} \right]$$

$$s_{\mathrm{maxT}} = \frac{R_2}{X_{1,\mathrm{eq}} + X_2}$$

gives

$$T = T_{\max}\left(\frac{2}{s/s_{\mathrm{maxT}} + s_{\mathrm{maxT}}/s}\right)$$

Defining the ratio of maximum torque to full-load torque

$$k \equiv \frac{T_{\max}}{T_{\mathrm{T}}} = 2.27$$

the full-load slip can then be found as

$$s_{\mathrm{maxT}} = s_{\mathrm{fl}}\left(k + \sqrt{k^2 - 1}\right) = 0.1292 = 12.92\%$$

Thus the motor speed at maximum torque is $n_{\max} = 1200(1 - s_{\mathrm{maxT}}) = 1045$ r/min.

Part (c): We want s_{maxT} to increase by a factor of $1/0.1292 = 7.74$. Thus the rotor resistance must increase by this factor. In other words

$$R_2 + R_{2,\mathrm{ext}} = 7.74 R_2$$

which gives $R_{2,\mathrm{ext}} = 2.29\ \Omega$.

Part (d): The 50-Hz voltage will be (5/6) that of 60-Hz. Thus the applied voltage will be 366.7 V, line-to-line.

Part (e): If the frequency and voltage are scaled from their rated value by a factor k_{f}, the torque expression becomes

$$T = \left(\frac{1}{k_{\mathrm{f}}\omega_{\mathrm{s0}}}\right)\left[\frac{n_{\mathrm{ph}}\,(k_{\mathrm{f}}V_{1,\mathrm{eq}})^2(R_2/s)}{(R_2/s)^2 + (k_{\mathrm{f}}(X_{1,\mathrm{eq}} + X_2))^2}\right]$$

where ω_{s0} is the rated-frequency synchronous speed of the motor. Clearly, the torque expression will remain constant if the slip scales invesely with k_{f}. Thus

$$s_{\mathrm{fl},50} = \left(\frac{60}{50}\right) s_{\mathrm{fl},60} = 0.0360$$

The synchronous speed at 50 Hz is 1000 r/min and thus

$$n_{\mathrm{fl},50} = 1000(1 - s_{\mathrm{fl},50}) = 964 \text{ r/min}$$

Problem 10-37

Part (a): From the data given in Problem 10.32, the motor inductances are:

$$L_1 = 1.56 \text{ mH}; \quad L_2 = 1.62 \text{ mH}; \quad L_m = 64.2 \text{ mH};$$

and thus

$$L_\mathrm{S} = L_1 + L_\mathrm{m} = 65.76 \text{ mH}$$

and

$$L_\mathrm{R} = L_2 + L_\mathrm{m} = 65.81 \text{ mH}$$

$R_\mathrm{a} = R_1 = 54.0 \text{ m}\Omega$ and $R_\mathrm{aR} = R_2 = 148 \text{ m}\Omega$. Finally, the rated motor torque
The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_\mathrm{rated} = \frac{\sqrt{2}\, V_\mathrm{base}}{\sqrt{3}\, \omega_\mathrm{base}} = \frac{\sqrt{2}\, 2400}{\sqrt{3}\, (120\pi)} = 5.19 \text{ Wb}$$

The required torque can be determined from the given power and speed as

$$T_\mathrm{mech} = \frac{P_\mathrm{mech}}{\omega_\mathrm{m}} = \frac{950 \times 10^3}{850\ \pi/30} = 10670 \text{ N} \cdot \text{m}$$

Setting $\lambda_\mathrm{DR} = \lambda_\mathrm{rated}$ gives

$$i_\mathrm{Q} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{L_\mathrm{R}}{L_\mathrm{m}}\right)\left(\frac{T_\mathrm{mech}}{\lambda_\mathrm{DR}}\right) = 350.8 \text{ A}$$

and

$$i_\mathrm{D} = \frac{\lambda_\mathrm{DR}}{L_\mathrm{m}} = 81.0 \text{ A}$$

Part (b):

$$I_\mathrm{a} = \sqrt{\frac{i_\mathrm{D}^2 + i_\mathrm{Q}^2}{2}} = 254.6 \text{ A}$$

Part (c):

$$\omega_\mathrm{me} = \omega_\mathrm{m}\left(\frac{\text{poles}}{2}\right) = 360.6 \text{ rad/sec}$$

$$\omega_\mathrm{e} = \omega_\mathrm{me} + \left(\frac{R_\mathrm{aR}}{L_\mathrm{R}}\right)\left(\frac{i_\mathrm{Q}}{i_\mathrm{D}}\right) = 365.8 \text{ rad/sec}$$

and thus

$$f_e = \frac{\omega_2}{2\pi} = 58.2 \text{ Hz}$$

Part (d):

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e(L_S - L_m^2/L_R)i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 1419 \text{ V, l-n} = 2458 \text{ V, l-l} \end{aligned}$$

Problem 10-38

Part (a): From the given data, the motor inductances are:

$$L_1 = 1.22 \text{ mH}; \quad L_2 = 1.25 \text{ mH}; \quad L_m = 68.78 \text{ mH};$$

and thus

$$L_S = L_1 + L_m = 67.00 \text{ mH}$$

and

$$L_R = L_2 + L_m = 67.03 \text{ mH}$$

$R_a = R_1 = 42.9$ mΩ and $R_{aR} = R_2 = 93.7$ mΩ.

The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_{\text{rated}} = \frac{\sqrt{2}\, V_{\text{base}}}{\sqrt{3}\, \omega_{\text{base}}} = \frac{\sqrt{2}\, 230}{\sqrt{3}\,(120\pi)} = 0.498 \text{ Wb}$$

The motor torque is $T_{\text{mech}} = 64(1275/1800) = 45.3$ N·m. Setting $\lambda_{DR} = \lambda_{\text{rated}}$, we can solve for i_Q and i_D

$$i_Q = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{L_R}{L_m}\right)\left(\frac{T_{\text{mech}}}{\lambda_{DR}}\right) = 30.9 \text{ A}$$

and

$$i_D = \frac{\lambda_{DR}}{L_m} = 7.57 \text{ A}$$

The motor mechanical velocity in electrical rad/sec is

$$\omega_{me} = \omega_m \left(\frac{\text{poles}}{2}\right) = 267.0 \text{ rad/sec}$$

and thus

$$\omega_e = \omega_{me} + \left(\frac{R_{aR}}{L_R}\right)\left(\frac{i_Q}{i_D}\right) = 272.7 \text{ rad/sec}$$

and

$$f_e = \frac{\omega_2}{2\pi} = 43.4 \text{ Hz}$$

Part (b):

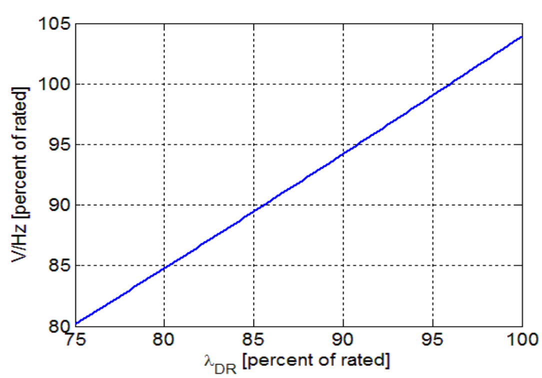
$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = 22.5 \text{ A}$$

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e(L_S - L_m^2/L_R i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 99.8 \text{ V, l-n} = 172.9 \text{ V, l-l} \end{aligned}$$

Part (c):

$$S_{in} = \sqrt{3}\, V_a I_a = 6.74 \text{ kVA}$$

Part (d): Rated V/Hz occurs at $\lambda_{DR} = 8.2\%$ of rated.



Problem 10-39

Part (a): From the given data, the motor inductances are:

$$L_1 = 1.22 \text{ mH}; \quad L_2 = 1.25 \text{ mH}; \quad L_m = 68.78 \text{ mH};$$

and thus

$$L_S = L_1 + L_m = 67.00 \text{ mH}$$

and

$$L_{\mathrm{R}} = L_2 + L_{\mathrm{m}} = 67.03 \text{ mH}$$

$R_{\mathrm{a}} = R_1 = 42.9 \text{ m}\Omega$ and $R_{\mathrm{aR}} = R_2 = 93.7 \text{ m}\Omega$.

The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_{\mathrm{rated}} = \frac{\sqrt{2}\, V_{\mathrm{base}}}{\sqrt{3}\,\omega_{\mathrm{base}}} = \frac{\sqrt{2}\, 230}{\sqrt{3}\,(120\pi)} = 0.498 \text{ Wb}$$

The motor torque is $T_{\mathrm{mech}} = 64(1425/1800) = 50.7$ N·m. Setting $\lambda_{\mathrm{DR}} = \lambda_{\mathrm{rated}}$, we can solve for i_{Q} and i_{D}

$$i_{\mathrm{Q}} = \left(\frac{2}{3}\right)\left(\frac{2}{\text{poles}}\right)\left(\frac{L_{\mathrm{R}}}{L_{\mathrm{m}}}\right)\left(\frac{T_{\mathrm{mech}}}{\lambda_{\mathrm{DR}}}\right) = 40.6 \text{ A}$$

$$i_{\mathrm{D}} = \frac{\lambda_{\mathrm{DR}}}{L_{\mathrm{m}}} = 6.43 \text{ A}$$

and

$$I_{\mathrm{a}} = \sqrt{\frac{i_{\mathrm{D}}^2 + i_{\mathrm{Q}}^2}{2}} = 29.1 \text{ A}$$

Part (b): The motor mechanical velocity in electrical rad/sec is

$$\omega_{\mathrm{me}} = \omega_{\mathrm{m}}\left(\frac{\text{poles}}{2}\right) = 298.5 \text{ rad/sec}$$

and thus

$$\omega_{\mathrm{e}} = \omega_{\mathrm{me}} + \left(\frac{R_{\mathrm{aR}}}{L_{\mathrm{R}}}\right)\left(\frac{i_{\mathrm{Q}}}{i_{\mathrm{D}}}\right) = 307.3 \text{ rad/sec}$$

and

$$f_{\mathrm{e}} = \frac{\omega_2}{2\pi} = 48.9 \text{ Hz}$$

$$\begin{aligned} V_{\mathrm{a}} &= \sqrt{\frac{(R_{\mathrm{a}} i_{\mathrm{D}} - \omega_{\mathrm{e}}(L_{\mathrm{S}} - L_{\mathrm{m}}^2/L_{\mathrm{R}} i_{\mathrm{Q}})^2 + (R_{\mathrm{a}} i_{\mathrm{Q}} + \omega_{\mathrm{e}} L_{\mathrm{S}} i_{\mathrm{D}})^2}{2}} \\ &= 96.2 \text{ V, l-n} = 166.5 \text{ V, l-l} \end{aligned}$$

Part (c): i_{Q} is now increased to 44.7 A and hence, with I_{DR} and hence λ_{DR} unchanged

$$T_{\mathrm{mech}} = \left(\frac{3}{2}\right)\left(\frac{\text{poles}}{2}\right)\left(\frac{L_{\mathrm{m}}}{L_{\mathrm{R}}}\right)\lambda_{\mathrm{DR}} i_{\mathrm{Q}} = 55.7 \text{ N}\cdot\text{m}$$

Thus the motor speed is

$$n = 1800\left(\frac{T_{\rm mech}}{64}\right) = 1568 \text{ r/min}$$

and $\omega_{\rm m} = n\pi/30 = 164.1$ rad/sec.

$$P_{\rm mech} = T_{\rm mech}\omega_{\rm m} = 8.32 \text{ kW}$$

Part (d): The terminal voltage is

$$\begin{aligned} V_{\rm a} &= \sqrt{\frac{(R_{\rm a}i_{\rm D} - \omega_{\rm e}(L_{\rm S} - \frac{L_{\rm m}^2}{L_{\rm R}})i_{\rm Q})^2 + (R_{\rm a}i_{\rm Q} + \omega_{\rm e}L_{\rm S}i_{\rm D})^2}{2}} \\ &= 107.6 \text{ V, l-n} = 186.4 \text{ V, l-l} \end{aligned}$$

The drive frequency can be found from

$$\omega_{\rm me} = \omega_{\rm m}\left(\frac{\text{poles}}{2}\right) = 328.3 \text{ rad/sec}$$

$$\omega_{\rm e} = \omega_{\rm me} + \left(\frac{R_{\rm aR}}{L_{\rm R}}\right)\left(\frac{i_{\rm Q}}{i_{\rm D}}\right) = 338.0 \text{ rad/sec}$$

and

$$f_{\rm e} = \frac{\omega_2}{2\pi} = 53.8 \text{ Hz}$$

Part (e):

$$S_{\rm in} = \sqrt{3}\, V_{\rm a}I_{\rm a} = 10.3 \text{ kVA}$$

Part (f): Iteration with a MATLAB script gives $\lambda_{\rm DR} = 95.3\%$ of $\lambda_{\rm rated}$.

Problem 10-40

Part (a): From the given data, the motor inductances are:

$$L_1 = 4.96 \text{ mH}; \quad L_2 = 6.02 \text{ mH}; \quad L_m = 118.3 \text{ mH};$$

and thus

$$L_{\rm S} = L_1 + L_{\rm m} = 123.3 \text{ mH}$$

and

$$L_{\rm R} = L_2 + L_{\rm m} = 124.3 \text{ mH}$$

$R_{\mathrm{a}} = R_1 = 212\ \mathrm{m}\Omega$ and $R_{\mathrm{aR}} = R_2 = 348\ \mathrm{m}\Omega$. Finally, the rated motor torque

The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_{\mathrm{rated}} = \frac{\sqrt{2}\, V_{\mathrm{base}}}{\sqrt{3}\, \omega_{\mathrm{base}}} = \frac{\sqrt{2}\, 4160}{\sqrt{3}\, (120\pi)} = 9.01\ \mathrm{Wb}$$

At a power output of 1135 kW and a speed of 836 r/min, $\omega_{\mathrm{m}} = 87.5$ rad/sec, $T_{\mathrm{mech}} = 1.30 \times 10^4$ N·m. Setting $\lambda_{\mathrm{DR}} = \lambda$rated gives

$$i_{\mathrm{Q}} = \left(\frac{2}{3}\right)\left(\frac{2}{\mathrm{poles}}\right)\left(\frac{T}{\Lambda_{\mathrm{PM}}}\right) = 252.0\ \mathrm{A}$$

$$i_{\mathrm{D}} = \sqrt{2I_{\mathrm{a,rated}}^2 + i_Q^2} = 76.2\ \mathrm{A}$$

$$I_{\mathrm{a}} = \sqrt{\frac{i_{\mathrm{D}}^2 + i_{\mathrm{Q}}^2}{2}} = 186.2\ \mathrm{A}$$

The terminal voltage is

$$\begin{aligned} V_{\mathrm{a}} &= \sqrt{\frac{(R_{\mathrm{a}} i_{\mathrm{D}} - \omega_{\mathrm{e}}(L_{\mathrm{S}} - \frac{L_{\mathrm{m}}^2}{L_{\mathrm{R}}}) i_{\mathrm{Q}})^2 + (R_{\mathrm{a}} i_{\mathrm{Q}} + \omega_{\mathrm{e}} L_{\mathrm{S}} i_{\mathrm{D}})^2}{2}} \\ &= 2516\ \mathrm{V,\ l\text{-}n} = 4357\ \mathrm{V,\ l\text{-}l} \end{aligned}$$

The drive frequency can be found from

$$\omega_{\mathrm{me}} = \omega_{\mathrm{m}} \left(\frac{\mathrm{poles}}{2}\right) = 350.2\ \mathrm{rad/sec}$$

$$\omega_{\mathrm{e}} = \omega_{\mathrm{me}} + \left(\frac{R_{\mathrm{aR}}}{L_{\mathrm{R}}}\right)\left(\frac{i_{\mathrm{Q}}}{i_{\mathrm{D}}}\right) = 359.4\ \mathrm{rad/sec}$$

and

$$f_{\mathrm{e}} = \frac{\omega_2}{2\pi} = 57.2\ \mathrm{Hz}$$

Part (b): The equivalent-circuit of Chapter 6 can be analyzed readily using MATLAB as follows:

- All the reactances must be scaled from their 60-Hz values to 57.2 Hz.
- The rms input voltage must be set equal to 2516 V, line-to-neutral.
- The slip must be calculated based upon a synchronous speed of $n_{\mathrm{s}} = 60 f_{\mathrm{e}}(2/\mathrm{poles}) = 858$ r/min.

If this is done, the equivalent circuit will give exactly the same results as those of part (a).